MA331 Homework 3 - Aparajita Rana

I pledge my honor that I have abided by the Stevens Honor System. - Aparajita Rana

Problem #1

(i) Moment estimator $\hat{\theta}_M$ when $x_1, x_2, \ldots, x_n$ is the random sample. $x \sim \text{uniform}(0, \theta)$

$\therefore f(x) = \frac{1}{\theta}$ Sample moment $\Rightarrow \frac{x_1 + x_2 \ldots + x_n}{n}$ & $\frac{x_1^2 + \ldots x_n^2}{n}$

population moment

$\int_0^\theta x f(x)\,dx \to \int_0^\theta \frac{x}{\theta}\,dx = \frac{\theta}{2}$ (1st)

$\frac{x_1^2 + \ldots + x_n^2}{n}$ (2nd)

$\frac{\theta}{2} = \frac{x_1 + \ldots x_n}{n}$

$$\boxed{\hat{\theta}_M = \frac{2}{n}\sum_{i=1}^{n} x_i}$$

(ii) Maximum likelihood estimator θ_2

the likelihood of the function gives us

$$L(\theta \mid x_n) = \begin{cases} \frac{1}{\theta^n}, & 0 \le x_y \le \theta \quad y = 1(1)n \\ 0 \end{cases}$$

so we get $\to \boxed{\hat{\theta}_L = x_n}$ because it is called maximum when $(x_1, \ldots x_n)$

(iii) $\hat{\theta}_M = (1 + 2.4 + 3.2 + 1.2 + 0.5 + 3.1 + 6.8)\frac{2}{7} = \boxed{5.2}$ 0

$x_7 \to \max(\text{all vals}) \to \hat{\theta}_L = \boxed{6.8}$

I believe $\hat{\theta}_M = 5.2$ is better because it more accurately represents the avg. values

iv) & v)

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## [1] 7.033513
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## [1] 6.775629
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## Warning in xy.coords(x, y): NAs introduced by coercion
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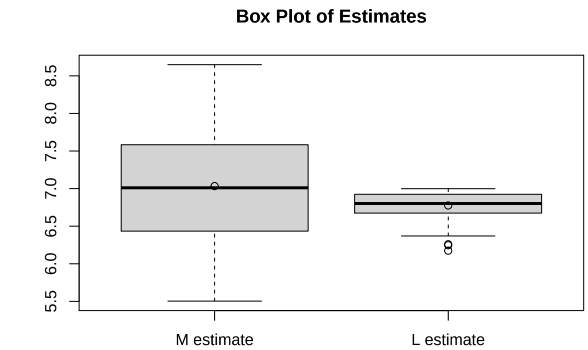
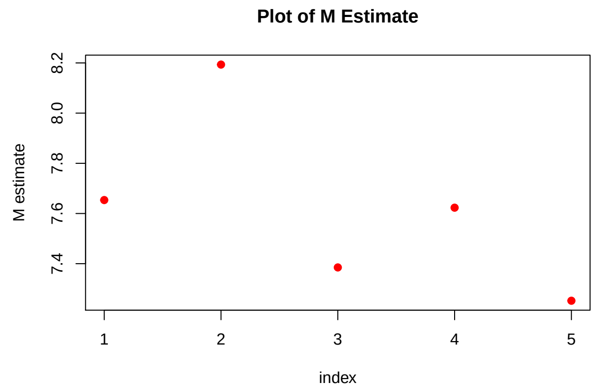
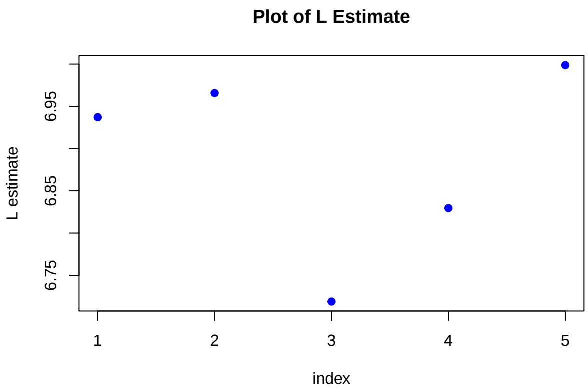
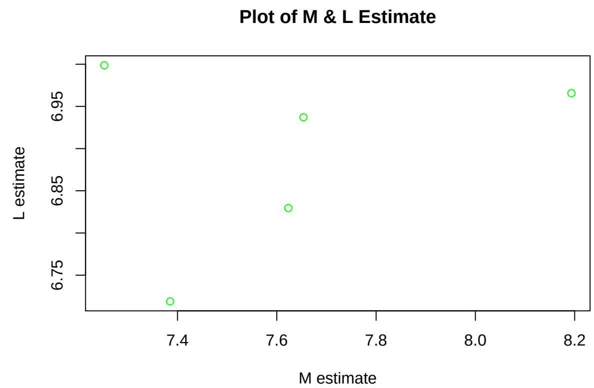

Box Plot of Estimates
5.5
6.0
6.5
7.0
7.5
8.0
8.5
M estimate
L estimate


Plot of M Estimate
M estimate
7.4
7.6
7.8
8.0
8.2
1
2
3
4
5
index


Plot of L Estimate
L estimate
6.75
6.85
6.95
1
2
3
4
5
index


Plot of M & L Estimate
L estimate
6.75
6.85
6.95
7.4
7.6
7.8
8.0
8.2
M estimate

Problem #2: Assume SRS $X_1, \ldots, X_n \rightarrow$ pop random var $X \sim N(\mu, \sigma^2)$

i) moment estimate $M_1 = E(X) = \mu$, $M_2 = E(X^2) = V(X) + E(X)^2$

$M_1 = \bar{X}$ & $m_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ ... ② $\rightarrow$ $= \sigma^2 + \mu^2$

① $\rightarrow$ $\hat{\mu} = \bar{X}$ $= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \hat{\mu}^2$ combining ① & ②

alternatively $\{\bar{X}$ & $\sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}\}$ $\boxed{\left(\bar{X}, \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)} \leftarrow \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

ii) $X \sim N(N, \sigma^2)$

$$L = \prod_{i=1}^{n}\left(\frac{1}{\sigma\sqrt{2\pi}}\right) \cdot \exp\ -\frac{1}{2\sigma^2}(x_i - N)^2$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \exp\ -\sum_{i=1}^{n}(x_i - N)^2/2\sigma^2)$$

$\log L = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{3}(x_i - N)^2$

$$\frac{\partial \log L}{\partial N} = 0, \ \frac{\partial \log L}{\partial \sigma} = 0 \rightarrow -\frac{1}{2\sigma^2} 2 \cdot \sum_{i=1}^{3}(x_i - N) = 0$$

$$\therefore n\bar{x} = nN \ \& \ \hat{N}_{MLE} = \bar{x}$$

$$\frac{\partial \log L}{\partial \sigma} = 0 \rightarrow -\frac{n}{2} \cdot \frac{1}{\sigma^2} 2\sigma = \frac{1}{2}\sum_{i=1}^{n}(x_i - N)^2 \cdot \frac{2}{\sigma^3} = 0$$

$$\sigma^2 = \frac{1}{n}\Sigma(x_i - \hat{N})^2$$

$$\sigma_{MLE} = \sqrt{1/n \sum_{i=1}^{5}(x_i - \hat{N})^2}$$

$$\therefore \boxed{\hat{N}_{MLE} = \bar{X} \ \& \ \hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

Problem 3: 6.17) a] $z_{0.975} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{2.3}{\sqrt{340}} = 0.24 \rightarrow CI = \hat{x} \pm z_{0.975} \cdot \frac{\sigma}{\sqrt{n}} =$

$\boxed{(5.156, 5.644)} \rightarrow \boxed{0.24}$

b] $\boxed{(5.079, 5.721)}$ Error Margin = 0.321

6.27) a] $CI = 11.5 \pm 1.96 \cdot \frac{8.3}{\sqrt{1200}} = \boxed{(11.02, 11.97)}$

b] No, this represents the range of vals + avg. time

c] Yes it's good because sample size is more accurate

6.28) a] $\boxed{\hat{x} = 690, \sigma = 498}$ $CI = 690 \pm 1.96 \cdot \frac{498}{\sqrt{1200}} \rightarrow \boxed{(661.82, 718.17)}$

b] (661.82, 718.17)

c] To directly calculate we could do multiply the old by 60?