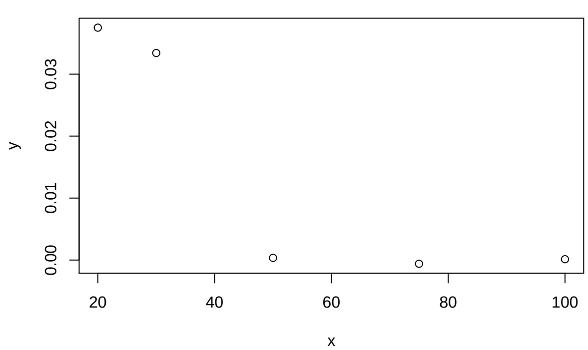
MA331 Homework 2 - Aparajita Rana

I pledge my honor that I have abided by the Stevens Honor System. - Aparajita Rana

Binomial Distribution using R function:

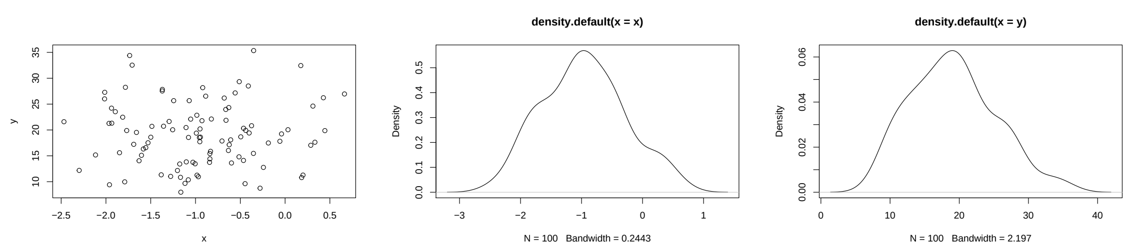
```
## [1] 0.5955987

## [1] 0.09401122

## [1] 0.0002305229

## [1] 1.826106e-08

## [1] 5.431127e-13
```

Using Laplace theorem:
P(N=20)=0.6331
P(N=30)=0.0606
P(N=50)=0.00058
P(N=75)=1.475e^-7
P(N=100)=8.91e^-11

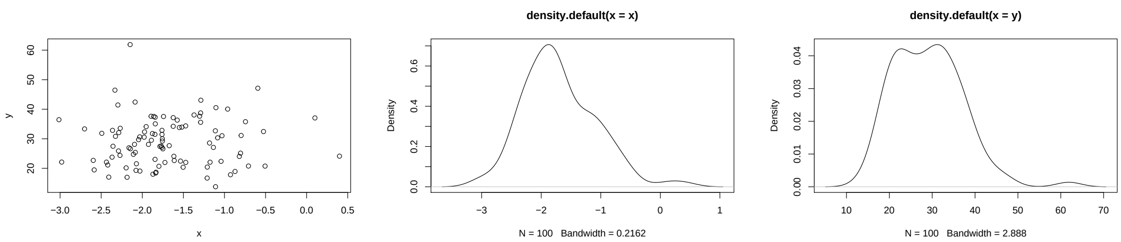


As N increases, the errors decrease and will be as close to 0 as possible but the error points will never hit 0.

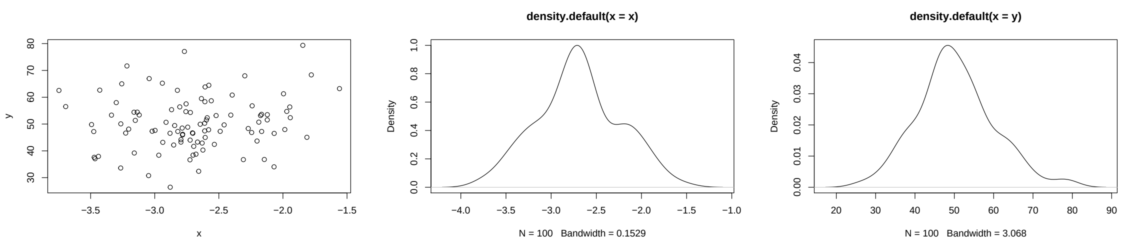
```r
func(20)
```



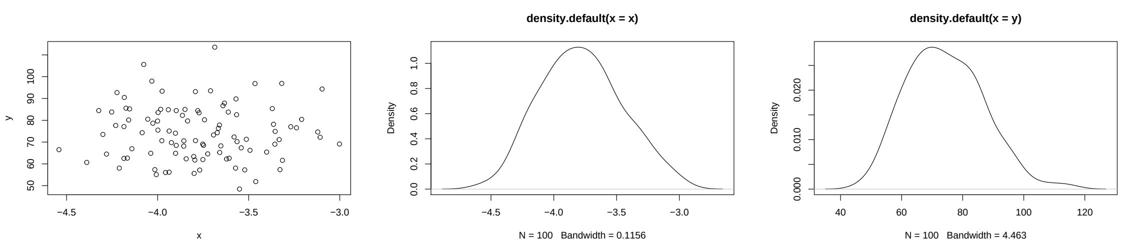
```r
func(30)
```




```r
func(50)
```




```r
func(75)
```



From the plots, the most significant finding is that the values find a peak around the middle point of the graph. We can see, though sample of size n is generated randomly for each graphs, their slopes/patterns are very similar.

Based on the scatter plots, we see the data is mostly uniformly randomly distributed with little correlation but slightly increased density in the center as well.

Problem 3: Show that $E[N] = np$ for $N \sim B(n,p)$

* the probability function of a binomial random vari $\rightarrow$ $b(N; n, p) = \binom{n}{N} p^N (1-p)^{n-N}$

$$E[N] = \sum_{N=0}^{n} N \binom{n}{N} p^N (1-p)^{n-N}$$

$$\sum_{M=0}^{n} N \frac{n!}{N!\,(n-N)!} p^N (1-p)^{n-N}$$

$$\sum_{M=1}^{n} \frac{n!}{(N-1)!\,(n-N)!} p^N (1-p)^{n-N}$$

x is gone so we let $y = N-1$ & $m = n-1$

putting $N = y+1$ and $n = m+1$ we get:

$$E[N] = \sum_{y=0}^{m} \frac{(m+1)!}{y!\,(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{n} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

$\rightarrow$ apply binomial theorem w/ $a = p$ & $b = 1-p$

$$= \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

$\hookrightarrow$ therefore: $E[N] = np$.

Problem 4.

$E[T] = 0 \rightarrow$ remember that the density function is symmetric around 0.

$$E(T) = \int_{-\infty}^{\infty} t f(t)\, dt \rightarrow \int_{-\infty}^{0} t f(t)\, dt + \int_{0}^{\infty} t f(t)\, dt$$

$$= \int_{\infty}^{0} (-t) f(-t)\, dt + \int_{0}^{\infty} t f(t)\, dt$$

$$= -\int_{\infty}^{0} (-t) f(-t)\, dt + \int_{0}^{\infty} t f(t)\, dt$$

$$= \int_{\infty}^{0} t f(-t)\, dt + \int_{0}^{\infty} t (f t)\, dt$$

$$= -\int_{0}^{\infty} t f(-t)\, dt + \int_{0}^{\infty} t f(t)\, dt$$

$$\therefore f(t) - f(-t) \rightarrow = 0$$