Staple here!
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Acknowledgement:

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Section: 1

Homework #4

Q1

Problem: Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

Proof. : The problem, $P(n)$, can be proved using induction.

Basis: Let $n = 1$. Then $P(1)$ is,

$$4^{1+1} + 5^{2-1} = 4^2 + 5 = 21$$

This value is obviously divisible by 21 so $P(n)$ then holds for the base case $n = 1$

Inductive Step: Assume that $P(n)$ is true for some arbitrary positive integer $n = k$. So $P(k)$ means that,

$$4^{k+1} + 5^{2k-1}$$

is divisible by 21.
Let $n = k + 1$. Under the assumption of the inductive hypothesis,

$$1 * 1! + ... + k * k! + (k + 1) * (k + 1)! = (k + 1)! - 1 + (k + 1) * (k + 1)!$$

$$= (k + 1)! * (1 + (k + 1)) - 1$$

$$= (k + 1)! * (k + 2) - 1 = (k + 2)! - 1$$

This equals the right hand side of the equation, $((k + 1) + 1)! - 1$. Therefore, through induction the proposition $P(n)$ holds for all positive integers n. □