Name: Jaime Arana-Rochel

Homework #4

1) Fitting Functions to Data

a) Using interpolation to fit the data using monomials as the basis function, I get the following coefficients (C_j) for the function of the form $f(t) = \sum_{j=0}^{13} C_j t^j$. Note: In fitting the data points, I changed the units of R to 1000's of km.

$$C_0 = 13.088500, C_1 = -1361.618631, C_2 = 8233.500025, C_3 = -20692.625254, C_4 = 29100.900029,$$

$$C_5 = -25824.355880, C_6 = 15372.284289, C_7 = -6338.070260, C_8 = 1834.096641,$$

$$C_9 = -371.455328, C_{10} = 51.554683, C_{11} = -4.670654, C_{12} = 0.248744, C_{13} = -0.005905$$

Fitting the data using the exponential functions, I get the following coefficients for the function of the form $f(t) = \sum_{j=0}^{13} C_j e^{-j*t}$ Note: I changed the units of R to 10,000's of km. Because of floating point rounding, the resulting coefficients make it so the function doesn't exactly describe the data, but it is a close approximation.

$$C0 = -55123799219.207520, C1 = 1012809726066.899658, C2 = -8558617899510.612305,$$

$$C3 = 44042422177217.234375, C4 = -153982824858716.343750,$$

$$C5 = 386246990022165.437500, C6 = -715119599461482.875000, C7 = 989473854207228.250000,$$

$$C8 = -1023150128301797.750000, C9 = 780885132203473.750000, C10 = -427576085308108.687500,$$

$$C11 = 159057702910939.562500, C12 = -36029994981379.492188, C13 = 3753463363136.949707$$

Given these fits,
Monomial: Density of Earth 1200 km from center estimate = 10.154332 g/cm^3
Exponential: Density of Earth 1200 km from center estimate = 3.503418 g/cm^3
Monomial: Density of Earth 6271 km from center estimate = 29.487595 g/cm^3
Exponential: Density of Earth 6271 km from center estimate = 5.710114 g/cm^3

b) If we were to repeat the fitting process using only the last four data points, then the estimates would be,
Monomial: Density of Earth 6271 km from center estimate = 1.908767 g/cm^3
Exponential: Density of Earth 6271 km from center estimate = 1.896475 g/cm^3

c) If instead we measure the earth's density based on the depth to the center from the surface of the earth, we find that our estimates don't change.
Monomial: Density of Earth 1200 km from center (Depth of 5171 km from surface) estimate

$= 10.154341\ g/cm^3$
Monomial: Density of Earth 6271 km from center (Depth of 100 km from surface) estimate $= 29.487524\ g/cm^3$

The estimates match up, except for some slight rounding errors.

2) Interpolations and believability

a) Using a monomial basis function to fit the data, I get the following function,

$$f(t) = 4.991610 - 0.033759t - 2.423947t^2 + 0.140661t^3 + 0.441540t^4 - 0.050535t^5 - 0.023476t^6 + 0.002956t^7$$

Using the method of least squares to get the coefficients a and b for a line $y = ax + b$, the coefficients are

$$a = 0.025732 \quad b = 2.265933$$

We then substitute these values into the fit equation of form $y(t) = Ae^{-\frac{t^2}{B}}$. I found that the resulting function needed to be scaled larger so as to better describe the data, so I scaled it by a factor of 190.

$$y(t) = 190Ae^{-\frac{t^2}{B}}$$

We get the following plot which suggests that the data follows a curve which does not intersect the x-axis as the monomial fit suggests.