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Homework #1

Q1: Exercise 1 Chapter 1

False. It is not the case that every instance of the Stable Matching Problem has a stable matching containing a pair (m, w) where m ranks w first, and w ranks m first.

Consider the set of men $M = \{m, m'\}$ and the set of women $W = \{w, w'\}$ with preference lists as follows

Man	1^{st}	2^{nd}
m	w	w'
m'	w'	w

Woman	1^{st}	2^{nd}
w	m'	m
w'	m	m'

There are two possible stable matchings given this information. One stable matching S has the pairs (m, w), (m', w'). In this matching, the men are as happy as possible. The other stable matching has pairs (m', w), (m, w'). The women are as happy as possible in this matching. However, in either stable matching, there is no pair such that both a man and a woman are as happy as possible, as described in the problem.

Q2: Exercise 3 Chapter 1

False. For every set of TV shows and ratings, there is not always a stable pair of schedules.

Suppose we have two networks A and B each with two shows. Network A has shows a_1 and a_2 with ratings 100 and 500 respectively. Network B has shows b_1 and b_2 with ratings 300 and 800 respectively. There are two possible scenarios: either a_1 competes with b_1 for a slot (meaning a_2 competes with b_2 for a slot) or a_1 competes with b_2 for a slot.

Case 1: a_1 competes with b_1

Let network A have schedule S with a_1 in slot 1 and a_2 in slot 2. Network B has schedule T with b_1 in slot 1 and b_2 in slot 2. Network A currently doesn't win any slots. Then the pair (S, T) is unstable because A can win a slot by switching the order of its shows. The same is true if a_1 and b_1 were competing for slot 2.

Case 2: a_1 competes with b_2

Let network A have schedule S with a_1 in slot 1 and a_2 in slot 2. Network B has schedule T with b_2 in slot 1 and b_1 in slot 2. Currently, network A wins slot 2 and network B wins slot 1. Then the pair (S, T) is unstable because B can win both slots by switching the order of its shows. The same is true if a_1 and b_2 were competing for slot 2.

Q3: Exercise 8 Chapter 1

True. A woman may end up better off by lying about her preferences.

Consider the set of men $M = \{m, m', m''\}$ and the set of women $W = \{w, w', w''\}$ with preference lists as follows

Man	1^{st}	2^{nd}	3^{rd}
m	w	w''	w'
m'	w	w'	w''
m''	w''	w	w'

Woman	1^{st}	2^{nd}	3^{rd}
w	m''	m	m'
w'	m'	m	m''
w''	m	m'	m''

Running the Gale-Shapley algorithm with these preferences we see that m proposes and gets engaged to w. m' proposes to w, gets rejected, then proposes to w' and gets engaged. Finally m'' proposes and gets engaged to w''. Thus the stable matching from running G-S is

$$S = \{(m, w), (m', w'), (m'', w'')\}.$$

In this case woman w is engaged to the man she ranked second. Now suppose that w lies about her preferences and ranks m in third and m' in second. Running G-S with this switch from w, we get the following stable matching

$$S = \{(m, w''), (m', w'), (m'', w)\}.$$

Clearly, we can see from this new matching that w is now better of because she is engaged with w'' who she ranks as first. So lying about her preference of m and m' helped w end up with a higher ranked man.

Q4

From the Wikipedia page on light, the speed of light l is approximately 3.00 x 10^8 m/s. Assuming we're using the classical model of an electron, the approximate radius of an electron is 2.8 x 10^{-15} m, according to Wikipedia. Thus the diameter d_e of an electron is $\approx$ 5.6 x

10^{-15} m. According to Wikipedia, the estimated age of the universe is $\approx$ 13.8 billion years = 13.8 x 10^9 years = 4.4 x 10^{17} seconds. Using this information

$$Unit\ Time\ = \frac{d_e}{l} = \frac{5.6\,x\,10^{-15}}{3.00\,x\,10^8} \approx 1.87\,x\,10^{-23}\ seconds$$

Given this unit time, if the time spent running the algorithm is the age of the universe, the algorithm will generate

$$\frac{4.4\,x\,10^{17}}{1.87\,x\,10^{-23}} \approx 2.35\,x\,10^{40}$$

total matchings.
Given n boys and n girls, there are a total of $n!$ possible matchings. So we need to find an n such that $n! \approx 2.35\,x\,10^{40}$. The closest fractional value for n is 35.23

$$35.23! \approx 2.35\,x\,10^{40}.$$

Of course, for the purpose of the algorithm which needs an integer n, the largest n would have to about 35.