Homework #8

Q1: Exercise 2 Chapter 8

Claim: *Diverse Subset* $\in NP$.
Given a set S of k customers, we can check in polynomial time that no two customers bought the same product.

Claim: *Independent Set* $\leq_p$ *Diverse Subset*
Given a graph G and k, we assign a customer for every vertex in G, and assign a product for every edge. We then create a 2-D array A where we specify that customer v bought product e if vertex v is incident to edge e.
With this construction, the array A has a diverse subset of size k if and only if G has an independent set of size k. First, if the array has a diverse subset of size k, then the associated vertices of G form an independent set of size k since no two vertices would be incident to the same edge. If G has an independent set of size k, then the corresponding customers in A form a diverse subset since no two customers would have bought the same product (they are not incident to the same product edge).

Having shown these two claims, we know that *DiverseSubset* is *NP-Complete*.

Q2: Exercise 7 Chapter 8

Claim: 4-*Dimensional Matching* $\in NP$.
Given a set of n 4-tuples, we can verify in polynomial time that each element from the union of these 4-tuples is disjoint.

Claim: 3-*Dimensional Matching* $\leq_p$ 4-*Dimensional Matching*
Given an instance of 3-*Dimensional Matching* with sets X,Y,Z and collection of ordered triples A, we construct an instance of 4-*Dimensional Matching*. We will have sets W,X,Y,Z and we create our collection of ordered 4-tuples B such that a 4-tuple (w_i, x_k, y_l, z_m) is defined for every triple (x_k, y_l, z_m) in A $(1 \leq i, k, l, m \leq n)$. Thus given either a triple of 4-tuple, we can derive either a 4-tuple or triple respectively.
So, we have 4-dimensional matching in our instance if and only if there is a 3-dimensional matching. Given a set of n disjoint triples in A, we can derive n disjoint 4-tuples in B. If we're given n disjoint 4-tuples in B, then we can derive n disjoint triples in A.

This completes the reduction, which means that 4-*Dimensional Matching* is indeed *NP-Complete*.

Q2: Exercise 22 Chapter 8

Since the arbitrary graph G might not be connected, we simply introduce an extra vertex u to G and add an edge from u to every other vertex in G. Now it is connected. Now we use our black-box algorithm A to ask whether our new graph G' has an independent set of size at least k and return the answer. Building G' takes polynomial time in construction. Also, it is clear to see that the original graph G has an independent set of size at least k if G' has an independent set.