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Equilibrium Index and Single-Stock Volatility Risk Premia

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Abstract

Writers of index options earn high returns due to a significant and high volatility risk premium, but writers of options in single-stock markets earn lower returns. Using an incomplete information economy, we develop a structural model with multiple assets where agents have heterogeneous beliefs about the growth of firms' fundamentals and a business-cycle indicator and explain the different volatility risk premia of index and single-stock options. The wedge between the index and individual volatility risk premium is mainly driven by a correlation risk premium which emerges endogenously due to the following model features: In a full information economy with independent fundamentals, returns correlate solely due to the correlation of the individual stock with the aggregate endowment ("diversification effect"). In our economy, stock return correlation is endogenously driven by idiosyncratic and systemic (business-cycle) disagreement ("risk-sharing effect"). We show that this effect dominates the diversification effect, moreover it is independent of the number of firms and a firm's share in the aggregate market. In equilibrium, the skewness of the individual stocks and the index differ due to a correlation risk premium. Depending on the share of the firm in the aggregate market, and the size of the disagreement about the business cycle, the skewness of the index can be larger (in absolute values) or smaller than the one of individual stocks. As a consequence, the volatility risk premium of the index is larger or smaller than the individual. In equilibrium, this different exposure to disagreement risk is compensated in the cross-section of options and model-implied trading strategies exploiting differences in disagreement earn substantial excess returns. We test the model predictions in a set of panel regressions, by merging three datasets of firm-specific information on analysts' earning forecasts, options data on S&P 100 index options, options on all constituents, and stock returns. Sorting stocks based on differences in beliefs, we find that volatility trading strategies exploiting different exposures to disagreement risk in the cross-section of options earn high Sharpe ratios. The results are robust to different standard control variables and transaction costs and are not subsumed by other theories explaining the volatility risk premia.

JEL Classification Codes: D80, G12, G13.

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THE AVERAGE IMPLIED VOLATILITY OF INDEX OPTIONS ON THE S&P 100 IS ABOUT 19.2% per year, but the realized volatility is only about 16.7%. The volatility risk premium, defined as the difference between the implied and realized volatility, is large and results into large returns for index option sellers. On the other hand, the average implied volatility of single-name options on all S&P 100 constituents is about 32.7% and the realized volatility is 31.8%, which yields a volatility risk premium of 1%. There are two noteworthy points here: First, implied volatilities of individual stock options are on average much higher than the one of index options. Second, the volatility risk premium is larger for index options. This paper aims at developing a structural model that explains endogenously these empirical facts using an incomplete information economy with multiple assets, in which investors have different perceptions of the volatility of expected future dividends and heterogeneous beliefs about the expected growth rate of firms' fundamentals. In equilibrium, uncertainty drives the second moments of asset prices and their co-movement even if the underlying fundamentals are independent. These features have implications for the absolute and the relative size of the volatility premium of both individual and index options, and for the excess returns of several well-established option trading strategies. We study these implications both theoretically and empirically.

The global financial crisis which has caused havoc across all markets the past months, has brought back asset volatility and correlations into the limelight of the financial press due to unusual high levels of volatility and correlation. As a little contested fact, plummeting returns come with a surge in both volatility and correlation. A well known volatility measure is the VIX, commonly also labeled fear gauge index, and it reached an intraday all time high of almost 90% in October 2008. Volatility measures derived from derivatives markets are said to provide useful information on investors' future expectations and its associated uncertainty. To motivate the relation between the volatility risk premium and uncertainty, we plot in Figure 1 (right panel) the volatility risk premium on the S&P 500 together with a proxy of a common uncertainty, calculated from differences in analysts' forecasts of future earnings for a cross-section of firms in the S&P 500.

[Insert Figure 1 approximately here.]

We note a strong comovement between the volatility risk premium and the uncertainty proxy. For instance, both after the LTCM collapse in late summer 1998 and after the terrorists' attacks in September 2001 (yellow bars), the volatility risk premium and the uncertainty proxy spike. To gauge this relationship in more detail, we find that a simple regression from the volatility risk premium on the uncertainty proxy yields a statistically highly significant standardized coefficient of 0.58 and an adjusted R^2 of 23%. Interestingly, the strong relationship between the volatility risk premium and uncertainty prevails also in the cross-section. In the left panel of Figure 1, we plot on the abscissa the average uncertainty proxy for 14 different sectors and on the ordinate the average volatility risk premia for these sectors.[1] The strong positive relationship is striking. Indeed, apart from the energy and media sector, there seems to be an almost linear relationship between the uncertainty proxies and the volatility risk premia which is indicated by the least-squares line in the Figure.

[1] The index volatility risk premium is defined as the difference between the VIX and the 30 day (annualized) realized volatility on the S&P 500. Similarly, the volatility risk premia on the individual sectors are calculated from implied volatilities of 30 day at-the-money options on individual stocks. We then average across time and across all firms within one sector. Realized volatility is calculated by summing daily log returns over the month.

The high levels of volatility and correlations have caused an increase in the trading volumes of volatility products, such as variance and correlation swaps, even if the aggregate trading volume of common assets such as stock or bonds have decreased drastically.[2] Over the past few years, volatility products have emerged as an asset class on their own right as more and more hedge funds engaged in so called volatility arbitrage strategies:[3] The core idea of volatility trading is that if the expected volatility is consistently higher than the realized volatility, then the seller of options locks in the premium. Buyers, on the other hand, go long in volatility because they want to hedge against a market downturn. The classical and most popular example of such a volatility strategy is a long position in a straddle. An alternative strategy involves delta-hedging an option position: If the investor is successful in hedging away the priced risk, then a major determinant of the profit or loss from this strategy is the difference between the realized volatility and the expected volatility of the option. Finally, to take advantage of the higher average weighted volatility of constituents versus the index volatility itself, banks and hedge funds enter so called dispersion trades. These trading strategies involve usually a short position in the index volatility and a long position in the constituents volatility.[4]

To illustrate these kind of trades in more detail, we plot in Figure 2 the returns of ten stocks in the basket of a fictive index. In the left panel, we show a day where all the returns of the constituents seem to be uncorrelated. The average pairwise constituents correlation is 4% and the index return of this equally weighted basket is 0.03%.[5] The measured dispersion for this day, defined as the difference between the sum of weighted constituents' variance and the index variance, is 41%. The investor had earned the gamma profits on this portfolio and earned the theta in the index, which she could use to pay for the theta in the single stocks. In the right panel, we plot the constituents returns one month later. The measured dispersion is only 2%, the average pairwise correlation between stock returns has reached 87%, but the index return is -1.54%. Stock return correlations increase when returns are low.[6] In this case, the investor left some money on the table, since she is paying for the theta in the stocks.[7] This example demonstrates that correlation is an important risk factor embedded in this kind of trades: Being long a dispersion trade means being short in correlation. It is natural to ask what happened to uncertainty in this period. Indeed we find that the estimated degree of disagreement about these ten firms in the sample increased more than three times (from 0.13 to 0.4) from June 6th, 2007 to July 6th, 2007.

[Insert Figure 2 approximately here.]

[2] The growth in variance trading is said to have increased by 100% in 2008 (see http://www.euromoney.com/Article/2059815).

[3] E.g., LTCM was labeled the "Central Bank of Volatility" (see Lowenstein, 2000).

[4] See Appendix B for details. Short dispersion trades, also called Chinese positions, are not that common. The reason is that shorting constituents volatility means shorting equity gamma. One could argue that the short equity gamma is covered by the long gamma position in the index. However, unforseen events such as mergers and acquisitions or company bankruptcies can cause big losses.

[5] By re-balancing the constituents deltas on the individual stocks, the investor would capture the volatility. However, in this case she does not have to re-balance the delta of the index, since the index has barely moved.

[6] It is a well documented fact that stock return correlations are increased at market downturns. See e.g. Erb, Harvey, and Viskanta (1994), Ledoit, Santa-Clara, and Wolf (2003), and Moskowitz (2003). Ribeiro and Veronesi (2002) build a model in which higher uncertainty in bad states induce excess comovement in the correlations of stock returns.

[7] As stylized this example might be, Barings PLC sustained huge losses from short positions in volatility on Nikkei futures as the Nikkei plunged in 1995. Similarly, when LTCM collapsed in late 1998, it had a 35 million short vega position due to the high index implied volatility. Many high-tech companies issued puts cheaply to manage their employee stock-option programs. LTCM bought heaps of these puts, issued by companies such as Microsoft and Dell, and hedged them by selling puts on the S&P 500 (see Lowenstein, 2000).

In this paper, we ask why dispersion in analysts' forecasts is so strongly related to implied volatility of index and single-name options and we explore in more detail the underlying drivers of the differential pricing of these assets. We extend the standard Lucas (1978) economy to a setting with two trees and disagreeing investors. The growth rate of the two firms' dividend stream is unknown to the agents. Hence, the growth rates have to be estimated. In our model, disagreement among investors is a key priced state variable that drives the second moments of stock returns, as well as the smile and the volatility risk premia of individual and index options. Stochastic volatilities and correlations of stock returns arise endogenously from the diverging optimal consumer policies of pessimistic and optimistic investors. Option implied volatility skews and volatility risk premia reflect the higher protection needs of pessimistic investors, who buy financial protection from optimistic agents in exchange for a premium. The more specific implications of our model are as follows.

First, a higher disagreement on the future dividends of one firm induces a higher stock volatility and a negative skewness in the distribution of the stock returns of that particular firm. At the same time, it also impacts on the second stock return through the stochastic discount factor. In our economy, stock return correlation is endogenously driven by the time-variability of disagreement. This finding gives theoretical support to the empirical evidence that correlations between asset returns vary over time.[8] In a standard Lucas economy with homogeneous agents, the state price density varies only due to dividend fluctuations. Stock returns are correlated due to the correlation of the individual stock with the aggregate endowment ("diversification effect"), a feature that is documented in Cochrane, Longstaff, and Santa-Clara (2008). In our economy, the state price density depends additionally on the cross-sectional wealth that is shifted across agents ("risk-sharing effect"). We show that this additional component dominates the two trees effect and is independent of the number of firms and their size in the aggregate market. Second, the risk-neutral skewness of the individual firm can be smaller or larger (in absolute terms) than the skewness of the index, depending on the share of the firm in the aggregate market and the size of the disagreement about the business cycle component. If the disagreement about the business cycle and the disagreement about firms' future dividends are large, then the risk-neutral skewness of the index can be more negative than the individual stock due to an additional correlation component. Vice versa, if the disagreement about the business cycle is zero and the share of the individual firm in the aggregate market is small, then the risk-neutral skewness of the individual firm tends to be more negative than the index. The differential pricing of index and single-stock options is empirically validated in Bakshi, Kapadia, and Madan (2003). Third, we provide an economic rationale for the different size of volatility risk premia in index and individual stock options. Volatility risk premia of individual and index options represent compensation for the priced disagreement risk. Hence, in the cross-section of options the volatility risk premium depends on the size of belief heterogeneity of this particular firm and the business cycle indicator. As the risk-neutral skewness, the volatility risk premium for index options can be larger or smaller depending on the size of disagreement and of the firm's share. Fourth, option excess returns reflect the different exposure to disagreement risk. Investors who buy options of firms which are more prone to heterogeneity in beliefs are compensated in equilibrium for holding this risk. We test these implications by running simulated option trading strategies which exploit the

[8]See Bollerslev, Engle, and Wooldridge (1988), and Moskowitz (2003), among many others.

difference between the volatility risk premia of index and individual options. We find that the excess returns of these strategies are substantial and the their annualized Sharpe ratio exceeds a short put index option strategy twice.

To validate our model empirically, we use data on S&P 100 index options and on single-stock options for all the index constituents in the period January 1996 to June 2007. We merge this dataset with analysts earning forecasts from the Institutional Brokers Estimate System (I/B/E/S) and stock return data from CRSP. We first compute a belief disagreement proxy for each individual firm and then construct a common factor that proxies for the overall belief disagreement across firms. We obtain a number of interesting results.

First, as predicted by our theoretical model, we find that belief disagreement increases the volatility risk premium of both index and individual stock options in a way that is remarkably robust with respect to the inclusion of other control variables. E.g., in a regression including the market volatility risk premium, the R^2 is about 12% higher when disagreement is accounted for. A simple regression of the market volatility risk premium on the volatility risk premium of single-stocks yields a R^2 of about 8%. To the best of our knowledge, there are only a few academic studies on the determinants of volatility risk premia. Carr and Wu (2009) find that classical risk factors, such as the market excess return, the Fama and French (1993) factors, a momentum factor, or two-bond market factors, cannot explain the variance risk premia of a limited set of individual options. Bollerslev, Gibson, and Zhou (2007) argue that the volatility risk premium in index options comes from time-varying risk aversion. In particular, they find that macro-finance variables have a statistically significant effect on the index volatility risk premium. In our model, the time-variability in the volatility risk premia of both index and single-stock options comes from the fluctuations of belief disagreement.

Second, we study simple option-based trading strategies aimed at exploiting the cross-sectional difference in volatility risk premia. This is a natural question in our context, since belief disagreement is theoretically linked to the option volatility risk premia. In particular, we study at-the-money straddle and put dispersion portfolios. Accordingly, each month we short the index straddles or puts, and buy individual straddles or puts, respectively. Ideally, the investor wishes to have the cheapest single name options in his portfolio. Since higher disagreement is linked to a higher volatility risk premium, we pick each month the quintile of firms with the highest belief disagreement and for these firms we buy straddles and puts, respectively. Both the at-the-money straddle and put portfolios generate statistically and economically significant returns. For example, the straddle portfolio yields an annualized Sharpe ratio of 1.96 and the put portfolio an annualized Sharpe ratio of 2.02. These Sharpe ratios are 2.5 times higher than the Sharpe ratio derived from investing all wealth into the index itself. Goyal and Saretto (2008) find similarly high returns and Sharpe ratios for straddle strategies, when sorting their portfolios of single-stock options according to the difference in implied and realized volatility, and interpret this as some form of volatility mispricing. While these features are indeed inconsistent with a single factor option-pricing model, such as the Black and Scholes (1973) and Merton (1973) model, in our economy they are fully compatible with the existence of a priced disagreement factor.

Third, we test whether the high returns of our portfolios are subsumed by other effects, which are typically used to account for differences in the cross-section of stock returns. We find that the results persist in any size, book-to-market, and momentum portfolio. We use a linear factor model including Fama and French (1993) factors and the Carhart (1997) momentum factor, and find that none of these factors contributes to the explanation of these option returns. Much of the recent literature on option returns has been devoted to the impact of trading frictions in form of transaction costs and margins on the profitability of option strategies. We test the impact of transaction costs on our trading strategies and find that transaction costs indeed lower the profitability of our trading strategies. However, the Sharpe ratio of the straddle portfolio exceeds the Sharpe ratio of a trading strategy that is known to be very profitable (see Bondarenko, 2003), shorting index put options, by 40% and the put portfolio exceeds the short index portfolio by 30%. Moreover, the CAPM alphas of our strategies still remain statistically significant.

Finally, our results are robust also with respect to other potential theories of volatility risk premia. Empirical evidence has shown that volatility risk premia tend to be high prior to an earning announcement. For example, when Cisco announced their earnings on August 10th, 2004, it managed to beat analysts' forecast with a 41% leap in net income, and a 26% increase in sales among other records for profitability. However, the share price lost more than 10% on the specific day of the announcement. Moreover, the share price sank more than 15% the next 60 trading days and the volatility risk premium increased by 43% from 0.10 to 0.14 prior to the announcement. Beber and Brandt (2006) study state-price densities of bond prices before and after macro announcements. They document a strong decrease in implied volatility and changes in skewness and kurtosis of the state-price density of bond option returns after a macro announcement. They attribute the changes in the higher moments of the state-price density to a time-varying risk aversion. Dubinsky and Johannes (2006) find similar effects for stock options and earning announcements. Implied volatilities of single-stock options increase prior to and decrease subsequent to an earning announcement. They argue that anticipated uncertainty surrounding the fundamental information about the firm causes the implied volatility to increase. Once the uncertainty is revealed, the implied volatility drops. This interplay of uncertainty, news revelation, and volatility risk premia is interesting also in our model. For instance, we find that in the previous month to the earning announcement of Cisco, belief disagreement rose by more than 30% from 0.68 to 0.95. These numbers suggest a non-trivial link between earning announcements, belief disagreement, and the volatility risk premium in individual options. We test this hypothesis and find that earning announcements have a significant impact on the volatility risk premium. An interaction term of belief disagreement and earnings announcements, however, is not statistically significant. This suggests that belief disagreement has a significant impact on volatility risk premia independent of the presence of earning announcements. Much more research is needed in this direction.

Related Literature.

Equilibrium models with multiple assets have been studied in the literature before, notably by Menzly, Santos and Veronesi (2004), Santos and Veronesi (2006), Pavlova and Rigobon (2007), Cochrane, Longstaff, and Santa-Clara (2008), and Martin (2009). Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006) study a multi-asset economy with external habit where the dividend shares of assets are mean-reverting processes. Pavlova and Rigobon (2007) study a two country two good economy with demand shocks and log-linear preferences. Martin (2009) studies an economy with a collection of Lucas trees, so called Lucas orchards and its impact on asset prices, risk premia, and the term structure. The model is able to replicate many salient features of asset returns such as momentum, mean-reversion, contagion, fight-to-quality, the value-growth effect, and excess volatility. Cochrane, Longstaff, and Santa-Clara (2008) study a Lucas (1978) economy with multiple assets and its implications for stock return moments, correlations, and the equity risk premium. We extend this literature by considering multiple agents who disagree on the expected growth rate of the firm's fundamentals. This implies a pricing kernel that is directly affected by the belief disagreement of both firms. Importantly, it implies an endogenous stochastic correlation that is driven by belief disagreement. We want to emphasize upfront that the intuition of our economy is completely different from the one in Cochrane, Longstaff, and Santa-Clara (2008). In their economy, the existence of a second asset induces correlation of stock returns. There is no trading among agents and the endogenous correlation comes from the fact that if tree two enjoys a positive dividend shock this raises asset two's return, and it also lowers asset one's share. As a consequence, expected returns typically rise with a tree's share of dividends, to attract investors to hold that larger share. Stock returns are correlated because a shock to any of the dividend streams is important for aggregate consumption. This is reflected in the state price density which is a function of the share dynamics and which represents the only source for stock return correlations. In addition to this effect, in our economy there is an increased correlation which comes from the optimal risk sharing among investors. The more pessimistic agent selects a relatively higher consumption in states of low dividends for firm 1, firm 2, or both. Similarly, the more optimistic agent selects a relatively higher consumption in states of high dividends. In order to finance the optimal consumption plan, the pessimistic investor asks financial protection, i.e., put options, from the optimist. This excess demand lowers the price of securities having positive exposure to dividend shocks of the two firms, and the risk implied by bad dividend states is transferred from the pessimist to the optimist. Individual put options offer financial protection against low dividend states of this firm. The higher price of these options reflects the desire of the pessimistic agent to buy protection against low dividend states of one of the two firms. Such a price is higher when the consumption share of this particular firm is more different from one, which happens when investors disagree more on the probability of the event that one firm will pay low dividends. If investors disagree on the joint occurrence of a low dividend for both firms, then their aggregate marginal utility out of the dividends of the two firms will differ even more. In this case, the pessimist requires protection against a joint bad state in dividends, which is best achieved by means of a index put option on the index.

The early literature on volatility risk premia is large and focuses mostly on index options. Fleming, Ostdiek, and Whaley (1995), Jackwerth and Rubinstein (1996), and Christensen and Prabhala (1998) observe that realized index volatilities tend to be substantially lower than implied volatilities of index options.[9] However, these papers mainly focus on the forecasting power of implied volatility for realized volatility, and study different measures of volatility. More recently, the literature on single-name options has found a more mixed evidence of the existence of a nonzero volatility risk premium. Bakshi and Kapadia (2003b) show that individual equity option prices embed a negative market volatility risk premium, although much smaller than for the index option, but their study is focussed on 25

[9]We note that there is some debate in the literature on the volatility risk premium being non-zero or not. Bates (2000), Benzoni (2002), Chernov and Ghysels (2000), Jones (2003), and Pan (2002) find large negative volatility risk premia using structural models. Broadie, Chernov, and Johannes (2007) find no evidence for a volatility risk premium on diffusive volatility risk, but possibly a risk premium on volatility jumps.

stocks only. Bollen and Whaley (2004) report that the average deviation between the Black and Scholes implied volatility and realized volatility is approximately zero for all the 20 individual stocks they study. Carr and Wu (2009) find evidence of a volatility risk premium, but also using only a subset of 35 stocks. Driessen, Maenhout, and Vilkov (2008) find insignificant differences between implied and realized volatilities studying average model-free volatility measures. The authors do find a significant difference in index options and interpret this as a correlation risk premium. A trading strategy that exploits this correlation risk premium yields high Sharpe ratios. Duarte and Jones (2007) study the impact of systematic risk on volatility risk premia and find evidence of a volatility risk premium that varies with the overall level of market volatility. Our empirical findings complement this literature, by showing that disagreement is potentially an important determining factor of volatility risk premia.

The literature aiming at giving a structural explanations for the emergence of volatility risk premia is sparse. Motivated by the empirical results in Bollen and Whaley (2004), who show that changes in implied volatility are correlated with signed option volume, Gârleanu, Pedersen, and Poteshman (2009) study the relationship between the level of end user option demand and the level and overall shape of implied volatility curves. They document that end users tend to have a net long index option position and a short equity-option position, thus helping to explain the relative expensiveness of index options.[10] They also show that there is a strong downward skew in the net demand of index but not equity options, which helps to explain the difference in the shapes of their overall implied volatility curves. However, their framework is more effective in explaining the steeper slope of index options due to excess demand of out-of-the-money puts, but less so in differentiating the pricing of individual options in the cross-section. Eraker (2007a) studies an equilibrium with long-run risk coupled with a highly persistent volatility process. Similarly, Drechsler and Yaron (2008) add infrequent but potentially large spikes in the level of volatility and infrequent jumps in the small, persistent component of consumption and dividend growth. While the volatility shocks from a standard long-run risk model have a market price of risk which is sufficiently large to generate a variance risk premium, second and third moments of the variance risk premium together with the short-horizon predictability of expected stock returns by the variance risk premium can only be generated in a setting with non-Gaussian shocks. In contrast to our paper, however, these works do not study the cross-section and volatility risk premia of individual options.

A vast literature has studied the risk-reward features of option strategies focussing to a large extent on the over-pricing of out-of-the-money index put options. Coval and Shumway (2001) report monthly Sharpe ratios of about 0.3 for zero beta straddle portfolios and Eraker (2007b) find an annualized Sharpe ratio of 1 for selling index out-of-the-money put options. Driessen and Maenhout (2008) find annualized Sharpe ratios of on average 0.72 for different index option strategies. Broadie, Chernov, and Johannes (2007) point out that index put options have large negative betas, which in turn yields large negative returns if the CAPM holds, and find that the high returns of out-of-the-money puts do not contradict the Black and Scholes and CAPM assumptions. More recent papers study the risk reward of option strategies that exploit differences in volatility risk premia. Goyal and Saretto (2008) study the cross-section of individual options and find that portfolios sorted according to the difference of implied and realized volatility earn high Sharpe ratios. They attribute these high returns to some form of volatility mispricing. Driessen,

[10] These findings are also complemented by the recent work of Lakonishok, Lee, Pearson, and Poteshman (2007) who document that for both individual equity calls and puts end users are more short than long.

Maenhout, and Vilkov (2008) argue that priced correlation risk is the main driving factor of index volatility risk premia. They find that a simple option-based trading strategy that locks in the correlation risk premium earns high Sharpe ratios. However, once they account for trading frictions, such as margins and transaction costs, they find that the correlation risk premium cannot be exploited to generate economically significant excess returns. They interpret this finding as a limit to arbitrage. We depart from these papers in the following dimensions: First, we explain volatility risk premia on individual and index stock options by belief disagreement. None of the aforementioned papers gives an economic rationale for the main drivers of volatility risk premia or the portfolio returns. Second, we show that the volatility risk premia compensate for the priced disagreement risk. Third, we empirically show that belief disagreement is indeed priced in the cross-section of option returns, even after transaction costs.

Finally, we are not the first to study the impact of belief disagreement on the implied volatility of options. Buraschi and Jiltsov (2006) demonstrate in a single asset heterogeneous agents economy that belief disagreement increases the implied volatility smile of index options. In this paper, we go one step beyond and provide a rationale for the difference in volatility risk premia of index and single-stock options, both theoretically and empirically. We study a multi-asset economy and focus on the impact of belief disagreement on volatility risk premia of index and individual options, and on option trading strategies. This is not possible in a single good model: Endogenous correlation and a priced correlation risk premium arise exactly due to the inclusion of additional assets.

The remainder of the paper is organized as follows: Section I. provides the model setup. Section II. gives theoretical model predictions. Section III. describes our panel data set and Section IV. presents the results of our empirical study. Finally, Section VI. concludes the paper.

I. The Economy with Uncertainty and Heterogeneous Beliefs

A. The Model

We extend the standard single-asset Lucas-tree pure-exchange framework to the case with two assets and two investors, and solve the model in semi-closed form. The economy has infinite horizon $[0, \infty)$ with uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ on which is defined a standard Brownian motion $W = \left(W_{D_1}, W_{D_2}, W_z, W_{\mu_{D_1}}, W_{\mu_{D_2}}, W_{\mu_z}\right)'$ in $\mathbb{R}^6$. All stochastic processes are assumed to be adapted to $\{\mathcal{F}_t; t \in [0, \infty)\}$, the augmented filtration is generated by the Brownian motion W.

There are two firms in the economy, which produce their perishable good. Dividends of firm $i = 1, 2$ have the following dynamics:

$$dD_i(t) = \mu_{D_i}(t)dt + \sigma_{D_i} dW_{D_i}(t).$$

Dividends are observable, but their expected growth rate is unobservable and has to be estimated given the available information. The growth rate of dividends of firm i has the following dynamics:

$$d\mu_{D_i}(t) = \left(a_{0D_i} + a_{1D_i}\mu_{D_i}(t)\right) dt + \sigma_{\mu_{D_i}} dW_{\mu_{D_i}}(t).$$

To capture a business-cycle indicator of firm-specific uncertainty, we allow the firm's future dividends to be related to a market-wide factor that affects the competitive landscape. We therefore consider a signal $z(t)$, which is related to the unknown dividend growth rate of each firm. It has the following dynamics:

$$\begin{aligned} dz(t) &= \left(\alpha_{D_1}\mu_{D_1}(t) + \alpha_{D_2}\mu_{D_2}(t) + \beta\mu_z(t)\right)dt + \sigma_z dW_z(t), \\ d\mu_z &= \left(a_{0z} + a_{1z}\mu_z(t)\right)dt + \sigma_{\mu_z} dW_{\mu_z}(t). \end{aligned}$$

The unobservable signal growth rate is μ_z, and σ_z is the volatility of the signal z, a_{0z} is the long-term growth rate of the expected change in the signal. $a_{1z} < 0$ the mean-reversion parameter and $\sigma_{\mu_z} > 0$ the volatility. For simplicity, we assume that the firm specific Brownian motions W_{D_1} and W_{D_2} are independent and that they are independent from the signal Brownian motion W_z. These assumptions allow us to focus on the additional impact of belief disagreement on option prices and variance risk premia in our economy.

Investors use both information on the dividends, $D_i(t)$, and the signal, $z(t)$ to make their inferences about the growth rates μ_{D_i} and μ_z. If $\beta = 0$, then note that the signal $z(t)$ contains information exclusively about the expected growth rate of dividends.

B. Disagreement

Uncertainty in our economy is modeled by two agents with different beliefs about the expected growth rates of dividends and signals. Agents update their beliefs using all available information according to Bayes' law. We consider learning dynamics, in which different steady-state posterior beliefs follow from different subjective volatility parameters, $\sigma^n_{\mu_{D_i}}, \sigma^n_{\mu_z}$, where $n = A, B$ indicates agents A and B. This assumption allows for a non-trivial steady-state distribution of the disagreement process. Moreover, since the dividends' expected growth rates μ_{D_i} are unobservable, the true value of $\sigma_{\mu_{D_i}}$ is unknown to all investors and cannot be recovered from the quadratic variations of the observable variables even if we sample at asymptotically high frequency.

Let $m^n(t) := \left(m^n_{D_1}(t), m^n_{D_2}(t), m^n_z(t)\right)' := E^n\left(\left(\mu_{D_1}(t), \mu_{D_2}(t), \mu_z(t)\right)' | \mathcal{F}^Y_t\right)$, and $\mu(t) := \left(\mu_{D_1}(t), \mu_{D_2}(t), \mu_z(t)\right)'$ and $\gamma^n(t) := E^n\left(\left(\mu(t) - m^n(t)\right)\left(\mu(t) - m^n(t)\right)' | \mathcal{F}^Y_t\right)$ where $\mathcal{F}^Y_t := \mathcal{F}^{D_1, D_2, z}_t$ is the information generated by $D_1(t)$, $D_2(t)$, and $z(t)$ up to time t. $E^n(\cdot)$ denotes expectation relative to the subjective probability of investor $n = A, B$. To specify the disagreement process in our model, let $Y(t) = (\log D_1(t), \log D_2(t), z(t))$, $b^A = \mathrm{diag}(\sigma^A_{\mu_{D_1}}, \sigma^A_{\mu_{D_2}}, \sigma^A_{\mu_z})$, $a_0 = (a_{0D_1}, a_{0D_2}, a_{0z})'$, $a_1 = \mathrm{diag}\left(a_{1D_1}, a_{1D_2}, a_{1z}\right)$, $B = \mathrm{diag}(\sigma_{D_1}, \sigma_{D_2}, \sigma_z)$ and $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha_{D_1} & \alpha_{D_2} & \beta \end{pmatrix}$. The posterior beliefs of agent A can be obtained in a standard fashion by using the Kalman-Bucy filter, and is given by:

$$dm^A(t) = (a_0 + a_1 m^A(t))dt + \gamma^A(t)A'B^{-1}dW^A_Y(t), \tag{1}$$

$$d\gamma^A(t)/dt = a_1\gamma^A(t) + \gamma^A(t)a_1' + b^A b^{A\prime} - \gamma^A(t)A'(BB')^{-1}A\gamma^A(t), \tag{2}$$

with initial conditions $m^A(0) = m_0^A$ and $\gamma^A(0) = \begin{pmatrix} \gamma^A_{D_1}(0) & \gamma^A_{D_1D_2}(0) & \gamma^A_{D_1z}(0) \\ \gamma^A_{D_1D_2}(0) & \gamma^A_{D_2}(0) & \gamma^A_{D_2z}(0) \\ \gamma^A_{D_1z}(0) & \gamma^A_{D_2z}(0) & \gamma^A_{z}(0) \end{pmatrix} = \gamma_0^A$, where $dW_Y^A(t) := B^{-1}\left(\left(dY(t) - Am^A(t)\right)dt\right)$ is the innovation process induced by the first investor's belief and filtration.[11] The agent specific parameter b^n in the dynamics of $\gamma^n(t)$ impacts on the distribution of $m^n(t)$ only indirectly by influencing the Riccati differential equation for $\gamma^n(t)$. If we assume that this parameter is perceived identically by all investors, then we can model a setting of rational Bayesian investors who disagree because of different initial priors at time zero. Hence, heterogeneous beliefs arise from agents' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables.

To specify the dynamic disagreement structure in our economy, we need a learning process for agent B. It is defined by the following three dimensional process:

$$\Psi(t) := \begin{pmatrix} \Psi_{D_1}(t) \\ \Psi_{D_2}(t) \\ \Psi_z(t) \end{pmatrix} = \begin{pmatrix} \left(m^A_{D_1}(t) - m^B_{D_1}(t)\right)/\sigma_{D_1} \\ \left(m^A_{D_2}(t) - m^B_{D_2}(t)\right)/\sigma_{D_2} \\ \left(m^A_{z}(t) - m^B_{z}(t)\right)/\sigma_{z} \end{pmatrix}.$$

Ψ_{D_i} (Ψ_z) measures the disagreement about the expected growth rate of dividend i (signal). The dynamics of $\Psi(t)$ is given by:

$$d\Psi(t) \quad = \quad B^{-1}\left(a_1 B + \gamma^B(t)A'B^{-1}\right)\Psi(t)dt + B^{-1}(\gamma^A(t) - \gamma^B(t))A'B^{-1}dW_Y^A(t), \tag{3}$$

with initial conditions

$$\Psi(0) = \left((m^A_{D_1}(0) - m^B_{D_1}(0))/\sigma_{D_1}, (m^A_{D_2}(0) - m^B_{D_2}(0))/\sigma_{D_2}, (m^A_{z}(0) - m^B_{z}(0))/\sigma_{z}\right)$$

and $\gamma^B(0) = \gamma_0^B$.

B.1. Uncertainty Co-movement

Note from equation (3) that if there is no common signal, the individual disagreement processes do not co-move. The reason is that the Riccati equation (2) for the steady-state volatilities only allows for the trivial solution that the steady-state covariance for the growth rate of firm 1 and 2 is equal to zero, i.e. $\gamma_{D_1D_2} = 0$. However, Figure 1 shows that there seems to be a strong counter-cyclical common component in Uncertainty-DiB across different sectors. A growing body of empirical finance has documented so called spillover effects from one market to another or from one sector to another even if the fundamentals are very weakly linked. The macro finance literature has mostly explained these effects via real linkages such as demand or supply shocks, see Pavlova and Rigobon (2007), however, empirical evidence for these links are rather weak (see e.g. Kaminsky and Reinhard, 2000). In our economy,

[11] A formal proof of this result can be found in Liptser and Shiryaev (2000); see also Buraschi, Trojani, and Vedolin (2007).

markets or different sectors are naturally related via the common uncertainty. To see this, we give in the following Proposition the instantaneous correlation between the different uncertainty proxies for each firm and the signal.

Proposition 1. *The instantaneous uncertainty correlation between firm 1 and firm 2 takes the following form:*

$$
\begin{aligned}
corr(d\Psi_{D_1} d\Psi_{D_2}) \;=\; & \Bigg(\left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}}\right)\left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right)\left(\frac{\gamma^A_{D_2} - \gamma^B_{D_2}}{\sigma^2_{D_2}}\right) \\
& + \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) \\
& \times \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \alpha_{D_2}\left(\gamma^A_{D_2} - \gamma^B_{D_2}\right) + \beta\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right)\Bigg) \times \\
& 1/\Bigg\{ \left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}} + \frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}} + \frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) \\
& \times \left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}} + \frac{\gamma^A_{D_2} - \gamma^B_{D_2}}{\sigma^2_{D_2}} + \frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \alpha_{D_2}\left(\gamma^A_{D_2} - \gamma^B_{D_2}\right) + \beta\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right)\Bigg\},
\end{aligned}
$$

where γ^n represent the steady state covariance/variance of firm's expected growth rates solving the Riccati equation 2 from the perspective of agent n. The instantaneous uncertainty correlation between firm 1 and the signal takes the following form:

$$
\begin{aligned}
corr\left(d\Psi_{D_1} d\Psi_z\right) \;=\; & \Bigg(\left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}}\right)\left(\frac{\gamma^A_{D_1z} - \gamma^B_{D_1z}}{\sigma_{D_1z}}\right) + \left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right)\left(\frac{\gamma^A_{D_2z} - \gamma^B_{D_2z}}{\sigma_{D_2\sigma_z}}\right) \\
& + \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) \\
& \times \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right) + \alpha_{D_2}\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right) + \beta\left(\gamma^A_z - \gamma^B_z\right)}{\sigma^2_z}\right)\Bigg) \\
& 1/\Bigg\{ \left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}} + \frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}} + \frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) \\
& \times \left(\frac{\gamma^A_{D_1z} - \gamma^B_{D_1z}}{\sigma_{D_1}\sigma_z} + \frac{\gamma^A_{D_2z} - \gamma^B_{D_2z}}{\sigma_{D_2}\sigma_z} + \frac{\alpha_{D_1}\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right) + \alpha_{D_2}\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right) + \beta\left(\gamma^A_z - \gamma^B_z\right)}{\sigma^2_z}\right)\Bigg\}.
\end{aligned}
$$

A similar expression arises for the instantaneous correlation between the disagreement about firm 2 expected growth rate and the disagreement about the signal growth rate.

In Figure 3 we plot the instantaneous correlation between uncertainty of firm 1 and firm 2 (left panel) and firm 1 and the exogenous signal (right panel) as a function of the weight given to each firm for the estimation of the signal growth rate. The rest of the economy is symmetrical in the sense that all firm-specific parameters are chosen such that firm 1 and firm 2 are the same.

[Insert Figure 3 approximately here.]

The correlation between the firm-specific disagreements is largest when the weights given to firm 1, 2 and the signal are all equal, i.e. $\alpha_{D_1} = \alpha_{D_2} = \beta = 1/3$. When zero weight is given to each individual firm for the estimation of the signal growth rate, i.e. $\alpha_{D_1} = \alpha_{D_2} = 0$, then the signal is non informative for the updating for the individual expected growth rates of each firm and the disagreement correlation is zero. The right panel plots the correlation between the disagreement about firm 1 and the common signal. We note that when the growth rate of the signal is solely a weighted function of both firm specific growth rates, then the correlation between the signal and the firm specific uncertainty is largest. This is intuitive as in this case the signal is most informative about the individual firm specific growth rates.

C. Investors' Preferences and Equilibrium

There are two investors in the economy with different subjective beliefs, but identical in all other aspects, such as preferences, endowments, and risk aversion. They maximize the life-time expected power utility subject to the relevant budget constraint:

$$V^n = \sup_{c_{D_1}^n, c_{D_2}^n} E^n \left(\int_0^\infty e^{-\delta t} \left(\frac{c_{D_1}^n(t)^{1-\gamma}}{1-\gamma} + \frac{c_{D_2}^n(t)^{1-\gamma}}{1-\gamma} \right) dt \,\Big|\, \mathcal{F}_0^Y \right), \tag{4}$$

where $c_{D_i}^n(t)$ is the consumption of agent n at time t of good i, $\gamma > 0$ is the relative risk aversion coefficient, and $\delta \geq 0$ is the time preference parameter. We assume time-separable utility functions. This not only simplifies the computation of the equilibrium, but also interpretations, since we can sum over individual beliefs without making any further assumptions on aggregation. Agents can trade in the risk-free bond, the firms' stocks and additionally on options written on the stocks and an index. We denote by $r(t)$ the risk free rate of the zero-coupon bond, assumed in zero net supply, by $S_i(t)$ the stock price of firm i, assumed in positive net supply, by $O_i(t)$ the price of a European option on the stock i, assumed in zero net supply, and by $I(t)$ the index option, also assumed in zero net supply.

Definition 1 (Equilibrium)**.** *An equilibrium consists of a unique stochastic discount factor such that (I) given equilibrium prices, all agents in the economy solve the optimization problem (4), subject to their budget constraint. (II) Good and financial markets clear.*

To solve for the equilibrium, we apply standard methods as introduced by Cox and Huang (1989), among many others. The stochastic discount factor $\xi^n(t)$ of agent n can be easily derived. Given the expressions for $\xi^n(t)$, we can price any contingent claim in the economy, by computing the expectations of its contingent claim payoff weighted by the state price density.

Since the investors' preferences are separable over the two goods, the representative investor's utility will also be separable. In this economy we need two representative investors to construct the equilibrium, because we have two goods. We write the representative agent's utility function as:

$$U^1 \left(c_{D_1}(t), \lambda(t) \right) = \sup_{c_{D_1}(t) = c_{D_1}^A(t) + c_{D_1}^B(t)} \left\{ \frac{c_{D_1}^A(t)^{1-\gamma}}{1-\gamma} + \lambda(t) \frac{c_{D_1}^B(t)^{1-\gamma}}{1-\gamma} \right\},$$

and the second one as:

$$U^2 \left(c_{D_2}(t), \lambda(t) \right) = \sup_{c_{D_2}(t) = c_{D_2}^A(t) + c_{D_2}^B(t)} \left\{ \frac{c_{D_2}^A(t)^{1-\gamma}}{1-\gamma} + \lambda(t) \frac{c_{D_2}^B(t)^{1-\gamma}}{1-\gamma} \right\},$$

where $\lambda(t) > 0$ is the stochastic weight that captures the impact of belief disagreement. From market clearing and the optimality condition in equation (4), we obtain the equilibrium consumption allocations and the corresponding individual stochastic discount factors. The stochastic discount factor and the individual optimal consumption policies of agent A and B are of the following form:

$$\xi^A(t) = \frac{e^{-\delta t}}{y_A} D_1(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^{\gamma}, \quad \xi^B(t) = \frac{e^{-\delta t}}{y_B} D_1(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^{\gamma} \lambda(t)^{-1},$$

and

$$c^A_{D_i}(t) \quad = \quad D_i(t) \left(1 + \lambda(t)^{1/\gamma}\right)^{-1}, \quad c^B_{D_i}(t) = D_i(t)\lambda(t)^{1/\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^{-1}.$$

The subjective state prices $\xi^n(t)$ are functions of $D_1(t)$ and the weighting process $\lambda(t)$. The separability of the utility function implies such a simple expression for $\xi^n(t)$. The weighting process $\lambda(t) = y_A \xi^A(t) / \left(y_B \xi^B(t)\right)$ follows the dynamics:

$$\frac{d\lambda(t)}{\lambda(t)} = -\left(\sum_{i=1}^{2} \Psi_{D_i}(t) dW^A_{D_i}(t) + \left(\sum_{i=1}^{2} \alpha_{D_i} \Psi_{D_i}(t) \frac{\sigma_{D_i}}{\sigma_z} + \beta \Psi_z(t)\right) dW^A_z(t)\right). \tag{5}$$

Finally, the relative price of the second good is of the form:

$$rp(t) = \left(\frac{D_2(t)}{D_1(t)}\right)^{-\gamma}.$$

In contrast to the single-good economy, the dynamics of $\lambda(t)$ depends on the disagreement about dividends related to both firms and the business cycle indicator: $d\lambda(t)$ is subject to shocks $W_{D_i}, W_z,\ i = 1, 2$, each weighted by the disagreement indices $\Psi_{D_i}(t)$ and $\Psi_z(t)$, respectively. It follows that the state price volatility is increasing in the disagreement about the cash flows and signals of each firm. The state-price $\xi^n(t)$ reflects the different optimal consumption policies of the two agents in the economy. Assume, for simplicity, that investor A is optimistic about both firms. Then, investor B will select a relatively higher consumption in states of low dividends for firm 1, firm 2, or both. Similarly, investor A will select a relatively higher consumption in states of high dividends. It follows that the relative consumption share in this economy is stochastic and its cyclical behavior is reflected in the dynamics of the stochastic weight $\lambda(t)$. In order to finance that optimal consumption plan, the pessimistic investor asks financial protection, i.e. put options, from the optimist. This excess demand lowers the price of securities having positive exposure to dividend shocks of the two firms and the risk implied by bad cash flow states is transferred from the pessimist to the optimist. It follows that if a negative dividend state occurs, the more optimistic agent is hit twice: First, because the aggregate endowment is lower, second, as a consequence of the protection agreement that makes her consumption share lower in those states. In the economy with two firms, individual put options on stocks of firm i offer financial protection against low dividend states of this firm. The higher price of these options reflects the desire of the pessimistic agent to buy protection against low dividend states of one of the two firms. Such a price is higher when the consumption share of good i is more different from one, which happens when investors disagree more on the probability of the event that one firm will pay low dividends. If investors disagree also on the joint occurrence of a low dividend for both firms, then their aggregate marginal utility out of the dividends of the two firms will differ even more in this case. The pessimist then requires protection against a joint bad state in dividends, which is best achieved by means of an index option on the two stocks. The larger difference of the marginal utility of the two investors in such a joint bad state requires the price of the index option to increase even more than the price of the individual options. This feature could potentially explain the larger size of volatility premia on index options relative to those of individual options.

D. Security Prices

For convenience, we give all the relevant expressions for the prices of financial assets from the perspective of agent A. Given the individual state-price densities, we can easily price any contingent claim in our economy. The equilibrium stock price of firm 1 and 2 are given by:

$$S_1(t) = D_1(t) E_t^A \left(\int_t^{\infty} e^{-\delta(u-t)} \frac{\xi^A(u)}{\xi^A(t)} \frac{D_1(u)}{D_1(t)} du \right), \tag{6}$$

and

$$S_2(t) = D_2(t) E_t^A \left(\int_t^{\infty} e^{-\delta(u-t)} \frac{\xi^A(u)}{\xi^A(t)} \left(\frac{D_2(u)}{D_2(t)} \right)^{1+\gamma} \left(\frac{D_1(u)}{D_1(t)} \right)^{-\gamma} du \right). \tag{7}$$

The stock index is simply the weighted sum of the two stocks:

$$ID(t) = \omega_1 S_1(t) + \omega_2 S_2(t), \tag{8}$$

where ω_1 and ω_2 are the market capitalizations of stock 1 and 2, respectively, at time t. The price of a European call option on stock i is:

$$O_i(t,T) = E_t^A \left(\frac{\xi^A(T)}{\xi^A(t)} (S_i(T) - K_i)^+ \right), \tag{9}$$

where K_i is the strike price of the option. Similarly, the price of the index call option is:

$$I(t,T) = E_t^A \left(\frac{\xi^A(T)}{\xi^A(t)} (ID(T) - K_{ID})^+ \right), \tag{10}$$

where K_{ID} is the corresponding strike price of the index option. To compute all these expectations, we need the joint density of $(D_1(t), D_2(t), \lambda(t))$, and the contingent payoff, because the stochastic discount factor is a function of either dividend and the stochastic weight $\lambda(t)$. This density is not available in closed form. However, we can compute its Laplace transform explicitly.

Proposition 2. *The joint Laplace transform of $D_1(t), D_2(t)$ and $\lambda(t)$ under the belief of agent A is given by:*

$$E_t^A \left(\left(\frac{D_1(T)}{D_1(t)} \right)^{\epsilon_{D_1}} \left(\frac{D_2(T)}{D_2(t)} \right)^{\epsilon_{D_2}} \left(\frac{\lambda(T)}{\lambda(t)} \right)^{\chi} \right) = F_{m^A} \left(m^A, t, T; \epsilon_{D_1}, \epsilon_{D_2} \right) \times F_{\Psi} \left(\Psi, t, T; \epsilon_{D_1}, \epsilon_{D_2}, \chi \right), \tag{11}$$

where

$$F_{m^A}(m^A, t, T; \epsilon_{D_1}, \epsilon_{D_2}) \quad = \quad \exp\left(A_{m^A}(\tau) + B_{m^A}(\tau) m^A\right), \tag{12}$$

with $\tau = T - t$ *and*

$$F_\Psi\left(\Psi, t, \epsilon_{D_1}, \epsilon_{D_2}, \chi, u\right) = \exp\left(A_\Psi(\tau) + B_\Psi(\tau)\Psi + \Psi' C_\Psi(\tau)\Psi\right).$$

for functions $A_{m^A}, B_{m^A}, A_\Psi, B_\Psi$ *and* C_Ψ *detailed in the proof in the Appendix.*

We compute all equilibrium quantities with respect to the steady-state distribution of beliefs, which is non-trivial when agents disagree about $\sigma_{\mu_{D_i}}$ in our model. Using the Laplace transform in Proposition 2, we can now price more efficiently any contingent claim in our economy using Fourier inversion methods. In this way, we can avoid to a large extent the use of Monte Carlo methods, which would be highly computationally demanding in our multi asset economy. In the following Proposition, we give semi-analytical expressions for asset prices in our economy.

Proposition 3. *Let*

$$G(t, T, x_{D_1}, x_{D_2}; \Psi) \equiv \int_0^\infty \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}}\right)^\gamma \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{\lambda(T)}{\lambda(t)}\right)^{-i\chi} F_\Psi\left(\Psi, t, T; x, i\chi\right) d\chi\right] \frac{d\lambda(T)}{\lambda(T)}.$$

1. *The equilibrium price of stock 1 is:*

$$\begin{aligned} S_1(t) \quad &:= \quad S_1\left(D_1, m^A, \Psi\right), \\ &= \quad D_1(t)\int_t^\infty e^{-\delta(u-t)} F_{m^A}(m^A, t, u; 1-\gamma, 0) G\left(t, u, 1-\gamma, 0;, \Psi\right) du. \end{aligned}$$

2. *The equilibrium price of stock 2 is:*

$$\begin{aligned} S_2(t) \quad &:= \quad S_2\left(D_1, D_2, m^A, \Psi\right), \\ &= \quad D_2(t)\int_t^\infty e^{-\delta(u-t)} F_{m^A}\left(m^A, t, u; -2\gamma, 1+\gamma\right) G\left(t, u, -2\gamma, 1+\gamma;, \Psi\right) du. \end{aligned}$$

3. *The equilibrium price of the index is:*

$$ID(t) := ID\left(D_1, D_2, m^A, \Psi\right) = \omega_1 S_1(t) + \omega_2 S_2(t).$$

4. *The equilibrium price of the European option on stock 1 is:*

$$\begin{aligned} O_1(t) \quad &:= \quad O_1\left(D_1, m^A, \Psi\right), \\ &= \quad E_t^A\left(e^{-\delta(T-t)}\left(\frac{D_1(t)}{D_1(T)}\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}}\right)^\gamma (S_1(T) - K_1)^+\right). \end{aligned}$$

   The formula for the option on stock 2 is identical with the corresponding replacements, and with $S_2(T)$ *and* K_2 *replacing* $S_1(T)$ *and* K_1*, respectively.*

5. The equilibrium price of the European option on the index is:

$$\begin{aligned} I(t) &:= I\left(D_1, D_2, m^A, \Psi\right), \\ &= E_t^A\left(e^{-\delta(T-t)}\left(\frac{D_1(t)}{D_1(T)}\frac{1+\lambda(T)^{1/\gamma}}{1+\lambda(t)^{1/\gamma}}\right)^{\gamma}(ID(T)-K_{ID})^+\right). \end{aligned}$$

From the above formulas, we obtain a semi-explicit dependence for the prices of stocks and options in our economy on the degree of disagreement among investors. Therefore, these formulas are very convenient to study how disagreement impacts on the prices and volatility risk premia of options. Moreover, they help us to understand more precisely the link between options excess returns, gains and losses of option strategies, and belief heterogeneity.

II. Model Predictions

Using the solutions of Proposition 2 and 3, we study the impact of belief disagreement on the volatility risk premia of individual stocks and the index. To this end, we calibrate the model to the dividend dynamics of the S&P 500 and assume for simplicity of exposition a symmetric economy. The parameters are summarized in Table 1. Risk aversion is set to 2 and the dividend volatility of firm 1 and 2 is 4%. We study comparative statistics for changes in belief disagreement Ψ_{D_i} between zero and 0.3.

[Insert Table 1 approximately here.]

A. Endogenous Stock Return Correlation

Returns are correlated even if the underlying fundamentals are weakly linked or not correlated at all. This is due to the correlation of the fundamentals with the aggregate endowment (see Cochrane, Longstaff, and Santa-Clara, 2008). In our economy, the stock return correlation has an additional component due to heterogeneous beliefs. To see this, note that the price of each stock is given by:

$$\frac{dS_i(t)}{S_i(t)} = m_{S_i}^A(t)dt + \sigma_{S_iD_1}(t)dW_{D_1}^A(t) + \sigma_{S_iD_2}(t)dW_{D_2}^A(t) + \sigma_{S_iz}(t)dW_z^A(t).$$

The conditional covariance between the returns on stock 1 and 2 is given in the following Proposition.

Proposition 4. *The conditional covariance of stock 1 and stock 2 returns is given by:*

$$Cov_t\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right) = \sigma_{S_1D_1}(t)\sigma_{S_2D_1}(t)dt + \sigma_{S_1D_2}(t)\sigma_{S_2D_2}(t)dt + \sigma_{S_1z}(t)\sigma_{S_2z}(t)dt,$$

where the coefficients, $\sigma_{S_1D_1}(t), \sigma_{S_2D_1}(t), \sigma_{S_1D_2}(t), \sigma_{S_2D_2}(t), \sigma_{S_1z}(t)$ and $\sigma_{S_2z}(t)$ are given in the Appendix.

Note that the equilibrium quantities, $\sigma_{S_iD_i}(t)$ and $\sigma_{S_iz}(t)$ depend on the disagreement process $\Psi(t)$. In Figure 4 we plot the correlation as a function of belief disagreement about the dividend growth rates, $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$.

Stock return correlation is an increasing function of both belief disagreement. An increase in belief disagreement from zero to 0.3 increases the correlation from 0.07 to approximately 0.4. In the two tree economy of Cochrane, Longstaff, and Santa-Clara (2008) stock return correlations are increasing because of a diversification effect: Stock returns are correlated because a shock to any of the dividend streams is important for aggregate consumption and this is reflected in the state price density which is a function of the share dynamics and which represents the only source for stock return correlations. In addition to this effect, in our economy there is an increased correlation which comes from the optimal risk sharing among investors ("risk-sharing effect"). The more pessimistic agent in our economy selects a relatively higher consumption in states of low dividends for firm 1, firm 2, or both. Similarly, the more optimistic agent selects a relatively higher consumption in states of high dividends. In order to finance the optimal consumption plan, the pessimistic investor asks financial protection from the optimist. This excess demand lowers the price of securities having positive exposure to dividend shocks of the two firms, and the risk implied by bad dividend states is transferred from the pessimist to the optimist.[12]

This feature of our model could also be interesting in light of the current crisis at whose heart lies an apparent puzzle: The mortgage sector is small relative to the overall economy, nevertheless, the mortgage sector has been the catalyst of the current crisis. Traditionally, financial contagion is viewed as being triggered via correlated defaults or demand and supply shocks in some sectors. However, the number of defaults cannot be one of the drivers of the current crisis.[13] As we argue, the reason could be a spreading of uncertainty as documented in Sector *B.1.*. Note that the impact of uncertainty is on the prices itself. If uncertainty in one sector increases, the systemic uncertainty in the overall economy will rise as well and thereby trigger returns to correlate even more. This feature of our model complements the work of Ribeiro and Veronesi (2002). In their economy, time variation in correlations of asset returns arises from the learning of the representative agent as each individual drift of the output processes is driven by a common business cycle indicator. Further, excess comovements in the correlations of asset returns emerge because of increased uncertainty in bad times. However, in their model, there is no country-specific component in the uncertainty. Therefore, contagion like spreading from one market to the other is not possible. Moreover, in a full information economy as in Cochrane, Longstaff, and Santa-Clara (2008), the return correlation hinges crucially on the size of the asset share. If an asset has a very small share in the overall endowment, the return correlation is nearly zero, only for symmetric economies, the return correlation reaches reasonable numbers. Martin (2009) introduces rare disasters which offers him an interesting leeway. Small assets in his economy co-move endogenously, despite independent fundamentals or a negligible size of the asset. Moreover, disasters spread in an asymmetric fashion, which means that if a large firm experiences a disaster, the price of the other small asset experiences a large negative shock in the price. Vice versa, if the small asset suffers a disaster, the price of the other asset jumps up. Our economy is more simplistic in the sense that independent of the size of an asset, stock returns correlate endogenously. Contagion-like effects emerge via the spreading of disagreement from one firm to the other. However,

[12]For a rigorous treatment of stock return correlations in an economy with heterogeneous beliefs, see also Ehling and Heyerdahl-Larsen (2008).

[13]The overall issuer weighted annual default rate – including both investment grade and speculative grade entities – was the lowest in 2007 since 1981, see Standard & Poor's (2008).

the effect is always symmetric: If a (small) asset suffers a large negative shock, then the impact on the other asset will be negative as well. HERE THINK MORE. BE CAREFUL.

[Insert Figure 4 approximately here.]

B. Implied Volatility and Risk-Neutral Skewness

Negative dividend shocks will not only increase the stock (index) volatility but will also depress the stock (index) price. As a consequence, the risk-neutral skewness of the return distribution is negative.[14] In Figure 5 (upper panel), we plot the risk-neutral skewness both for the individual stocks (upper left) and the index (upper right) for an increase in disagreement about firm's future dividends, $\Psi_{D_i}(t)$ if the business cycle indicator disagreement is set to 0.3. The average belief disagreement in our sample of firms is 0.3. For this value, belief disagreement decreases the risk-neutral skewness from zero to -0.3 for the individual stock and to -0.99 for the index. These are reasonable values, given the average risk-neutral skewness of -1.09 of S&P 100 options and -0.33 for individual options.

[Insert Figure 5 approximately here.]

To formalize the relationship between the market and individual stock skewness, assume that the stock return of firm i conforms according to the following consumption CAPM:[15]

$$r_i(t) = \beta_M r_M(t) + \beta_{\overline{\Psi}} \overline{\Psi(t)} + \epsilon_i(t), \tag{13}$$

where $\overline{\Psi}(t)$ is the consensus belief, $r_i(t)$ is the return on stock of firm i, and $r_M(t)$ is the return on the market. Assume that the idiosyncratic risk ϵ_i is Gaussian and independent of the market return, r_M, and the common disagreement, $\overline{\Psi}$. We can now state:

[14] See Buraschi and Jiltsov (2006).

[15] In equilibrium, the excess return of stock i can be written as:

$$m^A_{D_i} - r(t) = \gamma Cov\left(\frac{dS_i}{S_i}, d(D_1 + D_2)\right) - \frac{\lambda(t)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} Cov\left(\frac{dS_i}{S_i}, \frac{d\lambda}{\lambda}\right).$$

Rearranging terms yields that:

$$m_{D_i} - r(t) = \gamma Cov\left(\frac{dS_i}{S_i}, d(D_1 + D_2)\right) - \left(1 + \lambda(t)^{1/\gamma}\right)^{-1} \left(\frac{m_{D_i} - m^A_{D_i}}{\sigma_{D_i}}\right) \sigma_{S_i}(t) - \lambda(t)^{1/\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^{-1} \left(\frac{m_{D_i} - m^B_{D_i}}{\sigma_{D_i}}\right) \sigma_{S_i}(t).$$

We note that the excess return can be represented as a risk-tolerance weighted average of agents perceived growth rates. Assume now that we would construct a "consensus" agent. Jouini and Napp (2007) show that such an agent exists and under the assumption of power utility her "consensus" belief is proxied by the weighted average of the heterogeneous beliefs so that the market aggregate behavior is in principle a weighted average of heterogeneous individual behaviors (see also Chiarella, Dieci, and He, 2008).

Proposition 5. *If stock returns follow a consumption CAPM model as in equation (13), then the skewness of the individual stock is linked to the market skewness and the disagreement skewness as:*

$$
\begin{aligned}
SKEW(r_i) \quad = \quad & \beta_M^3 SKEW(r_M) + \beta_{\overline{\Psi}}^3 SKEW(\overline{\Psi}) - SKEW(m_M) + COSKEWS \\
& +3\beta_M \int_{-\infty}^{\infty} r_M q(r_M) dr_M + 3\beta_{\overline{\Psi}} \int_0^1 \overline{\Psi} q(\overline{\Psi}) d\overline{\Psi} + 3m_M(\beta_M - 1) \qquad (14) \\
& -6\beta_M \beta_{\overline{\Psi}} \int_{-\infty}^{\infty} \int_0^1 \int_{-\infty}^{\infty} r_M \overline{\Psi} m_M q(r_M, \overline{\Psi}, m_M) dr_M d\overline{\Psi} dm_M \qquad (15)
\end{aligned}
$$

where $COSKEW$ are the coskewness terms given in the appendix, $q(\cdot)$ the risk-neutral density, and m_M the growth rate of the market return under the risk-neutral measure of the consensus agent.

Clearly, equation (15) reveals that the conditions under which the individual skew is less negative than the index are not very intuitive, moreover, as the joint distribution of the expected return of the market, the common belief, and the market return itself cannot be calculated in closed form. However, we can derive two different scenarios: First, note that the diffusion term of the stochastic discount factor is related to agents' consumption shares: A negative dividend shock in one firm increases the volatility of the stochastic discount factor. However, if the second firm experiences a negative dividend shock as well, then the index volatility will increase even more. This is due to the decreasing marginal utility of consumption of the two investors, which generates endogenously the negative skewness in the market. The stronger negative skewness of the index follows exactly in those cases when both firms experience a negative dividend shock. Second, for the ease of exposition, assume that the common disagreement is set to zero, i.e. $\Psi_z(t) = 0$. In this case, a large increase of disagreement in one firm will induce a large negative skewness for this firm. At the same time, the additional impact on the index volatility due to correlation can potentially be very small depending on the weights given for the updating of the beliefs. As a consequence the impact on the index skewness is rather small and downweighted by the size of the share of this asset in the index. In Figure 5, lower panel, we plot the index (lower right) and individual (lower left) firm skewness in case of $\Psi_z(t) = 0$. Firm 1 experiences a large negative shock and the share of this firm in the overall index is set to 0.1. Moreover, the weight given to this firm for the updating of the signal growth rate is set to 0.1. We note that the large negative shock induces a large negative skewness of -0.62. The effect of the disagreement about the second firm, $\Psi_{D_2}(t)$ is nearly zero which is intuitive, as there is no co-movement between the firm-specific disagreement processes. However, the index skewness is less negative than for the individual firm: An increase in both disagreement processes, decreases the risk-neutral skewness from zero to -0.38 which is smaller than in the case for $\Psi_z(t) > 0$ due to the lack of the additional correlation.

In a standard Black and Scholes (1973) world, the implied volatility surface of single-stocks and index options is flat. Bakshi, Kapadia, and Madan (2003) report a one-to-one mapping between the risk-neutral skewness and the implied volatility smile. As a consequence, belief disagreement in our economy will generate an implied volatility smile. We consider the same two scenarios as for the risk-neutral skewness: In Figure 6, left panel, we plot the absolute difference between the implied volatility and the instantaneous volatility, i.e. the volatility risk premium for the case of high Dib (i.e. $\Psi_{D_1} = \Psi_{D_2} = \Psi_z = 0.3$) and low DiB (i.e. $\Psi_{D_1} = \Psi_{D_2} = \Psi_z = 0.1$) . In a standard Merton

(1974) world, the implied volatility and the instantaneous volatility would coincide, implying a zero volatility risk premium. In our economy, however, there is a difference, which represents compensation for the priced disagreement risk. The volatility risk premium of single-stock options for the low disagreement is smaller than for the index in the whole moneyness space. for instance, for an at-the-money option on the stock, the volatility risk premium for the low disagreement is around 1.1% and 1.6% for the high disagreement case. As for the index, the numbers are considerably larger: For low disagreement, the index volatility risk premium is 2% and 2.4% for the high disagreement. We note that the absolute difference between the index and individual stock volatility risk premia become smaller, the more out-of-the-money the option is: The difference between the index and individual volatility risk premium is not even 1%. For far in-the-money options, the difference between the index and individual volatility risk premium is as large as 3%. In the case the business cycle disagreement is set to zero, we note in Figure 6 (right panel), that the individual volatility risk premium of the index is smaller than for the individual stock. For an at-the-money option, the individual stock volatility risk premium for the low (high) disagreement is 1.5% (2.1%) and 1% (1.5%) for the index, respectively.

[Insert Figure 6 approximately here.]

C. Synthesizing Volatility and Correlation Risk Premia from Prices

To extract the volatility risk premia directly from prices, we use variance swaps. A long variance swap pays the difference between the realized variance over some time period and a constant called the variance swap rate:

$$(RV_i(t,T) - SW_i(t,T))\, L_i,$$

where L_i is the notional dollar value of the contract of firm i, $RV_i(t,T)$ is the realized variance of the stock price of firm i over some time period, and $SW_i(t,T)$ denotes the variance swap rate. Let $\mathbb{Q}$ denote the risk-neutral probability measure from the viewpoint of agent A. A swap has a market value of zero at entry:

$$SW_i(t,T) = E_t^{\mathbb{Q}}\left(RV_i(t,T)\right).$$

Under the physical measure the variance swap rate and the realized variance are linked by the following equation:

$$SW_i(t,T) = E_t^{\mathbb{P}}\left(\frac{\xi^A(T)}{\xi^A(t)} RV_i(t,T)\right) = E_t^{\mathbb{P}}\left(RV_i(t,T)\right) + Cov_t^{\mathbb{P}}\left(\frac{\xi^A(T)}{\xi^A(t)}, RV_i(t,T)\right).$$

The variance risk premium of firm i is now defined as the difference between the swap rate and the realized variance:

$$VARP_i(t) = SW_i(t,T) - E_t^{\mathbb{P}}\left(RV_i(t,T)\right) = Cov_t^{\mathbb{P}}\left(\frac{\xi^A(T)}{\xi^A(t)}, RV_i(t,T)\right) > 0. \tag{16}$$

Equation (16) makes clear why variance swaps are a good hedge for high disagreement in our economy. The conditional covariance between the stochastic discount factor and the realized variance is positive in our economy. [HERE

THINK!!!]. Similarly, the variance risk premium on the index is a weighted sum of the individual variance risk premia plus a covariance term and follows directly by using Itô's Lemma:

$$VARP_I(t) = \sum_{i=1}^{2} \bar{\omega}_i VARP_i(t) + 2\omega_1\omega_2 COVRP(t), \tag{17}$$

where $\bar{\omega}_i = \omega_i^2 + \sum_{j\neq i} \omega_i\omega_j \frac{Cov(S_i,S_j)}{\sigma_{S_i}}$ and $COVRP(t) = E_t^{\mathbb{Q}}(Cov(S_1,S_2)) - E_t^{\mathbb{P}}(Cov(S_1,S_2))$ denotes the covariance risk premium. The variance risk premium on the index is larger than the average individual variance risk premia due to the covariance risk premium term, which is non-zero in our economy. There are two main reasons for this. First, the covariance under the physical and risk-neutral dynamics are different, i.e. $Cov_t^{\mathbb{Q}}(S_1,S_2) \neq Cov_t^{\mathbb{P}}(S_1,S_2)$. Second, the dynamics of the economic fundamentals are different under the physical and risk-neutral measure, to this end, the drift adjustment leads to a non-zero difference, in other words $Cov_t^{\mathbb{Q}}(S_1,S_2) - E_t^{\mathbb{Q}}\left(Cov_{t+1}^{\mathbb{Q}}(S_1,S_2)\right) \neq Cov_t^{\mathbb{P}}(S_1,S_2) - E_t^{\mathbb{P}}\left(Cov_{t+1}^{\mathbb{P}}(S_1,S_2)\right)$. Moreover, in our economy, the covariance risk premium has the attractive feature that it is inherently counter-cyclical due to its dependence on the disagreement processes. Negative shocks in the underlying fundamentals cause uncertainty to increase

To make the variance swap rate and realized volatility more tractable, we use the standard industry approach and synthesize the variance risk premium from plain vanilla option prices. Under the assumption of no arbitrage, the variance swap rate can be synthesized using out-of-the-money put options. When the swap rate process is continuous, this relation is exact (see e.g. Carr and Madan, 1998 and Britten-Jones and Neuberger, 2000) and holds up to a small approximation error, when the swap rate process exhibits jumps (see e.g. Carr and Wu, 2009).

$$SW_i(t) = E_t^Q\left(RV(t,T)\right) = \frac{2}{(T-t)\,B(t,T)} \int_0^{\infty} \frac{P(K,T)}{K^2} dK, \tag{18}$$

where $B(t,T)$ is the price of a zero coupon bond with maturity T and $P(K,T)$ is a out-of-the-money put option with strike K and maturity T. For a variance swap with 30 days to maturity, we define the realized variance:

$$RV(t,t+30) = \frac{365}{30} \sum_{i=1}^{30} R_i(t_n)^2,$$

where we define a set of dates $t = t_0 < t_1 < \cdots < t_N = T$ and $R_i(t_n) = \log\left(S_i(t_n,T)/S_i(t_{n-1},T)\right)$. We can now calculate the (negative) volatility risk premium for firm i as:

$$VOLRP_i(t) = \left(\sqrt{SW_i(t,T)} - \sqrt{RV_i(t,T)}\right) \times 100, \tag{19}$$

and a similar expression arises for the index. Using a discretized version of equation (18), we plot in Figure 7 the volatility risk premium for an individual firm (left panel) and the correlation risk premium (right panel) as a function of belief disagreement about firm 1 and the business cycle indicator.

[Insert Figure 7 approximately here.]

Intuitively, we note that the increase of the individual volatility risk premia due to the firm specific disagreement is steeper than for the business cycle indicator. For example, an increase about the idiosyncratic (business cycle indicator) disagreement from zero to 0.3 increases the volatility risk premium from 0.2% to 0.7% (0.4%). The correlation risk premium increases with both the firm specific and systemic disagreement. This is clear, as a higher disagreement about the business cycle indicator implies a higher correlation among the individual stocks. An increase of both the disagreement processes from zero to 0.3 implies a correlation risk premium of 0.95%.

D. Simulated Option Trading Strategies

The previous subsection has shown that the volatility risk premia of index and individual options are driven by belief disagreement. The industry standard for exploiting the volatility risk premium in individual and index options is a straddle portfolio which involves buying a call and a put with the same moneyness. An alternative strategy involves delta-hedging an option position.[16] The idea is that if the investor is successful in hedging away the priced risk, then a major determinant of the profit or loss from this strategy is the difference between the realized and implied volatility. The advantages of a delta-hedged trade relative to a straddle are the potentially lower transaction costs since stock trading is cheaper than options trading.[17] The disadvantage is that straddle portfolios are more profitable than delta-hedged portfolios, because the former benefit from the difference in volatility risk premia of both calls and puts, while the latter benefit from only one option, either call or put. Moreover, at-the-money straddles are known to be very liquid (see Bondarenko, 2003) and have been analyzed extensively in the literature (see Coval and Shumway, 2001 and Driessen and Maenhout, 2007). Bakshi and Kapadia (2003a) study delta-hedged portfolios and find that delta-hedged call and put portfolios statistically under-perform zero, with losses that are most pronounced for at-the-money options. The authors argue that these findings are consistent with a negative volatility risk premium in index options. In addition they find that during periods of higher volatility the under-performance is worst, and this is due to the negative correlation of delta-hedged gains of at-the-money options and historical volatility. In the following, we employ two trading strategies, which exploit the difference in the volatility risk premia of individual and index options. These strategies are so called dispersion trades which involve a short option position on an index, against a long option position in the index constituents. The aim of the strategy is to realize a maximum difference between the index implied volatility and the average constituents volatility. In this case, the strategy will make money on both the long option position on the individual stocks and on the short option position on the index by earning theta. The success of the strategy lies in determining which component stocks to pick and it is, therefore, critical to make sure to buy the cheapest options. Determining cheap options is simple in our framework. When belief disagreement is large for one firm, the volatility risk premium for this particular firm should be large as well,

[16] Academic papers looking at option trading strategies include Jackwerth (2000), Coval and Shumway (2001), Aït-Sahalia, Wang, and Yared (2001), Bakshi and Kapadia (2003a), Bondarenko (2003), Bollen and Whaley (2004), Jones (2006), Driessen and Maenhout (2007), and Santa-Clara and Saretto (2007), among others. These papers focus on index options only. Option trading strategies for individual options are treated in Goyal and Saretto (2008) and Driessen, Maenhout, and Vilkov (2008).

[17] Delta-hedged positions are expensive. Strikes must be rolled which increases transaction costs. Moreover, it is not a priori clear which delta to choose. Branger and Schlag (2004) show that delta-hedged errors are not zero if the incorrect model is used or if rebalancing is discrete. This implies that if jumps or stochastic volatility are present and the Black-Scholes (1973) model is used to compute deltas, then the delta-hedged returns may be biased.

since the investor wants to be compensated for holding the risk. If investors disagree on the joint occurrence of a bad dividend state for both firms then their aggregate marginal utility will differ more than in the case if they disagree only on a bad state for one firm. In this case, the pessimist requires protection against a joint bad state in dividends by asking put index options from the more optimistic agent, since these options pay-off the most in bad states. Hence, if our predictions are correct, a short position in the index and a long position in the constituents should yield excess returns, because the index volatility risk premium is larger than the one of individuals. In the following, we simulate dispersion trades on individual and index straddle (put) options.

To test these strategies, we simulate two stocks[18] with equal weights which form the index. We use the formulas provided in Proposition 3 and apply a discrete-time version of the dividend dynamics. The Gaussian error terms for both dividend processes are drawn independently. The parameters used for the simulation are summarized in Table 1. To be consistent with the empirical work, the simulated sample path is taken to be 11 years and 5 months (2,877 days). At the beginning, at-the-money index straddles (puts) are sold and at-the-money straddles (puts) on the individual stock are bought such that the portfolio is vega neutral. We chose the stock with the highest belief disagreement at the end of the month. Making the portfolio vega neutral assures a volatility exposure, instead of a pure directional one. An equivalent amount of the particular stock is bought to delta neutral the portfolio and the remainder is invested in the risk-free bond. Across each simulation run, we generate 137 observations on gains and losses. In Table 2, we report the sample moments of each trading strategy over 1,000 simulations. The first point to note is that the annualized Sharpe ratio of the both dispersion trades are above 1: For the straddle portfolio the annualized Sharpe ratio is approximately 1.9 and for the put portfolio it is 1.5. The average return for an index put option seller is high at 11.7%, however, the Sharpe ratio is around 0.60 due to the high standard deviation. We also note that the dispersion trades involve much smaller skewness and a slightly smaller kurtosis than selling index puts.

[Insert Table 2 approximately here.]

III. Data

A. Options Data

We use option information from the OptionMetrics Ivy DB database, which is the most comprehensive database available. Data runs from January 1996 to June 2007. Our index option sample contains trades and quotes of S&P 100 index options traded on the Chicago Board Options Exchange (CBOE). The S&P 100 is a capitalization-weighted index with quarterly re-balancing. Options on the index are European style and expire on the third Friday of the contract month. Our sample also consists of trades and quotes of CBOE options on all constituents of the S&P 100. Individual stock options are American style. They usually expire on the Saturday following the third Friday of the contract month. Therefore, time to maturity is defined as the number of calendar days between the last trading date and expiration date. We apply a number of data filters to circumvent the problem of large outliers.

[18] For tractability reasons we confine ourselves to two firms.

First, we eliminate prices that violate arbitrage bounds, i.e. call prices are required not to fall outside the interval $\left(Se^{-rd} - Ke^{-\tau r}, Se^{-\tau d}\right)$, where S is the value of the underlying asset, K is the option's strike price, d is the dividend yield, r is the risk-free rate, and τ is the time to maturity. Second, we eliminate all observations for which (i) the ask is lower than the bid price, (ii) the bid is equal to zero, or (iii) the spread is lower than the minimum tick size (equal to USD 0.05 for options trading below USD 3 and USD 0.10 in any other cases). Importantly, to mitigate the impact of stale quotes we eliminate from the sample all the observations for which both the bid and the ask are equal to the previous day prices. We focus on short-term options which are known to be the most liquid with a time to maturity between 14 and 31 days.

B. *Stock Returns Data*

Stock data is retrieved from the CRSP database. To calculate the realized variance, we use daily returns from CRSP for single-stocks and from OptionMetrics for the index.[19] We calculate the realized variance over 21-day windows, requiring that the stock has at least 15 non-zero return observations.

C. *Difference in Beliefs Proxy*

To obtain a proxy of belief disagreement, we follow the procedure in Buraschi, Trojani, and Vedolin (2007). We use analyst forecasts of earnings per share from the Institutional Brokers Estimate System (I/B/E/S) database and compute for each firm the mean absolute difference scaled by an indicator of earnings uncertainty. In order to get a common belief disagreement factor for the index, we estimate a dynamic component using factor analysis for the analysts earning forecasts. Factor analysis has mainly been implemented for forecasting measures of macroeconomic activity and inflation (see, e.g., Stock and Watson 2002a, 2002b, 2004) and more recently in financial applications (see Ludvigson and Ng, 2007). Dynamic factor models allows us to escape the limitations of existing empirical analyzes in several dimensions. If comovements between individual difference in beliefs are strong, it makes sense to represent the overall belief disagreement in the economy by an index or a few factors, which describe the common behavior of these variables. Using dynamic factors instead of principal components has two reasons. First, we want it to be dynamic. Second, we want to allow for cross-correlation among the idiosyncratic components, because orthogonality is an unrealistic assumption in our setting. We estimate the common belief disagreement process according to Forni, Hallin, Lippi, and Reichlin (2000). To estimate the common component, we weight the individual belief disagreement processes of each firm by its market capitalization. This approach is quite natural in our context.

D. *Other Control Variables*

To focus on the additional explanatory power of belief disagreement, we include in our regressions several other determinants as controls. A natural variable is market volatility. A negative volatility risk premium increases the

[19] CRSP data only runs until December 2006. For the remaining six months, we rely on stock prices from OptionMetrics for the individual stocks.

option price, which results in an implied volatility that is higher than the expected future volatility. This is equal to say that the drift of the risk-neutral volatility process exceeds the drift under the physical probability measure. Since individual volatilities are generally positively correlated with market volatility, one might argue that individual volatilities will also exceed realized volatility. We calculate market volatility from CRSP and Optionmetrics prices as the 21-day historical realized volatility.

We also add a skewness measure to our regressions. Jump risk will change the underlying distribution of stock returns and hence it will also impact the volatility risk premium. Bakshi, Kapadia, and Madan (2003) provide a theoretical foundation how skewness is related to the implied volatility function. The more negative the risk-neutral skewness, the steeper the implied volatility function. We proxy skewness by the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios.

Systematic risk effects are captured by including a market excess return, the two Fama and French (1993) factors, and the Carhart (1997) momentum factor to our model. These data are available from Kenneth French's web page. Business-cycle effects are captured by macro factors. We proxy these factors by the price-earning ratio of the market, industrial production, housing start number, the producer price index, and non-farm employment to our regressions. Since the price-earning ratio for the S&P 100 does not exist, we use price-earnings data from the S&P 500. We estimate a macro factor with dynamic factor analysis using industrial production, housing start number, the producer price index, non-farm employment, and the S&P 500 P/E ratio. We retrieve S&P 500 price-earnings data from the S&P webpage, and the other macro variables we get from FRED.

We summarize the moments of the most important variables in Table 3. We find that both the index and individual volatility risk premia are negative. The volatility risk premium of the index is twice as large as the individual risk premia. Testing the null hypothesis that index implied and realized volatility are on average equal is very strongly rejected, based on a t-Test with Newey-West (1987) autocorrelation consistent standard errors for 22 lags. For the individual firms, we test for each individual firm the null hypothesis of a zero volatility risk premium, and find that at the 5% confidence level we cannot reject the null for 25% of all firms in our sample. Driessen, Maenhout, and Vilkov (2008) find that in their sample 2/3 of firms have a volatility risk premium which is not statistically different from zero using average model-free implied and average realized volatility. Bakshi and Kapadia (2003b) find that the difference between realized and implied volatility of single stock options is on average -1.5%. Their conclusions, however, apply to a subset of 25 firms for a short time period (January 1991 to December 1995). Carr and Wu (2009) use a similarly limited, though longer sample of 35 firms, and find a slightly stronger evidence of a variance risk premium in individual options. Duarte and Jones (2007) study the largest set of options with 5,156 stocks and find no evidence that the volatility risk premium is nonzero on average. However, they find strong evidence of a conditional risk premium that varies positively with the overall level of market volatility. Goyal and Saretto (2008) find on average a positive volatility risk premium in individual options.

[Insert Table 3 approximately here.]

IV. Empirical Analysis

In this section, we test the main predictions of our theoretical model. We analyze in a set of panel regressions the impact of belief disagreement on volatility risk premia of individual and index options together with the covariance risk premium. In a second step, we sort stocks according to the size of belief disagreement and study monthly portfolio returns of straddles and puts being short in the index and long in the constituents. We then study whether these returns are related to stock/option characteristics.

In our model, belief disagreement increases the volatility risk premia of both index and individual options. Since the volatility risk premia of individual and index options are both negative, we expect a negative sign in these regressions. If we can show that belief disagreement indeed impacts significantly the volatility risk premia, then a trading strategy that is stratified according to the size of belief disagreement, taking advantage of this priced risk premium, should yield excess returns.

A. Volatility Risk Premium

In this subsection, we examine several structural determinants of volatility risk premia. For each firm i, we denote by $RV_{i,t+\delta} - IV_{i,t+\delta}$ the volatility risk premium. We run the following regression for the individual risk premia:

$$\underbrace{RV_{i,t+\delta} - IV_{i,t+\delta}}_{\text{Volatility Risk Premium}} = \beta_0 + \beta_1 DIB_{i,t} + \beta_2 \overline{DIB}(t) + \sum_{j=3}^{7} \beta_j \text{Control}(j)_{i,t+\delta} + \sum_{k=1}^{2} \gamma_k \text{Control}_{t+\delta} + \epsilon_{i,t+\delta}, \tag{20}$$

where $DIB_{i,t}$ is the proxy of belief disagreement of each individual firm i at time t, $D\bar{I}B(t)$ the common disagreement estimated from the cross-section of individual disagreement proxies, $\text{Control}_{i,t+\delta}$ are the control variables of each firm i at time $t+\delta$, and $\text{Control}_{t+\delta}$ are time-series determinants, such as market volatility and the macro factor. For the index, we run the following regression:

$$\underbrace{RV_{t+\delta} - IV_{t+\delta}}_{\text{Index Volatility Risk Premium}} = \beta_0 + \beta_1 \overline{DIB_t} + \sum_{k=2}^{7} \beta_k \text{Control}_{t+\delta} + \epsilon_{t+\delta}. \tag{21}$$

To get a measure of the correlation risk premium, we use equation (17):[20]

$$CORRP(t) = \beta_0 + \beta_1 \overline{DIB_t} + \sum_{k=2}^{3} \beta_k \text{Control}_{t+\delta} + \epsilon_{t+\delta}, \tag{22}$$

where $CORRP(t)$ is the correlation risk premium calculated as the difference between the index and weighted individual variance risk premia.

One main prediction from the theoretical model is that options of stocks with larger belief disagreement have both larger volatility and correlation risk premia. The results for the baseline regressions are reported in Table 4, column

[20] Clearly, the relationship between the index and weighted individual volatility risk premia presented in equation (17) is only correct, if the weights ω_i are constant over time, which empirically, is not the case.

(1) for the individual risk premia and column (5) for the index risk premium. The coefficient of belief disagreement is economically and highly statistically significant for both the individual and index volatility risk premium. It also has the correct sign: Higher belief disagreement increases the volatility risk premia. Beliefs disagreement, a jump risk measure, and macro factors alone explain 26% of the variation in individual volatility risk premia and 22% of the variation in index volatility risk premia. The jump risk factor is not statistically significant at the standard significance levels and it has the wrong sign: Intuitively, we would expect that a higher jump risk would imply a higher volatility risk premium. Market volatility and the macro factor, however, are highly statistically significant. The loading of market volatility is negative, which is inline with the idea brought forward in Buraschi and Jackwerth (2001) and Bakshi and Kapadia (2003b): They suggest that one reason why implied volatilities are on average greater than realized volatilities is because market volatility is priced in equity options. The coefficient for the macro factor is positive and the intuition is that good business cycle states decrease the volatility risk premia of both individual and index options, while bad business cycle states increase the volatility risk premia. In a regression without belief disagreement, we find that the adjusted R^2 decreases to 10%, which indicates that it is the most important variable. We also note that the coefficient of the common belief disagreement index is more than three times larger than the coefficient of the individual disagreement proxies. This is interesting, since we would expect that the macro factor and the market volatility would be most important in explaining the volatility risk premia of index options, since the market volatility is by definition inherently linked to the volatility risk premium. Finally, in column (8) of Table 4, we regress the difference between the index variance risk premium and a weighted average of the constituents variance risk premia on the common disagreement proxy. The correlation risk premium is positively related to belief disagreement which is in line with our theoretical findings. The coefficient is statistically significant at the 5%. The market volatility increases the correlation risk premium, which is intuitive. At times of high market turmoil, which are associated with both higher market volatility and higher correlation among different stocks, risk-averse agents will demand a higher risk premium to holding a basket of stocks or options due to heightened correlation.

[Insert Table 4 approximately here.]

B. Trading Strategies

The results of the previous subsection show that the volatility risk premia of index and single-stock options are potentially related to disagreement about a firm's future dividends and an business cycle indicator. In the following, we seek to understand the extent to which belief disagreement can explain the apparent mispricing in option prices documented in the literature. The purpose of this section is to determine whether the difference in beliefs can generate abnormal trading opportunities by exploiting the existing structure of variance risk premia in index and single-stock options. We do so by carrying out a series of trading strategies that are often used in the industry. We focus on the following two benchmark strategies:

1. A (short-maturity) at-the-money straddle.
2. A (short-maturity) at-the-money put.

We test the performance of straddle and plain vanilla put dispersion strategies implied by so called generalized dispersion trades. In standard dispersion trades, a short volatility position in the index results in a long volatility position in all constituents. Since trading a whole book of constituents volatility is much too expensive[21], investment banks trade only a basket of single-stock volatilities. This may be either due to liquidity issues or because investors wish to buy single-stock volatility where they think the options are particularly cheap. The art of such a trade is to find the cheapest options to form the basket. In our economy, the firms with the highest dispersion have options which are under-priced and should hence yield the largest excess return. We first hedge the individual variance risk to get the individual weights for the options, by building a vega-neutral position. We then build a delta-neutral position to get the weights for the underlying stocks. The resulting strategy shorts all the wealth in index puts or straddles and invests a fraction in the individual puts or straddles, a fraction in the individual stock and the rest is invested in the risk-free bond.

Next, we construct a time series of our option strategies discussed above. To circumvent microstructure biases, we initiate option portfolio strategies on Tuesday, as opposed to the first trading day (Monday). The returns are constructed using, as a reference beginning price, the average of the closing bid and ask quotes. To compute the closing price, we use the terminal payoff of the option depending on the stock price at expiration and the strike price of the option. After expiration, a new option is chosen having the same characteristics and a new monthly return can be calculated. We use equally weighted monthly returns on calls and puts. This procedure is repeated for each month.

Table 5 provides summary statistics on the option portfolios. The average return on the straddle portfolio is almost 20% and for the put portfolio it is 2.7%. Both the straddle and put portfolio return attractive Sharpe ratios: The annualized Sharpe ratio for the Straddle portfolio is 1.96 and the one for the put portfolio it is 2.02.[22] To compare these numbers, investing all wealth into the index directly would have yielded a Sharpe ratio of 0.38 and a short index put portfolio would have produced a Sharpe ratio of 0.79.[23] Our numbers for the straddle portfolio are comparable to Goyal and Saretto (2008), who report a monthly return of 22% for a straddle portfolio and 2.6% for the delta-hedged puts. Coval and Shumway (2001) report 3% return per week for a zero-beta at-the-money straddle portfolio on the S&P 500.

We note that these findings also complement the work of Coval and Shumway (2001) and Bakshi and Kapadia (2003a) who find that negative delta hedged gains are consistent with a negative volatility risk premium. In our economy, higher belief disagreement results in a drop in market returns and an increase of the market volatility. This negative correlation yields the negative volatility risk premium. The negative risk premium is directly related

[21]Driessen, Maenhout, and Vilkov (2008) find that their correlation trading strategy does not yield significant abnormal returns after transaction costs and margins. In their study they employ the trading strategy on all constituents of the S&P 100 which is in general too costly. An industry standard is to reduce the number of constituents. The reason is the so called spread risk (or so called slippage). The spread risk is created by the liquidity and hence market makers; if an option is less actively traded, the market maker widens the volatility spread that she is prepared to trade for. In constructing a market or capitalization weighted single stock basket, the dispersion trader requires very tight spreads that actually reflect the degree of correlation that an underlying single stock may have with an index, as the spread deviates away from this range then the whole dispersion trade becomes less economically viable.

[22]We note, however, that Sharpe ratios can be highly misleading when analyzing derivatives (see Goetzmann, Ingersoll, Spiegel, and Welch, 2007).

[23]The annualized Sharpe ratio for the short put strategy is comparable to Bondarenko (2003).

to the delta-hedged gains. The volatility risk premium in our economy is $\theta_{\sigma_{S_i}} = -cov\left(\frac{d\lambda}{\lambda}, d\sigma_{S_i}\right)$. The risk premium is decreasing in the covariance between the stock volatility and the stochastic weight, $\lambda(t)$. When $\lambda(t)$ is high, the representative agent puts less weight on the first agent. Since stock volatility is negatively correlated with changes in $\lambda(t)$, the stock itself is more valuable to the first agent, and she requires a smaller risk premium.

Altogether the results confirm our theoretical model. Higher belief disagreement increases the spread between the implied volatilities of single-stock options and the spread between the implied volatility of single-stocks and index options. Both the straddle and the put dispersion portfolio yield economically meaningful returns.

[Insert Table 5 approximately here.]

C. Risk Adjusted Returns

Next, we examine whether the option trading strategy returns can be explained by a standard asset pricing model, such as the CAPM.[24] To this end, we regress the straddle and put option returns of the two strategies on various factors which are known to impact the cross-section of stock returns. We use the standard Fama and French (1993) factor, and the Carhart (1997) momentum factor.

We report the CAPM alpha and beta estimates in Table 5. The CAPM beta is insignificant for both the dispersion straddle and dispersion put portfolio, which means that the trading strategy is quite successful in hedging away the market risk. The CAPM beta for the short index put portfolio is much higher in value and statistically significant at the 5% level. The CAPM alpha is highly statistically significant and positive for all three strategies. The CAPM alpha of the straddle portfolio is approximately 16% with a t-value of 4.42. For the put portfolio the CAPM alpha is 2% with a t-value of 2.23. The short index put portfolio has a higher alpha coefficient, 11%, but it is only significant at the 10% level. The size of magnitude is approximately inline with the alphas estimated in Driessen, Maenhout, and Vilkov (2008) and Goyal and Saretto (2008). The size, book-to-market, and momentum factors load negatively on the strategy returns, but are all insignificant.

V. Robustness Checks

Our results support the hypothesis that belief disagreement is an important priced risk factor for volatility risk premia of individual and index options. We have also shown that an option trading strategy that exploits this priced disagreement risk yields excess returns. In this section, we assess the robustness of our results by studying the impact of transaction costs or whether other sources of risk capture the effect of belief disagreement.

A. Transaction Costs

There is a large body of literature that documents that transaction costs in the options market are quite large and that they are in part responsible for some pricing anomalies. However, we note that the literature has not reached

[24] Option prices are convex functions of the underlying stock price and therefore the linear relation implied by the CAPM introduces a misspecification. We acknowledge that as a consequence, the CAPM α can be strongly biased (see Broadie, Chernov, and Johannes, 2007).

complete consensus on the issue. Constantinides, Jackwerth, and Perrakis (2008) find that transaction costs do not eliminate the abnormal profits of index put and call strategies for different moneyness. Santa-Clara and Saretto (2007), on the other hand, find that transaction costs and margin requirements severely impact the profitability of index option trading strategies that involve writing out-of-the-money puts. The intuition is that these frictions limit arbitrageurs from supplying liquidity to the market. Therefore, trading strategies that involve providing liquidity to the market (writing options) have an exceptionally good performance. Goyal and Saretto (2008) find that the profits are higher for illiquid stock options than for liquid options. In particular, liquidity considerations reduce, but do not eliminate, the economically important profits of their portfolios of individual options. Driessen, Maenhout, and Vilkov (2008) find that transaction costs and margin requirements significantly reduce the profitability of their correlation trading strategy because of the larger bid-ask spread for individual options.

At first sight, one could expect that the results should not be explained by the transaction costs or bid-ask spreads. This is because we focus on buy-and-hold strategies that involve very little trading. In fact, options are assumed to be traded only once, namely at the beginning of each period. Moreover, our trading strategies do not involve the writing of out-of-the-money options, which are particularly prone to these trading frictions. The average bid-ask spread for the index options is approximately 6.23% and for the individual options it amounts to 8.29%. In the following, we study the impact of these spreads on the performance of our trading strategies. In the previous analysis, we have used mid quotes calculated from the bid-ask spread. Now, we calculate bid returns when options are written and ask returns when options are bought. The results are reported in Table 6. Indeed, we find that the bid-ask spreads lower the return of the straddle strategy by approximately 42% from 19% monthly return to 11%. For the put strategy, the average returns decreases from 2.7% to 1.2%. In line with the literature, the impact on the other strategies is small. The decrease in the short index put strategy is 13%, from a 37% to 32% monthly return. The annualized Sharpe ratio of the straddle strategy is still above one. For the put strategy the Sharpe ratio drops to 0.8. The CAPM alpha is still statistically significant, but now only at the 5% level.

[Insert Table 6 approximately here.]

B. Fundamental Uncertainty and Earning Announcements

When firms announce earnings every quarter, they reveal firm fundamentals which were, to some extent, unknown to investors prior to the announcement. One could argue that this uncertainty about the fundamentals of the firm is related to investor's expectations about future fundamentals of the firm such as earnings. Empirical evidence has shown that volatility risk premia tend to be high prior to an earnings announcement. It is therefore an interesting question whether the results implied by our proxy of belief disagreement are affected by the introduction of an uncertainty measure.

Ederington and Lee (1996) and Beber and Brandt (2006) document a strong decrease in implied volatility subsequent to major macroeconomic announcements in U.S. Treasury bond futures. While the first document that the implied volatility falls around announcements, the latter find in addition that the skewness and kurtosis of the options returns distribution change after announcements. Dubinsky and Johannes (2006) find the same effect for earning announcements: Implied volatilities of single stock options increase prior to and decrease subsequent to an earning announcement. In particular, the risk-neutral volatility of price jumps due to earning announcements, capturing the anticipated uncertainty for the equity price embedded in an earnings announcements, should be a priced risk factor. For single stock options, they find that there is no priced jump risk premium. However, there is evidence of a jump volatility risk premium.[25]

Dubinsky and Johannes (2006) develop an estimator of fundamental uncertainty surrounding announcement dates using option prices. In the following, we use their term-structure estimator which is defined as:[26]

$$\left(\sigma_{time}^{Q}\right)^{2} = T_i \left((\sigma_{t,T_i})^2 - (\sigma_{t+1,T_i-1})^2 \right),$$

where σ_{t,T_i} is the Black-Scholes implied volatility of an at-the-money option at time t with T_i days to maturity.

In a similar vein, Frazzini and Lamont (2007) find that there is a premium around scheduled earnings announcement dates, which is large, robust, and strongly related to the fact that volume surges around announcement dates. One explanation for the high volume around earnings announcements is differences of opinion about the meaning of the announcements (Kandel and Pearson, 1995). Hence, one explanation for the earnings announcement premium is that differences of opinion increase around earnings announcements, leading to a rise in price. Since volume and returns move together during and after the announcement, the volume hypothesis can explain both the event-day returns and the post-event drift in returns.

In the following, we want to test whether belief disagreement is subsumed by these earning announcement effects and whether our results are robust to the inclusion of the uncertainty measure proposed in Dubinsky and Johannes (2006).

B.1. Fundamental Uncertainty

Comparing our measure of belief disagreement to fundamental uncertainty, we find that the average belief disagreement is 50% higher than the fundamental uncertainty. Moreover, the proxy for belief disagreement has also more time-variation than the uncertainty proxy in terms of standard deviation.

We add fundamental uncertainty to our regression in Table 4, columns (4) and (7), respectively. We find that fundamental uncertainty is loading negatively on the volatility risk premium, which is intuitive: The higher the fundamental uncertainty, the more negative the volatility risk premium. It is interesting to note, however, that the slope coefficient of belief disagreement is not affected by the inclusion of fundamental uncertainty. The size of the belief disagreement coefficients is larger both for the individual and the index options volatility risk premium regression.

[25] An evidence of a risk premium attached to the volatility of jump sizes in index options is also reported in Broadie, Chernov, and Johannes (2007).

[26] Their term-structure estimator is less noisy than a time-series estimator, as it does not depend on implied volatilities at different dates.

B.2. Earning Announcements

Quarterly earning announcement dates are from the I/B/E/S database. As reported in DellaVigna and Pollet (2008) before 1995, a high number of earnings announcements was recorded with an error of at least one trading day. During the more recent years, the accuracy of the earnings date has increased substantially, and is almost perfect after December 1994. The variable earning announcement is a dummy variable which takes the value of 1 if there has been an earning announcement the previous month. The variable interaction is belief disagreement multiplied by the dummy earning surprise.

The results are summarized in Table 4, column (2). We note that the variable earning announcement is negative and highly significant, but the interaction term is not. Again, the inclusion of this variable does not affect the statistical significance of the belief disagreement coefficient. This suggests that belief disagreement has a significant impact on individual volatility risk premia independent of the presence of earning announcements.

C. Net-Buying Pressure

An alternative hypothesis which could possibly explain the volatility risk premium in single-stock and index options is the demand-based hypothesis in Bollen and Whaley (2004). As they argue, buying pressure in index put options drives the slope of the implied volatility, while buying pressure for calls on single-stocks appears to drive the shape of the implied volatility of single-stocks. Motivated by this evidence, we add buying pressure to our regression.

In accordance to Bollen and Whaley (2004), we define net buying pressure as the difference between the number of contracts traded during the day at prices higher than the prevailing bid/ask quote midpoint and the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint, times the absolute value of the option's delta and then scale this difference by the total trading volume across all option series.

Results are reported in Table 4, columns (3) and (6). Apart from the coefficient of demand pressure in puts for the index, the estimated coefficients are insignificant. Demand pressure in puts loads negatively on the volatility risk premium: Higher demand pressure in index puts increases the implied volatility, which leads to the increase in volatility risk premium. The estimated slope coefficient of belief disagreement is not affected by the inclusion of this variable.

VI. Conclusion

In this paper, we study theoretically and empirically the relation between belief disagreement among investors and the cross-sectional differences in option returns. Our model extends in a parsimonious way the standard Lucas (1978) model by considering an incomplete-market economy in which agents have heterogeneous beliefs about the firm's fundamentals and some signal growth rate. We model two firms whose expected dividend growth rates are unknown to the investors, and hence have to be estimated. This feature generates an additional risk factor due to disagreement, which is not priced in the standard multi-asset Lucas (1978) economy (see e.g., Cochrane, Longstaff, and Santa-Clara, 2008). This risk factor generates interesting asset pricing implications. We test our model using panel data consisting of data on professional earning forecasts, option returns on both index and individual options, and stock return data. We summarize our findings as follows.

First, we find that belief disagreement unambiguously increases the volatility risk premia of both index and individual options due to the optimal wealth shifting across investors. In our model, risk is transferred from the pessimistic agent to the optimistic investors. A negative dividend shock increases the volatility of the stock and decreases the stock price itself. This negative correlation between stock price and volatility induces a negative skewness. The skewness of the index can be larger or smaller than the skewness of the individual firm depending on the on the share of the firm in the aggregate market, the disagreement about the business cycle component, and the disagreement about both firms.

A negative dividend shock in both firms increases the volatility even more. This is due to the decreasing marginal utility of consumption of both agents, which generates endogenously the even stronger negative skewness in the market. The stronger negative skewness of the index follows from this doubling effect. Hence, the slope of the index implied volatility function is steeper than for the individual options. This is empirically supported e.g., by the findings in Bakshi, Kapadia, and Madan (2003). We empirically find strong support that belief disagreement impacts the volatility risk premia of both individual and index options. These results are robust to the inclusion of other risk factors.

Second, in our model volatility risk premia of individual and index options represent compensation for priced disagreement risk. Hence, in the cross-section the volatility risk premium will depend on the size of belief heterogeneity of this particular firm. The volatility risk premium of the index is larger due to the more negative slope of the implied volatility smile of index options. Moreover, if volatility risk premia indeed represent a compensation for disagreement risk, then a portfolio which is sorted according to the size of belief disagreement and aimed at exploiting this premia, should generate excess returns. To this end, we simulate trading strategies which are short index volatility and long individual volatility. We do this by selling index at-the-money straddles (puts) and buying individual at-the-money straddles (puts). We find that in our economy a straddle (put) trading strategy yields significant excess returns and a Sharpe ratio which is three times (twice) as large as being short the whole wealth in index put options – a strategy which has been shown to be very successful in the literature. We also test these implications empirically. We employ the same trading strategies on S&P 100 index options and individual options on all constituents of the index. We find that these strategies earn excess returns of 20% per month with a very high Sharpe ratio. These results persist in any size, book-to-market, and momentum portfolio. Taking into account transaction costs, the profitability of the trading strategies are lowered. However, the Sharpe ratios still exceed the Sharpe ratios of traditional option trading strategies.

There are several interesting avenues for future research. First, one could potentially study an international asset pricing setup where instead of multiple firms, we study different countries.

Second, the firms in our economy are all equity financed. One could potentially study a cross-section of leveraged firms and study in more detail the different implications of disagreement on the cross-section of bond versus stock volatility risk premia.

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Appendices

A Proofs

A-1 *Equilibrium*

The solution of the disagreement and the learning dynamics are given in detail in Buraschi, Trojani, and Vedolin (2007). The solution of the equilibrium is a straight-forward extension of the proofs to two assets; see Buraschi, Trojani, and Vedolin (2007).

A-2 *Joint Laplace Transform*

To save space, we defer all calculations to a separate technical Appendix, which is available on the authors' webpage.

A-3 *Stock Price Volatility, Correlation and Skewness*

The price of the stock satisfies a diffusion process which is given by:

$$\frac{dS_1}{S_1} = \mu^A_{S_1}(t)dt + \sigma_{S_1 D_1}(t)dW^A_{D_1}(t) + \sigma_{S_1 D_2}(t)dW^A_{D_2}(t) + \sigma_{S_1 z}(t)dW^A_z(t).$$

The diffusion term is characterized by:

$$\begin{aligned}
dS_1(t) - S_1(t)\mu^A_{S_1}(t)dt \quad = \quad & \frac{\partial S_1}{\partial D_1}\left(dD_1(t) - E^A_t\left(dD_1(t)\right)\right) + \frac{\partial S_1}{\partial m^A_{D_1}}\left(dm^A_{D_1}(t) - E^A_t\left(dm^A_{D_1}\right)\right) + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(d\Psi_{D_1}(t) - E^A_t\left(d\Psi_{D_1}(t)\right)\right) \\
& + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(d\Psi_{D_2}(t) - E^A_t\left(d\Psi_{D_2}(t)\right)\right) + \frac{\partial S_1}{\partial \Psi_z}\left(d\Psi_z - E^A_t\left(d\Psi_z(t)\right)\right), \\
= \quad & \frac{\partial S_1}{\partial D_1} D_1 \sigma_{D_1} dW^A_{D_1}(t) + \frac{\partial S_1}{\partial m^A_{D_1}}\left(\frac{\gamma^A_{D_1}}{\sigma_{D_1}} dW^A_{D_1}(t) + \frac{\gamma^A_{D_1 D_2}}{\sigma_{D_2}} dW^A_{D_2}(t) + \left(\frac{\alpha_{D_1}\gamma^A_{D_1} + \alpha_{D_2}\gamma^A_{D_1 D_2} + \beta\gamma^A_{D_1 z}}{\sigma_z}\right) dW^A_z(t)\right) \\
& + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_1 D_2} - \gamma^A_{D_1 D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) dW^A_{D_2}(t)\right. \\
& + \left.\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1 D_2} - \gamma^B_{D_1 D_2}\right) + \beta\left(\gamma^A_{D_1 z} - \gamma^B_{D_1 z}\right)}{\sigma_{D_1}\sigma_z}\right) dW^A_z(t)\right) \\
& + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\left(\frac{\gamma^A_{D_1 D_2} - \gamma^B_{D_1 D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_2} - \gamma^B_{D_2}}{\sigma^2_{D_2}}\right) dW^A_{D_2}(t)\right. \\
& + \left.\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1 D_2} - \gamma^B_{D_1 D_2}\right) + \alpha_{D_2}\left(\gamma^A_{D_2} - \gamma^B_{D_2}\right) + \beta\left(\gamma^A_{D_2 z} - \gamma^B_{D_2 z}\right)}{\sigma_{D_2}\sigma_z}\right) dW^A_z(t)\right) \\
& + \frac{\partial S_1}{\partial \Psi_z}\left(\left(\frac{\gamma^A_{D_1 z} - \gamma^B_{D_1 z}}{\sigma_{D_1}\sigma_z}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_2 z} - \gamma^B_{D_2 z}}{\sigma_{D_2}\sigma_z}\right) dW^A_{D_2}(t)\right. \\
& + \left.\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1 z} - \gamma^B_{D_1 z}\right) + \alpha_{D_2}\left(\gamma^A_{D_2 z} - \gamma^B_{D_2 z}\right) + \beta\left(\gamma^A_z - \gamma^B_z\right)}{\sigma^2_z}\right) dW^A_z(t)\right)
\end{aligned}$$

where

$$\begin{aligned}
\sigma_{S_1D_1}(t) &= \frac{1}{S_1(t)}\left(\frac{\partial S_1}{\partial D_1}D_1\sigma_{D_1} + \frac{\partial S_1}{\partial m^A_{D_1}}\frac{\gamma^A_{D_1}}{\sigma_{D_1}} + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1}-\gamma^B_{D_1}}{\sigma^2_{D_1}}\right) + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\gamma^A_{D_1z}-\gamma^B_{D_1z}}{\sigma_{D_1}\sigma_z}\right)\right),\\
\sigma_{S_1D_2}(t) &= \frac{1}{S_1(t)}\left(\frac{\partial S_1}{\partial m^A_{D_1}}\frac{\gamma^A_{D_1D_2}}{\sigma_{D_2}} + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_2}-\gamma^B_{D_2}}{\sigma^2_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\gamma^A_{D_2z}-\gamma^B_{D_2z}}{\sigma_{D_2}\sigma_z}\right)\right),\\
\sigma_{S_1z}(t) &= \frac{1}{S_1(t)}\Bigg(\frac{\partial S_1}{\partial m^A_{D_1}}\left(\frac{\alpha_{D_1}\gamma^A_{D_1}+\alpha_{D_2}\gamma^A_{D_1D_2}+\beta\gamma^A_{D_1z}}{\sigma_z}\right) + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1}-\gamma^B_{D_1}\right)+\alpha_{D_2}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right)\\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\alpha_{D_2}\left(\gamma^A_{D_2}-\gamma^B_{D_2}\right)+\beta\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right)\\
&\quad + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)+\alpha_{D_2}\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)+\beta\left(\gamma^A_z-\gamma^B_z\right)}{\sigma^2_z}\right)\Bigg).
\end{aligned}$$

Using the following the derivatives,

$$\begin{aligned}
\frac{\partial S_1}{\partial D_1} &= \int_t^\infty e^{-\delta(u-t)}F_{m^A}\left(m^A,t,T;\epsilon_{D_1},\epsilon_{D_2}\right)G\left(t,u,1-\gamma;\Psi\right)du,\\
\frac{\partial S_1}{\partial m^A_{D_1}} &= D_1\int_t^\infty e^{-\delta(u-t)}A\left(u-t\right)F_{m^A}\left(m^A,t,T;\epsilon_{D_1},\epsilon_{D_2}\right)G\left(t,u,1-\gamma;\Psi\right)du,\\
\frac{\partial S_1}{\partial \Psi} &= D_1\int_t^\infty e^{-\delta(u-t)}\left(B_\Psi+2C_\Psi\right)F_{m^A}\left(m^A,t,T;\epsilon_{D_1},\epsilon_{D_2}\right)G\left(t,u,1-\gamma;\Psi\right)du,
\end{aligned}$$

we can easily compute the stock volatility which is given by $\left(\sigma^2_{S_1D_1}+\sigma^2_{S_1z_1}+\sigma^2_{S_1D_2}+\sigma^2_{S_1z_2}\right)^{1/2}$. The corresponding coefficients for the volatility of stock 2 are:

$$\begin{aligned}
\sigma_{S_2D_1}(t) &= \frac{1}{S_2(t)}\left(\frac{\partial S_2}{\partial m^A_{D_2}}\left(\frac{\gamma^A_{D_1D_2}}{\sigma_{D_1}}\right) + \frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1}-\gamma^B_{D_1}}{\sigma^2_{D_1}}\right) + \frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_2}{\partial \Psi_z}\left(\frac{\gamma^A_{D_1z}-\gamma^B_{D_1z}}{\sigma_{D_1}\sigma_z}\right)\right),\\
\sigma_{S_2D_2}(t) &= \frac{1}{S_2(t)}\left(\frac{\partial S_2}{\partial D_2}D_2\sigma_{D_2} + \frac{\partial S_2}{\partial m^A_{D_2}}\frac{\gamma^A_{D_2}}{\sigma_{D_2}} + \frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_2}-\gamma^B_{D_2}}{\sigma^2_{D_2}}\right) + \frac{\partial S_2}{\partial \Psi_z}\left(\frac{\gamma^A_{D_2z}-\gamma^B_{D_2z}}{\sigma_{D_2}\sigma_z}\right)\right),\\
\sigma_{S_2z}(t) &= \frac{1}{S_2(t)}\Bigg(\frac{\partial S_2}{\partial m^A_{D_2}}\left(\frac{\alpha_{D_1}\gamma^A_{D_1D_2}+\alpha_{D_2}\gamma^A_{D_2}+\beta\gamma^A_{D_2z}}{\sigma_z}\right) + \frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1}-\gamma^B_{D_1}\right)+\alpha_{D_2}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right)\\
&\quad + \frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right) + \frac{\partial S_2}{\partial \Psi_z}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)+\alpha_{D_2}\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)+\beta\left(\gamma^A_z-\gamma^B_z\right)}{\sigma^2_z}\right)\Bigg).
\end{aligned}$$

The correlation between stock 1 and stock 2 can be calculated as follows:

$$\text{corr}\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right) = \frac{Cov\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right)}{\sqrt{\sigma^2_{S_1}(t)}\sqrt{\sigma^2_{S_2}(t)}},$$

where

$$\begin{aligned}
Cov\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right) &= E\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right) - E\left(\frac{dS_1}{S_1}\right)E\left(\frac{dS_2}{S_2}\right),\\
&= \left(\sigma_{S_1D_1}(t)\sigma_{S_2D_1}(t)+\sigma_{S_1z_1}(t)\sigma_{S_2z_1}(t)+\sigma_{S_1D_2}(t)\sigma_{S_2D_2}(t)+\sigma_{S_1z_2}(t)\sigma_{S_2z_2}(t)\right)dt.
\end{aligned}$$

Assume that the stock return is related to the market return via the following two factor representation:

$$r_i(t) = \beta_M r_M(t) + \beta_{\overline{\Psi}}\overline{\Psi}(t) + \epsilon_i(t), \tag{A-1}$$

where r_i is the return on the stock price of firm i, r_M is the return on the market, and $\overline{\Psi}$ is the common disagreement. The skewness of r_i is defined as:[27]

$$SKEW(r_i) = E\left(\left(r_i - E(r_i)\right)^3\right).$$

[27] For simplicity, we omit the standardization by the variance to the power 3/2.

Plugging relation (A-1) into this definition yields:

$$
\begin{aligned}
SKEW(r_i) &= E\Big(\beta_M^3 r_M^3 + \beta_{\overline{\Psi}}\overline{\Psi}^3 + \epsilon_i^3 + 3\beta_{\overline{\Psi}}\beta_M\overline{\Psi}^2 r_M + 3\beta_{\overline{\Psi}}\beta_M^2\overline{\Psi} r_M^2 + 3\beta_M r_M \epsilon_i^2 \\
&\quad + 3\beta_{\overline{\Psi}}\overline{\Psi}\epsilon_i^2 - 3\beta_M^2 r_M^2 m_M - 3\beta_{\overline{\Psi}}^2\overline{\Psi}^2 m_M - 3\epsilon_i^2 m_M - 6\beta_{\overline{\Psi}}\beta_M\overline{\Psi} r_M m_M + 3\beta_{\overline{\Psi}} r_M m_M^2 + 3\beta_{\overline{\Psi}}\overline{\Psi} m_M^2 - m_M^3\Big), \\
&= \beta_M^3 SKEW(r_M) + \beta_{\overline{\Psi}}^3 SKEW(\overline{\Psi}) - SKEW(m_M) + COSKEWS \\
&\quad + 3\beta_{\overline{\Psi}}\int_0^1 \overline{\Psi} q(\overline{\Psi}) d\overline{\Psi} + 3m_M(\beta_M - 1) - 6\beta_M\beta_{\overline{\Psi}}\int_0^1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\Psi} r_M m_M q(\overline{\Psi}, r_M, m_M) dr_M dm_M d\overline{\Psi},
\end{aligned}
$$

where $E(r_i) := m_M$. The last equality comes from the fact that $\epsilon_i \sim N(0,1)$ and that all co-skewness terms with ϵ_i are zero, because we assume independence between $\overline{\Psi}, m_M$ and ϵ_i.

$$
\begin{aligned}
COSKEWS &= 3\beta_{\overline{\Psi}}\beta_M^2\int_0^1\int_{-\infty}^{\infty}\overline{\Psi} r_M^2 q(\overline{\Psi}, r_M^2) dr_M d\overline{\Psi} - 3\beta_M^2\int_0^{\infty}\int_{-\infty}^{\infty} m_M r_M^2 q(r_M^2, m_M) dm_M dr_M \\
&\quad - 3\beta_{\overline{\Psi}}^2\int_0^1\int_{-\infty}^{\infty}\overline{\Psi}^2 m_M q(\overline{\Psi}^2 m_M) d\overline{\Psi} dm_M + 3\beta_M\int_{-\infty}^{\infty} r_M m_M^2 q(r_M, m_M^2) dr_M dm_M \\
&\quad + 3\beta_{\overline{\Psi}}\int_0^1\int_0^{\infty}\overline{\Psi} m_M^2 q(\overline{\Psi} m_M^2) dm_M d\overline{\Psi}.
\end{aligned}
$$

To synthesize the risk-neutral skewness, we follow Bakshi and Madan (2000) that the entire collection of twice-differentiable payoff functions with bounded expectation can be spanned algebraically. Applying this result to the stock price $S_1(t)$, we get

$$
G(S_1) = G(\tilde{S}_1) + \left(S - \tilde{S}\right) G_{S_1}\left(\tilde{S}\right) + \int_{\tilde{S}_1}^{\infty} G_{S_1 S_1}(K)\,(S_1 - K)^+\, dK + \int_0^{\tilde{S}_1} G_{S_1 S_1}(K)\,(K - S_1)^+\, dK,
$$

where G_{S_1} is the partial derivative of the payoff function $G(S_1)$ with respect to S_1 and $G_{S_1 S_1}$ the corresponding second-order partial derivative. By setting $\tilde{S}_1 = S_1(t)$, we obtain the final formula for the risk-neutral skewness of the stock, after mimicking the steps in Bakshi, Kapadia, and Madan (2003) (Theorem 1, p. 137).

Let $s(t,T) = \ln(S_1(t+T))$ be the firm value return between time t and T. The risk-neutral skewness of $s(t,T)$ of stock S_1 is given by

$$
skew_{S_1}(t,T) = \frac{e^{rT}W(t,T) - 2\mu(t,T)e^{rT}R(t,T) + 2\mu(t,T)^3}{(e^{rT}R(t,T) - \mu(t,T)^2)^{3/2}},
$$

where

$$
\begin{aligned}
R(t,T) &= \int_{S_1(t)}^{\infty}\frac{2\left(1 - \ln\left(\frac{K}{S_1(t)}\right)\right)}{K^2}(S_1(K) - K)^+ + \int_0^{S_1(t)}\frac{2\left(1 + \ln\left(\frac{S_1(t)}{K_1}\right)\right)}{K^2}(K - S_1(T))^+\, dK, \\
W(t,T) &= \int_{S_1(t)}^{\infty}\frac{6\ln\left(\frac{K}{S_1(t)}\right) - 3\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^2}{K^2}(S_1(t) - K)^+\, dK \\
&\quad - \int_0^{S_1(t)}\frac{6\ln\left(\frac{S_1(t)}{K}\right) - 3\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^2}{K^2}(K - S_1(T))^+\, dK,
\end{aligned}
$$

and

$$
\begin{aligned}
X(t,T) &= \int_{S_1(t)}^{\infty}\frac{12\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^2 - 4\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^3}{K^2}(S_1(T) - K)^+\, dK \\
&\quad - \int_0^{S_1(t)}\frac{12\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^2 - 4\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^3}{K^2}(K - S_1(T))^+\, dK, \\
\mu(t,T) &= E_t\left(\ln\left(\frac{S_1(t+T)}{S_1(t)}\right)\right) \approx e^{rT} - 1 - \frac{e^{rT}}{2}R(t,T) - \frac{e^{rT}}{6}W(t,T) - \frac{e^{rT}}{24}X(t,T).
\end{aligned}
$$

The skewness of stock 2 and of the index are equivalent.

This concludes the discussion about the stock volatility and skewness.

B Dispersion Trading

A-1 The Basics

Consider an index with n stocks. σ_i is the volatility of stock i, ω_i is the weight of stock i in the index, and ρ_{ij} is the correlation between stock i and stock j. The index itself has the following volatility:

$$\sigma^2_{Index} = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i \neq j}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}.$$

The average index variance is:

$$\bar{\sigma}^2_{Index} = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2.$$

We can now compare this average number to the actual index volatility. We define a dispersion spread, D, as:

$$D = \sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2 - \sigma^2_{Index}} = \sqrt{\bar{\sigma}^2_{Index} - \sigma^2_{Index}}.$$

The upper bound for the dispersion spread is now simply the average basket volatility and the lower bound is zero. A trading strategy which bets on the dispersion spread has two legs. If the investor is long dispersion, then she is long the volatility of the constituents and short index volatility. So, it would be desirable to have lots of volatility on the constituents and no volatility on the index. One of the main drivers is the exposure of this strategy to correlation. Particularly, it is quite easy to see that if one is long dispersion then she is also short in correlation, by considering the average correlation:

$$\bar{\rho} = \frac{\sigma^2_{Index} - \sum_{i=1}^{n} \omega_i^2 \sigma_i^2}{\sum_{i \neq j}^{n} \omega_i \omega_j \sigma_i \sigma_j}$$

A-2 P&L of a Dispersion Trade

For simplicity reasons, we study a case with constant volatility.[28] So, consider a delta-hedged portfolio which is long the stock options and short the index options. Remember, that the P&L of a delta-hedged option Π in a Black and Scholes framework is (see Hull, 2002)

$$P\&L = \theta \left[\left(\frac{dS}{S\sigma\sqrt{dt}} \right)^2 - 1 \right],$$

where the θ of the option is defined as the options sensitivity with respect to a change in the time to maturity.

Let the term $n = \frac{dS}{S\sigma\sqrt{dt}}$ represent the standardized move of the underlying stock S on the considered time period. Then, the P&L of the index can be written as

$$\begin{aligned}
P\&L &= \theta_I \left(n_I^2 - 1 \right), \\
&= \theta_I \left(\left(\sum_{i=1}^{n} \omega_i n_i \frac{\sigma_i}{\sigma_{Index}} \right)^2 - 1 \right), \\
&= \theta_I \left(\sum_{i=1}^{n} \left(\omega_i n_i \frac{\sigma_i}{\sigma_{Index}} \right)^2 + \sum_{i \neq j} \frac{\omega_i \omega_j \sigma_i \sigma_j}{\sigma_{Index}} n_i n_j - 1 \right), \\
&= \theta_I \sum_{i=1}^{n} \frac{\omega_i^2 \sigma_i^2}{\sigma^2_{Index}} \left(n_i^2 - 1 \right) + \theta_I \sum_{i \neq j} \frac{\omega_i \omega_j \sigma_i \sigma_j}{\sigma^2_{Index}} \left(n_i n_j - \delta_{ij} \right),
\end{aligned}$$

[28] Adding stochastic volatility yields analogous expressions with some additional terms which account for the Vega, Volga, and Vanna of the option.

Hence, a dispersion trade being short the index options and being long the individual options has the following P&L:

$$\begin{aligned} P\&L &= \sum_{i=1}^{n} P\&L_i - P\&L_I, \\ &= \sum_{i=1}^{n} \theta_i \left(n_i^2 - 1\right) + \theta_I \left(n_I^2 - 1\right). \end{aligned}$$

The short and long positions in the options are reflected in the sign of the θ_i. A long (short) position means a positive (negative) θ.

A-3 Weighting Scheme for Dispersion Trading

Since trading all constituents would be far too expensive, the investor has to ask herself which stock she should pick and then, how to weight them. There are the following weighting schemes, which are employed in the industry:

1. Vega Hedging:
   The investor will build her dispersion such that the vega of the index equals the sum of the vegas of the constituents.
2. Gamma Hedging:
   The gamma of the index is worth the sum of the gamma of the components. As the portfolio is already delta-hedged, this weighting scheme protects the investor against any move in the stocks, but leaves her with a vega position.
3. Theta Hedging:
   This strategy results in a short vega and a short gamma position.[29]

[29] To this end, remember that the relationship between the option's theta and gamma is as follows:

$$\Theta \approx -\frac{1}{2}\Gamma S^2 \sigma^2,$$

where S is the underlying spot price.

Table 1
Choice of Parameter Values and Benchmark Values of State Variables

This table lists the parameter values used for all figures in the paper. We calibrate the model to the mean and volatility of the dividends on the S&P 500. The average growth rate for the period 1996-2006 is 5.93% and the volatility is 3.52%. The initial values for the conditional variances are set to their steady-state variances.

Parameters for Fundamentals		
Long-term growth rate of dividend growth	a_{0D_i}	0.01
Mean-reversion parameter of dividend growth	a_{1D_i}	-0.01
Volatility of dividend	σ_{D_i}	0.07
Initial level of dividend	D_i	1.00
Initial level of dividend growth	$m^A_{D_i}$	0.01
Parameters for Signal		
Long-term growth rate of signal	a_{0z}	0.01
Mean-reversion parameter of signal	a_{1z}	-0.03
Volatility of signal	σ_z	0.06
Agent specific Parameters		
Relative risk aversion for both agents	γ	2.00
Time Preference Parameter	ρ	0.02

Table 2
Moments of Simulated Option Strategies

This table reports summary statics of the simulated strategy returns, average, standard deviation, skewness, kurtosis and Sharpe ratio. The Dispersion Straddle and Dispersion Put portfolios are formed by investing 100% of the wealth in shorting index straddles or puts, respectively, and investing a fraction of wealth into the options of the firm with the highest belief disagreement such that the portfolio is vega neutral. The remainder is invested in the individual stock of this particular firm such that the portfolio is delta neutral. The index consists of two equally weighted stocks. We simulate 2,877 trading days and 1,000 simulation runs. All options are at-the-money with a maturity of 28 trading days.

	Dispersion Straddle	Dispersion Put	Index	Short Index Put
Return	0.127	0.050	0.010	0.117
StDev	0.231	0.121	0.038	0.653
Skewness	-3.127	-4.125	-7.423	-8.239
Kurtosis	7.532	8.022	5.819	8.477
Ann. SR	1.867	1.482	0.683	0.607

Table 3
Summary Statistics of Most Important Variables

We report summary statistics of the main variables used in the analysis. The data runs from January 1996 to June 2007, with monthly frequency. Risk Premium is the difference between the realized and implied volatility. The realized volatility is calculated from stock return data retrieved from the CRSP database. It is calculated over a 21-day window, requiring that there are at least 15 nonzero observations per window. The implied volatility is calculated from option prices taken from the Optionmetrics database. Dispersion Individual is defined as the ratio of the mean absolute difference of analysts' forecasts and the standard deviation of these forecasts, retrieved from the I/B/E/S database. Dispersion Common is a common component estimated by dynamic factor analysis from the individual disagreement series. Market Volatility is defined as the historical volatility over a 21-day window. Corr Individual (Index) is the time-series average correlation with the individual (index) volatility risk premium.

	Mean	StDev	0.25 percentile	0.75 percentile	Corr Individual	Corr Index
Risk Premium Individual	-0.0110	0.0357	-0.0356	-0.0041	1.0000	0.6917
Risk Premium Index	-0.0261	0.0336	-0.0475	-0.0131	0.6917	1.0000
DIB Individual	0.3089	0.2038	0.1612	0.3787	0.5973	–
DIB Common	0.0410	0.0231	0.0264	0.0486	–	0.5252
Market Volatility	0.1663	0.0752	0.1070	0.2076	-0.0160	-0.6262

Table 4

Volatility and Correlation Risk Premium Regressions

Using data from January 1996 to June 2007, we run regressions from the volatility risk premium of individual and index options on a number of determinants. The volatility risk premium is defined as the difference between the options' 21 day realized and implied volatility. The correlation risk premium is approximated as the difference between the index volatility risk premium and a weighted average of the constituents volatility risk premia. DiB is our proxy for difference in beliefs for each firm, defined as the mean absolute difference among analysts forecasts standardized, Dib Common is our proxy for difference in beliefs for the market, Market Vola is the 21 day realized volatility of the index, Skewness is measured as the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios. Earning Announcement is a dummy variable which takes the value of 1 if there is an earning announcement scheduled for the respective month and zero else. Interaction is the variable Earning Announcement multiplied by DIB. Fundamental Uncertainty is defined as $\left(\sigma^{Q}_{time}\right)^2 = T_i\left(\left(\sigma_{t,T_i}\right)^2 - \left(\sigma_{t+1,T_i-1}\right)^2\right)$. DP is demand pressure and is defined as the difference between the number of contracts traded during the day at prices higher than the prevailing bid/ask quote midpoint and the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint, times the absolute value of the option's delta and then scale this difference by the total trading volume across all option series. Macro Factor is a common component estimated via dynamic factor analysis from Industrial production, Housing Starts, S&P 500 P/E ratio, and, Producer Price index (PPI). We use logarithmic changes over the past twelve months. $\star$ denotes significance at the 10% level, $\star\star$ denotes significance at the 5% level and $\star\star\star$ denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

	Individual				Index			Correlation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.002^{***}
	(-8.21)	(-7.37)	(-5.78)	(-6.48)	(-7.89)	(-6.11)	(-4.32)	(-2.71)
DiB	-0.037^{***}	-0.043^{***}	-0.042^{***}	-0.038^{***}	-0.099^{***}	-0.126^{***}	-0.172^{***}	
	(-22.18)	(-17.83)	(-19.82)	(-18.82)	(-17.83)	(-22.18)	(-17.83)	
Common DiB	-0.012^{**}	-0.009^{**}	-0.010^{**}	-0.019^{**}	-0.028^{***}	-0.038^{***}	-0.045^{***}	-0.098^{**}
	(-2.37)	(-2.21)	(-2.49)	(-2.53)	(-5.28)	(-6.28)	(-5.37)	(-2.12)
Market Vola	-0.028^{**}	-0.031^{**}	-0.030^{**}	-0.029^{*}	-0.097^{***}	-0.091^{**}	-0.136^{**}	-0.041^{**}
	(-2.45)	(-1.99)	(-2.17)	(-1.83)	(-3.01)	(-2.47)	(-2.33)	(-2.35)
Macro Factor	0.031^{**}	0.039^{**}	0.021^{**}	0.013^{**}	0.010^{**}	0.012^{*}	0.017^{***}	0.018
	(2.25)	(2.52)	(2.33)	(2.39)	(2.29)	(1.69)	(3.41)	(1.49)
Skewness	0.012^{*}	0.011	0.009	0.008	0.014^{**}	0.011	0.002	
	(1.71)	(1.12)	(0.83)	(1.16)	(1.89)	(1.32)	(1.17)	
DP Calls			0.041			-0.098		
			(1.61)			(-1.02)		
DP Puts			0.020			-0.029^{*}		
			(1.29)			(-1.71)		
Earning Announc.		-0.004^{***}						
		(-2.74)						
Interaction		-0.018						
		(-1.13)						
Fund. Uncertainty				-0.019^{***}			-0.121^{***}	
				(-2.99)			(-3.29)	
Adj. R^2	0.26	0.28	0.30	0.31	0.22	0.24	0.25	0.08

Table 5
Returns on Option Strategies

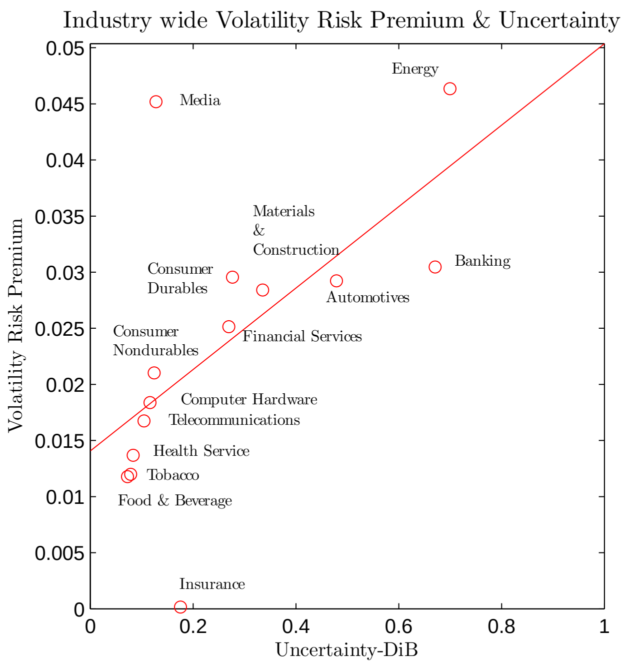
This table reports summary statics of the strategy returns, average, standard deviation, skewness, kurtosis and Sharpe ratio. The Dispersion Straddle and Dispersion Put portfolios are formed by investing 100% of the wealth in shorting index straddles or puts, respectively, and investing a fraction of wealth into the options of firms with the highest belief disagreement such that the portfolio is vega neutral. The remainder is invested in the individual stocks such that the portfolio is delta neutral. The index put is an equally-weighted portfolio of index put options with Black-Scholes deltas ranging from -0.8 to -0.2. Option returns of single-stocks and the index are sampled between January 1996 and June 2007. All statistics are monthly, except the Sharpe ratios, which are annualized.

	Dispersion Straddle	Dispersion Put	S&P 100	Short Index Put
Return	0.199	0.027	0.007	0.371
StDev	0.347	0.042	0.048	1.612
Skewness	-1.131	-2.876	-0.620	-3.499
Kurtosis	5.120	4.327	1.782	4.439
Ann. SR	1.962	2.027	0.381	0.792
CAPM Alpha	$0.162^{\star\star\star}$	$0.019^{\star\star}$		$0.110^{\star}$
t-Stat	4.42	2.23		1.65
CAPM Beta	0.231	0.012		$3.239^{\star\star}$
t-Stat	1.12	1.53		2.38

Table 6
Impact of Transaction Costs on Returns on Option Strategies

This table reports summary statics of the strategy returns, average, standard deviation, skewness, kurtosis and Sharpe ratio. We sort stocks based into quintiles based on the size of belief disagreement. Quintile 5 consists of stocks with the highest belief disagreement while quintile one consists of stocks with the lowest belief disagreement. The index put is an equally-weighted portfolio of 1-month index put options with Black-Scholes deltas ranging from -0.8 to -0.2. We use bid prices when options are written and ask prices when options are bought. Option returns of single-stocks and the index are sampled between January 1996 and June 2007. All statistics are monthly, except the Sharpe ratios, which are annualized.

	Mid Point			Bid-to-Ask		
	ATM Straddle	Put	Short Index Put	ATM Straddle	Put	Short Index Put
Return	0.199	0.027	0.371	0.115	0.012	0.321
StDev	0.347	0.042	1.612	0.356	0.040	1.752
Ann. SR	1.962	2.027	0.792	1.094	0.822	0.630
CAPM Alpha	0.162***	0.019**	0.110*	0.076**	0.009*	0.076*
t-Stat	4.42	2.23	1.64	2.11	1.78	1.64
CAPM Beta	0.231	0.012	3.239**	0.523	0.045	0.412
t-Stat	1.12	1.53	2.38	1.01	1.21	1.43



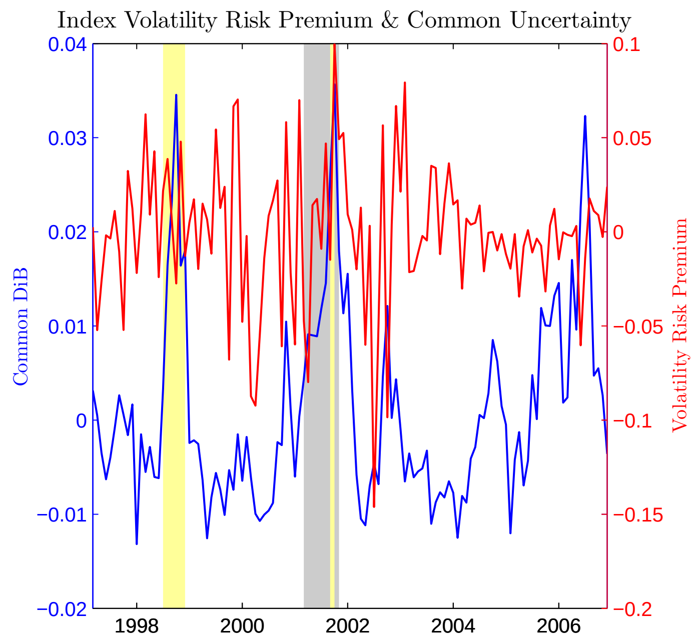
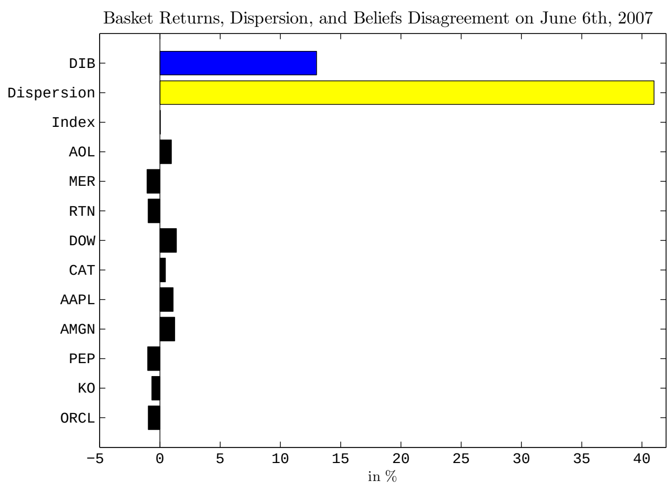


Figure 1. Industry and Index Volatility Risk Premia and Uncertainty-DiB

The left panel plots the average volatility risk premium defined as the difference between the 30 day at-the-money implied volatility and the 30 day realized volatility for different sectors together with the corresponding sector disagreement proxies. The right panel plots the Hodrick-Prescott filtered index volatility risk premium defined as the difference between the end-of-month VIX and the annualized 30 day realized volatility on the S&P 500 together with the Hodrick-Prescott filtered common disagreement proxy. Realized volatility is the square root of the sum of squared daily log returns on the S&P 500 over the month. Both volatility measures are of monthly basis and are available at the end of each observation month. The shaded areas represent financial or economic crisis defined according to the NBER.



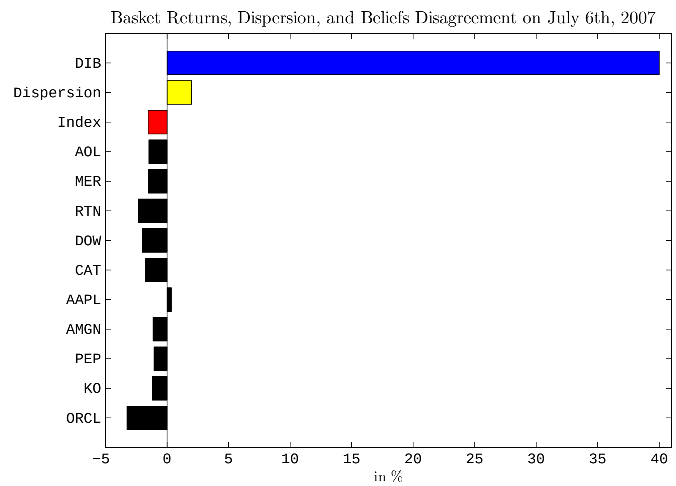


Figure 2. Constituents Returns of Fictive Portfolio

This figure plots the returns of ten stocks on June 6th 2007, and one month later, on July 6th. The dispersion, defined as $D = \sqrt{\sum_{i=1}^{N} \omega_i^2 \sigma_i^2 - \sigma_{\text{Index}}^2}$, where ω_i is the weight of stock i in the index, and $i = 1, \ldots, 10$ indexes the firms in the basket, is 41% in June and 2% in July. DIB is our proxy of belief disagreement and is 0.13 in June and 0.40 in July.

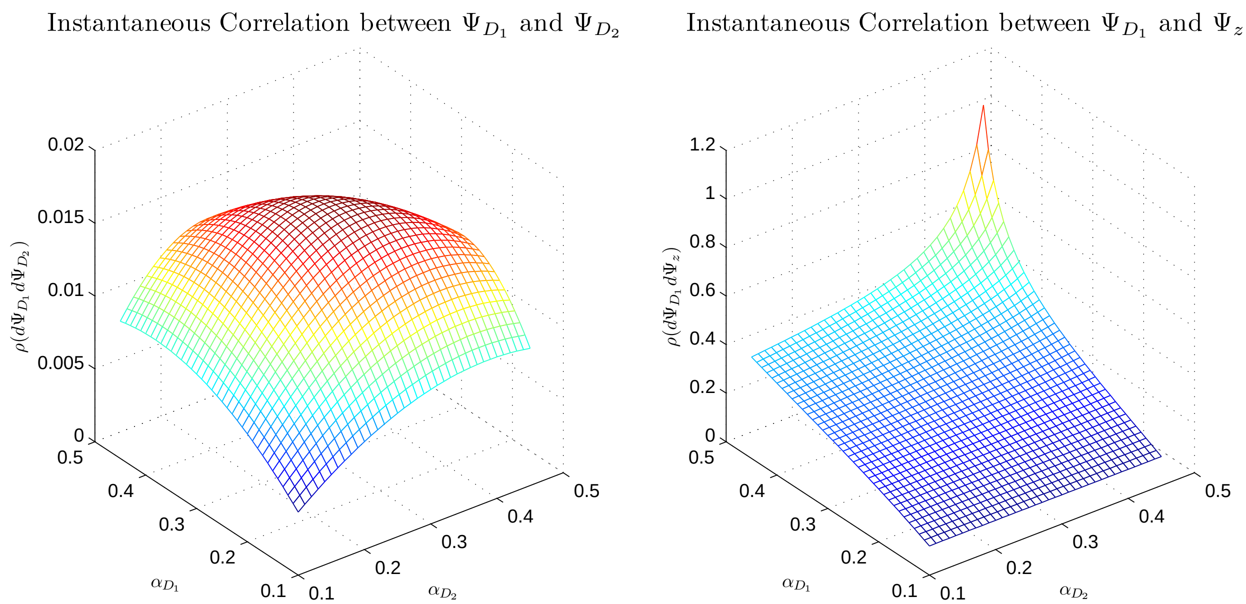


Figure 3. Uncertainty Correlation

The left panel plots the instantaneous correlation between the disagreement about firm 1 (Ψ_{D_1}) and firm 2 (Ψ_{D_2}) as a function of the weights α_{D_1} and α_{D_2}. The right panel plots the instantaneous correlation between the disagreement about firm 1 (Ψ_{D_1}) and the signal (Ψ_z). The parameters chosen are summarized in Table 1.

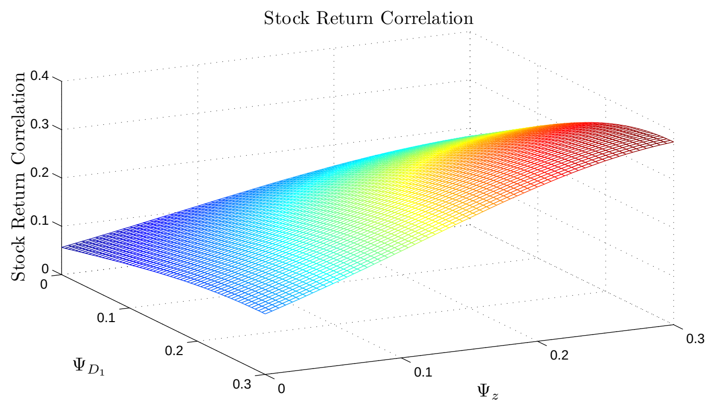


Figure 4. Stock Return Correlation

This figure plots the return correlation of stock 1 and stock 2 as a function of belief disagreement Ψ_{D_1} and Ψ_{D_2}. The parameters chosen are summarized in Table 1.

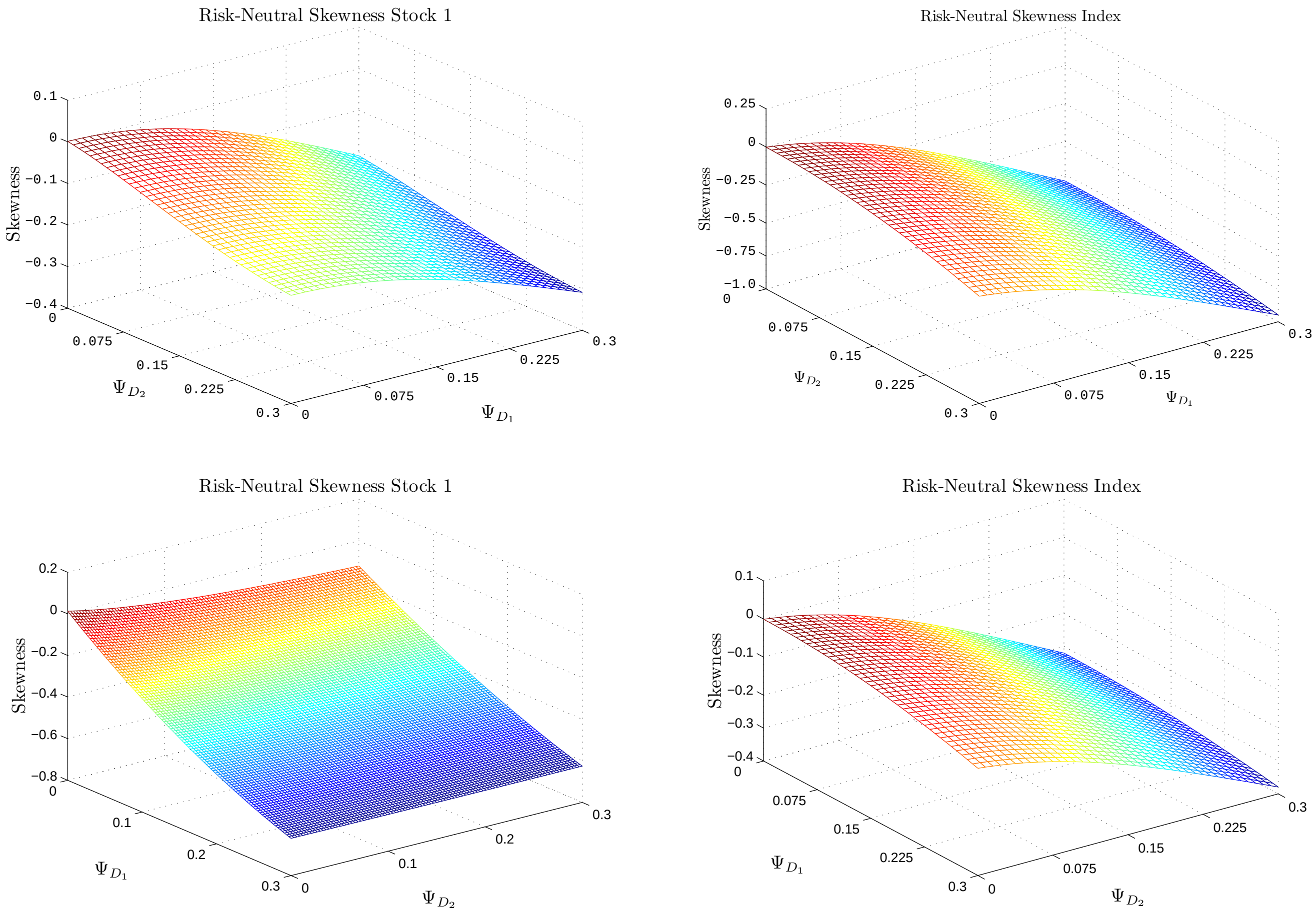
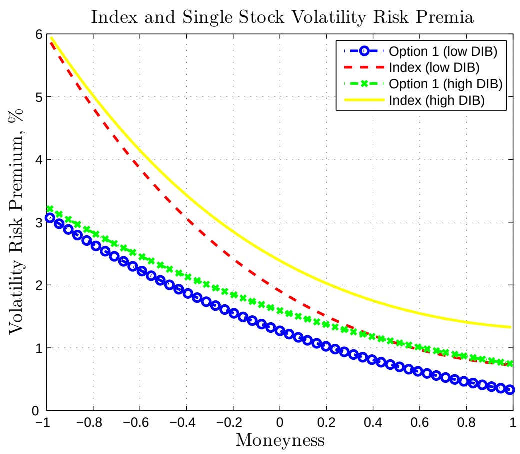


Figure 5. Risk-Neutral Skewness for Individual Stock and Index Options

These figures plot the risk-neutral skewness of the returns of stock 1 (left panel) and the index (right panel). The parameters chosen are summarized in Table 1.



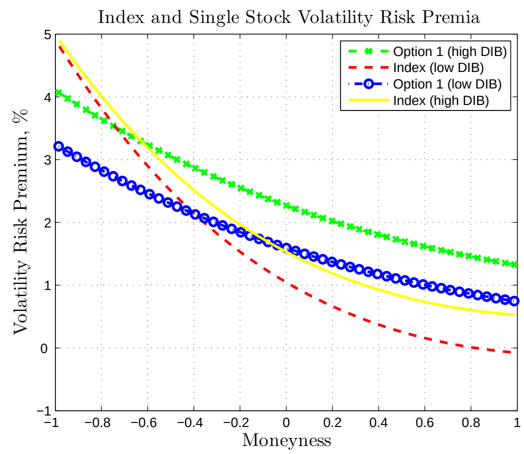
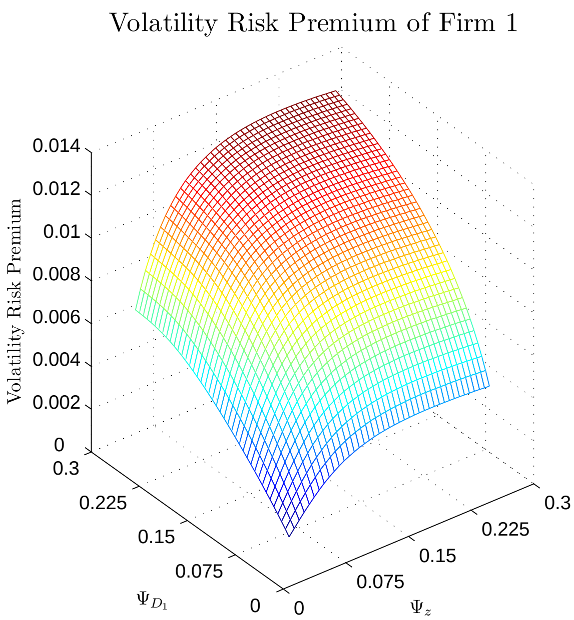


Figure 6. Index and Individual Stock Volatility Risk Premia

This figure plots the volatility risk premia for the individual stock and the index. In the left panel, we plot the risk premia for high disagreement, i.e. $\Psi_{D_1} = \Psi_{D_2} = \Psi_z = 0.3$ and low disagreement, i.e. $\Psi_{D_1} = \Psi_{D_2} = \Psi_z = 0.1$. The right panel plots the risk premia for high disagreement, i.e. $\Psi_{D_1} = \Psi_{D_2} = 0.3$ and $\Psi_z = 0$ and low disagreement, i.e. $\Psi_{D_1} = \Psi_{D_2} = 0.1$ and $\Psi_z = 0$. The volatility risk premium is defined as the difference between the implied volatility and the square root of the integrated variance $E_t^A\left(\int_t^T \sigma^2(s)ds\right)$ under the physical measure. The parameters chosen are summarized in Table 1. Moneyness, is defined as $\ln\left(\frac{S}{K}\right)$.



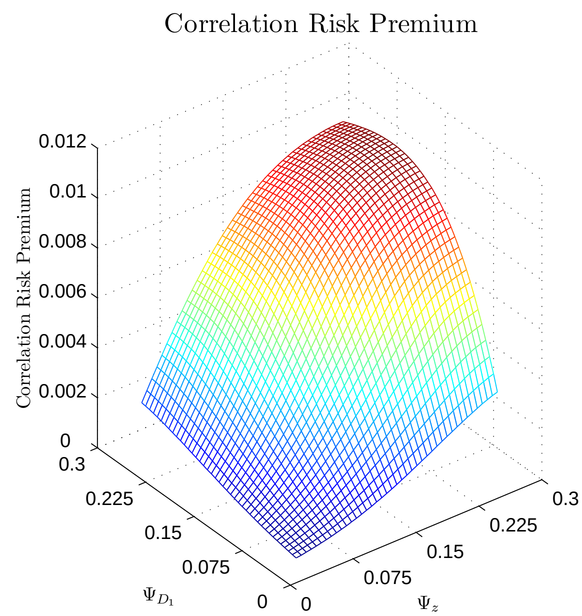


Figure 7. Volatility and Covariance Risk Premia

The left panel plots the volatility risk premium for firm 1 as a function of the disagreement about the growth rate of firm 1, Ψ_{D_1}, and the disagreement about the signal, Ψ_z. The volatility risk premium is calculated as the difference between the 30 day realized volatility and the volatility swap rate using a discretized version of equation 19. The 30 day realized volatility is calculated from running 10,000 simulations and averaging. The parameters chosen are summarized in Table 1.