Summary Report: An Optical Ray Tracer

***Abstract—* My simulation suggested that the optimal configuration of a plano-convex lens to reduce spherical aberration was one in which the curved surface was facing the incident rays. I also compared the RMS radius at the paraxial focus of a plano-convex lens with the RMS radius from an optimised lens. The results of my investigation suggest that a lens with both a positive curvature and a smaller negative curvature is better for reducing the effects of spherical aberration compared to a plano-convex lens.**

RAY AND OPTICAL ELEMENTS

In task 1 of the computing script, I created the 'ray_class' module which contained the 'Ray' class (line 14). This class was initialised with an initial position and direction. The class also included methods to return the current position, direction, append new positions and directions and to return all the vertices along the ray. For task 2, I made the 'ray_test' script to test the functionality of the class. I created an instance of a ray, r2, appended a point and then used the 'vertices' method to check whether the point had been successfully appended (line 17). Having appended a point, I initialised the ray and checked to see whether the vertices had updated accordingly.

For task 3, I created the 'optical_element_class' module which contained the 'OpticalElement' class (line 17) and the 'SphericalRefraction' class (line 34) which describes a spherical refracting surface and inherits from Optical Element. For task 4, I created the 'intercept' method (line 79) which returns the coordinates of the intercept between a ray and the spherical surface and returns None if there is no intercept. Intercepts with surfaces that had zero curvature were dealt with (lines 86 to 91) and the following equation was used for determining the distance between the starting point of a ray and its intercept with a spherical surface, l:

$$l = -\boldsymbol{r} \cdot \hat{\boldsymbol{k}} \pm \sqrt{(\boldsymbol{r} \cdot \hat{\boldsymbol{k}})^2 - (|\boldsymbol{r}|^2 - R^2)}\,, \qquad (1)$$

$\boldsymbol{r}$ is the vector distance between the centre of the spherical surface and the starting point of the ray, $\hat{\boldsymbol{k}}$ is the normalised direction of travel of the ray and R is the radius of the spherical surface [1]. As the equation could give up to two solutions, I devised a method to ensure that the correct solution was taken (lines 93 to 149).

The SphericalRefraction class also contained the method 'refracted_direction' (line 151), which determined the direction of the ray once it had propagated through the surface. This method used Snell's law in vector form to find the refracted direction, $\boldsymbol{i}_2$:

$$\boldsymbol{i}_2 = \sqrt{1 - \mu^2[1 - (\boldsymbol{i}_1 \cdot \boldsymbol{n})^2]}\boldsymbol{n} + \mu[\boldsymbol{i}_1 - (\boldsymbol{i}_1 \cdot \boldsymbol{n})\boldsymbol{n}]\,, \qquad (2)$$

μ is the ratio of the refractive indices, $\frac{n_1}{n_2}$, $\boldsymbol{i}_1$ is the incident direction of the ray and $\boldsymbol{n}$ is the normal to the surface. I created the 'OuputPlane' class (line 329) which has a method 'end_ray' that appends the intercept position to the ray. This class is useful for plotting the rays and to determine the RMS radius and spot diagram when investigating the ray/lens system.

Before testing the model, I created the 'ray_trace_2d' function (line 368), which took rays, refracting surfaces and an output plane as arguments and plotted the paths of the rays and the refracting surfaces. The function also took three additional optional arguments which returned the spot diagram, value of the root mean square (RMS) radius and the z-position of the focal point.

TESTING THE MODEL

I initially tested the model with the specifications given in task 9 of the project script: lens curvature of 0.03 mm^{-1} and z-intercept of 100 mm. Figure 1 shows a ray parallel to and 5 mm above the optical axis propagating through the lens (line 165, ray_test). I then tested multiple rays passing through the same refracting surface as shown in figure 2. Using the 'ray_bundle_2d' function (line 85, ray_class), I made a list of rays with the same direction but travelling at an angle to the axis and propagated these through the same surface which is demonstrated in figure 3 (line 206, ray_test). I then tested a ray travelling parallel and 0.1 mm above the optical axis in order to determine the position of the paraxial focus as demonstrated in figure 4. From the optional argument 'focal_point' in ray_trace_2d, I determined that the position of the paraxial focus was 200 mm.

Overall, the tests above seemed to confirm that my code was working correctly; figure 2 and figure 3 demonstrated that parallel rays were converging to a point as expected for a spherical lens. The paraxial focus was determined to be 200 mm which agreed with the following equation for the theoritcal focal length, f, for a refracting surface:

$$f = \frac{R}{n-1}\,, \qquad (3)$$

R is the radius of the surface and n is the refractive index within the surface, assuming that the refractive index is 1 outside the surface. The equation determines that the focal length is 100 mm which results in a focal position at 200 mm, matching my results.

To confirm that the rays were refracting correctly, I performed one more test as specified in task 11 of the lab script. I checked if the spherical surface could form an image from a point source (line 219). The diagram of this test is demonstrated in figure 5. Rays coming from a point source converge after passing through the spherical surface. This is expected as spherical surfaces are supposed to be able to form images.

INVESTIGATIONS

As required for task 12, I made a uniform collimated beam in my ray_class module. The function that makes this beam is called 'collimated_beam' (line 104, ray_class) and is initialised with a centre point, radius, number of rings and direction. I tested the collimated beam of radius 5 mm travelling parallel to the axis as demonstrated in figure 6. For task 13, I used the optional arguments ‘spot’ and ‘rms’ which plotted a spot diagram at the output plane and calculated the RMS radius of the points on the spot diagram relative to the optical axis. Using the same 5 mm beam, refracting surface and output plane, I was able to plot a spot diagram at the paraxial focus as demonstrated in figure 7 (line 254, ray_test). I also moved the output plane closer to 50 mm which was between the start of the ray beam and the lens to see the spot diagram of the beam before it was refracted, which is shown in figure 8 (line 270, ray_test). This cross-section of the beam seemed to match that of the ones in the project script which reassured me that the beam was uniform. The RMS radius at the paraxial focus of this setup was calculated to be 0.0162 mm (3 s.f.). The equation for the diffraction scale, s, is:

$$s = \lambda \frac{f}{D}, \tag{4}$$

λ is wavelength of the light, f is the focal distance, and D is the aperture diameter. Using this equation when assuming that the rays are within the visible light spectrum (400 nm to 700 nm), the diffraction scale is determined to be between 0.008 mm and 0.0014 mm, but the RMS radius of 0.0162 mm is greater than these values, which means that the lens system is not diffraction limited within the visible light spectrum.

For task 15, I tested a collimated beam propagating through a plano-convex lens, comparing the case when the curved face is facing the incident rays and then when the flat face is facing the rays. The first surface had a curvature 0 while the second had a curvature of 0.02 mm^{-1}. The collimated beam had a radius of 5 mm. Initially, I used a ray close to the paraxial axis to find the paraxial focus position, which was 196.75 mm (line 314, ray_test). Then I moved the output plane to this position and propagated a 5 mm radius beam through the plano-convex lens as demonstrated in figure 9 (line 329). The RMS radius for this configuration was 0.0333 mm. Then I flipped the lens such that the curved face was facing the rays and determined that the paraxial focus was now at 198.45 mm (line 360). I adjusted the position of the output plane and then repeated the same process. Figure 10 shows the diagram for beams propagating through this setup (line 382). The RMS radius of this setup was calculated to be 0.00836 mm which was 75% smaller than the first setup. Hence, my findings suggest that the plano-convex lens with the curved surface facing the incident beam results in a reduced RMS radius. This statement holds true for 10 mm, 2 mm and 0.1 mm beams too, as tested in my code (lines 300 to 408). The diffraction scale for a 5 mm beam, with a wavelength of 588 nm is 0.0116 mm, so the first lens system is not diffraction limited whereas the second configuration is diffraction limited for a wavelength of 588 nm, specified in the project script.

LENS OPTIMISATION

I used the ‘lens_optimisation.py’ module to contain the code that I used to optimise the curvatures of the lenses such that the RMS radius at the paraxial focus was minimised. The function that was used for optimisation was called ‘lens_optimise’ (line 20, lens_optimisation) and took parameters of the ray/lens system to optimise the curvatures of the lenses, using the scipy.optimize ‘minimize’ function (line 137). The ‘constraint’, ‘constraint1’ and ‘constraint2’ functions within lens_optimise are used to keep the focal position of the lens system within a specified range (lines 98 to 132). I set the initial guess for the lens curvatures equal to the curvatures for a plano-convex lens (line 65). I then optimised the plano-convex lens in task 15 (line 410, ray_test) to find the curvatures and RMS spread of the optimised lens system. According to my function, the optimal curvatures were 0.00423 mm^{-1} and -0.000633 mm^{-1} as shown in figure 11. I repeated these tests for different beam radii and compared between the plano-convex lens and the optimised lens with the results demonstrated in the table below:

Beam radius / mm	Plano-convex RMS radius / mm	Optimised RMS radius / mm	Percentage decrease in RMS radius
20	0.596	0.551	8.33
10	0.0683	0.0637	6.60
5	0.00836	0.00782	6.46
2	0.000531	0.000497	6.40
0.1	3.19×10^{-8}	3.16×10^{-8}	0.94
0.01	8.53×10^{-9}	8.46×10^{-9}	0.82

The table demonstrated that the optimised RMS radii were reduced by a greater proportion at larger beam radii. The contents of this table are plotted in figure 12 (line 455).

According to Thorlabs, a positive curvature for the first surface and a negative curvature for the second surface is optimal for reducing spherical aberration for collimated parallel beams [2]. So, for further testing, I decided to compare my optimising function to the Thorlabs LBF254-075-A lens which claimed to have a focal length of 75.0 mm, centre thickness of 5.0 mm and radii of curvature of 44.50 mm and -289.00 mm for the first and second surfaces respectively. I used my simulation to find the actual focal length which was 77.4 mm according to my code (line 496). Then I used my optimising function to find the optimal curvatures for this focal length (line 507). My results are demonstrated in the following table:

Lens	1st radius of curvature / mm	2nd radius of curvature / mm	RMS radius / mm
LBF254-075-A	44.5	-289.0	0.0127
Simulation	44.4	-294.9	0.0129

Although both lenses have similar radii of curvature, the Thorlabs lens still performs slightly better (line 503) which may be due to my optimising function choosing a local minimum for RMS radius rather than a global minimum. For further testing, I could have experimented with different initial guesses and different optimising functions, such as the scipy ‘fmin_tnc’ function which was recommended in the project script (rather than scipy ‘minimize’ function).

References

[1] *C Paterson, Imperial College London, “Project A: An Optical Ray Tracer”, Oct 2013,* [An Optical Ray Tracer, Project Script]

[2] *“Ø1" N-BK7 Best Form Spherical Lenses, AR Coating: 1050-1700 nm”, Thorlabs, n.d.,* [Thorlabs Best Form Lens]

GRAPHS

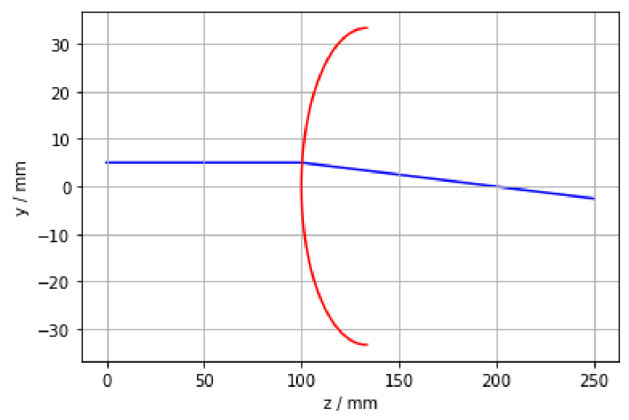

Figure 1: Ray propagating through spherical surface, 5mm above the optical axis.

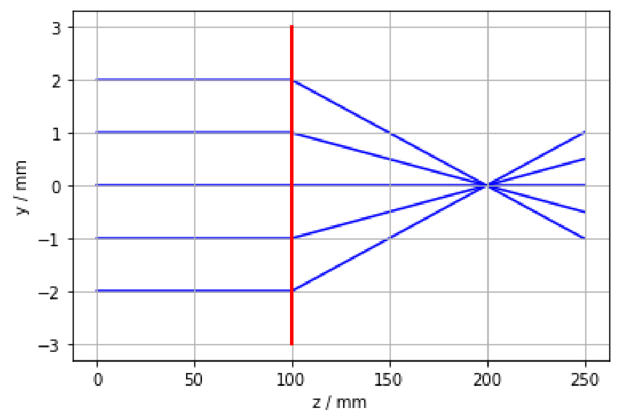

Figure 2: Multiple rays propagating through the spherical lens.

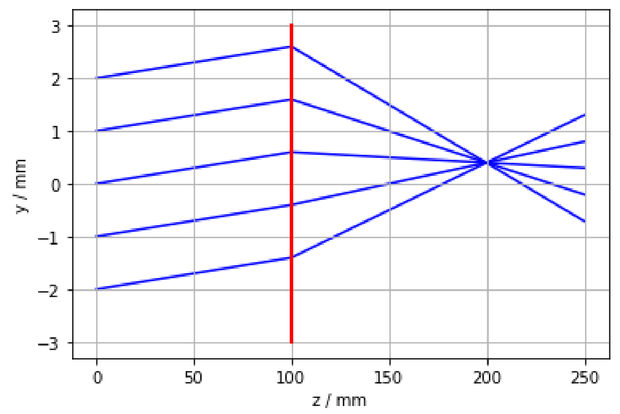

Figure 3: Rays approaching at an angle to the optical axis.

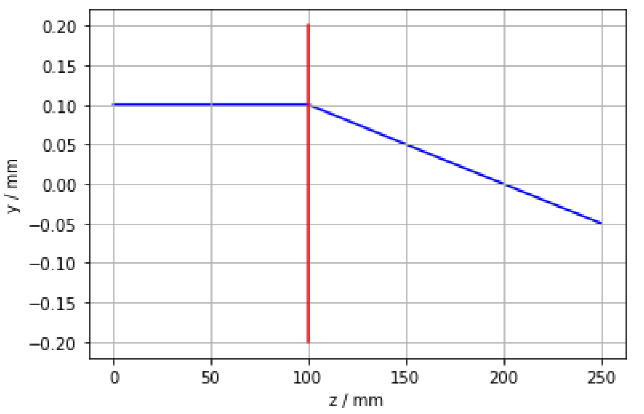

Figure 4: Ray used to find the position of the paraxial focus.

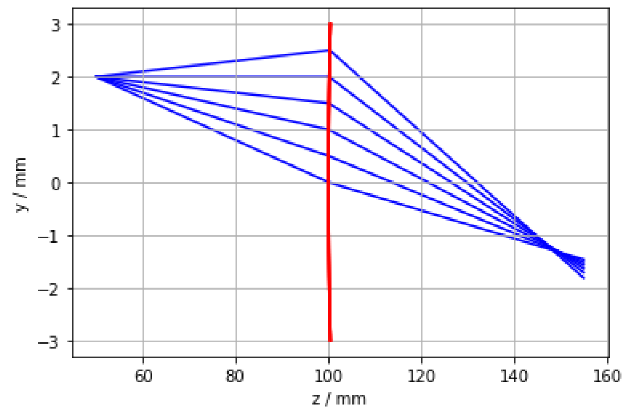

Figure 5: Rays travelling from a point converging to form an image.

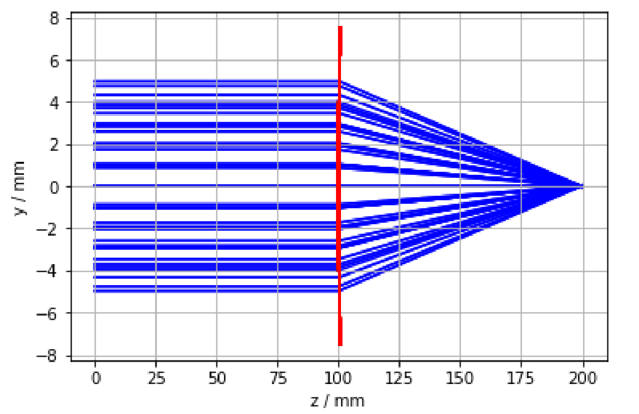

Figure 6: Ray beam with radius 5 mm converging to a point.

- More graphs on the next page.

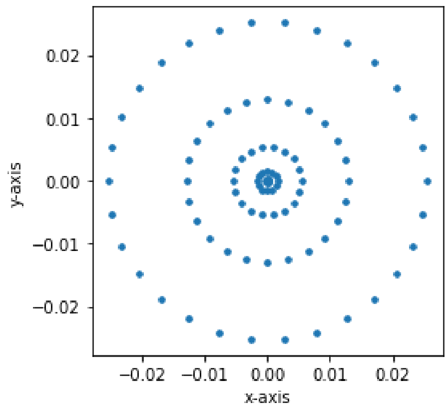

Figure 7: Spot diagram for the beam in figure 6.

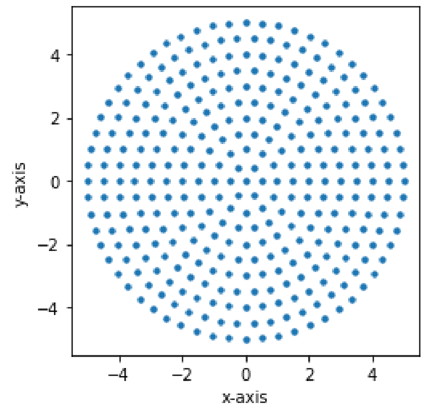

Figure 8: Cross-section of ray beam before propogating through the lens.

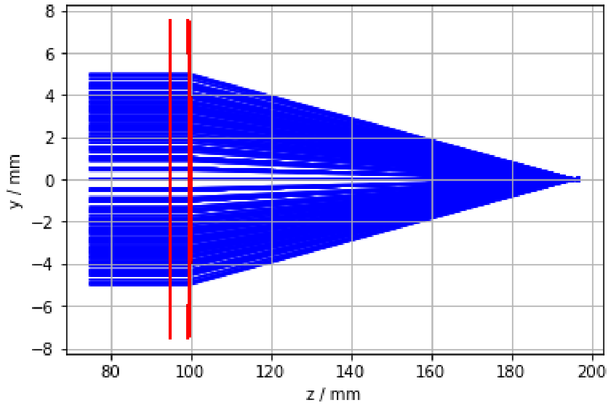

Figure 9: Plano-convex lens with flat surface facing incident beams.

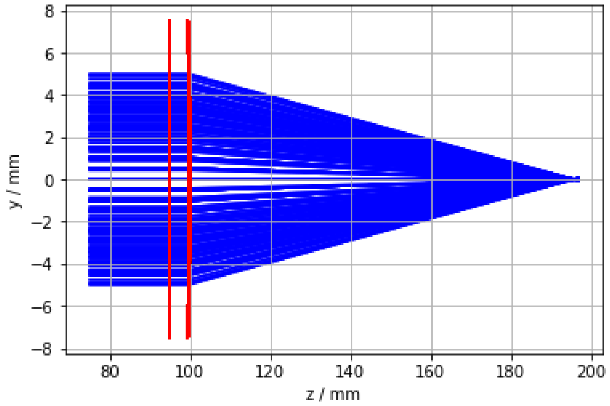

Figure 10: Plano-convex with curved surface facing incident beams.

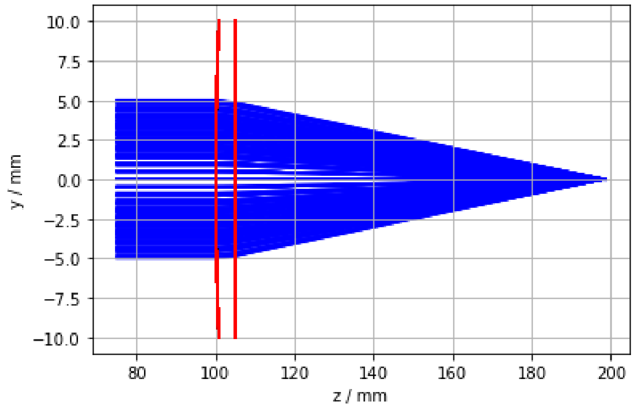

Figure 11: Ray beam passing through optimised lens configuration.

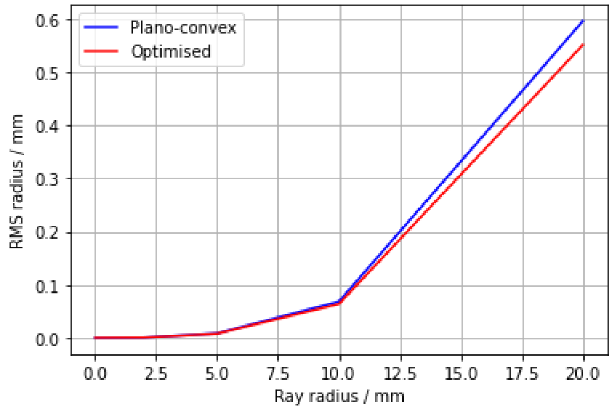

Figure 12: Comparing RMS radius between the plano-convex and optimised lens configurations.