9. Appendix I

All equations preceding Eq. 14 are referred to from the main draft. We will use the following identity in the below derivations and we use $d = N_w$.

$$\int_{\mathbf{x}} e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}+\mathbf{b}^T\mathbf{x}} \mathrm{d}^n\mathbf{x} = \sqrt{\frac{(2\pi)^n}{det(A)}} e^{\frac{1}{2}\mathbf{b}^T A^{-1}\mathbf{b}} \tag{14}$$

$\mathrm{d}^n\mathbf{x}$ is the n-dimensional volume differential.

9.1. Sampling distributions

Let $d = N_w$.

$$\begin{aligned} p(b_m|\mathbf{\Theta}\backslash\{a_m, b_m\},\mathbf{r}) &= \frac{\int_{a_m} p(\mathbf{\Theta},\mathbf{r})\mathrm{d}a_m}{\int_{a_m}\sum_{b_m} p(\mathbf{\Theta},\mathbf{r})\mathrm{d}a_m} \\ &= \frac{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})\mathrm{d}a_m}{\int_{a_m}\sum_{b_m} p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})\mathrm{d}a_m} \\ &= \frac{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(\mathbf{c})\mathrm{d}a_m}{\int_{a_m}\sum_{b_m} p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(\mathbf{c})\mathrm{d}a_m} \end{aligned}$$

canceling terms that are independent of a_m and b_m and compute $p(b_m = 1|\mathbf{\Theta}\backslash\{a_m, b_m\}, \mathbf{r})$

$$p(b_m = 1|\mathbf{\Theta}\backslash\{a_m, b_m\}, \mathbf{r}) = \frac{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m = 1)p(b_m = 1)\mathrm{d}a_m}{\int_{a_m}\sum_{b_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m)p(b_m)\mathrm{d}a_m} \tag{15}$$

$$= \frac{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m = 1)p(b_m = 1)\mathrm{d}a_m}{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m = 1)p(b_m = 1)\mathrm{d}a_m + \int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m = 0)p(b_m = 0)\mathrm{d}a_m} \tag{16}$$

$$p(b_m = 1|\mathbf{\Theta}\backslash\{a_m, b_m\}, \mathbf{r}) = \frac{NUM_1}{NUM_1 + DEN_1} \tag{17}$$

where

$$NUM_1 = \int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m = 1)p(b_m = 1)\mathrm{d}a_m \tag{18}$$

Let $A = diag(\mathbf{a})$, where $diag(\mathbf{a})$ indicates the constructing diagonal matrix with the elements from the vector $\mathbf{a}$. From eq. 4 and eq. 2.

$$NUM_1 = \lambda\int_{a_m} \frac{1}{\sqrt{(2\pi(\sigma^2)^d)}} e^{\frac{(\mathbf{r}-A\mathbf{b})^T(\mathbf{r}-A\mathbf{b})}{2\sigma^2}} \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_a} e^{-\frac{a_m^2}{2\sigma_a^2}} \mathrm{d}a_m$$

substituting $b_m = 1$ and ignoring the terms that are independent of a_m and b_m.

$$= \frac{\lambda C_1}{(2\pi)^{\frac{d+1}{2}}(\sigma^2)^{\frac{d}{2}}\sigma_a}\int_{a_m} e^{-\left[-\frac{1}{2}d_m a_m + \frac{1}{2}\left(\frac{1}{\sigma_a^2}+\frac{1}{\sigma^2}\right)a_m^2\right]}\mathrm{d}a_m$$

where $f = \{\frac{1}{\sigma^2} + \frac{1}{\sigma_a^2}\}$ and C_1 is constant. From eq. 14,

$$= \frac{\lambda C_1}{(2\pi)^{\frac{d+1}{2}}\sigma_a(\sigma^2)^{\frac{d}{2}}}\sqrt{\frac{2\pi}{f}} e^{\left(\frac{d_m^2}{2f}\right)} \tag{19}$$

$b_m = 0$, DEN_1 become as follows

$$= \frac{C_1(1-\lambda)}{(2\pi)^{\frac{d+1}{2}}(\sigma^2)^{\frac{d}{2}}\sigma_a}\int_{a_m} e^{-\left(\frac{a_m^2}{2\sigma_a^2}\right)}\mathrm{d}a_m \tag{20}$$

$$= \frac{C_1(1-\lambda)}{(2\pi)^{\frac{d}{2}}(\sigma^2)^{\frac{d}{2}}}. \tag{21}$$

Using eq. 19 and 21, eq. 17 can be written as

$$p(b_m = 1|\mathbf{\Theta}\backslash\{a_m, b_m\}, \mathbf{r}) = \frac{\lambda_{1,m}}{\lambda_{1,m} + 1 - \lambda} \tag{22}$$

where

$$\lambda_{1,m} = \frac{\lambda}{\sigma_a}\sqrt{\frac{1}{f}} e^{\left(\frac{d_m^2}{2f}\right)}$$

$$\begin{aligned} p(a_m|\mathbf{\Theta}\setminus\{a_m\}, \mathbf{r}) &= \frac{p(\mathbf{\Theta},\mathbf{r})}{\int_{a_m} p(\mathbf{\Theta},\mathbf{r})\mathrm{d}a_m} = \frac{p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})}{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})\mathrm{d}a_m} \\ &= \frac{p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(c)}{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(c)\mathrm{d}a_m} \end{aligned}$$

canceling the terms independent of a_m.

$$= \frac{p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m)}{\int_{a_m} p(\mathbf{r}|\mathbf{\Theta})p(a_m|b_m)\mathrm{d}a_m} = \frac{NUM_2}{DEN_2}$$

from eq. 18, $DEN_2 = \frac{NUM_1}{p(b_m=1)}$

$$p(a_m|\mathbf{\Theta}\setminus\{a_m\}, \mathbf{r}) = \frac{\frac{1}{(2\pi)^{\frac{d+1}{2}}\sigma_a(\sigma^2)^{\frac{d}{2}}} e^{\frac{1}{2}\left[2d_m a_m - \left(\frac{1}{\sigma_a^2}+\frac{1}{\sigma^2}\right)a_m^2\right]}}{\frac{1}{(2\pi)^{\frac{d+1}{2}}\sigma_a(\sigma^2)^{\frac{d}{2}}}\sqrt{\frac{2\pi}{f}} e^{\left(\frac{d_m^2}{8f}\right)}} \tag{23}$$

by completing the square.

$$\sqrt{\frac{f}{2\pi}} e^{\frac{f}{2}\left[\frac{1}{f}d_m a_m - a_m^2 - \frac{(d_m)^2}{4f^2}\right]} \tag{24}$$

$$= \mathcal{N}\left(\frac{d_m}{f}, \frac{1}{f}\right) \tag{25}$$

$$\begin{aligned} p(\sigma^2|\mathbf{\Theta}\backslash\{\sigma^2\}, \mathbf{r}) &= \frac{p(\mathbf{r},\mathbf{\Theta})}{\int_{\sigma^2} p(\mathbf{r},\mathbf{\Theta})\mathrm{d}\sigma^2} = \frac{p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})}{\int_{\sigma^2} p(\mathbf{r}|\mathbf{\Theta})p(\mathbf{\Theta})\mathrm{d}\sigma^2} \\ &= \frac{p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(c)}{\int_{\sigma^2} p(\mathbf{r}|\mathbf{\Theta})\prod_k p(b_k)p(a_k|b_k)p(\sigma^2)p(c)\mathrm{d}\sigma^2} \end{aligned}$$

cancelling the terms that are independent of the σ^2

$$= \frac{p(\mathbf{r}|\mathbf{\Theta})p(\sigma^2)}{\int_{\sigma^2} p(\mathbf{r}|\mathbf{\Theta})p(\sigma^2)\mathrm{d}\sigma^2}$$

$$= \frac{\frac{\beta^\alpha(\sigma^2)^{-\alpha-1-\frac{d}{2}}}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}} e^{-\frac{\mathbf{d_3}^T\mathbf{d_3}}{2\sigma^2}-\frac{\beta}{\sigma^2}}}{\frac{\beta^\alpha}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}}\int_{\sigma^2}(\sigma^2)^{-\left(\alpha+\frac{d}{2}\right)-1} e^{-\frac{\frac{1}{2}(\mathbf{d_3}^T\mathbf{d_3})+\beta}{\sigma^2}}\mathrm{d}\sigma^2}$$

Let $\mathbf{d_3} = \mathbf{r} - A\mathbf{b}$

(26)

From the definition of the $\mathcal{IG}$ distribution, the denominator can be expressed as

$$\begin{aligned} &\frac{\beta^\alpha}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}}\int_{\sigma^2}(\sigma^2)^{-\left(\alpha+\frac{d}{2}\right)-1} e^{-\frac{\frac{1}{2}(\mathbf{d_3}^T\mathbf{d_3})+\beta}{\sigma^2}}\mathrm{d}\sigma^2 \\ &= \frac{\beta^\alpha}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}}\frac{\Gamma(\alpha+\frac{d}{2})}{\left\{\frac{1}{2}(\mathbf{d_3}^T\mathbf{d_3})+\beta\right\}^{\alpha+\frac{d}{2}}}. \end{aligned}$$

Substituting in eq. 26,

$$p(\sigma^2|\mathbf{\Theta}\backslash\{\sigma^2\},\mathbf{r}) = \frac{\frac{\beta^\alpha\left(\sigma^2\right)^{-\alpha-1-\frac{d}{2}}}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}}e^{-\frac{\mathbf{d_3}^T\mathbf{d_3}}{2\sigma^2}-\frac{\beta}{\sigma^2}}}{\frac{\beta^\alpha}{\Gamma(\alpha)(2\pi)^{\frac{d}{2}}}\frac{\Gamma(\alpha+\frac{d}{2})}{\left\{\frac{1}{2}(\mathbf{d_3}^T\mathbf{d_3})-\beta\right\}^{\alpha+\frac{d}{2}}}}$$

$$= \frac{\left\{\frac{1}{2}(\mathbf{d_3}^T\mathbf{d_3})-\beta\right\}^{\alpha+\frac{d}{2}}\left(\sigma^2\right)^{-\left(\alpha+\frac{d}{2}\right)-1}e^{-\frac{\frac{1}{2}\mathbf{d_3}^T\mathbf{d_3}+\beta}{2\sigma^2}}}{\Gamma(\alpha+\frac{d}{2})}$$

$$= \mathcal{IG}(\alpha+\frac{d}{2},\frac{1}{2}\mathbf{d_3}^T\mathbf{d_3}+\beta)$$

10. Appendix II

The list of 29 voiced sentences is given below:

1. The game ended.
2. Bob banged the big board.
3. They wandered away.
4. Gold and diamond jewelery are worthy.
5. Gordan damaged the radio.
6. Granddad bid goodbye.
7. Joe grabbed Ginger.
8. Dad borrowed money.
9. Ian ironed the denim robe.
10. Maggie gave a monologue
11. On average women live longer than men.
12. Enter the grand ballroom.
13. Where are you going?
14. Bobby ran away.
15. The delivery boy died.
16. Download the movie now.
17. They drew the deer.
18. Earl dreaded the geology exam.
19. Emma and George are engaged.
20. The engine roared.
21. Learn grammar well.
22. Imagine you’re idle.
23. Lory made lemonade.
24. Manage your anger.
25. Model the red building.
26. Gabriel endured the boring drama.
27. Design your own wardrobe.
28. Abandon the rear wheel.
29. Unload the gun.