Indian Institute of Technology Kanpur
CS771 Introduction to Machine Learning, 2017-18-a

Assignment Number: 1
Student Name: Talla Aravind Reddy
Roll Number: 14746
Date: September 10, 2017

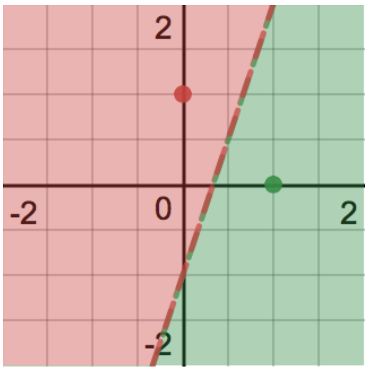
QUESTION
1

Part 1

Let $z = (x_1, y_1)$. We are given $z_r = (0, 1)$ and $z_g = (1, 0)$.
For the decision boundary, $d(z, z_r) = d(z, z_g)$
$\implies \langle z - z_r, U(z - z_r)\rangle = \langle z - z_g, U(z - z_g)\rangle$ where $U = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
$\implies 3x_1^2 + (y_1 - 1)^2 = 3(x_1 - 1)^2 + y_1^2$
$\implies 3x_1^2 - 3(x_1 - 1)^2 = y_1^2 - (y_1 - 1)^2$
$\implies 6x_1 - 3 = 2y_1 - 1$
$\implies 3x_1 = y_1 + 1$
$\implies 3x = y + 1$ is the decision boundary.

Part 2

For the decision boundary, $d(z, z_r) = d(z, z_g)$
$\implies \langle z - z_r, U(z - z_r)\rangle = \langle z - z_g, U(z - z_g)\rangle$ where $U = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
$\implies x_1^2 = (x_1 - 1)^2$
$\implies 2x_1 - 1 = 0$
$\implies x_1 = \frac{1}{2}$
$\implies x = \frac{1}{2}$ is the decision boundary.



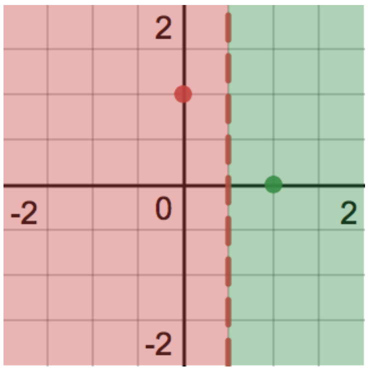


Figure 1: The figure on the left shows the decision boundary for part 1 i.e $3x = y + 1$.
The figure on the right shows the decision boundary for part 2 i.e $x = \frac{1}{2}$.

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QUESTION 2

One likelihood distribution which leads to $\hat{\mathbf{w}}_{cls}$ as the MAP estimate for the model is the gaussian distribution with mean as $\langle \mathbf{w}, \mathbf{x}^i \rangle$

$$\mathbb{P}[y|\mathbf{x}^i, w] = \mathcal{N}(\langle \mathbf{w}, \mathbf{x}^i \rangle, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(y - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2}{2\sigma^2} \right)$$

The prior distribution encapsulates the fact that $||\mathbf{w}||_2 \leq r$. This can be achieved by a uniform distribution over the d-dimensional ball with origin as centre and radius r.

$$\mathbb{P}[\mathbf{w}] = \mathcal{U}(\mathbf{0}, r) = \begin{cases} \dfrac{\Gamma(\frac{d}{2}+1)}{\pi^{\frac{d}{2}} r^d} & \text{if } ||\mathbf{w}||_2 \leq r \\ 0 & \text{if } ||\mathbf{w}||_2 > r \end{cases}$$

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QUESTION 3

The likelihood part of the estimate is same as the previous question, therefore one likelihood distribution is the gaussian distribution with mean as $\langle \mathbf{w}, \mathbf{x}^i \rangle$

$$\mathbb{P}[y|\mathbf{x}^i, w] = \mathcal{N}(\langle \mathbf{w}, \mathbf{x}^i \rangle, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(y - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2}{2\sigma^2} \right)$$

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QUESTION 4

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