Indian Institute of Technology Kanpur
CS771 Introduction to Machine Learning, 2017-18-a

QUESTION 1

Assignment Number: 1
Student Name: Talla Aravind Reddy
Roll Number: 14746
Date: September 10, 2017

Part 1

Let $z = (x_1, y_1)$. We are given $z_r = (0, 1)$ and $z_g = (1, 0)$.
For the decision boundary, $d(z, z_r) = d(z, z_g)$

$\implies \langle z - z_r, U(z - z_r)\rangle = \langle z - z_g, U(z - z_g)\rangle$ where $U = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$\implies 3x_1^2 + (y_1 - 1)^2 = 3(x_1 - 1)^2 + y_1^2$

$\implies 3x_1^2 - 3(x_1 - 1)^2 = y_1^2 - (y_1 - 1)^2$

$\implies 6x_1 - 3 = 2y_1 - 1$

$\implies 3x_1 = y_1 + 1$

$\implies 3x = y + 1$ is the decision boundary.

Part 2

For the decision boundary, $d(z, z_r) = d(z, z_g)$

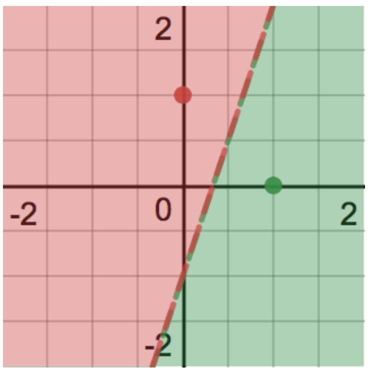
$\implies \langle z - z_r, U(z - z_r)\rangle = \langle z - z_g, U(z - z_g)\rangle$ where $U = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\implies x_1^2 = (x_1 - 1)^2$

$\implies 2x_1 - 1 = 0$

$\implies x_1 = \frac{1}{2}$

$\implies x = \frac{1}{2}$ is the decision boundary.



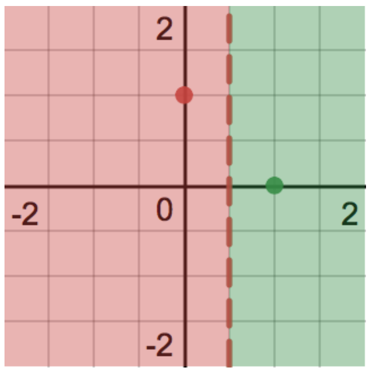


Figure 1: The figure on the left shows the decision boundary for part 1 i.e $3x = y + 1$.
The figure on the right shows the decision boundary for part 2 i.e $x = \frac{1}{2}$.

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QUESTION 2

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One likelihood distribution which leads to $\hat{\mathbf{w}}_{cls}$ as the MAP estimate for the model is the gaussian distribution with mean as $\langle \mathbf{w}, \mathbf{x}^i \rangle$

$$\mathbb{P}[y|\mathbf{x}^i, w] = \mathcal{N}(\langle \mathbf{w}, \mathbf{x}^i \rangle, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2}{2\sigma^2} \right)$$

The prior distribution encapsulates the fact that $||\mathbf{w}||_2 \leq r$. This can be achieved by a uniform distribution over the d-dimensional ball with origin as centre and radius r.

$$\mathbb{P}[\mathbf{w}] = \mathcal{U}(\mathbf{0}, r) = \begin{cases} \dfrac{\Gamma(\frac{d}{2} + 1)}{\pi^{\frac{d}{2}} r^d} & \text{if } ||\mathbf{w}||_2 \leq r \\ 0 & \text{if } ||\mathbf{w}||_2 > r \end{cases}$$

QUESTION

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The likelihood part of the estimate is same as the previous question, therefore one likelihood distribution is the gaussian distribution with mean as $\langle \mathbf{w}, \mathbf{x}^i \rangle$

$$\mathbb{P}[y|\mathbf{x}^i, w] = \mathcal{N}(\langle \mathbf{w}, \mathbf{x}^i \rangle, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\bigg(- \frac{(y - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2}{2\sigma^2} \bigg)$$

For the prior, we can take a multivariate gaussian distribution with mean at $\mathbf{0}$

$$\mathbb{P}[\mathbf{w}] = \mathcal{N}(\mathbf{w}; \mathbf{0}, \Sigma) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \exp\bigg(- \frac{1}{2} \mathbf{w}^T \Sigma^{-1} \mathbf{w} \bigg)$$

Here $\Sigma = \sigma^2 \begin{bmatrix} \frac{1}{\alpha_1} & 0 & \dots & \dots & 0 \\ 0 & \frac{1}{\alpha_2} & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0\dots & 0 \\ \vdots & \vdots & \vdots & \frac{1}{\alpha_{d-1}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\alpha_d} \end{bmatrix}$ i.e a diagonal matrix with it's i'th diagonal entry as $\frac{\sigma^2}{\alpha_i}$ where σ^2 is the variance of the likelihood distribution.

$$\log \mathbb{P}[\mathbf{w}|\mathbf{X}, \mathbf{y}] = C - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2 - \frac{1}{2\sigma^2} \sum_{j=1}^{d} \alpha_j \mathbf{w}_j^2$$

$$\implies \hat{\mathbf{w}}_{\text{MAP}} = \arg\min \sum_{i=1}^{n} (y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2 + \sum_{j=1}^{d} \alpha_j \mathbf{w}_j^2$$

Closed form expression:

We need to minimise

$$\mathcal{L} = \sum_{i=1}^{n} (y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2 + \sum_{j=1}^{d} \alpha_j \mathbf{w}_j^2$$

$$\mathcal{L} = ||\mathbf{X}\mathbf{w} - \mathbf{Y}||^2 + \sum_{j=1}^{d} \alpha_j \mathbf{w}_j^2$$

$\nabla_w \mathcal{L} = 2\mathbf{X}^T(\mathbf{X}\mathbf{w} - \mathbf{Y}) + 2\mathbf{D}_\alpha \mathbf{w}$ where D_α is the diagonal matrix with entries $\alpha_1, \alpha_2 \dots \alpha_d$

$$\nabla_w \mathcal{L} = 2((\mathbf{X^T X} + \mathbf{D}_\alpha)\mathbf{w} - \mathbf{X^T Y})$$

$$\nabla_w \mathcal{L} = 0$$

$$\iff (\mathbf{X^T X} + \mathbf{D}_\alpha)\mathbf{w} - \mathbf{X^T Y} = 0$$

$$\iff (\mathbf{X^T X} + \mathbf{D}_\alpha)\mathbf{w} = \mathbf{X^T Y}$$

$$\iff w = (\mathbf{X^T X} + \mathbf{D}_\alpha)^{\mathbf{-1}} \mathbf{X^T Y}$$

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QUESTION 4

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