13e CONGRÈS ANNUEL de la Société française de Recherche Opérationnelle et d'Aide à la Décision

Angers 11-12-13 avril 2012

ACTES

www.roadef2012.ima.uco.fr

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Probabilistic delay constrained shortest-path problem

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keywords: probabilistic constraint, shortest path, stochastic programming.

In this work, we introduce a new model for a resource (delay) constrained shortest-path (RCSP) problem [3, 4]. Our model intends to overcome the fact that, for some practical applications (e.g. routing in transportation networks), dealing with deterministic delays on network arcs may lead to solutions (paths) that, sometimes, violate in practice the required maximum delay constraint due to the increasing uncertain character of the arc delays, incurring in high penalties to service providers.

The idea is to take into account the risk that the delay of a path violates the maximum allowed for any solution. So, we aim to find the shortest-path (e.g. in distance) such that the probability that it does not violate the delay constraint is larger than or equal to $1 - \alpha$, with α being the risk level service providers consider in their quality of service strategies. The idea behind this technique appeared recently in [1] for problems having a more complex structure of uncertainty.

Let $G = (V, A)$ be a directed network. Let $c \in \mathbb{R}_+^{|A|}$ be a vector of coordinates c_{uv} representing the cost of arc $uv \in A$. Let ξ be a random parameter vector of components d_{uv}^{ξ}, for all arc $uv \in A$, representing the set of uncertain arc delays and assume we know its probability distribution function.

The problem is to find a minimum cost $(s-t)$-path in G from an origin node $s \in V$ to a destination node $t \in V$, where the probability that the path delay (the sum of the delays of each arc in the $(s-t)$-path) exceeds a maximum value $D \in \mathbb{N}^*$ is limited above by the risk parameter α. This problem is $\mathcal{NP} - hard$.

In our model, a set of possible realizations of the random parameter ξ represents a scenario of arc delays and we assume that we have a finite set K of scenarios. We define binary variables $z_k \in \{0, 1\}$, $\forall k \in K$ to denote the possibility of a scenario k to violate the probabilistic delay constraint. Let $\mathcal{N}$ be the node-arc adjacency matrix of the graph G and $\mathcal{E}_{|K|\times|A|}$ be a matrix in which each component is equal to the delay of a given arc in a given scenario. We represent an $(s-t)$-path in G by a vector $x \in \{0, 1\}^{|A|}$, where $x_{uv} = 1$ if uv belongs to the $(s-t)$-path, and $x_{uv} = 0$ otherwise. Finally, we associate with each scenario a probability of occurrence denoted by ρ_k, $k \in K$, with $\sum\limits_{k\in K} \rho_k = 1$.

The mathematical model is

$$(P) \min_{\substack{x \in \{0,1\}^{|A|} \\ z \in \{0,1\}^{|K|}}} \quad \mathbf{c}^T x \tag{1}$$

$$\begin{aligned} s.t. \quad \mathcal{N}x &= \mathbf{b} && (2) \\ \mathcal{E}x &\leqslant D\mathbf{e} + \mathcal{M}z && (3) \\ \boldsymbol{\rho} z &\leqslant \alpha && (4) \end{aligned}$$

where $\mathbf{b}_s = 1$, $\mathbf{b}_t = -1$ and $\mathbf{b}_i = 0\ \forall i \in V \setminus \{s,t\}$; $\mathbf{e} \in \{1\}^{|K|}$; and $\mathcal{M}$ is a very large positive constant. Observe that to obtain a feasible solution to (P), related to a given $\overline{z} \in \{0,1\}^{|K|}$ feasible to (4), we need to solve the problem

$$
\begin{aligned}
(\overline{P})_{\overline{z}} \quad \min_{x\in\{0,1\}^{|A|}} \quad & \mathbf{c}x && (5)\\
s.a. \quad \mathcal{N}x \ & = \mathbf{b} && (6)\\
\mathcal{E}x \ & \leqslant D\mathbf{e} + \mathcal{M}\overline{z} && (7)
\end{aligned}
$$

Our idea is to decompose (P) using the solutions (u,v) of the dual problem of $(\overline{P})_{\overline{z}}$, with the dual variables u and v associated to the constraints (6) and (7), respectively. If a dual solution $(\bar{u},\bar{v})$ is limited (i.e. a vertex of the dual of $(\overline{P})_{\overline{z}}$, denoted by $Vertex(Dual((\overline{P})_{\overline{z}}))$, it defines an optimality cut; otherwise, $(\hat{u},\hat{v})$ is unlimited (i.e. a ray of the dual of $(\overline{P})_{\overline{z}}$, denoted by $Ray(Dual((\overline{P})_{\overline{z}}))$, and thus defines a feasibility cut in the resulting L-shaped decomposition scheme

$$
\begin{aligned}
(M) \quad \min_{z\in\{0,1\}^{|K|}} \quad & w && (8)\\
s.t. \quad 0 \ & \geqslant \hat{u}\mathbf{b} + \hat{v}(D\mathbf{e} + \mathcal{M}z), \quad \forall\, (\hat{u},\hat{v}) \in Ray(Dual((\overline{P})_{\overline{z}})) && (9)\\
w \ & \geqslant \bar{u} + \bar{v}\mathcal{M}(z - \bar{z}), \quad \forall\, (\bar{u},\bar{v}) \in Vertex(Dual((\overline{P})_{\overline{z}})) && (10)\\
\boldsymbol{\rho} z \ & \leqslant \alpha && (11)
\end{aligned}
$$

(M) is the master problem and $(\overline{P})_{\overline{z}}$ is the slave problem.

We show how to solve (M) using a Branch-and-Bound (B&B) framework, providing a new branching strategy for 0-1 z variables and discuss a valid global inequality to avoid enumerating B&B nodes that surely will not improve the incumbent solution value found in the B&B tree.

Numerical experiments show that we can obtain solutions of good quality very fast in the B&B algorithm. However, the quality of lower bounds on optimal solutions need to be improved. Thus, we discuss how to use Lagrange relaxation to solve a new relaxation [2] of the B&B nodes in a relax-and-cut framework.

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