Name Surname: Aras Güngöre

Student ID: 2018401117

**1)**

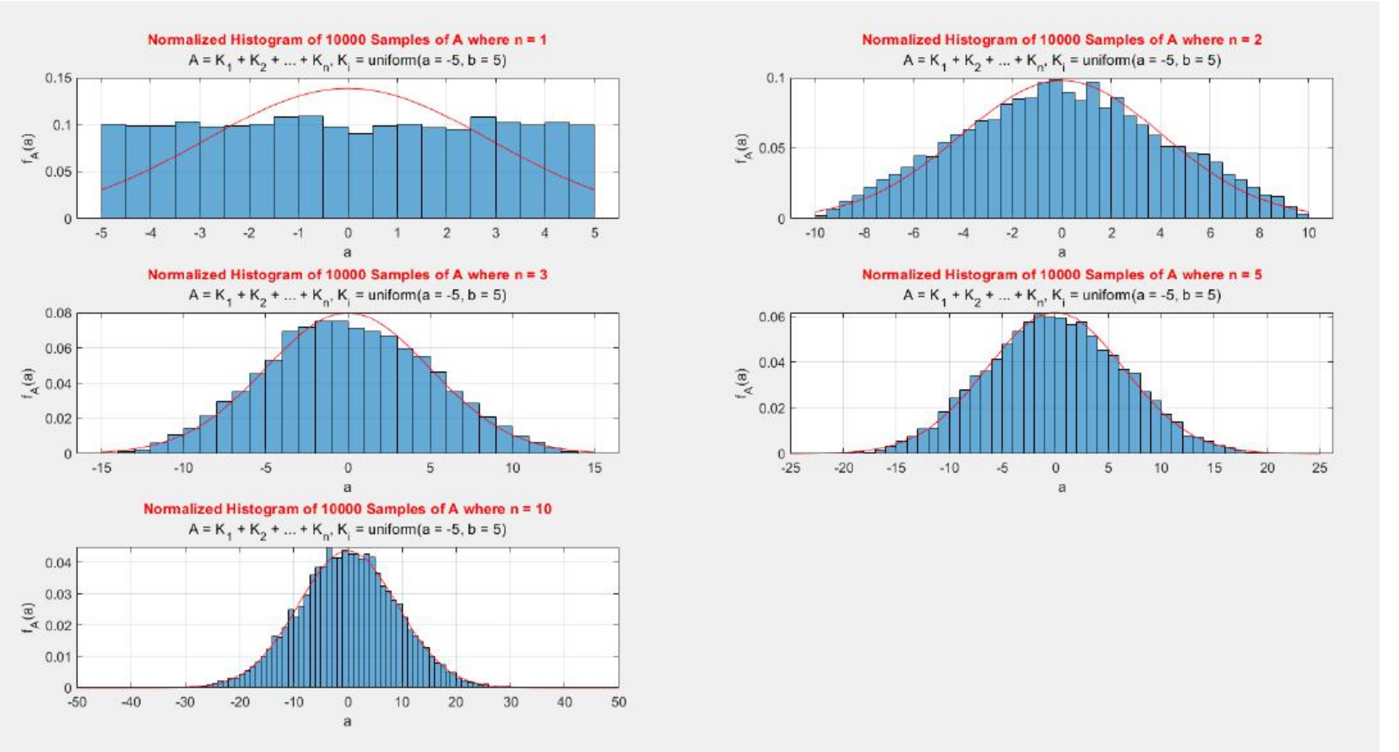
$$E[A] = E[K_1 + K_2 + \cdots + K_n] = E[K_1] + E[K_2] + \cdots + E[K_n] = E[K] + E[K] + \cdots + E[K]$$

$$=> E[A] = n \cdot E[K]$$

$$Var[A] = Var[K_1 + K_2 + \cdots + K_n] = Var[K_1] + Var[K_2] + \cdots + Var[K_n]$$
$$= Var[K] + Var[K] + \cdots + Var[K]$$

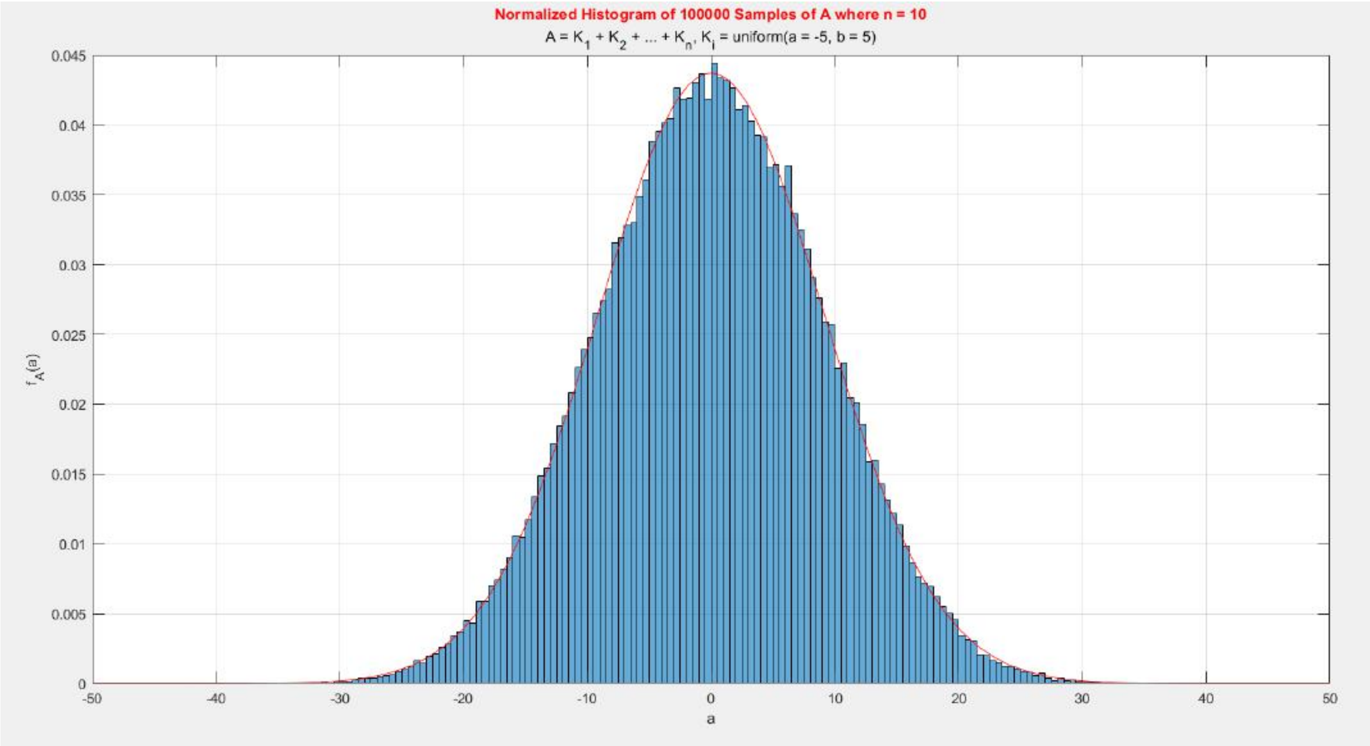
$$=> Var[A] = n \cdot Var[K]$$

**2.1)**



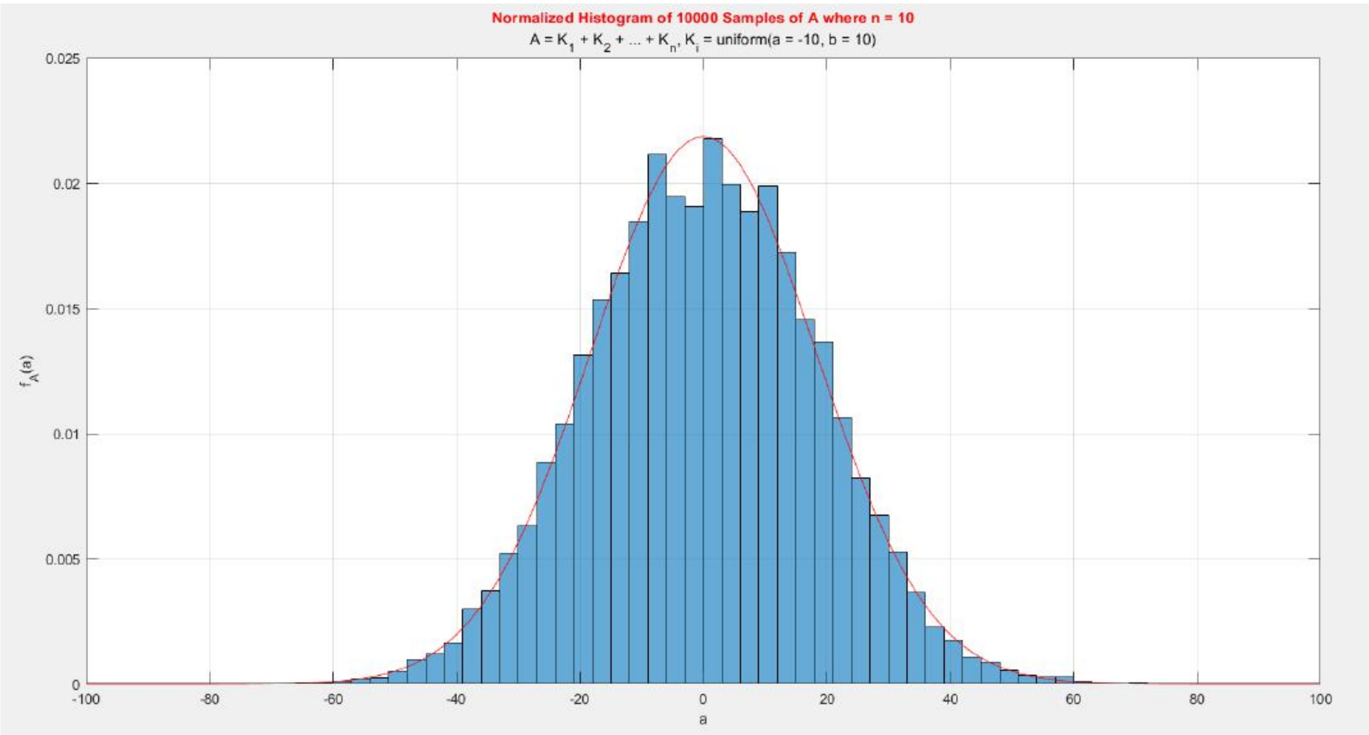

As n increases, the PDF of A $f_A(a)$ converges to Gaussian PDF according to Central Limit Theorem. For n = 2, the CLT approximation is off; however, for n ≥ 2 we can observe that the CLT approximations are pretty accurate. As n increases, the approximation becomes more and more accurate; and the graphs are extended on the x-axis since the variance is proportional to n.

**2.2)**




Increasing the sample size from 10000 to 100000 gives us a better defined Gaussian approximation.

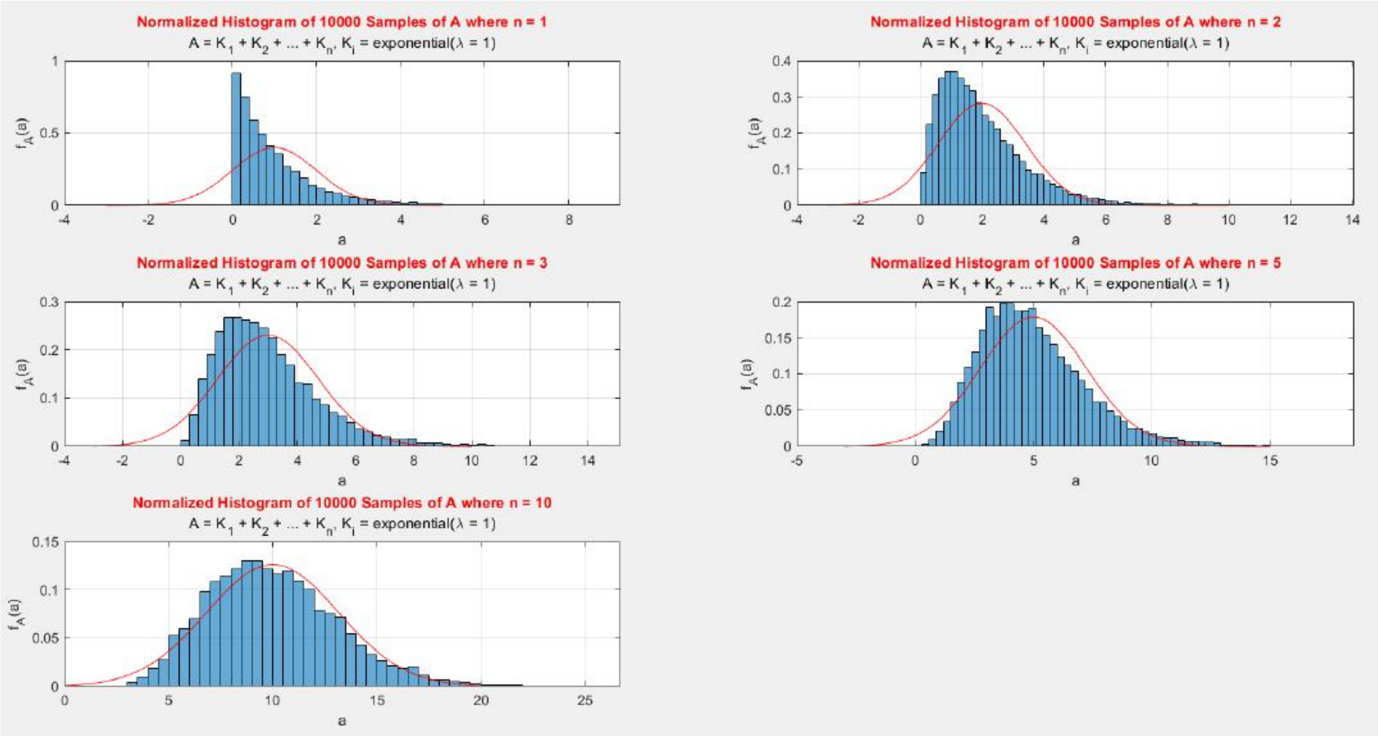
**2.3)**




By doubling the uniform range, standard deviation σ is doubled according to the formula:

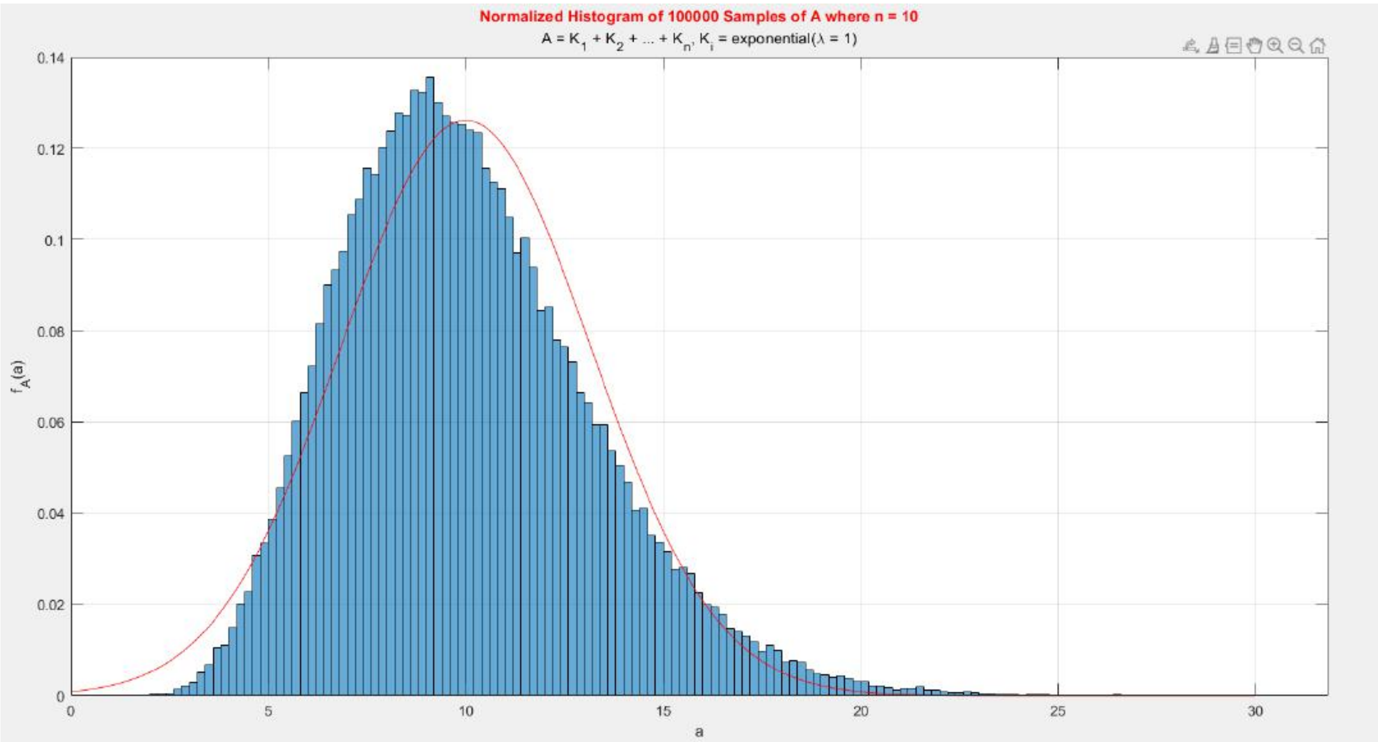
$Var[A] = \sigma_A^2 = n \cdot \frac{(b-a)^2}{12}$ . Consequently, the a values in the x-axis are doubled as well.

**3.1)**



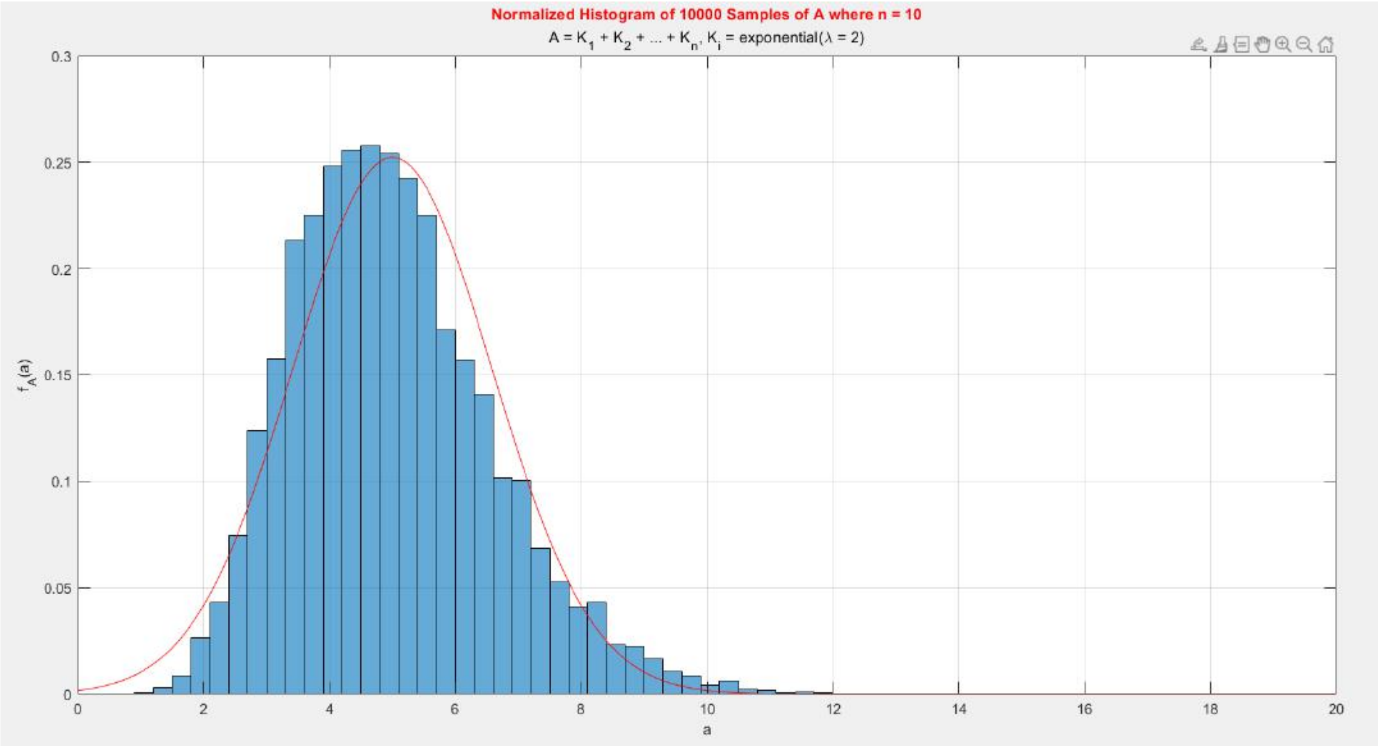

As n increases, the PDF of A $f_A(a)$ converges to Gaussian PDF according to Central Limit Theorem. For n = 2, the CLT approximation is not even close; even for n = 10 the CLT approximation is not accurate as the one in part 2.1. Unlike uniform PDF, exponential PDF doesn't get that close to Gaussian PDF for small values of n.

**3.2)**



Increasing the sample size from 10000 to 100000 doesn't give a much better Gaussian approximation, since the outline of the exponential PDF and the Gaussian PDF curves are still off.

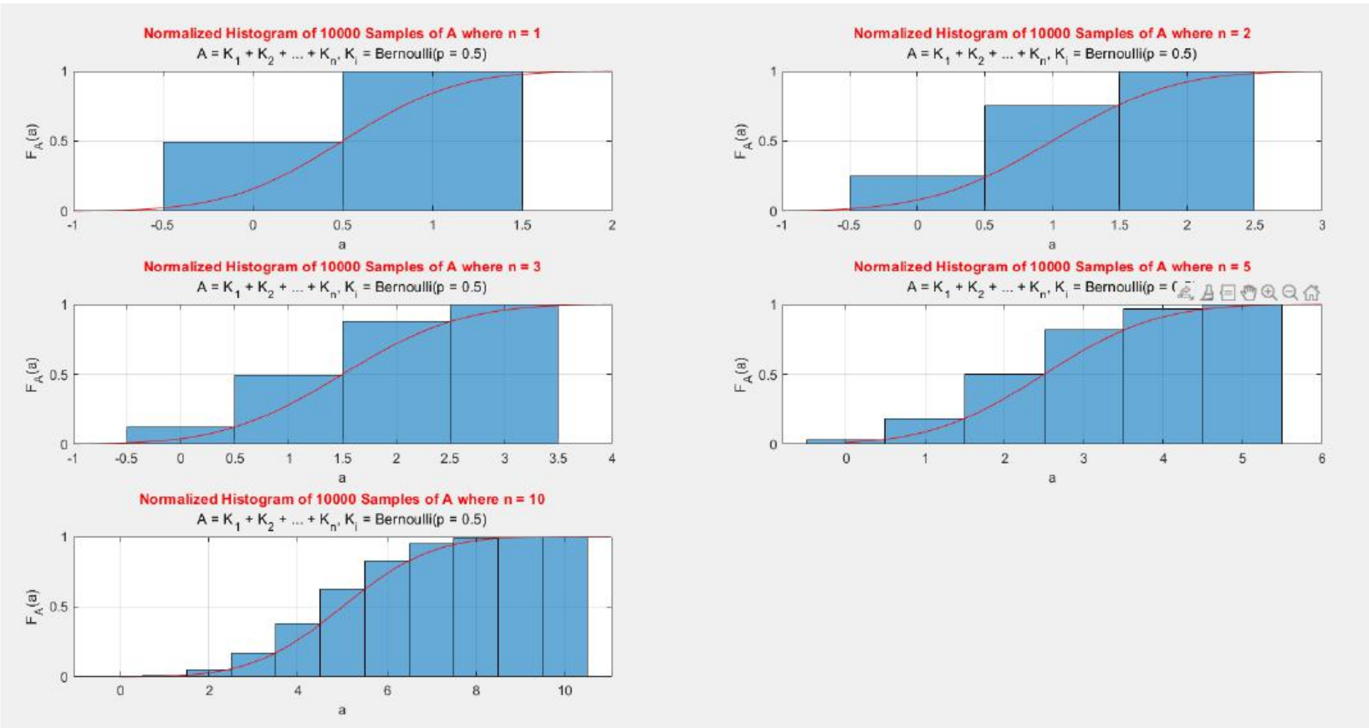
**3.3)**




By doubling the lambda, standard deviation σ is cut in half according to the formula:

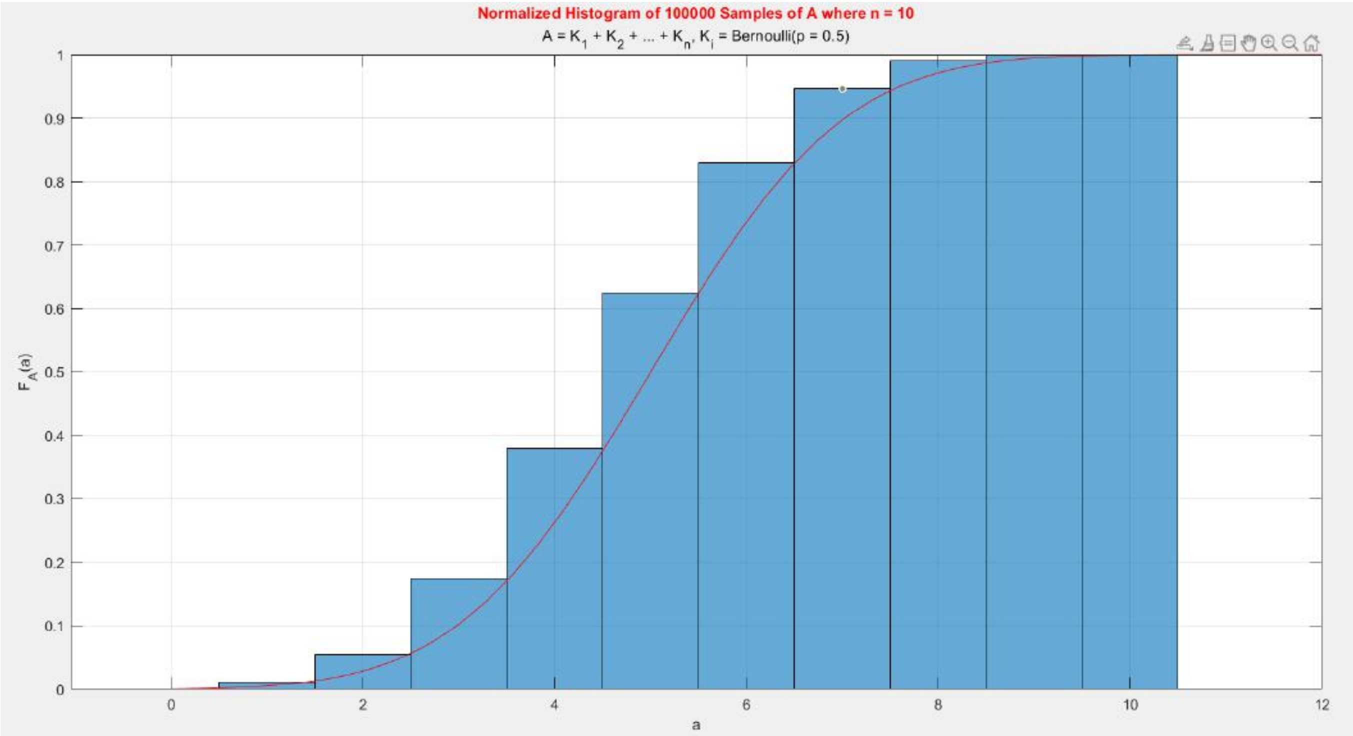
$Var[A] = \sigma_A^2 = n \cdot \frac{1}{\lambda^2}$. Consequently, the a values in the x-axis are cut in half as well.

**4.1)**



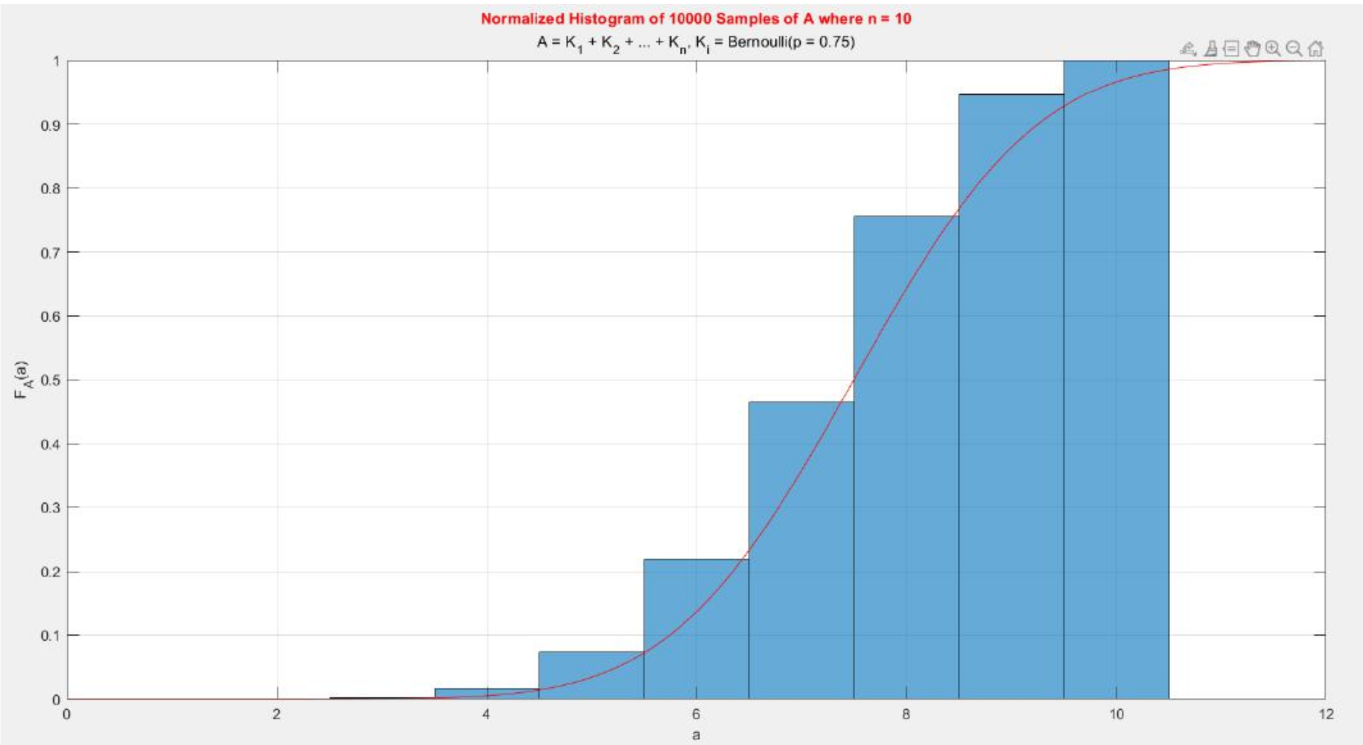
As n increases, the CDF of A $F_A(a)$ converges to Gaussian CDF according to Central Limit Theorem. For a = n, $F_A(a) = 1$; and for a = 0, $F_A(a)$ becomes equal to 0 as n increases. The right corners of the bars of the Bernoulli CDF are really close to the Gaussian CDF curve.

**4.2)**



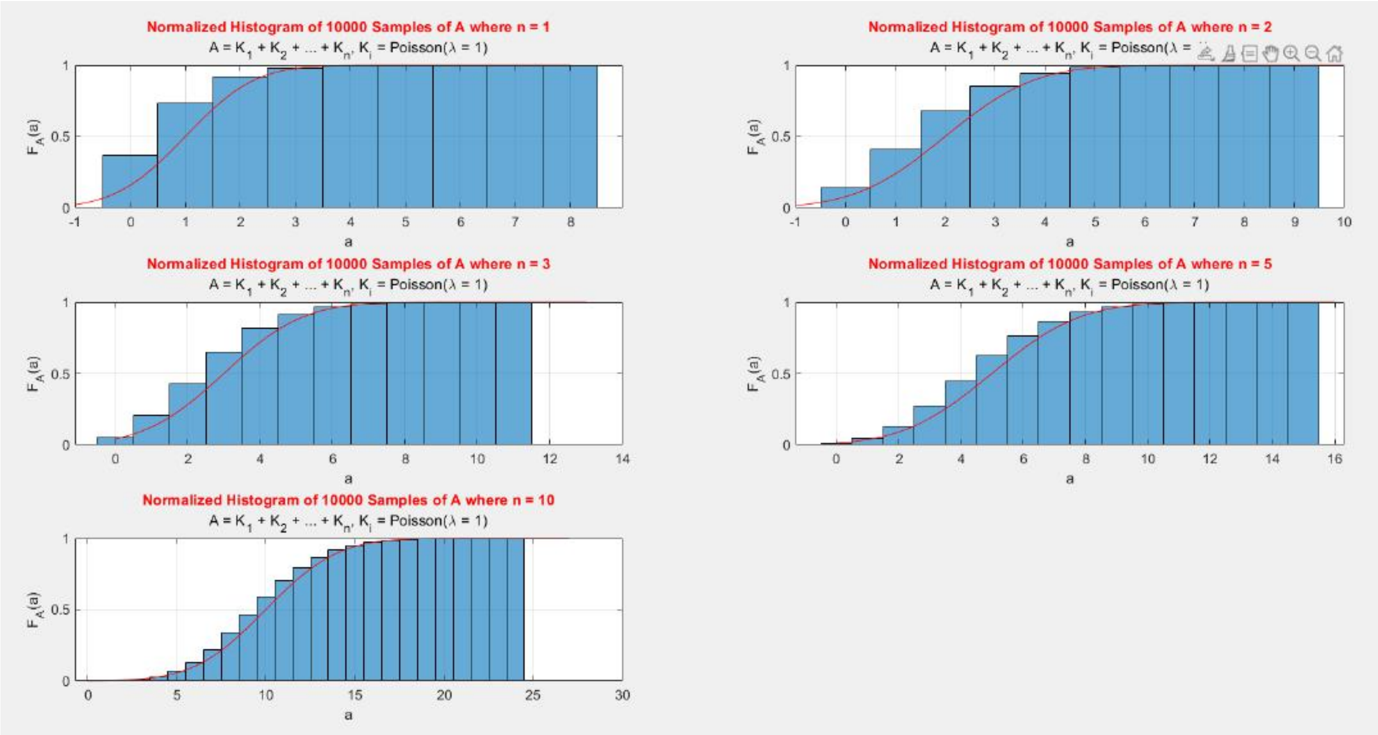

Increasing the sample size from 10000 to 100000 doesn't significantly change the graph, therefore it doesn't give a much better Gaussian approximation as the graph is also already close enough.

**4.3)**



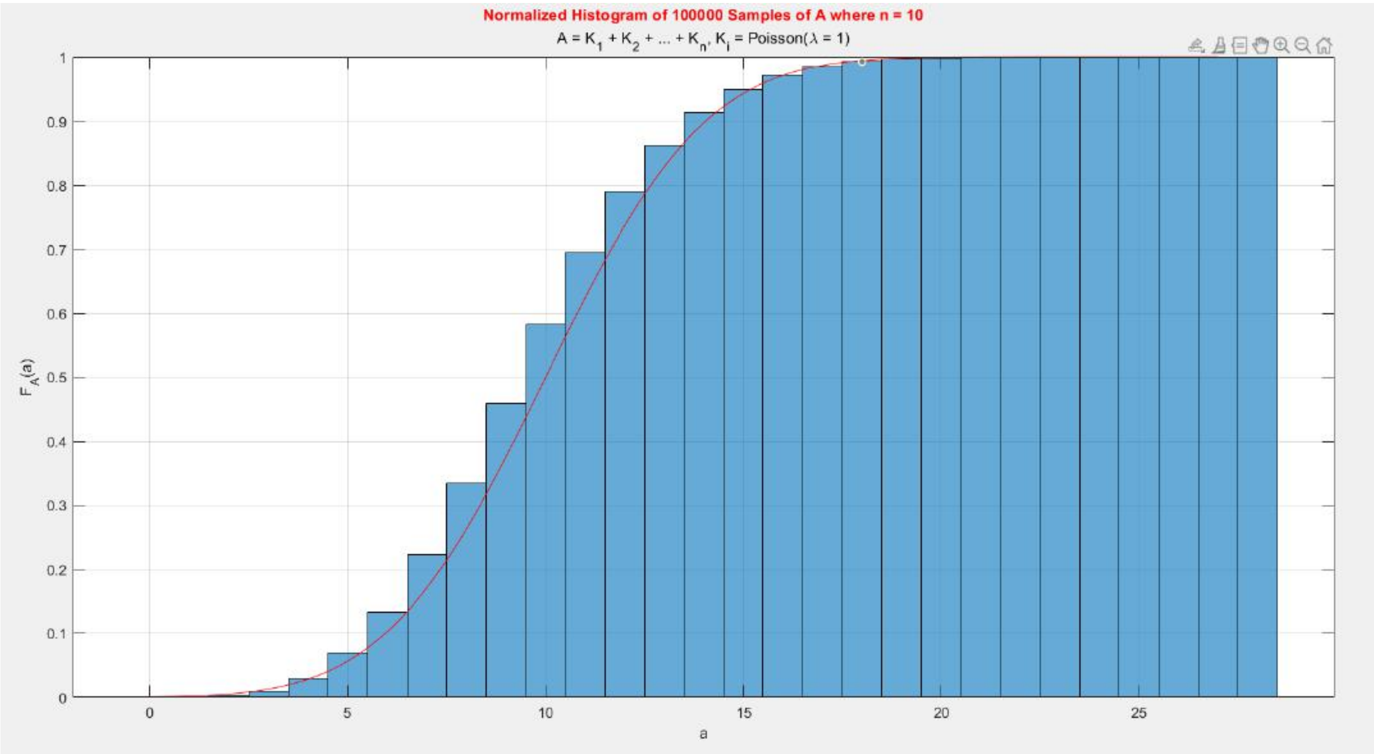
Increasing the p value from p = 0.5 to p = 0.75 favors the higher half of the a values so it increases the $F_A(a)$ for values n > 5, whereas it decreases $F_A(a)$ for values n < 5. Hence, the graph got steeper and the $F_A(a)$ for n < 4 is seemingly zero.

**5.1)**



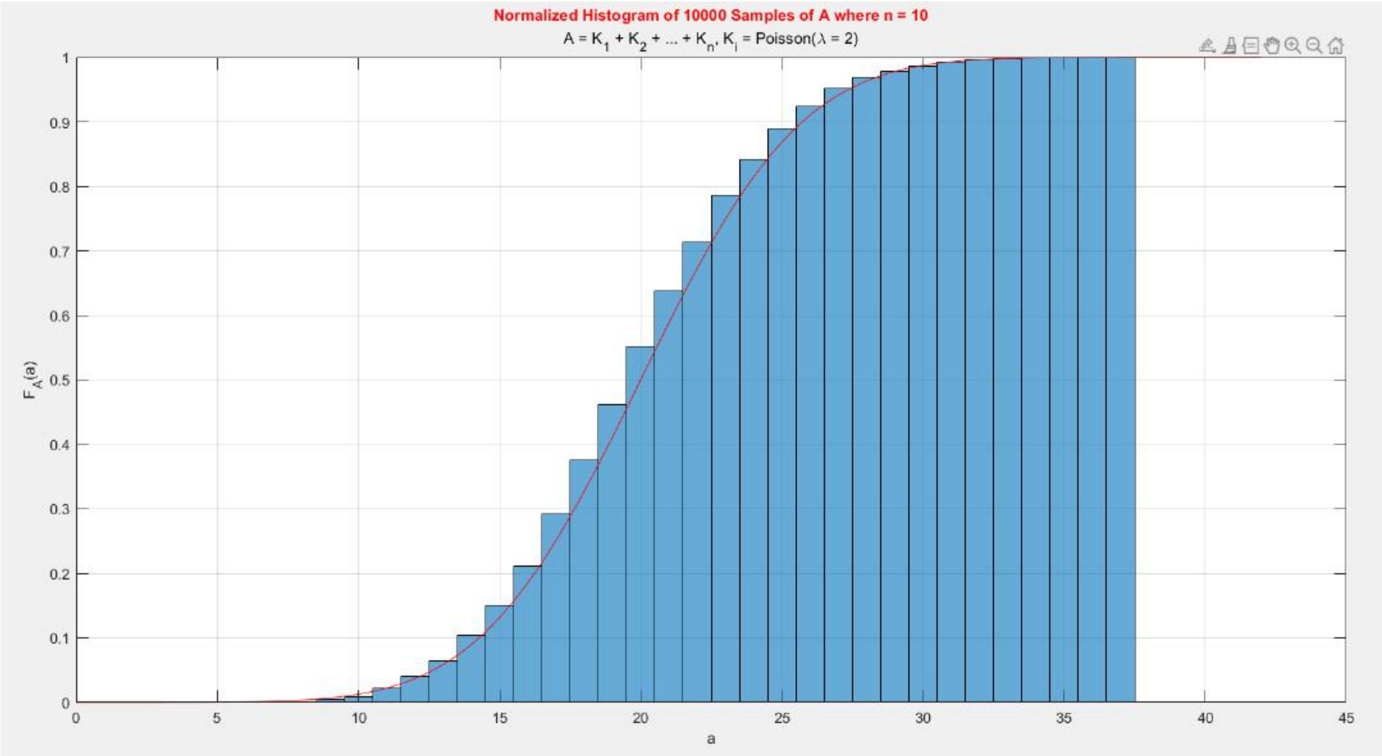

As n increases, the CDF of A $F_A(a)$ converges to Gaussian CDF according to Central Limit Theorem.

**5.2)**



Increasing the sample size from 10000 to 100000 gives us a better Gaussian approximation.

**5.3)**




Since $E[A] = Var[A] = \sigma_A^2 = n \cdot \lambda$, doubling the lambda means doubling the mean and the variance. Consequently, the graph is shifted to the right by an amount of n = 10 and is stretched on the x-axis by a factor that is equal to the increase in the standard deviation which is σ = √2 = 1.414.