BOGAZICI UNIVERSITY

ELECTRICAL & ELECTRONICS
ENGINEERING DEPARTMENT

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1 Introduction

The main purpose of the project is to analyze a 13.8 kV, 50 MVA, 0.9–power–factor–lagging, 60 Hz, four-pole Y–connected three-phase stand-alone synchronous generator and visualize the relations between its properties. One aspect analyzed in the project is how the field current I_F and power factor (PF) affect the operation of the given synchronous generator while we keep the terminal voltage V_T constant at rated conditions. Throughout the questions, we use limit circles and phasor diagrams to geometrically confirm our theoretical calculations since quiver plots are essential to visualize these dynamics.

2 Materials and Methods

As computing software, MATLAB 2022a was used to plot all the graphs and diagrams specified in the project, no extra packages were used. For phasor diagrams, the built-in function "quiver" was essential for plotting. Quiver's 'scale' property has been set to 0 so that phasors are adjacent in the phasor diagrams. The book "Electric Machinery Fundamentals" by Stephen J. Chapman was used for theoretical calculations and figures included in this report as well.

2.1 Per-phase Circuit: $\mathbf{E_A}$ and $\mathbf{I_A}$

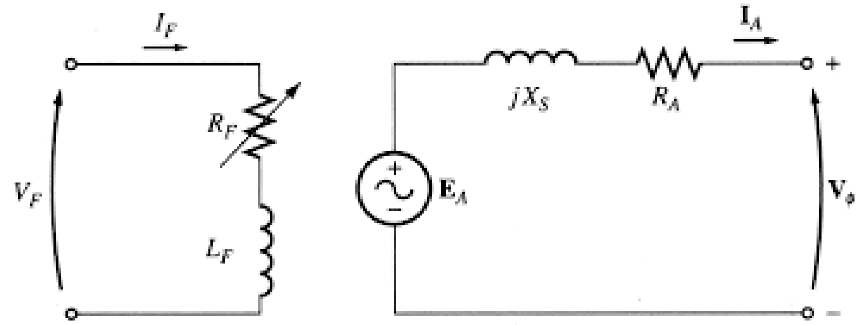

Figure 2.1: The per-phase equivalent circuit of a synchronous generator.

From the right-hand side of the circuit shown in figure 2.1, we derive the following equation:

$$\mathbf{E_A} = \mathbf{V_\Phi} + \mathbf{I_A} R_A + j\mathbf{I_A} X_S \tag{2.1.1}$$

Internal generated voltage $\mathbf{E_A}$, armature current $\mathbf{I_A}$ and output voltage $\mathbf{V_\Phi}$ is shown in bold format to indicate that they are phasors. Throughout the project, terminal voltage V_T is kept constant, so V_ϕ stays constant as well. The open-circuit terminal voltage $V_{T,OpenCircuit}$ can be calculated from the following equation given in the project:

$$V_{T,OpenCircuit}(I_F) = 20 \cdot \left(1.05 - e^{-(0.3 \cdot I_F)}\right) kV \tag{2.1.2}$$

which, as can be seen, is solely dependent on field current I_F.

2.2 Y-connected Circuit: V_T and V_ϕ

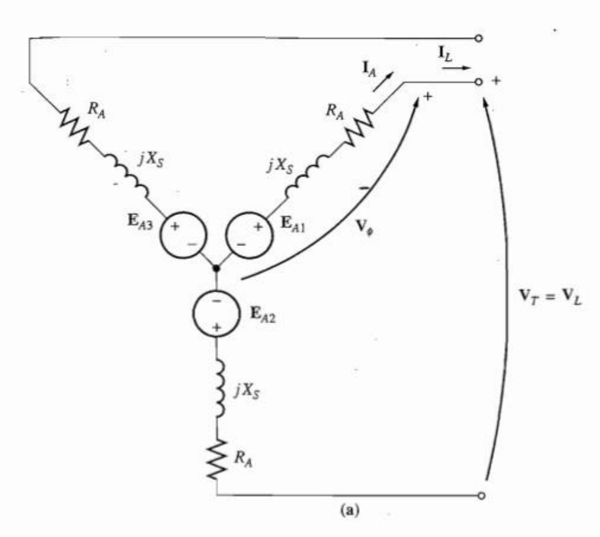

Figure 2.2: The generator equilavent circuit connected in Y.

Since the generator is Y-connected, from figure 2.2 we infer the equation between terminal voltage and the output voltage is:

$$V_T = \sqrt{3}V_\phi \tag{2.2}$$

From equation (2.2), we calculate the output voltage V_ϕ by plugging in the rated line voltage, which is given as 13.8 kV. Thus, $V_T = \sqrt{3}V_\phi = \sqrt{3} \cdot 13.8 = 23.9\ kV$.

2.3 Limit Circles: $|E_A|_{max}$ and $|I_A|_{max}$

$|E_A|_{max}$ is the maximum internal generated value which can be calculated by plugging in the given maximum field current $I_{F,max}$ = 10 A to the equation (2), as E_A is equal to $V_{T,OpenCircuit}$. We can calculate $|I_A|_{max}$ from the rated voltage $V_{T,rated}$ and rated apparent power S_{rated} as follows:

$$|I_A|_{max} = \frac{S_{rated}}{\sqrt{3} \cdot V_{T,rated}} \tag{2.3}$$

2.4 Calculation of $|E_A|$ from V_ϕ, I_A, R_A, X_S, and θ

By dividing equation (2.1.1) into real and imaginary parts, we respectively obtain:

$$E_A cos\delta = V_\phi + I_A R_A cos\theta - jI_A X_S sin\theta \tag{2.4.1}$$

$$E_A sin\delta = V_\phi + I_A R_A sin\theta + jI_A X_S cos\theta \tag{2.4.2}$$

in which δ and θ are phases of E_A and I_A respectively. Then, by taking the square of both sides of both equations and adding them, we get rid of δ and derive:

$$E_A^2 = (V_\phi + I_A R_A cos\theta - jI_A X_S sin\theta)^2 + (V_\phi + I_A R_A sin\theta + jI_A X_S cos\theta)^2 \quad (2.4.3)$$

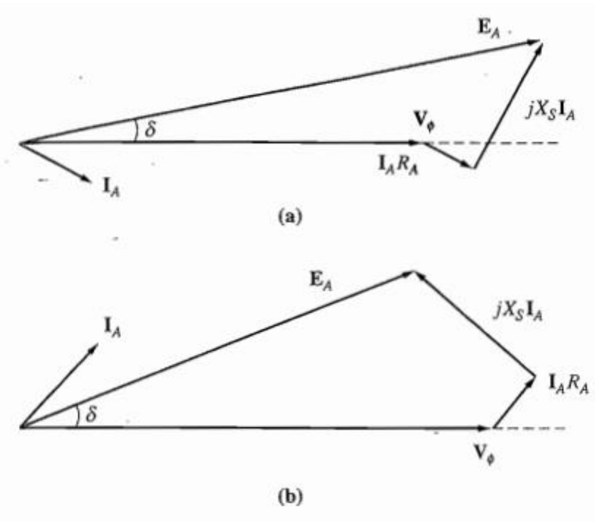

Figure 2.4: The phasor diagram of a synchronous generator at (a) lagging and (b) leading PF.

The phasor diagrams expected for valid field currents with different power factors (PFs) are shown in figure 2.4.

2.5 Real and Reactive Power: P_{out} and Q_{out}

Real and reactive power equations are given as:

$$P_{out} = 3V_\phi I_A cos\theta \quad (2.5.1)$$

$$Q_{out} = 3V_\phi I_A sin\theta \quad (2.5.2)$$

3 Discussion and Results

3.1 Phasor Diagram Plotting Function: plot_phasor_diagram

For question 1, by utilizing equation (2.1.1), I have built a function called "plot_phasor_diagram". This function takes armature current I_A, terminal voltage V_T, synchronous reactance X_S, armature resistance R_A, and power angle as function parameters respectively. The phasor diagram shows how the equation (2.1.1) is satisfied by representing the voltage components on the right side of the equation as magnitudes and phases, which adds up to E_A.

3.2 Plotting Limit Circles: plot_circle

For question 2, I have built a function called "plot_circle" to plot the limit circles of I_A and E_A at rated conditions. The phasor diagram with limit circles combined with phasors for questions 1 and 2 is shown in figure 3.2.

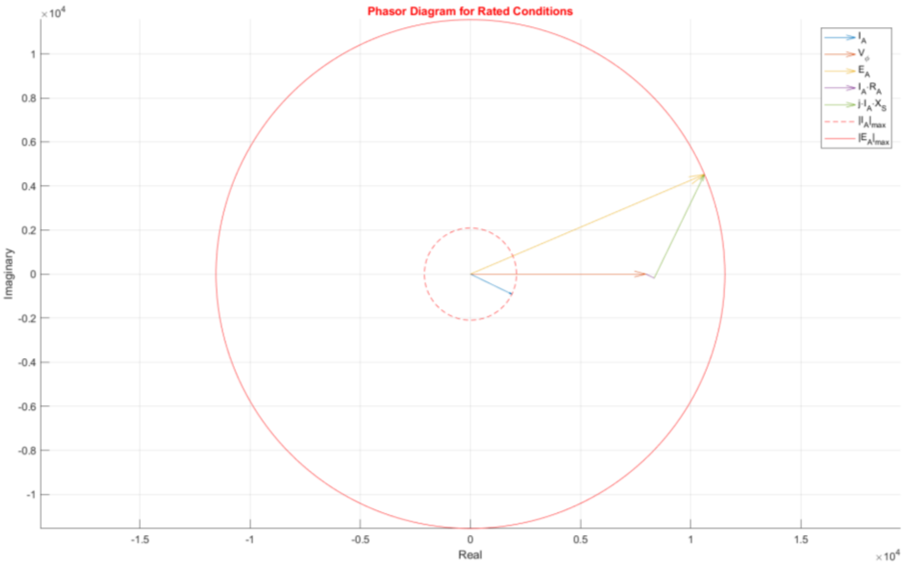

Figure 3.2: Phasor diagram for rated conditions.

3.3 Lagging and Leading PF: calculate_Ia_mag

For question 3, I have implemented a function named "calculate_Ia_mag". This function takes output voltage V_ϕ, field current I_F, synchronous reactance X_S, armature resistance R_A, and power angle θ as function parameters respectively. After finding E_A using I_F from the equations (2.1.1) and (2.1.2), the only unknown variable left in equation (2.4.3) is I_A. Thus, after symbolically solving the equation (2.4.3), the function returns I_A, magnitude of the armature current.

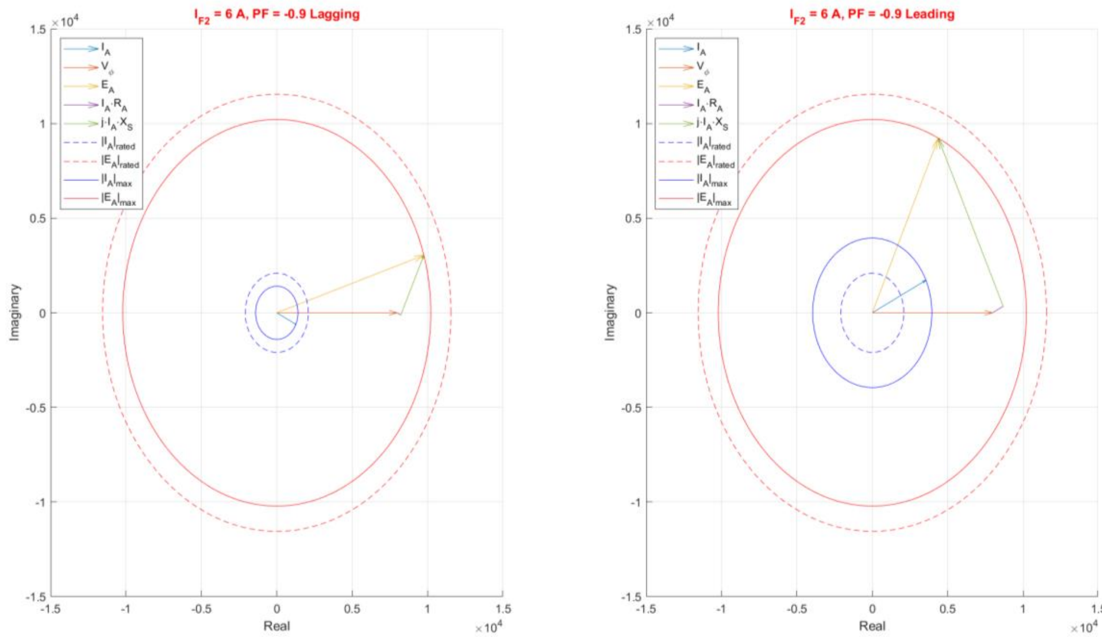

Figure 3.3: Phasor diagrams for I_{F2} = 6 A, PF = -0.9 and 0.9.

From equation (2.4.3), we find that I_{F1} = 2 A cannot be achieved because I_A is calculated to have a complex magnitude, which is unrealizable. So, only the phasor diagrams for I_{F2} = 6 A is included in figure 3.3. We observe that these plots coincide with the ones in figure 2.4.

3.4 Real and Reactive Power Comparison

For question 4, from equations (2.5.1) and (2.5.2) we infer that for constant V_ϕ and I_F, the real power P_{out} will stay positive for both lagging and leading power factors, whereas reactive power Q_{out} will be positive for leading PF and negative for lagging PF. Since I_F = 2 A is unrealizable, we only consider for I_F = 6 A. By plugging the equations (2.5.1) and (2.5.2) to MATLAB for I_F = 6 A, we fill the following table:

	PF = -0.9 Lagging	PF = 0.9 Leading
Real Power P_{out}	30.2 MVA	85.0 MVA
Reactive Power Q_{out}	14.6 MVAR	-41.2 MVAR

Table 3.4: Power comparison for PF = -0.9 lagging and 0.9 leading when I_{F2} = 6 A.

From table 3.4, we observe that for leading PF, the limiting factor of 50 MVA will be exceeded, and as a result, the excessive real power will burn the generator and the generator will not operate properly. However, for lagging PF, the generator will work as intended since the limiting factor of 50 MVA is not exceeded.

4 Appendix

4.1 main.m

```
%% Synchronous generator parameters

clc, clear, close all

V_rated = 13.8e3;                    % rated line voltage in volts (V)
S_rated = 50e6;                      % rated apparent power in volt-amperes (VA)
PF_rated = 0.9;                      % rated power factor
PA_rated = -acos(PF_rated);          % rated power angle in radians
freq_rated = 60;                     % rated frequency in hertz (Hz)  (unused)
X_S = 2.5;                           % synchronous reactance in ohms
R_A = 0.2;                           % armature resistance in ohms
windage_losses = 1e6;                % windage losses in watts (W)    (unused)
core_losses = 1.5e6;                 % core losses in watts (W)       (unused)
I_F_max = 10;                        % maximum field current in amperes (A)

%% 1 & 2) Simulation of the phasor_diagram function and plotting the |I_A|max and |E_A|max
at rated conditions

I_A_mag_rated = S_rated / (sqrt(3) * V_rated);              % magnitude of the armature
current I_A
fig_handle = figure('Position', get(0, 'Screensize'));
E_A_rated = plot_phasor_diagram(I_A_mag_rated, V_rated, X_S, R_A, PA_rated);
% The factor which limits internal generated voltage E_A is the field current,
% as the maximum field current I_F_max is 10 A.
```

```
E_A_mag_max = calculate_Vt_OC(I_F_max) / sqrt(3);
plot_circle(0, 0, I_A_mag_rated, 'r', '|I_A|_{max}', '--')
plot_circle(0, 0, E_A_mag_max, 'r', '|E_A|_{max}', '-')     % |E_A_rated| and |E_A_max|
coincide for I_F = I_F_max
title('Phasor Diagram for Rated Conditions', 'Color', 'r')
legend
axis equal
grid on

saveas(fig_handle, 'Plots/1.png','png');
%% 3) Plotting for 0.9-PF-lagging-and-leading E_A limit circles corresponding to I_F = 2
and 6 A

PA_leading = -PA_rated;         % leading (+) power angle in radians
PA_lagging = PA_rated;          % lagging (-) power angle in radians

I_F1 = 2;       % field current for case 1
I_F2 = 6;       % field current for case 2

V_phi = V_rated / sqrt(3);      % relation due to Y-connection



% There is no solution for I_F1 = 2 A.
% figure
%
% % for I_F1 = 2 A, PF = -0.9 lagging
% subplot(1, 2, 1)
% I_A_mag1 = calculate_Ia_mag(V_phi, I_F1, X_S, R_A, PA_lagging);
% E_A1 = plot_phasor_diagram(I_A_mag1, V_rated, X_S, R_A, PA_lagging);
% plot_circle(0, 0, I_A_mag_rated, 'b', '|I_A|_{rated}', '--')     % plot old |I_A|max
limit circle
% plot_circle(0, 0, E_A_mag_max, 'r', '|E_A|_{rated}', '--')       % plot old |E_A|max
limit circle
% plot_circle(0, 0, abs(I_A_mag1), 'b', '|I_A|_{max}', '-')        % plot new |I_A|max
limit circle
% plot_circle(0, 0, abs(E_A1), 'r', '|E_A|_{max}', '-')            % plot new |E_A|max
limit circle
% title('I_{F1} = 2 A, PF = -0.9 Lagging', 'Color', 'r')
% legend('Location', 'Northwest')
% grid on
%
% % for I_F1 = 2 A, PF = 0.9 leading
% subplot(1, 2, 2)
% I_A_mag2 = calculate_Ia_mag(V_phi, I_F1, X_S, R_A, PA_leading);
% E_A2 = plot_phasor_diagram(I_A_mag2, V_rated, X_S, R_A, PA_leading);
% plot_circle(0, 0, I_A_mag_rated, 'b', '|I_A|_{rated}', '--')     % plot old |I_A|max
limit circle
% plot_circle(0, 0, E_A_mag_max, 'r', '|E_A|_{rated}', '--')       % plot old |E_A|max
limit circle
% plot_circle(0, 0, abs(I_A_mag2), 'b', '|I_A|_{max}', '-')        % plot new |I_A|max
limit circle
% plot_circle(0, 0, abs(E_A2), 'r', '|E_A|_{max}', '-')            % plot new |E_A|max
limit circle
% title('I_{F1} = 2 A, PF = -0.9 Leading', 'Color', 'r')
% legend('Location', 'Northwest')
% grid on



fig_handle = figure('Position', get(0, 'Screensize'));

% for I_F2 = 6 A, PF = -0.9 lagging
subplot(1, 2, 1)
I_A_mag1 = calculate_Ia_mag(V_phi, I_F2, X_S, R_A, PA_lagging);
E_A1 = plot_phasor_diagram(I_A_mag1, V_rated, X_S, R_A, PA_lagging);
```

```
plot_circle(0, 0, I_A_mag_rated, 'b', '|I_A|_{rated}', '--')     % plot old |I_A|max limit
circle
plot_circle(0, 0, E_A_mag_max, 'r', '|E_A|_{rated}', '--')       % plot old |E_A|max limit
circle
plot_circle(0, 0, abs(I_A_mag1), 'b', '|I_A|_{max}', '-')        % plot new |I_A|max limit
circle
plot_circle(0, 0, abs(E_A1), 'r', '|E_A|_{max}', '-')            % plot new |E_A|max limit
circle
title('I_{F2} = 6 A, PF = -0.9 Lagging', 'Color', 'r')
legend('Location', 'Northwest')
grid on

% for I_F2 = 6 A, PF = 0.9 leading
subplot(1, 2, 2)
I_A_mag2 = calculate_Ia_mag(V_phi, I_F2, X_S, R_A, PA_leading);
E_A2 = plot_phasor_diagram(I_A_mag2, V_rated, X_S, R_A, PA_leading);
plot_circle(0, 0, I_A_mag_rated, 'b', '|I_A|_{rated}', '--')     % plot old |I_A|max limit
circle
plot_circle(0, 0, E_A_mag_max, 'r', '|E_A|_{rated}', '--')       % plot old |E_A|max limit
circle
plot_circle(0, 0, abs(I_A_mag2), 'b', '|I_A|_{max}', '-')        % plot new |I_A|max limit
circle
plot_circle(0, 0, abs(E_A2), 'r', '|E_A|_{max}', '-')            % plot new |E_A|max limit
circle
title('I_{F2} = 6 A, PF = -0.9 Leading', 'Color', 'r')
legend('Location', 'Northwest')
grid on

saveas(fig_handle, 'Plots/2.png','png');
%% 4) Analysis of generator behavior in terms of active and reactive power

% We use negative phases since complex apparent power S = V * I' where I' means conjugate
of I.
S_mag_lagging = abs(I_A_mag1 * 3 * V_phi);
S_mag_leading = abs(I_A_mag2 * 3 * V_phi);
P_lagging = round(S_mag_lagging * cos(-PA_lagging), 5)
Q_lagging = round(S_mag_lagging * sin(-PA_lagging), 5)
P_leading = round(S_mag_leading * cos(-PA_leading), 5)
Q_leading = round(S_mag_leading * sin(-PA_leading), 5)

%% Function definitions

function [Vt_OC] = calculate_Vt_OC(I_F)
%[Vt_OC] = calculate_Vt_OC(I_F):
%   I_F: field current in amperes (A)
%
%   Returns the open circuit terminal voltage for the given field current I_F.

    Vt_OC = 20 * (1.05 - exp(-0.3 * I_F)) * 1e3;
end



function plot_circle(xc, yc, r, marker, name, style)
%plot_circle(xc, yc, r, marker, name, style):
%   xc: x coordinate of the circle's center
%   yc: y coordinate of the circle's center
%   r: radius of the circle
%   marker: marker used on the plot
%   name: name of the plot
%   style: line style of the plot
%
%   Plots a circle with given center (xc, yc) and radius r.

    hold on
    angle = linspace(0, 2*pi, 200);     % angle array in range of [0, 2*pi]
```

```
    xp = r * cos(angle) + xc;           % x coordinates of the circle border
    yp = r * sin(angle) + yc;           % y coordinates of the circle border
    plot(xp, yp, marker, 'DisplayName', name, 'LineStyle', style);
    hold off
end



function [E_A] = plot_phasor_diagram(I_A, V_T, X_S, R_A, P_angle)
%[E_A] = plot_phasor_diagram(I_A, V_T, X_S, R_A, P_angle):
%   I_A: armature current in amperes (A)
%   V_T: terminal voltage in volts (V)
%   X_S: synchronous reactance in ohms
%   R_A: armature resistance in ohms
%   P_angle: power angle in radians
%
%   Plots the phasor diagram of the generator for given parameters.

    V_phi = V_T / sqrt(3);              % divide by sqrt(3) due to Y-connection
    I_A = I_A * exp(1i * P_angle);      % find armature current in phasor domain
    Ia_Ra = I_A * R_A;
    j_Ia_Xs = 1i * I_A * X_S;
    E_A = V_phi + Ia_Ra + j_Ia_Xs;
    Ia_x = real(I_A);           Ia_y = imag(I_A);           % get real and imaginary parts
of I_A
    Ia_Ra_x = real(Ia_Ra);      Ia_Ra_y = imag(Ia_Ra);      % get real and imaginary parts
of I_A * R_A
    j_Ia_Xs_x = real(j_Ia_Xs);  j_Ia_Xs_y = imag(j_Ia_Xs);  % get real and imaginary parts
of j * I_A * X_S
    Ea_x = real(E_A);           Ea_y = imag(E_A);           % get real and imaginary parts
of E_A

    hold on
    quiver(0, 0, Ia_x, Ia_y, 0, 'DisplayName', 'I_A')       % plot I_A
    quiver(0, 0, V_phi, 0, 0, 'DisplayName', 'V_\phi')      % plot V_phi
    quiver(0, 0, Ea_x, Ea_y, 0, 'DisplayName', 'E_A')       % plot E_A
    quiver(V_phi, 0, Ia_Ra_x, Ia_Ra_y, 0, 'DisplayName', 'I_A\cdotR_A')     % plot I_A *
R_A
    quiver(V_phi + Ia_Ra_x , Ia_Ra_y, j_Ia_Xs_x, j_Ia_Xs_y, 0, 'DisplayName',
'j\cdotI_A\cdotX_S')  % plot j * I_A * X_S
    xlabel('Real')
    ylabel('Imaginary')
    hold off
end



function [I_A_mag] = calculate_Ia_mag(V_phi, I_F, X_S, R_A, P_angle)
%[I_A_mag] = calculate_Ia_mag(V_phi, I_F, X_S, R_A, P_angle):
%   V_phi: output voltage in volts (V)
%   I_F: field current in amperes (A)
%   X_S: synchronous reactance in ohms
%   R_A: armature resistance in ohms
%   P_angle: power angle in radians
%
%   Returns the magnitude of the armature current |I_A| for given parameters.

    syms Ia;
    E_A_mag = calculate_Vt_OC(I_F) / sqrt(3);       % divide by sqrt(3) due to Y-connection
    I_A_mag = solve(((V_phi + Ia * R_A * cos(P_angle) + Ia * X_S * -sin(P_angle)) ^ 2 ...
        + (Ia * R_A * sin(P_angle) + Ia * X_S * cos(P_angle)) ^ 2) ...
        == (E_A_mag ^ 2), Ia);                      % solve the Pythagorean theorem
    I_A_mag = I_A_mag(2);
end
```