VOL. 88, NO. 1, FEBRUARY 2015 1

Max-h: A New Rule for Service Sports

Arash Dehghan, D. Marc Kilgour
Wilfrid Laurier University
mkilgour@wlu.ca

The recent work of Brams, Ismail, Kilgour, and Stromquist [**?**] (hereinafter, BIKS) drew attention to a class of contests with interesting features. A service sport is a two-sided competition that involves rallies in which one player (or team), called the server, hits some object, usually a ball, and the receiving player attempts to return it. The back-and-forth rally ends if the serve fails or, if it is successful, as soon as a return fails. Broadly speaking, service sports include tennis, table tennis, racquetball, squash, badminton, and volleyball.

Under the most common scoring rule, the opponent of the player whose serve or return fails scores one point. For most service sports, points accumulate and the winner of the game is the first player who attains some fixed number of points, often 11 or 21. In typical service sports it is an advantage to serve, in that the server is more likely to win the point when the competitors are equally skilled. Note that we do not consider tennis because of its unique system (game, set, and match) for determining winners, and because it allows missed serves to be repeated once [**?**].

In tennis and table tennis, the rules specify which player is to serve and for how long. In contrast to these fixed service rules, BIKS focused on service sports that use the

- Standard Rule (SR): The player who won the previous point is the next server.

Note that SR is a variable service rule—who serves when typically varies from game to game. BIKS also discussed three alternatives to the Standard Rule that are also variable:

- Catch-Up Rule (CR): The player who lost the previous point is the next server.
- Trailing Rule A (TRa): The player who is behind in accumulated points is the next server. In a tie, the server is the player who was ahead in total points prior to the tie.
- Trailing Rule B (TRb): The player who is behind in accumulated points is the next server. In a tie, the server is the player who was behind in total points prior to the tie.

CR was proposed in another context by Brams and Ismail [**?**]; TRa was proposed in another context by Anbarci et al. [**?**].

Assuming serves are independent events, BIKS calculated win probabilities and expected game lengths for these rules. Remarkably, they found that win probabilities under SR and CR are in fact identical, though the expected lengths of games are not. All three of SR, CR, and TRa are strategy-proof (or incentive-compatible)—it is never in a player's interest to deliberately lose a serve—but TRb is not.

In this article, we introduce a new family of service rules and contribute to the analysis of its properties. In a Best-of-$(2k+1)$ service sport, the winner is the first player to accumulate $k+1$ points. Let $1 < h \leq k$. The new service rule (based on a suggestion by Steven Brams) is

- Max-h Rule (Mh): The player who won the previous point is the next server unless that player has won h consecutive points.

Thus, if he or she continues to win, the initial server serves a maximum of h times; then the opponent, providing he or she keeps on winning, is allowed h consecutive serves, etc.

There are several reasons why the Max-h Rule is worthy of study. If $h = 1$ were allowed, Max-h would be identical to CR; if $h = k + 1$ were allowed, Max-h would be the identical to SR. Thus, because $1 < h < k + 1$, Max-h lies "between" CR and SR, which have equal win probabilities. Does Max-h have the same win probability as SR and CR? Similarly, one anticipates that expected game length under Max-h lies between the expected lengths under CR and SR. However, CR and SR are Markovian (they depend only on the outcome of the most recent serve), whereas Max-h is not—and, thus, Max-h has something in common with the non-Markovian TR rules. In our study, we minimize the complexity of the non-Markovian rule Max-h by restricting our attention to the case $h = k = 2$.

Win Probability

Let the two players be named A and B, with A serving first. Assume that A has probability p of winning the point when A serves (so B wins it with probability $1 - p$), and that B has probability q of winning the point when B serves (so A wins with probability $1 - q$). Assume further that $0 < p < 1$ and $0 < q < 1$, and that p and q are unrelated. We will consider only $k = 2$ in this paper, so the first player to reach 3 points wins.

Because we are considering a Best-of-5 game, there are a total of ten ways in which Player A can win. We have listed each possibility below (note that we represent Player A winning a point as $\underline{\text{A}}$ and Player B winning a point as $\underline{\text{B}}$):

[1] Player A wins the first three serves ($\underline{\text{AAA}}$)
[2] Player A wins the first, third, and fourth serves ($\underline{\text{ABAA}}$)
[3] Player A wins the first, second, and fourth serves ($\underline{\text{AABA}}$)
[4] Player A loses the first serve and wins the next three ($\underline{\text{BAAA}}$)
[5] Player A wins the first, second, and fifth serves ($\underline{\text{AABBA}}$)
[6] Player A wins the first, third, and fifth serves ($\underline{\text{ABABA}}$)
[7] Player A wins the first, fourth, and fifth serves ($\underline{\text{ABBAA}}$)
[8] Player A loses the first two serves and wins the final three ($\underline{\text{BBAAA}}$)
[9] Player A wins the second, fourth, and fifth serves ($\underline{\text{BABAA}}$)
[10] Player A wins the second, third, and fifth serves ($\underline{\text{BAABA}}$)

Every one of these ten 'win sequences' is possible under any service rule, but the sequence of servers differs according to the rule. In other words, the win sequence tells us the winner of each serve, but not the identity of the server. Thus, to study a particular service rule, we need ten 'outcome sequences' that record who served as well as who won each point. We use A and B to denote a server win by A and B, and $\overline{A}$ and $\overline{B}$ to denote wins by A and B after the opponent serves.

For example, consider the first win sequence in our list, in which A wins three consecutive points. Under SR, the outcome sequence is AAA, since A serves and wins each point, retaining the serve each time. Under CR, however, the outcome sequence is $A\overline{AA}$, since A wins the first serve, forfeits the next serve to B, who loses, and then B serves and loses again. Under Max-2, the outcome sequence must be $AA\overline{A}$, since A wins the first two serves and then forfeits the next serve to B, who serves and loses. The outcome sequences for the remaining nine win sequences are determined similarly.

To calculate the probability of an outcome sequence, simply multiply the probabilities for each outcome. The translation from outcome to probability is direct: A, B, $\overline{A}$, $\overline{B}$ correspond to probabilities p, q, $1 - q$, $1 - p$, respectively. For the first win sequence $\underline{\text{AAA}}$, the outcome sequences in the previous paragraph work out as follows:

- SR: AAA has probability p^3;
- CR: $A\overline{AA}$ has probability $p(1-q)^2$;
- Max-2: $AA\overline{A}$ has probability $p^2(1-q)$.

The probabilities for each outcome sequence are given in the table below.

TABLE 1: Player A Win Probabilities

Win Sequence	SR	CR	Max-2
AAA (3)	p^3	$p(1-q)^2$	$p^2(1-q)$
ABAA (4)	$p(1-p)(1-q)p$	$pqp(1-q)$	$p(1-p)(1-q)p$
AABA (4)	$p^2(1-p)(1-q)$	$p(1-q)qp$	$p^2q(1-q)$
BAAA (4)	$(1-p)(1-q)p^2$	$(1-p)p(1-q)^2$	$(1-p)(1-q)p(1-q)$
AABBA (5)	$p^2(1-p)q(1-q)$	$p(1-q)q(1-p)p$	p^2q^2p
ABABA (5)	$p(1-p)(1-q)(1-p)(1-q)$	$pqpqp$	$p(1-p)(1-q)(1-p)(1-q)$
ABBAA (5)	$p(1-p)q(1-q)p$	$pq(1-p)p(1-q)$	$p(1-p)qp^2$
BBAAA (5)	$(1-p)q(1-q)p^2$	$(1-p)^2p(1-q)^2$	$(1-p)qp^2(1-q)$
BABAA (5)	$(1-p)(1-q)(1-p)(1-q)p$	$(1-p)pqp(1-q)$	$(1-p)(1-q)(1-p)(1-q)p$
BAABA (5)	$(1-p)(1-q)p(1-p)(1-q)$	$(1-p)p(1-q)qp$	$(1-p)(1-q)pq(1-q)$

For each rule, the sum of the probabilities in its column gives the total probability that A wins the Best-of-5 game under that rule. After simplification, these probabilities are

$$Pr_{SR}(A) = p(6p^2q^2 - 6p^2q + p^2 - 9pq^2 + 12pq - 3p + 3q^2 - 6q + 3)$$

$$Pr_{CR}(A) = p(6p^2q^2 - 6p^2q + p^2 - 9pq^2 + 12pq - 3p + 3q^2 - 6q + 3)$$

$$Pr_{M2}(A) = p(-p^3q + 4p^2q^2 - 3p^2q + p^2 - pq^3 - 5pq^2 + 9pq - 3p + q^3 + q^2 - 5q + 3)$$

As BIKS observed, the probabilities in the SR and CR columns are all different, but their totals are equal. Thus, $Pr_{SR}(A) = Pr_{CR}(A)$, for all values of p and q. But $Pr_{M2}(A)$ is different.

Theorem 1. *Let $0 < p < 1$ and $0 < q < 1$. In a Best-of-5 game, $Pr_{M2}(A) \geq Pr_{SR}(A) = Pr_{CR}(A)$, with equality iff $p + q = 1$.*

Proof. To prove the inequality, we begin by showing that $Pr_{M2}(A) - Pr_{SR}(A) \geq 0$. Substituting the polynomials above and expanding gives

$$\begin{aligned} Pr_{M2}(A) - Pr_{SR}(A) &= p(-qp^3 - 2p^2q^2 + 3p^2q - pq^3 + 4pq^2 - 3pq + q^3 - 2q^2 + q) \\ &= pq(1-p)(p+q-1)^2 \end{aligned}$$

Since each factor in the product is non-negative, we conclude that $Pr_{M2}(A) \geq Pr_{SR}(A) = Pr_{CR}(A)$. To complete the proof, we note that the only values of p and q for which $pq(1-p)(p+q-1)^2 = 0$ are those that satisfy $p + q - 1 = 0$, or $p + q = 1$. □

Theorem 2.1 implies that Player A's probability of winning the Best-of-5 game is always greater under Max-2, unless $p + q = 1$, when the probabilities become equal. Below, we provide a graphical representation of this relation, in which we fix the value of p at $\frac{1}{2}$ and show how the win probabilities depend on the value of q.

Figure 2.1 also shows win probability for TRa, the Trailing Rule discussed above. It can be shown that

$$Pr_{TRa}(A) = p(-p^3q + p^3 + 4p^2q^2 - p^2q - 2p^2 - pq^3 - 4pq^2 + 5pq + q^3 - 3q + 2)$$

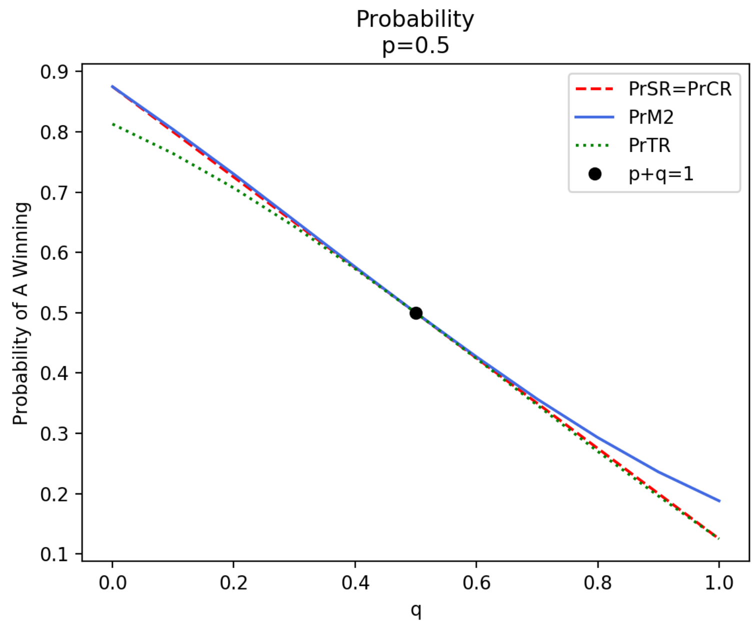

Figure 1 Win Probability Comparisons

We conjecture that $Pr_{TRa}(A) \leq Pr_{SR}(A)$, with equality iff $p+q=1$. Note that, except at $p=q=\frac{1}{2}$, $Pr_{SR}(A) = Pr_{CR}(A)$ is only slightly smaller than $Pr_{M2}(A)$ except when q is large. Clearly, the Max-2 rule benefits A particularly q is large—i.e. there is a high probability that B will win three consecutive games. As Figure 1 shows, A's win probability under TRa is substantially less than that under SR only when q is much smaller than p, when the extra serves that TRa gives to the trailing candidate, likely B, make a substantial difference.

The service rule affects not only a player's win probability, but also the expected length of the game. In the following section, we investigate the relation among our three rules with regard to expected length.

Expected Length of a Game

The expected length of a game refers to the expected number of serves prior to one side winning. Above, we found the total win probability for Player A by calculating the sum of the probabilities of its win sequences, probabilities that of course depend on the service rule. To calculate the expected length of a game, take into account the length of a win sequence as well as its probability—and do so for both players. For wins by A, the first win sequence in Table 1 has length 3; the second, third, and fourth have length 4, and the remainder have length 5.

For example, consider the expected length of a Best-of-5 game under SR. Taking our column of SR values in Table 1, we multiply each probability by the length of the corresponding game. Then we take the same product for the win sequences of player B shown in Table 2.

We find that

$$EL_{SR} = -3p^3q + p^3 + 3p^2q^2 + p^2q - 3p^2 - 3pq^2 + 4pq - 2q + 5$$
$$EL_{CR} = -3p^3q + 2p^3 + 3p^2q^2 + 2p^2q - 5p^2 - 3pq^2 + 3pq + 3p + 3$$

TABLE 2: Player B Win Probabilities

Win Sequence	SR	CR	Max-2
BBB (3)	$(1-p)q^2$	$(1-p)^3$	$(1-p)q(1-p)$
BABB (4)	$(1-p)(1-q)(1-p)q$	$(1-p)pq(1-p)$	$(1-p)(1-q)(1-p)q$
BBAB (4)	$(1-p)q(1-q)(1-p)$	$(1-p)^2pq$	$(1-p)qp(1-p)$
ABBB (4)	$p(1-p)q^2$	$pq(1-p)^2$	$p(1-p)q(1-p)$
BBAAB (5)	$(1-p)q(1-q)p(1-p)$	$(1-p)^2p(1-q)q$	$(1-p)qp^2q$
BABAB (5)	$(1-p)(1-q)(1-p)(1-q)(1-p)$	$(1-p)pqpq$	$(1-p)(1-q)(1-p)(1-q)(1-p)$
BAABB (5)	$(1-p)(1-q)p(1-p)q$	$(1-p)p(1-q)q(1-p)$	$(1-p)(1-q)pq^2$
AABBB (5)	$p^2(1-p)q^2$	$p(1-q)q(1-p)^2$	$p^2q^2(1-p)$
ABABB (5)	$p(1-p)(1-q)(1-p)q$	$pqpq(1-p)$	$p(1-p)(1-q)(1-p)q$
ABBAB (5)	$p(1-p)q(1-q)(1-p)$	$pq(1-p)pq$	$p(1-p)qp(1-p)$

$$EL_{M2} = -3p^3q + p^3 + 3p^2q^2 + p^2q - 2p^2 - 3pq^2 + 6pq - p + q^2 - 3q + 5$$

Theorem 2. *Let* $0 < p < 1$ *and* $0 < q < 1$. *In a Best-of-5 game,*

(i) $EL_{CR} > EL_{M2} > EL_{SR}$ *iff* $p + q > 1$
(ii) $EL_{CR} = EL_{M2} = EL_{SR}$ *iff* $p + q = 1$
(iii) $EL_{CR} < EL_{M2} < EL_{SR}$ *iff* $p + q < 1$

Proof. To begin, we compare EL_{CR} and EL_{M2}. By substituting and simplifying, we obtain

$$\begin{aligned} EL_{CR} - EL_{M2} &= p^3 + p^2q - 3p^2 - 3pq + 4p - q^2 + 3q - 2 \\ &= (p^2 - 2p - q + 2)(p + q - 1) \end{aligned}$$

Note that $p^2 - 2p - q + 2 = (1-p)^2 + (1-q)$ is always non-negative. Thus, the sign of $EL_{CR} - EL_{M2}$ is always the same as the sign of $p + q - 1$. Now we repeat this procedure for EL_{M2} and EL_{SR}, finding that

$$EL_{M2} - EL_{SR} = (p + q - 1)(p + q)$$

Again, $p + q$ is always non-negative, so the sign of $EL_{M2} - EL_{SR}$ is always the same as the sign of $p + q - 1$.

We have shown that, if $p + q > 1$, then $EL_{CR} > EL_{M2} > EL_{SR}$; if $p + q = 1$, then $EL_{CR} = EL_{M2} = EL_{SR}$; and if $p + q < 1$, then $EL_{CR} < EL_{M2} < EL_{SR}$. □

Theorem 2.2 implies that the expected length of a Max-2 game always lies between the expected lengths of games played under SR and CR. If the sum of the players' server win probabilities is less than 1, the shortest game is expected under CR and the longest game under SR, with Max-2 in between. But if the sum of these probabilities exceeds 1, the longest game is expected under CR and the shortest under SR, again with Max-2 in between. All expected lengths are equal the sum of the probabilities is 1.

In Figure 2.2, we illustrate Theorem 2 by again fixing the value of $p = \frac{1}{2}$ and graphing expected length as a function of q. Note that all expected lengths are equal at $q = \frac{1}{2}$, where $p + q = 1$. Observe that the expected lengths under SR and CR are quite far apart, whereas the expected length under Max-2 is a middling value that is surprisingly close to constant. Also, note that TRa, included here for completeness, is close to CR but slightly more extreme.

Now we investigate whether a Best-of-5 game under Max-2 rules is strategy-proof in the sense that it can never be in a server's interest to lose the point deliberately. Recall that BIKS showed that TRb may not be strategy-proof.

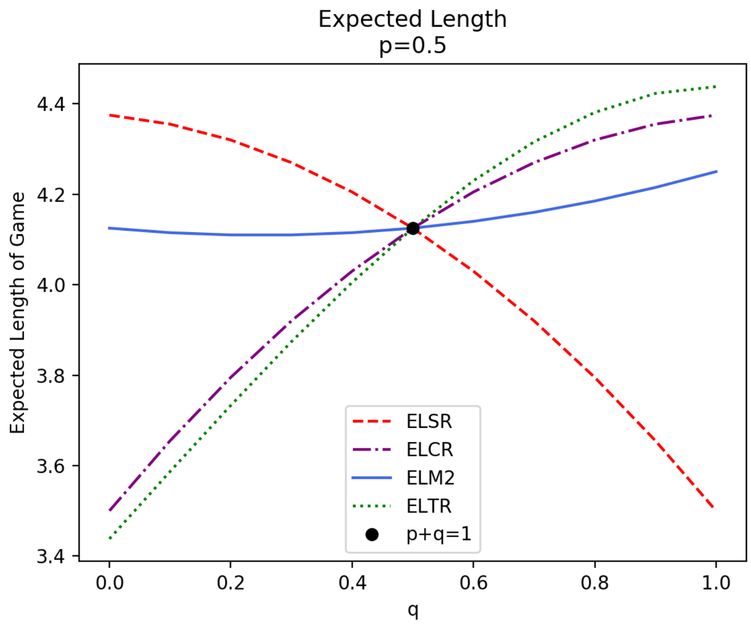

Figure 2 Expected Length Comparisons

Incentive Compatibility

A rule is strategy-proof, or incentive compatible, if no player can ever benefit from intentionally losing a serve; otherwise, it is strategy-vulnerable [**?**]. BIKS proved that SR and CR are strategy-proof, and that TRa is strategy-proof whenever $p + q > 1$. In this section, we assess whether Max-2 is incentive compatible in a Best-of-5 game.

Theorem 3. *Let* $0 < p < 1$ *and* $0 < q < 1$*. In a Best-of-5 game, Max-2 is strategy-proof.*

Proof. To show that Max-2 is strategy proof, we must verify that it is never advantageous for a player to lose a serve deliberately. We consider the probability tree shown in Figure 3, which contains all possible sequences of events in a Best-of-5 game under Max-2 rules. Terminal nodes, where one player has already won, are labeled with a score only (no player). Each non-terminal node is labeled (C, x, y), where C represents the player to serve, x the current score of Player A, and y the current score of Player B. Nodes with the same label have the same descendants, with the exception of two nodes with labels $(A, 1, 2)$ and two with labels $(B, 2, 1)$. This phenomenon occurs because, for example, at $(A, 1, 2)$ the previous winner may have been A or B; if it was A and A wins again, the serve must pass to B, whereas if it was B and A wins, then A serves again. The instances where loser of the point may become the next server are marked with an asterisk.

The tree is rooted at $(A, 0, 0)$ and is binary, meaning that all non-terminal nodes have exactly two children. We call the left-hand child of a non-terminal node Child A because that is the node that arises if A wins the point; similarly, the right-hand node is Child B.

Each node in Figure 3 is associated with a win probability for Player A. At a node where Player A serves, the win probability must equal p times the win probability at the Child A node (on the left) plus $1 - p$ times the win probability at the Child B node (on the right). Similarly, when Player B serves at the original node, the win probability

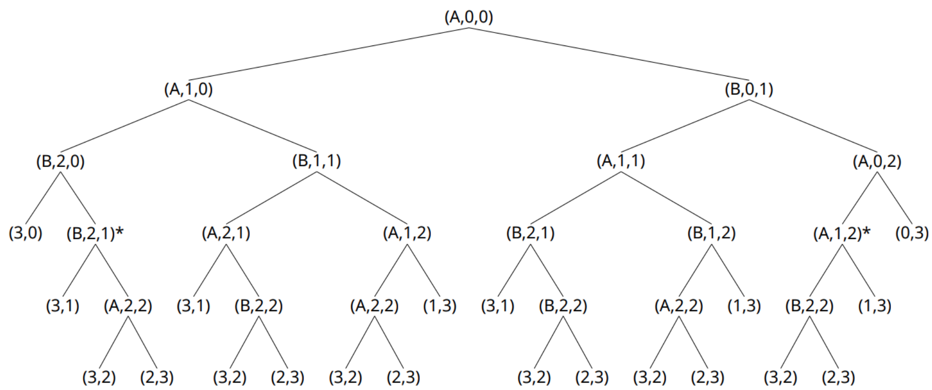

Figure 3 Best-of-5 Max-2 Probability Tree

TABLE 3: Win Probabilities

A Serving		B Serving	
Node	Probability	Node	Probability
(A,1,0)	$-p^3q+2p^2q^2-3pq^2+3pq+q^2-2q+1$	(B,0,1)	$p(1-q)(2pq-p-q^2-q+2)$
(A,1,1)	$p^2q-p^2-pq^2-pq+2p$	(B,2,0)	pq^2-q^2+1
(A,0,2)	$p^2(1-q)$	(B,1,1)	$p^2q-pq^2+pq+q^2-2q+1$
(A,2,1)	$pq-q+1$	(B,2,1)*	$pq-q+1$
(A,1,2)	p^2	(B,2,1)	$1-q^2$
(A,1,2)*	$p(1-q)$	(B,1,2)	$p(1-q)$
(A,2,2)	p	(B,2,2)	$1-q$

must equal $1-q$ times the win probability at Child A plus q times the win probability at Child B.

To calculate the win probability for A at each node, apply backward induction to Figure 3, beginning on the bottom row and working upward. For example, the win probability for A at $(A, 2, 2)$ (second from the left in the second-last row) is $p \times 1 + (1-p) \times 0 = p$. Similarly, the win probability for A at $(B, 2, 2)$ is $1-q$. All of these win probabilities for A are shown in Table 3. Note that the node (A,0,0) has been omitted from Table 3, since we know that A's win probability at $(A, 0, 0)$ is $Pr_{M2}(A)$, which we found earlier.

To demonstrate incentive compatibility, it is enough to show that, whenever Player A serves, Player A's win probability is always greater at Child A than at Child B. The inductive calculation of probabilities for each node is shown in Table 3. For each parent node in our tree (when A is serving), we compare A's win probability at Child A node to A's win probability at Child B. For example, consider the parent node (A,1,0), in which Player A is serving with score 1-0. For this node, Child A is $(B, 2, 0)$ and Child B is $(B, 1, 1)$, with win probabilities $W_A(B, 2, 0) = pq^2 - q^2 + 1$ and $W_A(B, 1, 1) = p^2q - pq^2 + pq + q^2 - 2q + 1$, respectively. To show that A's win probability at $(B, 2, 0)$ is greater than $(B, 1, 1)$, we show that $W_A(B, 2, 0) - W_A(B, 1, 1)$ is always non-negative.

$$\begin{aligned} W_A(B,2,0) - W_A(B,1,1) &= (pq^2 - q^2 + 1) - (p^2q - pq^2 + pq + q^2 - 2q + 1) \\ &= (1-p)q(p - 2q + 2) \end{aligned}$$

Since $1-p$, q, and $p - 2q + 2 = p + 2(1-q)$ are non-negative for all p and q, we

TABLE 4: Win Probability Differences

A Serving			
Parent Node	Child A	Child B	Win Probability (A-B)
(A,0,0)	(A,1,0)	(B,0,1)	$-p^3q+4p^2q^2-3p^2q+p^2-pq^3-3pq^2+6pq-2p+q^2-2q+1$
(A,1,0)	(B,2,0)	(B,1,1)	$(1-p)q(p-2q+2)$
(A,1,1)	(B,2,1)	(B,1,2)	$-p(1-q)-q^2+1$
(A,0,2)	(A,1,2)*	(0,3)	$p(1-q)$
(A,2,1)	(3,1)	(B,2,2)	q
(A,1,2)	(A,2,2)	(1,3)	p
(A,1,2)*	(B,2,2)	(1,3)	$(p)(1-q)$
(A,2,2)	(3,2)	(2,3)	1
B Serving			
Parent Node	Child A	Child B	Win Probability ((1-B)-(1-A))
(B,0,1)	(A,1,1)	(A,0,2)	$p(q-1)(2p-q-2)$
(B,2,0)	(3,0)	(B,2,1)*	$(1-p)q$
(B,1,1)	(A,2,1)	(A,1,2)	$(1-p)(p-q+1)$
(B,2,1)*	(3,1)	(A,2,2)	$1-p$
(B,2,1)	(3,1)	(B,2,2)	q
(B,1,2)	(A,2,2)	(1,3)	p
(B,2,2)	(3,2)	(2,3)	1

know that Player A has a greater probability of winning at $(B,2,0)$ than at $(B,1,1)$. Repeating this procedure for all nodes where A serves, we find that A always has a higher win probability at Child A than at Child B—in other words, when A wins the point on its serve. The probability differences are provided in Table 4.

To complete our proof, we must also show that, whenever Player B serves, Player B's win probability is always greater when it wins the point. To demonstrate this, we must show that Player B's win probability is always greater at Child B than at the Child A. First, at any node, the sum of Player A's win probability and Player B's win probability must equal 1. Since the win probability of A at any Child A node is always greater than at the corresponding Child B node, it follows that B's win probability at any Child B node is greater than at the corresponding Child A node.

For instance, consider the parent node $(B,0,1)$, in which Player B serves in a 0-1 game. From this parent node, the Child A node is $(A,1,1)$ and the Child B node is $(A,0,2)$; Table 3 now shows that A's win probabilities are $W_A(A,1,1)=p^2q-p^2-pq^2-pq+2p$ and $W_A(A,0,2)=p^2(1-q)$. Therefore

$$\begin{aligned} W_B(A,0,2))-W_B(A,1,1) &= \left[1-p^2(1-q)\right]-\left[1-p^2q+p^2+pq^2+pq-2p\right] \\ &= p(1-q)(2+q-2p) \end{aligned}$$

Since p, $1-q$, and $2+q-2p=q+2(1-p)$ are non-negative for all values of p and q, we conclude that Player B's win probability is greater at $(A,0,2)$ than at $(A,1,1)$. This same process can be repeated at every parent node where B serves, shown in Table 4. We conclude that Max-2 is strategy-proof in a Best-of-5 game, since winning a serve always gives a player a greater probability of winning the game. □

Conclusion

We defined a new service rule, Max-h, and analyzed its special case, Max-2, in a Best-of-5 game. This rule limits the number of consecutive serves a player may win to two. Under Max-2, Player A's win probability is at least equal to SR and CR, with equality exactly when $p+q=1$. However, calculations suggest that the advantage to Player A is usually very small, except when Player B's win probability on service is substantially greater than Player A's.

We also demonstrated that the expected length of a Max-2 game is always between the expected lengths of SR and CR games. If $p + q > 1$, the expected length of a SR game is smaller than that of a Max-2 game, which in turn is less than the expected length of a CR game. For example, when both players are strong servers, a Best-of-5 game is more competitive under Max-2 than under SR, but less competitive than under CR. If $p + q < 1$, the ordering of expected lengths is reversed. If $p + q = 1$, all three rules produce equal expected lengths.

Finally, we demonstrated that, just as with SR and CR, Max-2 is strategy proof in a Best-of-5 game: No player can ever gain by losing a serve deliberately.

REFERENCES

1. Brams, S.J., Ismail, M.S., Kilgour, D.M., Stromquist, W. (2018). *Catch-Up: A Rule That Makes Service Sports More Competitive*. *Amer. Math. Monthly*, 125: 771–796.
2. Yigal, G., Kilgour, D.M. (2017), "Serving Strategy in Tennis: Accuracy vs. Power," *Mathematics Magazine* 90 (2017), 188-196.
3. Brams, S.J., Ismail, M.S. (2018). *Making the rules of sports fairer*. *SIAM Review*. 60(1): 181–202.
4. Anbarci, N., Sun, C.-J, Ünver, M.U. (2015). *Designing fair tiebreak mechanisms: The case of FIFA penalty shootouts*. *http://ssrn.com/abstract=2558979*
5. Pauly, M. (2014). Can strategizing in round-robin sub-tournaments be avoided? *Social Choice and Welfare*. 43(1): 29–46.