Subject:

Year. Month. Date. ( )

Q1.

X	Y	Z	C
1	1	1	I
1	1	0	I
0	0	1	II
1	0	0	II

Entropy

$$E_C = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1$$

$$\cancel{E_{X,C=1} = -1\times\log 1 - 0\times\log 0 = 0}$$

$$\cancel{E_{X,C=2} = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2} = 1}$$

$$\cancel{E_X = \frac{2}{4}\times 0 +}$$

$$E_{X=1} = -\frac{2}{3}\log\frac{2}{3} - \frac{1}{3}\log\frac{1}{3}$$

$$\approx -\frac{2}{3}\times(-0.6) - \frac{1}{3}\times(-1.5) \approx 0.9$$

$E_{X=0} = 1$

$E_X = \frac{3}{4} \times 0.9 + \frac{1}{4} \times 1 = 0.925$

$E_{Y=1} = -1 \log(1) - 0 = 0$

$E_{Y=0} = -0 \times \log(0) - 1 \times \log(1) = 0$

$E_Y = \frac{1}{2} \times 0 + \frac{1}{2} \times 0 = 0$

$E_{Z=1} = -\frac{1}{2} \log \frac{1}{2} - \frac{1}{2} \log \frac{1}{2} = 1$

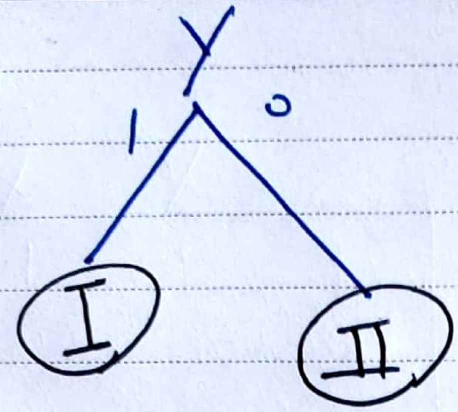
$E_{Z=0} = 1$

$E_Z = \frac{1}{2} \times 1 + \frac{1}{2} \times 1 = 1$

Info. Gain(IG)(X) $= E_c - E_X = 0.075$

$IG(Y) = 1$

$IG(Z) = 0$



Variance based: (let's say class I is 0 and class II is 1)

for $X=1$:

$P_I = \frac{2}{3}$, $P_{II} = \frac{1}{3}$

$\mu = \frac{2}{3} \times 0 + \frac{1}{3} \times 1 = \frac{1}{3}$

$Var = \left(0 - \frac{1}{3}\right)^2 \times \frac{2}{3} + \left(1 - \frac{1}{3}\right)^2 \times \frac{1}{3} = \frac{2}{27} + \frac{4}{27} = \frac{6}{27}$

for $X=0$:

$P_I = 0$, $P_{II} = 1$

$\mu = \frac{1}{3}$

$Var = \frac{4}{27}$

$Var(X) = \frac{3}{4} \times \frac{6}{27} + \frac{1}{4} \times \frac{4}{27} = \frac{22}{108}$

for $Y=1$:

$P_{I}=1, \quad P_{II}=0$

$\mu=0$

$Var=(0-0)^2\times 1+(0-1)^2\times 0=0$

for $Y=0$:

$P_{I}=0, \quad P_{II}=1$

$\mu=0\times P_{I}+1\times P_{II}=1$

$Var=(0-1)^2\times 0+(1-1)^2\times 1=0$

$Var(Y)=\frac{1}{2}\times 0+\frac{1}{2}\times 0=0$

for $Z=1$

$P_{I}=\frac{1}{2}, \quad P_{II}=\frac{1}{2}$

$\mu=0\times\frac{1}{2}+1\times\frac{1}{2}=\frac{1}{2}$

$Var=(0-\frac{1}{2})^2\times\frac{1}{2}+(1-\frac{1}{2})^2\times\frac{1}{2}=\frac{1}{4}$

$\cancel{Var(Z)=\frac{1}{2}\times}$

for $Z = 0$:

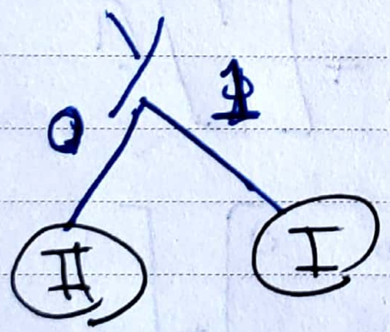
$P_I = \frac{1}{2}$, $P_{II} = \frac{1}{2}$

$\mu = \frac{1}{2}$

$Var = \frac{1}{4}$

$Var(Z) = \frac{2}{4} \times \frac{1}{4} + \frac{2}{4} \times \frac{1}{4} = \frac{1}{4}$

$Var(Y) < Var(Z) < Var(X)$



* Entropy and variance have very similar concepts (i.e., how diverse a data is). However, based on Entropy, features would be selected in the order of $(Y \rightarrow X \rightarrow Z)$.
using Variance: $Y \rightarrow Z \rightarrow X$.

8)

Q2.

a)

$S = \{S_1 S_1 S_1 S_1 S_2\}$

$P(S/\lambda_1) = P(S_1) \cdot P(S_1/S_1) \cdot P(S_1/S_1) \cdot P(S_1/S_1) \cdot$

$P(S_2/S_1) = 0.5 \times 0.1 \times 0.1 \times 0.1 \times 0.9 = 45 \times 10^{-5}$

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$P(S/\lambda_2) = 0.5 \times 0.4 \times 0.4 \times 0.4 \times 0.6 = 30 \times 4^3 \times 10^{-5}$

$30 \times 4^3 \times 10^{-5}$

The sequence is more likely to be form class 2; because: $P(S/\lambda_2) > P(S|\lambda_1)$.

b)

$P(S|\lambda_1) = 0.1 \times (0.1)^3 \times 0.9 = 9 \times 10^{-5}$

$P(S/\lambda_2) = 0.9 \times (0.4)^3 \times 0.6$

$= 54 \times 4^3 \times 10^{-5}$

$P(S|\lambda_2) > P(S/\lambda_1)$

c)

$a_{11} = \frac{2}{9}$ $a_{22} = \frac{1}{7}$ $a_{33} = \frac{1}{6}$

$a_{12} = \frac{4}{9}$ $a_{21} = \frac{4}{7}$ $a_{31} = \frac{5}{6}$

$a_{13} = \frac{3}{9}$ $a_{23} = \frac{2}{7}$ $a_{32} = 0$

5

$$a = \begin{bmatrix} \frac{2}{9} & \frac{4}{9} & \frac{3}{9} \\ \frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ 1/6 & 5/6 & 0 \end{bmatrix}^{5}$$

Q3.

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

a)

$$\mu = \frac{1}{3}(X_1 + X_2 + X_3) = \frac{1}{3}\begin{bmatrix} 1+2+3 \\ 1+3+2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$Cov = E(X-\mu)(X-\mu)^T$$

$$\hookrightarrow \frac{1}{3}\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}\right)$$

$$= \frac{1}{3}\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

b)

$$Cov - \lambda I = \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} = A$$

$$\det(A) = \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{4}{9} + \lambda^2 - \frac{4}{3}\lambda - \frac{1}{9}$$

$$= \frac{1}{3} - \frac{4}{3}\lambda + \lambda^2 = 3\lambda^2 - 4\lambda + 1$$

eigenvalues $\Rightarrow \lambda = \begin{cases} \frac{4+2}{6} = 1 \\ \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3} \end{cases}$

$$A = \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} \xrightarrow{\lambda=1} A_1 = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\xrightarrow{\lambda=1/3} A_2 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$A_1 \vec{x} = 0 \Rightarrow \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\frac{1}{3} \begin{bmatrix} -1 & 1 \\ +1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ +1 & -1 & 0 \end{array}\right] \Rightarrow x_1 = -x_2$$

$$\hookrightarrow \vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A_2 \vec{x} = 0 \Rightarrow \frac{1}{3} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right] \Rightarrow x_1 = x_2$$

$$\hookrightarrow \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigen vectors: $\longrightarrow \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\hookrightarrow \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

c)

$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ corresponds to $\lambda = 1$.

$\cancel{\vec{x} \cdot \vec{v}_1 = [2 \ 2]^T \cdot [1 \ -1]^T = 2 + (-2) = 0}$

Normalize $\vec{v}_1$:

$\|v_1\| = \sqrt{2} \longrightarrow \vec{v}_1' = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\vec{x} \cdot \vec{v}_1 = \frac{1}{\sqrt{2}} \left(2 \cdot 1 + 2 \cdot (-1) \right) = 0$