ECE407 Final Exam _Spring2020 Pattern Recognition

-You should show your work to get credit. You will not get any credit, if you only upload the answer.

-You should solve all the questions by yourself. You should not get any help from other people.

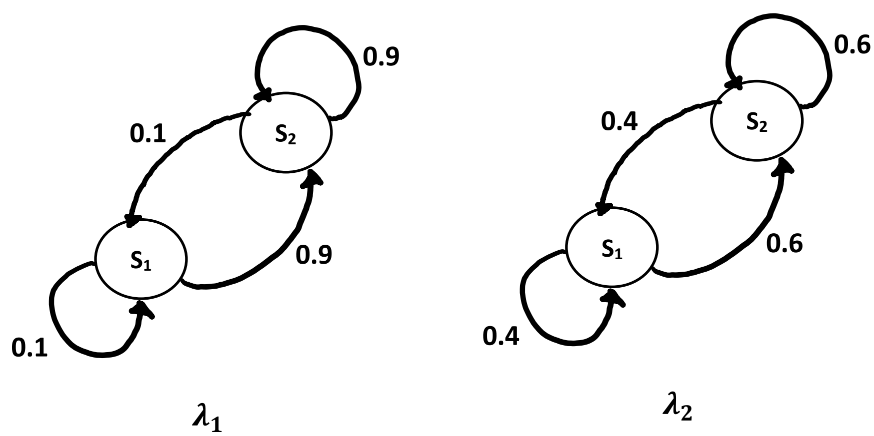
-Please solve each question to a different sheet(s) and you should upload your answers before 3:00 pm.

Q1 (30 pts) Consider a classification problem with three variables.

X	Y	Z	C
1	1	1	I
1	1	0	I
0	0	1	II
1	0	0	II

a) Draw a decision tree that can perfectly classify the two classes corresponding to the above decision tree using the concept of entropy.
b) Draw another decision tree that can perfectly classify the four examples corresponding to the above decision tree using the concept of variance.
c) Compare the results in part a and b.

Q2: (35 pts) Given the following Markov Models



a) We observe the sequence $S_1S_1S_1S_1S_2$. Is it from class 1 ((λ_1) or class 2 (λ_2) ? You can assume that initial state probabilities and class probabilities are equal: $P(\lambda_1) = P(\lambda_2) = 0.5$. Show your work.
b) Repeat part (a) with initial state probabilities $\pi_1 = 0.1$ *and* $\pi_2 = 0.9$ in both classes and the initial class probabilities are $P(\lambda_1) = 0.1,\ P(\lambda_2) = 0.9$.

c) Given the following four state sequences

$$S_1S_1S_1S_2S_2S_1S_2S_1S_3S_1S_3S_3.$$
$$S_2S_1S_3S_1$$
$$S_3S_1S_2S_3S_1$$
$$S_2S_1S_2S_3S_1$$

Estimate a **three** state Markov model based on this sequence. Estimate the initial state probabilities as well.

Q3: (30 pts) Given the observations $\left(x_1 = \begin{bmatrix}1\\1\end{bmatrix}\right), \left(x_2 = \begin{bmatrix}2\\3\end{bmatrix}\right), \left(x_3 = \begin{bmatrix}3\\2\end{bmatrix}\right)$.

a) Construct a covariance matrix from the observed data.

b) Find the eigenvalues and eigenvectors of the covariance matrix.

c) Given the vector x= [2 2]^T. Reduce the dimension of the vector x to 1 by projecting the vector x onto the eigenvector corresponding to the largest eigenvalue.