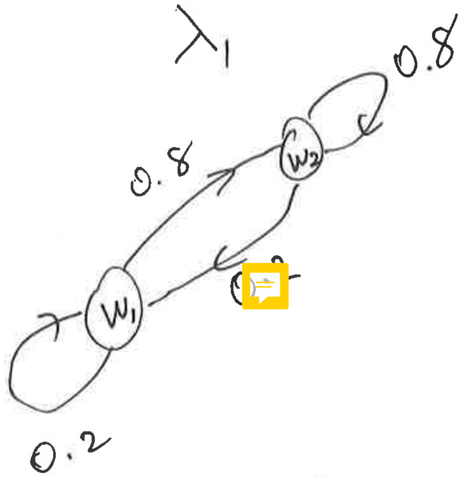
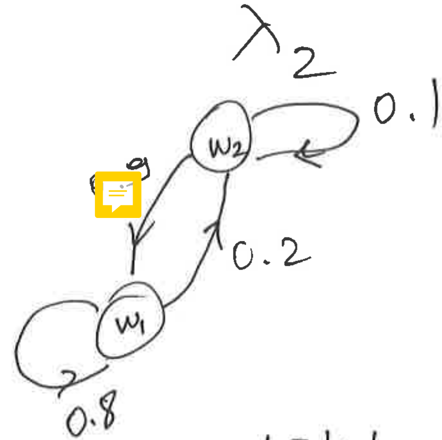
Ex1) Given the Markov Models





a) Determine the missing probabilities in λ_1 and λ_2

$\sum\limits_{\text{in a state}} \text{transition probabilities} = 1$

b) Given the observation sequence

$$O = \{ W_1, W_2, W_2 \}$$

Determine the model producing O with the highest probability. (Recognition problem)

$$P(O|\lambda_1) \quad \text{and} \quad P(O|\lambda_2)$$

Initial probabilities $\pi_1 = \pi_2 = 1/2$ in both models.

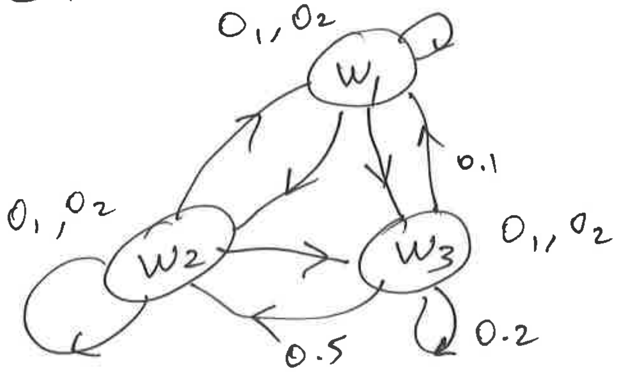
$$P(O|\lambda_1) = \frac{1}{2}\, a_{12,1} \cdot a_{22,1} = \frac{1}{2}\, 0.8 \times 0.8$$

$$P(O|\lambda_2) = \frac{1}{2}\, a_{12,2}\ a_{22,2} = \frac{1}{2}\, 0.2 \times 0.1$$

$$P(O|\lambda_1) > P(O|\lambda_2)$$

So we "recognize" λ_1. (Model 1).

Ex!! Given the Hidden (state) Markov Model



State transition matrix:

$$T = \begin{bmatrix} 0.5 & 1/4 & 1/4 \\ 0.1 & 0.8 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$$

$a_{33} = 0.2$

$a_{32} = 0.5$ $\qquad \sum_j a_{3j} = 1$

$a_{31} = 0.3$

$P(O_1|w_1) = b_1(O_1) = 1$; $P(O_1|w_2) = b_2(O_1) = 1/2$ $\qquad P(O_1|w_3) = 1/3$

$P(O_2|w_1) = b_1(O_2) = 0$; $P(O_2|w_2) = b_2(O_2) = 1/2$ $\qquad P(O_2|w_3) = 2/3$

$b_1(O_1) + b_1(O_2) = 1 \qquad b_2(O_1) + b_2(O_2) = 1 \qquad \frac{1}{3} + \frac{2}{3} = 1$

Initial probabilities $\pi_1 = P(w_1 \text{ at } t=1)$, $\pi_2 = P(w_2)$, $\pi_3 = P(w_3)$

$\pi_1 + \pi_2 + \pi_3 = 1$.

HMM: $\lambda = \{ a_{ij}, b_j(O), \pi_i \}$

We observe $O = \{O_1, O_2, O_2\}$ Calculate $P(O|\lambda)$

We may have many state possibilities generating $O_1 \; O_2 \; O_2$:

$w_1 \; w_2 \; w_3$
$w_1 \; w_3 \; w_2$
$w_1 \; w_2 \; w_2$
$w_1 \; w_3 \; w_3$
$w_2 \; w_2 \; w_2$
$w_2 \; w_3 \; w_2$
$w_2 \; w_3 \; w_3$

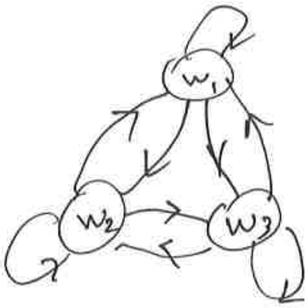
$$\begin{aligned} P(O|\lambda) = \; & \pi_1 b_1(O_1)\, a_{12}\, b_2(O_2)\, a_{23}\, b_3(O_2) \\ & + \pi_1 b_1(O_1)\, a_{13}\, b_3(O_2)\, a_{32}\, b_2(O_2) \\ & + \pi_1 b_1(O_1)\, a_{12}\, b_2(O_2)\, a_{22}\, b_2(O_2) \\ & + \pi_1 b_1(O_1)\, a_{13}\, b_3(O_2)\, a_{33}\, b_3(O_2) \\ & + \pi_2 b_2(O_1)\, a_{22}\, b_2(O_2)\, a_{22}\, b_2 O_2 \\ & \vdots \end{aligned}$$

Evaluation (Recognition) problem may be computationally complex

Example] We observe the state sequence:

$w_1\ w_1\ w_2\ w_2\ w_3\ w_1\ w_2\ w_3\ w_2\ w_1\ w_2\ w_2$

Estimate the state transition matrix.



$$a_{11} = \frac{\#\text{ of } 1-1 \text{ transitions}}{\text{Total } \#\text{ of transitions from state } 1}$$

$$\hat{a}_{11} = \frac{1}{4}$$

$$\hat{a}_{13} = \frac{0}{4} = 0 \quad , \quad \hat{a}_{12} = \frac{3}{4}$$

$$\sum_{j=1}^{3} a_{1j} = 1 = \frac{1}{4} + \frac{3}{4} = 1 \checkmark$$

$$\hat{a}_{21} = \frac{1}{5} \quad , \quad \hat{a}_{22} = \frac{2}{5} \quad , \quad \hat{a}_{23} = \frac{2}{5}$$

etc.

Training a Markov Model is easy!

Training a Hidden Markov " is not easy!

(Baum-Welch algorithm).