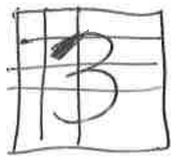
Classifying hand-written digits using Bernoulli based pixel modelling & Bayes Theorem

* Each hand-written digit is an image.
* In MNIST dataset



28X28

each image has 28x28 pixels.

* In Bernoulli modelling images are assumed to be "binary images"
* We represent each pixel using the Bernoulli distribution:

P.M.F. model p.d.f.

$$P(X) = p_i^{x_i}(1-p_i)^{(1-x_i)} = \begin{cases} p_i, & \text{when } x_i = 1 \\ 1-p_i, & \text{when } x_i = 0 \end{cases}$$

for the i-th pixel. $i = 1, 2, \dots, 784$

* Estimate p_i from the data set

Assume the digit is 3

$$P(X \mid Y=3) \qquad \hat{p}_{1j} = \frac{\#\text{ of } 1\text{'s in the } 1^{st}\ (i^{th}) \text{ pixel}}{n_j}$$

where $n_{j=3}$ is the # of "3"'s in the training set.

$Y = \{1, 2, \dots, 9, 0\}$ (There are 10 digits in the dataset.

* Estimate the probabilities for Bernoulli $P(X \mid Y=1), P(X \mid Y=2), \dots, P(X \mid Y=9)$ etc.

$i = 1, 2, \dots, 784.$
$j = 0, 1, \dots, 9$

p_{ij}

Estimate 7840 p_{ij}'s ; $p_{ij}^{x_{ij}}(1-p_{ij})^{x_{ij}}$

We assume that pixels are independent from each other (bad assumption but we don't have any other choice.) ②

$$P(X \mid Y=1) = \prod_{i=1}^{784} p_{i1}^{x_i} (1-p_{i1})^{(1-x_i)}$$

Given observation vector

$$P(X \mid Y=2) = \prod_{i=1}^{784} p_{i2}^{x_i} (1-p_{i2})^{(1-x_i)}$$

⋮

$P(X \mid Y=9)$

$$P(X \mid Y=0) = \prod_{i=1}^{784} p_{i0}^{x_i} (1-p_{i0})^{(1-x_i)}$$

(If U & V are ind. $P(UV) = P(U) \cdot P(V)$)

We obtained the pdf models from the training set.

Decision process: You observe a vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{784} \end{bmatrix}$

How can I decide (or recognize) X? $= \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ from an image.

$$\text{Solution} = \underset{j \in \{0,1,\ldots,9\}}{\arg\max}\ \pi_j\, P(X \mid Y=j)$$

(where π_j is the prior probability of digit j)

* We pick the model producing the highest probability as our answer.

$\pi_j = P(Y=j)$ (In this case $\pi_j \cong \frac{1}{10}$)

3 3

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