Region Covariance Matrix

and Pathological Image Classification

Region Covariance Matrix (ROC)

- ROC can describe image patterns
- Main reference:

Tuzel, O., Porikli, F., & Meer, P. (2006, May). Region covariance: A fast descriptor for detection and classification. In *European conference on computer vision* (pp. 589-600). Springer, Berlin, Heidelberg.

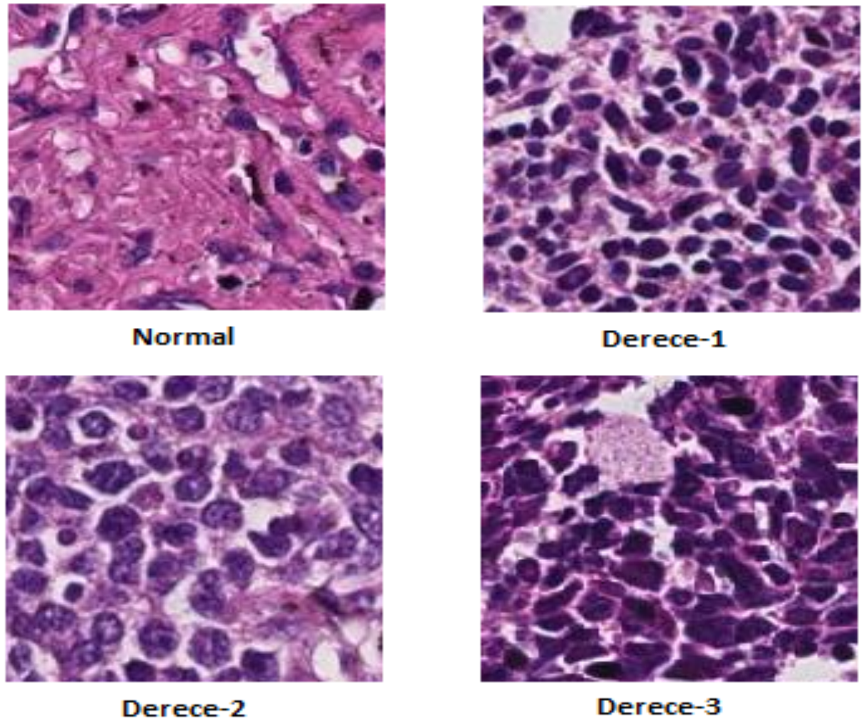
Tuzel, O., Porikli, F., & Meer, P. (2008). Pedestrian detection via classification on riemannian manifolds. *IEEE transactions on pattern analysis and machine intelligence*, *30*(10), 1713-1727.

Porikli, Fatih, and Tekin Kocak. "Robust license plate detection using covariance descriptor in a neural network framework." In *Video and Signal Based Surveillance, 2006. AVSS'06. IEEE International Conference on*, pp. 107-107. IEEE, 2006.

Liver Cancer Pathological Images

Class 0

Class 1



Class 3

Class 4

Four Classes: Regular, Degree 1, Degree 2, Degree 3 and Degree 4

Definition

- Let us assume that we have n=LxK pixels in a given image region R:

$$\mathbf{C}_R = \frac{1}{n-1}\sum_{k=1}^{n}(\mathbf{z}_k - \boldsymbol{\mu})(\mathbf{z}_k - \boldsymbol{\mu})^T$$

where z_k is the feature vector of the k-th pixel.

- We scan the image region whichever way we like

```
• For x=1,L;
    For y=1,K
        k = y+(x-1)*L
        z(k)=F(x,y)
  end; end
```

Feature vector F(x,y) of each pixel:

- Feature vector:

$$F(x,y) = \Big[x \quad y \quad R(x,y) \quad G(x,y) \quad B(x,y) \quad \left|\frac{\partial I(x,y)}{\partial x}\right| \left|\frac{\partial I(x,y)}{\partial y}\right| \left|\frac{\partial^2 I(x,y)}{\partial x^2}\right| \left|\frac{\partial^2 I(x,y)}{\partial y^2}\right| \Big]^T$$

where I(x,y) is the luminance (or gray) value of the image

R(x,y), G(x,y) and (B(x,y) are the red, green and blue pixel values

- C_R is a 9by9 matrix

Filters used in Derivative Computation

- 1st order derivative filter h= [-1 0 1]
- 2nd order derivative filter g=[-1 2 -1]

RGB color space to YUV color space

- Y is the luminance (or gray scale and U and V are called the color difference signals

$$\begin{bmatrix} Y' \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.14713 & -0.28886 & 0.436 \\ 0.615 & -0.51499 & -0.10001 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix},$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.13983 \\ 1 & -0.39465 & -0.58060 \\ 1 & 2.03211 & 0 \end{bmatrix} \begin{bmatrix} Y' \\ U \\ V \end{bmatrix}.$$

- I=Y’
- Transform each pixel to Y,U,V space
- YUV values are also used in region covariance matrix construction

Cost function for the Nearest-Neighbor Classifier

- Generalized Eigenvalue based computation:

$$\rho(\mathbf{C}_1, \mathbf{C}_2) = \sqrt{\sum_{i=1}^{n} \ln^2 \lambda_i(\mathbf{C}_1, \mathbf{C}_2)} \qquad (3)$$

where $\{\lambda_i(\mathbf{C}_1, \mathbf{C}_2)\}_{i=1...n}$ are the generalized eigenvalues of $\mathbf{C}_1$ and $\mathbf{C}_2$, computed from

$$\lambda_i \mathbf{C}_1 \mathbf{x}_i - \mathbf{C}_2 \mathbf{x}_i = 0 \qquad i = 1...d \qquad (4)$$

and $\mathbf{x}_i \neq 0$ are the generalized eigenvectors. The distance measure ρ satisfies the metric axioms for positive definite symmetric matrices $\mathbf{C}_1$ and $\mathbf{C}_2$

Or use any other matrix norm.

Other Matrix norms

- https://en.wikipedia.org/wiki/Matrix_norm
- Element-wise norms

$$||\mathbf{A}||_p = \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} |a_{ij}|^p \right\}^{1/p}$$

$$||\mathbf{A}||_1 = \sum_{i=1}^{M} \sum_{j=1}^{N} |a_{ij}|$$

Table : Success rate for *Nearest Neighbor*

	Class 1	Class 2	Class 3	Class 4	Success rate
Class 1	15/16	1/16	0/16	0/16	93.8% 6.2%
Class2	2/40	36/40	1/40	1/40	90.0% 10.0%
Class 3	0/24	0/24	21/24	3/24	87.5% 12.5%
Class 4	0/24	4/24	3/24	17/24	70.8% 29.2%
					85.6% 14.4%