Example Two classes salmon & seabass

Observed data:

$\omega_1 = sl$: Salmon: $\begin{bmatrix}10\\5\end{bmatrix}, \begin{bmatrix}11\\7\end{bmatrix}, \begin{bmatrix}12\\6\end{bmatrix}$, # of observations in ω_1, $m=3$

$\omega_2 = sb$: Seabass: $\begin{bmatrix}5\\10\end{bmatrix}, \begin{bmatrix}7\\11\end{bmatrix}, \begin{bmatrix}6\\12\end{bmatrix}$ # of observations in $\omega_2 \Rightarrow m=3$

$$\hat{\pi}_{sl} = \frac{3}{6} = \hat{\pi}_{sb} = \hat{\pi}_1 = \hat{\pi}_2$$ (Prior probabilities)

Mean vectors:

$$\hat{\mu}_1 = \frac{1}{3}\left(\begin{bmatrix}10\\5\end{bmatrix} + \begin{bmatrix}11\\7\end{bmatrix} + \begin{bmatrix}12\\6\end{bmatrix}\right) = \begin{bmatrix}11\\6\end{bmatrix}, \quad \hat{\mu}_2 = \frac{1}{3}\left(\begin{bmatrix}5\\10\end{bmatrix} + \begin{bmatrix}7\\11\end{bmatrix} + \begin{bmatrix}6\\12\end{bmatrix}\right) = \begin{bmatrix}6\\11\end{bmatrix}$$

Covariance matrices: Class 1:

$$\hat{\Sigma}_1 = \frac{1}{3-1}\sum_{i=1}^{3}(\vec{x}_1^{(i)} - \hat{\mu}_1)(\vec{x}_1^{(i)} - \hat{\mu}_1)^T = \frac{1}{2}\left(\begin{bmatrix}-1\\-1\end{bmatrix}[-1\ -1] + \begin{bmatrix}0\\1\end{bmatrix}[0\ 1] + \begin{bmatrix}1\\0\end{bmatrix}[1\ 0]\right)$$

$$\hat{\Sigma}_1 = \frac{1}{2}\left(\begin{bmatrix}1&1\\1&1\end{bmatrix} + \begin{bmatrix}0&0\\0&1\end{bmatrix} + \begin{bmatrix}1&0\\0&0\end{bmatrix}\right) = \begin{bmatrix}2&1\\1&2\end{bmatrix}\frac{1}{2} = \begin{bmatrix}1&0.5\\0.5&1\end{bmatrix}$$

$\sigma_{11}^2 = 1$ is the variance of the first entry of the feature vector for class 1.

class 1

$\sigma_{12}^2 = 1$ " " " " " 2nd " " for class 1.

" " " 2nd entry of the feature vector.

1st entry of the feature vector

$E[(x_1 - \mu_{11})(x_2 - \mu_{12})]$ is estimated as 0.5.

Class 2:

$$\hat{\Sigma}_2 = \frac{1}{3-1}\sum_{i=1}^{3}(\vec{x}_2^{(i)} - \hat{\mu}_2)(\vec{x}_2^{(i)} - \mu_2)^T = \frac{1}{2}\left(\begin{bmatrix}-1\\-1\end{bmatrix}[-1\ -1] + \begin{bmatrix}0\\1\end{bmatrix}[0\ 1] + \begin{bmatrix}1\\0\end{bmatrix}[1\ 0]\right)$$

$$\hat{\Sigma}_2 = \begin{bmatrix}1&0.5\\0.5&1\end{bmatrix}, \quad \sigma_{21}^2 = 1, \quad \sigma_{22}^2 = 1$$

Variance of the 1st entry of the feature vector for class 2.

It turns out that

$$\hat{\Sigma}_1 = \hat{\Sigma}_2$$

Generative Models based on Gaussians.

ω_1 has a multivariate " distribution

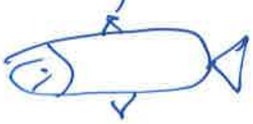
P_1 : $N\left(\begin{bmatrix}11\\6\end{bmatrix}, \begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix}\right)$

ω_2 " " " " " P_2: $N\left(\begin{bmatrix}6\\11\end{bmatrix}, \begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix}\right)$

This is the end of training!

Testing phase:

Catch a fish



$\rightarrow \vec{x} = \begin{bmatrix}7\\12\end{bmatrix}$

We calculate the likelihood of X for class 1 and 2.

$$P_1(\bar{x}) = P(x|\omega_1) = \frac{1}{\sqrt{(2\pi)^2|\hat{\Sigma}_1|}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix}7\\12\end{bmatrix} - \begin{bmatrix}11\\6\end{bmatrix}\right) \hat{\Sigma}_1^{-1} [-4 \quad 6]\right)$$

$$P_2(\bar{x}) = P(X|\omega_2) = \frac{1}{\sqrt{(2\pi)^2|\hat{\Sigma}_2|}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix}7\\12\end{bmatrix} - \begin{bmatrix}6\\11\end{bmatrix}\right) \hat{\Sigma}_2^{-1} [1 \quad 1]\right)$$

$\leftarrow$ Determinant of Σ_2 matrix.

where $\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}7\\12\end{bmatrix} - \begin{bmatrix}6\\11\end{bmatrix}$, $\begin{bmatrix}-4\\6\end{bmatrix} = \begin{bmatrix}7\\12\end{bmatrix} - \begin{bmatrix}11\\6\end{bmatrix}$

$$\pi_2 P_2(\vec{x}) > \pi_1 P_1(\bar{x}) \Rightarrow \text{class } 2$$

$$0.5 \times 0.094 > 0.5 \quad 1.8 \times 10^{-23}$$

In Practice ~~[illegible]~~ we also add cI to Σ matrices.

$$\Sigma_1 = \begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix} \rightarrow \Sigma_1 + cI = \begin{bmatrix}1+c & 0.5\\0.5 & 1+c\end{bmatrix}$$

small number: $c = 0.0001$

It should be smaller than σ_1, σ_2