Fuzzy inference systems

This document contains a method which can solve problems in a way that is closer to human logic; to express this method, we want to examine a hypothetical system for automatic car braking to describe the mechanism of this method. This document consists of 5 parts which explains the problem and defines linguistic terms, then the fuzzy sets defined according to those linguistic terms; the third part rules were determined and the output membership value is determined.

I. Problem description

Let's assume that we have a car with sensors for detecting the distance and amount of slippery ground and we want to program it using a fuzzy control system in such a way that it activates the car's brakes when the risk of an accident increases. It should be able to calculate accident risk with use of speed, obstacle distance and slippery amount. So, our linguistic terms include Speed ***s***, Slippery ***sl,*** Distance ***d.***

Main goal: Occupant's safety. (Stop on time)

II. Problem description

In this section membership functions for linguistic values are defined. First linguistic value divided into 3 membership functions, each of which represents as slow, medium and fast speed. Slow membership function determines as F_s which represents membership value of car current speed.

$$F_s(x) = (1 - x)^4 \tag{1}$$

In Eq.1, x represents speed value. for fast membership function we use F_f function which represent in Eq.2 and includes more membership space due to increasing the sensitivity of the model to high speed:

$$F_f(x) = 1 - (1 - x^2)^2 \tag{2}$$

and finally for medium speed value we represent F_m function which represent in Eq.3:

$$F_m(x) = -(x^2 - 1)^2 + 1 \tag{3}$$

Range of x values determined between [0,1] and Fig.1 represent this function:

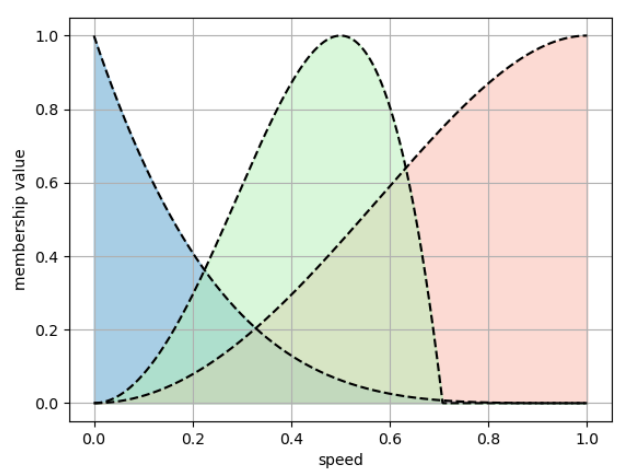


Figure 1 speed membership functions

Second linguistic value is divided into 3 membership functions, each of which represent as low, medium and high slippery level. Low membership function determines as F_l which represents membership value of road slippery level.

$$F_l(x) = \cos(3x) \tag{4}$$

In Eq.4, x represents slippery value. for medium membership function we use F_m function which represent in Eq.5:

$$F_m(x) = -3|x - 0.5| + 1 \tag{5}$$

and finally for medium speed value we represent F_h function which represent in Eq.6:

$$F_h(x) = x^3 \tag{6}$$

Range of x values determined between [0,1] and Fig.2 represent those function:

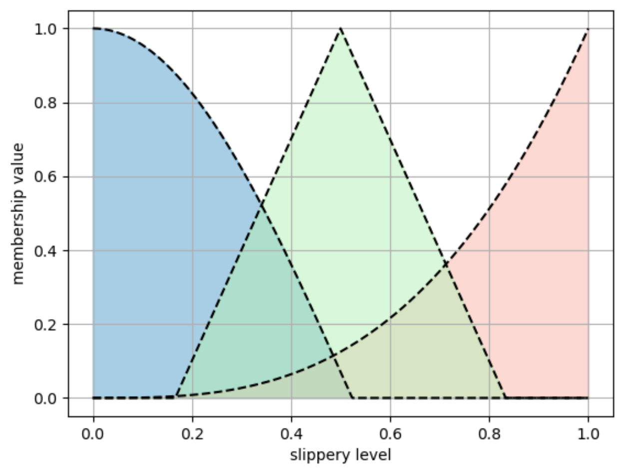


Figure 2 slippery membership functions

Third linguistic value is divided into 3 membership functions, each of which represent as far, middle and close distance level. far membership function determines as F_{fa} which represents membership value of far distance level.

$$F_{fa}(x) = \cos(3x - 3) \tag{7}$$

In Eq.7, x represents slippery value. for medium membership function we use F_{md} function which represent in Eq.8:

$$F_{md}(x) = \sin(5x - 1.2) \tag{8}$$

and finally for medium speed value we represent F_c function which represent in Eq.9:

$$F_c(x) = -(x - 0.1)^3 + 1 \tag{9}$$

Range of x values are determined between [0,1] and Fig.3 represent those function:

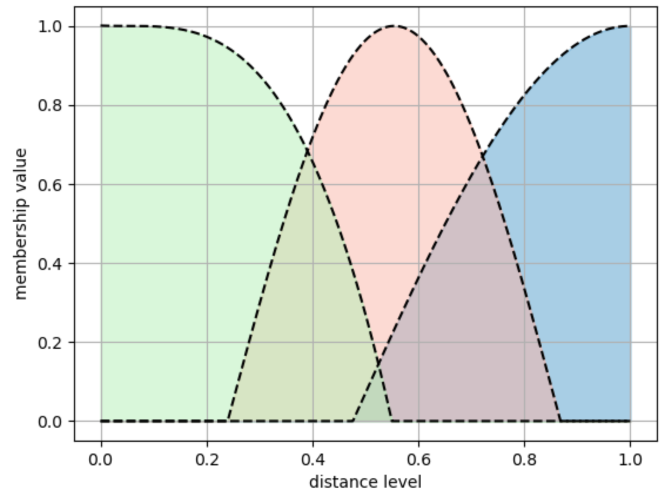


Figure 3 Distance membership functions

Last linguistic value which defines as output value for system, is divided into 3 membership functions and each of which represent as Low, medium and high risk. Very low membership function determines as F_{rl} which represents membership value of car accidence risk.

$$F_{rl}(x) = -|10x - 1| + 1 \tag{10}$$

In Eq.10, x represents risk value. for medium membership function we use F_{rm} function which represent in Eq.11:

$$F_{rm}(x) = -(5x - 2)^2 + 1 \tag{11}$$

and finally for high-risk value we represent F_{rh} function which represent in Eq.12:

$$F_{rh}(x) = -|4x - 3| + 1 \tag{11}$$

Range of x values determined between [0,1] and Fig.4 represent those function:

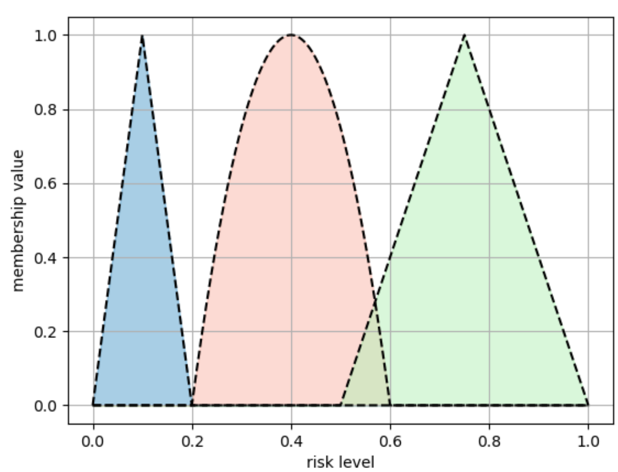


Figure 4 Risk membership function

III. Definition of rules

In this section, we can define several rules, some of which include the following list:

- **If speed is high and slippery is high and distance is close then risk is high.**
- **If speed is low and slippery is high and distance is medium then risk is medium.**
- **If speed is medium and slippery is low and distance is far then risk is low.**
- **If speed is low and slippery is medium and distance is close then risk is medium.**

By using these rules and membership functions defined in the previous section, a fuzzy inference system can be designed.

IV. Experimental results

Suppose the input of the fuzzy system is 0.8 for the speed value, 0.6 for the distance and 0.8 for the slippery level. The first step in the designed fuzzy system is to find the membership value of the inputs in each of the membership functions. For this we need to get the following values through the membership functions:

Speed	$\mu(x = low) = 0$ $\mu(x = medium) = 0.09$ $\mu(x = high) = 0.51$
Slippery	$\mu(x = low) = 0$ $\mu(x = medium) = 0.7$ $\mu(x = high) = 0.21$
Distance	$\mu(x = far) = 0.78$ $\mu(x = middle) = 0.14$ $\mu(x = close) = 0$

Then, through the proposed rules, the membership value is calculated for each risk level, and through the obtained values, the following diagram is obtained, on which we can estimate the amount of risk by calculating the ROG.

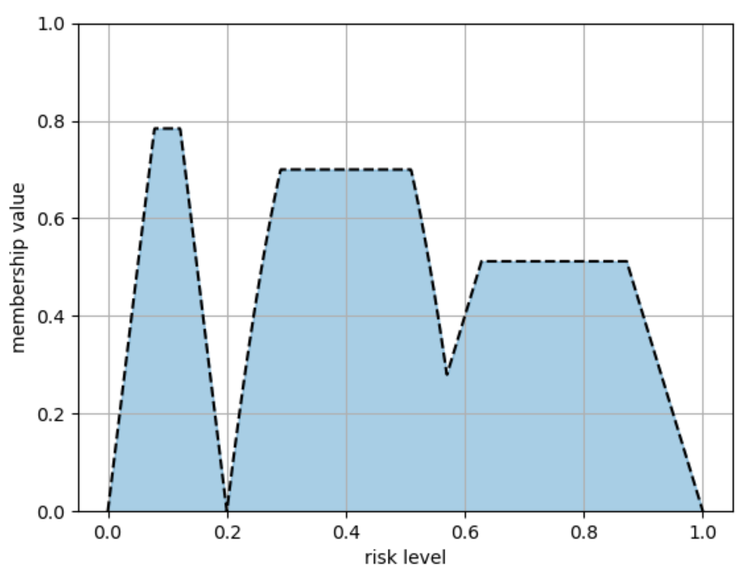
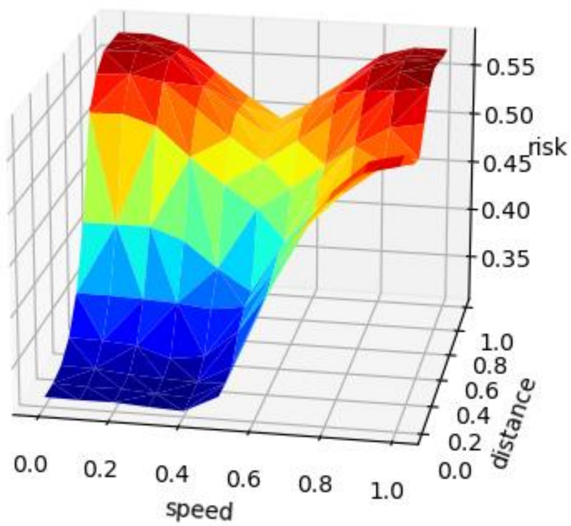
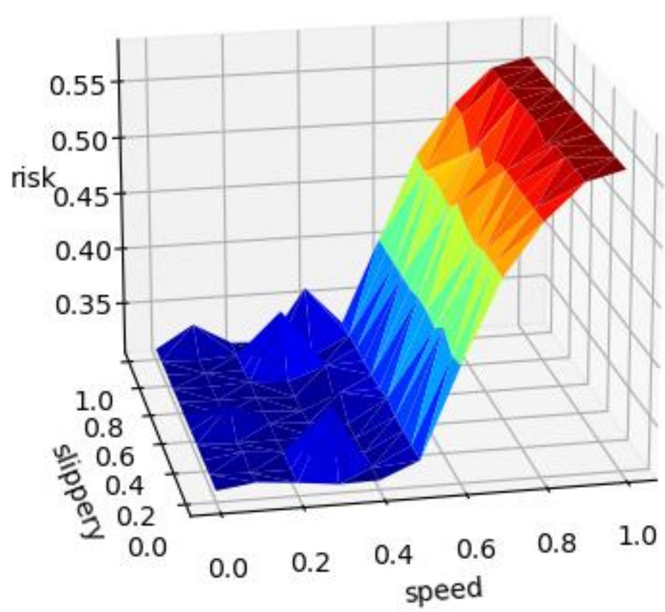


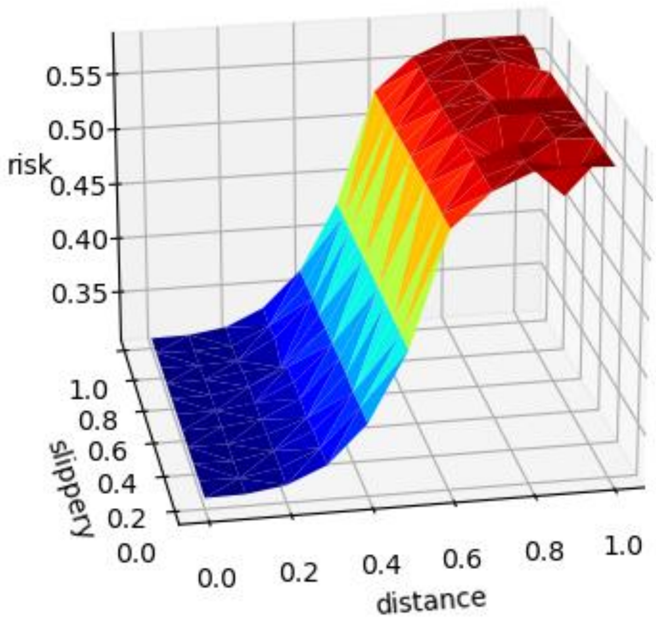
Figure 5 Final output

$$CoG = \frac{\int_b^a \mu_z(x) \, . \, x \, dx}{\int_b^a \mu_z(x) \, dx} \tag{11}$$

$$CoG = 0.47$$







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