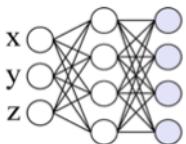


# Digital Geometry Processing Neural Fields

Andrea Tagliasacchi  
Simon Fraser University



# NEURAL FIELDS IN COMPUTER VISION

Full-Day Tutorial, June 20<sup>th</sup>, 2022



[neuralfIELDS.cs.brown.edu/cvpr22](http://neuralfIELDS.cs.brown.edu/cvpr22)



Yiheng Xie



Towaki Takikawa



Shunsuke Saito



Or Litany



James Tompkin

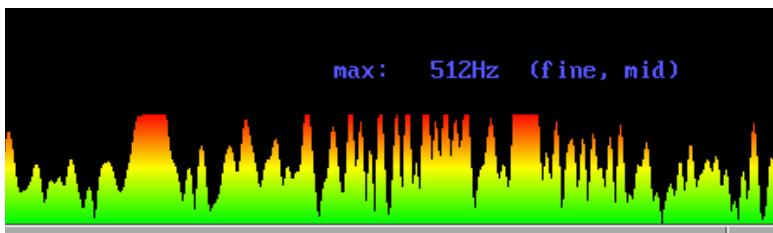
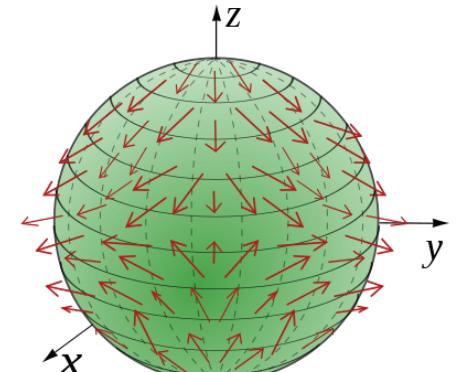
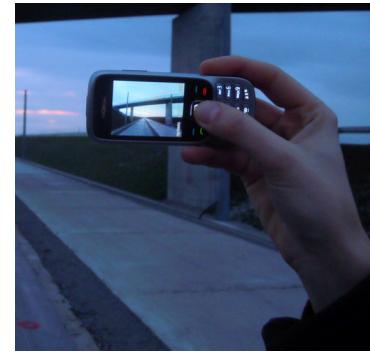
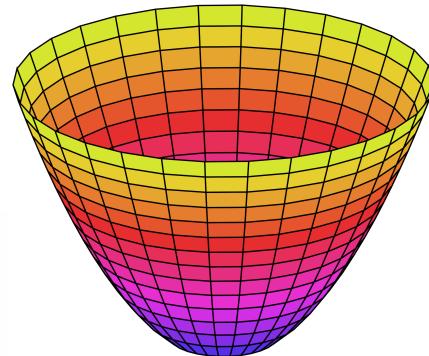


Vincent Sitzmann



Srinath Sridhar



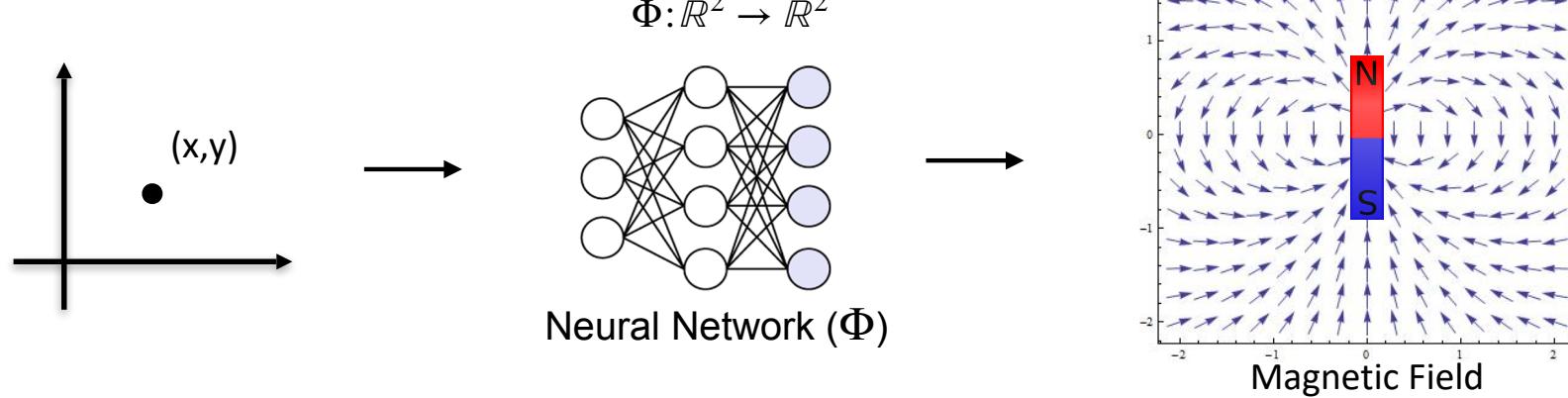


Audio  
(map 1D → 1D)

# Fields

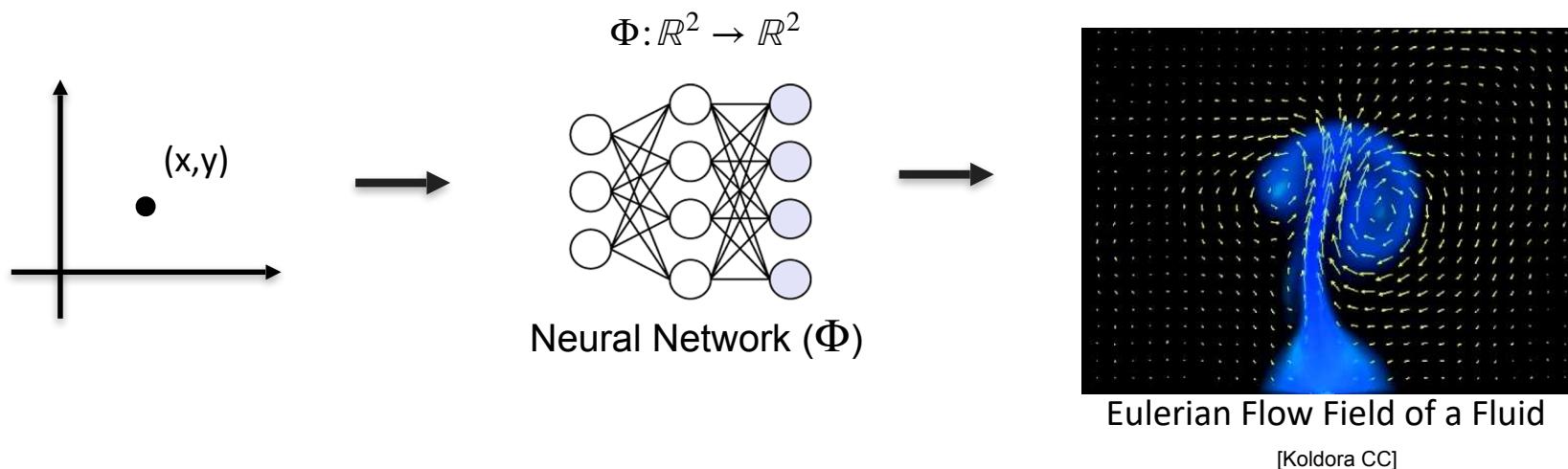
# What is a Neural Field

- Mapping between a coordinate and a physical quantity
  - e.g. the magnetic field induced by a magnet



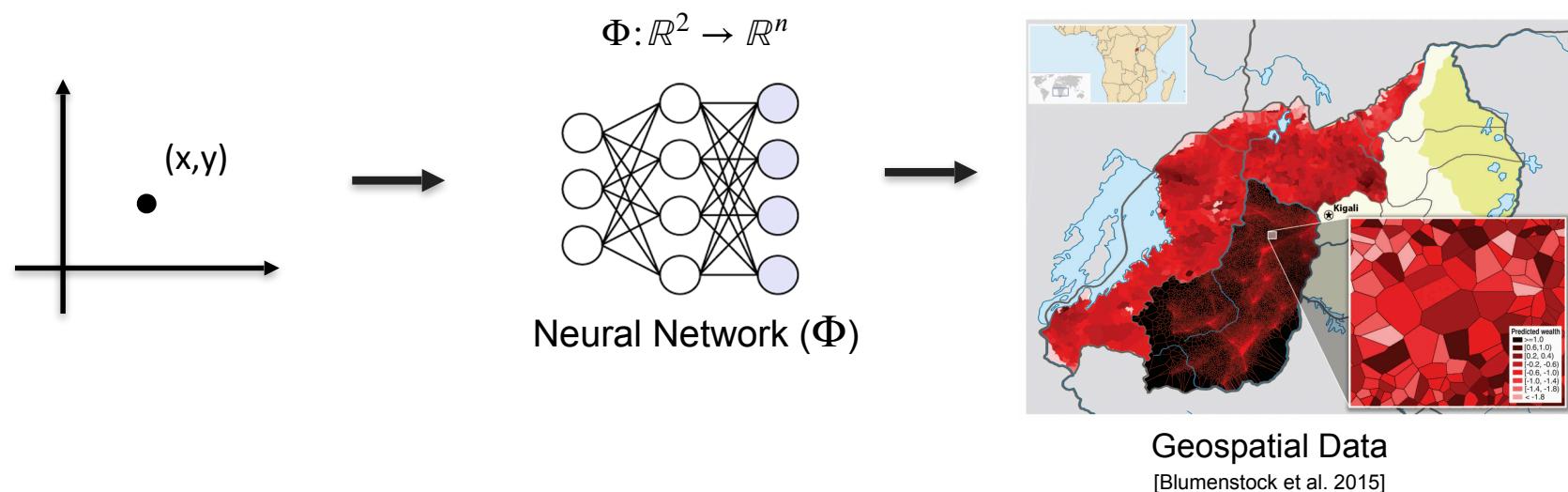
# What is a Neural Field

- Mapping between a coordinate and a physical quantity
  - e.g. the advection field within a fluid simulation package



# What is a Neural Field

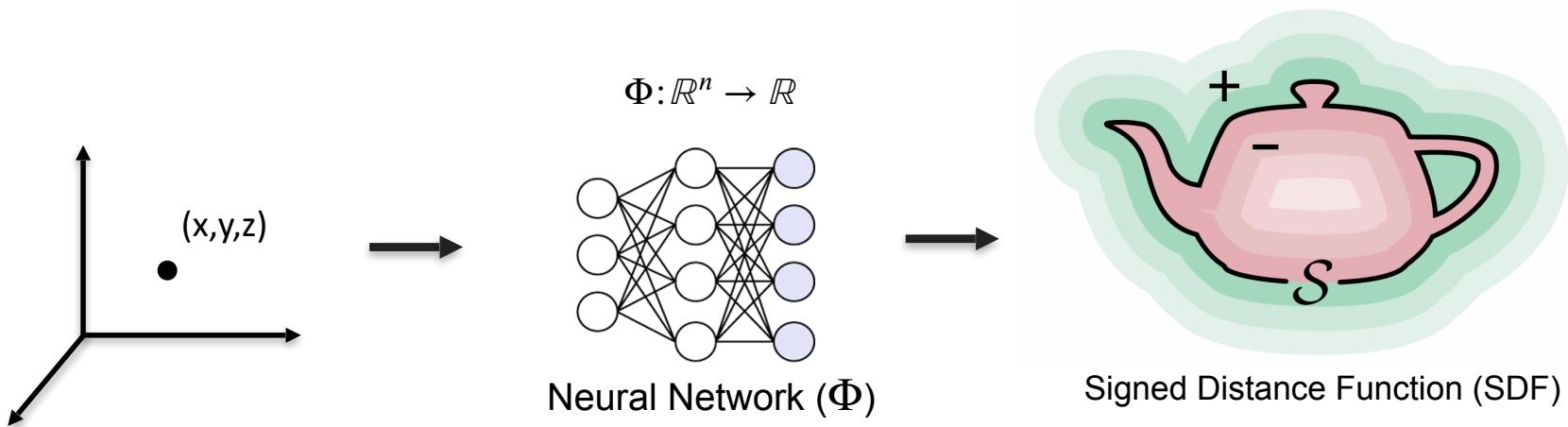
- Mapping between a coordinate and a physical quantity
  - e.g. the average wealth in a country



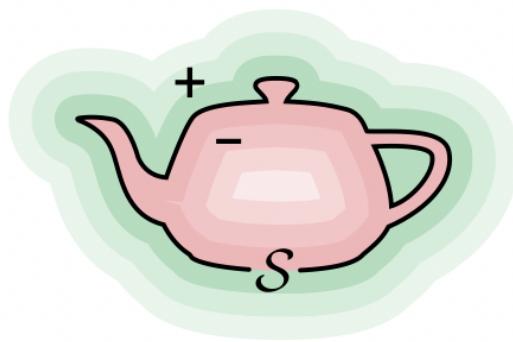
Geospatial Data  
[Blumenstock et al. 2015]

# What is a Neural Field

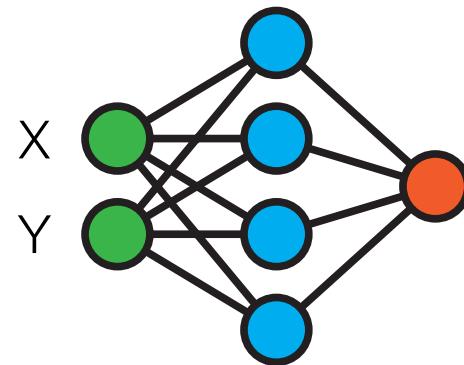
- Mapping between a coordinate and a physical quantity
  - e.g. the signed distance function to the boundary of an object



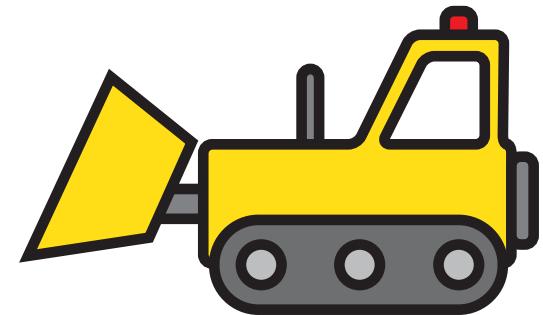
# Terminology & Misnomers



Implicit Neural  
Representations



Coordinate-based  
Neural Networks

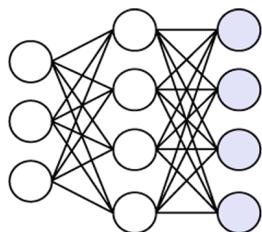


NeRFs

# Definitions

- Definition 1:  
A **field** is a quantity defined for all spatial and / or temporal coordinates.
- Definition 2:  
A **neural field** is a field that is [partially] parameterized by a neural network.

MLP  
Multi Layer Perceptron



coordinate input

$$z^{(1)} = x$$

nonlinearity

$$z^{(i+1)} = \sigma \left( W^{(i)} z^{(i)} + b^{(i)} \right), \underbrace{i = 1, \dots, k - 1}_{\text{induction}}$$

field output

$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$

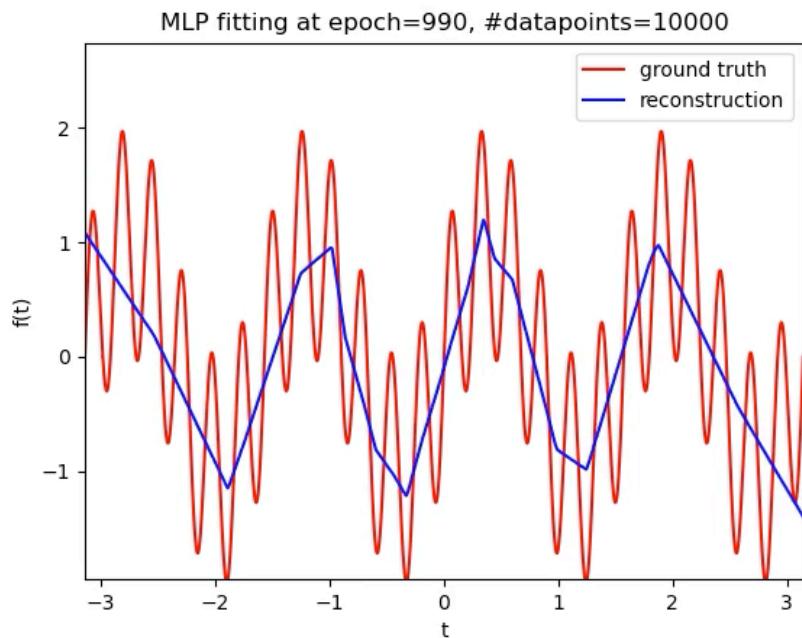
# Coding Exercise – Task 1

- Fit a signal by optimizing the MLP parameters
  - optimize MLP weights ( $W$ ) and biases ( $b$ )
  - training data:  $\{(x_k, f(x_k))\}_{k=1}^K$

$$z^{(1)} = x$$

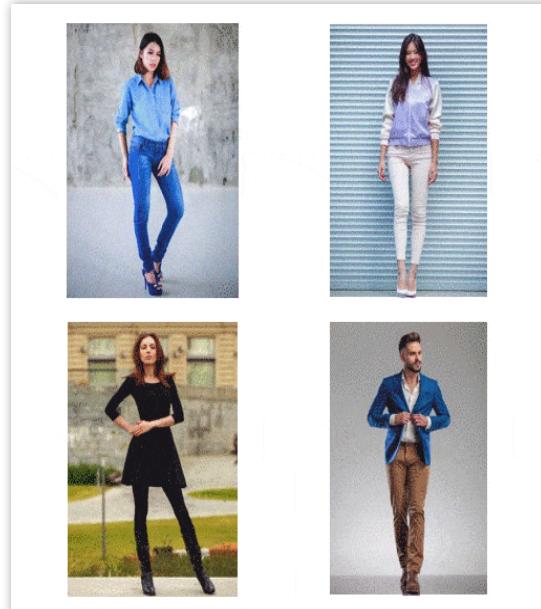
$$z^{(i+1)} = \sigma(W^{(i)} z^{(i)} + b^{(i)}), \quad i = 1, \dots, k-1$$

$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$



# Applications: Computer Vision

- e.g. lift 3D structures out of 2D images



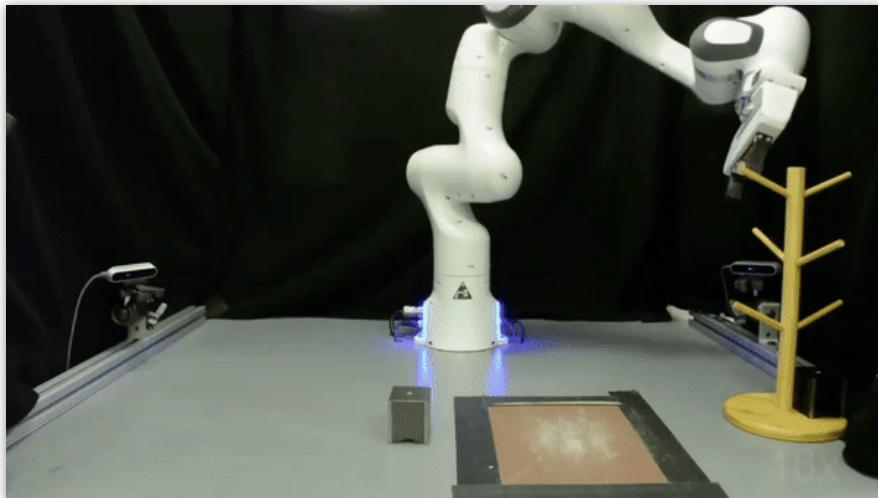
[Saito et al. 2020 PiFu]



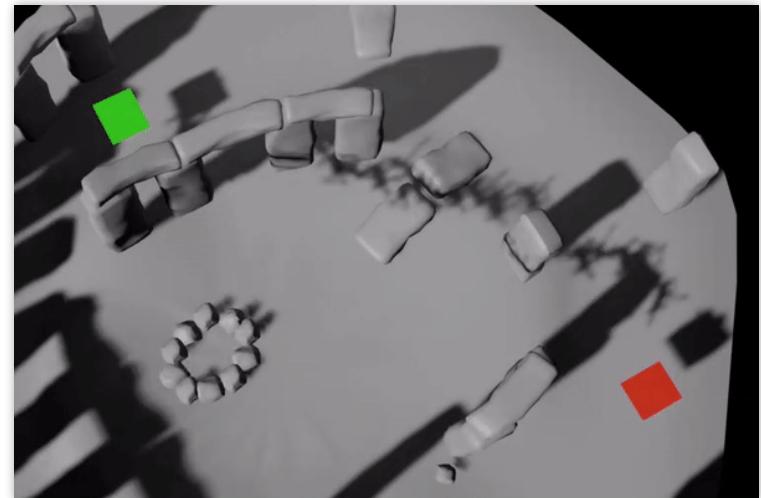
[Chan et al. 2021 EG3D]

# Applications: Robotics

- e.g. few-shot learning and motion planning

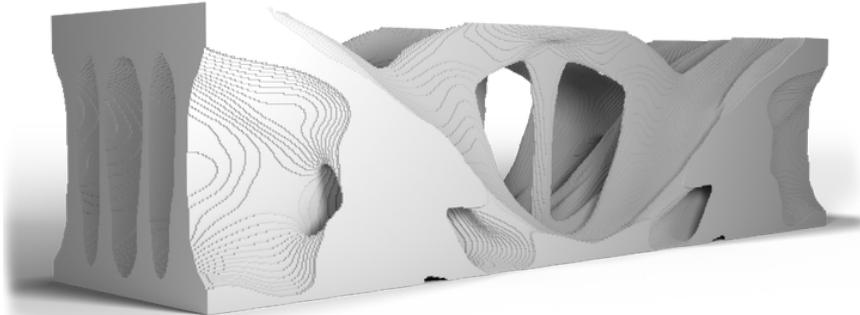


[Simeonov et al. 2021 (Neural Descriptor Fields)]

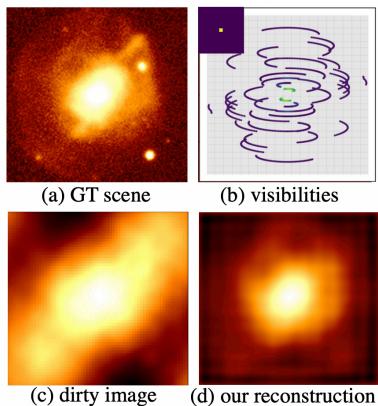


[Adamkiewicz, Chen et al. 2021]

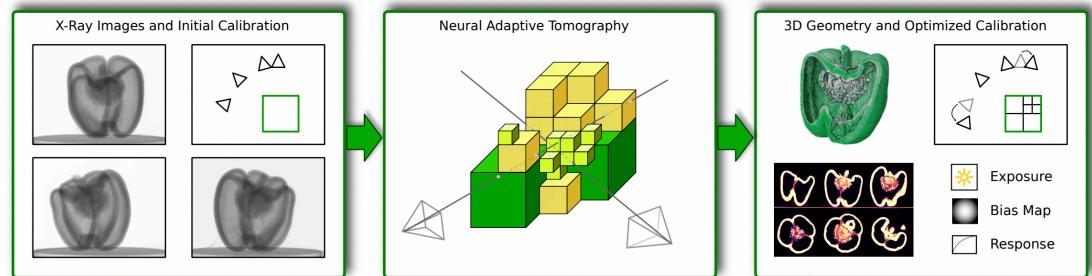
# Applications: Physics



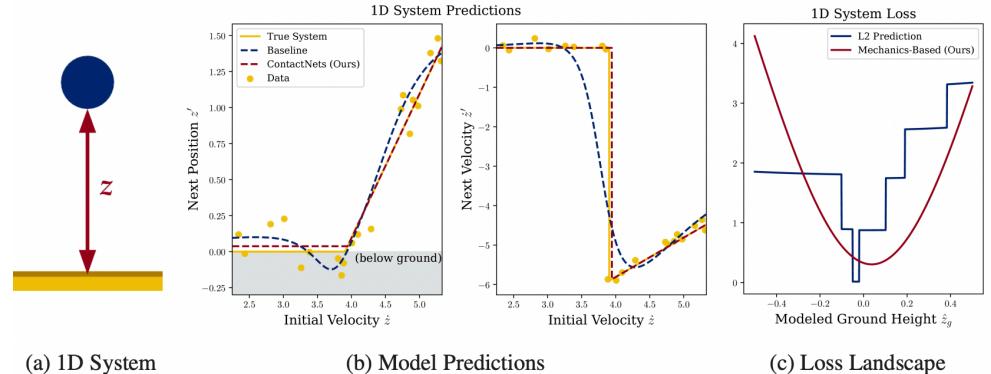
Topology Optimization [Doosti et al. 2021]



Astronomical Interferometry [Wu et al. 2021]



Tomographic Reconstruction [Ruckert et al. 2022]



Contact Dynamics [Pfrommer, Halm et al. 2022]

# Applications: ML / Signal Processing

## Rethinking positional encoding

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## Implicit Neural Representations with Periodic Activation Functions

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### Abstract

Implicitly defined, continuous, differentiable signal representations parameterized by neural networks have emerged as a powerful paradigm, offering many possible benefits over conventional representations. However, current network architectures for such implicit neural representations are incapable of modeling signals with fine detail, and fail to represent a signal's spatial and temporal derivatives, despite

## MULTIPLICATIVE FILTER NETWORKS

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Although deep learning has achieved state-of-the-art performance in various domains, it often requires high-dimensional input representations to approximate complex functions. This paper proposes a multiplicative filter network (MFN) that uses periodic activation functions to learn periodic features from low-dimensional inputs. MFN consists of two parallel branches that process the input in different dimensions. The outputs of these branches are multiplied together to produce the final output. This approach allows MFN to learn periodic features from low-dimensional inputs while maintaining the expressiveness of deep learning models.

### INTRODUCT

Neural networks are widely used in various applications, such as function approximation, regression, and classification. In recent years, there has been a significant interest in learning periodic functions from low-dimensional inputs. This is because periodic functions are common in many real-world applications, such as signal processing, image analysis, and computer graphics. One way to learn periodic functions from low-dimensional inputs is to use periodic activation functions, such as sine and cosine functions. These functions have the ability to capture periodic patterns in the input data. Another way is to use convolutional neural networks (CNNs), which are able to learn periodic features from low-dimensional inputs by using learned filters. However, CNNs require high-dimensional input representations to achieve good performance. This is because CNNs need to learn local features, which are represented by small receptive fields. As the receptive field size increases, the number of parameters required for the network also increases, leading to a curse of dimensionality.

## Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains

Matthew Tancik<sup>1,\*</sup> Pratul P. Srinivasan<sup>1,2,\*</sup> Ben Mildenhall<sup>1,\*</sup> Sara Fridovich-Keil<sup>1</sup>

Nithin Raghavan<sup>1</sup> Utkarsh Singhal<sup>1</sup> Ravi Ramamoorthi<sup>3</sup> Jonathan T. Barron<sup>2</sup> Ren Ng<sup>1</sup>

<sup>1</sup>University of California, Berkeley    <sup>2</sup>Google Research    <sup>3</sup>University of California, San Diego

### Abstract

We show that passing input points through a simple Fourier feature mapping enables a multilayer perceptron (MLP) to learn high-frequency functions in low-dimensional problem domains. These results shed light on recent advances in computer vision and graphics that achieve state-of-the-art results by using MLPs to represent complex 3D objects and scenes. Using tools from the neural tangent kernel (NTK) literature, we show that a standard MLP fails to learn high frequencies both in theory and in practice. To overcome this spectral bias, we use a Fourier feature mapping to transform the effective NTK into a stationary kernel with a tunable bandwidth. We suggest an approach for selecting problem-specific Fourier features that greatly improves the performance of MLPs for low-dimensional regression tasks relevant to the computer vision and graphics communities.

### 1 Introduction

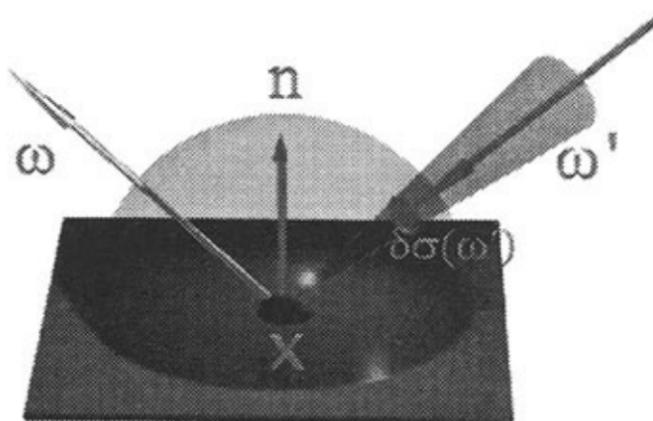
A recent line of research in computer vision and graphics replaces traditional discrete representations of objects, scene geometry, and appearance (*e.g.*, meshes and voxel grids) with continuous functions

# Applications: Computer Graphics

- Render a scene from a novel viewpoint (i.e. Novel View Synthesis)

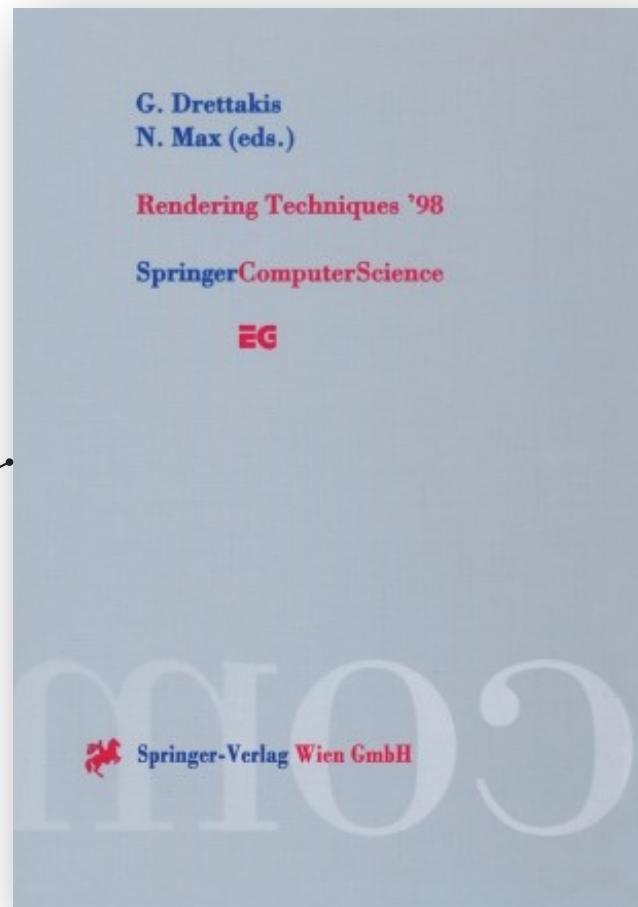


# Cambrian Explosion

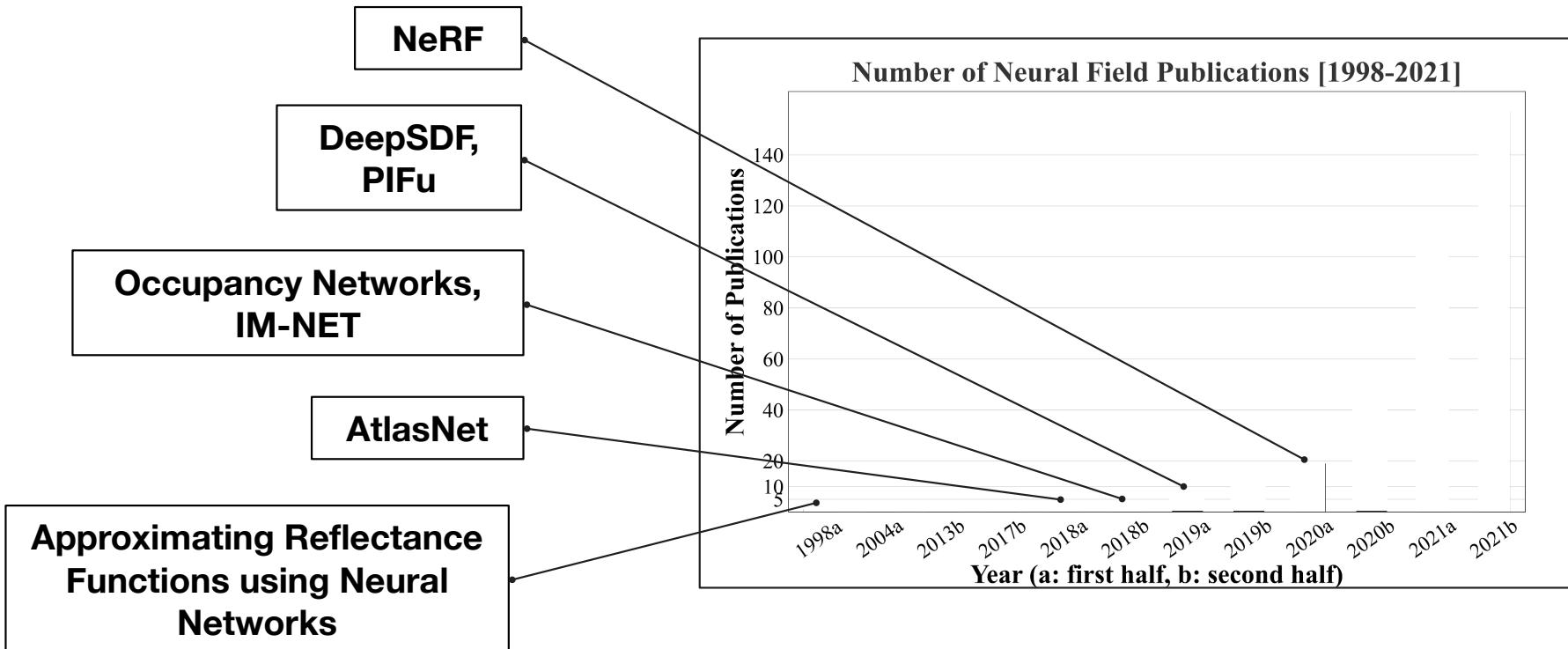


[Gargan and Neelamkavil 1998]

**Approximating Reflectance  
Functions using Neural  
Networks**



# Cambrian Explosion

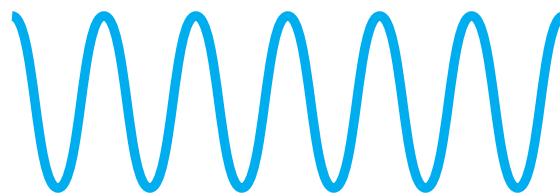


# Outline

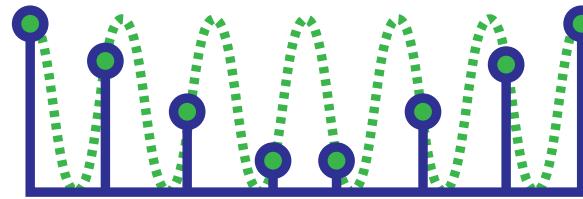
- Signals
- Forward map
- Architectures

# Signals – Representations

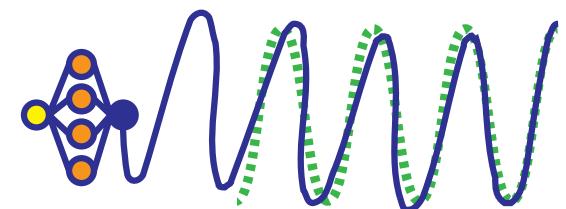
Many different types of signals:



Continuous

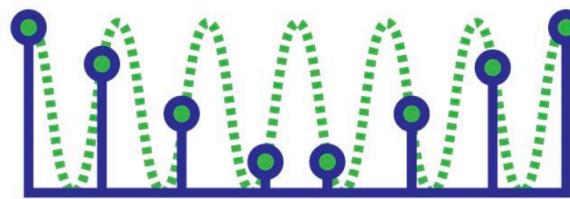


Discrete



Neural

# Signals – Discrete



$$f_x \in \mathbb{R}^m$$

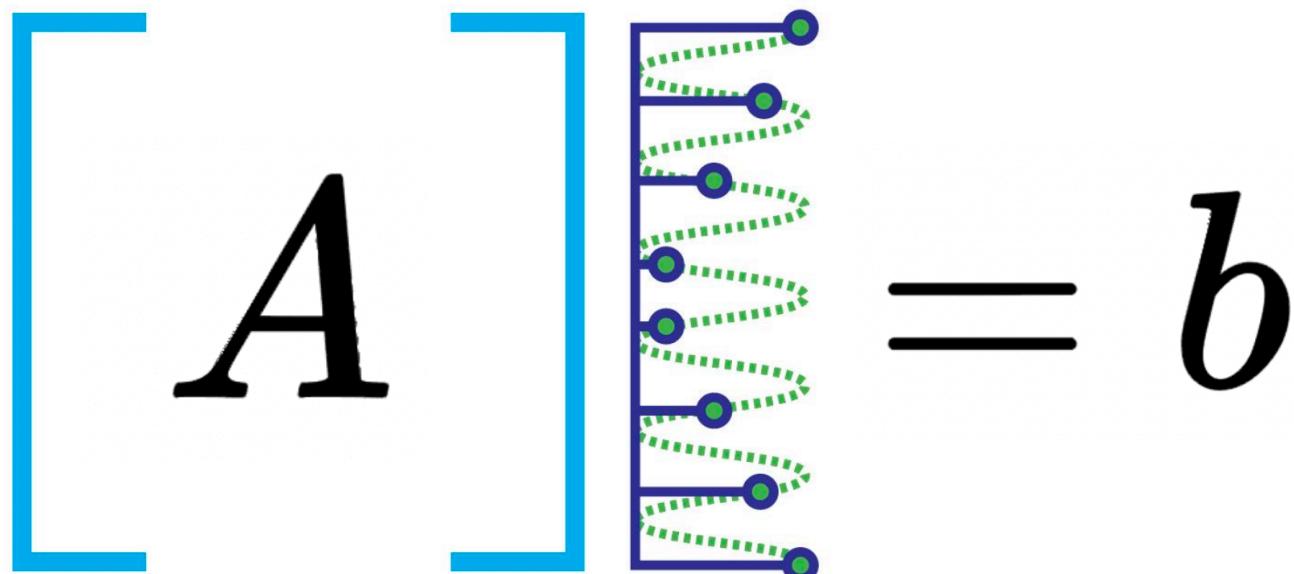
# Signals – Fourier Transform

- Ties together **discrete / continuous** representations



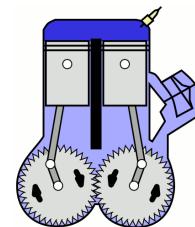
# Signals – Linear Algebra

- Convenient manipulation frameworks based on (sparse) linear algebra

$$A = b$$


# Signals – Properties

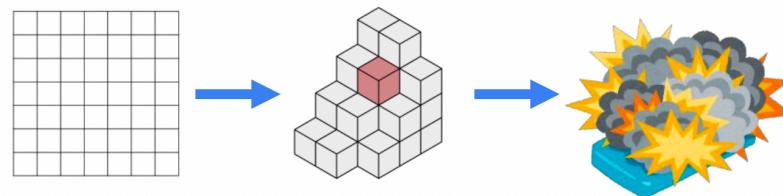
1. Compactness



2. Regularization

$$\operatorname{argmin}_x \|y - F(x)\| + \lambda P(x).$$

3. Domain Agnostic



# Signals – Compactness

- How much does it cost to represent a signal?
  - compression leverages coherence of data in space (“intra-frame” codecs)



[Chen et al.]

$$1024 \times 1024 \times 3 = 3 \text{ megabytes}$$

# Signals – Compactness

- How much does it cost to represent a signal? (2h movie @ 30fps)
  - compression also leverages coherence of frames in time (“inter-frame” codecs)

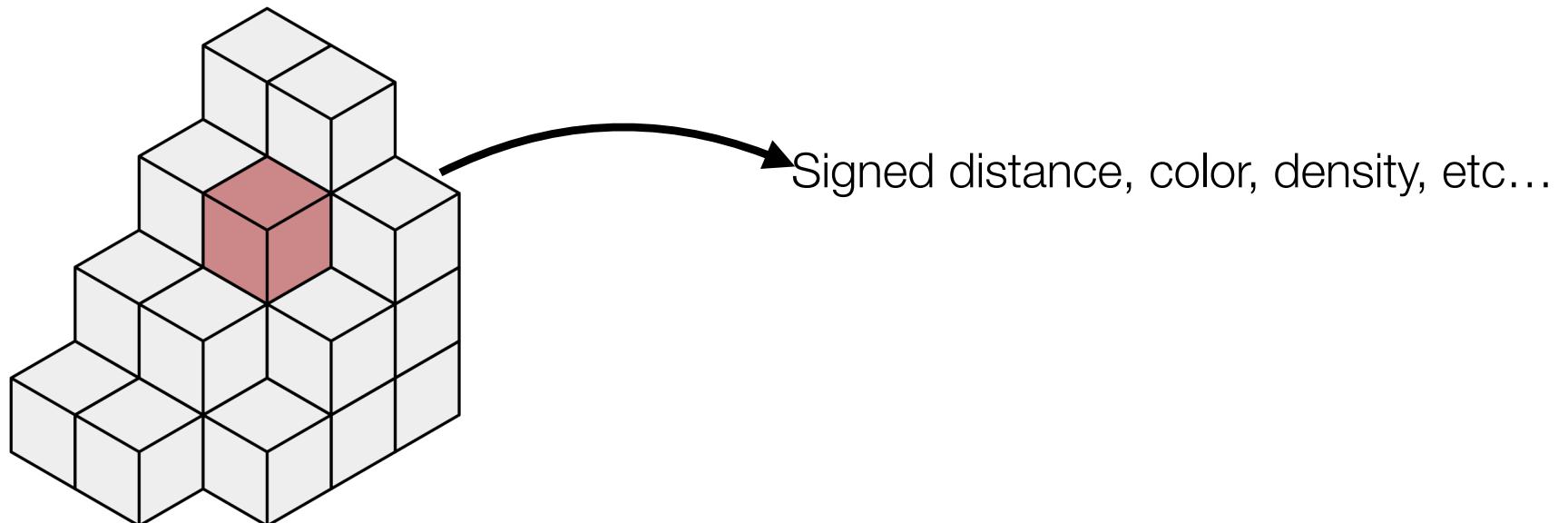


$$1024 \times 1024 \times 3 \times 7200 \text{ seconds} \times 30 \frac{\text{frames}}{\text{seconds}} = 0.68 \text{ terabytes}$$

[Chen et al.]

# Signals – Compactness

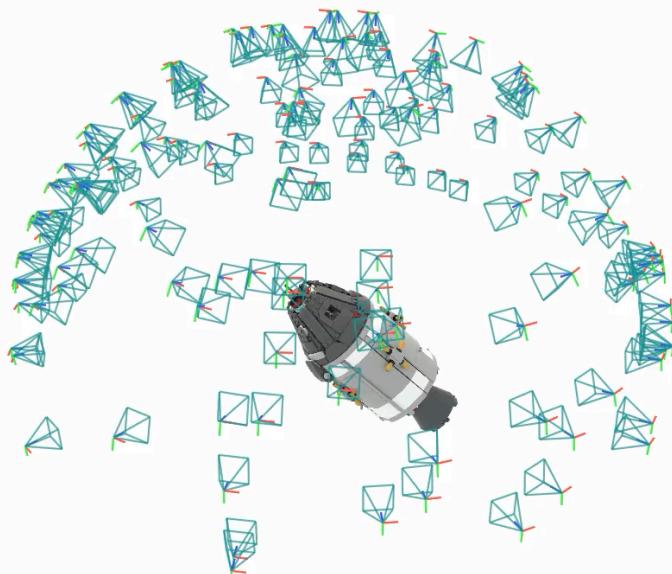
- Signals in 3D are generally sparse...
  - usually 2D manifolds embedded in 3D space
  - a dense 3D grid? **curse of dimensionality**



Very expensive in storage!

# Signals – Compactness

- Application: reconstructing 3D from multiview images



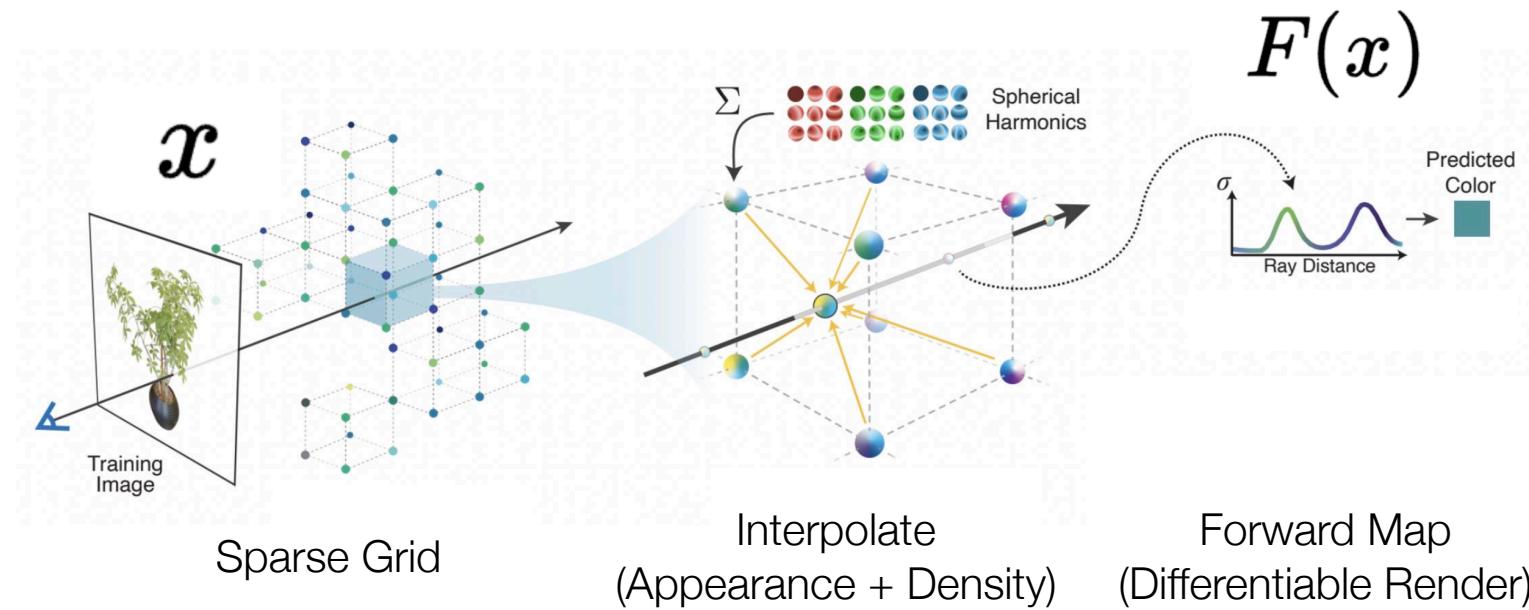
Signal of Interest: 3D Geometry/Appearance

Signal Available: Multiview Images

Inverse Problem:  $\operatorname{argmin}_x \|y - F(x)\|$

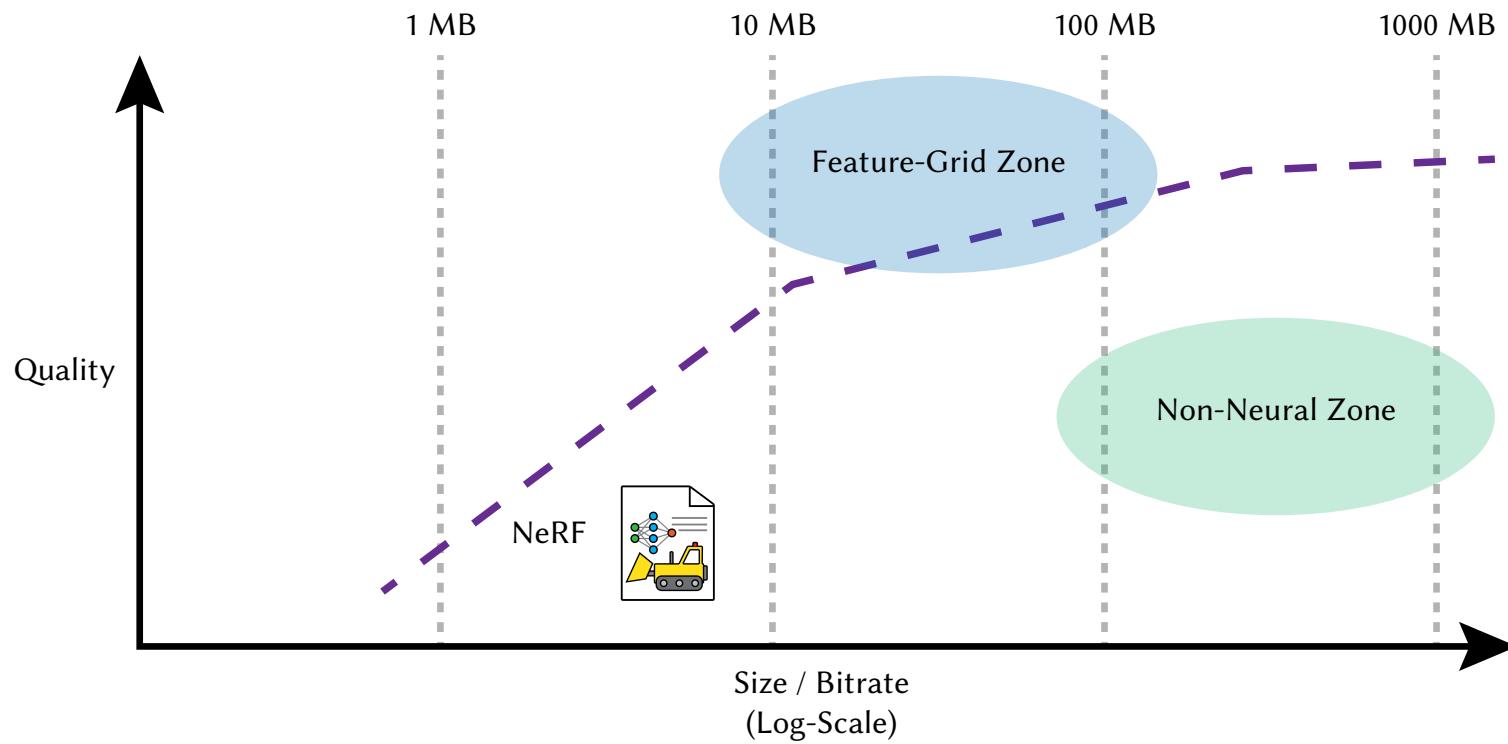
# Signals – Compactness

- Application: reconstructing 3D from multiview images
  - fast to train, but still **expensive in memory!**



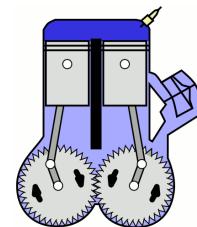
# Signals – Compactness

- Application: reconstructing 3D from multiview images
  - objective: get a better **rate-distortion** curve (closer to top-left of diagram)



# Signals – Properties

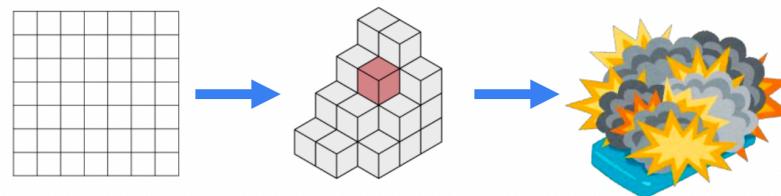
1. Compactness



2. Regularization

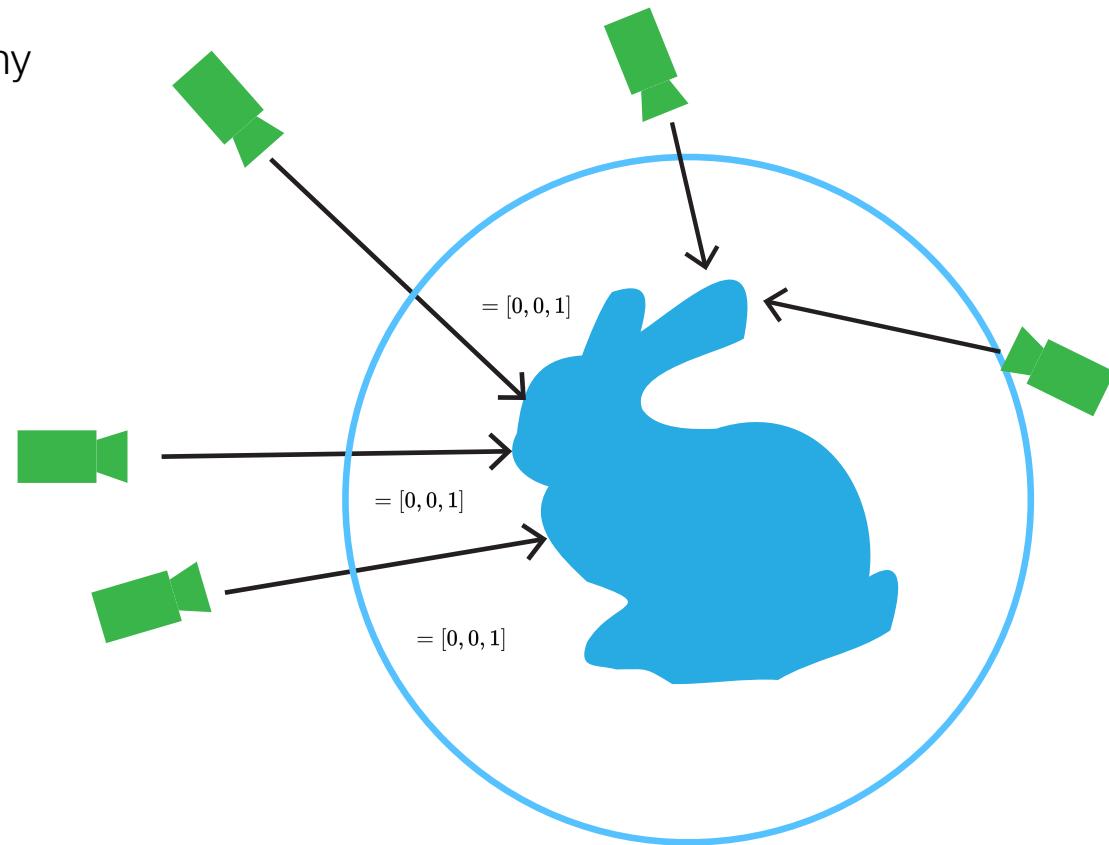
$$\operatorname{argmin}_x \|y - F(x)\| + \lambda P(x).$$

3. Domain Agnostic



# Signals – Regularization

- Many problems are under-constrained
  - simple 2D example: a solid blue 2D bunny
- How fix?
  - add more observations? maybe...
  - regularize the problem? yes!



# Signals – Regularization

- How to deal with underconstrained problems?
  - Bayesian optimization: Maximum a Posteriori (MAP)

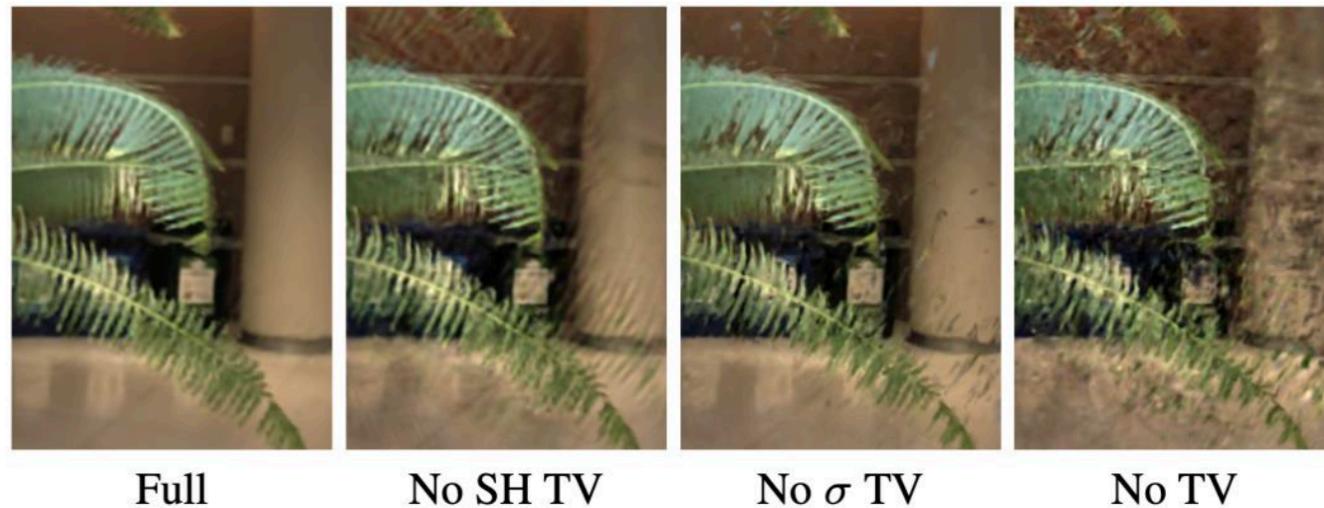
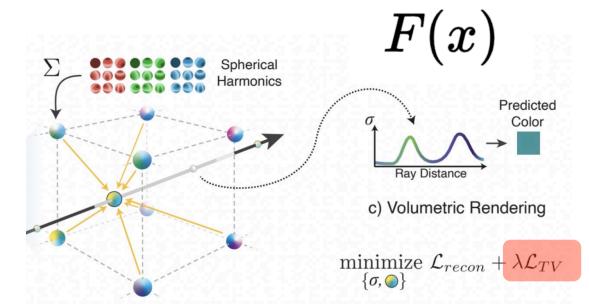
$$\operatorname{argmin}_x \|y - F(x)\| + \lambda P(x).$$

The diagram shows the MAP optimization equation with three red annotations:

- An upward arrow from the left side of the first term  $\|y - F(x)\|$  points to the text "find model parameters".
- An upward arrow from the right side of the second term  $P(x)$  points to the text "that reproduce my data".
- A downward arrow from the coefficient  $\lambda$  points to the text "controls balance data vs. prior".
- An upward arrow from the right side of the second term  $P(x)$  points to the text "so that the model parameters are plausible".

# Signals – Regularization

- How to deal with underconstrained problems?
  - Bayesian optimization: Maximum a Posteriori (MAP)
  - Example: Total Variation (TV) in Plenoxels

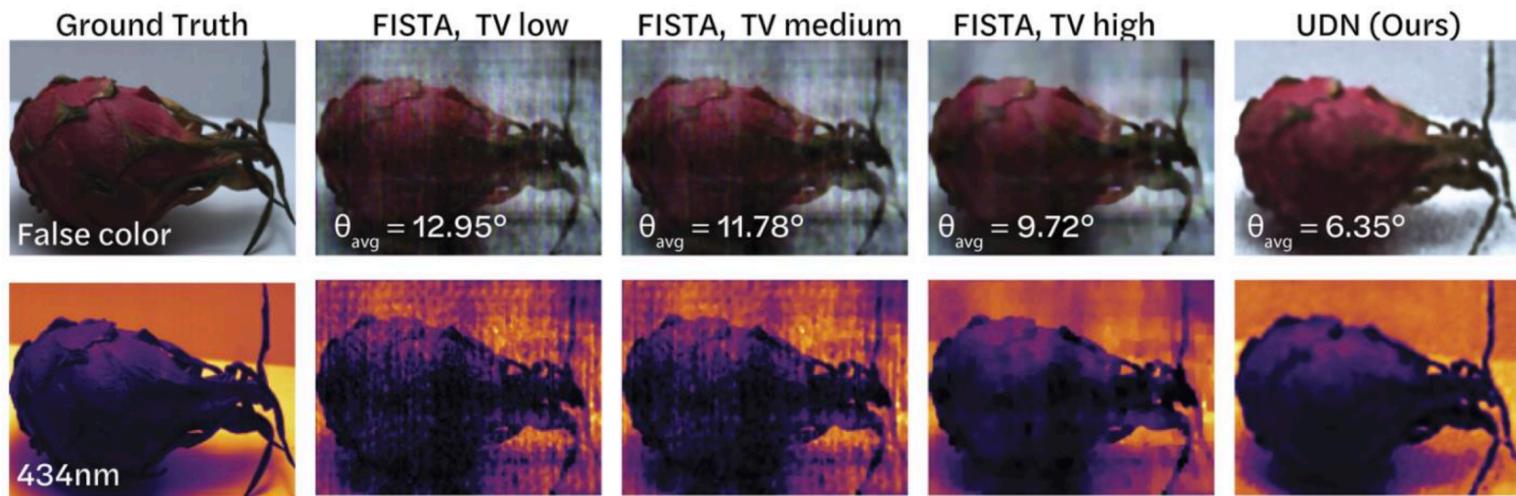


Plenoxels (Yu et al.) CVPR'22

# Signals – Regularization

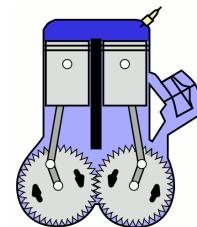
- How to deal with underconstrained problems?
  - Bayesian optimization: Maximum a Posteriori (MAP)
  - Example: Total Variation (TV) in Compressive Sensing

(neural field)  
does not require  
regularization!



# Signals – Properties

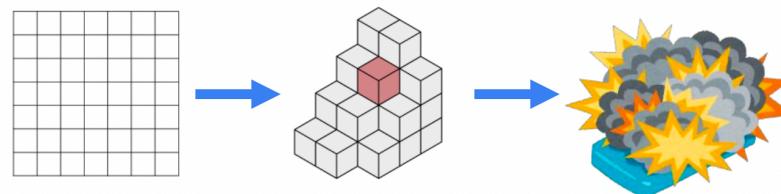
1. Compactness



2. Regularization

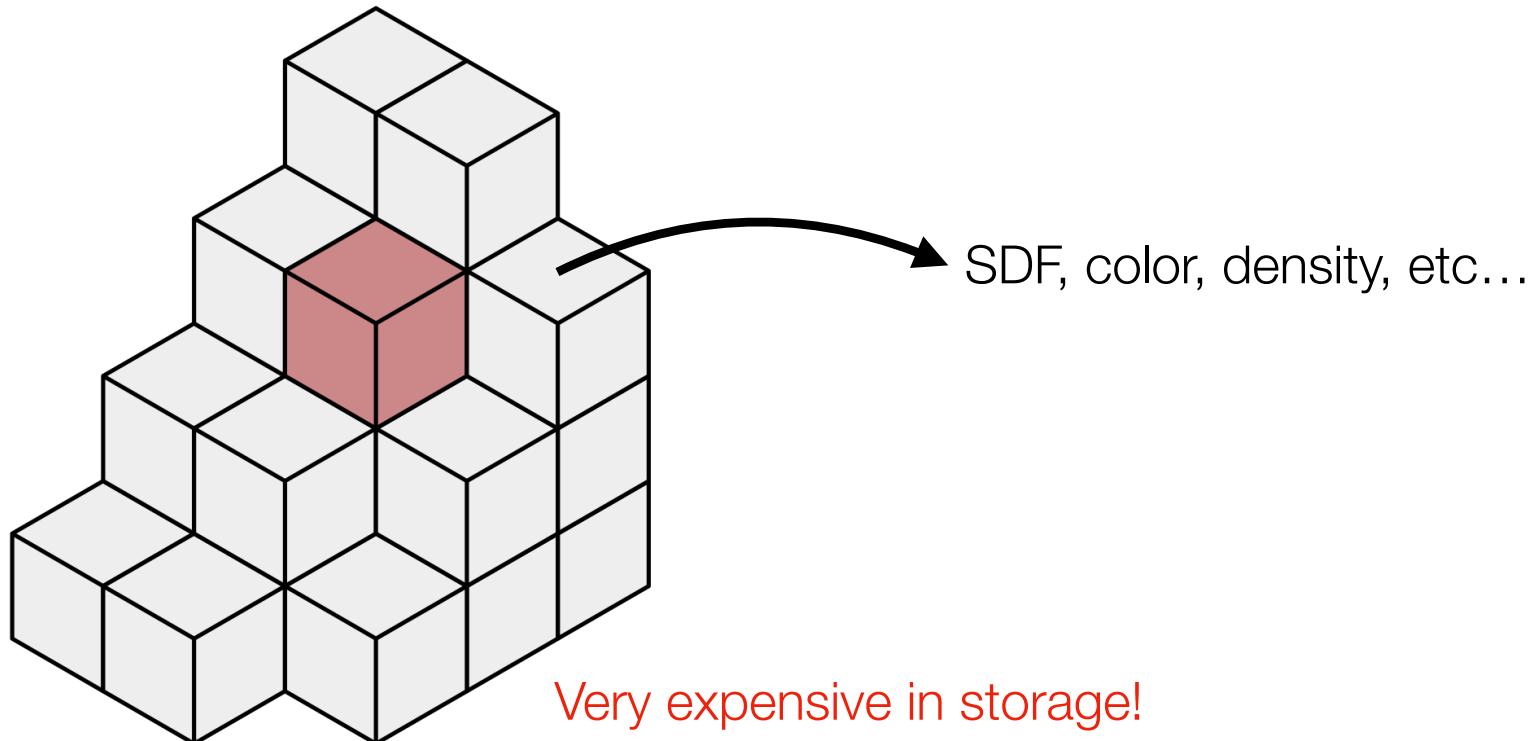
$$\operatorname{argmin}_x \|y - F(x)\| + \lambda P(x).$$

3. Domain Agnostic



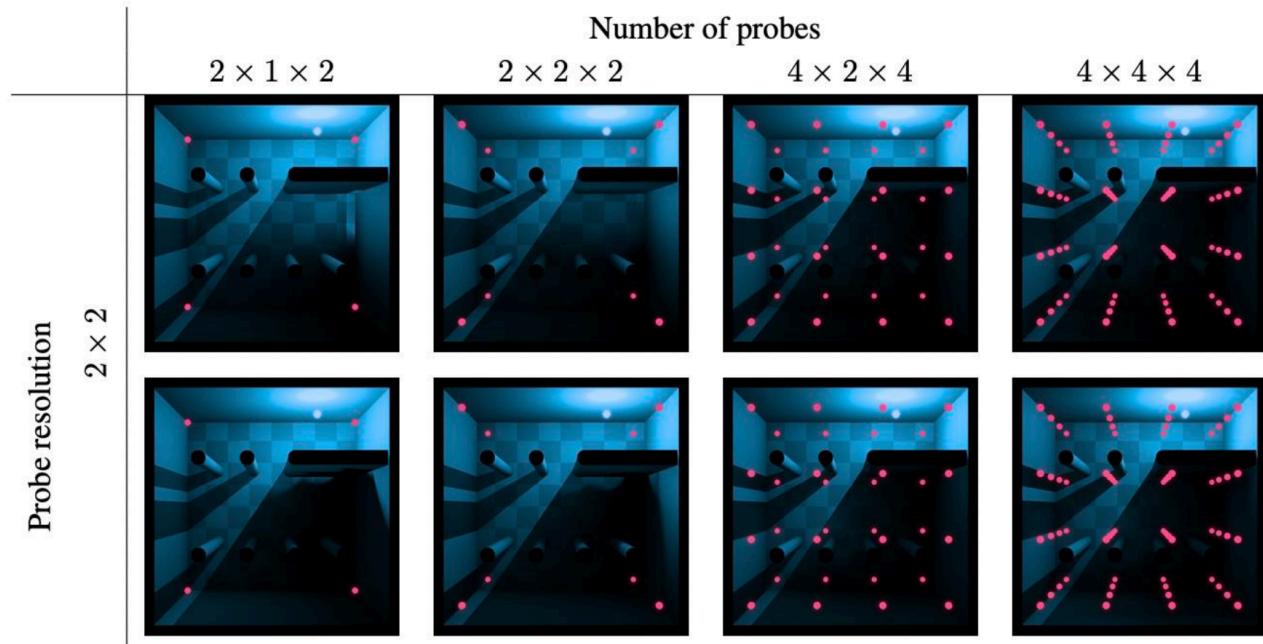
# Signals – Agnosticism

- Represent 3D signals for computer graphics



# Signals – Agnosticism

- Good practice: **don't over-tailor** representation to the problem at hand
  - e.g. 3D grids of 2D basis functions

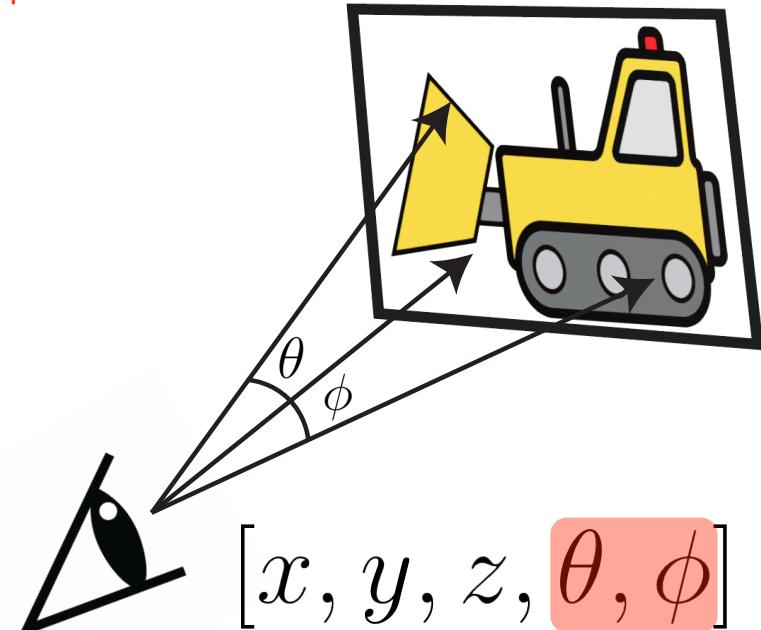


Majercik et al. “Dynamic diffuse global illumination with ray-traced irradiance fields” JCGT’19

# Signals – Agnosticism

- Good practice: **don't over-tailor** representation to the problem at hand
  - e.g. how to capture **view-dependent effects**? (e.g. NeRF)
  - ...neural networks are glorified **vector-matrix multiplications**

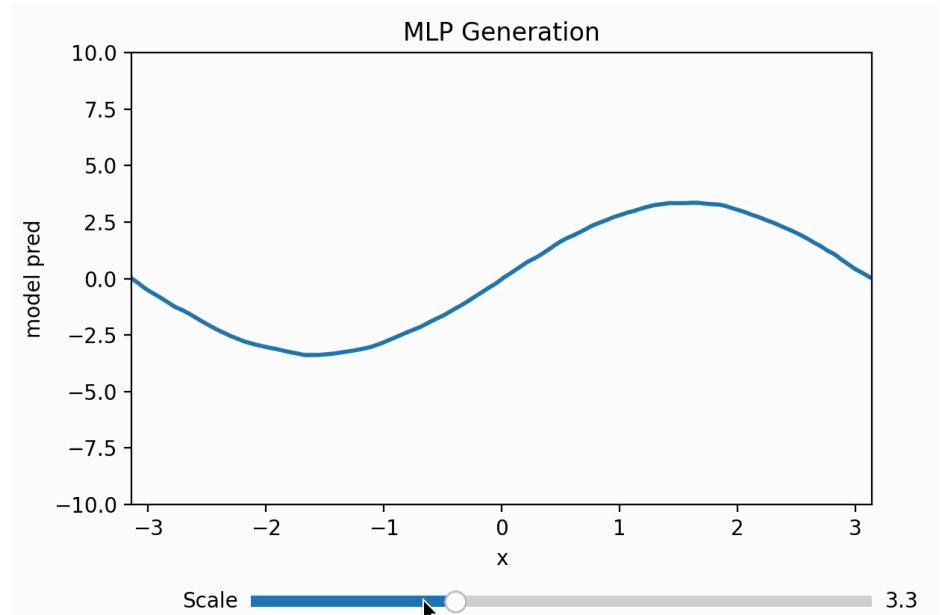
$$h = \begin{bmatrix} a & b & 1 \\ c & d & \mathbf{m} \\ e & f & \mathbf{n} \\ j & k & \mathbf{o} \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$



# Coding Exercise – Task 3

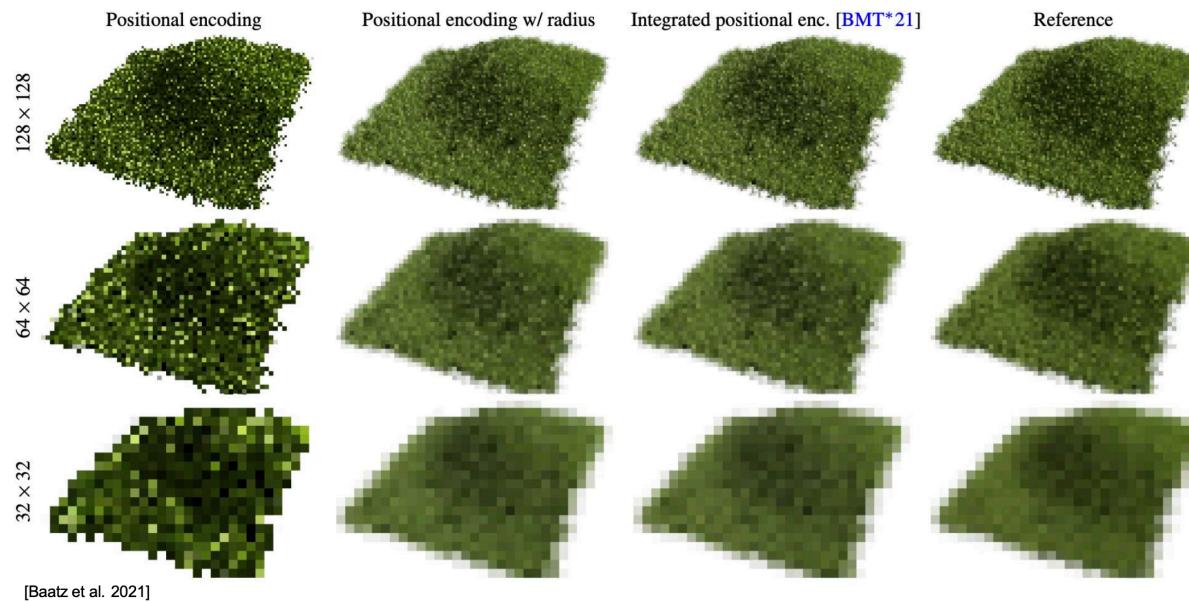
- Fit a signal by optimizing the MLP parameters
  - same network/objective as before (but fit to **multiple signals** vs. just one)
  - add a **conditioning signal** by concatenation

$$z^{(1)} = x$$
$$z^{(1)} = \gamma(x), s$$
$$z^{(i+1)} = \sigma(W^{(i)} z^{(i)} + b^{(i)}), \quad i = 1, \dots, k-1$$
$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$



# Signals – Agnosticism

- Good practice: **don't over-tailor** representation to the problem at hand
  - e.g. same representation can be **adapted to texture synthesis**



Baatz et al. "NeRF-Tex: Neural Reflectance Field Textures" EGSR'21

# Signals – Neural Fields

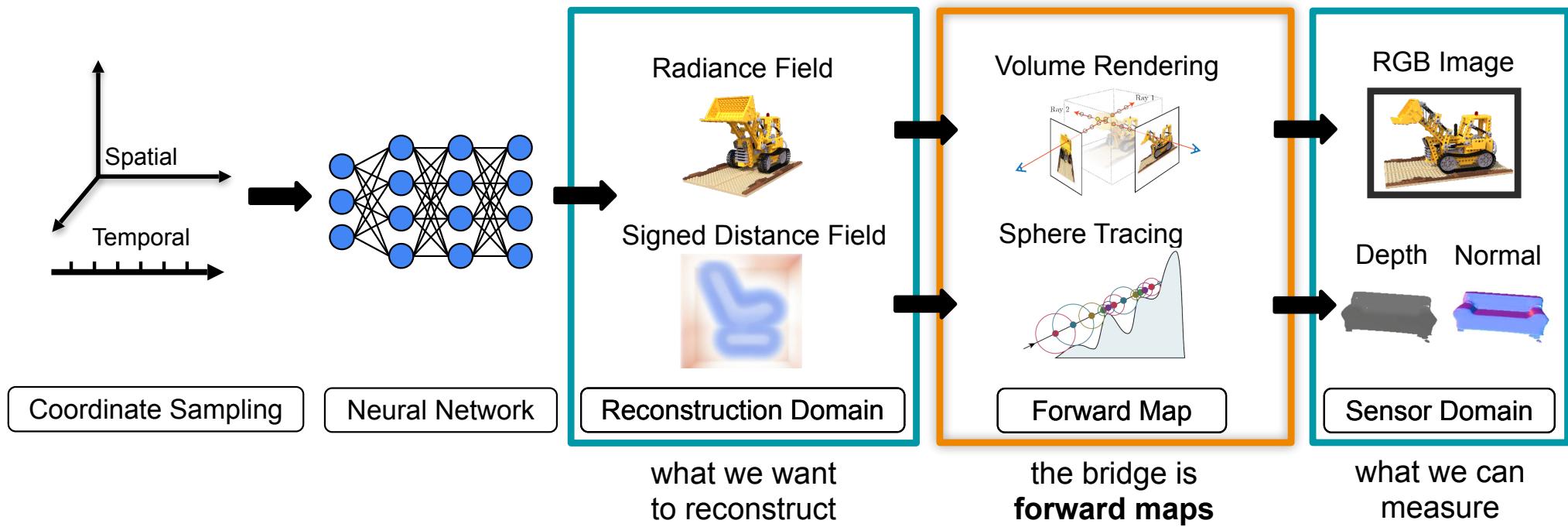
- Problems
  - Editability / manipulability
  - Computational complexity
  - Spectral bias

# Outline

- Signals
- Forward map
- Architectures

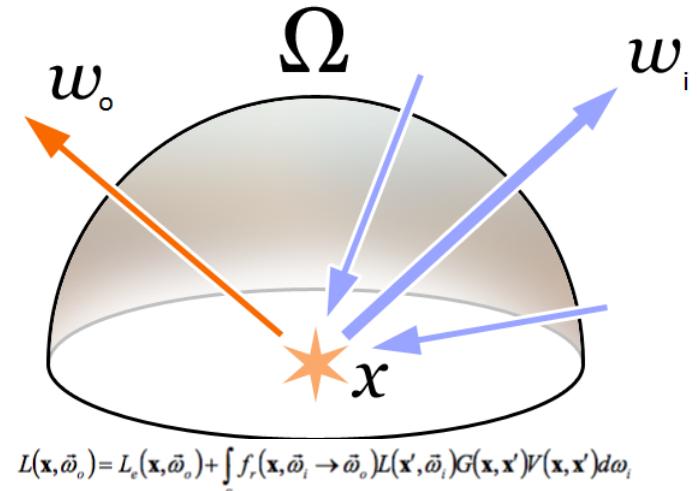
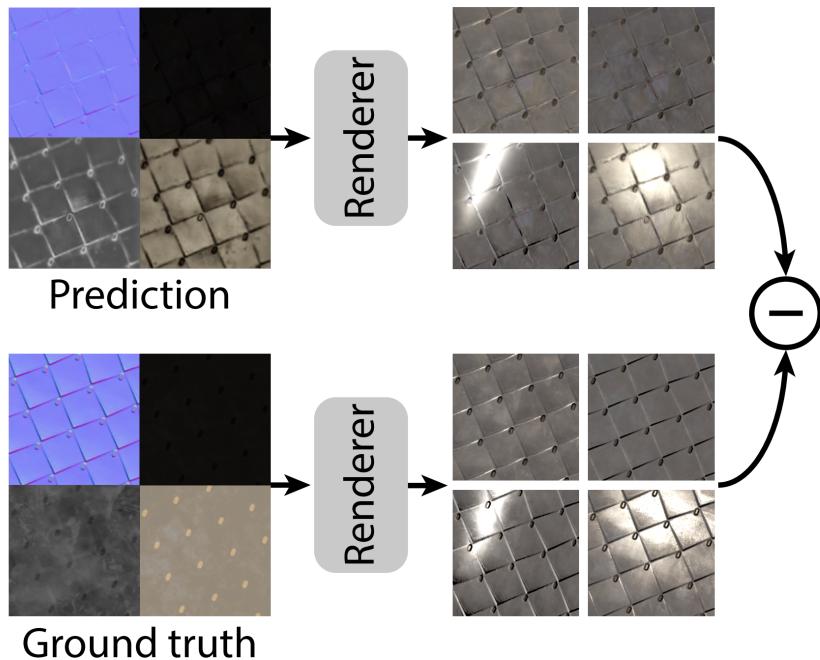
# Forward Maps

- Connect our neural representation to our measurement domain



# Forward Map

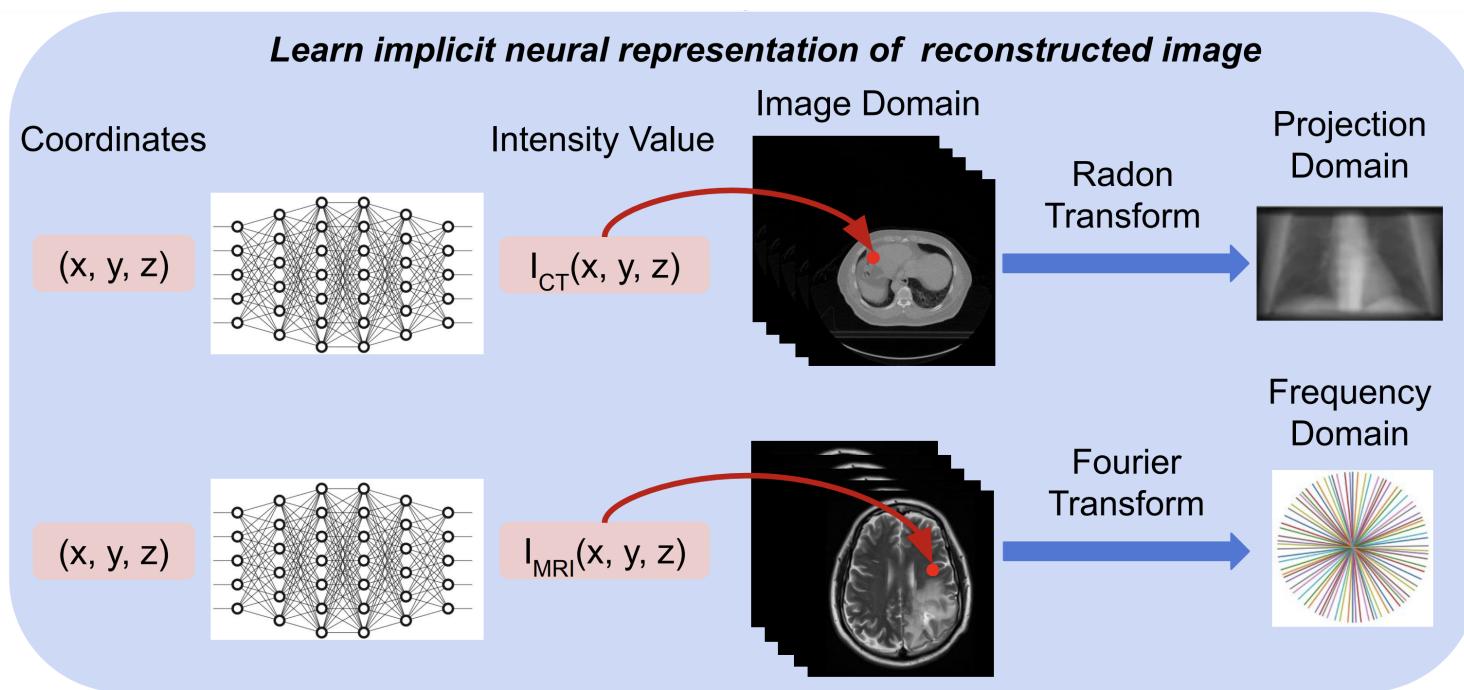
- Graphics: foward map is the **rendering equation** (BRDF modeling)



[Oechsle et al. 2018fr]

# Forward Maps

- Medical imaging: forward map is **Radon / Fourier** transforms

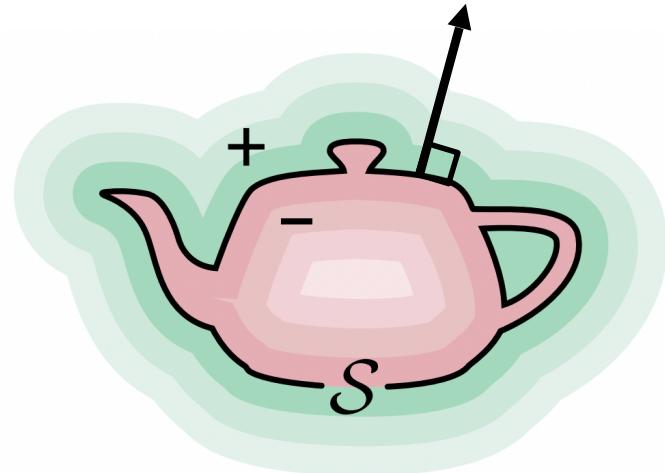


[Shen et al. 2020 (NeRP)]

# Forward Maps

- Geometry: forward map is **differential properties** of signal
  - e.g. **Eikonal loss** to make the field an SDF

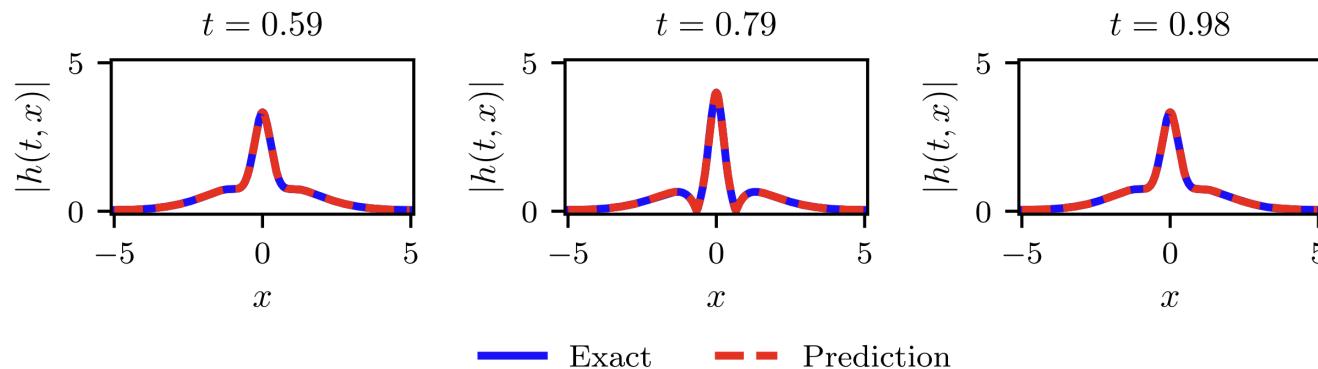
$$\mathcal{L} = \left\| \sum_{i=1}^n \left( \frac{\partial u}{\partial x_i} \right)^2 - 1 \right\|$$



# Forward Map

- Physics: a **differential equation**
  - e.g. Schrödinger's Equation

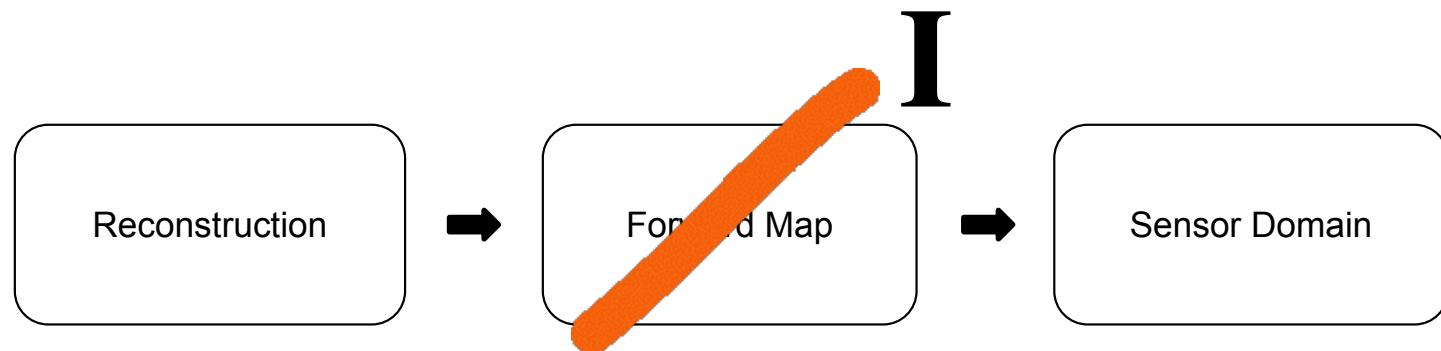
$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$



[Raissi et al. 2019 (PINN)]

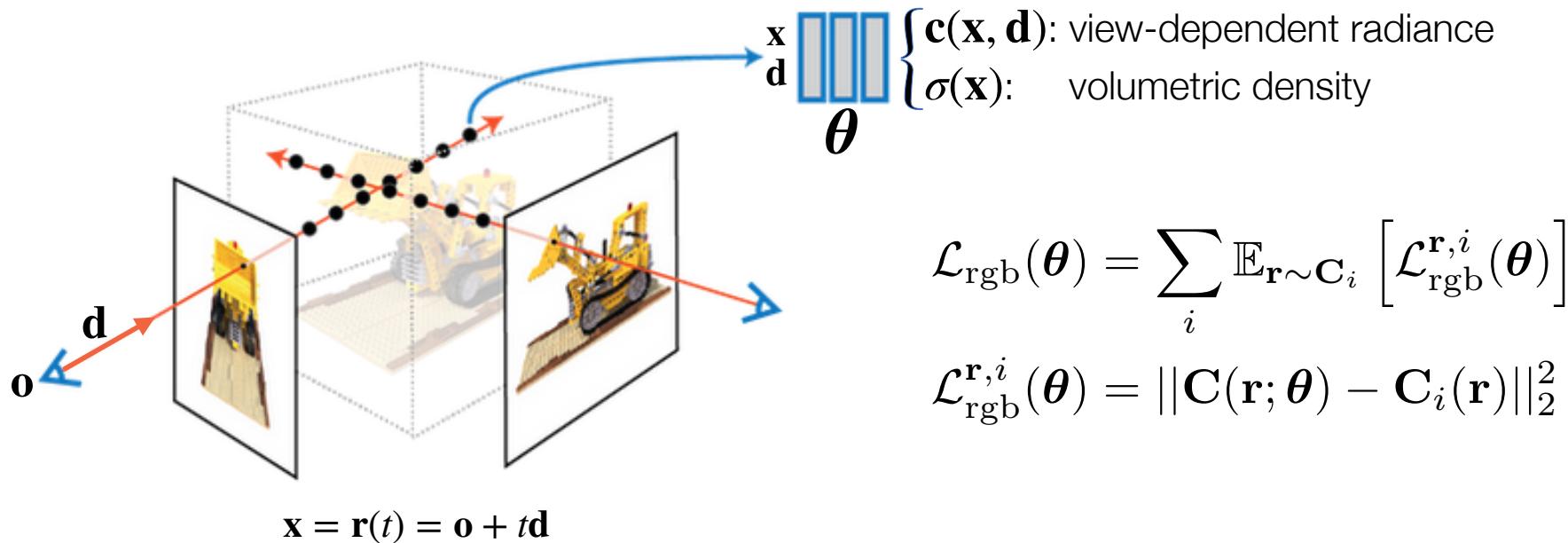
# Forward Map

- Image compression: the **identity** mapping



# Forward Map

- Volume rendering: optimize the neural field parameters  $\theta$ 
  - pick **random rays/pixels** from training images
  - minimize the **L2 hotometric reconstruction** loss



# Forward Map

- Volume rendering: pixel color is a **convex combination** of spatial radiance

Volume Rendering Digest (for NeRF)

Andrea Tagliasacchi<sup>1,2</sup> Ben Mildenhall<sup>1</sup>  
<sup>1</sup>Google Research   <sup>2</sup>Simon Fraser University

Neural Radiance Fields [3] employ simple volume rendering as a way to overcome the challenges of differentiating through ray-triangle intersections by leveraging a probabilistic notion of visibility. This is achieved by assuming the scene is composed by a cloud of light-emitting particles whose density changes in space (in the terminology of physically-based rendering, this would be described as a volume with absorption and emission but no scattering [4, See 11.1]. In what follows, for the sake of exposition simplicity, and without loss of generality, we assume the emitted light *does not* change as a function of view-direction. This technical report is a condensed version of previous reports [1, 2], but rewritten in the context of NeRF, and adopting its commonly used notation<sup>1</sup>.

**Transmittance.** Let the density field  $\sigma(\mathbf{x})$ , with  $\mathbf{x} \in \mathbb{R}^3$  indicate the differential likelihood of a ray hitting a particle (i.e. the probability of hitting a particle while travelling an infinitesimal distance). We reparameterize the density along a given ray  $\mathbf{r} = (\mathbf{o}, \mathbf{d})$  as a scalar function  $\sigma(t)$ , since any point  $\mathbf{x}$  along the ray can be written as  $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$ . Density is closely tied to the transmittance function  $T(t)$ , which indicates the probability of a ray traveling over the interval  $[0, t]$  without hitting any particles. Then the probability  $T(t+dt)$  of *not* hitting a particle when taking a differential step  $dt$  equal to  $T(t)$ , the likelihood of the ray reaching  $t$ , times  $(1 - dt \cdot \sigma(t))$ , the probability of not hitting anything during the step:

$$T(t+dt) = T(t) \cdot (1 - dt \cdot \sigma(t)) \quad (1)$$

$$\frac{T(t+dt) - T(t)}{dt} \equiv T'(t) = -T(t) \cdot \sigma(t) \quad (2)$$

This is a classical differential equation that can be solved as follows:

$$T'(t) = -T(t) \cdot \sigma(t) \quad (3)$$

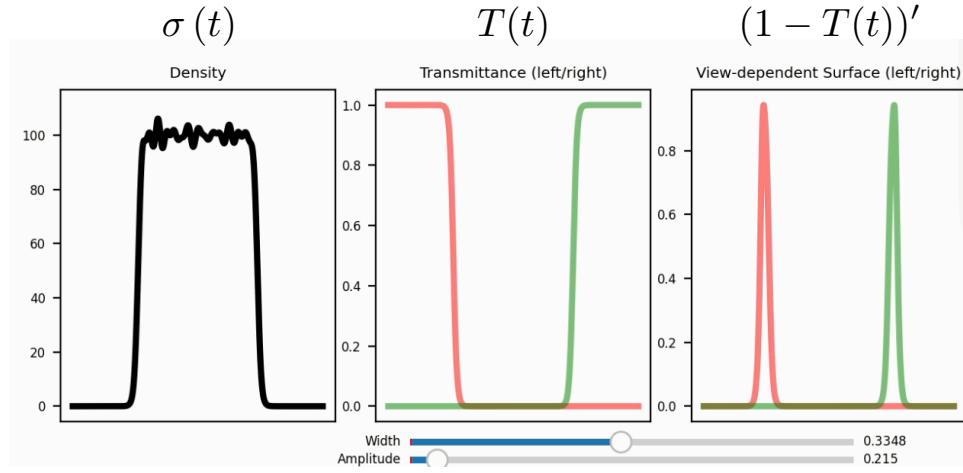
$$\frac{T'(t)}{T(t)} = -\sigma(t) \quad (4)$$

$$\int_a^b \frac{T'(t)}{T(t)} dt = - \int_a^b \sigma(t) dt \quad (5)$$

$$\log T(t)|_a^b = - \int_a^b \sigma(t) dt \quad (6)$$

$$T(a \rightarrow b) \equiv \frac{T(b)}{T(a)} = \exp \left( - \int_a^b \sigma(t) dt \right) \quad (7)$$

where we define  $T(a \rightarrow b)$  as the probability that the ray travels from distance  $a$  to  $b$  along the ray without hitting a particle, which is related to the previous notation by  $T(t) = T(0 \rightarrow t)$ .



$$\begin{aligned} \mathbf{C}(\mathbf{r}) &= \int_{t_n}^{t_f} T(t) \cdot \sigma(\mathbf{r}(t)) \cdot \mathbf{c}(\mathbf{r}(t), \mathbf{d}) dt \\ &= \int_{t_n}^{t_f} (1 - T(t))' \cdot \mathbf{c}(\mathbf{r}(t), \mathbf{d}) dt \end{aligned}$$

**geometry      appearance**

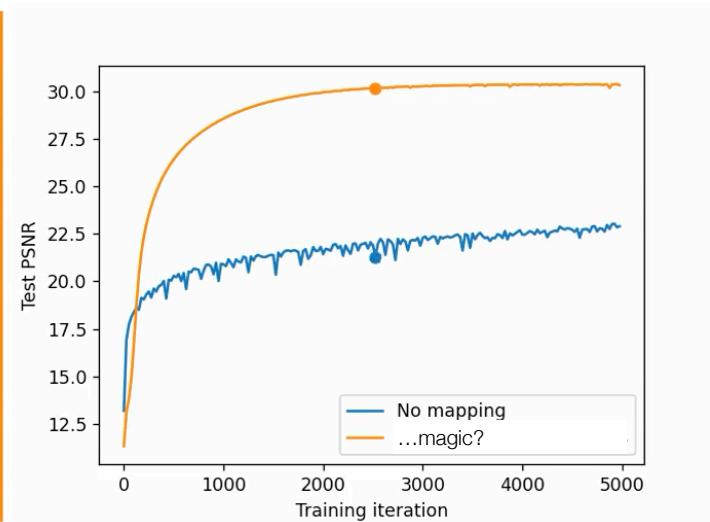
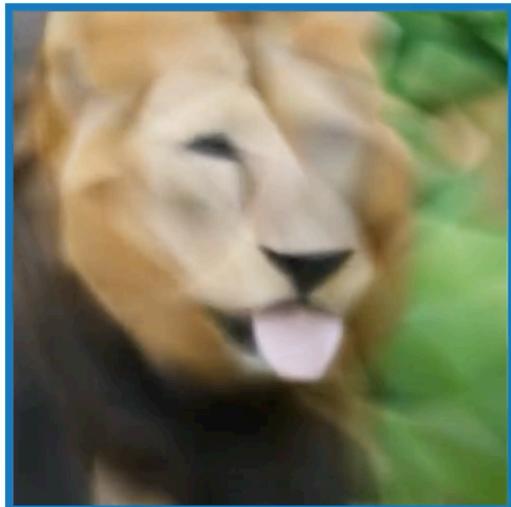
Tagliasacchi and Mildenhall “Volume Rendering Digest (for NeRF)” arXiv’2022

# Outline

- Signals
- Forward map
- Architectures

# Architecture: input encodings

- Spectral bias: neural networks are biased to fit lower frequency signals
  - difficult to model high-frequency details

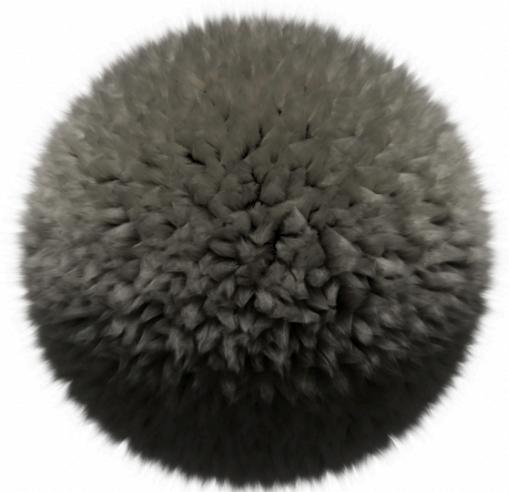


ReLU MLP

...magic?

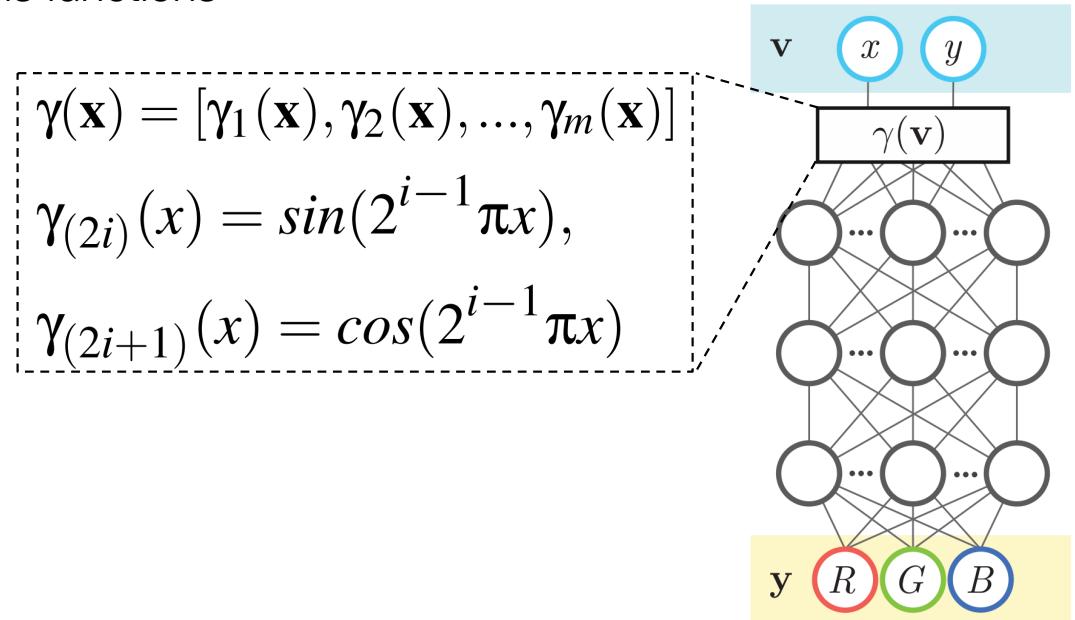
# Architecture: input encodings

- The signals we are interested in are **high frequency**
  - Are then MLP really the right way to represent them?



# Architecture: input encodings

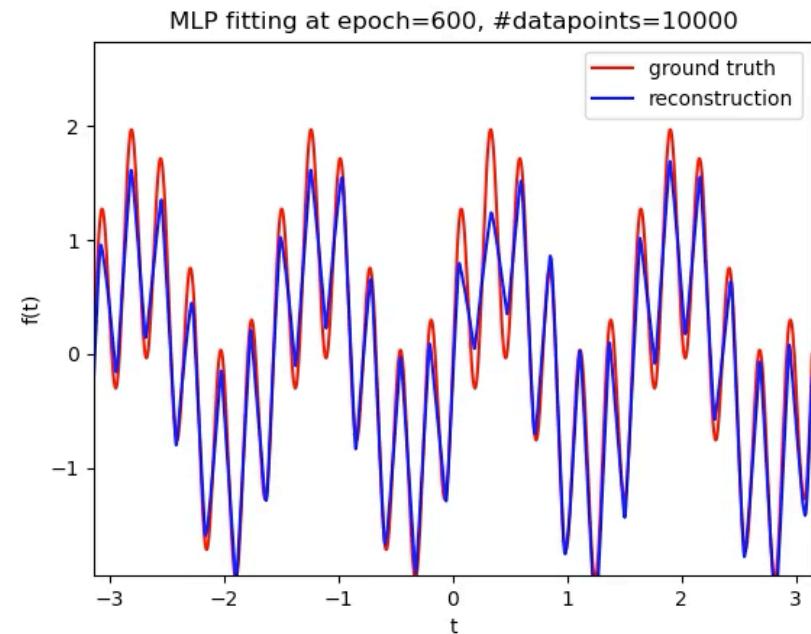
- Positional encodings
  - expand the dimensionality of the input (e.g.  $2 \rightarrow 2M$  dimensions)
  - via a collection of sinusoidal basis functions



# Coding Exercise – Task 2

- Fit a signal by optimizing the MLP parameters
  - same network/objective as before
  - add **positional encoding** at input layer

$$\begin{aligned} z^{(1)} &= x \\ z^{(1)} &= \gamma(x) \\ z^{(i+1)} &= \sigma(W^{(i)}z^{(i)} + b^{(i)}), \quad i = 1, \dots, k-1 \\ f(x) &= W^{(k)}z^{(k)} + b^{(k)} \end{aligned}$$



# Architecture: input encodings

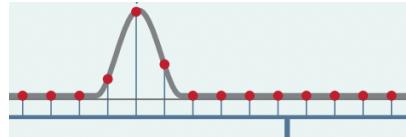
- Input encodings: many new variants emerging

$$\Phi(\mathbf{x}) = [\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, \phi_D(\mathbf{x})]^T,$$
$$\left[ e^{-\frac{(\mathbf{x} \cdot \alpha - t_i)^2}{2\sigma_x^2}} \right]^b$$

Super Gaussian [Ramasinghe et al. 2021]

$$\gamma(\mathbf{v}) = [\cos(2\pi \mathbf{B}\mathbf{v}), \sin(2\pi \mathbf{B}\mathbf{v})]^T$$

Random Fourier [Tancik et al. 2020]

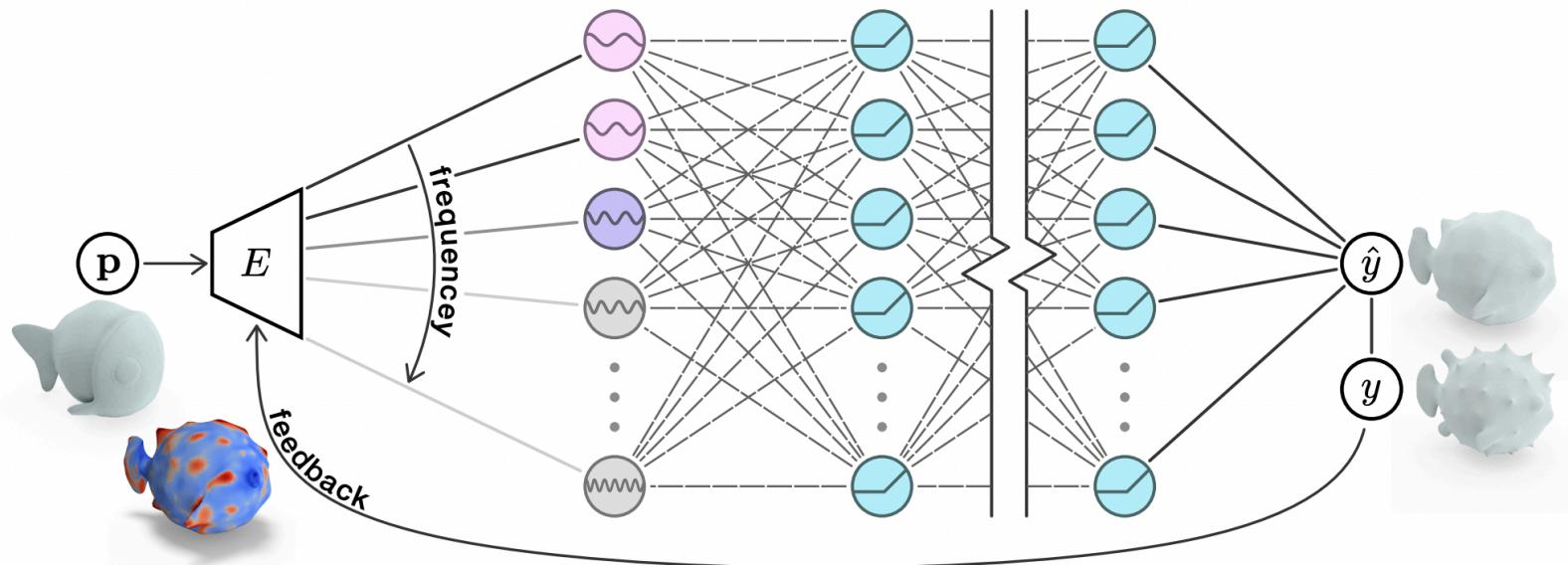


One-blob [Müller et al. 2020]



# Architecture: input encodings

- Progressive Positional Encodings (PPE)
  - often a good idea to optimize **multi-scale / coarse-to-fine**

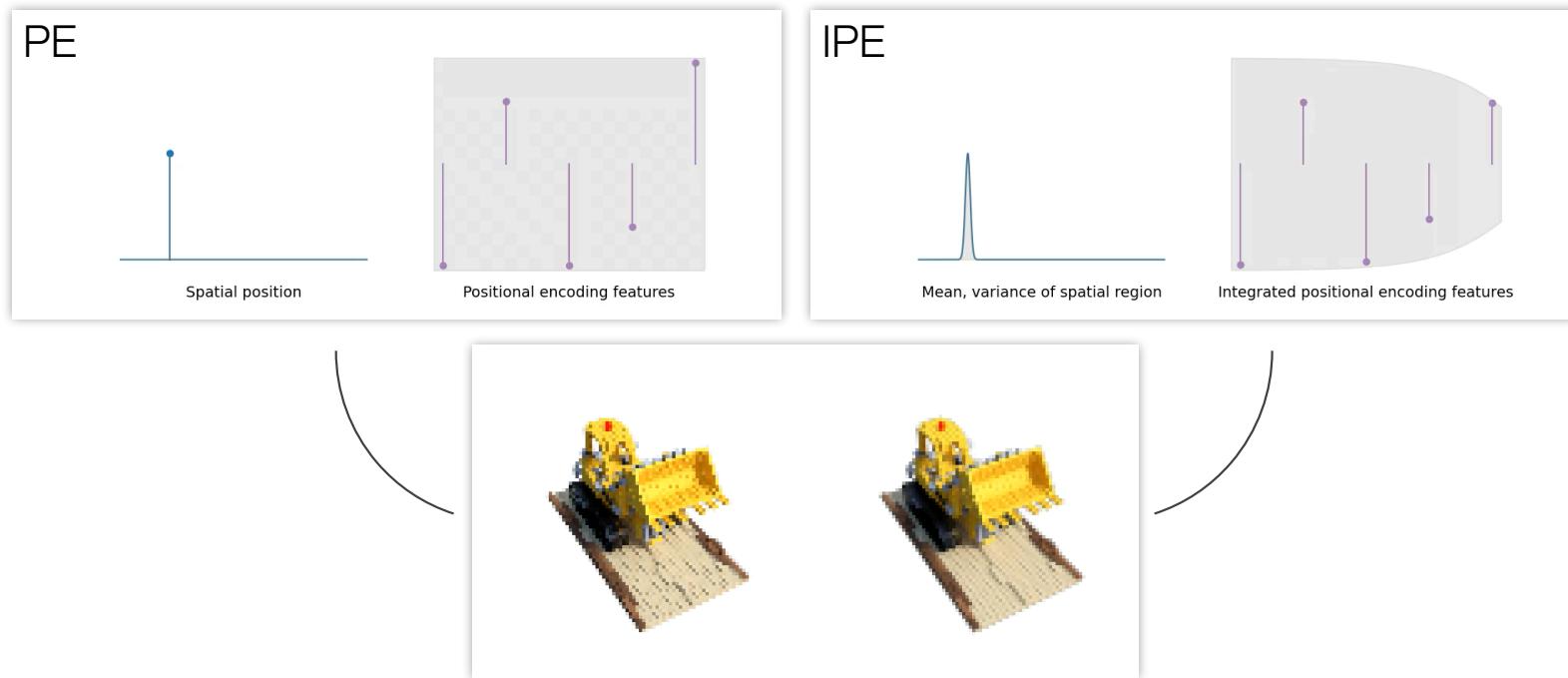


# Architecture: input encodings

- Integrated Positional Encodings (IPE)

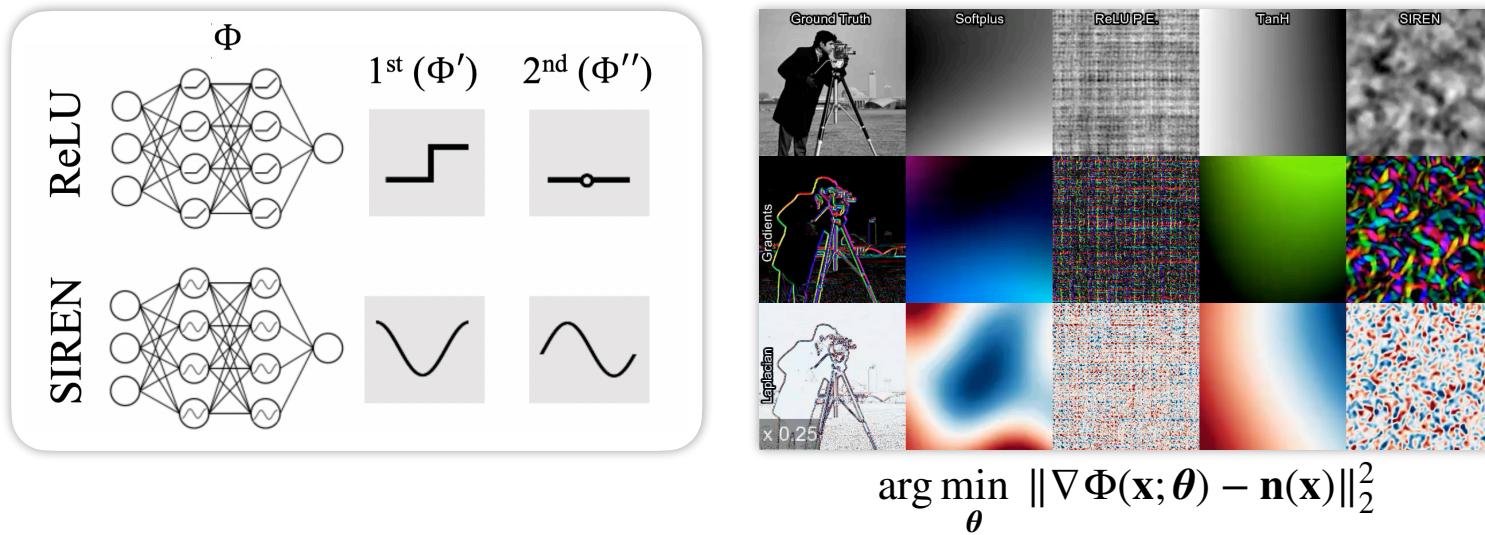
$$\gamma_\omega(x) = \sin(\omega x), \quad \gamma(x) = \left( \gamma_{2^\ell}(x) \right)_{\ell=0}^{L-1}$$

$$E_{x \sim \mathcal{N}(\mu, \sigma^2)} [\gamma_\omega(x)] = \sin(\omega\mu) \exp(-(\omega\sigma)^2/2)$$



# Architectures: activation functions

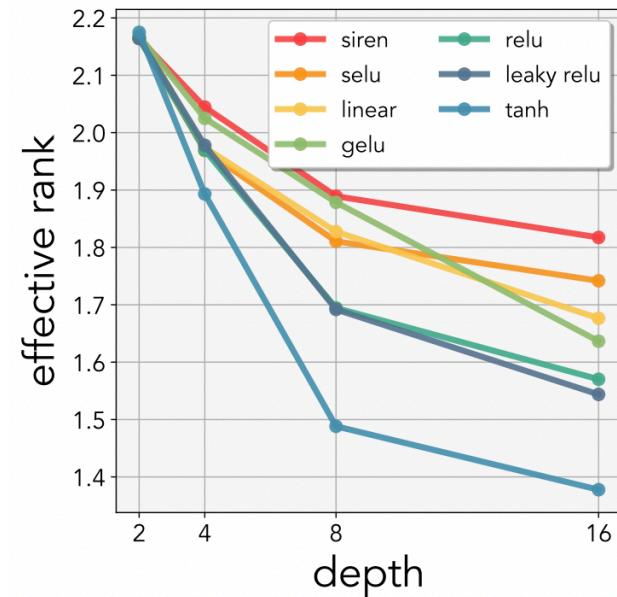
- Sinusoids at the input layer are “good”...
  - then why not try to apply them at **every layer**?
  - outcome: not only field is continuous, but also **all of its derivatives**



# Architectures: activation functions

- Smooth functions are “good” (sinusoids are smooth)
  - ... what **other smooth functions** can we try?

Gaussian	$e^{\frac{-0.5x^2}{a^2}}$
Quadratic	$\frac{1}{1+(ax)^2}$
Multi Quadratic	$\frac{1}{\sqrt{1+(ax)^2}}$
Laplacian	$e^{(\frac{- x }{a})}$
Super-Gaussian	$[e^{\frac{-0.5x^2}{a^2}}]^b$
ExpSin	$e^{-\sin(ax)}$



[Ramasininghe et al. 2021]

[Huh et al. 2021]

# Architectures: activation functions

- Smooth functions are “good” (sinusoids are smooth)
  - Can we modify the network architecture to decompose the signal?

**MLP**

$$z^{(1)} = x$$

non-linearity

$$z^{(i+1)} = \sigma(W^{(i)} z^{(i)} + b^{(i)}), \quad i = 1, \dots, k-1$$

$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$
  

**MFN**

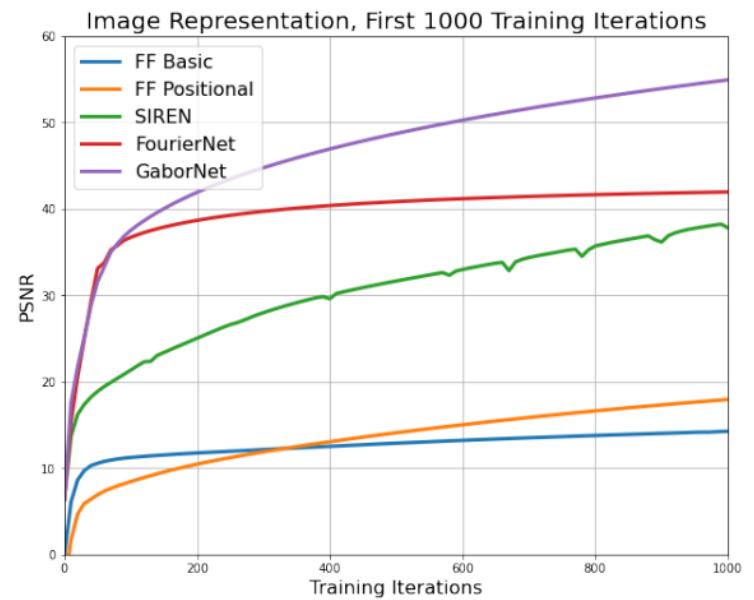
$$z^{(1)} = g(x; \theta^{(1)})$$

FourierNet

$$g(x; \theta^{(i)}) = \sin(\omega^{(i)}x + \phi^{(i)})$$

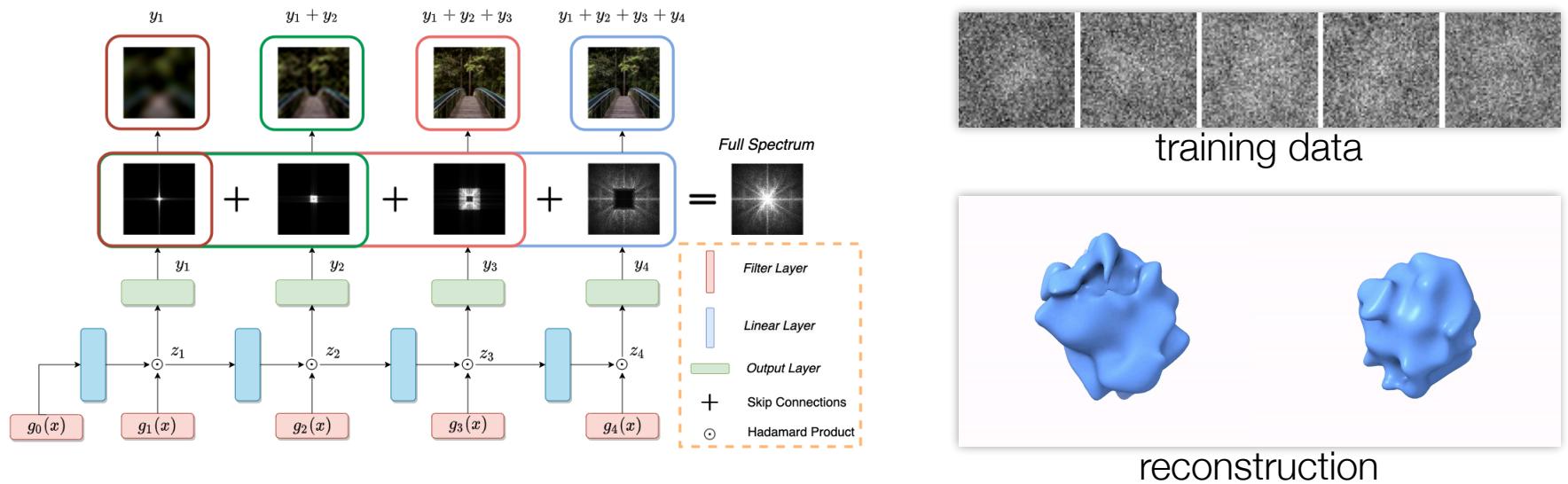
$$z^{(i+1)} = (W^{(i)} z^{(i)} + b^{(i)}) \circ g(x; \theta^{(i+1)}), \quad i = 1, \dots, k-1$$

$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$



# Architectures: activation functions

- Smooth functions are “good” (sinusoids are smooth)
  - Modify the network architecture to decompose the signal into **frequency bands**?

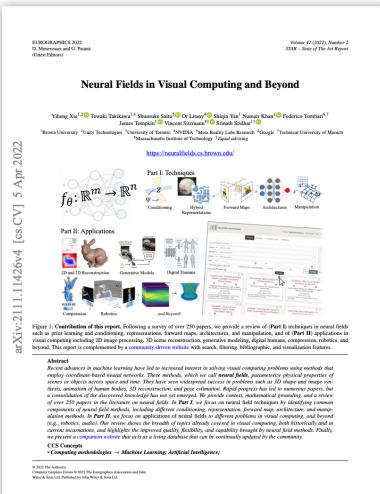


Shekarforoush et al. “Residual Multiplicative Filter Networks for Multiscale Reconstruction” NeurIPS’2022

# Summary

## • Resources

- Xie et al. "Neural Fields in Visual Computing and Beyond", Eurographics 2022
- Tagliasacchi et al. "Volume Rendering Digest (for NeRF)" arXiv'2022
- Tewari et al. "Advances in Neural Rendering" Eurographics 2022



Volume Rendering Digest (for NeRF)

Andrea Tagliasacchi<sup>1,2</sup> Ben Mildenhall<sup>1</sup>  
<sup>1</sup>Google Research <sup>2</sup>Simon Fraser University

Neural Radiance Fields [3] employ simple volume rendering as a way to overcome the challenges of differentiating through ray-triangle intersections by leveraging a probabilistic notion of visibility. This is achieved by assuming the scene is composed by a cloud of light-emitting particles whose density changes in space (in the terminology of physically-based rendering), this would be denoted as a volume with absorption and emission but not radiance [4]. See [5, 6] for what happens for the sake of expressivity, efficiency and without loss of generality, we assume the emitted light does *not* change as a function of view-direction. This technical report is a condensed version of previous reports [1, 2], but rewritten in the context of NeRF and adopting its commonly used notation.

**Transmittance.** Let the density field be  $\sigma(\mathbf{x})$ , with  $\mathbf{x} \in \mathbb{R}^n$  indicate the differential likelihood of a ray hitting a particle (i.e. the probability of hitting a particle while travelling an infinitesimal distance). We reparameterize the density field as a function of  $t$  ( $\sigma = \sigma(t)$ ) since any point  $\mathbf{x}$  along the ray can be written as  $\mathbf{x} = \mathbf{x}(t)$ . Denote  $\mathcal{T}(t)$  to be the transmittance of the ray at time  $t$ , then the likelihood that a ray of a ray travelling over the interval  $[0, t]$  without hitting any particles. Then the probability  $\mathcal{T}(t-dt)$  of not hitting a particle when taking a differential step  $dt$  is equal to  $\mathcal{T}(t)$ . Then the probability of the ray reaching  $t$ , times  $(1 - dt \cdot \sigma(t))$ , the probability of not hitting anything during the step:

$$\frac{\mathcal{T}(t+dt) - \mathcal{T}(t)}{dt} = \mathcal{T}'(t) = -\mathcal{T}(t) \cdot \sigma(t) \quad (1)$$

$$\frac{d\mathcal{T}}{dt} = -\mathcal{T}(t) \cdot \sigma(t) \quad (2)$$

This is a classical differential equation that can be solved as follows:

$$\mathcal{T}'(t) = -\mathcal{T}(t) \cdot \sigma(t) \quad (3)$$

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This state-of-the-art report discusses a large variety of neural rendering methods which have applications such as novel synthesis of static and dynamic scenes, generation of physics and image synthesis. We refer to the main text for more details on the various methods introduced. <https://arxiv.org/abs/2111.05849v2> [cs.GR] 30 Mar 2022

**Advances in Neural Rendering**

A. Tewari<sup>1,2\*</sup> J. Thorpe<sup>3</sup> B. Mildenhall<sup>4</sup> P. Srinivasan<sup>5</sup> E. Tancik<sup>6</sup> W. Yang<sup>7</sup> C. Lawrie<sup>8</sup> V. Kharlamov<sup>9</sup> S. Martin-Brualla<sup>10</sup> S. Lefebvre<sup>11</sup> T. Savva<sup>12</sup> C. Theobalt<sup>13</sup> M. Nitish<sup>14</sup> J. Barnes<sup>15</sup> G. Wetzstein<sup>16</sup> M. Zeltzer<sup>17</sup> V. Olsufyev<sup>18</sup>

<sup>1</sup>MIT Media Lab <sup>2</sup>MIT <sup>3</sup>Facebook Research <sup>4</sup>ETH Zurich <sup>5</sup>NVIDIA Research <sup>6</sup>NeRF Lab Research <sup>7</sup>MIT <sup>8</sup>Stanford University <sup>9</sup>Microsoft Research <sup>10</sup>Microsoft <sup>11</sup>Princeton University <sup>12</sup>Stanford University <sup>13</sup>Facebook <sup>14</sup>Microsoft <sup>15</sup>University of North Carolina at Chapel Hill <sup>16</sup>Stanford University <sup>17</sup>Microsoft <sup>18</sup>Microsoft

Figure 1: This state-of-the-art report discusses a large variety of neural rendering methods which have applications such as novel synthesis of static and dynamic scenes, generation of physics and image synthesis. We refer to the main text for more details on the various methods introduced. <https://arxiv.org/abs/2111.05849v2> [cs.GR] 30 Mar 2022

**Abstract**  
Synthesizing photorealistic images and videos is at the heart of computer graphics and has been the focus of decades of research. Traditionally, synthetic images or video are generated using rendering algorithms, such as rendering or ray tracing, which take a specifically defined representation of geometry and material properties as input. Collectively, these inputs are referred to as *scene representations*. In contrast, neural rendering methods learn a representation of the scene directly from raw images or raw video. Example scene representations are triangle meshes with accompanying textures (e.g., created as an artist), point clouds, and learned representations such as learned depth fields and learned radiance fields. The reconstruction of such a scene representation from observation using differentiable rendering is a key problem in computer graphics. We present a survey of the state-of-the-art in neural rendering, including learned computer graphics and machine learning to create algorithms for rendering images from real-world observations. Neural rendering methods have been applied to a wide range of applications, including novel view synthesis, image editing, and image enhancement. We also discuss how neural rendering has been used to solve other computer vision problems in the field through hundreds of publications. These new ways to represent scenes have opened up many opportunities for novel applications. In addition, we introduce the concept of learned scene representations, often also referred to as *neural scene representations*. A learned scene representation is a learned function that takes a query point and returns a learned representation of a query point. In addition to methods that handle static scenes, we cover novel scene representations for generating static, depth-deforming objects and scene editing and composition. Many of these approaches are scene specific, so we also discuss learned scene representations for novel applications such as novel view synthesis, image editing, and image enhancement. Finally, we provide an overview of theoretical concepts and definitions used in the current literature. We conclude with a discussion on open challenges and future directions.

**1. Introduction**  
Synthesis of controllable and photo-realistic images and videos is one of the fundamental goals of computer graphics. During the last decades, methods and representations have been developed

\* to mimic the image formation model of real cameras, including the handling of complex materials and global illumination. These methods have enabled the creation of images and videos that can transport from light sources to the virtual cameras for synthesis. To this end, all physical properties of the scene have to be known for

# Coding Exercises

- Task 1: fit a signal with an MLP
- Task 2: add positional encoding (remove spectral bias)
- Task 3: make the network conditional (learn families of functions)

[https://classroom.github.com/a/duduR83\\_](https://classroom.github.com/a/duduR83_)