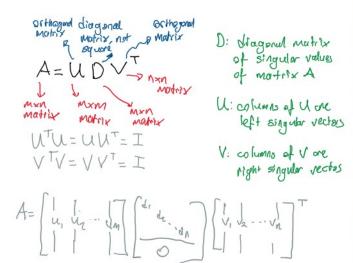
## 2.8. Singular Value Decomposition

Notes are created by Busra Tugce Gurbuz based on Deep Learning book (Goodfellow et al.)

- The singular value decomposition (SVD)
   provides another way to factorize a
   matrix, into singular vectors and singular
   values. SVD allows us to discover some of
   the same kind of information as the
   eigendecomposition.
- However, SVD is more generally applicable.
   Every real matrix has a SVD but the same is not true for eigendecomposition.
  - For example, if a matrix is not square, the eigendecomposition is not defined, and we must use a SVD instead.
- SVD is similar to eigendecomposition, but this time we will write it as a product of three matrices:



• We can actually interpret the SVD of A in terms of the eigendecomposition of functions of A. The left singular vectors of A are the eigenvectors of  $AA^T$ . The right singular vectors of A are the eigenvectors of  $A^TA$ . The non-zero singular values of A are the square roots of the eigenvalues of  $A^TA$ . The same is true for  $AA^T$ .

Proof for right singular vectors:

eigendecomposition of ATA:

ATA = [QNQT]T(QNQT)

= QNTQTQNQT

= QNTINQT = QNTNQT

right singular vector of A; U

UDVT = QNTNQT

Q=U QT=VT

NTN = D

- SVD is the first step in ML for dimensionality reduction.
  - It is also thought as data-driven generalization of the Fourier transform (FFT).
  - O It is also the basis of PCA.