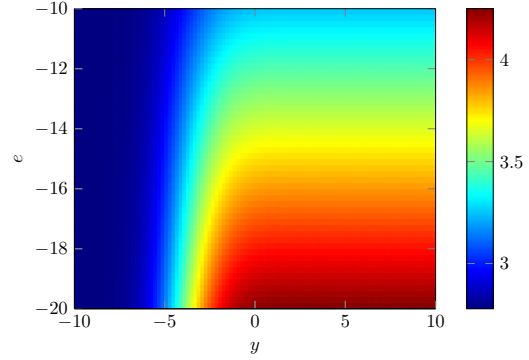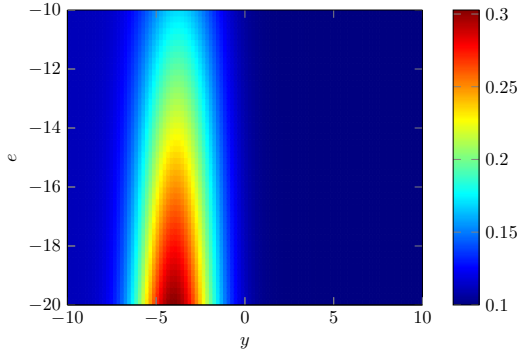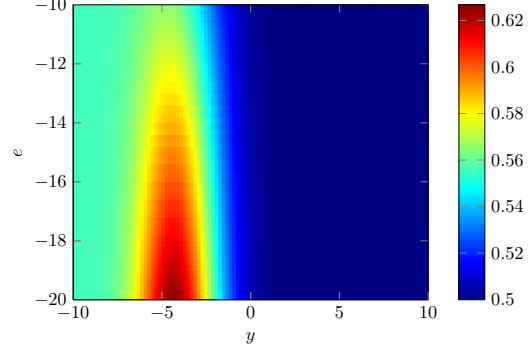(a) Terminal condition $V(T, \cdot)$

(b) Solution $V(0, \cdot)$

(a) Correction term $\tau = -(1-\gamma)V_y - (1-\varrho^{-1})(1 + V_e + (1-\gamma)V_y)_+$

(b) Optimal Control $\frac{1}{2\varrho}(1 + V_e + (1-\gamma)V_y)_+$

$$-\partial_t V - \frac{\gamma^2}{2}\partial_{yy}V - \mu\partial_y V - \frac{1}{4\rho}(1 + \partial_e V + (1-\gamma)\partial_y V)_+^2 = 0$$

The parameters of the problem:
$\gamma = 0.5$, $\alpha = 0.1$, $\beta = 1$, $\pi = q(1-q)$, $\eta(q) = \lambda(q) = q$, $T = 10$, $\mu = 0.1$ and $\varrho = 0.9$ $(a = 5)$, bounded domain for (t, y, e) is $[0, 10] \times [-20, 20] \times [0, 20]$ with transparent Dirichlet boundary condition

$$V(t, e, -20) = \frac{(T-t)}{4\varrho}, \quad V(t, e, 20) = -\alpha e + (1-\alpha)^2 \frac{(T-t)}{4\varrho}$$

and terminal condition $V(T, e, y) = -\alpha e 1_{\{y>0\}}$.