

Control problem

$$\inf_u J(u), \quad J(u) := \int_0^T C(s, x_s, u_s) ds + g(x_T), \quad dx_s = f(x_s, u_s) ds$$

Value function

$$V(t, x) := \inf_u \int_t^T C(s, x_s, u_s) ds + g(x_T), \quad dx_s = f(x_s, u_s) ds$$

HJ satisfied by value function

$$\begin{cases} 0 = V_t(t, x) + H(t, x, V_x(t, x)) \\ V(T, x) = g(x) \end{cases} \quad H(t, x, p) := \inf_u \{C(t, x, u) + p f(t, x, u)\}$$

PDE solver

Find  $V(t, x)$  and  $V_x(t, x)$  numerically or analytically

Optimal control using  $V_x(t, x)$

$$u^*(t, x) \in \operatorname{argmin}_u \left\{ C(t, x, u) + p \cdot f(t, x, V_x(t, x)) \right\}$$

Optimal trajectory, of state variable

$$dx_s^* = f(x_s^*, u^*(t, x_s^*)) ds$$