Control problem

 $\inf_{u} J(u), \quad J(u) := \int_{0}^{T} C(s, x_{s}, u_{s}) ds + g(x_{T}), \ dx_{s} = f(x_{s}, u_{s}) ds$

$$V(t,x) := \inf_{u} \int_{t}^{T} C(s,x_s,u_s) ds + g(x_T), \ dx_s = f(x_s,u_s) ds$$

$$,x,V_x(t,x)$$

$$\begin{cases} 0 = V_t(t, x) + H(t, x, V_x(t, x)) \\ V(T, x) = g(x) \end{cases} H(t, x, p) := \inf_{u} \{ C(t, x, u) + pf(t, x, u) \}$$

Find
$$V(t,x)$$
 and $V_x(t,x)$ numerically or analytically

Optimal control using
$$V_x(t,x)$$

$$u^*(t,x) \in \operatorname{argmin} \left\{ C(t,x,u) + V_x(t,x) \cdot f(t,x,u) \right\}$$

$$\mathrm{d}x_s^* = f\big(x_s^*, u^*(t, x_s^*)\big)\mathrm{d}s$$