## Control problem

 $\inf_{u} J(u), \quad J(u) := \int_{0}^{T} C(s, x_{s}, u_{s}) ds + g(x_{T}), \ dx_{s} = f(x_{s}, u_{s}) ds$ 

$$V(t,x) := \inf_{u} \int_{t}^{T} C(s,x_s,u_s) ds + g(x_T), \ dx_s = f(x_s,u_s) ds$$

$$V_x(t,x) = \inf_{u \in C(t,x,u)} V_x(t,x)$$

$$H(t,x,p) := \inf_{u \in C(t,x,u)} \{C(t,x,u) + pf(t,x,u)\}$$

$$\begin{cases} 0 = V_t(t, x) + H(t, x, V_x(t, x)) \\ V(T, x) = g(x) \end{cases}$$

PDE solver

Find 
$$V(t,x)$$
 and  $V_x(t,x)$  numerically or analytically

 $u^*(t,x) \in \operatorname{argmin} \left\{ C(t,x,u) + V_x(t,x) \cdot f(t,x,u) \right\}$ 

Optimal control using  $V_x(t,x)$ 

Optimal trajectory of state variable 
$$dx_{\circ}^{*} = f(x_{\circ}^{*}, u^{*}(t, x_{\circ}^{*}))ds$$