Control problem

$$\inf_{v \in \mathcal{V}} I(u) := \int_{0}^{T} C(e, r, u) de +$$

 $\inf_{u} J(u), \quad J(u) := \int_{0}^{T} C(s, x_{s}, u_{s}) ds + g(x_{T}), \ dx_{s} = f(x_{s}, u_{s}) ds$

$$V(t,x) := \inf_{u} \int_{t}^{T} C(s, x_s, u_s) ds + g(x_T), \ dx_s = f(x_s, u_s) ds$$

$$\begin{cases} 0 = V_t(t, x) + H(t, x, V_x(t, x)) \\ V(T, x) = g(x) \end{cases}$$

Optimal trajectory of state variable

Find
$$V(t,x)$$
 and $V_x(t,x)$ numerically or analytically

$$v_x(t,x)$$

Optimal control using
$$V_x(t,x)$$

$$u^*(t,x) \in \operatorname{argmin} \left\{ C(t,x,u) + V_x(t,x) \cdot f(t,x,u) \right\}$$

sing
$$V_x(t,x)$$

 $dx_s^* = f(x_s^*, u^*(t, x_s^*))ds$

$$H(t,x,p) := \inf_{u} \left\{ C(t,x,u) + pf(t,x,u) \right\}$$