Floating potentials for in-silico models: Enriched FEM as an exact and efficient approach

C8

Arash Golmohammadi*, Revathi Appali, Ursula van Rienen

Institute for General Electrical Engineering, University of Rostock, 18055 Rostock

Email: arash.golmohammadi@uni-rostock.de

INTRODUCTION

Deep brain stimulation electrodes are evolving toward more complex geometries, enabling clinicians to steer the stimulation pulse with more freedom in the brain. Nevertheless, this complicates the volume conductor modeling (VCM), which is essential for predicting the impact of stimuli. The current VCM implementations are limited; they differ between current- and voltage-controlled protocols and may need to solve VCM several times, once for each active contact. Additionally, relaxing or even ignoring some physical boundary conditions to ease computation is common. For instance, the floating potentials (FP) on the passive contacts are sometimes modeled (paradoxically) as perfect insulators. Or the fixed total currents are sometimes approximated by a uniform current injection (ignoring tissue-electrode interface). The Virtual Permittivity (VP) method attempts to resolve the former by ascribing a high conductivity to some wisely placed subdomains. Yet, even this method is problematic due to its high sensitivity to the chosen conductivity and the nontriviality of meshing modern electrodes for VP.

Current VCM approaches thus are insufficient for a patient-specific simulation. We instead used an enriched finite element method (eFEM) to tackle all the aforementioned problems. It is a unified formalism (does not distinguish between protocol input types), exactly satisfies all the physical boundary conditions (FP and interface), does not suffer from computational artifacts (no free parameter), is minimal in terms of additional costs, and more importantly, is compatible with the current FEM implementations. We used open-source packages (`FEniCS`, `multiphenics`) to show the effectiveness of this formalism.

METHODS

Like conventional FEM formulation, we solve the (possibly complex-valued and tensorial) Poisson equation on a certain region of interest Ω in the brain:

$$\nabla \cdot (\sigma \nabla u) = -f = 0 \quad on\ \Omega$$

bounded by $\partial\Omega$. However, we augment the space of solution (u) by an N-dimensional complex-valued vector representing the potential values on each contact. The complete solution thus is the (u, U) tuple.

A voltage-controlled protocol fixes the potential over a contact, whereas the current-controlled one fixes merely the total current injected from/to the contact surface. The tissue's voltage is coupled to the electrode's voltage by Ohm's law, regardless of the protocol in use. This coupling encapsulates the (usually non-trivial) tissue-electrode interfacing via impedance z. All N (passive or active) contacts whose potential is not fixed are endowed with an (unknown) constant FP. Mathematically, simultaneous imposition of both protocols is also possible, which leads to the following (interior) boundary conditions:

$$-\int_{\partial\Omega_i} \sigma \partial_n u = I_i \qquad on\ \partial\Omega_i \in \partial\Omega_I \qquad \textit{Fixed total injected current}$$

$$U_i = V_i \qquad on\ \partial\Omega_i \in \partial\Omega_V \qquad \textit{Unknown constant voltage}$$

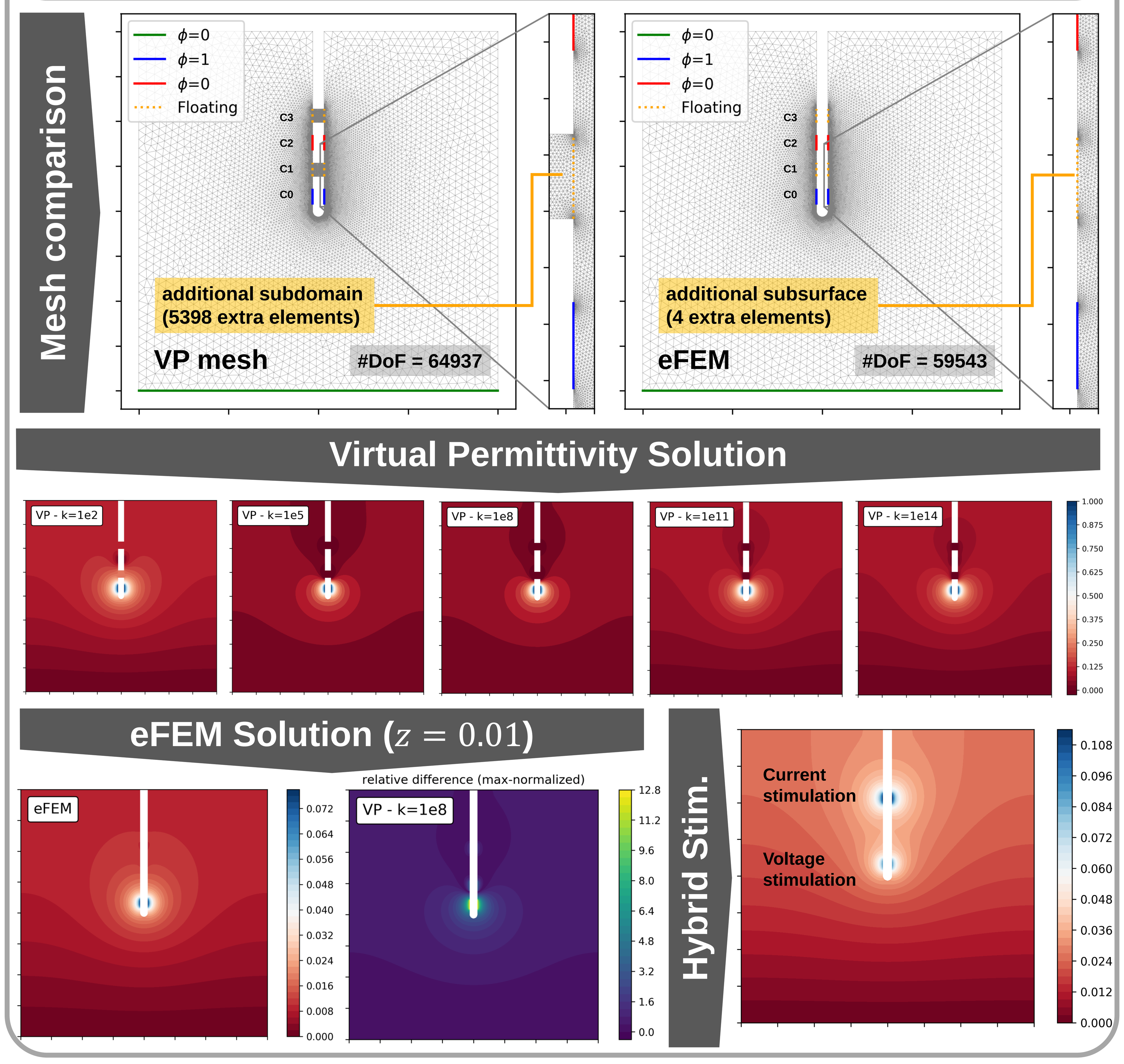
$$-\sigma \partial_n u = \frac{u - U}{z} \qquad on\ \partial\Omega_i \in \partial\Omega_I \cup \partial\Omega_V \qquad \textit{Ohm's law}$$

$$U_i = \phi_i \qquad on\ \partial\Omega_i \in \partial\Omega_V \qquad \textit{Fixed contact voltage}$$

Exteriors boundaries are treated as usual with Dirichlet and Neumann boundary conditions applied on $\partial\Omega_D, \partial\Omega_N$, respectively.

We followed the formalism proposed by [1] and extended it for simultaneous current and voltage inputs. We benchmarked our model (eFEM) with a comparable VP, with identical mesh (except on virtual subdomains introduced to avoid FP modeling). To only highlight the distinction between the formulations, both eFEM and VP are solved in normalized units. For VP, we swept the conductivity ratio k of the virtual subdomains to the tissue. We monitored the (dimensionless) currents and voltage distribution for each k, as well as the number of degrees of freedom (DoF) in each model. For real-world problems with complex 3D geometry, high DoF will be the main computational bottleneck.

RESULTS



DISCUSSION

VP only represents the actual VCM at a limited range of k. Low k does not model the conductors well, and very high values degrade the system solvability. Although an optimal k window exists, its range is unknown a priori and depends on the electrode and brain parameters. Additionally, VP can not incorporate the tissue-electrode interface and sacrifices reality to computational ease. On the contrary, eFEM captures all the physical boundary conditions and interfaces and does not corrupt the system solvability. Furthermore, by introducing only N(=4) additional DoFs in contrast to thousands of VP, inversion of eFEM matrix, the computational bottleneck of VCM, is faster. Unlike real-world (3D) scenarios, the total runtime of our simple problem (0.6 to 0.7s) is determined mainly by other operations rather than solving the system. Finally, If the parameters are real-valued, the mass block-matrix is also symmetric, which is desirable for preconditioning. Note that eFEM does not solve the geometrical degeneracies, like the jumps in conductivity, voltage, and current and refined mesh is still needed for obtaining accurate results.

CONCLUSION

eFEM provides a natural formalism to model volume conduction for electrodes with numerous contacts efficiently and physically consistently. The implementation is not complicated and, to a great extent, preserves the desirable properties of the original linear system. Its ease of utility via current open-source software makes it superior to other implementations of the VCM.

ACKNOWLEDGMENT & CODE AVAILABILITY

We would like to cordially thank Mr. Julius Zimmerman, who introduced us to the relevant packages and other related material. This work would not be possible without many fruitful discussions with him..

Scan to access the code repository.

REFERENCES

[1] Somersalo, Erkki, et al. "Existence and Uniqueness for Electrode Models for Electric Current Computed Tomography." *SIAM Journal on Applied Mathematics*, vol. 52, no. 4, Aug. 1992, pp. 1023–40, doi:10.1137/0152060.