Visual Exploration of the Transcendental Power Towers of e^i and i^e using Wolfram Alpha and Glowscript 3.2 VPython IDE

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Abstract

This paper explores the values found within the transcendental power towers of e^i and i^e in an online web application named Glowscript 3.2 VPython IDE. The number of values that are plotted are raised up to 18 exponentiations, starting with the values of e^i and i^e using the Wolfram Alpha web application, which is then used for the code in Glowscript to plot it in a complex graph. Results of this paper show that the values of the transcendental power towers of e^i and i^e are arbitrary in nature, but still shows symmetric order in the values of its negative values, reciprocals, and negatives of said reciprocals in the complex graph of real and imaginary values.

Keywords: *transcendentals, power tower, Glowscript IDE*

1. Introduction

When a given number is not a root of any non-zero integer polynomial of a finite degree with coefficients that can be presented rationally, then it is said to be a transcendental number. A real transcendental, as stated in Wolfram MathWorld, must be irrational. This correlates to the definition, which states that it cannot be a root on a finite degree, and a rational number, is always, at degree one.

However, the converse statement of this relationship is not true, as there can be still irrational numbers that are roots of polynomials, such as the root of any number n, which can be used to solve quadratic equations of the form $x^2 - n = 0$. Special numbers like the golden ratio are also not applicable, as it is still a root of a quadratic equation of the form $x^2 - x - 1 = 0$. Commonly known transcendentals that are noted by mathematicians include mathematical constants π and e. Other known transcendentals include values of natural logarithms, that may be paired with root values or another transcendental, such as $\ln 2$, $\ln 3 - \ln 2$, $\pi + \ln 2 + \sqrt{2} \ln 3$, and $\sqrt{2} + \ln(1 + \sqrt{2})$, also known as the universal parabolic constant.

In this paper, the author gives a visualization of the tetration or superexponentiation, more often known as a power tower, of the transcendental towers of e^i and i^e which are only described in a descriptive manner, which states that such values will never converge to a single point and will change infinitely across the complex plane. It is important to know that the imaginary value, i, makes the transcendental power towers to exist only in the complex plane and is not transcendental on its own as it is the solution to the quadratic equation of the form $x^2 + 1 = 0$. This makes the values for the power towers to be represented as $a + bi$, where a is the real part of the complex transcendental value of one of the given power towers, and b as the complex part of the said value.

2. Methodology

The computation for the values of the power towers of e^i and i^e are done automatically by Wolfram Alpha, a web application that is used for the computation of mathematics related operations using common words that a person will input into it. For this paper, the author only used the already available version of the website, hence the reason for the values of the power towers is only limited up to 18 values, as this is the only limit that the version can do for the author while doing it. The values for the transcendental power tower is approximated up to three significant figures only, but in special cases the values are so low that the author gets the non-zero value of the transcendental in order to place it properly to the code. It is unnecessary to do this, however, it makes the code closer to the approximations when it is executed on the program.

After listing down the computed values, the values are then coded as spheres, which are already available in the program. In the program, a complex graph in a two-dimensional representation is created to picture the real and the imaginary plane where the values of the transcendental power towers are plotted. For the tower of e^i, it has all its values in the first quadrant of the complex graph, whereas for the tower of $\boldsymbol{i^e}$ the points that represent the values can be found in the first and second quadrants of the complex graph as well as near the intersection of the real and imaginary axis, and almost within the imaginary axis. To shows a pattern, the author decided to put in the values of the negative values, reciprocals, and negatives of said reciprocals. Colors are assigned for the values of each group to avoid confusion to the author and to the viewer’s eyes when seeing the script.

3. Results and Discussion

Table 3.1 shows the guide for plotted data of the position components of the transcendental power towers of ei and ie along the real plane and the complex plane in the graph. It can be observed that the plotted points of the power towers of ei and ie can be described on where they are located at the complex graph. To properly visualize where these points are, the assigned colors are given to the said power towers along with their negative values, reciprocals, and negatives of said reciprocals.

Table 3.1: Guide for the plot of values of the power towers $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$.

Power Tower	Description	Color
e^i	At Q1	Red
$-e^i$	At Q3	Yellow
$\frac{1}{e^i}$	At Q4	Magenta
$-\frac{1}{e^i}$	At Q2	Bronze
i^e	At Q1 to Q2	Blue
$-i^e$	At Q3 to Q4	Plum
$\frac{1}{i^e}$	At Q2 to Q1	Navy Blue
$-\frac{1}{i^e}$	At Q4 to Q3	Indigo

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Tables 3.2 to 3.5 shows the approximations of the values transcendental power towers of $\boldsymbol{e^i}$ and its negative values, reciprocals, and negatives of said reciprocals. The location of the points too is termed in the table, with describing where can they be found in the quadrants or if they are almost near to the axis of the real plane or the complex plane in the graph.

Table 3.2: The approximations of the values of the power tower of e^i and the description of their locations in a complex graph.

Value	Approximation	Location
e^i	$0.54 + 0.84i$	Q1
e^{i^e}	$0.40 + 0.51i$	Q1
$e^{i^{e^i}}$	$1.17 + 0.24i$	Q1
$e^{i^{e^{i^e}}}$	$1.49 + 5.87i$	Q1
$e^{i^{e^{i^{e^i}}}}$	$0.66 + 0.52i$	Q1
$e^{i^{e^{i^{e^{i^e}}}}}$	$1 + 0.00007i$	Near +RE
$e^{i^{e^{i^{e^{i^{e^i}}}}}}$	$1.17 + 0.47i$	Q1

$e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}$	0.54 + 0.84i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}$	0.80 + 0.40i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}$	1.17 + 0.24i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}$	1.04 + 0.58i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}$	0.66 + 0.52i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}$	0.90 + 0.38i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}$	1.17 + 0.47i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}$	0.93 + 0.56i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}$	0.80 + 0.40i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}$	0.96 + 0.42i	Q1
$e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}}$	1.04 + 0.58i	Q1

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.3: The approximations of the values of the power tower of -e^i and the description of their locations in a complex graph.

Value	Approximation	Location
$-e^{i}$	– 0.54 – 0.84i	Q3
$-e^{i^{e}}$	– 0.40 – 0.51i	Q3
$-e^{i^{e^{i}}}$	– 1.17 – 0.24i	Q3
$-e^{i^{e^{i^{e}}}}$	– 1.49 – 5.87i	Q3
$-e^{i^{e^{i^{e^{i}}}}}$	– 0.66 – 0.52i	Q3
$-e^{i^{e^{i^{e^{i^{e}}}}}}$	– 1 – 0.00007i	Near –RE
$-e^{i^{e^{i^{e^{i^{e^{i}}}}}}}$	– 1.17 – 0.47i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}$	– 0.54 – 0.84i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}$	– 0.80 – 0.40i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}$	– 1.17 – 0.24i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}$	– 1.04 – 0.58i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}$	– 0.66 – 0.52i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}$	– 0.90 – 0.38i	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}$	– 1.17 – 0.47i	Q3

$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}$	$-0.93-0.56i$	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}$	$-0.80-0.40i$	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}$	$-0.96-0.42i$	Q3
$-e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}}$	$-1.04-0.58i$	Q3

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.4: The approximations of the values of the power tower of $\frac{1}{e^{i}}$ and the description of their locations in a complex graph.

Value	Approximation	Location
$\frac{1}{e^{i}}$	$0.54-0.84i$	Q4
$\frac{1}{e^{i^{e}}}$	$0.40-0.51i$	Q4
$\frac{1}{e^{i^{e^{i}}}}$	$1.17-0.24i$	Q4
$\frac{1}{e^{i^{e^{i^{e}}}}}$	$1.49-5.87i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i}}}}}}$	$0.66-0.52i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e}}}}}}}$	$1-0.00007i$	Near +RE
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}$	$1.17-0.47i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}$	$0.54-0.84i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}$	$0.80-0.40i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}$	$1.17-0.24i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}$	$1.04-0.58i$	Q4

$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}$	$0.66 - 0.52i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}$	$0.90 - 0.38i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}$	$1.17 - 0.47i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}$	$0.93 - 0.56i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}$	$0.80 - 0.40i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}$	$0.96 - 0.42i$	Q4
$\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}$	$1.04 - 0.58i$	Q4

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.5: The approximations of the values of the power tower of $-\frac{1}{e^{i}}$ and the description of their locations in a complex graph.

Value	Approximation	Location
$-\frac{1}{e^{i}}$	$-0.54 + 0.84i$	Q2
$-\frac{1}{e^{i^{e}}}$	$-0.40 + 0.51i$	Q2
$-\frac{1}{e^{i^{e^{i}}}}$	$-1.17 + 0.24i$	Q2
$-\frac{1}{e^{i^{e^{i^{e}}}}}$	$-1.49 + 5.87i$	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i}}}}}}$	$-0.66 + 0.52i$	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e}}}}}}}$	$-1 + 0.00007i$	Near –RE

$-\frac{1}{e^{i^{e^{i^{e^{i}}}}}}$	– 1.17 + 0.47i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e}}}}}}}$	– 0.54 + 0.84i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}$	– 0.80 + 0.40i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}$	– 1.17 + 0.24i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}$	– 1.04 + 0.58i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}$	– 0.66 + 0.52i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}$	– 0.90 + 0.38i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}$	– 1.17 + 0.47i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}$	– 0.93 + 0.56i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}$	– 0.80 + 0.40i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}$	– 0.96 + 0.42i	Q2
$-\frac{1}{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}$	– 1.04 + 0.58i	Q2

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Tables 3.6 to 3.9 shows the approximations of the values transcendental power towers of i^e and its negative values, reciprocals, and negatives of said reciprocals. The location of the points too is termed in the table, with describing where can they be found in the quadrants or if they are almost near to the axis of the real plane or the complex plane in the graph.

Table 3.6: The approximations of the values of the power tower of i^e and the description of their locations in a complex graph.

Value	Approximation	Location
i^{e}	$-0.43 + 0.90i$	Q2
$i^{e^{i}}$	$0.18 + 0.20i$	Q1
$i^{e^{i^{e}}}$	$1.80 + 1.32i$	Q1
$i^{e^{i^{e^{i}}}}$	$-0.18 + 0.66i$	Q2
$i^{e^{i^{e^{i^{e}}}}}$	$-0.000068 + 0.000071i$	Near OR
$i^{e^{i^{e^{i^{e^{i}}}}}}$	$0.23 + 0.38i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e}}}}}}}$	$0.00001 + i$	Near +IM
$i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}$	$-0.12 + 0.46i$	Q2
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}$	$0.18 + 0.20i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}$	$0.17 + 0.51i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}$	$-0.18 + 0.66i$	Q2
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}$	$-0.024 + 0.40i$	Q2
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}$	$0.23 + 0.38i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}$	$0.087 + 0.54i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}$	$-0.124 + 0.463i$	Q2
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}$	$0.042 + 0.41i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}$	$0.17+ 0.51i$	Q1
$i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}}$	$0.0355 + 0.52i$	Q1

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.7: The approximations of the values of the power tower of $-i^e$ and the description of their locations in a complex graph.

Value	Approximation	Location
$-i^{e}$	$0.43 - 0.90i$	Q4
$-i^{e^{i}}$	$-0.18 - 0.20i$	Q3
$-i^{e^{i^{e}}}$	$-1.80 - 1.32i$	Q3
$-i^{e^{i^{e^{i}}}}$	$0.18 - 0.66i$	Q4
$-i^{e^{i^{e^{i^{e}}}}}$	$0.000068 - 0.000071i$	Near OR
$-i^{e^{i^{e^{i^{e^{i}}}}}}$	$-0.23 - 0.38i$	Q3
$-i^{e^{i^{e^{i^{e^{i^{e}}}}}}}$	$-0.00001 - i$	Near –IM
$-i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}$	$0.12 - 0.46i$	Q4

$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}$	– 0.18 – 0.20*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}$	– 0.17 – 0.51*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}$	0.18 – 0.66*i*	Q4
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}$	0.024 – 0.40*i*	Q4
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}$	– 0.23 – 0.38*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}$	– 0.087 – 0.54*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}$	0.124 – 0.463*i*	Q4
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}$	– 0.042 – 0.41*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}$	– 0.17 – 0.51*i*	Q3
$-i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}}$	– 0.0355 – 0.52*i*	Q3

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.8: The approximations of the values of the power tower of $\frac{1}{i^{e}}$ and the description of their locations in a complex graph.

Value	Approximation	Location
$\frac{1}{i^{e}}$	– 0.43 – 0.90*i*	Q3
$\frac{1}{i^{e^{i}}}$	0.18 – 0.20*i*	Q4
$\frac{1}{i^{e^{i^{e}}}}$	1.80 – 1.32*i*	Q4
$\frac{1}{i^{e^{i^{e^{i}}}}}$	– 0.18 – 0.66*i*	Q3
$\frac{1}{i^{e^{i^{e^{i^{e}}}}}}$	– 0.000068 – 0.000071*i*	Near OR
$\frac{1}{i^{e^{i^{e^{i^{e^{i}}}}}}}$	0.23 – 0.38*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}$	0.00001– *i*	Near –IM
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}$	– 0.12 – 0.46*i*	Q3
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}$	0.18 – 0.20*i*	Q4

$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}$	0.17 – 0.51*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}$	– 0.18 – 0.66*i*	Q3
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}$	– 0.024 – 0.40*i*	Q3
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}$	0.23 – 0.38*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}$	0.087– 0.54*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}$	– 0.124 – 0.463*i*	Q3
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}$	0.042 – 0.41*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}}$	0.17 – 0.51*i*	Q4
$\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}}}$	0.0355 – 0.52*i*	Q4

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Table 3.8: The approximations of the values of the power tower of $-\frac{1}{i^{e}}$ and the description of their locations in a complex graph.

Value	Approximation	Location
$-\frac{1}{i^{e}}$	0.43 + 0.90*i*	Q1
$-\frac{1}{i^{e^{i}}}$	– 0.18 + 0.20*i*	Q2
$-\frac{1}{i^{e^{i^{e}}}}$	– 1.80 + 1.32*i*	Q2
$-\frac{1}{i^{e^{i^{e^{i}}}}}$	0.18 + 0.66*i*	Q1
$-\frac{1}{i^{e^{i^{e^{i^{e}}}}}}$	0.000068 + 0.000071*i*	Near OR
$-\frac{1}{i^{e^{i^{e^{i^{e^{i}}}}}}}$	– 0.23 + 0.38*i*	Q2

$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}$	– 0.00001 + i	Near +IM
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}$	0.12 + 0.46i	Q1
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}$	– 0.18 + 0.20i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}$	– 0.17 + 0.51i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}$	0.18 + 0.66i	Q1
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}$	0.024 + 0.40i	Q1
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}$	– 0.23 + 0.38i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}$	– 0.087 + 0.54i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}$	0.124 + 0.463i	Q1
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}$	– 0.042 + 0.41i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e}}}}}}}}}}}}}}}}}}$	– 0.17 + 0.51i	Q2
$-\frac{1}{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i^{e^{i}}}}}}}}}}}}}}}}}}}$	– 0.0355 + 0.52i	Q2

Guide: Q1 - Quadrant 1; Q2 - Quadrant 1; Q3 - Quadrant 3; Q4 - Quadrant 4; RE - Real Plane; IM - Imaginary Plane.

Figures 3.1 to 3.5 shows the output of the graph using Glowscript 3.2 VPython IDE. Take note that of the colors that are mentioned in Table 3.1 corresponds to the output that is shown by the program. The axis for the real and the imaginary plane are made to be transparent to show the position components of the spheres that represent the values in the said graph. Spheres that are close enough to their values may show up in almost the same point in the graph, so they are adjusted at the z-axis of the program. This is to avoid interference of the colors that are present in the said spheres.

With the given program, the author also deduced the pattern of the power towers for the values of the power towers of $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ that are computed automatically using the Wolfram Alpha web application. These patterns also show the patterns of the negative values, reciprocals, and negatives of said reciprocals of the transcendental power towers, which is shown on Figure 3.6.

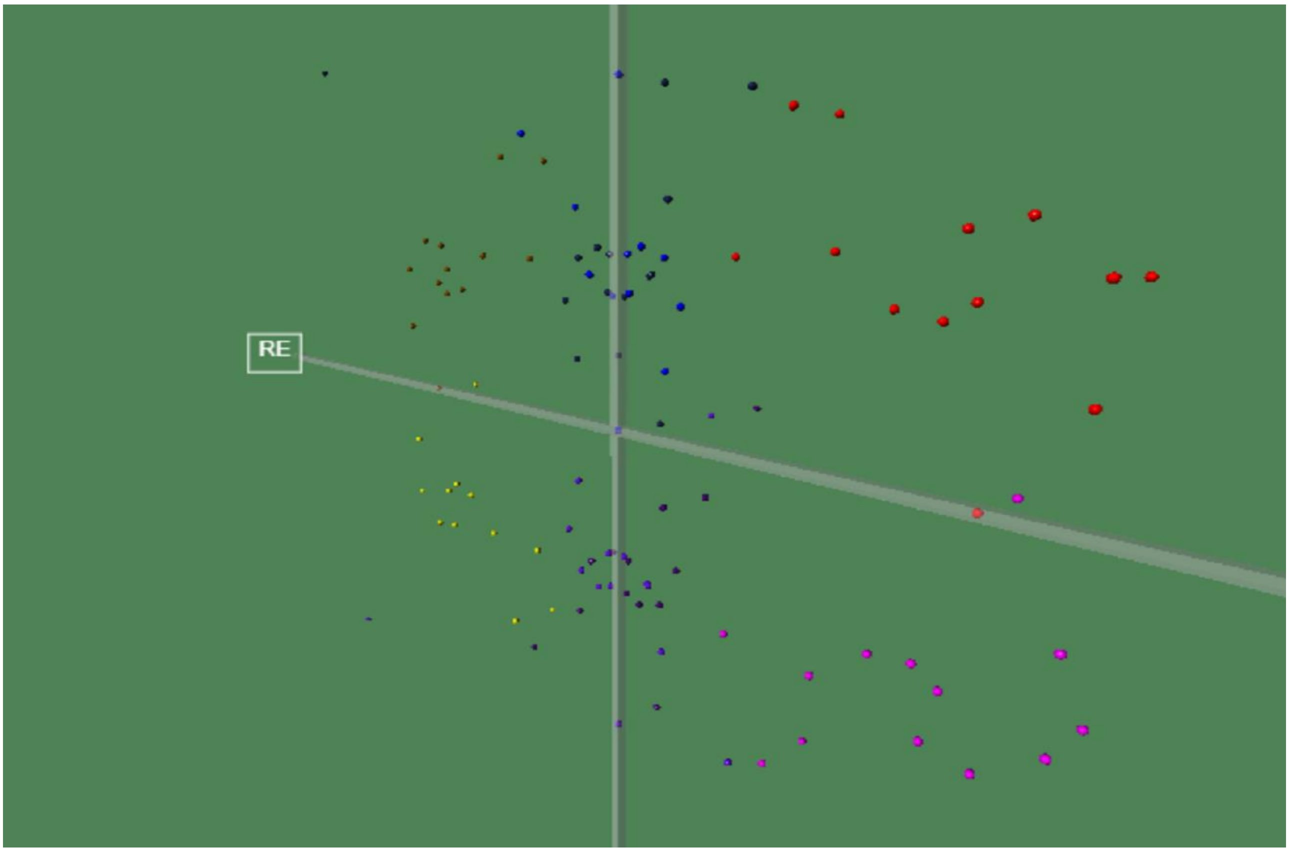

Figure 3.1: A diagonal view from the first and second quadrants of the axis of the real valued plane of the plotted values of the power towers of $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ using Glowscript 3.2 VPython IDE.

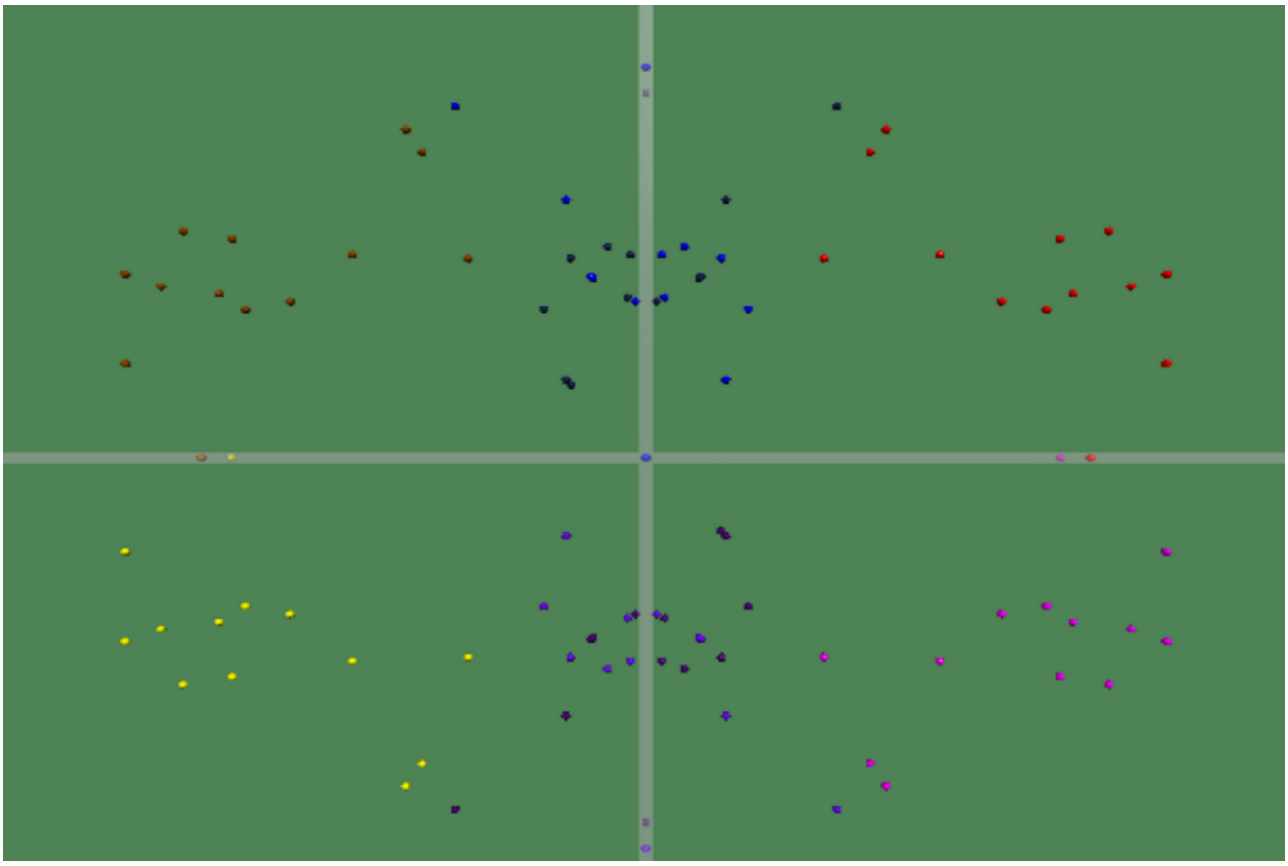

Figure 3.2: A front view coming the intersection of the real valued plane and the imaginary valued plane of the plotted values of the power towers of $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ using Glowscript 3.2 VPython IDE.

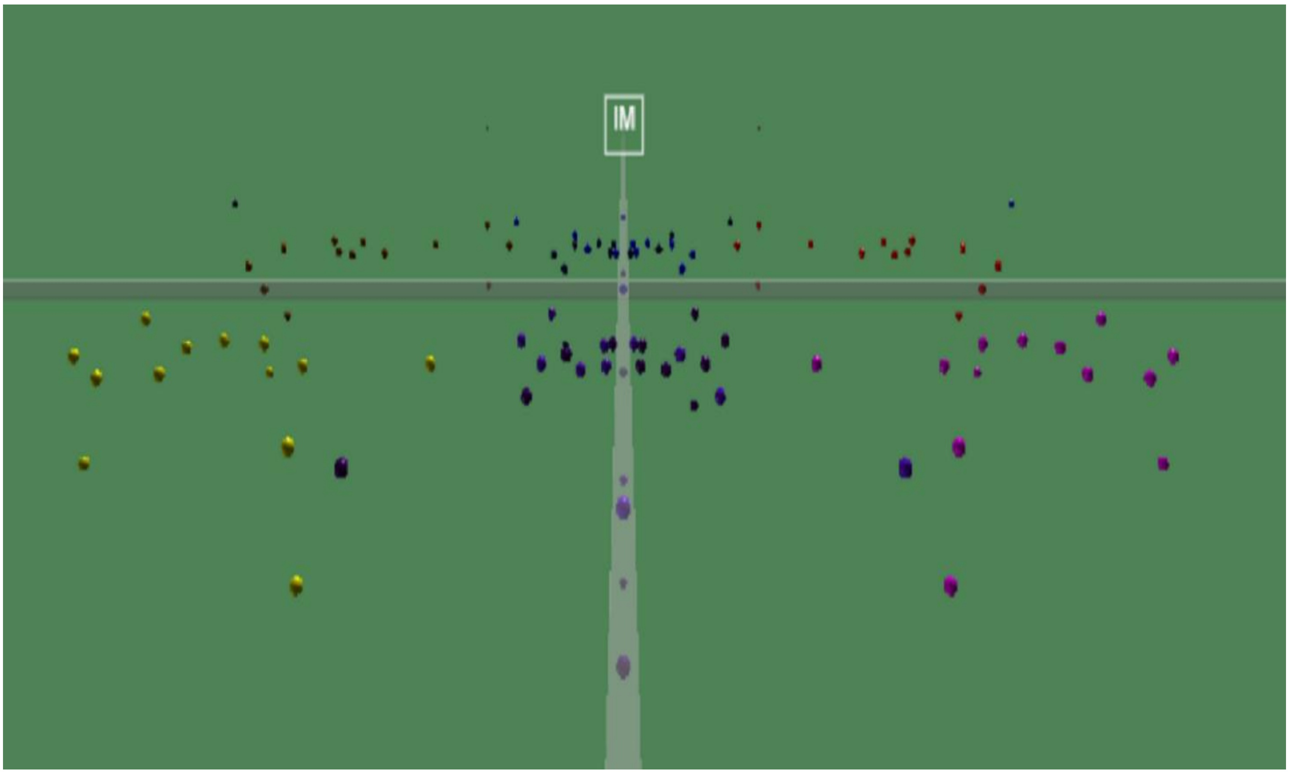

Figure 3.3: A downward view from the negative values of the imaginary valued plane that shows the plot of power towers e^i and i^e using Glowscript 3.2 VPython IDE.

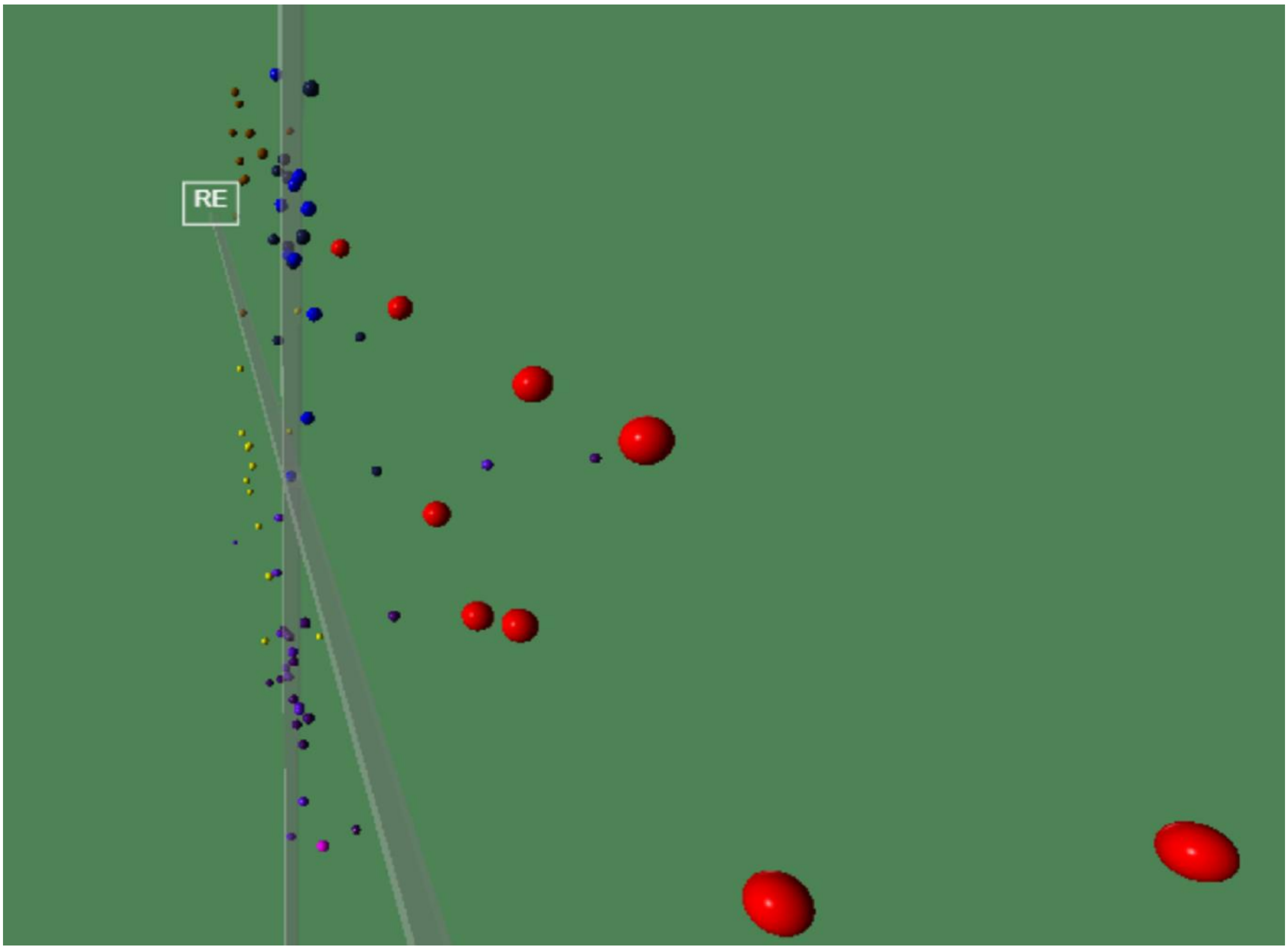

Figure 3.4: A sideward view from the postive values of the real valued plane that shows the plot of power towers e^i and i^e using Glowscript 3.2 VPython IDE.

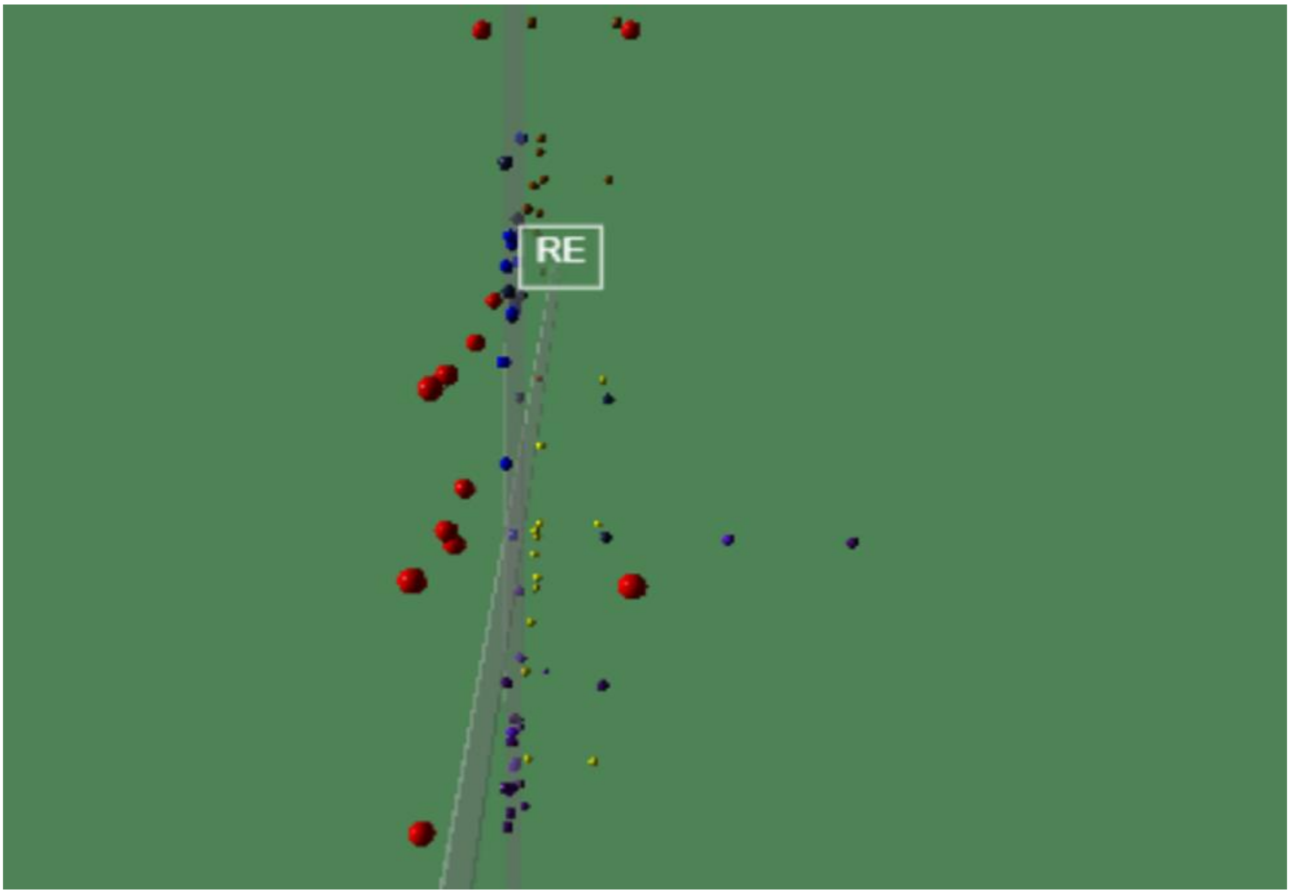

Figure 3.5: A sideward view from the postive values of the real valued plane that shows the plot of power towers $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ using Glowscript 3.2 VPython IDE.

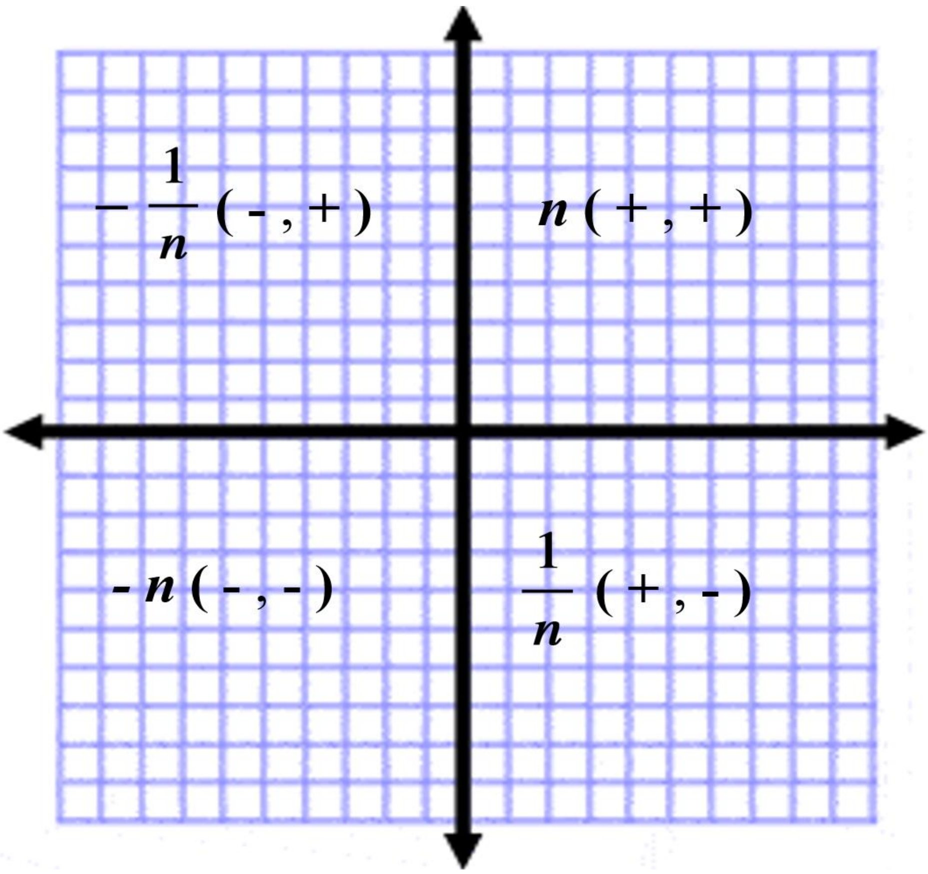

Figure 3.6: Illustration of the various symmetric order patterns found in power towers $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ using Glowscript 3.2 VPython IDE.

4 Conclusions and Recommendations

The illustrations of the power towers of $\boldsymbol{e^i}$ and $\boldsymbol{i^e}$ shows that the values of these transcendental power towers can only be found in the complex plane arbitrarily. Interestingly, the values of the said power towers still show symmetric order in the other quadrants in a complex graph, which is shown in Figure 3.6. It is noteworthy that, while this applies to both power towers, the values of the $\boldsymbol{i^e}$ transcendental power tower may be found near the origin or the intersection of the real valued plane and the imaginary valued plane.

A recommendation that the author wants to give in to this paper is that it can help other mathematicians in power towers, which is not for the faint of heart if an average person tried to do work on it. Another thing is that this paper is heavily reliant on the Wolfram Alpha web application, which the author thinks that many mathematicians only use for additional exploration, which is the point of this paper. The author does not think that people will be interested in looking at these things, but if anyone will, then another recommendation is to try and describe the values in a three-dimensional plot, under a given mathematical function.

Acknowledgements

I made this for fun, so I guess, thank you Jesus for letting me create this in my mind.

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