

## Problem B: Get a Clue!

*Shortest judge solution: 1871 bytes. Shortest team solution (during contest): 1841 bytes.*

*Python solutions by the judges: both Pypy and CPython*

This is a problem that can be solved by a brute-force search, but the implementation can be a bit messy, and depending on your exact approach, it may be important to have good constant factors.

The very simplest approach is just to generate all disjoint sets of cards for the four hands (well, one hand is already given so there's only one option for that) and then go through all the played rounds and check that they are consistent with assignment of cards to hands. However, this will probably not run in time unless it is fairly carefully implemented, so something a bit better is needed.

The next approach could be to first guess the murderer, weapon, and room, and then do the above-mentioned brute-force search for a partition of cards into hands, and break as soon as a valid solution is found. This turns out to run a bit faster and is definitely possible to make fast enough (because the worst case for the above is when pretty much all partitions of cards into hands yields a valid solution, and many of those partitions into hands will result in the same murderer/weapon/room).

A different algorithmic approach which essentially makes constant factor worries go away is to generate the possible hands separately: for player 2 we compute all possible subsets of 5 cards  $S_1, S_2, \dots$  that are compatible with all the played rounds, and similarly for player 3 all sets of 4 cards  $T_1, T_2, \dots$  and for player 4 all subsets of 5 cards  $U_1, U_2, \dots$ . Now for a given guess of murderer/weapon/room, let  $X$  be the set of remaining cards after removing the three answer cards and the hand of player 1. We are then trying to find  $i, j, k$  such that  $S_i \cup T_j \cup U_k = X$ . This can easily be done in time  $O(\#S \cdot \#T)$  – simply try all  $S_i$ 's and  $T_j$ 's and then check if  $X \Delta S_i \Delta T_j$  is one of the  $U_k$ 's (by keeping a dictionary of all  $U_k$ 's). The subsets are most conveniently represented by integers, which makes lookups quick. This solution is fast enough that it can even be done in CPython.

## Problem C: Mission Improbable

*Shortest judge solution: 818 bytes. Shortest team solution (during contest): 1178 bytes.*

*Python solutions by the judges: both Pypy and CPython*

If not for the top-down view, this problem would be really straightforward. Since solving easy problems is easier than solving harder problems, let's go over that first.

Consider the highest stack of crates in the input, and assume it has height  $H_1$ . It has to appear both in the front view and the side view at least once. Assume that it appears  $m_1$  times in the front view, and  $n_1$  times in the side view, and without loss of generality, assume  $n_1 \leq m_1$ . In that case, we will need at least  $m_1$  stacks of height  $H_1$ , and we can achieve the correct side and front view by arranging  $m_1$  stacks so that there is at least one in each of the  $m_1$  columns and  $n_1$  rows.

Then, proceed to the next height,  $H_2$ . Again, we have  $m_2$  columns and  $n_2$  rows that have to contain a stack with height  $H_2$ . The one thing that is different here is that it is possible for,

say,  $m_2$  to be zero – in which case we will put the stacks of height  $H_2$  in the column(s) already containing stacks of height  $H_1$ . Following this pattern, we will use up a total of

$$\sum H_i \cdot \max(m_i, n_i)$$

crates. After placing all the crates for all the heights, the side and front views are already correct, so (disregarding the existence of the top view) we can just leave the remaining spaces empty.

Now, the existence of the top view changes two things in that strategy. First, at the end, we might have to leave a single crate (instead of zero crates) in some of the remaining spaces, to prevent the top view from noticing the spaces are empty. This is easy – we just keep track of how many spaces we filled, and then add to the final answer the number of spaces seen in the top view minus the number of spaces already filled.

The more tricky part is that due to the top view seeing empty spaces in some spots, it might be impossible to put a stack of height  $H_i$  in each of the  $m_i$  columns and  $n_i$  rows using just  $\max(m_i, n_i)$  stacks. We want to have as many stacks as possible to cover both a row and a column, and then we can make the remaining columns and rows covered by just putting a stack of height  $H_i$  wherever it was in the original input. Notice that this is a bipartite matching problem – we have a set of rows and a set of columns, and we can connect a row to a column when the top view shows a non-empty stack. So, for each height  $H_i$  appearing in the input, we run bipartite matching to find out how many stacks can cover both a row and a column, and then replace  $\max(m_i, n_i)$  with  $m_i + n_i - \text{Bipartite}(i)$ . Note that this formula works fine even if one of the sides is zero. Since the runtime of bipartite matching is super-linear, the worst-case for this problem is if all the columns and rows in the front and side views are of the same height. With  $r, c \leq 100$ , this will easily run in time.

## Problem D: Money for Nothing

*Shortest judge solution: 1552 bytes. Shortest team solution (during contest): 1178 bytes.*

*Python solutions by the judges: only Pypy*

First, observe that this is at heart a geometry problem. We are given a set of lower-left and upper-right vertices, and we're asked for the area of the largest rectangle (with sides parallel to the axes) between some two chosen corners. One observation we make is that we can prune the input set. If there are two lower-left corners,  $(x_1, y_1)$  and  $(x_2, y_2)$ , with  $x_1 \leq x_2$  and  $y_1 \leq y_2$ , then we can remove  $(x_2, y_2)$  from the set. After this pruning, the set of lower-left corners forms a sequence  $L_1, L_2, L_3, \dots, L_n$ , with  $L_i = (x_i, y_i)$ , and  $x_i < x_{i+1}, y_i > y_{i+1}$ . We can perform similar pruning on the upper-right corner set, getting the sequence  $U_1, U_2, \dots, U_m$ , with  $U_i = (p_i, q_i)$ , and again  $p_i < p_{i+1}$  and  $q_i > q_{i+1}$ .

We'll cover two solutions to the problem. The first one is less geometric. Define  $u(i)$  to be the index of the optimum upper-right corner for  $L_i$  – that is, the rectangle  $L_i, U_{u(i)}$  has area no smaller than any other  $L_i, U_j$ . In the case of ties, choose the rightmost one. We're claiming that  $u(i)$  is non-decreasing. A geometric proof is to draw out the picture, and notice that for any  $i < j, k < l$ , the sum of areas of  $L_i U_k$  and  $L_j U_l$  is larger than the sum of areas  $L_i U_l$  and  $L_j U_k$ . You can also just write out the areas and see the inequality holds.

This allows a divide-and-conquer solution. Take the middle of all the  $L_i$ s, and find  $u(i)$  (through a linear scan). Then, for all  $j < i$ , we can consider only the upper right corners up to  $u(i)$ , and for all the  $j > i$  we can consider only the upper corners starting from  $u(i)$ , and recurse into both branches. Since we're halving the set of  $L$ s at each pass, we will have  $\log n$