

assign weights to those points so that the weighted average of  $S_1(v)$  is  $\leq 0.5$ , and the weighted average of  $S_2(v)$  is as high as possible (and vice versa).

This is a geometry problem. Note that all the points we can obtain by weighted averages of a set of points is exactly their convex hull; so we're looking for the highest  $y$ -variable value in the convex hull intersected with  $x \leq 0.5$ , which can be determined in  $O(n)$  in a number of ways.

## Problem D: Carl's Vacation

*Problem authors:*

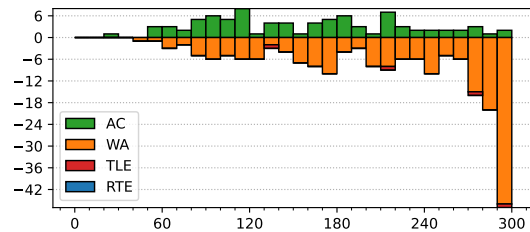
Arnav Sastry and the World Finals judges

*Solved by 86 teams.*

*First solved after 21 minutes.*

*Shortest team solution: 1537 bytes.*

*Shortest judge solution: 893 bytes.*



Clearly, the challenge in this problem is not about running time, but rather about how to represent the three-dimensional geometry problem in a convenient way.

On a plane, the shortest path between two points is always a line segment. So, the path will be a segment from the top of one pyramid to its base, then a segment across the ground to the base of the other pyramid, and then a segment from the base to the top of that pyramid.

We have 16 ways of selecting the faces of the two pyramids we will travel on, so we will check all of them. Now, we have two tilted triangles, each attached to the ground by the base, and we need to get from the top of one triangle to the top of the other using as short of a path as possible. The answer will be the same if we rotate each triangle around its base and flatten it out — so we now just need to find the shortest path between two points on the plane, under the condition that it passes through two specified segments. The shortest path between two points on a plane is just a segment. So, to summarize:

- We iterate over all selections of the faces on both pyramids.
- We rotate the top of the pyramid around the line containing the base of the selected face so that the top of the pyramid lands on the ground.
- We check if the segment between the two rotated tops intersects the two bases, in the right order.
- If yes, we take the distance between the rotated tops as a candidate distance.
- We output the smallest candidate distance.

Another approach is to observe that, for a pair of pyramid faces, we can parameterize the path taken by the ant by two angles  $\theta_1$  and  $\theta_2$ . The minimum of  $\text{dist}(\theta_1, \theta_2)$  can be found using golden section or ternary search.

## Problem E: A Recurring Problem

*Problem authors:*

Derek Kisman and Matthias Ruhl

*Solved by 0 teams.*

*Shortest judge solution: 2987 bytes.*

