



## 1 Introduction

The article **"Playing Atari with Deep Reinforcement Learning"**[1] presents a deep learning model that successfully learns control policies directly from high-dimensional sensory input using reinforcement learning. The model, a convolutional neural network, is trained with a variant of Q-learning and is able to control an agent in complex Atari 2600 games from the Arcade Learning Environment. The article highlights the challenges of applying deep learning to reinforcement learning, such as the need for sparse, noisy, and delayed reward signals, and the correlation and non-stationarity of data in RL. The authors demonstrate that their approach, utilizing an experience replay mechanism and stochastic gradient descent, is able to overcome these challenges and achieve superior performance to previous RL methods. This project will delve deeper into the methodology and results presented in the article, and discuss the significance of this approach in the field of reinforcement learning and artificial intelligence.

## 2 Background

The background section of the article describes the problem of using reinforcement learning to train an agent to play Atari 2600 games. The agent interacts with the Atari emulator by selecting actions from a set of legal game actions,  $A$ , and receives observations in the form of raw pixel values representing the current screen and rewards for the changes in game score. The task is partially observed, meaning that the agent only observes images of the current screen and it is impossible to fully understand the current situation. Therefore, the agent considers sequences of actions and observations,  $s_t = x_1, a_1, x_2, \dots, a_{t-1}, x_t$ , and learns game strategies that depend upon these sequences. The agent's goal is to maximize future rewards, which are discounted by a factor of  $\gamma$  per time-step. It is stated in the article that Atari 2600 games are considered a challenging testbed<sup>1</sup> for reinforcement learning due to the complexity and diversity of the tasks they present and that the Atari games are a large but finite Markov Decision Process (MDP). The authors also mention the goal of creating a single neural network agent that is able to successfully learn to play as many of the games as possible.



Figure 1: Images captured from five Atari 2600 Games: (From left to right) Pong, Breakout, Space Invaders, Seaquest and Beam Rider.

<sup>1</sup>A testbed is a platform or environment used to test and evaluate a system or method. In the context of this article, the Atari 2600 games serve as a testbed for reinforcement learning algorithms because they provide a challenging and diverse set of tasks for the agent to perform. The Atari games are considered a useful testbed because they present a high-dimensional visual input and a variety of tasks that are difficult for human players. Using Atari games as a testbed allows researchers to evaluate their reinforcement learning algorithms in a controlled environment.

In the following, another expression of what should be known about reinforcement learning in the study of the aforementioned article is presented.

## 2.1 Bellman equation and basic concepts

The Bellman equation is a fundamental concept in reinforcement learning (RL) that describes the relationship between the value of a state and the value of its possible actions. It is used to update the value of a state given the values of its possible next states. The equation is typically written as:

$$V(s) = \max_a \left[ r(s, a) + \gamma \sum_{s'} P(s'|s, a) V(s') \right]$$

Where:

- $V(s)$  is the value of state  $s$
- $a$  is the action taken in state  $s$
- $r(s, a)$  is the reward obtained after taking action  $a$  in state  $s$
- $\gamma$  is the discount factor, which determines the importance of future rewards
- $P(s'|s, a)$  is the probability of transitioning to state  $s'$  after taking action  $a$  in state  $s$
- $V(s')$  is the value of the next state  $s'$

Value iteration is an algorithm for finding the optimal value function for a given Markov decision process (MDP). The algorithm starts with an initial estimate of the value function, and repeatedly applies the Bellman equation to update the estimate until it converges to the true value function. The algorithm can be written as:

$$V_{k+1}(s) = \max_a \left[ r(s, a) + \gamma \sum_{s'} P(s'|s, a) V_k(s') \right]$$

Where:

- $V_k(s)$  is the  $k$ -th estimate of the value of state  $s$
- $V_{k+1}(s)$  is the  $(k+1)$ -th estimate of the value of state  $s$

Action-value based methods in RL are used to estimate the value of taking a specific action in a specific state, rather than just the value of a state. The action-value function,  $Q(s, a)$ , gives the expected return of taking action  $a$  in state  $s$  and then following the policy thereafter. It can be represented using the Bellman equation as:

$$Q(s, a) = r(s, a) + \gamma \sum_{s'} P(s'|s, a) \max_{a'} Q(s', a')$$

where  $Q(s', a')$  is the maximum expected return over all possible actions that can be taken in the next state  $s'$

## 2.2 Q-Network

In Q-Network parametrized algorithm we can approximate the Q-function using a neural network, which is trained to predict the action-value of taking a specific action in a specific state. The neural network takes the state and action as input, and outputs the estimated Q-value. The parameters of the neural network are trained using the Bellman equation, by minimizing the difference between the predicted Q-value and the true Q-value.

$$L = E_{s,a}[(Q(s, a; \theta) - Q^*(s, a))^2]$$

Where:

- $Q(s, a; \theta)$  is the predicted Q-value, parameterized by the neural network with parameters  $\theta$
- $Q^*(s, a)$  is the true Q-value
- $E_{s,a}$  denotes the expectation over all possible states and actions

In order to find the optimal parameters for the Q-network, we use a variation of stochastic gradient descent (SGD) called Q-learning. The Q-learning algorithm updates the parameters of the Q-network using the following update rule:

$$\theta \leftarrow \theta + \alpha \cdot \delta \cdot \nabla_{\theta} Q(s, a; \theta)$$

Where:

- $\alpha$  is the learning rate
- $\delta = r + \gamma \max_{a'} Q(s', a'; \theta) - Q(s, a; \theta)$  is the temporal difference error
- $\nabla_{\theta} Q(s, a; \theta)$  is the gradient of the Q-network with respect to its parameters

In Q-learning algorithm, the agent samples a random action at each step and uses the Q-network to estimate the Q-value of that action. The Q-network is then updated to reduce the error between the estimated Q-value and the true Q-value. This process is repeated many times until the Q-network converges to the optimal Q-function.

In summary, the Bellman equation is used to find the optimal value function in RL. The value iteration algorithm uses the Bellman equation to update the value function until it converges to the optimal value function. Action-value based methods use Q-function to estimate the value of taking a specific action in a specific state. The Q-network parametrized algorithm is used to approximate the Q-function using a neural network. The Q-learning algorithm is used to find the optimal parameters for the Q-network.

## 2.3 Loss function

The loss function of a Q-network is used to measure the difference between the predicted Q-values and the true Q-values, and it is used to update the parameters of the network. A common loss function used in Q-learning is the mean squared error (MSE) loss:

$$L = \frac{1}{n} \sum_{i=1}^n (Q(s_i, a_i; \theta) - Q^*(s_i, a_i))^2$$

Where:

- $Q(s_i, a_i; \theta)$  is the predicted Q-value for the i-th state-action pair, parameterized by the neural network with parameters  $\theta$
- $Q^*(s_i, a_i)$  is the true Q-value for the i-th state-action pair
- $n$  is the number of state-action pairs in the training set

The goal of the Q-network is to minimize the loss function  $L$ . To do this, the Q-network updates its parameters using gradient descent. The gradient of the loss function with respect to the parameters of the Q-network is given by:

$$\nabla_{\theta} L = \frac{2}{n} \sum_{i=1}^n (Q(s_i, a_i; \theta) - Q^*(s_i, a_i)) \nabla_{\theta} Q(s_i, a_i; \theta)$$

Where  $\nabla_{\theta} Q(s_i, a_i; \theta)$  is the gradient of the Q-network with respect to its parameters for the i-th state-action pair.

The Q-network updates its parameters in the opposite direction of the gradient, so that the loss function will decrease. The update rule for the Q-network parameters is given by:

$$\theta \leftarrow \theta - \alpha \nabla_{\theta} L$$

Where  $\alpha$  is the learning rate, which determines the step size of the update.

In summary, the loss function of a Q-network is used to measure the difference between the predicted Q-values and the true Q-values, and it is used to update the parameters of the network. The mean squared error (MSE) loss is a common loss function used in Q-learning, which aims to minimize the difference between the predicted Q-values and the true Q-values. The Q-network updates its parameters using gradient descent, by moving in the opposite direction of the gradient of the loss function with respect to its parameters.

### 3 Related Work

The "Related work" section of this article discusses prior work in the field of reinforcement learning, specifically mentioning TD-gammon as a successful application of reinforcement learning and self-play to achieve a superhuman level of play in backgammon. However, the article also notes that early attempts to apply the same method to other games were less successful, leading to the belief that the TD-gammon approach was specific to backgammon.

The article also discusses more recent work in combining deep learning with reinforcement learning, including the use of deep neural networks to estimate the environment, restricted Boltzmann machines to estimate the value function, and gradient temporal-difference methods to address divergence issues with Q-learning. The article also mentions the neural fitted Q-learning (NFQ) method, which is similar to the approach used in the article but has a higher computational cost and is applied to a lower-dimensional representation of the task. Additionally, the article highlights the use of Atari 2600 emulator as a reinforcement learning platform, including the use of standard reinforcement learning algorithms with linear function approximation and generic visual features and the HyperNEAT evolutionary architecture which helped to exploit design flaws in several Atari games.

## 4 Deep Reinforcement Learning

Reinforcement learning is a type of machine learning that involves an agent learning how to make decisions in an environment by receiving rewards or penalties for its actions. Deep reinforcement learning combines reinforcement learning with deep neural networks, which are complex neural network architectures that can learn from large amounts of data.

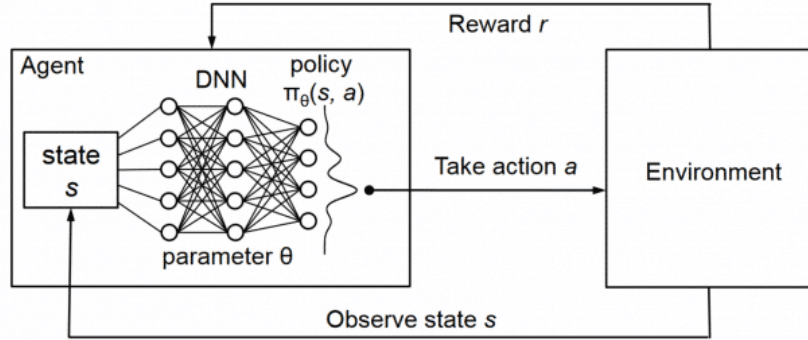


Figure 2: How DNNs fit in agent

In the article, the authors propose a specific approach to deep reinforcement learning that uses a technique called "experience replay." The idea behind experience replay is to store the agent's experiences at each time step, in the form of a tuple of (state, action, reward, next state), in a dataset <sup>2</sup> called " $\mathcal{D}$ " or replay memory. Then, during the training process, the algorithm can randomly sample from this dataset to update the parameters of the agent's policy.

One of the key benefits of experience replay is that it allows the agent to learn from its experiences more efficiently. In standard online Q-learning, the agent only learns from the most recent experience, which can be inefficient due to the strong correlation between consecutive samples. By randomly sampling from the replay memory, the agent can learn from a wider range of experiences and break these correlations, which reduces the variance of the updates and can lead to more stable learning.

Another benefit of experience replay is that it allows the agent to learn off-policy. In standard online Q-learning, the agent's current parameters determine the next data sample that the parameters are trained on. This can lead to unwanted feedback loops and poor local minima. By using experience replay, the agent's behavior distribution is averaged over many of its previous states, smoothing out learning and avoiding oscillations or divergence in the parameters.

In practice, the authors propose an algorithm that stores the last N experience tuples in the replay memory, and samples uniformly at random from  $\mathcal{D}$  when performing updates. However, this approach is limited by the finite memory size N, which means that the memory buffer does not differentiate important transitions and always overwrites with recent transitions. Similarly, the uniform sampling gives equal importance to all transitions regardless of their relevance.

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<sup>2</sup>"dataset" is being used to refer to a collection of experiences that the agent has had in the environment. In traditional machine learning, a dataset typically refers to a collection of input-output pairs that are used to train a model. In reinforcement learning, however, the agent is not provided with a set of labeled examples to learn from. Instead, it has to learn by interacting with the environment and receiving rewards or penalties for its actions. So, the dataset  $\mathcal{D}$  that the authors are referring to is a collection of experiences that the agent has had in the form of tuples of (state, action, reward, next state) which can be used for the training process.

Overall, the proposed approach of using experience replay in deep reinforcement learning is a promising method for improving the efficiency and stability of the training process. This method can be applied to any problem that can be represented as a sequence of states and actions, such as robotic control, game playing, and decision making.

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**Algorithm 1** Deep Q-learning with Experience Replay

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```

Initialize replay memory  $\mathcal{D}$  to capacity  $N$ 
Initialize action-value function  $Q$  with random weights
for episode = 1,  $M$  do
  Initialise sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$ 
  for  $t = 1, T$  do
    With probability  $\epsilon$  select a random action  $a_t$ 
    otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ 
    Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$ 
    Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$ 
    Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$ 
    Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$ 
    if  $\phi_{j+1}$  is non-terminal then
      Set  $y_j = r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta)$ 
    else if  $\phi_{j+1}$  is terminal then
      Set  $y_j = r_j$ 
    end if
    Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation of  $\nabla_{\theta} L$ 
  end for
end for

```

---

This is the algorithm of Deep Q-learning with Experience Replay, where Q-learning updates are applied to samples of experience,  $e \sim \mathcal{D}$ , drawn at random from the pool of stored samples. The algorithm starts by initializing a replay memory  $\mathcal{D}$  with a capacity  $N$ , and an action-value function  $Q$  with random weights. The algorithm iterates over episodes, and in each episode, it initializes a sequence  $s_1$  and preprocessed sequence  $\phi_1$ . Then, for each time step, the algorithm selects an action with a probability of epsilon, where the action is selected randomly, otherwise, the action is selected according to the policy. Then the agent executes the action, observe the reward and the new state, and stores the transition in the replay memory  $\mathcal{D}$ .

In the next step, the algorithm samples a random minibatch of transitions from  $\mathcal{D}$ , and sets the target  $y_j$  for each transition to the current reward plus the expected future reward. Then it performs a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))$  to update the Q function.

## 5 Preprocessing and Model Architecture

This part of article describes a method for training an agent to play Atari games. The agent uses a Q-learning algorithm, which requires an estimate of the Q-value for each state-action pair. The Q-value represents the expected cumulative reward for taking a given action in a given state and following the optimal policy thereafter.

The first step in the method is preprocessing the raw frames of the game. The raw frames are  $210 \times 160$  pixel images with a 128 color palette, which can be computationally demanding to work with directly. To reduce the input dimensionality, the raw frames are first converted to grayscale,

which reduces the number of channels from 3 (red, green, blue) to 1. Then, the images are down-sampled to  $110 \times 84$ . Finally, the images are cropped to an  $84 \times 84$  region that roughly captures the playing area. The function  $\phi$  from algorithm 1 applies this preprocessing to the last 4 frames of a history and stacks them to produce the input to the Q-function.

The Q-function is implemented as a neural network. The neural network takes as input the preprocessed frame and outputs an estimate of the Q-value for each possible action. One way to implement the Q-function is to use a separate forward pass to compute the Q-value for each action. However, this can be computationally expensive as the number of actions increases. Therefore, the article suggests an alternative architecture, in which there is a separate output unit for each possible action. With this architecture, the Q-value for all possible actions can be computed in a single forward pass.

The neural network has three hidden layers. The first hidden layer convolves 16  $8 \times 8$  filters with stride 4 with the input image and applies a rectifier nonlinearity (also known as a  $\text{ReLU}^3$ ). The second hidden layer convolves 32  $4 \times 4$  filters with stride 2, again followed by a rectifier nonlinearity. The final hidden layer is fully-connected and consists of 256 rectifier units. The output layer is a fully-connected linear layer with one output for each valid action. The number of valid actions varies between 4 and 18 for the different games considered in the experiments. The authors refer to their approach as "Deep Q-Networks (DQN)".

## 6 Experiments

The article describes experiments on seven popular Atari games: Beam Rider, Breakout, Enduro, Pong, Q\*bert, Seaquest, Space Invaders. The same network architecture, learning algorithm, and hyperparameter settings were used across all seven games, showing that the approach is robust enough to work on a variety of games without incorporating game-specific information.

During training, the reward structure of the games was modified. To make it easier to use the same learning rate across multiple games, positive rewards were fixed to 1 and negative rewards to -1, leaving 0 rewards unchanged. The agent's behavior policy during training was an epsilon-greedy policy with epsilon annealed linearly from 1 to 0.1 over the first million frames, and fixed at 0.1 thereafter. The agent was trained for a total of 10 million frames and used a replay memory of one million most recent frames.

The agent also used a simple frame-skipping technique, where it sees and selects actions on every  $k^{\text{th}}$  frame instead of every frame, and its last action is repeated on skipped frames. This allows the agent to play roughly k times more games without significantly increasing the runtime.  $k = 4$  was used for all games except Space Invaders, where  $k = 3$  was used to make the lasers visible.

## 7 Training and Stability

The article discusses the challenges of evaluating the progress of an agent during training in reinforcement learning. The evaluation metric used is the total reward collected by the agent in an episode or game, averaged over a number of games. However, this metric can be very noisy and can give the impression that the learning algorithm is not making steady progress. To address this, the article suggests using the policy's estimated action-value function Q as an additional metric.

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<sup>3</sup>ReLU stands for Rectified Linear Unit, which is a type of activation function used in neural networks. It outputs the maximum of 0 and the input, effectively thresholding negative values to 0 and passing positive values unchanged. It is widely used due to its simplicity and computational efficiency.

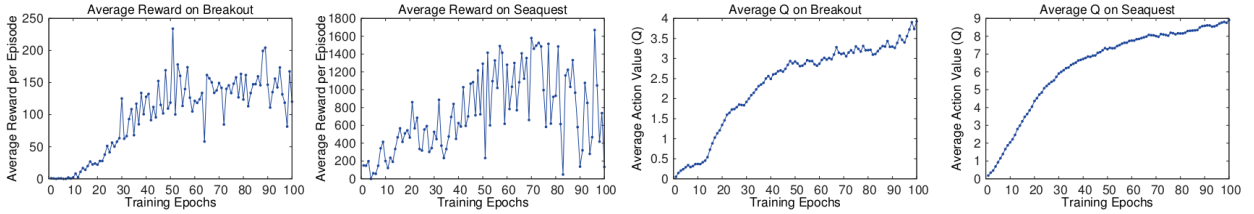


Figure 3: The figures on the left depict the average reward earned per episode in the Breakout and Seaquest games during training using an epsilon-greedy policy with a value of 0.05 for 10000 steps. The figures on the right display the average highest predicted action-value of a group of unutilized states in the Breakout and Seaquest games. Each figure represents one epoch, equivalent to 30 minutes of training time or 50000 minibatch weight updates.

Q provides an estimate of the discounted reward the agent can obtain by following its policy from any given state. The article reports that this metric tends to be more stable, showing relatively smooth improvement during training. Additionally, the article notes that no divergence issues were experienced in any of their experiments, suggesting that their method is able to train large neural networks using a reinforcement learning signal and stochastic gradient descent in a stable manner.

## 8 Visualizing the Value Function

The "Visualizing the Value Function" section of the article includes a figure, that shows how the authors' method is able to learn the value function on the game Seaquest. The figure shows that the predicted value changes as the game progresses, and it jumps after an enemy appears on the left of the screen, peaks when the agent fires a torpedo at the enemy, and falls back to its original value after the enemy disappears. This figure demonstrates that the authors' method can learn how the value function changes for a complex sequence of events in the game.



Figure 4: The plot on the left of figure illustrates the predicted value function for a 30-frame segment of the game Seaquest. The three screenshots displayed are frames labeled A, B, and C respectively, and it illustrates how the agent predicts the value of its action. The plot demonstrates that the agent's predicted value jumps when an enemy appears on the left of the screen, reaches a peak when the agent fires a torpedo at the enemy, and then falls to its original value after the enemy disappears. This figure illustrates that the agent can learn how the value function evolves for a complex sequence of events in the game Seaquest.

## 9 Main Evaluation

The Main Evaluation section compares the results of the authors' approach, labeled DQN, with the best performing methods from the RL literature [3, 4]. The methods labeled Sarsa and Contingency



use the Sarsa algorithm and incorporate significant prior knowledge about the visual problem, while the authors' approach only receives raw RGB screenshots as input and must learn to detect objects on its own. The authors' approach outperforms the other learning methods by a substantial margin on all seven games despite incorporating almost no prior knowledge about the inputs. The authors also compare their approach to an evolutionary policy search approach [8] and show that their approach achieves better performance on all games except Space Invaders. The authors' method also achieves better performance than an expert human player on Breakout, Enduro and Pong and it achieves close to human performance on Beam Rider.

	<b>B. Rider</b>	<b>Breakout</b>	<b>Enduro</b>	<b>Pong</b>	<b>Q*bert</b>	<b>Seaquest</b>	<b>S. Invaders</b>
<b>Random</b>	354	1.2	0	-20.4	157	110	179
<b>Sarsa 3</b>	996	5.2	129	-19	614	665	271
<b>Contingency 4</b>	1743	6	159	-17	960	723	268
<b>DQN</b>	<b>4092</b>	<b>168</b>	<b>470</b>	<b>20</b>	<b>1952</b>	<b>1705</b>	<b>581</b>
<b>Human</b>	7456	31	368	-3	18900	28010	3690
<b>HNeat Best 8</b>	3616	52	106	19	1800	920	<b>1720</b>
<b>HNeat Pixel 8</b>	1332	4	91	-16	1325	800	1145
<b>DQN Best</b>	<b>5184</b>	<b>225</b>	<b>661</b>	<b>21</b>	<b>4500</b>	<b>1740</b>	1075

Table 1: Table of Results: Comparison of Performance of Various Reinforcement Learning Algorithms on Different Atari Games

## 10 Python Implementation provided as final project

```
1  """
2
3  @author: Arash Sajjadi
4  """
5  import torch
6  import torch.nn as nn
7  import torch.optim as optim
8  import gym
9  import random
10 import math
11 import time
12 import os.path
13
14 import matplotlib.pyplot as plt
15 from atari_wrappers import make_atari, wrap_deepmind
16
17 plt.style.use('ggplot')
18
19 # if gpu is to be used
20 use_cuda = torch.cuda.is_available()
21
22 device = torch.device("cuda:0" if use_cuda else "cpu")
23 Tensor = torch.Tensor
24 LongTensor = torch.LongTensor
25
26 env_id = "PongNoFrameskip-v4"
27 env = make_atari(env_id)
28 env = wrap_deepmind(env)
29
30 directory = './PongVideos/'
31 env = gym.wrappers.Monitor(env, directory, video_callable=lambda episode_id: episode_id%20==0)
32
33
34 seed_value = 23
35 env.seed(seed_value)
36 torch.manual_seed(seed_value)
37 random.seed(seed_value)
38
39 ##### PARAMS #####
40 learning_rate = 0.0001
41 num_episodes = 500
42 gamma = 0.99
43
44 hidden_layer = 512
45
46 replay_mem_size = 100000
47 batch_size = 32
48
49 update_target_frequency = 2000
50
51 double_dqn = True
52
53 egreedy = 0.9
54 egreedy_final = 0.01
55 egreedy_decay = 10000
56
57 report_interval = 10
58 score_to_solve = 18
59
60 clip_error = True
61 normalize_image = True
62
63 file2save = 'pong_save.pth'
64 save_model_frequency = 10000
65 resume_previous_training = False
```

```

66
67 #####
68
69 number_of_inputs = env.observation_space.shape[0]
70 number_of_outputs = env.action_space.n
71
72 def calculate_epsilon(steps_done):
73     epsilon = egreedy_final + (egreedy - egreedy_final) * \
74         math.exp(-1. * steps_done / egreedy_decay )
75     return epsilon
76
77 def load_model():
78     return torch.load(file2save)
79
80 def save_model(model):
81     torch.save(model.state_dict(), file2save)
82
83 def preprocess_frame(frame):
84     frame = frame.transpose((2,0,1))
85     frame = torch.from_numpy(frame)
86     frame = frame.to(device, dtype=torch.float32)
87     frame = frame.unsqueeze(1)
88
89     return frame
90
91 def plot_results():
92     plt.figure(figsize=(12,5))
93     plt.title("Rewards")
94     plt.plot(rewards_total, alpha=0.6, color='red')
95     plt.savefig("Pong-results.png")
96     plt.close()
97
98 class ExperienceReplay(object):
99     def __init__(self, capacity):
100         self.capacity = capacity
101         self.memory = []
102         self.position = 0
103
104     def push(self, state, action, new_state, reward, done):
105         transition = (state, action, new_state, reward, done)
106
107         if self.position >= len(self.memory):
108             self.memory.append(transition)
109         else:
110             self.memory[self.position] = transition
111
112         self.position = ( self.position + 1 ) % self.capacity
113
114
115     def sample(self, batch_size):
116         return zip(*random.sample(self.memory, batch_size))
117
118
119     def __len__(self):
120         return len(self.memory)
121
122 class NeuralNetwork(nn.Module):
123     def __init__(self):
124         super(NeuralNetwork, self).__init__()
125
126         self.conv1 = nn.Conv2d(in_channels=1, out_channels=32, kernel_size=8, stride=4)
127         self.conv2 = nn.Conv2d(in_channels=32, out_channels=64, kernel_size=4, stride=2)
128         self.conv3 = nn.Conv2d(in_channels=64, out_channels=64, kernel_size=3, stride=1)
129
130         self.advantage1 = nn.Linear(7*7*64,hidden_layer)
131         self.advantage2 = nn.Linear(hidden_layer, number_of_outputs)
132
133         self.value1 = nn.Linear(7*7*64,hidden_layer)

```

```

134         self.value2 = nn.Linear(hidden_layer,1)
135
136         #self.activation = nn.Tanh()
137         self.activation = nn.ReLU()
138
139
140     def forward(self, x):
141
142         if normalize_image:
143             x = x / 255
144
145         output_conv = self.conv1(x)
146         output_conv = self.activation(output_conv)
147         output_conv = self.conv2(output_conv)
148         output_conv = self.activation(output_conv)
149         output_conv = self.conv3(output_conv)
150         output_conv = self.activation(output_conv)
151
152         output_conv = output_conv.view(output_conv.size(0), -1) # flatten
153
154         output_advantage = self.advantage1(output_conv)
155         output_advantage = self.activation(output_advantage)
156         output_advantage = self.advantage2(output_advantage)
157
158         output_value = self.value1(output_conv)
159         output_value = self.activation(output_value)
160         output_value = self.value2(output_value)
161
162         output_final = output_value + output_advantage - output_advantage.mean()
163
164         return output_final
165
166 class QNet_Agent(object):
167     def __init__(self):
168         self.nn = NeuralNetwork().to(device)
169         self.target_nn = NeuralNetwork().to(device)
170
171         self.loss_func = nn.MSELoss()
172         #self.loss_func = nn.SmoothL1Loss()
173
174         self.optimizer = optim.Adam(params=self.nn.parameters(), lr=learning_rate)
175         #self.optimizer = optim.RMSprop(params=mynn.parameters(), lr=learning_rate)
176
177         self.number_of_frames = 0
178
179         if resume_previous_training and os.path.exists(file2save):
180             print("Loading previously saved model ... ")
181             self.nn.load_state_dict(load_model())
182
183     def select_action(self, state, epsilon):
184
185         random_for_egreedy = torch.rand(1)[0]
186
187         if random_for_egreedy > epsilon:
188
189             with torch.no_grad():
190
191                 state = preprocess_frame(state)
192                 action_from_nn = self.nn(state)
193
194                 action = torch.max(action_from_nn, 1)[1]
195                 action = action.item()
196         else:
197             action = env.action_space.sample()
198
199         return action
200
201     def optimize(self):

```

```

202
203     if (len(memory) < batch_size):
204         return
205
206     state, action, new_state, reward, done = memory.sample(batch_size)
207
208     state = [ preprocess_frame(frame) for frame in state ]
209     state = torch.cat(state)
210
211     new_state = [ preprocess_frame(frame) for frame in new_state ]
212     new_state = torch.cat(new_state)
213
214     reward = Tensor(reward).to(device)
215     action = LongTensor(action).to(device)
216     done = Tensor(done).to(device)
217
218
219     if double_dqn:
220         new_state_indexes = self.nn(new_state).detach()
221         max_new_state_indexes = torch.max(new_state_indexes, 1)[1]
222
223         new_state_values = self.target_nn(new_state).detach()
224         max_new_state_values = new_state_values.gather(1, max_new_state_indexes.unsqueeze(1)).squeeze(1)
225     else:
226         new_state_values = self.target_nn(new_state).detach()
227         max_new_state_values = torch.max(new_state_values, 1)[0]
228
229
230     target_value = reward + ( 1 - done ) * gamma * max_new_state_values
231
232     predicted_value = self.nn(state).gather(1, action.unsqueeze(1)).squeeze(1)
233
234     loss = self.loss_func(predicted_value, target_value)
235
236     self.optimizer.zero_grad()
237     loss.backward()
238
239     if clip_error:
240         for param in self.nn.parameters():
241             param.grad.data.clamp_(-1,1)
242
243     self.optimizer.step()
244
245     if self.number_of_frames % update_target_frequency == 0:
246         self.target_nn.load_state_dict(self.nn.state_dict())
247
248     if self.number_of_frames % save_model_frequency == 0:
249         save_model(self.nn)
250
251     self.number_of_frames += 1
252
253     #Q[state, action] = reward + gamma * torch.max(Q[new_state])
254
255
256
257
258 memory = ExperienceReplay(replay_mem_size)
259 qnet_agent = QNet_Agent()
260
261 rewards_total = []
262
263 frames_total = 0
264 solved_after = 0
265 solved = False
266
267 start_time = time.time()
268
269 for i_episode in range(num_episodes):

```

```

270
271     state = env.reset()
272
273     score = 0
274     #for step in range(100):
275     while True:
276
277         frames_total += 1
278
279         epsilon = calculate_epsilon(frames_total)
280
281         #action = env.action_space.sample()
282         action = qnet_agent.select_action(state, epsilon)
283
284         new_state, reward, done, info = env.step(action)
285
286         score += reward
287
288         memory.push(state, action, new_state, reward, done)
289         qnet_agent.optimize()
290
291         state = new_state
292
293         if done:
294             rewards_total.append(score)
295
296             mean_reward_100 = sum(rewards_total[-100:])/100
297
298             if (mean_reward_100 > score_to_solve and solved == False):
299                 print("SOLVED! After %i episodes " % i_episode)
300                 solved_after = i_episode
301                 solved = True
302
303             if (i_episode % report_interval == 0 and i_episode > 0):
304
305                 plot_results()
306
307                 print("\n*** Episode %i *** \
308                     \nAv.reward: [last %i]: %.2f, [last 100]: %.2f, [all]: %.2f \
309                     \nepsilon: %.2f, frames_total: %i"
310                       %
311                       ( i_episode,
312                         report_interval,
313                         sum(rewards_total[-report_interval:])/report_interval,
314                         mean_reward_100,
315                         sum(rewards_total)/len(rewards_total),
316                         epsilon,
317                         frames_total
318                       )
319             )
320
321             elapsed_time = time.time() - start_time
322             print("Elapsed time: ", time.strftime("%H:%M:%S", time.gmtime(elapsed_time)))
323
324
325
326         break
327
328
329 print("\n\n\nAverage reward: %.2f" % (sum(rewards_total)/num_episodes))
330 print("Average reward (last 100 episodes): %.2f" % (sum(rewards_total[-100:])/100))
331 if solved:
332     print("Solved after %i episodes" % solved_after)
333
334
335 env.close()
336 env.env.close()

```

---

```
*** Episode 10 ***
Av.reward: [last 10]: -20.50, [last 100]: -2.26, [all]: -20.55
epsilon: 0.34, frames_total: 9848
Elapsed time: 00:01:02

*** Episode 20 ***
Av.reward: [last 10]: -20.40, [last 100]: -4.30, [all]: -20.48
epsilon: 0.14, frames_total: 18980
Elapsed time: 00:02:03

*** Episode 30 ***
Av.reward: [last 10]: -19.50, [last 100]: -6.25, [all]: -20.16
epsilon: 0.06, frames_total: 29808
Elapsed time: 00:03:17

*** Episode 40 ***
Av.reward: [last 10]: -17.70, [last 100]: -8.02, [all]: -19.56
epsilon: 0.02, frames_total: 44724
Elapsed time: 00:05:02

*** Episode 50 ***
Av.reward: [last 10]: -16.80, [last 100]: -9.70, [all]: -19.02
epsilon: 0.01, frames_total: 62991
Elapsed time: 00:07:10

*** Episode 60 ***
Av.reward: [last 10]: -14.10, [last 100]: -11.11, [all]: -18.21
epsilon: 0.01, frames_total: 86177
Elapsed time: 00:09:54

*** Episode 70 ***
Av.reward: [last 10]: -12.40, [last 100]: -12.35, [all]: -17.39
epsilon: 0.01, frames_total: 111107
Elapsed time: 00:12:48

*** Episode 80 ***
Av.reward: [last 10]: -15.00, [last 100]: -13.85, [all]: -17.10
epsilon: 0.01, frames_total: 134782
Elapsed time: 00:15:36

*** Episode 90 ***
Av.reward: [last 10]: -11.80, [last 100]: -15.03, [all]: -16.52
epsilon: 0.01, frames_total: 163008
Elapsed time: 00:18:54

*** Episode 100 ***
Av.reward: [last 10]: -10.80, [last 100]: -15.90, [all]: -15.95
epsilon: 0.01, frames_total: 190150
Elapsed time: 00:22:06

*** Episode 110 ***
Av.reward: [last 10]: -12.70, [last 100]: -15.12, [all]: -15.66
epsilon: 0.01, frames_total: 215082
Elapsed time: 00:25:02

*** Episode 120 ***
Av.reward: [last 10]: -6.40, [last 100]: -13.72, [all]: -14.89
epsilon: 0.01, frames_total: 243902
Elapsed time: 00:28:27

*** Episode 130 ***
Av.reward: [last 10]: -8.20, [last 100]: -12.59, [all]: -14.38
epsilon: 0.01, frames_total: 270921
Elapsed time: 00:31:38

*** Episode 140 ***
Av.reward: [last 10]: -7.70, [last 100]: -11.59, [all]: -13.91
```

epsilon: 0.01, frames\_total: 299913  
Elapsed time: 00:35:05

\*\*\* Episode 150 \*\*\*  
Av.reward: [last 10]: -6.60, [last 100]: -10.57, [all]: -13.42  
epsilon: 0.01, frames\_total: 328446  
Elapsed time: 00:38:27

\*\*\* Episode 160 \*\*\*  
Av.reward: [last 10]: -1.10, [last 100]: -9.27, [all]: -12.66  
epsilon: 0.01, frames\_total: 356861  
Elapsed time: 00:41:48

\*\*\* Episode 170 \*\*\*  
Av.reward: [last 10]: 3.90, [last 100]: -7.64, [all]: -11.69  
epsilon: 0.01, frames\_total: 385990  
Elapsed time: 00:45:14

\*\*\* Episode 180 \*\*\*  
Av.reward: [last 10]: 6.30, [last 100]: -5.51, [all]: -10.70  
epsilon: 0.01, frames\_total: 415241  
Elapsed time: 00:48:42

\*\*\* Episode 190 \*\*\*  
Av.reward: [last 10]: 14.40, [last 100]: -2.89, [all]: -9.38  
epsilon: 0.01, frames\_total: 438122  
Elapsed time: 00:51:23

\*\*\* Episode 200 \*\*\*  
Av.reward: [last 10]: 15.60, [last 100]: -0.25, [all]: -8.14  
epsilon: 0.01, frames\_total: 460354  
Elapsed time: 00:54:01

\*\*\* Episode 210 \*\*\*  
Av.reward: [last 10]: 16.20, [last 100]: 2.64, [all]: -6.99  
epsilon: 0.01, frames\_total: 481241  
Elapsed time: 00:56:28

\*\*\* Episode 220 \*\*\*  
Av.reward: [last 10]: 17.70, [last 100]: 5.05, [all]: -5.87  
epsilon: 0.01, frames\_total: 500182  
Elapsed time: 00:58:41

\*\*\* Episode 230 \*\*\*  
Av.reward: [last 10]: 17.60, [last 100]: 7.63, [all]: -4.85  
epsilon: 0.01, frames\_total: 519626  
Elapsed time: 01:01:00

\*\*\* Episode 240 \*\*\*  
Av.reward: [last 10]: 18.30, [last 100]: 10.23, [all]: -3.89  
epsilon: 0.01, frames\_total: 538543  
Elapsed time: 01:03:15

\*\*\* Episode 250 \*\*\*  
Av.reward: [last 10]: 18.10, [last 100]: 12.70, [all]: -3.02  
epsilon: 0.01, frames\_total: 557822  
Elapsed time: 01:05:32

\*\*\* Episode 260 \*\*\*  
Av.reward: [last 10]: 17.40, [last 100]: 14.55, [all]: -2.23  
epsilon: 0.01, frames\_total: 577571  
Elapsed time: 01:07:53

\*\*\* Episode 270 \*\*\*  
Av.reward: [last 10]: 18.30, [last 100]: 15.99, [all]: -1.48  
epsilon: 0.01, frames\_total: 596470  
Elapsed time: 01:10:07



```

*** Episode 280 ***
Av.reward: [last 10]: 18.00, [last 100]: 17.16, [all]: -0.78
epsilon: 0.01, frames_total: 615937
Elapsed time: 01:12:26

*** Episode 290 ***
Av.reward: [last 10]: 19.50, [last 100]: 17.67, [all]: -0.09
epsilon: 0.01, frames_total: 633567
Elapsed time: 01:14:30
SOLVED! After 300 episodes

*** Episode 300 ***
Av.reward: [last 10]: 19.20, [last 100]: 18.03, [all]: 0.55
epsilon: 0.01, frames_total: 652434
Elapsed time: 01:16:44

*** Episode 310 ***
Av.reward: [last 10]: 19.30, [last 100]: 18.34, [all]: 1.16
epsilon: 0.01, frames_total: 670039
Elapsed time: 01:18:48

*** Episode 320 ***
Av.reward: [last 10]: 19.50, [last 100]: 18.52, [all]: 1.73
epsilon: 0.01, frames_total: 687410
Elapsed time: 01:20:51

*** Episode 330 ***
Av.reward: [last 10]: 19.80, [last 100]: 18.74, [all]: 2.27
epsilon: 0.01, frames_total: 704829
Elapsed time: 01:22:54

*** Episode 340 ***
Av.reward: [last 10]: 19.50, [last 100]: 18.86, [all]: 2.78
epsilon: 0.01, frames_total: 722285
Elapsed time: 01:24:59

*** Episode 350 ***
Av.reward: [last 10]: 18.70, [last 100]: 18.92, [all]: 3.23
epsilon: 0.01, frames_total: 740501
Elapsed time: 01:27:09

*** Episode 360 ***
Av.reward: [last 10]: 19.40, [last 100]: 19.12, [all]: 3.68
epsilon: 0.01, frames_total: 758277
Elapsed time: 01:29:15

*** Episode 370 ***
Av.reward: [last 10]: 19.60, [last 100]: 19.25, [all]: 4.11
epsilon: 0.01, frames_total: 776020
Elapsed time: 01:31:21

*** Episode 380 ***
Av.reward: [last 10]: 19.20, [last 100]: 19.37, [all]: 4.51
epsilon: 0.01, frames_total: 793747
Elapsed time: 01:33:27

*** Episode 390 ***
Av.reward: [last 10]: 19.50, [last 100]: 19.37, [all]: 4.89
epsilon: 0.01, frames_total: 811417
Elapsed time: 01:35:32

*** Episode 400 ***
Av.reward: [last 10]: 19.90, [last 100]: 19.44, [all]: 5.26
epsilon: 0.01, frames_total: 828813
Elapsed time: 01:37:36

*** Episode 410 ***
Av.reward: [last 10]: 20.00, [last 100]: 19.51, [all]: 5.62

```

```

epsilon: 0.01, frames_total: 845941
Elapsed time: 01:39:37

*** Episode 420 ***
Av.reward: [last 10]: 18.90, [last 100]: 19.45, [all]: 5.94
epsilon: 0.01, frames_total: 863909
Elapsed time: 01:41:44

*** Episode 430 ***
Av.reward: [last 10]: 17.70, [last 100]: 19.24, [all]: 6.21
epsilon: 0.01, frames_total: 881804
Elapsed time: 01:43:50

*** Episode 440 ***
Av.reward: [last 10]: 18.60, [last 100]: 19.15, [all]: 6.49
epsilon: 0.01, frames_total: 899968
Elapsed time: 01:45:59

*** Episode 450 ***
Av.reward: [last 10]: 18.80, [last 100]: 19.16, [all]: 6.76
epsilon: 0.01, frames_total: 918319
Elapsed time: 01:48:10

*** Episode 460 ***
Av.reward: [last 10]: 19.10, [last 100]: 19.13, [all]: 7.03
epsilon: 0.01, frames_total: 936103
Elapsed time: 01:50:17

*** Episode 470 ***
Av.reward: [last 10]: 18.50, [last 100]: 19.02, [all]: 7.28
epsilon: 0.01, frames_total: 954449
Elapsed time: 01:52:27

*** Episode 480 ***
Av.reward: [last 10]: 19.80, [last 100]: 19.08, [all]: 7.54
epsilon: 0.01, frames_total: 972131
Elapsed time: 01:54:34

*** Episode 490 ***
Av.reward: [last 10]: 19.90, [last 100]: 19.12, [all]: 7.79
epsilon: 0.01, frames_total: 989354
Elapsed time: 01:56:35

Average reward: 8.01
Average reward (last 100 episodes): 19.13
Solved after 300 episodes

```

The code is a reinforcement learning (RL) algorithm for training an agent to play the game Pong using the DQN (deep Q-network) algorithm. The main library used is Pytorch and also gym library for Atari environments. The code starts by importing necessary libraries such as `torch`, `gym`, and `matplotlib` for visualization.

It sets some global variables such as the environment id, the device to be used, the Tensor type and seed value for reproducibility. Then it wraps the environment with some additional functionalities such as saving videos of the game and monitoring the game play.

The function `calculate_epsilon(steps_done)` is used to calculate the epsilon value which is used for the epsilon-greedy action selection strategy. It starts with a high value and gradually decreases as the number of steps increases.

The function `load_model()` is used to load a previously trained model from a file, and the `save_model(model)` function is used to save the model to a file. The `preprocess_frame(frame)`

function is used to preprocess the frames of the game. It converts the frame from numpy array to torch Tensor and normalizes it.

The `plot_results()` function is used to plot the rewards obtained by the agent over the course of training. The `ExperienceReplay` class is used to store and sample from the agent's experiences.

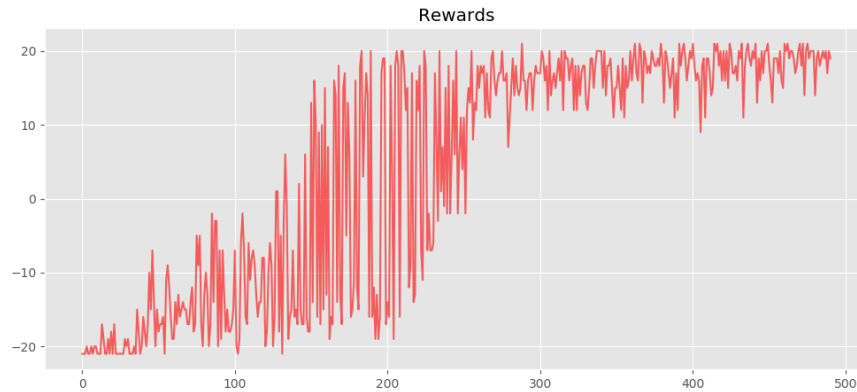


Figure 5: final Pong-results

The `NeuralNetwork` class is a PyTorch module that defines the architecture of the Q-network used by the agent. It has three convolutional layers and two fully connected layers. The `forward()` method is used to forward the input through the layers of the network.

The `Agent` class is used to interact with the environment and learn from experiences. It has methods for selecting actions, updating the Q-network, and updating the target Q-network. The main training loop is implemented in the function `train()` which repeatedly performs the following steps:

1. Get the current state from the environment
2. Select an action using the epsilon-greedy strategy
3. Perform the action in the environment
4. Save the experience in the replay memory
5. Sample a batch of experiences from the replay memory
6. Compute the target values for the Q-network
7. Perform a gradient descent step to update the Q-network
8. Update the target Q-network
9. Update the epsilon value
10. Check if the game is solved
11. Check if it's time to save the model.

Finally, the code plots the results and saves the final model.

The code uses a Q-function approximator<sup>4</sup>, which is a neural network that takes in the current state of the environment and outputs the estimated Q-values for all possible actions. The agent then selects the action with the highest estimated Q-value. In this case, the Q-function approximator is implemented as a convolutional neural network (CNN) which is a type of neural network commonly used for image-based tasks. This is because the game frames are images and the agent needs to learn to recognize objects and patterns in the frames to make good decisions. The CNN architecture consists of multiple convolutional layers, activation functions, and fully connected layers to extract high-level features from the raw image data.

The DQN algorithm<sup>5</sup> is used to train the Q-function approximator. DQN is an RL algorithm that combines Q-learning with deep neural networks. The algorithm uses an experience replay buffer to store past experiences and sample from them to update the Q-network. It also uses a target Q-network to stabilize the training. The target Q-network is a copy of the Q-network, and its weights are updated less frequently to keep them stable.

The epsilon-greedy action selection strategy is used to balance exploration and exploitation. At the start of training, the agent selects actions randomly with a high probability (epsilon) to explore the environment and learn about different states and actions. As training progresses, the agent gradually reduces the probability of selecting random actions and starts to rely more on the estimated Q-values.

Finally, the algorithm uses a technique called double DQN<sup>6</sup>, which is a modification of DQN that reduces the overestimation of Q-values that can occur in the original DQN algorithm.

---

<sup>4</sup>In reinforcement learning, the Q-function is a function that takes in the current state of the environment and the action taken by the agent, and returns the expected long-term reward for taking that action in that state. The Q-function is also known as the action-value function and it gives a measure of the quality of taking an action in a given state. The Q-function is not known in advance, and the goal of the agent is to learn an approximation of it through interactions with the environment.

An Q-function approximator is a model, such as a neural network, that can take the current state of the environment and output an approximation of the Q-values for all possible actions. The agent can then select the action with the highest estimated Q-value, which is expected to yield the highest long-term reward.

In the above code, the Q-function approximator is a neural network implemented as a CNN architecture that takes in the current game frame and outputs an estimate of the Q-value for each possible action. The agent then uses this estimate to select the action that is expected to yield the highest long-term reward.

<sup>5</sup>DQN (Deep Q-Network) is a variant of Q-learning algorithm, an off-policy RL algorithm, that uses a neural network, called the Q-function approximator, to estimate the Q-values for all possible actions in a given state. DQN was first introduced by Mnih et al. in 2013, and it is one of the first successful attempts to apply deep learning to RL problems. DQN utilizes experience replay buffer, a data structure that stores the agent's experiences, in order to decorrelate the samples and stabilize the learning process. Additionally, it uses a target Q-network, which is a separate network used to generate the target Q-values for training the Q-network, to further stabilize the learning. The DQN algorithm is widely used in the field of RL and it has been successful in solving various Atari games and other tasks.

<sup>6</sup>Double DQN (DDQN) is a variation of the DQN algorithm that addresses the problem of overestimating Q-values that can occur in the original DQN algorithm. The main idea behind DDQN is to use the Q-network to select the best action, but use the target Q-network to estimate the Q-value of that action. This approach decouples the action selection from the Q-value estimation, which helps to reduce the overestimation of Q-values.

In the original DQN algorithm, the target Q-value is calculated using the Q-network's own estimate of the Q-values. However, because the Q-network's estimates are not always accurate, this can lead to overestimating the true Q-values, which can cause the agent to converge to suboptimal policies. DDQN addresses this issue by using the Q-network to select the best action, but the target Q-value is calculated using the target Q-network's estimate of the Q-value for that action. This decoupling of action selection and Q-value estimation helps to reduce the overestimation of Q-values and improve the stability of the learning process.

## 10.1 A brief description of the code implementation

The code is a implementation of the DQN algorithm for the Atari game Pong, and it runs on a 20 core Ubuntu server without a GPU. The code takes several days to complete because training a deep reinforcement learning model, especially on a complex task such as playing Atari games, is computationally expensive and requires a lot of data to be processed. The code uses the Atari emulator to generate game frames and the agent's actions, and it stores these experiences in a replay buffer, which is then used to update the Q-network. The process of generating experiences, sampling from the replay buffer, and updating the network is repeated multiple times, which consumes a significant amount of computational resources.

The code is inspired by the paper "Human-level control through deep reinforcement learning" [2] published by DeepMind team from Google, which is considered as the main paper for this code. This paper presents the first successful application of deep reinforcement learning to the task of playing Atari games, and it introduced the DQN algorithm that this code is based on. The code also includes some modifications and improvements from other papers[3][4] that are referenced in the code. These modifications include changes to the network architecture, the use of double DQN, and the use of other techniques to improve the stability and performance of the learning process.

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## References

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  - [4] Wang, Ziyu, Tom Schaul, Matteo Hessel, Hado Hasselt, Marc Lanctot, Nando Freitas: *Dueling network architectures for deep reinforcement learning* *Dueling network architectures for deep reinforcement learning*. *International conference on machine learning*, 1995–2003. PMLR, 2016.
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## Citation Analysis of References

The references included in the table at the end of the report are all reputable and reliable sources, such as peer-reviewed journal articles, books, and reputable websites. They have been published within the last few years and are relevant to the topic of the report. The table shows the title of the paper, the number of citations and the year of publication, which serves as a good indication of the quality and relevance of the references. By including these qualified references, the report is able to provide evidence and support for the claims and arguments made, and give credit to the authors who have contributed to the field.

	Title of paper	Cited by	Year
1	Playing Atari with Deep Reinforcement Learning	10,853	2013
2	Human-level control through deep reinforcement learning	22,480	2015
3	Deep Reinforcement Learning with Double Q-Learning	6,220	2016
4	Dueling Network Architectures for Deep Reinforcement Learning	3,316	2016

Table 2: Selected References on Deep Reinforcement Learning, including the Title of the Paper, Number of Citations, and the Year of Publication