C3W1_Hidden_State_Activation

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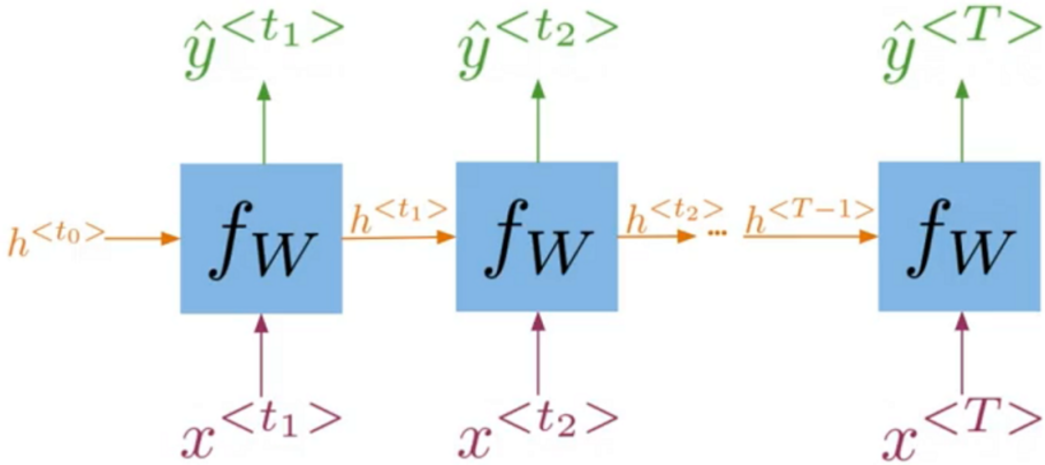
1 Hidden State Activation : Ungraded Lecture Notebook

In this notebook you'll take another look at the hidden state activation function. It can be written in two different ways.

You will see, step by step, how to implement each of them and then how to verify whether the results produced by each of them are the same.

1.1 Background



Here you can see an image of a recurrent neural network (RNN). The hidden state activation function for a vanilla RNN can be written as an equation in two ways. You can either write it like this:

$h^{<t>} = g(W_h[h^{<t-1>}, x^{<t>}] + b_h)$

Or you can write it like this:

$h^{<t>} = g(W_{hh}h^{<t-1>} + W_{hx}x^{<t>} + b_h)$

The difference between the formulas is that in the first case you concatenate the matrices together and perform the multiplication only once, while in the second case you perform two separate multiplications and then sum them. More specifically:

- W_h in the first formula denotes the *horizontal* concatenation of weight matrices W_{hh} and W_{hx} from the second formula.
- W_h in the first formula is then multiplied by $[h^{<t-1>}, x^{<t>}]$, another concatenation of parameters from the second formula but this time in a different direction, i.e *vertical*! In the second formula the two (non-concatenated) matrices are multiplied by its own respective parameter vector.

Below, you will calculate both options using NumPy

1.2 Imports

[1]:
```
import numpy as np
```

1.3 Joining (Concatenation)

1.3.1 Weights

A join along the vertical boundary is called a *horizontal concatenation* or *horizontal stack*.

Visually, it looks like this:- $W_h = [W_{hh} \mid W_{hx}]$

You will see two different ways to achieve this using numpy.

Note: The values used to populate the arrays, below, have been chosen to aid in visual illustration only. They are NOT what you'd expect to use building a model, which would typically be random variables instead.

- Try using random initializations for the weight arrays.

[2]:
```
# Create some dummy data

w_hh = np.full((3, 2), 1)  # illustration purposes only, returns an array of␣
 ↪size 3x2 filled with all 1s
w_hx = np.full((3, 3), 9)  # illustration purposes only, returns an array of␣
 ↪size 3x3 filled with all 9s


### START CODE HERE ###
# Try using some random initializations, though it will obfuscate the join. eg:␣
 ↪uncomment these lines
# w_hh = np.random.standard_normal((3,2))
# w_hx = np.random.standard_normal((3,3))
### END CODE HERE ###

print("-- Data --\n")
print("w_hh :")
print(w_hh)
print("w_hh shape :", w_hh.shape, "\n")
print("w_hx :")
print(w_hx)
```

```
print("w_hx shape :", w_hx.shape, "\n")

# Joining the arrays
print("-- Joining --\n")
# Option 1: concatenate - horizontal
w_h1 = np.concatenate((w_hh, w_hx), axis=1)
print("option 1 : concatenate\n")
print("w_h :")
print(w_h1)
print("w_h shape :", w_h1.shape, "\n")

# Option 2: hstack
w_h2 = np.hstack((w_hh, w_hx))
print("option 2 : hstack\n")
print("w_h :")
print(w_h2)
print("w_h shape :", w_h2.shape)
```

```
-- Data --

w_hh :
[[1 1]
 [1 1]
 [1 1]]
w_hh shape : (3, 2)

w_hx :
[[9 9 9]
 [9 9 9]
 [9 9 9]]
w_hx shape : (3, 3)

-- Joining --

option 1 : concatenate

w_h :
[[1 1 9 9 9]
 [1 1 9 9 9]
 [1 1 9 9 9]]
w_h shape : (3, 5)

option 2 : hstack

w_h :
[[1 1 9 9 9]
 [1 1 9 9 9]
```

```
 [1 1 9 9 9]]
w_h shape : (3, 5)
```

1.3.2 Hidden State & Inputs

Joining along a horizontal boundary is called a vertical concatenation or vertical stack. Visually it looks like this:

$[h^{<t-1>}, x^{<t>}] = \left[\frac{h^{<t-1>}}{x^{<t>}}\right]$

You will see two different ways to achieve this using numpy.

Try using random initializations for the hidden state and input matrices.

[3]:
```
# Create some more dummy data
h_t_prev = np.full((2, 1), 1)  # illustration purposes only, returns an array␣
 ↪of size 2x1 filled with all 1s
x_t = np.full((3, 1), 9)       # illustration purposes only, returns an array␣
 ↪of size 3x1 filled with all 9s

# Try using some random initializations, though it will obfuscate the join. eg:␣
 ↪uncomment these lines

### START CODE HERE ###
# h_t_prev = np.random.standard_normal((2,1))
# x_t = np.random.standard_normal((3,1))
### END CODE HERE ###

print("-- Data --\n")
print("h_t_prev :")
print(h_t_prev)
print("h_t_prev shape :", h_t_prev.shape, "\n")
print("x_t :")
print(x_t)
print("x_t shape :", x_t.shape, "\n")

# Joining the arrays
print("-- Joining --\n")

# Option 1: concatenate - vertical
ax_1 = np.concatenate(
    (h_t_prev, x_t), axis=0
)  # note the difference in axis parameter vs earlier
print("option 1 : concatenate\n")
print("ax_1 :")
print(ax_1)
print("ax_1 shape :", ax_1.shape, "\n")

# Option 2: vstack
```

```
ax_2 = np.vstack((h_t_prev, x_t))
print("option 2 : vstack\n")
print("ax_2 :")
print(ax_2)
print("ax_2 shape :", ax_2.shape)
```

```
-- Data --

h_t_prev :
[[1]
 [1]]
h_t_prev shape : (2, 1)

x_t :
[[9]
 [9]
 [9]]
x_t shape : (3, 1)

-- Joining --

option 1 : concatenate

ax_1 :
[[1]
 [1]
 [9]
 [9]
 [9]]
ax_1 shape : (5, 1)

option 2 : vstack

ax_2 :
[[1]
 [1]
 [9]
 [9]
 [9]]
ax_2 shape : (5, 1)
```

1.4 Verify Formulas

Now you know how to do the concatenations, horizontal and vertical, lets verify if the two formulas produce the same result.

Formula 1: $h^{<t>} = g(W_h[h^{<t-1>}, x^{<t>}] + b_h)$

Formula 2: $h^{<t>} = g(W_{hh}h^{<t-1>} + W_{hx}x^{<t>} + b_h)$

To prove: **Formula 1** ⇔ **Formula 2**

You will ignore the bias term b_h and the activation function $g(\ )$ because the transformation will be identical for each formula. So what we really want to compare is the result of the following parameters inside each formula:

$W_{h}[h^{{},x}{}]   W_{hh}h^{} + W_{hx}x^{} $

You will do this by using matrix multiplication combined with the data and techniques (stacking/concatenating) from above.

- Try adding a sigmoid activation function and bias term to the checks for completeness.

[4]:
```python
# Data

w_hh = np.full((3, 2), 1)  # returns an array of size 3x2 filled with all 1s
w_hx = np.full((3, 3), 9)  # returns an array of size 3x3 filled with all 9s
h_t_prev = np.full((2, 1), 1)  # returns an array of size 2x1 filled with all 1s
x_t = np.full((3, 1), 9)       # returns an array of size 3x1 filled with all 9s


# If you want to randomize the values, uncomment the next 4 lines

# w_hh = np.random.standard_normal((3,2))
# w_hx = np.random.standard_normal((3,3))
# h_t_prev = np.random.standard_normal((2,1))
# x_t = np.random.standard_normal((3,1))

# Results
print("-- Results --")
# Formula 1
stack_1 = np.hstack((w_hh, w_hx))
stack_2 = np.vstack((h_t_prev, x_t))

print("\nFormula 1")
print("Term1:\n",stack_1)
print("Term2:\n",stack_2)
formula_1 = np.matmul(np.hstack((w_hh, w_hx)), np.vstack((h_t_prev, x_t)))
print("Output:")
print(formula_1)

# Formula 2
mul_1 = np.matmul(w_hh, h_t_prev)
mul_2 = np.matmul(w_hx, x_t)
print("\nFormula 2")
print("Term1:\n",mul_1)
print("Term2:\n",mul_2)

formula_2 = np.matmul(w_hh, h_t_prev) + np.matmul(w_hx, x_t)
```

```
print("\nOutput:")
print(formula_2, "\n")

# Verification
# np.allclose - to check if two arrays are elementwise equal upto certain␣
 ↪tolerance, here
# https://numpy.org/doc/stable/reference/generated/numpy.allclose.html

print("-- Verify --")
print("Results are the same :", np.allclose(formula_1, formula_2))

### START CODE HERE ###
# # Try adding a sigmoid activation function and bias term as a final check
# # Activation
# def sigmoid(x):
#     return 1 / (1 + np.exp(-x))

# # Bias and check
# b = np.random.standard_normal((formula_1.shape[0],1))
# print("Formula 1 Output:\n",sigmoid(formula_1+b))
# print("Formula 2 Output:\n",sigmoid(formula_2+b))

# all_close = np.allclose(sigmoid(formula_1+b), sigmoid(formula_2+b))
# print("Results after activation are the same :",all_close)
### END CODE HERE ###
```

```
-- Results --

Formula 1
Term1:
 [[1 1 9 9 9]
 [1 1 9 9 9]
 [1 1 9 9 9]]
Term2:
 [[1]
 [1]
 [9]
 [9]
 [9]]
Output:
[[245]
 [245]
 [245]]

Formula 2
Term1:
 [[2]
```

```
 [2]
 [2]]
Term2:
 [[243]
 [243]
 [243]]

Output:
[[245]
 [245]
 [245]]

-- Verify --
Results are the same : True
```

1.5 Summary

That's it! You have verified that the two formulas produce the same results, and seen how to combine matrices vertically and horizontally to make that happen. You now have all the intuition needed to understand the math notation of RNNs.

```
[ ]:
```