C4W2_Positional_Encoding

August 19, 2025

1 Positional Encoding

In this lab, you will learn how to implement the positional encoding of words in the transformer.

[1]:
```python
import os
os.environ['TF_CPP_MIN_LOG_LEVEL'] = '3'
import numpy as np
import matplotlib.pyplot as plt
import tensorflow as tf
```

1.1 1. Positional Encoding

In sequence to sequence tasks, the relative order of your data is extremely important to its meaning. When you were training sequential neural networks such as RNNs, you fed your inputs into the network in order. Information about the order of your data was automatically fed into your model. However, when you train a Transformer network using multi-head attention, you feed your data into the model all at once. While this dramatically reduces training time, there is no information about the order of your data. This is where positional encoding is useful - you can specifically encode the positions of your inputs and pass them into the network using these sine and cosine formulas:

$$PE_{(pos,2i)} = sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \tag{1}$$

$$PE_{(pos,2i+1)} = cos\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \tag{2}$$

- d is the dimension of the word embedding and positional encoding
- pos is the position of the word.
- k refers to each of the different dimensions in the positional encodings, with i equal to k // 2.

To develop some intuition about positional encodings, you can think of them broadly as a feature that contains the information about the relative positions of words. The sum of the positional encoding and word embedding is ultimately what is fed into the model. If you just hard code the positions in, say by adding a matrix of 1's or whole numbers to the word embedding, the semantic meaning is distorted. Conversely, the values of the sine and cosine equations are small enough (between -1 and 1) that when you add the positional encoding to a word embedding, the word embedding is not significantly distorted, and is instead enriched with positional information.

Using a combination of these two equations helps your Transformer network attend to the relative positions of your input data.

1.1.1 1.1 - Sine and Cosine Angles

Notice that even though the sine and cosine positional encoding equations take in different arguments (`2i` versus `2i+1`, or even versus odd numbers) the inner terms for both equations are the same:

$$\theta(pos, i, d) = \frac{pos}{10000^{\frac{2i}{d}}} \tag{3}$$

Consider the inner term as you calculate the positional encoding for a word in a sequence. $PE_{(pos,0)} = sin\left(\frac{pos}{10000^{\frac{0}{d}}}\right)$, since solving `2i = 0` gives `i = 0` $PE_{(pos,1)} = cos\left(\frac{pos}{10000^{\frac{0}{d}}}\right)$, since solving `2i + 1 = 1` gives `i = 0`

The angle is the same for both! The angles for $PE_{(pos,2)}$ and $PE_{(pos,3)}$ are the same as well, since for both, `i = 1` and therefore the inner term is $\left(\frac{pos}{10000^{\frac{2}{d}}}\right)$. This relationship holds true for all paired sine and cosine curves:

k	0	1	2	3	...	d - 2	d - 1
encoding(0) =	$[sin(\theta(0,0,d))$	$cos(\theta(0,0,d))$	$sin(\theta(0,1,d))$	$cos(\theta(0,1,d))$	...	$sin(\theta(0,d//2,d))$	$cos(\theta(0,d//2,d))]$
encoding(1) =	$[sin(\theta(1,0,d))$	$cos(\theta(1,0,d))$	$sin(\theta(1,1,d))$	$cos(\theta(1,1,d))$	...	$sin(\theta(1,d//2,d))$	$cos(\theta(1,d//2,d))]$

... | encoding(pos) = | $[sin(\theta(pos,0,d))|$ $cos(\theta(pos,0,d))|$ $sin(\theta(pos,1,d))|$ $cos(\theta(pos,1,d))|$... $|sin(\theta(pos,d//2,d))|$ $cos(\theta(pos,d//2,d))]|$

[2]:
```python
def get_angles(position, k, d_model):
    """
    Computes a positional encoding for a word

    Arguments:
        position (int): position of the word
        k (int): refers to each of the different dimensions in the positional␣
 ↪encodings, with i equal to k//2
        d_model(int): the dimension of the word embedding and positional␣
 ↪encoding

    Returns:
        _ (float): positional embedding value for the word
    """
    i = k // 2
    angle_rates = 1 / np.power(10000, (2 * i) / np.float32(d_model))
    return position * angle_rates
```

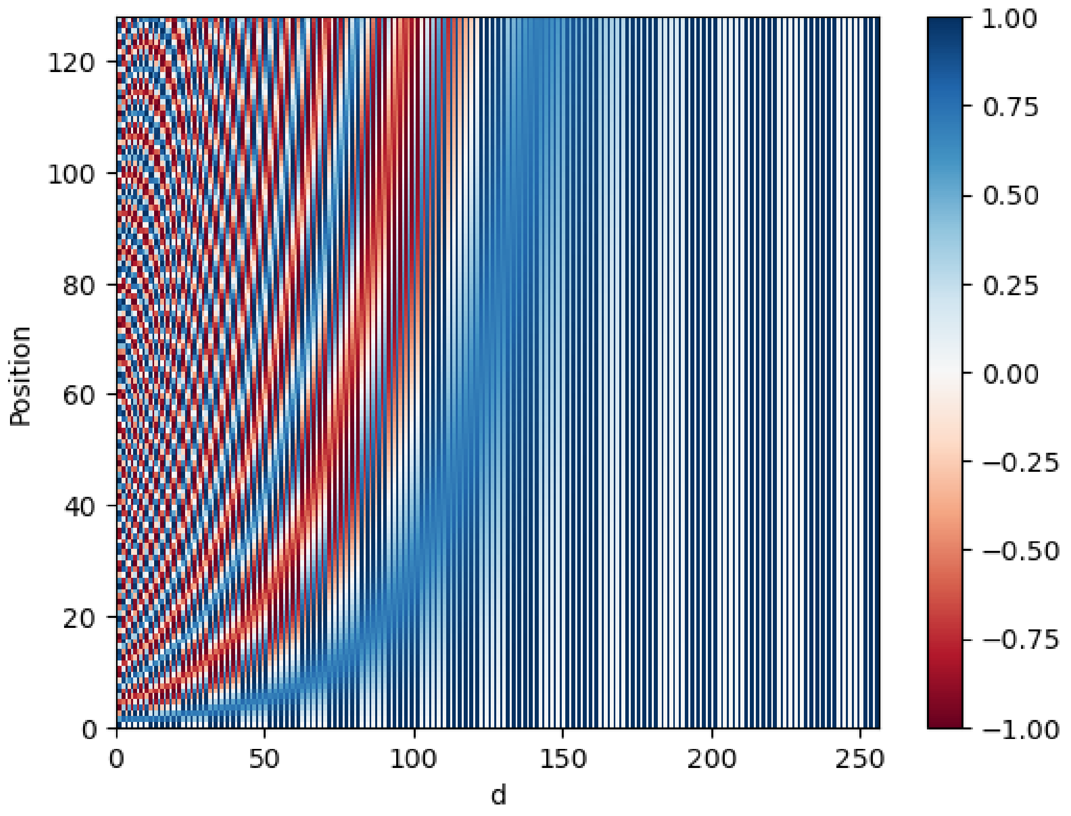
1.1.2 1.2 - Sine and Cosine Positional Encodings

Now you can use the angles you computed to calculate the sine and cosine positional encodings, shown in equations (1) and (2).

[3]:
```
def positional_encoding(positions, d):
    """
    Precomputes a matrix with all the positional encodings

    Arguments:
        positions (int): Maximum number of positions to be encoded
        d (int): Encoding size

    Returns:
        pos_encoding (tf.Tensor): A matrix of shape (1, position, d_model) with␣
 ↪the positional encodings
    """
    # initialize a matrix angle_rads of all the angles
    angle_rads = get_angles(np.arange(positions)[:, np.newaxis],
                            np.arange(d)[np.newaxis, :],
                            d)

    # apply sin to even indices in the array; 2i
    angle_rads[:, 0::2] = np.sin(angle_rads[:, 0::2])

    # apply cos to odd indices in the array; 2i+1
    angle_rads[:, 1::2] = np.cos(angle_rads[:, 1::2])

    pos_encoding = angle_rads[np.newaxis, ...]

    return tf.cast(pos_encoding, dtype=tf.float32)
```

Now you can visualize the positional encodings.

[4]:
```
pos_encoding = positional_encoding(128, 256)

plt.pcolormesh(pos_encoding[0], cmap='RdBu')
plt.xlabel('d')
plt.xlim((0, 256))
plt.ylabel('Position')
plt.colorbar()
plt.show()
```



Each row represents a positional encoding - notice how none of the rows are identical! You have created a unique positional encoding for each of the words.

Congratulations on finishing this Lab! Now you should have a better understanding of the positional encoding in the transformer and this will surely help you with this week's assignment!

Keep it up!

```
[ ]:
```