M1M3 Actuator Control

Contents

1. Introduction

The M1M3 actuators, as part of the Active Optics System (AOS), are responsible for controlling the M1M3 figure error. The ultimate goal of M1M3 actuator control is to optimize the image quality across the focal plane. But before involving all the other elements of the AOS, it is useful to look into how the actuators can optimize the M1M3 surface, i.e., how good the optimized surface can be, and how much forces are needed to achieve that optimized shape. In this note, we use the general terms "surface" or "surface shape" to refer to the surface sag (see Document-16390 for the definition of surface sag, and the surface sag bending modes and influence function used in the analysis presented in this note).

Because of the gravity of the mirror, the actuators actually need to do two things – to hold the mirror, i.e., to counter the gravity, and to control the mirror shape. Our mirror shape optimization therefore has to take into account the weight of the mirror, whose density is not uniform across the substrate due to the variation in its inner structure. The effect of gravity can be taken into account using Finite Element Analysis (FEA). We can dial into the FEA model an arbitrary set of forces that sums up to the mirror weight, and let the FEA determine how the mirror is deformed. This is our initial state. Let u_0 stand for the initial force vector, and x_0 for the surface shape associated with u_0, the surface shape x when the force vector is u can be written as

$$x = x_0 + B(u - u_0) \tag{1}$$

where B is the influence matrix, which can be found in Document-16389 (the description is in Document-16390). The initial forces u_0 and surface sag x_0 are also discussed in Document-16390.

If each actuator can create any force that we want it to create, the surface optimization problem is simply a least square minimization:

$$u = u_0 - B^{-1} x_0 \, . \tag{2}$$

However, in real world, there are constraints on how much forces can be generated by the actuators without loosing accuracy. For pneumatic actuators, there is a limit on the air pressure that can be supplied. The current design is 700lb. On the other hand, the even tighter constraint is actually the rated capacity of the load cells, which is 500lb, i.e., about 2500N. So the optimization of the mirror surface has to take into account the constraints on actuator forces. One of the major motivations for the analysis described in this note is to estimate whether the current design of the actuators will be able to control the mirror to meet the allocated error budget.

The x_0 and u_0 used in this analysis comes out of the FEA model, where it is assumed that the mirror has been polished to its perfect shape (as defined by the LSST optical model) in outer space, and then placed under the effect of gravity and the actuators. This means there is zero polishing error, a zenith print through is not polished into the mirror, no forces is needed to bend out the low-frequency modes due to polishing, and no additional forces is necessary to correct the mirror shape if the supporting actuators during polishing has not be optimized. These additional factors have no effect on the analysis method described in this note, but do affect our x_0. However, at the time this note is written, the M1M3 monolithic mirror is still being polished. The polishing related information is either not available or not finalized. We are not able to include them here in the results described in this note. In Section 4, we use the best information available to try to scope the effect of these factors. These factors are unlikely to affect the conclusion in this note.

2. Mathematics

The mathematics behind this analysis is the optimized control algorithm. The general formalism is described by George Angeli in Document-15312. For our application, the actuator forces are not allowed to be all zeros. The control law takes a slightly different form.

Given a system defined by Eq. (1), and the noise in the measurement of the system state (x) is negligible[1], to minimize the cost function

$$J = x^T Q x + \rho u^T H u \,, \tag{3}$$

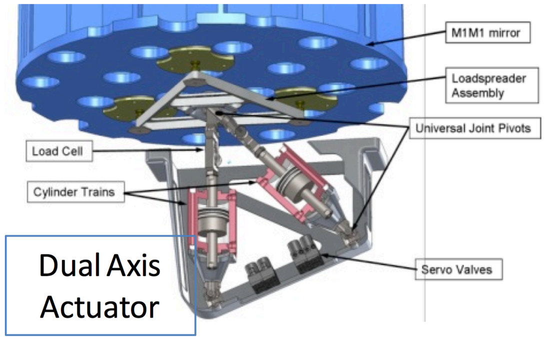
the optimal input to the system is

$$u = -(B^T Q B + \rho H)^{-1} B^T Q (x_0 - B u_0) \,. \tag{4}$$

By including the forces in the cost function together with the system state, the control law punishes large forces. The relative weight between the forces and the surface shape is controlled by the weighting factor ρ. When ρ is large, even a small increase in the forces leads to a large increase of the cost function. The larger ρ is, the more the control focuses on making sure the forces are not too large, which often means a larger surface error.

It is a natural choice to use the RMS of the surface and the forces in the cost function. The matrices Q and H depend on the definition of the influence matrix B. The influence matrix B we use is the one that relates the Fy and Fz forces to the surface sag (see Document-16390 and 16389). This means the matrix Q is a unit matrix. The matrix H, however, is not. This is because the input to B is the Fy and Fz forces, while it is the RMS of the cylinder forces that we need to constrain. The cylinder forces are always along the length of the actuators, as shown in Figure 1.

[1] This corresponds to the matrix A being unit matrix and no measurement noise (η=0) in Document-15312.



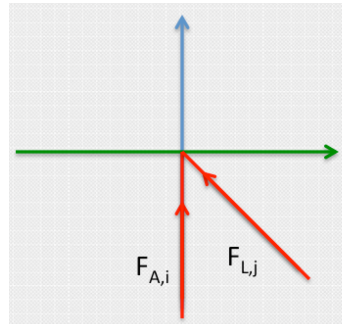


Figure 1 Left: A figure showing an axial actuator and a lateral actuator supporting the same position at the back surface of the mirror. Right: A schematic drawing with blue showing the positive z-axis, green showing the positive y-axis (along mirror back surface) and red showing the cylinder forces.

Let F_A and F_L stand for the axial and lateral cylinder forces, we have

$$F_y = \frac{F_L}{\sqrt{2}} \quad \text{for all 100 lateral actuators}$$

$$F_z = \begin{cases} \frac{F_L}{\sqrt{2}} + F_A & \text{for the 100 axial actuators that are colocated with lateral actuators} \\ F_A & \text{for the 56 axial actuators that are not colocated with lateral actuators} \end{cases}$$

(5)

Eq. (3) can also be written as

$$J = x^T Qx + \rho(\sum_{i=1}^{156} F_{A,i}^2 + \sum_{j=157}^{256} F_{L,j}^2) = x^T Qx + \rho u^T Hu \cdot$$

(6)

The matrix H is shown in Figure 2. The first 156 diagonal elements, corresponding to the 156 Fz's, are equal to 1. The rest 100 diagonal elements, corresponding to the 100 Fy's, are equal to 3. The off-diagonal elements between the 100 Fy's and the 100 Fz's that are collocated with the Fy's are equal to -2. The rest are all zero.

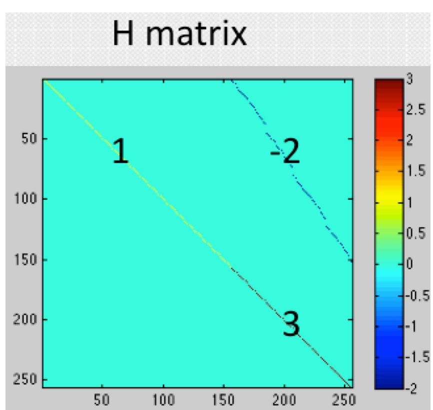


Figure 2 The H matrix.

Using nanometer as the unit of x and Newton as the unit of u, the underlying control law says that 1/√ρ Newtons of increase in RMS cylinder forces is as bad as 1nm of increase in the surface RMS.

One caveat in applying Eq. (4) is that the matrix B is a rank-deficient. If we use pseudo-inverse as the inverse of a matrix, $(B^TB)^{-1}B^T \neq B^{-1}$. This is because $(B^T)^{-1}B^T \neq I$. We therefore need to break the first bracket on the right hand side of Eq. (4) using the Kailath Variant, which can be found on page 153 of Ref. (Bishop 1995)

$$(\mathbf{A}+\mathbf{BC})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{I}+\mathbf{CA}^{-1}\mathbf{B})^{-1}\mathbf{CA}^{-1} \tag{7}$$

3. Results

3.1. Zenith Pointing

Figure 3 shows the results of the optimization at horizon. Note that what comes out of the optimization directly, as defined in Eq. (4), is the Fy and Fz forces. Here we have converted them to the cylinder forces. The top plot shows the maximum cylinder force vs. the mirror surface RMS. Each circle corresponds to a different weighting factor ρ. For example, when ρ=0, the forces disappear from the cost function J. This is what we get using Eq. (2). The rightmost circle corresponds to ρ=16. The bottom plot shows the RMS cylinder force vs. the mirror surface RMS.

To achieve the best surface possible at zenith pointing, the cylinder actuator forces only need to go up to about 2100N.

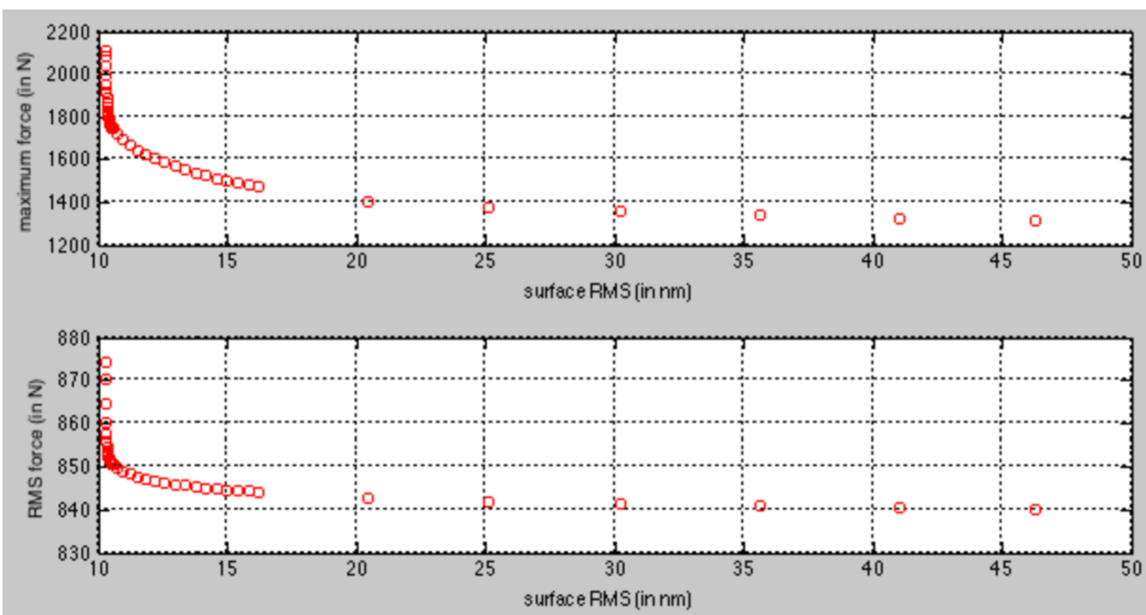


Figure 3 M1M3 control optimization result for zenith pointing. Top plot shows maximum actuator force vs. the mirror surface RMS. Bottom plot shows RMS actuator force vs. the mirror surface RMS.

3.2. Horizon Pointing

Using the same method that we've just described for zenith pointing, we also looked at the optimization at horizon.

Horizon pointing is an extreme case, because we rarely point at horizon. Nevertheless, it gives us a good feeling of the scope of the problem if we don't optimize the forces. The fact that the M1M3 weights about 166,600N, and we use 100 lateral actuators to counter the mirror weight means that 2356N is required from each lateral actuator on average to counter gravity, while the rated capacity is 2500N. So the lateral actuators cannot do much to help with the mirror shape.

On the other hand, for other teams of the project who need to model the M1M3 print through at any zenith angle without dealing with the actuators directly, the print through at zenith and horizon are needed. We will talk about this in more detail in Sections 3.3 and 5.

Figure 4 shows the results of the optimization at horizon. Here we only plot the maximum cylinder forces, instead of the RMS force, because the maximum force is what matters most. Blue circles in the top plot of Figure 4 shows the maximum cylinder force vs. the mirror surface RMS. Each circle corresponds to a different weighting factor ρ. When ρ=0, the cylinder forces will go up to 6000N in order to produce the best surface with RMS of about 8 nanometers, in which case the actuators with 2500N rated capacity will not be good to use. On the other hand, plateaus are formed at places, for example, between 30nm and 55nm surface RMS, which means that, compared to 55nm of surface RMS, 30nm can be achieved at almost no additional cost to the maximum actuator force.

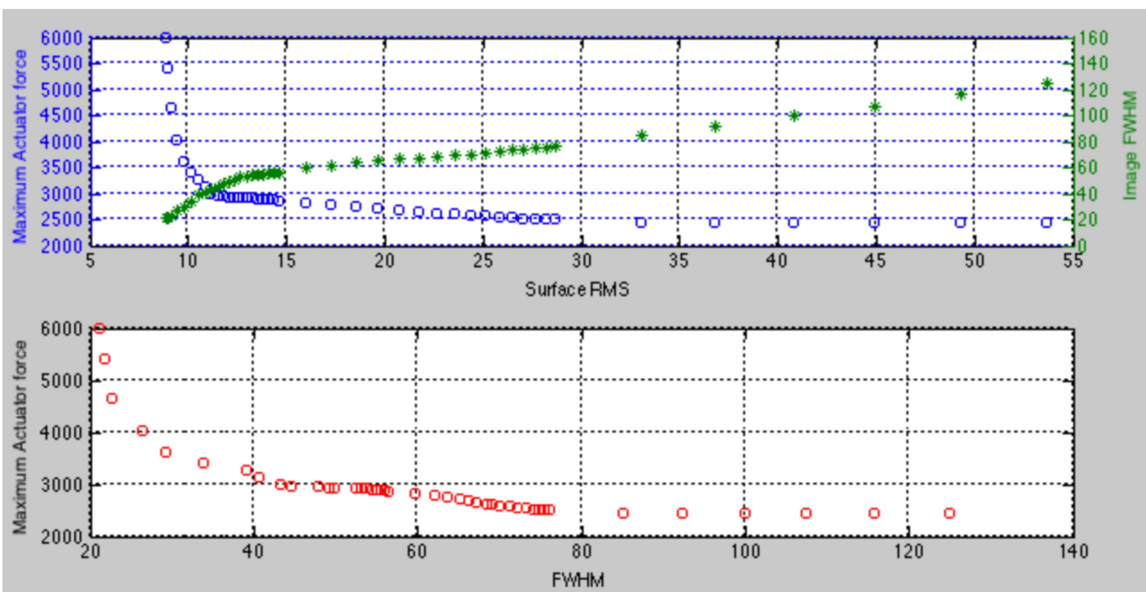


Figure 4 M1M3 control optimization result for horizon pointing. Top plot: blue circles show maximum actuator force vs. the mirror surface RMS; green stars show the weighted RMS of the image FWHM across the focal plane vs. the mirror surface RMS. Bottom plot: maximum actuator force vs. the weighted RMS of the image FWHM.

Although we are not optimizing the image quality directly, we know that to first order the image quality is a monotonous function of the mirror surface RMS. The green stars in the top plot of Figure 4 shows the weighted RMS of the FWHM of PSFs at 31 field positions as shown in Figure 5.[2] Figure 4 bottom plot shows the maximum actuator force vs. the weighted RMS of image FWHM. The PSF at each field position is the FFT PSF obtained by inserting the mirror print through into the LSST ZEMAX model. The FWHM calculation for each image is done by first

[2] Per email communications with Brian Bauman.

masking off the outside of the image, then calculating the RMS of the image and multiplying by 1.665, assuming it to be a 2D Gaussian. The center of the mask is the intensity centroid of the image stamp. The radius of the stamp is determined using the farthest pixel that has intensity no less than 2% of the peak intensity. The FWHM due to telescope intrinsic aberration has been subtracted in quadrature.

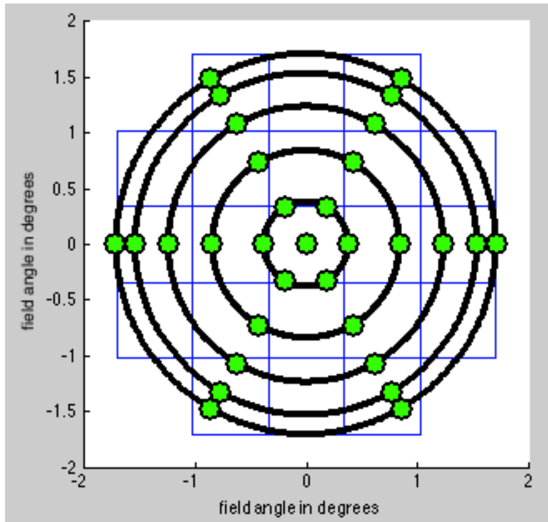


Figure 5 Distribution of field points in LSST focal plane. The blue squares show the science rafts.

3.3. Between Horizon and Zenith

To do the optimization at any zenith angle between horizon and zenith, we need the initial forces u_0 and surface sag x_0 at any zenith angle. Using the subscripts z and h for zenith and horizon, we have

$$u_0 = u_{0_z} \cos\alpha + u_{0_h} \sin\alpha$$
$$x_0 = x_{0_z} \cos\alpha + x_{0_h} \sin\alpha$$

(8)

where α is the zenith angle. As verification, we take one initial state of the mirror at horizon pointing. In the FEA model, when we cut the actuators forces but half, and let g=4.9m/s^2, the output surface is found to be exactly half of the original surface in terms of x-, y-, and z-displacements.

Figure 6 shows an altitude histogram from an example run of the LSST Operation Simulator. Note that it is the altitude angle that is shown, which is 90^o – α. The peak is at about α=45^o. To be conservative, we look at α=60^o to examine whether or not the current rated capacity of the actuators will be able to meet the error budget allocation for gravity print through. The optimization results are shown in Figure 7, which is formatted the same way as Figure 4.

From the bottom plot of Figure 7, it is seen that the maximum actuator force gets below 2500N when the weighted FWHM is about 50mas. This corresponds to a mirror surface RMS of about 13nm. Figure 3 in LTS-123 shows that at 60^o zenith angle the error budget for gravity is about 135mas at 500nm, for M1 and M3 respectively. The RSS of the two is about 135*√2=190mas.

Although we have used 770nm in our ZEMAX model, even after the adjustment due to wavelength, we are still well within the error budget. We actually have a few hundred Newtons of margin. So we conclude that the current actuator design has no problem meeting the error budget allocation.

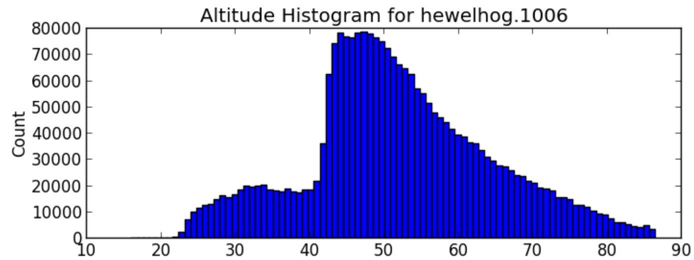


Figure 6 Altitude histogram from an example run of LSST Operation Simulator.

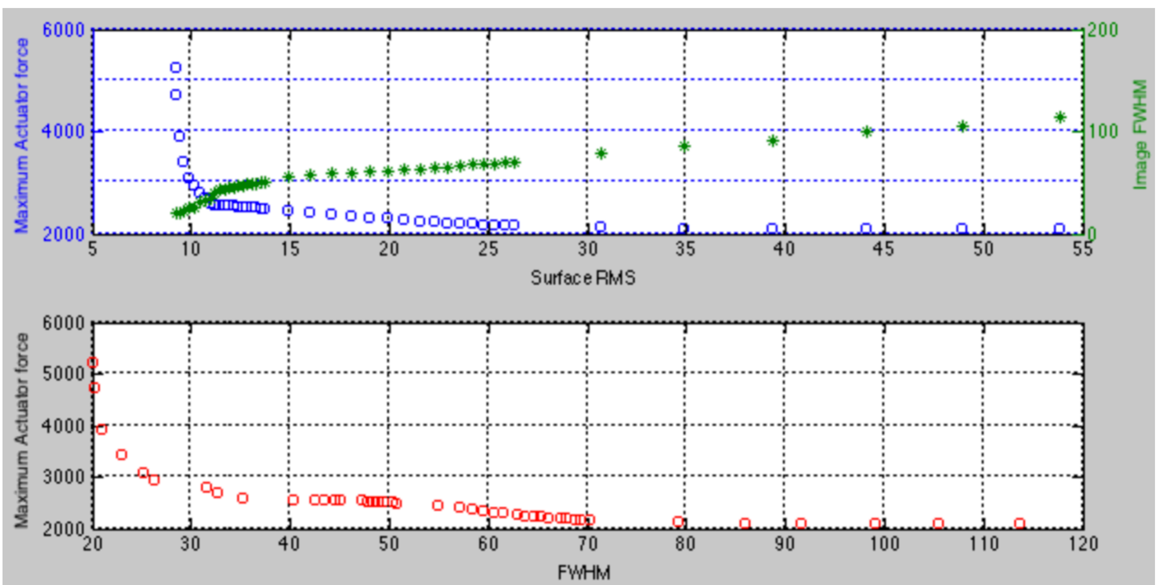


Figure 7 M1M3 control optimization result for 60° zenith angle. Top plot: blue circles show maximum actuator force vs. the mirror surface RMS; green stars show the weighted RMS of the image FWHM across the focal plane vs. the mirror surface RMS. Bottom plot: maximum actuator force vs. the weighted RMS of the image FWHM.

4. Additional Factors that Could Affect Results of this Analysis

As we noted at the end of Section 1, there are a few things related to the polishing of the mirror that we have not taken into account in the preceding sections. Now we talk about the effects of these additional factors and how they could affect our conclusion.

(1) M1M3 polishing error. This is the final product we get from the M1M3 optical testing data processing. Because this is the residual after bending mode corrections[3], it is mostly of high spatial frequency. This means we won't need much additional forces to correct it,

[3] We follow the SOML procedure and use the first 20 bending modes for this correction.

because we won't be able to correct much. On the other hand, although this does affect our assessment of the image quality, it is allocated separately in the error budget (105mas, see LTS-123 and 124). So the polishing error can be neglected in this analysis.

(2) Bending mode correction applied during M1M3 optical testing data processing. This is also negligible because these are mostly low order modes, which is very easy to bend out. It is seen that the forces needed for these bending mode corrections are mostly no more than a few tens of Newtons.

(3) Zenith print through that is polished into the mirror. This is even higher frequency than the polishing error. So just like the polishing error, additional forces won't help much. But this does affect our estimation of the image quality. When we are close to zenith, it makes the print through close to zero. As we move away from zenith, this starts to have a negative effect. It is possible that when we don't take this into account we are within error budget, but when we do the image quality won't be acceptable. So this needs to be checked quantitatively.

(4) Support actuator forces not perfectly optimized. Based on the most recent data from SOML, the forces that have been used to support the mirror during polishing is quite different from what we think the forces should be based on our FEA model.

(3) and (4) are actually very closely related. (4) can be considered a special case of (3). When the support forces are not optimized, it introduces an additional shape error, the negative of which is polished into the mirror. To check the effects of these two things together, we use our linear model as defined by Eq. (1), to determine the shape of the mirror under these non-ideal support forces. The negative of this is added as a static term to the mirror shape, specifically, we add it to x_0. Then we rerun the control optimization. For 60^o zenith angle, the results are shown in Figure 8. Compared to Figure 7, the results do not change much. With the actuators of rated capacity 2500N, we can still control the mirror well within the allocated budget.

Note that the polishing related data used in this section are not considered final. That is why we do not include these results in the main framework of the analysis.

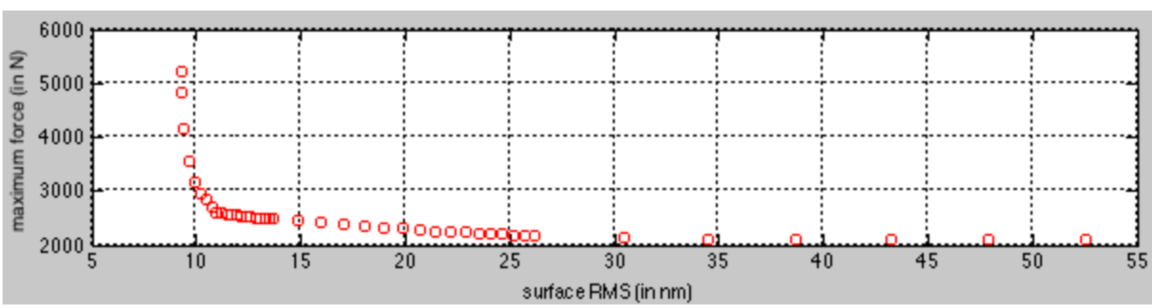


Figure 8 M1M3 control optimization result for 60^o zenith angle with polished-in print through and non-optimal support forces taken into account.

5. M1M3 Print Through Data

Based on the results presented in Section 3, we provide the zenith and horizon print through data and show how to use them.

The zenith print through is more straightforward. Based on Figure 3, we can simply take the first data point from the left, corresponding to ρ=0, since the maximum cylinder force is well below 2500N. This print through map is given in Document-16407 and Document-16463, and shown in Figure 9 left. The surface RMS is 10.3 nanometers, with a maximum actuator force of 2104N. For the horizon print through, we want the first data point from the left that is below 2500N in Figure 4. The print through map is shown in Figure 9 right, and also given in Document-16407 and Document-16463. This corresponds to a surface RMS of 28.2 nanometers, a maximum actuator force of 2494.4N, and the weighing factor ρ=0.9025.

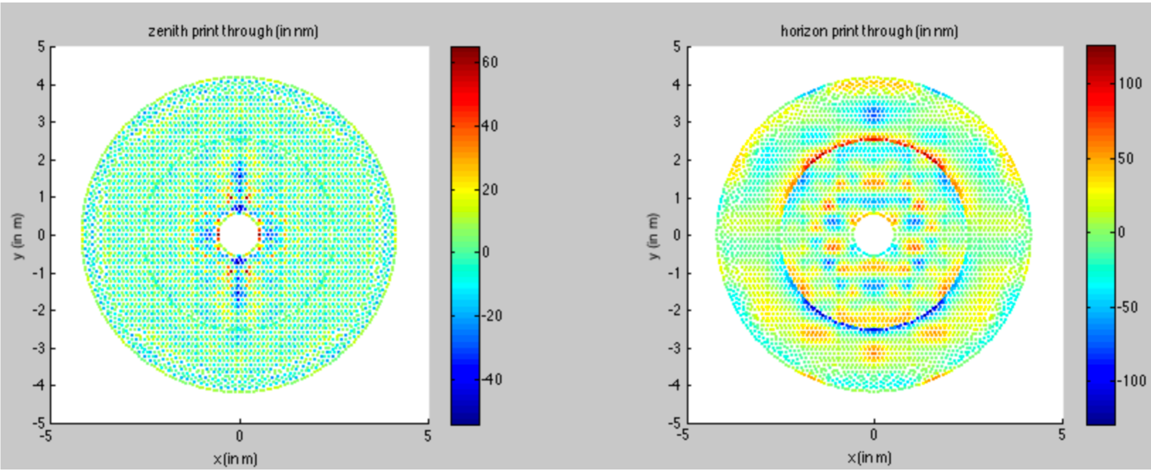


Figure 9 M1M3 zenith (left) and horizon (right) print through maps. The numerical data is given in Document-16407.

There are four columns of data in Document-16407. The first two columns are the x- and y-coordinates of the FEA nodes in meter. The third and fourth columns are the surface sag for the zenith and horizon print throughs in nanometer.

Document-16463 is a MATLAB .mat file. The variables it contains are:

- x and y are the coordinates of the FEA nodes in meter.
- sz and sh are the surface sag for the zenith and horizon print throughs in nanometer.
- uz and uh are the y and z forces (in Newton) that produces sz and sh.
- Fz and Fh are cylinder forces (in Newton) converted from uz and uh.
- xf and yf are the coordinates of the actuator load cells in meter.
- actID are the actuator ID numbers.

The M1M3 surface at zenith angle α can be obtained using

$$x = x_z \cos\alpha + x_h \sin\alpha + x_{pe} + x_{pp} \tag{9}$$

x_{pe} is the polishing error, i.e., the end product of the M1M3 optical testing data processing. x_{pp} is the print through that is polished into the mirror. The additional component that is also polished into the mirror due to the support actuators during polishing not being optimized is also included in x_{pp}. Before we have further knowledge about the mirror polishing, we recommend that $x_{pp} = -x_z$ is used together with some preliminary data for x_{pe}. Finally, note that the print through map by combining zenith and horizon using sinα and cosα will not give identical results as one gets by applying the control optimization at zenith angle α, because of the inclusion of the forces in the cost function. But since these print throughs are all on ~10 nanometer RMS level, Eq. (9) is good enough for use for most practical purposes.

6. Summary

In this technical note, we have discussed why we need to control the M1M3 actuators by optimizing the surface sag and the actuator forces at the same time, and how we do it. We described the control law for a linear system, and the mathematics. We looked at the optimization results at zenith and horizon, and used 60^o zenith angle as an example of a more general and representative case. We concluded that the current rated capacity of the actuators (2500N) is good enough for controlling the mirror to stay within its allocated error budget. We also considered the various polishing related factors that could affect the analysis. Although the input data are still preliminary, mostly likely these won't affect our conclusion.

The zenith and horizon print through data for M1M3 are given in Document-16407 and Document-16463. We described how to use them in Section 5.

Works Cited

Bishop, Christoffer. *Neural Networks for Pattern Recognition.* Oxford University Press, 1995.