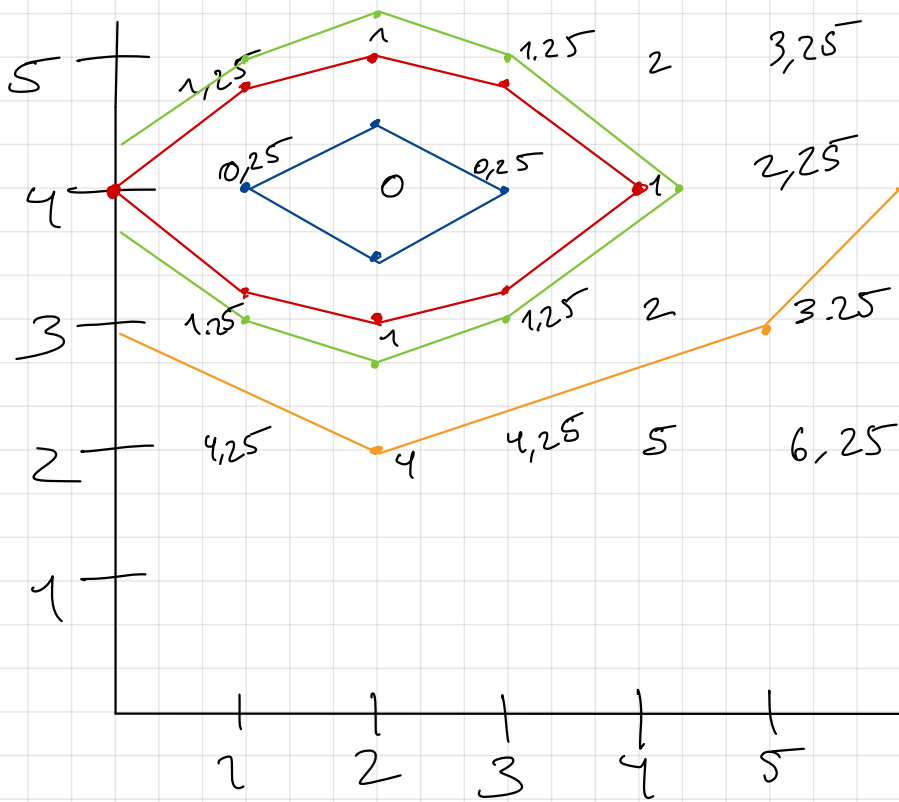


Exercise 2:

Isolines of $f(x,y) = \frac{(x-2)^2}{4} + (y-4)^2$

1.



• $f(x,y) = 0,25$

• $f(x,y) = 1$

• $f(x,y) = 1,25$

• $f(x,y) = 4$

$$f(x,y) = \frac{(x-2)^2}{4} + (y-4)^2 = 0 \Leftrightarrow \frac{(x-2)^2}{4} = 0$$

$$\wedge (y-4)^2 = 0$$

$$\frac{(x-2)^2}{4} = 0 \Leftrightarrow (x-2)^2 = 0 \Leftrightarrow x-2=0 \Leftrightarrow x=2$$

$$(y-4)^2 = 0 \Leftrightarrow y-4=0 \Leftrightarrow y=4$$

zero f-insoline is a point $(2,4)$

2. Sei $(x, y) = (2, 4, 4, 2)$ which direction can you climb up the function fastest?

Using gradient

$$f(x, y) = \frac{(x-2)^2}{4} + (y-4)^2$$

$$\frac{\partial f(x, y)}{\partial x} = \frac{\partial \left(\frac{x^2}{4} - \frac{4x}{4} + 1 \right)}{\partial x} =$$

$$\frac{\partial \left(\frac{x^2}{4} - x - 1 \right)}{\partial x} = \frac{2}{4}x - 1 = \frac{x}{2} - 1$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{\partial (y^2 - 8y + 16)}{\partial y} = 2y - 8$$

$$\Rightarrow \nabla f\left(\frac{x}{2} - 1, 2y - 8\right)$$

$$\nabla f(2, 4, 4, 2) = \left(\frac{2}{2} - 1, 2(4, 2) - 8 \right) = (0, 2, 0, 4)$$

$$\|(0, 2, 0, 4)\| = \sqrt{0^2 + 2^2 + 0^2 + 4^2} = 4.5$$

Direction $\vec{v} = \left(\frac{0,2}{0,45}, \frac{0,4}{0,45} \right) = (0,44, 0,89)$

3. Along which 2 directions could you move down faster if the maximum allowed slope is 0,2?

Slope of the Direction

$$\nabla \vec{v} f(2,4,4,2) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot \begin{bmatrix} 0,2 \\ 0,4 \end{bmatrix} \leq 0,2$$

$$\wedge \|\vec{v}\| = 1$$

$$\Rightarrow u_1(0,2) + u_2(0,4) = 0,2 \quad \begin{array}{l} \text{choose } 0,2 \\ \text{to maximise} \\ \text{the directional} \\ \text{derivative} \end{array}$$

$$\sqrt{u_1^2 + u_2^2} = 1$$

$$\sqrt{u_1^2 + u_2^2} = 1$$

$$\stackrel{\text{set } x \in \mathbb{R}}{\Rightarrow} \sqrt{(1-2u_2)^2 + u_2^2} = 1$$

$$\Rightarrow \sqrt{1-4u_2+4u_2^2+u_2^2} = 1$$

$$\Rightarrow \sqrt{1-4u_2+5u_2^2} = 1$$

$$\Rightarrow 1-4u_2+5u_2^2 = 1$$

$$\Rightarrow u_2(-4+5u_2) = 0$$

$$\Rightarrow u_2 = 0 \quad \vee \quad (-4+5u_2) = 0$$

$$u_1(0,2) + u_2(0,4) = 0,2$$

$$\Leftrightarrow u_1 + 2u_2 = 1$$

$$\Leftrightarrow u_1 = 1 - 2u_2 \quad (1)$$

$$\text{Fall 1. } u_2 = 0$$

$$\Rightarrow u_1 = 1$$

$$\text{Fall 2. } -4 + 5u_2 = 0$$

$$\Rightarrow u_2 = \frac{4}{5}$$

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$$u_1 = 1 - 2 \cdot \left(\frac{4}{5}\right) = -\frac{3}{5}$$

Directions:

1. $\vec{u}_a (1, 0)$

2. $\vec{u}_b \left(-\frac{3}{5}, \frac{4}{5}\right)$

