# 2D Vision and Deep Learning Assignment 1

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## Exercise 1

$$x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, y = \begin{bmatrix} 6 \\ 7 \end{bmatrix}, z = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

### 1.1

Inner product $\langle x, y \rangle = 4 * 6 + 5 * 7 = 59$ ✓

### 1.2

Outer product $x \otimes y = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 6 & 7 \end{bmatrix} = \begin{bmatrix} 24 & 28 \\ 30 & 35 \end{bmatrix}$ ✓

### 1.3

Determine $(x \otimes y)z = \begin{bmatrix} 24 & 28 \\ 30 & 35 \end{bmatrix} * \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 444 \\ 555 \end{bmatrix}$ ✓

### 1.4

Rank of $x \otimes y \Longrightarrow \begin{bmatrix} 24 & 28 \\ 30 & 35 \end{bmatrix} = \begin{bmatrix} 30 & 35 \\ 30 & 35 \end{bmatrix} = \begin{bmatrix} 24 & 28 \\ 0 & 0 \end{bmatrix} \Longrightarrow Rank(x \otimes y) = 1$ (✓) ~~4~~

not equal!

## Exercise 2

$$R_1 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}, R_2 = \begin{bmatrix} \sin\alpha & \cos\alpha \\ \cos\alpha & -\sin\alpha \end{bmatrix}$$

### 2.1

Determine determinant of $R_1$
$\det R_1 = \cos^2\alpha + \sin^2\alpha = 1$ ✓

Determine determinant of $R_2$
$\det R_2 = -\sin^2\alpha - \cos^2\alpha = -1$ ✓

### 2.2

Matrices orthogonal when

$$A * A^T = E \tag{1}$$

$R_1 * {R_1}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Longrightarrow R_1$ is orthogonal ✓

$$R_2 * {R_2}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Longrightarrow R_2$ is orthogonal ✓

### 2.3

Calculate inverse of $R_1$

$$\left(\begin{array}{cc|cc} cos\alpha & -\sin\alpha & 1 & 0 \\ \sin\alpha & \cos\alpha & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} \cos\sin\alpha & -\sin^2\alpha & \sin\alpha & 0 \\ 0 & 1 & -\sin\alpha & \cos\alpha \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} \cos\sin\alpha & 0 & \sin\alpha - \sin^3\alpha & \cos\sin^2\alpha \\ 0 & 1 & -\sin\alpha & \cos\alpha \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & \cos\alpha & \sin\alpha \\ 0 & 1 & -\sin\alpha & \cos\alpha \end{array}\right)$$

$$\Longrightarrow {R_1}^{-1} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$ ✓

Calculate inverse of $R_2$

$$\left(\begin{array}{cc|cc} \sin\alpha & \cos\alpha & 1 & 0 \\ \cos\alpha & -\sin\alpha & 0 & 1 \end{array}\right) \longrightarrow \ldots \longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & \sin\alpha & \cos\alpha \\ 0 & 1 & \cos\alpha & -\sin\alpha \end{array}\right)$$

$$\longrightarrow {R_2}^{-1} = \begin{pmatrix} \sin\alpha & \cos\alpha \\ \cos\alpha & -\sin\alpha \end{pmatrix}$$ ✓

### 2.4

Difference between $R_1$ and $R_2$

$$R_D = \begin{pmatrix} \cos\alpha - \sin\alpha & -\sin\alpha - \cos\alpha \\ \sin\alpha - \cos\alpha & \cos\alpha + \sin\alpha \end{pmatrix}$$

0,5

3,5

Difference in what $R_1$ and $R_2$ do
not litteral difference

## Exercise 3

$$\begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ u_2 & u_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_{11} & \cdots & v_{1k} \\ v_{21} & \cdots & v_{2k} \\ v_{31} & \cdots & v_{3k} \end{bmatrix}$$

$$\Rightarrow \begin{array}{ll} -u_3 v_{21} + u_2 v_{31} & | \cdot u_1 \\ u_3 v_{11} - u_1 v_{31} & | \cdot u_2 \\ -u_2 v_{11} + u_1 v_{21} & | \cdot u_3 \end{array} \longrightarrow \left. \begin{array}{l} -(u_3 u_1 v_{21}) + (u_2 u_1 v_{31}) \\ (u_3 u_2 v_{11}) \mp (u_1 u_2 v_{31}) \\ -(u_2 u_3 v_{11}) + (u_1 u_3 v_{21}) \end{array} \right\} +$$

$$\downarrow$$

$$\left. \begin{array}{l} -(u_3 u_1 v_{21}) + (u_2 u_1 v_{31}) \\ (u_3 u_2 v_{11}) - (u_1 u_2 v_{31}) \\ -(u_1 u_2 v_{31}) + (u_1 u_3 v_{21}) \end{array} \right\} +$$

$$\downarrow$$

$$\begin{pmatrix} -(u_3 u_1 v_{21}) + (u_2 u_1 v_{31}) \\ (u_3 u_2 v_{11}) - (u_1 u_2 v_{31}) \\ 0 \end{pmatrix}$$

analog für alle $v_{1k} .. v_{3k}$

$\Rightarrow \text{Rang}([u]_\times) = 2$ für alle $\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \in \mathbb{R}^3$

Figure 1: Aufgabe 3 Beweis

## Exercise 4

Draw the regions corresponding to vectors x $\in \mathbb{R}^2$, where $||x|| \leq 1$ with the following norms.

### 4.1

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### 4.2

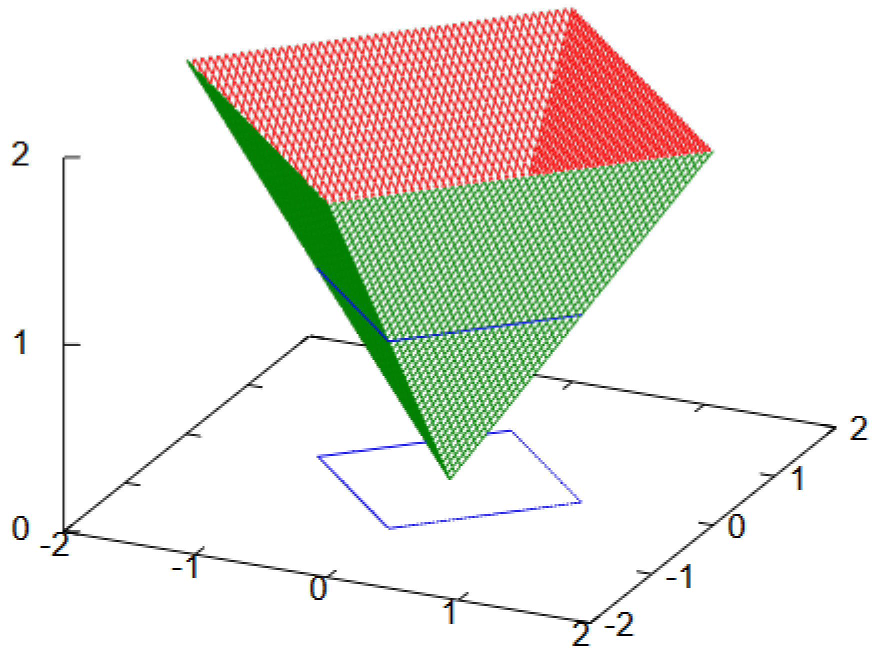


Abb. 2: Summennorm alias 1-Norm

**4.3**

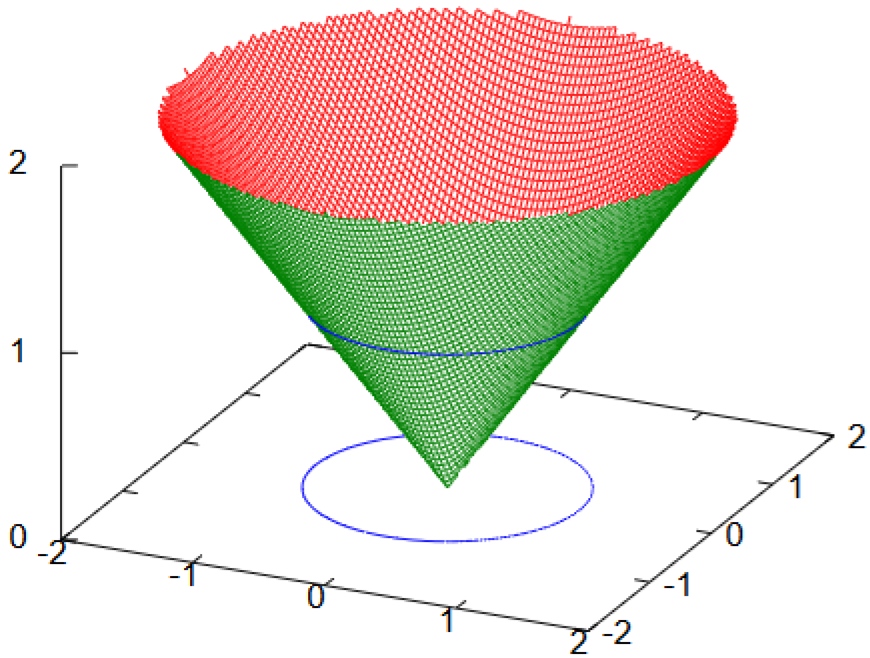


Abb. 3: Euklidische Norm alias 2-Norm

**4.4**

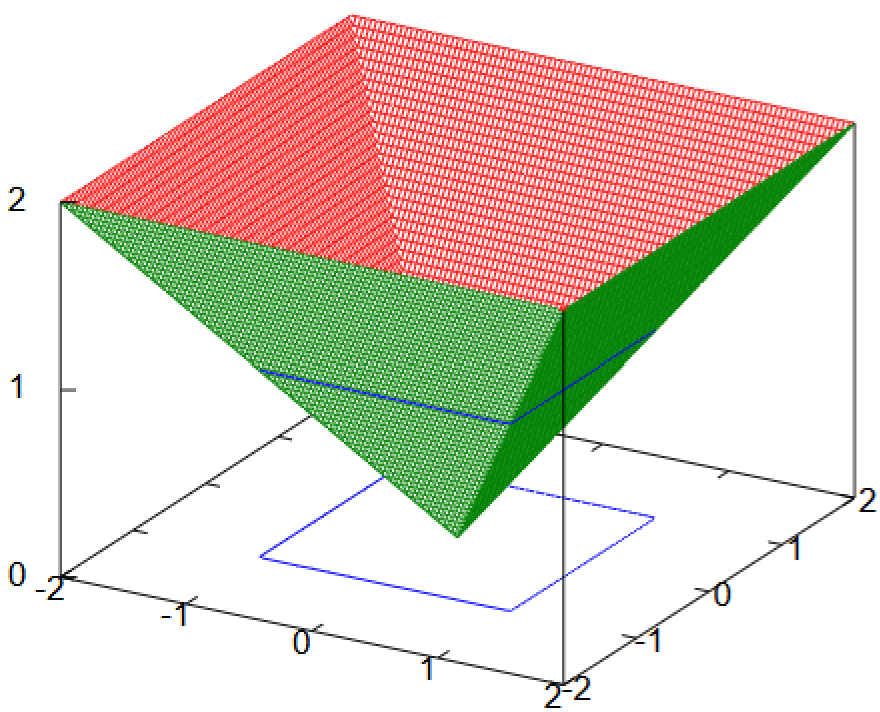


Abb. 4: Maximum Norm alias Tschebyschew-Norm

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# Exercise 5

## 5.1

Derive $y = \frac{1}{1+e^{-x}}$

$\longrightarrow y' = \frac{d}{dx}[\frac{1}{1+e^{-x}}]$

$= -\frac{\frac{d}{dx}[e^{-x}+1]}{(e^{-x}+1)^2}$

$= -\frac{\frac{d}{dx}[e^{-x}]+\frac{d}{dx}[1]}{(e^{-x}+1)^2}$

$= -\frac{-e^{-x}}{(e^{-x}+1)^2} = \frac{e^{\cancel{-}x}}{(e^{\cancel{-}x}+1)^2}$ (✓)

## 5.2

Derive $y = |x|$

$\longrightarrow y' = \frac{d}{dx}[|x|]$

$= \frac{x}{|x|}$ ✓

## 5.3

Derive $y = w^T x(x, w \in \mathbb{R}^n)$ with respect to x

$\longrightarrow y' = \sum_{i=1}^{n} w_i$ f

## 5.4

Derive $y = w^T x(x, w \in \mathbb{R}^n)$ with respect to w

$\longrightarrow y' = \sum_{i=1}^{n} x_i$ f

## 5.5

Derive $y = Mx(M \in \mathbb{R}^{m\times n}, x \in \mathbb{R}^n)$ with respect to x

$$\longrightarrow y' = \begin{bmatrix} \sum_{i=1}^{n} M_{1,i} \\ \sum_{j=1}^{n} M_{2,j} \\ \dots \\ \sum_{k=1}^{n} M_{m,k} \end{bmatrix}$$ f

## 5.6

Derive $y_i$ with respect to $m_{ij}$, where $y = Mx(M \in \mathbb{R}^{m\times n}, x \in \mathbb{R}^n)$

$\longrightarrow y_i' = x_j$ ✓

2,5

# Exercise 6

$$S = 0, 0, 1, 1, 1, 1 \tag{2}$$

### 6.1

Sample mean of S
$\overline{x} = \frac{4}{6} = \frac{2}{3}$ ✓

### 6.2

Sample variance of S
$\frac{0-\frac{2}{3}+0-\frac{2}{3}+1-\frac{2}{3}+1-\frac{2}{3}+1-\frac{2}{3}+1-\frac{2}{3}}{6}$
$= \frac{0}{6} = 0$ f

### 6.3

Probability of S if p(x=0) = p(x=1) = 0.5
$(\frac{1}{2})^6 = \frac{1^6}{2^6} = \frac{1}{2^6}$ (✓) exact order

### 6.4

Probability of S if p(x=1) = 0.6
$0.4^2 \cdot 0.6^4$ = ? (✓) exact order

### 6.5

For which value of p(x=1) is the probability of S maximized?
p(S) $= (1-p)^2 \cdot p^4$
$= (1-2p+p^2) \cdot p^4$
$= p^4 - 2p^5 + p^6$

p'(S) $= 6p^5 - 10p^4 + 4p^3 | : 6p^3$ (p can't be 0, because then S would be impossible)
$= p^2 - \frac{10}{6}p + \frac{4}{6}$

$p^2 - \frac{10}{6}p + \frac{4}{6} = 0$
$\longrightarrow p_1 = \frac{2}{3}$
$\longrightarrow p_2 = 1 \leftarrow$ can't be 1 because then S would be impossible

Therefore for p(x=1) $= \frac{2}{3}$ the probability of S is maximzed. ✓ /4

## Exercise 7

### 7.1

What is $p(y = T, x = b)$?
$\longrightarrow p(y = T, x = b) = p(y = T \cap x = b) = 0.1$ ✓

### 7.2

What is $p(y = T|x = b)$?
$\longrightarrow p(y = T|x = b) = \frac{p(x=b|y=T) \cdot p(y=T)}{p(x=b)} = \frac{p(y=T \cap x=b)}{p(x=b)} = \frac{0.1}{0.3} = 0.\overline{3}$ ✓

### 7.3

What is $p(y = T)$?
$\longrightarrow p(y = T) = p(y = T, x = a) + p(y = T, x = b) + p(y = T, x = c) = 0.5$ ✓

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