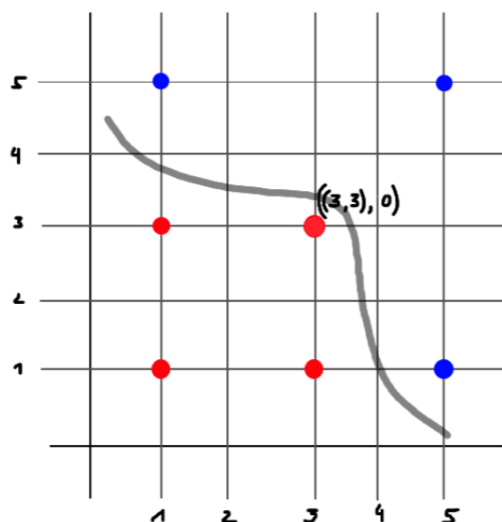


2.1:

Feature vector $(3, 3)$ belongs to class 0 since its euclidean distance to vector $((1, 3), 0)$ is lower than its distance to $((1, 5), 1)$. This can be derived from the triangle inequality $|b| > |a|$. The same can be calculated for any other vector in the vicinity, e.g. vectors $((3, 1), 0)$ and $((5, 1), 1)$. Leading to the conclusion that $(3, 3)$ is element of class 0.



2.2:

As depicted in the graphic on the left there is no possibility for linear separation between the two vector-class sets.

Or, more mathematically speaking:

The two subsets of class $0 \subseteq \mathbb{R}^n$ and class $1 \subseteq \mathbb{R}^n$ are not linearly separable since there are no $n + 1$ real numbers w_1, \dots, w_{n+1} so that for all $a = (a_1, \dots, a_n) \in 0$, $b = (b_1, \dots, b_n) \in 1$ holds:

$$\sum_{i=1}^n w_i a_i \leq w_{n+1} < \sum_{j=1}^n w_j b_j$$

Meaning that there is no set of points $x = (x_1, \dots, x_n)$ in \mathbb{R}^n for which $\sum_{i=1}^n w_i x_i = w_{n+1}$ holds. There is no separating hyperplane.

3 Task

3.1 Exercise

$$g(x) = \sum_{i=1}^2 x_i w_i + w_0 \cdot b = 0 \cdot (-2) + 0 \cdot 1 + (-1) \cdot 1 = -1 \quad (1)$$

$$\Rightarrow f(g(x)) = 0, \text{ because } \theta = 0 \text{ and } g(x) < \theta \quad (2)$$

3.2 Exercise

To generate output according to the AND-function the following weights are applicable:

$$w_0 = -2, w_1 = 1, w_2 = 1$$

Leading to an output $f(g(x)) = 1$ where $g(x) \geq \theta$ (which is only the case for $x = (1, 1)$) and 0 otherwise (where $x = (0, 0) \vee (0, 1) \vee (1, 0)$)

Task 4:

1. $\frac{e^{1h} - e^{-1h}}{e^{1h} + e^{-1h}} = \frac{2}{1 + e^{-21h}} - 1$

2. $\frac{e^{1h}}{e^{1h} + e^{1h}} - \frac{e^{-1h}}{e^{1h} + e^{-1h}} \quad (x = e^{1h}) \quad \frac{x}{x + \frac{1}{x}} - \frac{\frac{1}{x}}{x + \frac{1}{x}}$

$= \frac{\frac{x^2}{x^2 + 1}}{\frac{x^2 + 1}{x^2 + 1}} - \frac{\frac{1}{x^2 + 1}}{\frac{x^2 + 1}{x^2 + 1}} = \frac{x^2 - 1}{x^2 + 1} = \frac{e^{21h} - 1}{e^{21h} + 1}$

3. $\frac{2}{1 + e^{21h}} - 1 = \frac{2 \cdot e^{21h}}{e^{21h} + 1} - 1 = \frac{2e^{21h} - (e^{21h} + 1)}{e^{21h} + 1}$

$= \frac{e^{21h} - 1}{e^{21h} + 1}$

$\Rightarrow 2. = 3. \Rightarrow 1.$