Rastiello
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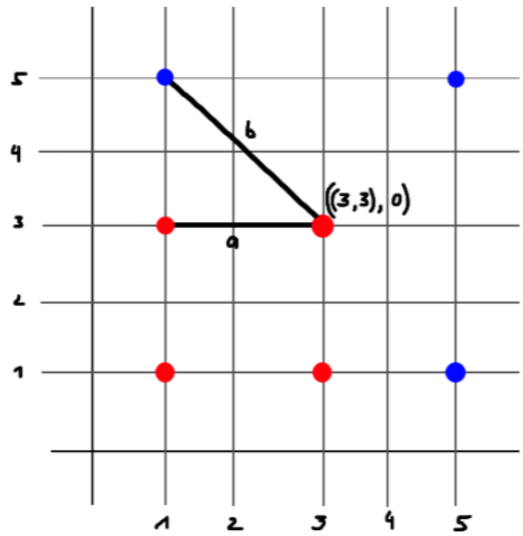
Assignment 07

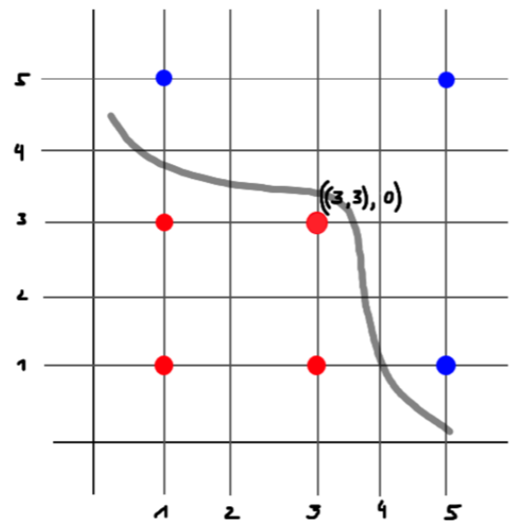
02.01.2021



2.1:

Feature vector (3, 3) belongs to class 0 since its euclidean distance to vector ((1, 3), 0) is lower than its distance to ((1, 5), 1). This can be derived from the triangle inequality |b| > |a|. The same can be calculated for any other vector in the vicinity, e.g. vectors ((3, 1), 0) and ((5, 1), 1). Leading to the conclusion that (3, 3) is element of class 0.

✓



2.2:

As depicted in the graphic on the left there is no possibility for linear seperation between the two vector-class sets.

Or, more mathematically speaking:
The two subsets of class 0 ⊆ R^n and class 1 ⊆ R^n are not linearly seperable since there are no n + 1 real numbers $w_1, \ldots, w_{n+1}$ so that for all $a = (a_1, \ldots, a_n) \in 0$, $b = (b_1, \ldots, b_n) \in 1$ holds:

$$\sum_{i=1}^{n} w_i a_i \leq w_{n+1} < \sum_{j=1}^{n} w_j b_j$$

Meaning that there is no set of points $x = (x_1, \ldots, x_n)$ in R^n for which $\sum_{i=1}^{n} w_i x_i = w_{n+1}$ holds.
There is no seperating hyperplane.

✓

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3 Task

3.1 Exercise

$$g(x) = \sum_{i=1}^{2} x_i w_i + w_0 \cdot b = 0 \cdot (-2) + 0 \cdot 1 + (-1) \cdot 1 = -1 \quad ff \tag{1}$$

$$\implies f(g(x)) = 0, \text{ because } \theta = 0 \text{ and } g(x) < \theta \quad ff \tag{2}$$

w_2 is used for bias

/0,5

3.2 Exercise

To generate output according to the AND-function the following weights are applicable:

$w_0 = -2$, $w_1 = 1$, $w_2 = 1$ ff

Leading to an output $f(g(x)) = 1$ where $g(x) \geq \theta$ (which is only the case for $x = (1,1)$) and 0 otherwise (where $x = (0,0) \vee (0,1) \vee (1,0)$)

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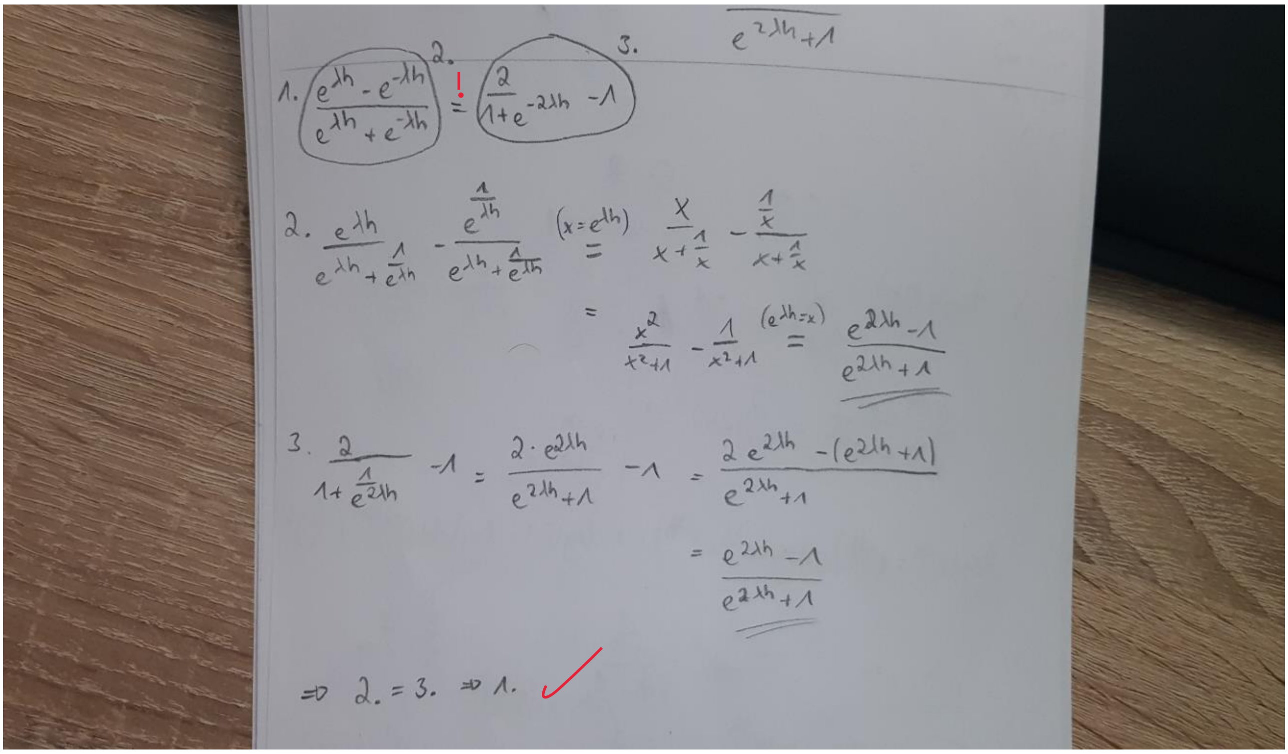
Task 4:



$e^{2\lambda h}+1$

1. $\dfrac{e^{\lambda h} - e^{-\lambda h}}{e^{\lambda h} + e^{-\lambda h}} \overset{!}{=}$ 2. $\dfrac{2}{1+e^{-2\lambda h}} - 1$ 3.

2. $\dfrac{e^{\lambda h}}{e^{\lambda h} + \frac{1}{e^{\lambda h}}} - \dfrac{\frac{1}{e^{\lambda h}}}{e^{\lambda h} + \frac{1}{e^{\lambda h}}} \overset{(x = e^{\lambda h})}{=} \dfrac{x}{x + \frac{1}{x}} - \dfrac{\frac{1}{x}}{x + \frac{1}{x}}$

$= \dfrac{x^2}{x^2+1} - \dfrac{1}{x^2+1} \overset{(e^{\lambda h} = x)}{=} \dfrac{e^{2\lambda h} - 1}{e^{2\lambda h} + 1}$

3. $\dfrac{2}{1 + \frac{1}{e^{2\lambda h}}} - 1 = \dfrac{2 \cdot e^{2\lambda h}}{e^{2\lambda h} + 1} - 1 = \dfrac{2 e^{2\lambda h} - (e^{2\lambda h} + 1)}{e^{2\lambda h} + 1}$

$= \dfrac{e^{2\lambda h} - 1}{e^{2\lambda h} + 1}$

$\Rightarrow$ 2. = 3. $\Rightarrow$ 1. ✓

MLP[f] ≙ MLP[g] ???

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