Assignment 7

Welcome to the seventh assignment of the lecture *2D Vision and Deep Learning*. **Please read all instructions carefully!** This assignment covers some basic concepts of classification. Submission is due on Monday, January 4th, 2021 at 2pm. Please note that late assignments will receive zero (0) marks, so you are strongly encouraged to start the assignment early. If you have any questions please contact Tim Heußler (`theussle@students.uni-mainz.de`).

Exercise 1 (6 points)**.** Write a python program `bayesColorSegmentation.py`, that segments skin (or other) colored objects in a live color video using a bayesian classifier. Your program should generate a live mask image showing white pixel for the segmented skin regions (see right image below). For this segmented pixel it should hold

$$P(\text{Skin}|\text{color}) > P\left(\overline{\text{Skin}}\middle|\,\text{color}\right).$$

As explained in the lecture, using the chromaticity space and Bayes' theorem this results in

$$\frac{P(\text{color}|\text{Skin}) \cdot P(\text{Skin})}{P\left(\text{color}|\overline{\text{Skin}}\right) \cdot P\left(\overline{\text{Skin}}\right)} = \frac{P(\text{color}|\text{Skin}) \cdot P(\text{Skin})}{\frac{1}{256^2} \cdot (1 - P\,(\text{Skin}))} > 1$$

as condition that a pixel belongs to the skin class.

In your program you should set $P(\text{Skin})$ as the percentage number of pixels in a prescribed image area (such as the green area shown in the left image below). Also use this aera to learn $P(\text{color}|\text{Skin})$ as described in the lecture.

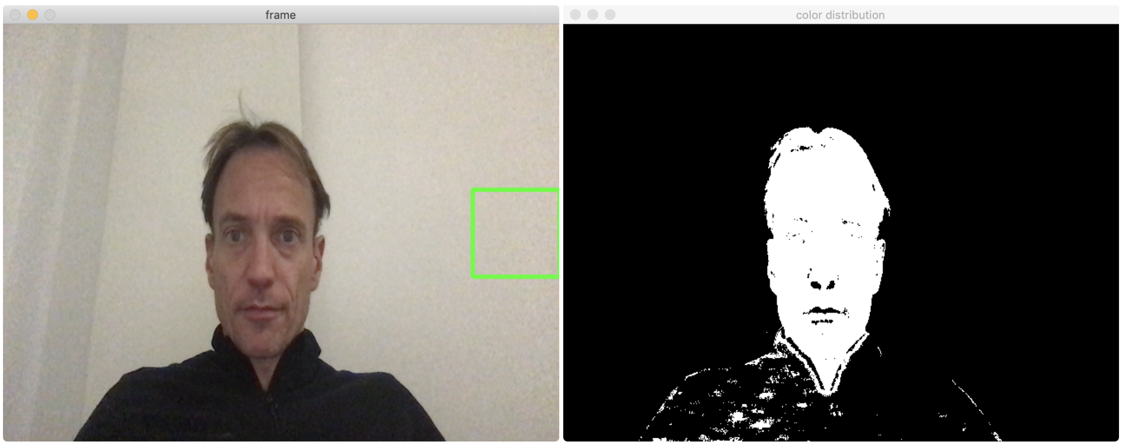

Figure 1: Images taken from the live stream of a webcam. left: Color image. Right: Segmentation mask.

Exercise 2 (2 points)**.** Given the following set of pairs $(\mathbf{p}_i, k_i)$, with *prototype feature vectors* $\mathbf{p}_i \in \mathbb{R}^2$ and *class indices* $k_i \in \{0, 1\}$:

$$\{((1,1),0)\,,((1,3),0)\,,((1,5),1)\,,((3,1),0)\,,((5,1),1)\,,((5,5),1)\}\,.$$

1. (1 point) Which class index would have the observed feature vector (3.3) using a *nearest neighbor classifier*? Justify your answer.
2. (1 point) Is the classification problem linearly separable? Justify your answer.

Exercise 3 (2 points)**.** Given a McCulloch-and-Pitts neuron with a simple threshold function with threshold $\theta = 0$ as *activation function*, *two inputs* and *one output.*

1. (1 point) Which output is generated by the input vector $\mathbf{x} = (0, 0)$ if the weights are $w_0 = -1, w_1 = -2, w_2 = 1$ and the bias $b = 1$?
2. (1 point) Determine values for w_0, w_1 and w_2 such that the neuron reproduces the logical AND function.

Exercise 4 (2 points)**.** The most commonly used form of a sigmoidal activation function for artifical neural networks is

$$f(h) = \frac{1}{1 + \exp(-\lambda h)} \quad \text{with } \lambda \in (0, \infty),$$

which saturates at 0 and 1. Another activation function often used is

$$g(h) = \tanh(\lambda h) = \frac{\exp(\lambda h) - \exp(-\lambda h)}{\exp(\lambda h) + \exp(-\lambda h)} \quad \text{with } \lambda \in (0, \infty),$$

which is also a sigmoid function and saturates at ± 1. Show that $g(h) = 2f(2h) - 1$ and use this to show that for every multi layer perceptron (MLP) using f as an activation function there is an exactly equivalent MLP using g as an activation function.

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