Problem Chosen
B

2024 MCM/ICM Summary Sheet

Team Control Number
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Searching for Lost Submersibles

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Abstract

Searching for a moving object requires a significant amount of knowledge about that object's motion to recover it efficiently—especially when that object is lost in a region as vast as the Ionian Sea. In this case, it is critical to be aware of the magnitude and direction of the water's currents. This is because such an object may not be able to propel itself to its destination. To solve this issue, we have developed a model that predicts where tourism submersibles may be when they lose propulsion and communication.

This model uses data from surface buoys to estimate trends in deep sea currents of the Mediterranean. These trends are used to simulate the approximate motion of a deactivated submersible. Fluctuations in deep sea currents are then simulated to generate a heat map of where the submersible is most likely to be at any given time. This heat map can help optimize the search route for the submersible to increase the chance of it being found. Finding the submersible quickly is vital, as our model indicates that the probability of finding the submersible decreases significantly with time.

Contents

1 Introduction **1**
1.1 Approach 1
1.2 Assumptions 2
1.3 Definitions 2

2 Retrieving Information **3**
2.1 Analyzing Topographical Data 3
2.2 Analyzing Current Data 4
2.3 Calculating Buoyant Force 8

3 Saving Vessels **10**
3.1 Preparation 10
3.2 Essential Equipment 10
3.3 Simulations in the Ionian Sea 11
3.4 Performing Optimal Searches 12

4 Results **14**
4.1 Probability of Finding the Submersible 14
4.2 Other Simulations 16

5 Limitations **17**

6 Potential For Improvement **17**

7 Conclusion **17**

8 Memo **19**

1 Introduction

For much of human history, the ocean and seas have remained an enigma in terms of their current patterns and ever-changing topographical compositions. Man has made continued attempts to mitigate the mystery of the water through a myriad of research projects. Evidence of these projects can be deduced from all that we know about the topography of the ocean floor and all that we know about the currents of water. All this to say, humans are making progress in taming the beast that is water. With this in mind, man is not yet invulnerable, situations arise that triumph against even humanity's best efforts. A prime example of the ocean putting to shame mankind's fancy models and science is Malaysia Airlines Flight 370, where a crashed plane in the ocean managed to evade detection from any authority for over a year. More examples can be found in the ever-growing list of missing ships around the world that have yet to be found.

To focus on the problem at hand, a more restricted area must be considered, namely the Ionian Sea. The model represented in this paper will lay out effective safety measures for submersibles belonging to the Maritime Cruises Mini-Submarines (MCMS). This company is looking to host tours around the Ionian Sea to allow tourists to explore shipwrecks on the sea floor and perhaps other important landmarks. These tourists need confirmation of a degree of safety being represented by the company if the deep-sea submersible loses communication with the ship or even worse loses all propulsion control entirely.

All simulated forces in this model account for the laws of physics, including the drag of real-world submersibles. The model uses the initial position, velocity, and depth of the submersible at the time of failure to calculate a distribution of its probable location at any given time, as well as a set of its most likely points of collision with the ocean floor. The model had been optimized for small time frames (less than 24 hours), as well as adapted to fit the conditions of the Ionian Sea.

1.1 Approach

One of the first necessary is to be able to predict the path of a deactivated submersible, our model analyzes the currents along with the topographical data of the Ionian Sea. This model recognizes that this data is not necessarily reflective of the incredibly variable current patterns of the sea in real life. To account for this a margin of error is included in the predicted path and creates many possible variations of the submersible path. This error also allows the model to predict the possible unaccounted-for forces in the currents such as **thermohaline circulation** [2] while maintaining a representation of the uncertainty of conditions in the Ionian Sea. Using the array of many predicted paths of the submersible the model creates a heat map for the most likely position of the submersible over time. Using this heat map the searching strategies of the search vessels can optimize the likelihood of the submersible being found.

1.2 Assumptions

While developing a model that predicts where a submersible might be, it is critical to make assumptions so a potential location can be found efficiently. The models of this paper do not take into account the curvature of the Earth, assuming that the ocean has a flat surface.

In addition, the models assume that all sea vessels are cubes experiencing forces on their center of mass. Thus, they do not rotate, which prevents them from gaining rotational kinetic energy and losing translational kinetic energy.

The model assumes that the force of buoyancy is negligible in the calculations. This is due to the magnitude of the force being incredibly small compared to the mass of the submarine (explored further in Section 2.3).

The model also assumes that as soon as the incident occurs where the submersible loses communication with the host ship the propulsion is simultaneously lost. This results in the submersible having an initial velocity and location at the moment of the incident. This is used in the following sections to determine the eventual location of the submersible as a function of time.

This function of time is then used to calculate where the submersible impacts the ocean floor, and the model assumes that the submersible comes to a complete stop once it impacts the ocean floor. In reality, the submersible may continue to shift or roll once it has impacted the ocean floor, however, this model assumes the displacement to be negligible when considering a search strategy for the submersible.

Lastly, the model assumes that search vessels have access to cameras and flashlights that are strong enough to reach a distance of 50 meters from the center of the search vessel.

1.3 Definitions

Pitometer: A technology that is used to judge the speed of a submersible at any given time using a pressure reading.

Ballast Tanks: A feature of submersibles and submarines that allow for depth control. They are filled with water to increase the density of the submersible and therefore to submerge and decrease depth. To ascend these tanks are emptied utilizing compressed air already stored in the submersible and eventually decrease the density of the submersible. This makes it more buoyant and allows the submersible to climb the depths of the water.

Bathymetry: The study of ocean floor topography and depth

Thermohaline circulation: Large-scale currents caused by a difference in density between different parts of the ocean. Effects are greater in open waters but still present in the Ionian sea.

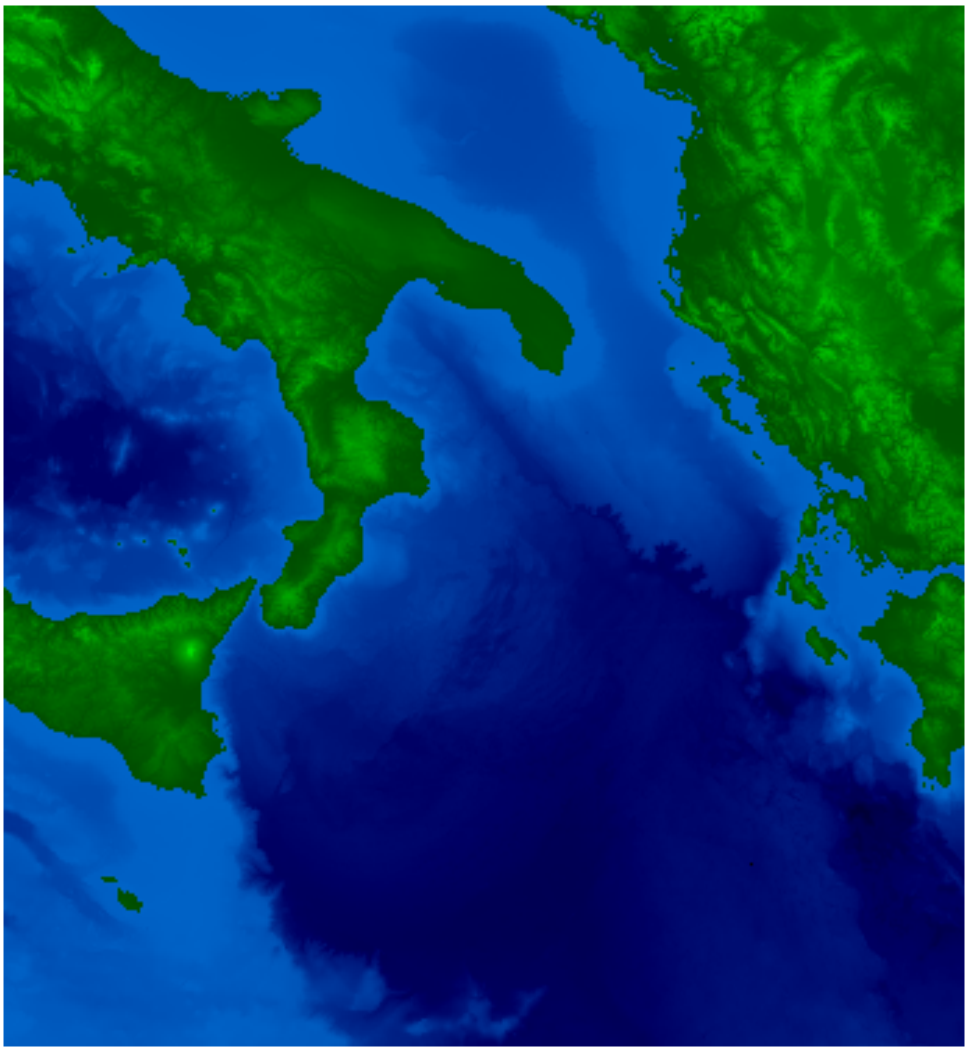

Figure 1: Topography of Ionian Sea

2 Retrieving Information

2.1 Analyzing Topographical Data

One of the first things that was necessary in the creation of our model was a representation of the depth of the Ionian Sea. Because the model is attempting to simulate the possible trajectory of the submersible after it loses communication with the host ship it would make sense that it should predict whether the submersible crashes or not. For example, if the model predicted that the submersible at a certain point is at a location and depth that is in reality underneath the searchable surface of the sea, the reliability of the model is greatly decreased.

To achieve an acceptable level of accuracy in the model it must know when the submersible is going to crash into the sea floor or perhaps a slope of descending depth. This is where the topographical data can be utilized.

To model the depth of the water, data was extracted from the General Bathymetric Chart of the Ocean (GEBCO). This is a publicly available dataset that is overseen by the Intergovernmental Hydro-graphic Commission (IHC). GEBCO provides extremely accurate depth measurements of the entire ocean floor, however, due to computational limitations this model limits the resolution of depth measurements into 2.6 km^2 regions. Figure 1 shows an image representing the model's depth measurement.

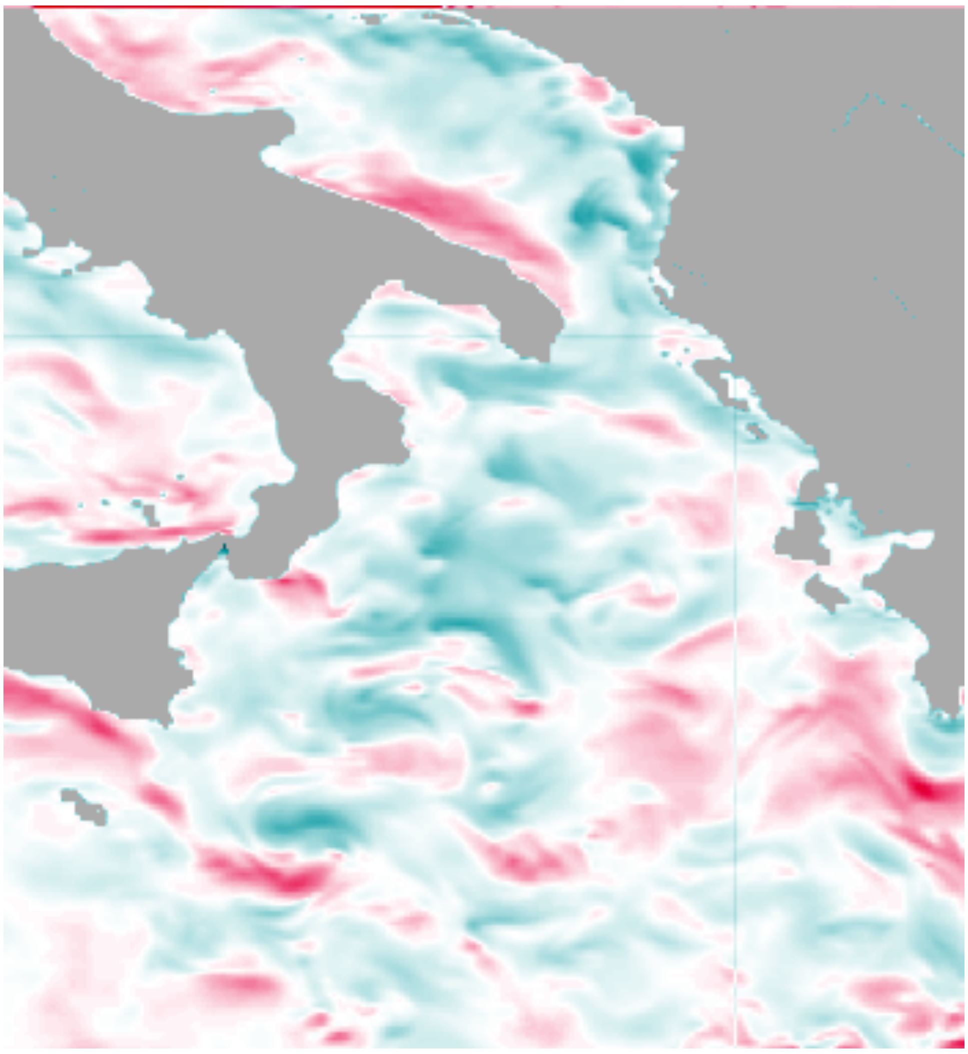

Figure 2: Currents directed east and west. Red represents currents going east and blue represents currents going west.

This data is constantly cross-referenced as the submersible completes its predicted path to further predict if it has crashed into the sea bed. In such scenarios, the simulation stops and tracks the location of the submersible (This will be represented in later sections)

2.2 Analyzing Current Data

In the effort to track the location of a submersible that has lost all propulsion ability and communication with the host ship, it is first imperative that a solution model contains an analysis of currents in the Ionian Sea. Considering a submersible that is touring various shipwrecks with a crew full of tourists it is reasonable to assume that the submersible will traverse the sea floor. Although these deep-sea currents differ greatly from those found at the surface of the Ionian Sea, this model assumes that the deep-sea currents flow in similar directions and with similar magnitude to surface currents in the Ionian Sea (as mentioned in Section 1.2).

This model utilizes two heat map representations of the currents of the Ionian Sea, one representing the intensity of current flow in the Eastward direction with the other representing the Northward current intensity. These are shown in Figures 2 and 3 respectively. This data was found from [8].

These heat maps were taken from a worldwide map where the latitude

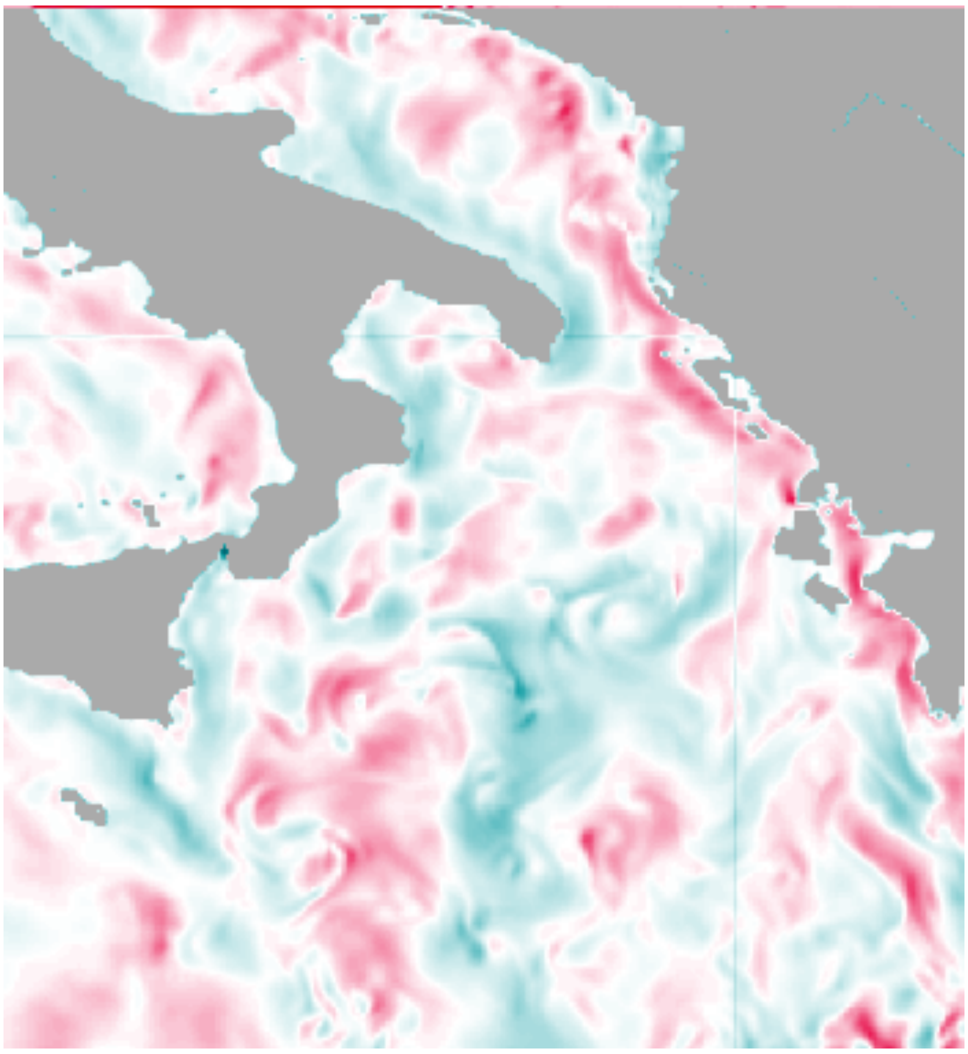

Figure 3: Currents directed north and south. Red represents currents going north and blue represents currents going south.

and longitude have been lined up for consistency within the pixels around the Ionian Sea. The magnitudes of the current velocities for each pixel were extrapolated from the scale represented in the source of the heat maps. Using this strategy this model can find the magnitude and direction of the current for each pixel represented in the map of the Ionian Sea. Combining the analysis of these heat maps, the model creates a slope field representation of the currents of the Ionian Sea in Figures 4 and 5. This slope field can be used to determine the change of velocity of the submersible along its path due to the strength of the currents.

For the analysis of how this current affects the acceleration of the submersible, an equation for drag force has to be utilized. The force of drag $\mathrm{F_D}$ is a value that can be calculated from Equation 1.

$$\mathrm{F_D} = \frac{Av^2\rho}{2} \tag{1}$$

Where A is the relative area of the surface that is being affected by the current (i.e. the surface area of the submarine that is being affected by the currents), v is the relative velocity of the submersible in the medium (the water that it is traveling through), and ρ is the density of the liquid in kg/m^3. This model attempts to analyze the drag force on a submersible traversing the ocean's depths. Equations 2 to 7 reference a model made in [3]. This model uses a very complex system of equations that take in two parameters, D and φ, which represent the depth and latitude at a certain point of the water. These equations are used to figure out the density of the water the submersible is traversing at a certain depth and as it traverses the latitudes of the sea.

$$\rho(D,\varphi) = 1000 \ + \ \alpha(\varphi)\left[\mu(\varphi) + \frac{1-\mu(\varphi)}{2}\chi(D)\right] \ + \ \beta(\varphi)D^{\theta(\varphi)} \tag{2}$$

$$\mu(\varphi) = 0.928 - 0.079\cos(0.053\varphi) \tag{3}$$

$$\chi(D) = 1 + \tanh(0.00988D - 1.01613) \tag{4}$$

$$\alpha(\varphi) = 27.91 - 2.06e^{-(0.0161|\varphi|)^5} \tag{5}$$

$$\beta(\varphi) = 0.00637 + 0.00828e^{-(0.017|\varphi|)^{4.66}} \tag{6}$$

$$\theta(\varphi) = 0.964 - 0.091e^{-(0.016|\varphi|)^5} \tag{7}$$

Using this value along with Equation 9 (Drag Force) the model can deduce the drag force affecting the submersible at every point it travels through the Ionian Sea. Using this equation for drag force the model can, utilizing Newton's second law, $\Sigma F = ma$, where m is the mass of the submersible and a is its acceleration, figure out the acceleration that the submersible is experiencing due to the drag force. Combining Equations 1 - 7 along with some assumptions referenced in Section 1.2 an equation for the instantaneous change of velocity can be compiled, represented in Equation 8. Note the estimated drag coefficient of 0.25 was assumed from [4] and varies greatly depending on the type of submarine. According to [4] the drag coefficient for submarines ranges from 0.1 to 0.3, and since the exact model of the MCMS submarine is unknown, assuming it is of similar build to that of the OceanGate Titan deep-sea submersible, a value of 0.25 is appropriate. Of course, once a specific analysis of the submersible is completed this equation can be updated to further increase the accuracy of the model. As for the surface area value, this also can be updated following the analysis of the real submersible, but this value was just estimated again using what we know about existing deep-sea submersibles (i.e. Titan Submersible).

$$a = \frac{(7)(0.25)v^2\rho(D,\varphi)}{2m} \tag{8}$$

The model can calculate the instantaneous acceleration for each point along its trajectory throughout the sea. With these equations in place along with the assumption that the location and velocity of the submersible at the moment of the incident are known the model can predict the future path of the submersible. An example of this model in use can be seen in Figure 4. This figure shows 10 likely paths of a submersible that loses propulsion at 35°52'10.0"N 17°03'37.6"E, with a depth of 1000 m and an initial velocity of 1 m/s due east. These initial conditions are used

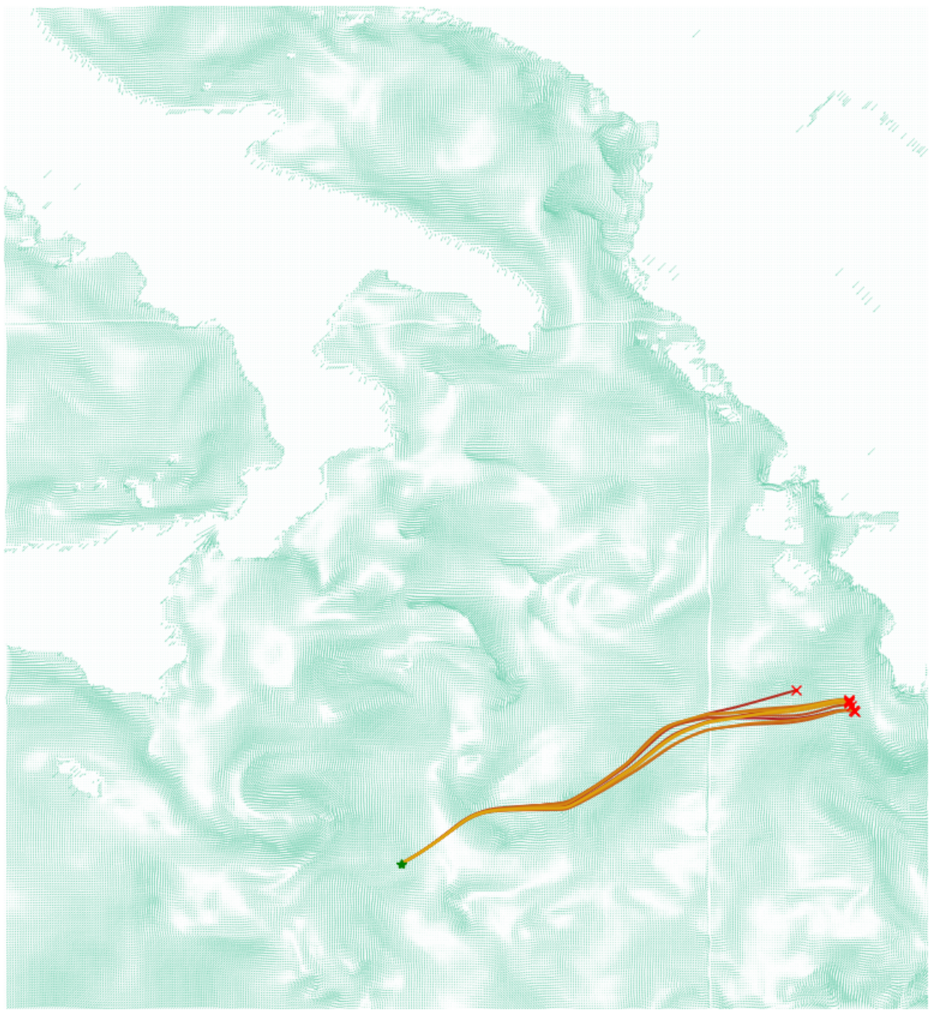

Figure 4: 10 Simulated Paths of a Submersible with the Same Initial Conditions

to calculate 10 possible routes for the submersible, using a simulation of currents. It is also cross-referencing the topography of the region to see if the submersible has crashed, and also incorporating an error variable to attempt to account for the difference of currents between depths.

To account for differences between deep-sea currents and surface currents, a 60% error has been accounted for in the motion of the submersible. This number was chosen because it resulted in the most realistic results. Error values much more than 60% resulted in the submersible disregarding trends in the current such as thermohaline circulation [2] as well as convections in the Ionian caused by changes in depth. Values much lower than 60% resulted in points that were clumped unrealistically close together. The model produces the most accurate results for error values between 50% and 65%. In the simulation, the error is implemented as a force vector that acts on the submersible with a random direction and magnitude. To calculate a 60% error, our model first calculates the average magnitude of all of the currents in the sea. This magnitude is then multiplied by a constant of 0.6 to get an average magnitude. The magnitude of the random vector is then calculated randomly between 0 and twice the average magnitude as seen in Equation 9, to ensure that random vectors are, on average, 60% of the magnitude of the current.

$$||\vec{R}|| = 0.60\bar{f}_c r, \quad 0 \leq r \leq 2 \tag{9}$$

Using the random vector found in Equation 9, the model can represent slight discrepancies in the predicted path of the submersible and model a few complete possible paths of the submersible with the error accounted for, as seen in Figure 4. To represent larger trends in the movement of submersibles, the model runs 5,000 simulations using different random vectors, to create a scatter plot shown in Figure 5. Both of these figures are using techniques referenced in Section 2.1 to constantly check to see if the submersible in its trajectory has crashed into the sea floor. In the event of the submersible crashing it is referenced in the figures as a red x on the map at the crash location. Remember that as per Section 1.2 the submersible does not move when it crashes and rather stays at a constant position. Note that due to the random variation of the simulation, two distinct search zones have been identified. A slight northward drift early in the route can cause ships to crash into an underwater ridge, resulting in the smaller red cluster seen in Figure 5. Although the majority of collisions occur further east, a separate search team should still be deployed to the left cluster. This search strategy might be overlooked without the insight of this model, and possible implementations of these search methods are discussed in Section 3.3.

2.3 Calculating Buoyant Force

Along with the Cartesian estimate of the submersible's location, it is important to consider the change of depth that may be experienced as the submersible follows along its path. Using the known workings of existing deep-sea submersibles, it is reasonable to say that during the duration of the tour, the submersible is at a neutral buoyancy. Considering the context of the situation this assumption can be further verified. This tour is meant to explore shipwrecks on the sea floor. It would only make sense that for the viewing pleasure of the tourists aboard, the depth of the submersible is constant throughout the exploration. It would also make sense that the submersible changes its depth in the water to adjust for the exploration of other shipwrecks. To understand what this means in the context of buoyancy, a beginner's understanding of the working of submersibles is required.

Submersibles control their depth in the water using a variety of methods. Most commonly, ballast tanks are used to change the buoyancy of the submersible and subsequently change the neutral buoyancy point. In this model, it is assumed that following the incident where communication and propulsion are lost, the ability to control the ballast tanks is also lost. This leads to the conclusion that the buoyancy of the submersible does not change at any point following the incident.

The fact that the buoyancy is constant following the incident removes a variable in calculating the predicted height of the submersible. Using a previous model from [3], the density of water can be computed through Equation 2, where D is the depth and φ is the latitude at that point.

With the density equation, even further calculation is required to find the buoyancy force acting on the submersible. Namely, Archimedes's Principle, which is represented by Equation 10.

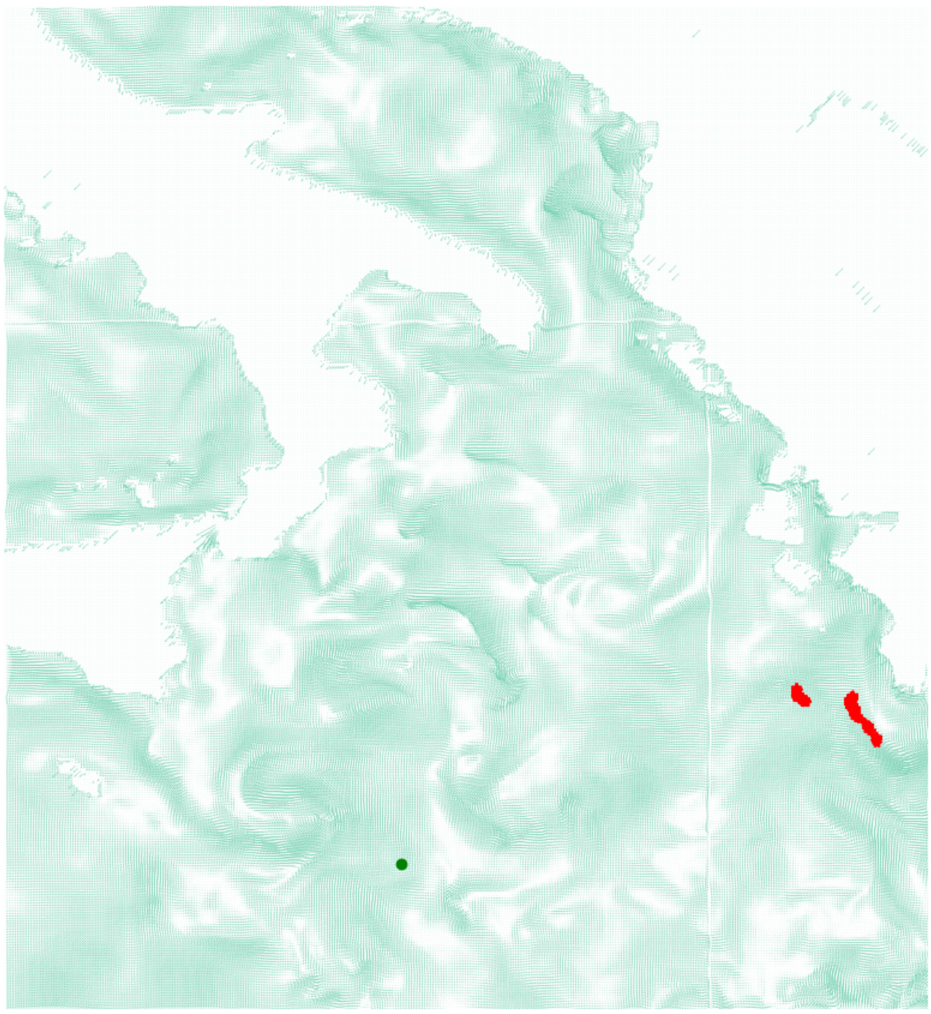

Figure 5: 5000 Simulated Collision Points of Submersible

$$F_B = -\rho g V \tag{10}$$

Where ρ is the density of the liquid, g is the gravitational acceleration constant and V is the volume of the submersible. We know at the last point of contact (as per the previous assumptions) that the submersible was at a neutral buoyancy. So as the water density changes as the submersible moves along its path, the model can calculate the change in buoyant force. With the buoyant force, much like the drag force in the current, Newton's second law can be utilized to find the instantaneous vertical acceleration of the submersible due to the buoyant force.

$$\begin{gathered}\sum F = ma \\ F_G + F_B = ma \\ mg - \rho g V = ma \\ a = g(1 - \frac{\rho V}{m})\end{gathered} \tag{11}$$

In attempting to utilize this equation in the model, a realization was reached that the vertical acceleration experienced by the submersible as it travels around the Ionian Sea is quite negligible. The density of the water (assuming a constant depth) only changes by a factor of 0.3% due to the latitude of the location. Because the mass of the submersible is so great, this minimal change in the density of the water is barely going to affect the submersible's path at all. So for the sake of the model, the effect of buoyant force was neglected and the possible vertical movement was accounted for with the 60% error referenced in Section 2.2.

3 Saving Vessels

3.1 Preparation

There are multiple precautions the MCMS should consider before tours of the Ionian Sea begin. Firstly, it is worth having a protocol for when submersibles lose communication. If this were to occur, pilots should be trained to immediately stop controlling their submersible. This will limit the variability between instances of communication loss and propulsion loss because searchers can assume that lost submersibles are just flowing with the current regardless of what the issue is. Additionally, the shipwrecks that tourists will be exploring will not move. Thus, it would be worthwhile to create predetermined paths for submersibles to travel along since search vessels would be able to predict the direction which a malfunctioning submersible momentarily traveled after losing communication.

3.2 Essential Equipment

Implementing a variety of safety features will be vital for the success of the MCMS. To prevent catastrophes from unfolding, each submersible should be equipped with a depth gauge to know its depth, a pitometer to measure its speed, and a GPS-like machine to keep track of its coordinates. However, a typical GPS that transmits messages through radio waves will not be cost-efficient to use [5]. Instead, one alternative that could be explored is utilizing electrolocation. Not only is it comparatively cheaper than using a stronger version of a GPS that would work underwater, but it also scales with effectiveness when salt is added to water. Due to the Ionian Sea's high salinity levels, this would be an optimal alternative to a traditional GPS as electricity is able to travel easily through salty water [9]. This information must be sent to the host ships every ten minutes so it can react and send a search vessel to save the missing submersible. These vessels should have cameras and flashlights facing not only in each of the cardinal directions, but also above and below the vessel to maximize their vision. Pairing cameras with flashlights would work well in this part of the ocean since the salinity levels prevent algae from growing, making the water much more clear than in other parts of the world. Additionally, search vessels should also have a GPS that relies on electrolocation so these watercraft can move in the correct direction and also be able to be saved if necessary. Lastly, submersibles and search vessels should come with powerful magnets of opposite charges on the top and bottom of them. This way, search vessels can attach themselves to broken submersibles and decrease their own density by emptying their

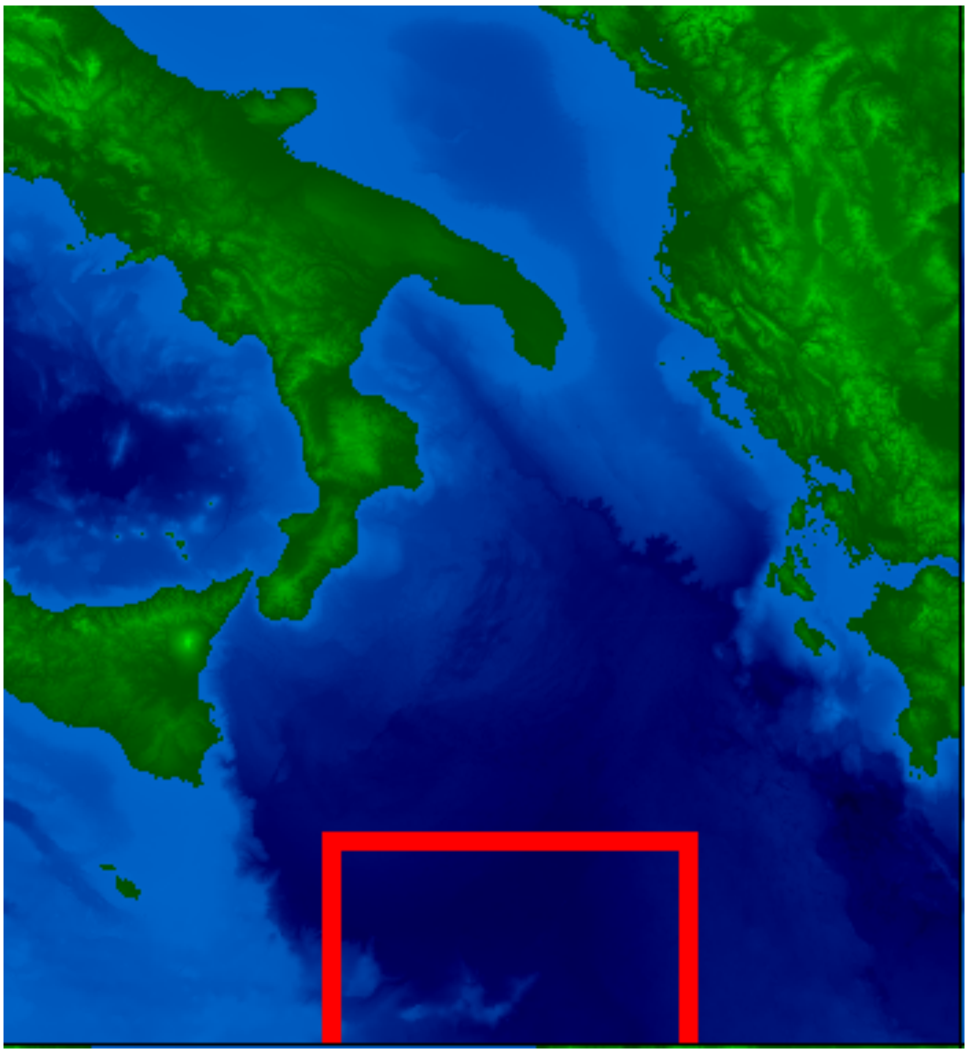

Figure 6: Region of Simulation

ballast tanks, allowing them to ascend with the submersible and eventually reach the host ship.

While submersibles must be prepared for communication and propulsion issues, a host ship must also have equipment that will help them react swiftly in the case of an emergency. All host ships should come with a computer that has access to a database of updated currents in the Ionian Sea, which is what our model uses in order to run. In addition, these computers should also be powerful enough to run simulations with the model in the first place. This way, pilots know what direction to search in and can expect how long it will take to find lost submersibles.

3.3 Simulations in the Ionian Sea

Figure 6 outlines a region that is likely to have high traffic of submersibles due to its depth. In order to designate search areas, we have divided the region shown in Figure 6 into hexagons. Figures 7 and 8 show a zoomed-in view of the region in Figure 6. Each hexagon in these diagrams represents a 4.8 km^2 zone, with its color representing how many simulated submersibles are in that zone at a given time. The number of simulated submersibles in a zone directly correlates with the probability of the real submersible being in that zone, with the probability being measured as $n_{simulated}/500$. After 3 hours, the possible positions of the submersible are clustered fairly close together,

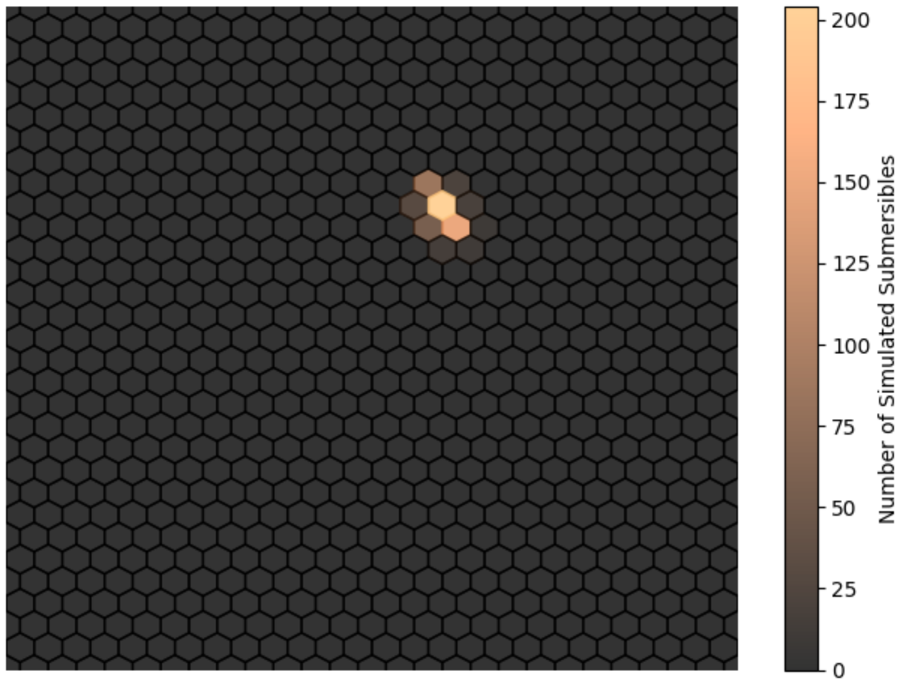

Figure 7: The Probable Locations of Submersible 3 hours after Faliure

as seen in Figure 7. Searching can be concentrated on just two 4.8 km^2 zones both with a high probability of containing the submersible. The feasibility of searching this region is explored in section 4. After an additional 7 hours, the search zone is much more spread out, as seen in Figure 8. In addition to a wider search area, the probability of the submersible being in any given hexagon is considerably lower(note that the colors in this diagram have been scaled from 7 for visibility).

As time progresses, the probable locations for the submarine become increasingly more spread out. We analyzed the spread of this data using its deviation. Deviation is a measure of how much a set of data varies from the mean of that data, shown in equation 12. Although standard deviation is a more common unit to measure the variance of data, there is no need to standardize our deviation measurement because we are comparing the same 500 observational units at different times. Figure 9 shows the deviation of 500 simulated submersibles as a function of time. These are the same 500 simulated submersibles represented in Figures 7 and 8. The curve follows an exponential trend, before leveling off as submersibles start to collide with the ocean floor. Bumps in the curve are caused by fluctuations in the currents, which amplify gaps between submersibles as they are influenced by the currents. This shows the importance of quickly finding lost submersibles using the most optimal search routes.

$$\text{Deviation} = \sum_{j}^{n} \left(y_j - \bar{y}\right)^2 \tag{12}$$

3.4 Performing Optimal Searches

As seen in Figure 4, our model is capable of showing the paths a submersible may follow before crashing. When observing the possible positions of a submersible at a certain time, we can generate heat maps that reveal where

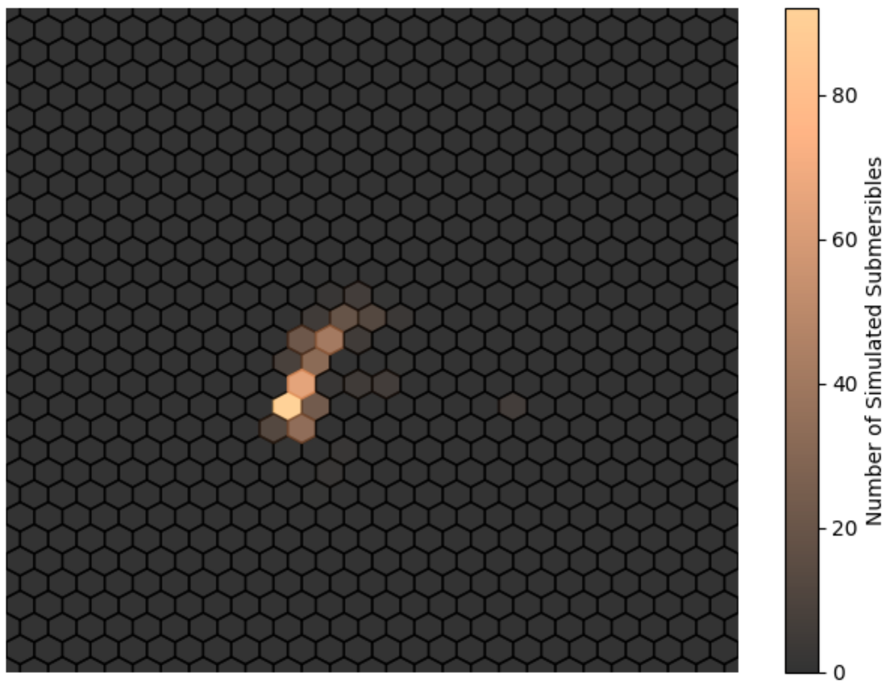

Figure 8: The Probable Locations of Submersible 10 hours after Failure

the submersible will most likely be. These heat maps will be updated every hour so pilots of search vessels can efficiently look for the missing submersible in its probable location. While searching, pilots should avoid outliers that our algorithm creates. These are made by random force vectors generated to simulate currents that may not be represented in our model due to possible inaccuracies. While the current may not be perfectly reflected by our model, the model is not severely inaccurate and thus it is unlikely for the submersible to significantly deviate from predicted paths.

Using information from [1], it is reasonable to assume that search vessels can travel at speeds of 10 m/s. In addition, using our assumptions that search vessels can see 25 m ahead in all directions from their centers, search vessels can clear up to 2000 m^2 at once because

$$A = \pi r^2 = \pi 25^2 \approx 2000\ m^2.$$

Also, the distance between the centers of two adjacent circles is twice the radius of a single circle, indicating that this distance is $2(25) = 50m$. Considering that search vessels can travel at speeds of 10 m/s, this distance can be traversed in $\frac{50}{10} = 5$ seconds. An example of this method of searching is shown in Figure 10.

Using the area of a hexagon in our heat map—which is approximately $4.82\ km^2 = 4.82 \times 10^6\ m^2$ —there are $\frac{4.82\times10^6}{2000} = 2,410$ circles for search vessels to move to. Because it takes 5 seconds to reach a new circle, $2410(5) = 12,050$ seconds or around 200 minutes are needed to get to every circle. Thus, on average, only a third of these hexagons can be searched before a new probable location is generated since the model runs hourly. So, in an hour, the model can find a lost submersible with a third of the probability of the hexagon it searches in. The average position of a search vessel at the end of a simulation at a particular time is the center of the hexagon. So, the distance between the center of a hexagon and one of its sides must be calculated in order to find out how long it takes to get to a new

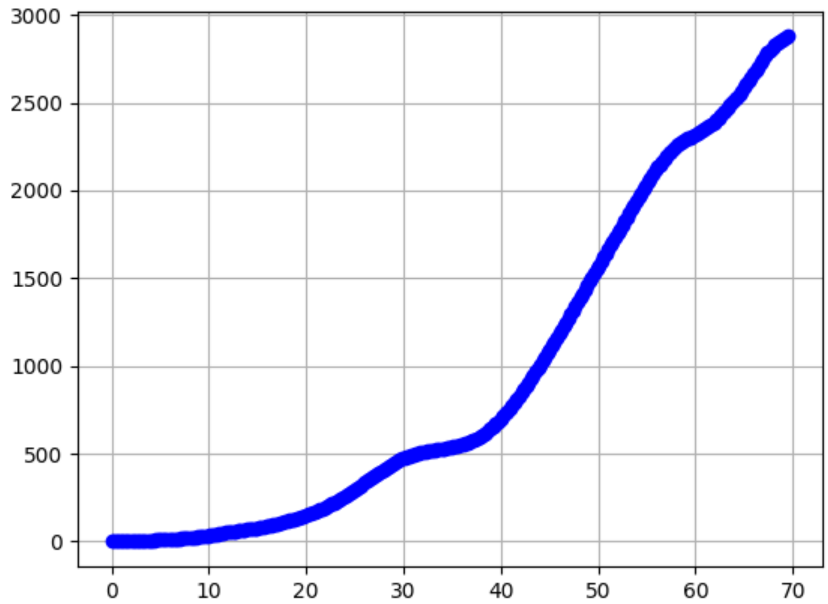

Figure 9: Deviation of Predicted Location over time (measured in Δkm^2/hr)

hexagon. Using the Pythagorean Theorem, this distance is

$$a = \sqrt{s^2 - (\frac{s}{2})^2} = \sqrt{2.4^2 - (\frac{2.4}{2})^2} \approx 2\ km = 2000\ m.$$

So, it will take 200 seconds to reach a new hexagon. This time is insignificant because it is only around 5.5% of the time before the model is ran again and the search vessel will still be looking for the lost submersible as it heads toward a new hexagon. Due to this, these 200 seconds are not considered in our calculations.

4 Results

4.1 Probability of Finding the Submersible

In the model, as it stands, the time elapsed is proportional to the size of the possible area of the submersible. This means that as time progresses from the initial time of the incident the area that needs to be searched gets bigger. Because of restrictions discussed in Section 3.4 there is not enough time to thoroughly search all of the possible locations of the submarine. So, as the search area grows, it becomes harder to cover it in a timely matter. If we consider each step of the heat map simulation referenced in Figures 7 and 8 to have an inherent probability of finding the submersible, we can represent the total probability of finding the submersible as a function of time and subsequently area searched. Said function is represented in Equation 13.

$$p(t) = 1 - \prod_{n=1}^{t}(1 - \frac{p_n}{3}) \tag{13}$$

Note in this equation p is the total probability of finding the submersible after t hours of searching. In this equation, p_n represents the probability

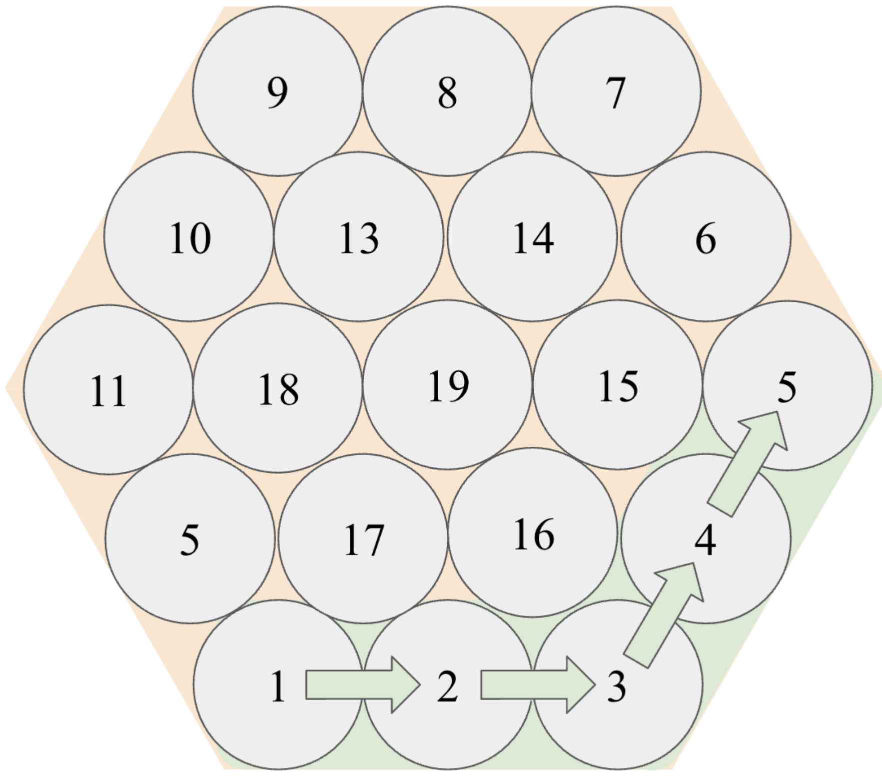

Figure 10: A representation of a search vessel traveling through the area of a hexagon in a heat map. The vessel begins at one of the outer circles and spirals inwards to end up at the center of the hexagon, starting at the first circle and then traveling to the next number. The green arrows and path represent circles that the vessel has already traveled to at a particular instance.

that the submersible is in our 4.8 km^2 zone of search. Because we are only able to search a third of this zone in an hour, $\frac{p_n}{3}$ represents the probability of finding the submersible in that hour.

Time	p_n
3	0.586
4	0.456
5	0.398
6	0.376
7	0.316
8	0.31
9	0.314
10	0.214
11	0.202
12	0.05
13	0.024
14	.028

Table 4.1 shows a set of values for p_n over time, where time is measured in hours after failure. This data correlates to the simulation in Figures 7 and 8. For this dataset, the total probability of finding the submersible is 0.69 if the search starts 3 hours after failure and searching for 11 hours . These numbers were chosen because it would take a search vessel about 3

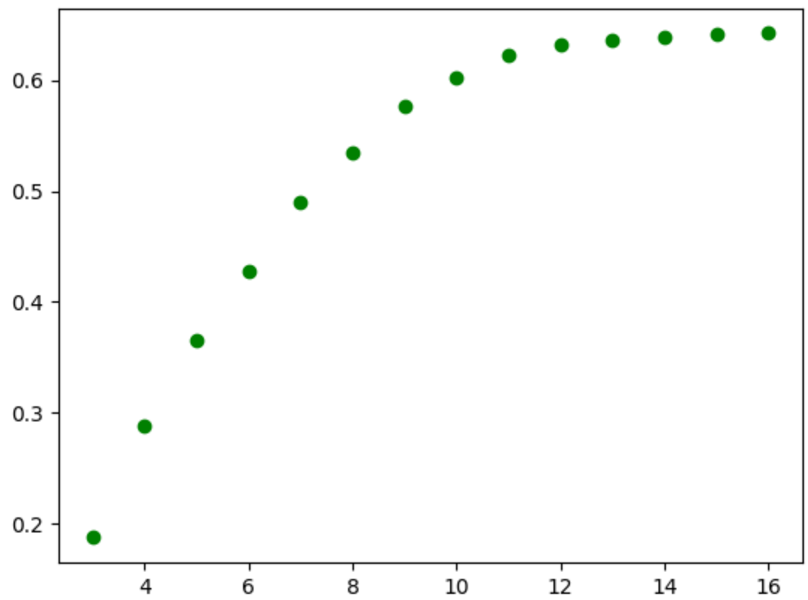

Figure 11: Probability of Finding Submersible Based on Search Time in Hours

hours to reach the search zone, and the odds of finding the submersible do not increase significantly after 11 hours as seen in Figure 11. If the search started two hours later, the chance of success drops to 0.54. This shows that response time is very important when dealing with lost submersibles.

4.2 Other Simulations

Even though the Ionian Sea has some notable characteristics such as its high salinity, our model still functions in other parts of the ocean as it recognizes the topography and current patterns of different places. Given proper datasets, the same method can be used to model the motion of the submersible through any environment. Additional adjustments can be made to the equations for drag and buoyancy to account for differences in environment and submersible specifications. The 60% error can also be adjusted to fine-tune the model to different scenarios.

When multiple submersibles are sent along the same route, no alterations must be made. Although increasing the number of simulations increases the accuracy of results, these results will not vary between different submersibles traveling along the same route. Thus, more runs of our model would not be necessary because it would increase the amount of time needed to gather the same results. Additional simulations would need to be run to account for submersibles that are traveling at different locations or velocities, however, commercial submersibles would likely follow a series of predetermined routes, which allows for search areas and strategies to be calculated for all submersibles along a given route.

5 Limitations

Due to the assumptions made by this model, it is only effective when used for small distances and small time periods. This model considered changes in buoyancy to be negligible, which is true for the Ionian Sea but could be a major oversight if the model were applied to a larger scale. In an environment with more significant differences in salinity and temperature, buoyancy could be a major factor in predicting the collision of a submersible with the ocean floor. This model is therefore not suited to larger environments. Additionally, the model is not accurate when run for long time periods. Due to a lack of data relating to underwater current patterns, this model assumes that the underwater currents are approximately equal to surface currents with a 60% of error. This helps to reflect larger trends in the convection of the ocean while accounting for the fact that underwater currents are fairly unknown. Although this approach is effective for short timescales, this high percentage of error quickly makes the data produced by the model impractically inaccurate. Figure 9 demonstrates how quickly the deviation of predictions increases after 24 hours of simulation of the model. Between 24 and 48 hours, the deviation triples. Due to this weakness, the model should only be applied for short time frames.

6 Potential For Improvement

While our model is effective, there are still things that could be adjusted. Currently, there is only one host ship. Having multiple host ships scattered throughout the sea would increase the safety of the MCMS' operations because when a submersible gets lost, it may be far from the host ship that deployed it. So, another host ship might be closer and thus it could locate the missing submersible faster.

In addition, our model only considers a single search vessel being sent to find a lost submersible. If more search vessels are dispatched, our model's logic suggests that the probability of the submersible being found will increase. However, increasing the amount of search vessels will decrease effectiveness each time this is done since the extra vessels will search improbable locations. It would be beneficial to implement the possibility of sending multiple search vessels to find a single submersible to find the optimal size of a search team as time proceeds.

7 Conclusion

Predicting the possible locations of a lost submersible is a challenging feat. While our model may be able to determine the probable position of a vessel, the actual location can become harder to find as time passes—primarily due to the various currents of the sea. Fortunately, however, a malfunctioning submersible eventually moves at the speed of ocean currents due to a drag force opposing its motion. This helps the pilots of search vessels because it implies that they will eventually be able to find lost watercraft if they search long enough.

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8 Memo

Individuals from all over the world flock to Greece every summer to enjoy its breathtaking landscapes, delicious food, and vibrant nightlife. However, Greece has more to offer than what tourists currently get to experience. The MCMS desires to expand these attractions and allow people to appreciate Greece from a new perspective.

Our company has come up with the idea of exploring shipwrecks in the Ionian Sea using well-equipped submersibles. Each of these watercraft will contain location systems that utilize electrolocation to periodically send their locations to a host ship located on the surface of the sea. This way, if a submersible loses propulsion and communication with the host ship, the host ship will be able to dispatch a search vessel to find and bring back the defunct watercraft.

However it is not just about being able to return the lost submersible. The more pressing matter is attempting to predict the motion of this submersible as it loses communication with the host ship. Three bright mathematicians in our safety committee have devised a method of predicting the possible locations of missing submersibles so we would be able to utilize our safety measures during an emergency. They have coded a model that can analyze current patterns of the sea and use them to predict the location of the submersible over time without any information from the submersible. This model can also create heat maps to represent the most probable locations for the submersible to be at any given time. Consequentially, host ships will have general ideas of where to search for missing submersibles. The model the mathematicians have created considers the variability of the ocean by incorporating the fluctuating velocities of the ocean's currents and the different densities throughout the Ionian Sea. So there can be a multitude of different simulations running at the same time to provide a large search area, of course incorporating the unpredictability of the sea. This model also considers where islands are located and the depth of the ocean at certain coordinates. Thus, if the submersible crashes and stops at a location, our model will be able to predict the most likely places for that crash to have occurred.

To maximize the safety of tourists and the efficacy of our team's primary model, we plan to develop predetermined routes for submersibles to travel throughout the Ionian Sea. This consistency in the tours means that in between the periods where the submersible sends information to the host the general planned path of the submersible is known. This not only makes sure that pilots are comfortable traversing the waters along the predetermined path, but also the model can incorporate sections where acceleration is likely to occur for instance. In summary, the usage of predetermined paths greatly increases the safety of the submersible and its crew as the model is more likely to be able to replicate their movement.

While exploring the depths of the Ionian Sea will be an astonishing experience for tourists and locals alike, our team is also able to expand the model to other seas and tourist destinations as well. A pretty relevant example could be the Aegean Sea, these historic waters I am sure are full of Ancient Greek history at the sea bed. Given that the Aegean will be given the same interest as the Ionian in terms of the desire to explore the sea floor, the model will be ready for any catastrophic event. The Aegean Sea has significantly more islands and variability in topography, nonetheless, our model will be able to effectively predict where a submersible might crash into the underwater mountain ranges of the Aegean. Thus, we will be able to plan many more safe and impressive trips in Greek waters. Furthermore, allowing us to build our company will not only generate life-long memories and engaging stories but will greatly stimulate the Greek economy. Opportunities to explore the ancient history of one of the world's most revered empires through a medium that is not commonly available to the public (i.e. submersible tours) would be an extraordinary draw for tourism. For a country that focuses most on the industries of tourism, this opportunity to provide very safe life-changing adventures should not be dismissed.