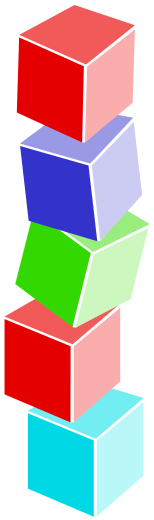
Problem 17: Perfect Cubes[1]

Source filename: `cubes.(cpp|java)`
Input filename: `cubes.in`
Output filename: `cubes.out`



Fermat's Last Theorem stated that for $n > 2$ there exists no integers $a, b, c > 1$ such that $a^n = b^n + c^n$. This theorem was one of the most famous conjectures in all of mathematics; and it remained unproven for over 350 years until Andrew Wiles' dramatic proof was released in October 1994.

It is perhaps surprising then to learn that it *is* possible to find integers $a, b, c, d \geq 1$ that satisfy the "perfect cube" equation: $a^3 = b^3 + c^3 + d^3$. For example a quick calculation will confirm that $6^3 = 3^3 + 4^3 + 5^3$.

Objective

Write a program that finds all sets of integers $\{a,b,c,d\}$ which satisfy the equation $a^3 = b^3 + c^3 + d^3$ where the range of values for a is specified in the input file and $b, c, d \geq 1$.

Input File Specifications (`cubes.in`)

The input file contains one or more test cases. The data for each test case is a single line that contains two positive integers, m and n, separated by a single space, where $0 < m \leq n \leq 200$. The end of the input file is marked by a line that contains 2 zeroes separated by a single space.

Output Specifications (`cubes.out`)

For each test case list ***all*** of the perfect cubes, one per line, in non-decreasing order of a (where $m \leq a \leq n$) using the format: `a = f(b,c,d)` (One could think of the function `f` as defined by $\sqrt[3]{b^3 + c^3 + d^3}$)

There should be exactly one space before and after the equal sign. No other spaces should appear in the output. After each test case, output a line that contains 3 plus signs, '+++', even if there are no perfect cubes for a found in the range from m to n. There should be no blank lines in the output.

The values for b, c, and d should also be listed in non-decreasing order on each line. That is, the output for $a = 6$ should be reported as `6 = f(3,4,5)` instead of `6 = f(4,3,5)` or any of the other 4 permutations of 3, 4 and 5.

There are several values of a that are associated with multiple distinct sets of b, c and d triples. In these cases the triples with the smaller b values should be listed first (Note the ordering in the example below for the lines associated with $a = 18$ & $a = 36$.) If, in these cases, the b values are also equal, list the triples with the smaller c values first.

Example Input File

```
1 20
21 23
24 36
0 0
```

Corresponding Output

```
6 = f(3,4,5)
9 = f(1,6,8)
12 = f(6,8,10)
18 = f(2,12,16)
18 = f(9,12,15)
19 = f(3,10,18)
20 = f(7,14,17)
+++
+++
24 = f(12,16,20)
25 = f(4,17,22)
27 = f(3,18,24)
28 = f(18,19,21)
29 = f(11,15,27)
30 = f(15,20,25)
36 = f(4,24,32)
36 = f(18,24,30)
+++
```

[1] This problem appeared at the CCSC:MidSouth Student Programming Contest April 3, 2009.