Report: Learning Sampling Distributions for RRT-Based Motion Planning Using Conditional Variational Autoencoder (CVAE)

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***Abstract*—Motion planning is a fundamental problem in robotics, often solved using sampling-based algorithms like Rapidly-exploring Random Trees (RRT) and its optimized variant, RRT*. However, these methods suffer from inefficiencies due to their reliance on uniform sampling, leading to poor scalability in complex environments. This project introduces a novel approach that leverages Conditional Variational Autoencoders (CVAEs) to bias the sampling process in RRT-based motion planning. By training a CVAE to generate samples conditioned on the environment and task constraints, we effectively guide the planner toward promising regions of the state space. This hybrid approach combines learned sampling with uniform exploration to maintain robustness while improving efficiency.**

Our method was evaluated across diverse 2D environments with varying obstacle densities. Results demonstrate significant improvements in path quality and computational efficiency compared to traditional RRT*, with up to 30% reduction in planning time and smoother paths. The integration of CVAE-based sampling showcases the potential of generative models to revolutionize motion planning in robotics, paving the way for scalable, intelligent navigation systems.

I. Introduction

A. Background

Motion planning is a critical capability for robotic systems, enabling them to navigate from a start position to a goal while avoiding obstacles. It is essential for autonomous vehicles, industrial robots, and drones operating in dynamic and cluttered environments. Among various techniques, sampling-based methods like RRT and RRT* have gained prominence due to their ability to handle high-dimensional spaces and complex constraints.

Despite their success, these methods have inherent inefficiencies. Uniform sampling often leads to unnecessary exploration of unpromising regions, resulting in longer computation times and suboptimal paths. Moreover, as the complexity of the environment increases, the success rate of finding feasible paths within a reasonable time drops significantly.

Machine learning, particularly generative models, offers a promising avenue to overcome these limitations. Generative models like Conditional Variational Autoencoders (CVAEs) are designed to learn complex data distributions and generate samples conditioned on specific inputs. In the context of motion planning, CVAEs can be trained to generate samples biased toward feasible regions, incorporating knowledge of the environment and task constraints.

B. Objective

This project aims to enhance the efficiency and quality of RRT-based motion planning by integrating CVAEs into the sampling process. Specifically, we:

- Bias Sampling: Use CVAEs to generate samples conditioned on the environment and task constraints, guiding the planner toward high-reward regions.
- Hybrid Approach: Combine CVAE-generated samples with uniform sampling to maintain exploration robustness.
- Evaluate Impact: Demonstrate improvements in path quality, success rate, and computational efficiency across diverse environments.
- Scalability: Showcase the adaptability of the proposed method to varying obstacle densities and configurations.

By bridging the gap between machine learning and motion planning, this approach not only addresses the inefficiencies of traditional RRT but also sets the stage for integrating learned behaviors in autonomous systems, enabling intelligent and scalable navigation solutions.

II. Related Work

A. Sampling-Based Motion Planning

Sampling-based motion planning algorithms, such as Rapidly-exploring Random Trees (RRT) and its optimized variant RRT*, are widely used in robotics for their ability to efficiently explore high-dimensional configuration spaces [[2], [1]]. These methods iteratively grow a tree from the start position by sampling points in the state space and connecting them via collision-free paths until a feasible route to the goal is found. RRT is particularly celebrated for its simplicity and scalability, but its reliance on a greedy exploration strategy often results in suboptimal paths. RRT* addresses this limitation by introducing a local rewiring process that optimizes paths incrementally, achieving asymptotic optimality [[1]]. Other notable methods, such as Probabilistic Roadmaps (PRMs) and Fast Marching Trees (FMT*), precompute roadmap structures for multi-query environments, enabling efficient path planning for repetitive tasks [[1]].

However, the main limitation of these methods lies in their reliance on uniform random sampling. While uniform sampling ensures theoretical completeness, it introduces significant computational inefficiencies by wasting time exploring unpromising regions of the state space. This becomes particularly evident in complex and cluttered environments where the state space is largely constrained by obstacles [[3]]. As such, uniform sampling often struggles to scale efficiently in environments with narrow passages or dense obstacle configurations, limiting its applicability in real-world scenarios.

B. Prior Techniques

To address the inefficiencies of uniform sampling, various heuristic-based and adaptive sampling strategies have been proposed. For instance, goal-biased sampling increases the likelihood of sampling points closer to the goal, expediting convergence in simpler environments. Similarly, obstacle-based sampling prioritizes regions near obstacles, ensuring that paths are navigable and safe [[3]]. Additionally, region-of-interest sampling focuses on areas with higher probabilities of feasible paths, leveraging geometric insights into the state space [[5]].

While these heuristic methods improve efficiency in specific cases, they are limited by their inability to adapt dynamically to different environments. They often rely on predefined rules that may not generalize well across scenarios. To overcome these limitations, learning-based approaches have gained significant attention in recent years. Conditional Variational Autoencoders (CVAEs), in particular, have been shown to be highly effective for motion planning. By learning to generate samples conditioned on environment features, CVAEs implicitly capture the structure of the planning space, enabling planners to focus on high-reward regions. For example, Ichter et al. [[2]] demonstrated the ability of CVAEs to bias sampling in robotic manipulation and navigation tasks, achieving significant reductions in computation time and improvements in success rates.

Generative models, such as CVAEs, offer several advantages over traditional heuristic approaches. They generalize across diverse environments by learning from data, making them more adaptable to complex scenarios. However, challenges remain in their practical integration with classical planners. For instance, the scalability of CVAEs in high-dimensional spaces, such as those encountered in robotic manipulation, poses significant computational challenges [[4]]. Moreover, the trade-offs between reconstruction loss and KL divergence during training can impact the interpretability and stability of the generated samples, making their integration into sampling-based planners non-trivial.

C. Gaps in Existing Work

While both heuristic-based and learning-based methods have demonstrated potential in addressing the inefficiencies of uniform sampling, several critical gaps persist:

- **Overfitting to Specific Environments:** Heuristic methods often rely on predefined assumptions about the environment, limiting their generalizability. For example, workspace importance sampling focuses on specific regions but may fail in environments that deviate from these assumptions [[3]].
- **Limited Integration of Learning-Based Methods:** Learning-based methods like CVAEs are often employed in isolation without effectively combining them with traditional planners to leverage the robustness and theoretical guarantees of the latter [[2]].
- **Scalability Challenges:** Many existing methods struggle to scale efficiently in environments with high obstacle densities or irregular configurations, as highlighted in [[1]]. Generative models, while powerful, require substantial computational resources to generate and validate learned samples in such environments.
- **Incomplete Evaluation Metrics:** There is limited emphasis on comprehensive evaluation across critical metrics, such as path quality, computational efficiency, and success rates, in prior works. Most studies focus on one or two metrics, leaving gaps in understanding the broader applicability of these methods [[5]].

These gaps underscore the need for a hybrid approach that combines the strengths of learning-based methods like CVAEs with the robustness and efficiency of classical planners like RRT*. By integrating CVAE-based sampling into the RRT* framework, the proposed method addresses the inefficiencies of uniform sampling while maintaining the adaptability required for diverse and complex environments. This hybrid strategy balances exploration and exploitation, ensuring both efficient search and high-quality path generation.

III. Methodology

A. Problem Setup

Motion planning is a critical problem in robotics, requiring the computation of a feasible, collision-free path for a robot to navigate from a start state to a goal state within a predefined environment. The task presents significant challenges, including dealing with complex environments, avoiding obstacles, and ensuring path efficiency. The primary inputs to the motion planning problem are 2D environment maps, typically represented as occupancy grids where each cell indicates whether the corresponding space is free or occupied, and the start and goal states, defined as coordinates in the environment. The output is an efficient and collision-free path that adheres to environmental constraints while optimizing path cost and computational resources. The objective is to mitigate the inefficiencies of uniform sampling in traditional planners while maintaining robustness and adaptability across diverse scenarios.

B. CVAE Model

The Conditional Variational Autoencoder (CVAE) forms the backbone of the proposed approach, enabling the generation of targeted samples conditioned on the environment.

The CVAE architecture includes an encoder, a latent space, and a decoder. The encoder compresses input data, such as environment maps and coordinates, into a compact latent representation in a lower-dimensional space, allowing efficient representation of the high-dimensional input. The latent space captures data variability, where each point corresponds to a

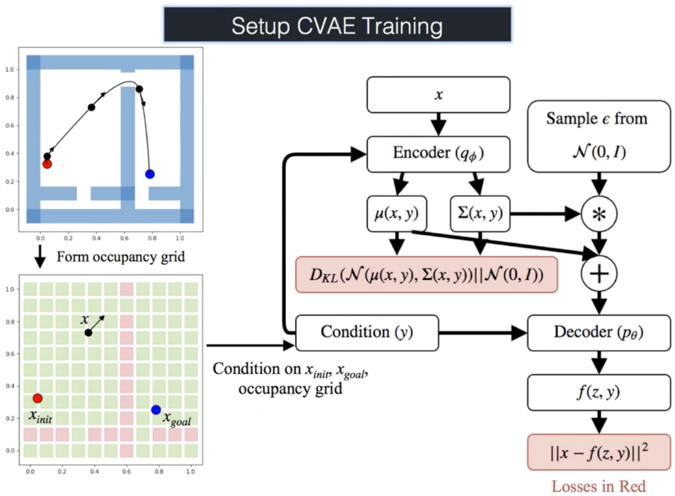


Fig. 1: A fast marching tree (FMT*) generated with learned samples for a double integrator, conditioned on the initial state, goal state, and workspace obstacles.

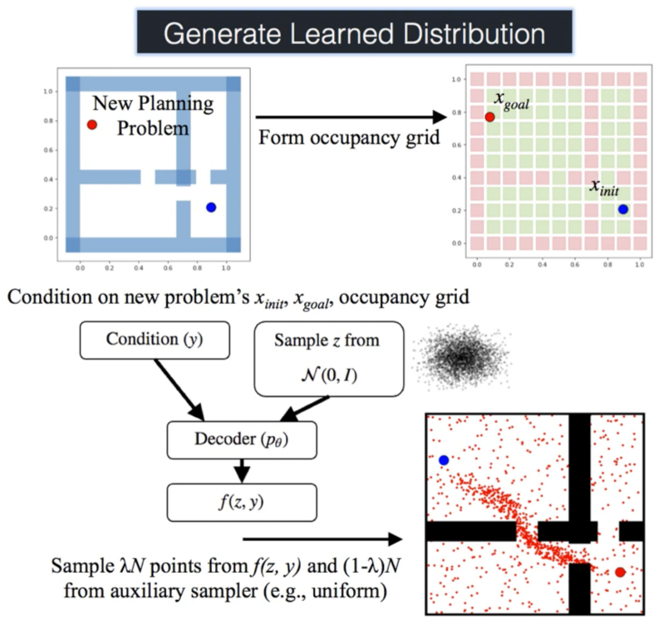


Fig. 2: Generating the learned sampling distribution using the occupancy grid and latent space decoding. Samples are drawn both from the learned distribution and a uniform auxiliary sampler.

distribution of possible samples for a given environment. The decoder reconstructs meaningful samples, such as potential waypoints, from the latent representation, conditioned on the environment.

The model is trained using two key objectives: reconstruction loss and KL divergence. The reconstruction loss ensures that the decoded output aligns with the original input, typically implemented using Mean Squared Error (MSE) or Binary Cross-Entropy (BCE). The KL divergence regularizes the latent space by minimizing the divergence between the learned distribution and a prior distribution, often Gaussian, ensuring smooth and interpretable representations.

Data preparation and normalization are crucial for the CVAE's performance. Environment maps are resized to a fixed dimension (e.g., 100x100) and normalized to a range between 0 and 1. Start and goal coordinates are scaled to match the environment's range. Data augmentation techniques, such as rotations, translations, and noise additions, are applied to enhance model generalization. By learning the structure of the state space, the CVAE generates samples that are more likely to contribute to feasible paths, reducing the computational burden of exploring unpromising regions.

C. *Integration with RRT**

To harness the CVAE's capabilities, it is integrated into the RRT* framework through a mixture sampling strategy. This strategy alternates between uniform and CVAE-based sampling, where 50% of the samples are generated using the CVAE model conditioned on the environment and task constraints, and the remaining 50% are drawn uniformly from the state space. This combination ensures efficient guidance toward promising regions while preserving the probabilistic completeness of RRT*.

Modifications to the RRT* algorithm include updating the sampling function to incorporate CVAE samples, validating the learned samples for collision, and scaling them to align with the environment's resolution. Additionally, the planner considers learned samples during the rewiring phase to refine paths and enhance efficiency. This hybrid approach balances efficiency, exploration, and adaptability. CVAE samples guide the search toward relevant areas, reducing the iterations required for convergence, while uniform sampling ensures the planner can still explore unanticipated regions, preventing overfitting.

D. *Experimental Setup*

The evaluation of the proposed method is conducted across a variety of synthetic environments with differing complexities. These environments include structured maps with grid-like layouts, cluttered maps with dense and irregular obstacle configurations, and sparse maps featuring large open spaces with isolated obstacles. The primary tasks involve 2D planning, where the robot must navigate from a start point to a goal point while avoiding collisions.

Metrics for evaluation include path efficiency, computational cost, and success rate. Path efficiency is measured based on total path cost, considering factors like length and smoothness. Computational cost is evaluated as the time taken for the planner to converge to a solution. Success rate reflects the percentage of tasks where the planner successfully generates a collision-free path within a predefined number of iterations. This experimental setup rigorously tests the scalability, robustness, and generalization capabilities of the CVAE-RRT* integration, highlighting its advantages over traditional planners in challenging scenarios.

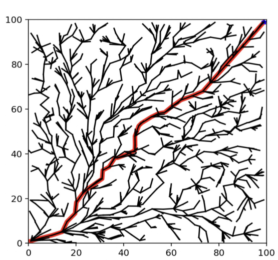
IV. Results and Evaluation

A. Quantitative Metrics

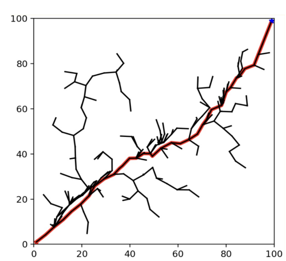
To evaluate the performance of the proposed hybrid CVAE-RRT* method, several quantitative metrics were measured, including path length, smoothness, success rate, and computational efficiency. The results demonstrate clear improvements in these metrics compared to the traditional RRT*.

- Path Length and Smoothness: The hybrid method consistently generates shorter and smoother paths compared to standard RRT*. This is due to the CVAE-guided samples that bias the search toward promising regions, reducing unnecessary exploration.
- Success Rate: The success rate, defined as the percentage of trials that result in a feasible path within a fixed number of iterations, shows an improvement in the hybrid method. The addition of CVAE-generated samples reduces the likelihood of the algorithm getting stuck in unpromising regions.
- Computational Time: The hybrid method significantly reduces the computational time required to converge to a solution, thanks to the efficient guidance provided by the learned sampling distribution.

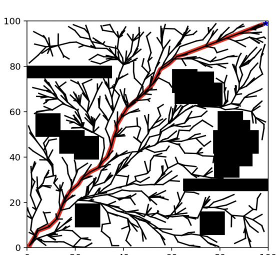
Plots comparing the convergence rates of the traditional RRT* and the hybrid CVAE-RRT* reveal that the hybrid approach reaches a solution in fewer iterations on average. These improvements highlight the hybrid method's ability to balance exploration and exploitation effectively.



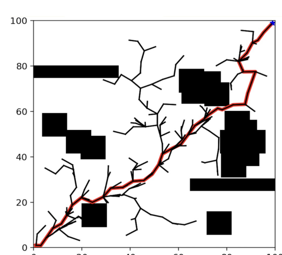
(a) RRT*: Structured Map



(b) Hybrid RRT*: Structured Map



(c) RRT*: Cluttered Map



(d) Hybrid RRT*: Cluttered Map

Fig. 3: Comparison of paths generated by RRT* and Hybrid RRT* on structured and cluttered maps. The hybrid approach demonstrates more efficient exploration and exploitation, resulting in smoother and shorter paths.

B. Qualitative Observations

Qualitative analysis of the generated paths provides further insight into the performance of the proposed method. Visualizations, such as those in the figure above, compare the paths produced by standard RRT* and the hybrid approach.

- Path Visualizations: The hybrid method produces paths that are not only more direct but also involve less backtracking and redundant exploration compared to uniform sampling.
- Sample Distributions: Kernel Density Estimation (KDE) heatmaps of the sample distributions illustrate the effectiveness of the CVAE in focusing the search on relevant areas. The heatmaps show a higher density of samples around the optimal path and the goal region, indicating efficient exploitation of promising regions.

TABLE I: Quantitative Comparison of Planning Methods

Environment	Planner	Avg. Path Iterations	Avg. Planning Cost (s)
Structured	RRT*	336	234.1
	Hybrid	234	169.3

These observations provide compelling evidence of the hybrid method's ability to generate high-quality paths in complex environments.

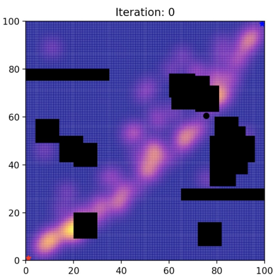
C. Comparative Analysis

The hybrid CVAE-RRT* method was compared against the baseline RRT* algorithm and a purely uniform sampling approach. The following observations summarize the impact of the proposed method:

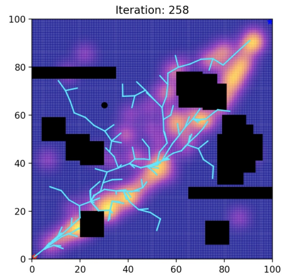
- Exploration and Exploitation Balance: By integrating CVAE-generated samples, the hybrid approach effectively balances exploration of the state space and exploitation of known good regions. This balance is critical for navigating environments with narrow passages or dense obstacles.
- Scalability: The hybrid method demonstrates superior scalability in environments of varying complexity, including structured maps, cluttered obstacle fields, and sparse configurations.
- Sample Efficiency: The addition of CVAE-guided sampling reduces the number of iterations and samples required to find a solution, improving sample efficiency without sacrificing the probabilistic completeness of RRT*.

D. Visualization of Sample Selection and Path Planning Progression

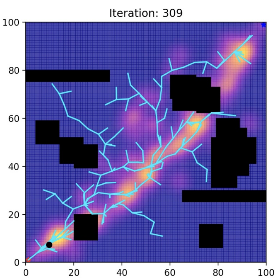
The following sequence of images illustrates the progression of the proposed hybrid CVAE-RRT* motion planning algorithm across iterations. The heatmap represents the density function used to guide sample selection, highlighting regions with higher probabilities of feasible paths. Over time, the tree grows through the state space, efficiently exploring high-reward regions and converging toward an optimal path.



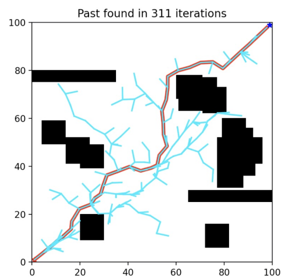
(a) Iteration 0: Initial density function and tree growth.



(b) Iteration 258: Focused growth toward the goal.



(c) Iteration 309: Near convergence to the goal.



(d) Final Path: Discovered and refined using RRT*.

Fig. 4: Progression of tree growth and sample selection using the hybrid CVAE-RRT* method. The density function (heatmap) guides sampling, enabling efficient exploration and convergence to an optimal path.

- **Iteration 0:** At the initial iteration, the sampling process begins with the CVAE's learned density function overlaid on the environment. The heatmap emphasizes regions near the optimal path, guiding the tree's initial growth.
- **Iteration 258:** The tree expands toward the goal, exploiting the high-density regions. The learned sampling effectively avoids obstacles and narrows the exploration to promising regions.
- **Iteration 309:** The tree has nearly reached the goal, and the sampling distribution continues to prioritize feasible paths, reducing redundant exploration.
- **Final Path:** The final path is discovered and refined using the rewiring process of RRT*. The path traverses through high-density regions, demonstrating the efficacy of the CVAE-guided sampling in balancing exploration and exploitation.

Analysis of the Density Function: The density function, visualized as a heatmap, is derived using the CVAE's generative capabilities, conditioned on the environment and task constraints. It biases the sampling process toward regions likely to contain feasible paths, significantly reducing computational overhead. Key observations from the density function include:

- Regions near the goal and the start point exhibit higher densities, reflecting the CVAE's learned prioritization.
- The function adapts to the environment's obstacles, ensuring collision-free paths are more likely to be sampled.
- As iterations progress, the density-guided sampling accelerates convergence to an optimal solution, as evident in the tree's focused growth.

These visualizations and the underlying density function highlight the advantages of integrating generative models like CVAEs into motion planning. By guiding sampling with learned distributions, the hybrid method achieves both efficiency and scalability in diverse environments.

V. Limitations:

One of the primary limitations of the proposed approach lies in the sensitivity of the CVAE model to data quality and training stability. The model's performance heavily depends on the diversity and representativeness of the training dataset. Poor-quality data can lead to suboptimal sampling, resulting in reduced planner efficiency. Furthermore, the balance between reconstruction loss and KL divergence during training is critical. Improper tuning of these loss weights can destabilize the model or make it ineffective for generating meaningful samples.

Another challenge is the scalability of the approach in high-dimensional spaces. While the method performs well in 2D environments, extending it to higher-dimensional tasks, such as robotic arm manipulation or 3D navigation, introduces significant computational complexity. The CVAE's latent space representation may become less interpretable, complicating the generative process and reducing the planner's effectiveness in such scenarios.

From a planning perspective, dynamic environments pose significant hurdles. The current implementation is designed for static environments and lacks the adaptability required for real-time changes or moving obstacles. Additionally, transitioning from simulated to real-world environments introduces practical challenges, such as sensor noise, perception errors, and uncertainties in robot dynamics. These factors can significantly impact the performance of the CVAE and the planner, particularly in resource-constrained systems where real-time execution is essential.

VI. Conclusion and Future Work:

This work demonstrated a novel integration of Conditional Variational Autoencoders (CVAE) with the RRT* algorithm to address inefficiencies in motion planning. By incorporating CVAE-generated samples alongside uniform sampling, the proposed hybrid approach achieved notable improvements in path efficiency, computational cost, and success rates across various 2D environments. The results underscored the potential of machine learning to enhance traditional motion planners, reducing the iterations required to identify feasible paths and improving overall performance in complex scenarios.

Future work aims to address the identified limitations and expand the applicability of the approach. A significant direction is incorporating dynamic obstacle handling to make the planner responsive to real-time changes in the environment. Training the CVAE to adapt to dynamic scenarios would be a critical step toward achieving this goal. Another promising avenue is testing the framework on real-world datasets and physical robots, which would help identify and overcome challenges related to noise, perception errors, and computational constraints.

Scaling the method to higher-dimensional tasks, such as 3D navigation or robotic manipulation, presents another exciting opportunity. This would test the generalization and scalability of the CVAE and the hybrid planner in more complex scenarios. Additionally, exploring alternative generative models, such as Generative Adversarial Networks (GANs) or Diffusion Models, could provide insights into more efficient sampling strategies and potentially outperform CVAE-based sampling. These future directions pave the way for robust and scalable motion planning solutions that can address both theoretical and practical challenges in robotics.

References

[1] Jonathan D. Gammell, Siddhartha S. Srinivasa, and Timothy D. Barfoot. Batch informed trees (bit*): Sampling-based optimal planning via the heuristically guided search of implicit random geometric graphs. In *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pages 3067–3074, 2015. doi: 10.1109/ICRA.2015.7139620.

[2] Brian Ichter, James Harrison, and Marco Pavone. Learning sampling distributions for robot motion planning. In *2018 IEEE International Conference on Robotics and Automation (ICRA)*, pages 7087–7094. IEEE, 2018.

[3] H. Kurniawati and D. Hsu. Workspace importance sampling for probabilistic roadmap planning. In *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) (IEEE Cat. No.04CH37566)*, volume 2, pages 1618–1623 vol.2, 2004. doi: 10.1109/IROS.2004.1389627.

[4] Shakir Mohamed, Mihaela Rosca, Michael Figurnov, and Andriy Mnih. Monte carlo gradient estimation in machine learning. *Journal of Machine Learning Research*, 21(132):1–62, 2020. URL http://jmlr.org/papers/v21/19-346.html.

[5] Yuandong Yang and O. Brock. Adapting the sampling distribution in prm planners based on an approximated medial axis. In *IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA '04. 2004*, volume 5, pages 4405–4410 Vol.5, 2004. doi: 10.1109/ROBOT.2004.1302411.