Lecture 4
DIP
27/08/2021

Manipulating pixels in spatial domain

1. Point- Point [Prev Class]

For power-law, first normalise 0-1 and then do transform later scale it back.

Histogram & Contrast:

$$\underbrace{\frac{I_{max}-I_{min}}{I_{max}+I_{min}}}$$ → One of the ways to describe contrast (has issues as its not robust because high/low values can derail the value)

Obtained from image's histogram

To make it more robust, we can try to avoid extreme % (top, bottom 3% or so) can be removed then contrast can be calculated

Contrast Stretching:

$$f_{ac}(a) = a_{min} + (a-a_{low})\left[\frac{a_{max}-a_{min}}{a_{high}-a_{low}}\right]$$

where Original Range: $[a_{low} - a_{high}]$

New intensity range: $[a_{min} - a_{max}]$

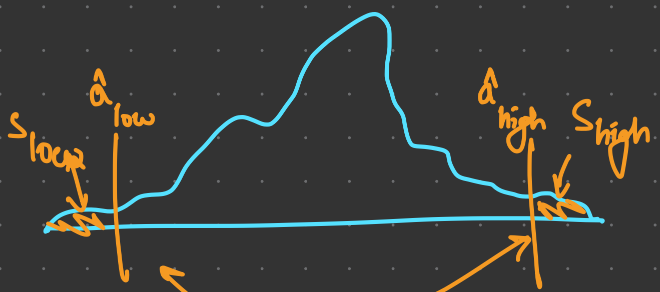
For 8 bit example as input, [0,255]

$$f_{ac}(a) = (a-a_{low})\left(\frac{255}{a_h - a_l}\right)$$

Sample:

⇒ Not robust if there's a single extreme pixel then stretching may not happen properly

Modified Robust Version of Contrast Stretching:



Cropped out then Stretched

$$\hat{a}_{low} = \min\{i \mid H(i) \geq M \cdot N \cdot s_{low}\}$$

$$\hat{a}_{high} = \max\{i \mid H(i) \leq M \cdot N \cdot (1 - s_{high})\}$$

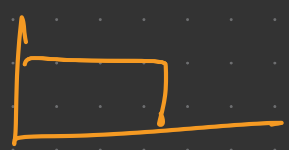
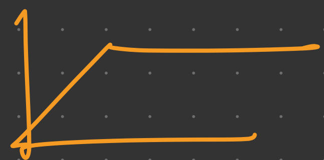
$$\Downarrow$$

$$f_{mac}(a) = \begin{cases} a_{min} & , a < \hat{a}_{low} \\ a_{min} + (a - \hat{a}_{low}) \dfrac{a_{max} - a_{min}}{\hat{a}_{high} - \hat{a}_{low}} & , \hat{a}_{low} < a < \hat{a}_{high} \\ a_{max} & , a > \hat{a}_{high} \end{cases}$$

Histogram Equalisation

Dark $\longrightarrow$ Bright [May work better than contrast stretching]

Issue with contrasting:

PDF  $\Rightarrow$ CDF 

Use CDF of histogram as transform

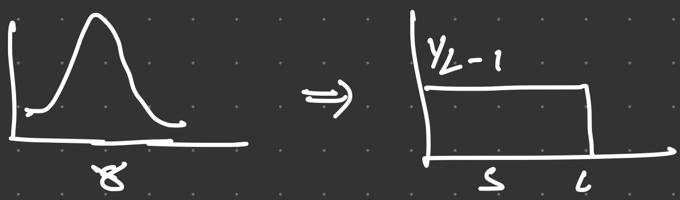
s, r are two random variables

$$s = T(r)$$

↳ Single valued function & monotonically increasing

$$P_s(s) = P_r(r) \cdot \frac{dr}{ds} \quad \text{--- (1)} \qquad 0 \leq T(r) \leq 1 \Rightarrow 0 \leq r \leq 1$$

We want T() to be able to made into uniform distribution.



$$\left(\frac{1}{L-1}\right) ds = P_r(r) \cdot dr$$

Sum up all freq. of intensities till r

$$s = (L-1)\int_0^r P_r(w)\,dw$$ → for each value of s, we get uniform output

If you transform histogram by its CDF, we get histogram equalised version.

$$F_X(x) = \int_{-\infty}^{x} f(t)\,dt \rightarrow \text{CDF} \quad \text{--- (2)}$$

$$f(x) = \frac{d}{dx}\left[F(x)\right]$$

Using both $P_s(s) = 1$

Example:

If $s = 1.33 \Rightarrow$ round to $s = 1$

all r with $s = 1$ we map to 1