Forward Kinematics

$p = f(\theta)$

$\hookrightarrow$ Joint space $\theta \in \mathbb{R}^n$

Task space $p \in \mathbb{R}^m$

$f : R^n \to R^m \to$ Forward kinematics

$f^{-1} : R^m \to R^n \to$ Inverse kinematic

Vector valued non-linear fn

For 2R manipulator,

$JA = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, L.L $= \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ $\quad p = (e_x, e_y, \phi)^T$

$$e_x = l_1 \cos\theta_1 + l_2 \cos\theta_2$$

$$e_y = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$\phi = \theta_1 + \theta_2$$

Workspace:

Set of end-effector config

$$W = \{g(\theta) \mid \theta \in Q\} \subset SE(3)$$

Tough to interpret

$$g : Q \to SE(3)$$

config space

Reachable workspace:

$$W_R = \{p(\theta) \mid \theta \in Q\} \subset \mathbb{R}^3$$

Non-linear mapping b/w config & workspace

→ Straight lines in C-space maps to curves in workspace

2R workspace:

→ Torus $[l_1 - l_2, l_1 + l_2]$

3R Workspace:

DH Parameters

Attach ref frames to each link of open chain and then derive forward kinematics from knowledge of relative displacement

thumb - x
middle - z

2 link parameters: Link length, Link twist

2 joint parameters: Joint Angle, Link offset

Steps: 1) $\hat{z}_i$ coincides with joint axis $\{i\}$ and $\hat{z}_{i-1}$ with $\{i-1\}$
RH determines direction of rotation

2) Find a line-segment a_{i-1} that mutually intersects $\hat{z}_i$ & $\hat{z}_{i-1}$

3) $\hat{x}_{i-1}$ is chosen to be in the direction of mutually $\perp^{\text{lar}}$ pointing from $\hat{z}_{i-1}$ & $\hat{z}_i$

Link length $[a_{i-1}]$: Length of mutually $\perp^{\text{lar}}$ b/w $\hat{z}_{i-1}$ and $\hat{z}_i$

Link Twist $[\alpha_{i-1}]$: Angle from $\hat{z}_{i-1}$ to $\hat{z}_i$ measured about $\hat{x}_{i-1}$

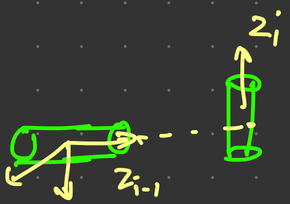
Link Offset $[d_i]$: Distance from intersection of $\hat{x}_{i-1}$ to $\hat{x}_i$ about $\hat{z}_i$

Joint Angle $[\theta_i]$: Angle from $\hat{x}_{i-1}$ to $\hat{x}_i$ measured about $\hat{z}_i$

$$^{i-1}_{\ \ i}[T] = R(\hat{x}_{i-1}, \alpha_{i-1})\, T(\hat{x}_{i-1}, a_{i-1})\, T(\hat{z}_i, d_i)\, R(z_i, \theta_i)$$

Special Cases:

1 When $\hat{z}_i \perp \hat{z}_{i-1}$, line segment fails to exist



$a_{i-1} = 0$
$\alpha_{i-1} = \pi/2$

2 When $\hat{z}_{i-1} \parallel \hat{z}_i$, orthogonal line segment won't be unique