Dynamics

Linear Momentum:

① $p = mv = m\dot{r} = m\frac{dr}{dt}$ → Single particles

② $p = p_1 + p_2 = m_1 v_1 + m_2 v_2$ → Two particles

③ $p = \sum m_i v_i$ — n-particles

Newton's Law:

$$F = ma$$

$$F = m\frac{dv}{dt} = \frac{dp}{dt}$$

$$= m\ddot{r} + \dot{m}\cancel{\dot{r}}$$

0

$$F = \dot{p}$$

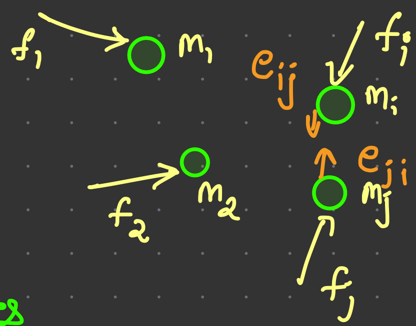
$$r_{COM} = \sum_i \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{m_1 + m_2 + m_3} = \sum \frac{m_i r_i}{M}$$

$$\boxed{p = M\dot{r}_{COM}}$$

Dynamics of System of Particles

Rigid body's external force

$$\dot{p} = \sum_i m_i \frac{d^2 r_i}{dt^2} = \sum_i \left(f_i + \sum e_{ij}\right)$$

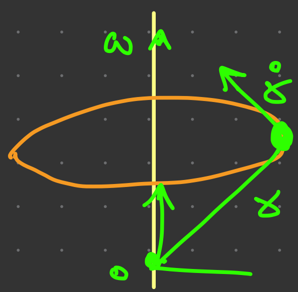


$$\sum e_{ij} = 0 \implies \dot{p} = \sum f_i$$

Rate of change of L.M = Sum of external forces

$$\dot{p} = Ma_{COM} = \sum_i f_i$$

Angular Momentum



→ Particle at r, rotating about O.

→ Ang momentum.

$$H_o = r \times p = r \times m\dot{r}$$

Inertia Tensor

Total momentum:

$$H_o = \sum r_i \times p_i = \sum r_i \times m\dot{r}_i$$

$$= \sum r_i \times m_i(\omega \times r_i)$$

$$= \sum m_i r_i \times (-r_i \times \omega)$$

$$= \sum -m_i [\hat{r}_i][\hat{r}_i]\omega$$

$a \times b = [a]b$

$$H = \sum -m_i \hat{r}_i \hat{r}_i \omega$$

For continous distribution

$$m = \rho dv$$

$$H = \left(\int_V -\rho \hat{r}_i \hat{r}_i \, dv \right) d\omega \implies H = I\omega$$

If object is massive, more inertia. $\implies$ If more massive object than m has more inertia
$\Downarrow$
greater its resistance to change velocity

→ Mass conc. near axis, less inertial
Mass conc. away axis, more inertia