Robotics Dynamics & Control
Quiz - 1
Quick Note

Robot
- Perception ← Proprioceptive [Internal states] / Exteroceptive [Vision, LIDAR]
- Cognition
- Action

Laws of Robotics:
1) Ro. may not injure a human being or harm them
2) Must obey orders given by human being unless it conflicts with 1st law
3) Must protect its own existence unless conflict

Develop Robot
- Fabrication
- Measurement
- Calibration
- Control

Differential Kinematics:
Relationship b/w joint motion & end-effector motion in terms of velocities

Manipulators
- Serial
- Parallel
- Hybrid

Industrial Robots → Mechanical Barrier, Safety Light Curtains

Cobots: → Compact, light weight

Mobile Robots
- Locomotion
- Terrain
  - Aerial
  - Underwater

Manipulators
- Serial
- Parallel
- Hybrid

Passive Stability:

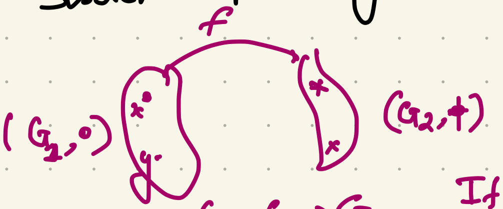
Active Stability:

Groups:
Finite/∞ set of elements together with binary option

Properties:
- Closure
- Associativity
- Identity
- Inverse

Check exercise in slides for example

Homomorphism:
→ f b/w groups G_1, G_2 that identifies similarities b/w them
→ Structure preserving like (two rings, two grps, two vector spaces)



$f: G_1 \to G_2$
$x \to f(x)$
$y \to f(y)$
$z \to f(z)$

If $x \circ y = z$
$f(x) + f(y) = f(z)$
$= f(x \circ y)$

Rigid Body:
Distance b/w two particles remain fixed
$\|p - q\| = \text{Constant}$

Rigid Displacements:
- $\|p(t) - q(t)\| = \text{Constant}$
- Single mapping $g: O \to \mathbb{R}^3$

→ for n unconstrained, $2n$ coordinates

Config Space:
Implicit param.:
n-dim space embedded in Euc. space of $> n$ subject to constraints

$S^1 \times S^1 \times S^1 = SO(3)$

Vector in $\mathbb{R}^n$
$v = q - p$
$|v| = \sqrt{v_1^2 + v_2^2 \dots v_n^2}$

DOF = (Σ freedom of joints) − (No. of ind. constraints)

→ Prone to singularity

Explicit:
→ for n-dof, n no. of coordinates
↳ Original c-space

Joint Space:
Complete spec. of positions of every point of robot

Implicit:
→ for n-dof, m no. of coordinates where $m \geq n$ with m−n constraint
→ Work with constraints (true C-space)
→ Generalized coordinates
↳ singularity

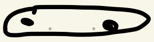
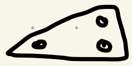
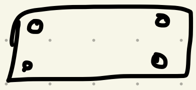
Task Space:
Config space of robot's end-effector

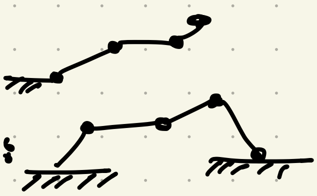
Rigid Motion:
$2n \to 4 \Rightarrow \mathbb{R}^2$
$3n \to 4 \Rightarrow \mathbb{R}^n$

Topology:
Can be transformed without stitching / cutting

Config Space
→ Space of all possible placements of robot
→ Continuous space (often non-linear)

Config. of system
→ Location of every point in system
→ System: Set of point
→ Dof: No. of real numbers to specify config

Links:
↳ Binary link: 2 nodes 
→ Ternary link: 3 nodes 
→ Quaternary link: 4 nodes 

Chains:
↳ Open chain
↳ Closed chain:

l_i is attached to l_{i+1} by joint j_{i+1}

Joints:

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

Mechanism:
→ Planar: → Parallel to certain plane
→ axes of hinges are $\perp^{lar}$ to base plane
→ Spherical → Axes of all joints intersect at point (Gyro)
→ Spatial → All joint axes are skew to each other

Dof = 0 ⇒ Statically determinant structure
" Indeterminant structure
(Last few slides examples)

DOF ≤ 0 ⇒ Zero mobility / structure

Grubler's Equation:
$$dof = K(N-1) - \sum_{i=1}^{j} (K - f_i)$$
$$= K(N-1-j) + \sum_{i=1}^{j} f_i$$
K = 3 (or) 6
N = No. of links
j = No. of joints
f_i = freedom of j^{th} joint
(compound mech)

Overconstrained mechanism:

Synthesis vs Analysis

Topology:
Shape of config space & its representation
$A \times B \neq B \times A$ (not comm)

Joint space & Task space: (Lec 4)

↓ C-space of a robot (n) dim(JS)

↘ C-space of end-effector (m) dim(TS)

If $n > m$

↘ Redundant robot

Robot examples:

- ↳ Cartesian Robot
- ↳ Cylindrical Robot
- ↳ Polar Robot
- ↳ SCARA
- ↳ Articulated ARM

Manipulators:

Serial
- ↳ Large W.S
- ↳ Errors accumulate
- ↳ Chained effort
- ↳ Less payload capacity
- ↳ Motors at each joint

||el
- ↳ Limited W.S
- ↳ Errors average
- ↳ Distributed effort (high rigidity)
- ↳ High payload capacity
- ↳ Motors at base

TRANSFORMATIONS

Rigid Body Transformations:

Mapping $g: \mathbb{R}^3 \to \mathbb{R}^3$ is RBT if:

1) Length is preserved

$$\|p-q\| = \|g(p)-g(q)\|$$

2) Cross product is preserved:

$$g \cdot (v \times w) = g(v) \times g(w)$$

} Need to be rotational or translational

⇒ Implications:

1) Inner product is preserved

$$g_*(v \cdot w) = g_*(v) \cdot g_*(w)$$

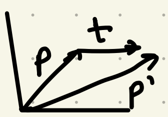
2) Angles b/w vectors are preserved

3) RH frame transformed into RH frame

Translation:

$$p' = p + t$$

↳ Translation vector



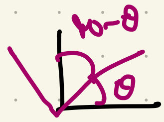
Points & Coordinate Frames:

$${}^{A}p = {}^{A}x\,\hat{x}_A + {}^{A}y\,\hat{y}_A$$

$$\hat{x}_A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \hat{y}_A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Orientation:

Coordinate Transformation



Axes: $\hat{x}_A, \hat{y}_A$; $\hat{x}_B, \hat{y}_B$

$${}^{A}p = \underbrace{({}^{B}x\cos\theta - {}^{B}y\sin\theta)}_{{}^{A}x}\hat{x}_A + \underbrace{({}^{B}x\sin\theta + {}^{B}y\cos\theta)}_{{}^{A}y}\hat{y}_A$$

Coordinates of any point in B (^Bp) can be written in A frame as

$$\begin{bmatrix} A_x \\ A_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix}$$

$$^A p = {}^A_B[R]\,{}^B p$$

↳ Rotation matrix

R-Matrix Properties:

① $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ with columns q_1, q_2

$\|q_1\| = \|q_2\| = 1$

$q_1^T q_2 = 0$

$R^T R = I = R R^T$

IMP Read About

② $GL(n) \Rightarrow$ set of all non-singular matrix

General linear gp. $\rightarrow GL(2) \supset O(2) \supset SO(2)$

$SO(2)$

↳ special orthogonal

Determinant (+1)

Orthogonal matrix having determinant +1, −1

Reflect vector so not preferred

SO(2) Properties:

↳ Lie group - set under multiplication

→ 1) Closure : $\rightarrow R_1 \in SO(2),\ R_2 \in SO(2) \Rightarrow R_1R_2 \in R \in SO(2)$

2) Associative $\rightarrow R_1, R_2, R_3 \in SO(2) \Rightarrow R_1(R_2 \cdot R_3) = (R_1 \cdot R_2) \cdot R_3$

3) Identity $\rightarrow R \in SO(2) \Rightarrow R \cdot I = I \cdot R = R$

4) Inverse $\rightarrow R \in SO(2),\ R^{-1} \in SO(2)$ $\quad RR^{-1} = I = R^{-1}R$

$\Rightarrow R^T = R^{-1}$

→ Smooth manifold

$$SO(2) = \{R \in GL(2) \mid RR^T = I,\ \det(R) = 1\}$$

$GL(2)$ or $\mathbb{R}^{2\times 2}$ → Only for planar rotation

→ Forms an abelian group (commutative property)

$R_1 R_2 = R_2 R_1$

Only for SO(2) but not higher dimension → Verified using $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$

SO(2) with complex numbers:

$z = x + iy$

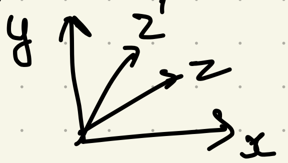
$z' = e^{i\theta} z$

$z' = (c\theta + i s\theta)(x + iy)$

$= (x\cos\theta - y\sin\theta) + i(x\sin\theta + y\cos\theta)$

$z_1 z_2 = z_2 z_1$

$e^{i\theta} = \cos\theta + i\sin\theta = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix}$



Length invariance:

$z' = Rz$

$z'^T z' = (Rz)^T (Rz)$

$= z^T R^T R z$

$= z^T I z$

To bring same place

$R(-\theta) = \begin{bmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{bmatrix} = R^T = R^{-1}$

Inverse problem

Finding $\theta \in S^1$ from $z \in \mathbb{C}$ and from $R \in \mathbb{R}^{4\times 4}$

$\theta = \tan^{-1}(y, x)$ (or) $\arctan(r_{21}, r_{11})$ (or) $\arctan(-r_{12}, r_{11})$

$R_1 R_2 = R_2 R_1$

Lecture 6

$n = \dim(JS)$

$m = \dim(TS)$

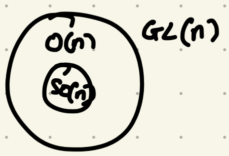
$n > m \Rightarrow$ Redundant system

$$SO(2) = \{R \in GL(2) \mid R^T R = I, \det(R) = 1\}$$

$$|q_1| = |q_2| = 1$$

↳ Smooth manifold

↳ Locally looks like $\mathbb{F}$



3D Rotation

$$^A_B[R] = R(y_A, \theta) = \begin{array}{c} \begin{array}{ccc} \hat{x}_B & \hat{y}_B & \hat{z}_B \end{array} \\ \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \end{array}$$

Cosine Transformation

3D Rotation Properties:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$SO(3) = \{R \in \mathbb{R}^{3\times 3} \mid R^T R = I, \det = 1\}$$

If $R_1, R_2 \in SO(3) \Rightarrow R_1 R_2 \neq R_2 R_1$,

$R^T R = I$

Composition of Rotations:

For vector ${}^1p$ in frame {1} transformed under ${}^1A$ into ${}^1q$ in same frame

$$ {}^1p = {}^1A\,{}^1q \quad (1) $$

$$ {}^0p = {}^0_1[R]\,{}^1p \quad (2) \qquad {}^0q = {}^0_1[R]\,{}^1q \quad (3) $$

$$ {}^0_1[R]^T\,{}^0p = {}^1A\,{}^0_1[R]^T\,{}^0q $$

Multiply by ${}^0_1[R]$,

$$ {}^0p = \underbrace{{}^0_1[R]\,{}^1A\,{}^0_1[R]^T}_{R\,{}^1A\,R^T}\,{}^0q $$

$$ {}^0({}^1A) = {}^0_1[R]\,{}^1A\,{}^0_1[R]^T $$

If ${}^1A = {}^1_2[R]$

$$ {}^0({}^1_2[R]) = {}^0_1[R]\,{}^1_2[R]\,{}^0_1[R]^T $$

$$ \Rightarrow {}^a({}^b_c[R]) = {}^a_b[R]\,{}^b_c[R]\,({}^a_b[R])^T $$