16-11-21

3 major topics

1) Inverse Dynamics based control / computed torque method
2) Lyapunov Stablity
3) Controller Design via Lyapunov

Euler Lagrange System

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(q) + G(q) = \tau$$

q = Generalised coordinates / Position
$\dot{q}$ = Velocity
$\ddot{q}$ = Acceleration

$q \in \mathbb{R}^{n \times m}$

M = Mass matrix ($M \in \mathbb{R}^{n \times n}$)
C = Coriolis & Centri ($C \in \mathbb{R}^{n \times n}$)
F = Damping forces ($F \in \mathbb{R}^{n \times 1}$)
(or) Friction forces
G = gravity ($G \in \mathbb{R}^{n}$)
$\tau \in \mathbb{R}^{n}$

Properties:

$(\dot{M} - 2C)$ is a skew symmetric matrix

$M \rightarrow$ positive definite matrix [can be singular]

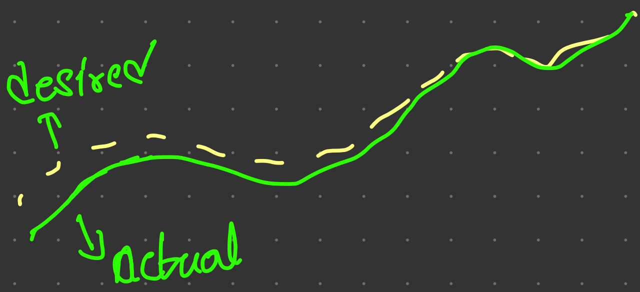
$C(q,\dot{q}) \rightarrow \bar{C}(q,\dot{q})\dot{q}$

↳ Tutorial: Adaptive control of robots tutorial 'Spong'

Robot: Dynamical & Control [Vidhyasagar]

No unique way to find the matrix [Coriolis]

All are like a tracking problem



$$\begin{matrix} q^d & \dot{q}^d & \ddot{q}^d \\ \uparrow & \uparrow & \uparrow \\ q & \dot{q} & \ddot{q} \end{matrix}$$

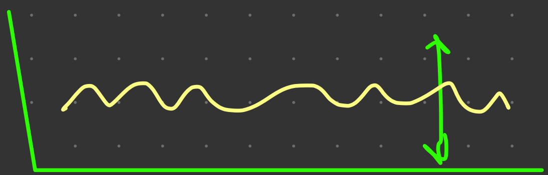
→ Tracking
↳ Regulation $[q^d = 0]$ → Not tracking but rather attains like a equilibrium

Higher order → Smoother trajectory

Tracking error:

$e = q - q^d$ → should tend to 0 as soon as possible

Error should be bounded



Error remains in a finite bound. Bound should be as small as possible depending on application

More accuracy ∝ More control input

$$M\ddot{q} + \underbrace{C\dot{q} + F + G}_{H} = \tau$$

$q \to q^d$

$$M\ddot{q} + H = \tau \quad \Big\} \text{ Design } \tau$$

There is no explicit dependence on time

↳ Autonomous System

If $H(q, \dot{q}, t) \rightarrow$ Non-autonomous system

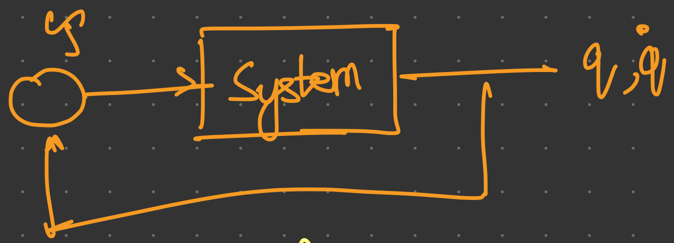
$$\tau = M\ddot{q} + H$$

As per second law of motion,

$$M\ddot{q} = \tau$$

$$e = q - q^d$$
$$\dot{e} = \dot{q} - \dot{q}^d$$
$$\ddot{e} = \ddot{q} - \ddot{q}^d$$

$$\tau = M(\quad) + H$$



Idea is to give acceleration

$$\tau = M\ddot{q}^d + H$$

$$M\ddot{q} + H = \tau = M\ddot{q}^d + H$$
$$\Rightarrow \ddot{q} = \ddot{q}^d \Rightarrow e = 0$$

↳ No error is in this design

$e = 0 \longrightarrow e(0) \neq 0$

$\dot{e}(0) \neq 0$

Error Dynamics: $\ddot{x} + a\dot{x} + bx = 0$

↳ Gives finite value when $x(0) \neq 0$, $\dot{x}(0) \neq 0$

$$s^2 + as + b = 0$$

$$s_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

Graphs based on type of roots

$$\ddot{e} + k_1 \dot{e} + k_2 e = 0$$

$k_1, k_2 > 0$ [Matrices that are +ve definite]

Guaranteed to go down exponentially / oscillate

$$M\ddot{q} + H = \tau \quad (1)$$

$$q \to q^d$$

option 1:

$$D\tau = M\ddot{q}^d + H \quad (2)$$

$$e = q - q^d$$

(2) in (1),

$$\ddot{e} = 0$$
$$\dot{e} = c$$
$$e = ct + c_0$$

$$\ddot{e} + k_1 \dot{e} + k_2 e = 0$$

$k_1, k_2 > 0 \in \mathbb{R}^{n \times n}$ Positive definite matrices

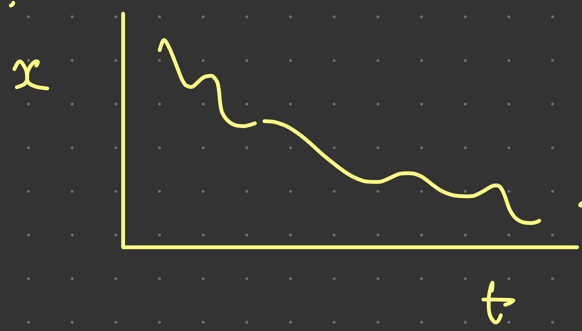
Solve like 2nd ODE

$$\ddot{x} + a\dot{x} + bx = 0$$

$a, b > 0$ scalars

Underdamped

$x = ?$

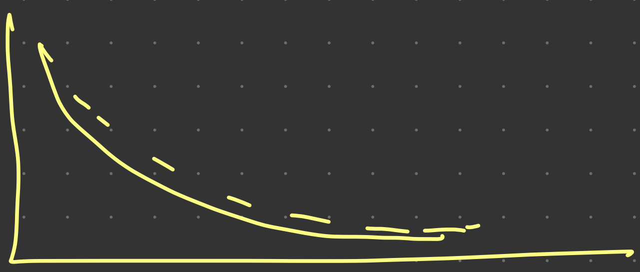


How fast it dampens

$$x = e^{-\alpha t}\left[\cos(\quad) + \sin(\quad)\right]$$

Real & Same values → Frequency gives freq of oscillat

Overdamped



$$x = e^{-\alpha t}$$

Real & Diff value

$x \to 0$ as $t \to \infty$ stabilises

$\Downarrow$

$e \to 0$ as $t \to \infty$

$q = q^d$ ~~as~~ $t \to \infty$

$$M\ddot{q} + H = \tau \qquad \ddot{e} = 0,\ \ddot{q} = \ddot{q}^d$$

$$\tau = Mu + H \qquad \tau = M\ddot{q} + H$$

$\ddot{q} = u$

$$\ddot{e} + k_1\dot{e} + k_2 e = 0$$

$$\ddot{e} = -k_1\dot{e} - k_2 e$$

$$\ddot{q} = \ddot{q}^d - k_1\dot{e} - k_2 e$$

If

$$y = q_1^2 + q_2^2 - - - - + q_n^2$$

$$y^d = q_1^{d2} + q_2^{d2} - - - + q_n^{d2}$$

$$e = y - y^d$$

Inp/Out linearisation tries to track function as much as possible

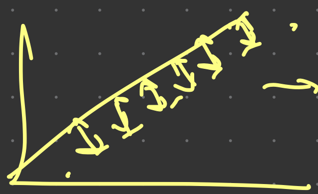
$\boxed{\text{Stotin \& Ree}}$

$\ddot{e} + k_1\dot{e} + k_2 e = 0$ $\qquad e \to 0 \Rightarrow t \to \infty$

$\qquad y \to y^d$

Depending on how much steps we consider for, it will not be exact [there will always be a non-zero value unless fully $t \to \infty$

$$q_1 \Rightarrow q_2 \to q_1^d - q_2^d$$

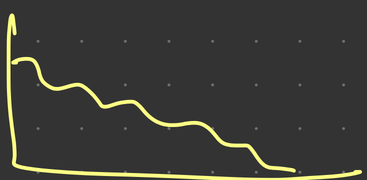


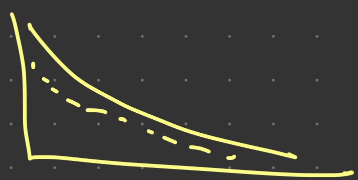
→ Bounded difference [Error will be there]

$$\tau = M[\ddot{q}^d - k_1\dot{e} - k_2 e] + H$$

$$\ddot{e} + k_1\dot{e} + k_2 e = 0$$

$\boxed{\text{Revise 21:00}}$





$e_{1,2} = -k$

Underdamped:

Critically damped: → Robotic (precision imp) → (Accuracy imp)

Overdamped: Not used much

$$M\ddot{q} + H = \tau \quad \Big| \quad \begin{aligned} \tau &= Mu + H \\ u &= \ddot{q}^d - k_1\dot{e} - k_2 e \end{aligned}$$

Known

$$M = \hat{M} + \Delta M \quad \text{unknown}$$

$$H = \hat{H} + \underbrace{\Delta H}_{\text{Variation}}$$

$$M\ddot{q} + H = \hat{M}u + \hat{H} \leftarrow \text{on substitute}$$

$$M\ddot{q} = \hat{M}u + (\hat{H} - H) = \hat{M}u - \Delta H$$

$$\underline{\ddot{q} = M^{-1}\hat{M}u - \Delta H}$$

$$\ddot{q} = M^{-1}\hat{M}[\ddot{q} - k_1\dot{e} - k_2 e] - \Delta H$$

$$\ddot{q} = M^{-1}\hat{M}u + u - u - \Delta H$$

$$\ddot{q} = u + (M^{-1}\hat{M} - I)u - \Delta H$$

$$\ddot{q} = (\ddot{q}^d - k_1\dot{e} - k_2 e) + (M^{-1}\hat{M} - I)u - \Delta H$$

$$\ddot{e} + k_1\dot{e} + k_2 e = (M^{-1}\hat{M} - I)u - \Delta H \rightarrow \text{Models all uncertainty}$$

→ CI + PI

complementary function

[Nominal design]

$$\ddot{e} + k_1\dot{e} + k_2 e = 0$$

Everything is non-linear, take small part and consider as linear

So, to avoid this $\boxed{\text{Lyapunov}}$ stability

System is stable when it dissipates energy

$\ddot{x} = f(x)$ Equilibrium points: Invariant point [Dead situation]

$f(x)|_{x^*} = 0 \rightarrow$ Can be stable/unequilib

Stable equilibrium: Comes back to equilibrium point after disturbance

Unstable equilibrium: Comes to equilibrium point after disturbance

converging asymptotically stable & if it always $\dot{V}(x)<0$

How to check stability/unstability?

Lyapunov function candidate

$V(x)>0$ ⇐ continuously differentiable (C_1)

C_0 → differentiable but not continuous

Losing energy means diff ≤ 0

Key

↳ Cannot guarantee it'll go to eq. 0. But when

If $\dot{V}(x)\leq 0$ [Negative semi-definite] ⟶ STABLE

If $\dot{V}(x)>0$

Note:
Resemble KE

Revise 1:02

Learn asymptotically vs exponential decrease

↳ Guarantees minimum decay unlike asymptotically