ICP

Iterative Closest Point Algorithm

Tries to minimise errors in 3D

$$\underset{R \in SO(3),\, t \in \mathbb{R}^3}{\operatorname{argmin}} \|(RP+t) - Q\|^2$$

General algorithm considers closest points in both PCD's are correspondences following which it finds [R(t)] that best aligns 2 PC by minimising euclidean distance b/w corresponding points

B/w to PC P_i & Q_i

$$F(t) = \sum_{i=1}^{n} \|(RP_i + t) - Q_i\|^2$$

Even if precise, noise may contribute to get very close

Step 1: Assume R is known, solve for t

$$\frac{\partial F}{\partial t} = 2 \sum (RP_i + t) - Q_i = 0$$

$$t = \underbrace{\frac{1}{n} \sum_{i=1}^{n} Q_i}_{\text{Average location of this point cloud}} - R \underbrace{\frac{1}{n} \sum_{i=1}^{n} P_i}_{\text{Average location of P's pcd}}$$

$$t = \bar{Q} - R\bar{P}$$

$$\hookrightarrow R = \underset{R \in SO(3)}{\operatorname{argmin}} \|(R(P_i - \bar{P}) - (Q_i - \bar{Q})\|^2$$ → due to back substitut

For ease,

Take $X = P_i - \bar{P}$ $\quad Y = Q_i - \bar{Q}$

$X' = RX$

So,
$$\sum_{i=1}^{n} \|x_i' - y_i\|^2 = Tr\left((X'-Y)^T (X'-Y)\right)$$
$$= Tr(X'^T X') + Tr(Y^T Y) - 2Tr(Y^T X')$$
$$= \underbrace{\sum_{i=1}^{n} \left(|x_i|^2 + |y_i|^2\right)}_{\text{Doesn't get involved why?}} - 2\,Tr(Y^T X')$$

IMP (only matters due to R) imp

So,
$$\boxed{R = \operatorname{argmax} Tr(Y^T X')}$$

$$Tr(Y^T X') = Tr(Y^T R X) = Tr(\underset{(3\times n)}{X}\,\underset{(n\times 3)}{Y^T}\,\underset{(3\times 3)}{R})$$

Using SVD $\quad XY^T = UDV^T$

$$Tr(XY^T R) = \underbrace{Tr(UDV^T R) = Tr(DV^T R U)}_{\text{Property of trace}} = \sum_{i=1}^{3} d_i v_i^T R u_i$$

$$D = \begin{bmatrix} d_1 & & 0 \\ 0 & d_2 & \\ & & d_3 \end{bmatrix}$$

If $M = V^T R U$

Orthonormal matrix with $det(M) = +1/-1$

Length of each column vector is equal to one and each component of vector is less than 1

$$Tr(V^T X) = \sum_{i=1}^{3} d_i M_{ii} \le \sum_{i=1}^{3} d_i$$

$\hookrightarrow \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$ as constraint

$$\Rightarrow \boxed{R = VU^T}$$

Orientation Rectification? 4O:

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ICP doesn't give exact transform of motion

If initially $\vec{x}_{0j}$, $\hat{\vec{x}}_{1j}$, $\hat{\vec{x}}_{2j}$ ..

↓

Predicted/Estimated after registering in frame ⓪

Let $\hat{x}_{0j} = T_{0i} x^{i}_{j}$

↳ point x_j seen in i^{th} frame

$\hat{x}_{0j}$ → ex where we expect to see in {0} through ICP

↳ We will not exactly be apply to get $\hat{x}_{0j} \neq x_{0j}$