UAV: Dynamics & Control

Lecture 1

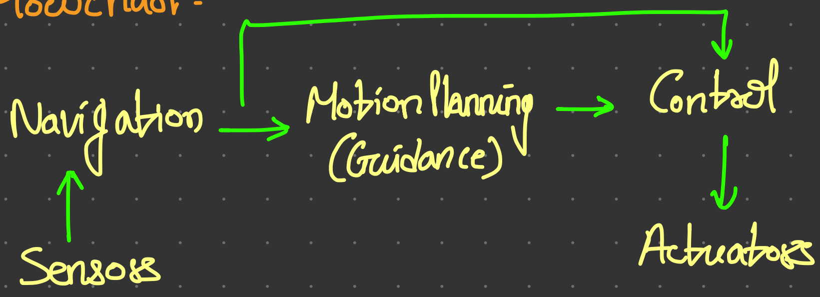
Types of UAV:

1. Fixed wing UAV
2. Rotary wing UAV
3. Hybrid UAV
4. Flapping wing UAV

Applications:

1. Civilial Applications
2. Defence Applications

Flowchart:



1. Navigation
2. Guidance
3. Control

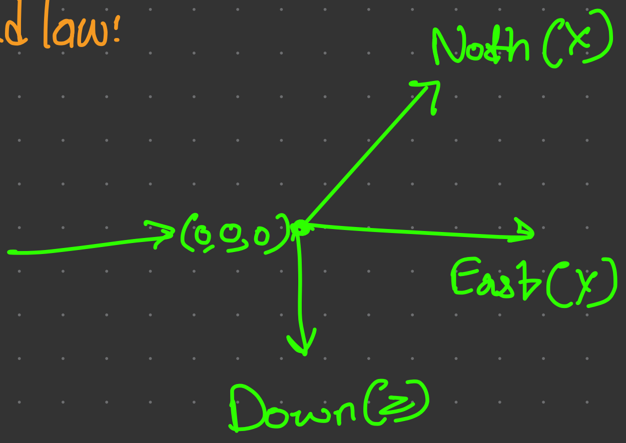
Control System:

→ Objective: To achieve desired velocity, acc., ang. pos, des. altitude set by guidance by changing actuator inputs

UAV mathematical model based on second law:

→ Frame of references:

- Inertial frame of reference
- Body frame
- Intermediate vehicle frame



Also called NED frame

Variables:

Linear Motion

1. Position: (x, y, z)
2. Velocity: $(\dot{x}, \dot{y}, \dot{z}) = (v_x, v_y, v_z)$
3. Acceleration: $(\ddot{x}, \ddot{y}, \ddot{z}) = (a_x, a_y, a_z)$

Newton's Law:

when $\omega_x = \omega_y = \omega_z = 0$

$m\dot{v}_x = ma_x = F_{xb}$

$m\dot{v}_y = ma_y = F_{yb}$

$m\dot{v}_z = ma_z = F_{zb}$

Rotational Motion:

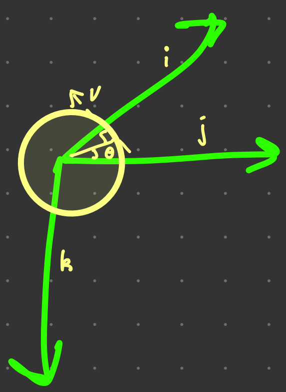
Ang. Vel: $\omega = \frac{d\theta}{dt} = \dot{\theta}$

$v = r \times \omega$, $a = r \times v$

$\vec{r} = x\hat{i} + y\hat{j}$

$v = v_x\hat{i} + v_y\hat{j}$

$\omega = -|\omega|\hat{k}$



So, $\left.\begin{aligned} v_x &= -y|\omega| \\ v_y &= x|\omega| \end{aligned}\right\} v \cdot r = 0$

Represent orientation b/w two frames using Euler Angles

→ Yaw (ψ)
↳ Rotation about z-axis

→ Pitch (θ)
↳ Rotation about y-axis

→ Roll (ϕ)
↳ Rotation about x-axis

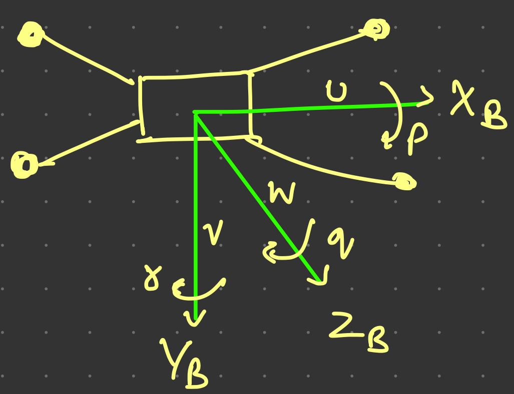
Lecture 2

Variables in body frame:

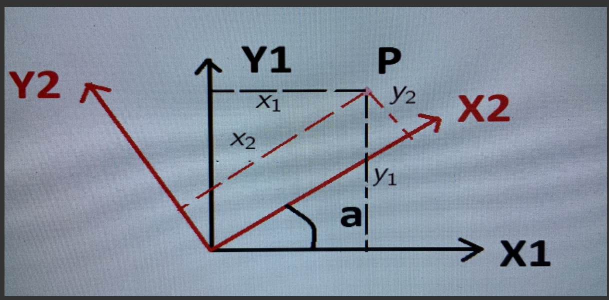
$V_B = [u, v, w]^T$

Acceleration: $(\dot{u}, \dot{v}, \dot{w})$

Ang. Velocity: $\Omega = [p, q, r]^T$



Rotation Matrix:



$$P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \rightarrow \text{for } x_1\text{-}y_1\text{-}z_1 \text{ frame}$$

$$P_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \rightarrow \text{for } x_2\text{-}y_2\text{-}z_2 \text{ from}$$

If $z_1 = z_2$ [Rotation about Z-axis

$$P_2 = \begin{bmatrix} ca & sa & 0 \\ -sa & ca & 0 \\ 0 & 0 & 1 \end{bmatrix} P_1$$

Euler Angle Representation:

1st rot	i to v_1	
2nd rot	v_1 to v_2	
3rd rot	v_2 to b	

Roll $\dot{\phi}$	Pitch $\dot{\theta}$	$\dot{\psi}$ → Yaw
0	0	$\neq 0$
0	$\neq 0$	0
$\neq 0$	0	0

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R(\phi)^b_{v_2} \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R(\phi)^b_{v_2} R(\theta)^{v_2}_{v_1} \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix}$$

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\phi c\theta \end{pmatrix}}_{R} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{bmatrix} 1 & s\phi \tan\theta & c\phi \tan\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi \sec\theta & c\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

→ Useful because ϕ, θ, ψ can't be obtained directly but p, q, r from gyro can be.

Connecting inertial & body frames

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underbrace{R(\phi)^{b}_{v_2} \, R(\theta)^{v_2}_{v_1} \, R(\psi)^{v_1}_{i}}_{B} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = B^{-1} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

→ How is it obtained?

Coriolis Equation:

$V_B = [u, v, w]^T$

$\Omega = [p, q, r]^T$

$F = [f_{x_b}, f_{y_B}, f_{z_B}]^T$

Rate of change of vector in the inertial is the sum of [Rate of chang of vector in rotating frame and change due to relative angular velocity b/w inertial frame and rotating frame

$$\left(\frac{dV_B}{dt}\right)_{Iner} = \left(\frac{\delta V_B}{\delta t}\right)_{Body} + \left(\Omega \times V_B = \frac{F}{m}\right)$$

$$\dot{u} = rv - qw + \frac{f_{xb}}{m}$$

$$\dot{v} = pw - ru + \frac{F_y}{m}$$

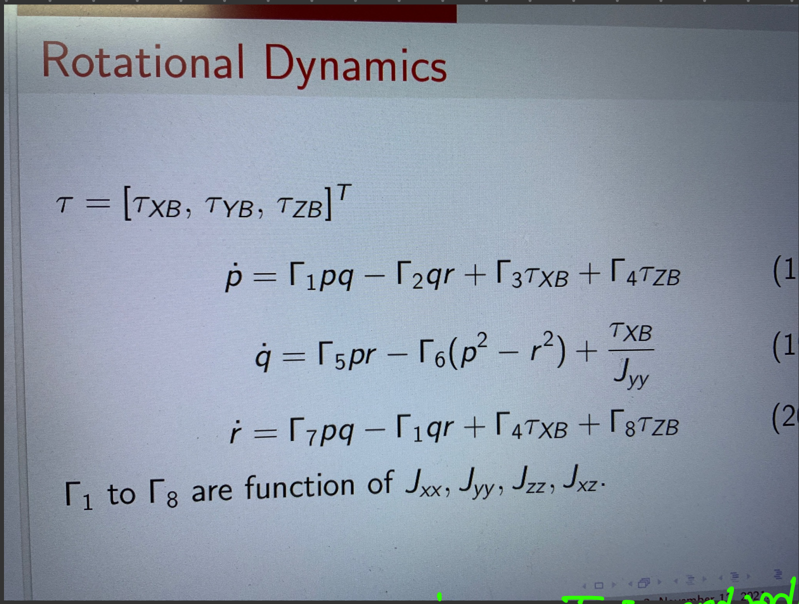
$$\dot{w} = qu - pv + \frac{f_{zB}}{m}$$

Rotational Dynamics:

Rate of change of ang. momentum = Applied torque

$$J = \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

$$J\left(\frac{d\Omega}{dt}\right)_{inte} = J\left(\frac{d\Omega}{\delta t}\right)_{Body} + \Omega \times J\Omega = \tau$$



; won't all

(To go forward → increase T_3, T_4 and reduce T_1, T_2)

→ To go straight

Drones

$t_{XB} = (T_1+T_4)I - (T_2+T_3)I$

$t_{YB} = (T_1+T_2)b - (T_3+T_4)b$ [↑ T_1+T_4 ↓ T_2+T_3]

→ To move sideways

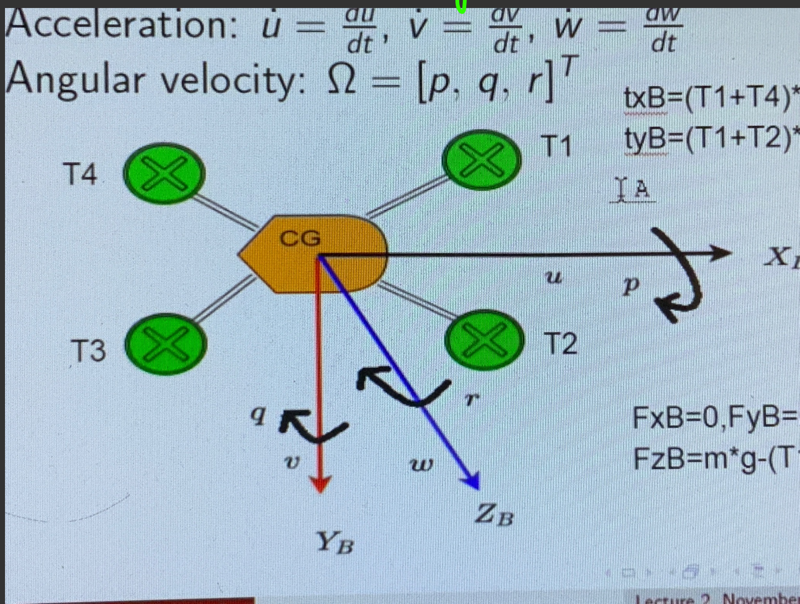
$mg\sin(\theta)\cos(\phi)$

$F_{XB} = 0$, $F_{YB} = 0$ $\cos(\theta)\sin(\phi)$

$F_{ZB} = mg - (T_1+T_2+T_3+T_4)$ $\cos(\theta)\cos(\phi)$

$t_{ZB} = -t_1 - t_3 + t_2 + t_4$

To change orientation (chang t_1+t_3 by $-(\ )\ t_2+t_4$



→ Hovering condition:
$mg = T_1+T_2+T_3+T_4$
$T_1 = T_2 = T_3 = T_4 = mg/4$

} usually there will be linear, rotational drag

→ If motor rotating clockwise, torque is created anti-clockwise

→ Usually diagonal propellers are kept in same direction of rotation