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DIP Lecture 11

Image filtering in Fourier Domain & Morphological Processing

Ideal lowpass filter: [In freq domain]

$$H(u,v) = \begin{cases} 1, & D(u,v) \leq D_0 \\ 0, & D(u,v) > D_0 \end{cases}$$

where $D(u,v) = [(u-M/2)^2 + (v-N/2)^2]^{1/2}$

↳ Cutoff frequency

We can do convolution for implementation,

$I(x,y) \xrightarrow{DFT} G[u,v]$

$h(x,y) \xrightarrow{DFT} H[u,v]$

$G[u,v] \otimes H[u,v] \xrightarrow{IDFT} I_{LPF}(x,y) = I(x,y) * h(x,y)$

↑ This multiplication is same as performing convolution

We use circle to be center as its center shifted

$$F^{-1}\{G,H\}$$

↓ Low filtered image

HIGH PASS

→ Eliminates center
→ Compliment of low pass
→ Sharpening
→ Large cutoff – More information removed

HIGH CUTOFF
→ Edge detection
→ Shouldn't be too high if we want to do sharpening
→ Larger circle

BUTTERWORTH FILTER

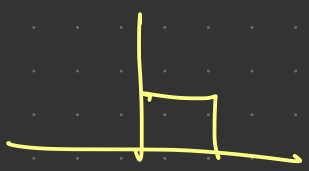
→ Gentler cutoff slope → Reduces ringing effect

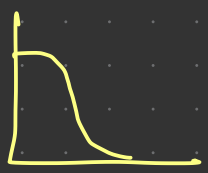
→ $f(x) = \frac{1}{1+(x/D)^{2n}}$ → Order of filter that controls sharpness of cutoff

↳ Threshold

Ideal → Butterworth

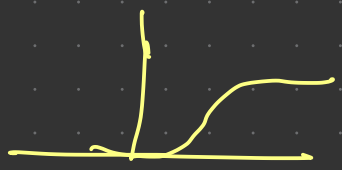
LOW PASS





$$f(x) = \frac{1}{1+(x/D)^{2n}}$$

HIGH PASS



$$f(x) = \frac{1}{1+(D/x)^{2n}}$$

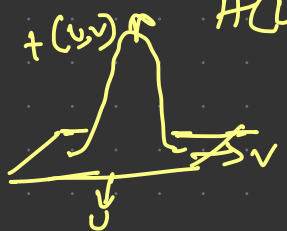
Gaussian Filtering:

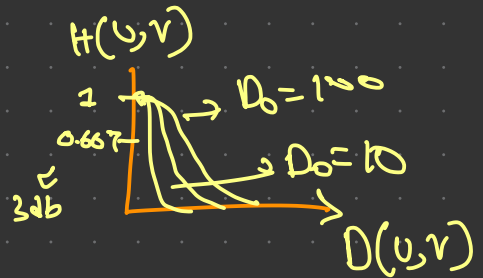
In $\sigma = 10$, we have many low freq so more blurring

In σ high, more clear

LOW PASS:

$$H(U,V) = e^{-D^2(U,V)/2D_0^2}$$

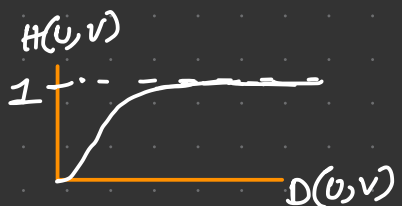


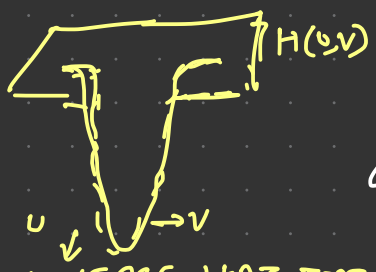


→ Used to remove blemishes in photograph

HIGH PASS:

$$H(U,V) = 1 - e^{-D^2(U,V)/2D_0^2}$$





INVERSE HAT TYPE

↑ 1 − [ ] is done because it contains all the frequencies thus subtracting with low pass filtered freq leaving behind high pass only

With increasing D_0, GHPF, we get finer edges

Laplacian in freq domain:

$$F\left[\frac{d^n f(x)}{dx^n}\right] = (ju)^n F(u)$$

So for 2-D,

$$F\left\{\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right\} = (ju)^2 F(u,v) + (jv)^2 F(u,v)$$

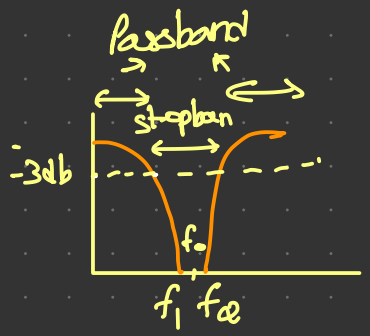
$$= -(u^2+v^2) F(u,v)$$

Notch filters:

→ Eliminate sharp range of freq.

→ Band reject filter

→ Removes periodic noises



→ Higher freq noise

For removal of periodic image:

Notch filter

→ Sets rows, columns of DFT corresponding to noise = 0

→ Removes much of periodic noise

Band reject filter

→ Creates filter with 0's at radius of noise from center, 1 elsewhere

→ Apply filter to DFT

→ Homomorphic Filtering:

Modelled using object physics

$$I(x,y) = L(x,y) \cdot R(x,y)$$

→ Applied in face images

L → Reflectance [Modelled as high pass]

[Modelled as low pass] Luminance

→ Circular convolution causes a wraparound error when it exceeds boundary

Recipe for transform domain processing:

Step 1: Pad image

Step 2: Multiply fp by $(-1)^{x+y}$ → To get centered shifted output

Step 3: Compute $F_p = DFT(f_p)$

Step 4: Centered Gaussian lowpass filter H

Step 5: $G_p = HF_p$

Step 6: Compute $Re[IDFT[G_p]](-1)^{(x+y)}$

$$f(x)e^{\frac{j2\pi u_0 x}{M}} \longleftrightarrow F[u-u_0]$$

where $u_0 = \left(\frac{M}{2}\right)$

Morphological processing:

→ Set of non-linear operations related to shape or morphology of image

→ Application: Plant phenotyping, document image analysis, background subtraction

Morphological operators:

→ Object/Region seen as set of pixels

0 - foreground

1 - background

→ Set operations

A, B

A∪B

A∩B

A-B

B-A

→ Logical operations

A&B

A|B

A|B'

A⊗B

→ Structuring element (Kernel)

Small binary image matrix having

→ fits entire image

→ Intersects image

→ Neither intersects nor hits image

→ Specific size

→ Patterns of ones & zeros specify shape