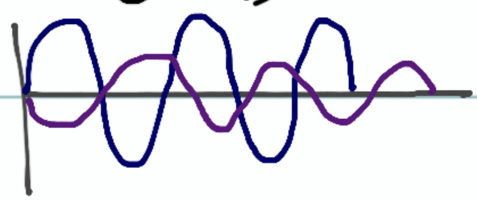
13/10/20

Osmo Adaptation 1

We try to identify frequency response model

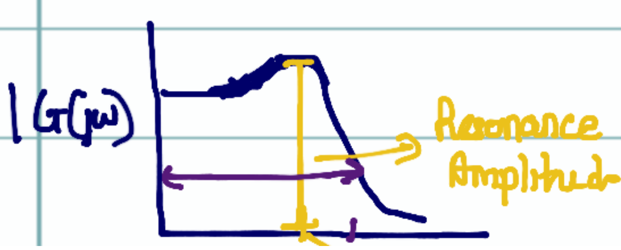
Input = $X \sin(\omega t)$

Output = $Y \sin(\omega t + \phi)$



$$|G(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|} = \text{Amplitude Ratio}$$

$$\angle G(j\omega) = \angle \frac{Y(j\omega)}{X(j\omega)} = \text{Phase Shift of Output wrt Input}$$

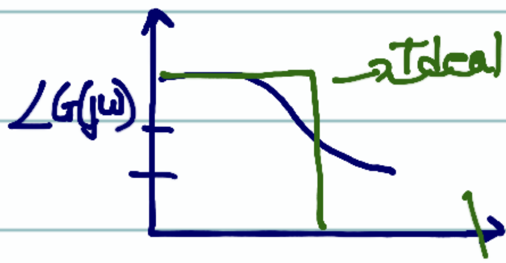


$$G(s) = \frac{\omega_n^2}{s^2 + 2s\xi\omega_n + \omega_n^2}$$

$\omega \to \omega_r$ Resonant Frequency $\Downarrow$ $|G(j\omega)|$

Bandwidth $= \frac{1}{\sqrt{2}} |G(j\omega)|_{\omega=0}$

The amplitude is dependent on damping frequency

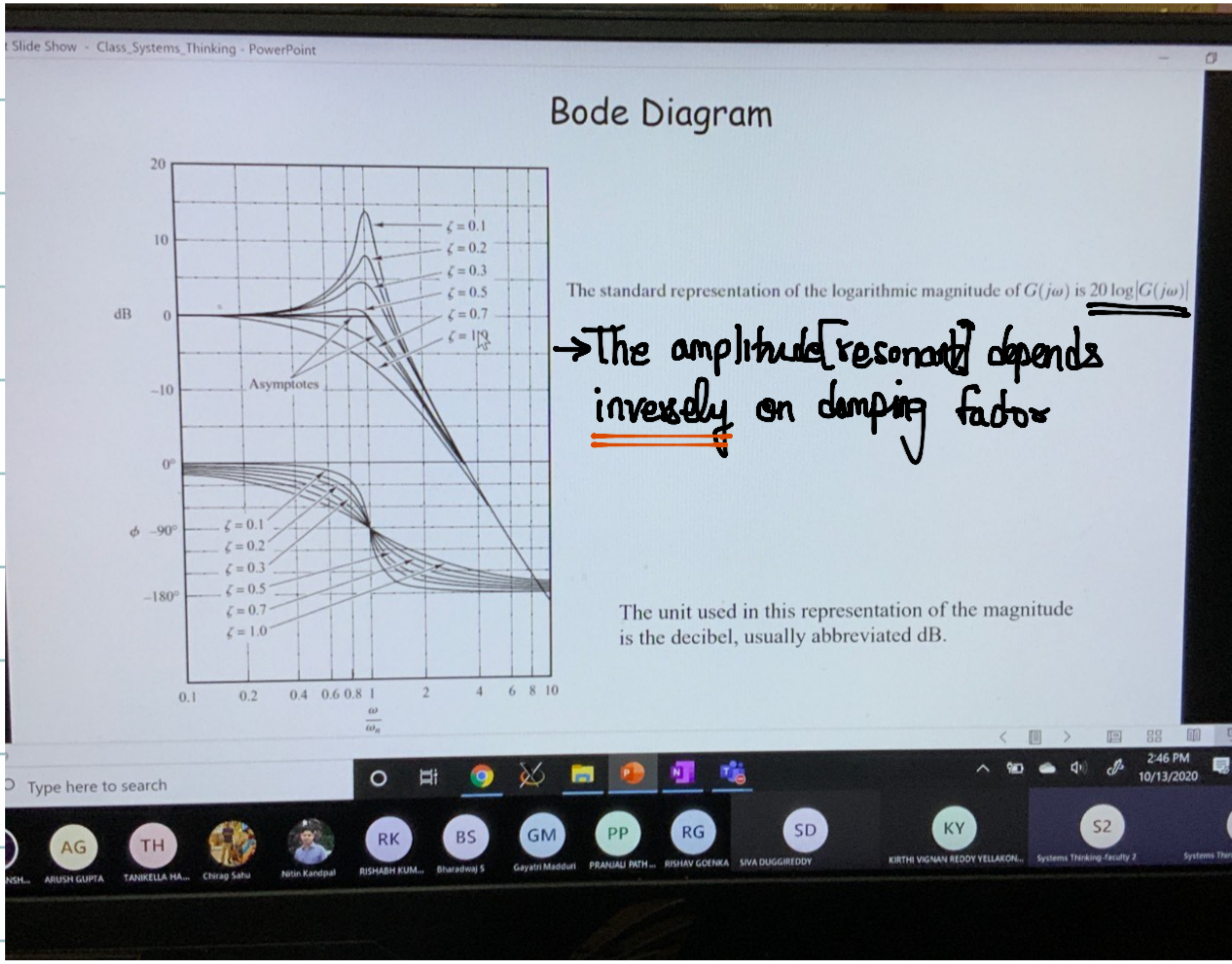


In form of $\tan^{-1}$

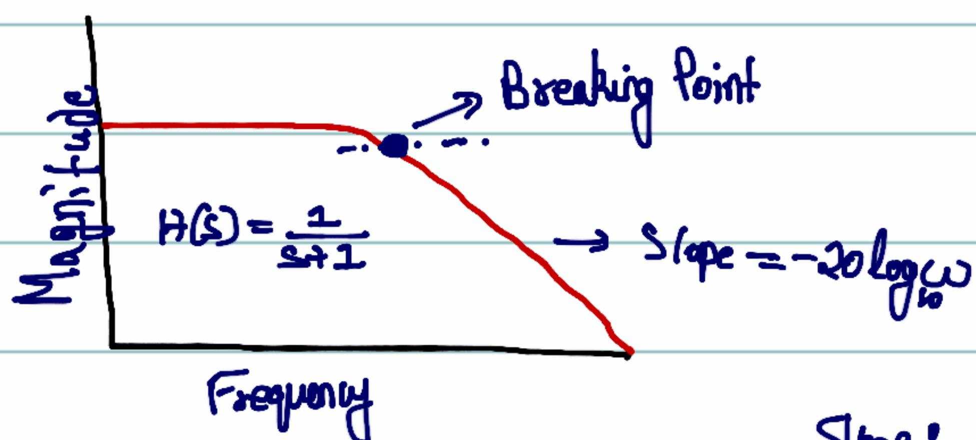
Magnitude is plotted in decibel scale,

$$dB = 20 \log_{10} |G(j\omega)|$$

Bode's Diagram



→The amplitude[resonant] depends inversely on damping factor



Slope:
-20db/dec → 10 fold change
-6db/Octave → 2 fold change

To find breaking point.

First Order,

$$|G(j\omega)| = \sqrt{(\omega^2+1)}$$

Wild Type – No change made, base time system

→ In the plot we see that as the damping factor increased, amplitude decreased.

→ Response time / Efficiency may increase with damping factor.

→

When we write the differential equations

$$\dot{y} = \underbrace{[A_0 u - x]}_{\text{Fed back to system}} - \gamma y \quad ; \quad \dot{x} = \alpha(A_0 u - x) + \beta y$$

Here, $\boxed{\text{Sln1} \mid \text{Sho1} \rightarrow \text{Pbs2} \rightarrow \underset{\text{Effector}}{\text{Hog1}}}$

→ Osmolyte is to balance pressure preventing water from flowing inside to output.

→ Called MAPK cascade due to presence of 3 proteins which work together to sense error and make correction. Error is fed back to controller i.e. osmolyte export

→ Hog1 also helps in synthesis of glycerol as per necessity

- Production (Gpd1, Gpd2) → Slower
- Feedback [Osmolyte Export] → Faster

We can directly identify as we know effect of Pbs2 as it affected response time, it [feedback] must be a faster process than the production of glycerol [Gpd1, Gpd2]

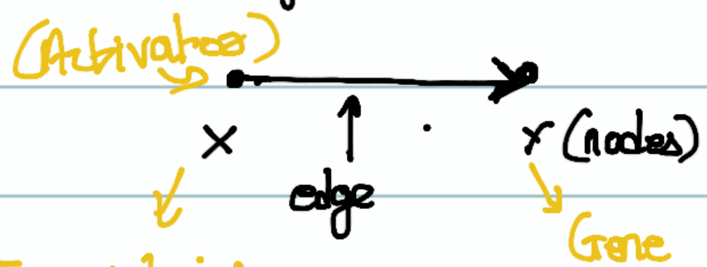
→ Purpose of the slow feedback production, has a role
If we give repeated stimulus, it slowly activates response which means adaptation is evolving, increasing response time

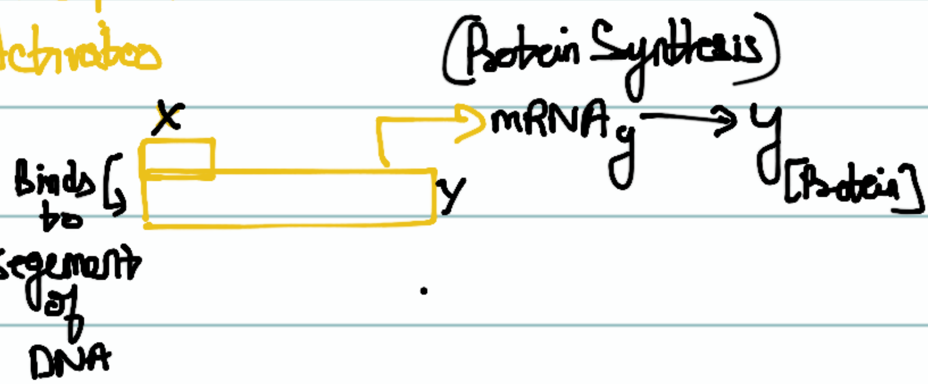
→ Given input is sinusoidal, ~~so~~ output is also sinusoidal

Modelling Biological Systems in time domain

Bigger networks can be broken down into simpler parts

Two component system,



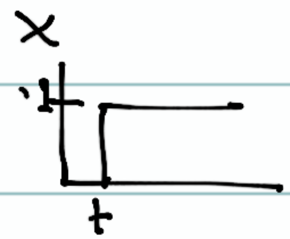


To write,

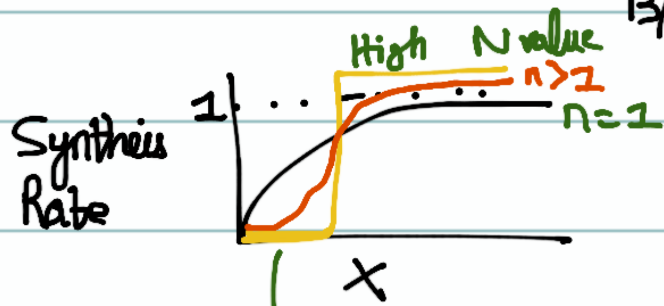
$$\frac{dy}{dt} = \text{Synthesis} - \text{Degradation}$$

$$\frac{dy}{dt} = k_s X - k_d y$$

If X is a step function, we can leave it for simplicity.



Synthesis Rate $= R_s' = \dfrac{k_s X^n}{\underbrace{X^n + K^n}_{\text{B/N } 0 \,\&\, 1}} = k_s \Theta(x > k)$



If $X = K$,

$R_{s\,max} = \dfrac{k_s}{2}$

If N is very high, it can behave like a step/switch function

Directed Graph

$X \dashv Y$

Represses the gene

X inhibits Y

(Inhibitor)

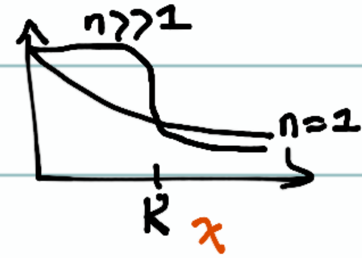
Binds with Y and doesn't let other functions like making of mRNA etc to happen.

Write equation

$$\dot{y} = \frac{k_s K^n}{K^n + X^n} - k_d y$$

$$\dot{y} = \frac{k_s K^n}{K^n + x^n} - k_d y$$

Synthes. Rate can be written as k_s



$$\dot{y} = k_s \underbrace{\theta(x<K)}_{0 \text{ or } 1} - k_d y$$

$$x \rightarrow y \quad \left\{ \begin{array}{l} \dot{y} = k_s X - k_d y \\ \dot{y} = \dfrac{k_s x^n}{x^n + K^n} - k_d y \\ \dot{y} = k_s \theta(x>K) - k_d y \end{array} \right\} \text{Activator}$$

$$x \dashv y = \left\{ \begin{array}{l} \dot{y} = k_s X - k_d y \\ \dot{y} = \dfrac{k_s K^n}{K^n + x^n} - k_d y \\ \dot{y} = k_s \theta(x<K) - k_d y \end{array} \right\} \text{Repressor}$$

$$\dot{y} = k_s - k_d y$$

If $k_s \neq 0$,

$$\frac{dy}{k_s - k_d y} = dt$$

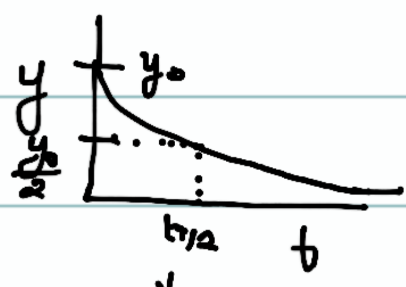
$$\ln(k_s - k_d y) = t$$

$$y = y_{st}(1 - e^{-k_d t}$$

If $k_s = 0$,

$$\dot{y} = -k_d y$$

$$\underbrace{y = y_0 e^{-k_d t}}_{\text{Protein decay}}$$



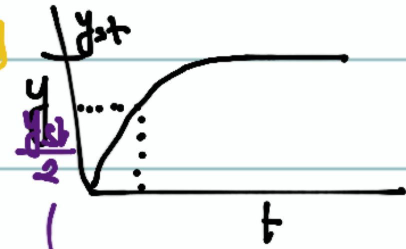
where $y_{st} = \frac{K_s}{K_d}$

To find half life :

1 $\boxed{t_{1/2} = \frac{\log 2}{K_d}}$

[Response Time]

For half life



For faster response time, $t_{1/2}$ should be very small

Parameters for 1st order in Laplace,

→ τ [Time constant as $t_{1/2}$]