

Least Squares in Matrix Form

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} \dots + \beta_k x_{ki} + \varepsilon_i \quad [i=1, 2, \dots, n]$$

$$y = (y_1 \ y_2 \ \dots \ y_n)^T$$

$$\beta = (\beta_1 \ \beta_2 \ \dots \ \beta_k)^T$$

$$\varepsilon = (\varepsilon_1 \ \varepsilon_2 \ \dots \ \varepsilon_n)^T$$

$$X = \begin{pmatrix} 1 & x_{21} & \dots & x_{k1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{2n} & \dots & x_{kn} \end{pmatrix}$$

$$y = X\beta + \varepsilon$$

$$e = y - Xb$$

Residuals

$$S(b) = \sum e_i^2 = e'e = (y - Xb)'(y - Xb)$$

$$S(b) = y'y - y'Xb - b'X'y + b'X'Xb$$

$$\frac{\partial S}{\partial b} = -2X'y - 2X'Xb \Rightarrow b = (X'X)^{-1}X'y$$

$$n \geq k$$