Velocity Kinematics

→ $V = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$ → Linear Velocity

→ $\vec{V} = \omega \times \vec{r}$

$\vec{\omega} = \dot{\theta}\hat{k}$

→ $\dot{\hat{i}} = \dot{\theta}\hat{j}$, $\dot{\hat{j}} = -\dot{\theta}\hat{i}$

→ Prismatic: $v = \dot{q}\hat{k}$

$\omega = 0$

→ $p = f(\theta)$ → Non-linear

$\dot{p} = J(\theta)\dot{\theta}$ → Linear

↳ $\dot{p} \in \mathbb{R}^m, \dot{\theta} \in \mathbb{R}^n, J : \mathbb{R}^n \to \mathbb{R}^m$

→ Revolute: $v = \dot{q}\hat{k} \times \vec{r}$

$\omega = \dot{q}\hat{k}$

J_v

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1(\theta)}{\partial \theta_1} & \frac{\partial f_1(\theta)}{\partial \theta_2} \\ \frac{\partial f_2(\theta)}{\partial \theta_1} & \frac{\partial f_2(\theta)}{\partial \theta_2} \\ --- & --- \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

↳ Homogenous matrix

→ $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c\theta_1 L_1 + c(\theta_1+\theta_2) L_2 \\ s\theta_2 L_1 + s(\theta_1+\theta_2) L_2 \end{pmatrix} = \begin{pmatrix} f_1(\theta_1, \theta_2) \\ f_2(\theta_1, \theta_2) \end{pmatrix}$$

$\theta = \theta_1 + \theta_2 = f_3(\theta_1, \theta_2)$

$$\dot{\theta} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \theta_2 \end{pmatrix}$$

↳ J_ω

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \dot{V}_{tip} = \begin{pmatrix} J_1(\theta) & J_2(\theta) \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

→ $J_1(\theta), J_2(\theta)$ are ⊥ to line joining v_{tip} & joint

→ Singularity of 2R is if $\theta_2 = 0$ (or) π

$J_2 = 2J_1$, $J_1 = 0$

→ If $J_1(\theta)$ & $J_2(\theta)$ are collinear tool-tip is unable to generate velocity in desired direction

→ Inverse Jacobian:

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = J_v^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

$\dot{p} = J(\theta)\dot{\theta}$ $\quad \dot{\theta} = J^{-1}\dot{p}$

$b = Ax$ → Least Squares

$J_v^{-1} = \frac{1}{\det(J)} \operatorname{adj}(J)$

↳ $l_1 l_2 s\theta_2 = 0$

$\theta_2 = 0, \pi$

Condition Numbers:

$\det(A) = \lambda_1 \lambda_2 \cdots \lambda_n$

$C.N = \frac{\sigma_l}{\sigma_s} = \frac{\sqrt{\lambda_l}}{\sqrt{\lambda_s}}$

Away from sing

→ Move in any direction

If $K = 1$ ⇒ Isotrophic condition

Amplification is higher val direction

$K \geq 1$, If K close to 1, well condition

If K is large, its ill condition → Closer to sing