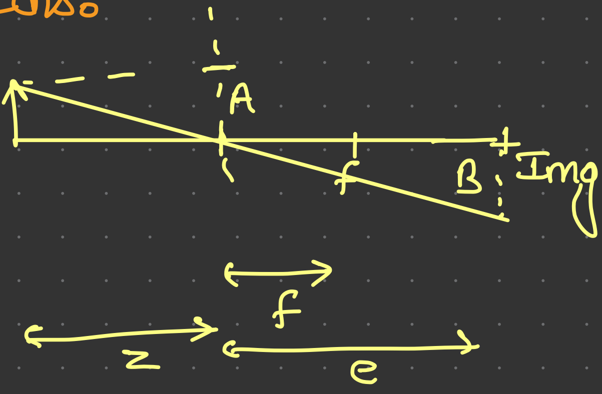
Pinhole Camera:

- → Barrier reduces blurring
- → Opening is called aperture
  - ↳ More aperture, more blurring
  - ↳ If too small, less light gets through
    diffraction effects

Thin Lens:



$\frac{B}{A} = \frac{e}{z}$, $\frac{e}{f} = 1 + \frac{e}{z} \Rightarrow \frac{1}{f} = \frac{1}{e} + \frac{1}{z}$

→ Specific dist for 'in focus'

→ Blur Circle: $\frac{L\delta}{2e} = R$ (L → Length of lens/pinhole)

- → Minimal $L \Rightarrow$ Minimal R
- → R is smaller than image resoluti
  for good image

→ If obj too far ($z >> f$),
img formed at f.

→ Perspective: Dependence of apparent
size of object on depth (z)
- → Far objects are smaller
- → Can percieve depth

→ Projective:
- → Straight lines are still
  straight
- → Loses length & angles

→ Vanishing point:
Int. of $\parallel^{lel}$ lines

→ Depth of field [DOF]
Dist b/w nearest and farthest objects
that are acceptably sharp

→ Effect of aperture on DOF

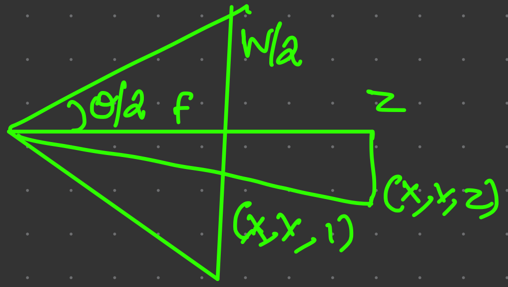
Aperture ↓ ⇒ ↑ objects in
focus but
reduces amount
of light

Field of view:

$\uparrow f \Rightarrow$ Wider field of view [more world points]

$\downarrow f \Rightarrow$ Narrow field of view [less world points]

Small FOV $\Rightarrow \uparrow$ focus

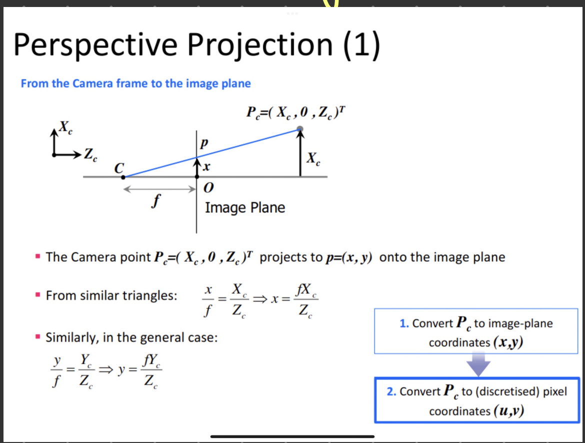


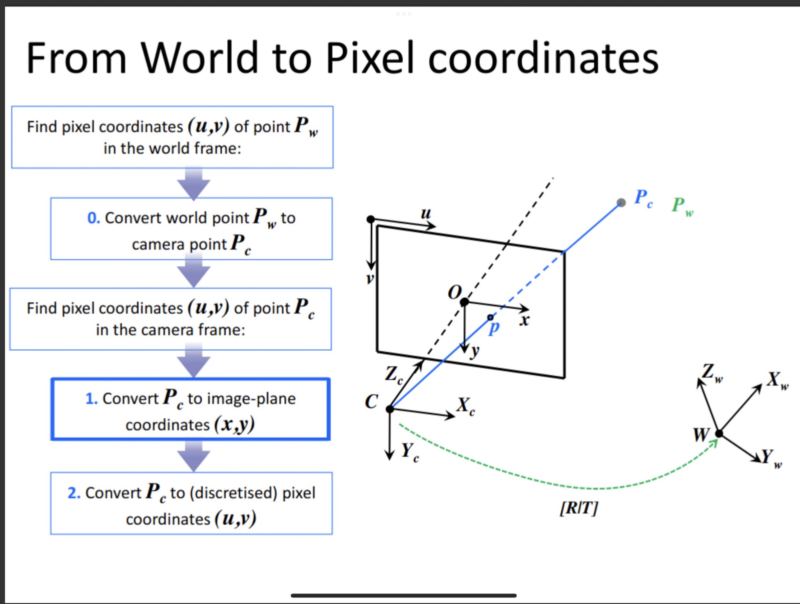
$$\tan\left(\frac{\theta}{2}\right) = \frac{W}{2f} \Rightarrow f = \frac{W}{2}\left[\tan\left(\frac{\theta}{2}\right)\right]^{-1}$$

Perspective Camera:

→ Camera measures angles & not distances

→ Camera plane → Image Plane:

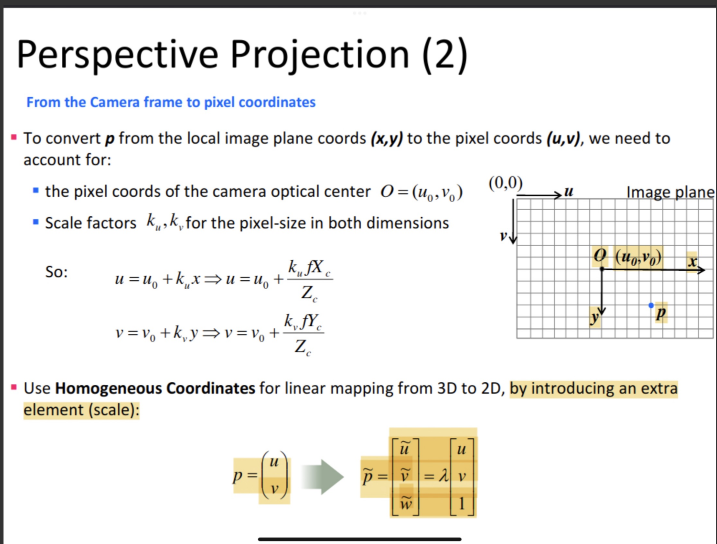


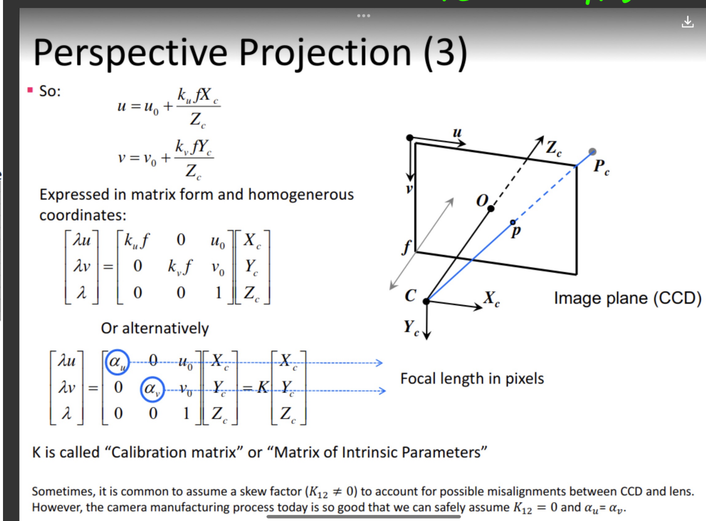


$$x = \frac{fX_c}{Z_c}, \quad y = \frac{fY_c}{Z_c}$$

k_u, k_v are scale factors taking f in cms to pixels

$$k_u, k_v = \frac{\text{Image-Width/Height}}{\text{Sensor width/height}}$$

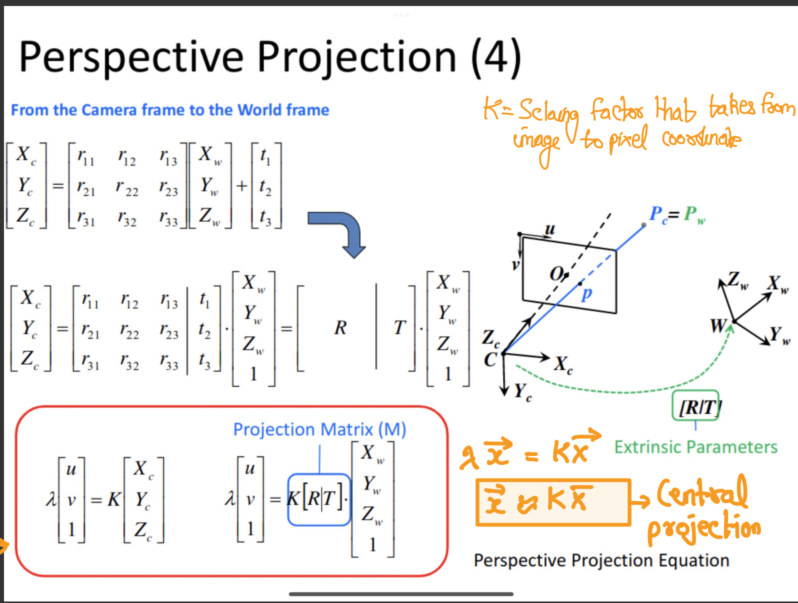




$$K = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$ is calibration matrix (or) Matrix of intrinsic parameters

where α_u, α_v are focal length in pixels

→ Proof of vanishing point can be showed by taking above equation



IMP →

λ = Depth ($\lambda = Z_c$) of scene point

$\vec{x} \propto K\vec{X} \Rightarrow K^{-1}\lambda\vec{x} = \vec{X}$

$\lambda K^{-1}\vec{x} = \vec{X}$ i.e. $K^{-1}\vec{x}$ is $\parallel^{lel}$ to or points to direction of 3D vector $\vec{X}$

$$\lambda \vec{x}_c = K T_w^c \vec{X}_w$$

$$= K[R|t]\vec{X}_w$$

extrinsics, intrinsics

$\Rightarrow \vec{x}_c \propto K[R|t]\vec{X}_w$ (3×1)

$\vec{x}_c = K[R|t]\vec{X}_w$

(or)

$\vec{x}_c \parallel^{el}$ to $K\vec{x}_c$

(or)

$cross(\vec{x}_c, K\vec{x}_c) = 0$

Distortions:

→ Trans. of ideal coordinates (u, v) to real observable coordinates

→ Based on non-linear function

Ex: $\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2)\begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$ $r^2 = (u - u_0)^2 + (v - v_0)^2$

Camera Calibration

Divide each v_i by v_{12}. or $P_{34} = 1$ and $P_{ij} = \frac{P_{ij}}{P_{34}}$

is the final solution for P.

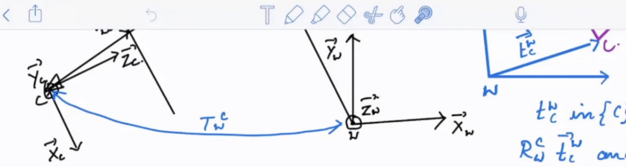
How to get K from P?

Before that let us look at $AP = 0_{2M\times1}$.

Since it is overdetermined and the observations are noisy $\nexists$ NO P for which $AP = 0$.

Hence find the best P for which $\|AP\|_2^2$ is minimized or $P^T A^T A P$ is minimized such that $\|P\| = 1$.

The solution is by SVD.



t_c^w in {c} is $R_w^c t_c^w$ and t_w^c is simply $-R_w^c t_c^w$

$$\underset{(3\times1)}{\vec{X}_c} = \underset{(3\times4)}{[R_w^c \;\; t_w^c]}\, \vec{X}_{w\,(4\times1)} \longrightarrow (1)$$

$$\underset{(3\times1)}{\vec{x}_c} = K_{3\times3}\, \vec{X}_{c\,(3\times1)} \longrightarrow (2).$$

$$= K[R_w^c \;\; t_w^c]\, \vec{X}_w \longrightarrow (3) = P_{3\times4}\vec{X}_w \longrightarrow (4).$$

$P_{3\times4} \longrightarrow$ Camera Projection Matrix.

Camera Calibration: Process of estimating P.

Often (1) is rewritten as:

$$\vec{X}_c = R_w^c \vec{X}_w + t_w^c \to (5) = R_w^c \vec{X}_w - R_w^c t_c^w \to (6)$$

$$= R_w^c [I - t_c^w]\, \vec{X}_{w\,(4\times1)} \longrightarrow (7)$$

or $\vec{x}_c = K R_w^c [I - t_c^w]\, \vec{X}_{w\,(4\times1)} \longrightarrow (8)$

or $\boxed{\vec{x}_c = P_{3\times4}\vec{X}_w \to 9}$

or rather $\lambda \vec{x}_c = P\vec{X}_w$ or $\vec{x}_c \approx P\vec{X}_w$

or simply $\boxed{x \approx PX}$

$\vec{x}_c \approx P\vec{X}_w$ or
$\vec{x}_c = P X_w$
$\vec{x}_c \approx K\vec{X}_c$
$\vec{x}_c = K\vec{X}_c$

To estimate P:

↳ as a first step towards estimating K.

$P_{3\times4} = 12$ parameters.

Every pair of correspondence $X_i \leftrightarrow x_i$ gives two equations (Why?).

Hence 6 pairs of correspondences are needed to solve for P.

As a matter of fact one needs to solve for only 11 parameters of P as $\vec{x} \approx P\vec{X}$ is a homogeneous equation, which means any $\gamma P\vec{X}$ also projects to same $\vec{x}$.

Hence every P_{ij} can be divided by P_{34} for example and P_{34} made 1.

Hence 6 pairs of correspondences $X_i \leftrightarrow x_i$ are still required to solve for the 11 parameters of P.

How does one solve?

$$x_i = P_{11}X_i + P_{12}Y_i + P_{13}Z_i + P_{14} \longrightarrow (9)$$

$$y_i = P_{21}X_i + P_{22}Y_i + P_{23}Z_i + P_{24} \longrightarrow (10)$$

$$z_i = P_{31}X_i + P_{32}Y_i + P_{33}Z_i + P_{34} \longrightarrow (11).$$

$$x_i = \frac{x_i}{z_i}, \quad y_i = \frac{y_i}{z_i} \longrightarrow (12).$$

as $\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = z_i \begin{bmatrix} x_i/z_i \\ y_i/z_i \\ 1 \end{bmatrix}$

Then $x_i = \dfrac{P_{11}X_i + P_{12}Y_i + P_{13}Z_i + P_{14}}{P_{31}X_i + P_{32}Y_i + P_{33}Z_i + P_{34}}$.

or $X_iP_{11} + Y_iP_{12} + Z_iP_{13} + P_{14} - P_{31}X_ix_i - P_{32}Y_ix_i - P_{33}Z_ix_i - P_{34}x_i = 0 \longrightarrow (13)$

Illy $X_iP_{21} + Y_iP_{22} + Z_iP_{23} + P_{24} - P_{31}X_iy_i - P_{32}Y_iy_i - P_{33}Z_iy_i - P_{34}y_i = 0 \longrightarrow (14)$

or
$$\underbrace{\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -X_ix_i & -Y_ix_i & -Z_ix_i & -x_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -X_iy_i & -Y_iy_i & -Z_iy_i & -y_i \end{bmatrix}}_{2\times12} \begin{bmatrix} P_{11} \\ P_{12} \\ \vdots \\ P_{34} \end{bmatrix} = 0_{2\times1} \longrightarrow (15)$$

For every $\vec{X}_i \leftrightarrow \vec{x}_i$ we get 2 eqns of the form (15)

If there are $M >> 6$ correspondences

We have $A_{2M\times12}\, P = 0 \longrightarrow (16)$

How to solve for P?

Overdetermined set of equations

Avoid the trivial solution $P_{3\times4} = 0_{3\times4}$.

$SVD(A) = UDV^T$.

Last column of $V_{12\times12}$, a 12×1 column vector is the solution for P.

Divide each v_i by v_{12}. or $P_{34} = 1$ and $P_{ij} = \frac{P_{ij}}{P_{34}}$

is the final solution for P.