EC5.201 Signal Processing

Final Project Report

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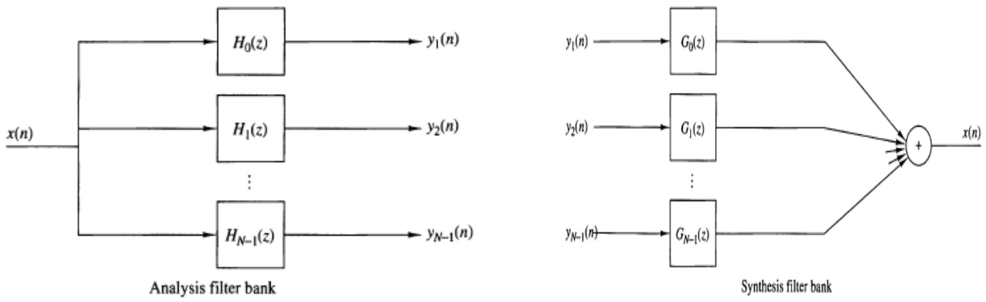
Abstract:

Our project aims to discuss multirate signal processing and suggests methods for achieving low complexity in two channel multirate filter banks. The base filter bank discussed in this project would be a Two-Channel Quadrature Mirror Filter Bank as shown in figure below. We will be demonstrating the purpose of each component in this QMF bank which will then be extended to better filter design techniques and optimizations in the analysis and synthesis banks.

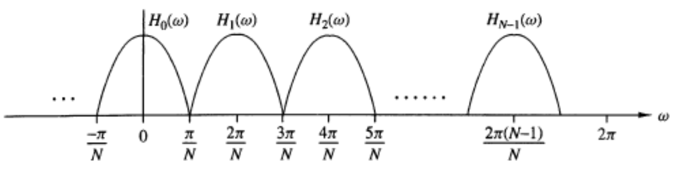
Background Theory:

Digital Filter Banks:

Filter banks are generally categorized as two types, analysis filter banks and synthesis filter banks. An analysis filter bank consists of a set of filters, with system functions $H_k(z)$, arranged in a parallel bank as illustrated in figure 1. The frequency response characteristics of this filter bank split the signal into a corresponding number of sub bands. On the other hand, a synthesis filter bank consists of a set of filters with system functions $G_k(z)$, arranged as shown in Figure below:



The outputs of the filters are summed up to form the synthesized signal x(n). These filter banks are often used in spectrum and signal synthesis. In both the analysis and synthesis filters, $H_0(z)$, and $G_0(z)$, are considered to be prototypes filters as the rest form a frequency shifted version of this filter by multiples of 2π/N.



Frequncy bands of a N-Channel Bank

$$H_k(\omega) = H_0\left(\omega - \frac{2\pi k}{N}\right), \qquad k = 1, 2, \ldots, N-1$$

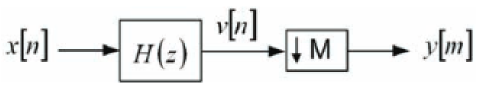
$$H_k(z) = H_0(ze^{-j2\pi k/N}), \qquad 1 \le k \le N-1$$

$$h_k(n) = h_0(n)e^{j2\pi nk/N}, \qquad k = 0, 1, \ldots, N-1$$

In the above equations, the h0(n) give the impulse response of the prototype analysis filter which could be FIR or IIR depending upon use case.

DECIMATION:

Down-sampling with the factor of M is the operation of removing every M-1 samples of a signal while retaining the M^{th} sample. It is Time Dependent.



In Frequency Domain we can write Y(z) as:

$$Y(z) = \sum_{m=-\infty}^{\infty} x[Mm]z^{-m} = \sum_{m=-\infty}^{\infty} y_s[Mm]z^{-m} = \sum_{k=-\infty}^{\infty} y_s[k]z^{-k/M}$$
$$= Y_s\left(z^{1/M}\right)$$

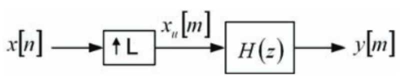
The main drawback of down-sampling is the aliasing effect. To remove aliasing, the process of Decimation is carried out in two steps:

- Band limiting the Original signal to π/M
- Down-sampling by a factor of M.

Here H(z) is called the antialiasing filter. H(z) is not capable of suppressing aliasing to 0 but can suppress it to an acceptable value. In MATLAB, the inbuilt function “decimate” is used for Decimation.

UPSAMPLING:

Up-sampling with a factor of L is the operation of adding L-1 zeroes between every consecutive sample. It is Time Dependent



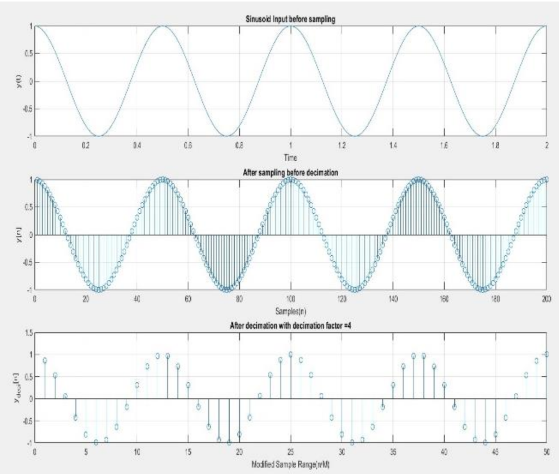
In the Frequency Domain we can write Y(z) in terms of X(z) as:

$$Y(z) = \sum_{m=-\infty}^{\infty} y[m]z^{-m} \sum_{\substack{n=-\infty \\ n=mL}}^{\infty} x[n/L]z^{-n} = \sum_{m=-\infty}^{\infty} x[m]z^{-Lm} = X\left(z^{L}\right)$$

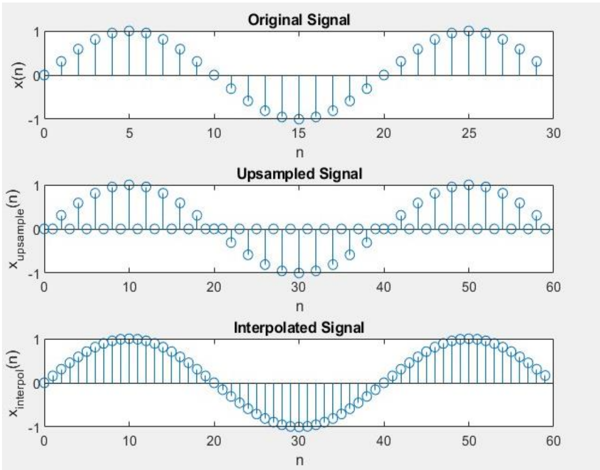
The main disadvantage of Up-sampling is the unwanted L-1 images formed due to inserting the L-1 zeroes when Up-sampling by a factor of L. To remove these images, the process of Interpolation is carried out in two steps:

- Up-sampling the input signal by inserting L-1 zero samples between consecutive samples.
- Removal of the resultant L-1 images from the spectrum of the Up-Sampled signal.

H(z) also known as the anti-imaging filter is used to remove images from the spectrum of the up-sampled signal. Removal of these values from the spectrum causes Interpolation in time domain and hence the zero samples are filled in with interpolated values. In MATLAB we use the inbuilt function "interp" to carry out interpolation.

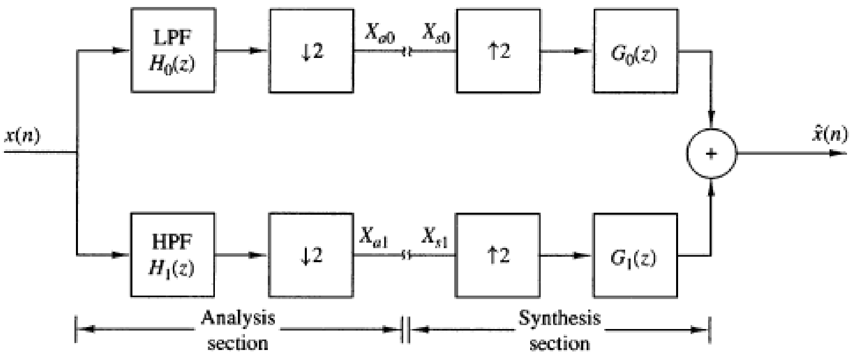


Decimation



Interpolation

Two-Channel Quadrature Mirror Filter Bank:



Here we take decimation factor D and up sampling factor I are equal.
As we discussed in general digital filter banks, here there are only 2 bands, high-pass and low-pass. So, if H0(z) is taken as a low pass filter, its shifted version would produce a high-pass filter. The Fourier transforms of outputs after decimation are as follows,

$$X_{a0}(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right) H_0\left(\frac{\omega}{2}\right) + X\left(\frac{\omega - 2\pi}{2}\right) H_0\left(\frac{\omega - 2\pi}{2}\right)\right]$$

$$X_{a1}(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right) H_1\left(\frac{\omega}{2}\right) + X\left(\frac{\omega - 2\pi}{2}\right) H_1\left(\frac{\omega - 2\pi}{2}\right)\right]$$

Now if we assume that there is no operation taking place between analysis and synthesis filters, then $X_{a0}(\omega)$, $= X_{s0}(\omega)$ and $X_{a1}(\omega)$, $= X_{s1}(\omega)$. So that would imply:

$$\hat{X}(\omega) = X_{s0}(2\omega)G_0(\omega) + X_{s_1}(2\omega)G_1(\omega)$$

In the frequency domain, this can be written as follows:

$$\hat{X}(\omega) = \frac{1}{2}[H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega)]\,X(\omega) + \frac{1}{2}[H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega)]\,X(\omega-\pi)$$

The z-transform of output can be written as follows:

$$\hat{X}(z) = \frac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]\,X(z) + \frac{1}{2}[H_0(-z)G_0(z) + H_1(-z)G_1(z)]\,X(-z)$$

We can simplify the following function as: $Y(z) = V_0(z)X(z) + V_1(z)X(-z)$

Here the values of V0(z) and V1(z) are given by:

$$V_0(z) = \frac{H_0(z)G_0(z) + H_1(z)G_1(z)}{2}$$
$$V_1(z) = \frac{H_0(-z)G_0(z) + H_1(-z)G_1(z)}{2}.$$

In Z-transform of output, $V_0(z)$ is the distortion function of the QMF bank. $V_1(z)$ represents the aliasing transfer function which is to be eliminated.

PERFECT RECONSTRUCTION:

The aliasing term $V_1(z) = 0$ is a must for perfect construction to occur. Apart from this, the distortion function must also not change the signal x[n] apart from an arbitrary delay for all possible inputs. This is possible when $V_1(z) = Cz^{-k}$. This would imply that X(ω) would have a linear phase which inturn means that output x[n] is simply a delayed version of the input sequence x[n] i.e Cx[n-k].

PERFECT MAGNITUDE RECONSTRUCTION:

A PMR output is said to be obtained if the magnitude of the distortion function is a constant ($V_0(z)$ =C) and the aliasing function is 0 ($V_1(z) = 0$). So, in this case, we have no magnitude distortion but have phase distortion.

PROBLEM STATEMENT:

As these multi-rate filter banks have huge applications in DACs etc, it is imperative to reduce the overall complexity of the filters. Many design techniques available for the hybrid analog/digital usually are of higher order and complexity than necessary as these filters are all optimised and fixed digital filters which could lead to accuracy issues. So due to a lot of factors out of which a few are stated above, we need to design new filter banks and design techniques. This project will look into some aspect of optimisation.

SOLUTION APPROACH:

In both the digital and hybrid analog/digital filter banks, we design the analysis filters with very low-complexity. We can also do the same method for the synthesis filters by simply interchanging them as they are related to each other.

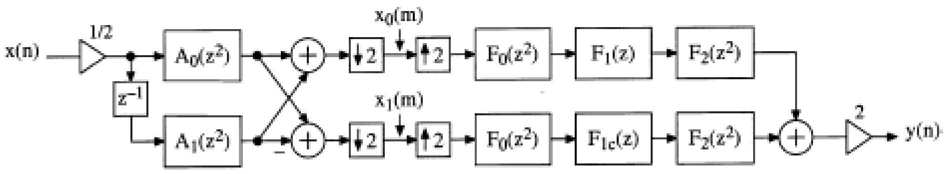
Proposed solutions for digital filter banks are approximately PR filter banks (having magnitude distortion but not phase distortion). The filter bank design is performed by first optimizing the digital or analog analysis filters and then optimizing the digital synthesis filters.

By designing the analysis and synthesis filters separately, it is possible to obtain analysis filters of very low order and complexity. The overall complexity is also low. Further, the proposed filter banks are, in all cases, easy to design by making use of well-known and reliable optimization techniques.

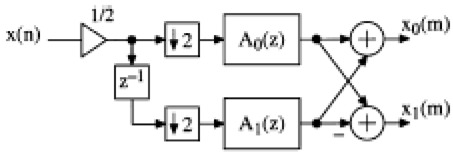
The digital filter banks contain filters that are Equi-ripple Elliptic half-band IIR filters which are designed using closed-form solutions. In the hybrid analog/digital filter banks, they are also equiripple half-band IIR filters with a fixed zero pair. For synthesis filters, they are FIR filters in both the digital and hybrid filter banks. Linear programming is used for optimal synthesis filters.

DIGITAL FILTER BANKS:

Proposed Filter Bank:



ANALYSIS FILTER



We take a half- band IIR low pass and high pass filters which can be written in polyphase form as follows:

$$H_0(z) = \frac{A_0(z^2) + z^{-1}A_1(z^2)}{2}$$
$$H_1(z) = \frac{A_0(z^2) - z^{-1}A_1(z^2)}{2}$$

Here, we take $A_0(z^2)$ and $A_0(z^2)$ are both real, causal all-pass filters. So, we know that the low-pass can be expressed in the form as follows:

$$H_0(e^{j\omega}) = \frac{e^{j\phi_0(\omega)} + e^{j\phi_1(\omega)}}{2}.$$

Here $\phi_0(w)$ and $\phi_1(w)$ are phase responses whose passband differs by pi in the stop band which would imply that $H_0(e^{jw})$, $H_1(e^{jw})$ would behave like a low-pass response and high-pass response respectively. We notice that elliptical filters are of this form. So, we use low and high pass Elliptical filters for this purpose to achieve a low complexity.

NECESSARY CONDITIONS TO EXPRESS DIGITAL FILTERS AS SUM OF ALL PASS FILTERS:

Odd- order low pass filters like Elliptic are derived from analog filters from bilinear transformation. When we express each function as N(Z)/D(Z) and N_c (z^2)/D(z^2) for H_0(z) and H_1(z) respectively.

We will observe that both the functions have to be mirror images of each other. This is seen in the plots as well due to the condition that the high pass and low pass will have to cover the entire bandwidth

FILTER DESIGN CONDITIONS:

The analysis filter banks must have certain conditions for efficient implementation. For Low-pass filter H_0(z),

$$1 - \delta_s^2 \leq |H_0(e^{j\omega T})|^2 \leq 1, \qquad \omega T \in [0, \pi - \omega_s T]$$

$$|H_0(e^{j\omega T})|^2 \leq \delta_s^2, \qquad \omega T \in [\omega_s T, \pi]$$

For High-pass filter H_1(z),

$$1 - \delta_s^2 \leq |H_1(e^{j\omega T})|^2 \leq 1, \qquad \omega T \in [\omega_s T, \pi]$$

$$|H_1(e^{j\omega T})|^2 \leq \delta_s^2, \qquad \omega T \in [0, \pi - \omega_s T]$$

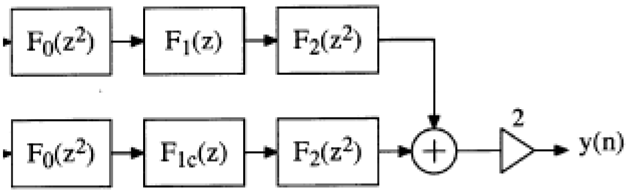
SYNTHESIS FILTER:

As per given analysis filters, To get PMR, we need to take G_0(z) = 2H_0(z) and G_1(z) = -2H_1(z) so that the distortion function and aliasing function as follows:

$$V_0(z) \ = \ z^{-1}A_0(z^2)A_1(z^2) \quad V_1(z) \ = \ 0$$

This is a preferred solution for cases where the linear phase is tolerable.

Modifications made to obtain a linear phase synthesis filter:



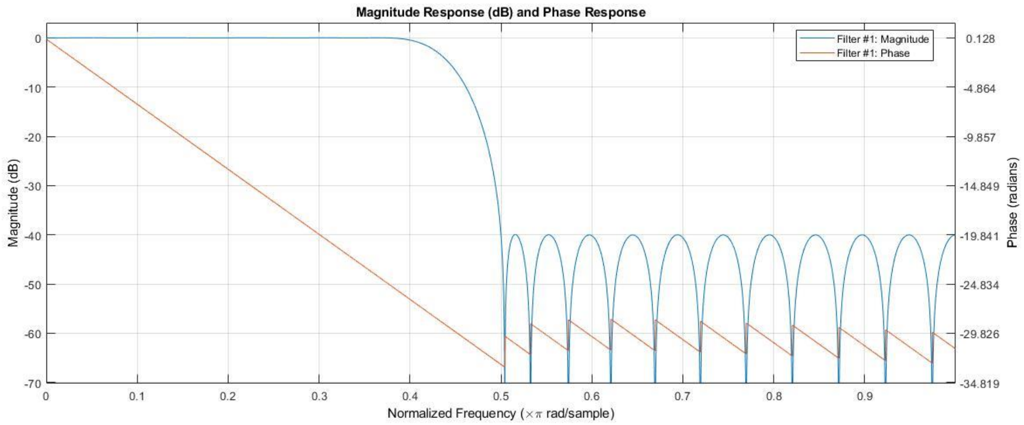
$$G_0(z) = 2F_0(z^2)F_1(z)F_2(z^2)$$
$$G_1(z) = 2F_0(z^2)F_{1c}(z)F_2(z^2)$$

Here we take F_0(z) to reduce/eliminate the phase distortion. This is needed because though the numerator in the analysis filter is linear-phase FIR that won't contribute to phase distortion. But there is still phase distortion from the denominator D(z^2). To cancel this out, F_0(z) = D(z) So that linear phase is maintained.

While here, F_1(z), F_{1c}(z) and $F_2(z)$ help in reducing aliasing and to maintain the magnitude distortion. There are two cases which can be taken as per solution i.e. Aliasing function can be 0 or Aliasing Function is made very small.

The synthesis filter banks, based on both the cases i.e. aliasing function can be 0 or aliasing Function is made very small. In the case 1, we would have to use Parks-McClellan Algorithm for achieving the required optimisation.

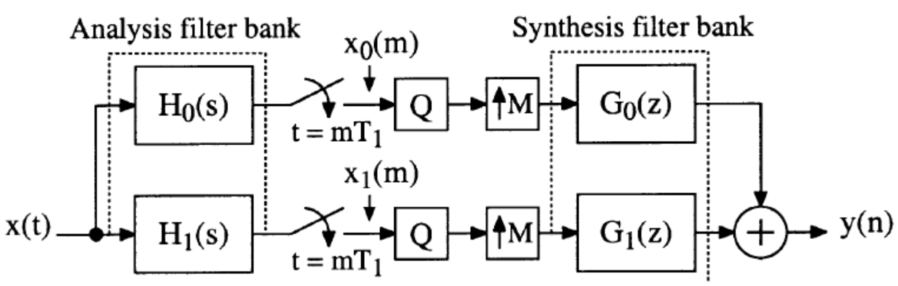
Parks-McClellan Algorithm is an iterative algorithm used for finding optimal FIR Filter. It's used for optimisation purposes of FIR filters and is also indirectly used to find optimal filter coefficients. Minimise the error in passband and stopband by using Chebyshev approximation.



HYBRID ANALOG/DIGITAL FILTER BANKS:

In this type of filter bank, we take the analysis filter as analog while the synthesis filter is a digital FIR filter.

In this filter bank, we consider that the output is bandlimited to π/T. This is done so that the Nyquist value for sampling would be 1/T and there won't be any aliasing in ideal scenario.



The frequency response of y(n) is written as follows:

$$Y(e^{j\omega T}) = V_0(j\omega)\frac{1}{T}X(j\omega) + V_1(j\omega)\frac{1}{T}X\left(j\omega - j\frac{\pi}{T}\right)$$

Where,

$$V_0(j\omega) = \frac{1}{2}H_0(j\omega)G_0(e^{j\omega T}) + \frac{1}{2}H_1(j\omega)G_1(e^{j\omega T})$$

$$V_1(j\omega) = \frac{1}{2}H_0\left(j\omega - j\frac{\pi}{T}\right)G_0(e^{j\omega T}) + \frac{1}{2}H_1\left(j\omega - j\frac{\pi}{T}\right)G_1(e^{j\omega T})$$

Similar to the digital filters, V_0(jw) represents the distortion function while V_1(jw) refers to the aliasing function component. The cases for obtaining PR and PMR are also similar to the digital filter banks stated above. But for hybrid, only approximate PR and PMR banks are considered.

ANALYSIS FILTER:

The low pass and high pass filters in the analysis filter banks are expressed as a sum/ difference like all pass stable filters can be written as follows in the s-domain.

$$H_0(s) = \frac{A_0(s) + A_1(s)}{2}, \quad H_1(s) = \frac{A_0(s) - A_1(s)}{2}$$

$$H_0(j\omega) = e^{j\frac{\Phi_0(\omega) + \Phi_1(\omega)}{2}} H_{0R}(\omega) \qquad H_1(j\omega) = je^{j\frac{\Phi_0(\omega) + \Phi_1(\omega)}{2}} H_{1R}(\omega)$$

$$H_{0R}(\omega) = \cos\left(\frac{\Phi_0(\omega) - \Phi_1(\omega)}{2}\right)$$

$$H_{1R}(\omega) = \sin\left(\frac{\Phi_0(\omega) - \Phi_1(\omega)}{2}\right)$$

We again use elliptic filters to achieve this configuration. As we express them as sum of two all-pass filters, the number of parameters required to individually describe them reduces hence less storage requirement. Although this is not preferred to keep in parallel directly, so a ladder structure is preferred.

SYNTHESIS FILTER:

Here we take the synthesis filters as follows:

$$G_0(z) = 2F_0(z)F_1(z) \quad G_1(z) = 2F_0(z)F_{1c}(z)$$

Where F_0(z) is a K_{F0} order non-linear phase FIR filter while F_1(z) and F_{1c}(z) have a K_{F1} order which are linear phase-filters with separate use cases. F0(z) is used for equalising the phase distortion while the other two are used to shape the magnitude response.

$$V_{0R}(\omega T) = H_{0R}(\omega)F_{1R}(\omega T)|F_0(e^{j\omega T})|$$
$$-H_{1R}(\omega)F_{1cR}(\omega T)|F_0(e^{j\omega T})|$$
$$V_{1R}(\omega T) = H_{0R}\left(\omega - \frac{\pi}{T}\right)F_{1R}(\omega T)|F_0(e^{j\omega T})|$$
$$-H_{1R}\left(\omega - \frac{\pi}{T}\right)F_{1cR}(\omega T)|F_0(e^{j\omega T})|$$

CONDITIONS FOR IMPLENTING FILTER:

For Low-pass filter H0(z),

$$1 - \delta_s^2 \le |H_0(j\omega)|^2 \le 1, \qquad \omega \in [0, \pi/T - \omega_s]$$

$$|H_0(j\omega)|^2 \le \delta_s^2, \qquad \omega \in [\omega_s, \pi/T]$$
$$|H_0(j\omega)| = 0, \qquad \omega = \pi/T$$

For High-pass filter H1(z),

$$1 - \delta_s^2 \le |H_1(j\omega)|^2 \le 1, \quad \omega \in [\omega_s, \pi/T]$$

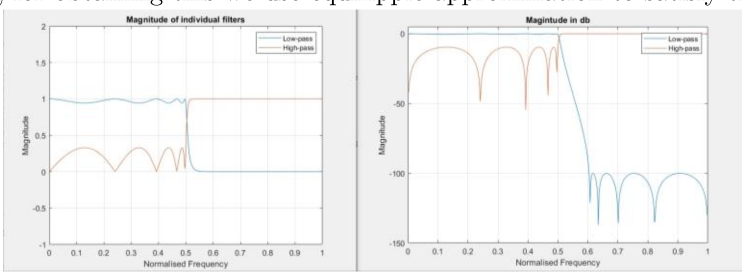
$$|H_1(j\omega)|^2 \le \delta_s^2, \quad \omega \in [0, \pi/T - \omega_s]$$
$$|H_1(j\omega)| = 0, \qquad \omega = 0$$

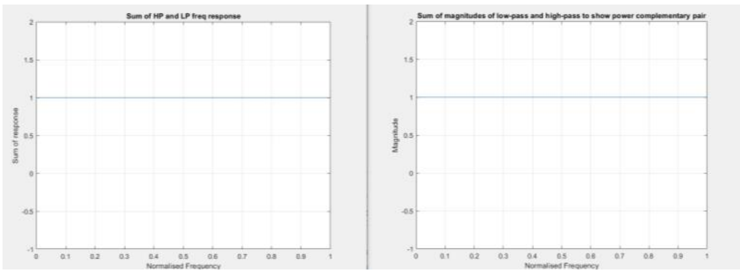
The first two conditions are similar to what is taken in the digital filter. The extra condition for the magnitude to be zero is due to the following:

We have that f1[n] is antisymmetric, so $V_{1R}(0) = H_{0R}$(-jπ/T) $F_{0R}(0)$ as $F_{1R}(0) = 0$. This will lead to aliasing at every + jπ/T and -jπ/T as F(z) due to which we prefer to keep a zero-pair at those points. We can see that H_0 (jw) and H_1(jw) are power complementary FIR filters. Hence we can write:

$$|H_0(j\omega)|^2 + |H_1(j\omega)|^2 = 1$$

So, for obtaining this we use equiripple approximation to satisfy this.





CONCLUSION:

This project focuses on introducing multirate signal processing and Quadrature Mirror Filter Banks. Both digital and hybrid analog/digital filter banks have been discussed in brief along certain filter design techniques and optimization methods used to obtain a lower complexity. The reference research paper considered uses both the

We discuss some filter design techniques and optimisation methods to obtain a lower complexity for application. The synthesis filters in both the cases are digital FIR filters. Only the analysis filter types differ in both the cases i.e. using digital IIR filters and analog filters in digital and hybrid analog/digital fillers.

First the improvements to be done as done for analysis filters then we move on to synthesis filters. By reducing the individual complexity of each aspect, we can obtain a computationally less complex filter design in digital. We do not go into detail for hybrid analog/digital filters due to the less availability of papers in this field.

References:

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