~~Answ~~

Answers to Theoretical Questions

Q2.1b) We know that $F\{rect(T)\}$ gives a sinc function function which is fully real $\therefore$ The imaginary part is entirely 0 throughout.

In the phase plot, we notice a shift from $\pi/2$ and $-\pi/2$ continously, due to the shift in the $X(j\omega)$ values from positive to negative in a oscillatory fashion.

c) As T changes from T=2 to T=1, they are less space which gets widened and there is also a change in the amplitude.

As T changes from T=2 to T=4, they become more compressed due to more sampling and this change is also in the amplitude

They can be explained by the scaling property where

if $x(t) \longleftrightarrow X(j\omega) \Rightarrow x(at) = \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$

d) i) $x(t) = e^{jt}$

$$X(j\omega) = \frac{2\sin(\pi(\omega+1))}{\omega+1}$$ as per

fourier transform formula

Explanation for plots

→ We notice that the transform is fully real due to which $img(X(j\omega)) = 0$

phase plot ~~example~~ is same type as seen for previous question

ii) $x(t) = \cos(t)$

$$X(\omega) = \frac{-2\omega\sin(\pi\omega)}{\omega^2-1}$$

The transform is again purely real due to which ~~x(jω)~~ $img(X(j\omega)) = 0$

We can also notice the symmetry showing that it is even.

iii) $x(t) = \sin(t)$

$$X(j\omega) = \frac{-2j\sin(j\omega)}{\omega^2-1}$$

The plot is purely imaginary due to which $Im(X(j\omega)) = 0$

Q2.2

a) $H(\omega)$ = Frequency Response

The relationship between input and output coffs is as follows

$$b_k = a_k H(jk\omega_0)$$

As per the relationship we see that there is no change in the periodicity of the output signal. It remains same as input

b) For $\cos(t)$,

$$F\{\cos(t)\} = \pi(\delta(\omega-1)+\delta(\omega+1))$$

$$\cos t = \frac{e^{j}}{2}+\frac{e^{-j}}{2}$$

$A=[1,0,1]$ as we can see from the fourier transform $a_0=0$ as $\cos(t)$ is even. Then $a_1=1, a_{-1}=1$. The other coefficients are also zero $\forall k>1$ or $k<-1$

d) $\omega_c=2, \omega_0=1$

For LPF, output is 0 if $\omega_0>\omega_c$ and only if $\omega_0<\omega_c$ the signal is passed.

$\therefore$ For $\omega_c=2$, signal passes unaffected as $\omega_0<\omega_c$

For $\omega_c=0.5$, signal output is 0 as $\omega_0>\omega_c$

e) For high pass filter,

output is 0 if $\omega_0<\omega_c$ and is passed only when $\omega_0>\omega_c$.

So when $\omega_c=2$, signal output is 0, as $\omega_0<\omega_c$

$\omega_c=0.5$, signal is passed as $\omega_0>\omega_c$

c) $H(jw) = \dfrac{G}{a+jw}$

We know that a complex valued nature corresponds to a phase shift in f domain. When inverse fourier transform is taken, this corresponds to a time scale shift in time domain.

∴ The complex valued nature of LTI system corresponds to a time shift in the output of signal in the time domain.