Residual

Let $x_i = (x^{(i)}, y^{(i)})$, $X_i = (X^{(i)}, Y^{(i)}, Z^{(i)}, 1)$

and $P_K X_i = (p^{(i)}, q^{(i)}, r^{(i)})$ → Homogeneous

then our residual is

$$r = \left[\frac{p^{(1)}}{r^{(1)}} - x^{(1)}, \frac{q^{(1)}}{r^{(1)}} - y^{(1)}, \frac{p^{(2)}}{r^{(2)}} - x^{(2)}, \ldots, \frac{p^{(n)}}{r^{(n)}} - x^{(n)}, \frac{q^{(n)}}{r^{(n)}} - y^{(n)} \right]^T$$

Jacobian

The variables we are working with are the elements of P.

$\therefore$ $K = [P_{11}, P_{12}, P_{13}, P_{14}, P_{21}, P_{22} \text{ —— } P_{33}, P_{34}]$

$\Delta K = [\Delta P_{11}, \Delta P_{12}, \Delta P_{13} \text{ —— } \Delta P_{33}, \Delta P_{34}]$

hence Jacobian will be of the form

$$J = \left[\frac{\partial f}{\partial P_{11}}, \frac{\partial f}{\partial P_{12}}, \frac{\partial f}{\partial P_{13}}, \ldots, \frac{\partial f}{\partial P_{33}}, \frac{\partial f}{\partial P_{34}} \right]$$

in our residual we have a function for x coordinate & y coordinate

$$f_{xi} = \frac{p^{(i)}}{r^{(i)}} - x^{(i)}$$

Only $p^{(i)}$ & $r^{(i)}$ are functions of elements of P

$$f_{yi} = \frac{q^{(i)}}{r^{(i)}} - y^{(i)}$$

here also only $q^{(i)}$, $r^{(i)}$ are fns of elements of P.

$$p^{(i)} = P_{11} X^{(i)} + P_{12} Y^{(i)} + P_{13} Z^{(i)} + P_{14}$$
$$q^{(i)} = P_{21} X^{(i)} + P_{22} Y^{(i)} + P_{23} Z^{(i)} + P_{24}$$
$$r^{(i)} = P_{31} X^{(i)} + P_{32} Y^{(i)} + P_{33} Z^{(i)} + P_{34}$$

$$\therefore J_{xi} = \left[\frac{\partial f_{xi}}{\partial P_{11}} \quad \frac{\partial f_{xi}}{\partial P_{12}} \quad \cdots\cdots \quad \frac{\partial f_{xi}}{\partial P_{34}} \right]^T$$

$$= \left[\frac{X^{(i)}}{r^{(i)}}, \frac{Y^{(i)}}{r^{(i)}}, \frac{Z^{(i)}}{r^{(i)}}, \frac{1}{r^{(i)}}, 0, 0, 0, 0, \frac{-p^{(i)} X^{(i)}}{r^{(i)2}}, \frac{-p^{(i)} Y^{(i)}}{r^{(i)2}}, \frac{-p^{(i)} Z^{(i)}}{r^{(i)2}}, \frac{-p^{(i)}}{r^{(i)2}} \right] \quad — \quad (1)$$

$$J_{yi} = \left[\frac{\partial f_{yi}}{\partial P_{11}}, \frac{\partial f_{yi}}{\partial P_{12}} \quad \cdots\cdots \quad \frac{\partial f_{yi}}{\partial P_{34}} \right]^T$$

$$= \left[0, 0, 0, 0, \frac{X^{(i)}}{r^{(i)}}, \frac{Y^{(i)}}{r^{(i)}}, \frac{Z^{(i)}}{r^{(i)}}, \frac{1}{r^{(i)}}, \frac{-q^{(i)} X^{(i)}}{r^{(i)2}}, \frac{-q^{(i)} Y^{(i)}}{r^{(i)2}}, \frac{-q^{(i)} Z^{(i)}}{r^{(i)2}}, \frac{-q^{(i)}}{r^{(i)2}} \right] \quad — \quad (2)$$

Now final jacobian will look like

$$J = \begin{bmatrix} J_{x1} \\ J_{y1} \\ J_{x2} \\ J_{y2} \\ \vdots \\ J_{xn} \\ J_{yn} \end{bmatrix} \Rightarrow \underline{\underline{2n \times 12}}$$

J_{xi} & J_{yi} from ① & ②

$$J = \begin{bmatrix} \frac{X^{(1)}}{r^{(1)}}, \frac{Y^{(1)}}{r^{(1)}}, \frac{Z^{(1)}}{r^{(1)}}, 1, 0, 0, 0, 0, \frac{-p^{(1)}}{r^{(1)2}} X^{(1)}, \frac{-p^{(1)}}{r^{(1)2}} Y^{(1)}, \frac{-p^{(1)}}{r^{(1)2}} Z^{(1)}, \frac{-p^{(1)}}{r^{(1)2}} \\ 0, 0, 0, 0, \frac{X^{(1)}}{r^{(1)}}, \frac{Y^{(1)}}{r^{(1)}}, \frac{Z^{(1)}}{r^{(1)}}, 1, \frac{-q^{(1)}}{r^{(1)2}} X^{(1)}, \frac{-q^{(1)}}{r^{(1)2}} Y^{(1)}, \frac{-q^{(1)}}{r^{(1)2}} Z^{(1)}, \frac{-q^{(1)}}{r^{(1)2}} \\ \frac{X^{(2)}}{r^{(2)}}, \frac{Y^{(2)}}{r^{(2)}}, \ldots \ldots \frac{-p^{(2)}}{r^{(2)2}} \\ 0, 0, \ldots \ldots \frac{-q^{(2)}}{r^{(2)2}} \\ \vdots \qquad \vdots \\ \frac{X^{(n)}}{r^{(n)}}, \frac{Y^{(n)}}{r^{(n)}}, \ldots \ldots \frac{-p^{(n)}}{r^{(n)2}} \\ 0, 0, \ldots \ldots \frac{-q^{(n)}}{r^{(n)2}} \end{bmatrix}$$

$$\boxed{\text{Update step} = J^T J \, \Delta k = -J^T r}$$