Assignment10

Aravind A Anil

April 22, 2021

Problem Statement: Two independent random variables X and Y are uniformly distributed in the interval [-1,1].The probability that max [X,Y] is less than $\frac{1}{2}$ is

a) $\frac{3}{4}$ b) $\frac{9}{16}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

X $\sim$ U(-1,1)

$$\begin{aligned} F_X(x) &= P(X < x) \\ &= \int_{-1}^{x} \frac{1}{2} dx \\ &= \frac{1}{2}(x+1) \end{aligned}$$

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{1}{2}(x+1) & -1 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

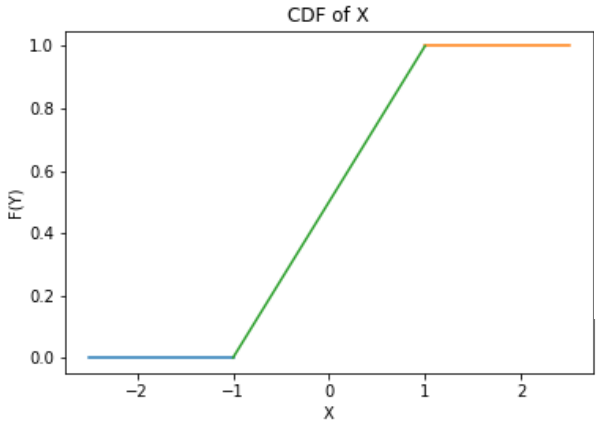

Figure 1: CDF of X

Y $\sim$ U(-1,1)

$$\begin{aligned} F_Y(y) &= P(Y < y) \\ &= \int_{-1}^{x} \frac{1}{2} dy \\ &= \frac{1}{2}(y+1) \end{aligned}$$

$$F_Y(y) = \begin{cases} 0 & y \leq -1 \\ \frac{1}{2}(y+1) & -1 < y < 1 \\ 1 & y \geq 1 \end{cases}$$

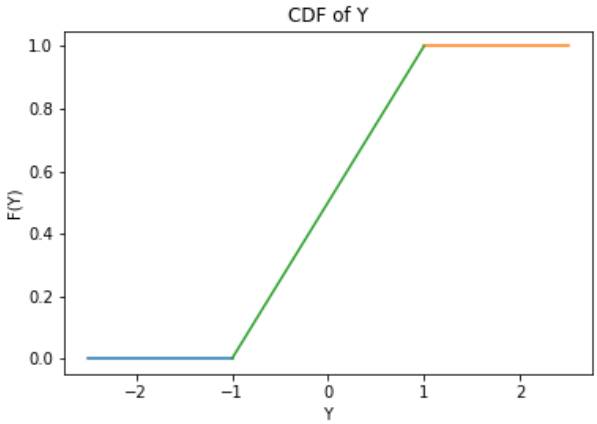

Figure 2: PDF OF Y

$Pr\Big(\max(X,Y)\Big) < \frac{1}{2}$ implies that

$X < \frac{1}{2}$ & $Y < \frac{1}{2}$

Since X and Y are independent

$$
\begin{aligned}
&Pr\left(X < \frac{1}{2}, Y < \frac{1}{2}\right) \\
&\qquad = Pr\left(X < \frac{1}{2}\right) \times Pr\left(Y < \frac{1}{2}\right) \\
&\qquad = F_X\left(\frac{1}{2}\right) \times F_Y\left(\frac{1}{2}\right) \\
&\qquad = \frac{3}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \\
&\qquad = \frac{9}{16}
\end{aligned}
$$

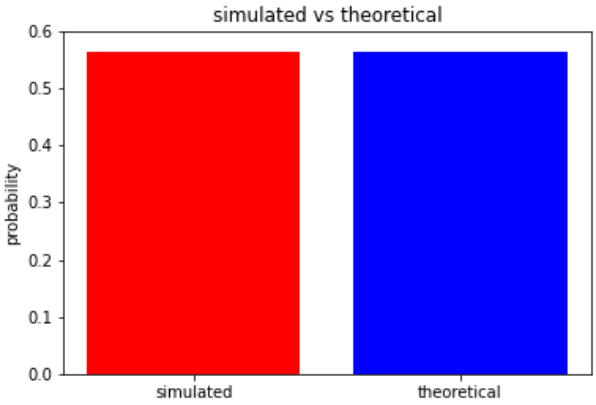

Figure 3: simulated vs theoretical