## Assignment10

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Statement: Two independent random variables X and Y are uniformly distributed in the interval [-1,1]. The probability that max [X,Y] is less than  $\frac{1}{2}$  is

a) 
$$\frac{3}{4}$$
 b)  $\frac{9}{16}$  c)  $\frac{1}{4}$  d)  $\frac{2}{3}$ 

c) 
$$\frac{1}{4}$$

d) 
$$\frac{2}{3}$$

 $X \sim U(-1,1)$ 

$$F_X(x) = P(X < x)$$
$$= \int_{-1}^{x} \frac{1}{2} dx$$
$$= \frac{1}{2}(x+1)$$

$$F_X(x) = \begin{cases} 0 & x \le -1\\ \frac{1}{2}(x+1) & -1 < x < 1\\ 1 & x \ge 1 \end{cases}$$

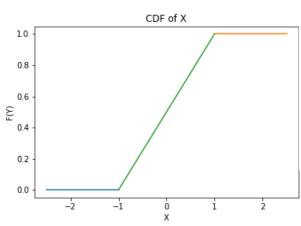


Figure 1: CDF of X

$$Y \sim U(-1,1)$$

$$F_Y(y) = P(Y < y)$$
$$= \int_{-1}^{y} \frac{1}{2} dy$$
$$= \frac{1}{2} (y+1)$$

$$F_Y(y) = \begin{cases} 0 & y \le -1\\ \frac{1}{2}(y+1) & -1 < y < 1\\ 1 & y \ge 1 \end{cases}$$

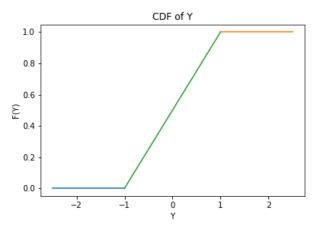


Figure 2: PDF OF Y

$$Pr\left(\max(X,Y)\right) < \frac{1}{2} \text{ implies that}$$
  
 $X < \frac{1}{2} \& Y < \frac{1}{2}$ 

Since X and Y are independent

$$Pr\left(X < \frac{1}{2}, Y < \frac{1}{2}\right)$$

$$= Pr\left(X < \frac{1}{2}\right) \times Pr\left(Y < \frac{1}{2}\right)$$

$$= F_X\left(\frac{1}{2}\right) \times F_Y\left(\frac{1}{2}\right)$$

$$= \frac{3}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2}$$

$$= \frac{9}{16}$$

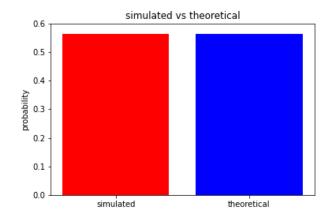


Figure 3: simulated vs theoretical