Assignment4-Probability and Random variables

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Problem Statement: The probability that a bulb produced by a factor will fuse after 150 days of use is .05.The probability that out of 5 such bulbs

i none

ii not more than one

iii more than one

iv at least one

will fuse after 150 days of work

Solution: Let X be the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials.Each trials are Bernoulli trial

P=.05(Probability that the bulb will fuse after 150days)

q=.95(probability that bulb will not fuse after 150days)

X has a binomial distribution with p=.05 and n=5

$$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x} \tag{1}$$

1 P(None)

Here x=0,therefore from equation..(1)

$$\begin{aligned} P(X = 0) &= {}^{5}C_{0}(.05)^{0}(.95)^{5} \\ &= .95^{5} \\ &= .7738 \end{aligned}$$

2 P(not more than one)

Here x=0,1,from equation..(1)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= {}^{5}C_{0}(.05)^{0}(.95)^{5} + {}^{5}C_{1}(.05)^{1}(.95)^{4} \\ &= .95^{5} + 5 \times .05 \times .95^{4} \\ &= .95^{4}[.95 + .25] \\ &= .95^{4} \times 1.20 \\ &= .9774075 \end{aligned}$$

3 P(more than one)

Here x=2,3,4,from equation..(1)

$$\begin{aligned} P(X > 1) &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= 1 - P(X \leq 1) \\ &= 1 - {}^{5}C_{0}(.05)^{0}(.95)^{5} + {}^{5}C_{1}(.05)^{1}(.95)^{4} \\ &= 1 - (.95^{5} + 5 \times .05 \times .95^{4}) \\ &= 1 - .95^{4}[.95 + .25] \\ &= 1 - .95^{4} \times 1.20 \\ &= 1 - .9774075 \\ &= .0225925 \end{aligned}$$

4 P(at least one)

Here x=1,2,3,4,5,from equation...(1)

$$\begin{aligned} P(X \leq 1) &= 1 - P(X = 0) \\ &= 1 -^{5} C_0 (.05)^0 (.95)^5 \\ &= 1 - .95^5 \\ &= .22621 \end{aligned}$$