

# Assignment 3—probability and Random Variable

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**problem statement:** Suppose we have four box A,B,C and D containing coloured marbles as given below: one of the box has

Table 1:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

been selected at random and a single marble is drawn from it.If the marble is red.What is the probability that it was drawn from box A?Box B?Box C?

**Solution:** Here we are having 4 boxes with 10 balls each.There is a equal likelihood of selecting four boxes

$X \in (0, 1, 2, 3)$  where 0 represents Box A,1 represents Box B,2 represents Box C,3 represents Box D

	A	B	C	D
X	0	1	2	3

$Y \in (0, 1, 2)$  where 0 represents Red marble,1 represents White marble,2 represents Black marble

	Red	White	Black
Y	0	1	2

$$Pr(X=0) = \frac{1}{4}$$

$$Pr(Y=0|X=0)=\frac{1}{10}$$

$$Pr(X=1)=\frac{1}{4}$$

$$Pr(Y=0|X=1)=\frac{6}{10}$$

$$Pr(X=2)=\frac{1}{4}$$

$$Pr(Y=0|X=2)=\frac{8}{10}$$

$$Pr(X=3)=\frac{1}{4}.$$

$$Pr(X=0|Y=3) = 0$$

Since red balls are in all the three boxes.The probability that selected ball is red is given by  $Pr(Y = 0)$ :Probability of getting a Red marble

$$= Pr(X = 0)Pr(Y = 0|X = 0) + Pr(X = 1)$$

$$Pr(Y = 0|X = 1) + Pr(X = 2)Pr(Y = 0|X = 2)$$

$$+ Pr(X = 3)Pr(Y = 0|X = 3)$$

$$= \frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10}$$

$$= \frac{1}{4} \left( \frac{1}{10} + \frac{6}{10} + \frac{8}{10} \right)$$

$$= \frac{1}{4} \times \frac{3}{2}$$

## 1 Part A

$Pr(X = 0|Y = 0)$ :probability that marble is drawn from box A given it is Red marble.By using bayes theorem

$$= \frac{Pr(Y = 0|X = 0).Pr(X = 0)}{Pr(Y = 0)}$$

$$= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}}$$

$$= \frac{1}{15}$$

## 2 Part B

$Pr(X = 1|Y = 0)$ :probability that marble is drawn from box B given it is Red marble.By

using bayes theorem

$$\begin{aligned} &= \frac{Pr(Y = 0|X = 1).Pr(X = 1)}{Pr(Y = 0)} \\ &= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\ &= \frac{2}{5} \end{aligned}$$

### 3 part C

$Pr(X = 2|Y = 0)$ :probability that marble is drawn from box C given it is Red marble.By using bayes theorem

$$\begin{aligned} &= \frac{Pr(Y = 0|X = 2).Pr(X = 2)}{Pr(Y = 0)} \\ &= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\ &= \frac{8}{15} \end{aligned}$$