Assignment5-Probability and Random variables

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Problem-Statement:A die is thrown again and again until three sixes are obtained.Find the probability of obtaining third six in the sixth throw of the die

Solution: Here we need to find the probability of obtaining third six in the sixth throw of die
P(getting 3rd six in 6th throw)

$$\begin{aligned}&= \text{P(getting 2 sixes in 5 throws)}\\&\quad \times \text{P(getting head in sixth throw)}\\&= \text{P(getting 2 sixes in 5 throws)} \times \frac{1}{6}\end{aligned}$$

Calculating P(getting 2 sixes in 5 throws)

variables	Description
X	number of sixes in 5 throws
p	probability of getting six
q	probability of not getting six
n	number of times die is thrown

n=5, p = $\frac{1}{6}$ and q = $\frac{5}{6}$
Here checking whether six is obtained or not in a given throw is a Bernoulli's trail.
Since we are throwing the die 5 times
X has a Binomial Distribution
P(X=x)=$^nC_x p^x q^{n-x}$

We need to find P(getting 2 sixes in 5 throws)

$$\begin{aligned}P(X=2) &= {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3\\&= \frac{5\times 4}{2}\times\left(\frac{1}{6}\right)^2\times\left(\frac{5}{6}\right)^3\\&= 10\times 5^3\times\left(\frac{1}{6}\right)^5\\&= \frac{10\times 5^3}{6^5}\end{aligned}$$

Hence the required probability is,

$$\begin{aligned}&= P(X=2)\times\frac{1}{6}\\&= \frac{10\times 5^3}{6^6}\times\frac{1}{6}\\&= \frac{10\times 5^3}{6^6}\\&= \frac{10\times 125}{46656}\\&= \frac{625}{23328}\\&= .0267\end{aligned}$$