

Assignment4-Probability and Random variables

Aravind A Anil

February 15, 2021

Problem Statement: The probability that a bulb produced by a factor will fuse after 150 days of use is .05. The probability that out of 5 such bulbs

i none

ii not more than one

iii more than one

iv at least one

will fuse after 150 days of work

Solution:

Variable	Description
X	Number of bulbs fused
p	Probability of getting fused bulb
q	probability that bulb is not fused
n	number of bulb picked

Table 1:

Here, $X=0,1,2,3,4,5$

$p=.05$

$q=.95$

Picking a bulb is a Bernoulli trial, so

X has a binomial distribution with $p=.05$ and

$n=5$

$X \sim \text{Bin}(p, n)$

$$P(X = x) = {}^n C_x p^x q^{n-x} \quad (1)$$

If $n=1$, then X will be Bernoulli Distribution

$X \sim \text{Bin}(p, 1)$

This can also be written as,

$X \sim \text{Ber}(p)$

1 P(None)

Here $x=0$, therefore from equation..(1)

$$\begin{aligned} P(X = 0) &= {}^5 C_0 (.05)^0 (.95)^5 \\ &= .95^5 \\ &= .7738 \end{aligned}$$

2 P(not more than one)

Here $x=0,1$, from equation..(1)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= {}^5 C_0 (.05)^0 (.95)^5 + {}^5 C_1 (.05)^1 (.95)^4 \\ &= .95^5 + 5 \times .05 \times .95^4 \\ &= .95^4 [.95 + .25] \\ &= .95^4 \times 1.20 \\ &= .9774075 \end{aligned}$$

3 P(more than one)

Here x=2,3,4,from equation..(1)

$$\begin{aligned}P(X > 1) &= P(X = 2) + P(X = 3) \\&\quad + P(X = 4) + P(X = 5) \\&= 1 - P(X \leq 1) \\&= 1 - {}^5C_0(.05)^0(.95)^5 + {}^5C_1(.05)^1(.95)^4 \\&= 1 - (.95^5 + 5 \times .05 \times .95^4) \\&= 1 - .95^4[.95 + .25] \\&= 1 - .95^4 \times 1.20 \\&= 1 - .9774075 \\&= .0225925\end{aligned}$$

4 P(at least one)

Here x=1,2,3,4,5,from equation...(1)

$$\begin{aligned}P(X \leq 1) &= 1 - P(X = 0) \\&= 1 - {}^5C_0(.05)^0(.95)^5 \\&= 1 - .95^5 \\&= .22621\end{aligned}$$