Assignment7

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Problem Statement:(6.17):A person plays a game of tossing a coin thrice.For each head,he is given Rs 2 by the organiser of the game and for each tail,he has to give Rs 1.50 to the organiser.Let X denotes the amount gained or lost by the person.show that X is random variable and exhibit it as a function on the sample space of the experiment.

Solution:,here we are tossing a coin three times so,
.Let $X_i, i = 0, 1, 2,$ be the value at the end of each toss.
Then $X_i = X_{i-1} + Y$,Where $Y \in \{2, -1.5\}$
$X_0 = Y$
$X_1 = X_0 + Y$
$X_2 = X_1 + Y$
$Z \in 0, 1$,where 0 represent getting a tail and 1 represents getting a head

	head	tail
Z	1	0

1. **X_0 can have two values**
   (a) Case-i
   - When **Z=0**,
     $X_0 = -1.5$
     $Pr(X_0|Z = 0) = \frac{1}{2}$

   (b) Case-ii
   - When **Z=1**,
     $X_0 = 2$
     $Pr(X_0|Z = 1) = \frac{1}{2}$

2. **X_1 can have four values**
   (a) Case-i
   - When $X_0 = -1.5$ & $Z = 0$

$$X_1 = -1.5 - 1.5$$
$$= -3$$

$$Pr(X_1|Z = 0, X_0 = -1.5) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

   (b) Case-ii
   - When $X_0 = 2\&Z = 0$

$$X_1 = 2 - 1.5$$
$$= .5$$

$$Pr(X_1|Z = 0, X_0 = 2) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

   (c) Case-iii
   - When $X_0 = -1.5\&$ $Z = 1$

$$X_1 = -1.5 + 2$$
$$= .5$$

$$Pr(X_1|Z = 1, X_0 = -1.5) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

(d) Case-iv

- When $X_0 = 2\&\ Z = 1$

$$\begin{aligned} X_1 &= 2 + 2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z = 1, X_0 = 2) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

3. **X_2 can have eight values**

(a) Case-i

- When $X_1 = -3$ & $Z = 0$

$$\begin{aligned} X_2 &= -3 - 1.5 \\ &= -4.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = -3, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

(b) Case-ii

- When $X_1 = -3$ & $Z = 1$

$$\begin{aligned} X_2 &= -3 + 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = -3, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

(c) Case-iii

- When $X_1 = .5$ & $Z = 0$

$$\begin{aligned} X_2 &= .5 + -1.5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = .5, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

(d) Case-iv

- When $X_1 = .5$ & $Z = 1$

$$\begin{aligned} X_2 &= .5 + 2 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = .5, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

(e) Case-v

Case-v will be same as Case-iii since $X_1 = .5$ is occuring two times

$$\begin{aligned} X_2 &= .5 - 1.5 \\ &= -1 \end{aligned}$$

$$Pr(X_2|X_1 = .5, Z = 0) = \tfrac{1}{8}$$

(f) Case-vi

Case-vi will be same as Case-iv since $X_1 = .5$ is occuring two times

$$\begin{aligned} X_2 &= .5 + 2 \\ &= 2.5 \end{aligned}$$

$$Pr(X_2|X_1 = .5, Z = 1) = \tfrac{1}{8}$$

(g) Case-vii

- When $X_1 = 4$ & $Z = 0$

$$\begin{aligned} X_2 &= 4 - 1.5 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

(h) Case-*viii*

- When $X_1 = 4$ & $Z = 0$

$$\begin{aligned} X_2 &= 4 + 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

Here values of X_2 can be 6,2.5,-1,-4.5
All these are real values
Hence,X_2=**{6,2.5,-1,-4.5}**
& X_2 is a **Random Variable**

X_2	-4.5	-1	2.5	6
$Pr(X_2)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Table 1: Probability Distribution of X_2

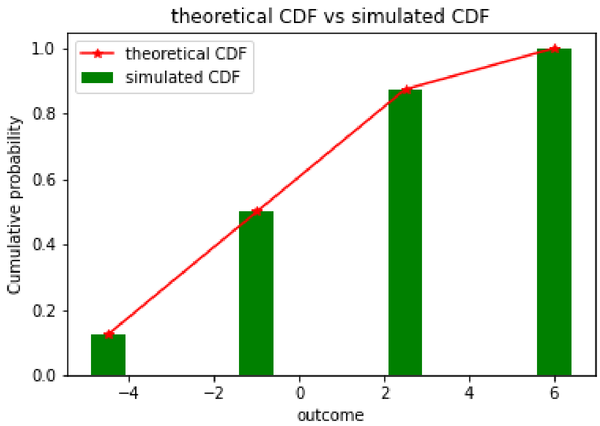

Figure 1: CDF

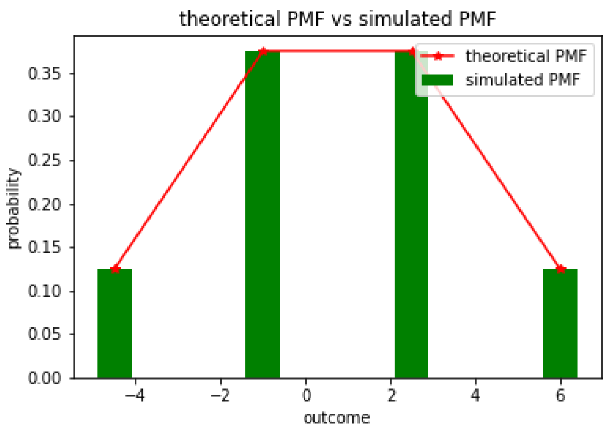

Figure 2: PMF