

# Assignment7

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**Problem Statement:** Let  $X_i, i = 0, 1, 2$ , be the value at the end of each toss. Then  $X_i = X_{i-1} + Y$ , Where  $Y \in \{2, -1.5\}$  and 2 for head and -1.5 for tail. You are supposed to find the distribution of  $X_2$ . Use this framework to solve the problem.

**Solution:** here we are tossing a coin three times so,

$$X_0 = Y$$

$$X_1 = X_0 + Y$$

$$X_2 = X_1 + Y$$

$Z \in 0, 1$ , where 0 represent getting a tail and 1 represents getting a head

	head	tail
Z	1	0

## 1 $X_0$ can have two values

### 1.1 Case-i

- When  $Z=0$ ,  
 $X_0 = -1.5$   
 $Pr(X_0|Z=0) = \frac{1}{2}$

### 1.2 Case-ii

- When  $Z=1$ ,  
 $X_0 = 2$   
 $Pr(X_0|Z=1) = \frac{1}{2}$

## 2 $X_1$ can have four values

### 2.1 Case-i

- When  $X_0 = -1.5$  &  $Z = 0$

$$\begin{aligned} X_1 &= -1.5 - 1.5 \\ &= -3 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z=0, X_0=-1.5) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

### 2.2 Case-ii

- When  $X_0 = 2$  &  $Z = 0$

$$\begin{aligned} X_1 &= 2 - 1.5 \\ &= .5 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z=0, X_0=2) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

### 2.3 Case-iii

- When  $X_0 = -1.5$  &  $Z = 1$

$$\begin{aligned} X_1 &= -1.5 + 2 \\ &= .5 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z=1, X_0=-1.5) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

## 2.4 Case-iv

- When  $X_0 = 2$  &  $Z = 1$

$$\begin{aligned}X_1 &= 2 + 2 \\ &= 4\end{aligned}$$

$$\begin{aligned}Pr(X_1|Z = 1, X_0 = 2) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4}\end{aligned}$$

## 3 $X_2$ can have eight values

### 3.1 Case-i

- When  $X_1 = -3$  &  $Z = 0$

$$\begin{aligned}X_2 &= -3 - 1.5 \\ &= -4.5\end{aligned}$$

$$\begin{aligned}Pr(X_2|X_1 = -3, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

### 3.2 Case-ii

- When  $X_1 = -3$  &  $Z = 1$

$$\begin{aligned}X_2 &= -3 + 2 \\ &= -1\end{aligned}$$

$$\begin{aligned}Pr(X_2|X_1 = -3, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

### 3.3 Case-iii

- When  $X_1 = .5$  &  $Z = 0$

$$\begin{aligned}X_2 &= .5 + -1.5 \\ &= -1\end{aligned}$$

$$\begin{aligned}Pr(X_2|X_1 = .5, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

### 3.4 Case-iv

- When  $X_1 = .5$  &  $Z = 1$

$$\begin{aligned}X_2 &= .5 + 2 \\ &= 2.5\end{aligned}$$

$$\begin{aligned}Pr(X_2|X_1 = .5, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

### 3.5 Case-v

Case-v will be same as Case-iii since  $X_1 = .5$  is occurring two times

$$\begin{aligned}X_2 &= .5 - 1.5 \\ &= -1\end{aligned}$$

$$Pr(X_2|X_1 = .5, Z = 0) = \frac{1}{8}$$

### 3.6 Case-vi

Case-vi will be same as Case-iv since  $X_1 = .5$  is occurring two times

$$\begin{aligned}X_2 &= .5 + 2 \\ &= 2.5\end{aligned}$$

$$Pr(X_2|X_1 = .5, Z = 1) = \frac{1}{8}$$

### 3.7 Case-vii

- When  $X_1 = 4$  &  $Z = 0$

$$\begin{aligned} X_2 &= 4 - 1.5 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

### 3.8 Case-viii

- When  $X_1 = 4$  &  $Z = 0$

$$\begin{aligned} X_2 &= 4 + 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

$X_2$	-4.5	-1	2.5	6
$Pr(X_2)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Table 1: Probability Distribution of  $X_2$

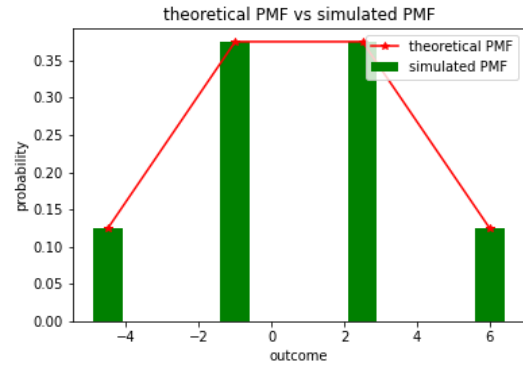


Figure 2: PMF

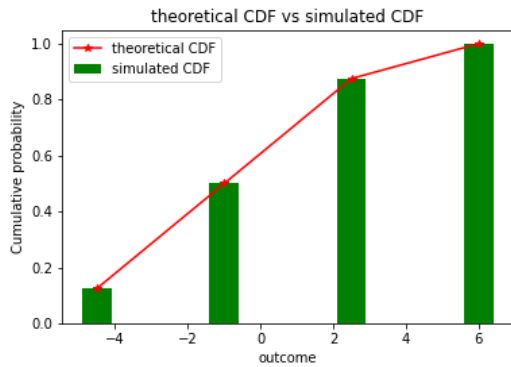


Figure 1: CDF