Assignment 3–probability and Random Variable

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problem statement: Suppose we have four box A,B,C and D containing coloured marbles as given below: one of the box has been selected at random and a single marble is drawn from it.If the marble is red.What is the probability that it was drawn from box A?Box B?Box C?

Table 1:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

Solution: Here we are having 4 boxes with 10 balls each.There is a equal likelihood of selecting four boxes

$X \in (0,1,2,3)$ where 0 represents Box A,1 represents Box B,2 represents Box C,3 represents Box D

$Y \in (0,1,2)$ where 0 represents Red marble,1 represents White marble,2 represents Black marble

$Pr(X=0) = \frac{1}{4}$

$Pr(Y=0|X=0)=\frac{1}{10}$

$Pr(X=1)=\frac{1}{4}$

$Pr(Y=0|X=1)=\frac{6}{10}$

$Pr(X=2)=\frac{1}{4}$

$Pr(Y=0|X=2)=\frac{8}{10}$

$Pr(X=3)=\frac{1}{4}$.

$Pr(X=0|Y=3) =0$

Since red balls are in all the three boxes.The probability that selected ball is red is given by

$Pr(Y=0)$:Probability of getting a Red marble

$$
\begin{aligned}
&= Pr(X=0)Pr(Y=0|X=0) + Pr(X=1) \\
&\quad Pr(Y=0|X=1) + Pr(X=2)Pr(Y=0|X=2) \\
&\quad + Pr(X=3)Pr(Y=0|X=3) \\
&= \frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} \\
&= \frac{1}{4}(\frac{1}{10} + \frac{6}{10} + \frac{8}{10}) \\
&= \frac{1}{4} \times \frac{3}{2}
\end{aligned}
$$

1 Part A

$Pr(X=0|Y=0)$:probability that marble is drawn from box A given it is Red marble.By using bayes theorem

$$
\begin{aligned}
&= \frac{Pr(Y=0|X=0).Pr(X=0)}{Pr(Y=0)} \\
&= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\
&= \frac{1}{15}
\end{aligned}
$$

2 Part B

$Pr(X=1|Y=0)$:probability that marble is drawn from box B given it is Red marble.By

using bayes theorem

$$= \frac{Pr(Y=0|X=1).Pr(X=1)}{Pr(Y=0)}$$
$$= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}}$$
$$= \frac{2}{5}$$

3 part C

$Pr(X = 2|Y = 0)$:probability that marble is drawn from box C given it is Red marble.By using bayes theorem

$$= \frac{Pr(Y=0|X=2).Pr(X=2)}{Pr(Y=0)}$$
$$= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}}$$
$$= \frac{8}{15}$$