

# Assignment 3—probability and Random Variable

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**problem statement:** Suppose we have four box A,B,C and D containing coloured marbles as given below: one of the box has

Table 1:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

been selected at random and a single marble is drawn from it.If the marble is red.What is the probability that it was drawn from box A?Box B?Box C?

**Solution:** Here we are having 4 boxes with 10 balls each.There is a equal likelihood of selecting four boxes

$X \in (0, 1, 2, 3)$  where 0 represents Box A,1 represents Box B,2 represents Box C,3 represents Box D

$Y \in (0, 1, 2)$  where 0 represents Red marble,1 represents White marble,2 represents Black marble

Suppose  $P(X=0)$ : probability that Box A is selected  $= \frac{1}{4}$

We are having 1 red balls in box A,so probability of getting the red ball from box A is given by

$P(Y=0/X=0)$ : probability that Red marble is selected given it is from Box A  $= \frac{1}{10}$

$P(X=1)$ : probability that Box B is selected  $= \frac{1}{4}$

We are having 6 red balls in box B,so

probability of getting the red ball from box B is given by

$P(Y=0/X=1)$ : probability that Red marble is selected from Box B  $= \frac{6}{10}$

$P(X=2)$ : probability that Box C is selected  $= \frac{1}{4}$

We are having 8 red balls in box C,so

probability of getting the red ball from box C is given by

$P(Y=0/X=2)$ : probability that Red marble is selected from Box C  $= \frac{8}{10}$

$P(X=3)$ : probability that Box D is selected  $= \frac{1}{4}$ .

We are having 0 red balls in box D,so

probability of getting the red ball from box D is given by

$P(X=0/Y=3)$ : probability that Red marble is selected from Box D  $= 0$

Since red balls are in all the three boxes.The probability that selected ball is red is given by

$P(Y=0)$ :Probability of getting a Red marble      bayes theorem

$$\begin{aligned}
 &= P(X=0)P(Y=0/X=0)+P(X=1) \\
 &\quad P(Y=0/X=1) + P(X=2)P(Y=0/X=2) \\
 &\quad + P(X=3)P(Y=0/X=3) \\
 &= \frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} \\
 &= \frac{1}{4} \left( \frac{1}{10} + \frac{6}{10} + \frac{8}{10} \right) \\
 &= \frac{1}{4} \times \frac{3}{2}
 \end{aligned}
 \qquad
 \begin{aligned}
 &= \frac{P(Y=0/X=1).P(X=1)}{P(Y=0)} \\
 &= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\
 &= \frac{2}{5}
 \end{aligned}$$

### 3 part C

$P(X=2/Y=0)$ :probability that marble is drawn from box C given it is Red marble.By using bayes theorem

## 1 Part A

$P(X=0/Y=0)$ :probability that marble is drawn from box A given it is Red marble.By using bayes theorem

$$\begin{aligned}
 &= \frac{P(Y=0/X=0).P(X=0)}{P(Y=0)} \\
 &= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\
 &= \frac{1}{15}
 \end{aligned}
 \qquad
 \begin{aligned}
 &= \frac{P(Y=0/X=2).P(X=2)}{P(Y=0)} \\
 &= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{2}} \\
 &= \frac{8}{15}
 \end{aligned}$$

## 2 Part B

$P(X=1/Y=0)$ :probability that marble is drawn from box B given it is Red marble.By using