Assignment4-Probability and Random variables

Aravind A Anil

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Problem Statement: The probability that a bulb produced by a factor will fuse after 150 days of use is .05.The probability that out of 5 such bulbs

i none

ii not more than one

iii more than one

iv at least one

will fuse after 150 days of work

Solution:

Variable	Description
X	Number of bulbs fused
p	Probability of getting fused bulb
q	probability that bulb is not fused
n	number of bulb picked

Table 1:

Here, X=0,1,2,3,4,5
p=.05
q=.95
Picking a bulb is a Bernoulli trial,so
X has a binomial distribution with p=.05 and n=5
X∼Bin(p,n)

$$P(X = x) =^{n} C_x p^x q^{n-x} \tag{1}$$

If n=1,then X will be Bernoulli Distribution
X∼Bin(p,1)
This can also be written as,
X∼Ber(p)

1 P(None)

Here x=0,therefore from equation..(1)

$$\begin{aligned} P(X = 0) &=^{5} C_0 (.05)^0 (.95)^5 \\ &= .95^5 \\ &= .7738 \end{aligned}$$

2 P(not more than one)

Here x=0,1,from equation..(1)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &=^{5} C_0 (.05)^0 (.95)^5 +^{5} C_1 (.05)^1 (.95)^4 \\ &= .95^5 + 5 \times .05 \times .95^4 \\ &= .95^4 [.95 + .25] \\ &= .95^4 \times 1.20 \\ &= .9774075 \end{aligned}$$

3 P(more than one)

Here x=2,3,4,from equation..(1)

$$\begin{aligned} P(X>1) &= P(X=2)+P(X=3) \\ &\quad + P(X=4)+P(X=5) \\ &= 1-P(X\leq 1) \\ &= 1-{}^{5}C_0(.05)^0(.95)^5+{}^{5}C_1(.05)^1(.95) \\ &= 1-(.95^5+5\times .05\times .95^4) \\ &= 1-.95^4[.95+.25] \\ &= 1-.95^4\times 1.20 \\ &= 1-.9774075 \\ &= .0225925 \end{aligned}$$

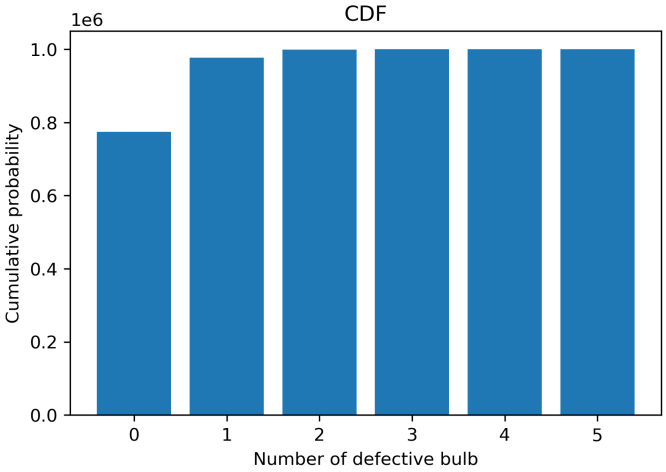

Figure 1: simulated CDF plot

4 P(at least one)

Here x=1,2,3,4,5,from equation...(1)

$$\begin{aligned} P(X\leq 1) &= 1-P(X=0) \\ &= 1-{}^{5}C_0(.05)^0(.95)^5 \\ &= 1-.95^5 \\ &= .22621 \end{aligned}$$

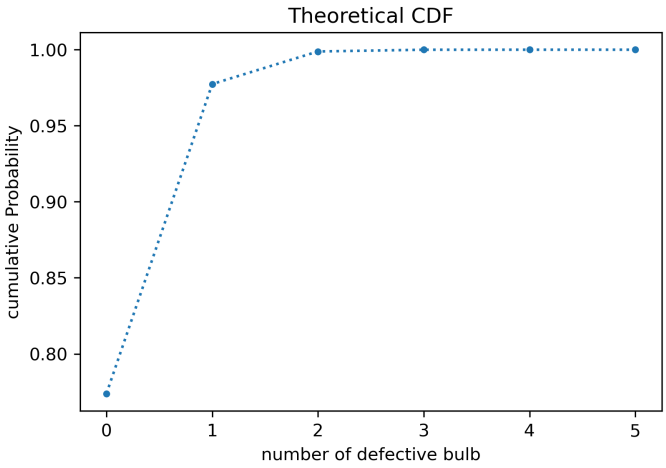

Figure 2: theoretical CDF plot

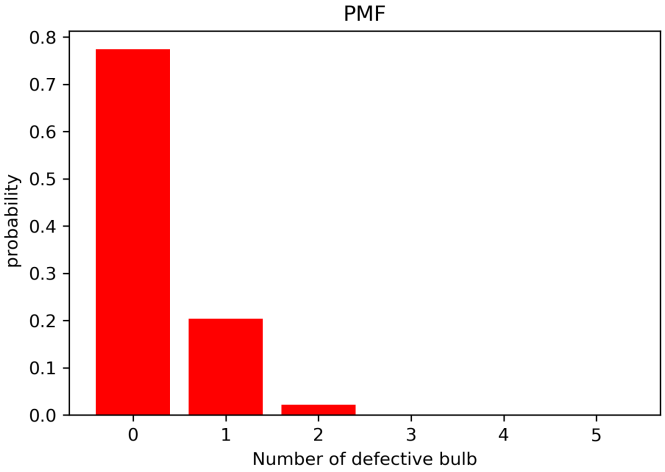

Figure 3: simulated PMF plot

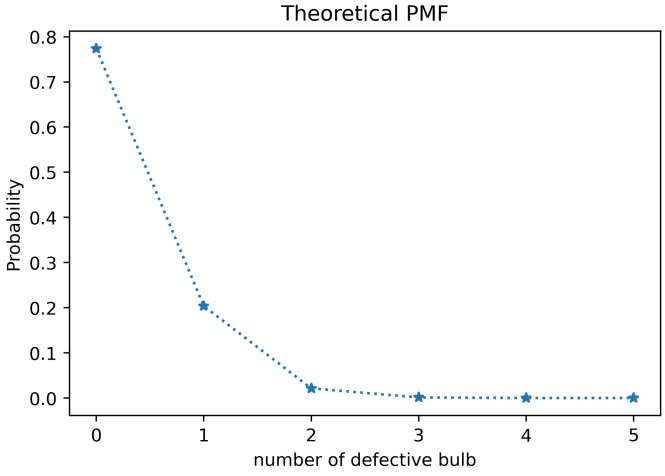

Figure 4: theoretical PMF plot