Assignment7

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Problem Statement:(6.17):A person plays a game of tossing a coin thrice.For each head,he is given Rs 2 by the organiser of the game and for each tail,he has to give Rs 1.50 to the organiser.Let $X_i, i = 0, 1, 2,$ be the value at the end of each toss.Then $X_i = X_{i-1} + Y$,Where $Y \in \{2, -1.5\}$ find the distribution of X2.

Solution:,here we are tossing a coin three times so,

$X_0 = Y$

$X_1 = X_0 + Y$

$X_2 = X_1 + Y$

$Z \in 0, 1$,where 0 represent getting a tail and 1 represents getting a head

	head	tail
Z	1	0

1 X_0 can have two values

1.1 Case-i

- When **Z=0**,
  $X_0 = -1.5$
  $Pr(X_0|Z = 0) = \frac{1}{2}$

1.2 Case-ii

- When **Z=1**,
  $X_0 = 2$
  $Pr(X_0|Z = 1) = \frac{1}{2}$

2 X_1 can have four values

2.1 Case-i

- When $X_0 = -1.5$ & $Z = 0$

$$\begin{aligned} X_1 &= -1.5 - 1.5 \\ &= -3 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z = 0, X_0 = -1.5) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

2.2 Case-ii

- When $X_0 = 2 \& Z = 0$

$$\begin{aligned} X_1 &= 2 - 1.5 \\ &= .5 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z = 0, X_0 = 2) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

2.3 Case-iii

- When $X_0 = -1.5 \&\ Z = 1$

$$\begin{aligned} X_1 &= -1.5 + 2 \\ &= .5 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z = 1, X_0 = -1.5) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

2.4 Case-iv

- When $X_0 = 2$& $Z = 1$

$$\begin{aligned} X_1 &= 2 + 2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} Pr(X_1|Z = 1, X_0 = 2) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

3 X_2 can have eight values

3.1 Case-i

- When $X_1 = -3$ & $Z = 0$

$$\begin{aligned} X_2 &= -3 - 1.5 \\ &= -4.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = -3, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3.2 Case-ii

- When $X_1 = -3$ & $Z = 1$

$$\begin{aligned} X_2 &= -3 + 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = -3, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3.3 Case-iii

- When $X_1 = .5$ & $Z = 0$

$$\begin{aligned} X_2 &= .5 + -1.5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = .5, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3.4 Case-iv

- When $X_1 = .5$ & $Z = 1$

$$\begin{aligned} X_2 &= .5 + 2 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = .5, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3.5 Case-v

Case-v will be same as Case-iii since $X_1 = .5$ is occuring two times

$$\begin{aligned} X_2 &= .5 - 1.5 \\ &= -1 \end{aligned}$$

$Pr(X_2|X_1 = .5, Z = 0) = \frac{1}{8}$

3.6 Case-vi

Case-vi will be same as Case-iv since $X_1 = .5$ is occuring two times

$$\begin{aligned} X_2 &= .5 + 2 \\ &= 2.5 \end{aligned}$$

$Pr(X_2|X_1 = .5, Z = 1) = \frac{1}{8}$

3.7 Case-*vii*

- When $X_1 = 4$ & $Z = 0$

$$\begin{aligned} X_2 &= 4 - 1.5 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 0) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3.8 Case-*viii*

- When $X_1 = 4$ & $Z = 0$

$$\begin{aligned} X_2 &= 4 + 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} Pr(X_2|X_1 = 4, Z = 1) &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

X_2	-4.5	-1	2.5	6
$Pr(X_2)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Table 1: Probability Distribution of X_2

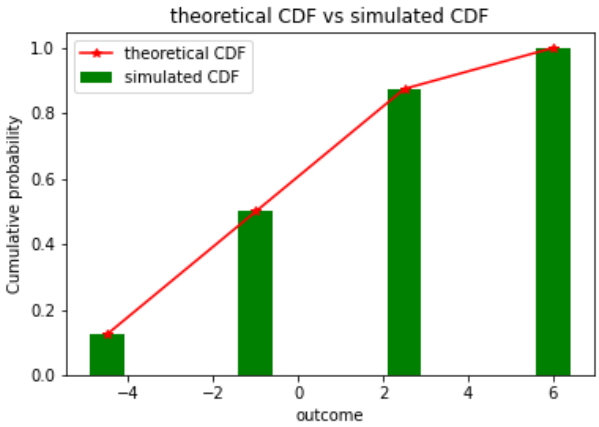

Figure 1: CDF

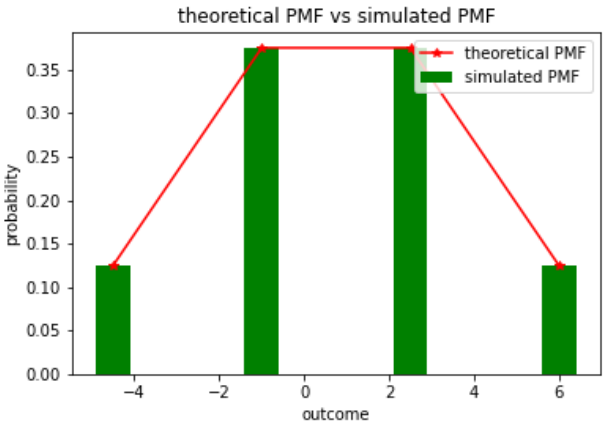

Figure 2: PMF