

# Assignment10

Aravind A Anil

April 16, 2021

**Problem Statement:** Two independent random variables X and Y are uniformly distributed in the interval  $[-1,1]$ . The probability that  $\max[X,Y]$  is less than  $\frac{1}{2}$  is

a)  $\frac{3}{4}$     b)  $\frac{9}{16}$     c)  $\frac{1}{4}$     d)  $\frac{2}{3}$

X and Y are having uniform distribution

$$f_X(x) = \begin{cases} \frac{1}{2} & \text{if } -1 < X < 1 \\ 0 & \text{otherwise} \end{cases}$$

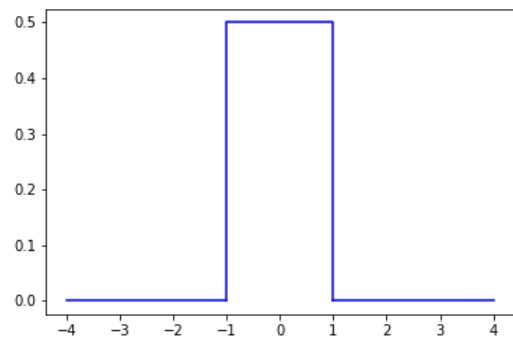


Figure 2: PDF OF Y

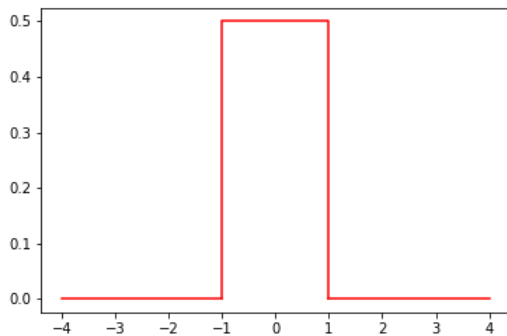


Figure 1: PDF of X

$$f_Y(y) = \begin{cases} \frac{1}{2} & \text{if } -1 < Y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$Pr(\max(X,Y)) < \frac{1}{2}$  implies that  
 $X < \frac{1}{2}$  &  $Y < \frac{1}{2}$

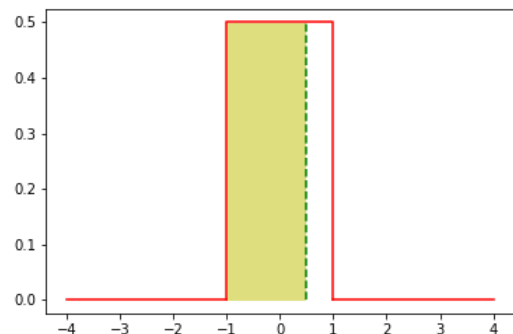


Figure 3:  $X < \frac{1}{2}$

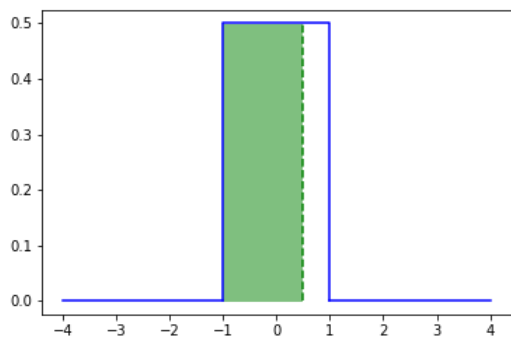


Figure 4:  $Y < \frac{1}{2}$

Since  $X$  and  $Y$  are independent

$$\begin{aligned}
 Pr\left(X < \frac{1}{2}, Y < \frac{1}{2}\right) &= Pr\left(X < \frac{1}{2}\right)Pr\left(Y < \frac{1}{2}\right) \\
 &= \text{area}\left(X < \frac{1}{2}\right) \times \text{area}\left(Y < \frac{1}{2}\right) \\
 &= \frac{3}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \\
 &= \frac{9}{16}
 \end{aligned}$$

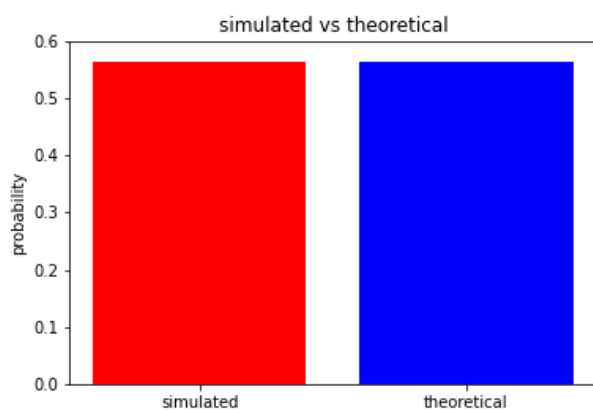


Figure 5: simulated vs theoretical