Assignment10

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April 12, 2021

Problem Statement: Two independent random variables X and Y are uniformly distributed in the interval [-1,1].The probability that max [X,Y] is less than $\frac{1}{2}$ is

a) $\frac{3}{4}$ b) $\frac{9}{16}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

X and Y are having uniform distribution

$$f_X(x) = \begin{cases} \frac{1}{2} & \text{if} -1 < X < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{2} & \text{if} -1 < Y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$Pr(\max(X,Y) < \frac{1}{2}$ implies that

$X < \frac{1}{2}$ & $Y < \frac{1}{2}$

Since X and Y are independent

$$\begin{aligned} Pr(X < \frac{1}{2}, Y < \frac{1}{2}) \\ &= \int_{-1}^{\frac{1}{2}} \int_{-1}^{\frac{1}{2}} \frac{1}{2}.\frac{1}{2} dxdy \\ &= \frac{1}{4} \int_{-1}^{\frac{1}{2}} \left[\frac{1}{2} + 1\right] dy \\ &= \frac{1}{4}.\frac{3}{2}\left[\frac{1}{2} + 1\right] \\ &= \frac{1}{4} \times \frac{3}{2} \times \frac{3}{2} \\ &= \frac{9}{16} \end{aligned}$$

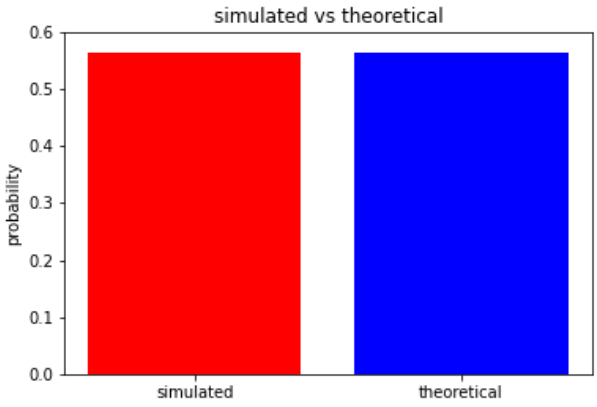

Figure 1: simulated vs theoretical