Assignment7

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Problem Statement:(6.17):A person plays a game of tossing a coin thrice.For each head,he is given Rs 2 by the organiser of the game and for each tail,he has to give Rs 1.50 to the organiser.Let X denotes the amount gained or lost by the person.show that X is random variable and exhibit it as a function on the sample space of the experiment.

1 Condition-1

Solution:,here we are tossing a coin three times so,
.Let $X_i, i = 0, 1, 2$, be the value at the end of each toss.
Then $X_i = X_{i-1} + Y$,Where $Y \in \{2, -1.5\}$
$X_0 = Y$
$X_1 = X_0 + Y$
$X_2 = X_1 + Y$
$Z \in 0, 1$,where 0 represent getting a tail and 1 represents getting a head

	head	tail
Z	1	0

Let,
Z_0 represents toss 1
Z_1 represents toss 2
Z_2 represents toss 3

There can be eight cases

1. Case-i
   - when $Z_0 = 0$,$Z_1 = 0$,$Z_2 = 0$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 - 1.5 \\ &= -3 \\ X_2 &= -3 - 1.5 \\ &= -4.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 0, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

2. Case-ii
   - when $Z_0 = 0$,$Z_1 = 1$,$Z_2 = 1$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + 2 \\ &= .5 \\ X_2 &= -.5 + 2 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 1, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3. Case-iii

- when $Z_0 = 0$,$Z_1 = 0$,$Z_2 = 1$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + -1.5 \\ &= -3 \\ X_2 &= -3 + 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 0, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

4. Case-iv

- when $Z_0 = 1$,$Z_1 = 1$,$Z_2 = 1$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 + 2 \\ &= 4 \\ X_2 &= 4 + 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 1, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

5. Case-v

- when $Z_0 = 1$,$Z_1 = 0$,$Z_2 = 1$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 - 1.5 \\ &= .5 \\ X_2 &= .5 + 2 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 0, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

6. Case-vi

- when $Z_0 = 1$,$Z_1 = 0$,$Z_2 = 0$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 - 1.5 \\ &= .5 \\ X_2 &= .5 + -1.5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 0, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

7. Case-vii

- when $Z_0 = 1$,$Z_1 = 1$,$Z_2 = 0$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 + 2 \\ &= 4 \\ X_2 &= 4 + -1.5 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 1, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

8. Case-viii

- when $Z_0 = 0$,$Z_1 = 1$,$Z_2 = 0$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + 2 \\ &= .5 \\ X_2 &= .5 + -1.5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 1, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

Here values of X_2 can be 6,2.5,-1,-4.5
All these are real values
Hence,X_2=**{6,2.5,-1,-4.5}**
& X_2 is a **Random Variable**

X_2	-4.5	-1	2.5	6
$Pr(X_2)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Table 1: Probability Distribution of X_2

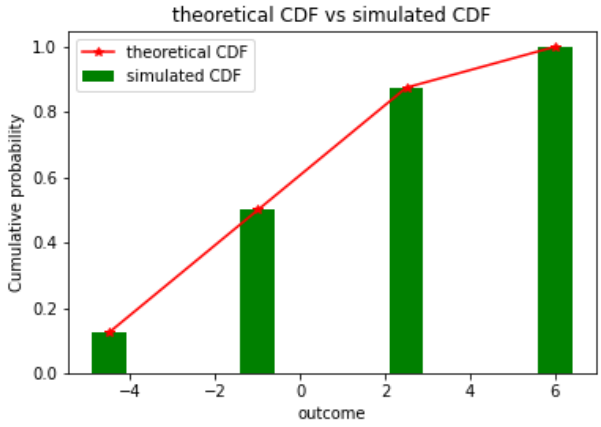

Figure 1: CDF

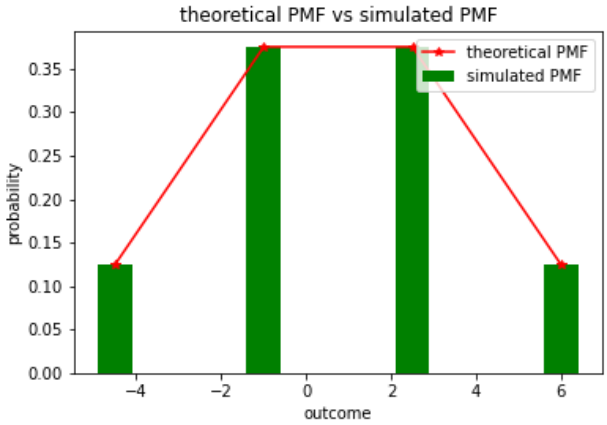

Figure 2: PMF

2 Condition-2

$X_0 = Y$
$X_1 = X_0 + Y$
$X_2 = X_1 + X_0 + Y$
There can be eight cases

1. Case-i

   - when $Z_0 = 0$,$Z_1 = 0$,$Z_2 = 0$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 - 1.5 \\ &= -3 \\ X_2 &= -3 - 1.5 - 1.5 \\ &= -6 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 0, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

2. Case-ii

   - when $Z_0 = 0$,$Z_1 = 1$,$Z_2 = 1$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + 2 \\ &= .5 \\ X_2 &= -.5 + -1.5 + 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 1, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

3. Case-iii

- when $Z_0 = 0$,$Z_1 = 0$,$Z_2 = 1$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + -1.5 \\ &= -3 \\ X_2 &= -3 + -1.5 + 2 \\ &= -2.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 0, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

4. Case-iv

- when $Z_0 = 1$,$Z_1 = 1$,$Z_2 = 1$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 + 2 \\ &= 4 \\ X_2 &= 4 + 2 + 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 1, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

5. Case-v

- when $Z_0 = 1$,$Z_1 = 0$,$Z_2 = 1$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 - 1.5 \\ &= .5 \\ X_2 &= .5 + 2 + 2 \\ &= 4.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 0, Z_2 = 1) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

6. Case-vi

- when $Z_0 = 1$,$Z_1 = 0$,$Z_2 = 0$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 - 1.5 \\ &= .5 \\ X_2 &= .5 + 2 + -1.5 \\ &= 1 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 0, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

7. Case-vii

- when $Z_0 = 1$,$Z_1 = 1$,$Z_2 = 0$

$$\begin{aligned} X_0 &= 2 \\ X_1 &= 2 + 2 \\ &= 4 \\ X_2 &= 4 + 2 + -1.5 \\ &= 4.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 1, Z_1 = 1, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

8. Case-viii

- when $Z_0 = 0$,$Z_1 = 1$,$Z_2 = 0$

$$\begin{aligned} X_0 &= -1.5 \\ X_1 &= -1.5 + 2 \\ &= .5 \\ X_2 &= .5 + -1.5 - 1.5 \\ &= -2.5 \end{aligned}$$

$$\begin{aligned} Pr(Z_0 = 0, Z_1 = 1, Z_2 = 0) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

X_2	6	-2.5	1	4.5	8
$Pr(X_2)$	$\frac{1}{8}$	$\frac{2}{8}=\frac{1}{4}$	$\frac{2}{8}=\frac{1}{4}$	$\frac{2}{8}=\frac{1}{4}$	$\frac{1}{8}$

Table 2: Probability Distribution of X_2

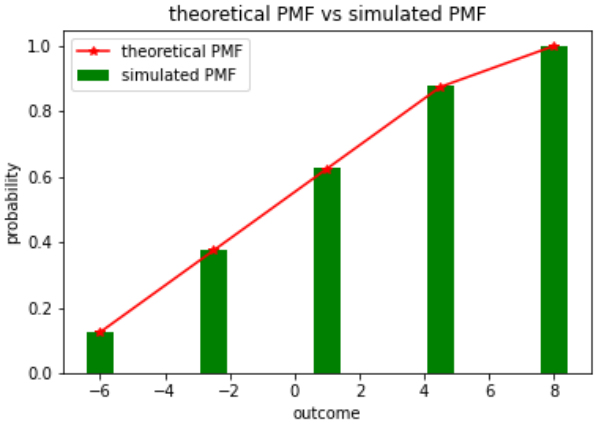

Figure 3: CDF

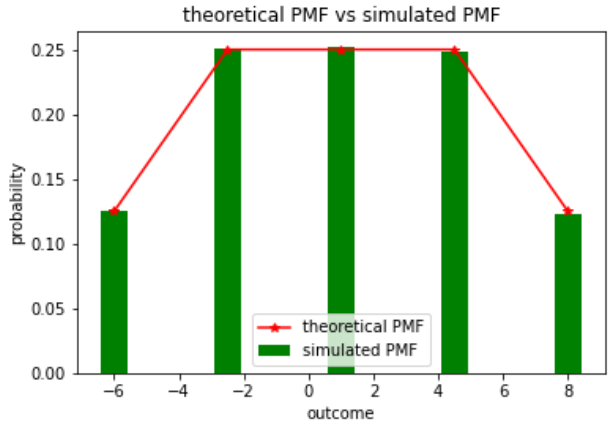

Figure 4: PMF