

1. Descriptive Statistics

Descriptive statistics summarise and describe the main characteristics of a dataset.

Key Measures and Real-World Examples

a. Measures of Central Tendency

1. Mean (Average)

Formula:

$$\text{Mean} = \sum X / n$$

Example:

Monthly sales (in units) for a store: [120, 150, 180, 200, 250]

Calculation:

$$\text{Mean} = (120 + 150 + 180 + 200 + 250) / 5 = 180$$

Interpretation: The store sells an average of 180 units per month.

2. Median

Example:

Sales data: [120, 150, 180, 200, 250]

Sorted order: [120, 150, 180, 200, 250]

The middle value is 180.

3. Mode

The most frequent value in the dataset.

Example:

Customer satisfaction ratings: [4, 4, 5, 3, 4, 3, 5, 4]

Mode = 4 (most frequent).

b. Measures of Dispersion

1. Range

Formula:

$$\text{Range} = \text{Maximum Value} - \text{Minimum Value}$$

Example:

Sales data: [120, 150, 180, 200, 250]

$$\text{Range} = 250 - 120 = 130$$

2. Variance and Standard Deviation

Variance Formula:

$$\text{Variance } (\sigma^2) = \sum (X_i - \bar{X})^2 / n$$

Standard Deviation Formula:

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

Example:

Daily temperatures: [30, 32, 34, 36, 38]

- Mean = 34

- Variance = $[(30 - 34)^2 + (32 - 34)^2 + (34 - 34)^2 + (36 - 34)^2 + (38 - 34)^2] / 5 = 8$

- Standard Deviation = $\sqrt{8} = 2.83$

2. Inferential Statistics

Inferential statistics help draw conclusions about a population based on sample data.

Key Concepts and Real-World Examples

a. Hypothesis Testing

- Null Hypothesis (H_0): There is no difference/effect.
- Alternative Hypothesis (H_a): There is a difference/effect.

Example:

Does a new marketing strategy increase sales?

- Sales before strategy: [200, 210, 215, 220, 230]
- Sales after strategy: [250, 260, 270, 280, 300]

Interpretation: If $p < 0.05$, the difference is statistically significant.

b. Confidence Intervals (CI)

Formula:

$$CI = \bar{X} \pm Z \times (\sigma / \sqrt{n})$$

Example:

- Sample mean = 180, Std Dev = 15, $n = 25$, $Z = 1.96$
- $CI = 180 \pm 1.96 \times (15 / \sqrt{25})$
- $CI = (174.12, 185.88)$

3. Probability

Probability quantifies the likelihood of events.

Key Concepts and Real-World Examples

a. Simple Probability

Example:

A bag contains 5 red, 3 blue, and 2 green balls.

- Total balls = $5 + 3 + 2 = 10$
- Probability (P) = Favourable Outcomes / Total Outcomes
- $P(\text{Blue}) = 3 / 10 = 0.3$

b. Conditional Probability

Formula:

$$P(A | B) = P(A \cap B) / P(B)$$

Example:

A factory produces 60% A-grade items and 40% B-grade items. If 5% of A-grade and 10% of B-grade items are defective, what is the probability that a defective item is A-grade?

We are given:

- $P(A) = 0.60$ (probability that an item is A-grade)
- $P(B) = 0.40$ (probability that an item is B-grade)
- $P(D | A) = 0.05$ (probability that an A-grade item is defective)
- $P(D | B) = 0.10$ (probability that a B-grade item is defective)

We want to find $P(A | D)$, the probability that a defective item is A-grade.

Bayes' Theorem:

Bayes' Theorem states:

$$P(A | D) = (P(D | A) * P(A)) / P(D)$$

Where:

- $P(A | D)$ is the probability that the item is A-grade given that it is defective.
- $P(D | A)$ is the probability that an A-grade item is defective.
- $P(A)$ is the probability that an item is A-grade.
- $P(D)$ is the total probability that an item is defective.

To calculate $P(D)$ (the total probability of a defective item), we use the law of total probability:

$$P(D) = P(D | A) * P(A) + P(D | B) * P(B)$$

Step 1: Calculate P(D)

$$P(D) = (0.05 * 0.60) + (0.10 * 0.40) \quad P(D) = 0.03 + 0.04 \quad P(D) = 0.07$$

Step 2: Apply Bayes' Theorem

Now, we can calculate $P(A | D)$:

$$P(A | D) = (0.05 * 0.60) / 0.07 \quad P(A | D) = 0.03 / 0.07 \quad P(A | D) \approx 0.4286$$

Conclusion:

The probability that a defective item is A-grade is approximately 0.4286 or 42.86%.

c. Bayes' Theorem

Formula:

$$P(A | B) = [P(B | A) \times P(A)] / P(B)$$

4. Frequency Distribution

Frequency distribution shows how often each value occurs.

Example: Exam scores: [70, 80, 90, 70, 80, 90, 90, 100, 70]

Score	Frequency
70	3
80	2
90	3
100	1