

# ELEC-E8101 Group project:

## Lab A report

### Group 05

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#### Reporting of Task 4.1

The given Linearized equations of Motions are:

$$(I_b + m_b l_b^2) \ddot{\theta}_b = m_b l_b g \theta_b - m_b l_b \ddot{x}_w - \frac{2K_t}{R_m} v_m + \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right)$$

$$\left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) \ddot{x}_w = -m_b l_b l_w \ddot{\theta}_b + \frac{2K_t}{R_m} v_m - \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right)$$

Solving the above equations (as suggested in the hint) we get

$$\begin{pmatrix} m_b l_b & (I_b + m_b l_b^2) \\ \left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) & m_b l_b l_w \end{pmatrix} \begin{pmatrix} \ddot{x}_w \\ \ddot{\theta}_b \end{pmatrix} = \begin{pmatrix} 0 & \left( \frac{2K_e K_t}{R_m} \right) + \frac{b_f}{l_w} & m_b l_b g & -\frac{2K_e K_t}{R_m} - b_f \\ 0 & -\left( \frac{2K_e K_t}{R_m} \right) - \frac{b_f}{l_w} & 0 & \frac{2K_e K_t}{R_m} + b_f \end{pmatrix} \begin{pmatrix} x_w \\ \dot{x}_w \\ \theta_b \\ \dot{\theta}_b \end{pmatrix}$$

4.1.1 The choice of X is

$$\begin{bmatrix} x_w \\ \dot{x}_w \\ \theta_b \\ \dot{\theta}_b \end{bmatrix}$$

The input to the system is

$$u = v_m$$

and the output of the system is

$$y = \theta_b$$

4.1.2 The equations are represented in the State space form

The matrices are

$$\alpha = \begin{bmatrix} 0 & \left( \frac{2K_e K_t}{R_m} \right) + \frac{b_f}{l_w} & m_b l_b g & -\frac{2K_e K_t}{R_m} - b_f \\ 0 & -\left( \frac{2K_e K_t}{R_m} \right) - \frac{b_f}{l_w} & 0 & \frac{2K_e K_t}{R_m} + b_f \end{bmatrix}$$

$$\beta = \begin{bmatrix} -\frac{2K_t}{R_m} \\ \frac{2K_t}{R_m} \end{bmatrix}$$

$$\gamma = \begin{bmatrix} m_b l_b & (I_b + m_b l_b^2) \\ (\frac{I_w}{l_w} + l_w m_b + l_w m_w) & m_b l_b l_w \end{bmatrix}$$

Now, with the above mentioned matrices, we get

$$A = \gamma^{-1} \alpha$$

$$B = \gamma^{-1} \beta$$

Also we chose the matrix X as 4\*1 matrix so we assume ,

$$\dot{X} = \begin{bmatrix} \dot{x}_w \\ \ddot{x}_w \\ \dot{\theta}_b \\ \ddot{\theta}_b \end{bmatrix}$$

After derivations , we get the new matrices as

$$\gamma_{new} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_b l_b & (I_b + m_b l_b^2) & 0 \\ 0 & (\frac{I_w}{l_w} + l_w m_b + l_w m_w) & m_b l_b l_w & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\alpha_{new} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & (\frac{2K_e K_t}{R_m}) + \frac{b_f}{l_w} & m_b l_b g & -\frac{2K_e K_t}{R_m} - b_f \\ 0 & -(\frac{2K_e K_t}{R_m}) - \frac{b_f}{l_w} & 0 & \frac{2K_e K_t}{R_m} + b_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\beta_{new} = \begin{bmatrix} 0 \\ -\frac{2K_t}{R_m} \\ 0 \\ \frac{2K_t}{R_m} \end{bmatrix}$$

And the updated matrices for A, B, C and D are

$$A = \gamma_{new}^{-1} \alpha_{new}$$

$$B = \gamma_{new}^{-1} \beta_{new}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

4.1.3 :The numeric forms of A,B,C and D matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -773.8 & -6.6 & 16.2 \\ 0 & 0 & 0 & 1 \\ 0 & 3313.2 & 63.1 & -69.6 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 36.5980 \\ 0 \\ -156.7072 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

## Reporting of Task 4.2

4.2.1 : The transfer function of the system is computed from the matrices A,B,C and D using Matlab's inbuilt functions

$$G(s) = -156.7 \frac{s}{(s + 843.4)(s + 5.64)(s - 5.679)}$$

For the calculation of the transfer function of the system we used two methods,

Method 1 : `sys = ss(A,B,C,D); H = tf(sys);` where the above method gave us the result mentioned in 4.2.1

Method 2: `[num den]= ss2tf(A,B,C,D); H = tf(num,den);`

The result we got is

$$G(s) = \frac{-156.7s^2 + 1.866e^{-11} + 3.48e^{-14}}{s^4 + 843.3s^3 - 63.07s^2 - 2.702e04s}$$

When we use this  $G(s)$  in the next step (4.2.2) and plot the step response of the closed loop system, we weren't getting a proper result. The step response curve was very sharp. We aren't sure what difference does this make of generating the transfer function in two different methods. Since the first method had a proper smooth step response, we are proceeding with the first method and taking that result.

4.2.2: There wasn't many problems with the matlab numeric form , but it wasn't considering the  $e^{-11}$ ,  $e^{-14}$  terms to zero. we had considered those terms as minimal .i.e to zero.

## Reporting of Task 4.3

4.3.1. Equation of PID was considered as:

$$C(s) = K_p + K_i \frac{1}{s} + K_d s = \frac{K_p s + K_i + K_d s^2}{s} = \frac{N}{D}$$

The poles of the plant transfer function were:

$$P_1 = -843.3571, P_2 = -5.6416, P_3 = 5.679$$

The pole  $P_3$  lies on the right hand side of the s-plane, this causes our system to be unstable. We have to find the value of  $K_p$ ,  $K_i$  and  $K_d$  such that the closed loop transfer function is stable. We have to replace  $P_3$  with a pole that lies on the LHS of the s-plane. Here we

choose our new pole to be  $p = -3$ . It is chosen in such a way that, the new pole does not effect the existing dominant pole and also lie in the RHS of s-plane.

$$\text{Closed loop transfer function } P(s) = \frac{CG}{1 + CG}$$

4.3.2.Hence Using pole placement method  $K_p$ ,  $K_i$  and  $K_d$  were calculated as  $K_p = -47.2363$ ,  $K_i = -264.7467$ ,  $K_d = -0.0556$

4.3.3.The resulting closed loop transfer function is:

$$Cs = \frac{s(s + 843.4189)(s + 5.6421)}{s(s + 843.3571)(s + 5.6416)(s + 3.0014)}$$

## Reporting of Task 4.4

4.4.1: The equations of motion after adding the disturbance d is

$$(I_b + m_b l_b^2) \ddot{\theta}_b = m_b l_b g \sin \theta_b - m_b l_b \ddot{x}_w \cos \theta_b - \frac{2K_t}{R_m} v_m + \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right) + l_b \cos \theta_b d$$

and

$$\left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) \ddot{x}_w = -m_b l_b \ddot{\theta}_b \cos \theta_b + m_b l_b l_w \dot{\theta}_b^2 \sin \theta_b + \frac{2K_t}{R_m} v_m - \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right) + l_w d$$

4.4.2: Linearizing the equations above we consider

$$\sin \theta_b = \theta_b; \theta_b^2 = 0; \ddot{\theta}_b \cos \theta_b = \ddot{\theta}_b; \ddot{x}_w \cos \theta_b = \ddot{x}_w$$

After considering the above equations we get

$$(I_b + m_b l_b^2) \ddot{\theta}_b = m_b l_b g \theta_b - m_b l_b \ddot{x}_w - \frac{2K_t}{R_m} v_m + \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right) + l_b d$$

$$\left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) \ddot{x}_w = -m_b l_b l_w \ddot{\theta}_b + \frac{2K_t}{R_m} v_m - \left( \frac{2K_e K_t}{R_m} + b_f \right) \left( \frac{\dot{x}_w}{l_w} - \dot{\theta}_b \right) + l_w d$$

Matrices in numeric form:

$$\alpha_d = \begin{bmatrix} 0 & \left( \frac{2K_e K_t}{R_m} \right) + \frac{b_f}{l_w} & m_b l_b g & -\frac{2K_e K_t}{R_m} - b_f \\ 0 & -\left( \frac{2K_e K_t}{R_m} \right) - \frac{b_f}{l_w} & 0 & \frac{2K_e K_t}{R_m} + b_f \end{bmatrix}$$

$$\beta_d = \begin{bmatrix} -\frac{2K_t}{R_m} & l_b \\ \frac{2K_t}{R_m} & l_w \end{bmatrix}$$

$$\gamma_d = \begin{bmatrix} m_b l_b & (I_b + m_b l_b^2) \\ \left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) & m_b l_b l_w \end{bmatrix}$$

The updated matrices after the 4\*4 constraint is

$$\gamma_{dnew} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_b l_b & (I_b + m_b l_b^2) & 0 \\ 0 & \left( \frac{I_w}{l_w} + l_w m_b + l_w m_w \right) & m_b l_b l_w & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\alpha_{dnew} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & (\frac{2K_e K_t}{R_m}) + \frac{b_f}{l_w} & m_b l_b g & -\frac{2K_e K_t}{R_m} - b_f \\ 0 & -(\frac{2K_e K_t}{R_m}) - \frac{b_f}{l_w} & 0 & \frac{2K_e K_t}{R_m} + b_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\beta_{dnew} = \begin{bmatrix} 0 & 0 \\ -\frac{2K_t}{R_m} & l_b \\ 0 & \\ \frac{2K_t}{R_m} & l_w \end{bmatrix}$$

The final matrices value for disturbance input is

$$A = \gamma_{dnew}^{-1} \alpha_{dnew}$$

$$B = \gamma_{dnew}^{-1} \beta_{dnew}$$

$$C = [0 \quad 0 \quad 1 \quad 0]$$

$$D = 0$$

Values for matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -773.8 & -6.6 & 16.2 \\ 0 & 0 & 0 & 1 \\ 0 & 3313.2 & 63.1 & -69.6 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 36.5980 & 1.8795 \\ 0 & 0 \\ -156.7072 & 1.0797 \end{bmatrix}$$

$$C = [0 \quad 0 \quad 1 \quad 0]$$

$$D = 0$$

## Reporting of Task 4.5

4.5.1: The simulink model we used is shown in Figure 1 :

We initially had some trouble with setting the values of kp,ki,kd in PID block, because when we tried with the same negative values what we got in the previous steps, our system response was negative. So we had to change the values of PID parameters to positive.

4.5.2: The realization of  $d(t)$ ,  $v_m(t)$ ,  $x_m(t)$ ,  $\theta_b(t)$  are shown in Figure 2, Figure 3, Figure 4, Figure 5 respectively

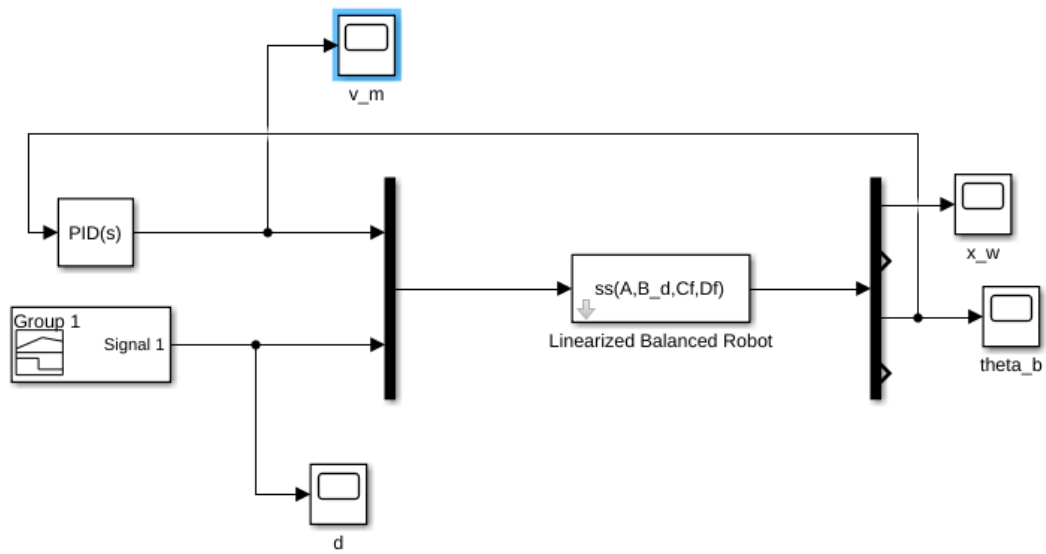


Figure 1: Simulink model



Figure 2:  $d(t)$

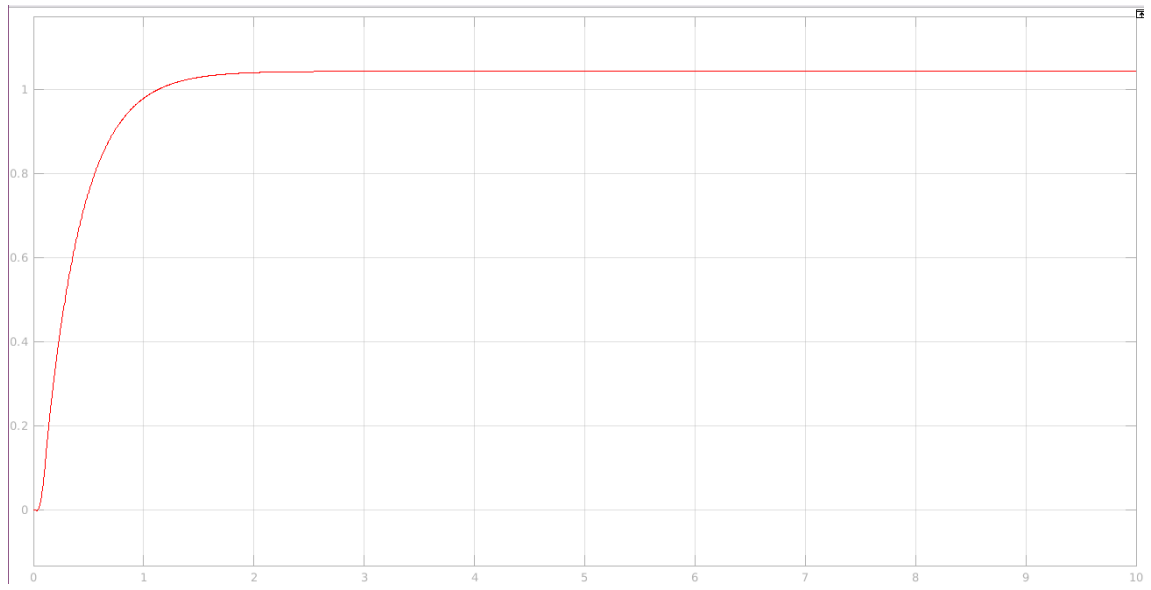


Figure 3:  $v_m(t)$

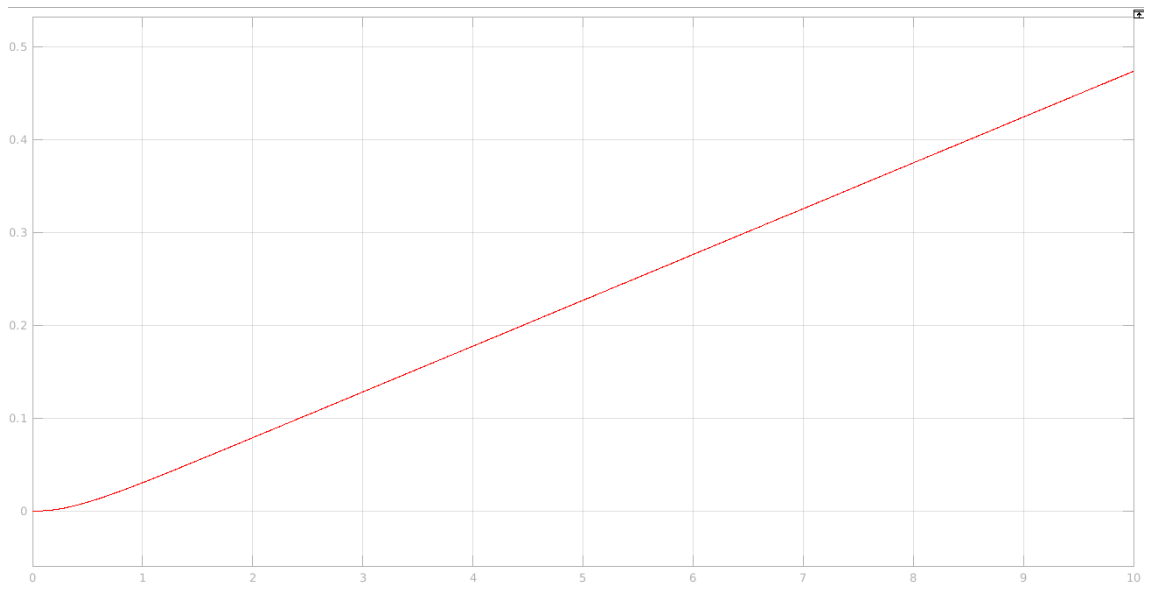


Figure 4:  $x_w(t)$



Figure 5:  $\theta_b(t)$

## Reporting of Task 4.6

4.6.2: The sampling period we calculated is  $108Hz \approx 110Hz$ .

4.6.3: Justification for 4.6.1 and 4.6.2; We were initially confused with the question as in what to proceed with, to compute the bandwidth of the system or compute the bandwidth of open loop system. We discussed with research group and concluded that we have to get the bandwidth of the open loop system. We get the bode plot of the open loop system(plant) response in Figure 6. So Bandwidth is as follows: Lower Freq =  $2.31rad/s$  Upper Freq =  $13.8rad/s$  BW = Upper freq - Lower freq =  $11.4rad/s = 1.8287Hz$

To get the sampling period, with Nyquist Theorem, also considering the Arduino's Processing speed as mentioned in the lab book, we take 30 times the value we got from Nyquist theorem.

So we compute BW =  $2 * 30 * 1.8287 = 108Hz \approx 110Hz$

So the sampling time we chose is  $T_s = 1/BW = 0.01s$

4.6.4: The final discrete time transfer function is :

$$H(Z) = \frac{-0.001639z^2 + 0.001419z + 0.0002199}{z^3 - 2.004z^2 + 1.001z - 0.0002174}$$

## Reporting of Task 4.7

4.7.1: The smallest values of  $\bar{d}$  for which the robot falls : in continuous case: 0.584 in discrete case : 0.584 (Both the cases, it was same , thought it may differ) The only difference we observe is In the Continuous case, the robot falls at 3.84s. But at the discrete case, it falls early 3.57s as shown in Figure 7

4.7.2 : Realizations for Continuous case:  $\theta_b(t)vs\theta_b^{lin}(t), v_m(t)vs v_m^{lin}(t)$  are found in Figure 8, Figure 9 respectively.

Realizations for Discrete case:  $\theta_b(t)vs\theta_b^{lin}(t), v_m(t)vs v_m^{lin}(t)$  are found in Figure 10, Figure 11 respectively.

4.7.3: These experiments were used to relate how the linearization of the equations of motion helped in stabilizing the robot. From the Figure 7, we can see that linearizing



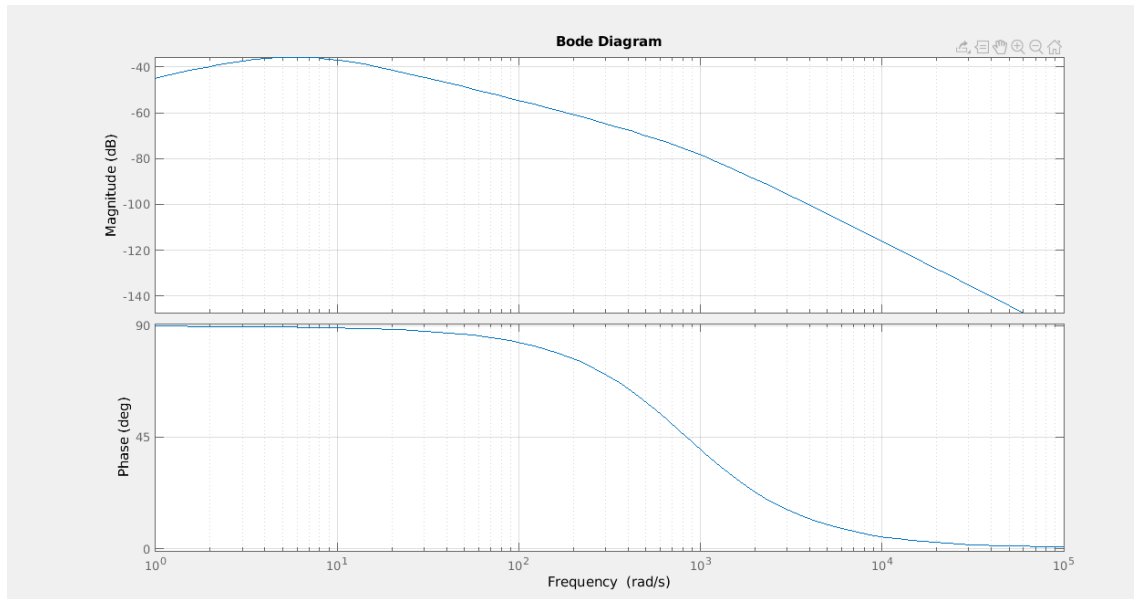


Figure 6: bode plot for the transfer function

the robot will still have the robot in stable position. But anyways, this is simulating the robot, we don't know, how it works in the real case.

Also to know if we can tune the PID controller, we are not sure how to tune the PID controller with this setup.

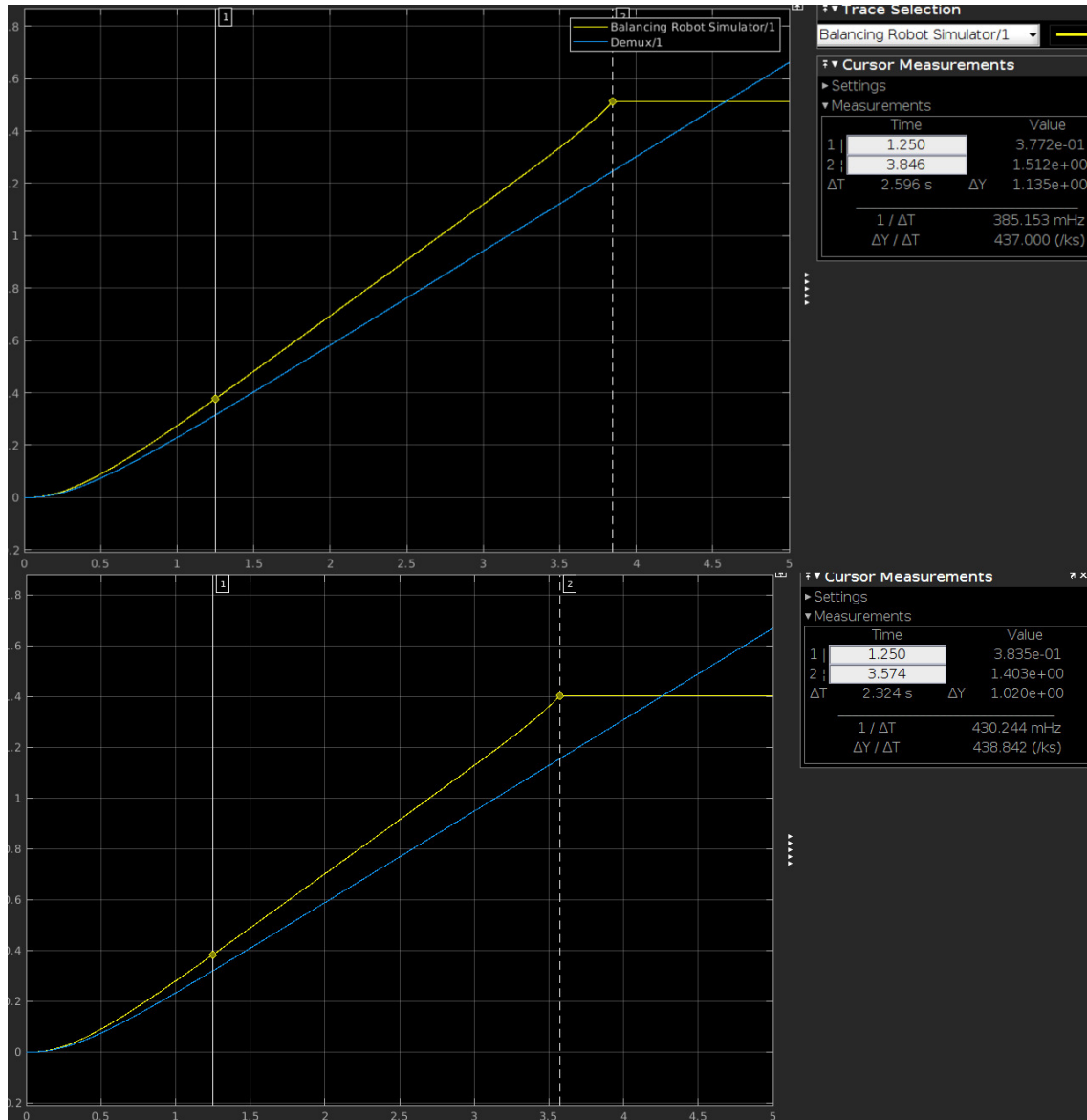


Figure 7: Difference in robot fall time value for continuous and discrete

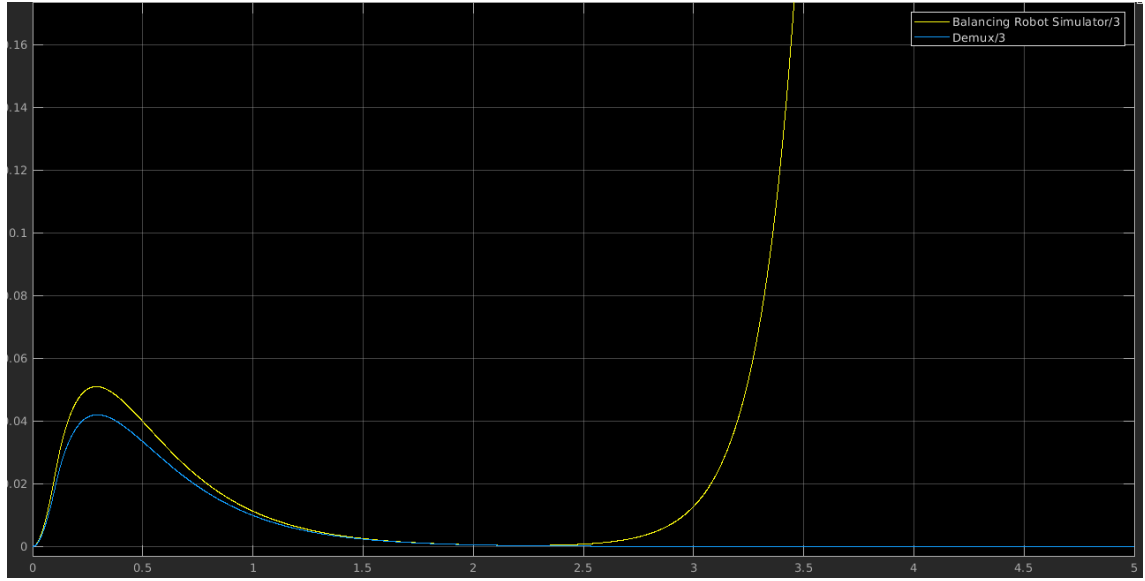


Figure 8:  $\theta_b(t)vs\theta_b^{lin}(t)$

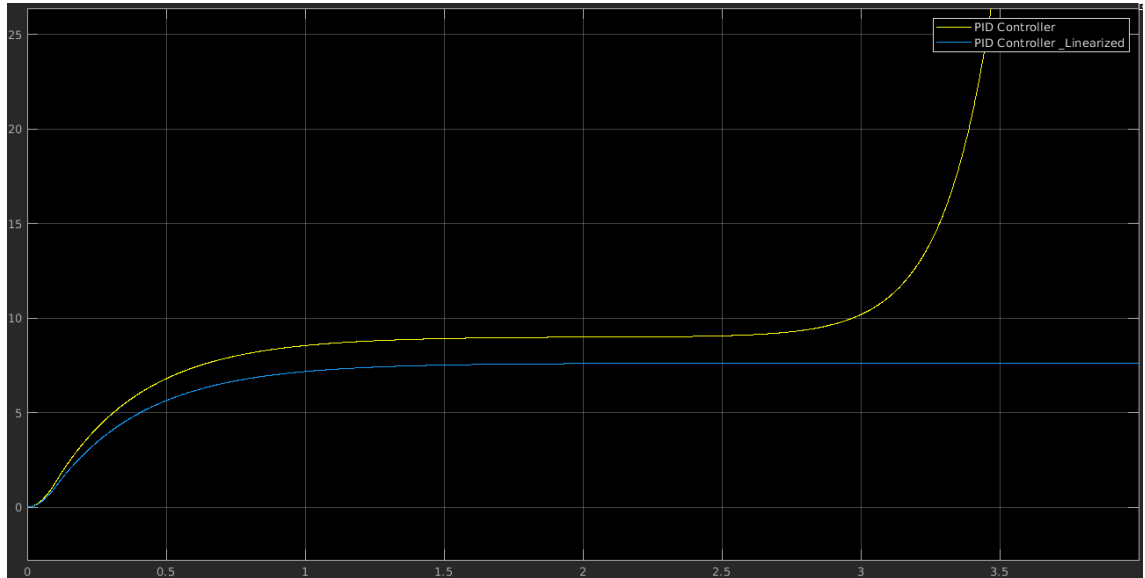


Figure 9:  $v_m(t)vs v_m^{lin}(t)$

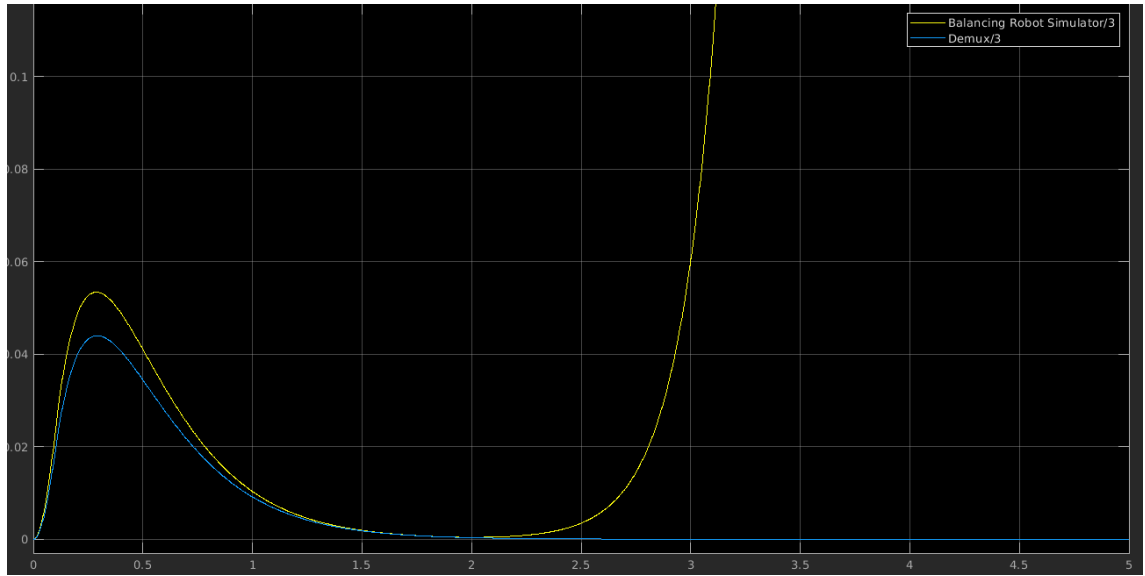


Figure 10:  $\theta_b(t)vs\theta_b^{lin}(t)$

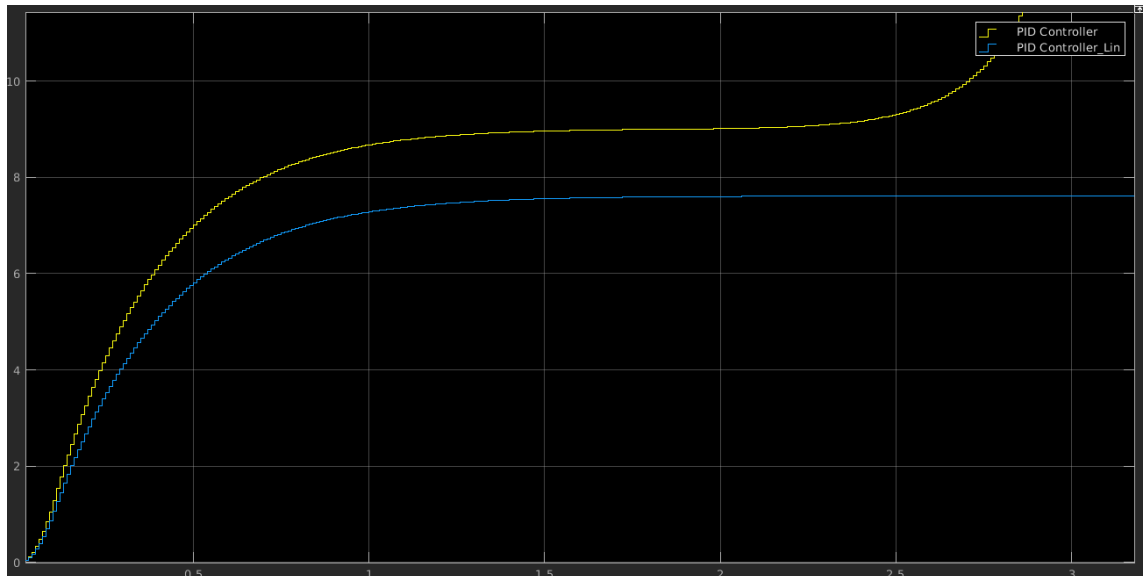


Figure 11:  $v_m(t)vs v_m^{lin}(t)$