ELEC-E8101 Group project: Lab A report Group #21

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Instructions: *For this lab report there is no size limit on your report, but try to be concise.*

Reporting 4.1

Equations of Motion (EOM) are derived using Newton's third law as follows:
Equations for the body:

- The Newtons law for the vertical movement of the body

$$m_b\ddot{y_b} = F_y + m_b g$$

- The Newtons law for the horizontal movement of the body

$$m_b\ddot{x_b} = F_x$$

- The law for the angular movement of the body

$$I_b\ddot{\theta_b} = T_f - T_m - F_x l_b \text{cos}\theta_b + F_y l_b \text{sin}\theta_b$$

Equations of wheel:

- The Newtons law for the vertical movement of the wheel

$$F_y + m_\omega g = N$$

- The Newtons law for the vertical movement of the wheel

$$m_\omega\ddot{x_\omega} = F_t - F_x$$

- The law for the angular movement of the wheel

$$I_w\ddot{\theta_\omega} = T_m - T_f - F_t l_\omega$$

we can substitute Eq. 1.1 and Eq. 1.2 in Eq.1.3, we obtain:

$$I_b\ddot{\theta_b} = -T_f + T_m - F_x l_b \cos\theta_b + F_y l_b \sin\theta_b$$

we should eliminate θ_ω by substituting below equation in equation 1.3:

$$\ddot{\theta_\omega} = \frac{\ddot{x_\omega}}{l_\omega}$$

As a result we derive the following equation:

$$I_b\ddot{\theta_b} = T_f - T_m - m_b\ddot{x_b} l_b \cos\theta_b + (m_b\ddot{y_b} - m_b g) l_b \sin\theta_b$$

The final equations before Linearization:

$$\ddot{\theta_b} = \frac{\left(\frac{K_t K_e \dot{x_\omega}}{\mathrm{R_m}\, l_m} - \frac{K_t K_e \dot{\theta_b}}{R_m} - \frac{K_t V_m}{R_m} + m_b l_b g \sin\theta_b - m_b l_b \ddot{x_\omega} \cos\theta_b\right)}{I_b + m_b l_b^2}$$

$$\ddot{x_\omega} = \frac{\left(-\frac{l_\omega K_t K_e \dot{x_\omega}}{\mathrm{R_m\ l_\omega}} + \frac{l_\omega K_t V_m}{R_m} + \frac{l_\omega K_t K_e \dot{\theta_b}}{R_m} - m_b l_\omega^2 (\ddot{\theta_b} l_b \cos\theta_b - \dot{\theta_b}^2 l_b \sin\theta_b)\right)}{I_\omega + m_\omega l_\omega^2 + m_b l_\omega^2}$$

Reporting 4.1:
We need to linearize $\sin\theta_b$ and $\cos\theta_b$, $\dot{\theta}_b^2$ terms. Let linearization point be $\theta_b = 0$. The normal position of the robot will bw at $\theta_b = 0$ and we need to model the movement around the vertical position of the robot. Hence, we choose $\theta_b = 0$

$$\sin\theta_b = \sin\theta_{\mathrm{b0}} =$$

After linearisation we derive:

$$\ddot{\theta_b} = \frac{\left(\frac{K_t K_e \dot{x_\omega}}{\mathrm{R_m}\, l_m} - \frac{K_t K_e \dot{\theta_b}}{R_m} - \frac{K_t V_m}{R_m} + m_b l_b g \theta_b - m_b l_b \ddot{x_\omega}\right)}{I_b + m_b l_b^2}$$

$$\ddot{x_\omega} = \frac{\left(-\frac{K_t K_e \dot{x_\omega}}{\mathrm{R_m}} + \frac{l_\omega K_t V_m}{R_m} + \frac{l_\omega K_t K_e \dot{\theta_b}}{R_m} - m_b l_\omega^2 \ddot{\theta_b} l_b\right)}{I_\omega + m_\omega l_\omega^2 + m_b l_\omega^2}$$

We chose the state variables for the system as

Reporting 4.2

We chose the state variables for the system as,

$$x = \begin{bmatrix} x_w \\ \dot{x_w} \\ \theta_b \\ \dot{\theta}_b \end{bmatrix}$$

And the input to the system is u which is given by

$$u = V_m$$

From the system of equations of motion, we get the parametric form

$$\gamma = \begin{bmatrix} I_w + m_w l_w^2 + m_b l_w^2 & m_b l_w^2 l_b \\ m_b l_b & I_b + m_b l_b^2 \end{bmatrix}$$

$$\beta = \begin{bmatrix} \frac{l_w K_t}{R_m} \\ \frac{-K_t}{R_m} \end{bmatrix}$$

$$\alpha = \begin{bmatrix} 0 & \frac{-K_e \mathrm{Kt}}{R_m} & 0 & \frac{l_\omega K_e K_t}{R_m} \\ 0 & \frac{K_e K_t}{R_m l_\omega} & m_b l_b g & \frac{-K_e K_t}{R_m} \end{bmatrix}$$

From the parametric form, we can easily get to the state space representation by using the following equations.

$$A = \gamma^{-1}\beta$$

$$\alpha = \begin{bmatrix} 0 & \frac{-K_e \mathrm{Kt}}{R_m} & 0 & \frac{l_\omega K_e K_t}{R_m} \\ 0 & \frac{K_e K_t}{R_m l_\omega} & m_b l_b g & \frac{-K_e K_t}{R_m} \end{bmatrix}$$

The C and D matrices are pretty straightforward and are as follows -

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

And on putting the values of all the constants into the above equations and simplyfing, we get the numerical parametric form -

$$A = \begin{bmatrix} 0 & -0.4350 & -0.0061 & 0.0091 \\ 0 & 1.9034 & 0.0620 & -0.0400 \end{bmatrix}$$

$$B = \begin{bmatrix} 20.5759 \\ -90.0275 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

Reporting 4.3

1. The transfer function of the system can be created from the A,B,C and D matrices which we evaluated previously. The transfer function is -

$$G(s) = -90.03 \frac{s}{(s + 475.1)(s + 5.657)(s - 5.72)}$$

2. We did not face any major numerical problems with Matlab as we did not use any hard-coded values in the equations.

Reporting 4.4

1)Equation of PID

$$C(s) = k_p + K_I \frac{1}{s} + K_D s = \frac{K_p s + K_I + K_D s^2}{s} = \frac{N}{D}$$

From Plant transfer function

poles are $P_1 = -475.060$ $P_2 = -5.6571$ $P_3 = 5.7195$

As we can see, there is an unstable pole at $P_3 = 5.7195$ We need to find values of Kp, Ki, and Kd in such way that the closed loop transfer function eqn
$P(s) = \dfrac{CG}{1 + CG}$
is stable. For that we need to remove the pole and place it in at $z = -3$ We chose -3 because its in between another pole -5.7 and origin. Its far from 5.7 and doesn't interfere with that dominant pole. Its under dampening features are useful. Too close to the zero and it will be be close to Right hand plane and potential instability Its also possible to move the pole from origin, it could cause undue stress on the actuators.

2)Hence Using pole placement method K_p, K_i and K_d are calculated. $K_p = -46.5603$ $K_i = -260.2962$ $K_d = -0.0969$

3)

$$\text{Cs} = \frac{91387s(s + 453.3)(s + 5.657)}{s(s + 9991)(s + 475.5)(s + 5.657)(s + 3s)}$$

Reporting 4.5

1. EOM in parametric form

$$\ddot{\theta}_b\left(I_b + m_b l_b^2\right) + \ddot{x}\left(m_b l_b\right) = x_\omega\left(\frac{K_t K_e}{R_m l_m}\right) - \theta\left(\frac{K_t K_e}{R_m}\right) - \theta\left(m_b l_b g\right) - V_m\left(\frac{K_t}{R_m}\right) + d\, l_b$$

$$\ddot{x}_w\left(I_w + m_w l_w^2 + m_b l_w^2\right) + \theta_b\left(m_b l_w^2 l_b\right) = V_m\left(\frac{l_w K_t}{R_m}\right) - \dot{x}_w\left(\frac{l_w K_t K_e}{R_m}\right) + \theta_b\left(\frac{l_w K_t K_e}{R_m}\right) + d\left(l_w^2\right)$$

The novel equations in numerical state space form are -

$$A = \begin{bmatrix} 0 & -0.4350 & -0.0061 & 0.0091 \\ 0 & 1.9034 & 0.0620 & -0.0400 \end{bmatrix}$$

$$B = \begin{bmatrix} 20.5759 & 2.1054 \\ -90.0275 & 2.0256 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

Reporting 4.6

The Simulink model for task 4.7 is shown in Figure 1.

The realisations for $\theta_b(t), x_w(t)$, $v_m(t)$, *and* $d(t)$ are shown in Figure 2

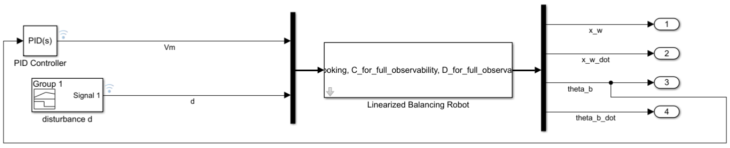

Figure 1: Simulink model for task 4.7

Reporting 4.7

The bode plot for the system is show in Figure 3.

Bandwidth is as follows..
LowerFreq = 5.446
UpperFreq = 13.73 rad/s
Converting to Hertz
UpperFreq = 2.1852 Hz
By Nyquists Theorem, to avoid anti-aliasing it should be twice the the UpperFreq Also

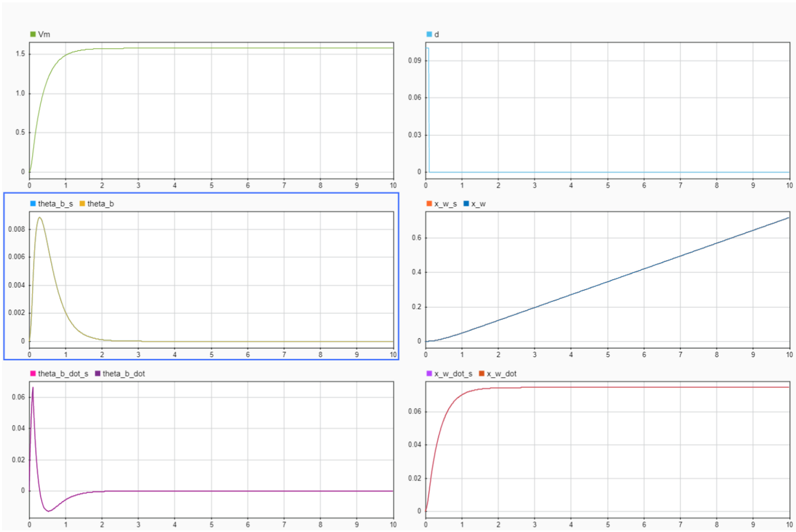

Figure 2: Signals for task 4.7

considering Arduino's processing speed, we can sampling frequency as 25 times the value we got which satisfies Nyquist Theorem. Therefore
Fs = 2*25*2.1852 = 109.26   110
In terms of sampling Period
Ts = 1/Fs = 0.01

Reporting 4.8

1. For both the continuous and the discrete cases, the smallest value of d at which the robot fell down was 0.535.
2. The plot of $\theta_b(t), x_w(t)$, $v_m(t)$, *and* $d(t)$ are shown in Figure4

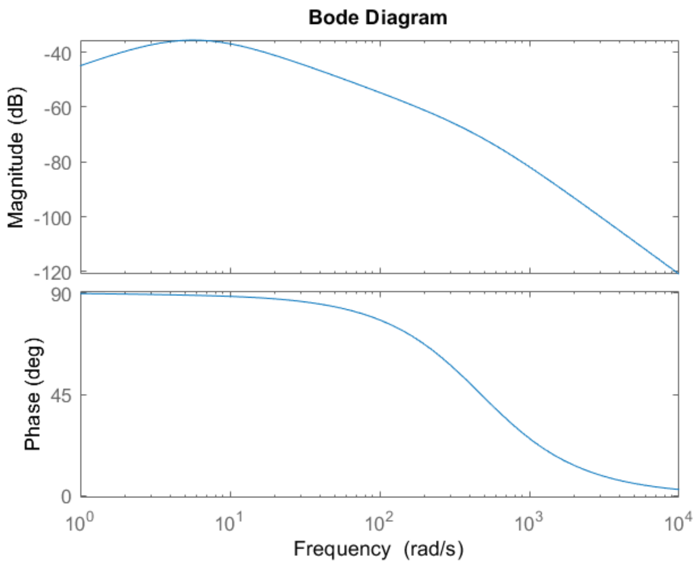

Figure 3: Bode plot

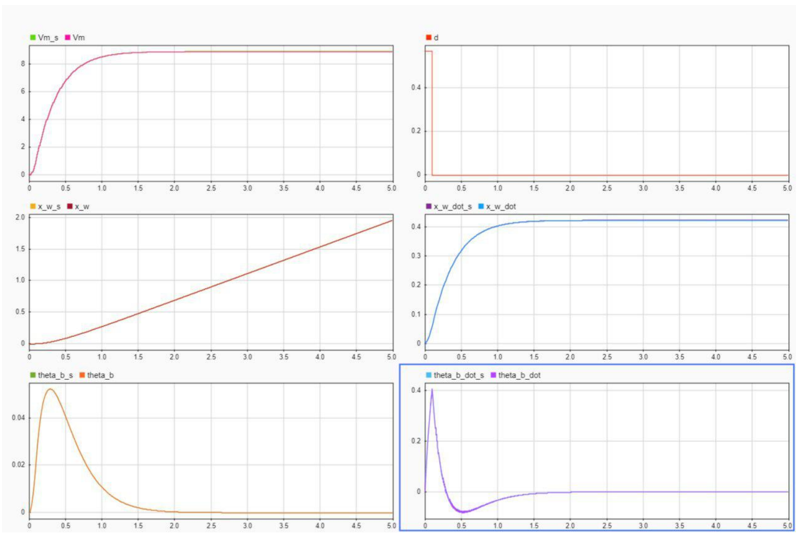

Figure 4: Discrete signals for task 4.9