Homework-3

1.)

$$1 + G(z) = 1 + \frac{k(0.2z + 0.5)}{z^2 - 1.2z + 0.2} = 0.$$

$$= \frac{z^2 - 1.2z + 0.2 + k(0.2z + 0.5)}{z^2 - 1.2z + 0.2} \Rightarrow$$ Characteristic eqn.

$$\phi(z) = z^2 - 1.2z + 0.2 + k\,0.2z + k\,0.5 = 0$$

$$\Rightarrow z^2 + z(0.2k - 1.2) + 0.2 + k(0.5) = 0.$$

It is of second order, so we can use triangle rule.

$a_2 < 1$, $a_2 > -1 + a_1$, $a_2 > -1 - a_1$. for second order equation $z^2 + a_1 z + a_2 = 0$.

$\boxed{a_1 = 0.2k - 1.2}$ $\boxed{a_2 = 0.2 + 0.5k}$

To find values of k.

$\underline{a_2 < 1}$:-

$0.2 + 0.5k < 1$

$0.5k < 0.8$

$\boxed{k < 1.6}$

$a_2 > -1 + a_1$

$0.2+0.5k > -1 + 0.2k - 1.2.$

$0.3k > -2.4$

$\boxed{k > -8}$

$a_2 > -1 - a_1$

$0.2+0.5k > -1 - 0.2k + 1.2.$

$0.7k > 0.$

$\boxed{k > 0}$

$\therefore$ The values of k are in range.

$\boxed{0 < k < 1.6}$ because it was

given as $\boxed{k > 0}$,

$\boxed{0 < k < 1.6;}$

2.) characteristic equation:-

$$\chi(z) = z^3 - 2z^2 + 1.4z - 0.1 = 0$$

Jury's stability criterion.

$a_0 = 1$ $a_1 = -2$ $a_2 = 1.4$ $a_3 = -0.1$.

1	-2	1.4	-0.1.
-0.1	1.4	-2	1
0.99.	-1.86	1.2.	
1.2	-1.86	0.99	
-0.4645	0.3945		
0.3945	-0.4645		
-0.1292			

$b_3 = \frac{-0.1}{1} = -0.1.$

$b_2 = 1.2121$

$b_1 = -0.8493.$

Since, $a_0^1 < 0$ and $a_0^0 < 0$, the system is unstable. Since there are two -ve values, there are two poles outside unit circle.

3.)

Given

$$P(z) = \frac{-1}{z^2+z+2}.$$

3.a

z^2+z+2 :-

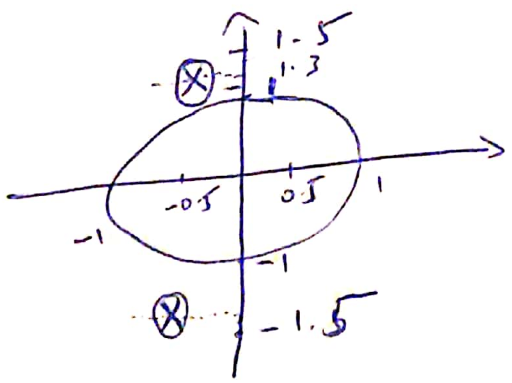
$$\text{Poles} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1-4(2)}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{7}\, i}{2}$$

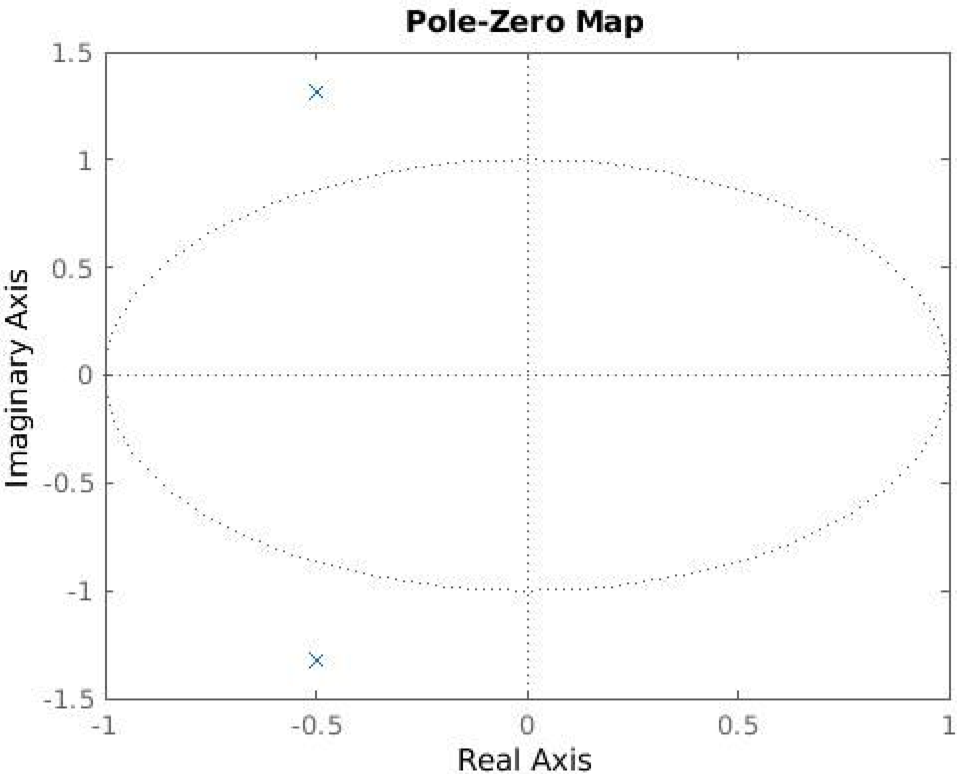
$$= -\frac{1}{2} \pm 1.3229\, i$$

$$= -0.5 + 1.3229i, \quad -\cancel{0.5}\ 1.3229i$$



⊗ Poles of system as

Please refer the matlab plot as well.


Pole-Zero Map
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
Imaginary Axis
Real Axis

3.b

To

$$G(z) = -\frac{k}{z^2+z+2-k}.$$

Now, from the figure.

$$U(z) = k\,E(z) \quad \text{--- (1)}$$

and

$$E(z) = R(z) - Y(z) \quad \text{--- (2)}.$$

Substitute (2) in (1).

$$U(z) = k\,[R(z) - Y(z)].$$

$$\boxed{U(z) = k\,R(z) - k\,Y(z)} \quad \text{--- (3)}.$$

The output

$$Y(z) = P(z) \cdot U(z)$$

from (3)

$$Y(z) = P(z)\,[k\,R(z) - k\,Y(z)].$$

$$Y(z) = k\,P(z)\,R(z) - k\,P(z)\,Y(z)$$

$$Y(z) + k\,P(z)\,Y(z) = k\,P(z)\,R(z).$$

$$Y(z)\,[1 + k\,P(z)] = k\,P(z)\,R(z).$$

$$\boxed{G(z) = \frac{Y(z)}{R(z)} = \frac{k\,P(z)}{1 + k\,P(z)}}.$$

i.e $G(z) = \dfrac{k\left(\dfrac{-1}{z^2+z+2}\right)}{1+k\left(\dfrac{-1}{z^2+z+2}\right)} = \dfrac{\dfrac{-k}{z^2+z+2}}{\dfrac{z^2+z+2-k}{z^2+z+2}}$

$$\boxed{G(z) = \frac{-k}{z^2+z+2-k}}$$

3.c :-

The characteristic equation from 3.b is

$\chi(z) = z^2 + z + 2 - k$.

Comparing with triangle rule because,

it is a second order system. $z^2 + a_1 z + a_2 = 0$.

$a_2 = 2 - k$. $a_1 = 1$.

$a_2 < 1$.	$a_2 > a_1 - 1$.	$2 - k > -a_1 - 1$
$2 - k < 1$	$2 - k > 1 - 1$	$2 - k > -1 - 1$.
$1 < k$.	$2 - k > 0$	$2 + 2 > k$.
$\boxed{k > 1}$	$\boxed{k < 2}$	$\boxed{k < 4}$

Therefore combining

$\boxed{1 < k < 2}$

3.d).

by Finite - Value Theorm,

$$\lim_{k \to \infty} y[k] = \lim_{z \to 1} (z-1)\, Y(z)$$

Now, input is $r[k] = u[k]$

$$R(z) = U(z) = \left(\frac{z}{z-1}\right)$$

$$Y(z) = G(z)\, R(z)$$

$$\lim_{k \to \infty} y[k] = \lim_{z \to 1} (z-1) \cdot Y(z)$$

$$= \lim_{z \to 1} \cancel{(z-1)} \frac{-k}{z^2+z+2-k} \left(\frac{z}{\cancel{z-1}}\right)$$

$$= \lim_{z \to 1} \frac{-kz}{z^2+z+2-k}$$

$$= \frac{-k}{1+1+2-k}$$

$$= \frac{-k}{4-k}$$

The value $\boxed{k \neq 4}$ will have finite value in output.

3.e

The characteristic equation is given by,

$$\chi^2(z) = z^2 + z + 2 - k.$$

$$k = 1.5$$

$$\chi^2(z) = z^2 + z + 0.5$$

from the above equation, we can conclude that, the closed loop system has unstable by the fact that it has 2 zeros outside unit circle in terms of chacteristic equation.

$$\boxed{Z = 2}$$

$$\boxed{P = 0}$$

$$\boxed{N = Z - P = 2 - 0 = 2}$$

$$\boxed{N = 2}$$

Therefore it encircles -1 two times.

So option <u>B</u> is the appropriate option.