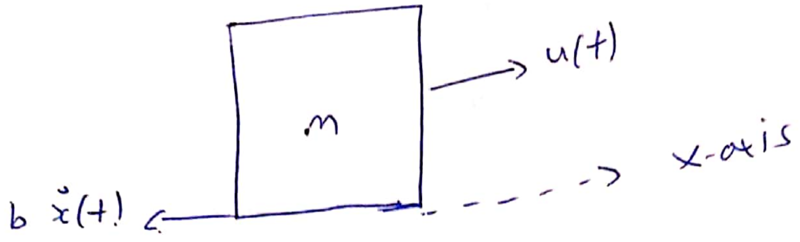
Home work -2

1.)

a.)



Given,

$v(t) = \dot{x}(t)$

by equating the force

$m\dot{v}(t) = u(t) - b\,v(t)$

$m\dot{v}(t) + b\,v(t) = u(t)$

Divide by m

$\dot{v}(t) + \frac{b}{m} v(t) = \frac{u(t)}{m}$

b.) Given, $m = 1000\ kg$,

$b = 100$,

overshoot $< 20\%$.

settling time $(t_s) = 8s$

$\xi = 0.6\left(1 - \frac{M.P}{100}\right)$

$= 0.6\left(1 - \frac{20}{100}\right)$

$\boxed{\xi = 0.48}$

i.e $\boxed{\xi \geq 0.48}$

$t_s = \frac{4.6}{\xi \omega_n}$

$\omega_n = \frac{4.6}{\xi t_s}$

$= \frac{4.6}{(0.48)\,8}$

$\boxed{\omega_n = 1.198}$

For step 1, we assume $\boxed{\xi = 0.48}$

[Can be changed based on experiments to achieve design specification]

From (1.a)

$$\dot{v}(t) + \frac{b}{m} v(t) = \frac{1}{m} u(t)$$

Taking Laplace Transform

$$sV(s) + \frac{b}{m} V(s) = \frac{1}{m} U(s)$$

$$V(s) \left(s + \frac{b}{m} \right) = \frac{1}{m} (U(s))$$

$$G(s) = \frac{V(s)}{U(s)} = \frac{1}{ms+b} = \frac{1}{1000s+100}$$

The closed loop transfer function is

$$H(s) = \frac{PID(s)\, G(s)}{1 + PID(s)\, G(s)}$$

where $PID(s) = k_p + \frac{k_i}{s}$

$$H(s) = \frac{\left(\frac{k_p s + k_i}{m} \right)}{s^2 + \left(\frac{b+k_p}{m} \right) s + \frac{k_i}{m}}$$

For second order system

$$H(s) = \frac{\cancel{w_n^2}}{s^2 + \left(\frac{b+k_p}{m} \right) + k_i}$$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

equating, we get

$$\frac{k_i}{m} = \omega_n^2.$$

$$\boxed{k_i = m\,\omega_n^2}$$

$$\boxed{k_i = 1435}$$

$$b + k_p = 2\xi\omega_n m.$$

$$\boxed{k_p = 2\xi\omega_n m - b}$$

$$\boxed{k_p = 1050}$$

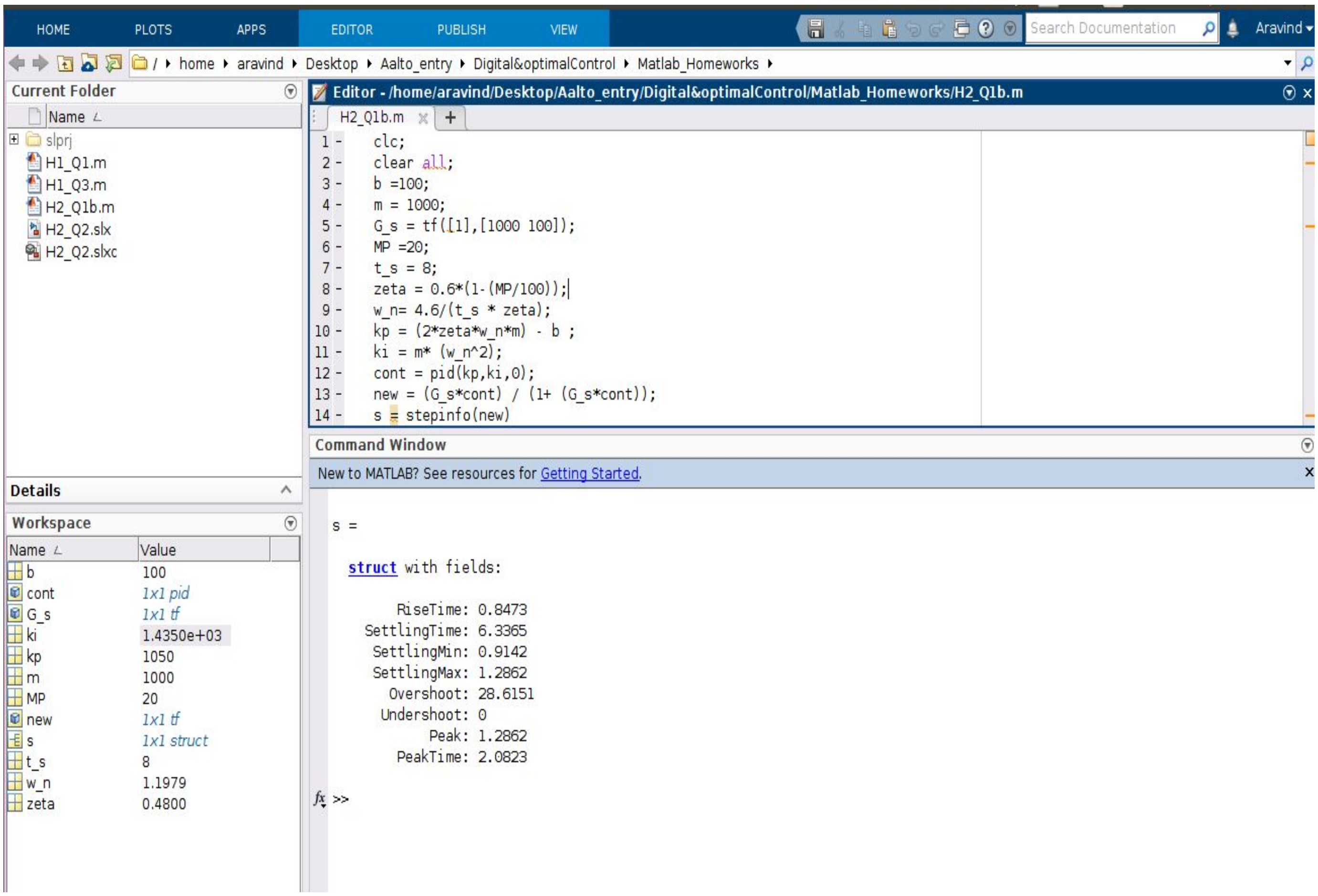
The values for zeta calculated as 0.48 and the following results are obtained:

Kp = 1050;

Ki = 1435;

Settling time = 6.33s

Overshoot = 28 (Which is not desired)



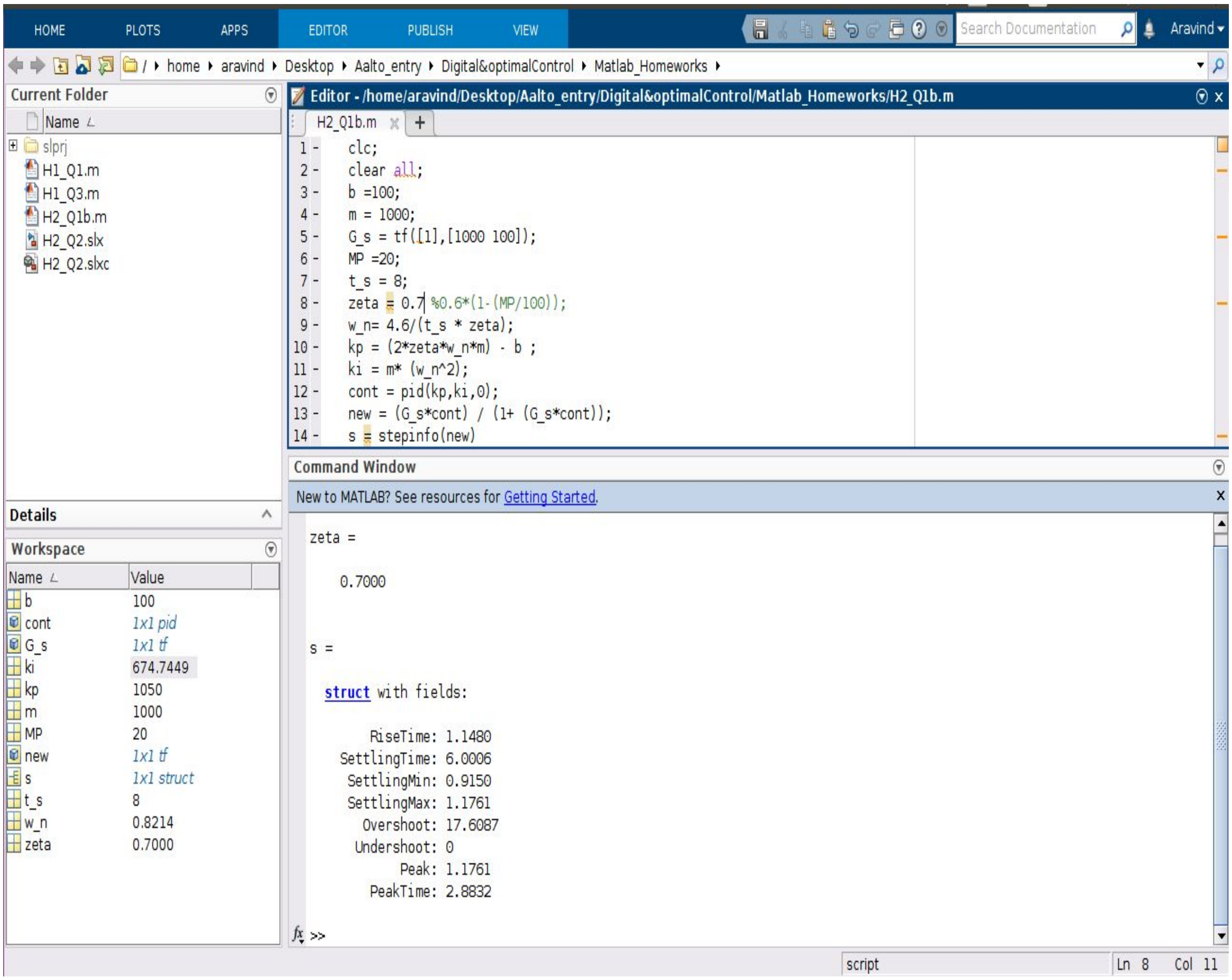
Since the design specifications were not met and it is observed that we need to increase the damping factor to achieve the design specifications, we make zeta=0.7(increased and checked the value which meets our specs). And the following results are obtained:

Kp = 1050;

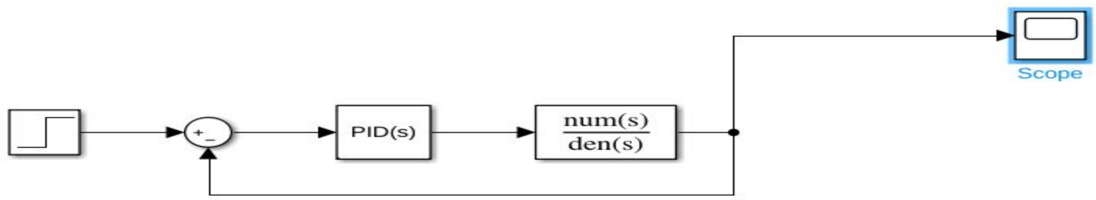
Ki = 670;

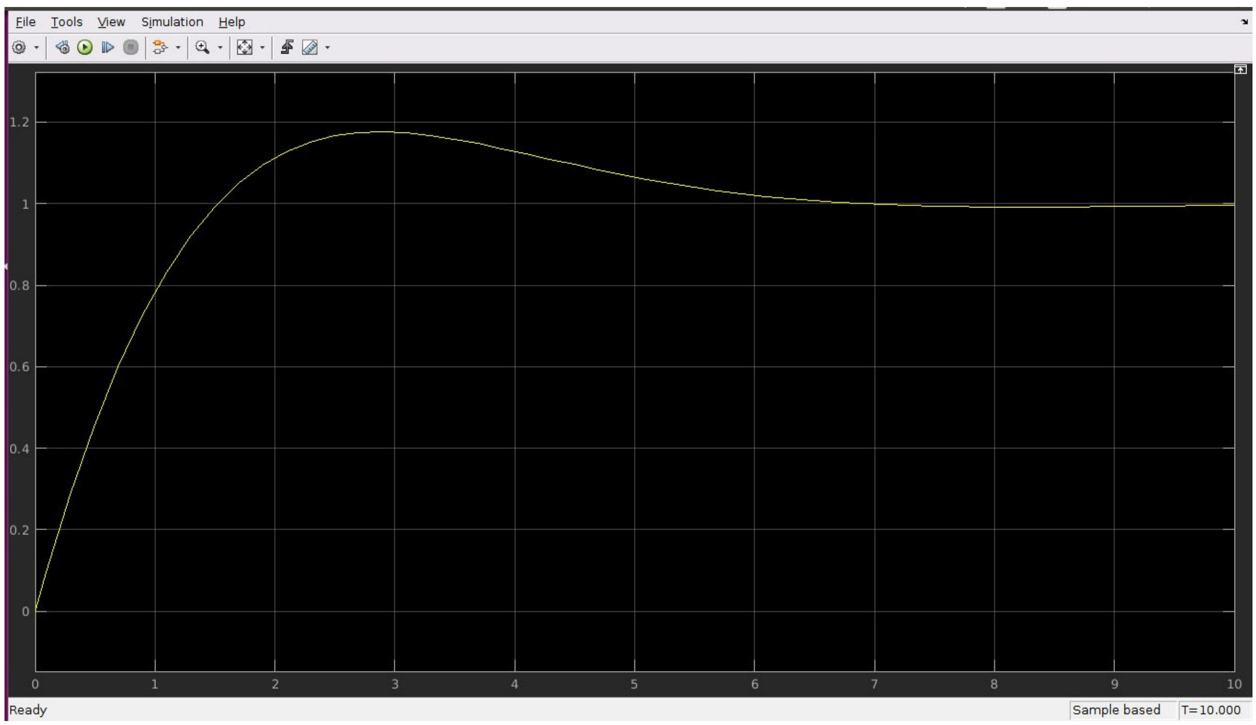
Settling time = 6s

Overshoot = 18%(less than 20%)



The simulink verification is as follows

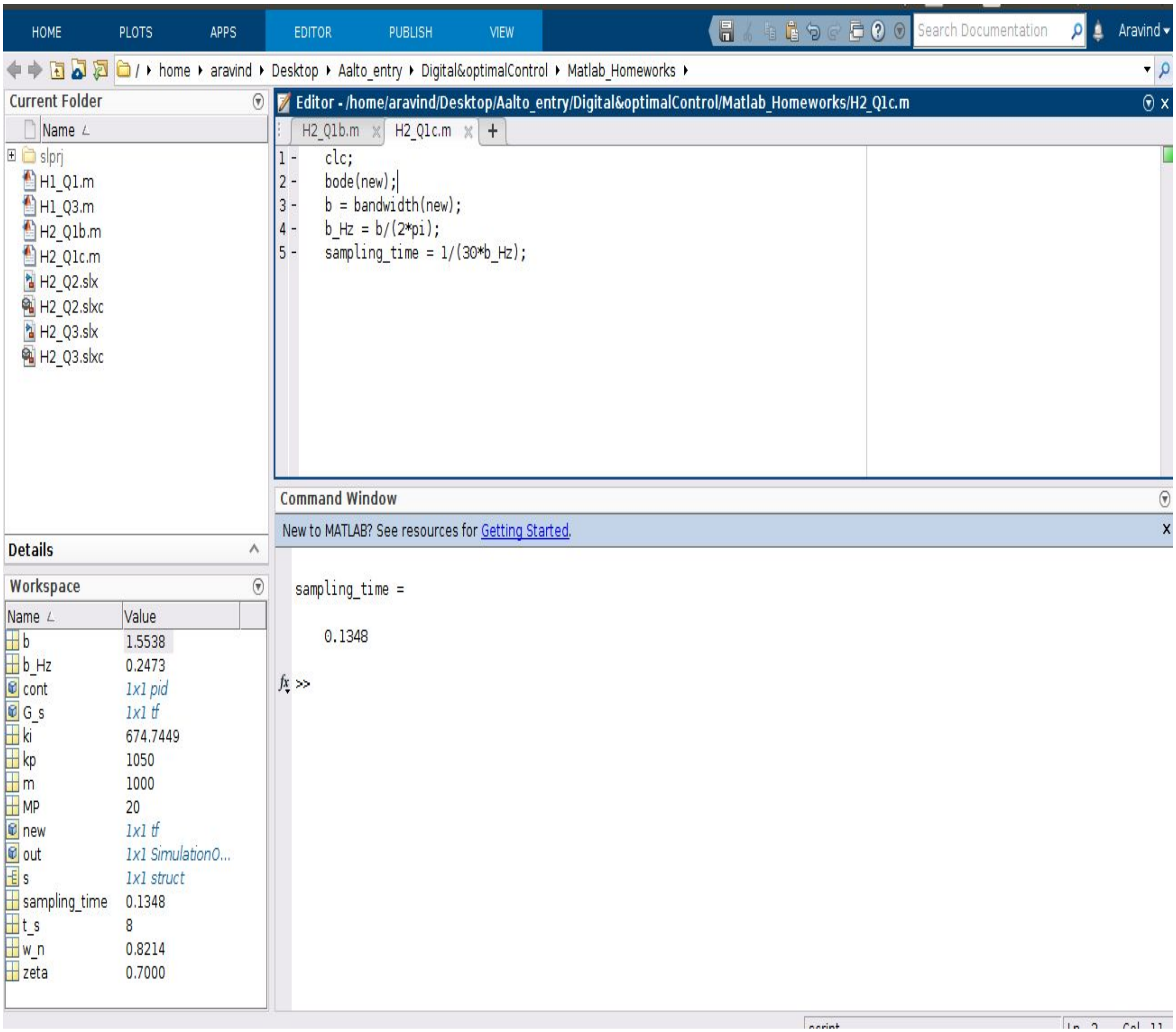




As we can see the output is settled at close to 6s and maximum overshoot is between(1.17-1.20)

1.c

The calculation of Sampling time can be found in the screenshot of the code below and value is Sampling time = 0.1s



1.C

$T = 0.1s$

To discretize the controller

$$PID(s) = \frac{1050s + 670}{s}$$

$$PID(z) = PID(s)\Big|_{s = \frac{2}{0.1}\left(\frac{z-1}{z+1}\right)}$$

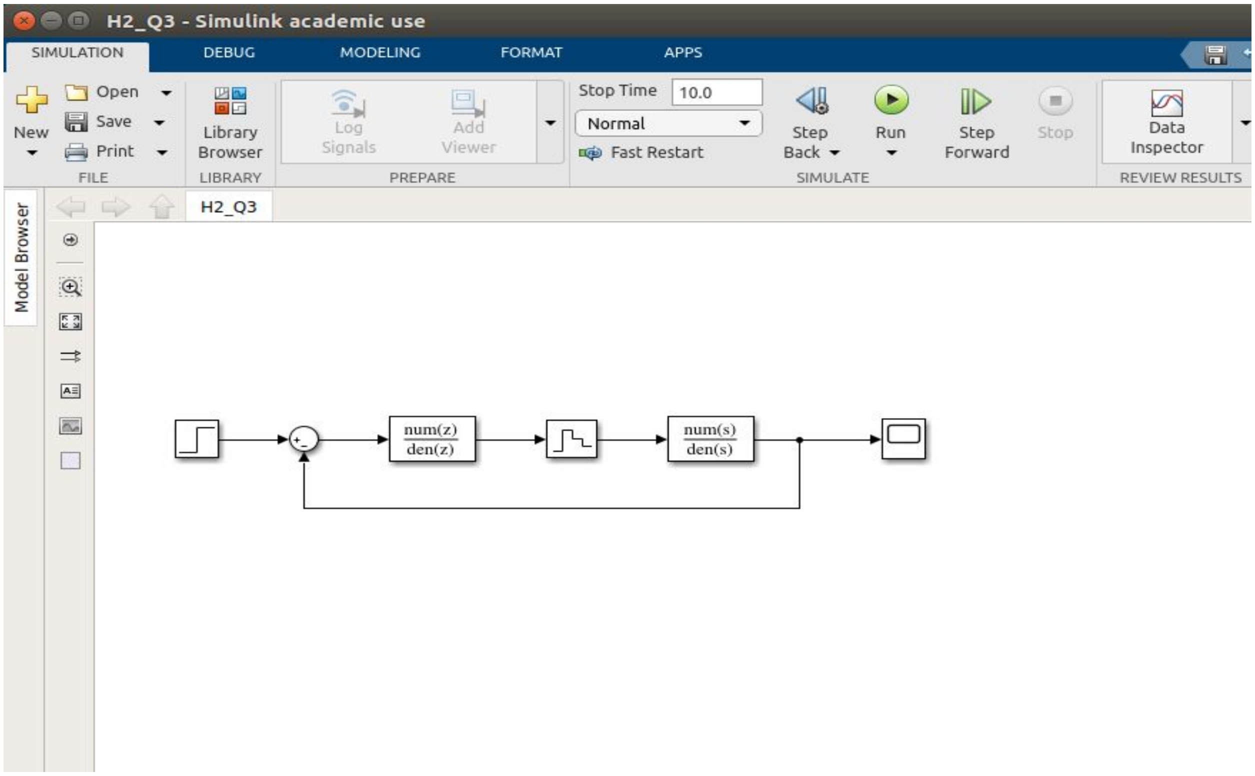
$$= \frac{1050 \cdot 20\left(\frac{z-1}{z+1}\right) + 670}{20\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{1050(z-1) + 33.5(z+1)}{(z-1)}$$

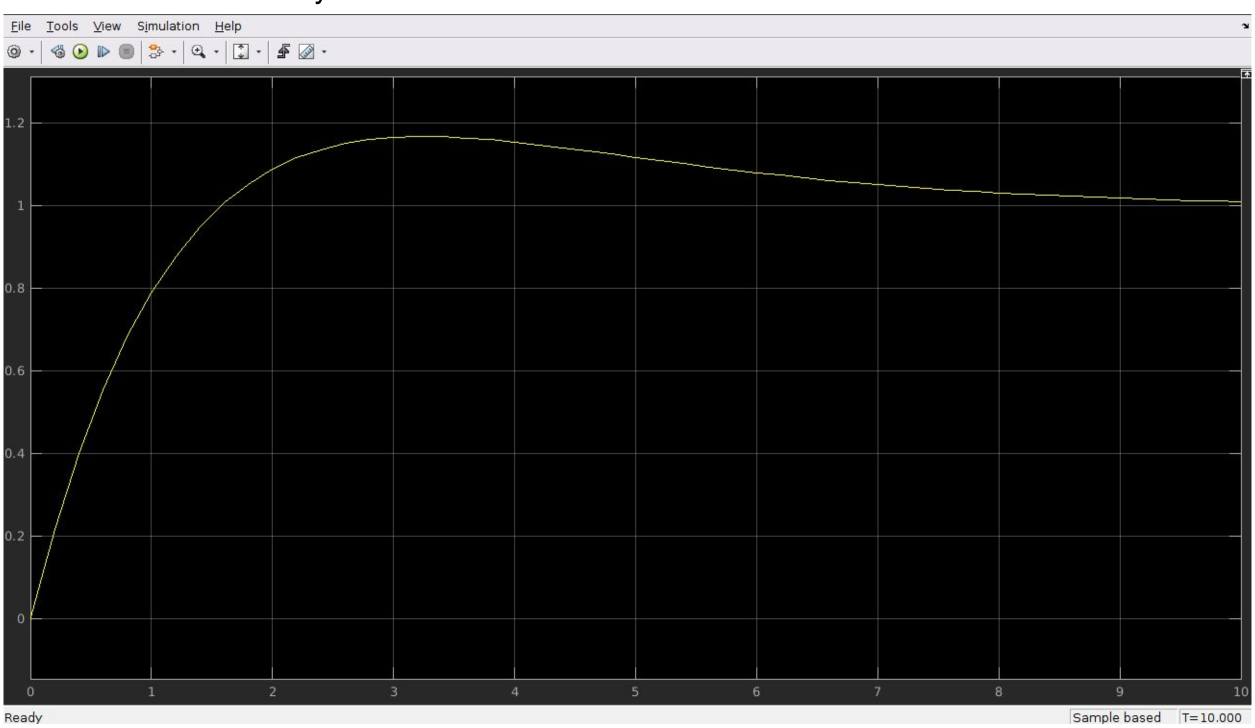
$$PID(z) = \frac{1083.5z - 1016.5}{z-1}$$

Now, since our controller is discrete & our process is continuous, we use ZOH for controller output and check the result in simulation.

The model in simulink is as follows:

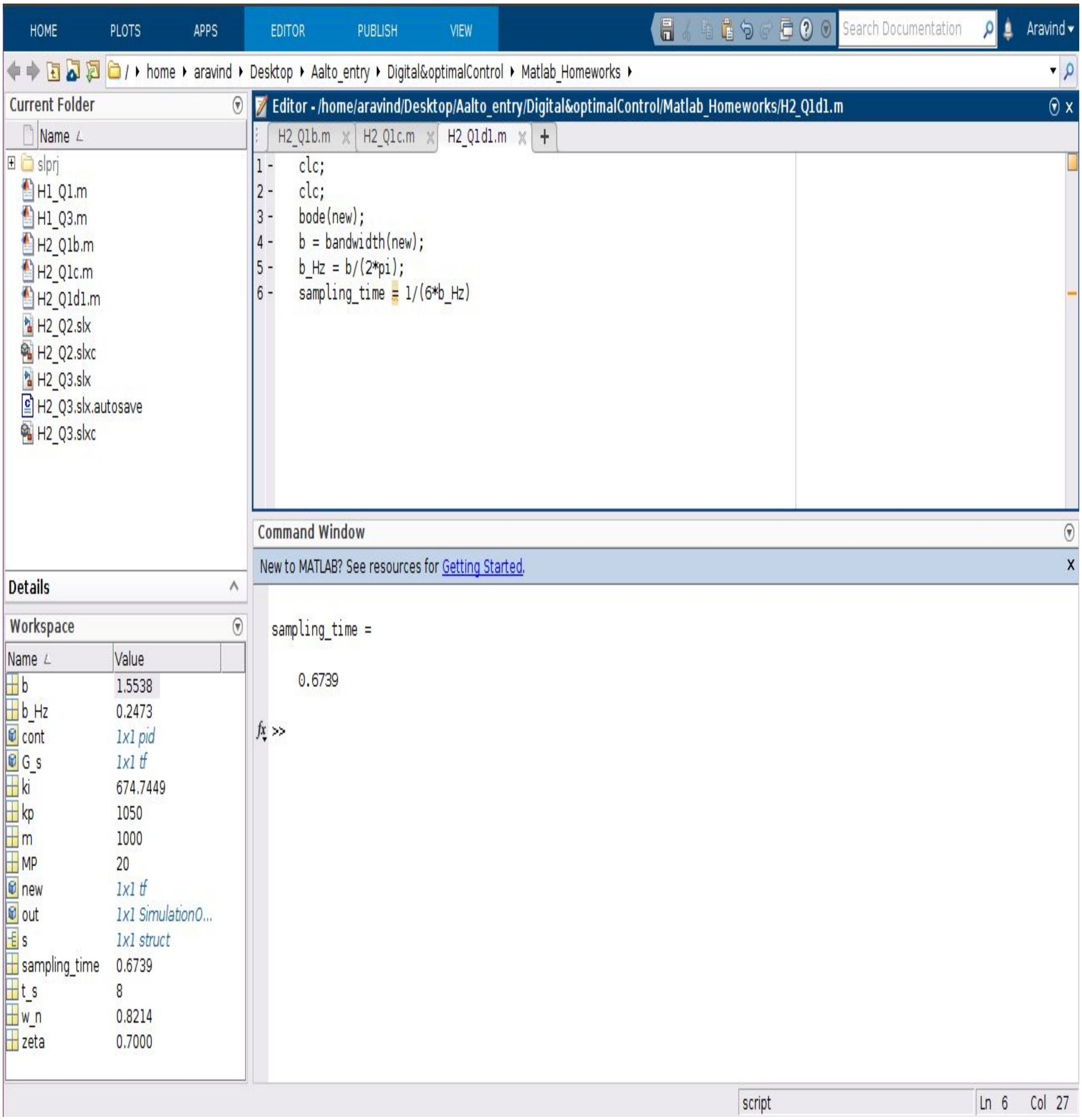


And the solution of the system above is

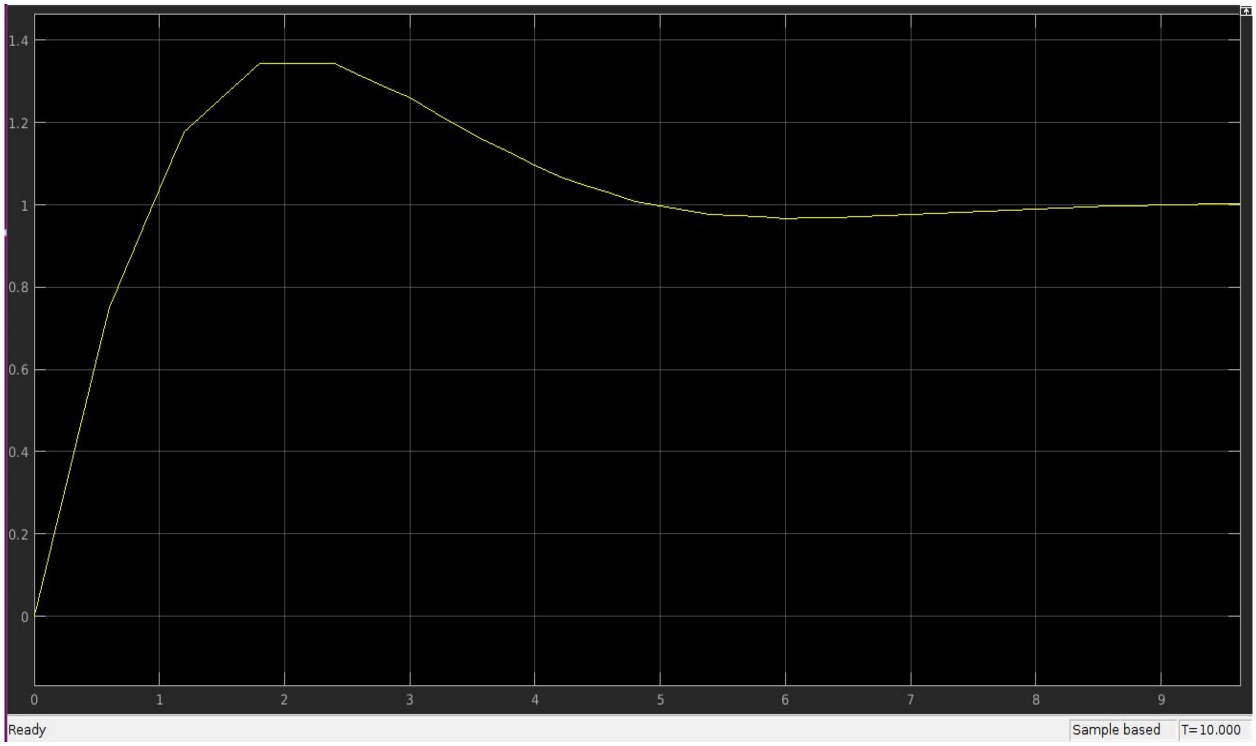


1.d:

The Sampling time required to calculate at 6 times the bandwidth is as follows and the sampling time is calculated as : Sampling time =0.6s



The simulink model is same as that of the 1.c , but just that the Controller parameters are different.
The output of the simulink model at Sampling time = 0.6s is as follows



1.d

(ii). Discretize the Plant.

Since our plant is an LTI-system and we consider our i/p is step-input to cas., so we use step-invariance method for calculating discretized function

$$G(z) = \frac{z-1}{z} Z\left(L^{-1}\left(\frac{G(s)}{s}\right)_{t=nT_s}\right)$$

$$= \frac{z-1}{z} Z\left(L^{-1}\left(\frac{1}{s(1000s+100)}\right)\right)$$

$$L^{-1}\left(\frac{1}{s(1000s+100)}\right) :- L^{-1}(T(s))$$

Taking Partial fraction

$$T(s) = \frac{A}{s} + \frac{B}{s+0.1} = \frac{A(s+0.1)+B(s)}{s(s+0.1)} = \frac{\frac{1}{1000}}{s(s+0.1)}$$

$$A(s+0.1) + B(s) = \frac{1}{1000}$$

$s = -0.1$ $\quad B(-0.1) = \frac{1}{1000} \Rightarrow B = -\frac{1}{100} = -0.01$.

$s = 0$ $\quad A(0.1) = \frac{1}{1000} \Rightarrow A = \frac{1}{100} = 0.01$

$$L^{-1}(T(s)) = L^{-1}\left(\frac{0.01}{s} - \frac{0.01}{s+0.1}\right) = 0.01\,u(t) - 0.01\,e^{-0.1t}$$

$$L^{-1}(T(s)) = 0.01\left(u(t) - e^{-0.1t}\right)$$

$\boxed{t = n \times 0.6}$

$$\left[L^{-1}(T(s))\right]_{t=0.6n} = 0.01\left(u(0.6n) - e^{-0.1\times0.6\,n}\right)$$

$$Z\left[L^{-1}(T(s))\right] = \frac{z}{z-1}\,0.01 - \frac{z}{z - e^{-0.1\times 0.6}} \times 0.01$$

$$= 0.01\left[\frac{z}{z-1} - \frac{z}{z - e^{-0.1\times 0.6}}\right]$$

$$\left(\frac{z-1}{z}\right) Z\left[L^{-1}\,T(s)\right]_{t=nT_s} = \frac{z-1}{z} \times 0.01\left[\frac{z}{z-1} - \frac{z}{z-0.9417}\right].$$

$$= 0.01 - \left(\frac{(z-1)\,0.01}{z-0.9417}\right)$$

$$\left(\frac{z-1}{z}\right)\left(Z\left[L^{-1}(T(s))\right]_{t=nT_s}\right) = \frac{0.00058}{z-0.9417}.$$

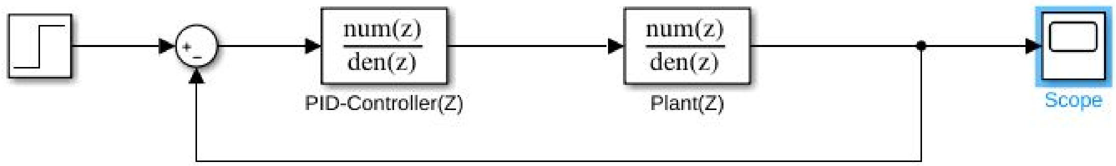
This is the new discretized plant equation at 6 times the bandwidth sampling.

$$\boxed{G(z) = \frac{0.00058}{z-0.9417}}$$

and the discretized PID at 6 times bandwidth sampling is found in 1.d (i) solution.

Scanned by CamScanner

The model for the above values in simulink is as follows:



And the output of the above simulink simulation is :