ELEC-E8101 Digital and Optimal Control

Homework 4

Homework 4 is given on Monday 25.11.2019. To be returned electronically by Thursday, 05.12.2019 at 23:55 in MyCourses portal ("Assignments").

The homeworks are not mandatory (compulsory), but they give 4 points per homework. For more information, see Course PM. It is highly recommended to do them. Solutions must be delivered in pdf-form (not Word, no Latex files, etc.). The whole solution must be written in one document and set in one file, including calculations, program codes, figures, etc. The solution file must have enough information, so that it becomes clear, how you have solved the problem. For example, the Matlab program codes and Simulink diagrams must be included in the solution document. If you want to use handwriting (and then change the document to pdf) you can do it, provided that the document can be read without difficulty. It is allowed to discuss and do the problems in groups. However, everybody must prepare and deliver his/her solutions individually. Copying directly somebody else's solution is not considered group work and is prohibited.

The above information concerns all 4 homework assignments to be given during the course.

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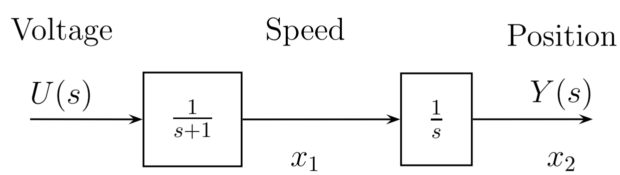
1. A DC/DC motor can be described by the simple block diagram below, where $U(s)$ is the input voltage and $Y(s)$ the shaft position.



   a) Show that the state-space representation of the system (by using the state variables x_1 and x_2 in the figure) is given by [0.5p]

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x}(t)$$

   b) Sample the state-space model with sampling time h, assuming ZOH and determine the discrete state-space representation of the form: [0.5p]

$$\mathbf{x}(kh+h) = \Phi(h)\mathbf{x}(kh) + \Gamma(h)\mathbf{u}(kh)$$
$$\mathbf{y}(kh) = C\mathbf{x}(kh) + D\mathbf{u}(kh)$$

   Hint:

$$\Phi(h) = e^{Ah} = e^{At}\big|_{t=h} = \mathcal{L}^{-1}\left\{(sI-A)^{-1}\right\}\big|_{t=h}$$
$$\Gamma(h) = \int_0^h e^{As} ds B$$

c) Let $h = 1$. Design for the discretized system a state feedback control law of the form

$$u[k] = my_{\text{ref}} - L\mathbf{x}[k],$$

where m is constant y_{ref} is the reference and L a gain vector. Assume that the states are measurable, the desired closed loop poles are $0.5 \pm j0.5$ and the static gain of the closed loop system is 1. [0.5p]

d) Suppose now that only the input u and output y are measured (i.e., x_1 is not measured). Design a discrete-time controller to the process, which consists of a combination of state feedback and state observer as

$$u[k] = my_{\text{ref}} - L\hat{\mathbf{x}}[k].$$

The desired closed loop poles due to state feedback are, as before, $0.5 \pm j0.5$ and the observer has the dead-beat characteristics. Choose the pre-compensator coefficient m such that the output of the controlled process follows the reference. Verify the performance by constructing a Simulink program. [1p]

e) Let a step load disturbance act at the output of the process (still inside the loop). Simulate and see what happens. Explain. [0.5p]