ELEC-E8101 Digital and Optimal Control

Homework 1

Homework 1 given on Monday 07.10.2019. To be returned electronically by Monday, 21.10.2019 at 23:55 in MyCourses portal ("Assignments").

The homeworks are not mandatory (compulsory), but they give 4 points per homework. For more information, see Course PM. It is highly recommended to do them. Solutions must be delivered in pdf-form (not Word, no Latex files, etc.). The whole solution must be written in one document and set in one file, including calculations, program codes, figures, etc. The solution file must have enough information, so that it becomes clear, how you have solved the problem. For example, the Matlab program codes and Simulink diagrams must be included in the solution document. If you want to use handwriting (and then change the document to pdf) you can do it, provided that the document can be read without difficulty. It is allowed to discuss and do the problems in groups. However, everybody must prepare and deliver his/her solutions individually. Copying directly somebody else's solution is not considered group work and is prohibited.

The above information concerns all 4 homework assignments to be given during the course.

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1. Consider the difference equation

$$x[k+2] - 1.5x[k+1] + 0.54x[k] = u[k] \tag{1}$$

with initial conditions x[0]=x[1]=0 and

$$u[k] = \begin{cases} 1, & k = 1, \\ 0, & k = 0, 2, 3, 4, \ldots \end{cases}$$

a) By using z-transforms calculate an analytical solution for $x[k]$. [0.5p]

b) Write a MATLAB code (script or function, as you wish), which uses difference equation (1) and solves the solution for x pointwise as $k = 0, 1, 2, 3, \ldots, 10$. Verify that the solution is in accordance to that obtained in part a). [0.5p]

2. Consider the following difference equation:

$$y[k+2] - 1.3y[k+1] + 0.4y[k] = u[k+1] - 0.4u[k].$$

a) Determine the pulse transfer function. [0.3p]

b) Is the system stable? Justify your answer. [0.3p]

c) Determine the step response. [0.4p]

3. Your manager has given a task for you. Your assignment is to replace the old analog PID (Proportional - Integral - Derivative) controller with a new digital PID controller. The transfer function of the continuous time process is:

$$P(s) = \frac{e^{-0.7s}}{s^2 + 0.8s + 0.5}.$$

The old analog PID controller is presented in the form:

$$G_{\text{PID}}(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right),$$

where $K = 1$, $T_i = 1.5$ and $T_d = 1$. Develop a discrete PID controller approximation for the continuous-time PID controller $G_{\text{PID}}(s)$:

a) using backward difference approximation. [0.3p]

b) using Tustin's transformation. [0.3p]

Now, a practical continuous PID controller (use $N = 10$) is considered, given by

$$\hat{G}_{\text{PID}}(s) = K\left((Y_{\text{ref}}(s) - Y(s)) + \frac{1}{T_i s}(Y_{\text{ref}}(s) - Y(s)) - \frac{T_d s}{1 + T_d s/N} Y(s)\right).$$

Develop a discrete PID controller approximation for the continuous-time PID controller $\hat{G}_{\text{PID}}(s)$:

c) by replacing the integral by summation and using backward difference approximation in the derivative action. [0.4p]

d) What are the facts that affect the choosing of the sampling rate h? By simulations, compare how the different approximations work. In the simulations use the continuous time process and the discrete PID controllers which you have defined in (a)–(c). Compare the control signals and the responses of the controlled systems. Use different reference signals. What is the best approximation? Try using different sampling periods in the discrete controllers. What happens if the sampling period changes? [1p]