Homework-1

1.) $x[k+2] - 1.5\,x[k+1] + 0.54\,x[k] = u[k]$

$x[0] = x[1] = 0$

$u[k] = \begin{cases} 1 & , k=1 \\ 0 & , k=0,2,3,\ldots \end{cases}$

$u[k] = \delta(k-1)$

1.a

$z^2 X(z) - z^2 x_0 - z\,x_1 - 1.5[z\,X(z) - z\,x_0]$

$+ 0.54\,X(z) = U(z)$

$z^2 X(z) - 1.5\,z\,X(z) + 0.54\,X(z) = z^{-1}$.

$X(z)\,[z^2 - 1.5z + 0.54] = z^{-1}$.

$$X(z) = \frac{z^{-1}}{(z^2 - 1.5z + 0.54)}$$

$$X(z) = \frac{z^{-1}}{(z-0.9)(z-0.6)}$$

$$= \frac{1}{z(z-0.9)(z-0.6)}$$

$$\frac{X(z)}{z} = \frac{1}{z^2(z-0.9)(z-0.6)}$$

$$= \frac{A}{z^2} + \frac{B}{z} + \frac{C}{z-0.9} + \frac{D}{z-0.6}$$

$$A(z-0.9)(z-0.6) + B(z)(z-0.9)(z-0.6) + C(z^2)(z-0.6) + D(z^2)(z-0.9) = 1.$$

put $z = 0.9$

$$C(0.9)^2(0.3) = 1 \Rightarrow \boxed{C = 4.12}$$

Put $z = 0.6.$

$$D(0.6)^2(-0.3) = 1. \Rightarrow \boxed{D = -9.259}$$

Put $z = 0.$

$$A(-0.9)(-0.6) = 1, \Rightarrow \boxed{\cancel{A = 3.70}} \quad \boxed{A = 1.851}$$

Put $z = 1.$

$$A(0.1)(0.4) + B(0.1)(0.4) + C(0.4) + D(0.1) = 1. \Rightarrow \boxed{B = 5.14.}$$

$$\frac{X(z)}{z} = \frac{1.851}{z^2} + \frac{5.14}{z} + \frac{4.12}{(z-0.9)} - \frac{9.26}{(z-0.6)}.$$

$$X(z) = \frac{1.851(z)}{z^2} + \frac{5.14}{z}(z) + 4.12\left(\frac{z}{z-0.9}\right) - 9.26\left(\frac{z}{z-0.6}\right)$$

Take Inverse z-transform & solving

$x(k) \; z$

$X(z) = 1.851 \, z^{-1} + 5.14 + 4.12 \left(\frac{z}{z-0.9}\right) - 9.26 \left(\frac{z}{z-0.6}\right)$

$x(k) = 1.851 \, \delta(k-1) + 5.14 \, \delta(k) + 4.12 \cdot (0.9)^k - 9.26 \, (0.6)^k$

1.b matlab code.

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1.b:

Matlab code :

```
clc;
clear;

syms z
S = z/(z^2*(z-0.9)*(z-0.6));
A = iztrans(S); % result in the next step
B =@(n) (50*kroneckerDelta(n - 1, sym(0)))/27 - (250*(3/5)^n)/27 + (1000*(9/10)^n)/243 +
(1250*kroneckerDelta(n, sym(0)))/243;
R = zeros(1,10);
for c = 0:1:9
        R(c+1) = B(c);
end

R
x= zeros(1,10);
x(1) = 0;
x(2) = 0;
for n = 1:7
        x(n+3)=(1.5*x(n+2))-(0.54*x(n+1))+ (kroneckerDelta(n-1,sym(0)));
end
X
```

Output off the above code:

```
R =

        0         0         0         1.0000 1.5000 1.7100 1.7550 1.7091 1.6160 1.5010

x =

        0         0         0         1.0000 1.5000 1.7100 1.7550 1.7091 1.6159 1.5010
```

Given,

2). $y[k+2) - 1.3\, y[k+1] + 0.4\, y[k] = u[k+1] - 0.4\, u[k]$

2.a Take z-transforms on both-sides.

$z^2 Y(z) - 1.3\, z\, Y(z) + 0.4\, Y(z) = z U(z) - 0.4\, U(z)$

$\because$ All initial conditions are assumed zero. And $U(z)$ for pulse transfer function.

$Y(z)\left[z^2 - 1.3z + 0.4\right] = U(z)\left[z - 0.4\right]$

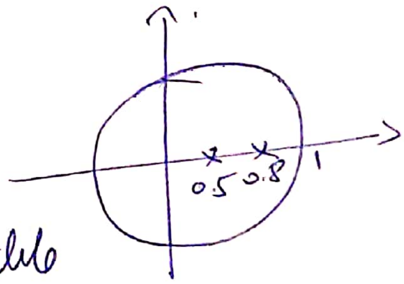
$$G(z) = \left[\frac{Y(z)}{U(z)}\right] = \frac{z - 0.4}{z^2 - 1.3z + 0.4} = \frac{z - 0.4}{(z - 0.5)(z - 0.8)}$$

2.b: The poles of system are.

$z = 0.5,\ 0.8$

are in-side the unit circle.

$\therefore$ the system is stable



2.c Step response of system.

$$U(z) = \frac{z}{z-1}$$

$Y(z) = G(z) \cdot U(z)$

Now, we need to compute the initial conditions

$y[k+2] - 1.3\, y[k+1] + 0.4\, y[k] = u[k+1] - 0.4\, u[k]$.

$u[k] = \begin{cases} 1, & k = 0, 1, 2 \ldots \\ 0, & \text{otherwise} \end{cases}$

Put $k = -2$.

$y[0] - 1.3\, y[-1] + 0.4\, y[-2] = u[-1] - 0.4\, u[-2]$.

$\boxed{y[0] = 0}$.

Put $\boxed{k = -1}$

$y[1] - 1.3\, y[0] + 0.4 \cdot y[-1] = u[0] - 0.4\, u[-1]$.

$\boxed{y[1] = 1}$

Put $k = 0$.

$y[2] - 1.3 \cdot y[1] + 0.4 \cdot y[0] = u[1] - 0.4\, u[0]$.

$y[2] = 1 - 0.4 + 1.3 = 1.9$.

$y[2] = 1.9$.

Now, taking z-transforms, we get.

$[z^2 Y(z) - z^2 y_0 - z y_1] - 1.3 [z Y(z) - z y_0]$
$+ [0.4 \cdot Y(z)] = [z \cdot U(z) - z U[0]]$
$- 0.4\, U(z)$

$[z^2 Y(z) - z] - 1.3 [z \cdot Y(z)] + 0.4 \cdot Y(z) = z(U(z) - 1)$
$- 0.4\, U(z)$

$z^2 Y(z) - 1.3 z Y(z) + 0.4\, Y(z) - z = z U(z) - 0.4\, U(z) - z$

$z^2 Y(z) - 1.3 z Y(z) + 0.4\, Y(z) = z U(z) - 0.4\, U(z) \cancel{-z} + z$.

$Y(z) [z^2 - 1.3 z + 0.4] = U(z) [z - 0.4] - z + z$.

$$\frac{Y(z)}{U(z)} = \frac{z - 0.4}{z^2 - 1.3 z + 0.4}$$

$$Y(z) = \frac{z-0.4}{(z-0.5)(z-0.8)} \quad U(z)$$

$$Y(z) = \frac{z-0.4}{(z-0.5)(z-0.8)} \left(\frac{z}{z-1}\right)$$

$$\frac{Y(z)}{z} = \frac{z-0.4}{(z-1)\ (z-0.5)\ (z-0.8)}$$

By partial fraction

$$\frac{z-0.4}{(z-1)(z-0.5)(z-0.8)} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)} + \frac{C}{z-0.8}$$

Put $z = 1$, $\boxed{A = 6}$

put $z = 0.5$, $\boxed{B = 0.666}$

Put $z = 0.8$, $\boxed{C = 6.666}$

$$\frac{Y(z)}{z} = \frac{6}{(z-1)} + \frac{0.666}{(z-0.5)} - \frac{6.666}{(z-0.8)}$$

$$Y(z) = 6 \cdot \left(\frac{z}{z-1}\right) + 0.666\left(\frac{z}{z-0.5}\right) - 6.666 \cdot \left(\frac{z}{z-0.8}\right)$$

Taking, inverse z-transform

$$\boxed{y(k) = 6\, u(k) + 0.666\, (0.5)^k - 6.666\, (0.8)^k}$$

(or)

$$y(k) = \begin{cases} 6 + 0.666\,(0.5)^k - 6.666\,(0.8)^k, & k = 0, 1, 2, 3 \ldots \\ 0, & \text{otherwise.} \end{cases}$$

3.)

Given,

$$P(s) = \frac{e^{-0.75s}}{s^2+0.8s+0.5}.$$

$$G_{PID}(s) = k\left(1 + \frac{1}{T_i s} + T_d s\right)$$

3.a To create PID approximation using backward difference approximation

$$G_{PID}(z) = G(s)\Big|_{s=\frac{1-z^{-1}}{h}}.$$

$$G_{PID}(z) = k\left(1 + \frac{1}{T_i\left(\frac{1-z^{-1}}{h}\right)} + T_d\left(\frac{1-z^{-1}}{h}\right)\right.$$

$$= k\left(1 + \frac{h}{T_i(1-z^{-1})} + T_d\left(\frac{1-z^{-1}}{h}\right)\right.$$

$$= k\left[1 + \frac{h}{T_i}\left(\frac{z}{z-1}\right) + \frac{T_d}{h}\left(\frac{z-1}{z}\right)\right] \quad (1)$$

Now given. $k=1$, $T_i = 1.5$ & $T_d = 1$.

Equation (1) becomes

$$G_{PID}(z) = 1 + \frac{h}{1.5}\left(\frac{z}{z-1}\right) + \frac{1}{h}\left(\frac{z-1}{z}\right)$$

$$\boxed{G_{PID}(z) = 1 + \frac{h}{1.5}\left(\frac{z}{z-1}\right) + \frac{1}{h}\left(\frac{z-1}{z}\right)} \quad (2)$$

is approximation using backward difference approximation

3.b using Tustin's transformation.

$$G(z) = G(s)\Big|_{s=\frac{2}{h}\left(\frac{z-1}{z+1}\right)}$$

$$G_{PID}(z) = 1 + \frac{1}{1.5 \cdot \left(\frac{2}{h}\left(\frac{z-1}{z+1}\right)\right)} + \frac{2}{h}\left(\frac{z-1}{z+1}\right)$$

$$= 1 + \frac{h(z+1)}{3(z-1)} + \frac{2}{h}\left(\frac{z-1}{z+1}\right)$$

$$\boxed{G_{PID}(z) = 1 + \frac{h}{3}\left(\frac{z+1}{z-1}\right) + \frac{2}{h}\left(\frac{z-1}{z+1}\right)}$$

3.C

$$G_{PID}(s) = k\left[Y_{ref}(s) - Y(s) + \frac{1}{T_i s}(Y_{ref}(s) - Y(s)) - \frac{T_d s}{1 + \frac{T_d s}{N}} (Y(s)) \right]$$

taking piecewise.

$$k(Y_{ref}(s) - Y(s)) \overset{z}{\Rightarrow} k[Y_{ref}(z) - Y(z)]$$

$$\frac{K_i}{T_i s}[Y_{ref}(s) - Y(s)] \Rightarrow$$ Taking integral term.

inverse laplace transform.

$$\left(\frac{1}{s}\right)\frac{k}{T_i} \Longrightarrow u(t)\,\frac{K_I}{T_i}$$

Now sampling it.

$$u(kh).\frac{K_i}{T_i} \xrightarrow{z\text{-transform}} \left[\sum_{k=-\infty}^{\infty} u(kh).\frac{k_i}{T_i} z^{-k}\right]$$

$$= \frac{k_i}{T_i}\left[\sum_{k=-\infty}^{\infty} u(kh)\, z^{-k}\right]$$

$$= \frac{k_i}{T_i}\left(\frac{h.z}{z-1}\right)$$

$$= \frac{z\,h.k}{T_i(z-1)}$$

$$\frac{-k_d s}{1 + \frac{k_d s}{N}}(Y(s)) \xrightarrow{z\text{-transform}} Y(s)\Big|_{s=\frac{z-1}{zh}} = \frac{-k_d\,\frac{z-1}{zh}\,Y(z)}{1 + \frac{k_d}{N}\left(\frac{z-1}{zh}\right)}$$

Substituting above terms.

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$$G_{PID}(z) = k\left[Y_{ref}(z) - Y(z)\right] + \frac{1}{T_i}\left[\frac{k \cdot h \cdot z}{(z-1)}\left[Y_{ref}(z) - Y(z)\right]\right]$$

$$- T_d\left[\frac{k \cdot N \cdot (z-1)\, Y(z)}{z \cdot h \cdot N + k(z-1)}\right]$$

by substituting values of k, T_i, T_d, N.

$$= Y_{ref}(z) - Y(z) + \frac{1}{1.5}\left[\frac{hz}{(z-1)}\left(Y_{ref}(z) - Y(z)\right)\right]$$

$$- \frac{10\,(z-1)}{10zh + (z-1)}\, Y(z)$$

$$G_{PID}(z) = Y_{ref}(z) - Y(z) + \frac{h}{1.5}\left(\frac{z}{z-1}\right)\left[Y_{ref}(z) - Y(z)\right]$$

$$- \frac{10\,(z-1)}{z(10h+1) - 1}\, Y(z)$$

(2)

is the final solution for 3.C

3.d

$$P(s) = \frac{e^{-0.7s}}{s^2 + 0.8s + 0.5} \Rightarrow \cancel{e^{-0.7}}$$

$$P(z) = P(s)\Big|_{s=\frac{z-1}{zh}} = \frac{e^{-0.7\left(\frac{z-1}{zh}\right)}}{\left(\frac{z-1}{zh}\right)^2 + 0.8\left(\frac{z-1}{zh}\right) + 0.5}$$

$$= \frac{e^{-0.7\left(\frac{z-1}{zh}\right)}(zh)^2}{(z-1)^2 + 0.8(z-1)(zh) + 0.5(zh)^2}.$$

$$P(z) = \frac{e^{-0.7\left(\frac{z-1}{zh}\right)}(zh)^2}{(z-1)^2 + 0.8(z-1)(zh) + 0.5(zh)^2}.$$