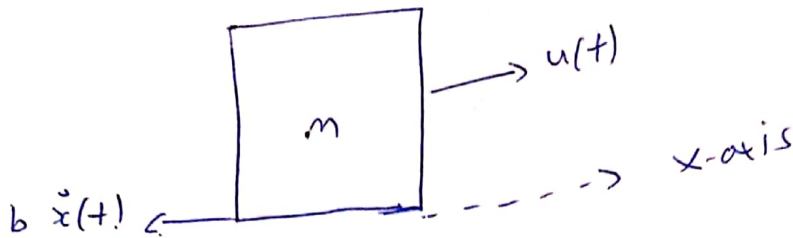


Home work-2

1.)

a.)



Given,

$$v(t) = \dot{x}(t)$$

by equating the force

$$m \ddot{v}(t) = u(t) - b v(t)$$

$$m \dot{v}(t) + b v(t) = u(t)$$

divide by m

$$\dot{v}(t) + \frac{b}{m} v(t) = \frac{u(t)}{m}$$

b.)

Given, $m = 1000 \text{ kg}$,

$b = 100$,

overshoot $< 20\%$.

settling time $c(t_s) = 8s$

$$\xi = 0.6 \left(1 - \frac{m.p}{100} \right)$$

$$= 0.6 \left(1 - \frac{20}{100} \right)$$

$$\boxed{\xi = 0.48}$$

i.e.

$$\boxed{\xi \geq 0.48}$$

$$t_s = \frac{4.6}{\xi \omega_n}$$

$$\omega_n = \frac{4.6}{\xi t_s}$$

$$= \frac{4.6}{(0.48) 8}$$

$$= 1.198$$

$$\boxed{\omega_n = 1.198}$$

For step 1, we assume $\boxed{\xi = 0.48}$

[Can be changed based on experiments to achieve design specification]

From (1.9)

$$\ddot{v}(t) + \frac{b}{m} \dot{v}(t) = \frac{1}{m} u(t)$$

Taking Laplace Transform

$$sV(s) + \frac{b}{m} V(s) = \frac{1}{m} U(s)$$

$$V(s) \left(s + \frac{b}{m} \right) = \frac{1}{m} U(s)$$

$$G(s) = \frac{V(s)}{U(s)} = \frac{1}{ms + b} = \frac{1}{1000s + 100}$$

The closed loop transfer function is

where

$$H(s) = \frac{PID(s) G(s)}{1 + PID(s) G(s)}$$

$$PID(s) = k_p + \frac{k_i}{s}$$

$$H(s) = \frac{\left(\frac{k_p s + k_i}{m} \right)}{s^2 + \left(\frac{b + k_p}{m} \right) s + \frac{k_i}{m}}$$

For second order system

$$H(s) = \frac{\omega_n^2}{s^2 + \left(\frac{b + k_p}{m} \right) s + \frac{k_i}{m}}$$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

equating, we get

$$\frac{k_i}{m} = \omega_n^2$$

$$k_i = m \omega_n^2$$

$$k_i = 1435$$

$$b + k_p = 2\xi\omega_n m$$

$$k_p = 2\xi\omega_n m - b$$

$$k_p = 1050$$

The values for zeta calculated as 0.48 and the following results are obtained:

$K_p = 1050$;

$K_i = 1435$;

Settling time = 6.33s

Overshoot = 28 (Which is not desired)

The image shows a MATLAB workspace with the following variables:

Name	Value
b	100
cont	1x1 pid
G_s	1x1 tf
ki	1.4350e+03
kp	1050
m	1000
MP	20
new	1x1 tf
s	1x1 struct
t_s	8
w_n	1.1979
zeta	0.4800

The Command Window displays the following structure for the PID controller:

```
s =  
  
struct with fields:  
  
    RiseTime: 0.8473  
    SettlingTime: 6.3365  
    SettlingMin: 0.9142  
    SettlingMax: 1.2862  
    Overshoot: 28.6151  
    Undershoot: 0  
    Peak: 1.2862  
    PeakTime: 2.0823
```

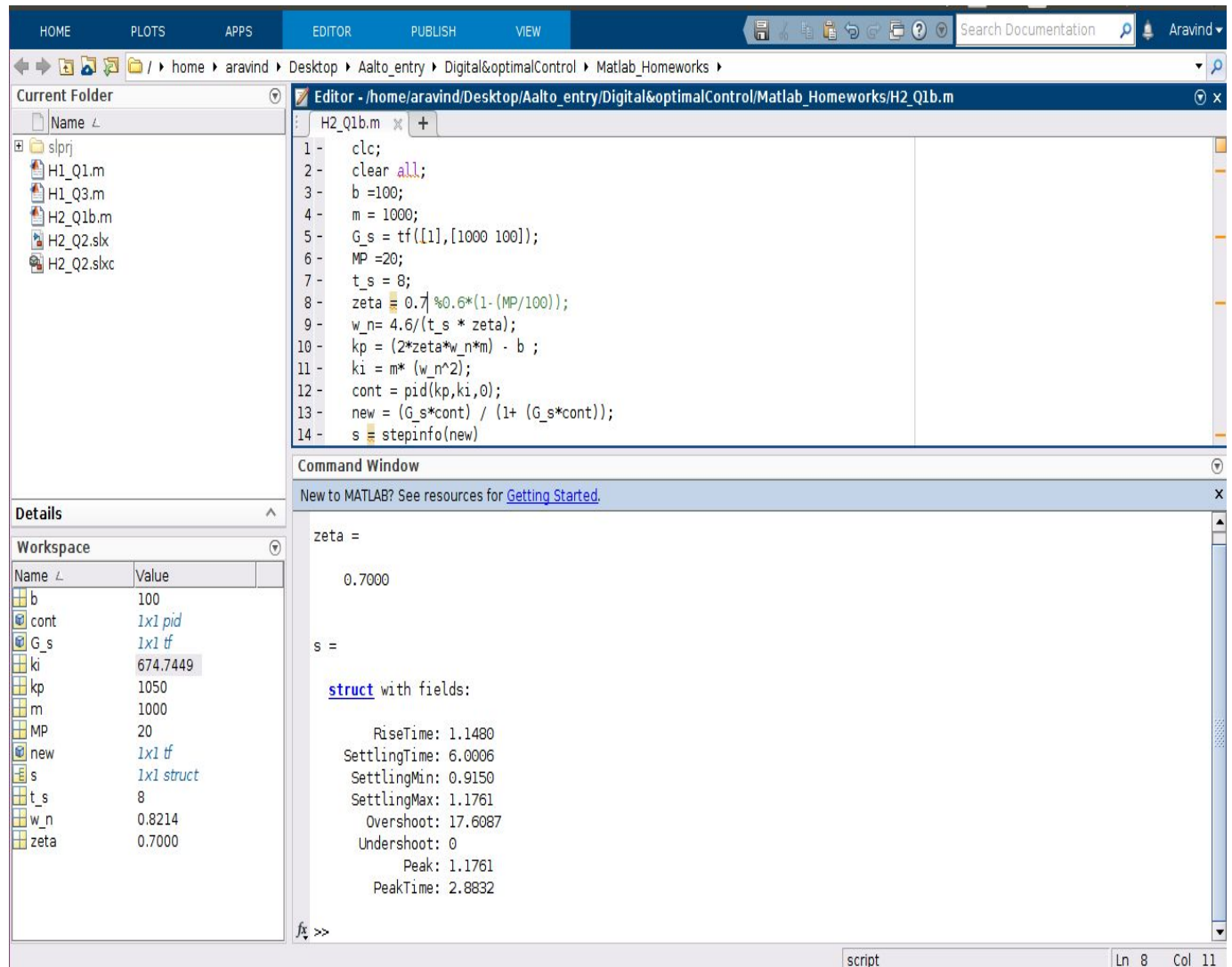
Since the design specifications were not met and it is observed that we need to increase the damping factor to achieve the design specifications, we make $\zeta=0.7$ (increased and checked the value which meets our specs). And the following results are obtained:

$K_p = 1050$;

$K_i = 670$;

Settling time = 6s

Overshoot = 18% (less than 20%)



The screenshot displays the MATLAB environment with the following components:

- Current Folder:** Contains files `slprj`, `H1_Q1.m`, `H1_Q3.m`, `H2_Q1b.m`, `H2_Q2.slx`, and `H2_Q2.slxc`.
- Editor:** Shows the script `H2_Q1b.m` with the following code:

```
1 - clc;
2 - clear all;
3 - b = 100;
4 - m = 1000;
5 - G_s = tf([1],[1000 100]);
6 - MP = 20;
7 - t_s = 8;
8 - zeta = 0.7 % 0.6*(1-(MP/100));
9 - w_n = 4.6/(t_s * zeta);
10 - kp = (2*zeta*w_n*m) - b ;
11 - ki = m*(w_n^2);
12 - cont = pid(kp,ki,0);
13 - new = (G_s*cont) / (1+ (G_s*cont));
14 - s = stepinfo(new)
```
- Command Window:** Displays the execution results:

```
zeta =
    0.7000

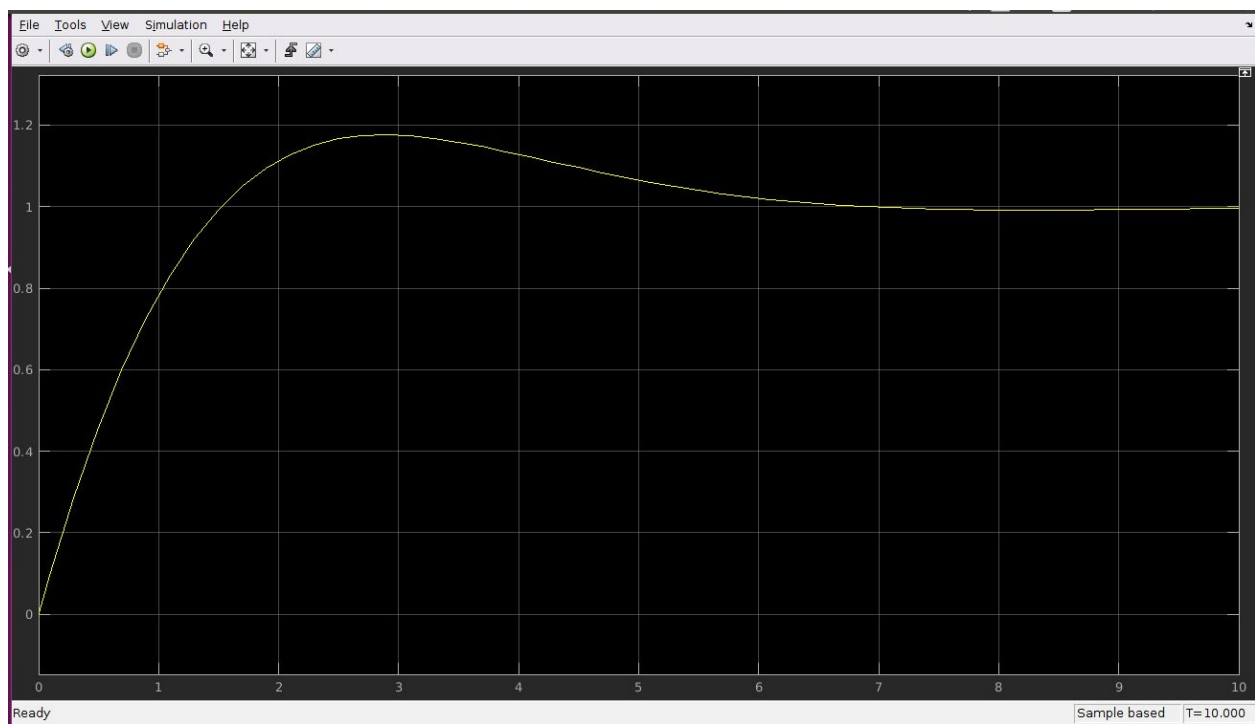
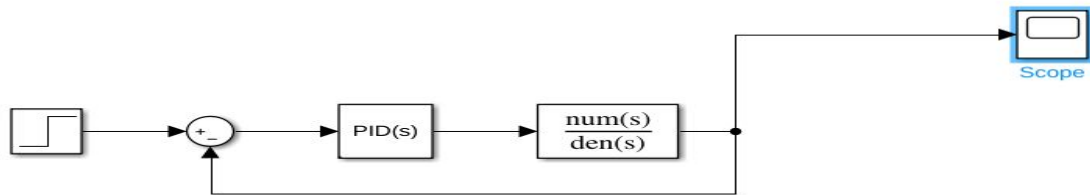
s =

 struct with fields:
    RiseTime: 1.1480
    SettlingTime: 6.0006
    SettlingMin: 0.9150
    SettlingMax: 1.1761
    Overshoot: 17.6087
    Undershoot: 0
    Peak: 1.1761
    PeakTime: 2.8832
```
- Workspace:** Lists variables and their values:

Name	Value
b	100
cont	1x1 pid
G_s	1x1 tf
ki	674.7449
kp	1050
m	1000
MP	20
new	1x1 tf
s	1x1 struct
t_s	8
w_n	0.8214
zeta	0.7000

The status bar at the bottom indicates the current position is at line 8, column 11.

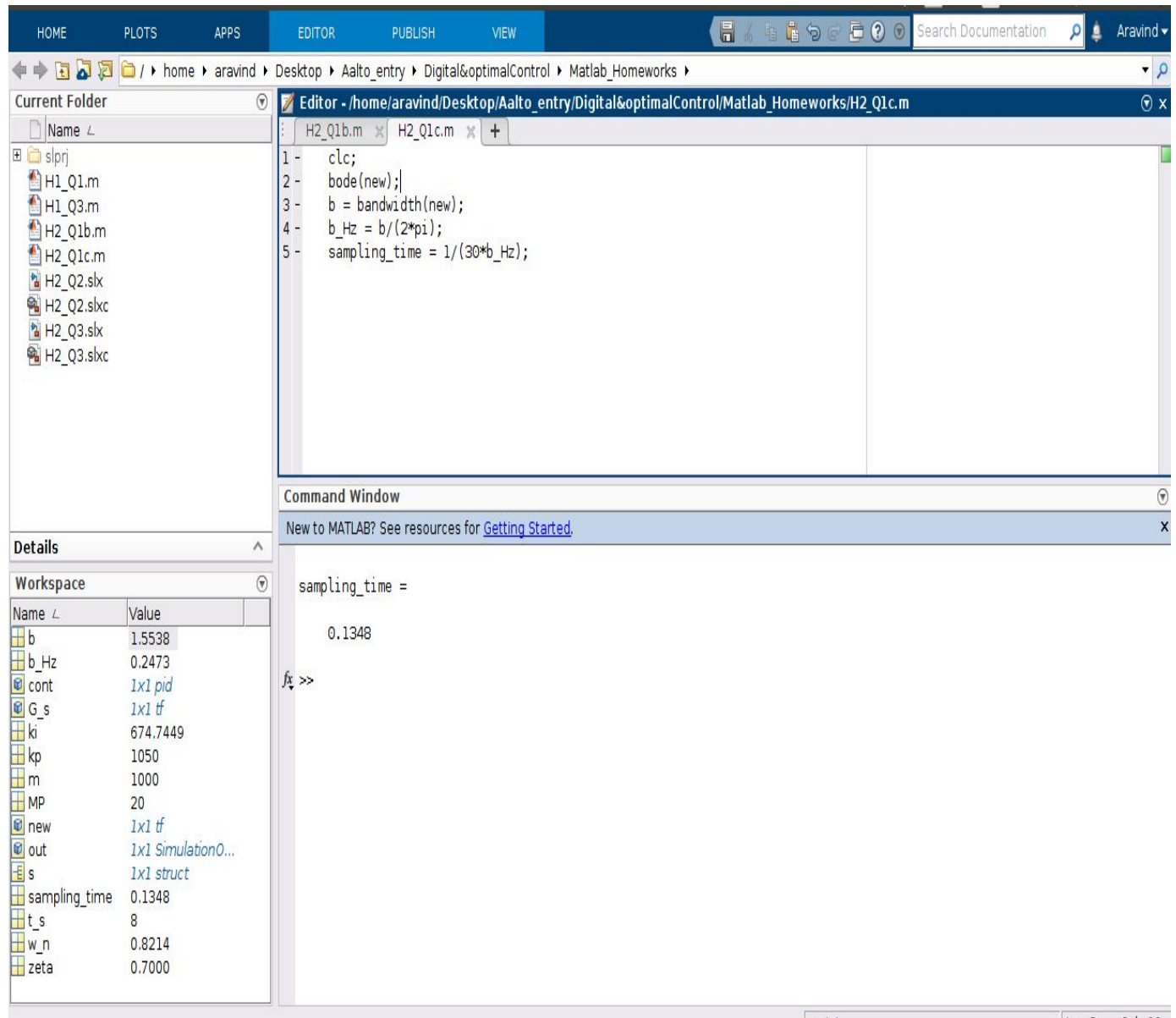
The simulink verification is as follows



As we can see the output is settled at close to 6s and maximum overshoot is between(1.17-1.20)

1.c

The calculation of Sampling time can be found in the screenshot of the code below and value is
Sampling time = 0.1s



1.C

$T = 0.1s$
To discretize the controller

$$PID(s) = \frac{1050s + 670}{s}$$

$$PID(z) = PID(s) \left| \begin{array}{l} s = \frac{z-1}{0.1} \end{array} \right.$$

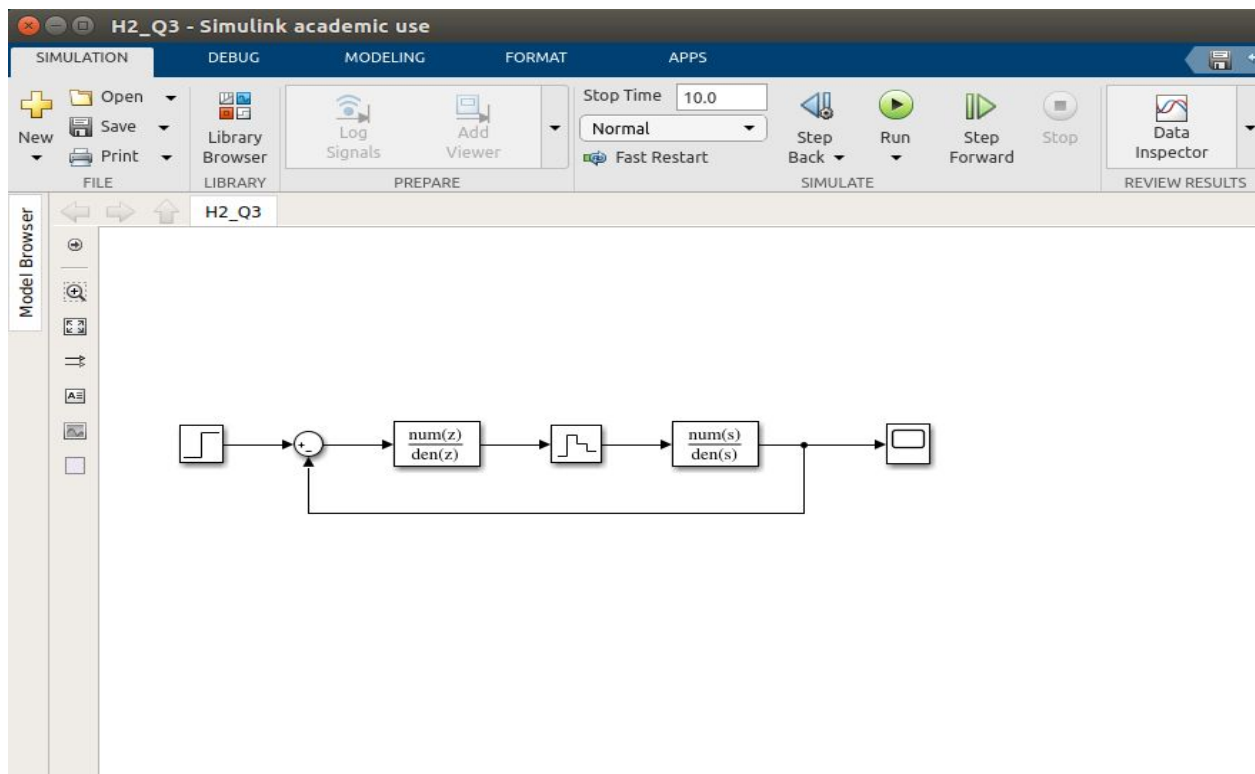
$$= \frac{1050 + 20 \left(\frac{z-1}{z+1} \right) + 670}{20 \left(\frac{z-1}{z+1} \right)}$$

$$= \frac{1050 \left(\frac{z-1}{z+1} \right) + 33.5}{20(z-1)}$$

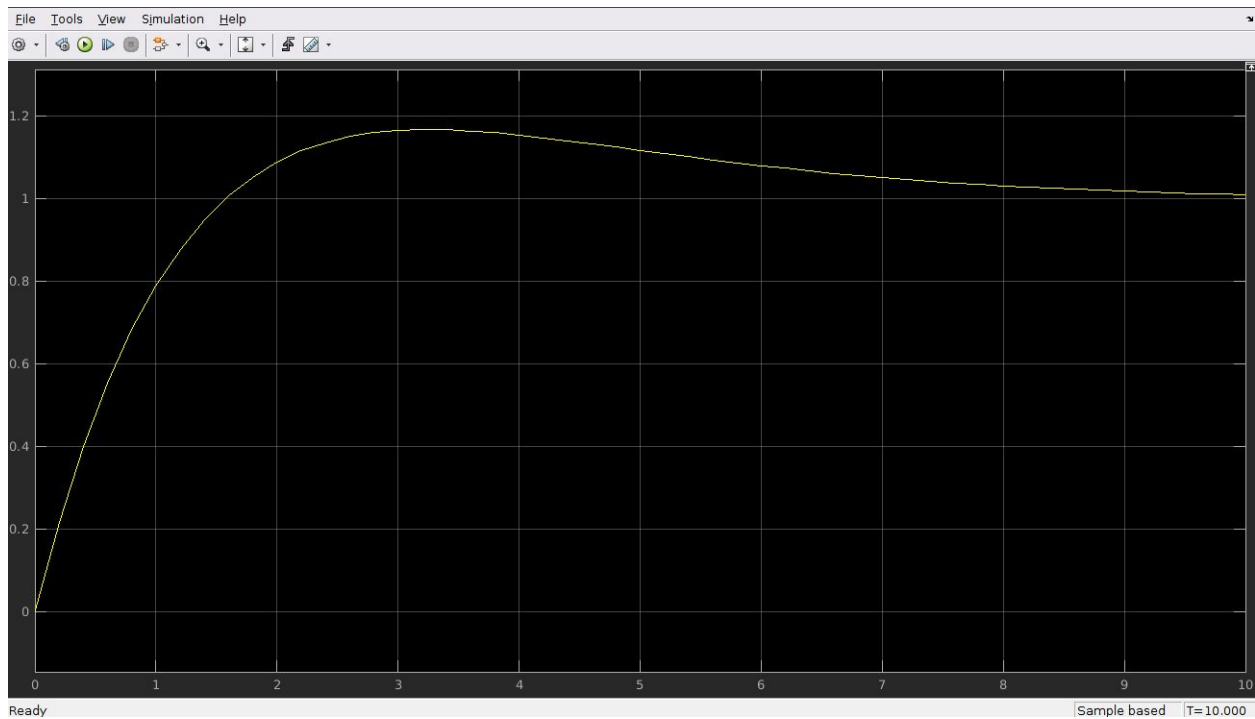
$$PID(z) = \frac{1083.5z - 1016.5}{z-1}$$

Now, Since our controller is discrete & our process is continuous, we use ZOH for controller output and check the result in simulation.

The model in simulink is as follows:



And the solution of the system above is



1.d:

The Sampling time required to calculate at 6 times the bandwidth is as follows and the sampling time is calculated as : Sampling time =0.6s

The screenshot displays the MATLAB environment with the following components:

- Editor:** The file `H2_Q1d1.m` contains the following code:

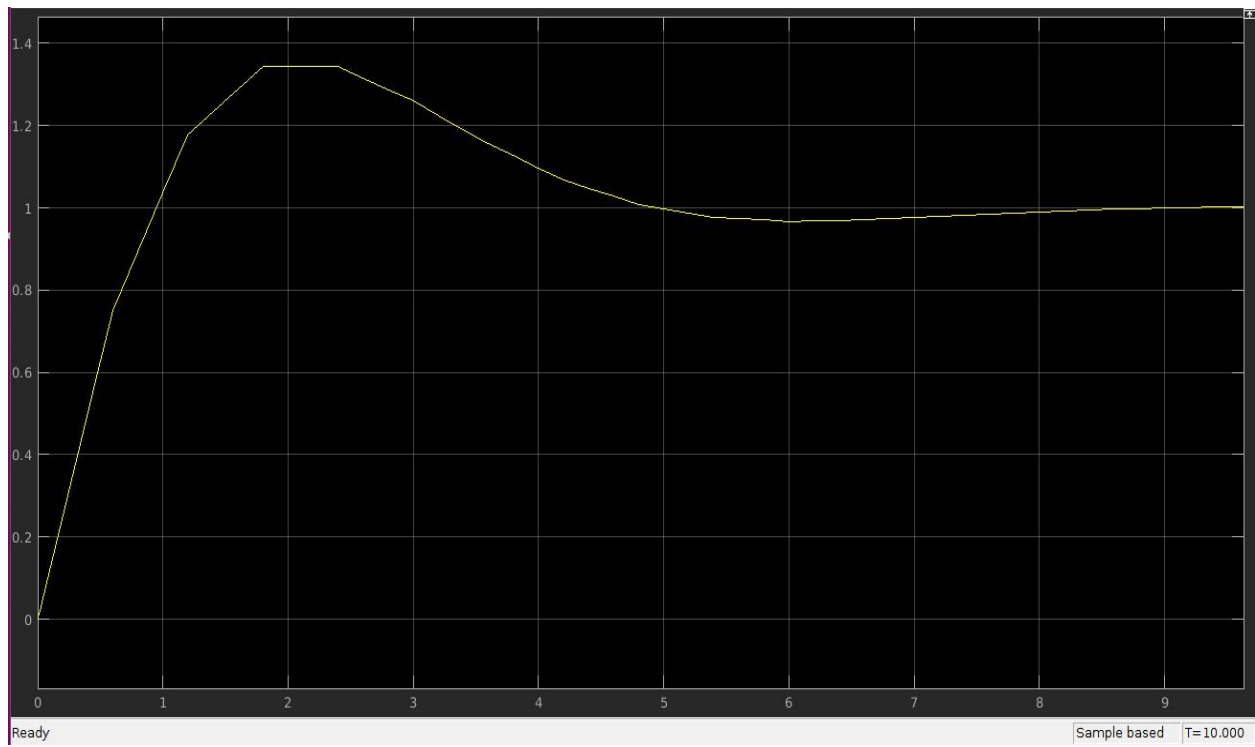
```
1 - clc;
2 - clc;
3 - bode(new);
4 - b = bandwidth(new);
5 - b_Hz = b/(2*pi);
6 - sampling_time = 1/(6*b_Hz)
```
- Current Folder:** Lists files in the directory `/home/aravind/Desktop/Aalto_entry/Digital&optimalControl/Matlab_Homeworks`, including `H1_Q1.m`, `H1_Q3.m`, `H2_Q1b.m`, `H2_Q1c.m`, `H2_Q1d1.m`, `H2_Q2.slx`, `H2_Q2.slx`, `H2_Q3.slx`, `H2_Q3.slx.autosave`, and `H2_Q3.slx`.
- Command Window:** Shows the execution of the script, resulting in `sampling_time = 0.6739`. The prompt `>>` is visible.
- Workspace:** A table listing variables and their values:

Name	Value
b	1.5538
b_Hz	0.2473
cont	1x1 pid
G_s	1x1 tf
ki	674.7449
kp	1050
m	1000
MP	20
new	1x1 tf
out	1x1 Simulation0...
s	1x1 struct
sampling_time	0.6739
t_s	8
w_n	0.8214
zeta	0.7000

The status bar at the bottom indicates the current position is `Ln 6 Col 27`.

The simulink model is same as that of the 1.c , but just that the Controller parameters are different.

The output of the simulink model at Sampling time = 0.6s is as follows



1. d
(ii).

Discretize the plant.

Since our plant is an LTI-system and we consider our i/r is step-input to our, so we use step-invariance method for calculating discretized function

$$G(z) = \frac{z^{-1}}{z} \geq \left(L^{-1} \left(\frac{G(s)}{s} \right)_{t=nT_s} \right)$$

$$= \frac{z^{-1}}{z} \geq \left(L^{-1} \left(\frac{1}{s(1000s+100)} \right) \right)$$

$$L^{-1} \left(\frac{1}{s(1000s+100)} \right) :- L^{-1} \left(T(s) \right)$$

Taking Partial fraction

$$T(s) = \frac{A}{s} + \frac{B}{s+0.1} = \frac{A(s+0.1) + B(s)}{s(s+0.1)} = \frac{1}{1000}$$

$$A(s+0.1) + B(s) = \frac{1}{1000}$$

$$s = -0.1 \quad B(-0.1) = \frac{1}{1000} \Rightarrow B = -\frac{1}{100} = -0.01$$

$$s = 0 \quad A(0.1) = \frac{1}{1000} \Rightarrow A = \frac{0.1}{100} = 0.001$$

$$L^{-1} \left(T(s) \right) = L^{-1} \left(\frac{0.001}{s} - \frac{0.001}{s+0.1} \right) = 0.001 u(t) - 0.001 e^{-0.1t}$$

$$L^{-1} \left(T(s) \right) = 0.001 \left(u(t) - e^{-0.1t} \right)$$

$$t = n \times 0.6$$

$$\left[L^{-1} \left(T(s) \right) \right]_{t=0.6n} = 0.001 \left(u(0.6n) - e^{-0.1 \times 0.6n} \right)$$

$$z \left[L^{-1}(T(s)) \right] = \frac{z}{z-1} 0.01 - \frac{z}{z - e^{-0.1 \times 0.6}} \times 0.01$$

$$= 0.01 \left[\frac{z}{z-1} - \frac{z}{z - e^{-0.1 \times 0.6}} \right]$$

$$\left(\frac{z-1}{z} \right) z \left[L^{-1}(T(s)) \right]_{t=nT_s} = \frac{z-1}{z} \times 0.01 \left[\frac{z}{z-1} - \frac{z}{z - 0.9417} \right]$$

$$= 0.01 - \left(\frac{(z-1) 0.01}{z - 0.9417} \right)$$

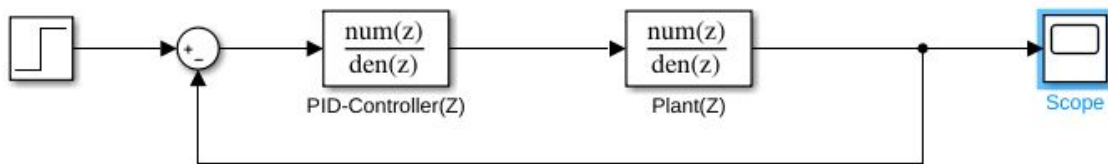
$$\left(\frac{z-1}{z} \right) \left(z \left[L^{-1}(T(s)) \right]_{t=nT_s} \right) = \frac{0.00058}{z - 0.9417}$$

This is the new discretized plant equation at 6 times the bandwidth sampling.

$$G(z) = \frac{0.00058}{z - 0.9417}$$

and the discretized PID at 6 times bandwidth sampling is found in 1.d (i) solution.

The model for the above values in simulink is as follows:



And the output of the above simulink simulation is :

