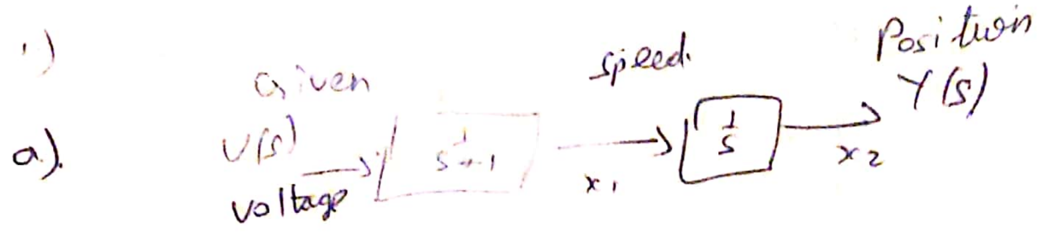
Homework-4

1)

a)



Variables.

$x_1 \Rightarrow$ speed (velocity)

$x_2 \Rightarrow$ Position.

The transfer function

$$\frac{Y(s)}{U(s)} = \left(\frac{1}{s} \cdot \frac{1}{(s+1)}\right)$$

$$= \frac{1}{s^2+s}$$

$$Y(s)\left[(s^2+s)\right] = U(s)$$

$$Y(s)\, s^2 + s \cdot Y(s) = U(s)$$

Taking inverse laplace transform and assuming zero initial conditions. ie, when there is no input voltage, the shaft position is at zero position.

$$\boxed{\ddot{y}(t) + \dot{y}(t) = u(t)} \quad (1)$$

Now, we assume state variables

$y(t) = x_2(t) \Rightarrow$ Position (A)

$\dot{y}(t) = x_1(t) \Rightarrow$ velocity

i.e. $\boxed{\dot{x}_2(t) = x_1(t)}$ (2)

Now from ①, we get

$$\dot{x}_1(t) + x_1(t) = u(t)$$

$$\dot{x}_1(t) = -x_1(t) + u(t) \quad — ③$$

Now, in state space representation

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

From equation ② and ③,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u. \quad ④$$

and also from equation ④

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

∴ The given state space representation is achieved. i.e.

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

b.) To sample the state space equation

Now
$$\phi(h) = L^{-1}\{(sI - A)^{-1}\}\Big|_{t=h}$$
$$\Gamma(h) = \int_0^h e^{As}\,ds\,B.$$

$\underline{\underline{\phi(h)}}$:-

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} s+1 & 0 \\ -1 & s \end{bmatrix}.$$

$$(sI - A)^{-1} = \frac{1}{s(s+1)} \begin{bmatrix} s & 0 \\ 1 & s+1 \end{bmatrix}.$$
$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{1}{s(s+1)} & \frac{1}{s} \end{bmatrix}.$$

$$L^{-1}\{sI - A\}^{-1} = \begin{bmatrix} e^{-t} & 0 \\ 1 - e^{-t} & 1 \end{bmatrix}$$

$\underline{\underline{\frac{1}{s(s+1)}}}$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$
$$1 = A(s+1) + Bs$$
Put $s = 0$ | $s = -1$.
$\boxed{A = 1}$ | $B = -1$
$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$
$\therefore$ Inverse
$$= 1 - e^{-t}.$$

Now
$$\phi(h) = L^{-1}\{(sI - A)^{-1}\}\Big|_{t=h}$$
$$= \begin{bmatrix} e^{-h} & 0 \\ 1 - e^{-h} & 1 \end{bmatrix}.$$

$\underline{\Gamma(h)} =$

$$\Gamma(h) = \int_0^h e^{As}\, ds\, B.$$

$$e^{Ah} = \begin{bmatrix} e^{-h} & 0 \\ 1-e^{-h} & 1 \end{bmatrix}.$$

$$e^{As} = \begin{bmatrix} e^{-s} & 0 \\ 1-e^{-s} & 1 \end{bmatrix}.$$

$$\Gamma(h) = \int_0^h \begin{bmatrix} e^{-s} & 0 \\ 1-e^{-s} & 1 \end{bmatrix} ds \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \int_0^h \begin{bmatrix} e^{-s} \\ 1-e^{-s} \end{bmatrix} ds$$

Now

$$\underline{\int_0^h e^{-s} ds} = \left[-e^{-s}\right]_0^h$$
$$= -e^{-h} + 1$$
$$= 1 - e^{-h}.$$

$$\int_0^h 1 - e^{-s}\, ds = [s]_0^h + [e^{-s}]_0^h$$
$$= h + e^{-h} - 1$$

$\therefore$

$$\Gamma(h) = \begin{bmatrix} 1 - e^{-h} \\ h + e^{-h} - 1 \end{bmatrix}.$$

State space form.

$$x(kh+h) = \begin{bmatrix} e^{-h} & 0 \\ 1-e^{-h} & 1 \end{bmatrix} x(kh) + \begin{bmatrix} 1-e^{-h} \\ h+e^{-h}-1 \end{bmatrix} u(kh)$$

$$y(kh) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(kh)$$

c.)   Given $h=1$.

So,

$$\phi(h) = \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix}$$

$$\Gamma(h) = \begin{bmatrix} 0.6321 \\ 0.3679 \end{bmatrix}.$$

To design feedback control,

$$W_c = [\Gamma \quad \Gamma\phi].$$

$$\Gamma\phi = \begin{bmatrix} 0.2325 \\ 0.7674 \end{bmatrix}.$$

$$W_c = \begin{bmatrix} 0.6321 & 0.2325 \\ 0.3679 & 0.7674 \end{bmatrix}.$$

$$\text{Det}(W_c) = 0.4 \neq 0.$$

∴ W_c rank is 2.

The system is controllable.

To model the feedback controller, we take Ackerman formula.

$$L = [0 \quad 1]\, W_c^{-1}\, p(\phi)$$

$$W_c^{-1} = \frac{1}{0.4}\begin{bmatrix} 0.7674 & -0.2325 \\ -0.3679 & 0.6321 \end{bmatrix}$$

$$= \begin{bmatrix} 1.9207 & -0.5818 \\ -0.9209 & 1.5822 \end{bmatrix}.$$

$\underline{\underline{P(\phi)}}$:-

Given Poles
$0.5 + 0.5i, \quad 0.5 - 0.5i,$

$$\therefore P(z) = \cancel{z+0} \cdot (z - (0.5 + 0.5i))(z - (0.5 - 0.5i))$$

$$= z^2 - z(0.5 - 0.5i) - z(0.5 + 0.5i) + [(0.5 + 0.5i)(0.5 - 0.5i)]$$

$$= z^2 - 0.5z - 0.5\cancel{z}i - 0.5z + 0.5\cancel{z}i + [(0.5)^2 + (0.5)^2]$$

$$= z^2 - z + 0.5.$$

$$P(\phi) = \phi^2 - \phi + 0.5I.$$

$$\phi^2 = \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix} \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1353 & 0 \\ 0.8646 & 1 \end{bmatrix}$$

$$\phi^2 - \phi + 0.5I = \begin{bmatrix} 0.1353 & 0 \\ 0.8646 & 1 \end{bmatrix} - \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$P(\phi) = \begin{bmatrix} 0.2674 & 0 \\ 0.2325 & 0.5 \end{bmatrix}$$

Now

$$L = [0 \quad 1] \; W_c^{-1} \; P(\phi).$$

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$$L = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1.9207 & -0.5818 \\ -0.9209 & 1.5822 \end{bmatrix} \begin{bmatrix} 0.2674 & 0 \\ 0.2325 & 0.5 \end{bmatrix}.$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3783 & -0.2909 \\ 0.1216 & 0.7911 \end{bmatrix}.$$

$$L = \begin{bmatrix} 0.1216 & 0.7911 \end{bmatrix}$$

i.e. $l_1 = 0.1216$ $\quad l_2 = 0.7911$

To compute m:-

$\therefore$ all poles are inside unit circle, and y_{ref} is given as feedback

by FVT

$$\lim_{k \to \infty} y[k] = \lim_{z \to 1} (z-1)\, Y(z)$$

$$x[k+1] = \phi\, x[k] + \Gamma(u[k] + m y_{ref}) \qquad u[k]$$

$$= (\phi - \Gamma L)\, x[k] + \Gamma m y_{ref}.$$

Taking Z-transform

$$z X(z) = (\phi - \Gamma L) X(z) + \Gamma m Y_{ref}(z).$$

$$\Rightarrow \boxed{(zI - \phi + \Gamma L) X(z) = \Gamma m Y_{ref}(z).} \quad \text{— (1)}$$

Also

$$y[k] = C x[k] \Rightarrow Y(z) = C X(z)$$

Take from (1)

$$Y(z) = C\left[(zI - \phi + \Gamma L)\right]^{-1}\Gamma M\, Y_{ref}(z)$$

Now $Y_{ref}(z) = y_{ref} \cdot u(z)$

i.e y_{ref} is feedback signal so

$$Y_{ref}(z) = y_{ref} \cdot \frac{z}{z-1}.$$

$$Y(z) = C\left[zI - \phi + \Gamma L\right]^{-1}\Gamma M\, y_{ref}\frac{z}{z-1}$$

$$\lim_{k\to\infty} y[k] = \lim_{z\to 1} C\left[zI - \phi + \Gamma L\right]^{-1}\Gamma M\, y_{ref}\, z$$

$$zI - \phi + \Gamma L = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix} + \begin{bmatrix} 0.6321 \\ 0.3679 \end{bmatrix}\begin{bmatrix} 0.1216 & 0.7911 \end{bmatrix}.$$

$$= \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix} + \begin{bmatrix} 0.07686 & 0.5 \\ 0.04473 & 0.2910 \end{bmatrix}$$

$$\lim_{z\to 1} \therefore = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix} + \begin{bmatrix} 0.07686 & 0.5 \\ 0.04473 & 0.2910 \end{bmatrix}$$

Further calculations are done in matlab

From matlab code, it is computed

$$y[k] = 1.2641\, M\, y_{ref}$$

For $y[k] \doteq y_{ref}$ $\quad M = \frac{1}{1.2641} = 0.7910$

$$\boxed{M = 0.7910}$$

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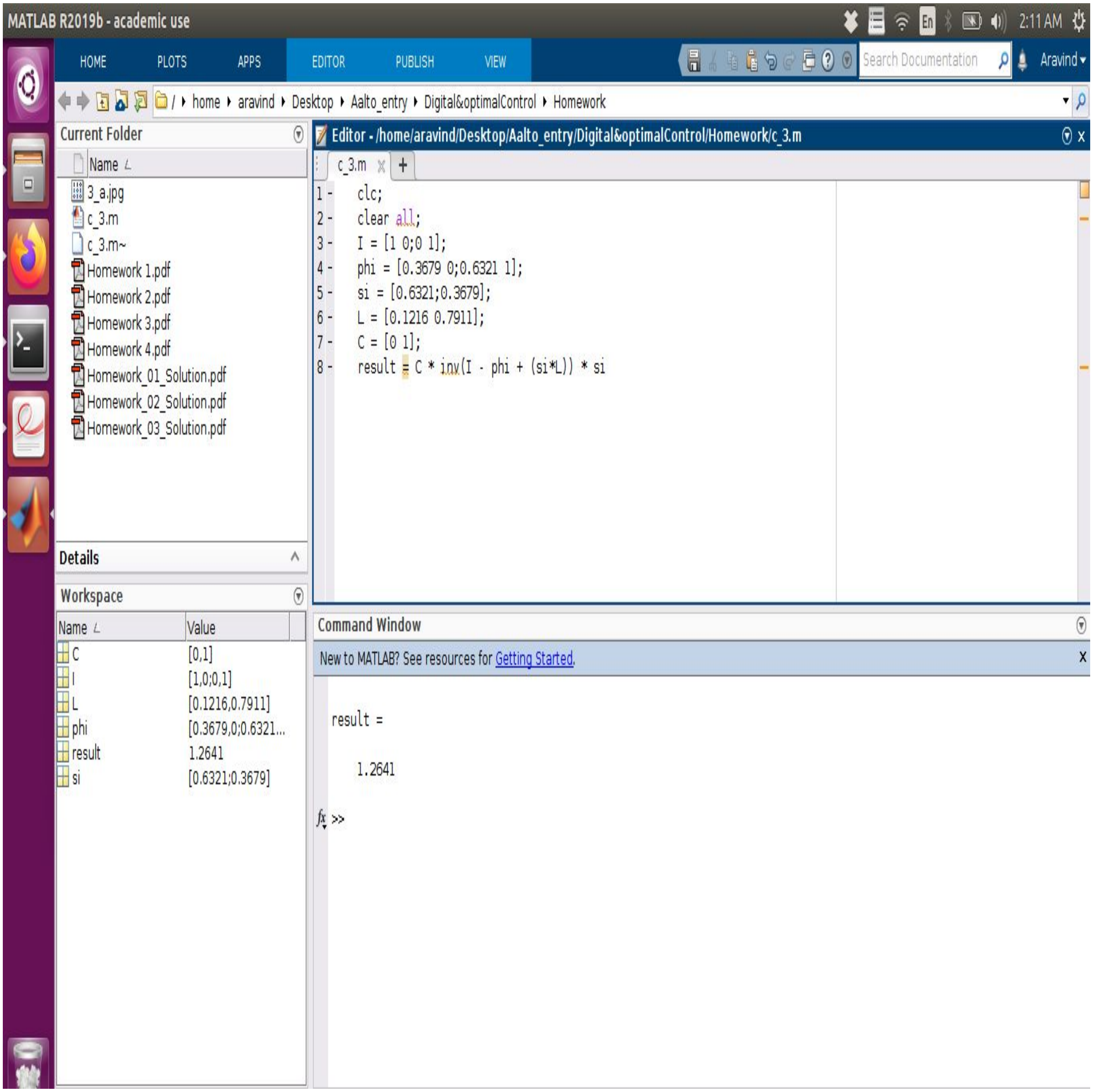
MATLAB R2019b - academic use
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APPS
EDITOR
PUBLISH
VIEW
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/ home aravind Desktop Aalto_entry Digital&optimalControl Homework
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Homework 1.pdf
Homework 2.pdf
Homework 3.pdf
Homework 4.pdf
Homework_01_Solution.pdf
Homework_02_Solution.pdf
Homework_03_Solution.pdf
Details
Workspace
Name
Value
C [0,1]
I [1,0;0,1]
L [0.1216,0.7911]
phi [0.3679,0;0.6321...
result 1.2641
si [0.6321;0.3679]
Editor - /home/aravind/Desktop/Aalto_entry/Digital&optimalControl/Homework/c_3.m
c_3.m
clc;
clear all;
I = [1 0;0 1];
phi = [0.3679 0;0.6321 1];
si = [0.6321;0.3679];
L = [0.1216 0.7911];
C = [0 1];
result = C * inv(I - phi + (si*L)) * si
Command Window
New to MATLAB? See resources for Getting Started.
result =
1.2641
>>

d.)

To design controller with state feedback & state observer.

Observer gain is K.

Also observer has dead-beat characteristics $\boxed{\phi_o = \phi - KC}$

$$\det(zI - \phi_o) = \det(zI - \phi + KC)$$

$$zI - \phi + KC = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 0 \end{bmatrix} + \begin{bmatrix} 0 & k_1 \\ 0 & k_2 \end{bmatrix}$$

$$= \begin{bmatrix} z - 0.3679 & k_1 \\ -0.6321 & z - 1 + k_2 \end{bmatrix}.$$

$$= z^2 - z + zk_2 - 0.3679z - 0.3679 - 0.3679k_2 + 0.6321k_1$$

$$= z^2 + z(k_2 - 1.3679) + (0.6321k_1 - 0.3679k_2 - 0.3679)$$

Since it has dead-beat characteristics

$\boxed{k_2 = 1.3679}$

$0.3679k_2 = 0.3679 + 0.6321k_1$

$k_1 = 0.3679(k_2 - 1)$

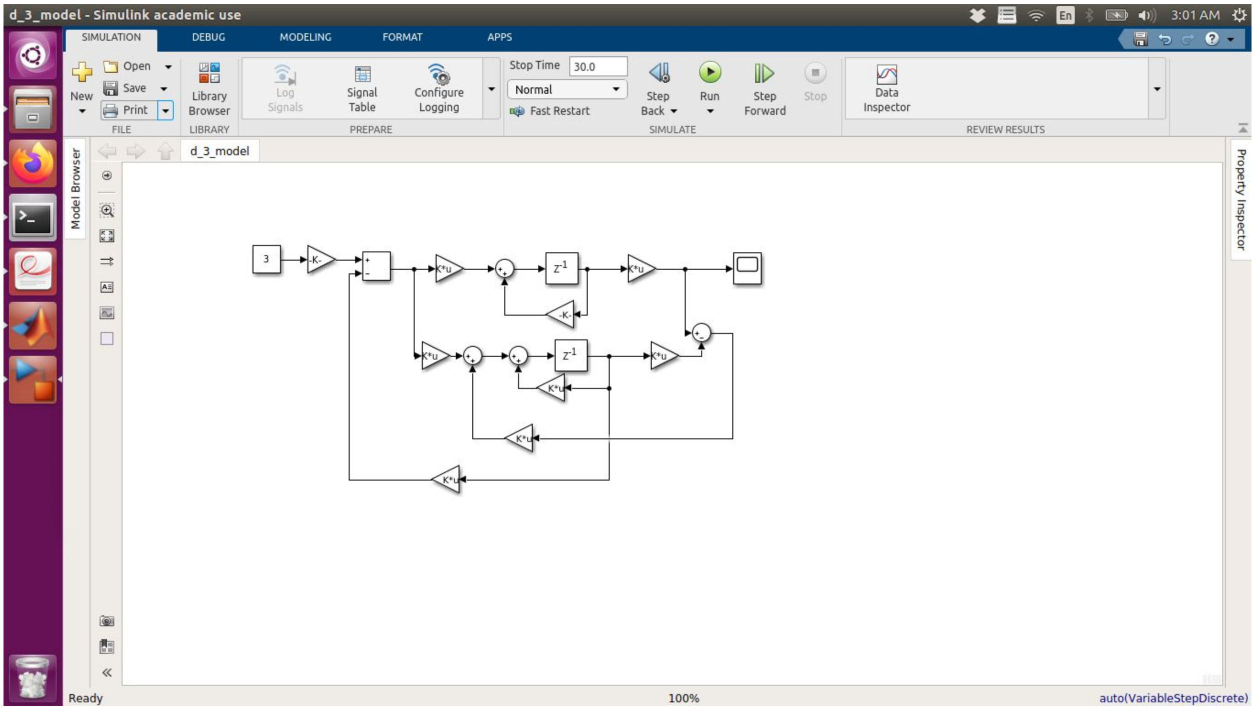
$k_1 = (0.3679)^2$

$\boxed{k_1 = 0.1354}$

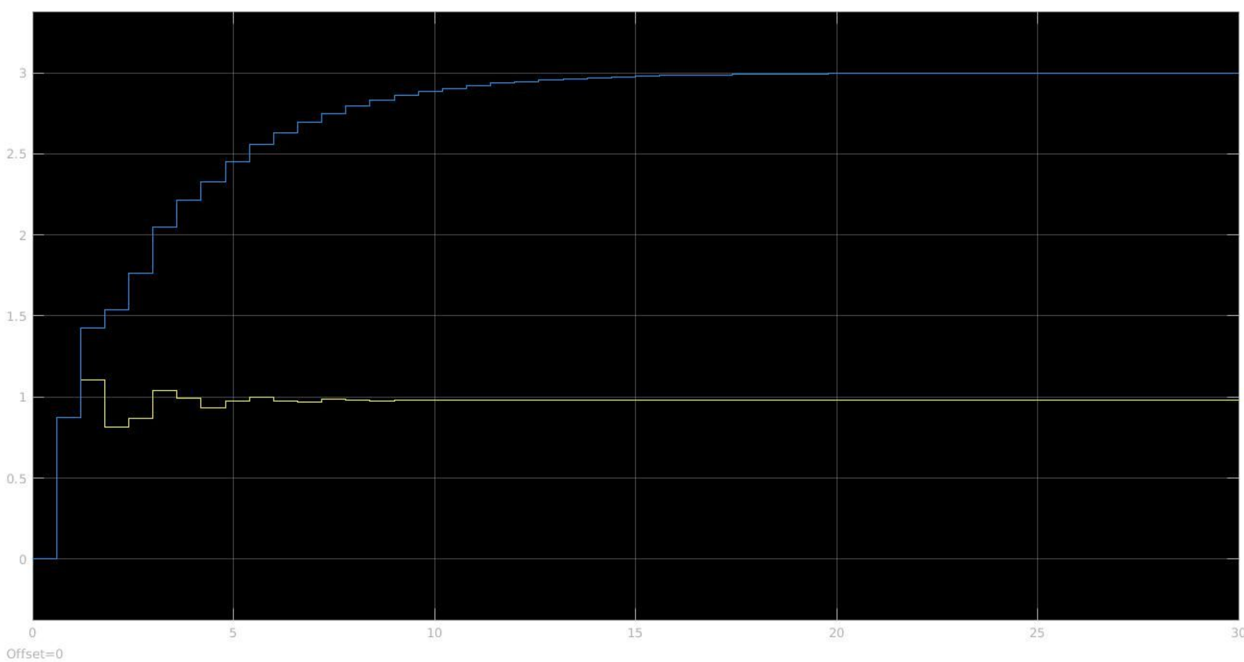
$$[k_1, k_2] = [0.1354, 1.3679]$$

1.d

The model in simulink designed was: [yRef = constant ,3]



The output in the scope was found to be :



As you can see, the output value reaches to 3, which was given as yref value in the model

1.e

The disturbance was added in output and output was observed. It is seen , some kind of fluctuation in noticed in reaching the desired value. But I was not able to find much difference in the output.