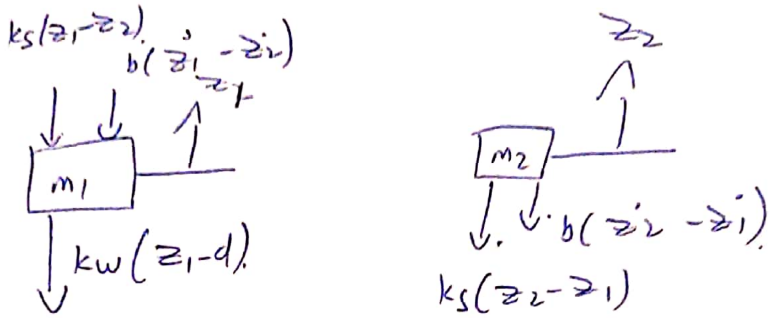
Assignment - 2

1.) Free Flow diagram:-



1.a

Outputs :- z_2 => displacement of mass m_2.

Inputs :- d => displacement of road reference.

Constants :- m_1, m_2 => mass of objects (unspring and spring)

k_s, k_w => spring constant

b => Damper coefficient

Variables :- z_1 => displacement of mass m_1

z_2 => displacement of mass m_2

1.b :-

For mass 1

$$m_1 \ddot{z}_1 = -k_w(z_1-d) - k_s(z_1-z_2) - b(\dot{z}_1-\dot{z}_2)$$

$$m_1 \ddot{z}_1 = k_w d - k_w z_1 - k_s(z_1-z_2) - b(\dot{z}_1-\dot{z}_2) \quad (1)$$

$$m_2 \ddot{z}_2 = -k_s (z_2 - z_1) - b(\dot{z}_2 - \dot{z}_1) \quad -(2)$$

Now

$$x_1 = z_1 \qquad x_3 = z_2$$

$$x_2 = \dot{z}_1 \qquad x_4 = \dot{z}_2$$

So equation (1) & (2) becomes

$$m_1 \dot{x}_2 = k_w d - k_w x_1 - k_s(x_1 - x_3) - b(x_2 - x_4)$$

$$m_2 \dot{x}_4 = -k_s(x_3 - x_1) - b(x_4 - x_2)$$

i.e

$$\dot{x}_2 = \frac{1}{m_1}\left[k_w d - k_w x_1 - k_s(x_1 - x_3) - b(x_2 - x_4)\right] \quad -(3)$$

$$\dot{x}_4 = \frac{1}{m_2}\left[-k_s(x_3 - x_1) - b(x_4 - x_2)\right] \quad -(4)$$

using (3), (4) & assumptions of $z, x \Rightarrow$ relation

$$\frac{d}{dx}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{m_1}\left[k_w d - k_w x_1 - k_s(x_1 - x_3) - b(x_2 - x_4)\right] \\ x_4 \\ \frac{1}{m_2}\left[-k_s(x_3 - x_1) - b(x_4 - x_2)\right] \end{bmatrix}$$

This is the final differential equation (5)

1.c & 1.d are found in Matlab codes and model file.

2.) Given,

2.a & 2.b

$$m.\frac{dv}{dt} = \alpha_n uT(\alpha_n v) - m.g.C_r \, sgn(v) - \frac{1}{2}\rho C_d . A v^2 - m.g.\sin\theta . \quad -(1)$$

is a first order differential equation

Now $$T(\alpha_n v) = T_m\left(1 - \beta\left(\frac{\alpha_n . v}{\omega_m} - 1\right)^2\right).$$

Substituting in (1).

$$m.\frac{dv}{dt} = \alpha_n . u . T_m . \left[1 - \beta\left(\frac{\alpha_n . v}{\omega_m} - 1\right)^2\right] - m.g.C_r sgn(v) - \frac{1}{2}\rho . C_d . A . v^2 - m.g.\sin\theta . \quad -(2)$$

Given.

α_3 is used $\alpha_3 = 16$, $m = 800$ kg. & substituting all constants in "constants 2.m" file. $\boxed{\theta = 0^\circ}$

Please find Matlab ".mdl" file for the Simulink model.

2.c

Considering $\boxed{\theta = 5^\circ}$

To find the optimum values of α_n & input u.

	$u=0.7$	$u=0.8$	$u=0.9$	$u=1.0$
α_1	25.5083	25.5	25.7	25.8589
α_2	36.81084	37.6098	38.23	38.7359
α_3	44.1152	46.6144	48.6152	$\boxed{50.2597}$
α_4	39.3190	44.5661	48.3255	$\boxed{51.3635}$
α_5	18.0562	35.4232	42.3894	47.0238

The values above from the table which are close to the speed of vehicle when it is flat ~~are~~ i.e 50.95 m/s are.

α_3 gear ratio & $\boxed{u = 1.0}$ assuming max-throttle

α_4 gear ratio & $\boxed{u=1.0}$

Finally concluding.

$\boxed{u=1.0}$, increasing the throttle input and gear ratio α_4 is more closer to same value