Assignment-1

1.) System :

$$\dot{q} + 18.2q = 435.5u - 5.1v \quad \text{— (1)}$$
$$0.3q - \dot{v} = 36.7u. \quad \text{— (2)}$$
$$y = -0.2q.$$

From (1) and (2)

$$\dot{q} = -18.2q - 5.1v + 435.5u. \quad \text{— (3)}$$
$$\dot{v} = +0.3q + (-36.7u) \quad \text{— (4)}$$

State-space variables

$x_1(t) = q$, $\quad u(t) = u$

$x_2(t) = v$, $\quad y(t) = y$

So 3 & 4 becomes,

$$\dot{x}_1(t) = -18.2\,x_1(t) - 5.1\,x_2(t) + 435.5\,u(t)$$
$$\dot{x}_2(t) = +0.3\,x_1(t) - 36.7\,u(t)$$
$$y(t) = -0.2\,x_1(t)$$

Solution for 1.a

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -18.2 & -5.1 \\ +0.3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 435.5 \\ -36.7 \end{bmatrix} u$$

$\dot{x} = Ax + BU$

$Y = Cx + DU$..

$$Y = \begin{bmatrix} -0.2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$$A = \begin{bmatrix} -18.2 & -5.1 \\ +0.3 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 435.5 \\ -36.7 \end{bmatrix}; \quad C = \begin{bmatrix} -0.2 & 0 \end{bmatrix}; \quad D = 0$$

Solution for 1.b

Provided in matlab script "Tf.m".

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2.) Given System Parameters

$A(t) \Rightarrow$ Reactant concentration
$B(t) \Rightarrow$ Product "
$A_i(t) \Rightarrow$ inflow Reactant ~~rate~~ Concentration
$q_i, q_o \Rightarrow$ volumetric flowrate of inflow/outflow.
$r \Rightarrow$ rate of transformation

Solution

2.a

Inputs:

$A_i(t) \Rightarrow$ inflow reactant concentration.

Outputs:-

$A(t)$ and $B(t) \Rightarrow$ Concentration of Reactant and Product respectively.

Constants:-

V - Volume of tank
T - Temperature inside tank.
q_i, q_o - volumetric flow rate of inflow/outflow channel.

Variables:-

$A(t) \Rightarrow$ Reactant concentration
$B(t) \Rightarrow$ Product concentration.
$A_i(t) \Rightarrow$ inflow Reactant concentration.

2.b

The amount/concentration of reactant inside the system is given as.

$$V \cdot \frac{dA(t)}{dt} = q_i A_i(t) - q_o A_o(t) - r.V.$$

$$\frac{dA(t)}{dt} = \frac{q_i}{V} A_i(t) - \frac{q_o}{V} A_o(t) - r$$

$$\frac{dA(t)}{dt} = \frac{q_i}{V} A_i(t) - \frac{q_o}{V} A(t) - k_o e^{-\frac{E_a}{RT}} A(t) \quad -(1)$$

Similarly for concentration of Product $B(t)$

$$\frac{dB(t)}{dt} = -\frac{q_o}{V} B(t) + k_o e^{-\frac{E_a}{RT}} A(t) \quad -(2)$$

Equation (1) and (2) represent the differential equations for system

2.c

From equations (1) and (2)

consider.

$x_1(t) = A(t)$,

$x_2(t) = B(t)$,

$A_i(t) = u(t)$.

So,

$$\dot{x}_1(t) = \frac{q_i}{V} u(t) - \left(\frac{q_o}{V} + k_o e^{-\frac{E_a}{RT}} \right) x_1(t)$$

$$\dot{x}_2(t) = -\frac{q_o}{V} x_2(t) + k_o e^{-\frac{E_a}{RT}} x_1(t)$$

$$y(t) = x_1(t) + x_2(t)$$

$\dot{x} = Ax + BU$

$Y = Cx + DU$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{q_o}{V} - k_o e^{-\frac{E_a}{RT}} & 0 \\ k_o e^{-\frac{E_a}{RT}} & -\frac{q_o}{V} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{q_i}{V} \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

2.d

Solution found in "ChemReactor.m" Matlab code.

3.) System,

current => $i(t)$
coil Inductance => L
Ball displacement => $z(t)$
Force, magnetic => $F_m(t)$

3.a

Inputs :-

$i(t)$ => current controlled by amplifier.

Outputs :-

$z(t)$ = displacement of ball.

Constants :- m => mass of ball.
L => Inductance.
g => acceleration of gravity
a => ~~m~~agnetic force constant.

Variables :-
$i(t)$ => input current.
$z(t)$ => displacement of ball.
$F_m(t)$ => magnetic force due to coil.

3.b

so the resultant Force acting on ball is

$F_a = F_g + F_m$. (direction of forces not considered)

$$F_a = mg - \frac{L}{2a} i(t)^2 e^{-\frac{z(t)}{a}} \quad \text{— (1)}$$

Where $\boxed{F_g = mg}$

$$\boxed{F_m = -\frac{L}{2a} i(t)^2 e^{-\frac{z(t)}{a}}}$$

Now $F_a = ma$

$= m\ddot{z}$

$$\boxed{F_a = m\frac{d^2 z(t)}{dt^2}}$$

So equation (1) becomes

$$m\frac{d^2 z(t)}{dt^2} = mg - \frac{L}{2a} i^2(t) e^{-\frac{z(t)}{a}}$$

$$\boxed{\frac{d^2 z(t)}{dt^2} = g - \frac{L}{2am} i^2(t) e^{-\frac{z(t)}{a}}} \quad \text{--- (2)}$$

Equation (2) becomes the basic differential equation for system.

3.C

State variables

$\boxed{x_1(t) = z(t)}$ => displacement of ball.

$x_2(t) = \dot{z}(t)$ => velocity of ball.

i.e.

$$\boxed{x_2(t) = \frac{dz(t)}{dt}}$$

The system is a non-linear system, because the $i(t)$ [input of system] has squared-term in the system equation. So, the system is non-linear.

3.d) equillibrium at $z = d = 1\,cm = 0.01\,m$.

at equillibrium

$$\frac{dz(t)}{dt} = \frac{d^2 z(t)}{dt^2} = 0.$$

So, ② becomes.

$$0 = g - \frac{L}{2am} i^2(t) e^{-\frac{z_1(t)}{a}}$$

$z_1(t)$ at equillibrium $= 0.01\,m$.

$$g = \frac{L}{2am} i^2(t) e^{-\frac{0.01}{a}}$$

$$\frac{g \cdot 2am}{L} \cdot e^{+\frac{0.01}{a}} = i^2(t)$$

$$i(t) = \sqrt{\frac{g \cdot 2am}{L}} \, e^{\frac{0.01}{2a}}$$

$$i(t) = \sqrt{\frac{9.8 \times 2 \times 6.06 \times 10^{-3} \times 0.8}{0.229}} \cdot e^{\frac{0.01}{2 \times 6.66 \times 10^{-3}}}$$

$$\boxed{i(t) = 1.431\ A}$$ at equillibrium

Position.

3.e) 3.e Linearization of Magnetic Force :

$$F_m(z,i) = \frac{-L}{2a} i^2 e^{-\frac{z}{a}}.$$

$$\Delta F_m(z,i) = F_m(z,i) + \frac{d}{dz}(F_m(z,i)) \cdot \Delta z + \frac{d}{di}(F_m(z,i)) \Delta i$$

$$= \frac{-L}{2a} i^2 e^{-\frac{z}{a}} + \left(\frac{-1}{a}\right)\left(-\frac{L}{2a} i^2 e^{-\frac{z}{a}}\right) \cdot \Delta z$$

$$+ \left(-\frac{L}{2a} \cdot 2i \cdot e^{-\frac{z}{a}}\right) \cdot \Delta i$$

$$= -\frac{L}{2a} i^2 e^{-\frac{z}{a}} + \frac{L}{2a^2} \cdot i^2 e^{-\frac{z}{a}} \cdot \Delta z - \frac{L}{2a} \cdot i \cdot e^{-\frac{z}{a}} \cdot \Delta i$$

$$\boxed{\Delta F_m(z,i) = -\frac{L}{2a} i^2 e^{-\frac{z}{a}} + \frac{L}{2a^2} i^2 e^{-\frac{z}{a}} \cdot \Delta z - \frac{L}{a} \cdot i \cdot e^{-\frac{z}{a}} \cdot \Delta i}$$

— (3)

Equation (3) is linearized Form of magnetic force using Taylor series expansion.

where,

$\Delta F_m(z, i) \Rightarrow$ Linearized magnetic force.

Now From equation (1)

$F_a = F_g + F_{m\,\text{Linearized}}$

$$F_a = \underbrace{mg - \frac{L}{2a} i^2 e^{-\frac{z}{a}}}_{\substack{\text{at equillibrium point}\\ \text{this term is zero.}}} + \frac{L}{2a^2} i^2 e^{-\frac{z}{a}} \cdot \Delta z - \frac{L}{a} i \cdot e^{-\frac{z}{a}} \cdot \Delta i$$

So, also considering $\Delta z = z(t)$ $\Delta \Delta i = i(t)$

$$m \cdot \frac{d^2 z(t)}{dt^2} = \frac{L}{2a^2} i^2 e^{-\frac{z(t)}{a}} \cdot z(t) - \frac{L}{a} \cdot i \cdot e^{-\frac{z(t)}{a}} \cdot i(t)$$

$$\boxed{\frac{d^2 z(t)}{dt^2} = \frac{L}{2a^2 m} \cdot i^2 e^{-\frac{z(t)}{a}} \cdot z(t) - \frac{L}{am} \cdot i e^{-\frac{z}{a}} \cdot i(t)}$$

— (4)

(4) becomes final equation of the system.

3.f :-

$$x_1(t) = z(t) = \Delta z \quad , \quad u(t) = i(t) = \Delta i$$

$$x_2(t) = \dot{z}(t)$$

So

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{L \cdot i^2}{2a^2 m} \cdot e^{-\frac{z}{a}} \cdot x_1(t) - \frac{L \cdot i}{am} \cdot e^{-\frac{z}{a}} \cdot u(t)$$

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{L \cdot i^2}{2a^2 m} \cdot e^{-\frac{z}{a}} & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{L \cdot i}{am} e^{-\frac{z}{a}} \end{bmatrix} \Delta u. \quad -(5)$$

and

$$y(t) = z(t) = x_1(t)$$

So

$$\Delta Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \quad -(6)$$

Equation (5) and (6) are state space equations is operating point.

$$A = \begin{bmatrix} 0 & 1 \\ \frac{L \cdot i^2}{2a^2 m} e^{-\frac{z}{a}} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -\frac{L \cdot i}{a \cdot m} e^{-\frac{z}{a}} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

3.g :-

Solution in "MagLevit.m" matlab code

3.h :-

Solution in "magLevit.m" matlab code. The system is not stable. As it has Poles on right half of complex-plane.