1. (max. 1.5 points)
A system is described by the following differential equations:

$$\begin{cases} \dot{q} + 18.2q = 435.5u - 5.1v \\ 0.3q - \dot{v} = 36.7u \\ y = -0.2q \end{cases}$$

where q and v are time-varying functions, u is an input, and y is an output.

a) Put the equations above in state-space form. **(0.5 points)**

b) Write the transfer function of the system in MATLAB and provide the script of the solution. Name the script “TF.m”. **(1 point)**

2. (max. 3 points)
Consider the chemical reactor represented in Figure 1. The reactor converts a reactant into a product and is composed of the following parts:

- **A tank in which the chemical reaction takes place. This tank has the following parameters: a constant volume V, a homogeneous temperature T, the concentration of the reactant $A(t)$ (mol/L), and the concentration of the product $B(t)$ (mol/L);**
- **An inflow channel through which the reactant flows into the tank. In the inflow channel, the concentration of the reactant and the volumetric flow rate are denoted by A_i (mol/L) and q_i (L/s), respectively;**
- **An outflow channel through which the product and reactant flow out of the tank. The volumetric flow rate of the outflow channel is denoted by q_o (L/s).**

Inside the tank, the reactant $A(t)$ is transformed into the product $B(t)$ at the following rate:

$$r = kA(t),$$

where r is the reaction rate expressed in $\mathrm{mol}/(s \cdot L)$, and $k = k_0 e^{-\frac{E_a}{RT}}$ is the rate constant given by the Arrhenius equation.
We are interested in the concentrations A and B in the tank controlled by A_i.

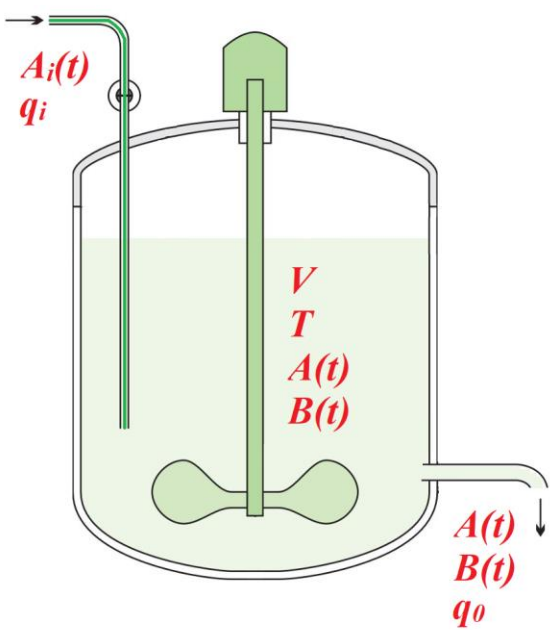

Figure 1 – Chemical reactor.

a) Based on the description of the system, structure the problem. You are supposed to answer these questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system? **(0.5 points)**

b) Set up the basic differential equations of the system using first principles. These equations describe the evolution of the concentrations of the reactant and product in inside the tank. **(1 point)**

Tips: 1. Use the general mole balance equation:

$$\begin{Bmatrix} \text{rate of accumulation} \\ \text{or depletion} \\ \text{of component j} \end{Bmatrix} = \begin{Bmatrix} \text{rate of inflow} \\ \text{of component j} \end{Bmatrix} - \begin{Bmatrix} \text{rate of outflow} \\ \text{of component j} \end{Bmatrix} +/- \begin{Bmatrix} \text{rate of transformation} \\ \text{of component j} \\ \text{by chemical reactions} \end{Bmatrix}$$

2. Make sure that the units $(\mathrm{mol/s})$ of both sides of the equations match.

c) Form the state-space model of the system. **(0.5 points)**

d) Write a MATLAB script named “ChemReactor.m”. In the script, compute the state-space equations of the system using the values from Table 1 below.

Symbol	Value	Unit	Description
V	12	L	Volume of the tank
q_o, q_i	0.15	$\mathrm{L \cdot s^{-1}}$	Volumetric flow rates
k_0	9.4	$\mathrm{s^{-1}}$	Frequency of collisions
E_a	2500	$\mathrm{J \cdot mol^{-1}}$	Activation energy
R	8.31	$\mathrm{J \cdot k^{-1} \cdot mol^{-1}}$	Gas constant
T	293	K	Temperature

Table 1 – System parameters.

Assume that the concentration of the reactant in the inflow channel becomes $1\ \mathrm{mol/L}$ at time 0 (e.g., after opening the reactant inflow valve). Consider it as a step function. Plot the corresponding step response of the system, i.e., the concentration of the product and reactant in the tank, in the MATLAB script. **(1 point)**

3. (max. 5.5 points)
Consider the magnetic levitation system shown in Figure 2a. An electromagnet is located in the upper part of the system and is used to suspend the steel ball with the magnetic force F_m. The magnetic force depends on the distance between the ball and the electromagnet $z(t)$ (the air gap), and the current in the electromagnet $i(t)$, which is controlled by the amplifier input voltage. The air gap is sensed by an optical transducer comprising a light source and a photodetector. The gravitational force acting on the ball is denoted by mg, where m is the mass of the ball and g the acceleration of gravity. The electromagnet has inductance L.

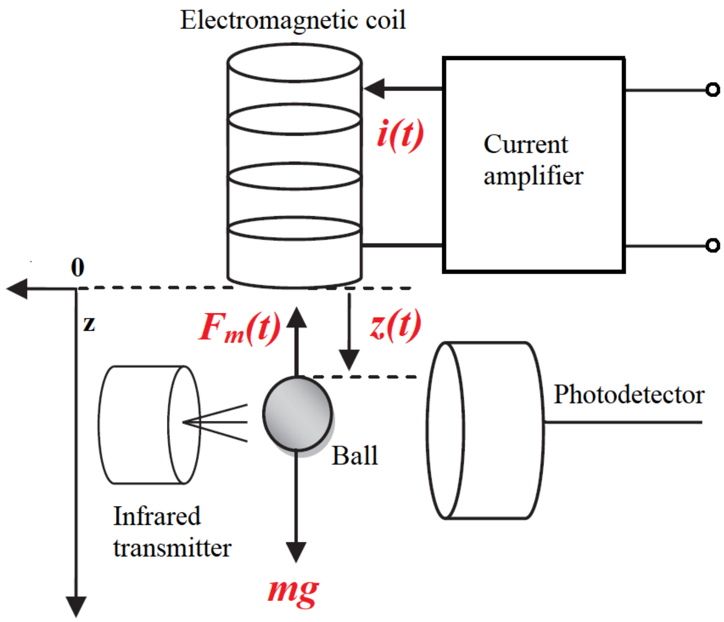

Figure 2a – Schematic diagram of the magnetic levitation system.

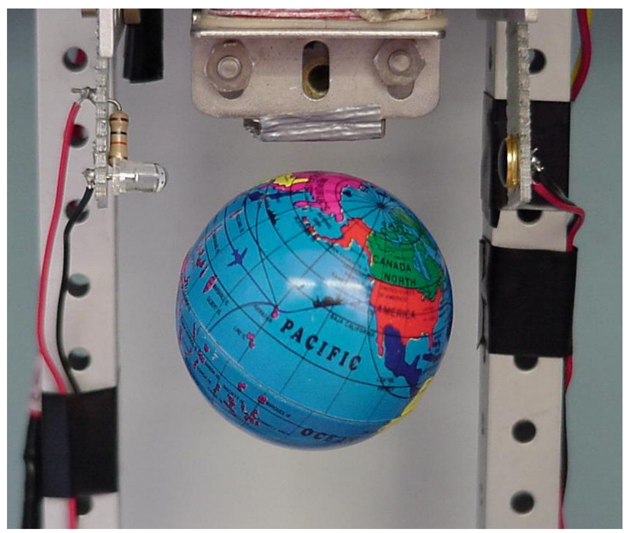

Figure 2b – Close-up view of the magnetically suspended ball.

a) Based on the description of the system, structure the problem. You are supposed to answer these questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system? **(0.5 points)**

b) Set up the basic equations of the system. You are supposed to derive the underlying differential equation describing the motion of the suspended ball using first principles. Assume that the magnetic force acting on the ball along the z-axis is given by

$$F_m(x,i) = -\frac{L}{2a} i^2 e^{-\frac{z}{a}},$$

where a is a constant depending on the diameter of the iron core of the electromagnet and the mass of the ball. **(0.5 points)**

c) What are the state variable(s) of the system? Is the system linear or nonlinear? Explain. **(0.5 points)**

d) Calculate the input for the operation point at $z = d$, where the ball reaches its equilibrium position, using the system parameters given in Table 2 below. **(0.5 points)**

Symbol	**Value**	**Unit**	**Description**
g	9.81	m/s^2	Acceleration of gravity
a	6.66	mm	Constant
L	0.229	H	Inductance
m	0.8	kg	Mass of the ball
d	1	cm	Equilibrium position of the ball

Table 2 – System parameters.

e) Linearize the magnetic force around the operation point by a Taylor series expansion. **(1.5 points)**

f) Form the state-space model of the system around the operation point. **(1 point)**

g) Write a MATLAB script named “MagLevit.m”. In the script, compute the state-space equations and transfer function of the system. **(0.5 points)**

h) Assume that the coil current suddenly rises and drops due to disturbances. Consider the sudden change as an impulse disturbance, i.e., the Dirac delta function. Plot the response of the system (position of the ball) to the current disturbance in the MATLAB script. Is the system stable? **(0.5 points)**

What to return?

You are supposed to submit your assignment to the related link for Assignment 1 in MyCourses. Your submission should include one zip file “Assign01_student number.zip” consisting of a PDF file “Assign01_student number.pdf” and three MATLAB files, TF.m, “ChemReactor.m” and “MagLevit.m”.

The hard deadline for submission of this assignment is 22.09.2019 at 23:59.