# Homework problems 1

The answers to the homework problems should be returned to the corresponding folder on the Robotics-course MyCourses-platform. The answer files should be named as "solutions_homework problems 1_Firstname_Lastname.pdf". Deadline for returning the solutions is Thursday 3.10, 12:00 am, (*before the lectures when the solutions are presented*).

**1.** The rotation matrix $R$ describes the orientation of a new coordinate frame {N} with respect to

the base frame {B}: $^{B}R_{N} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$
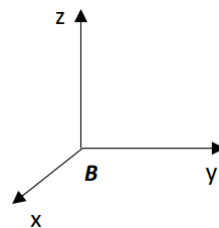


**a)** Illustrate the directions of the coordinate axes for the new frame {N} in relation to the base frame {B} shown in the drawing above. You can for example copy the image of the base frame {B} shown above and draw the axes of the new frame on top of that. (4 points)

**b)** Does $R$ describe <u>a right-handed coordinate frame</u>? If it does or does not, explain how did you come to the conclusion? (2 points)

**2**. The task is to solve the unknown relative locations of coordinate frames, marked with the dashed arrow lines, by utilizing the known relative locations marked with the solid arrow lines. The setup is illustrated in the figure below.
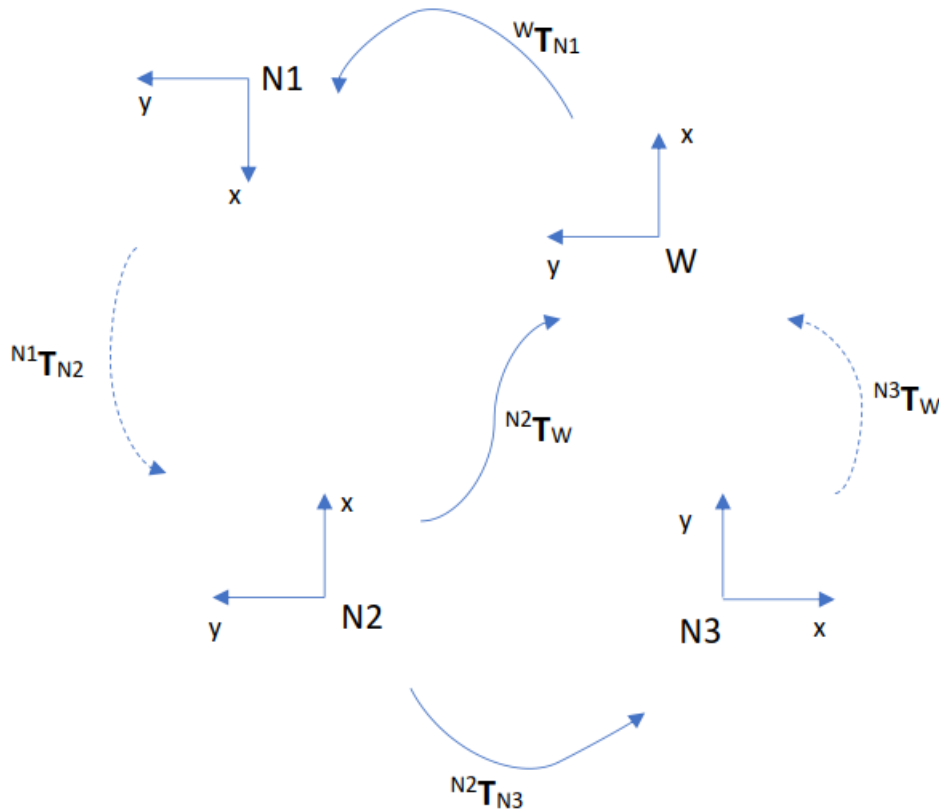


**a )** Determine first, on the matrix symbol level, the equations for the unknown transformation matrices $^{N1}T_{N2}$ and $^{N3}T_W$. (*7 points*)

**b)** Thereafter calculate the numerical forms of the 4x4 homogenous transformation matrices $^{N1}T_{N2}$ and $^{N3}T_W$. (*7 points*)

The known transformation matrices are:

$$^{W}T_{N1}= \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{N2}T_{W}= \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{N2}\mathbf{T}_{N3}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*Hint: in order to calculate the <u>inverse of a numerical form homogenous transformation matrix</u>, we must use the equation (2.25) of Corke's text book:*

$$T^{-1}=\begin{pmatrix} R & t \\ 0_{1\times 3} & 1 \end{pmatrix}^{-1}=\begin{pmatrix} R^{T} & -R^{T}t \\ 0_{1\times 3} & 1 \end{pmatrix} \qquad (2.25)$$

**3**. The 3D-frame {B} is initially located coincident with the frame {A}. We first rotate frame {B} 30 degrees around its z-axes. Then we translate the rotated frame 5 units in its z-direction, and finally, rotate the translated frame 60 degrees around its x-axes.

**a)** Give the 4x4 homogenous transformation matrix that describes the position and orientation of frame {B} with respect to frame {A}. (5 points)

**b)** The coordinates of a target point <u>with respect to frame {A}</u> are $^{A}$**P1**= [x=0,y=0,z=6]. What are the coordinates of the target point given with respect to frame {B} ?  (5 points)

**c)** The coordinates of another target point, <u>given with respect to frame {B}</u>, are $^{B}$**P2**= [x=0,y=0,z=6]. What are the coordinates of this target point given with respect to frame {A} ?  (5 points)

**4**. We are supposed to create a new coordinate frame {B} by rotating **30** degrees around the **z**-axis of the original coordinate frame {A}.

Instead of using the "rotations about principal axis-formulas of slide 15, p. 34 Corke's text book" use the **Rodrigues' rotation formula** (slide 30, p. 42-43 of Corke's text book) to form the corresponding 3x3 rotation matrix $^A\boldsymbol{R}_B$. (*5 points*)

**5**. **a)** For the same **30** degrees rotation around the **z**-axis as in problem 4, create the corresponding unit quaternion by applying the formula (2.22). (compare slide 33 and pp. 44-45 of Corke's text book). Check if you get the same result with the Robotics toolbox (by using the UnitQuaternion constructor of the Corke's Robotics toolbox) (*5 points*)

**b)** The task is to compound two consecutive rotations expressed with UnitQuaternions. You are supposed to compound the two rotations by using the <u>matrix-vector multiplication formula</u> (see p. 44 of the text book or lecture slide 34).

Use the UnitQuaternion, constructed in part **a)** and compound it with itself to create a rotation of 60° around the **z**-axis of the original frame.

Check with UnitQuaternion constructor if you get the same result with the toolbox. (*5 points*)

*Hint: If you face problems with the toolbox, try using the commands **R**= rotz(angle), UnitQuaternion(**R**) to check your result with the toolbox. Other ways of creating the same matrix **R** may not, for some unknown reasons, necessarily give the correct result with the UnitQuaternion(**R**) command.*