# Homework problems 2

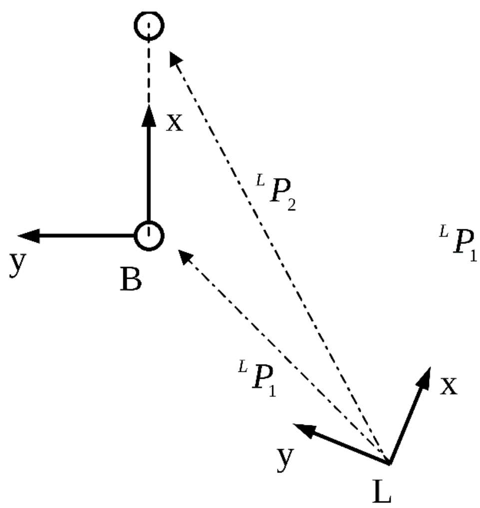
The answers to the homework problems should be returned to the corresponding folder on the Robotics-course MyCourses-platform. The answer files should be named as "solutions_homework problems 2_Firstname_Lastname.pdf". Deadline for returning the solutions is Thursday 17.10, 12:00 am, (*before the lectures when the solutions are presented*).

**1.** The positions of two landmarks are measured by a laser range meter. The position vectors given with respect to frame {L} of the laser range finder are denoted as ${}^{L}\boldsymbol{P}_1$ ja ${}^{L}\boldsymbol{P}_2$ in the figure below.

An orthogonal right-handed coordinate system {B} is formed (w.r.t frame {L}) based on the positions of the landmarks. For that, we have the following rules:

- The origin of frame {B} (*w.r.t frame* {L}) is assigned to the location of the first landmark
- The x-axis of frame {B} is pointing from the first landmark towards the second landmark along the xy-plane of frame {L} (*i.e. the projection of the unit vector of the x-axis of frame* {B} *with respect to the z-axis of frame* {L}*is zero*)
- The z-axis of frame {B} is pointing upwards (along the z-direction of frame {L}) and the y-axis comes from the right hand rule.

The mathematical steps to form a 4x4 homogenous transformation matrix ${}^{L}\boldsymbol{T}_B$ describing the pose (position and orientation) of frame {B} with respect to frame {L} as a function of vector components of the position vectors ${}^{L}\boldsymbol{P}_1$ ja ${}^{L}\boldsymbol{P}_2$ are as follows:




$${}^{L}P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \qquad {}^{L}P_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

First, apply the given rules to determine the homogenous transformation matrix ${}^{L}\boldsymbol{T}_{B}$

$${}^{L}\boldsymbol{T}_{B} = \begin{bmatrix} {}^{L}\boldsymbol{X}_{B} & {}^{L}\boldsymbol{Y}_{B} & {}^{L}\boldsymbol{Z}_{B} & {}^{L}\boldsymbol{P}_{B} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ where

${}^{L}\boldsymbol{P}_{B} = {}^{L}\boldsymbol{P}_{1}$ , i.e. the origin of frame {B} is at the position $P_1$

and the direction of the x-axis of frame *B* (*given w.r.t. frame L*) points from $P_1$ towards $P_2$, parallel to the xy-plane of frame *L*

$${}^{L}\boldsymbol{X}_{B} = \begin{bmatrix} \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} & \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} & 0 \end{bmatrix}^T$$

The z-axis of frame {B} is pointing to the same direction as the z-axis of frame L

$${}^{L}\boldsymbol{Z}_{B} = [0 \quad 0 \quad 1]^T$$

then, the direction of the y-axis is determined by applying the cross-product rule (Corke, p. 31)

$${}^{L}\boldsymbol{Y}_{B} = {}^{L}\boldsymbol{Z}_{B} \times {}^{L}\boldsymbol{X}_{B}$$ (*see Corke's text book p. 43 for the calculation of the cross-product*)

## An alternative way of determining the ${}^{L}\boldsymbol{X}_{B}$ and ${}^{L}\boldsymbol{Y}_{B}$ direction vectors:

First determine the rotation angle θ of frame {B} around the z-axis of frame {L} by considering the direction of the x-axis of frame B w.r.t frame {L}.

$$\theta = atan2(y_2 - y_1, x_2 - x_1)$$

then ${}^{L}\boldsymbol{R}_{B}$ becomes $${}^{L}\boldsymbol{R}_{B} = \begin{bmatrix} \cos\theta & -sin\,\theta & 0 \\ \sin\theta & cos\,\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And now we have all the required vector/matrix components for forming the homogenous transformation matrix ${}^{L}\boldsymbol{T}_{B}$

**a)** Calculate the numerical from of ${}^{L}\boldsymbol{T}_{B}$ when the position vectors are

${}^{L}\boldsymbol{P}_{1}=[5\ \ 15\ \ 3\ \ 1]^T$ and ${}^{L}\boldsymbol{P}_{2}=[10\ \ 20\ \ 5\ \ 1]^T$ (*10 points*)

**b)** Let's assume that the first position vector ${}^{L}\boldsymbol{P}_{1}$ is the same as in part **a)**. About the 2nd position vector ${}^{L}\boldsymbol{P}_{2}$ we know that its x-coordinate is the same as the x-coordinate of the first position vector, i.e. $x_1 = x_2$ and the y-coordinate of the second position vector has a smaller value than the y-coordinate of the first position vector, i.e. $y_1 > y_2$. Determine the numerical form of ${}^{L}\boldsymbol{T}_{B}$.

(*10 points*)

**2.** Integration of rotational motion when using rotation matrix for representing orientation.
Imagine that an Inertial Measurement Unit (IMU) has been mounted on board of a mobile robot. The gyroscopes and accelerometers of the IMU are constantly measuring rotational speed and linear acceleration of the robot. The problem consists of two parts:

**a)** First, the initial orientation of the mobile robot (or the IMU) needs to be determined. The initial orientation is described with three consecutive rotations, first $\theta_y$ (*yaw angle*) around z-axis followed by $\theta_p$ (*pitch angle*) around y-axis and $\theta_r$ (*roll angle*) around x-axis (compare Corke's text book, p. 84). Let's assume that the heading of the robot corresponds to zero yaw-angle, $\theta_y$=0.

The tilt angles of the robot, $\theta_p$ and $\theta_r$, are determined by measuring the direction of gravitational acceleration seen by the acceleration sensors of the IMU. The tilt angles can be computed by utilizing the equations (3.20) and (3.21) of Corke's text book (p. 84). As an answer give the corresponding 3x3 orthonormal rotation matrix (*10 points*).

$\boldsymbol{a}_{IMU}$=$[a_x \;\; a_y \;\; a_z]^T$ =$[0 \;\; -3.53 \;\; 9.15]^T$; % accelerometer measurements before robot starts to move m/s$^2$

g = 9.81 m/s$^2$;  % gravitational acceleration

**b)** Then, compute an update of the rotation matrix ***R***, which describes the orientation of the robot w.r.t. the reference (inertial) frame, as a function of the angular speed measurement, $\boldsymbol{\omega}$ acquired from the IMU.

To solve the problem, you can use Eq. (3.7) of Corke's text book. With the equation you can compute an approximation of the rotation matrix at a given time instant $\delta_t$ into the future (here 60 ms).

The sum of an orthonormal matrix and another matrix is **not** an orthonormal matrix, but if the added term is small then it will be "close" and we can normalize it. You can use the equations, presented on slide 19 (of Lecture_slides_3) to do the normalization.

Repeat the step 5 times, **with and without** normalizing the updated rotation matrix after each of the five steps. As an answer to the problem, give the numerical forms of the final rotation matrix (*acquired after repeating the update step 5 times*) with and without doing the normalization after each update step.

Is there any significant difference between the final rotation matrices corresponding to the two cases ? (*10 points*)

The numerical value of the angular velocity vector $\boldsymbol{\omega}_{IMU}$ is

$\boldsymbol{\omega}_{IMU}$ = $[\omega_x \;\; \omega_y \;\; \omega_z]^T$ = $[0.7 \;\; 0.8 \;\; 0]^T$; % gyroscope measurements in radians/sec

Assume that the gyro measurement of the angular velocity remains constant during the 5 integration/update steps.

*Hint: A similar case has been discussed as an example problem at the end of Lecture slides_3.*

**3.** A single degree of freedom robot should move smoothly from the initial position $\Theta(0) = -10^o$ to the final position $\Theta(t_f) = 65^o$ in 4 seconds. The robot starts and stops the motion at a standstill. Create a linear segment with parabolic blend type of trajectory to describe the motion.

As a solution show your MATLAB code and give a plot of the joint position, velocity and acceleration of the robot joint as a function of time.

See Craig's text book, pages 210-213 for more details. This problem is actually very similar to the example 7.3 in the book.

The acceleration and deacceleration of the motion of the joint during the parabolic blend regions is set to 30 deg/sec$^2$.   (10 points)

*HINT: Your can build the trajectories for display simply by implementing commands like:*

t = 0:0.01:4;

x = a0 + a1*t + a2*t.^2 + a3*t.^3;

*where for example t.^ is element-wise power.*

*You can start solving the problem by first determining the duration of the acceleration and deacceleration phases by means of Eq. 7.22 of Craig's text book*