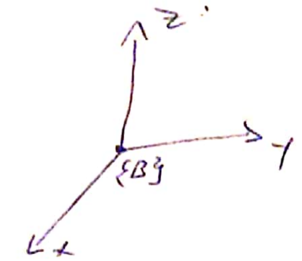
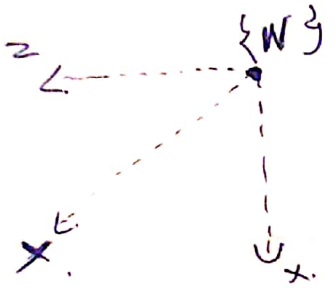
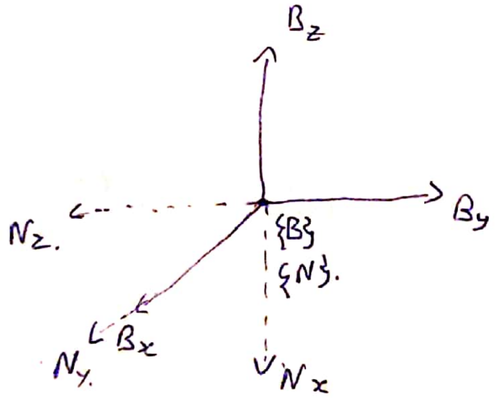
1.) **1.a** Given frame:



$${}^{B}R_{N} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



by Combing.



1.b

Rule =

$\hat{z} = \hat{x} \times \hat{y}$, $-(1)$ $\hat{x} = \hat{y} \times \hat{z}$ $-(2)$, $\hat{y} = \hat{z} \times \hat{x}$. $-(3)$

$\hat{x} = [0 \quad 0 \quad -1]$, $\hat{y} = [1 \quad 0 \quad 0]$ $\hat{z} = [0 \quad -1 \quad 0]$.

$$\hat{x} \times \hat{y} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = \hat{i}(0-0) - \hat{j}(1) + \hat{k}(0-0) = -\hat{j} = [0 \quad -1 \quad 0] = \hat{z}$$

(1) is Proved.

$$\hat{y} \times \hat{z} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-1-0) = -\hat{k} = [0 \quad 0 \quad -1] = \hat{x}$$

(2) is Proved.

$$\hat{z} \times \hat{x} = \begin{bmatrix} i & j & k \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \hat{i}(1-0) - \hat{j}(0-0) + \hat{k}(0-0) = \hat{i} = [1\ \ 0\ \ 0] = \hat{y}.$$

(3) is proved.

As all above equations are satisfied

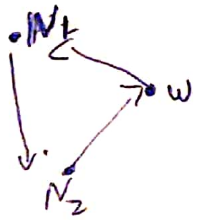
Yes, R describes a Right-handed co-ordinate frame.

2.) Given,

${}^{W}T_{N_1}$, ${}^{N_2}T_{W}$, ${}^{N_2}T_{N_3}$ are known transformations.

To find ${}^{N_1}T_{N_2}$, ${}^{N_3}T_{W}$

2.a From combining. $\{N_2\}$, $\{W\}$, $\{N_1\}$.



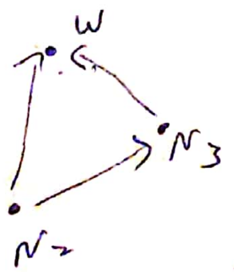
$$\Rightarrow \quad ({}^{N_1}T_{N_2})^{-1} = {}^{N_2}T_{W} \cdot {}^{W}T_{N_1}.$$

$${}^{N_2}T_{W} \cdot {}^{W}T_{N_1} \cdot {}^{N_1}T_{N_2} = I$$

$${}^{W}T_{N_1} \cdot {}^{N_1}T_{N_2} = ({}^{N_2}T_{W})^{-1}.$$

$$\boxed{{}^{N_1}T_{N_2} = ({}^{W}T_{N_1})^{-1} ({}^{N_2}T_{W})^{-1}} \quad \text{— (1)}$$

From combining $\{N_2\}$, $\{N_3\}$, $\{W\}$



$${}^{N_2}T_{W} = {}^{N_2}T_{N_3} \cdot {}^{N_3}T_{W}.$$

$$\boxed{{}^{N_3}T_{W} = ({}^{N_2}T_{N_3})^{-1} \cdot {}^{N_2}T_{W}} \quad \text{— (2)}$$

$\therefore$ The unknowns from (1) & (2) are

$${}^{N_1}T_{N_2} = ({}^{W}T_{N_1})^{-1} ({}^{N_2}T_{W})^{-1}.$$

$${}^{N_3}T_{W} = ({}^{N_2}T_{N_3})^{-1}\ {}^{N_2}T_{W}$$

2.b

$$ {}^{N_1}T_{N_2} = \left({}^{W}T_{N_1}\right)^{-1} \left({}^{N_2}T_{W}\right)^{-1} $$

$$ = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boxed{ {}^{N_1}T_{N_2} = \begin{bmatrix} -1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} } \quad -(3) $$

Now,

$$ {}^{N_3}T_{W} = \left({}^{N_2}T_{N_3}\right)^{-1} \, {}^{N_2}T_{W} $$

$$ = \begin{bmatrix} 0 & -1 & 0 & -7 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boxed{ {}^{N_3}T_{W} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} } $$

$-(4)$

(3) & (4) represent the transformation matrices for ${}^{N_1}T_{N_2}$ & ${}^{N_3}T_{W}$ respectively.

Scanned by CamScanner

3). Given

{A} & {B} are co-incident frame.

Translational or rotation along z-axis for 30 degrees:

$$\text{transform } A_{(z\text{ axis} - 30^\circ)} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{transform } B \; \begin{matrix}(\text{translate. z axis}) \\ 5 \text{ units}\end{matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{transform } C \; \begin{matrix}(\text{rotation x-axis}) \\ 60^\circ\end{matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 & 0 \\ 0 & \sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3.a :

$${}^{A}T_{B} = \oplus \text{transform } A \oplus \text{transform} B \oplus \text{transform } C$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.866 & 0 \\ 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.866 & 0 \\ 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}T_{B} = \begin{bmatrix} 0.866 & -0.25 & 0.433 & 0 \\ 0.5 & 0.433 & -0.7499 & 0 \\ 0 & 0.866 & 0.5 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.b

Given.

$$^{A}P_1 = [0 \quad 0 \quad 6]$$

$$^{B}P_1 = ?$$

$$^{B}P_1 = {}^{B}T_A \cdot {}^{A}P_1$$

we know that

$$^{B}T_A = \ominus\, {}^{A}T_B.$$

$$^{B}T_A = \begin{bmatrix} 0.866 & 0.5 & 0 & 0 \\ -0.25 & 0.433 & 0.866 & -4.33 \\ 0.433 & -0.7499 & 0.5 & -2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{B}P_1 = {}^{B}T_A \cdot {}^{A}P_1$$

$$= \begin{bmatrix} 0.866 & 0.5 & 0 & 0 \\ -0.25 & 0.433 & 0.866 & -4.33 \\ 0.433 & -0.7499 & 0.5 & -2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.866 \\ 0.5 \\ 1 \end{bmatrix}$$

$$^{B}P_1 = [0, 0.866, 0.5] \text{ i.e } [x=0, y=0.866, z=0.5]$$

3.c

Given,

$$^{B}P_2 = [0 \quad 0 \quad 6].$$

$$^{A}P_2 = {}^{A}T_B \cdot {}^{B}P_2$$

$$= \begin{bmatrix} 0.866 & -0.25 & 0.433 & 0 \\ 0.5 & 0.433 & -0.7499 & 0 \\ 0 & 0.866 & 0.5 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \end{bmatrix}.$$

$$^{A}P_2 = \begin{bmatrix} 2.598 \\ -4.4994 \\ 8 \\ 1 \end{bmatrix} = [x = 2.598,\ y = -4.4994,\ z = 8]$$

$$^{A}P_2 = [2.598 \quad -4.4994 \quad 8]$$

4.) Given A & B are incident frames rotation of 30° around z-axis.

$\theta = 30°$

vector $= [0 \quad 0. \quad 1]$.

<u>4.a</u>

$$[\hat{V}]_x = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin 30 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (1 - \cos 30) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -0.133 & 0 & 0 \\ 0 & -0.133 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R. = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.)

Given

30° around z-axis.

5.a

$\theta = 30°$

vector = [0 0 1]

$\hat{q} = \left(\cos\frac{\theta}{2}\right) \langle \hat{u} \sin\frac{\theta}{2} \rangle.$

$\hat{q} = \cos 15 \langle [0 \quad 0 \quad 1] \sin 15 \rangle.$

$= 0.9659 \langle [0 \quad 0 \quad 0.2588] \rangle.$

Matlab screenshots are attached.

5.b :-

Compounding

$$\mathring{q} \oplus q_0' = \begin{pmatrix} s & -v_1 & -v_2 & -v_3 \\ v_1 & s & -v_3 & v_2 \\ v_2 & v_3 & s & -v_1 \\ v_3 & -v_2 & v_1 & s \end{pmatrix} \begin{pmatrix} s' \\ v_1' \\ v_2' \\ v_3' \end{pmatrix}.$$

$$= \begin{bmatrix} 0.9659 & 0 & 0 & -0.2588 \\ 0 & 0.9659 & -0.2588 & 0. \\ 0 & 0.2588 & 0.9659 & 0 \\ 0.2588 & 0 & 0 & 0.9659 \end{bmatrix} \begin{bmatrix} 0.9659 \\ 0 \\ 0 \\ 0.2588 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8659 \\ 0 \\ 0 \\ 0.4999 \end{bmatrix} = 0.866 \langle 0, 0, 0.5 \rangle.$$

Matlab Screenshots are attached.

HOME PLOTS APPS

New Script | New Live Script | New | Open | Find Files | Compare — FILE
Import Data | Save Workspace | New Variable | Open Variable | Clear Workspace — VARIABLE
Favorites | Analyze Code | Run and Time | Clear Commands — CODE
Simulink — SIMULINK
Layout | Preferences | Set Path | Add-Ons — ENVIRONMENT
RESOURCES

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Current Folder

Name

Tf.m

Details

Workspace

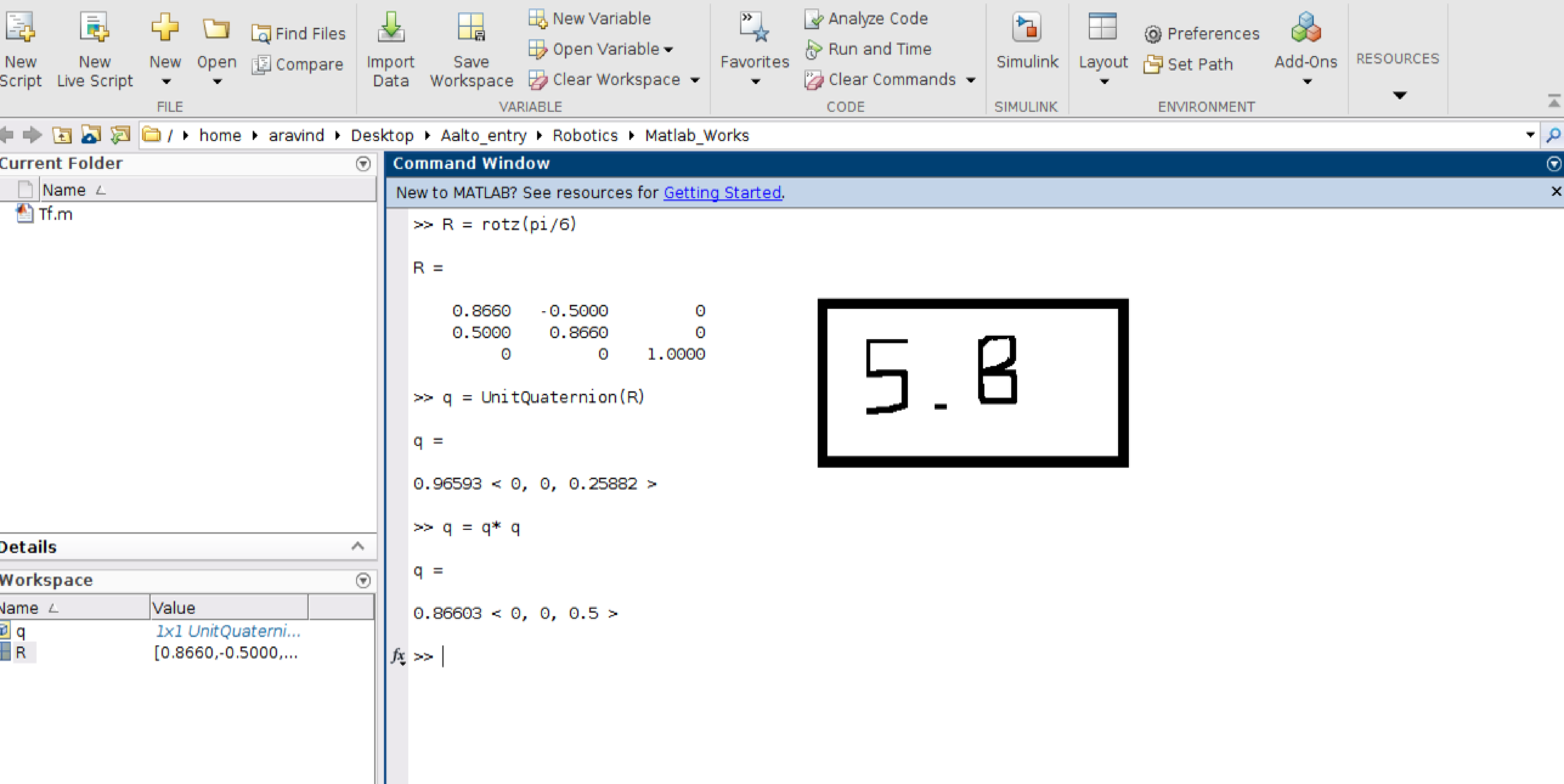
Name	Value
ans	1x1 UnitQuaterni...
R	[0.8660,-0.5000,...

Command Window

New to MATLAB? See resources for Getting Started.

```
>> R = rotz(pi/6)

R =

    0.8660   -0.5000         0
    0.5000    0.8660         0
         0         0    1.0000

>> UnitQuaternion(R)

ans =

0.96593 < 0, 0, 0.25882 >

>>
```

5.A


New Script
New Live Script
New
Open
Find Files
Compare
FILE
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
VARIABLE
Favorites
Analyze Code
Run and Time
Clear Commands
CODE
Simulink
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Current Folder
Name
Tf.m
Details
Workspace
Name
Value
q
1x1 UnitQuaterni...
R
[0.8660,-0.5000,...
Command Window
New to MATLAB? See resources for Getting Started.
>> R = rotz(pi/6)
R =
0.8660 -0.5000 0
0.5000 0.8660 0
0 0 1.0000
>> q = UnitQuaternion(R)
q =
0.96593 < 0, 0, 0.25882 >
>> q = q* q
q =
0.86603 < 0, 0, 0.5 >
>>
5.8