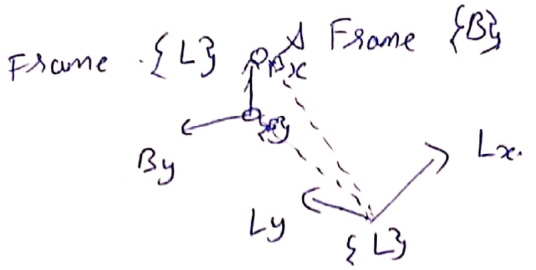
Assignment - 2

1.) Given,



$$^{L}P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}, \quad ^{L}P_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

1.a

$$^{L}T_B = \begin{bmatrix} ^{L}X_B & ^{L}Y_B & ^{L}Z_B & ^{L}P_{1B} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x-axis of {B} = $\overrightarrow{P_1 P_2} = \dfrac{P_2 - P_1}{norm(\overrightarrow{P_1 P_2})}$

$\{^{L}X_B\}$

$$= \begin{bmatrix} \dfrac{x_2 - x_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}} & \dfrac{(y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} \dfrac{5}{\sqrt{5^2+5^2}} & \dfrac{5}{\sqrt{5^2+5^2}} & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} \dfrac{5}{5\sqrt{2}} & \dfrac{5}{5\sqrt{2}} & 0 \end{bmatrix}^T = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \end{bmatrix}^T$$

z-axis of B $= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

$\{^{L}Z_B\}$

y axis of B $= {}^{L}Z_B \times {}^{L}X_B$

$$= \begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} = i\left(-\frac{1}{\sqrt{2}}\right) - j\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \begin{bmatrix} -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \end{bmatrix}^T$$

Now to get L_{TB}

$$L_{T_B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 5 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 15 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.71 & -0.71 & 0 & 5 \\ 0.71 & 0.71 & 0 & 15 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1. \end{bmatrix} \quad -(1)$$

Equation (1) represents the solution for.

1.a

1.b Given, $x_1 = x_2$. $y_1 > y_2$.

$$L_{x_B} = \begin{bmatrix} \frac{0}{\sqrt{0+\sqrt{(y_2-y_1)^2}}} & \frac{y_2-y_1}{\sqrt{0+(y_2-y_1)^2}} & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0. & \frac{y_2-y_1}{\sqrt{(y_2-y_1)^2}} & 0 \end{bmatrix}^T$$

Since $y_1 > y_2$.

$$L_{xB} = \begin{bmatrix} 0 & -1. & 0 \end{bmatrix}^T$$

$$L_{zB} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$L_{y_B} = \begin{bmatrix} i & j & k. \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = i(0+1) - j(0-0) + k[0-0]$$

$$= i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T.$$

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$$L_{T_B} = \begin{bmatrix} 0 & 1 & 0 & 5 \\ -1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad - (2)$$

Equation (2) represents the solution for <u>1.b</u>.

2.) Given To compute the rotational matrix of IMU.

2.a

$$a_{IMU} = \begin{bmatrix} 0 \\ -3.53 \\ 9.15 \end{bmatrix}, \quad g = 9.81\ m/s^2.$$

$$\theta_y = 0^\circ$$

Now

$$\sin\theta_p = -\frac{a_x}{g} = 0.$$

$$\sin\theta_p = 0$$

$$\theta_p = \sin^{-1}(0)$$

$$\boxed{\theta_p = 0^\circ} \quad -(1)$$

$$\tan\theta_r = \frac{a_y}{a_z} = -\frac{3.53}{9.15}$$

$$\theta_r = \tan^{-1}\left(\frac{-3.53}{9.15}\right)$$

$$\boxed{\theta_r = -21.1^\circ}$$

Orthonormal rotation matrix R

$$R = \begin{bmatrix} \cos\theta_y & -\sin\theta_y & 0 \\ \sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_p & 0 & \sin\theta_p \\ 0 & 1 & 0 \\ -\sin\theta_p & 0 & \cos\theta_p \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin\theta_r \\ 0 & \sin\theta_r & \cos\theta_r \end{bmatrix}$$

Now $\theta_y = 0^\circ$, $\theta_p = 0^\circ$, $\theta_r = -21.1^\circ$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.93 & 0.36 \\ 0 & -0.36 & 0.93 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.93 & 0.36 \\ 0 & -0.36 & 0.93 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.93 & 0.36 \\ 0 & -0.36 & 0.93 \end{bmatrix}.$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.933 & 0.36 \\ 0 & -0.36 & 0.933 \end{bmatrix}. \quad - (1)$$

(1) is solution for $\underline{\underline{Z}}$.a