Group Discussion: Game Theory

Let there be a situation where A and B together can create a pie with 9 slices.
But A alone can only make 1 slice and B alone can only make 2 slices.
And everybody involved knows all these information.
How should they divide the pie?

We will return to this problem later tonight...

Group Discussion: Game Theory

Yeah yeah, it’s about Fortnite

Random Numbers

Pick any number between 0 and 100 (both inclusive)

The person whose number lies closest to the square root of the average of all numbers, wins

https://forms.gle/vsbiyQh53vHujhLV8

“I’m an optimist in the sense that I believe humans are noble and honourable, and some of them are really smart… I have a somewhat more pessimistic view of people in groups”

~ Steve Jobs

The Stock Market

An investor sells a stock to another

One convinced it is headed down

The other convinced it is going up

I think I know what you think, but you have no idea what you think I think

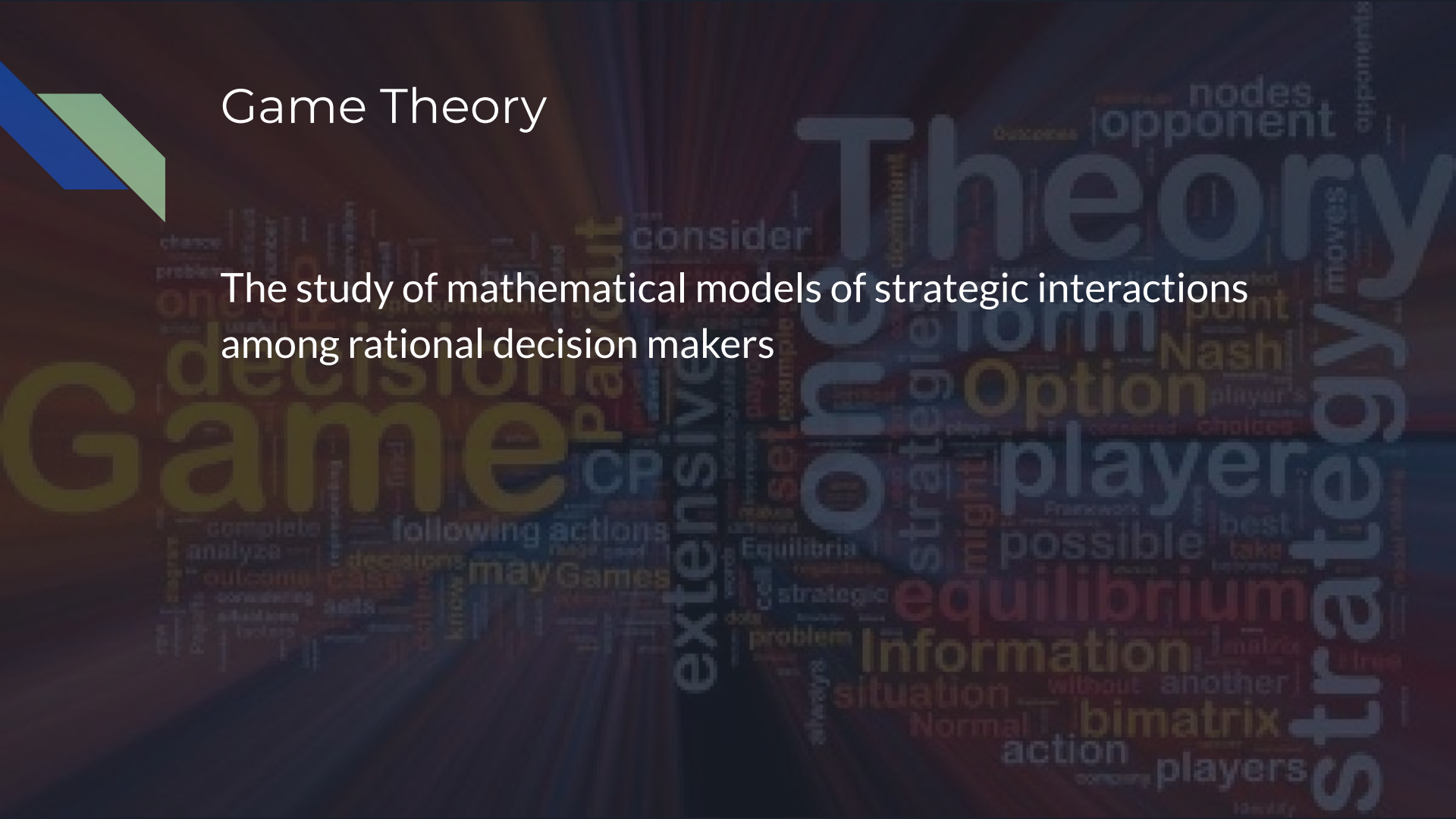
Human Interactions

What do humans interact with?

1. Human vs Nature
2. Human vs Self
3. Human vs Human
4. Human vs Society

Game Theory

The study of mathematical models of strategic interactions among rational decision makers

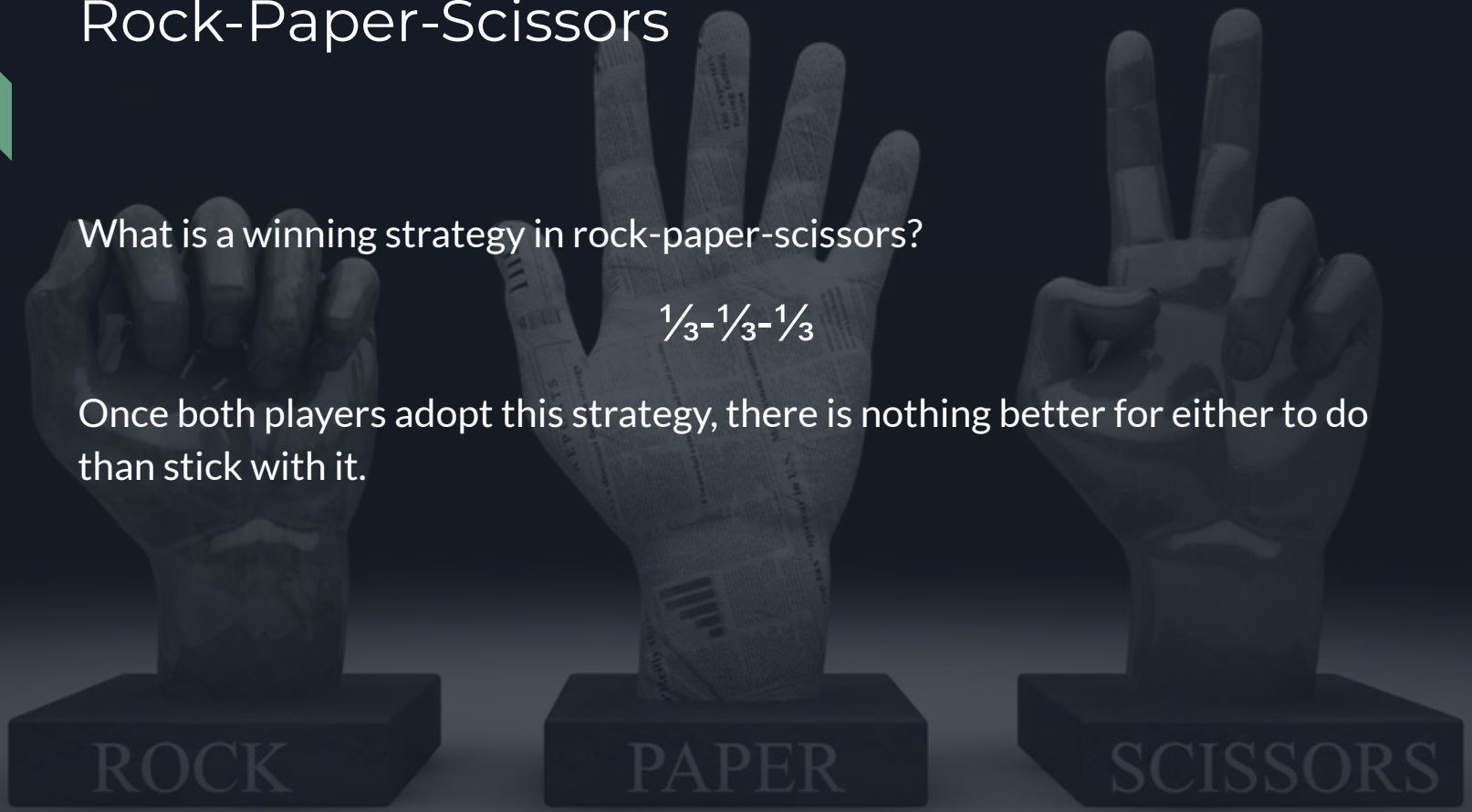


Rock-Paper-Scissors

What is a winning strategy in rock-paper-scissors?

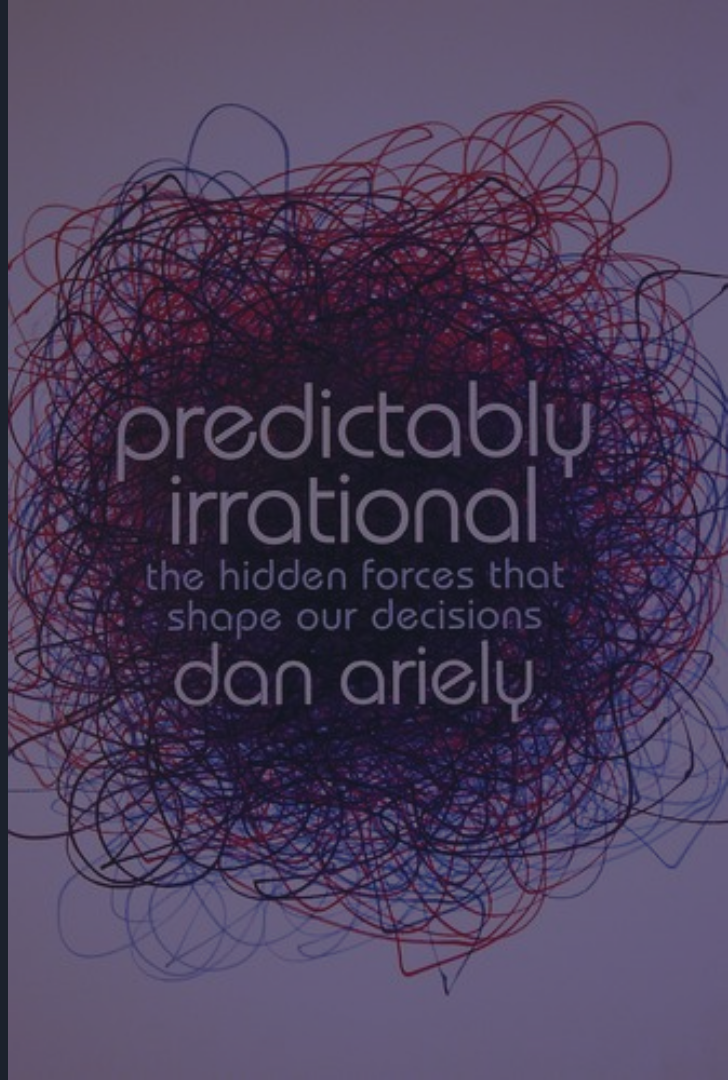
$$\frac{1}{3}\text{-}\frac{1}{3}\text{-}\frac{1}{3}$$

Once both players adopt this strategy, there is nothing better for either to do than stick with it.



But humans are predictably irrational

1. People who win tend to repeat their strategy
2. Losers change



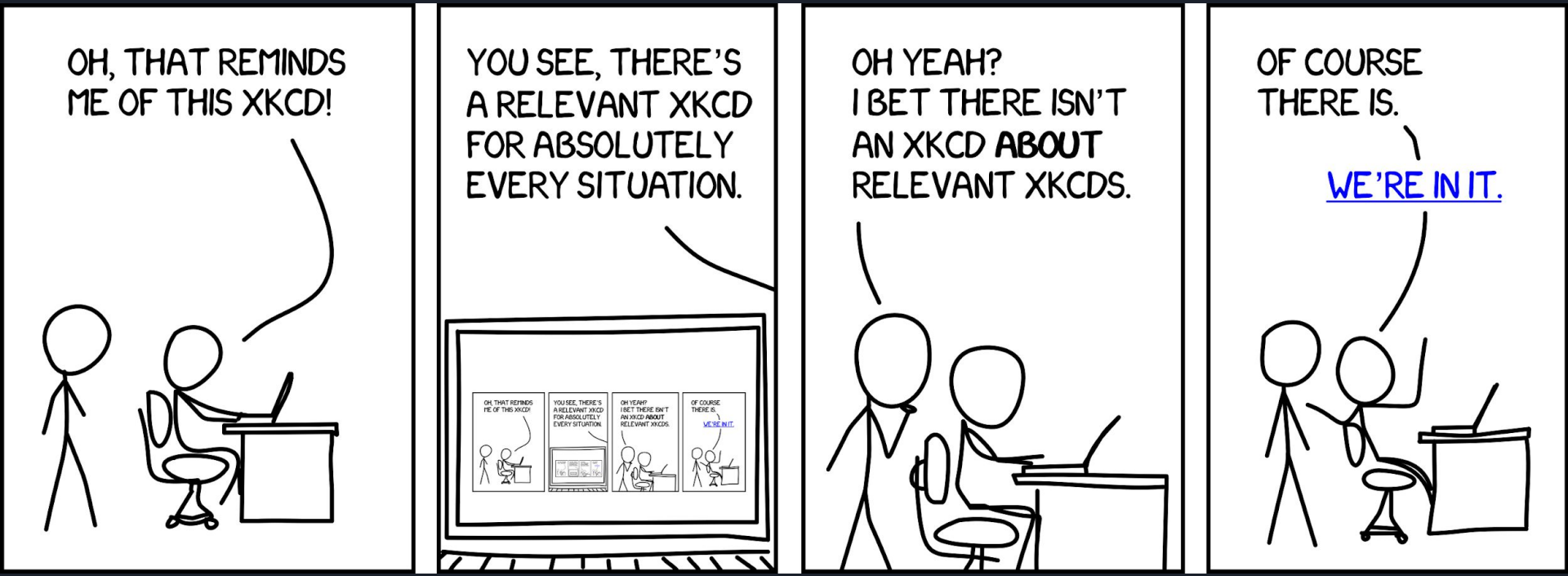
Recursion

Back To The Stock Market

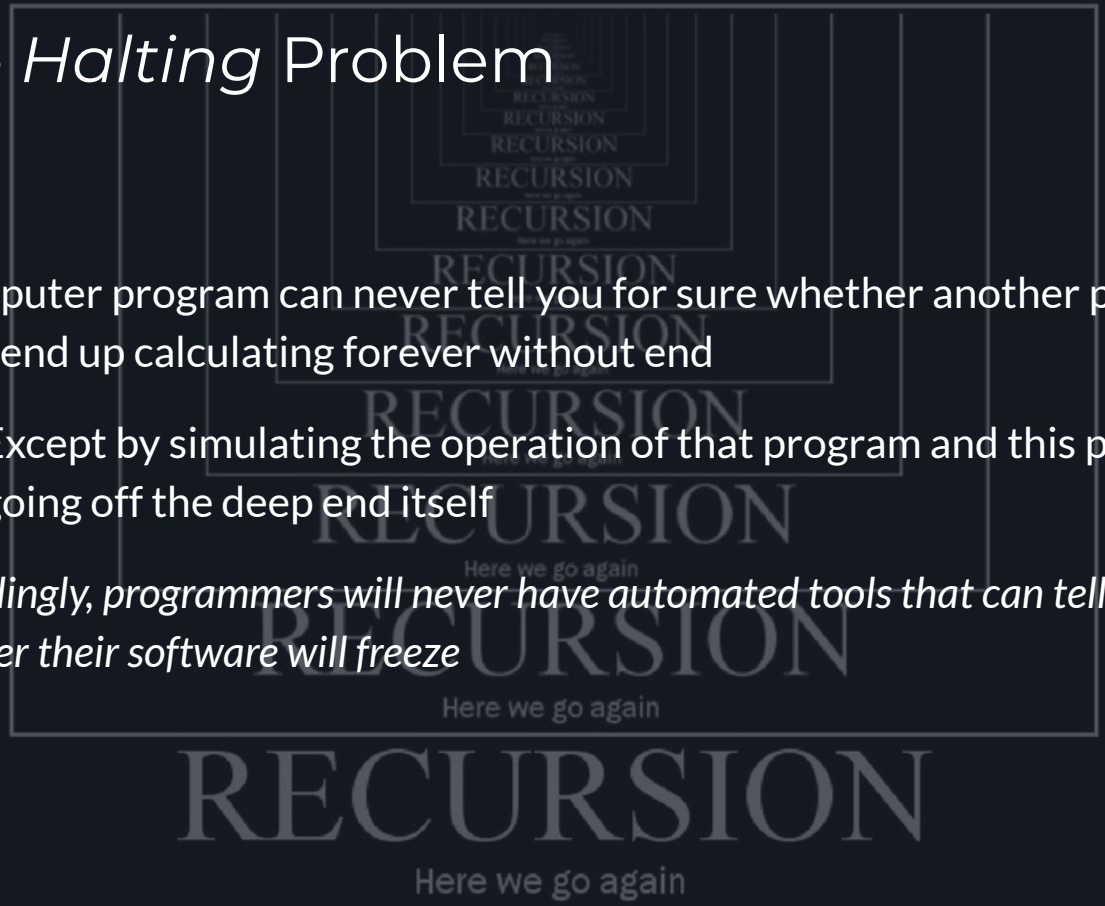
“Successful investing is anticipating the anticipations of others”

In this way, the value of a stock isn't what people think it's worth but what people think people think it's worth

In fact, even that's not going far enough


OH, THAT REMINDS ME OF THIS XKCD!
YOU SEE, THERE'S A RELEVANT XKCD FOR ABSOLUTELY EVERY SITUATION.
OH YEAH? I BET THERE ISN'T AN XKCD ABOUT RELEVANT XKCDS.
OF COURSE THERE IS.
WE'RE IN IT.

The *Halting* Problem



A computer program can never tell you for sure whether another program might end up calculating forever without end

> Except by simulating the operation of that program and this potentially going off the deep end itself

Accordingly, programmers will never have automated tools that can tell them whether their software will freeze

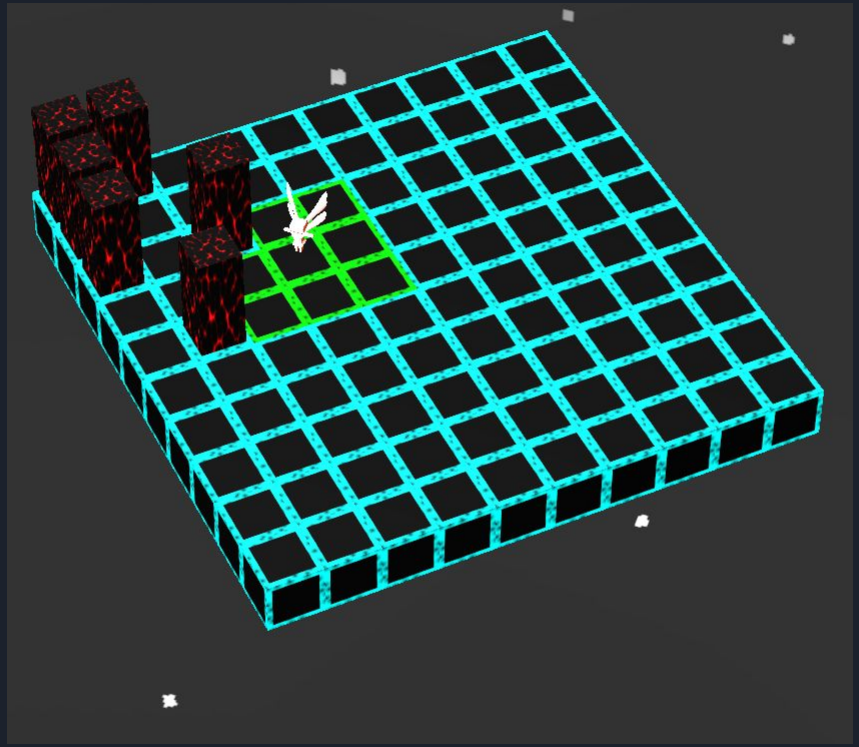
Combinatorial Game Theory

You are an Angel in an infinite grid, each turn you can jump upto K spaces away

Let us say K=1, then you can jump anywhere within the shown boundary

But your arch-nemesis, the Devil is watching. On the Devil's turn, they can remove one square anywhere on the infinite grid and the Angel can no longer land there

Can the Devil eventually trap the Angel? Or does the Angel have a strategy to avoid capture forever



"In poker, you never play your hand, you play the man across from you."
~James Bond, in the Casino Royale.

An Infinite Recursion?

Level I - ‘I know’

Level 2 - ‘You know that I know’

Level 3 - ‘I know that you know that I know’

Instead, *gamers* try to find a base level of theoretical understanding in most situations…

What *is* the Nash?

Reaching Equilibrium

Game theory covers an incredibly broad spectrum of scenarios of cooperation and competition

And this field began with the latter

A player in a game has found nash equilibrium when they make the choice that leaves them better off no matter what their opponents decide to do

When you're **competing** with others, it makes sense to choose the course of action that benefits **you** the most no matter what everyone else decides to do

Was it actually about Random Numbers...?

Pick any number between 0 and 100 (both inclusive)

The person whose number lies closest to the square root of the average of all numbers, wins

The maximum average = 100

The maximum square root of such an average value = 10

So there's no real point picking a number above 10

Truth vs Complexity

It isn't enough to have a solution if that problem is intractable

We know that an equilibrium exists

But not how to get there

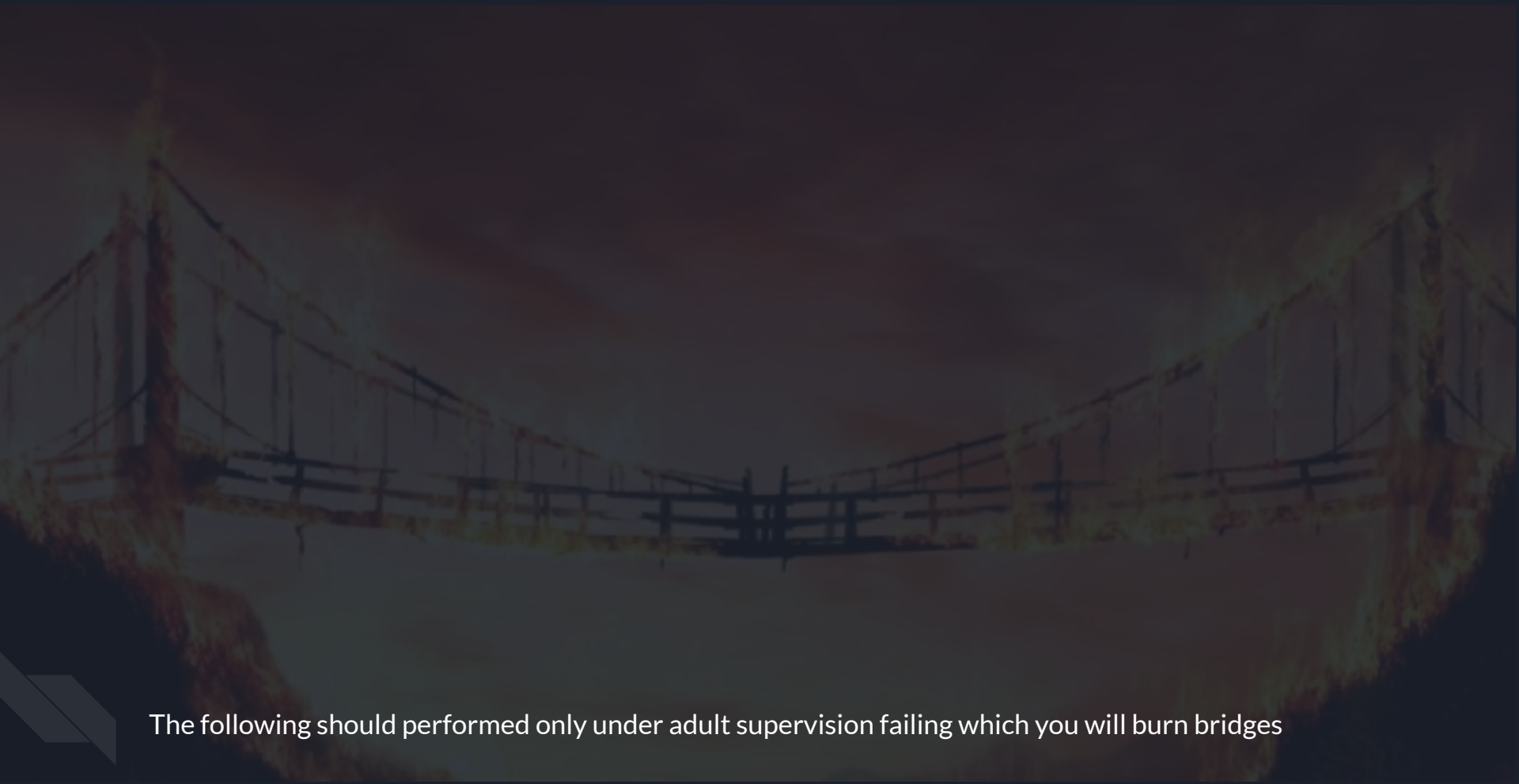
If your laptop cannot find it, neither can the economic market

The strategy profile s* is a Nash equilibrium if:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall \quad s_i \in S_i$$

Ofcourse, I didn't even attempt to prove this here

Game Theory and Negotiation...


The following should performed only under adult supervision failing which you will burn bridges

Rubinstein Bargaining Model

Two players

Complete information

Unlimited offers—the game keeps going until one player accepts an offer

Alternating offers—the first player makes an offer in the first period, if the second player rejects, the game moves to the second period in which the second player makes an offer, if the first rejects, the game moves to the third period, and so forth

*Delays are costly**

Let there be a situation where A and B together can create a pie with 9 slices.
But A alone can only make 1 slice and B alone can only make 2 slices.
And everybody involved knows all these information.
How should they divide the pie?

Someway Different...

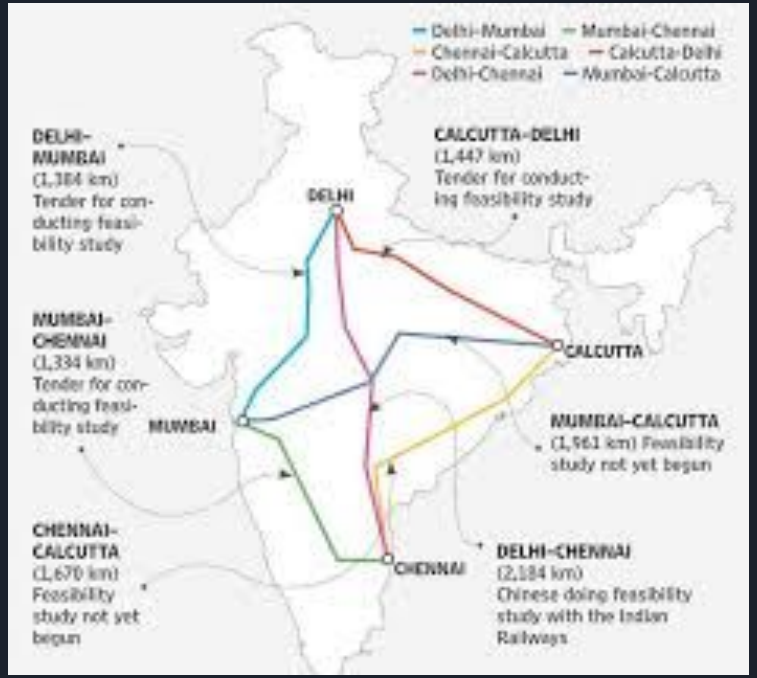
A needs B as much as B needs A to create the extra 6 slices

A Similar Problem

Two of your friends want you to visit them - You live in Delhi, one of your friends lives in Mumbai and the other in Kolkata

Route	Distance	Cost
Mumbai - Delhi	1137km	Rs.2800
Delhi - Kolkata	1311km	Rs.3500
Kolkata - Mumbai	1655km	Rs.4300

How would your friends split the cost of your travel assuming you don't have to pay anything?



Splitting the Money

Consider the following example, say someone owes money to two different people. $100 to one and $50 to the other. The problem is, they only have $100 to their name. How would you split the money (and thereby leave that person completely broke :P)?

The *Alternate* Reasons

Why you shouldn’t

1. Divide Proportionately: Proportional division doesn't account for what a party could get on its own
   And using this perspective will help you justify a solution that is different from proportional division.
2. Also remember, negotiation problems don't come to you all framed and tied up in a bow.

Shapley Value

We started off with a two person problem

We could divide the net savings 50-50

But with three or more people, what each person contributes isn't always equal

You could instead use this:

The Shapley value or the amount that player **i** gets given a coalitional game ***(v,N)*** is

$$\frac{|S|!\,(n-|S|-1)!}{n!}(v(S\cup\{i\})-v(S))$$

The Devil in the Detail

A last question for the night. I offer you and your friend $100 if the two of them can agree on how to divide it up. One side gets to make a proposal and the other can only say yes or no

For example, the proposer can offer a 90/10 split. If the receiver says yes, the receiver receives $10 while the proposer gets $90. If the receiver says no, they both get zero and that's it

In theory, a fully rational receiver should say yes to a dollar. Anticipating this, the proposer can offer just a dollar

Dozens have studies have shown that most people would reject offers that small

So what is game theory at the end of the day?
It is the mathematics of looking at decisions and strategies until some human element creeps in and collapses the whole rational framework

Some fancy references:
Rubinstein Bargaining Model research paper: http://arielrubinstein.tau.ac.il/papers/11.pdf
Shapley Value research paper: https://econpapers.repec.org/article/inmormnsc/v_3a20_3ay_3a1973_3ai_3a3_3ap_3a370-372.htm