



Propositional Logic

Next Chapter:

The Inference Symbol

| = or ASSES

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GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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Next Topic:

The Inference Symbol

G GO
|- or A |- S E S

KB: knowledge Base
(set of premises)

$KB \models Y$ means

(logically) infers

KB infers Y

means

$$\frac{KB}{\therefore Y} \quad \begin{cases} \text{Valid} \\ \text{Ans.} \end{cases}$$



P_1
 P_2
 P_3

 $\therefore c$

Valid Arg iff $P_1 \wedge P_2 \wedge P_3 \rightarrow c$ is a Tautology.

iff $P_1, P_2, P_3 \models c$

P_1 P_2 P_3 $\therefore c$

Valid Arg

Way 1

iff

 $P_1 \wedge P_2 \wedge P_3 \rightarrow c$

is a Tautology.

iff

 $P_1, P_2, P_3 \models c$

Way 3

Way 1 = Way 2 = Way 3



$$P_1, P_2, P_3 \models \gamma$$

means

$$\frac{P_1 \\ P_2 \\ P_3}{\therefore \gamma}$$

Valid

means

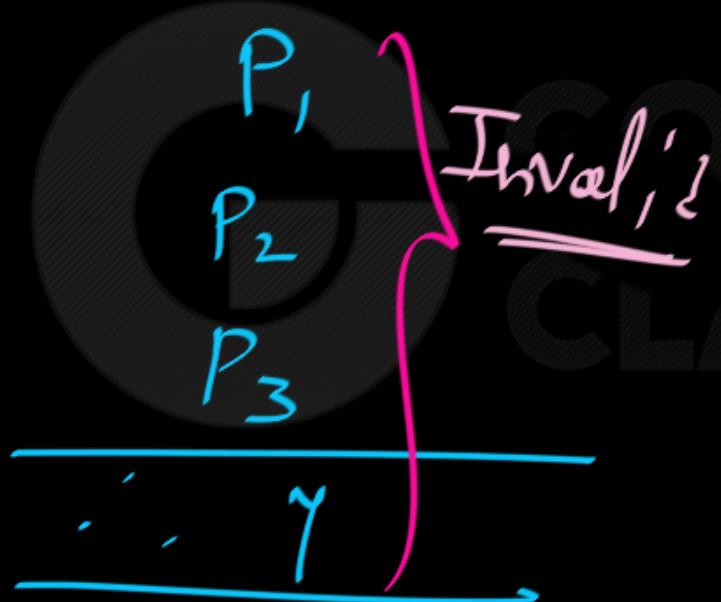
$$P_1 \wedge P_2 \wedge P_3 \rightarrow_C$$

is a Tautology.



$P_1, P_2, P_3 \not\models \gamma$

means



means

$$P_1 \wedge P_2 \wedge P_3 \rightarrow_C$$

is a Tautology.
Not



$A, B, C \vdash \gamma$ iff



$$\frac{A \\ B \\ C}{\vdash \gamma} \text{ valid}$$

$A, B, C \not\models \gamma$ iff

$$\frac{A \\ B \\ C}{\therefore \gamma}$$

Invalid

Does not infer

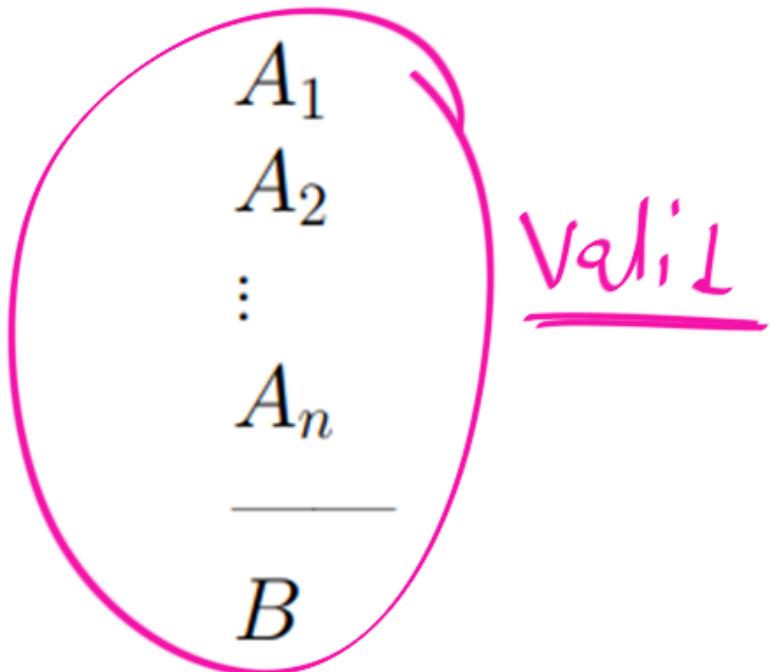
$KB \models \gamma$

KB infers γ

KB entails γ

KB implies γ

γ is a Consequence
of KB .



Read this as “from A_1, \dots, A_n , infer B .”

– Sometimes written “ $A_1, \dots, A_n \vdash B$ ”



- **Entailment:** $KB \models Q$
 - Q is entailed by KB (a set of premises or assumptions) if and only if there is no logically possible world in which Q is false while all the premises in KB are true.
 - Or, stated positively, Q is entailed by KB if and only if the conclusion is true in every logically possible world in which all the premises in KB are true.

$$\frac{KB}{\therefore Q} \quad \overbrace{\hspace{1cm}}^{\text{Valid Arg.}}$$



Logical inference:

X (logically)infers Y, iff " $X \rightarrow Y$ " is a tautology.

$$X \models Y$$

iff

$X \rightarrow Y$ is a Tautology.



Logical Consequence

Logical inference

Logical implication

Logical Entailment

Same



Logical consequence:

Y is a (logical) consequence of X, iff " $X \rightarrow Y$ " is a tautology.

$$X \models Y$$

Logical inference:

X (logically)infers Y, iff " $X \rightarrow Y$ " is a tautology.

$$X \Vdash Y$$

Logical implication:

X (logically)implies Y, iff " $X \rightarrow Y$ " is a tautology.

$$X \not\models Y$$



Q:

$$KB = \{p \rightarrow q, p, s \rightarrow r\}$$

Which of p, q, r, s are inferred by(entailed by) KB?

- $p \rightarrow q, p, s \rightarrow r \models p$
- $p \rightarrow q, p, s \rightarrow r \models q$
- $p \rightarrow q, p, s \rightarrow r \models r$
- $p \rightarrow q, p, s \rightarrow r \models s$

Q: \rightarrow set of premises (knowledge Base)

KB = { $p \rightarrow q, p, s \rightarrow r$ }

Which of p, q, r, s are inferred by (entailed by) KB?

i.e. which of the following is True?

- $p \rightarrow q, p, s \rightarrow r \models p$ ✓
- $p \rightarrow q, p, s \rightarrow r \models q$ ✓
- $p \rightarrow q, p, s \rightarrow r \models r$ ✗
- $p \rightarrow q, p, s \rightarrow r \models s$ ✗



(a)

$$\boxed{P \rightarrow Q, P, S \rightarrow R \models P}$$

True /
false)

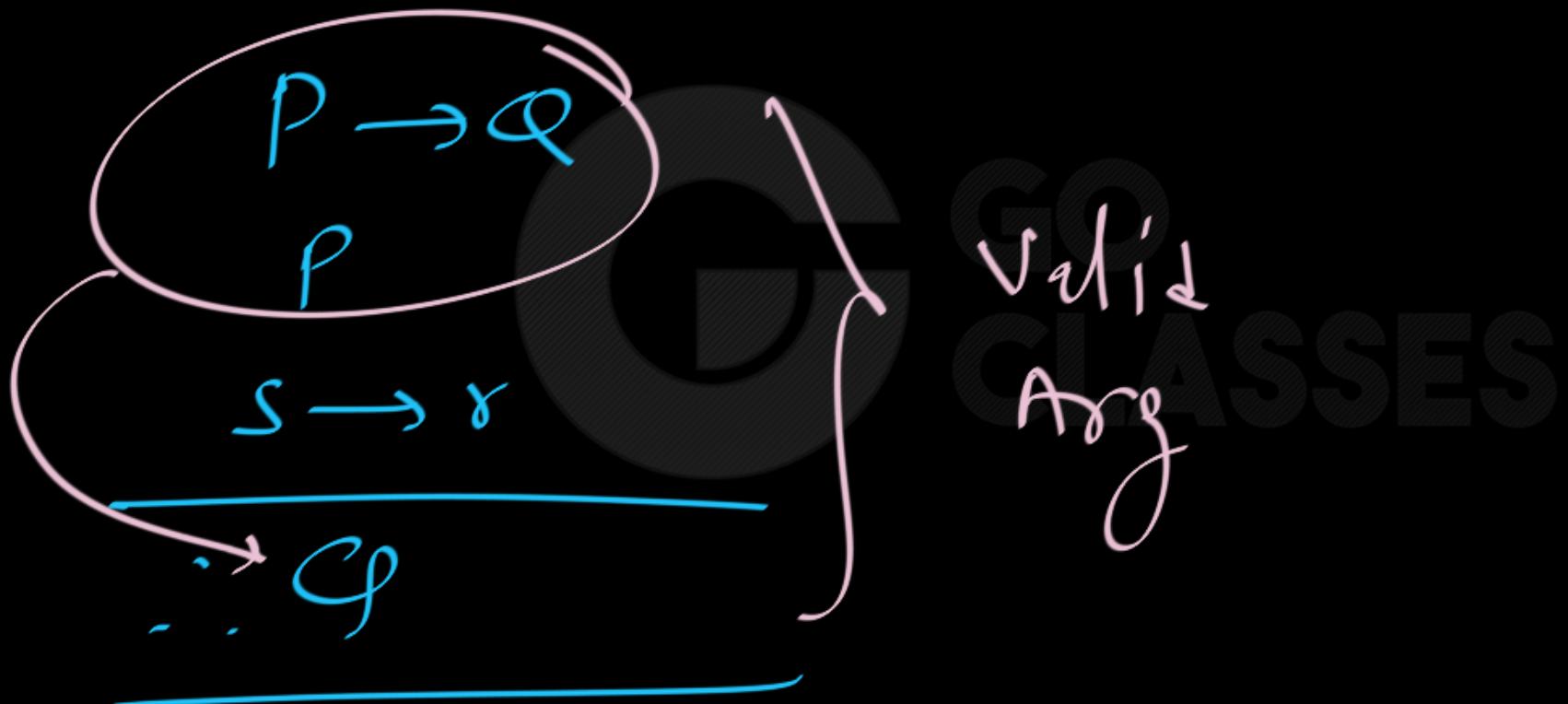
$$\begin{array}{c} P \rightarrow Q \\ \textcircled{P} \\ S \rightarrow R \\ \hline \textcircled{P} \end{array}$$

Valid Arg



(a)

$$P \rightarrow Q, P, S \rightarrow R \models Q$$



Valid Arg

(a)

$$P \rightarrow Q, P, S \rightarrow R \models S$$

✓ ① $P \rightarrow Q = \text{True}$ → false

✓ ② $P = \text{True}$

✓ ③ $S \rightarrow R = \text{True}$

$$\frac{1. 2. 3.}{\therefore S \text{ false}}$$

Involves Any.



(a)

$$P \rightarrow Q, P, S \rightarrow R$$

$$| = > R$$

$$\begin{aligned} ① \quad & P \rightarrow Q = \text{True} \\ ② \quad & P = \text{True} \\ ③ \quad & S \rightarrow R = \text{True} \\ \hline \therefore & R = \text{false} \end{aligned}$$

Invalid

~~False~~