Digital Logic :

Next Topic :

Representations of Signed Binary Numbers

2's Complement Number System

2's Complement Numbers

In the 2's complement number system, a positive number, N, is represented by a 0 followed by the magnitude of N as in the sign and magnitude system; however,

a negative number, $-N$, is represented by its 2's complement, N^*. If the word length is n bits, the 2's complement of a positive integer N is defined as

N^* is obtained by complementing N bit-by-bit and then adding 1. An alternative way to form the 2's complement of N is to start at the right and leave any 0's on the right end and the first 1 unchanged, then complement all bits to the left of the first 1. In

Twos compliment

- Compliment bits in positive value and add 1 to create negative value
- Most significant bit still a sign bit

$$0110 = (6)_{10} \qquad 1001 + 1 = 1010 = (-6)_{10}$$

- One representation of zero

$$0000 = (0)_{10} \qquad 1000 = (-8)_{10} \qquad 1111 = (-1)_{10}$$

- One more negative number than positive

$$\text{MIN: } 1000 = (-8)_{10} \qquad \text{MAX: } 0111 = (7)_{10}$$

SIGNED BINARY NUMBERS

Positive integers (including zero) can be represented as unsigned numbers. However, to represent negative integers, we need a notation for negative values. In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign. Because of hardware limitations, computers must represent everything with binary digits. It is customary to represent the sign with a bit placed in the leftmost position of the number. The convention is to make the sign bit 0 for positive and 1 for negative.

It is important to realize that both signed and unsigned binary numbers consist of a string of bits when represented in a computer. The user determines whether the number is signed or unsigned. If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number. If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number. For example, the string of bits 01001 can be considered as 9 (unsigned binary) or as +9 (signed binary) because the leftmost bit is 0. The string of bits 11001 represents the binary equivalent of 25 when considered as an unsigned number and the binary equivalent of −9 when considered as a signed number. This is because the 1 that is in the leftmost position designates a negative and the other four bits represent binary 9. Usually, there is no confusion in interpreting the bits if the type of representation for the number is known in advance.

As an example, consider the number 9, represented in binary with eight bits. +9 is represented with a sign bit of 0 in the leftmost position, followed by the binary equivalent of 9, which gives 00001001. Note that all eight bits must have a value; therefore, 0's are inserted following the sign bit up to the first 1. Although there is only one way to represent +9, there are three different ways to represent −9 with eight bits:

signed-magnitude representation:	10001001
signed-1's-complement representation:	11110110
signed-2's-complement representation:	11110111

In signed-magnitude, −9 is obtained from +9 by changing only the sign bit in the leftmost position from 0 to 1. In signed-1's-complement, −9 is obtained by complementing all the bits of +9, including the sign bit. The signed-2's-complement representation of −9 is obtained by taking the 2's complement of the positive number, including the sign bit.

using 5-bits (in 2's System)

for signed Numbers

10 → +10

0	1	0	1	0

−10

1	0	1	1	0

+10 = 01010 $\xrightarrow{\text{2's Comp}}$ 10110

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2's Complement Number System

As a Weighted Number System

Weighted Number System:

→ with every "position", some weight is Associated.

Eg: binary NS

$$\underset{2^4}{1}\;\underset{2^3}{0}\;\underset{2^2}{1}\;\underset{2^1}{1}\;\underset{2^0}{0}$$

Decimal NS

$$\underset{10^2}{9}\;\underset{10^1}{0}\;\underset{10^0}{2}$$

Base-r NS $\quad r \geq 2$

"2's Comp System" is a Weighted Ns;

$$\underset{(-2^4)}{1}\ \overset{2^3}{0}\ \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{0} = -2^4 + 2^2 + 2^1$$

$$= -10$$

In 2's Comp system:

MSB

$$a_n \quad a_{n-1} \quad a_{n-2} \; \cdots \; a_1 \; a_0$$

$$-2^n \quad \underbrace{2^{n-1} \quad 2^{n-2} \quad \cdots \quad 2^1 \quad 2^0}$$

In 2's Comp System:

$$0\,1\,0\,1\,0 \longrightarrow 0\cdot(-2^4) + 1(2^3) + 0(2^2) + 1(2^1) + 0(2^0)$$

-2^4

$$= 10$$

$$\underbrace{0\,1\,1\,0\,1}_{} \longrightarrow 13$$

+ve

In 2's system

$$1\,0\,\underline{1\,1\,0\,1} \xrightarrow{\text{way 1}} -(010011) = -19$$

1 → -ve

1 → -2^5

way 2

$$-2^5 + 13 = -32 + 13$$

$$= -19$$

In 2's Comp system:

MSB

$-2^0 \leftarrow 1 \longrightarrow 1 \cdot (-2^0) = -1$ ✓

$10 \longrightarrow -2$

$100 \longrightarrow -4$

$1000 \longrightarrow -8$

-2^3

In 2's Comp system:

$101 \longrightarrow -3$

$1101 \longrightarrow -3$

$11101 \longrightarrow -3$

$111101 \longrightarrow -3$

$\boxed{1111101} \longrightarrow -3$

$-2^6 + 2^5 + 2^4$
$+ 2^3 + 2^2$
$+ 1$
$= -3$

In 2's Comp system:

$$\underbrace{1111111}_{\text{Do not matter}}\underline{101011} \longrightarrow -21$$

$$-2^5 + 11 = -32 + 11$$

$$\underbrace{11111111}\underbrace{101} \longrightarrow -2^2 + 1 = -3$$

In 2's Comp system:

$$0\underbrace{111111}_{} \longrightarrow 2^6 - 1 = 63$$

+ve

$$\underbrace{111111}_{}1 \longrightarrow -1$$

Non-weighted Number Systems:

Sign-magnitude
1's Comp system

$101011 \xrightarrow{\text{in SM}} -11$

$101011 \xrightarrow{\text{in 1's sys}} -(010100) = -20$

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Representations of Signed Binary Numbers

Range of each Number System for n-bits

3 bits → 8 Combinations

Ideal:

Should Represent 8 Different Numbers.

Unsigned ✓

2's Comp ✓

SM system, 1's Comp Sys

Represent only 7 Different Numbers.

Signed

			Unsigned	Sign-mag	1's Comp	2's Comp
0	0	0	0	0	0	0
0	0	1	1	1	1	1
0	1	0	2	2	2	2
0	1	1	3	3	3	3
1	0	0	4	−0	−3	−4
1	0	1	5	−1	−2	−3
1	1	0	6	−2	−1	−2
1	1	1	7	−3	−0	−1

$\underbrace{1\,0\,1}$ in 1's Comp system $\rightarrow -(010) = -2$

$1 \rightarrow$ -ve

in 2's Comp system $\rightarrow -(011) = -3$

Note: using 4 bits

In Sign mag	In 1's Comp sys
0 = 0 000	0 = 0 000
0 = 1 000	0 = 1 111

Note: using n bits

In Sign mag	In 1's Comp sys
$0 = 0\underbrace{000\ldots0}$	$0 = 0\underbrace{000\ldots0}$
$0 = 1\underbrace{000\ldots0}$	$0 = 1\underbrace{111\ldots1}$

Note:

In 2's Comp system, Every number has Unique Representation.

Range of Sign-magnitude Representation:

Using 3 bits: $\rightarrow$ $\boxed{-3 \text{ to } +3}$ 7 Numbers

one Combination wasted

min: $\underline{1}\ \underline{1}\ \underline{1} = -3$

max: $\underline{0}\ \underline{1}\ \underline{1} = +3$

sign

magnitude

Range of Sign-magnitude Representation:

n-bits :

min : $\underbrace{1}_{-ve}\ \underbrace{111\ldots1}_{\text{magnitude}} = -(2^{n-1}-1)$

max : $\underbrace{0}_{+ve}\ \underbrace{1111\ldots1}_{(n-1)\ 1's} = (2^{n-1}-1)$

$2^n : 1\underbrace{00 - - 0}_{n \text{ zeroes}}$

$2^4 = 16 = 1\underbrace{0000}$

$\underline{2^n - 1} = \underbrace{11 - - 1}_{n \text{ 1's}}$

$7 = 2^3 - 1 = 111$

$15 = 2^4 - 1 = \underbrace{1111}$

Range of Sign-magnitude Representation:

Using n-bits:

$$-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)$$

Eg: using 4 bits:

$$-(7) \text{ to } +7$$

Range of 1's Complement Representation:

Using 3 bits: $\boxed{-3 \text{ to } +3}$

-ve

min : 1 0 0 = -3

max : 0 1 1 = +3

+ve

7 Numbers

one Combination wasted.

Range of 1's Complement Representation:

using n-bits:

min: $1\underbrace{00---0}$ $= -(2^{n-1}-1)$

max: $0\underbrace{111--1}_{(n-1)\ 1's}$ $= +(2^{n-1}-1)$

In 1's Comp:

$$\underbrace{10000} = -(01111)$$

$$= -(2^4 - 1)$$

Range of 1's Complement Representation:

Using n-bits:

$$-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)$$

Eg: Using 5 bits:

$-(15)$ to $+15$

Range of 2's Complement Representation:

weighted Ns

for 3 bits:

min: 1 0 0 = -4 (-2^2)

max: 0 1 1 = $+3$

$\boxed{-4 \text{ to } +3} \Rightarrow$ 8 Numbers

Range of 2's Complement Representation:

for n bits:

min : 1 0 0 - - 0 = -2^{n-1}

max : 0 1 1 - - - 1 = $+(2^{n-1}-1)$

Range of 2's Complement Representation:

$$-2^{n-1} \text{ to } +(2^{n-1}-1)$$

Eg: for 4 bits:

-8 to $+7$ ✓

Range of all Representation:

using 4 bits

		Signed		
	Unsigned	Sign-mag	1's Comp	2's Comp
min:	0000 = 0	1111 = −7	1000 = −7	1000 = −8
max:	1111 = 15	0111 = +7	0111 = +7	0111 = +7
Range	0 to 15	−7 to +7	−7 to +7	−8 to +7

Range of all Representation:

using n bits

	Unsigned	Sign-mag	1's Comp	2's Comp
		Signed		
min:	0000...0	1111...1	1000...0	10000...0
max:	1111...1	0111...1	0111...1	01111...1
Range	0 to $2^n - 1$	$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$		-2^{n-1} to $+(2^{n-1}-1)$

Sign and Magnitude Numbers

In an n-bit sign and magnitude system, a number is represented by a sign bit, 0 for positive and 1 for negative, followed by $n - 1$ bits that represent the magnitude of the number. With $n - 1$ bits the magnitude can be 0 to $2^{(n-1)} - 1$. With the sign bit, numbers in the range $-(2^{(n-1)} - 1)$ to $+(2^{(n-1)} - 1)$ are represented including a positive and negative 0. This is illustrated in Table 1-1 for $n = 4$. For example, 0011 represents +3 and 1011 represents –3. Note that 1000 represents minus 0.

$+N$	Positive Integers (all systems)	$-N$	Negative Integers: Sign and Magnitude	Negative Integers: 2's Complement N^*	Negative Integers: 1's Complement $\overline{N}$
+0	0000	−0	1000	——	1111
+1	0001	−1	1001	1111	1110
+2	0010	−2	1010	1110	1101
+3	0011	−3	1011	1101	1100
+4	0100	−4	1100	1100	1011
+5	0101	−5	1101	1011	1010
+6	0110	−6	1110	1010	1001
+7	0111	−7	1111	1001	1000
		−8	——	1000	——

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—