Propositional Logic

Next Topic:

Converse, Inverse, Contrapositive

(Of Conditional Statements)

Website : https://www.goclasses.in/

Q :

Consider the conditional statement:

"If the weather is nice, then I'll wash the car." ; Y : $\boxed{w \rightarrow c}$

w c

Conversely

"If I'll wash the car, then the weather is nice." ; Converse of Y

"If the weather is not nice, then I'll not wash the car." ; Inverse of Y

"If I'll not wash the car, then the weather is not nice." ; Contrapositive of Y

Y: If w then c

Converse of Y:
If c then w

Contrapositive of Y:
If not c then not w

Inverse of Y:
If ¬w then ¬c

Y: $\boxed{w \rightarrow c}$

Converse of Y:
$c \rightarrow w$

Contrapositive of Y
$\neg c \rightarrow \neg w$

Inverse of Y
$\neg w \rightarrow \neg c$

Converse

Contrapositive

"If the weather is nice, then I'll wash the car."

$$p = \underline{the\ weather\ is\ nice} \qquad q = \underline{I'll\ wash\ the\ car}$$

Now the statement is: if p, then q, which can also be written as $p \to q$.

We can also make the negations, or "nots," of p and q. The symbolic version of "not p" is \(\sim p.

$$\sim p = the\ weather\ is\ not\ nice \qquad \sim q = I\ won't\ wash\ the\ car$$

Using these "nots" and switching the order of p and q, we can create three new statements.

$$Converse \qquad q \to p \qquad \underbrace{If\ I\ wash\ the\ car}_{q}, \underbrace{then\ the\ weather\ is\ nice}_{p}.$$

$$Inverse \qquad \sim p \to \sim q \qquad \underbrace{If\ the\ weather\ is\ not\ nice}_{\neg p}, \underbrace{then\ I\ won't\ wash\ the\ car}_{\neg q}.$$

$$Contrapositive \qquad \sim q \to \sim p \qquad \underbrace{If\ I\ don't\ wash\ the\ car}_{\neg q}, \underbrace{thentheweatherisnotnice}_{\neg p}.$$

Conditional Statement:

$P \rightarrow Q$

Converse of $P \rightarrow Q$

$Q \rightarrow P$

Inverse of $P \rightarrow Q$

$\neg P \rightarrow \neg Q$

Contrapositive of $P \rightarrow Q$

$\neg Q \rightarrow \neg P$

CONVERSE, CONTRAPOSITIVE, AND INVERSE:

The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.
The contrapostive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.
The inverse of $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$.

Converse of $q \rightarrow p$: $p \rightarrow q$

" " $\neg q \rightarrow \neg p$: $y \rightarrow x$

$x \rightarrow y$

$\neg p \rightarrow \neg q$

$\boxed{\text{Converse of } \alpha \rightarrow \beta} \equiv \beta \rightarrow \alpha$

$\boxed{\text{Inverse of } \alpha \rightarrow \beta} \equiv \neg \alpha \rightarrow \neg \beta$

CONVERSE, CONTRAPOSITIVE, AND INVERSE:

We can form some new conditional statements starting with a conditional statement $p \to q$. In particular, there are three related conditional statements that occur so often that they have special names.

The proposition $q \to p$ is called the converse of $p \to q$.

The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$.

The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

We will see that of these three conditional statements formed from $p \to q$, only the contrapositive always has the same truth value as $p \to q$.

Q :
What are the contrapositive, the converse, and the inverse of the conditional statement
"The home team wins whenever it is raining?"

$\underbrace{\text{The home team wins}}_{W}$ whenever $\underbrace{\text{it is raining}}_{R}$

: Y : $\boxed{R \to W}$

Y : $\underbrace{W \text{ if } R}$

If R, W $\equiv \boxed{R \to W}$

Converse $W \to R$

inverse $\neg R \to \neg W$

Contrapositive $\neg W \to \neg R$

Conditional Statement: “The home team wins whenever it is raining?”

Solution: Because “q whenever p” is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as

“If it is raining, then the home team wins.” $R \to W$

Consequently, the contrapositive of this conditional statement is

“If the home team does not win, then it is not raining.” $\neg W \to \neg R$

The converse is

“If the home team wins, then it is raining.” $W \to R$

The inverse is

“If it is not raining, then the home team does not win.” $\neg R \to \neg W$

Only the contrapositive is equivalent to the original statement.

Implication:

If this book is interesting, then I am staying at home.

- **Converse:** If I am staying at home, then this book is interesting.
- **Inverse:** If this book is not interesting, then I am not staying at home.
- **Contrapositive:** If I am not staying at home, then this book is not interesting.

1.9. Terminology. For the compound statement $p \to q$

- p is called the **premise, hypothesis,** or the **antecedent**.
- q is called the **conclusion** or **consequent**.
- $q \to p$ is the **converse** of $p \to q$.
- $\neg p \to \neg q$ is the **inverse** of $p \to q$.
- $\neg q \to \neg p$ is the **contrapositive** of $p \to q$.

We will see that the converse and the inverse are not equivalent to the original implication, but the contrapositive $\neg q \rightarrow \neg p$ is.

In other words, $p \rightarrow q$ and its contrapositive have the exact same truth values.

p	q	original statement $p \to q$	Contrapositive $\neg q \to \neg p$	Converse $q \to p$	Inverse $\neg p \to \neg q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

$\underbrace{P \to Q}_{} \equiv \underbrace{\neg Q \to \neg P}_{}$

Conditional Statement $\equiv$ its Contrapositive

Conditional Statement $\not\equiv$ its Converse

Converse of Condition Statement $\equiv$ Inverse of that statement

$\underbrace{P \rightarrow Q}$

false only when

$\left.\begin{array}{l} P = T \\ Q = F \end{array}\right\}$

$\neg Q \rightarrow \neg P$

false only when

$\neg Q = T$; $\neg P = F$

$\left\{\begin{array}{l} Q = F \\ P = T \end{array}\right.$

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$$

Direct proof

To prove

If p then q

p

$\left.\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix}\right\}$ Correct knowledge

q

Proof by Contraposition (Contrapositive)

To prove

If p then q

$\neg q$

$\left.\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix}\right\}$

$\neg p$

If p then q

|||

If $\neg q$ then $\neg p$

$p \to q$ — original Conditional Statement

Converse $q \to p$

Inverse $\neg p \to \neg q$

Contrapositive $\neg q \to \neg p$

p	q	original $p \to q$	Converse $q \to p$	Inverse $\neg p \to \neg q$	Contrapositive $\neg q \to \neg p$
F	F	T	T	T	T
F	T	T	F	F	T
T	F	F	T	T	F
T	T	T	T	T	T

Contrapositive of $\neg q \to \neg p$

$\equiv p \to q$

inverse

$q \to p$

converse

$\neg p \to \neg q$

Eg: What is the Converse of

$$\boxed{\neg p \to \neg q}$$

Answer:

$\neg q \to \neg p$

Converse

Contrapositive

$q \to p$

Inverse

$p \to q$

$\boxed{\text{Any}}$ Conditional Statement

$\equiv$ It's Contrapositive

DEFINITIONS

Let **p** and **q** be statement variables which apply to the following definitions.

Conditional:	The conditional of **q** by **p** is "If **p** then **q**" or "**p** implies **q**" and is denoted by **p** $\longrightarrow$ **q**. It is false when **p** is true and **q** is false; otherwise it is true.
Contrapositive:	The contrapositive of a conditional statement of the form "If **p** then **q**" is "If ~**q** then ~**p**". Symbolically, the contrapositive of **p** $\longrightarrow$ **q** is ~**q** $\longrightarrow$ ~**p**. A conditional statement is logically equivalent to its contrapositive.
Converse:	Suppose a conditional statement of the form "If **p** then **q**" is given. The converse is "If **q** then **p**." Symbolically, the converse of **p** $\longrightarrow$ **q** is **q** $\longrightarrow$ **p**. A conditional statement is **not** logically equivalent to its converse.
Inverse:	Suppose a conditional statement of the form "If **p** then **q**" is given. The inverse is "If ~**p** then ~**q**." Symbolically, the inverse of **p** $\longrightarrow$ **q** is ~**p** $\longrightarrow$ ~**q**. A conditional statement is **not** logically equivalent to its inverse.
Only if :	**p** only if **q** means "if not **q** then not **p**, " or equivalently, "if **p** then **q**."
Biconditional (**iff**):	The biconditional of **p** and **q** is "**p** if, and only if, **q**" and is denoted **p** $\longleftrightarrow$ **q**. It is true if both **p** and **q** have the same truth values and is false if **p** and **q** have opposite truth values.
Sufficient condition:	**p** is a sufficient condition for **q** means "if **p** then **q**."
Necessary condition:	**p** is a necessary condition for **q** means "if not **p** then not **q**."

Propositional Logic

Next Topic:

Logic-English Translation

Translating Between English and Propositional Logic

Website : https://www.goclasses.in/

Negation is easy to recognize because it almost always includes the word *not*, as in *it's not the case that E* or *it's not true that E*. Other instances include declarative expressions containing and embedded *not*. Example:

(1) a. Clint went to the Chatterbox Cafe.
b. Clint did not go to the Chatterbox Cafe.

If (1a) is translated as φ, then (1b) is translated as $\neg\varphi$.

Negation:

The sun is out. : p

The sun is not out. — $\neg p$

It is Not the case that the sun is out. — $\neg p$

It is Not true that the sun is out. — $\neg p$

It is false that the sun is out. — $\neg p$

The sun is out, is false. — $\neg p$

Negation of p

Negation:

p: This book is interesting.

This book is Not interesting. $\neg p$

It is Not the case that this book is interesting. $\neg p$

It is Not true that this book is interesting. $\neg p$

It is false that this book is interesting. $\neg p$

This book is interesting, is false. $\neg p$

Negation of p

Negation:

p

It is Not the case that p. $\neg p$

It is Not true that p. $\neg p$

It is false that p. $\neg p$

p is false. $\neg p$

Negations of p

The sun is out. : p

same

The sun is out, is true. p

We are students. q

same

We are students, is true. q

Delhi is capital of India. — True

Delhi is capital of India, is true. - True

Same

Kolkata is capital of India. — False

Kolkata is capital of India, is true. - false

Same

$\boxed{p \text{ is false}} \equiv \neg p$

$\boxed{p \text{ is True}} \equiv p$

$\neg\alpha$ is false $\equiv \alpha$

$\neg\alpha$ is True $\equiv \neg\alpha$

$\underbrace{\text{P is true}}_{p}$ and $\underbrace{\text{Q is true}}_{q}$. $\equiv p \wedge q$

Same

P and Q.

P is false But Q is true. $\equiv \neg P \wedge Q$

$\underbrace{\text{P is false}}_{\neg P}\ \underbrace{\text{But}}_{\wedge}\ \underbrace{\text{Q is true}}_{Q}$

Not P, But Q. $\equiv \neg P \wedge Q$

But $\equiv$ AND

P is false and Q is false. $\equiv$ $\neg P \wedge \neg Q$

$\neg P$ $\wedge$ $\neg Q$

Not P, and Not Q. $\equiv$ $\neg P \wedge \neg Q$

$P \rightarrow Q$

If P is true then Q is true.

P   Q

$P \rightarrow Q$

Same

If P then Q. $\equiv$ $P \rightarrow Q$

If P is false then Q is true.

$\neg P \quad\quad Q$

$\neg P \rightarrow Q$

Same

If Not P then Q. $\equiv \neg P \rightarrow Q$

Comma $\equiv$ AND

$P, Q \equiv P \wedge Q$

intelligent, smart $\equiv$ Int $\wedge$ smart

If P is false then Q is false. $\equiv \neg P \rightarrow \neg Q$

If $\neg P$ then $\neg Q$

If Not P then Not Q. $\equiv \neg P \rightarrow \neg Q$

Q is True $\equiv$ Q

How to write formulas/Expressions:

$(p \to q)$ is True $\equiv$ p is false

OR

q is True

$p \to q \equiv \neg p \vee q \equiv \overline{p} + q$

$P \oplus Q$ is True $\equiv$ $P=T, Q=F$

OR

$P=F, Q=T$

$$\boxed{P \oplus Q \equiv P\overline{Q} + \overline{P}Q}$$

$P \longleftrightarrow Q \equiv$ $P = T, Q = T$

OR

$P = F, Q = F$

$$\boxed{P \longleftrightarrow Q \equiv PQ + \bar{P}\bar{Q}}$$

$$q \quad \overline{P} \oplus Q \equiv PQ + \overline{P}\,\overline{Q}$$

$$\overline{P} \oplus Q \equiv PQ + \overline{P}\,\overline{Q} \equiv P \leftrightarrow Q$$

$$\boxed{\overline{P} \oplus Q \equiv P \leftrightarrow Q}$$

$$\overline{P} \leftrightarrow \overline{Q} \equiv \alpha\beta + \overline{\alpha}\,\overline{\beta}$$

α β

$$\equiv \overline{P}\,\overline{Q} + PQ$$

$$\boxed{\overline{P} \leftrightarrow \overline{Q} \equiv P \leftrightarrow Q}$$

p is false $\equiv \bar{p}$

p " True $\equiv p$

P	Q	R	α
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	F

$$\alpha \equiv \overline{P}QR + P\overline{Q}\,\overline{R} + PQ\overline{R}$$

P	Q	R	α
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	T

$$\alpha \equiv \bar{P}Q\bar{R} + \bar{P}QR + P\bar{Q}\bar{R} + PQ\bar{R} + PQR$$

$$P \oplus \overline{Q} \equiv PQ + \overline{P}\,\overline{Q}$$

$$P \oplus \overline{Q} \equiv P \leftrightarrow Q$$

$$\overline{P} \oplus \overline{Q} \equiv P\overline{Q} + \overline{P}Q$$

$$\overline{P} \oplus \overline{Q} \equiv P \oplus Q$$

Conjunction sometimes involves the word *and*, but not always. If E and F are English declaratives, then *E and F*, *E but F*, *E nonetheless F*, *E however F*, *E nevertheless F*, and *E moreover F* are all translated as PL conjunctions. For example:

(2) Pastor Ingqvist is a Lutheran but Father Wilmer is not.

If *Pastor Ingqvist is a Lutheran* is translated as φ and *Father Wilmer is a Lutheran* as ψ, then (2) is translated as $\varphi \wedge \neg\psi$.

NOTE : In Logic, we have -

AND = But = Although = Though = Even Though = However = Yet = Still = Moreover = Nevertheless = Nonetheless = Comma

P and Q $\equiv$ $P \wedge Q$

P is true and Q is true. $\equiv$ $P \wedge Q$

P But Q $\equiv$ $P \wedge Q$

P Yet Q $\equiv$ $P \wedge Q$

P Still Q $\equiv$ $P \wedge Q$

P However Q $\equiv$ $P \wedge Q$

P, Q $\equiv$ $P \wedge Q$

Although P, Q. $\equiv$ $P \wedge Q$

P though Q $\equiv$ $P \wedge Q$

P nonetheless Q $\equiv$ $P \wedge Q$

Disjunction usually involves the word *or* (but is inclusive in PL). Sentences like E *or* F and *either* E *or* F are translated using or as $\varphi \vee \psi$.

Implication:

If P then Q

Different Ways of Expressing $p \rightarrow q$

- **if p, then q**
- **if p, q**
- **q unless $\neg p$**
- **q if p**
- **q whenever p**
- **q follows from p**
- **p implies q**
- **p only if q**
- **q when p**
-
- **p is sufficient for q**
- **q is necessary for p**

- **a necessary condition for p is q**
- **a sufficient condition for q is p**

If P, then Q.
Q if P.
Q whenever P.
Q, provided that P.
Whenever P, then also Q.
P is a sufficient condition for Q.
For Q, it is sufficient that P.
Q is a necessary condition for P.
For P, it is necessary that Q.
P only if Q.

$$P \Rightarrow Q$$

Consider the statement:

$\underbrace{\text{Being 18+}}_{P}$ is necessary for $\underbrace{\text{joining}}_{Q}$ army. $\equiv Q \rightarrow P$

What does it mean?

If you $\underbrace{\text{are in army}}_{Q}$ then $\underbrace{\text{you are 18+}}_{P}$. ✓ $Q \rightarrow P$

If you are 18+ then you are in army. ✗

P is Necessary for Q ; $Q \to P$

$\equiv$ Q is Sufficient for P

$$\boxed{Q \to P}$$

Consider the statement:

Being Rajasthani is sufficient to be Indian. $\equiv P \rightarrow Q$

P Q

What does it mean?

If you are Rajasthani then you are Indian. ✓ $P \rightarrow Q$

P Q

If you are Indian then you are Rajasthani. ✗

α is sufficient for β

$\equiv \alpha \rightarrow \beta$

β is Necessary for α

$\equiv \alpha \rightarrow \beta$

equi-valent

NOTE :

We have seen in previous lectures that:

P is sufficient condition for Q. $\equiv$ $P \rightarrow Q$

P is necessary condition for Q $\equiv$ $Q \rightarrow P$

Q is sufficient for P $\equiv$ $Q \rightarrow P$

Consider the statement:

$\underbrace{\text{You can join army}}_{A}$ Only If $\underbrace{\text{You are 18+}}_{18+}$. : $A \rightarrow 18+$

What does it mean?

If $\underbrace{\text{you are in army}}_{A}$ then $\underbrace{\text{you are 18+}}_{18+}$. ✓ $A \rightarrow 18+$

If you are 18+ then you are in army. ✗

Consider the statement:

You can join army Only If You are 18+.

It means that

Without being 18+, you can't be in army.

Being 18+ is necessary condition for being in army. $\equiv$ $A \rightarrow 18+$

NOTE :

We have seen in previous lectures that:

P Only If Q.

So,

Without Q, P cannot happen.

So,

Q is necessary condition for P. $\equiv$ $p \rightarrow q$

NOTE :

We have seen in previous lectures that:

If P is necessary condition for Q then Q is sufficient condition for P.

So,

P is sufficient condition for Q. $\equiv p \rightarrow q$

Is same as

Q is necessary condition for P. $\equiv p \rightarrow q$

Consider the statement:

Being 18+ is necessary for joining army. $\equiv$ $p \rightarrow q$

q p

Means

Without being 18+, you cannot join army.

If you are not 18+, you cannot join army. $\equiv$ $\neg q \rightarrow \neg p$

$\neg q$ $\neg p$

Contrapositive

NOTE :

Q is necessary condition for P. : $p \rightarrow q$

Means

Without Q, P cannot happen.

So,

If Not Q then Not P.

$\neg q \rightarrow \neg p$

Contrapositive

Provided that $\equiv$ If

Provided that P, Q $\equiv P \to Q$

Q Provided that P $\equiv P \to Q$

If

P If Q

$\equiv$ If Q, P

$\equiv Q \rightarrow P$

$\boxed{P}$ If $\boxed{Q}$

$P \leftarrow Q$

P $\boxed{\text{only if}}$ Q

$P \rightarrow Q$

$\boxed{\alpha}$ only if $\boxed{\beta}$

$\alpha \rightarrow \beta$

$\boxed{\text{Only if } p, \ q} \equiv q \rightarrow p$

$\equiv q \ \boxed{\text{only if}} \ p$

$\longrightarrow$

p is Necessary for q

$\equiv$ q is sufficient for p

$\equiv q \rightarrow p$

Implication is used to capture conditionality. English sentences like *if E then F*, *F provided that E*, *assuming E, F*, *E only if F*, *F if E* and *F given E* are all translated using PL implication.

(3) Wally eats Powdermilk biscuits only if Evelyn makes them.

With *Wally eats Powdermilk biscuits* as φ and *Evelyn makes them* as ψ, we translate (3) into PL as $\varphi \rightarrow \psi$.

Biimplication makes a stronger claim than the conditional. It's used to translate English sentences of the form *E if and only if F*

There are some other common ways to express p ↔ q:

"*p* is necessary and sufficient for *q*"
"if *p* then *q*, and conversely"
"*p* iff *q*."

$p \leftrightarrow q$

$p \leftrightarrow q$

$p \leftrightarrow q$

If q then p

Neither p Nor q

$\equiv$ p is false and q is false

$\equiv$ $\overline{p} \wedge \overline{q}$

$\equiv$ $\overline{p \vee q}$

$\equiv$ $p \downarrow q$

- English sentences like *neither ... nor ...* are essentially a negated disjunction, a negative version of *either ... or ...*.

(4) Florian neither washed the car nor went to the mercantile.

With *Florian washed the car* as φ and *Florian went to the mercantile* as ψ, we translate (4) as $\neg(\varphi \vee \psi)$.

- Sometimes we also negate conjunctions in English. This kind of sentence usually takes the form *it's not true that both E and F* or *not E and F*.

(5) It's not true that Clint owns both a Ford and a Chevy dealership.

Given that *Clint owns a Ford dealership* is translated as φ and *Clint owns a Chevy dealership* is translated as ψ, a translation of (5) would be $\neg(\varphi \wedge \psi)$.

$$\neg(p \wedge q) \equiv \overline{p} \vee \overline{q}$$

Some Sample Propositions

a: There is a velociraptor outside my apartment.

b: Velociraptors can open windows.

c: I am in my apartment right now.

d: My apartment has windows.

e: I am going to be eaten by a velociraptor

I won't be eaten by a velociraptor if there isn't a velociraptor outside my apartment.

Some Sample Propositions

a: There is a velociraptor outside my apartment.

$\neg e$ if $\neg a$

b: Velociraptors can open windows.

c: I am in my apartment right now.

$\neg a \rightarrow \neg e$ ≡

d: My apartment has windows.

$e \rightarrow a$

e: I am going to be eaten by a velociraptor

$\neg a$

$-e$ I won't be eaten by a velociraptor if there isn't a velociraptor outside my apartment.

"p if q"

translates to

$$q \rightarrow p$$

It does **not** translate to

$$p \rightarrow q$$

If q, p

$q \rightarrow p$

Precedence is for logical formulas

NOT for English statement.

intuition / expected meaning

(man की बात)

$P \to Q \wedge R$

$\equiv P \to (Q \wedge R)$

Some Sample Propositions

a: There is a velociraptor outside my apartment.

b: Velociraptors can open windows.

c: I am in my apartment right now.

d: My apartment has windows.

e: I am going to be eaten by a velociraptor

If there is a velociraptor outside my apartment, but it can't open windows, I am not going to be eaten by a velociraptor.

$\to a$

$\to \neg b$

$\neg e \leftarrow$

If (a and ¬b) then ¬e ✓

$\equiv \quad a\bar{b} \rightarrow \bar{e}$

If a then ¬b, ¬e ✗

"p, but q"

translates to

$$p \land q$$

Some Sample Propositions

a: There is a velociraptor outside my apartment.

b: Velociraptors can open windows.

c: I am in my apartment right now.

d: My apartment has windows.

e: I am going to be eaten by a velociraptor

I am only in my apartment when
there are no velociraptors outside.

Some Sample Propositions

a: There is a velociraptor outside my apartment.

b: Velociraptors can open windows.

c: I am in my apartment right now.

d: My apartment has windows.

e: I am going to be eaten by a velociraptor

I am only in my apartment when there are no velociraptors outside.

c

I am in my apartment only when there is no velo outside. $\neg a$

$\equiv a \to \neg c$

c only when $\neg a \equiv c \to \neg a$

$\neg a$ is Necessary for $c \equiv c \to \neg a$

c " sufficient " $\neg a \equiv c \to \neg a$

“p only when q”

translates to

$$p \rightarrow q$$

HW

EXERCISE 1.8.1. *Which of the following statements are equivalent to "If x is even, then y is odd"? There may be more than one or none.*

(1) y is odd only if x is even.
(2) x is even is sufficient for y to be odd.
(3) x is even is necessary for y to be odd.
(4) If x is odd, then y is even.
(5) x is even and y is even.
(6) x is odd or y is odd.

Hw

EXERCISE 1.10.1. *p is the statement "I will prove this by cases", q is the statement "There are more than 500 cases," and r is the statement "I can find another way."*

1. *State $(\neg r \vee \neg q) \rightarrow p$ in simple English.*
2. *State the converse of the statement in part 1 in simple English.*
3. *State the inverse of the statement in part 1 in simple English.*
4. *State the contrapositive of the statement in part 1 in simple English.*

(Propositional Logic-English Translation)

Next Sub-Topic:

Unless Word in English

Website : https://www.goclasses.in/

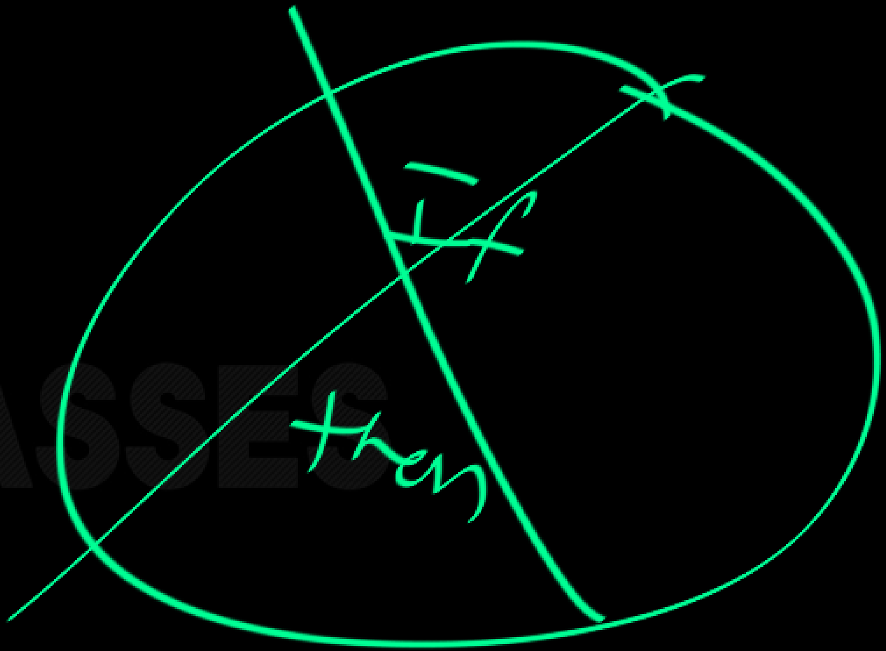
Unless in English:

There are numerous ways to express conditionals in English.
We have already seen several conditional-forming expressions, including 'if', 'provided', 'only if'.

In the present section, we consider a further conditional-forming expression – 'unless'.

Provide that P, Q

When P, Q



Of course there are other grammatical constructions that mean exactly the same thing as $P \Rightarrow Q$. Here is a summary of the main ones.

$$\left.\begin{array}{l}\text{If } P\text{, then } Q.\\ Q \text{ if } P.\\ Q \text{ whenever } P.\\ Q\text{, provided that } P.\\ \text{Whenever } P\text{, then also } Q.\\ P \text{ is a sufficient condition for } Q.\\ \text{For } Q\text{, it is sufficient that } P.\\ Q \text{ is a necessary condition for } P.\\ \text{For } P\text{, it is necessary that } Q.\\ P \text{ only if } Q.\end{array}\right\} \quad P \Rightarrow Q$$

Unless in English:

There are a lot of ways of saying "if P then Q" in English(in previous slide). We can even say it without using "if…then…"

Here is an example: (imagine a stubborn child "Jen")
Jen won't go to the party UNLESS Mary goes to the party.

P Q

If Not Q , P

Jen won't go to the party unless Mary goes to the party.

P $\qquad$ Q

means

If Mary doesn't go to the party then Jen won't go to the party.

$\neg Q$ $\qquad$ P

$\neg Q \rightarrow P$

Jen won't go to the party unless (Mary goes to the party).

$\underbrace{\text{Jen won't go to the party}}_{p}$ unless ($\underbrace{\text{Mary goes to the party}}_{q}$).

means

If not-(Mary goes to the party) then Jen won't go to the party.

If not-q then p

$\neg q \rightarrow p$

Unless in English:

(again imagine a stubborn child)

I won't study UNLESS You complete my demand.

Unless in English:

(again imagine a stubborn child)

I won't study UNLESS You complete my demand.

p q

If $\neg q$ then p

$\neg q \rightarrow p$

Russia :

we won't stop war unless you don't Join NATO.

p ≡ intension

q

Just a chance (not a guarantee)

$\equiv$ If $\neg q$ then p

$\equiv \neg q \rightarrow p$

Russia:

We won't stop war unless you don't Join NATO.

Does it mean that

If Ukrain doesn't Join NATO, Rus will stop war.

Unless in English:

I'll eat cake, unless I'm full.

$\underbrace{\text{I'll eat cake}}_{P}$, unless $\underbrace{\text{I'm full}}_{Q}$.

If $\neg Q$ then P : $\neg Q \to P$

Here the idea is that you fully intend to eat cake and you will do so if you are not full;
that is, if you are not full, then you'll eat cake,
or "¬Full → Eat Cake".

P unless Q

intension

Just a chance

$\equiv$ If not

If not Q , P $\equiv$ $\boxed{\neg Q \to P}$

You won't crack GATE unless you appear in GATE.

p

q

$q \rightarrow \neg p$ ✗

$\neg q \rightarrow p$ ✓

Unless $\equiv$ $\boxed{\text{If not}}$

$\boxed{\text{provides that}}$ $\equiv$ If

Unless in English:

P unless Q $\equiv$ P If not Q

$\equiv \neg Q \rightarrow P$

Unless in English:

'unless' is equivalent to 'if not'

"P unless Q" is same as "P if not Q"

Here, 'if not' is short for 'if it is not true that'. Notice that this principle applies when 'unless' appears at the beginning of the statement, as well as when it appears in the middle of the statement.

Unless in English:

'unless' is equivalent to 'if not'

"Unless P, Q" is same as "If not P then Q"

Here, 'if not' is short for 'if it is not true that'. Notice that this principle applies when 'unless' appears at the beginning of the statement, as well as when it appears in the middle of the statement.

Unless p, q

If not p, q

$\neg p \to q$

$\mathcal{A}$ unless $\mathcal{B}$ is equivalent to $\mathcal{A}$ if not $\mathcal{B}$ which is symbolized $\sim\mathcal{B} \to \mathcal{A}$	unless $\mathcal{A}$, $\mathcal{B}$ is equivalent to if not $\mathcal{A}$, then $\mathcal{B}$ which is symbolized $\sim\mathcal{A} \to \mathcal{B}$

p unless q ≡ p if not q

≡ ¬q → p

≡ q + p ≡ p OR q

$\boxed{\text{Unless} \equiv \text{OR}}$

Unless ≡ OR

Unless ≡ If not

P unless Q ≡ P or Q

≡ If not Q, P ≡ $P \vee Q$

≡ $\neg Q \rightarrow P$

NOTE: Don't Apply Precedence in English Statements.

we understand the feeling

$\neg > \wedge > \vee > \rightarrow > \leftrightarrow$

for logical operators

Unless is also used with conditional statements.

Example :

If you don't leave early, you'll miss the train unless train is late.

Unless is also used with conditional statements.

L : leave early
M : miss Train
late : Train late

Example :

If you don't leave early, you'll miss the train unless train is late.

2

main intention / statement

Just a chance

(If $\neg L$ then M) unless late $\equiv (\neg L \to M) + \text{late}$

$\neg \text{late} \rightarrow (\neg L \rightarrow m)$

$\neg \text{late} \rightarrow (\neg L \rightarrow m)$

$\equiv (\neg L \rightarrow m) \vee \text{Late}$

equivalent

Unless is also used with conditional statements.

Example :

If you study well, you will crack GATE exam unless you make silly mistakes.

Unless is also used with conditional statements.

Example :

If you study well, you will crack GATE exam unless you make silly mistakes.

Unless is also used with conditional statements.

English Statement: If p then q unless r.

(If p then q) unless r

$\equiv (p \to q) \vee r$

$\equiv \neg r \to (p \to q)$

- One of the more confusing English words to translate is *unless*. This word expresses a dependency between two propositions, but one which is not always as straightforward as the conditional with *if* ... *then* .... For example:

  (6) Myrtle doesn't cook a Walleye unless Clint catches it.

  If *Myrtle cooks a Walleye* is φ and *Clint catches a Walleye* is ψ, then (6) can be translated as either $\neg\psi \to \neg\varphi$, $\varphi \to \psi$, or $\neg\varphi \vee \psi$.

$q \rightarrow p$: If q then p

$\underbrace{p \rightarrow q}$ $\equiv$ $\underbrace{q \leftarrow p}$

Condition

$\alpha \longrightarrow \beta$: If α then β

$\alpha \longleftarrow \beta$: If β then α

$P \to Q$: whenever P is True then Q is True

P True $\longrightarrow$ Q True

P false — Q True
or
Q false

$x \rightarrow y$

i) x is sufficient for y.

ii) y is Necessary for x.

$y \rightarrow x$

i) y is suff for x

ii) x is Necessary for y.