Revision & Practice

Expressing Boolean Functions using Minterms, Maxterms

Digital Logic :

Recap :

Basics So Far

Three Representations of Logic Functions

	AND	OR	NOT
1. Logic Expression	X.Y	X+Y	X' $\overline{X}$ ~X !X

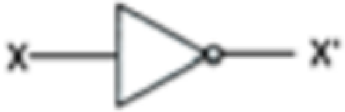
2. Truth Table

X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

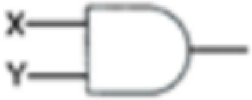
X	X'
0	1
1	0

3. Circuit Diagram / Schematic



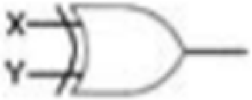
Logic Gate Symbols

- AND denoted by X.Y

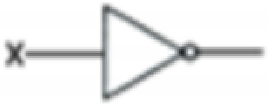


- OR denoted by X + Y

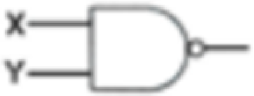
- XOR denoted by $X \oplus Y$



- NOT denoted by X' or X



- NAND denoted by $\overline{X.Y}$



- NOR denoted by $\overline{X + Y}$

- XNOR denoted by $\overline{X \oplus Y}$

Variables and Literals

We start with the idea of a Boolean variable. It is a simple variable that can take one of only two values: 0 (False) or 1 (True).

Following standard digital design practice, we use the values 0 and 1.

Following standard teaching practice, we denote all Boolean variables by single letters; normally "A", "B", "C", "D", or "W", "X", "Y", "Z".

A literal is either a Boolean variable or its complement.

Literals based on the variable X: X and $\overline{X}$.

Literals based on the variable Y: Y and $\overline{Y}$.

NOTE: X and $\overline{X}$ represent the same variable, but they are not the same literal.

X and $\overline{Y}$ represent different variables.

Digital Logic :

Coming Back :

Standard Forms of Boolean Expressions

Minterm, Maxterm, SOP, POS

$$f = \sum m(f=1) \rightarrow \text{1-minterms}$$

$$\overline{f} = \sum m(\overline{f}=1) = \sum m(f=0)$$

1-minterms of $\overline{f}$

0-minterms of f

- **Any Boolean function can be expressed as a sum (OR) of its 1-minterms:**

$$F(\text{list of variables}) = \Sigma(\text{list of 1-minterm indices})$$

order matters

Example:

Row Numbers	Variable Values x	y	z	Function Values F_1	F_1'
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	0

$$\begin{aligned} F_1(x, y, z) &= \Sigma(3, 5, 6, 7) \\ &= m_3 + m_5 + m_6 + m_7 \\ &= x'yz + xy'z + xyz' + xyz \end{aligned}$$

$$\begin{aligned} F_1'(x, y, z) &= \Sigma(0, 1, 2, 4) \\ &= m_0 + m_1 + m_2 + m_4 \\ &= x'y'z' + x'y'z + x'yz' + xy'z' \end{aligned}$$

$$f_1(a,b,c) = \sum m(1,3,4)$$

$$= \boxed{\bar{a}\bar{b}c + \bar{a}bc + a\bar{b}\bar{c}}$$

$\underset{\bar{a}}{0}\,\underset{\bar{b}}{0}\,\underset{c}{1}$

$\underset{\bar{a}}{0}\,\underset{b}{1}\,\underset{c}{1}$

$$f_2(b,c,a) = \sum m(1,3,4)$$

$$= \bar{b}\bar{c}a + \bar{b}ca + b\bar{c}\bar{a}$$

$$\underbrace{f_1 \neq f_2}$$

- **Maxterms can be defined as the complement of minterms:**

$$M_i = m_i' \text{ and } M_i' = m_i$$

x	y	z	Maxterms	Designation
0	0	0	$x + y + z$	M_0
0	0	1	$x + y + z'$	M_1
0	1	0	$x + y' + z$	M_2
0	1	1	$x + y' + z'$	M_3
1	0	0	$x' + y + z$	M_4
1	0	1	$x' + y + z'$	M_5
1	1	0	$x' + y' + z$	M_6
1	1	1	$x' + y' + z'$	M_7

Maxterms for Three Binary Variables

$f = \prod m(f=0)$ → false-maxterms

0- ''

$\overline{f} = \prod m(\overline{f}=0) = \prod m(f=1)$

0-maxterms of $\overline{f}$ = 1-maxterms of f

- Any Boolean function can be expressed as a product (AND) of its 0-maxterms:

$$F(\text{list of variables}) = \Pi(\text{list of 0-maxterm indices})$$

Example:

Row Numbers	Variable Values x	y	z	Function Values F_1	F_1'
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	0

Equation Table

$$\begin{aligned} F_1(x, y, z) &= \Pi(0, 1, 2, 4) \\ &= M_0 M_1 M_2 M_4 \\ &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \end{aligned}$$

$$\begin{aligned} F_1'(x, y, z) &= \Pi(3, 5, 6, 7) \\ &= M_3 M_5 M_6 M_7 \\ &= (x + y' + z')(x' + y + z')(x' + y' + z)(x' + y' + z') \end{aligned}$$

$$f(A,B,C) = \underbrace{ABC}_{\text{minterm}} + \underbrace{\overset{1}{A}\,\overset{1}{B}\,\overset{0}{\bar{C}}}_{\text{minterm}} + \underbrace{\overset{0}{\bar{A}}\,\overset{1}{B}\,\overset{1}{C}}_{\text{minterm}}$$

$f(A,B,C)$ → In Canonical SOP

= Sum of minterm (True minterms)

$$f(A,B,C) = m_7 + m_6 + m_3$$

$$f(a,b,c) = \sum(3,6,7)$$

$$f(A,B,C) = ABC + AB\overline{C} + \overline{A}BC$$

Row Number	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

– *f* can be written as the sum of row numbers having TRUE minterms

$$f = \sum(3,6,7)$$

Sum-of-minterms and Product-of-maxterms

Two mechanical ways to translate a function's truth table into an expression:

X	Y	**Minterm**	**Maxterm**	F
0	0	$\overline{X}\,\overline{Y}$	$X+Y$	0
0	1	$\overline{X}\,Y$	$X+\overline{Y}$	1
1	0	$X\,\overline{Y}$	$\overline{X}+Y$	1
1	1	$X\,Y$	$\overline{X}+\overline{Y}$	0

The sum of the minterms where the function is 1:

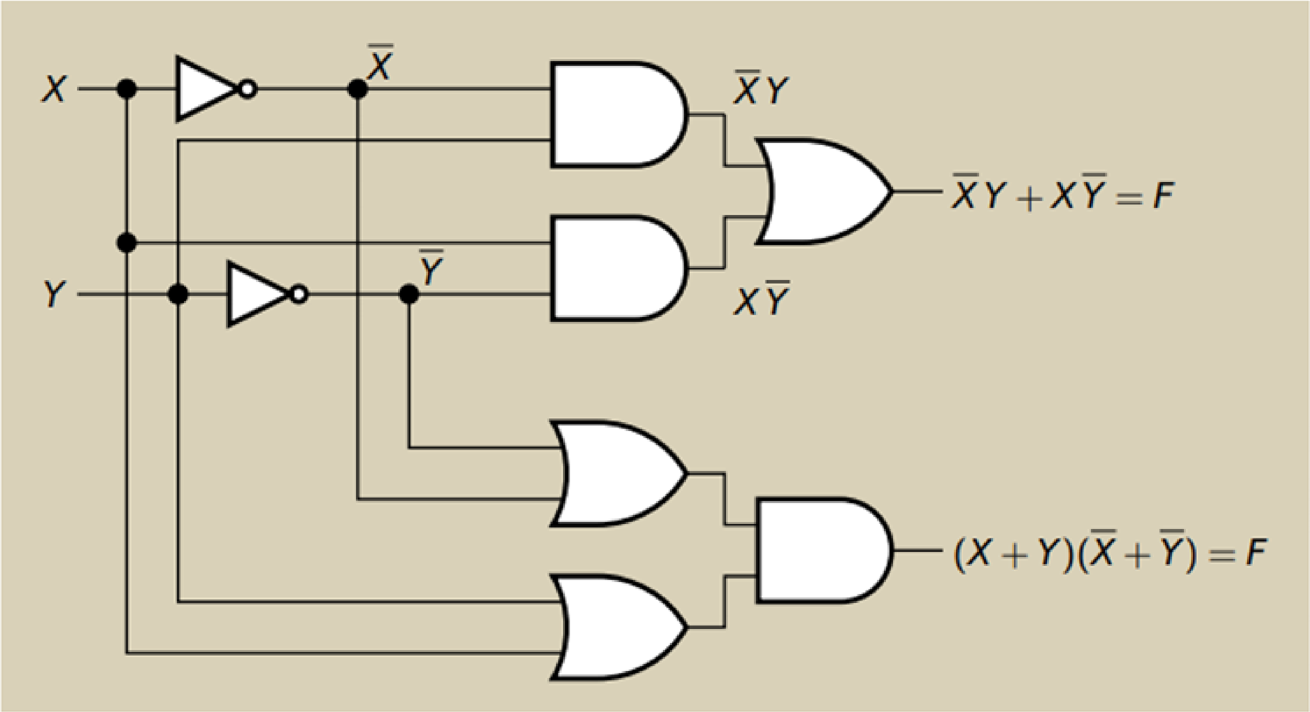
$$F = \overline{X}\,Y + X\,\overline{Y}$$

The product of the maxterms where the function is 0:

$$F = (X+Y)(\overline{X}+\overline{Y})$$

Expressions to Schematics

$$F = \overline{X}Y + X\overline{Y} = (X+Y)(\overline{X}+\overline{Y})$$



Minterms and Maxterms: Another Example

The minterm and maxterm representation of functions may look very different:

X	Y	**Minterm**	**Maxterm**	F
0	0	$\overline{X}\,\overline{Y}$	$X+Y$	0
0	1	$\overline{X}\,Y$	$X+\overline{Y}$	1
1	0	$X\,\overline{Y}$	$\overline{X}+Y$	1
1	1	$X\,Y$	$\overline{X}+\overline{Y}$	1

The sum of the minterms where the function is 1:

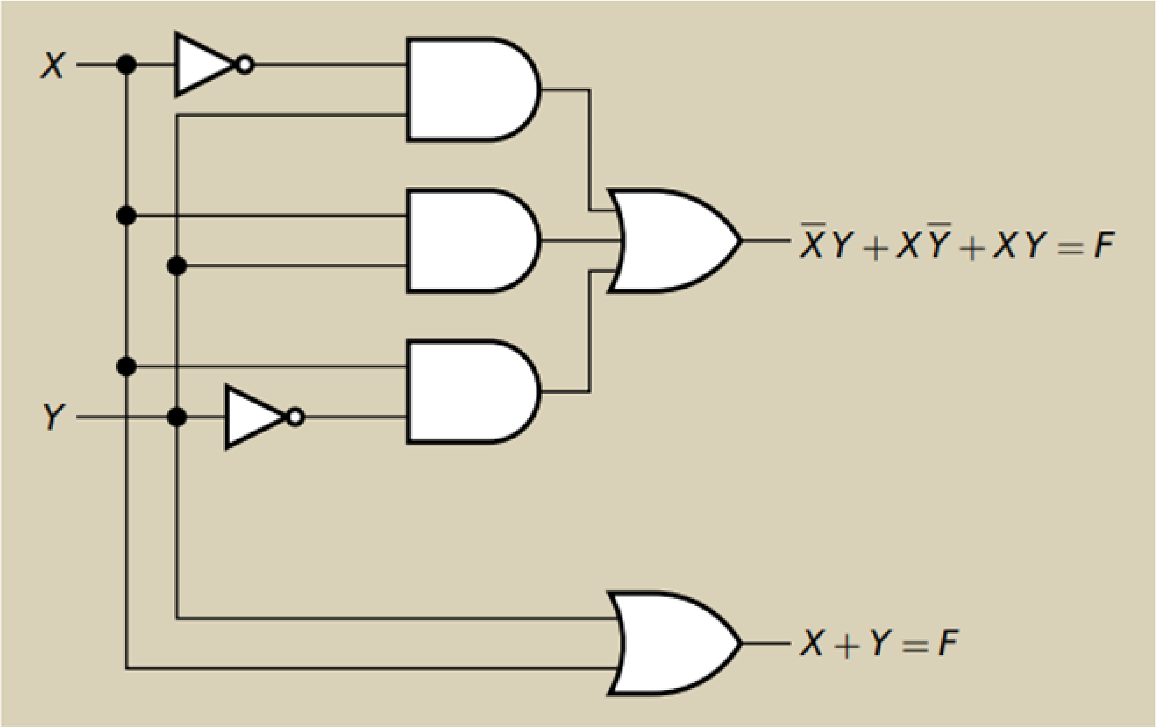
$$F = \overline{X}\,Y + X\,\overline{Y} + X\,Y$$

The product of the maxterms where the function is 0:

$$F = (X + Y)$$

Expressions to Schematics 2

$$F = \overline{X}Y + X\overline{Y} + XY = X + Y$$



Propositional logic = Boolean Algebra

Normal forms (NF)

- DNF = SOP
- CNF = POS

SOP ✓

POS ✓

CNF : Conjunctive NF.

DNF : Disjunctive Normal form

Propositional logic

Canonical DNF = CSOP

(Principle DNF)

Canonical CNF = CPOS

Q:

Express the Boolean function $F = A + B'C$ as a sum of minterms.

$$F = (A) + (\bar{B}C)$$ — In SOP ✓

Convert into Sum of $\boxed{\text{minterms}}$

Canonical SOP

$$= (A \cdot 1 \cdot 1) + (1 \cdot \bar{B}C)$$

$$= \left(A(B+\bar{B})(C+\bar{C}) \right) + (A+\bar{A})\bar{B}C$$

$B+\bar{B}=1$

Q: $a + a = a$ ✓ SOP ✓ $\alpha + \alpha = \alpha$ ✓

Express the Boolean function $F = A + B'C$ as a sum of minterms.

SOP (Canonical SOP)?

$B'C$ → Not a minterm

minterms → Contains All Variables (Product term)

Method 1: (Inefficient)

$$F = (A \cdot 1 \cdot 1) + (1 \cdot \bar{B} C)$$

$$= A \cdot (B + \bar{B}) \cdot (C + \bar{C}) + (A + \bar{A}) \bar{B} C$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$f = \Sigma(7, 6, 5, 4, 1)$$ ✓

Q :

Express the Boolean function $F = A + B'C$ as a sum of minterms.

Not a minterm

$F_{(A,B,C)} = F = (A) + (B'C)$ SOP

over

$= 1\,\_\,\_$

$\_\,0\,1$

$f = \sum(1,4,5,6,7)$

$\begin{cases} 100 - m_4 \\ 101 - m_5 \\ 110 - m_6 \\ 111 - m_7 \end{cases}$

$\begin{cases} 001 - m_1 \\ 101 - m_5 \end{cases}$

Express the Boolean function $F = A + B'C$ as a sum of minterms.

$$F = A'B'C + AB'C + AB'C + ABC' + ABC$$
$$= m_1 + m_4 + m_5 + m_6 + m_7$$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

Express the Boolean function $F = xy + x'z$ as a Sum of minterm

method 1: (Inefficient)

$F = (xy) + (x'z) \longrightarrow$ In sop ✓

$= xy(z+z') + x'z(y+\bar{y})$

$= xyz + xyz' + x'yz + \bar{x}\bar{y}z$ — In csop

$F = \Sigma m(7, 6, 3, 1)$

Express the Boolean function $F = xy + x'z$ as a sum of minterm

method 2 : (efficient)

$F(x,y,z) = xy + x'z$ → In SOP ✓

$xy\_$

$\begin{cases} 110 - m_6 \\ 111 - m_7 \end{cases}$

0_1

$\begin{cases} 001 - m_1 \\ 011 - m_3 \end{cases}$

Sum terms

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

$f = \sum m(1, 3, 6, 7)$

$000 = (x+y+z)$

$f = \prod m(0, 2, 4, 5) = (M_0)(M_2)(M_4)(M_5)$

$= (x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$

Express the Boolean function $F = \underbrace{xy + x'z}$ as a product of maxterms.

In SOP

find minterms

Best method;

Remaining will be (false) maxterms

$$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')$$
$$= M_0 M_2 M_4 M_5$$

CPOS

A convenient way to express this function is as follows:

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

Summary of SOP and POS

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(1,3,5,6)$	$\sum m(0,2,4,7)$
Product of sums (POS)	$\prod m(0,2,4,7)$	$\prod m(1,3,5,6)$

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(1,3,5,6)$	$\sum m(0,2,4,7)$
Product of sums (POS)	$\prod M(0,2,4,7)$	$\prod M(1,3,5,6)$

Converting between canonical forms

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

DeMorgans

Minterms

- A minterm can be defined as a product term that is 1 in exactly one row of the truth table.
- *n* variable minterms are often represented by *n*-bit binary integers.

- The following table gives the minterms for a **three-input** system

			m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
A	**B**	**C**	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$	$\overline{A}B\overline{C}$	$\overline{A}BC$	$A\overline{B}\,\overline{C}$	$A\overline{B}C$	$AB\overline{C}$	ABC
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Minterms

- A minterm can be defined as a product term that is 1 in exactly one row of the truth table.
- *n* variable minterms are often represented by *n*-bit binary integers.

- How to associate minterms with integers?
  - State an ordering on the variables
  - Form a binary number
    - Set bit *i* of the binary number to 1 if the i^{th} variable appears in the minterm in an uncomplemented form
    - Set bit *i* to 0 if the variable appears in the complemented form.

Minterm Examples

- Assume a 3-variable expression,

$$F(x,y,z) = \overline{x}\,\overline{y}\,\overline{z} + \overline{x}yz + xyz$$

$$\overline{x}\,\overline{y}\,\overline{z} = minterm\,000 = m_{000} = m_0$$
$$\overline{x}yz = minterm\,011 = m_{011} = m_3$$
$$xyz = minterm\,111 = m_{111} = m_7$$

$$\begin{aligned} F(x,y,z) &= \overline{x}\,\overline{y}\,\overline{z} + \overline{x}yz + xyz \\ &= m_0 + m_3 + m_7 \\ &= \sum(m_0, m_3, m_7) \\ &= \sum(0,3,7) \end{aligned}$$

Maxterms

- A maxterm can be defined as a sum term that is 0 in exactly one row of the truth table.
- *n* variable maxterms are also represented by *n*-bit binary integers.
- How to associate maxterms with integers?
  - State an ordering on the variables
  - Form a binary number
    - Set bit *i* of the binary number to 0 if the i^{th} variable appears in the maxterm in an uncomplemented form
    - Set bit *i* to 1 if the variable appears in the maxterm in the complemented form.

- The following table gives the maxterms for a **three-input** system

A	B	C	M_0 $A+B+C$	M_1 $A+B+\overline{C}$	M_2 $A+\overline{B}+C$	M_3 $A+\overline{B}+\overline{C}$	M_4 $\overline{A}+B+C$	M_5 $\overline{A}+B+\overline{C}$	M_6 $\overline{A}+\overline{B}+C$	M_7 $\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

Maxterm Examples

- Assume a 3-variable expression,

$$F(x,y,z) = (x+y+z)(x+y+\bar{z})(\bar{x}+y+z)$$

$$\begin{aligned}
x+y+z &= max\,term\,000 &= M_{000} &= M_0 \\
x+y+\bar{z} &= max\,term\,001 &= M_{001} &= M_1 \\
\bar{x}+y+z &= max\,term\,100 &= M_{100} &= M_4
\end{aligned}$$

$$\begin{aligned}
F(x,y,z) &= (x+y+z)(x+y+\bar{z})(\bar{x}+y+z) \\
&= M_0 \cdot M_1 \cdot M_4 \\
&= \prod(M_0, M_1, M_4) \\
&= \prod(0,1,4)
\end{aligned}$$

Summary of Minterms and Maxterms

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

A Sample Three Variable Function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6)$$
$$= \sum(1, 4, 5, 6)$$

$$f(x_1, x_2, x_3) = \prod(M_0, M_2, M_3, M_7)$$
$$= \prod(0, 2, 3, 7)$$

Some Definitions

- The ***canonical sum of products (CSOP)*** form of an expression refers to rewriting the expression as a sum of minterms.
  - *Examples for 3-variables:* $\bar{a}bc + abc$ is a CSOP expression; $\bar{a}b + c$ is not.
- The ***canonical product of sums (CPOS)*** form of an expression refers to rewriting the expression as a product of maxterms.
  - *Examples for 3-variables:* $(\bar{a}+b+c)(a+b+c)$ is a CPOS expression; $(\bar{a}+b)c$ is not.
- There is a close correspondence between the truth table and minterms and maxterms.

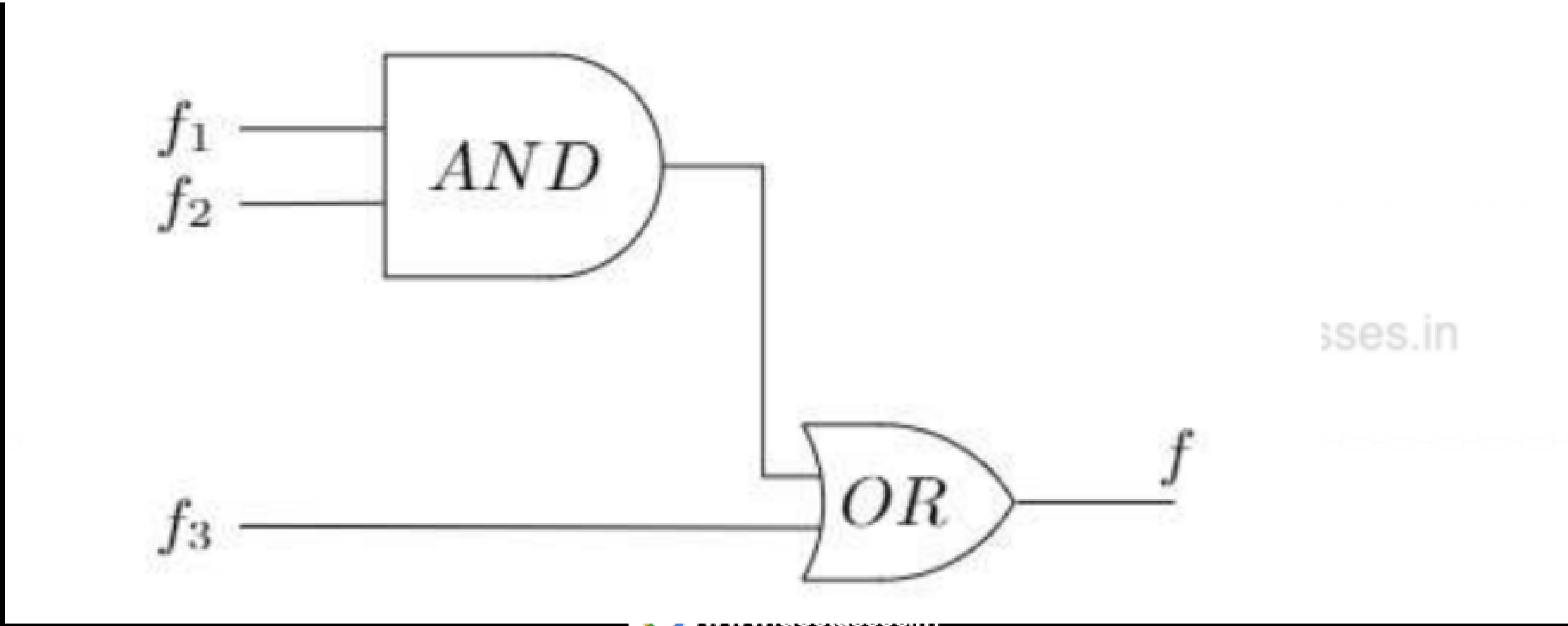
4.24.1 Min Sum Of Products Form: GATE CSE 1988 | Question: 2-v top

https://gateoverflow.in/91685

Three switching functions f_1, f_2 and f_3 are expressed below as sum of minterms.

- $f_1(w, x, y, z) = \sum 0, 1, 2, 3, 5, 12$
- $f_2(w, x, y, z) = \sum 0, 1, 2, 10, 13, 14, 15$
- $f_3(w, x, y, z) = \sum 2, 4, 5, 8$

Express the function f realised by the circuit shown in the below figure as the sum of minterms (in decimal notation).



4.24.1 Min Sum Of Products Form: GATE CSE 1988 | Question: 2-v top

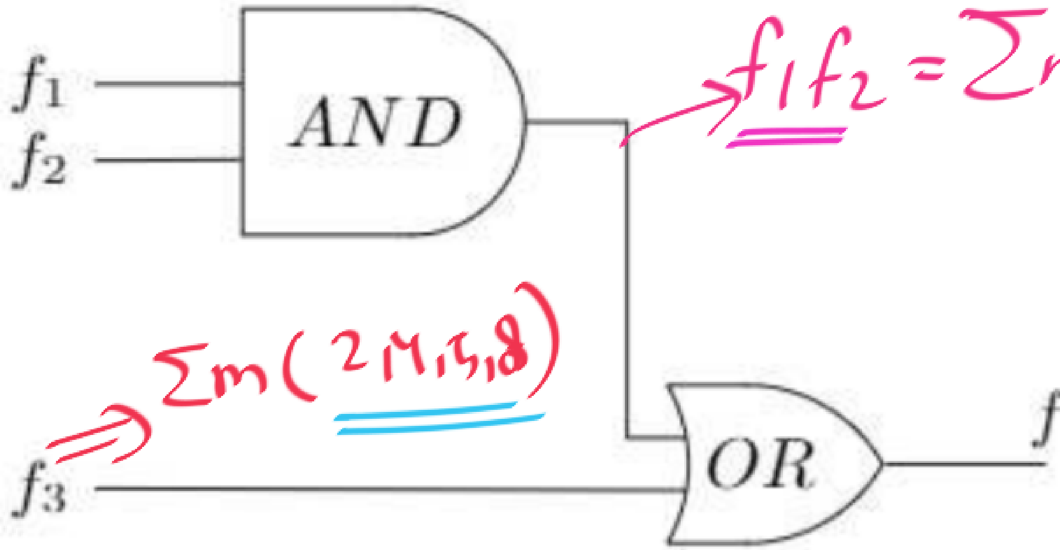
https://gateoverflow.in/91685

Three switching functions $f_1,\ f_2$ and f_3 are expressed below as sum of minterms.

- $f_1(w,x,y,z) = \sum 0,1,2,3,5,12$
- $f_2(w,x,y,z) = \sum 0,1,2,10,13,14,15$
- $f_3(w,x,y,z) = \sum 2,4,5,8$

$f_1 f_2 = 1$ iff $f_1 = 1, f_2 = 1$

Express the function f realised by the circuit shown in the below figure as the sum of minterms (in decimal notation).



$f_1 f_2 = \sum m(0,1,2 \quad)$

$f_3 \Rightarrow \sum m(2,4,5,8)$

$f = f_3 + (f_1 f_2)$

$= \sum m(0,1,2,4,5,8)$

Note:

$$f_1 = \sum m(\_ \underset{P}{\_} \_ \_)$$

$$f_2 = \sum m(\_ \_ \underset{Q}{\_} \_)$$

$$f_1 f_2 = \sum m(\text{intersection } P \cap Q)$$

$$f_1 + f_2 = \sum m(\text{union } P \cup Q)$$

4.24.5 Min Sum Of Products Form: GATE CSE 2005 | Question: 18 top

https://gateoverflow.in/1354

The switching expression corresponding to $f(A, B, C, D) = \Sigma(1, 4, 5, 9, 11, 12)$ is:

A. $BC'D' + A'C'D + AB'D$

B. $ABC' + ACD + B'C'D$

C. $ACD' + A'BC' + AC'D'$

D. $A'BD + ACD' + BCD'$

4.24.5 Min Sum Of Products Form: GATE CSE 2005 | Question: 18 top

https://gateoverflow.in/1354

The switching expression corresponding to $f(A,B,C,D) = \Sigma(1,4,5,9,11,12)$ is:

A. $BC'D' + A'C'D + AB'D$

B. $ABC' + ACD + B'C'D$

C. $ACD' + A'BC' + AC'D'$

D. $A'BD + ACD' + BCD'$

B.

$A B \bar{C} + A C D + \bar{B} \bar{C} D$

$= 110\_$ → 1100 — m_{12}, 1101 — m_{13}

1_11 → 1011 — m_{11}, 1111 — m_{15}

$\_001$ → 0001 — m_1, 1001 — m_9

4.24.5 Min Sum Of Products Form: GATE CSE 2005 | Question: 18 top https://gateoverflow.in/1354

The switching expression corresponding to $f(A,B,C,D) = \Sigma(1,4,5,9,11,12)$ is:

order

A. $BC'D' + A'C'D + AB'D$

B. $ABC' + ACD + B'C'D$

C. $ACD' + A'BC' + AC'D'$

D. $A'BD + ACD' + BCD'$

C. $ACD' + A'BC' + AC'D' \rightarrow$ SOP ✓

1_10

1010 — m_{10}

1110 — m_{14}

0101 — m_5

0100 — m_4

1000 — m_8

1100 — m_{12}

4.24.5 Min Sum Of Products Form: GATE CSE 2005 | Question: 18 top https://gateoverflow.in/1354

mcq

The switching expression corresponding to $f(A, B, C, D) = \Sigma(1, 4, 5, 9, 11, 12)$ is:

mcq

A. $BC'D' + A'C'D + AB'D$

B. $ABC' + ACD + B'C'D$

C. $ACD' + A'BC' + AC'D'$

D. $A'BD + ACD' + BCD'$ → HW

A. $B\bar{C}\bar{D} + A'C'D + AB'D$

$= \_100$; 0_01 ; 10_1

$\begin{cases} 0100 - m_4 \\ 1100 - m_{12} \end{cases}$ $\begin{cases} 0001 - m_1 \\ 0101 - m_5 \end{cases}$ $\begin{cases} 1001 - m_9 \\ 1011 - m_{11} \end{cases}$

Digital Logic :

Recap :

Minterms, Maxterms, CSOP, CPOS

Review: Minterm

- A product term in which all the variables appear exactly once, either complemented or uncomplemented, is called a minterm
- A minterm represents exacly one combination of the binary variables in a truth table. It has the value of 1 for that combination and 0 for the others

X	Y	Z	Product Term	Symbol	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	$\overline{X}\,\overline{Y}\,\overline{Z}$	m_0	1	0	0	0	0	0	0	0
0	0	1	$\overline{X}\,\overline{Y}Z$	m_1	0	1	0	0	0	0	0	0
0	1	0	$\overline{X}Y\overline{Z}$	m_2	0	0	1	0	0	0	0	0
0	1	1	$\overline{X}YZ$	m_3	0	0	0	1	0	0	0	0
1	0	0	$X\overline{Y}\,\overline{Z}$	m_4	0	0	0	0	1	0	0	0
1	0	1	$X\overline{Y}Z$	m_5	0	0	0	0	0	1	0	0
1	1	0	$XY\overline{Z}$	m_6	0	0	0	0	0	0	1	0
1	1	1	XYZ	m_7	0	0	0	0	0	0	0	1

Table 2-6 Minterms for Three Variables

Review: Maxterm

- A sum term in which all the variables appear exactly once, either complemented or uncomplemented, is called a maxterm
- A maxterm represents exacly one combination of the binary variables in a truth table. It has the value of 0 for that combination and 1 for the others

X	Y	Z	Sum Term	Symbol	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	$X+Y+Z$	M_0	0	1	1	1	1	1	1	1
0	0	1	$X+Y+\overline{Z}$	M_1	1	0	1	1	1	1	1	1
0	1	0	$X+\overline{Y}+Z$	M_2	1	1	0	1	1	1	1	1
0	1	1	$X+\overline{Y}+\overline{Z}$	M_3	1	1	1	0	1	1	1	1
1	0	0	$\overline{X}+Y+Z$	M_4	1	1	1	1	0	1	1	1
1	0	1	$\overline{X}+Y+\overline{Z}$	M_5	1	1	1	1	1	0	1	1
1	1	0	$\overline{X}+\overline{Y}+Z$	M_6	1	1	1	1	1	1	0	1
1	1	1	$\overline{X}+\overline{Y}+\overline{Z}$	M_7	1	1	1	1	1	1	1	0

Table 2-7 Maxterms for Three Variables

- A minterm and maxterm with the same subscript are the complements of each other, i.e., $M_j = m'_j$

Review: Sum of Minterms

- A Boolean function can be represented algebraically from a given truth table by forming the logical sum of all the minterms that produce a 1 in the function. This expression is called a sum of minterms

(a) X	Y	Z	F	$\overline{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$$F = X'Y'Z' + X'YZ' + XY'Z + XYZ$$

$$= m_0 + m_2 + m_5 + m_7$$

$$F(X,Y,Z) = \Sigma m(0,2,5,7)$$

Review: Product of Maxterms

- A Boolean function can be represented algebraically from a given truth table by forming the logical product of all the maxterms that produce a 0 in the function. This expression is called a product of maxterms

(a) X	Y	Z	F	$\overline{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$F = (X+Y+Z')(X+Y'+Z')(X'+Y+Z)(X'+Y'+Z)$

$= M_1 . M_3 . M_4 . M_6$

$F(X,Y,Z) = \prod M(1,3,4,6)$

- To convert a Boolean function F from SoM to PoM:
  - Find F' in SoM form
  - Find F= (F')' in PoM form

Review: Important Properties of Minterms

- There are 2^n minterms for n Boolean variables. These minterms can be evaluated from the binary numbers from 0 to 2^n-1
- Any Boolean function can be expressed as a logical sum of minterms
- The complement of a function contains those minterms not included in the original function

$$F(X,Y,Z)= \Sigma m(0,2,5,7) \Rightarrow F'(X,Y,Z)= \Sigma m(1,3,4,6)$$

- A function that includes all the 2^n minterms is equal to logic 1

$$G(X,Y)= \Sigma m(0,1,2,3)= 1$$

Review: Sum-of-Products

- The sum-of-minterms form is a standard algebraic expression that is obtained from a truth table
- When we simplify a function in SoM form by reducing the number of product terms or by reducing the number of literals in the terms, the simplified expression is said to be in Sum-of-Products form
- Sum-of-Products expression can be implemented using a two-level circuit

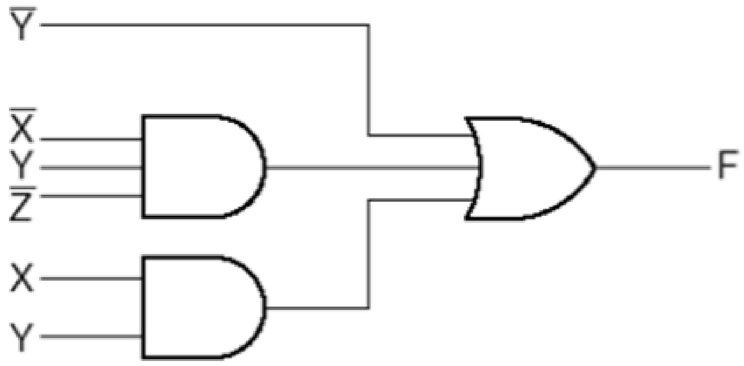


Fig. 2-5 Sum-of-Products Implementation

$F = \Sigma m(0,1,2,3,4,5,7)$ (SoM)

$= Y' + X'YZ' + XY$ (SoP)

Review: Product-of-Sums

- The product-of-maxterms form is a standard algebraic expression that is obtained from a truth table
- When we simplify a function in PoM form by reducing the number of sum terms or by reducing the number of literals in the terms, the simplified expression is said to be in Product-of-Sums form
- Product-of-Sums expression can be implemented using a two-level circuit

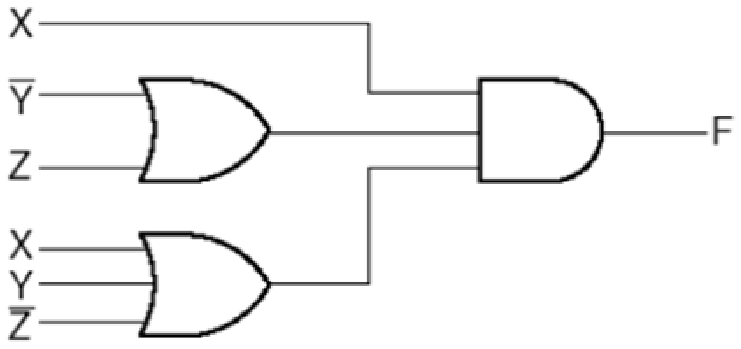


Fig. 2-7 Product-of-Sums Implementation

$F = \prod M(0,2,3,4,5,6)$ (PoM)

$= X(Y' + Z)(X + Y + Z')$ (PoS)

$$f(A,B,C) = ABC + AB\overline{C} + \overline{A}BC$$

Row Number	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

– *f* can be written as the sum of row numbers having TRUE minterms

$$f = \sum (3,6,7)$$

Finding the Complement of a Boolean Expression

$$F = (a + c)(a + b')(a + b + c')$$

Finding the Complement of a Boolean Expression

$$\overline{F} = \overline{(a+c)(a+b')(a+b+c')}$$

$\overline{F} = \bar{a}\bar{c} + \bar{a}b + \bar{a}\bar{b}c$ — In SOP

$\overline{F}(a,b,c)$ order

0 _ 0
000 — m_0
010 — m_2

01 _
010 — m_2
011 — m_3

001 — m_1

$\overline{F}(a,b,c) = \sum m(0,1,2,3)$ for these $\overline{F}=1$

$\overline{F}(a,b,c) = \prod m(4,5,6,7)$ false maxterms of $\overline{F}$

$F(a,b,c) = \sum m(4,5,6,7)$ for these $F=1$

$F(a,b,c) = \prod m(0,1,2,3)$ false maxterms

NOTE:

If we say that Minterms of function F(a,b,c) are 2, 5,6,7 then it IMPLICITELY means that these are the Minterms where F is 1 i.e. True Minterms.

NOTE:

If we say that Maxterms of function F(a,b,c) are 2, 5,6,7 then it IMPLICITELY means that these are the Maxterms where F is 0 i.e. False Maxterms.

Q:

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

a) $(x+y'+z)(x'+z')(x+y)$
b) $(AB'+C)D'+E$

Q:

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

a) (x+y'+z)(x'+z')(x+y) $\longrightarrow$ POS
b) (AB'+C)D'+E

a

(a) $F = (x+y'+z)(x'+z')(x+y)$

method 1: In POS

find (false) "maxterms" of F:

$(x+y'+z)$

$0+1+0$ — M_2

$(x'+z')$

$1+\_+1$

$1+0+1$ — M_5

$1+1+1$ — M_7

$(x+y)$

$0+0+\_$

$0+0+0 = M_0$

$0+0+1 = M_1$

$$F = \prod_m (0, 1, 2, 5, 7)$$

for these $f = 0$

$$F = \sum_m (3, 4, 6)$$

$$\overline{F} = \sum (0, 1, 2, 5, 6)$$

$$\overline{F} = \prod (3, 4, 6)$$

(a) $f = (x+y'+z)(x'+z')(x+y)$

In POS

method 2:

We are more habitual of SOP.

$$\overline{F} = \overline{x}y\overline{z} + xz + \overline{x}\,\overline{y} \quad \text{— In "SOP"}$$

$\overline{x}y\overline{z}$: 010 — m_2

xz: 1 _ 1 → 101 — m_5, 111 — m_7

$\overline{x}\,\overline{y}$: 00 _ → 000 — m_0, 001 — m_1

$\checkmark\ \overline{F} = \sum m(\underline{\underline{0, 1, 2, 5, 7}})$ for these $\overline{F} = 1$

$\checkmark\ F = \sum m(3, 4, 6)$ $\checkmark$

② $F = (AB' + C)D' + E$

$F_{(ABCDE)} = A\bar{B}\bar{D} + C\bar{D} + E$

order

In SOP

16 minterms

10_0_

_ _10_

$$\left\{\begin{array}{l} 10000 - 16 \\ 10001 - 17 \\ 10100 - 20 \\ 10101 - 21 \end{array}\right.$$

8 minterms

Note:

$$F = \sum m(F=1)$$

$$F = \prod m(F=0)$$

$$\overline{F} = \sum m(\overline{F}=1) = \sum m(F=0)$$

$$\overline{F} = \prod m(\overline{F}=0) = \prod m(F=1)$$

Q: $F = (a+b')(\bar{a}+b)$ — In POS

$\bar{F} = G = \underbrace{\bar{a}b + a\bar{b}}$

$G = \sum m(1, 2) \rightarrow G = 1$

$F = \sum m(0, 3)$

Q: HW

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

Given the Boolean function

$$F = xy + x'y' + y'z$$

Q: HW

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

3. Express the following function in sum of minterms and product of maxterms:

a) F(A,B,C,D) = B'D + A'D + BD
b) F = (AB+C)(B+C'D)

Q:

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

4. Express the complement of the following function in sum of minterms:

a) $F(A,B,C,D) = \Sigma(0,2,6,11,13,14)$
b) $F(x,y,z) = \Pi(0,3,6,7)$

Q:

Find out the minterms of following function which are used in the Canonical SOP expression of the function?

4. Express the complement of the following function in sum of minterms:

a) $F(A,B,C,D) = \Sigma(0,2,6,11,13,14)$ → $F=1$

b) $F(x,y,z) = \Pi(0,3,6,7)$ → $F=0$

(a) $\overline{F} = \Sigma(1,3,4,5,7,8,9,10,12,15)$

(b) $\overline{F} = \Sigma(0,3,6,7) = \Pi(1,2,4,5)$