Group Theory

Next Topic:

Practice Questions

Let $G = \langle \{e, a, b, c, d\}, * \rangle$ be a group with the following Cayley table partially completed.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a		e	b	
b	b				
c	c				
d	d				

(a) Demonstrate $a * d = c$ and $a * a = d$.

(4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$.

(4 marks)

(c) Implement group axiom, for each element of G, find its inverse.

(5 marks)

(d) Complete the above Cayley table of G.

(5 marks)

(e) Is G abelian?

(2 marks)

Some Group Properties:

(1) In Cayley Table of a Group,

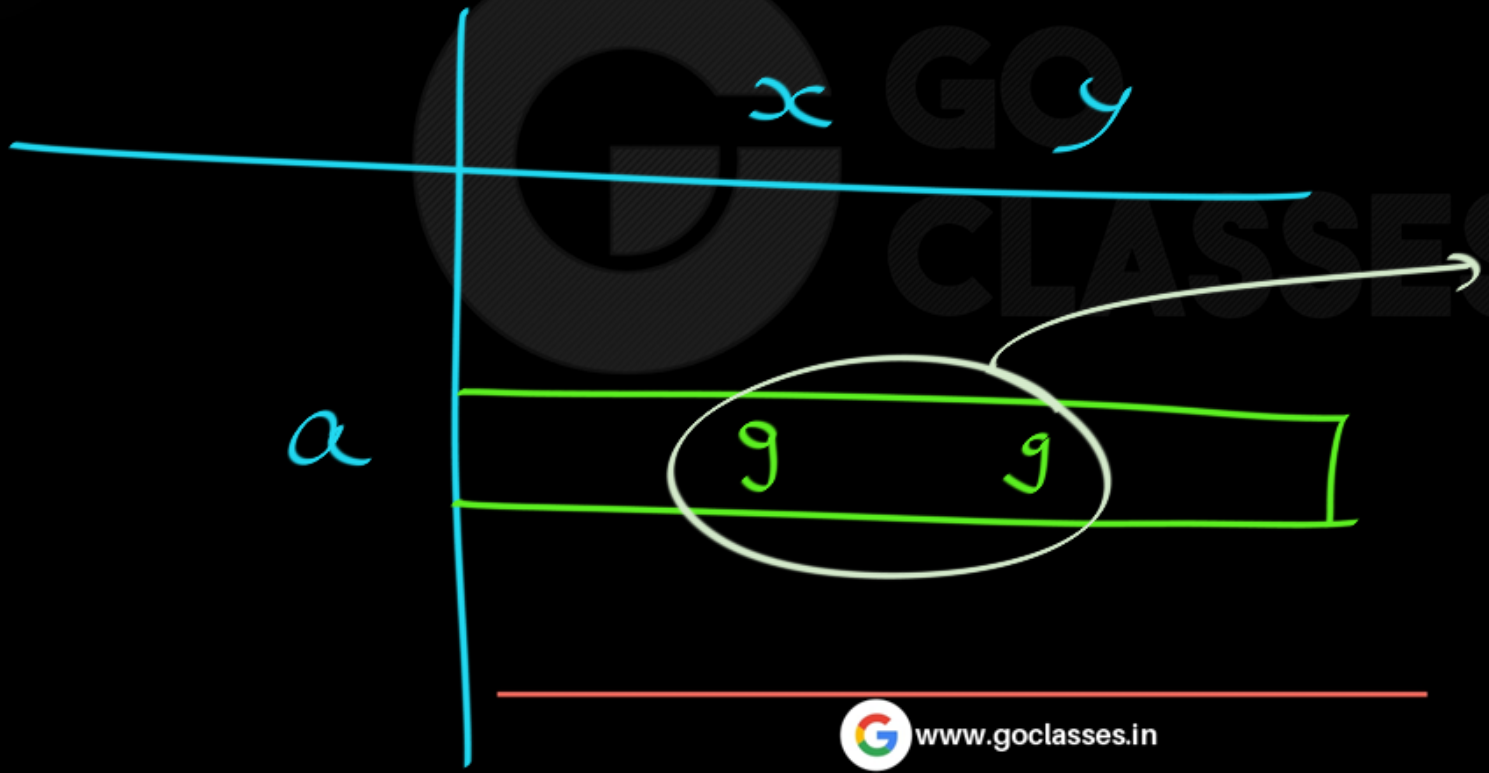
No element can repeat in any Row

No " " " " " Column.

i.e. Every element appears Exactly Once in each Row & in each Column.

Some Group Properties :

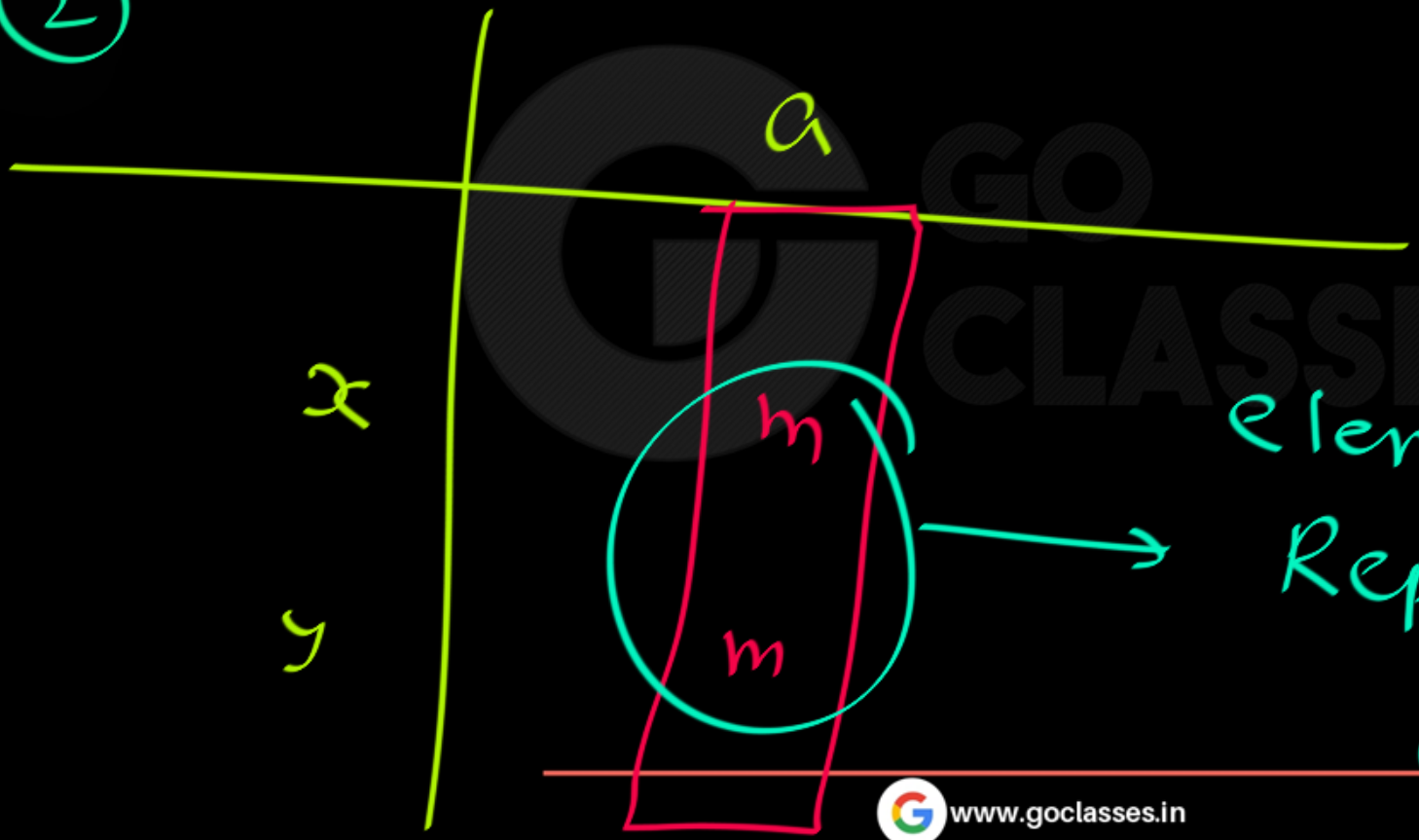
(2)



This Repetition Can Not happen.

Some Group Properties:

(2)



element can Not Repeat in any Column.

Some Group Properties:

(2) If $xy = e$ → identity element

$x^{-1} = y$

$y^{-1} = x$

$xy = e$

$x^{-1} x\, y = x^{-1} e$

$e\, y$

$y = x^{-1}$

$xy = e$

$x(y\, y^{-1}) = e y^{-1}$

e

$x = y^{-1}$

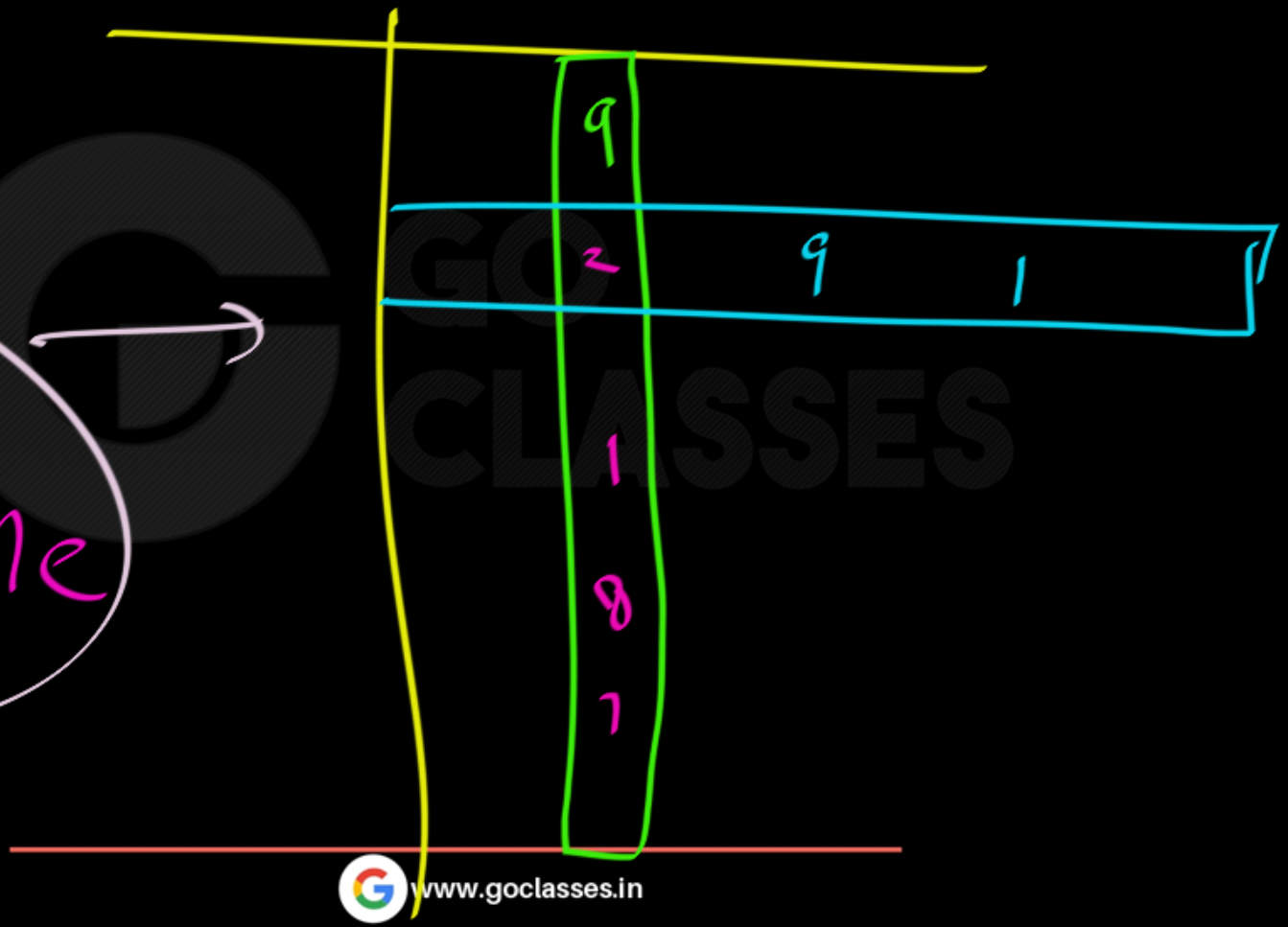
Group $(G, *)$ → binary op^{n}

If $ab = e$ → identity element

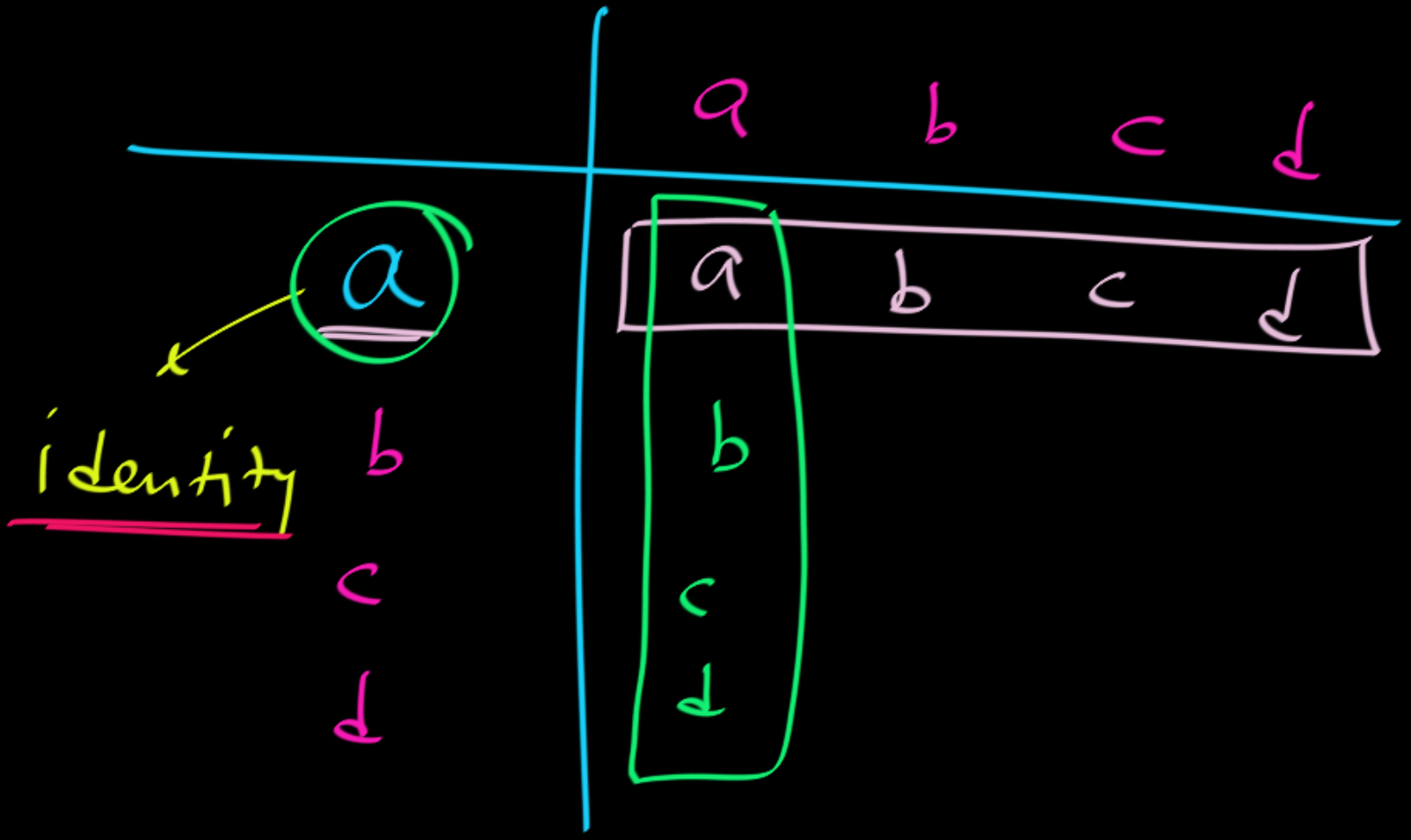
$a^{-1} = b$; $b^{-1} = a$

Sudoku

Group
Cayley Table



Group :



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$*$	e	a	b	c	d
e	e	a	b	c	d
a	a		e	b	
b	b				
c	c				
d	d				

(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

(c) Implement group axiom, for each element of G, find its inverse. (5 marks)

(d) Complete the above Cayley table of G. (5 marks)

(e) Is G abelian? (2 marks)

Let $G = \langle \{e, a, b, c, d\}, * \rangle$ be a group with the following Cayley table partially completed.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b				
c	c				
d	d				

(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

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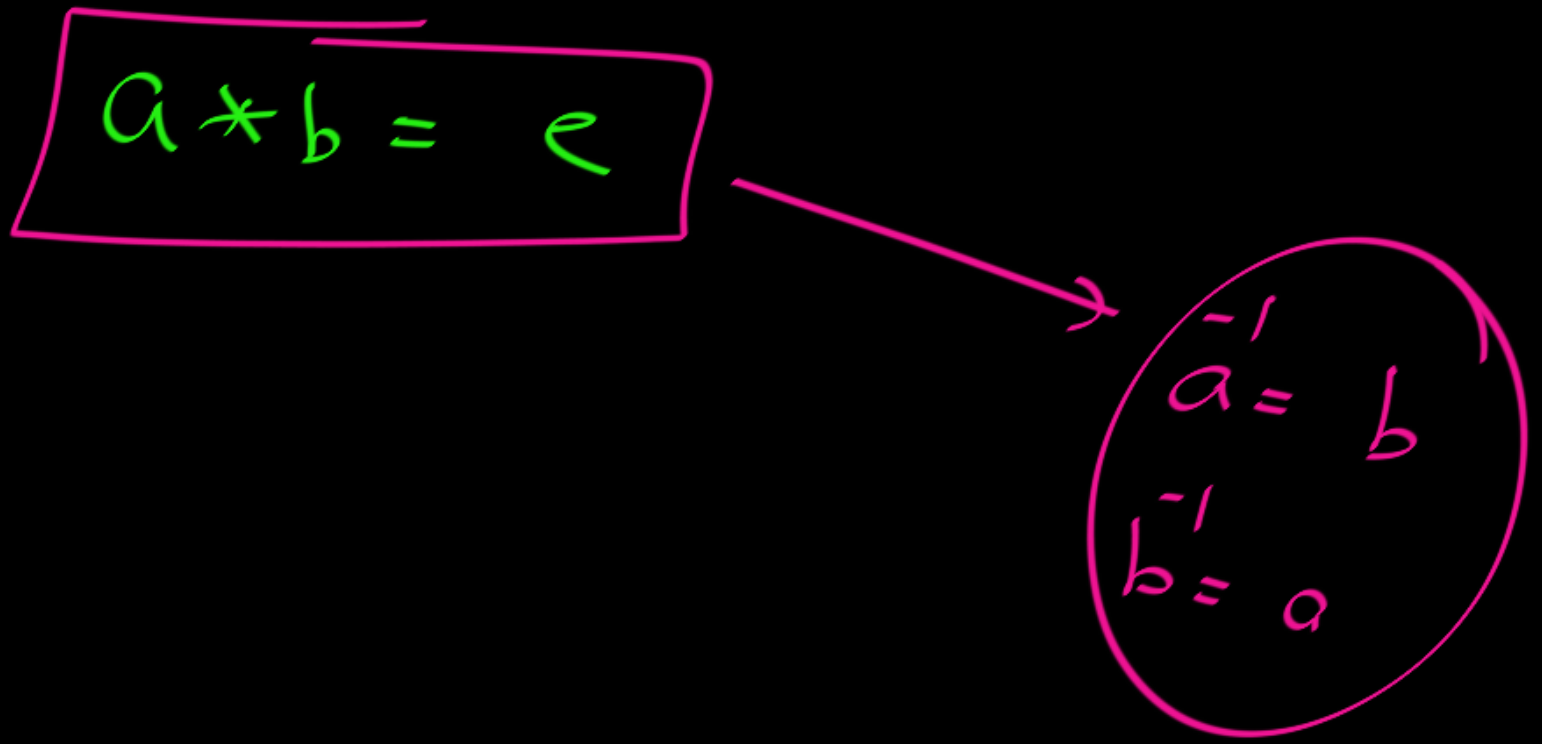

a * b = e
$a^{-1} = b$
$b^{-1} = a$

Let $G = \langle \{e, a, b, c, d\}, * \rangle$ be a group with the following Cayley table partially completed.

① identity : e

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e			
c	c	b			
d	d	c			

b, c

(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

$b^{-1} = a$

(c) Implement group axiom, for each element of G, find its inverse. (5 marks)

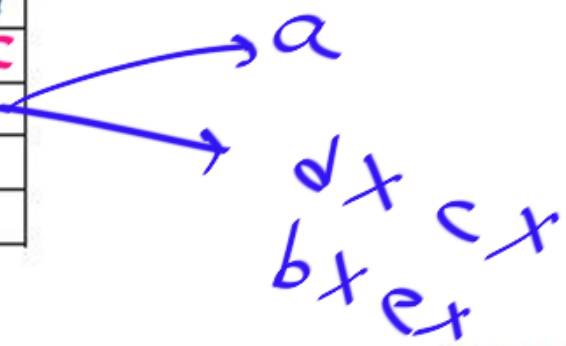
(d) Complete the above Cayley table of G. (5 marks)

$b * a = e$

(e) Is G abelian? (2 marks)

Let $G = \langle \{e, a, b, c, d\}, * \rangle$ be a group with the following Cayley table partially completed.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b			
d	d	c			



(a) Demonstrate $a * d = c$ and $a * a = d$.

(4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$.

(4 marks)

(c) Implement group axiom, for each element of G, find its inverse.

(5 marks)

(d) Complete the above Cayley table of G.

(5 marks)

(e) Is G abelian?

(2 marks)

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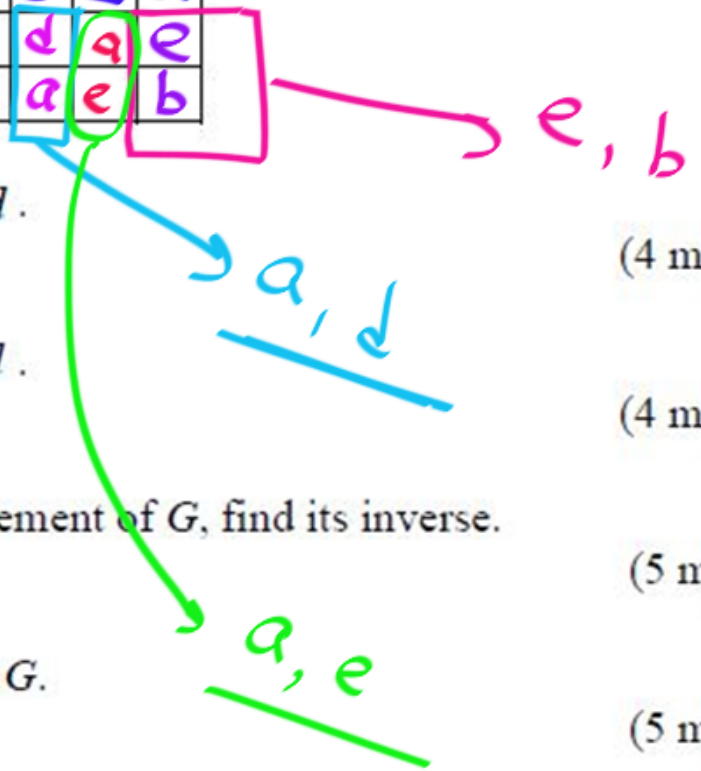
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b



(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

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(d) Complete the above Cayley table of G. (5 marks)

(e) Is G abelian? (2 marks)

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$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

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$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

(a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)

(b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)

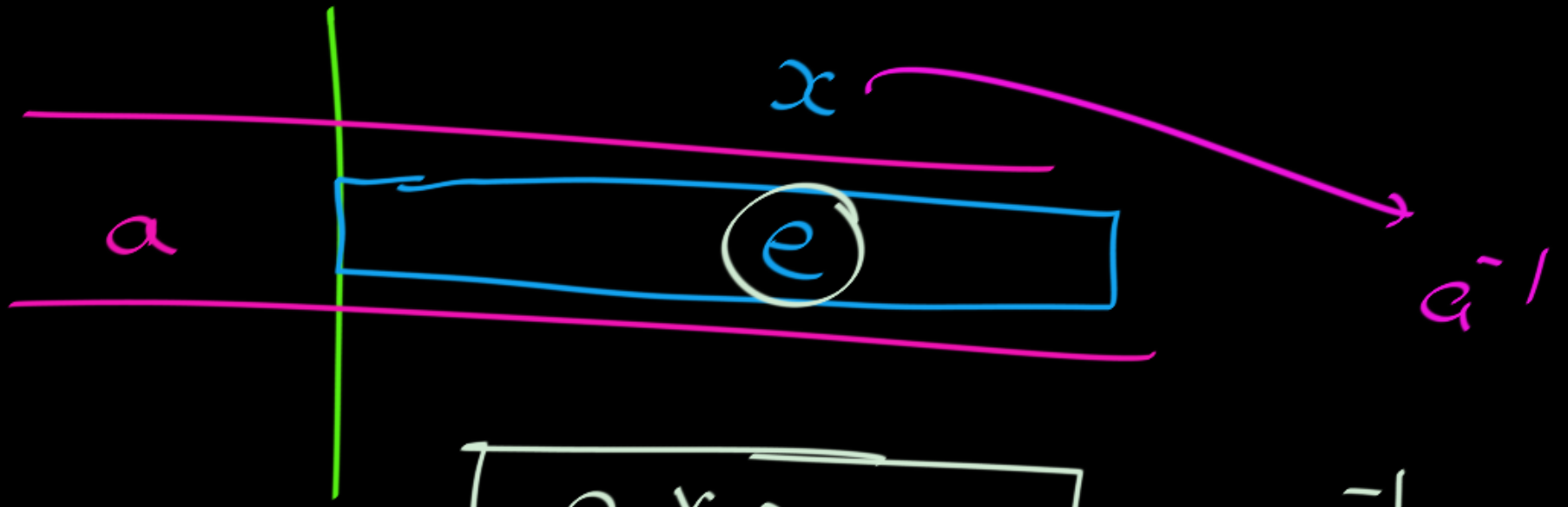
(c) Implement group axiom, for each element of G, find its inverse. (5 marks)

(d) Complete the above Cayley table of G. (5 marks)

(e) Is G abelian? (2 marks)

Group Cayley Table:

e : Identity element find a^{-1}



$$a * x = e \rightarrow x = a^{-1}$$

Group Cayley Table:

Abelian iff Table is Symmetric

$\forall_i$ i^{th} Row $=$ i^{th} Column

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e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

(a) Demonstrate $a * d = c$ and $a * a = d$.

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