Question1: Solve the following summation

$$\sum_{i=1}^{4} 8$$

Answer 1

$$\sum_{i=1}^{4} 8 = 8 + 8 + 8 + 8 = 8(4)$$

$$= 32$$

Question 2: True/False

$$\sum_{j=i}^{n} j = \sum_{k=0}^{n-i} i + k$$

Answer 2: True

j=i+k

Let $k = j - i$. When $j = i, k = 0$ and when $j = n, k = n - i$. So we can write

$$\sum_{j=i}^{n} j = \sum_{k=0}^{n-i} i + k$$

Question 3: True/False

$$\sum_{k=2}^{\infty} k = -1 + \sum_{k=1}^{\infty} k$$

Answer 3: True

$$\sum_{k=2}^{\infty} k \quad = 2 + 3 + 4 + \cdots \infty$$

$$-1 + \sum_{k=1}^{\infty} k \quad = -1 + 1 + 2 + 3 + 4 + \cdots \infty \qquad = 2 + 3 + 4 + \cdots \infty$$

We can easily see that both summations are same

Question 4

Which of the following is same as given summation ? $\sum_{i=1}^{5} i^2$

A. $\sum_{i=0}^{4} (i+1)^2$

B. $\sum_{i=2}^{6} (i-1)^2$

C. Both

D. None

Answer 4

All of the below 3 represent same summation.

One way to check is to unroll them. Or you can use method given in class

$$\boxed{\sum_{i=1}^{5} i^2} = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\boxed{\sum_{i=0}^{4} (i+1)^2} = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\boxed{\sum_{i=2}^{6} (i-1)^2} = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

Question 5

Use your intuition to write the long sum in sigma notation

$$1 + 2 + 3 + 4 + 5$$

$$\sum_{i=1}^{5} i$$

Answer

$$1 + 2 + 3 + 4 + 5 = \sum_{k=1}^{5} k\,.$$

Question 6

Use your intuition to write the long sum in sigma notation

$$1 + 4 + 9 + 16 + 25 + 36$$

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1 2 3^2 4^2 5^2

Answer

$$1 + 4 + 9 + 16 + 25 + 36 = \sum_{k=1}^{6} k^2 .$$

Question 7

True/False

$$\sum_{k=1}^{n}(k+c) = nc + \sum_{k=1}^{n} k$$

Answer: True

$$\sum_{k=1}^{n}(k+c) = \sum_{k=1}^{n} k + \sum_{k=1}^{n} c = \sum_{k=1}^{n} k \; + \; cn$$

Bonus Question

Use your intuition to write the long sum in sigma notation

$$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots + \frac{1}{100}$$

Answer

In this example, the first term -1 can also be written as a fraction $-\frac{1}{1}$. We also notice that the signs of the terms alternate, with a minus sign for the odd-numbered terms and a plus sign for the even-numbered terms. So we can take care of the sign by using $(-1)^k$, which is -1 when k is odd, and $+1$ when k is even. We can therefore write the sum as

$$(-1)^1\tfrac{1}{1} + (-1)^2\tfrac{1}{2} + (-1)^3\tfrac{1}{3} + (-1)^4\tfrac{1}{4} + \ldots + (-1)^{100}\tfrac{1}{100}\,.$$

We can now see that k-th term is $(-1)^k\,1/k$, and that there are 100 terms, so we would write the sum in sigma notation as

$$\sum_{k=1}^{100}(-1)^k\frac{1}{k}\,.$$