Number System

even prime

2, 3, 5, 7, 11, 13 ...

9

1 Not a Prime Number

$6k \pm 1$ is a prime N.

293 → 49x6 − 1

Natural Numbers

Positive Int.

1 2 3 ... ∞

Whole Numbers

0 and N

0, 1, 2, 3 ...

Prime Numbers

3 = 1 × 3

Not divisible 1 3

25 till 100

Composite numbers

No. of factors

24 → 8 × 3

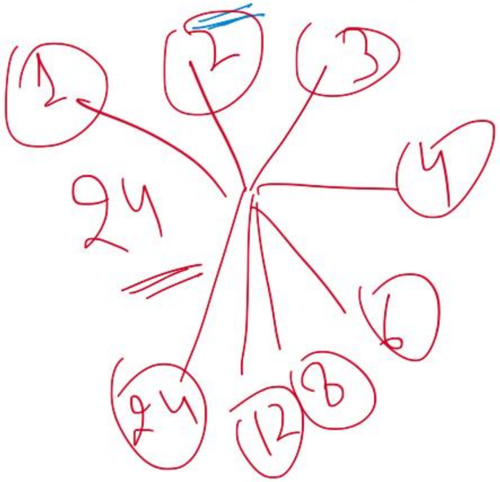
$2^3 \times 3^1$

$(3+1) \times (1+1)$

$= 4 \times 2 = 8$

293 → 289 17

2 3 5 7 11 13 17 19



Rational Numbers p/q

$p > q$ Mixed fraction

$p = q$ Integer

$p < q$ Proper fraction

Real Numbers

I Nom fraction

Irrational Numbers Surds π, e

i^{4n}

$\sqrt{2} \to 0.414213562 3$

$x \to 0.2532323232$

$100x = 25.323232$

$99x = 25.07$

$x = \frac{2507}{9900}$

Complex Numbers $a + ib$

$i = \sqrt{-1}$

$i^2 = -1$

$\boxed{i^4 = 1}$

Divisibility Rules

25

$20 + 5$

$\frac{\cancel{20} + 5}{2} \Rightarrow 1$

$\frac{25}{2} \Rightarrow 1$

Divisible by 2
Unit digit should be even

by 4
Last 2 digits should be divisible by 4

$\frac{abc}{4}$

$\frac{100a + 10b + c}{4}$

ab

$\frac{10a + b}{2}$

Rules

by 2 → unit digit by 2

by 3, 9 → sum of all digits by 3, 9

by 4 → Last 2 digits by 4

by 5 → Last digit 0 or 5

by 6 → divisible by 2 & 3 both

by 7 → unit digit × 5 and add it in remaining digits

by 8 → Last 3 digits by 8

10

by 11 → add alternate digits and diff

1331

4 − 4 ⇒ 0

343

34
15
49

Divisibility of algebraic expressions

1. $a^n + b^n$ — $a+b$ — n is odd

2. $a^n - b^n$ → $a-b$ — Always

$a^n - b^n$ → $a+b$ — n is Even

Which one of the following numbers is exactly divisible by $(11^{13} + 1)$?

A. $11^{26} + 1$

B. $11^{33} + 1$

C. $11^{39} - 1$

D. $11^{52} - 1$

GATE 2021 EE

11^{11}

$11^{13} + 1$

$n = 4$

$a + b$

11^{13} 1

Even & Odd number

2, 4, 6, 8, 10

1, 3, 5, 7, 9, 11, 13

Result

Even ⇒ $e \pm e$

$o \pm o$

$E/O \times E$

Odd → $e \pm O$

$O \times O$

Eg. There are two, 2-digit numbers ab and cd, ba is the another two digit number prepared by reversing the digits of ab, if ab × cd = 493, ba × cd = 2059, what is value 'g' sum of (ab + cd) = ?

(a) 43
(b) 45
(c) 47
(d) 46

Ans

If a and b are integers and $a - b$ is even, which of the following must always be even?

A. ab e ✓ o ✗

B. $a^2 + b^2 + 1$ ✗ o

C. $a^2 + b + 1$ ✗ o

D. $ab - b$ e ✓

Gate 2017 ME

HCF → GCD

12 , 18

6

2 , 3

$12 \rightarrow 2 \times 2 \times 3$

$18 \rightarrow 2 \times 3 \times 3$

$2 \times 3 \Rightarrow 6$

LCM

Least

12 , 18

36

18 36 54 72

$12 \rightarrow 2 \times 2 \times 3$

$18 \rightarrow 2 \times 3 \times 3$

$2 \times 3 \times 2 \times 3$

$= 36$ LCM

HCF

$\frac{N}{D}$

$$\frac{HCF(N)}{LCM(D)}$$

LCM of a fraction

$$\frac{LCM(N)}{HCF(D)}$$

HCF

LCM

$\frac{2}{3}$ $\frac{3}{5}$ $\frac{7}{4}$

HCF $\frac{1}{60}$

LCM = $\frac{42}{1}$

Unit Digit of big powers

Eg. Find the unit digit of 2^{754} 4 Ans

188

$\cancel{754}$

4

2

752

2^2

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^8 = 128$

$2^{10} = 256$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$4^{25} \rightarrow 4$

Cyclicity table

1	:	1
2	:	2, 4, 6, 8
3	:	3, 9, 7, 1
4	:	4, 6
5	:	5
6	:	6
7	:	7, 9, 3, 1
8	:	8, 4, 2, 6
9	:	9, 1
0	:	0

Find remainder

$$\frac{\cancel{25}^{1}}{\cancel{2}} = 1$$

$$\frac{\cancel{20}^{0} + \cancel{5}^{1}}{2} \Rightarrow 1$$

$$\frac{\cancel{25}^{0} + \cancel{34}^{4} + \cancel{327}^{2}}{5} = \frac{6}{5} = 1$$

$$\frac{\cancel{10}^{5} \times \cancel{15}^{2} \times \cancel{21}^{8}}{13} = \frac{\cancel{80}^{2}}{13} \; 2$$

$$\frac{(x+a)^n}{x} = \frac{a^n}{x}$$

$$(x+a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + \cdots + x^0 a^n$$

$$\frac{7^{99}}{6} = \frac{(6+1)^{99}}{6} = \frac{1}{6} \Rightarrow 1$$

$$\frac{5^{100}}{7} = \frac{(25)^{50}}{7} = \frac{(7\times3+4)^{50}}{7} = \frac{4^{50}}{7} = \frac{(16)^{25}}{7}$$

$$= \frac{(7\times2+2)^{25}}{7} = \frac{2^{25}}{7} = \frac{2\cdot2^{24}}{7} = \frac{2\cdot(8)^{8}}{7}$$

$$= 2\cdot\frac{(7+1)^{8}}{7} = 2\cdot1$$

$$= 2 \text{ Ans}$$

1 $\dfrac{\text{Number}}{a \text{ or } b \text{ or } c} \rightarrow K \Rightarrow (n-1) \times \mathrm{LCM}(a,b,c) + K$

2 $\dfrac{\text{Number}}{a \text{ or } b \text{ or } c} = \rightarrow \begin{array}{l} a-K \\ b-K \\ c-K \end{array} \Rightarrow n \times \mathrm{LCM}(a,b,c) - K$

$n \geq$

$n=1$

3 $\dfrac{\text{Number} + K}{a \text{ or } b \text{ or } c}$

If a prime number on division by 4 gives a remainder of 1, then that number can be expressed as

A. sum of squares of two natural numbers
B. sum of cubes of two natural numbers
C. sum of square roots of two natural numbers
D. sum of cube roots of two natural numbers

GATE 2012 AE

$4k+1 \rightarrow 5 \rightarrow 4+1$

$\rightarrow 13 \rightarrow 9+4$

$\rightarrow 17 \rightarrow 16+1$

9 4

3 + 2

27 8

3 9

4 16

Base System

Binary (2) 0, 1

Decimal 0, 1 - - - 9

Base 8 0, 1, 2, 3 . . . 7

Base 6 0 5

Any Base ⟶ Decimal

$$(abcd.efg)_B \longrightarrow a\times B^3 + b\times B^2 + c\times B^1 + d + e\times B^{-1} + f\times B^{-2} + g\,B^{-3}$$

$$\begin{array}{r|l|l} 8 & 124 & \\ \hline 8 & 15 & 4 \\ \hline 8 & 1 & 7 \\ \hline & 0 & 1 \end{array}$$

Decimal ⟶ Any Base

$$(124)_{10} \longrightarrow (174)_8$$

Base 7

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 436 \\ +\,356 \\ \hline 1125 \end{array}$$ Ans

$\boxed{12}$ $7+5$

$\boxed{7}+1$

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 436 \\ +\,356 \\ \hline 1125 \end{array}$$ Ans

0 1 2 3 4 5 6

Consider the equation: $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

A. 1634
B. 1737
C. 3142
D. 3162

Gate 2014 CSE

7526
y
4364

0 1 2 3 4 5 6 7

7526
−4364

Y ⇒ 3142 Ans

Some more

GATE PYQs

Q.1 The product of three integers X, Y and Z is 192. Z is equal to 4 and P is equal to the average of X and Y. What is the minimum possible value of P?

A. 6
B. 7
C. 8
D. 9.5

Gate 2019 ME

$P = \frac{X+Y}{2}$

$XYZ = 192$

$Z = 4$

$XY = 48$

8×6

12×4

16×3

24×2

1×48

$P = 7$

Q.2 The sum and product of two integers are 26 and 165 respectively. The difference between these two integers is ______

A. 2
B. 3
C. 4
D. 6

Ans

Gate 2019 ME

$x - y = ?$

$x + y = 26$

$xy = 165$

$(x-y)^2 = x^2 + y^2 - 2xy$

$= x^2 + y^2 + 2xy - 4xy$

$= (x+y)^2 - 4xy$

$= 26^2 - 4 \times 165 \Rightarrow 676 - 660$

$= 16$

Q.3 Given two sets $X = \{1, 2, 3\}$ and $Y = \{2, 3, 4\}$, we construct a set Z of all possible fractions where the numerators belong to set X and the denominators belong to set Y. The product of elements having minimum and maximum values in the set Z is _____.

A. 1/12
B. 1/8
C. 1/6
D. 3/8

Gate 2019 EE

$$Z = \frac{X \downarrow}{Y \uparrow}$$

$$\frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$$

Q.4 A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged. What is the number?

A. 63

B. 72

C. 81

D. 90

Gate 2018 ME

ab

$10a+b$

$-(10b+a)$

45

$9a-9b=45$

$a-b=5$

$10a+b$

-45

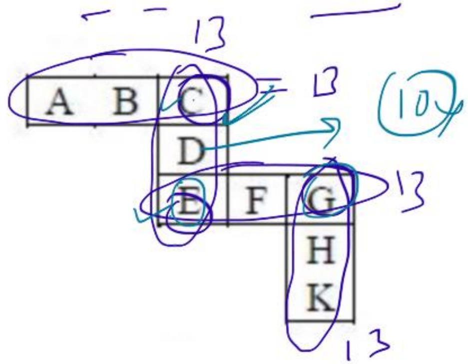
$10b+a$

$a+b=9$

72

Q.5 Each of the letters in the figure below represents a unique integer from 1 to 9. The letters are positioned in the figure such that each of $(A + B + C), (C + D + E), (E + F + G)$ and $(G + H + K)$ is equal to 13. Which integer does E represent?

Gate 2018 CE



A. 1
B. 4 Ans
C. 6
D. 7

$(A \text{ to } K) + C + E + G = 52$

$45 + C + E + G = 52$

$C + E + G = 7$

$= 1, 2, 4$

4

Q.6 The sum of the digits of a two digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number.

A. 39
B. 57
C. 66
D. 93

Gate 2016 CE

12

ab

$$10b + a$$
$$-(10a + b)$$
$$54$$

$$a + b = 12$$

$$9b - 9a = 54$$

$$b - a = 6$$

$$a = 3$$
$$b = 9$$

Q.7 A positive integer m in base 10 when represented in base 2 has the representation p and in base 3 has the representation q. We get $p - q = 990$ where the subtraction is done in base 10. Which of the following is necessarily true:

A. $m \geq 14$
B. $9 \leq m \leq 13$
C. $6 \leq m \leq 8$
D. $m < 6$

Gate 2010 MN

m	p	q	p-q
9	1001	100	901
10	1010	101	909
11	1011	102	909
12	1100	110	990
13	1101	111	990

Q.8 X is a 30 digit number starting with the digit 4 followed by the digit 7. Then the number X^3 will have

A. 90 digits
B. 91 digits
C. 92 digits
D. 93 digits

Gate 2017 CSE

$X = 4\underbrace{7777}_{29 \text{ times}} \Rightarrow X = 4.7 \times 10^{29}$

x^3

$X^3 = (4.7)^3 \times 10^{29 \times 3}$

3 digits $\times 10^{87}$

Q.9 Find the smallest number y such that $y \times 162$ is a perfect cube.

A. 24
B. 27
C. 32
D. 36

Ans

Gate 2017 CSE

90%

$y \times 162$

81×2

$27 \times 3 \times 2 \times 6 \times 6$