Digital Logic :

Next Topic :

Decimal & Binary Number Systems

Website : https://www.goclasses.in/

Number System:

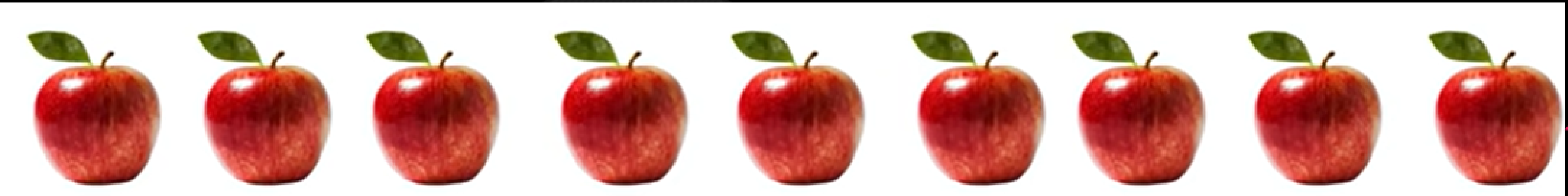
A System of Counting

Number systems are simply ways to count things.

Or ways to represent numbers.

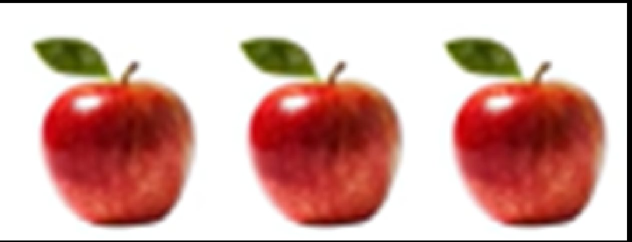
Number System:

0. Roman System



= IX

= V



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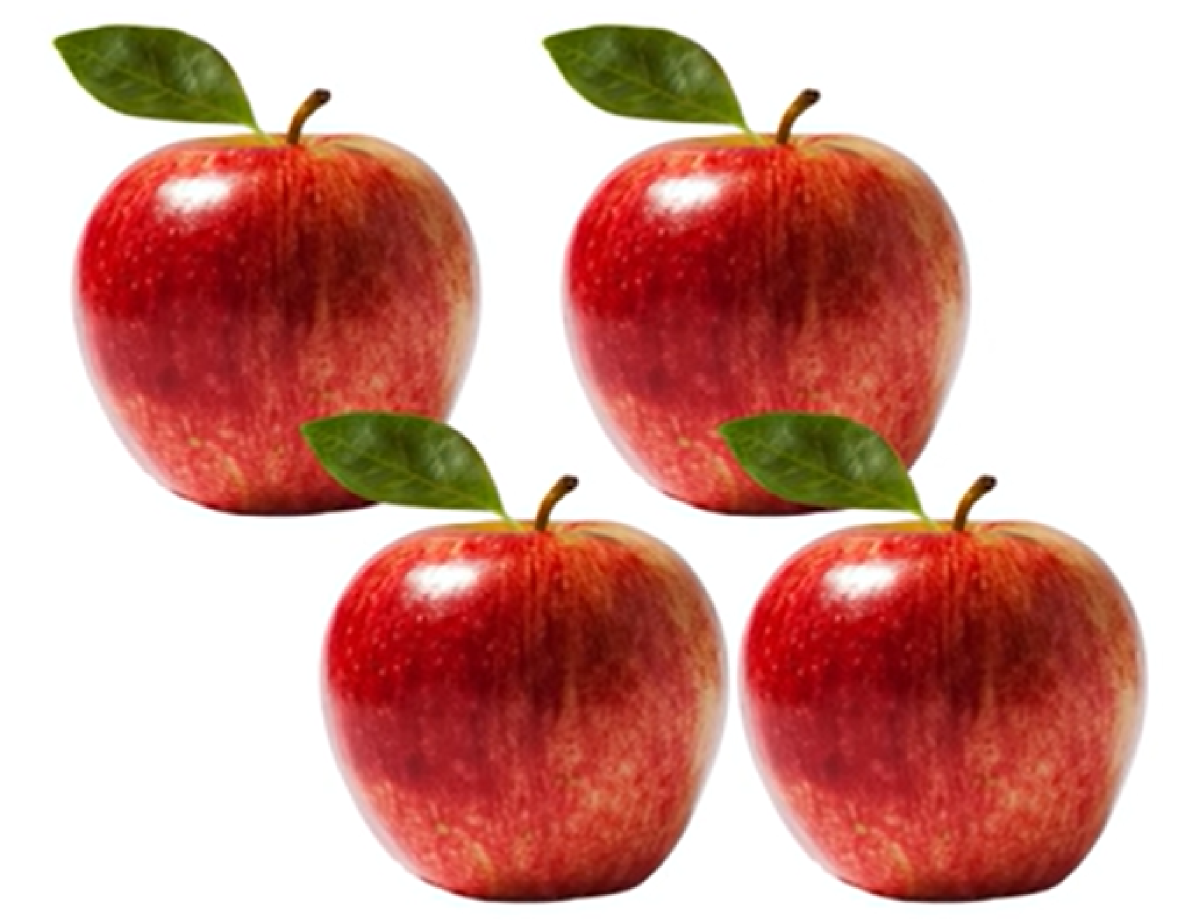
Number System:

1. Unary System

How small babies count?

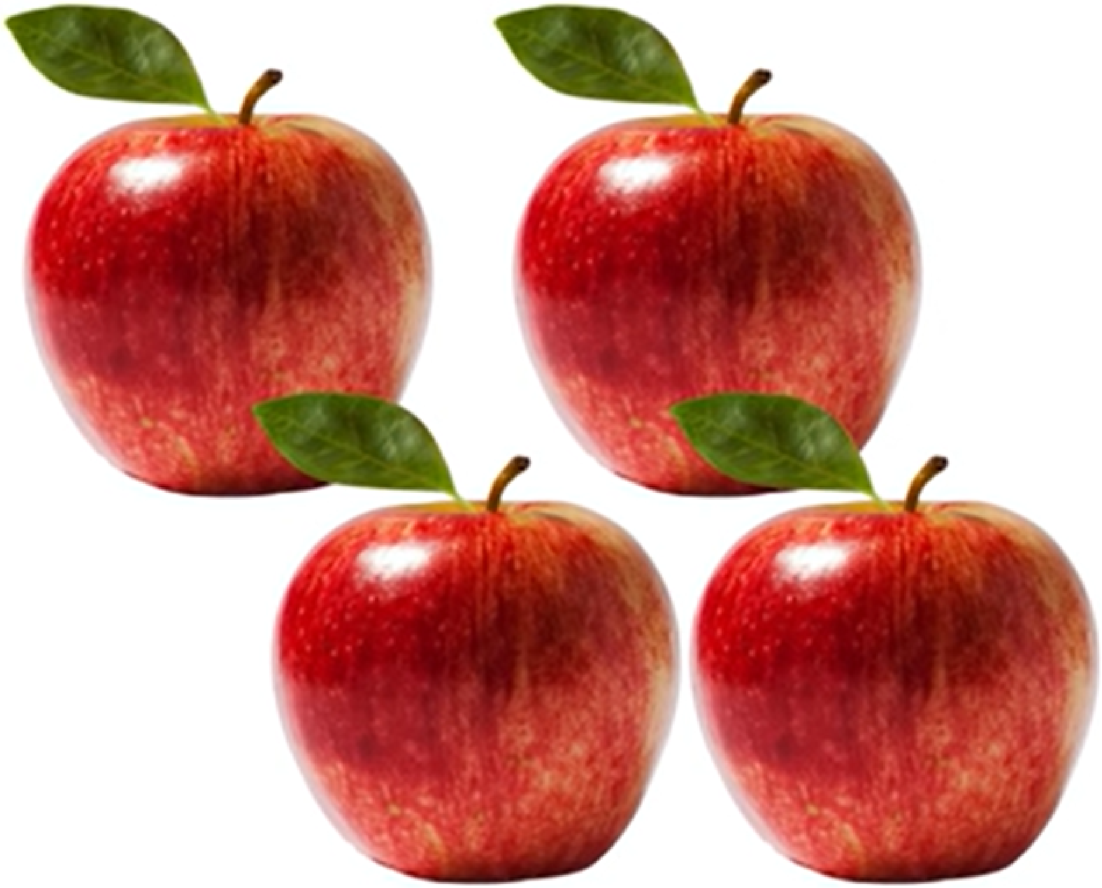
3 = "111"

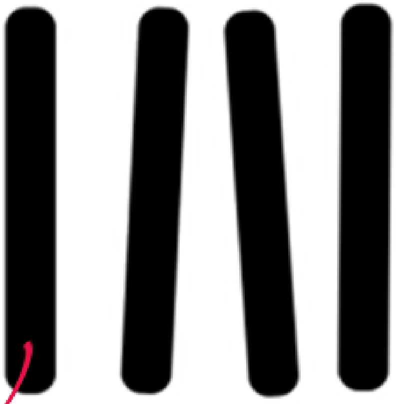
One Symbol = {1}



= 1111

= 11





symbol

Number System:

2. Decimal System

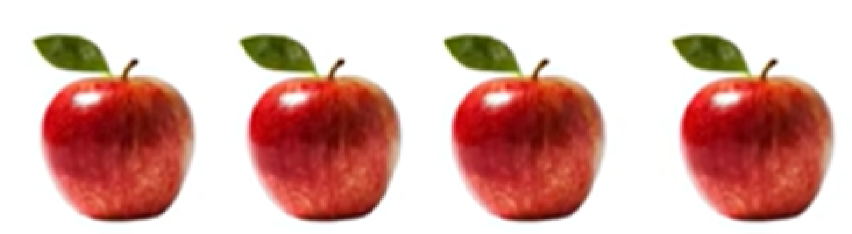
How a normal human counts?

10 symbols (0,1,2,3,...,9)

- **Decimal system** uses 10 symbols *(digits)*
  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- The ***decimal system*** is the number system that we use everyday

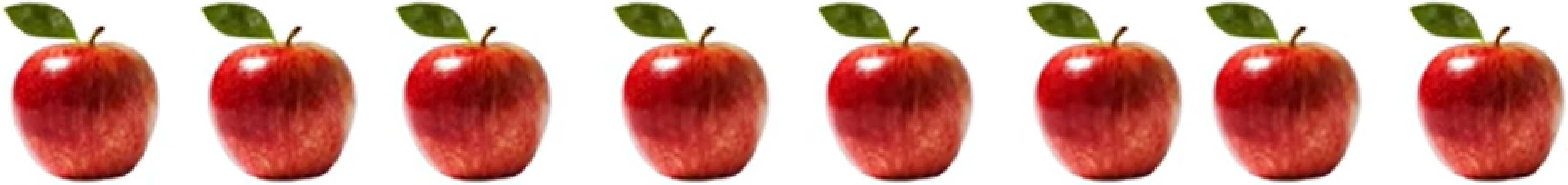
= 9



= 4

= 1

9= 

8= 

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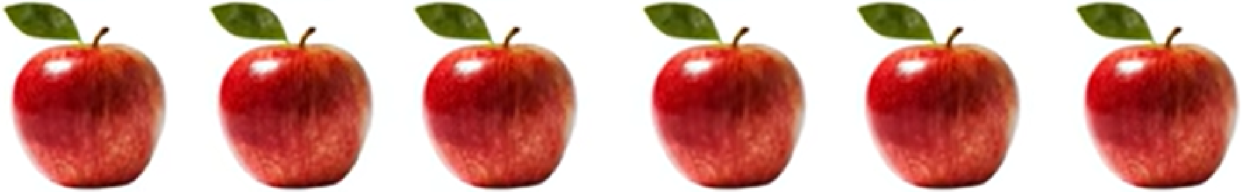
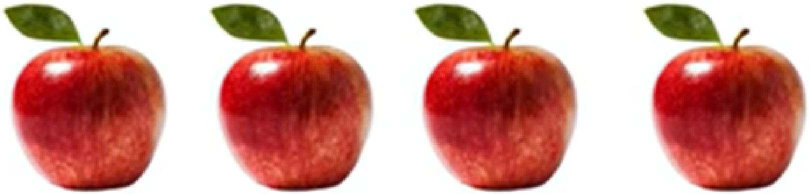
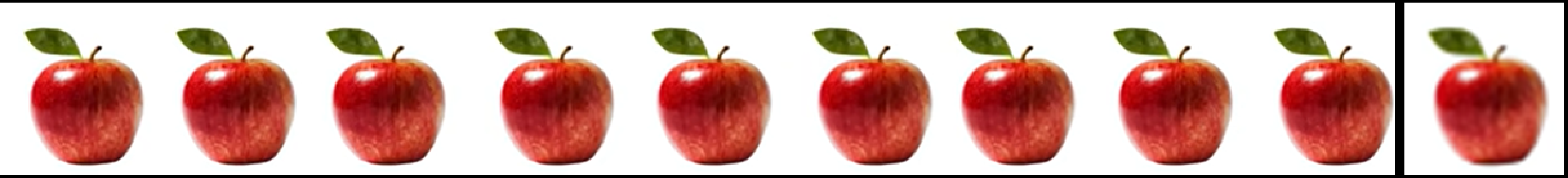
6= 

5= 

4= 



1 0

Decimal system:

Symbols: $\underbrace{0, 1, 2, \text{---}, 9}$

Digits

0
1
2
.
.
9

Repeat

10
11
12
.
.
19

20
21
.
.
29

30
31
.
.
39

- - - -

90
91
.
.
99

100

The base of the number system is defined as the total number of symbols(digits) available in the number system.

Decimal system ⇒ Base = 10

The base value of a number system is the number of different values the set has before repeating itself. For example, decimal has a base of ten values, 0 to 9.

$$\underbrace{329}_{} = 3\times10^2 + 2\times10 + 9\times10^0$$

3	2	9
10^2	10^1	10^0

second symbol (2)

after 10

Counting 10 items.

$\underbrace{10^2 \quad 10^1 \quad 10^0}_{} \rightarrow$ weights

$\underbrace{329}$ = $3\times10^2 + 2\times10 + 9\times10^0$

3	2	9
10^2	10^1	10^0

Third Symbol(3)

after 100

Counting

100 items.

weights

2	2	7	0	6
10^4	10^3	10^2	10^1	10^0

Digital Logic

$$(22706)_{10} = 2\times10^4 + 2\times10^3 + 7\times10^2 + 0\times10^1 + 6\times10^0$$

$= \underline{\underline{22706}}$

10 → tell us the number system

$$(\underline{\underline{1000}})_{10} = \underline{1\times10^3 + 0\times10^2 + 0\times10^1 + 0\times10^0}$$

$= \underline{\underline{1000}}$

$(abcde)_{10} \rightarrow$ Decimal system

$= e\times10^0 + d\times10^1 + c\times10^2 + b\times10^3 + a\times10^4$

2 5 6

→ second symbol we are using after counting 10 items.

→ 5 tells us How many 10's we have counted? ⇒ 5 ✓

22

$$\underbrace{0 - 9 \qquad 10 - 19}_{2\ \text{tens}}$$

2

1 2 5 5 =

How many 100's we have counted? = 2

How many 1000's we have counted? = 1

$$\begin{array}{cccc} 1 & 2 & 5 & 5 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 10^3 & 10^2 & 10^1 & 10^0 \end{array} = \underline{1\times10^3 + 2\times10^2 + 5\times10^1 + 5\times10^0}$$

Number System:

3. Binary System: The Computer Number System

How a computers counts?

2 symbols (0,1) (bits)

Binary	Decimal
0	0
1	1
No symbol	2
1 0	2
1 1	3

Binary	Decimal
1 0 0	4
1 0 1	5
1 1 0	6
1 1 1	7
1 0 0 0	8

Base Ten:

Greater than 9=New Digit

Binary:

Greater than 1=New Digit

Binary Number

0

Amount of Things

Binary Number

1

Amount of Things

Binary Number

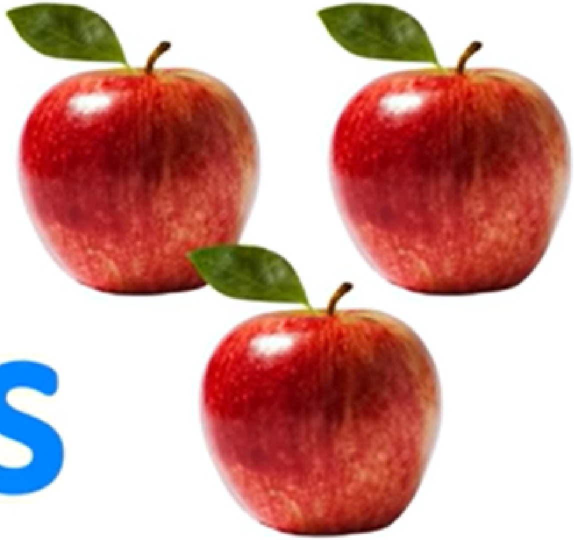
10

Amount of Things

Binary Number

11

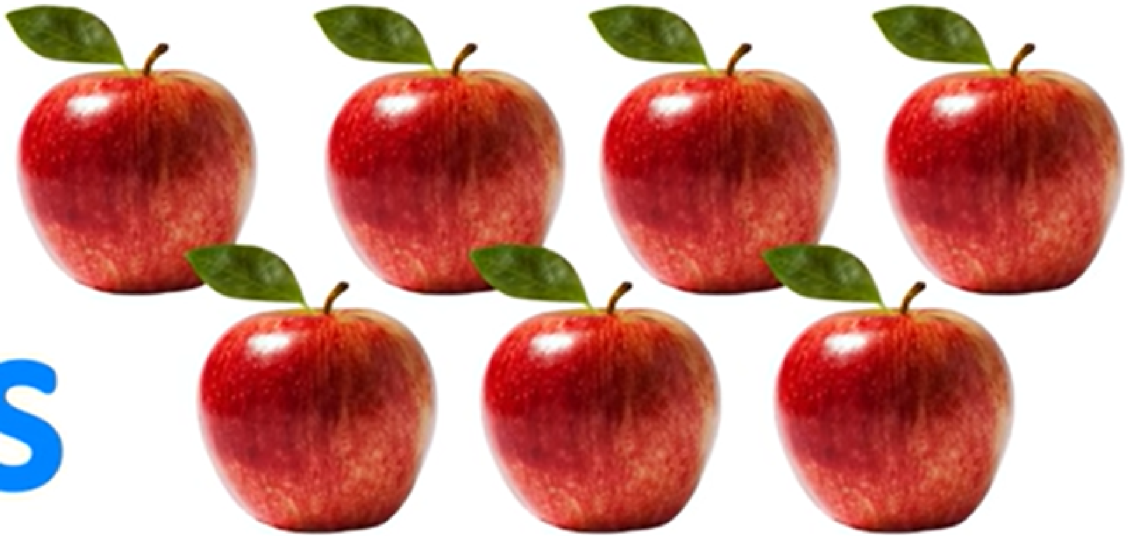
Amount of Things



Binary Number

111

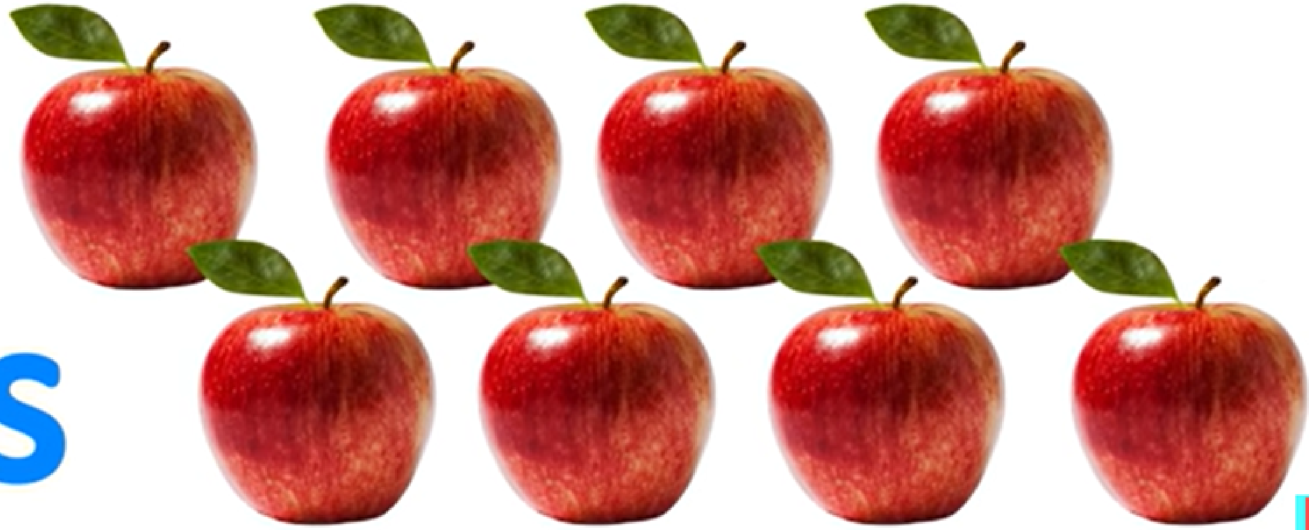
Amount of Things



Binary Number

1000

Amount of Things



Binary System:

find out the weights

1 0 1 1

second symbol used after counting 2 items.

4th symbol used after counting 8 items.

$(a_2\ a_1\ a_0)_2$

$a_2 \to 2^2$, $a_1 \to 2^1$, $a_0 \to 1$

1 0 1 1

2^3 2^2 2^1 2^0

0	1	1	0	0	1

2^5 2^4 2^3 2^2 2^1 2^0

$(1011)_2 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 8+2+1 = 11$

$(11)_{10}$

2 tells us Binary system

$(11)_2 = 1\times2^1 + 1\times2^0 = (3)_{10}$

Number Systems and Conversion

When we write decimal (base 10) numbers, we use a positional notation; each digit is multiplied by an appropriate power of 10 depending on its position in the number. For example,

$(22003)_{10} = 2 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$

$10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0$

Number Systems and Conversion

Similarly, for binary (base 2) numbers, each binary digit is multiplied by the appropriate power of 2:

$(1\,0\,1)_2 = (5)_{10}$?

$2^2 \quad 2^1 \quad 2^0$

$= 1 \times 2^2 + 1 \times 2^0 = 5$

$= 4 + 1 = 5$

Digital Logic :

Next Sub-Topic :

Binary to Decimal Conversion

(Same as you read a decimal number - Easy Peasy)

$(1011011)_2 = (\ ?\ )_{10}$?

$1 \times 2^6 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0$

$= 64 + 16 + 8 + 2 + 1 = 91$ Apples

Binary numbers to Decimal Number

2^6

$$(N)_2 = (11100110)_2$$

decimal value is given by,

$$(N)_2 = \underline{1 \times 2^7} + \underline{1 \times 2^6} + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 = \underline{(230)}_{10}$$

$(14)_{10} = (\ ?\ )_2$

		Remainder
2	14	0
2	7	1
2	3	1
2	1	1
	0	

$(14)_{10} = (1110)_2$

$(14)_{10} \rightarrow (1110)_2$

14 Apples

Digital Logic :

Next Sub-Topic :

Decimal to Binary Conversion

(Do you understand or have you by-hearted?)

Decimal → Binary

Method : Successive Division by 2.

write Remainders in Reverse order.

$(27)_{10} \longrightarrow (\ ?\ )_2 = (\underline{11011})_2$

binary of 27.

		Remainder
2	27	1
2	13	1
2	6	0
2	3	1
2	1	1
	0	

$(32)_{10} \longrightarrow (\ ?\ )_2 = (1000\ 00)_2$

binary of 32.

		Remainders
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(a\ b\ c\ d)_2 = (N)_{10}$

$a, b, c, d \in \{0, 1\}$

$N = \boxed{a \times 2^3 + b \times 2^2 + c \times 2^1 + d}$

How to find "d" ?

$d = N \bmod 2$

$$N = ax2^3 + bx2^2 + cx2^1 + d$$

$\left(\frac{N}{2}\right) \to$ Quotient $q_1 = ax2^2 + bx2^1 + c$ ✓

$\to$ Remainder $r_1 = d$

$(a\ b\ c\ d)_2 = (N)_{10}$

$N = \boxed{a \times 2^3 + b \times 2^2 + c \times 2^1 + d}$

find c ?

$Q_1 = \text{Quotient}\left(\frac{N}{2}\right)$

$c = Q_1 \bmod 2$

$$Q_1 = \boxed{ax_2^2 + bx_2^1 + c}$$

$$\left.\frac{Q_1}{2}\right\} \begin{cases} \rightarrow \text{Quotient } Q_2 = ax_2 + b \\ \rightarrow \text{Remainder } R_2 = c \end{cases}$$

$$Q_2 = a \times 2 + b$$

find b:

$$b = Q_2 \bmod 2$$

This process is continued until we finally obtain a_n. Note that the remainder obtained at each division step is one of the desired digits and the least significant digit is obtained first.

$(27)_{10} = ?_2 = (a_4 a_3 a_2 a_1 a_0)$

Remainder

2	27 = N	
2	13 = Q_1	$R_1 = 1 = a_0$
2	6 = Q_2	$R_2 = 1 = a_1$
2	3 = Q_3	$R_3 = 0 = a_2$
2	1 Q_4	$R_4 = 1 = a_3$
	0 Q_5	$R_5 = 1 = a_4$

a_0

$(11011)_2$

a_4

2	N = 44	
2	22	Q_1
2	11	Q_2
2	5	Q_3
2	2	Q_4
2	1	Q_5
	0	Q_6

Remainder

$\boxed{R_1 = 0 = a_0}$

$R_2 = 0 = a_1$

$R_3 = 1 = a_2$

$R_4 = 1 = a_3$

$R_5 = 0 = a_4$

$R_6 = 1 = a_5$

$(N = 44)_{10} = (a_5 a_4 a_3 a_2 a_1 a_0)$

$= (101100)$

Convert 53_{10} to binary.

IV

a_1 a_2

$(a_5 a_4 a_3 a_2 a_1 a_0)_2$

1 1 0 1 0 1

$$
\begin{array}{rll}
2 \,\underline{|53} & & \\
2 \,\underline{|26} & \text{rem.} = 1 = a_0 & \\
2 \,\underline{|13} & \text{rem.} = 0 = a_1 & \\
2 \,\underline{|6} & \text{rem.} = 1 = a_2 & 53_{10} = 110101_2 \\
2 \,\underline{|3} & \text{rem.} = 0 = a_3 & \\
2 \,\underline{|1} & \text{rem.} = 1 = a_4 & \\
0 & \text{rem.} = 1 = a_5 &
\end{array}
$$

	Quotient	Remainder
N= 156 ÷ 2	78	0
78 ÷ 2	39	0
39 ÷ 2	19	1
19 ÷ 2	9	1
9 ÷ 2	4	1
4 ÷ 2	2	0
2 ÷ 2	1	0
1 ÷ 2	0	1

a_0

a_1

$(156)_{10} = (10011100)_2$

a_0

Binary:

$(1101)_2$ → 4th symbol we use after counting 8 items.

2^3

weight $= 2^1$ = why? = We use second symbol after counting 2 items.

 $= (5)_{10} = (101)_2$

Apples

$= (V)$ Roman $= (11111)$ unary

$= (\text{FIVE})$ English

Summary :-

$$(abcde)_2 = (\; ? \;)_{10}$$

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$= a\times 2^4 + b\times 2^3 + c\times 2^2 + d\times 2^1 + e\times 2^0$$

Digital Logic :

Next Sub-Topic :

Decimal <---> Binary Conversion

(Do Quickly; Don't apply the previous methods)

① Remember Binary of 0 — 15.

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101

Decimal	Binary
6	110
7	111
8	1000
9	1001
10	1010

① Remember Binary of 0 — 15.

Decimal	Binary
11	1011
12	1100
13	1101
14	1110
15	1111

Decimal	Binary
6	110
7	111
8	1000
9	1001
10	1010

② Power of 2 :

Decimal	Binary
$2^0 = 1$	1
$2^1 = 2$	10
$2^2 = 4$	100
$2^3 = 8$	1000

$$(2^4)_{10} = (10000)_2$$

$$(2^n)_{10} = (1\underbrace{00\ldots0}_{n\ \text{zeros}})_2$$

$$(2^{10})_{10} = (1024)_{10} = (10000000000)_2$$

3 $(17)_{10} = (?)_2$

$2^4 + 1$

$\rightarrow 16 + \rightarrow 1$

$= 1\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{1}$

$(22)_{10}$

$1\ 0\ 1\ 1\ 0$

$16 + 6$

weight

2^4

$(80)_{10} = 1\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}$

$\rightarrow 64 + 16 = 80$

$16 \rightarrow 2^4$

$2^6 = 64$

6 space

$(11)_{10} = 8 + 3$

1 0 1 1

weight 2^3

$(165)_{10} =$

$\rightarrow 128 + \rightarrow 32$

$1\ 0\ \_\ 1\ 0\ 0\ 1\ 0\ 1 \rightarrow 5$

$2^7 = 128$

7 space

5 space

$= (10100101)_2$

$$(521)_{10} = 1\,\underbrace{0000\,0100\,1}_{9\text{ space}}$$

$512 + 8 + 1$

$512 = 2^9$

$40 = \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{0}$

32 8

$41 = \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{1}$

32 8 1

$(14)_{10} = 1\ 1\ 1\ 0$

$8 + 6$

Bit Permutations

1 bit	2 bits	3 bits	4 bits	
0	00	000	0000	1000
1	01	001	0001	1001
	10	010	0010	1010
	11	011	0011	1011
		100	0100	1100
		101	0101	1101
		110	0110	1110
		111	0111	1111

Each additional bit doubles the number of possible permutations

0

|

8 = 1000 — milestone ✓

9 = 1001

10 = 1010 — milestone ✓

11 = 1011

12 = 1100 — milestone

13 = 1101

14 = 1110

15 = 1111 — milestone

16 = 10000 — "

17 = 10001

Digital Logic :

Recap :

Decimal & Binary Number Systems

Website : https://www.goclasses.in/

$(600)_{10} = (\quad ? \quad)_2 = (1001011000)_2$

$\underbrace{600}$

$512 + 64 + 16 + 8$

1 0 0 1 0 1 1 0 0 0

$\underbrace{\quad\quad\quad\quad}$ 6 space

$\underbrace{512 = 2^9}$; $2^6 = 64$

$(194)_{10} = (\ ?\ )_2 = (11000010)_2$

$128 + 64 + 2$

$\underline{1}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{0}$

$\underbrace{\underline{1}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{0}}_{7}$

7 space

$\underbrace{128 = 2^7}$

$(128)_{10} = (1000\ 0000)_2$

$(250)_{10} = (1111\ 1010)_2$

lower closest

$128 = 2^7$

$256 = 2^8$

Not the lower closest

$$\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\quad \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}$$

$128 + 64 + 32 + 16$

8

2

$2^3 = 8 = 1\ 0\ 0\ 0$

$2^4 = 16 = 1\ 0\ 0\ 0\ 0$

$2^n = 1\ \underbrace{0\ 0\ 0\ 0 \ldots 0}_{n \text{ zeros}}$

$2^n - 1 = \underbrace{1\ 1\ 1 \ldots 1}_{n \text{ ones}}$

$2^3 - 1 = 7 = 111$

$2^4 - 1 = 15 = 1111$

$$(1\,1\,0\,1\,1\,0\,0\,1)_2 = (\,?\,)_{10}$$

$$1\times 2^7 + 1\times 2^6 + 1\times 2^4 + 1\times 2^3 + 1\times 2^0$$

$$= 128 + 64 + 16 + 8 + 1$$

$(1\,0\,1\,1\,0\,0\,0\,1\,0)_2 = (\ ?\ )_{10}$

$2^8 \quad 64 \quad 32 \quad 2$

$$\boxed{256 + 64 + 32 + 2}$$