Partial Order Relations

Next Topic:

Sub-lattice

Deletion of elements from a POSET, Lattice.

Website : https://www.goclasses.in/

Deletion of an element from Base Set in any POSET:

$(N, \leq)$ — Poset

$(E, \leq)$ — Poset

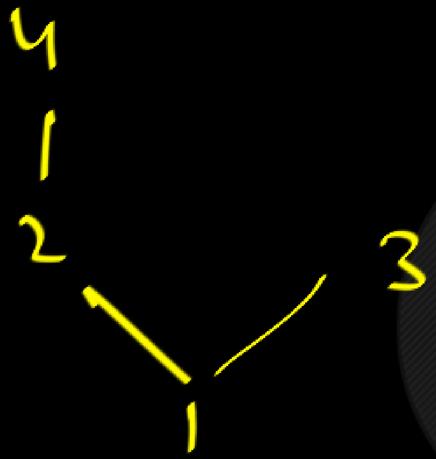
$(\{1,2,3\}, \leq)$ — Poset

Q: POR R on set $A = \{a, b\}$
from R, if I delete some element, can I guarrate that result will be POR? — NO.

$R = \{(a,a), (b,b)\}$; $R_2 = \{(b,b)\}$
└ POR

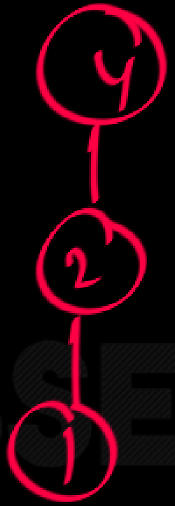
NOT POR on A.

Q: $(\{1, 2, 3, 4\}, |)$



Delete 3 $\longrightarrow$

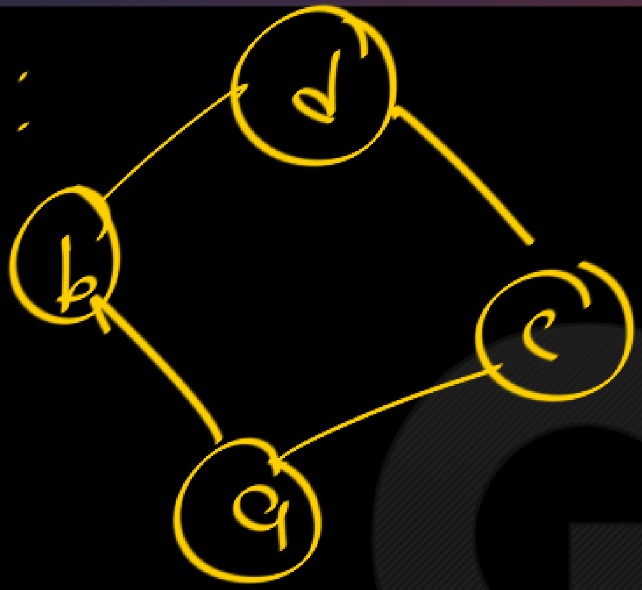
$(\{1, 2, 4\}, |)$



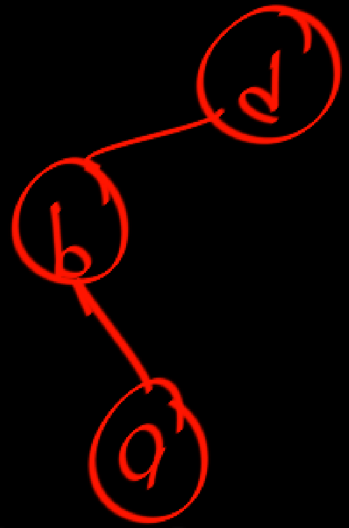
$|R| = 4 + 2 + 1 + 1$
$= 8$

$|R_2| = 6$

Q :



Delete c $\longrightarrow$



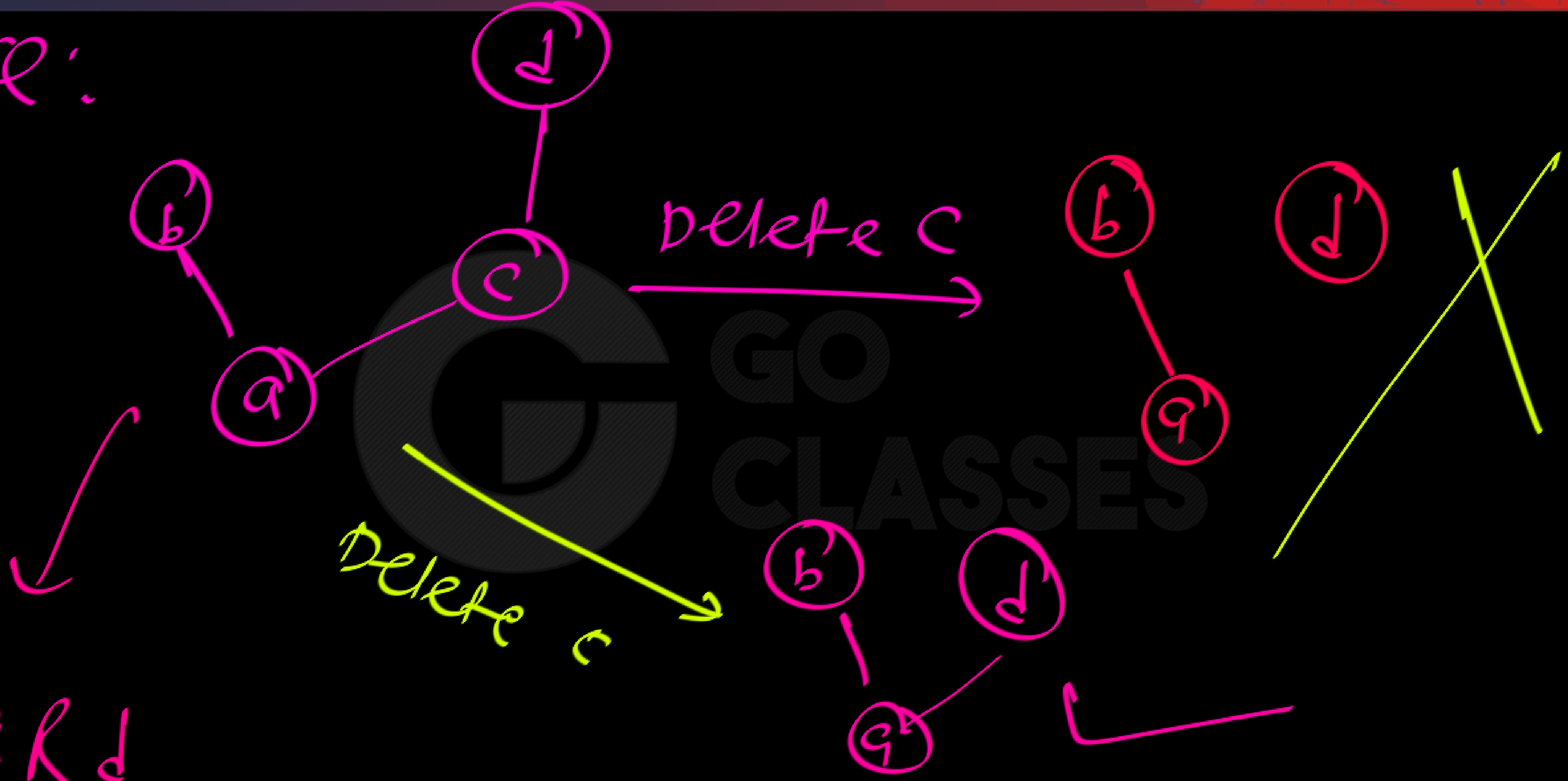
$|R| = 4+2+2+1$

$= 9$

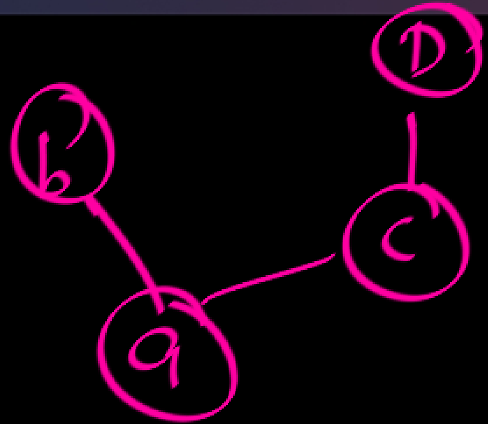
$|R_2| = 6$

$(c,c), (a,c), (c,d)$ gone

Q:



a R d



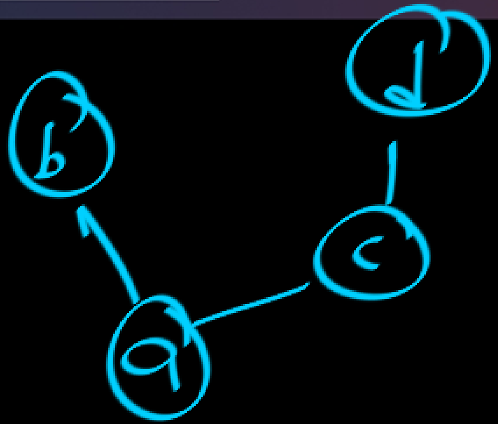
R

$$R = \{(a,a), (a,b), (b,b), (a,c), (a,d), (c,c), (d,d), (c,d)\}$$

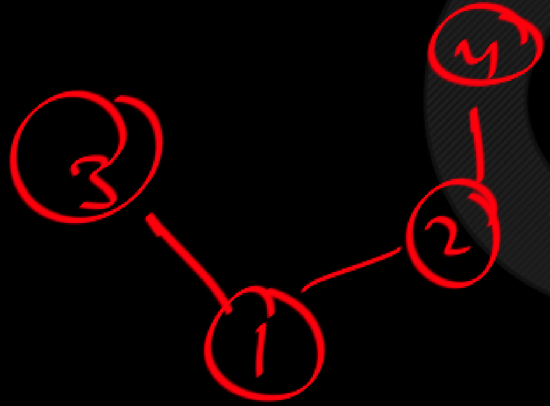
Base set $= \{a, b, c, d\}$

Delete c:

$$R_2 = \{(a,a), (a,b), (b,b), (a,d), (d,d)\}$$

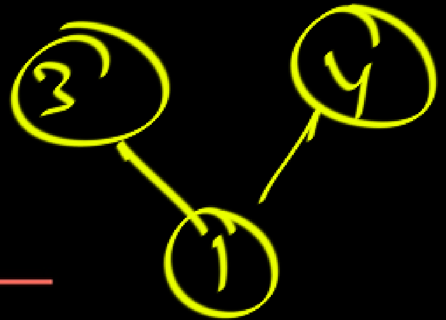


$(\{1, 3, 2, 4\}, |)$ — Poset



Delete 2:

$(\{1, 3, 4\}, |)$ — Poset



1 R 4

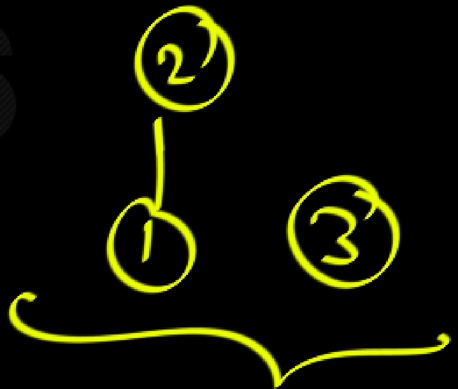
Q: R on $\{1,2,3\}$

$R = \{(1,1),(2,2),(3,3),(1,2)\}$

poset

$R - (2,x),(x,2)$

$= \{(1,1),(3,3)\}$ is poset on $\{1,3\}$



HD of R

Q:

aRf

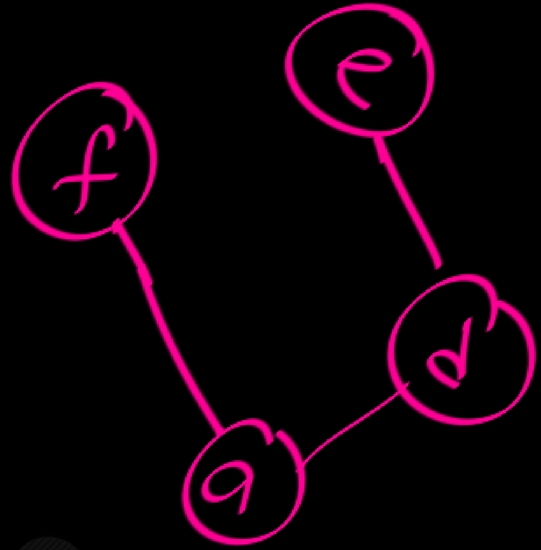
HD of R

PoR

Base Set:

$(\{a, b, c, d, e, f\}, R)$ — Poset

Delete b, c



Base set:

$\{a, d, e, f\}$

Note: If (A, R) is Poset

then (B, R) is also Poset,

$\forall\ B \subseteq A$.

$(\mathbb{Z}, \leq)$ — Poset

$(\mathbb{N}, \leq)$ — Poset

$(E, \leq)$ — Poset

$(\{1,2\}, \leq)$ — Poset

Q: $A = \{R_1, R_2, G_1, G_2, B\}$

R on A; xRy iff x, y have Same Color.

Eq. Relation

R on A —— Eq. Relation

Q: $B = \{ R_1, R_2, \quad G_2, \} \subseteq A$

R on B ; xRy iff x, y have Same Color.

Eq. Relation

R on B —— Eq. Relation

Note: If (A, R) is Eq. Relation then $\forall B \subseteq A$, (B, R) is also Eq. Relation.

Q: $A = \{1,2,3,4\}$

R on A; xRy iff $x \equiv_3 y$

Eq. Relation

Eq. Relation

<u>Eq. Classes</u>: $[1] = \{1,4\}$ $[3] = \{3\}$

$[2] = \{2\}$

Q: $B = \{1, 2, 4\}$ $B \subseteq A$

R on B; xRy iff $x \equiv_3 y$

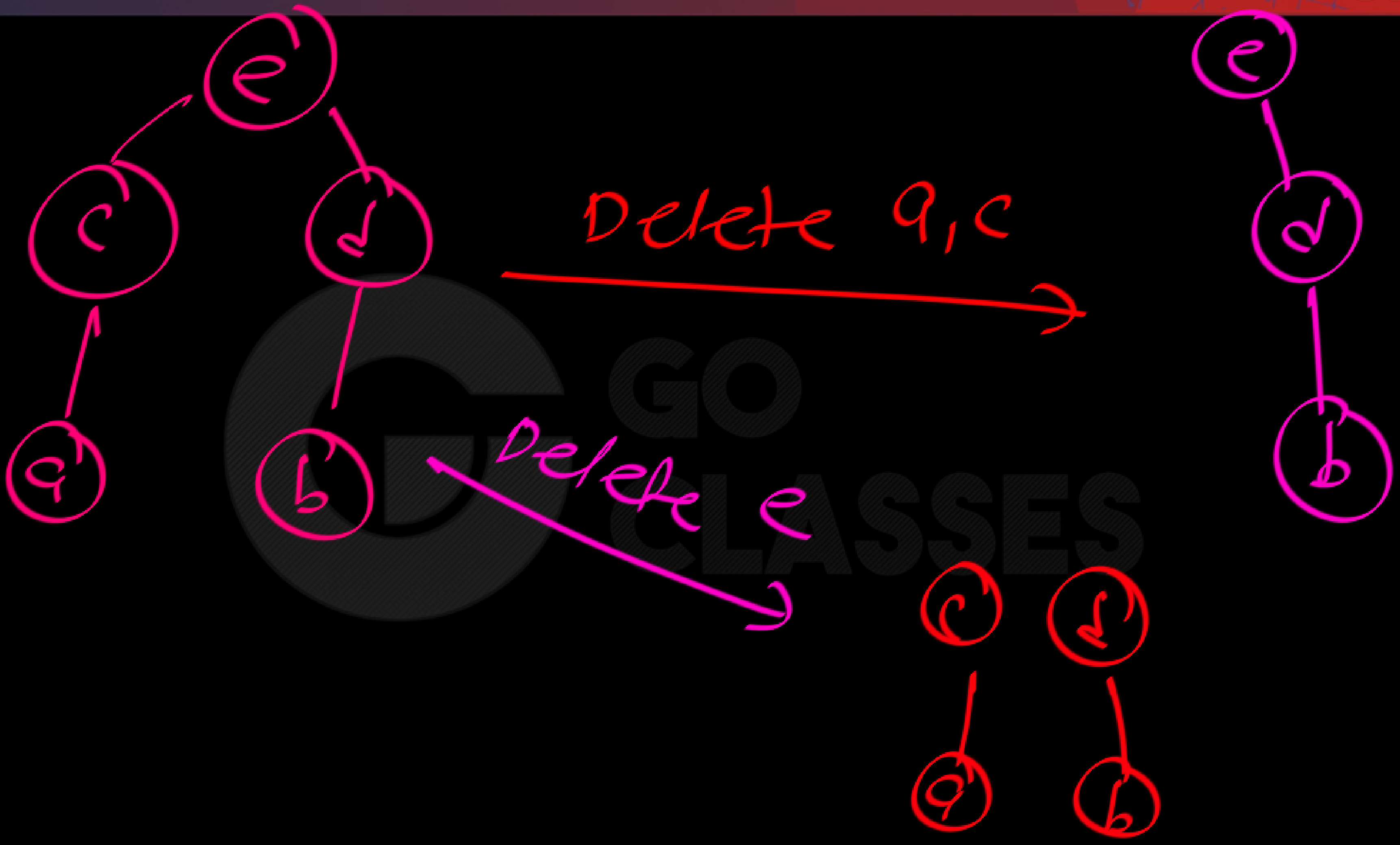
Eq. Relation

Eq. Relation

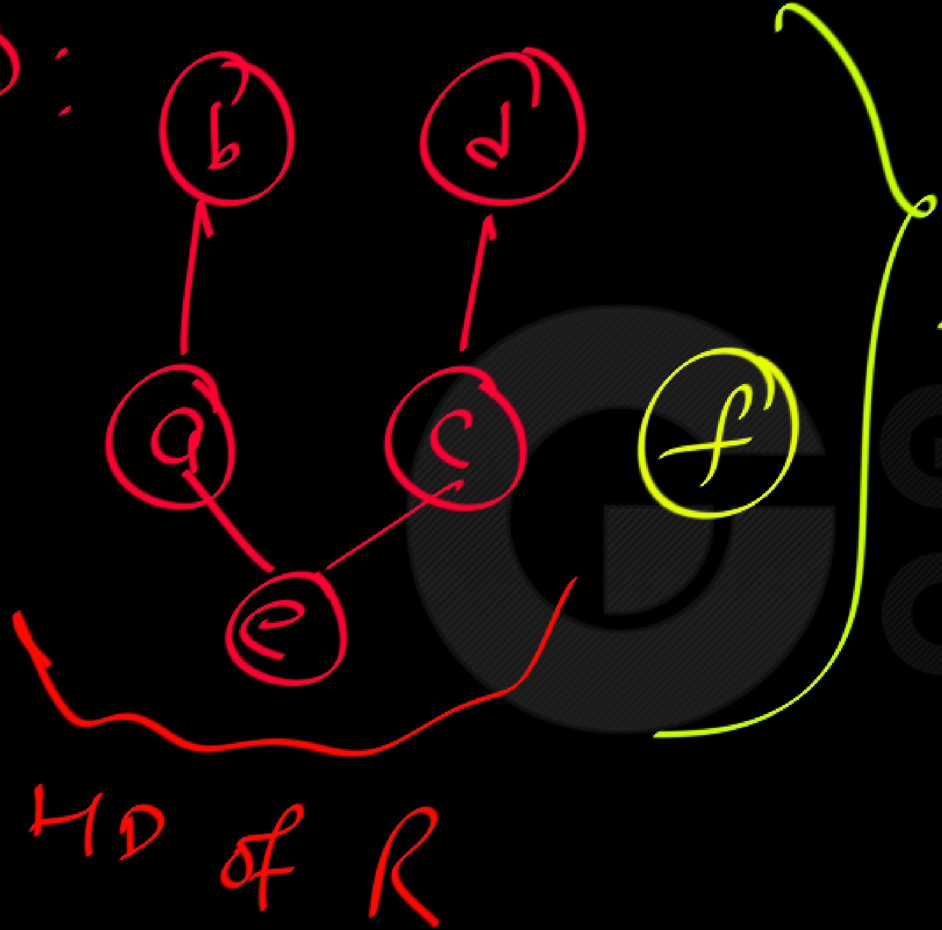
Eq. Classes: $[1] = \{1, 4\}$

$[2] = \{2\}$

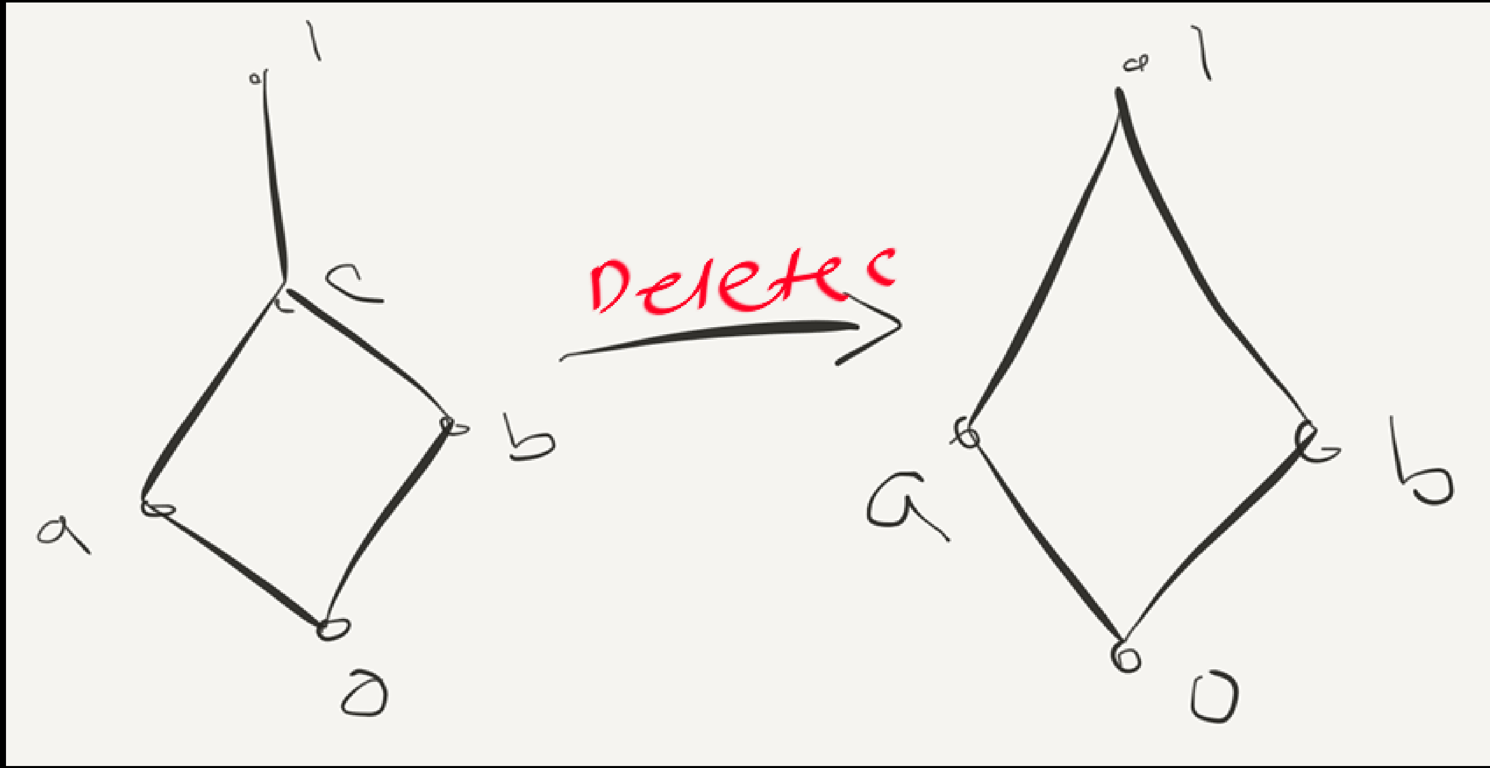
Q:



ϕ :



$\Rightarrow$ $R_2 = R \cup \{(f,f)\}$

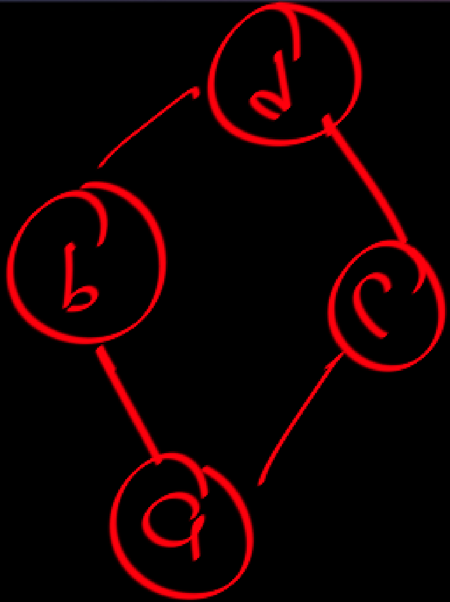


aR1, bR1

Sub-lattice: Lattice "L"

Sub-lattices of L

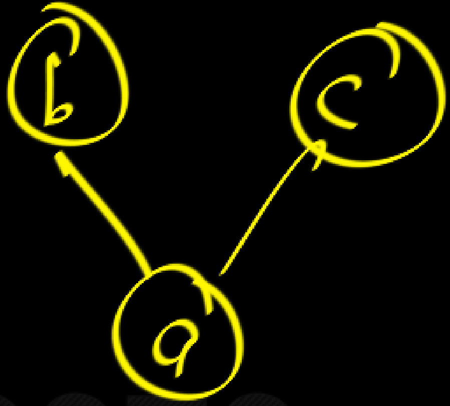
1. subset of L ✓
2. S = lattice ✓
3. GLB, LUB for any a, b ∈ S must be same in L, S.



Lattice
L

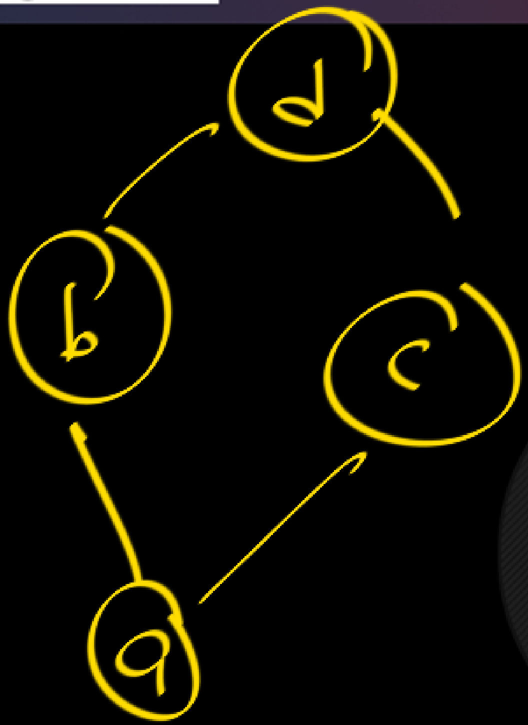
Subset {a,b,c}

means
Delete d



not a sublattice of L

Not even lattice.



lattice
L

Subset $\{a, b, d\}$

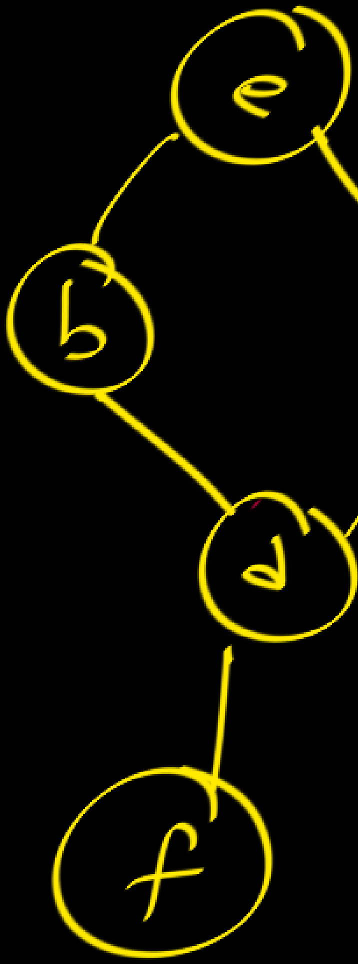
means
Delete c

Subset of L ✓

lattice ✓

S

	L	S
$a \wedge b$	a	a
$a \vee b$	b	b
$a \wedge d$	a	a

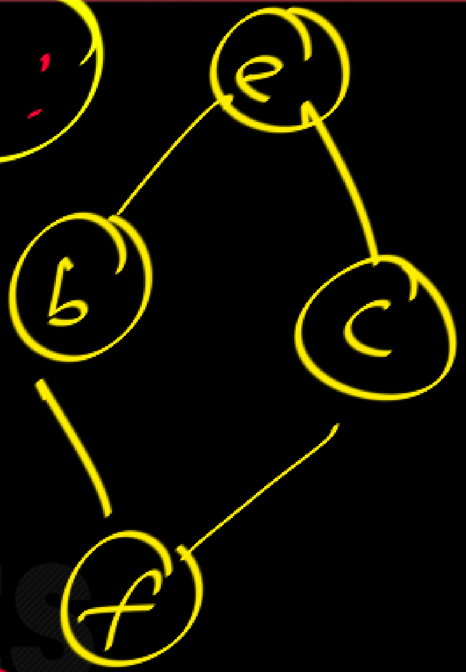


Lattice L

Subset $\{b, c, e, f\}$

Deletion of d

S :



Not Sublattice of L.

	L	S
$b \wedge c$	d	f

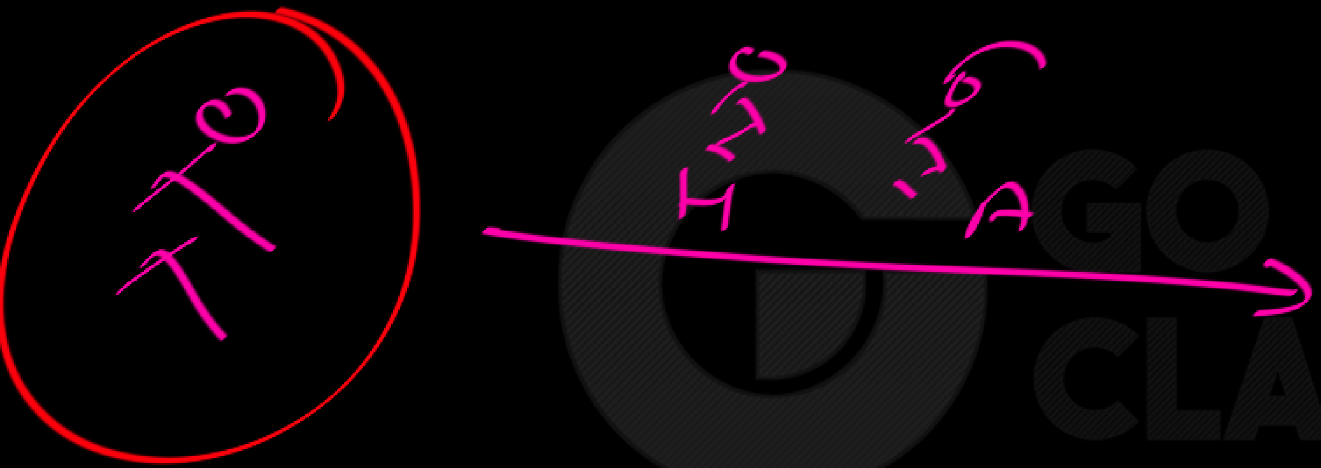
Intuition behind Sublattice : ⇒

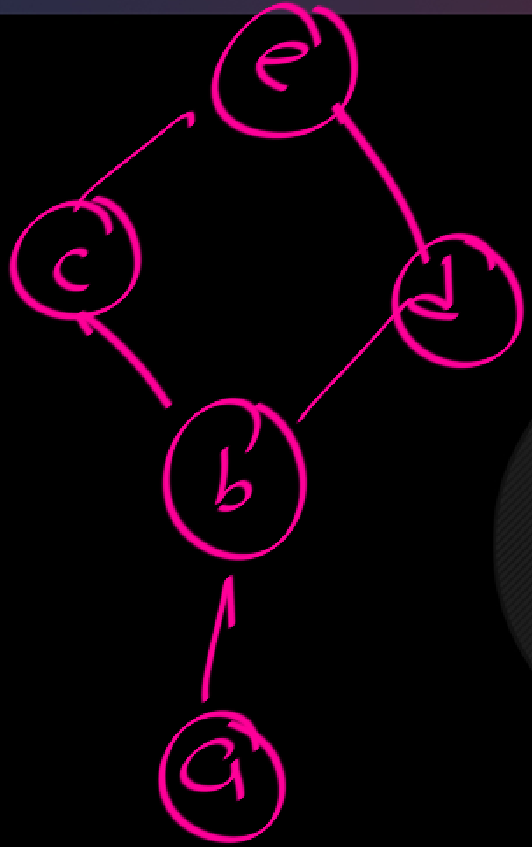
Subset $\{a, b, \ldots\}$

Lattice L

Taking a, b then take their GLB, LUB also.

Hindi Serial :

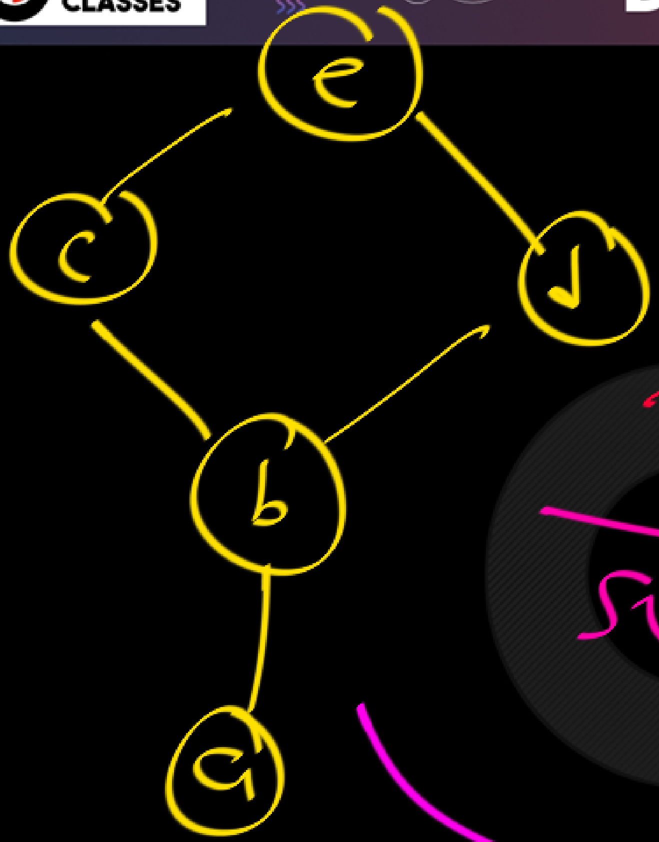




lattice L

subset {a, c, d, e} — Not sublattice of L

because we are taking d, c but not taking d ∧ c.



subset {a, e} — Sublattice

{c, d} Not sublattice

subset {b, c, d, e} Sublattice

subset {a, d, e} Sublattice

Sublattices [edit]

A *sublattice* of a lattice L is a subset of L that is a lattice with the same meet and join operations as L. That is, if L is a lattice and M is a subset of L such that for every pair of elements a, b in M both $a \wedge b$ and $a \vee b$ are in M, then M is a sublattice of L.[2]

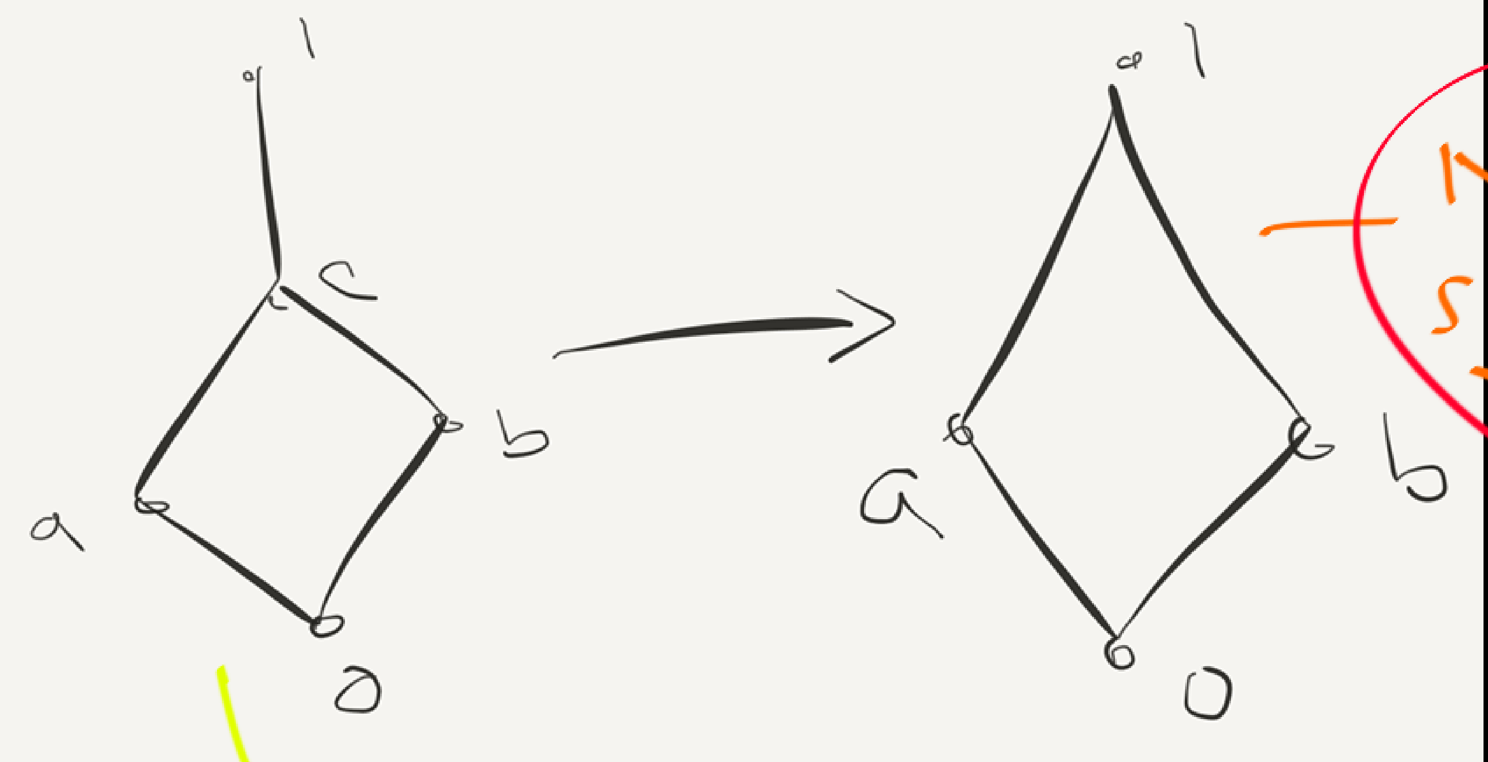
A sublattice of a lattice L is a nonempty subset of L that is a lattice with the same meet and join operations as L.

If S is sublattice of L

then $\boxed{\forall\ a, b \in S}$

	L	S
$a \wedge b$	x	x
$a \vee b$	y	y

$a \vee b = c$



Not Sublattice

$a \vee b = 1$

Subset $\{a, b, 0, 1\}$

taking a, b → Not taking $a \vee b$

The Intuition behind Sublattice :

Lattice L ; you want to take some elements from L and want to create sublattice ;

{ take those elements & take their GLB, LUB also }

Sub-lattices and lattices.

Asked 9 years, 2 months ago Modified 8 years, 3 months ago Viewed 3k times

subset

5

I have read in a textbook that $\mathcal{P}(X)$, the power-set of X under the relation 'contained in' is a lattice. They also said that $S := \{\varnothing, \{1,2\}, \{2,3\}, \{1,2,3\}\}$ is a lattice but not a sub-lattice. Why is it so?

lattice-orders

Sub-lattices and lattices.

Asked 9 years, 2 months ago Modified 8 years, 3 months ago Viewed 3k times

$L = (P(X), \leq)$

subset

5

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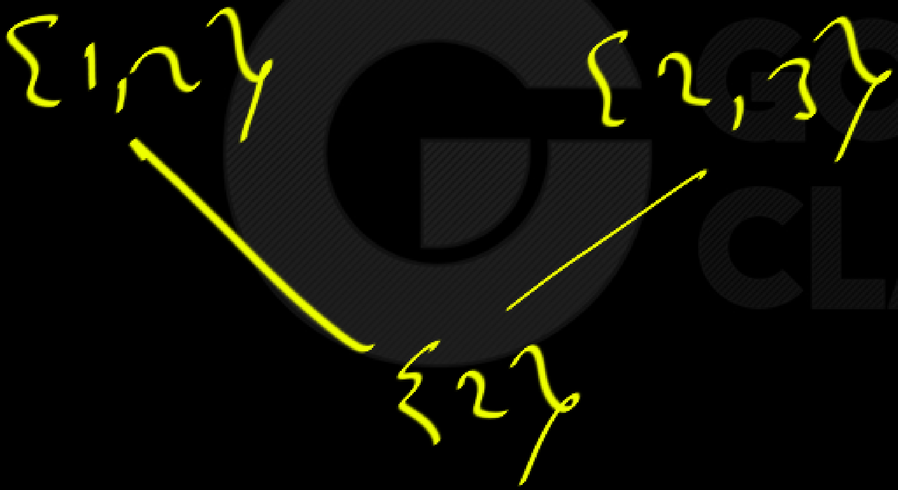
not sublattice of L.

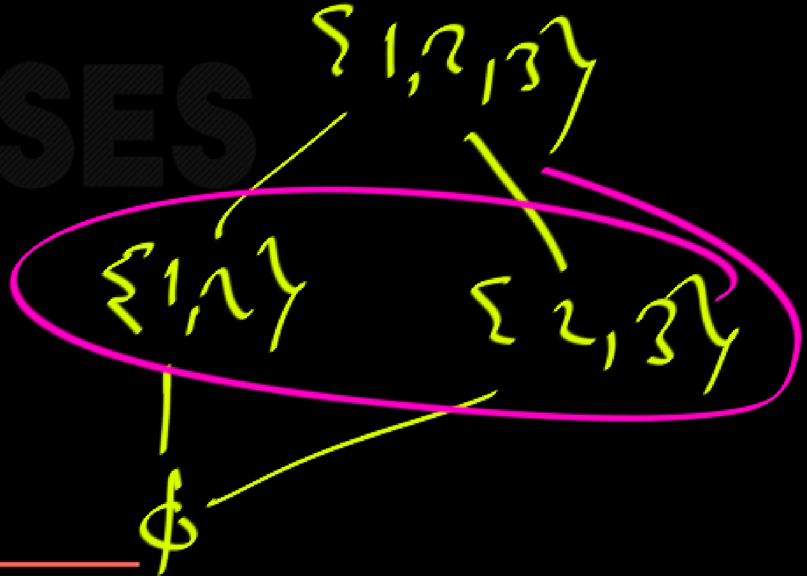
lattice-orders

	L	S
$\{1,2\} \wedge \{2,3\}$	$\{2\}$	ϕ

$(P(\{1,2,3\}), \subseteq)$

$\{\phi, \{1,2\}, \{2,3\}, \{1,2,3\}\}, \subseteq$





Q: R is a partial order relation on some set A (which can be either finite or infinite). Which of the following statements are true?

1) (A,R) has at least one minimal element and at least one maximal element.

2) If (A,R) has a smallest and largest elements, then every two element of A are comparable.

3) If (A,R) has no maximal elements, then A is infinite.

4) if (A,R) has a single minimal element, then it is a smallest element.

5) If every two elements are comparable, then there is a smallest and largest elements.

Q: R is a partial order relation on some set A (which can be either finite or infinite). Which of the following statements are true?

1) (A,R) has at least one minimal element and at least one maximal element. — false $(\mathbb{Z}, \leq)$

2) If (A,R) has a smallest and largest elements, then every two element of A are comparable. — false $(\{1, 3, 5, 15\}, |)$

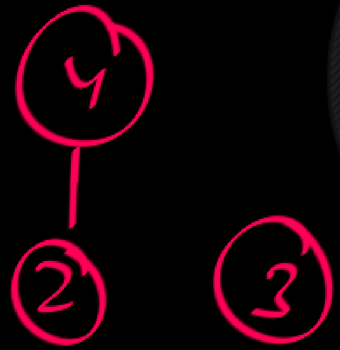
3) If (A,R) has no maximal elements, then A is infinite. — True

4) if (A,R) has a single minimal element, then it is a smallest element. — false

5) If every two elements are comparable, then there is a smallest and largest elements. — false $(\mathbb{Z}, \leq)$

A — false:

$(\{2,3,4\}, |)$



minimal = 2, 3

maximal = 3, 4

$(\mathbb{Z}, \leq)$ — No minimal, No maximal

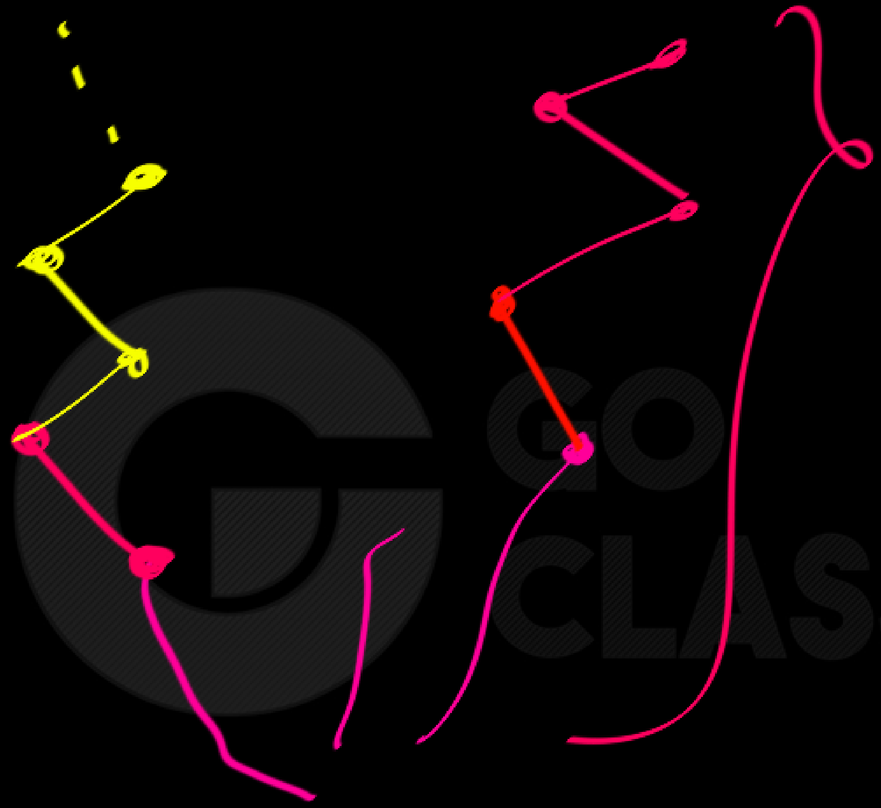
$(\mathbb{N}, \leq)$ — No maximal

$(\mathbb{N}, \geq)$ — no minimal

(2) finite Poset without maximal element: — Not possible

without maximal element: means

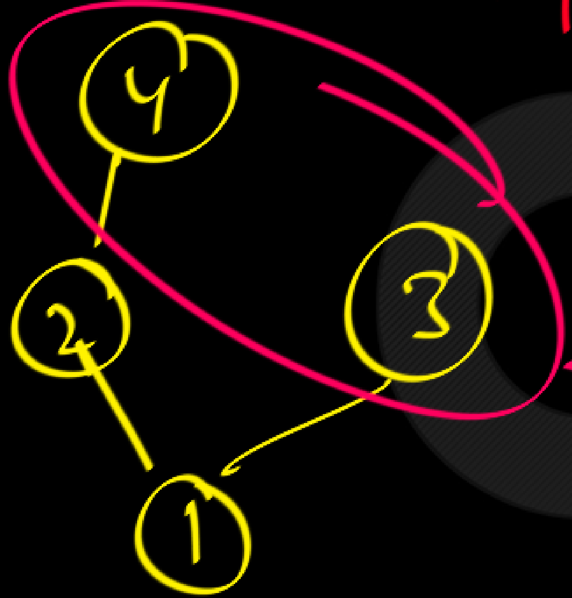
$\neg \exists m$, $m \not< \text{anyone else}$



If no maximal then infinite Poset.

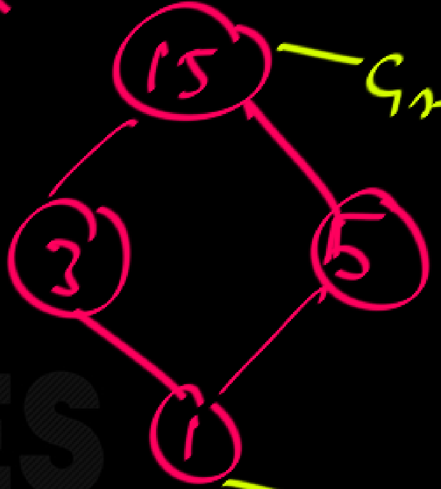
Every finite Poset has at least one maximal, at least one minimal element.

$(\{1, 2, 3, 4\}, |)$



maximal

$(\{1, 3, 5, 15\}, |)$

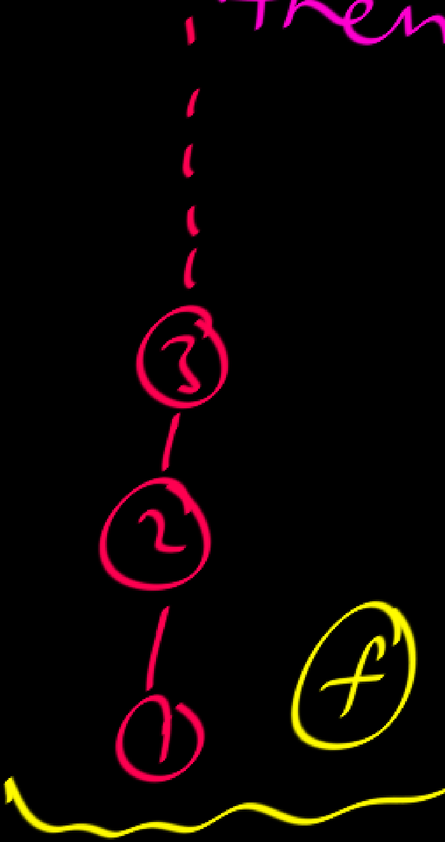


Greatest

least = smallest

Greatest = Largest = maximum = DNE

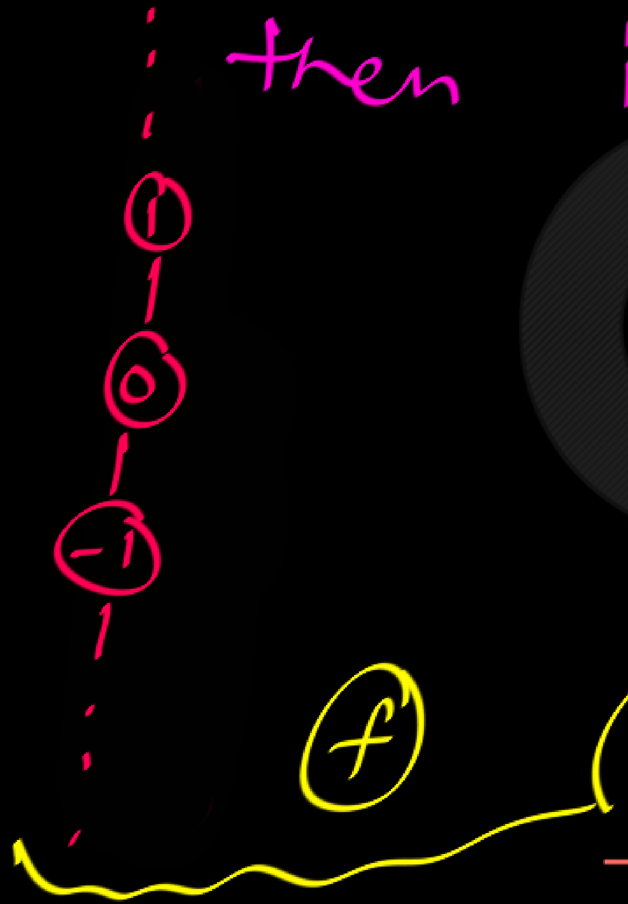
Q: If unique maximal element then it is Greatest "? — NO



maximal = f

Greatest = DNE

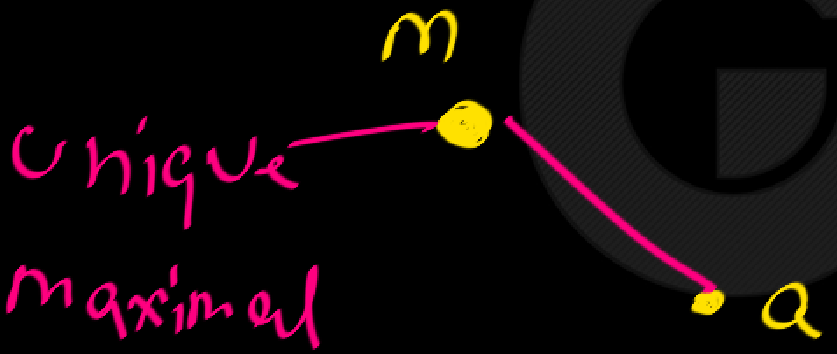
Q: If unique minimal element then it is least " ? — **NO**



minimal = f

least = DNE

Q: In finite Poset, unique maximal then it is Greatest.



$a \not R m$

In finite poset :

Unique maximal = Greatest
" minimal = Least