Next Topic:

Properties Satisfied by ALL Lattices

Properties Satisfied by EVERY Lattice:

PROPOSITION 7.2-2: ***Basic Operational Properties of Meet and Join***

Let $\mathcal{A}$ be a lattice with order relation $\leq$. Then the following properties hold:

a) ***Commutativity***: $x \wedge y = y \wedge x$; $\quad x \vee y = y \vee x$.

b) ***Associativity***: $(x \wedge y) \wedge z = x \wedge (y \wedge z)$; $\quad (x \vee y) \vee z = x \vee (y \vee z)$.

c) ***Idempotence***: $x \wedge x = x$; $\quad x \vee x = x$.

d) ***Absorption***: $x \wedge (x \vee y) = x$; $\quad x \vee (x \wedge y) = x$.

Proof: Proved in the previous lectures.

Proposition Every Lattice Satisfies these:

Any lattice has the following properties:

1. *Commutativity:* $a \cap b = b \cap a$ *and* $a \cup b = b \cup a$.
2. *Associativity:* $(a \cap b) \cap c = a \cap (b \cap c)$ *and* $(a \cup b) \cup c = a \cup (b \cup c)$.
3. *Idempotent law:* $a \cap a = a$ *and* $a \cup a = a$.
4. *Absorption law:* $(a \cup b) \cap a = a$ *and* $(a \cap b) \cup a = a$.

Properties Satisfied by EVERY Lattice:

PROPOSITION 7.2-1: ***Basic Order Properties of Meet and Join***

Let $\langle \mathcal{A}, \leq \rangle$ *be a lattice. Then*

a) $x \wedge y \leq \{x, y\} \leq x \vee y$.

b) $x \leq y$ *iff* $x \wedge y = x$.

c) $x \leq y$ *iff* $x \vee y = y$.

In Any Lattice L;

let $a, b \in L$; $a \neq b$

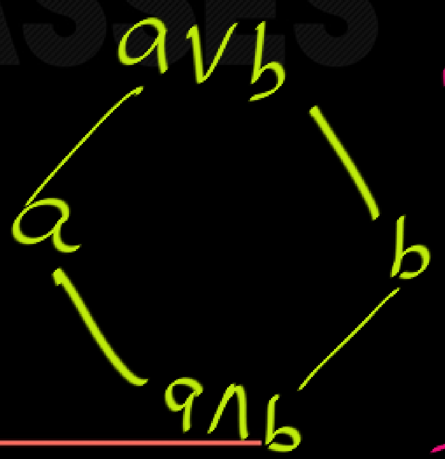
Case 1: a, b Comparable

Assume aRb

$b = a \vee b$

$a = a \wedge b$

Case 2: a, b NOT Comparable



In Any Lattice L;

let $a, b \in L$; $a \neq b$

Case 1: a, b Comparable

Assume aRb

$(a \wedge b) R b$

$(a \wedge b) R a$

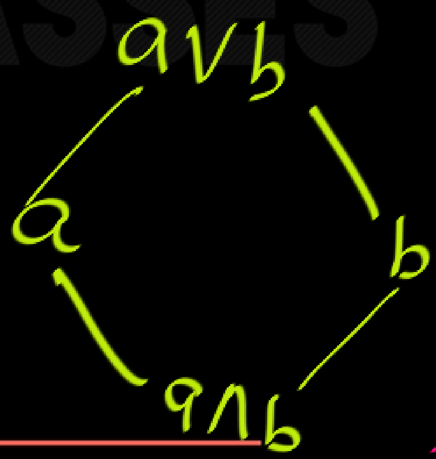
$aR(a \vee b)$

$bR(a \vee b)$

$b = a \vee b$

$a = a \wedge b$

Case 2: a, b NOT Comparable



$aR(a \vee b)$

$bR(a \vee b)$

$bR(a \wedge b)$

$(a \wedge b) R a$

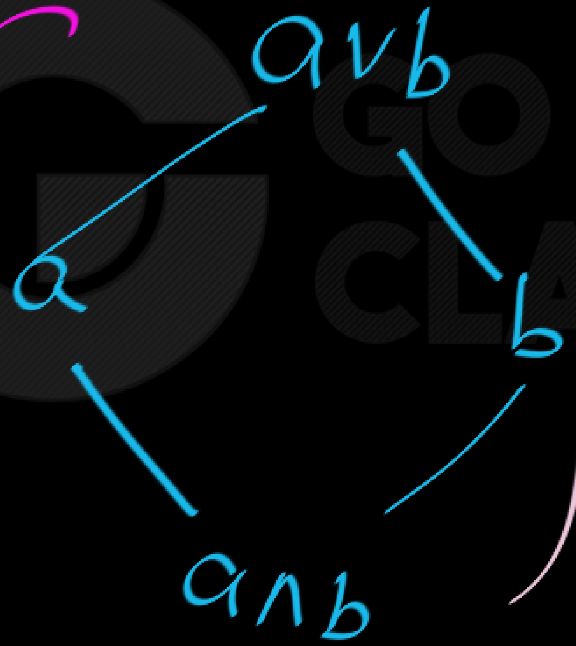
$(a \wedge b) R b$

In Every lattice L :

$a, b \in L$

$b \; R \; (a \vee b)$

$(a \wedge b) \; R \; b$



$a \; R \; (a \vee b)$

$(a \wedge b) \; R \; a$

$(a \wedge b) \; R \; (a \vee b)$

Lattice L;

$$\forall\, x, y \in L \; \left(\begin{array}{l} x \, R \, (x \vee y) \;\; \& \\ (x \wedge y) \, R \, x \;\; \& \\ (x \wedge y) \, R \, (x \vee y) \end{array} \right)$$

Properties Satisfied by EVERY Lattice:

PROPOSITION 7.2-1: ***Basic Order Properties of Meet and Join***

Let $\langle \mathcal{A}, \leq \rangle$ *be a lattice. Then* symbol for POR R

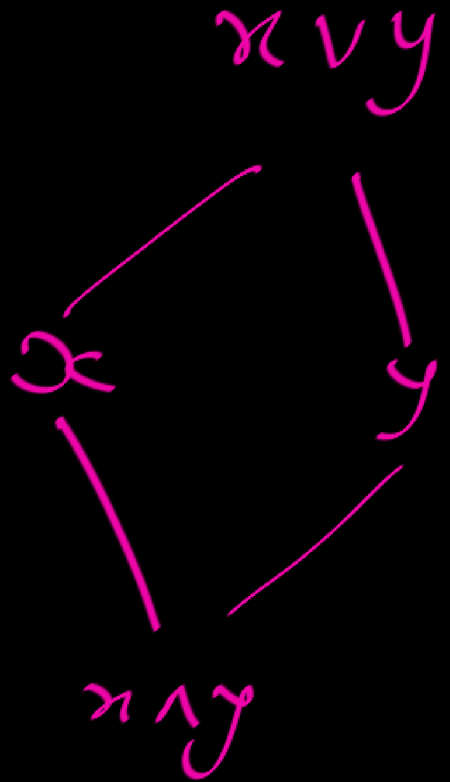
a) $x \wedge y \leq \{x, y\} \leq x \vee y$.

$(x \wedge y) \; R \; x \; R \; x \vee y$

$(x \wedge y) \; R \; y \; R \; x \vee y$

Lattice L;

$\forall x, y \in L$



$(x \wedge y) \, R \, x \, R \, (x \vee y)$

$(x \wedge y) \, R \, y \, R \, (x \vee y)$

$\forall$ Lattice L

$$(x \wedge y) \, R \, x \quad R \, (x \vee y)$$

$$(x \wedge y) \, R \, y \quad R \, (x \vee y)$$

Properties Satisfied by EVERY Lattice:

PROPOSITION 7.2 - 1: ***Basic Order Properties of Meet and Join***

Let $\langle \mathcal{A}, \leq \rangle$ be a lattice. Then

a) $x \wedge y \leq \{x, y\} \leq x \vee y$.

b) $x \leq y$ *iff* $x \wedge y = x$.

(b)

xRy iff $x \wedge y = x$

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(b) $xRy \xrightarrow{(1)} x \wedge y = x$

Proof: If xRy

$y = x \vee y$

$x = x \wedge y$

If xRy then $x \wedge y = x$

(b) xRy $\leftarrow$ $x \wedge y = x$

If $(x \wedge y) = x$

In every lattice:

$(x \wedge y) R y$

$\Rightarrow$ xRy

$\boxed{xRy}$

$(x \wedge y) R y$

$\boxed{xRy}$

In every lattice:

$(x \wedge y) \; R \; (y) \; R \; (x \vee y)$

$\boxed{(x \wedge y) R (y)}$

Properties Satisfied by EVERY Lattice:

PROPOSITION 7.2-1: ***Basic Order Properties of Meet and Join***

Let $\langle \mathcal{A}, \leq \rangle$ be a lattice. Then

a) $x \wedge y \leq \{x, y\} \leq x \vee y.$

b) $x \leq y$ *iff* $x \wedge y = x.$

c) $x \leq y$ *iff* $x \vee y = y.$

xRy

(c) xRy iff $(x \vee y) = y$

(1)

(2)

(c) $xRy \rightarrow (x \vee y) = y$

If xRy

$\boxed{y = x \vee y}$

$x = x \wedge y$

If xRy then $y = x \vee y$

(c) $xRy \leftarrow (x \vee y) = y$

If $(x \vee y) = y$

In every lattice:

$xR(x \vee y)$

xRy

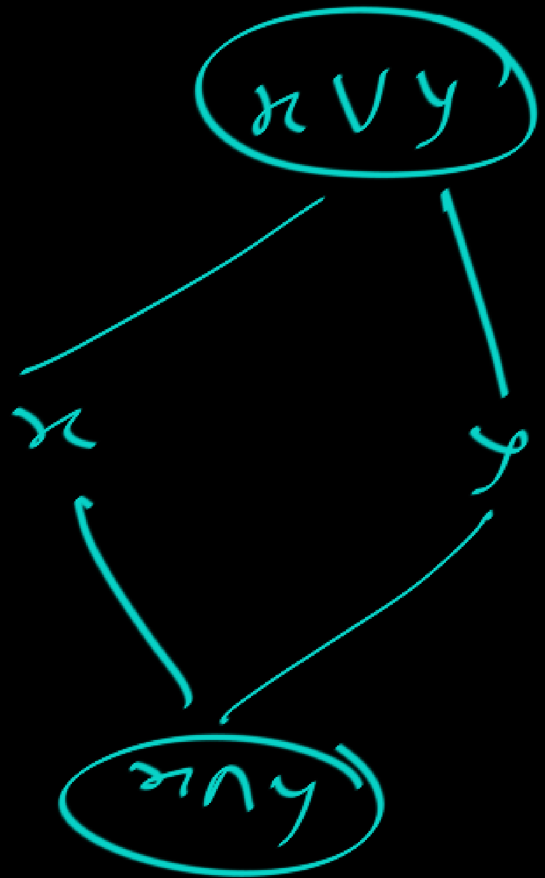
$\boxed{xRy}$

In every Lattice

$xR(x \vee y)$

xRy

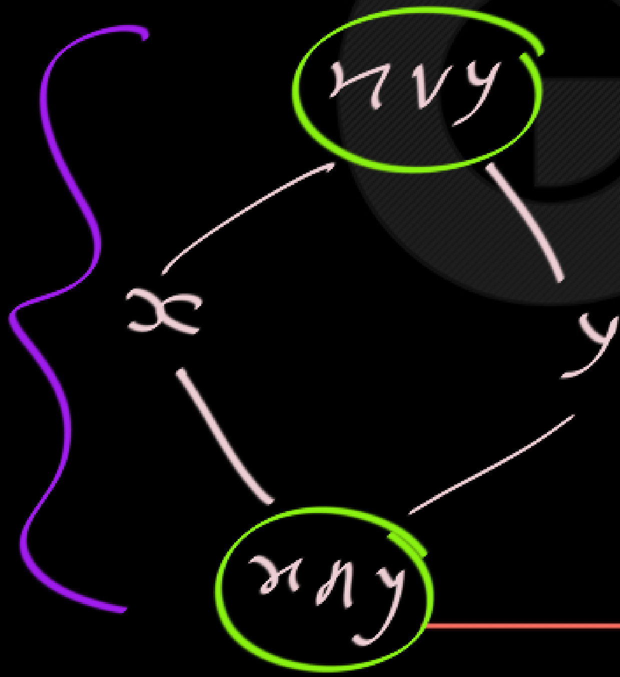
Every
Lattice:



$x \, R \, (x \vee y)$

Note: In Every Lattice L

$\forall x, y \in L$



$x \, R \, (x \vee y)$

$y \, R \, (x \vee y)$

$(x \wedge y) \, R \, x$

$(x \wedge y) \, R \, y$

$(x \wedge y) \, R \, (x \vee y)$