First Order Logic

Homework 1

Quantifiers

- By Deepak Poonia (IISc Bangalore)

Instructor:

Deepak Poonia
IISc Bangalore
GATE CSE AIR 53; AIR 67; AIR 107; AIR 206;

Discrete Mathematics Complete Course:

https://www.goclasses.in/courses/Discrete-Mathematics-Course

On "GATE-Overflow" Website

Join GO Classes **GATE CSE Complete Course** now:

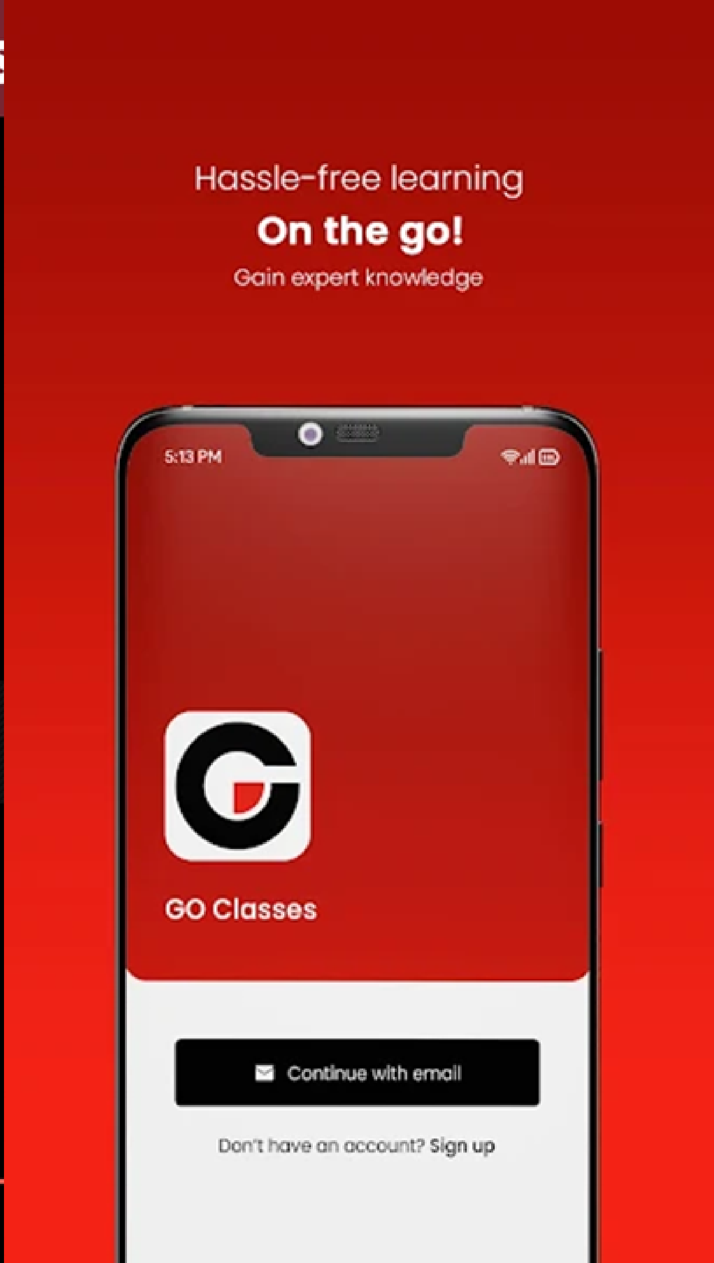
https://www.goclasses.in/s/pages/gatecompletecourse

1. Quality Learning: No Rote-Learning. Understand Everything, from basics, In-depth, with variations.
2. Daily Homeworks, Quality Practice Sets, Weekly Quizzes.
3. Summary Lectures for Quick Revision.
4. Detailed Video Solutions of Previous ALL GATE Questions.
5. Doubt Resolution, Revision, Practice, a lot more.

Download the GO Classes Android App:

https://play.google.com/store/apps/details?id=com.goclasses.courses

Search “GO Classes” on Play Store.



NOTE :

Complete Discrete Mathematics & Complete Engineering Mathematics Courses, by GO Classes, are **FREE** for ALL learners.

Visit here to watch : https://www.goclasses.in/s/store/

SignUp/Login on Goclasses website for free and start learning.

We are on Telegram. **Contact us** for any help.

Link in the Description!!

Join GO Classes **Doubt Discussion** Telegram Group :

Username:

@GATECSE_GOCLASSES

We are on Telegram. **Contact us** for any help.

Join GO Classes Telegram Channel, Username: @GOCLASSES_CSE

Join GO Classes **Doubt Discussion** Telegram Group :

Username: @GATECSE_Goclasses

(Any doubt related to Goclasses Courses can also be asked here.)

Join GATEOverflow Doubt Discussion Telegram Group :

Username: @GateOverflow_CSE

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction
Oscar Levin, 3rd Edition, Exercise 0.2.12

12. Let $P(x)$ be the predicate, $\boxed{\text{“}3x + 1 \text{ is even.”}}$

(a) Is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

$P(5)$: True

Domain

$5, \ldots$

$P(5) = \text{True}$

$\exists x\, P(x)$: True

At least one element satisfies property P → YES

True

12. Let $P(x)$ be the predicate, "$3x + 1$ is even."

(a) Is $\boxed{P(5) \text{ true}}$ or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

Can't Say anything

12. Let $P(x)$ be the predicate, $\boxed{\text{"}3x + 1 \text{ is even."}}$

(a) Is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

No

Can't say Anything

$P(x)$: $\boxed{3x+1 \text{ is even}}$

(a) $P(5)$: $\boxed{3\times 5+1 \text{ is even}}$ True

True

(b) $\boxed{\exists_x P(x)}$: True ; witness : 5

(c) $\forall_x P(x)$: Can't say anything

Q1:

Domain

a, b, c

P(x) Predicate

Given: P(a) : True

$\exists x P(x)$: True

$\forall x P(x)$: Can't Say Anything

Q1:

Domain

a, b, c
- - -

P(x) Predicate

Given: P(a) : false

$\exists x\, P(x)$: Can't say Anything

$\forall x\, P(x)$: False

Q1:

a, b, c
- - - .

Domain

$P(x)$ Predicate

Given: $P(a)$:

$\boxed{P(5) : \text{True}}$ ✓ ;

$P(x) : \boxed{3x+1 \text{ is even}}$

$\boxed{\exists x\, P(5) \equiv P(5) = T}$ $\exists x\, P(x) \equiv \text{True}$

$\boxed{\forall x\, P(5) \equiv P(5) = T}$ $\forall x\, P(x)$ Can't Say Anything

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction
Oscar Levin, 3rd Edition, Exercise 0.2.13

13. Let $P(x)$ be the predicate, "$4x + 1$ is even."

(a) Is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

13. Let $P(x)$ be the predicate, $\boxed{\text{"}4x + 1 \text{ is even."}}$

(a) Is $P(5)$ true or false? P(5) : false

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

false

Domain

5, - - -

$\boxed{P(5) : \text{false}}$

$\boxed{\forall x P(x)}$: false

$\boxed{\exists x P(x)}$: Can't say Anything ✓

Note:

Given: $\boxed{P(a) : \text{True}}$

Domain

$a, b, c, \_\_\_\_$

$P(x)$: Predicate

1. $\forall x P(x)$: Can't determine
2. $\exists x P(x)$: True
3. $\forall x \neg P(x)$: False
4. $\exists x \neg P(x)$: Can't determine

Note:

Given: $\boxed{P(a) : \text{false}}$

Domain

a, b, c,

_ _

_ _

P(x) : Predicate

1. $\exists x P(x)$: Can't Determine
2. $\exists x \neg P(x)$: True
3. $\forall x P(x)$: false
4. $\forall x \neg P(x)$: Can't Determine

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Kenneth H. Rosen,

Discrete Mathematics and Its Applications,

Seventh Edition, Exercise 1.4 Question 13

13. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n+1>n)$

b) $\exists n(2n=3n)$

c) $\exists n(n=-n)$

d) $\forall n(3n\leq 4n)$

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.14

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

Ans

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.16

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

No element is both P and Q:

$\equiv$ for every elements x, $\boxed{\boxed{P(x) \wedge Q(x)} \text{ is false.}}$

$\equiv \forall x \left(\neg \left(P(x) \wedge Q(x) \right) \right)$

$\equiv \forall x \left(\overline{P(x)} \vee \overline{Q(x)} \right) \equiv \forall x \left(P(x) \rightarrow \overline{Q(x)} \right)$

No element is both P & Q.

$$\boxed{\forall x \left(P(x) \oplus Q(x) \right)}$$

WRONG

$P ✓, Q ✗$

$P ✗, Q ✓$

Allow-ed

$\boxed{a : P = F, \; Q = F}$

No person has both car & bus both.

Allowed

a : car ✓ bus ✗

b : car ✗ bus ✓

c : car ✗ bus ✗

d : car ✓ bus ✓

Not Allowed

No element is both P & Q.

① $\forall x \left(P(x) \oplus Q(x) \right)$ ✗

② $\forall x \left(P(x) \longrightarrow \overline{Q}(x) \right)$ ✓

③ $\forall x \left(\overline{P(x)} \longrightarrow Q(x) \right)$ ✗

No element is both P & Q.

$$\forall x \left(P(x) \longrightarrow \neg Q(x) \right) \checkmark$$

$$\forall x \left(Q(x) \longrightarrow \neg P(x) \right) \checkmark$$

Same

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

Represent this statement

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

No number is both even & odd.

$$\forall x \left(E(x) \longrightarrow \neg O(x) \right)$$ ✓

There does not exist an element x, such that $E(x) \wedge O(x)$ is True.

$$\neg \exists x \left(E(x) \wedge O(x) \right)$$

$\boxed{\text{Some integer is prime}}$

$\exists x \left(\text{int}(x) \wedge \text{prime}(x) \right)$

$\boxed{\text{2 is prime}}$

Template:

$\boxed{\text{No element is both } P \text{ \& } Q.}$

(1) $\forall x \left(P(x) \rightarrow \overline{Q(x)} \right)$

(2) $\forall x \left(Q(x) \rightarrow \neg P(x) \right)$

(3) $\neg \exists x \left(P(x) \wedge Q(x) \right)$

Correct

Template:

$\boxed{\text{No element is both P \& Q.}}$

① $\forall x \left(P(x) \oplus Q(x) \right)$ ✗

② $\forall x \left(\neg Q(x) \rightarrow P(x) \right)$ ✗

WRONG

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

Represent This in FOL

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

(b) $\forall x \left(E(x) \rightarrow O(x+1) \right)$ ✓

(c) $\exists x \left(Prime(x) \wedge Even(x) \right)$ ✓

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) $\boxed{\text{No number is both even and odd.}}$

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

a $\boxed{}$ a+1 → No number

D : b/w any two different numbers, there is a third different number.

True for real

false for integers

D : b/w any two different numbers,
there is a third different number.

$\forall_x \forall_y \Big((x \neq y) \longrightarrow$ there is some z b/w x, y $\Big)$

D : b/w any two different numbers, there is a third different number.

$$\forall x \forall y \left((x \neq y) \longrightarrow \exists z \left((x < z < y) \vee (y < z < x) \right) \right)$$

(e) No number b/w x, $x+1$.

$$\forall_x \left(\boxed{\text{No } z \text{ such that } x < z < x+1} \right)$$

$$\neg \exists_z \left(x < z < x+1 \right)$$

(e) No number b/w x, $x+1$.

$$\forall_x \left(\neg \exists_z \left(x < z < x+1 \right) \right)$$

$$\equiv \forall_x \forall_z \neg \left(x < z < x+1 \right)$$

Same

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.17

17. Translate into English:

(a) $\forall x(E(x) \rightarrow E(x+2))$.

(b) $\forall x \exists y(\sin(x) = y)$.

(c) $\forall y \exists x(\sin(x) = y)$.

(d) $\forall x \forall y(x^3 = y^3 \rightarrow x = y)$.

for every even x, $x+2$ is also even.

17. Translate into English:

(a) $\forall x(E(x) \rightarrow E(x+2))$.

(b) $\forall x \exists y(\sin(x) = y)$.

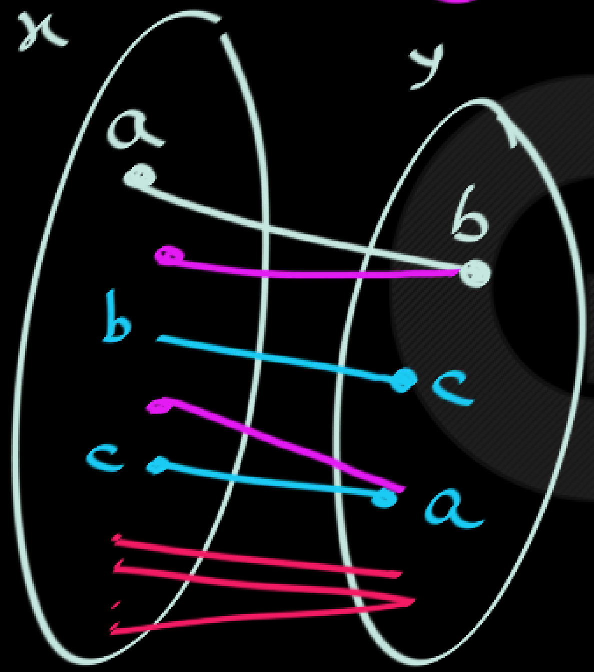
(c) $\forall y \exists x(\sin(x) = y)$.

(d) $\forall x \forall y(x^3 = y^3 \rightarrow x = y)$.

for every pair (x,y) if $x^3 = y^3$ then $x = y$

for every pair (x,y), if $x \neq y$ then $x^3 \neq y^3$.

(b) $\forall_x \exists_y \left(\sin(x) = y \right)$



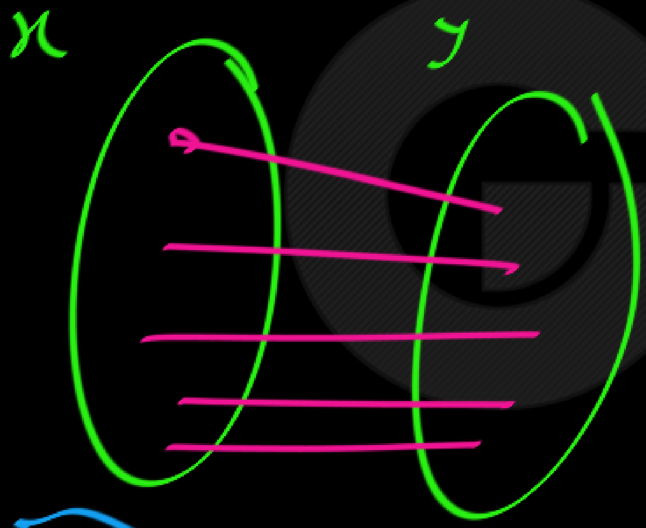
$\sin(a) = b$

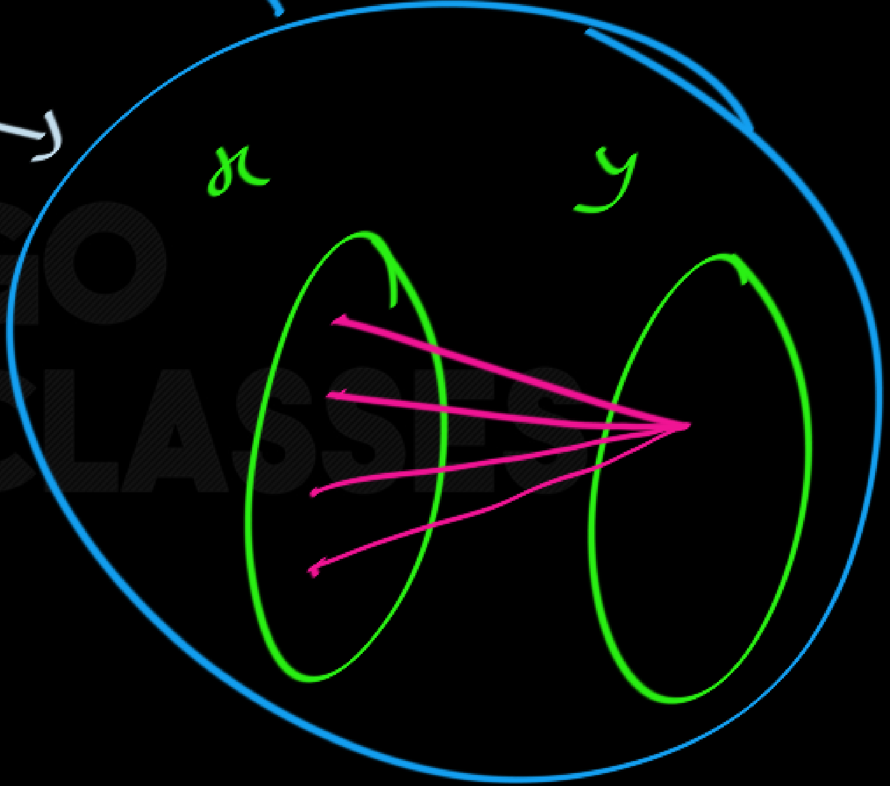
$\sin(b) = c$

$\sin(c) = a$

$\forall_x \exists_y$

$\forall x \, \exists y$

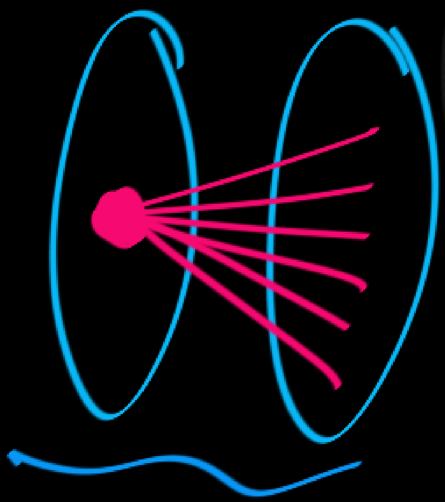




General picture

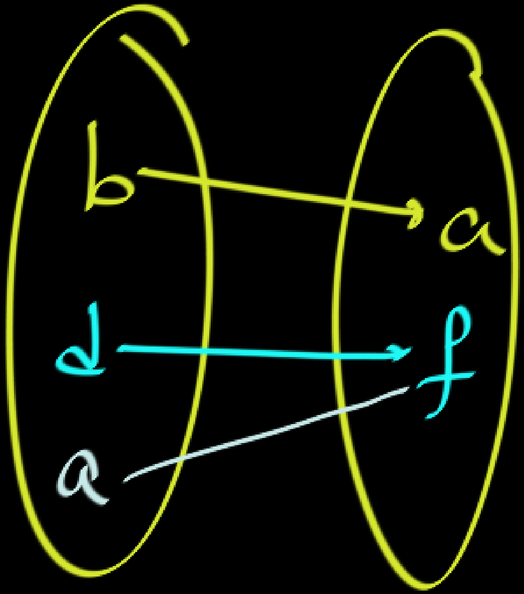
General Pictures:

$\exists \forall$



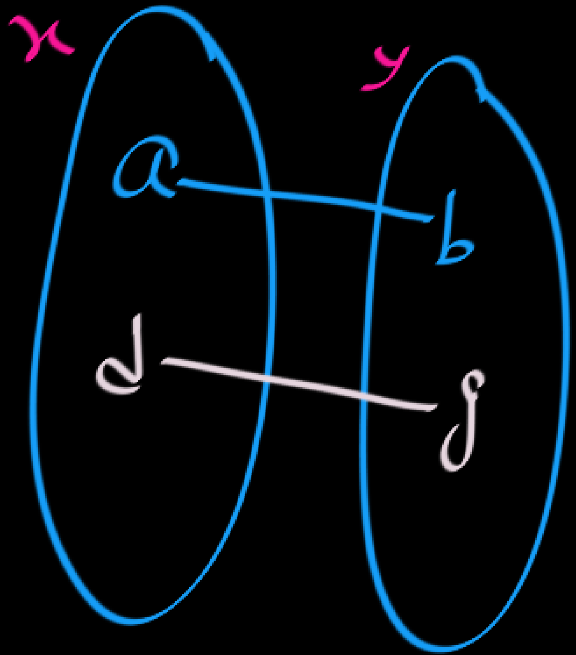
$\forall \exists$

(c) $\forall_y \exists_x (\sin(x) = y)$



$$\left.\begin{aligned}\sin(a) &= b \\ \sin(f) &= d \\ \sin(f) &= a\end{aligned}\right\}$$

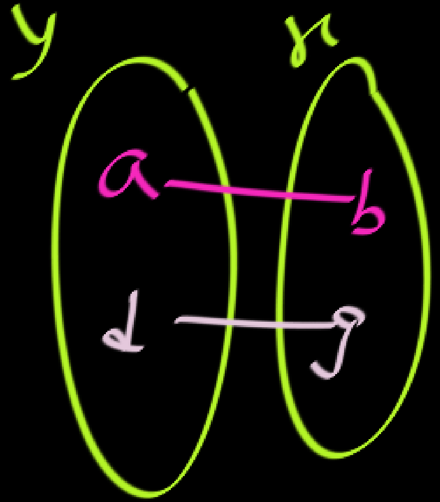
$\forall x \, \exists y \; P(x,y)$



$P(a,b) = T$

$P(d,g) = T$

$\forall y \, \exists x \; P(x,y)$



$P(b,a) = T$

$P(g,d) = T$

$\forall x\, \exists y\; P(x,y)$

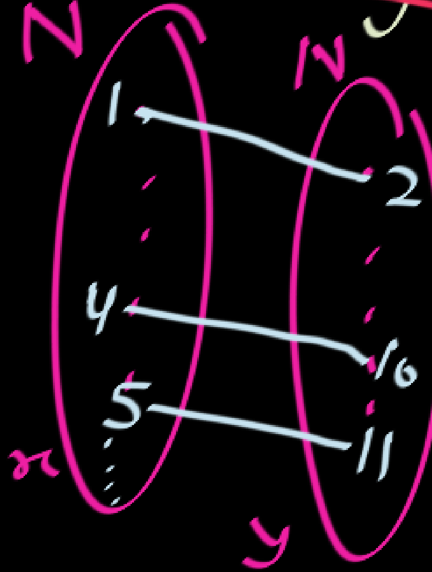
T/F ?

$\forall y\, \exists x\; P(x,y)$

T/F ?

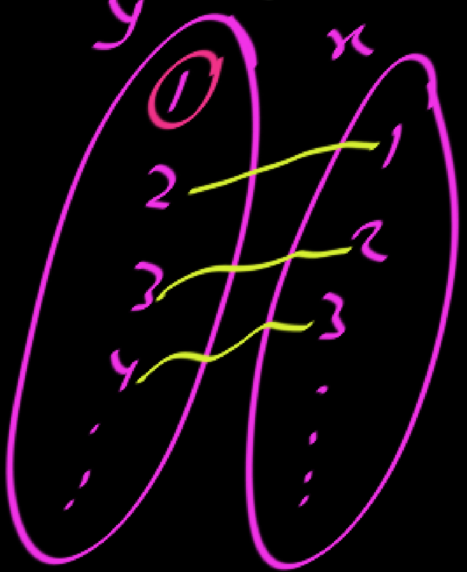
$\boxed{P(x,y) : x < y}$ Domain : $N = \{1, 2, 3, --\}$

$\boxed{\forall x \exists y \; P(x,y)}$: True



$x<y$

$\forall y \exists x \; P(x,y)$: False



$x<y$

No one for 1

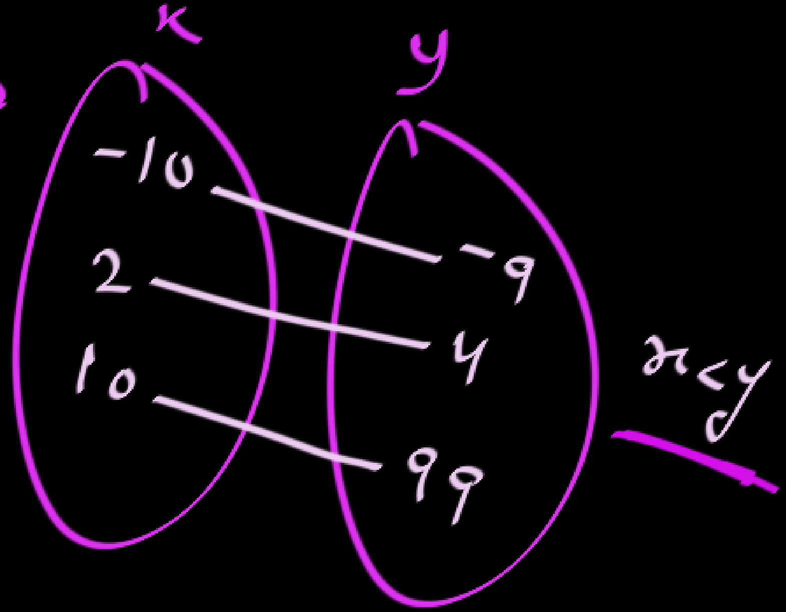
$\boxed{P(x,y) : x<y}$

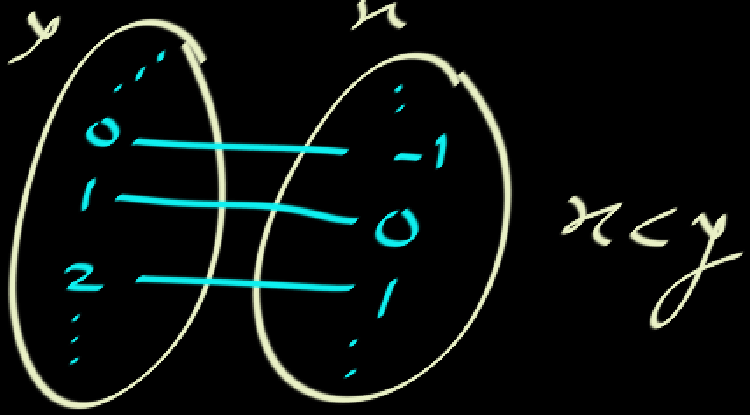
Domain : $N = \{1,2,3,--\}$

Domain: $\mathbb{Z}$; $P(x,y) : x < y$

$\boxed{\forall x\, \exists y\, P(x,y)} = T$

$\boxed{\forall y\, \exists x\, P(x,y) = T}$





Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.18

18. Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? In other words, is the statement $\forall x P(x) \rightarrow \exists x P(x)$ always true? Is the converse always true? Assume the domain of discourse is non-empty.

YES

18. Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? In other words, is the statement $\forall x P(x) \rightarrow \exists x P(x)$ always true? Is the converse always true? Assume the domain of discourse is non-empty.

$\boxed{\forall x P(x) \longrightarrow \exists x P(x)}$: $\boxed{\text{Always True}}$

$\boxed{\forall x\, P(x) : \text{True}}$ Given

$\boxed{\exists x\, P(x)} = ? \rightarrow$ True

Domain

$\boxed{\text{If } \forall x\, P(x) = T \text{ then } \exists x\, P(x) = T}$: TRUE

Correct

$\forall x P(x)$: True

$\exists x P(x) = ?$

for empty Domain :

$\forall x P(x)$: T

$\exists x P(x)$: F

Note:

Domain, in FOL, is Never Empty.

Unless the $\boxed{\text{Question Explicitly gives it.}}$

Q:

$\boxed{\text{if } \forall_x P(x) = T \text{ then } \exists x P(x) = T}$

$\rightarrow$ True

Q:

$$\boxed{\text{if } \forall_x P(x) = T \text{ then } \exists x P(x) = T}$$

Assume Empty Domain.

Ans: False

Note:

$\boxed{\text{Never Consider Empty Domain.}}$

Unless Question explicitely tells you to consider.

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction
Oscar Levin, 3rd Edition, Exercise 0.2.19

19. For each of the statements below, give a domain of discourse for which the statement is true, and a domain for which the statement is false.

(a) $\forall x \exists y (y^2 = x)$.

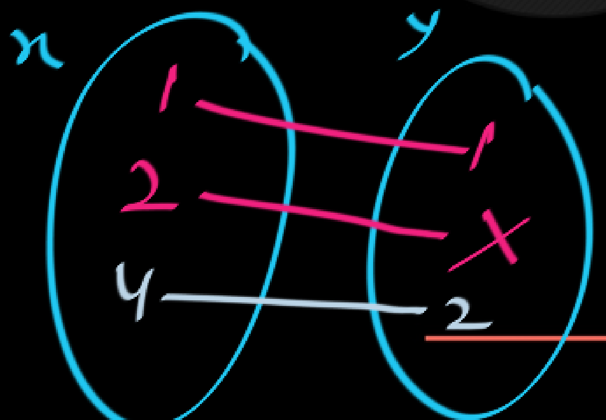
(b) $\forall x \forall y (x < y \rightarrow \exists z (x < z < y))$.

(c) $\exists x \forall y \forall z (y < z \rightarrow y \leq x \leq z)$.

(a) $\boxed{\forall x \, \exists y \, (y^2 = x)}$

Give a Domain, where (a) is false:

Domain: $\mathbb{N} = \{1, 2, 3, \ldots\}$ $\rightarrow$ (a): false

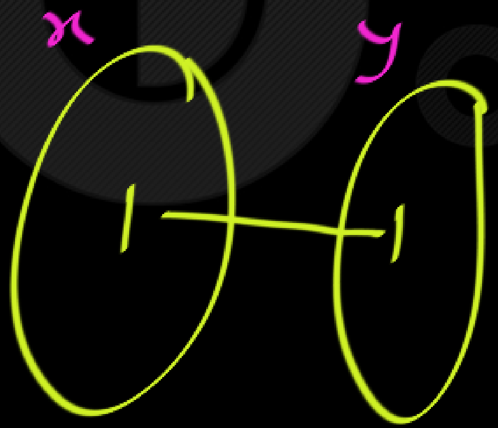


$\boxed{y^2 = x}$

(a) $\forall x \exists y \, (y^2 = x)$

Domain in which (a) True :

Domain: $\{1\}$ $\longrightarrow$ (a) True

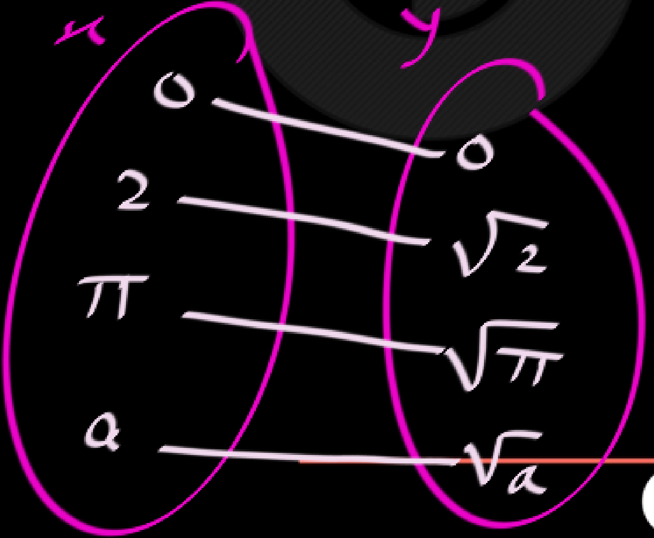


$1^2 = 1$

(a) $\forall_x \exists_y \left(y^2 = x \right)$

Domain in which (a) True :

Domain: $\boxed{\text{Non-negative Real numbers.}}$



$y^2 = x$

(a) True

(b) $\forall x \forall y \left(x < y \longrightarrow \exists z (x < z < y) \right)$

for every pair (x, y); If $x < y$ then there is a number z b/w x, y

$x < z < y$

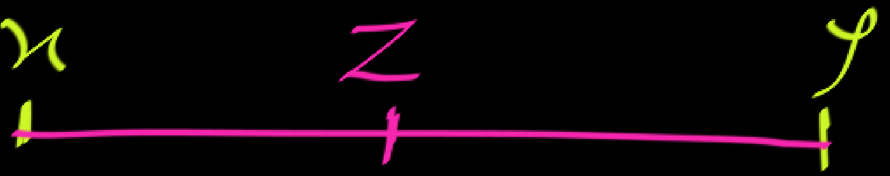
(b) $\forall x \forall y \left(x < y \longrightarrow \exists z (x < z < y) \right)$

Domain: Real : (b) : True

Domain: Z : (b) : False

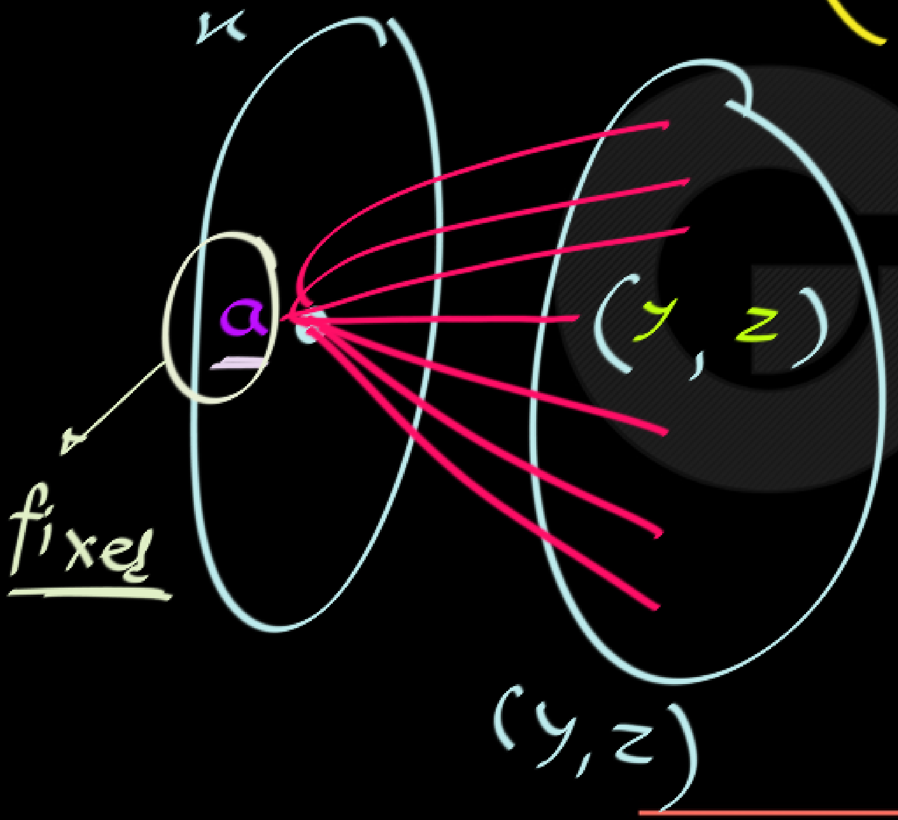
Real :

$x < y$



(b) $\forall x \forall y \left(x < y \longrightarrow \exists z (x < y < z) \right)$

(c) $\exists_x \forall_y \forall_z \left(y < z \longrightarrow y \le x \le z \right)$



If $y < z$ then

$y \le a \le z$

$y < z$

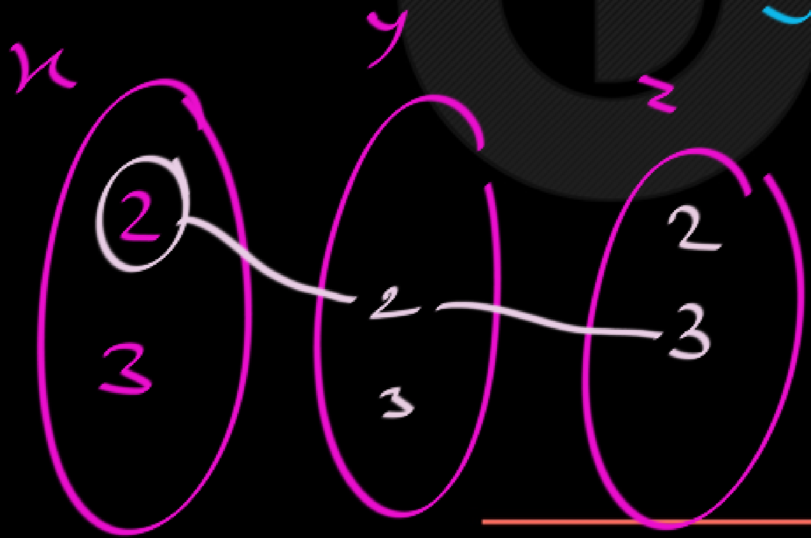
(c) $\exists x \forall y \forall z \left(y < z \rightarrow y \leq x \leq z \right)$

Domain: Real $\longrightarrow$ (c) False

any a $\;\not\longrightarrow\;$ $a+1,\ a+2$

(c) $\exists x \forall y \forall z \left(y < z \longrightarrow y \le x \le z \right)$

Domain : $\{2, 3\}$ $\longrightarrow$ $\boxed{\text{(c) True}}$



if $\boxed{y < z}$ then $y \le x \le z$

(c) $\exists_x \forall_y \forall_z \left(y < z \longrightarrow y \leq x \leq z \right)$

Domain : $\{ 2 \}$ $\longrightarrow$ $\boxed{\text{(c) True}}$

if $\boxed{y < z}$ then $\underline{y \leq x \leq z}$

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.20

20. Consider the statement, "For all natural numbers n, if n is prime, then n is solitary." You do not need to know what *solitary* means for this problem, just that it is a property that some numbers have and others do not.

(a) Write the converse and the contrapositive of the statement, saying which is which. Note: the original statement claims that an implication is true for all n, and it is that implication that we are taking the converse and contrapositive of.

(b) Write the negation of the original statement. What would you need to show to prove that the statement is false?

(c) Even though you don't know whether 10 is solitary (in fact, nobody knows this), is the statement "if 10 is prime, then 10 is solitary" true or false? Explain.

(d) It turns out that 8 is solitary. Does this tell you anything about the truth or falsity of the original statement, its converse or its contrapositive? Explain.

(e) Assuming that the original statement is true, what can you say about the relationship between the *set* P of prime numbers and the *set* S of solitary numbers. Explain.

20. Consider the statement, "For all natural numbers n, if n is prime, then n is solitary." You do not need to know what *solitary* means for this problem, just that it is a property that some numbers have and others do not.

(a) Write the converse and the contrapositive of the statement, saying which is which. Note: the original statement claims that an implication is true for all n, and it is that implication that we are taking the converse and contrapositive of.

(b) Write the negation of the original statement. What would you need to show to prove that the statement is false?

(c) Even though you don't know whether 10 is solitary (in fact, nobody knows this), is the statement "if 10 is prime, then 10 is solitary" true or false? Explain.

(d) It turns out that 8 is solitary. Does this tell you anything about the truth or falsity of the original statement, its converse or its contrapositive? Explain.

(e) Assuming that the original statement is true, what can you say about the relationship between the *set P* of prime numbers and the *set S* of solitary numbers. Explain.

Given: Domain: Natural no.

$$\forall x \left(P(x) \rightarrow S(x) \right)$$

(a) Converse: $\forall x \left(S(x) \rightarrow P(x) \right)$

Contrapositive: $\forall x \left(\neg S(x) \rightarrow \neg P(x) \right)$

 : Every politician is bad.

Claim

Negation : Some politician is not bad.

Given:

Domain: Natural no.

$m: \forall_x \left(P(x) \longrightarrow S(x) \right)$

m: Every prime is solitary.

(b) $\neg m$: Some prime is not solitary.

$\neg m: \exists_x \left(\text{Prime}(x) \land \neg S(x) \right)$

Given:

Domain: Natural no.

$m: \forall_x \left(P(x) \longrightarrow S(x) \right)$

(b) To prove m is false:

Given one natural no n,

$P(n)$: True ; $S(n)$: false

Given:

Domain: Natural no.

$\forall x \left(P(x) \rightarrow S(x) \right)$

(c) m: $\boxed{\text{If 10 prime then 10 solitary.}}$

True (because Given)

Given:

Domain: Natural no.

$\forall_x \left(P(x) \longrightarrow S(x) \right)$

(c) M: $\boxed{\text{If } 7 \text{ prime then } 7 \text{ solitary.}}$

$\longrightarrow$ True (because Given)

Given:

Domain: Natural no.

$M: \forall x \left(P(x) \longrightarrow S(x) \right)$

(D) Given: $\boxed{S(8) : \text{True}}$ → M: Can't Determine

→ Converse: Can't Determine.

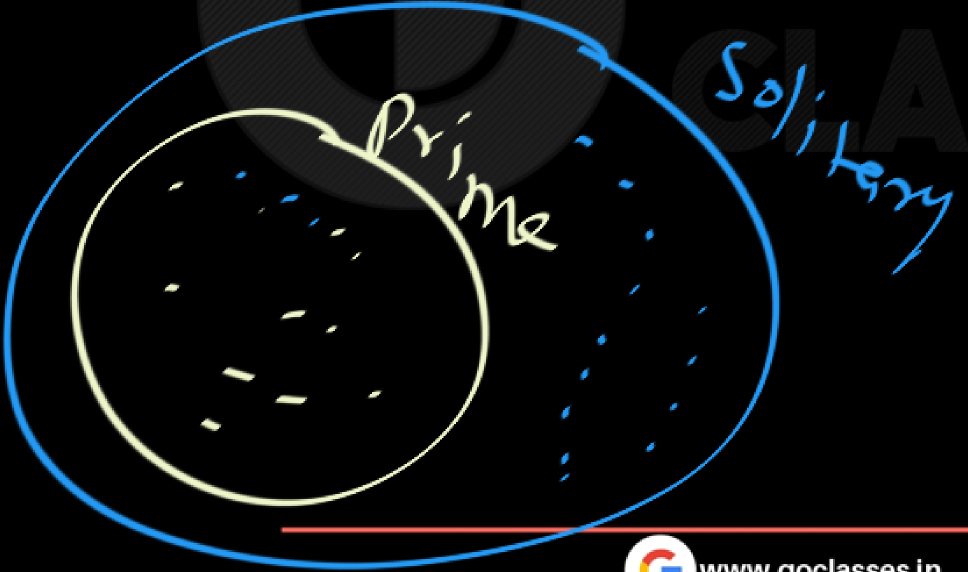
Converse: $\boxed{\forall x \left(S(x) \longrightarrow P(x) \right)}$

Given:

Domain: Natural no.

$$\forall x \left(P(x) \rightarrow S(x) \right)$$

e)



$\boxed{P \subseteq S}$

set of prime

set of Solitary

Daily Practice

Standard Questions

Topic: First Order Logic

Q5 Let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x, y)$ denote the following predicates with domain $\mathbb{Z}$:

$$\begin{aligned} P(x)&:\ x^2 = x, \\ Q(x)&:\ x \leq 0, \\ R(x)&:\ x^2 = x + 1, \\ S(x)&:\ x \text{ is even}, \\ T(x, y)&:\ (x < y) \wedge (y < x^2) \end{aligned}$$

Determine whether each of the following statements is true or false, and give brief reasons.

(a) $\forall x \in \mathbb{Z}, P(x) \rightarrow Q(x)$

(b) $\forall x \in \mathbb{Z}, P(x) \rightarrow \sim Q(x)$

(c) $\forall x \in \mathbb{Z}, R(x) \rightarrow P(x)$

(d) $\forall x \in \mathbb{Z}, P(x) \rightarrow R(x)$

(e) $\forall x \in \mathbb{Z}, (P(x) \wedge S(x)) \rightarrow Q(x)$

(f) $\forall x \in \mathbb{Z}, (P(x) \wedge Q(x)) \rightarrow S(x)$

(g) $\exists x \in \mathbb{Z}$ such that $R(x)$

(h) $\exists x \in \mathbb{Z}$ such that $S(x) \wedge Q(x)$

(i) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $T(x, y)$

(j) $\forall x \in \mathbb{Z}, (\sim P(x) \rightarrow \exists y \in \mathbb{Z}$ such that $T(x, y))$

Q5 Let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x, y)$ denote the following predicates with domain $\mathbb{Z}$:

$$P(x):\ x^2 = x,$$
$$Q(x):\ x \leq 0,$$
$$R(x):\ x^2 = x + 1,$$
$$S(x):\ x \text{ is even},$$
$$T(x, y):\ (x < y) \wedge (y < x^2)$$

Determine whether each of the following statements is true or false, and give brief reasons.

(a) $\forall x \in \mathbb{Z}, P(x) \rightarrow Q(x)$ F

(b) $\forall x \in \mathbb{Z}, P(x) \rightarrow \sim Q(x)$ F

(c) $\forall x \in \mathbb{Z}, R(x) \rightarrow P(x)$ T

(d) $\forall x \in \mathbb{Z}, P(x) \rightarrow R(x)$ F

(e) $\forall x \in \mathbb{Z}, (P(x) \wedge S(x)) \rightarrow Q(x)$ T

(f) $\forall x \in \mathbb{Z}, (P(x) \wedge Q(x)) \rightarrow S(x)$ T

(g) $\exists x \in \mathbb{Z}$ such that $R(x)$ F

(h) $\exists x \in \mathbb{Z}$ such that $S(x) \wedge Q(x)$ T — witness: 0

(i) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $T(x, y)$ — F

(j) $\forall x \in \mathbb{Z}, (\sim P(x) \rightarrow \exists y \in \mathbb{Z}$ such that $T(x, y))$ — T

$P(x)$; $\boxed{x^2 = x} \longrightarrow x = 0 ; x = 1$

$x^2 - x = 0$

$x(x-1) = 0 \Rightarrow x = 0, 1$

$R(x)$; $x^2 = x + 1$

$x^2 - x - 1 = 0 \longrightarrow x = \frac{1 \pm \sqrt{1+4}}{2}$

$\boxed{\text{In } Z\text{, no } x \text{ satisfies } R.}$ ✓

(a) $\forall_{x \in z},\ P(x) \rightarrow Q(x)$

$x \le 0$

false :

Counter eg : $x = 1$

$P(1) = \text{True}$

$Q(1) = \text{False}$

B

$\forall_{x \in Z}, \left(P(x) \rightarrow \neg Q(x) \right)$

false

Countex eg: $x = 0$

$P(0)$: True

$\neg Q(0)$: false

(c) $\forall_{x \in Z}, R(x) \rightarrow P(x)$

True

false for every $x \in Z$.

$F \rightarrow \text{Any} \equiv \text{True}$

D

$\forall_{x \in D}$, $P(x) \rightarrow R(x) \equiv$ Always false

$\boxed{\forall_{x \in D} \neg P(x)}$: for every $x \in Z$, $P(x)$ false

false

$\boxed{A \rightarrow F \equiv \neg A}$

Counterep : 0, 1

(e) $\forall_{x \in Z}, \ (P(x) \wedge S(x)) \rightarrow Q(x)$

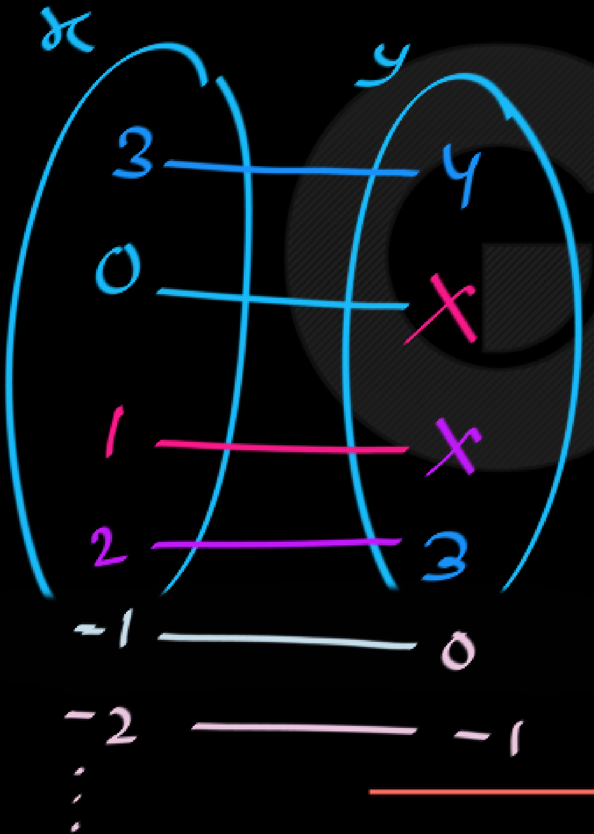
True

True only for $x = 0$

$Q(0)$: True

(i) $\forall_{x \in Z} \exists_{y \in Z} \left(x<y \text{ and } y<x^2 \right)$

false



i : False

Counterex: $x = 0$, $x = 1$ only two Counterex.

j $\forall_{x \in Z} \left(\underbrace{\neg P(x) \rightarrow \overbrace{\exists_y (x < y \wedge y < x^2)}^{N}}_{m} \right)$

$x = 1 \longrightarrow m = T$

$x = 0 \longrightarrow m = T$

$x = 2, 3, \ldots \longrightarrow N = T \longrightarrow m = T$

$x = -1, -2, \ldots \longrightarrow N = T \longrightarrow m = T$

J: True