First Order Logic

Homework 1

Quantifiers

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GATE CSE AIR 53; AIR 67; AIR 107; AIR 206;

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Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.12

12. Let $P(x)$ be the predicate, "$3x + 1$ is even."

(a) Is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.13

13. Let $P(x)$ be the predicate, "$4x + 1$ is even."

(a) Is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Kenneth H. Rosen,

Discrete Mathematics and Its Applications,

Seventh Edition, Exercise 1.4 Question 13

13. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$

b) $\exists n(2n = 3n)$

c) $\exists n(n = -n)$

d) $\forall n(3n \leq 4n)$

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.14

14. For a given predicate $P(x)$, you might believe that the statements $\forall x P(x)$ or $\exists x P(x)$ are either true or false. How would you decide if you were correct in each case? You have four choices: you could give an example of an element n in the domain for which $P(n)$ is true or for which $P(n)$ if false, or you could argue that no matter what n is, $P(n)$ is true or is false.

(a) What would you need to do to prove $\forall x P(x)$ is true?

(b) What would you need to do to prove $\forall x P(x)$ is false?

(c) What would you need to do to prove $\exists x P(x)$ is true?

(d) What would you need to do to prove $\exists x P(x)$ is false?

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Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.16

16. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

Daily Practice

Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.17

17. Translate into English:

(a) $\forall x(E(x) \longrightarrow E(x+2))$.

(b) $\forall x \exists y(\sin(x) = y)$.

(c) $\forall y \exists x(\sin(x) = y)$.

(d) $\forall x \forall y(x^3 = y^3 \longrightarrow x = y)$.

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Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.18

18. Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? In other words, is the statement $\forall x P(x) \rightarrow \exists x P(x)$ always true? Is the converse always true? Assume the domain of discourse is non-empty.

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Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.19

19. For each of the statements below, give a domain of discourse for which the statement is true, and a domain for which the statement is false.

(a) $\forall x \exists y (y^2 = x)$.

(b) $\forall x \forall y (x < y \rightarrow \exists z (x < z < y))$.

(c) $\exists x \forall y \forall z (y < z \rightarrow y \leq x \leq z)$.

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Standard Books' Questions

Topic: First Order Logic

Source:

Discrete Mathematics, An Open Introduction

Oscar Levin, 3rd Edition, Exercise 0.2.20

20. Consider the statement, "For all natural numbers n, if n is prime, then n is solitary." You do not need to know what *solitary* means for this problem, just that it is a property that some numbers have and others do not.

(a) Write the converse and the contrapositive of the statement, saying which is which. Note: the original statement claims that an implication is true for all n, and it is that implication that we are taking the converse and contrapositive of.

(b) Write the negation of the original statement. What would you need to show to prove that the statement is false?

(c) Even though you don't know whether 10 is solitary (in fact, nobody knows this), is the statement "if 10 is prime, then 10 is solitary" true or false? Explain.

(d) It turns out that 8 is solitary. Does this tell you anything about the truth or falsity of the original statement, its converse or its contrapositive? Explain.

(e) Assuming that the original statement is true, what can you say about the relationship between the *set* P of prime numbers and the *set* S of solitary numbers. Explain.

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Q5 Let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x,y)$ denote the following predicates with domain $\mathbb{Z}$:

$$\begin{aligned} P(x)&:\ x^2 = x, \\ Q(x)&:\ x \le 0, \\ R(x)&:\ x^2 = x+1, \\ S(x)&:\ x \text{ is even}, \\ T(x,y)&:\ (x<y)\wedge(y<x^2) \end{aligned}$$

Determine whether each of the following statements is true or false, and give brief reasons.

(a) $\forall x \in \mathbb{Z}, P(x) \to Q(x)$

(b) $\forall x \in \mathbb{Z}, P(x) \to \sim Q(x)$

(c) $\forall x \in \mathbb{Z}, R(x) \to P(x)$

(d) $\forall x \in \mathbb{Z}, P(x) \to R(x)$

(e) $\forall x \in \mathbb{Z}, (P(x) \wedge S(x)) \to Q(x)$

(f) $\forall x \in \mathbb{Z}, (P(x) \wedge Q(x)) \to S(x)$

(g) $\exists x \in \mathbb{Z}$ such that $R(x)$

(h) $\exists x \in \mathbb{Z}$ such that $S(x) \wedge Q(x)$

(i) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $T(x,y)$

(j) $\forall x \in \mathbb{Z}, (\sim P(x) \to \exists y \in \mathbb{Z}$ such that $T(x,y))$