Combinatorics

Questions:

Combinatorial Arguments

Website : https://www.goclasses.in/

Hockey-Stick Identity

For $n, r \in \mathbb{N}, n > r, \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$.

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n-1}{k} + \binom{n-2}{k} + \ldots + \binom{k+1}{k} + \binom{k}{k}.$$

Q :

a combinatorial proof of $k\binom{n}{k} = n\binom{n-1}{k-1}$

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Side 1 = Select k people, then select one captain. $\binom{n}{k} * k$

Side 2 = Select a captain first, then select k-1 people from remaining n-1 people. $= (n)\binom{n-1}{k-1}$

Eg: a, b, c, d ; k = 3 select 3, one of them is Captain.

side 1 → $\binom{4}{3} \times 3$

a, b, c
a, b, c
a, b, c
a, c, d
a, c, d

side 2

a, b, c
a, c, d
b, a, b

b, a, c

$(4)\binom{3}{2}$

a combinatorial proof of $k\binom{n}{k} = n\binom{n-1}{k-1}$

To select a committee of k people with a president from a group of n people, you can

1. select the k committee members, and then select the president from the members, or
2. select the president from the n people, and the $k-1$ remaining committee members from the remaining $n-1$ people.

You could also choose $k-1$ ordinary committee members from the group of n, and then a president from the remaining $n-k+1$, so $(n-k+1)\binom{n}{k-1}$ should also give the same value – Henry Oct 5, 2014 at 0:09

Q:

Combinatorial Proof - $\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$

$$r\binom{n}{r} = (n)\binom{n-1}{r-1}$$

$$= \boxed{\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}}$$

$**$

$$\boxed{{}^{n}C_{r} = \frac{n}{r} \times {}^{n-1}C_{r-1}}$$

$$\binom{50}{30} = \frac{50}{30}\binom{49}{29}$$

$$\binom{48}{21} = \frac{48}{21}\binom{47}{20}$$

Prove the identity $\binom{k}{2} + \binom{k}{k-2} + k^2 = \binom{2k}{2}$, where $k \geq 2$ using a combinatorial proof.

$$\binom{2k}{2} = k^2 + 2\binom{k}{2} = k^2 + \binom{k}{2} + \binom{k}{k-2}$$

$$\boxed{\binom{n}{r} = \binom{n}{n-r}}$$

n boys , n girls

select 2

$$\binom{n}{2}$$

$$\binom{2n}{2} = \underbrace{\binom{n}{2}}_{\text{both boys}} + \underbrace{\binom{n}{n-2}}_{\text{both girls}} + \underbrace{\binom{n}{1}\binom{n}{1}}_{\text{one boy one girls}}$$

Sum of $k\binom{n}{k}$ is $n2^{n-1}$

Proof that $\sum_{k=1}^{n} k\binom{n}{k}$ for $n \in \mathbb{N}$ is equal to $n2^{n-1}$.

Story: make a Committee, with one of them as president.

$$\underbrace{\sum_{k=1}^{n} k \binom{n}{k}}_{\displaystyle\downarrow} = \underline{(n)\, 2^{n-1}}$$

a subset of n people, one of them is President

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☛ https://gateoverflow.in/302843

Let $U = \{1, 2, \dots, n\}$ Let $A = \{(x, X) \mid x \in X, X \subseteq U\}$. Consider the following two statements on $|A|$.

I. $|A| = n2^{n-1}$

II. $|A| = \Sigma_{k=1}^{n} k \binom{n}{k}$

Which of the above statements is/are TRUE?

A. Only I
B. Only II
C. Both I and II
D. Neither I nor II

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Same

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$U = \{1, 2, 3, --, n\}$

(x, A)

President

Committee

$A \subseteq U$

$\boxed{x \in A}$

$(1, U)$ ✓

$(2, \{2, 3\})$ ✓

$(1, \{1\})$ ✓