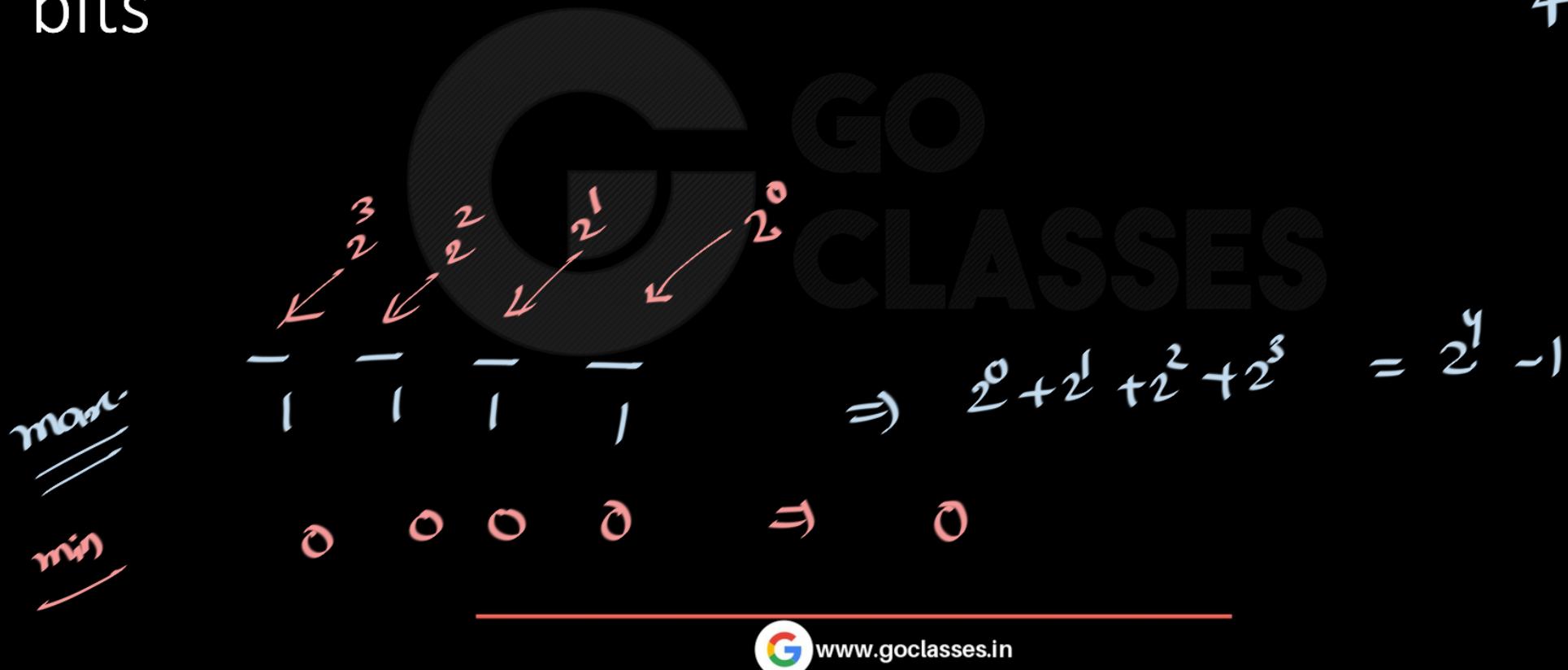




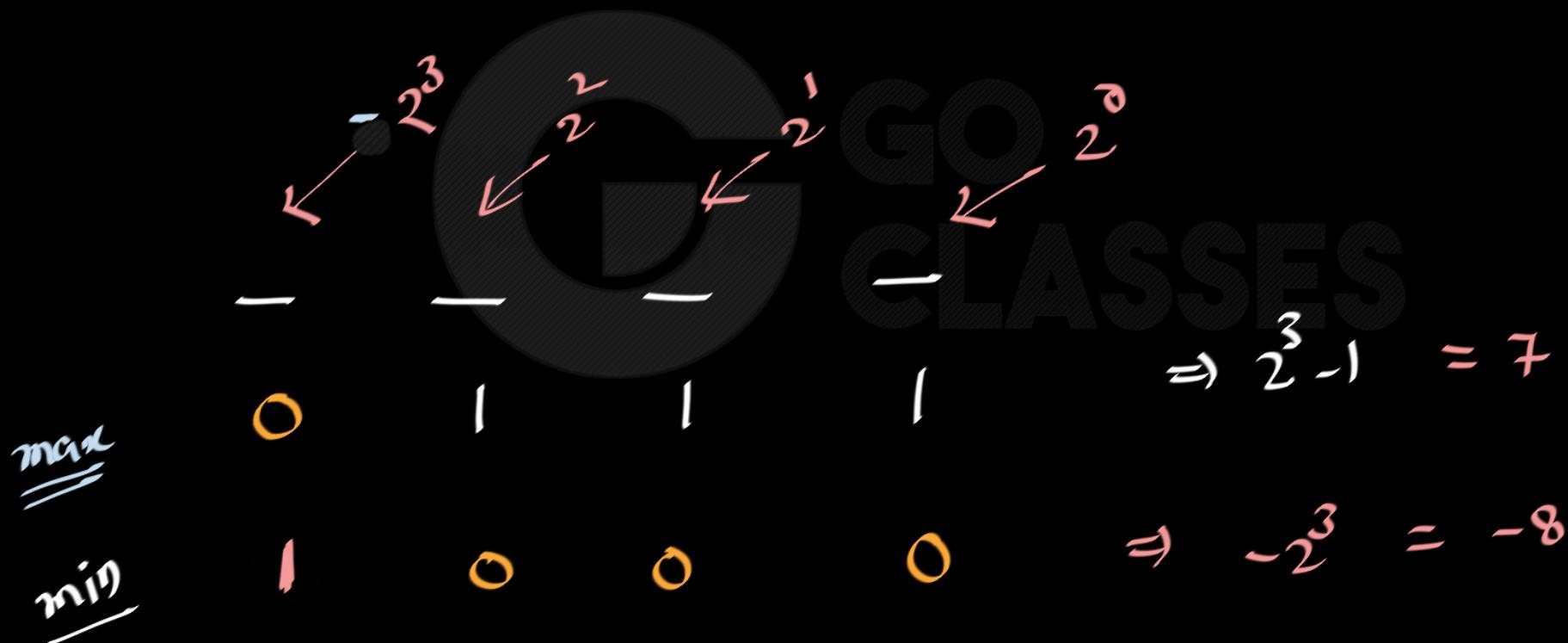
Min and Max number in Unsigned form with k bits

4





Min and Max number in signed (2C) form  
with k bits



Signed No.

in 4 bits

Can

we represent 8 ?

$8 \equiv 01000$

$8 \equiv 1000$   
WRONG



## Question 5

GATE CSE 2013

The smallest integer that can be represented by an 8-bit number in 2's complement form is

- A. -256
- B. -128
- C. -127
- D. 0

CLASSES



## Question 5

GATE CSE 2013

The smallest integer that can be represented by an 8-bit number in 2's complement form is

- A. -256
- B. -128
- C. -127
- D. 0

$$\begin{array}{r} \cancel{-2}^1 = -128 \\ \cancel{-}^1 \quad - \quad - \quad - \quad - \quad - \end{array}$$



## Question 6

GATE CSE 1994

$$\begin{array}{l} \text{min: } -2^{n-1} \\ \text{max: } 2^{n-1} - 1 = 2^n - 1 \end{array}$$

Consider  $n$ -bit (including sign bit) 2's complement representation of integer numbers. The range of integer values,  $N$ , that can be represented is \_\_\_\_\_  
 $\leq N \leq$ \_\_\_\_\_.

$$2^0 + 2^1 + \dots + 2^{k-1} = 2^k - 1$$

- - - - -

- - - -

$2^{n-1}$

$2^{n-2}$



## Question 6

GATE CSE 1994

$$\begin{array}{ll} \text{min: } & -2^{n-1} \\ \text{max: } & \sum_{(n-2)+1}^{n-1} -1 = 2^n - 1 \end{array}$$

Consider  $n$ -bit (including sign bit) 2's complement representation of integer numbers. The range of integer values,  $N$ , that can be represented is  $\underline{-2^{n-1}} \leq N \leq \underline{2^n - 1}$



## Question 7

GATE CSE 2005 ISRO2009-18, ISRO2015-2

The range of integers that can be represented by an  $n$  bit 2's complement number system is:

- A.  $-2^{n-1}$  to  $(2^{n-1} - 1)$
- B.  $-(2^{n-1} - 1)$  to  $(2^{n-1} - 1)$
- C.  $-2^{n-1}$  to  $2^{n-1}$
- D.  $-(2^{n-1} + 1)$  to  $(2^{n-1} - 1)$

IES



# Homework 1



# CLASSES



## Question 1

$$2^6 + \dots + 2^0 = 2^7 - 1$$

If we have seven (7) bits to represent integers, what is largest unsigned number and what is largest 2s complement signed number we can represent (in decimal **and** binary)?

---

$$2^7 - 1 = 128 - 1 = 127$$

Largest unsigned:

---

$$2^6 - 1 = 64 - 1 = 63$$

Most positive signed:



## Solution:

Largest unsigned:

1111 111 (127)

Most positive signed:

0111 111 (63)



## Question 2

Complete the following table. Use the fewest number of bits on each case.

Decimal	2's complement
-256	
-154	
-107	
135	

01000 00000  
10111 00001  
11000 00000



## Solution:

Complete the following table. Use the fewest number of bits on each case.

Decimal	2's complement
-256	100000000
-154	101100110
-107	10010101
135	010000111



## Question 3

- Convert the decimal number -67 to 8-bit, signed 2's complement binary. Hint:  $67 = 0100\ 0011$
- Convert the 16-bit signed 2's complement binary number 1001 0001 1111 1010 to decimal Hint: We can apply method1 or method 2 to get answer
- Give the 8 bit 2's complement representation of the integer -23.
- Give the 32 bit 2's complement representation of the integer -23.



## Solution

- Convert the decimal number -67 to 8-bit, signed 2's complement binary.

$$-67 = 1011\ 1100 + 1 = 1011\ 1101$$

- Convert the 16-bit signed 2's complement binary number 1001 0001 1111 1010 to decimal

We can apply method1 or method 2 to get answer

- Give the 8 bit 2's complement representation of the integer -23.

11101001

- Give the 32 bit 2's complement representation of the integer -23.

111111111111111111111111 1 1 1 0 1 0 0 1



# Question 4

GATE CSE 2003

Assuming all numbers are in 2's complement representation, which of the following numbers is divisible by 11111011?

- A. 11100111
- B. 11100100
- C. 11010111
- D. 11011011

$$\begin{array}{r} \overbrace{1\ 1\ 1\ 1\ 1\ 0\ 1\ 1}^{\text{-5}} \\ \downarrow \\ | 0 | \rightarrow -5 \\ \swarrow \\ -2 \end{array}$$



# Solution

GATE CSE 2003

Assuming all numbers are in 2's complement representation, which of the following numbers is divisible by 11111011?

=-5

- A. 11100111 =-25
- B. 11100100 =-28
- C. 11010111 =-41
- D. 11011011 =-37

A is correct



## Question 5

GATE CSE 2016 Set 1

The 16-bit 2's complement representation of an integer is

1111 1111 1111 0101; its decimal representation is \_\_\_\_\_



# Solution

GATE CSE 2016 Set 1

The 16-bit 2's complement representation of an integer is

1111 1111 1111 0101; its decimal representation is \_\_\_\_\_

We can apply method1 or method 2 to get answer

Using Method2-

MSb is 1 => number is negative

Take 2's complement of given binary.

=1000 0000 0000 1011

=11 (in decimal)

Hence answer is -11

Note:- we can also ignore leading one's except first one before solving using any method



## Question 6

Suppose you are given the following 4-bit binary number: **0101**.

You're not told whether or not the number is signed or unsigned. Is this information important in knowing what the value of the number is, in decimal? That is, do you need to know if it's signed or unsigned to say what the decimal value is? Why or why not?

0 1 0 1

↳ 5



## Solution

Suppose you are given the following 4-bit binary number: **0101**.

You're not told whether or not the number is signed or unsigned. Is this information important in knowing what the value of the number is, in decimal? That is, do you need to know if it's signed or unsigned to say what the decimal value is? Why or why not?

**Since MSb is 0 hence it does not matter if we treat this number as signed or unsigned.  
We will get same decimal**



## Question 7

In 5 bits, what is the most negative value and the most positive value representable in signed form, using two's complement? Express your answers in both binary and decimal.

Same question as above, but representable in unsigned form?

$$-2^4 = -16$$

1 0000

most neg.

$$2^{4-1} = 15$$

0 (111)

most pos



# Solution

In 5 bits, what is the most negative value and the most positive value representable in signed form, using two's complement? Express your answers in both binary and decimal.

**Most negative: 10000, -16**

**Most positive: 01111, 15**



Same question as above, but representable in unsigned form?

**You cannot represent negative numbers with unsigned binaries.**

**Most positive: 11111, 31**



## Question 8

Convert these unsigned binary numbers into decimal.

- a. 1010
- b. 10101010
- c. 11000001

Convert these signed binary numbers into decimal.

- a. 1010
- b. 00110100
- c. 11000001

(2 methods here)



# Solution

Convert these unsigned binary numbers into decimal.

- a. 1010      **10**
- b. 10101010    **170**
- c. 11000001    **193**

Convert these signed binary numbers into decimal.

- a. 1010      **-6**
- b. 00110100    **52**
- c. 11000001    **-63**



# Bonus Question GATE CSE 2022

Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation R1 + R2, which one of the following values of R1 and R2 gives an arithmetic overflow?

- A. R1 = 1011 and R2 = 1110
- B. R1 = 1100 and R2 = 1010
- C. R1 = 0011 and R2 = 0100
- D. R1 = 1001 and R2 = 1111

$$\begin{array}{l} \text{min} \\ \hline -2^3 \\ \text{max: } 2^3 - 1 = 7 \end{array}$$

<https://gateoverflow.in/371928/Gate-cse-2022-question-8>



2's complement range for 4 bits = [-8, 7]

Option A

R1 = 1011 = -5 and R2 = 1110 = -2.

$$R_1 + R_2 = -7 \quad \checkmark$$

Option B

R1 = 1100 = -4 and R2 = 1010 = -6.

$$R_1 + R_2 = -10$$

Option C

R1 = 0011 = 3 and R2 = 0100 = 4.

$$R_1 + R_2 = 7$$

Option D

R1 = 1001 = -7 and R2 = 1111 = -1.

$$R_1 + R_2 = -8$$



2's complement range for 4 bits = [-8, 7]

Option A

$R1 = 1011 = -5$  and  $R2 = 1110 = -2$ .  $R1+R2 = -7$  No overflow

Option B

$R1 = 1100 = -4$  and  $R2 = 1010 = -6$ .  $R1+R2 = -10$  Overflow

Option C

$R1 = 0011 = 3$  and  $R2 = 0100 = 4$ .  $R1+R2 = 7$  No overflow

Option D

$R1 = 1001 = -7$  and  $R2 = 1111 = -1$ .  $R1+R2 = -8$  No overflow