Digital Logic :

Lecture 4 :

Boolean Algebra, Logic gates

Algebraic Simplification


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Digital Logic :

Recap :

Dual of an expression,

Boolean Algebra Laws

Laws of Boolean Algebra

Duality: A dual of a Boolean expression is derived by interchanging OR and AND operations, and 0s and 1s (literals are left unchanged).

$$\{F(x_1, x_2, \ldots, x_n, 0, 1, +, \bullet)\}^D = \{F(x_1, x_2, \ldots, x_n, 1, 0, \bullet, +)\}$$

Any law that is true for an expression is also true for its dual.

Operations with 0 and 1:

1. $x + 0 = x \qquad x * 1 = x$
2. $x + 1 = 1 \qquad x * 0 = 0$

Idempotent Law:

3. $x + x = x \qquad x\ x = x$

Involution Law:

4. $(x')' = x$

Laws of Complementarity:

5. $x + x' = 1 \qquad x\ x' = 0$

Commutative Law:

6. $x + y = y + x \qquad x\ y = y\ x$

Laws of Boolean Algebra (cont.)

Associative Laws:

$(x + y) + z = x + (y + z)$ $\quad$ $x\,y\,z = x\,(y\,z)$

Distributive Laws:

$x\,(y + z) = (x\,y) + (x\,z)$ $\quad$ $x + (y\,z) = (x + y)(x + z)$

"Simplification" Theorems:

$x\,y + x\,y' = x$ $\quad$ $(x + y)\,(x + y') = x$

$\boxed{x + x\,y = x}$ $\quad$ $x\,(x + y) = x$

DeMorgan's Law:

$(x + y + z + \ldots)' = x'y'z'$ $\quad$ $(x\,y\,z\,\ldots)' = x' + y' + z'$

Single Variable Theorems

- x · 0 = 0
- x + 1 = 1
- x · 1 = x
- x + 0 = x
- x · x = x
- x + x = x
- x + !x = 1
- x · !x = 0
- x · x · x · … x = x
- !!x = x

Two Variable Theorems

- $x \cdot y = y \cdot x$
- $x + y = y + x$
- Both are commutative

Three Variable Theorems

- Associative Laws
  - $x \cdot (y \cdot z) = (x \cdot y) \cdot z$
  - $x + (y + z) = (x + y) + z$
- Distributive Law
  - $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
- $x + (y \cdot z) = (x + y) \cdot (x + z)$

Theorem for Multiplying and Factoring:

$$(x + y)(x' + z) = xz + x'y$$

Consensus Theorem:

$$xy + yz + x'z = (x + y)(y + z)(x' + z)$$

$$xy + x'z = (x + y)(x' + z)$$

Boolean Algebra — Analogy — Number Theory

AND ⟵ Analogous ⟶ multiplication

OR ⟵ Analogous ⟶ Addition

$$\left.\begin{aligned} 0 \cdot x &= 0 \\ 1 + x &= 1 \end{aligned}\right\} \quad \begin{aligned} 1 \cdot x &= x \\ 0 + x &= x \end{aligned}$$

Boolean Algebra

$0 \cdot 0 = 0$; $1 \cdot 1 = 1$

$1 \cdot 0 = 0$; $0 \cdot 1 = 0$

AND

OR

OR

$0 + 0 = 0$

$0 + 1 = 1 = 1 + 0$

$1 + 1 = 1$

Analogy

Number Theory

$0 \times 0 = 0$ $\quad 1 \times 1 = 1$

$0 \times 1 = 0$

multiplication

$0 + 0 = 0$

$0 + 1 = 1 = 1 + 0$

$1 + 1 = 2$

Addition

Q: Number of Boolean functions over a, b, c, d, such that $f = \overline{f}$?

Q: Number of Boolean functions over a, b, c, d, such that $f = \overline{f}$?

Ans: 0

f is NEVER equal to its complement.

Boolean Algebra :

Next Topic :

Simplification of Boolean Functions

Algebraic Simplification

= Simplification using Boolean Algebra

1. $$\underbrace{ab + ac}_{a(b+c)} = a(b+c)$$ Distributive property

Eg: $$a + ac = a(\underbrace{1+c}_{1}) = a \cdot 1 = a$$

2

$$a(b+c) = ab + ac$$

$$(a+b)(c+d) = ac + ad + bc + bd$$

Simplify

$$A\overline{B}D + A\,\overline{B}\,\overline{D} =$$

$$A\overline{B}D + A\,\overline{B}\,\overline{D} =$$

$$A\overline{B}(\underbrace{D + \overline{D}}_{1})$$

$$= A\overline{B}$$

$$A\overline{B}D + A\overline{B}\,\overline{D} = \quad A\overline{B}\left(\mathrm{D} + \overline{D}\right)$$

$$A\overline{B}\,(1)$$

$$A\overline{B}$$

$$(A + B)(A + C)$$

$$(A+B)(A+C)$$

$$AA + AC + BA + BC$$

$$= A + \cancel{AC} + \cancel{AB} + BC$$

$$A + BC \checkmark = (A+B)(A+C)$$

$AA = A$

$A + A\alpha$
$= A$

$A + A\alpha$
$= A(1+\alpha)$
$= A$

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$

$$(\overline{A} + B)(A + B) =$$

$$(\overline{A} + B)(A + B) =$$

$0 \Leftarrow \overline{A}A + \overline{A}B + BA + BB = B$

$\boxed{B + AB} + B\overline{A}$

$= \boxed{B + B\overline{A}} = B$ ✓

$0 + \alpha = \alpha$

$B + B\alpha$

$= B$

$$(\overline{A} + B)(A + B) = B(\overline{A} + A)$$

$$B(1)$$

$$B$$

Absorption Law:

$$\alpha + \cancel{\alpha\beta} + \cancel{\alpha\gamma} + \cancel{\alpha Z}$$

$$= \alpha \checkmark$$

Eg: $a\bar{b}c + a\bar{b}cd + acde\bar{b}$

$$= a\bar{b}c \checkmark$$

$$A\overline{C} + AB\overline{C} =$$

$$A\overline{C} + AB\overline{C} =$$

$$A\overline{C}(1+B)$$

$$= A\overline{C}$$

$$\underbrace{A\overline{C}}_{\alpha} + \underbrace{A\overline{C}}_{\alpha}\underbrace{B}_{\beta}$$

$$\alpha + \alpha\beta = \alpha$$

$$= A\overline{C} \checkmark$$

$$A\overline{C} + AB\overline{C} = \quad A\overline{C}\,(1 + B)$$

$$A\overline{C}\,(1)$$

$$A\overline{C}$$

Demorgan Law:

Exp E

find $\overline{E}$.

$$\overline{(a\bar{b})} = \bar{a} + b$$

1. Simplify the following expressions using Boolean Algebra.

a. $A + \overline{A}$
b. $A(\overline{A} + B)$
c. $(\overline{A} + \overline{B})(\overline{A} + B)$
d. $\overline{A} + \overline{A}B$
e. $\overline{A}(\overline{ABCD})$

1. Simplify the following expressions using Boolean Algebra.

a. $A + \overline{A} = 1$

b. $A(\overline{A} + B) = 0 + AB = AB$

c. $(\overline{A} + \overline{B})(\overline{A} + B) = \overline{A} + \overline{A}B + \overline{A}\,\overline{B} + 0 = \overline{A}$

d. $\overline{A} + \overline{A}B = \overline{A}$

e. $\overline{A}(\overline{ABCD}) = \overline{A}(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

$= \overline{A} + \overline{A}\,\overline{B} + \overline{A}\,\overline{C} + \overline{A}\,\overline{D} = \overline{A}(1 + \cdots)$

$= \overline{A}$ ✓ $\quad = \overline{A}$

(c) $(\bar{A}+\bar{B})(\bar{A}+B)$

$= (\overline{AB})(\overline{A\bar{B}})$

Absorption Law: $\alpha + \alpha\beta + \alpha\gamma + \alpha z$

$= \alpha$

a. $F = (XY + Z)(Y + X\overline{Z})$

b. $F = (AB + C)(B + \overline{C}D)$

c. $F = A(\overline{B}CD + \overline{B}\,\overline{D} + D(\overline{A} + AB))$

HW

1. Simplify the Boolean expression using Boolean algebra.

a. $(A+B)+\overline{B}$.

b. $AA+BC+B\overline{C}$.

c. $\overline{A}+C+AB$.

d. $\overline{A}(B+AC)$.

1. Simplify the Boolean expression using Boolean algebra.

a. $(A + B) + \overline{B}$ $= A + (B + \overline{B}) = 1$

b. $AA + BC + B\overline{C}$ $= A + B$

A

$\underline{BC + B\overline{C}} \to B$

c. $\overline{A} + C + \cancel{A}B$ $= \overline{A} + B + C$

d. $\overline{A}(B + AC)$ $= \overline{A}B + \overline{A}AC$

$= \overline{A}B$

$\overline{A}AC = 0$

$A + \overline{A}B$

$= A + B$

Simplify $A.(\overline{A}+B)+A.\overline{B}$. Show each stage of your working.

Simplify $A.(\bar{A}+B)+A.\bar{B}$. Show each stage of your working.

$$0 = A\bar{A} + AB + A\bar{B}$$

$$= AB + A\bar{B} = A(\underbrace{B+\bar{B}}_{=1})$$

$$= A$$

Using Boolean algebra, simplify the following expressions:

(d) $(A + \overline{A})(AB + AB\overline{C})$

Using Boolean algebra, simplify each expression:

(b) $\overline{A}\,\overline{B}C + \overline{(A + B + \overline{C})} + \overline{A}\,\overline{B}\,\overline{C}D$

(d) $ABCD + AB(\overline{CD}) + (\overline{AB})CD$

Using Boolean algebra, simplify the following expressions:

(d) $\underbrace{(A + \overline{A})}_{1}\underbrace{(AB + AB\overline{C})}_{\text{Absorption}} = AB$

Using Boolean algebra, simplify each expression:

(b) $\overline{A}\,\overline{B}C + \overline{(A + B + \overline{C})} + \overline{A}\,\overline{B}\,\overline{C}D$

(d) $ABCD + AB(\overline{CD}) + (\overline{AB})CD$

(b) $\bar{A}\bar{B}C + \underbrace{\overline{(A+B+\bar{C})}} + \bar{A}\bar{B}\bar{C}D$

$= \underline{\underline{\bar{A}\bar{B}C}} + \underline{\underline{\bar{A}\bar{B}C}} + \bar{A}\bar{B}\bar{C}D$

$= \underline{\bar{A}\bar{B}}C + \underline{\bar{A}\bar{B}}\bar{C}D$

$= \bar{A}\bar{B}(C + \cancel{\bar{C}}D) = \underline{\underline{\bar{A}\bar{B}}}(C+D)$

$$\boxed{\alpha + \alpha\beta = \alpha}$$ ✓

$$\underbrace{\bar{A}\bar{B}C}_{\alpha} + \underbrace{\bar{A}\bar{B}\bar{C}D}_{\alpha ?}$$

Absorption Law ✗

Problem_#01] Using Boolean algebra techniques, simplify as much as possible.

(a) $A(A + B)$ **(b)** $A(\overline{A} + AB)$ **(c)** $BC + \overline{B}C$

(d) $A(A + \overline{A}B)$ **(e)** $A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C$

Problem_#01] Using Boolean algebra techniques, simplify as much as possible.

(a) $A(A + B)$ (b) $A(\overline{A} + AB)$ (c) $BC + \overline{B}C$

(d) $A(A + \overline{A}B)$ (e) $A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C$

(a) A (b) AB (c) C (D) A

(e) $A\overline{B}C + \overline{A}C = C(\overline{A} + \cancel{A}\overline{B})$

$= C\overline{A} + C\overline{B}$

$$\alpha + \alpha' \beta = \alpha + \beta$$

$$\alpha + \alpha \beta = \alpha$$

$$\boxed{\alpha + \overline{\alpha} = 1}$$

$$\boxed{\alpha \overline{\alpha} = 0}$$

Using Boolean algebra techniques, simplify the following expressions as much as possible:

(a) $A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + A\bar{B}CDE$

(b) $(B + \bar{B})(BC + BC\bar{D})$

(c) $BCDE + BC(\overline{DE}) + \left(\overline{BC}\right)DE$

(d) $\bar{B}\bar{C}D + \left(\overline{B + C + D}\right) + \bar{B}\bar{C}\bar{D}E$

Using Boolean algebra techniques, simplify the following expressions as much as possible:

(a) $A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + A\bar{B}CDE$

(b) $(B+\bar{B})(BC+BC\bar{D})$

(c) $BCDE + BC(\overline{DE}) + (\overline{BC})DE = BC + DE$

(d) $\bar{B}\bar{C}D + (\overline{B+C+D}) + \bar{B}\bar{C}\bar{D}E$

(a) $A\bar{B} + AB\bar{C} = A(\bar{B} + \cancel{B}\bar{C}) = A(\bar{B}+\bar{C})$

(b) BC

(c) $BC(\underbrace{DE + \overline{DE}}_{1}) + \overline{BC}\,DE = BC + \cancel{\overline{BC}}\,DE$

$$BC + \overline{BC}\,DE = \alpha + \cancel{\overline{\alpha}}\,DE$$

α $\quad$ $\overline{\alpha}$

$$= \alpha + DE$$

(D) $\overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D} + \overline{B}\,\overline{C}\,\overline{D}E$

$$= \overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D} = \overline{B}\,\overline{C}(D+\overline{D}) = \overline{B}\,\overline{C}$$

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HW

Yuvraj

Question 3 (21 marks)

Using Boolean algebra, simplify the following expressions. Please state the Boolean identity or the Boolean theorem applied there, next to each simplification step.

i. $Z=\underbrace{(A+\overline{A})}_{1}.\underbrace{(A.B+A.B.\overline{C})}_{AB} = AB$ ✓

ii. $F=(A+\overline{B}.C).(A+B.\overline{C})$

iii. $X= \overline{A}.B.\overline{C}+\overline{A}.C+\overline{A}.\overline{B}.\overline{C}+\overline{A}.\overline{C}$

iv. $Z=(A.B.C).[A.B+\overline{C}.(B.C+A.C)]$

v. $F=(A+\overline{B}).(A+C)$

vi. $Y=\overline{A+\overline{B.\overline{C}}+C.D}+B.C$

vii. $Z=\overline{X.\overline{Y}.(\overline{\overline{W}+\overline{Y}})}$

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Question 3 (21 marks)

Using Boolean algebra, simplify the following expressions. Please state the Boolean identity or the Boolean theorem applied there, next to each simplification step.

HW

Yuvraj

i. $Z=\underbrace{(A+\overline{A})}_{1}.\underbrace{(A.B+A.B.\overline{C})}_{AB} = AB$

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HW

Yuvraj

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iv. $Z=(A.B.C).[A.B+\overline{C}.(B.C+A.C)]$

v. $F=(A+\overline{B}).(A+C)$

vi. $Y=\overline{A+\overline{B.\overline{C}}+C.D}+B.C$

vii. $Z=\overline{X.\overline{Y}.(\overline{\overline{W}+\overline{Y}})}$

i. $Z = (A + \overline{A}) \cdot (A \cdot B + A \cdot \overline{B} \cdot \overline{C})$

1. **Expression**: $Z = (A + \overline{A}) \cdot (A \cdot B + A \cdot \overline{B} \cdot \overline{C})$
2. **Simplification**:
   - The term $A + \overline{A}$ always equals 1. This is known as the **Complement Law** in Boolean algebra.
   - Therefore, the expression simplifies to:

$$Z = 1 \cdot (A \cdot B + A \cdot \overline{B} \cdot \overline{C})$$

   - Simplifying further, $Z = A \cdot B + A \cdot \overline{B} \cdot \overline{C}$.

ii. $F = (A + B \cdot \overline{C}) \cdot (A + \overline{B} \cdot C)$

1. **Expression**: $F = (A + B \cdot \overline{C}) \cdot (A + \overline{B} \cdot C)$
2. **Simplification**:
   - Apply the **Distributive Law** (multiply out the terms):

$$F = A \cdot (A + \overline{B} \cdot C) + B \cdot \overline{C} \cdot (A + \overline{B} \cdot C)$$

   - Simplify the expression using the **Idempotent Law** and **Absorption Law**:

$$F = A + A \cdot \overline{B} \cdot C + B \cdot \overline{C} \cdot A + B \cdot \overline{C} \cdot \overline{B} \cdot C$$

   - Further simplification:

$$F = A + B \cdot \overline{C} \cdot C$$

   - Since $\overline{B} \cdot B = 0$, the expression simplifies to:

$$F = A + 0 = A$$

iii. $X = \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot C + A \cdot \overline{B} \cdot C$

1. **Expression**: $X = \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot C + A \cdot \overline{B} \cdot C$
2. **Simplification**:
   - Apply the **Distribution**:

$$X = \overline{A} \cdot (B \cdot \overline{C} + C) + A \cdot \overline{B} \cdot C$$

   - Simplify:

$$X = \overline{A} \cdot (C + B \cdot \overline{C}) + A \cdot \overline{B} \cdot C$$

   - Further simplification gives:

$$X = \overline{A} \cdot C + A \cdot \overline{B} \cdot C$$

iv. $Z = (\overline{A} + \overline{B} \cdot C) \cdot [A + B \cdot \overline{C}] \cdot [B \cdot C + A \cdot \overline{C}]$

This expression requires multiple steps of simplification, applying the Distributive, Absorption, and other Boolean identities. Here is the general approach:

1. **Expression**: $Z = (\overline{A} + \overline{B} \cdot C) \cdot [A + B \cdot \overline{C}] \cdot [B \cdot C + A \cdot \overline{C}]$
2. **Simplification**:
   - First, distribute:

$$Z = (\overline{A} + \overline{B} \cdot C) \cdot (A + B \cdot \overline{C}) \cdot (B \cdot C + A \cdot \overline{C})$$

   - This step involves careful application of Boolean identities and distribution until you reach the simplest form.

v. $F = (A + \overline{B}) \cdot (A + C)$

1. **Expression**: $F = (A + \overline{B}) \cdot (A + C)$
2. **Simplification**:
   - Apply **Distributive Law**:

$$F = A \cdot A + A \cdot C + \overline{B} \cdot A + \overline{B} \cdot C$$

   - Simplify using **Idempotent Law**:

$$F = A + A \cdot C + \overline{B} \cdot A + \overline{B} \cdot C$$

   - Further simplification gives:

$$F = A + \overline{B} \cdot C$$

vi. $Y = A + \overline{B} \cdot \overline{C} + C \cdot D + B \cdot C$

1. **Expression**: $Y = A + \overline{B} \cdot \overline{C} + C \cdot D + B \cdot C$
2. **Simplification**:
   - Combine like terms:

$$Y = A + (C \cdot D + B \cdot C) + \overline{B} \cdot \overline{C}$$

   - Simplify using Boolean identities.

vii. $Z = X \cdot \overline{Y} \cdot (W + \overline{Y})$

1. **Expression**: $Z = X \cdot \overline{Y} \cdot (W + \overline{Y})$
2. **Simplification**:
   - Distribute:

$$Z = X \cdot \overline{Y} \cdot W + X \cdot \overline{Y} \cdot \overline{Y}$$

   - Since $\overline{Y} \cdot \overline{Y} = \overline{Y}$, it simplifies to:

$$Z = X \cdot \overline{Y} \cdot W + X \cdot \overline{Y}$$

   - Finally, apply the **Absorption Law**:

$$Z = X \cdot \overline{Y}$$

First Image: Left Side

(a) $x = ABC + AC$

1. **Expression:** $x = ABC + AC$
2. **Factor out** the common term AC:

$$x = AC(B + 1)$$

3. **Simplify** using the **Complement Law** $B + 1 = 1$:

$$x = AC$$

(b) $y = (Q + R) \cdot (\overline{Q} + \overline{R})$

1. **Expression:** $y = (Q + R) \cdot (\overline{Q} + \overline{R})$
2. **Distribute** to expand:

$$y = Q \cdot \overline{Q} + Q \cdot \overline{R} + R \cdot \overline{Q} + R \cdot \overline{R}$$

3. **Simplify** using the **Complement Law**:
   - $Q \cdot \overline{Q} = 0$
   - $R \cdot \overline{R} = 0$

$$y = Q \cdot \overline{R} + R \cdot \overline{Q}$$

   - This is the final simplified expression.

(c) $w = ABC + \overline{A}BC + A$

1. **Expression:** $w = ABC + \overline{A}BC + A$
2. **Factor out** common terms:

$$w = BC(A + \overline{A}) + A$$

3. **Simplify** using the **Complement Law** $A + \overline{A} = 1$:

$$w = BC \cdot 1 + A = BC + A$$

(d) $q = RST \cdot (R + S + T)$

1. **Expression**: $q = RST \cdot (R + S + T)$
2. **Distribute** the terms:

$$q = R \cdot S \cdot T \cdot (R + S + T)$$

3. **Simplify** by distributing RST over $R + S + T$:

$$q = RST + RST + RST$$

4. Since RST is common across all terms, the expression simplifies to:

$$q = RST$$

(e) $x = \overline{A}B \cdot \overline{C} + \overline{A}BC + A\overline{B}C + ABC$

1. **Expression**: $x = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$
2. **Combine terms**:
   - Group terms to simplify:

$$x = \overline{A}B(\overline{C} + C) + AC(\overline{B} + B)$$

   - Use **Complement Law** $\overline{C} + C = 1$ and $\overline{B} + B = 1$:

$$x = \overline{A}B + AC$$

(f) $z = (B + \overline{C}) \cdot (B + C)$

1. **Expression**: $z = (B + \overline{C}) \cdot (B + C)$
2. **Distribute** the terms:

$$z = B \cdot B + B \cdot C + \overline{C} \cdot B + \overline{C} \cdot C$$

3. Simplify:
   - $B \cdot B = B$
   - $\overline{C} \cdot C = 0$

$$z = B + B \cdot C + \overline{C} \cdot B$$

   - Further, since B is common:

$$z = B + BC + B\overline{C}$$

   - Finally, using **Absorption Law**:

$$z = B$$

(f) $z = (B + \overline{C}) \cdot (B + C)$

1. **Expression:** $z = (B + \overline{C}) \cdot (B + C)$
2. **Distribute** the terms:

$$z = B \cdot B + B \cdot C + \overline{C} \cdot B + \overline{C} \cdot C$$

3. Simplify:
   - $B \cdot B = B$
   - $\overline{C} \cdot C = 0$

$$z = B + B \cdot C + \overline{C} \cdot B$$

   - Further, since B is common:

$$z = B + BC + B\overline{C}$$

   - Finally, using **Absorption Law**:

$$z = B$$

(g) $y = (C + D) + \overline{A}CD + A\overline{B}C + ABCD$

1. **Expression:** $y = (C + D) + \overline{A}CD + A\overline{B}C + ABCD$
2. Simplify by factoring and combining terms as above.

(h) $x = AB(CD) + ABD + ABCD$

1. **Expression:** $x = AB(CD) + ABD + ABCD$
2. Simplify by distributing and factoring as necessary.

(a) $(\overline{A}+B)(A+C)$

1. **Expression:** $(\overline{A}+B)(A+C)$
2. **Distribute** the terms:

$$(\overline{A}+B)(A+C)=\overline{A}A+\overline{A}C+BA+BC$$

3. **Simplify** using Boolean identities:
   - $\overline{A}A=0$ (Complement Law)

$$0+\overline{A}C+BA+BC$$

   - This simplifies to:

$$\overline{A}C+BA+BC$$

(b) $AB\overline{B}+ABCD+\overline{A}BCDE$

1. **Expression:** $AB\overline{B}+ABCD+\overline{A}BCDE$
2. **Simplify** each term:
   - $AB\overline{B}=0$ (Complement Law)

$$0+ABCD+\overline{A}BCDE$$

3. **Final Simplified Expression:**

$$ABCD+\overline{A}BCDE$$

(c) $BC+B\overline{C}D+BCD$

1. **Expression:** $BC+B\overline{C}D+BCD$
2. **Combine** like terms:
   - $BC+BCD=BC(1+D)=BC$ (Absorption Law)

$$BC+B\overline{C}D$$

3. **Final Simplified Expression:**

$$BC+B\overline{C}D$$

(d) $BCDE+BC(DE)+(BC)DE$

1. **Expression:** $BCDE+BC(DE)+(BC)DE$
2. **Factor** out the common terms:
   - All terms have $BCDE$ in common.

$$BCDE+BCDE+BCDE=BCDE(1+1+1)=BCDE$$

3. **Final Simplified Expression:**

$$BCDE$$

(a) $x = ABC + \bar{A}C = C(\bar{A} + \cancel{A}B) = C(\bar{A} + B)$

Hw

(b) $y = (Q + R)(\bar{Q} + \bar{R})$

(c) $w = ABC + A\bar{B}C + \bar{A}$

Garima

(d) $q = \overline{RST}(\overline{R + S + T})$

(e) $x = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}BC + ABC + A\,\bar{B}\,\bar{C} + A\bar{B}C$

(f) $z = (B + \bar{C})(\bar{B} + C) + \overline{\bar{A} + B + \bar{C}}$

(g) $y = (\overline{C + D}) + \bar{A}C\bar{D} + A\bar{B}\,\bar{C} + \bar{A}\,\bar{B}CD + AC\bar{D}$

(h) $x = AB(\overline{\bar{C}D}) + \bar{A}BD + \bar{B}\,\bar{C}\,\bar{D}$

Using Boolean algebra, simplify the following expressions:

(a) $(\overline{A} + B)(A + C)$

(b) $A\overline{B} + A\overline{B}C + A\overline{B}CD + A\overline{B}CDE$

(c) $BC + \overline{BCD} + B$

(d) $(B + \overline{B})(BC + BC\overline{D})$

(e) $BC + (\overline{B} + \overline{C})D + BC$

Pawan

HW

Arpon

Using Boolean algebra, simplify the following expressions:

(a) $CE + C(E + F) + \overline{E}(E + G)$

(b) $\overline{B}\,\overline{C}D + (\overline{B + C + D}) + \overline{B}\,\overline{C}\,\overline{D}E$

(c) $(C + CD)(C + \overline{C}D)(C + E)$

(d) $BCDE + BC(\overline{DE}) + (\overline{BC})DE$

(e) $BCD[BC + \overline{D}(CD + BD)]$

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$\overline{x}y\,\underbrace{(z+\overline{z})}_{1} + xz$$

$$= \overline{x}y + xz \checkmark$$

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$\overline{X}Y(Z + \overline{Z}) + XZ$ (by reverse distribution)

$\overline{X}Y1 + XZ$ (by complementarity)

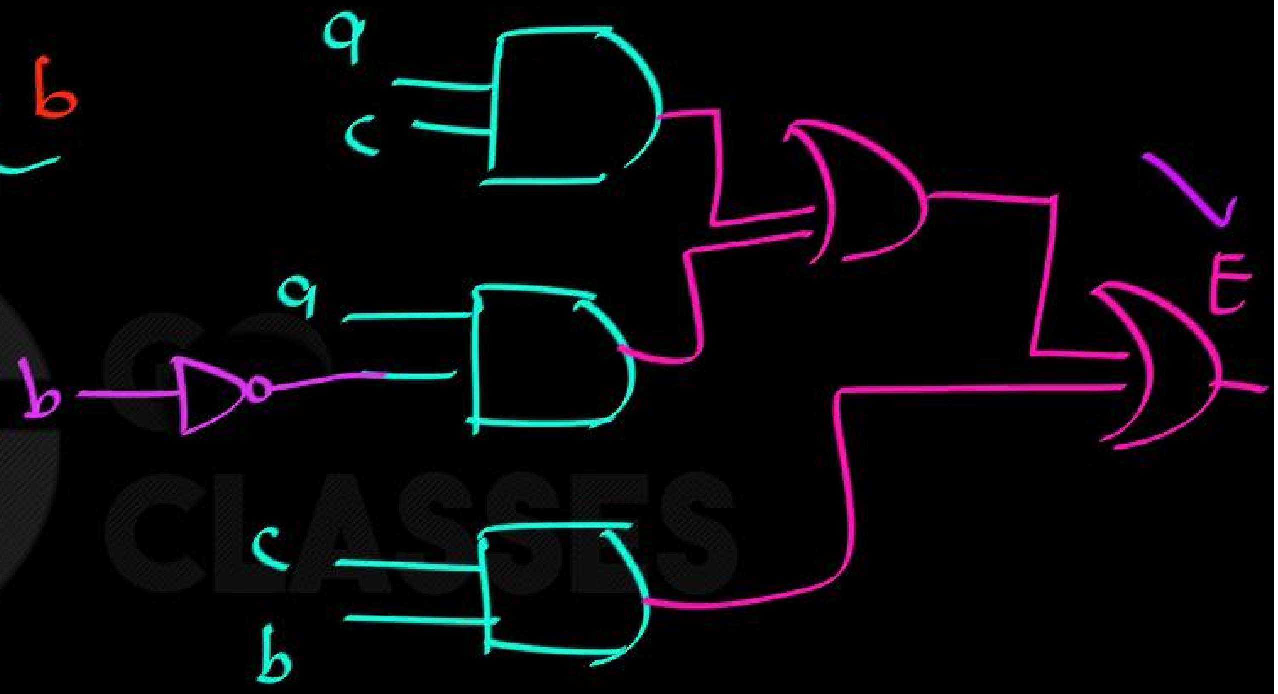
$\overline{X}Y + XZ$ (by identity)

Boolean Algebra :

Next Topic :

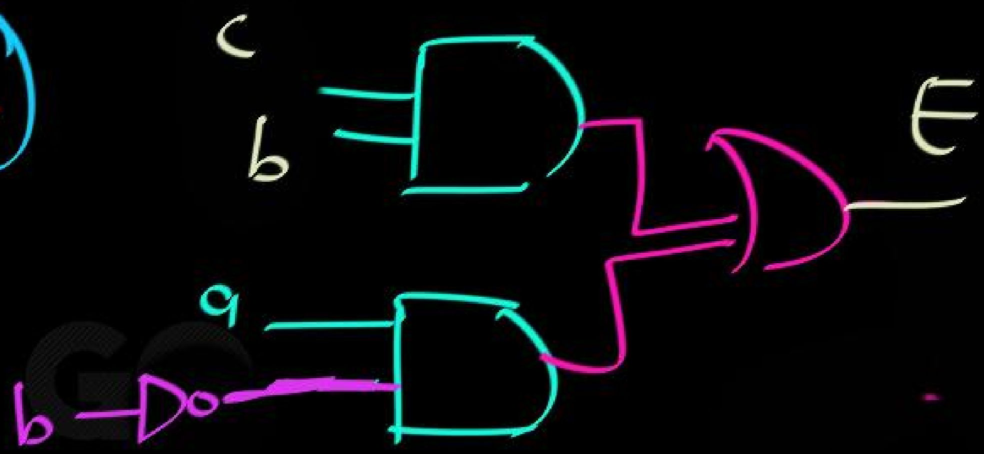
Why Simplify a Boolean Function?

$$E = \underbrace{ac + a\bar{b} + cb}$$



$E = ac + a\bar{b} + cb$

$E = a\bar{b} + cb$



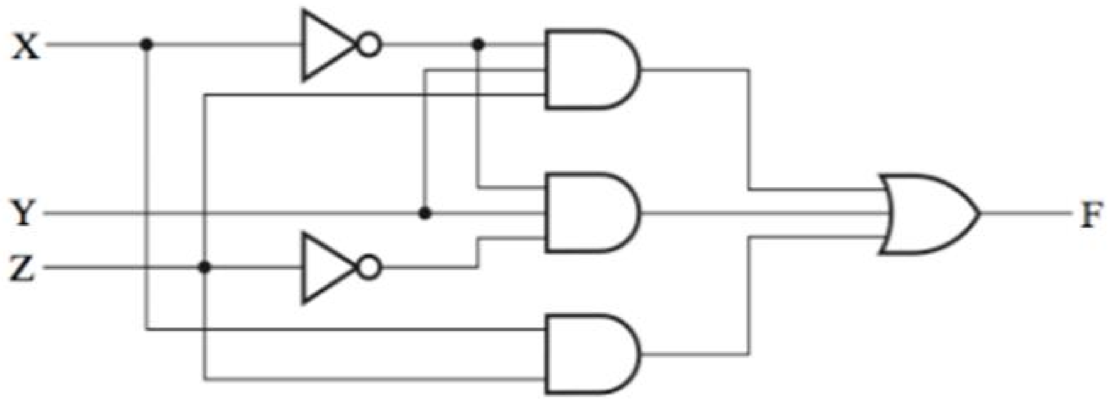
In each case, one expression can be replaced by a simpler one. Since each expression corresponds to a circuit of logic gates, simplifying an expression leads to simplifying the corresponding logic circuit.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

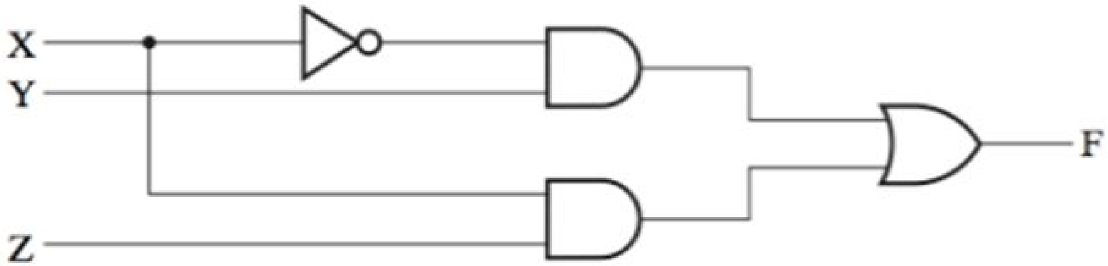
$\overline{X}Y(Z + \overline{Z}) + XZ$ (by reverse distribution)

$\overline{X}Y1 + XZ$ (by complementarity)

$\overline{X}Y + XZ$ (by identity)



(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



(b) $F = \overline{X}Y + XZ$

Literal: x, $\bar{x}$, y, $\bar{y}$, z, $\bar{z}$

Variable: x, y, z

$x \Rightarrow$ Variable

$\bar{x} \Rightarrow$ Not Variable

Literal: a variable in its original or complementery form is called literal.

Literal Count (LC) tells you how big is your expression. eg: $f = \underbrace{a+a+a+b}$ ⇒ LC=4

Each appearance of a variable or its complement in an expression will be referred to as a *literal*. Thus, the following expression, which has three variables, has 10 literals:

$$ab'c + a'b + a'bc' + b'c'$$ ⟶ #Variables = 3

$f = a+b$ ⇒ LC=2

$f = \underbrace{ab'c + a'b + a'b'c' + b'c'}$

LC = 10 ✓

$E: \underbrace{a + \bar{a} + a + \bar{a}}$ → over only one variable.

Literal count of $E = 4$

$\underbrace{E_2 : 1}$

$LC(E_2) = 0$

$$E_1 : \underbrace{a+b+b}_{LC=3} \equiv E_2 : \underbrace{a+b}_{LC=2}$$

Simplification / minimization of an Expression:

(minimize) Reducing the Litral Count.

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C$$

Literal Count = 8

How big is the Expression

#Variables = 3

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C$$

$$\underbrace{B + \overline{B}} = 1$$

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C$$

$B(\overline{A} + A) + \overline{B}(\overline{C}+C)$ (by distribution)

$B + \overline{B}$ (by complementarity)

1 (by complementarity)

Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\overline{X} + Y = \overline{X}\,\overline{Y} + \overline{X}Y + XY$$

Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\bar{X} + Y = \bar{X}\bar{Y} + \bar{X}Y + XY$$

$$\cancel{\bar{x}\bar{y}} + y$$

$$= \bar{x} + y$$

Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\overline{X} + Y = \overline{X}\,\overline{Y} + \overline{X}Y + XY$$

$\overline{X}\,\overline{Y} + \overline{X}Y + \overline{X}Y + XY$ (by idempotence)

$\overline{X}(\overline{Y} + Y) + Y(\overline{X} + X)$ (by distribution)

$\overline{X}\,1 + Y\,1$ (by null)

$\overline{X} + Y$ (by identity)

Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$\overline{X}Y(Z + \overline{Z}) + XZ$ (by reverse distribution)

$\overline{X}Y1 + XZ$ (by complementarity)

$\overline{X}Y + XZ$ (by identity)

Example

HW

- Using the Theorems and Laws of Boolean algebra, Prove the following.

$$\boxed{(A+B).(A+A'B').C + (A'.(B+C'))' + A'.B + A.B.C} = A+B+C$$

using
by Case method ✓
Boolean Algebra HW

Case 1: $A = 0$ ✓

$LHS = B + (B + C')'$

$= B + \cancel{\overline{B}}C = B + C$

$RHS = B + C$ ✓

Same

Case 2: $A = 1$

$LHS = C + 1 + - + -$

$= 1$

$RHS = 1$

Same

Algebraic Simplification

HW

- Using Boolean algebra techniques, simplify this expression: $AB + A(B + C) + B(B + C)$

Algebraic Simplification

- Using Boolean algebra techniques, simplify this expression: AB + A(B + C) + B(B + C)
- Solution

=AB + AB + AC + BB + BC	(Distributive law)
=AB + AB + AC + B + BC	(B.B=B)
= AB + AC + B + BC	(AB+AB=AB)
= AB + AC + B	(B+BC =B)
=B+AC	(AB+B =B)

Algebric Simplification

- Minimize the following Boolean expression using Algebric Simplification

$F(A,B,C)=A'B+BC'+BC+AB'C'$

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Algebric Simplification

GO Classes

- Minimize the following Boolean expression using Algebric Simplification

$F(A,B,C)=A'B+BC'+BC+AB'C'$

- Solution

$=A'B+(BC'+BC')+BC+AB'C'$ [indeponent law]

$= A'B+(BC'+BC)+(BC'+AB'C')$

$= A'B+B(C'+C)+C'(B+AB')$

$=A'B + B.1+ c' (B+A)$

$= B(A'+1)+C'(B+A)$

$=B + C'(B+A)$ [A'+1=1]

$= B+BC'+AC'$

$= B(1+C')+AC'$

$= B+AC'$ [1+C' = 1]

Algebric Simplification

- Simplify: C + (BC)’

Algebric Simplification

- Simplify: C + (BC)'

=C + (BC)'	Original Expression
=C + (B' + C')	DeMorgan's Law.
=(C + C') + B'	Commutative, Associative Laws.
=1 + B'	Complement Law.
=1	Identity Law.

NOTE:

Cancellation Laws do NOT hold in Boolean Algebra.

Eg: If $ab = ac$ then $b = c$? No.

$$\underbrace{\cancel{0}\cdot 1}_{0} = \underbrace{\cancel{0}\cdot 0}_{0} \Rightarrow 1 \neq 0$$

If $a+b = a+c$ then $b = c$? No.

$$\cancel{1}+0 = \cancel{1}+1 \Rightarrow 0 \neq 1$$

If $a \oplus b = a \oplus c \Longrightarrow b = c$?

If $a \odot b = \overline{a} \odot c \Longrightarrow$?

If $a \oplus b = a \oplus c \implies b = c$? ✓

$\underline{a=0}$ $b = c$

$\underline{a=1}$ $\bar{b} = \bar{c} \implies b = c$

If $a \odot b = \bar{a} \odot c \implies \boxed{b = \bar{c}}$?

$\underline{a=0} \implies \bar{b} = c$ ✓

$\underline{a=1} \implies b = \bar{c} \implies \bar{b} = c$

If $a \odot b = a \odot c \Longrightarrow b = c$?

If $a + b = a + c$ and $\overset{a=0}{\Longrightarrow}$ $b = c$?

If $a \odot b = a \odot c \Longrightarrow b = c$? ✓

$a=0 \Rightarrow \bar{b} = \bar{c}$

$a=1 \Rightarrow b = c$

If $a + b = a + c$ and $a=0 \Longrightarrow b = c$? ✓

As we have previously observed, some of the theorems of Boolean algebra are not true for ordinary algebra. Similarly, some of the theorems of ordinary algebra are *not* true for Boolean algebra. Consider, for example, the cancellation law for ordinary algebra:

$$\text{If } x + y = x + z, \quad \text{then} \quad y = z \tag{3-31}$$

The cancellation law is *not* true for Boolean algebra. We will demonstrate this by constructing a counterexample in which $x + y = x + z$ but $y \neq z$. Let $x = 1$, $y = 0$, $z = 1$. Then,

$$1 + 0 = 1 + 1 \text{ but } 0 \neq 1$$

In ordinary algebra, the cancellation law for multiplication is

$$\text{If } xy = xz, \quad \text{then} \quad y = z \tag{3-32}$$

This law is valid provided $x \neq 0$.

In Boolean algebra, the cancellation law for multiplication is also *not* valid when $x = 0$. (Let $x = 0, y = 0, z = 1$; then $0 \cdot 0 = 0 \cdot 1$, but $0 \neq 1$). Because $x = 0$ about half of the time in switching algebra, the cancellation law for multiplication cannot be used.

Even though Statements (3-31) and (3-32) are generally false for Boolean algebra, the converses

$$\text{If } y = z, \quad \text{then} \quad x + y = x + z \tag{3-33}$$

$$\text{If } y = z, \quad \text{then} \quad xy = xz \tag{3-34}$$

$X \oplus 0 = ? = X$

$0 \longrightarrow 0$

$1 \longrightarrow 1$

Associativity of $\odot$

Prove using by-case method

HW

Q: $a(b \oplus c) = ab \oplus ac$

Case 1: $a = 0$

$LHS = 0$

$RHS = 0 \oplus 0 = 0$

Same

Case 2: $a = 1$

$LHS = b \oplus c$

$RHS = b \oplus c$

Same

$$\underbrace{a \oplus b = a' \oplus b'}$$ $\neq$ $\underbrace{\overline{a \oplus b}}$

$\equiv$

$a \odot b$

Prove:

$a = 0$

LHS $= b$; RHS $= b$

$a = 1$

LHS $= \overline{b}$; RHS $= \overline{b}$

The following theorems apply to exclusive OR:

$$X \oplus 0 = X$$

$$X \oplus 1 = X'$$

$$X \oplus X = 0$$

$$X \oplus X' = 1$$

$$X \oplus Y = Y \oplus X \text{ (commutative law)}$$

$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z \text{ (associative law)}$$

$$X(Y \oplus Z) = XY \oplus XZ \text{ (distributive law)}$$

$$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$$

$$\overline{(a \oplus b)} = a \odot b = a' \oplus b = a \oplus b'$$

$$= a' \odot b'$$

Note:

$a \oplus b = a' \oplus b'$

$a \neq b \longrightarrow$ Diff

$a = b \longrightarrow$ Same

$a \oplus b = a' \oplus b$

$a \neq b \longrightarrow$ Same

$a = b \longrightarrow$ Different

NOTE:

$$a \oplus b = a' \oplus b'$$

$$a \odot b = a' \odot b'$$

$$a' \oplus b = a \oplus b' = a \odot b = \overline{(a + b)}$$

$$a' \odot b = a \odot b' = a \oplus b = \overline{a \odot b}$$

Any of these theorems can be proved by using a truth table or by replacing $X \oplus Y$ with one of the equivalent expressions from Equation (3-7). Proof of the distributive law follows:

$$\begin{aligned} XY \oplus XZ &= XY(XZ)' + (XY)'XZ = XY(X' + Z') + (X' + Y')XZ \\ &= XYZ' + XY'Z \\ &= X(YZ' + Y'Z) = X(Y \oplus Z) \end{aligned}$$

Equivalence is the complement of exclusive OR:

$$\begin{aligned}(X \oplus Y)' &= (X'Y + XY')' = (X + Y')(X' + Y) \\ &= XY + X'Y' = (X \equiv Y)\end{aligned} \qquad (3\text{-}18)$$

Just as for exclusive OR, equivalence is commutative and associative.

$$a \oplus b = \overline{a \odot b} = \overline{ab + \bar{a}\bar{b}} = (\bar{a} + \bar{b})(a + b)$$

$$a \odot b = ab + \bar{a}\bar{b}$$

$$a \oplus b = \bar{a}b + a\bar{b} = (a + b)(\bar{a} + \bar{b}) \checkmark$$

$$a \odot b = ab + \bar{a}\bar{b} = (a + b')(\bar{a} + b) \checkmark$$

$$a \odot b = \overline{a \oplus b} = \overline{\left(\bar{a}b + a\bar{b}\right)}$$

$$= (a+\bar{b})(\bar{a}+b)$$

$$a \oplus b = \bar{a}b + a\bar{b}$$

13a.	$(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)$	Consensus Law
13b.	$X Y + \overline{X} Z + Y Z = X Y + \overline{X} Z$	Consensus Law
14a.	$X \oplus Y = (X + \overline{Y}) \cdot (\overline{X} + Y)$; 14b. $X \oplus Y = \overline{X} Y + X \overline{Y}$	XOR Gate
15a.	$X \odot Y = (X + Y) \cdot (\overline{X \cdot Y})$; 15b. $X \odot Y = \overline{X}\,\overline{Y} + X Y$	XNOR Gate
15c.	$X \odot Y = (X + Y) \cdot (\overline{X} + \overline{Y})$	XNOR Gate

DeMorgan's Theorem

$$\overline{(X_1 + X_2 + \ldots + X_n)} = \overline{X_1} \cdot \overline{X_2} \cdot \ldots \cdot \overline{X_n}$$

$$\overline{(X_1 \cdot X_2 \cdot \ldots \cdot X_n)} = \overline{X_1} + \overline{X_2} + \ldots + \overline{X_n}$$

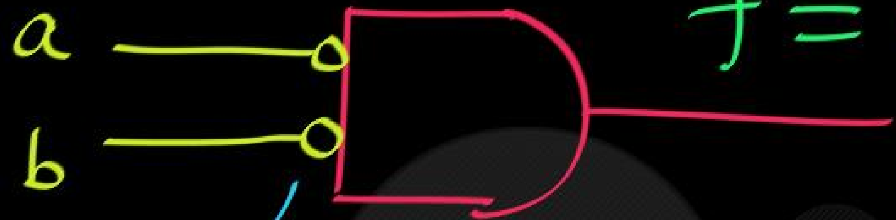
Remember:

$$\overline{X \cdot Y} \neq \overline{X} \cdot \overline{Y}$$

$$\overline{X + Y} \neq \overline{X} + \overline{Y}$$

Note:

①

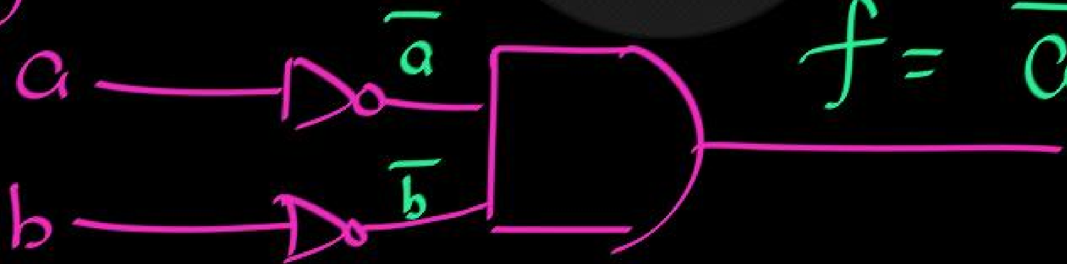


$f = \bar{a}\,\bar{b}$

||

bubble ≡ NOT

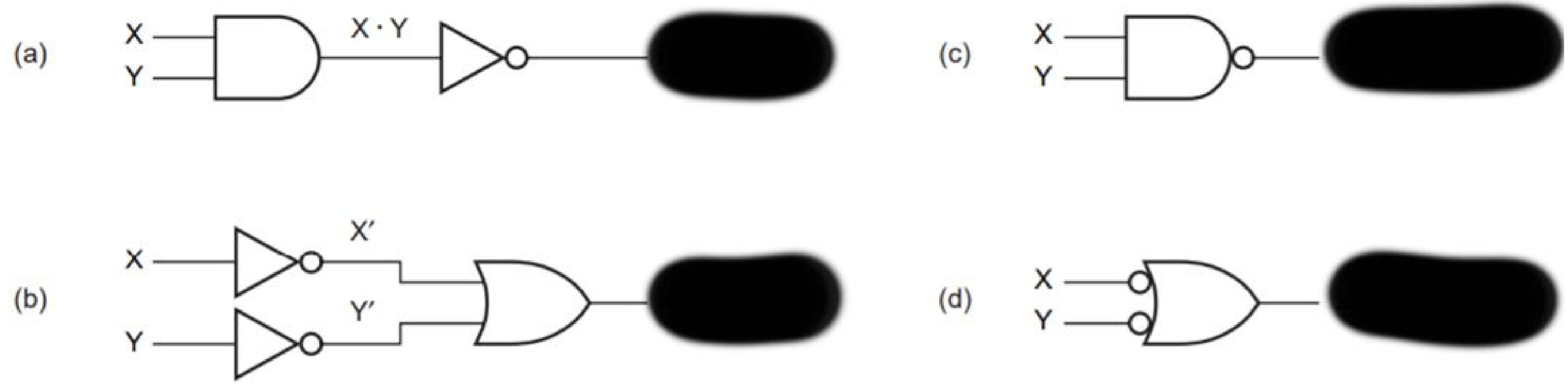
②



$f = \bar{a}\,\bar{b}$

DeMorgan's theorems

$(X \cdot Y)' = X' + Y'$:

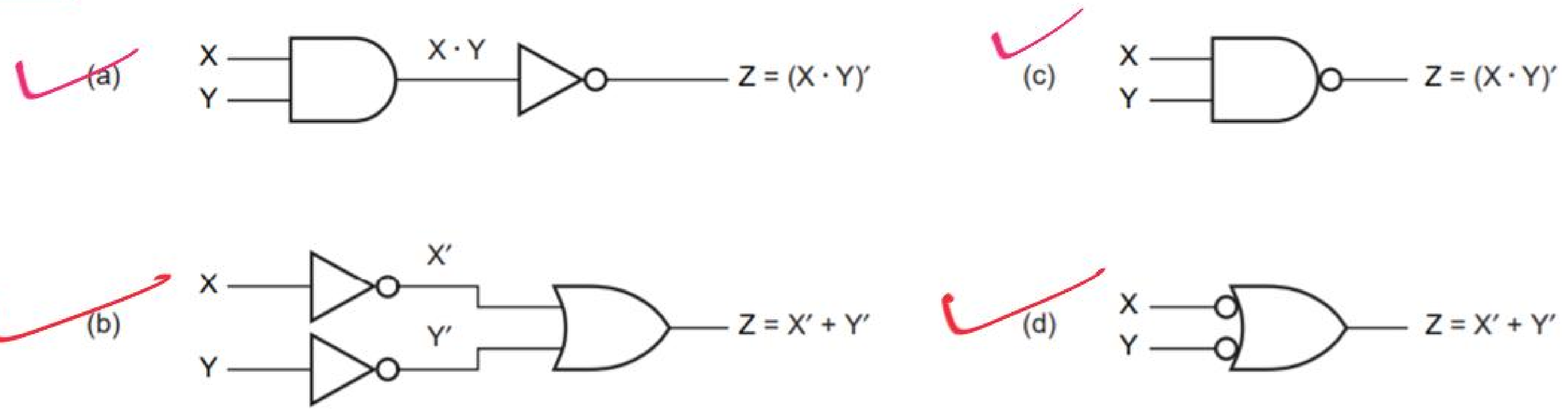


$\overline{(xy)} = \bar{x} + \bar{y}$

DeMorgan's theorems

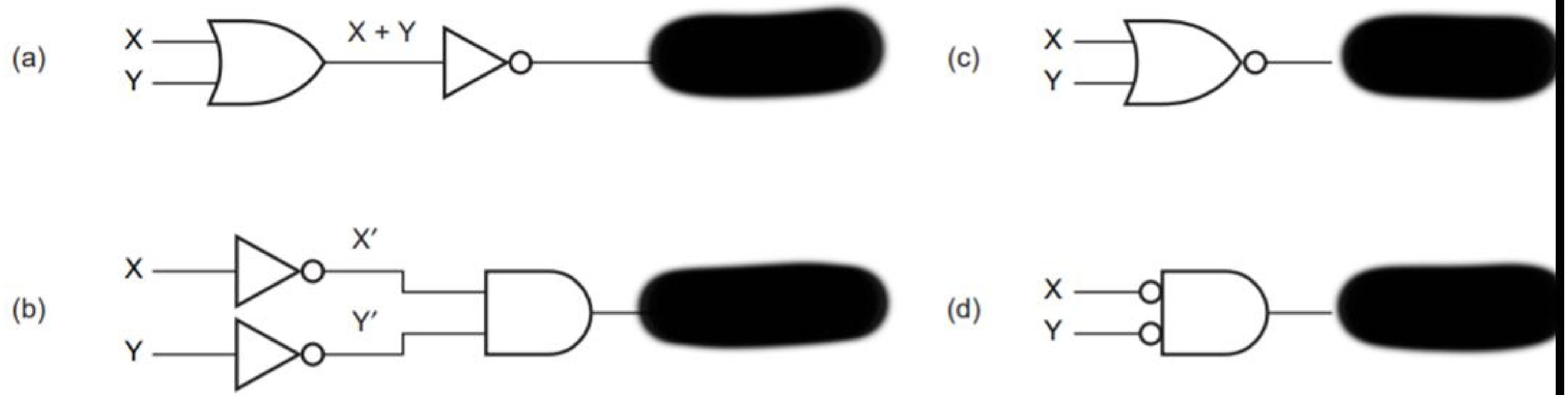
NAND = NOT of AND

$(X \cdot Y)' = X' + Y'$:



[redacted]

$$(X + Y)' = X' \cdot Y':$$

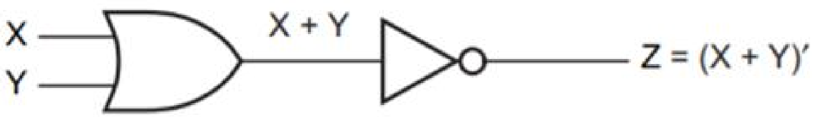


[redacted]

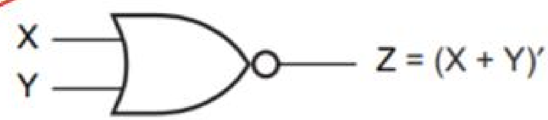
NOR = NOT of OR = $\overline{x+y} = \bar{x}\,\bar{y}$

$$(X + Y)' = X' \cdot Y':$$

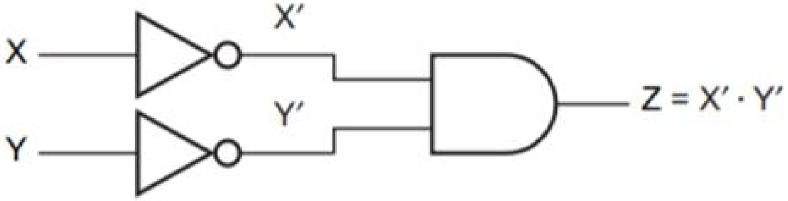
(a)



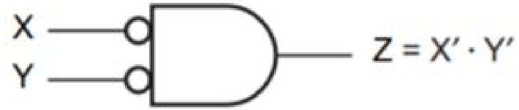
(c)



(b)

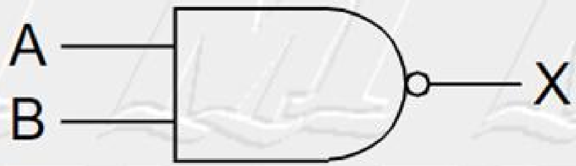


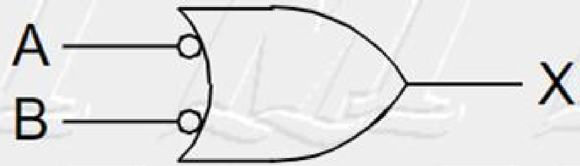
(d)



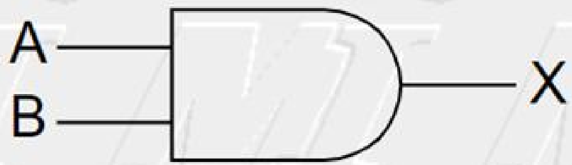
Gates

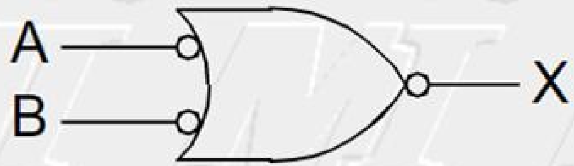
	Standard	DeMorgan's
NAND	$X = \overline{A \cdot B}$	$X = \overline{A} + \overline{B}$



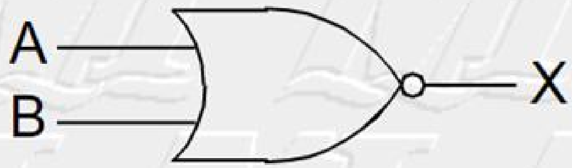


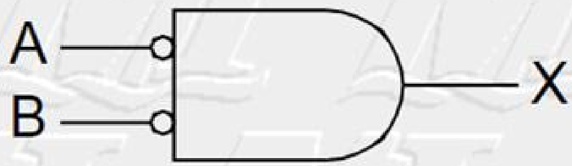
	Standard	DeMorgan's
AND	$X = A \cdot B$	$X = \overline{\overline{A} + \overline{B}}$





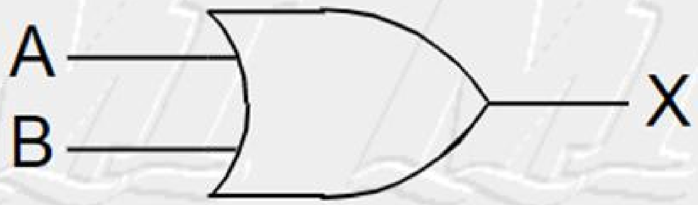
	Standard	DeMorgan's
NOR	$X = \overline{A + B}$	$X = \overline{A} \cdot \overline{B}$





OR

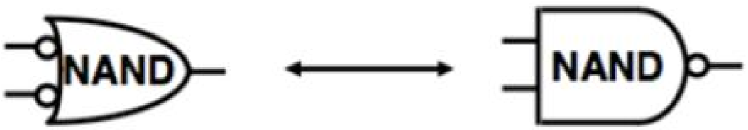
$X = A + B$



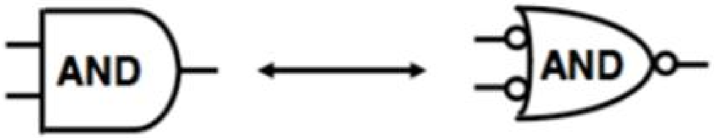
$X = \overline{\overline{A} \cdot \overline{B}}$

$A + B = \overline{\overline{A}\,\overline{B}}$

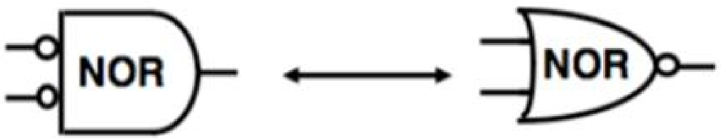
$$\overline{X} + \overline{Y} = \overline{XY}$$



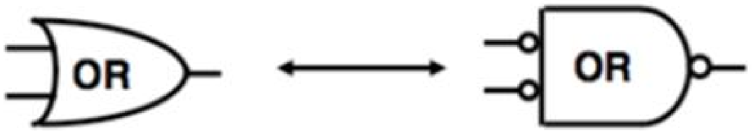
$$XY = \overline{\overline{X} + \overline{Y}}$$



$$\overline{X}\,\overline{Y} = \overline{X + Y}$$



$$X + Y = \overline{\overline{X}\,\overline{Y}}$$



$$a = a+a = a+0 = a \oplus 0 = a \odot 1$$

$$a' = a'+\bar{a} = \bar{a}\,\bar{a} = \overline{a+0} = a \oplus 1 = a \odot 0$$

$$a \oplus b = a' \oplus b' = \overline{a \odot b} = a'b + a\bar{b}$$

$$0 = a \cdot 0 = a\bar{a} = a \oplus a = a \odot \bar{a}$$

$f(a,b)$

a	b	f
0	0	0/1
0	1	0/1
1	0	0/1
1	1	0/1

for every Row (input Combination) we have 2 choices for $f(a,b)$

Q:

Number of Different Boolean Functions on 1 Boolean variables?

Q:

$f(a)$

Number of Different Boolean Functions on 1 Boolean variables?

a	f_0	f_1	f_2	f_3
0	0	0	1	1
1	0	1	0	1

Ans: 4

$f_0 = 0$

$f_1 = a$

$f_2 = \bar{a}$

$f_3 = 1$

Diff

$$a \oplus (a + a) = a \oplus a = 0 = f_0$$

$$a \odot (a \oplus \bar{a}) = a \odot 1 = a = f_1$$

$$(a \oplus 1) \odot (\bar{a} + 1) = \bar{a} = f_2$$

$$a + \bar{a} = 1 = f_3 \qquad a\bar{a} = 0 = f_0$$

$f(a)$ = **Boolean function**

$$\text{Domain} \xrightarrow{f(a)} \text{CoDomain} \rightarrow \text{fix} = \{0, 1\}$$

Domain → input Combi-nations

Domain: 0, 1

CoDomain: 0, 1

#functions $= 2^2 = 4$

Q:

Number of Different Boolean Functions on 0 Boolean variables?

Q:

Number of Different Boolean Functions on 0 Boolean variables?

2 ⟶ Constant Boolean functions

$f_0 = 0$

$f_1 = 1$

Q:

Number of Different Boolean Functions on n Boolean variables?

Q:

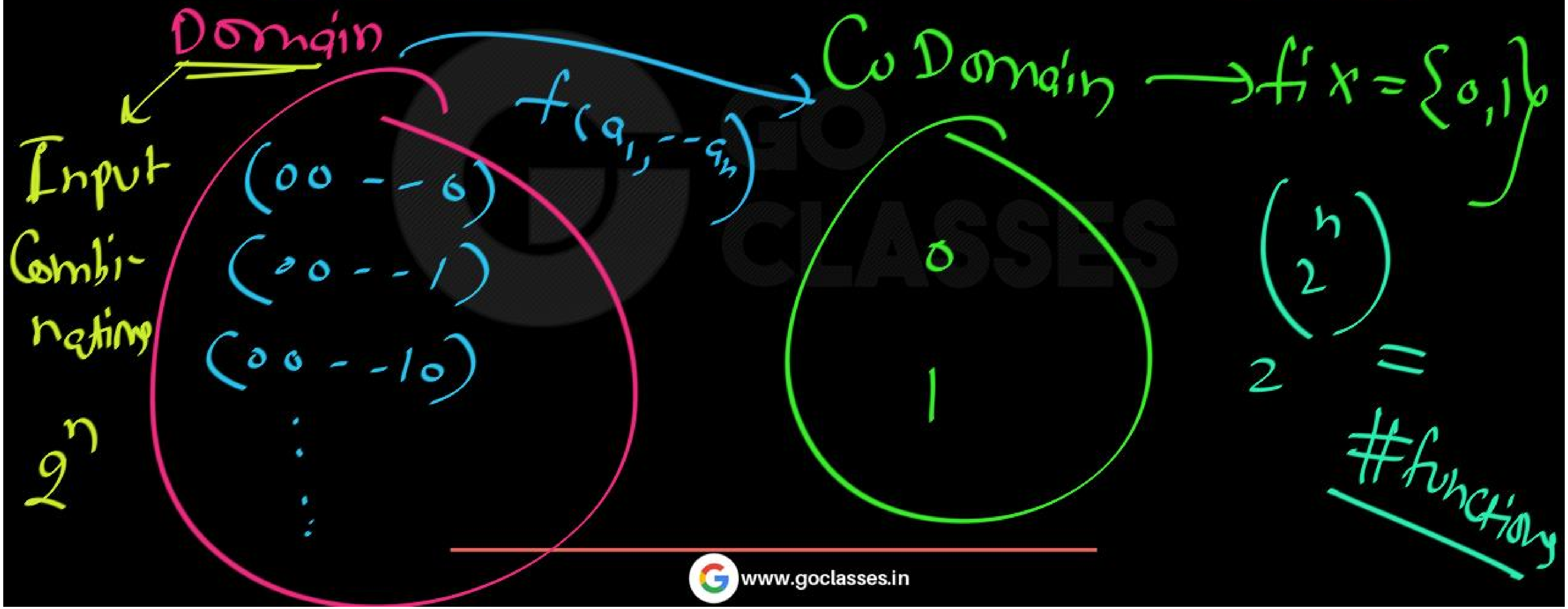
Number of Different Boolean Functions on n Boolean variables?

$a_1, a_2 - - a_n$	$f(a_1, a_2, - - a_n)$
2^n Rows	→ 2 choices
	→ 2 "
	→ 2 "
	⋮
	→ 2 "

#functions = $(2)^{(2^n)}$

Q:

Number of Different Boolean Functions on n Boolean variables?



On n boolean variables ;

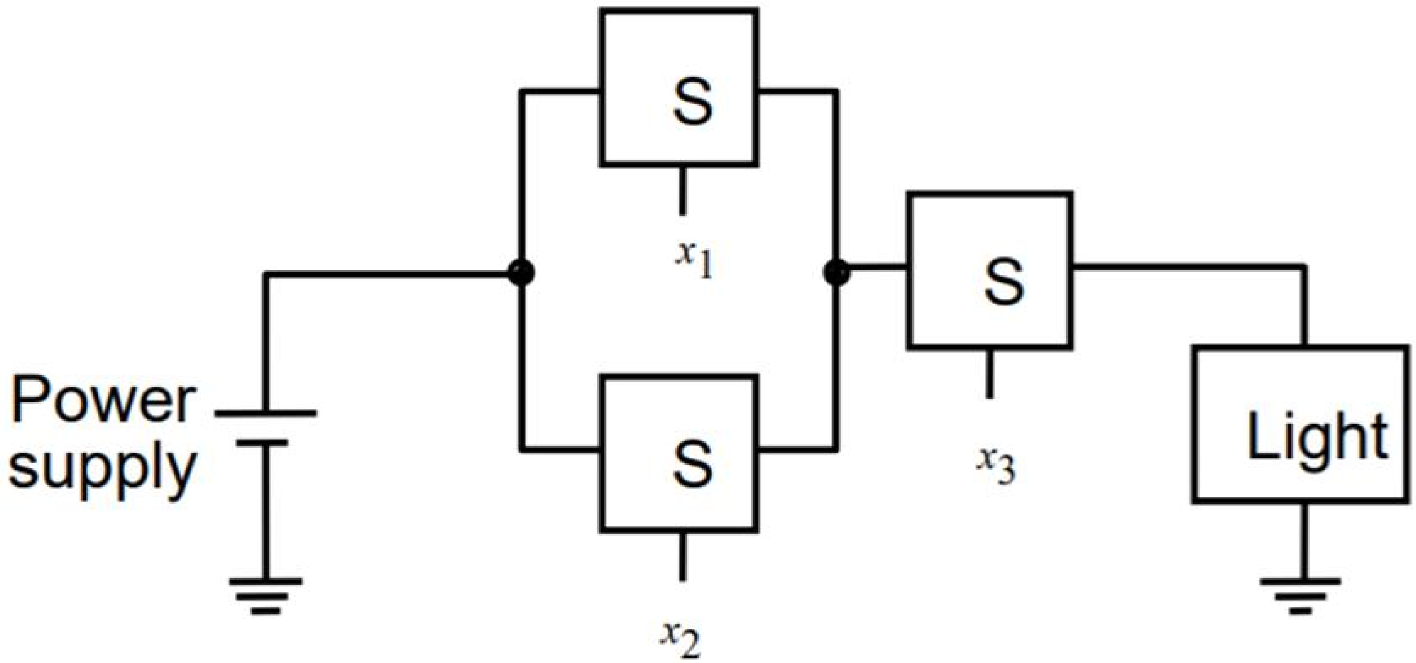
$$\#\text{Boolean functions} = 2^{(2^n)}$$

Boolean Algebra :

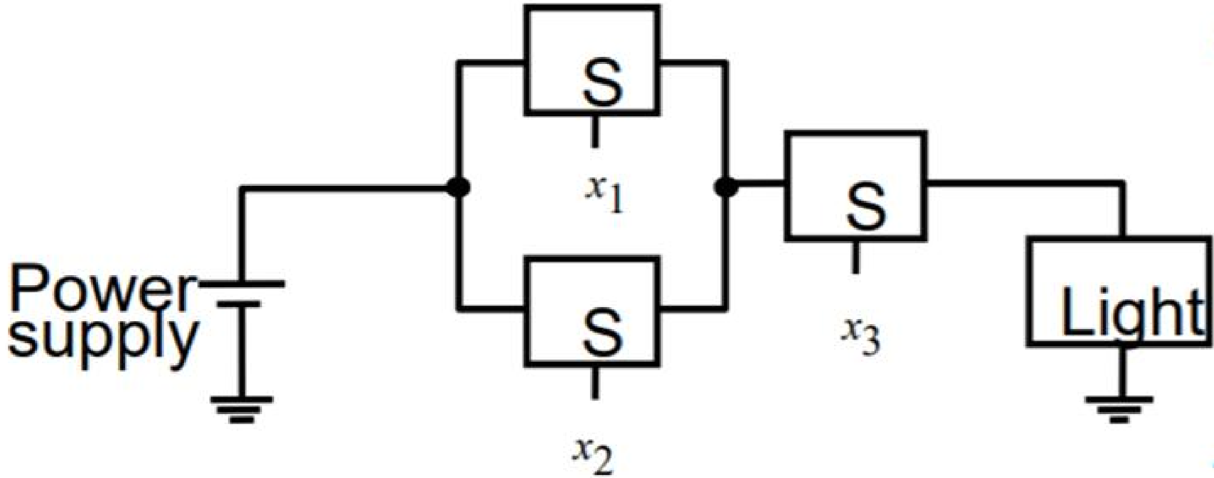
Next Topic :

Switches, Boolean Expressions & Logic gate circuit implementation

A Series Parallel Circuit



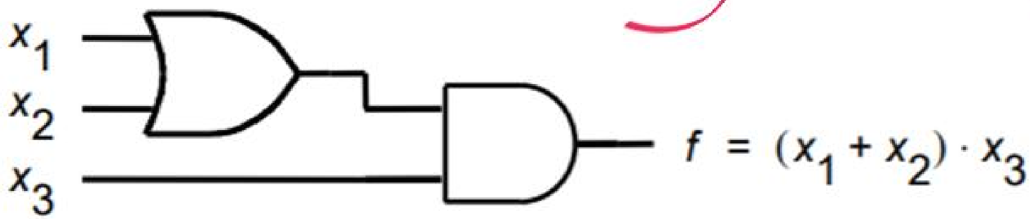
Boolean Functions



physical circuit

$f = (x_1 \text{ OR } x_2) \text{ AND } (x_3)$
$= (x_1 + x_2) \, . \, x_3$

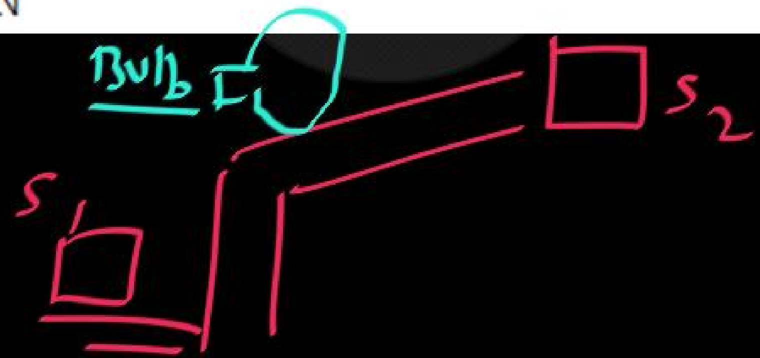
logical behaviour



logic gate implementati

Design Problem 1

- Design a circuit for staircase light control using a two-way switch
- `If both switches are OFF or ON, light is off`
- `If one of them is ON and another is OFF, the light is ON`




S_1
S_2
$\oplus$
Bulb

Design Problem 1

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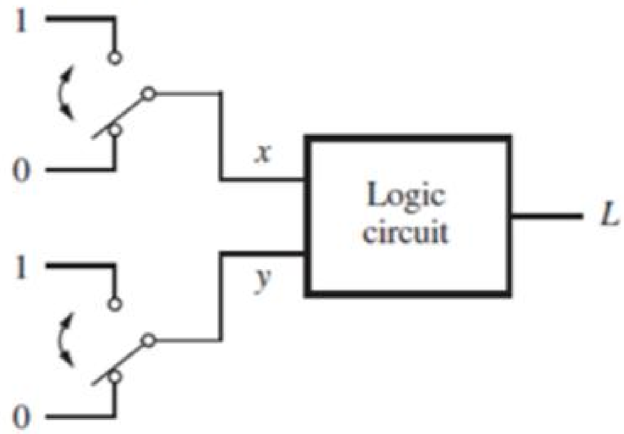


(a) Two switches that control a light

x	y	L
0	0	0
0	1	1
1	0	1
1	1	0

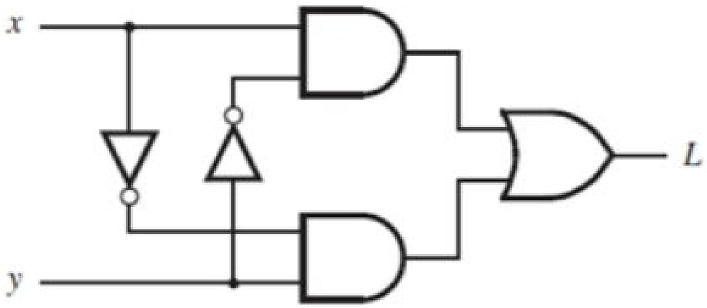
(b) Truth table

Boolean Expression

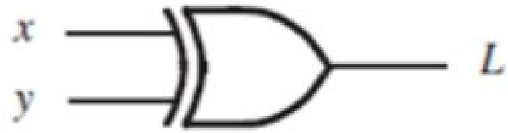
- Lamp is ON when
  - Either x is ON and y is OFF
  - Or x is OFF and y is ON

Boolean Expression

- Lamp is ON when
  - Either x is ON and y is OFF
  - Or x is OFF and y is ON
- L = x . y'
  + x' . y



(c) Logic network



(d) XOR gate symbol

Design Problem 2

- Design a circuit which realizes the following truth table

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

f = ?

Design Problem 2

- Design a circuit which realizes the following truth table

$x=0 \Rightarrow \overline{x}$

$x=1 \Rightarrow x$

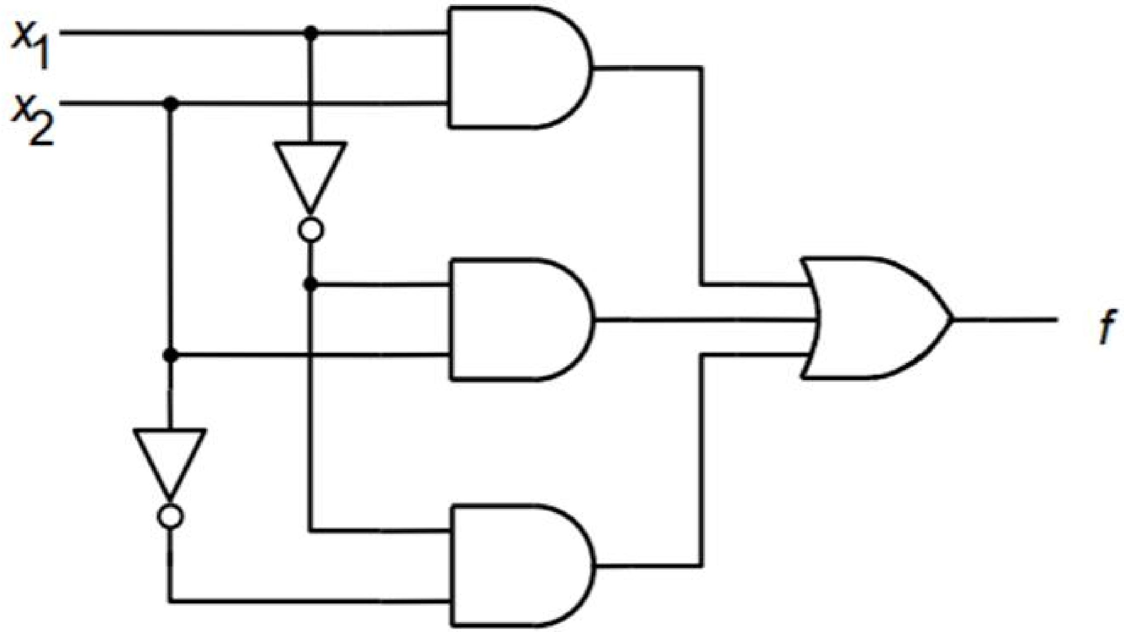
Next lecture

x_1	x_2	$f(x_1, x_2)$
0	0	1 ✓
0	1	1 ✓
1	0	0
1	1	1 ✓

f = ? $= \overline{x_1}\,\overline{x_2} + \overline{x_1}\,x_2 + x_1 x_2$

Design Problem 2 (contd.)

- $f = x_1'.x_2' + x_1'.x_2 + x_1.x_2$



Power of Simplification

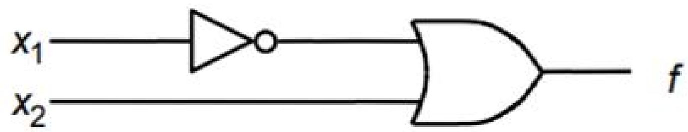
- $f = x_1'.x_2' + x_1'.x_2 + x_1.x_2$

  $= x_1'.x_2' + x_1'.x_2 + x_1'.x_2 + x_1.x_2$

  $= x_1'.(x_2' + x_2) + (x_1' + x_1).x_2$

  $= x_1'.1 + 1.x_2$

  $= x_1' + x_2$



Minimal-cost realization