LOGICAL CONNECTIVES

Implica-->tion

Bi<-->implication

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DISCRETE MATHS

HW

Propositional Logic

Homework 2

Implication, Bi-Implication

Necessary & Sufficient Conditions

NOTES :

Necessary Conditions

If we say that "x is a necessary condition for y," we mean that if we don't have x, then we won't have y. Or put differently, without x, you won't have y. To say that x is a necessary condition for y does **NOT** mean that x guarantees y.

Example :

Having oxygen in the earth's atmosphere is a necessary condition for human life. Certainly, having oxygen will not guarantee human life. There are many other conditions needed for human life other than oxygen in the atmosphere.

Sufficient Conditions

If we say that "x is a sufficient condition for y," then we mean that if we have x, we know that y must follow.
In other words, x guarantees y.

Example :

Rain pouring from the sky is a sufficient condition for the ground to be wet.
“Having 200 rupees” is sufficient(enough) to “have at least 100 rupees”.

NOTES :

In Logic, In the conditional statement, “If A, then B”, the antecedent(A) refers to a sufficient condition for the consequent(B).
If you have A, then B must follow.

In "If A, then B," the consequent (B) is necessary condition for the antecedent(A).
Without B being true, A cannot be true.

NOTE :
Mathematically speaking, necessity and sufficiency are dual to one another.

For any statements S and N, the assertion that "N is necessary for S" is equivalent to the assertion that "S is sufficient for N".

If P is sufficient for Q, then Q is necessary for P.

NOTES :

If x is a necessary condition for y, then y is a sufficient condition for x.

And, equivalently,

If y is a sufficient condition for x, then x is a necessary condition for y.

•"Being a father is a sufficient condition for being male, and being male is a necessary condition for being a father." (But being a father is not a necessary condition for being a male; and being a male is not a sufficient condition for being a father.)

Implication

Theorems can usually be written in the form

if a collection of *assumptions* holds,
then so does some *conclusion*

or, in other words,

a collection of *assumptions* **implies** some *conclusion*

or, in symbols,

a collection of *hypotheses* $\Longrightarrow$ some *conclusion*

Bi-implication

P is equivalent to Q

or, in other words,

P implies Q, and vice versa

or

Q implies P, and vice versa

or

P if, and only if, Q

P iff Q

or, in symbols,

$$P \iff Q$$

Conjunction

Conjunctive statements are of the form

P **and** Q

or, in other words,

both P and also Q hold

or, in symbols,

$P \wedge Q$ or P & Q

Question 1: **For each of the following, say whether it is true or false.**

x's being a square is a sufficient condition for x's being a rectangle.

x's being a square is a necessary condition for x's being a rectangle.

x's being a rectangle is a sufficient condition for x's being a square.

x's being a rectangle is a necessary condition for x's being a square.

x's being a mother is a sufficient condition for x's being a female.

x's being a mother is a necessary condition for x's being a female.

x's being a female is a sufficient condition for x's being a mother.

x's being a female is a necessary condition for x's being a mother.

x's being greater than 15 is a sufficient condition for x's being less than 20.

x's being greater than 15 is a necessary condition for x's being less than 20.

Some answers may depend on how one interpret the statements. So, except for mathematical statements, DO NOT worry about the answer of other questions.

Question 1(Contd.): For each of the following, say whether it is true or false.

x's being less than 20 is a sufficient condition for x's being greater than 15.

x's being less than 20 is a necessary condition for x's being greater than 15.

x's being less than 12 is a sufficient condition for x's being less than 20.

x's being less than 12 is a necessary condition for x's being less than 20.

x's being less than 20 is a sufficient condition for x's being less than 12.

x's being less than 20 is a necessary condition for x's being less than 12.

x's having two arms is a sufficient condition for x's being a human being.

x's having two arms is a necessary condition for x's being a human being.

x's wanting to do a is a sufficient condition for x's doing a.

x's wanting to do a is a necessary condition for x's doing a.

Some answers may depend on how one interpret the statements. So, except for mathematical statements, DO NOT worry about the answer of other questions.

Solution 1 :

1. true
2. false
3. false
4. true
5. true
6. false
7. false
8. true
9. false
10. false
11. false
12. false
13. true
14. false
15. false
16. true

17. false (A chimpanzee, for example, may have two arms, but it is not a human being.)

18. false (A war veteran may have lost both his arms, but he is a human being even so.)

19. false (A person may want to win a foot race, but his/her wanting to do so, does not guarantee he/she can or will do so.)

20. false (A person may do something without wanting to do so, e.g. without wanting to hurt another's feelings, we sometimes do.)

Question 2 :

6. Let proposition p be T and proposition q be F. Find the truth values for the following:

(a) $p \vee q$
(b) $q \wedge p$
(c) $\neg p \vee q$
(d) $p \wedge \neg q$
(e) $q \rightarrow p$
(f) $\neg p \rightarrow q$
(g) $\neg q \rightarrow p$

Question 3 :

7. Let proposition p be T, proposition q be F, and proposition r be T. Find the truth values for the following:

   (a) $p \vee q \vee r$
   (b) $p \vee (\neg q \wedge \neg r)$
   (c) $p \rightarrow (q \vee r)$
   (d) $(q \wedge \neg p) \leftrightarrow r$

   (e) $\neg r \rightarrow (p \wedge q)$
   (f) $(p \rightarrow q) \rightarrow \neg r$
   (g) $((p \wedge r) \rightarrow (\neg q \vee p)) \rightarrow (q \vee r)$

Question 4 :

(e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:

i. Jill passed math?

ii. Jill did not pass math?

4. Determine whether each molecular statement below is true or false, or whether it is impossible to determine. Assume you do not know what my favorite number is (but you do know that 13 is prime).

(a) If 13 is prime, then 13 is my favorite number.

(b) If 13 is my favorite number, then 13 is prime.

(c) If 13 is not prime, then 13 is my favorite number.

(d) 13 is my favorite number or 13 is prime.

(e) 13 is my favorite number and 13 is prime.

(f) 7 is my favorite number and 13 is not prime.

(g) 13 is my favorite number or 13 is not my favorite number.

3. Let p, q be primitive statements for which the implication $p \dashrightarrow q$ is false. Determine the truth values for each of the following.

a) $p \wedge q$ **b)** $\neg p \vee q$ **c)** $q \rightarrow p$ **d)** $\neg q \rightarrow \neg p$

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