



Propositional Logic

Doubts Session-2

Homework 1,2 Discussion

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GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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GATE 2023 Result:

Go Classes Complete Course Enrolled Students
of 2023 Batch

AIR 11, 13, 16, 29, 34, 34, 34, - - -

14+

In Top-100 (Still Counting)



Next Topic:

Homework 1,2

Discussion



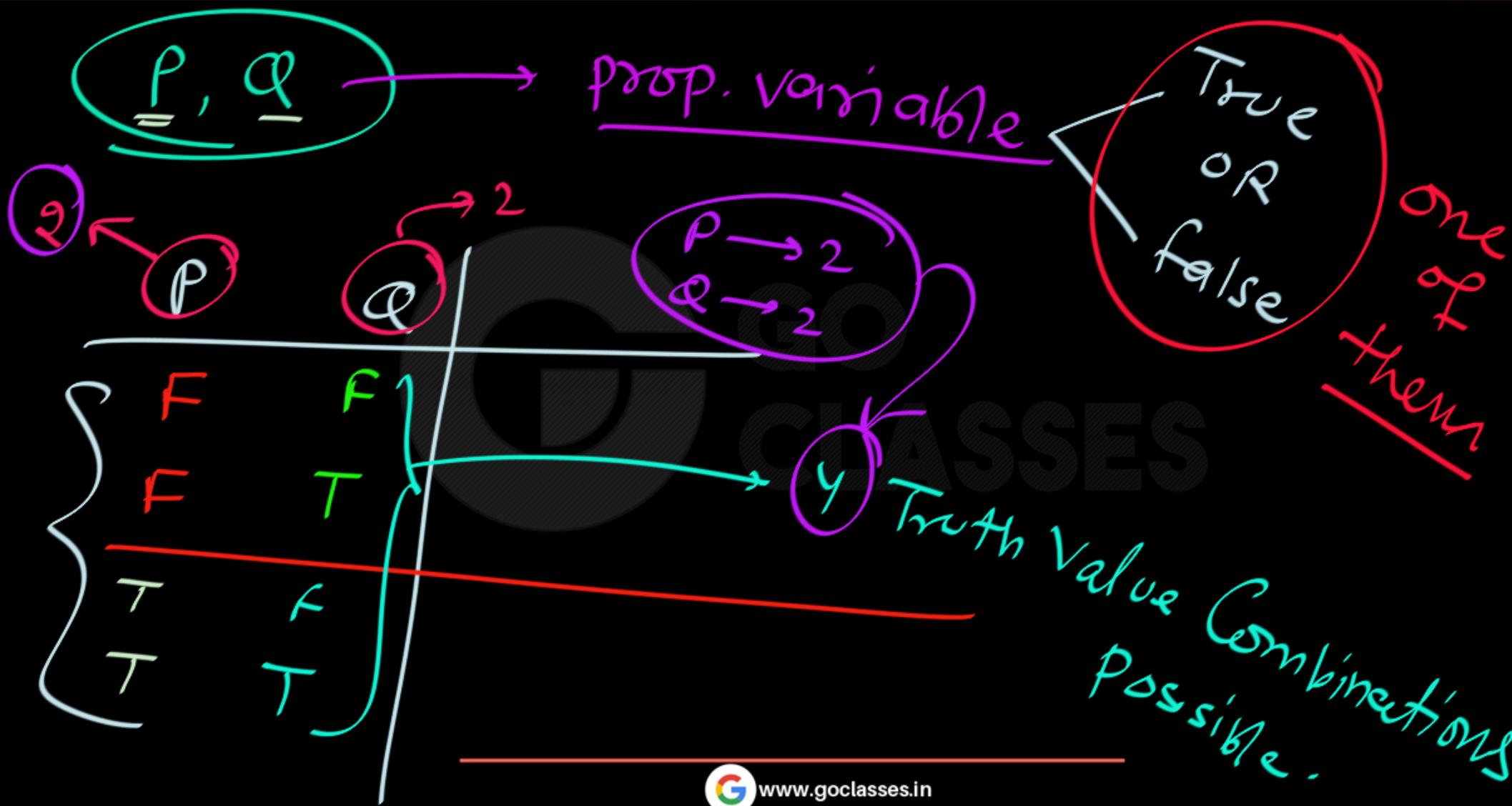
✓ HW 1 Q 2:

We have seen that for two propositional variables, we have 4 rows in truth table (because with two propositional variables, total 4 combinations of truth values are possible).

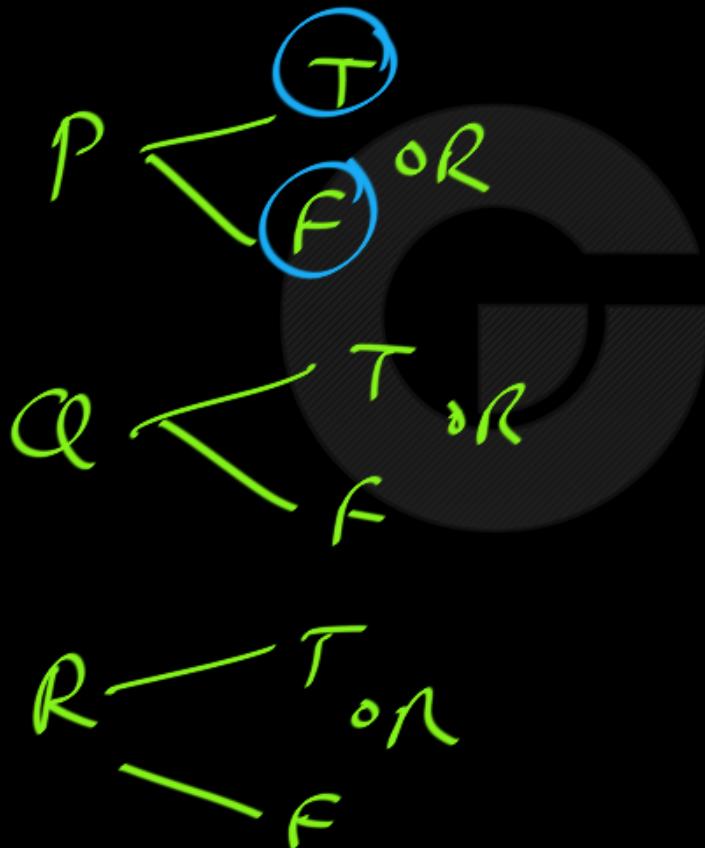
For 3 propositional variables, in the truth table, how many rows we have ?

For 4 propositional variables, in the truth table, how many rows we have ?

For n propositional variables, in the truth table, how many rows we have ?



P, Q, R → Prop. Variables



$$\left. \begin{array}{l} P \rightarrow 2 \text{ choices} \\ Q \rightarrow 2 \text{ choices} \\ R \rightarrow 2 \text{ choices} \end{array} \right\} \begin{array}{l} 2 \times 2 \times 2 \\ = 8 \\ \text{Combinations} \end{array}$$

P, Q, R → Prop. Variables

P	Q	R	
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

8 Combinations

P, Q, R, S

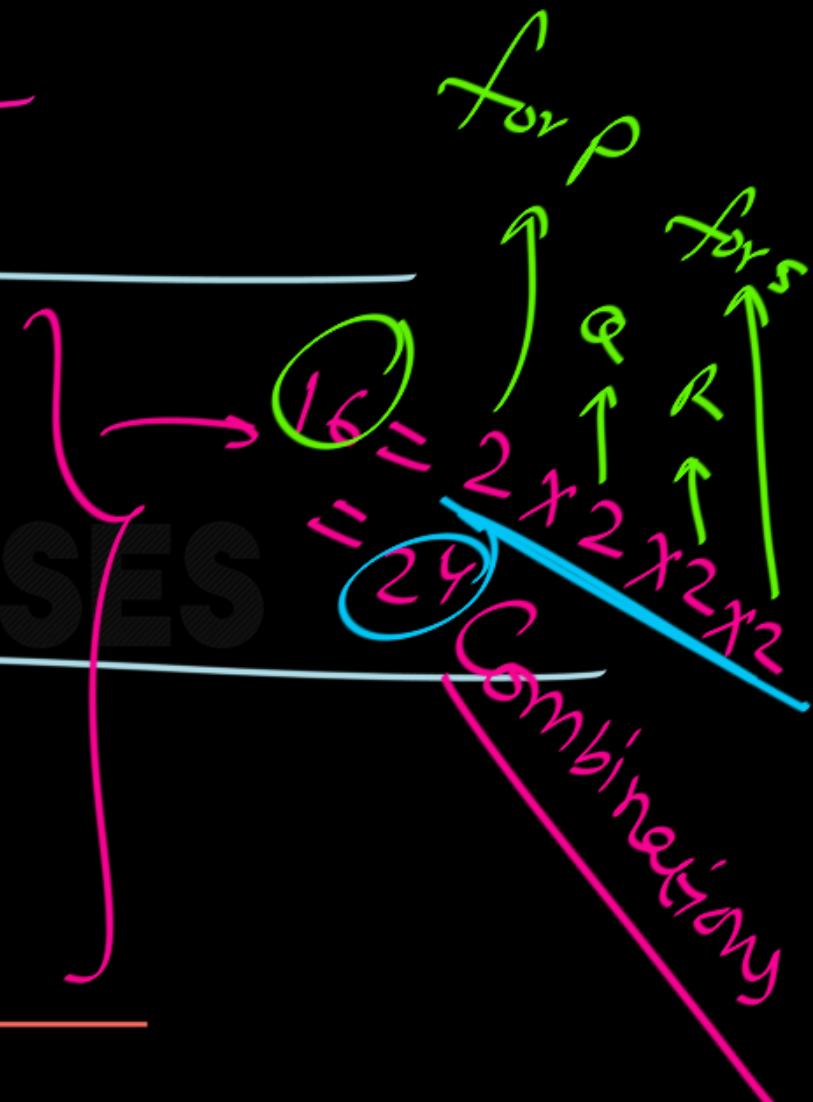
- Prop. Variables

$$\underline{P=F}$$

P	Q	R	S
F	F	F	F
T	T	T	T

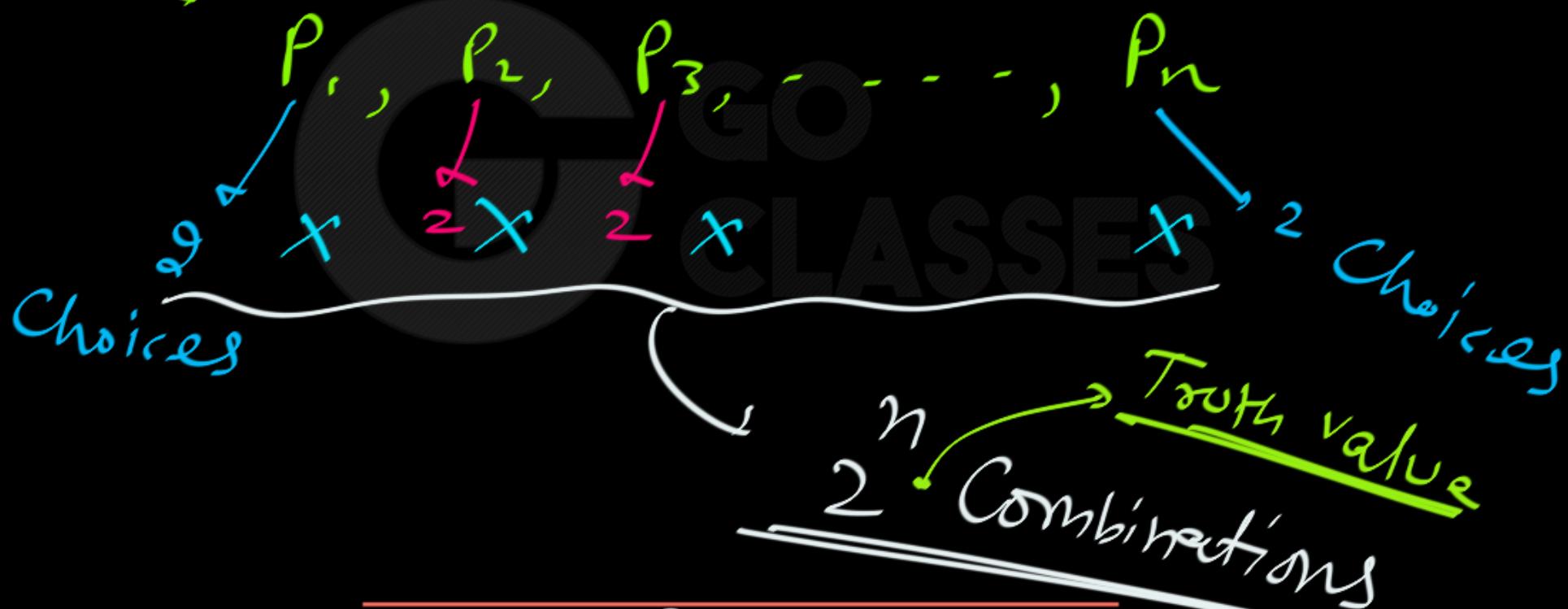
8 ✓
Combo.

$$\underline{P=T}$$





n prop. variables:



n prop. variables;

$\rho_1 \rho_2 \dots \rho_n$

$\left\{ \begin{matrix} 2^n \\ \text{Rows} \end{matrix} \right.$



HW 1 Question 8:

It is known that p is true.

What can we say about $(\neg p) \wedge q$?

- A. True
- B. False.
- C. Nothing can be said
- D. Multiple Values

HW 1 Question 8:

It is known that p is true.

What can we say about $(\neg p) \wedge q$?

- A. True
- B. False. ✓
- C. Nothing can be said
- D. Multiple Values

$$\boxed{F \wedge \text{anything} = F}$$

$P = \text{True}$

Given

$(\neg P)$

$$\begin{cases} F \wedge F = F \\ F \wedge T = F \end{cases}$$

$\text{False} \wedge q$ = $F \wedge q$



HW 1 Q 9:

It is known that p is true.

What can we say about $\neg(p \wedge q)$?

- A. True
- B. False.
- C. Nothing can be said
- D. Multiple Values



HW 1 Q 9:

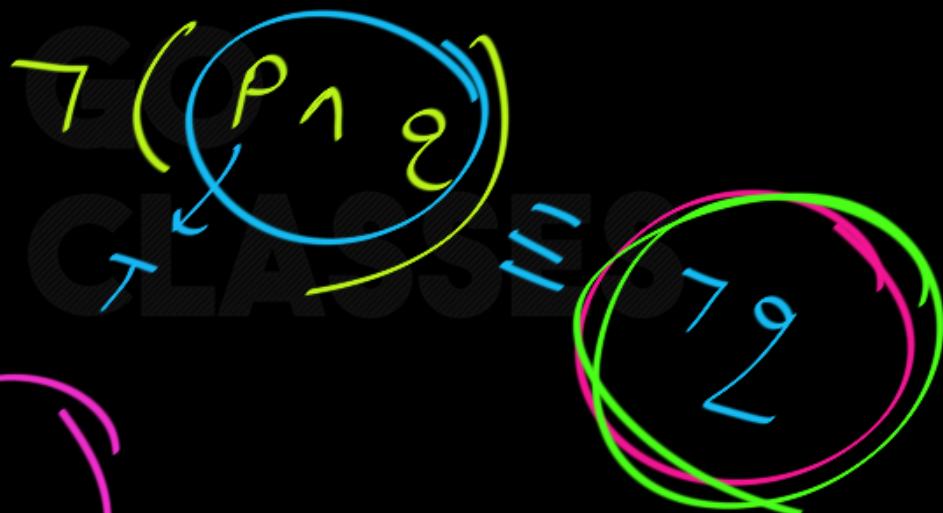
$p = T$

Given

It is known that p is true.

What can we say about $\neg(p \wedge q)$?

- A. True ~~X~~
- B. False. ~~X~~
- C. Nothing can be said ✓
- D. Multiple Values ~~X~~



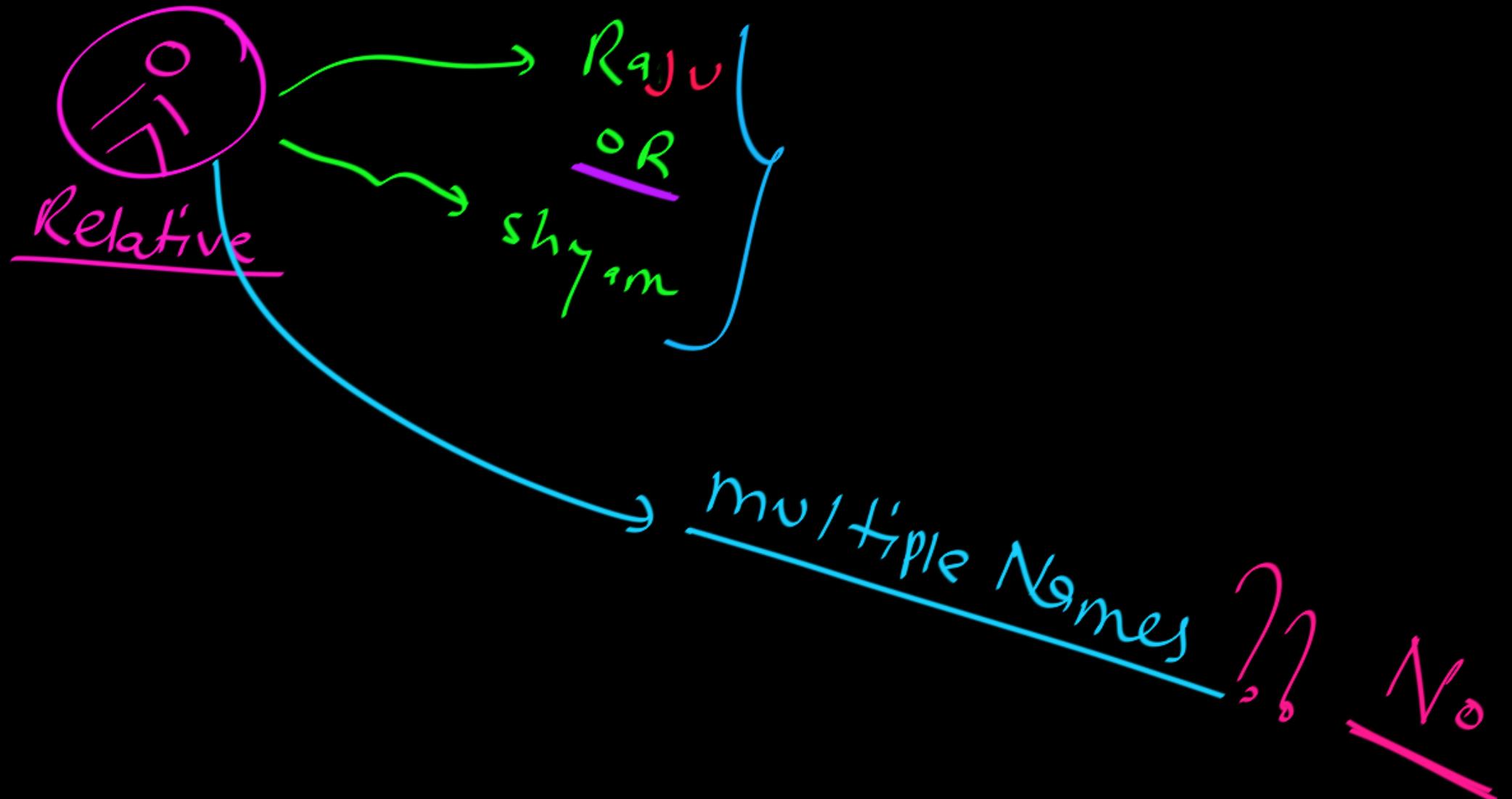
Any
Proposition

T

on

f

Not both



$$T \wedge q = ?$$

Constant

$$T \wedge q =$$

F \boxed{F}

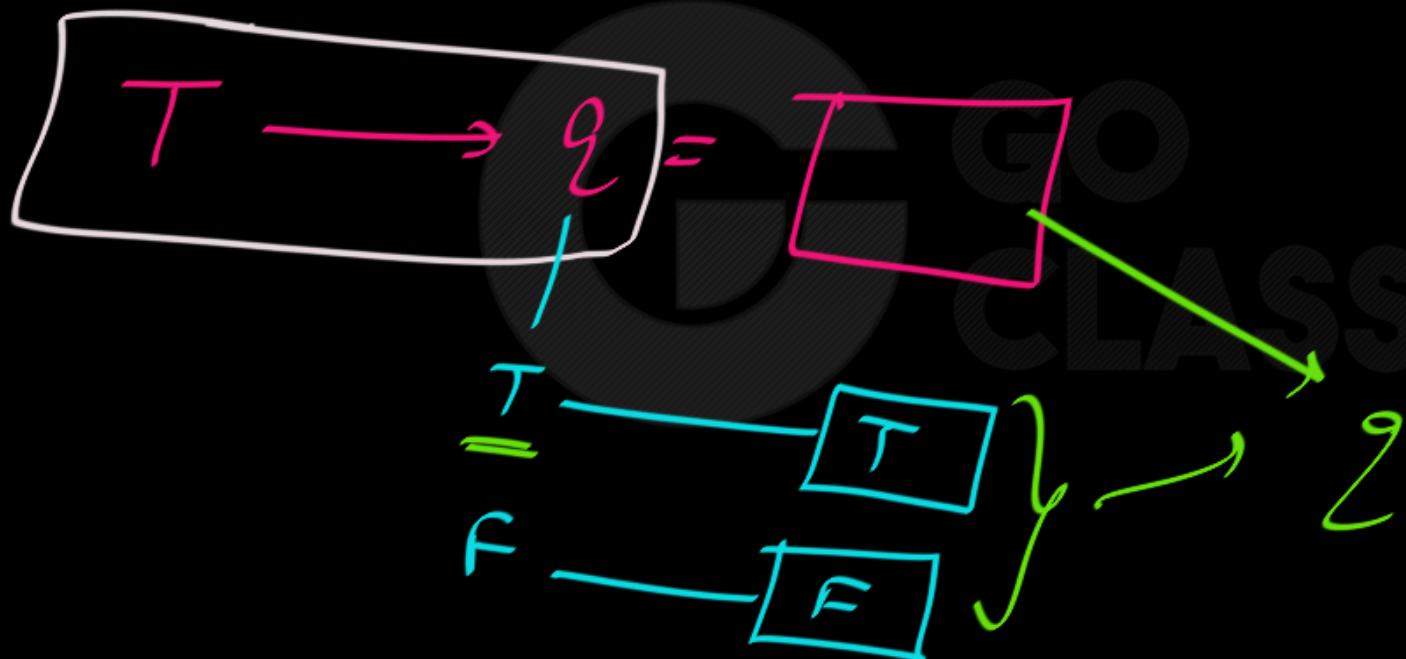
T \boxed{T}

\boxed{q}

The diagram illustrates the expression $T \wedge q =$ followed by three options in boxes: F, T, and \boxed{q} . A green arrow points from the F box to a circled \boxed{q} at the bottom right.

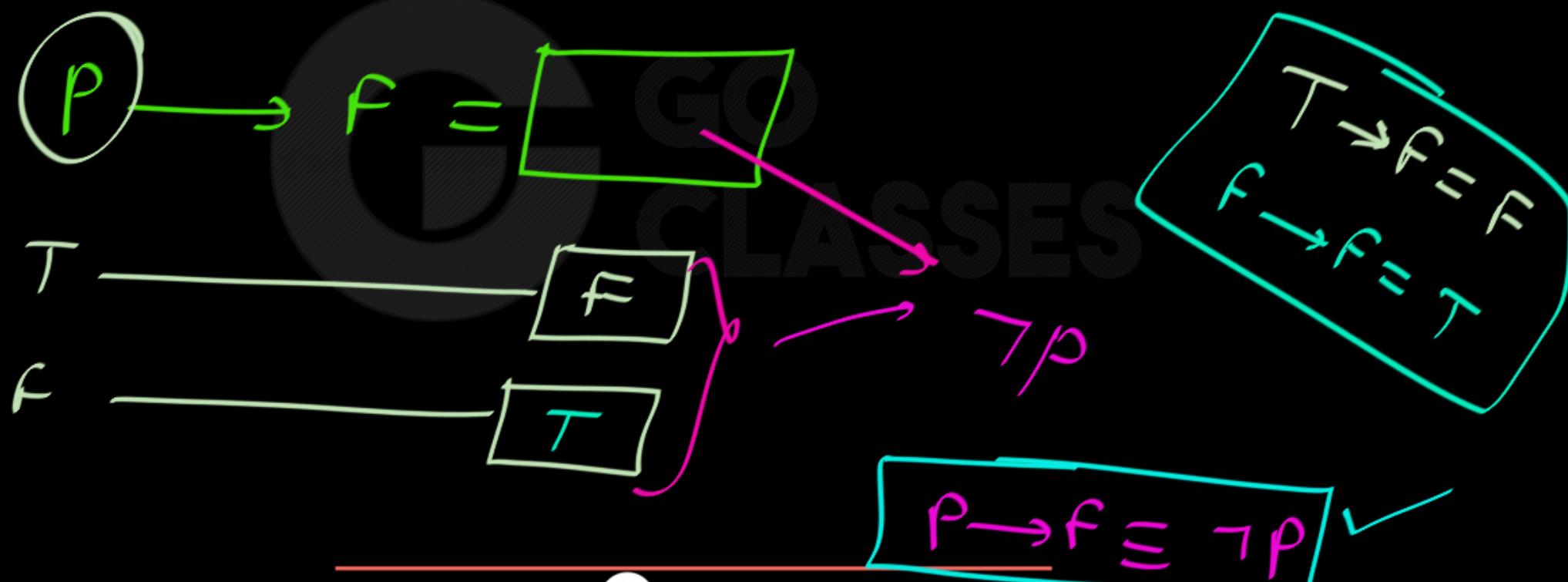
$$T \wedge q = q$$

$$T \rightarrow q = ? = q \xrightarrow{\text{Constant}}$$



$$\boxed{T \rightarrow T = T}$$
$$\boxed{T \rightarrow F = F}$$

$P \rightarrow F = ?$ *Constant 1*





$$T \oplus P = ?$$

A diagram illustrating the operation $T \oplus P$. It shows two sets, T and F , each containing two elements. A mapping arrow points from the element T in set T to the result $\neg P$. Another mapping arrow points from the element F in set F to the same result $\neg P$.

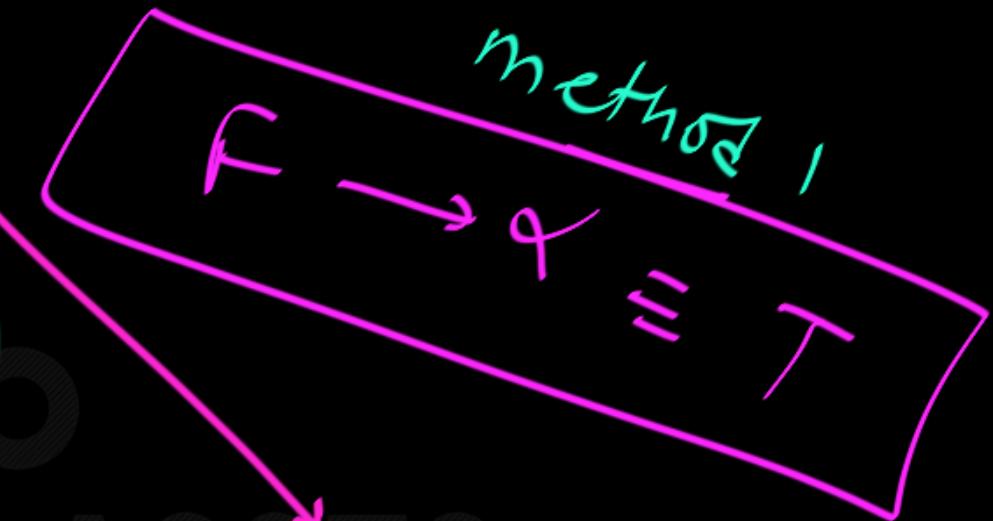
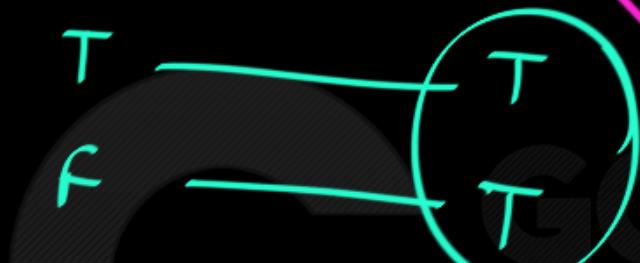
$$F \oplus P = ?$$

A diagram illustrating the operation $F \oplus P$. It shows two sets, F and T , each containing two elements. A mapping arrow points from the element F in set F to the result P . Another mapping arrow points from the element T in set T to the same result P .

$$\begin{array}{l} T \oplus F = T \\ F \oplus F = F \end{array}$$



$$F \rightarrow P = ?$$



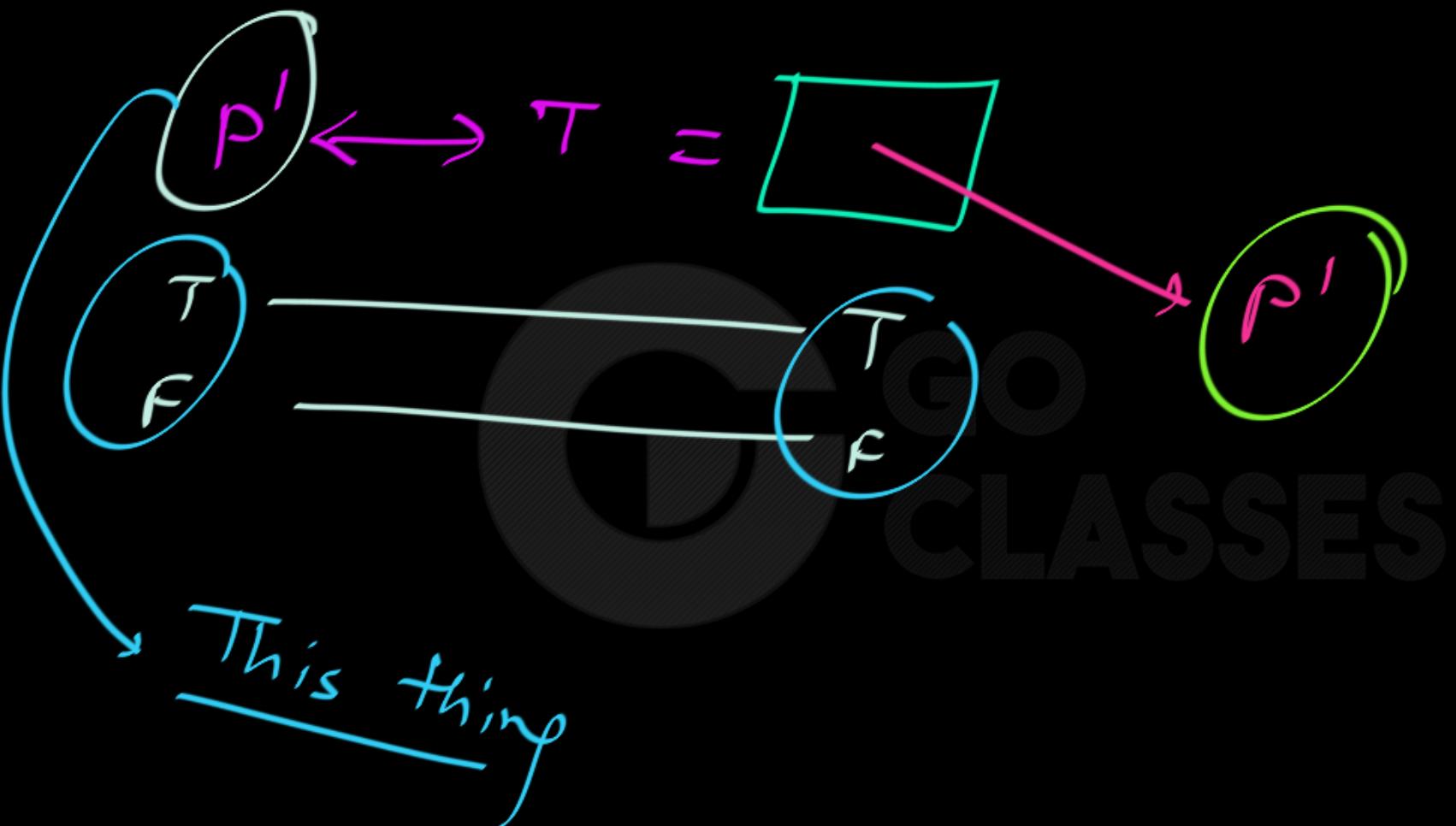
$$F \leftrightarrow P = ?$$

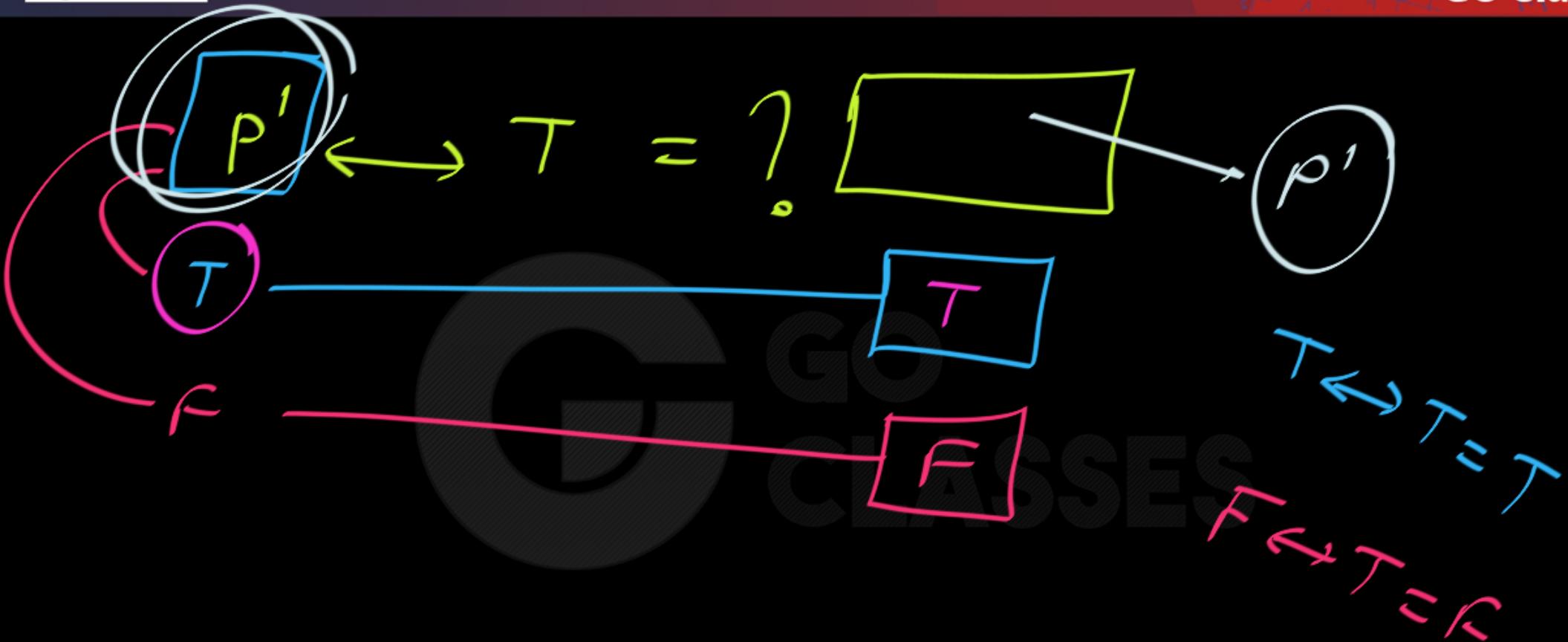
P

F

T

P'







$$\rho' \oplus f = ? = \rho' \quad \left| \begin{array}{l} \rho \hookrightarrow \rho' = f \\ \end{array} \right.$$

$$\rho' \oplus t = ? = \rho$$

$$\rho \oplus \rho' = ?$$

\top	f	\top	t
f	\top	\top	t



P is sufficient for Q

$$\boxed{P \rightarrow Q}$$

(P) is Necessary for Q

without P, Q can not happen

$$\neg P \rightarrow \neg Q \equiv Q \rightarrow P$$

P

is Necessary

for

Q

111

Q is sufficient for P

Some

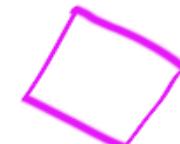
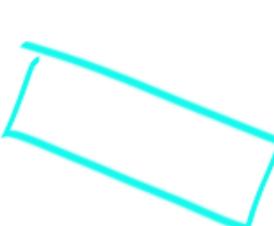
$Q \rightarrow P$



HW 2 Question 1: For each of the following, say whether it is true or false.

x's being a square is a sufficient condition for x's being a rectangle.
enough

T •



22

10

x's being a square is a necessary condition for x's being a rectangle.

F

x's being a rectangle is a sufficient condition for x's being a square.

F

x's being a rectangle is a necessary condition for x's being a square.

T •

x's being a mother is a sufficient condition for x's being a female.

T

x's being a mother is a necessary condition for x's being a female.

F

x's being a female is a sufficient condition for x's being a mother.

F

x's being a female is a necessary condition for x's being a mother.

T

x's being greater than 15 is a sufficient condition for x's being less than 20.

F

x's being greater than 15 is a necessary condition for x's being less than 20.

F

Some answers may depend on how one interpret the statements. So, except for mathematical statements, DO NOT worry about the answer of other questions.



Square is suff for Rectangle

Rect is suff for Square

Some



HW 2 Question 1(Contd.): For each of the following, say whether it is true or false.

x's being less than 20 is a sufficient condition for x's being greater than 15. F

x's being less than 20 is a necessary condition for x's being greater than 15. F

x's being less than 12 is a sufficient condition for x's being less than 20. T

Some answers may depend on how one interpret the statements. So,

x's being less than 12 is a necessary condition for x's being less than 20. F

x's being less than 20 is a sufficient condition for x's being less than 12. F

except for mathematical statements, DO NOT

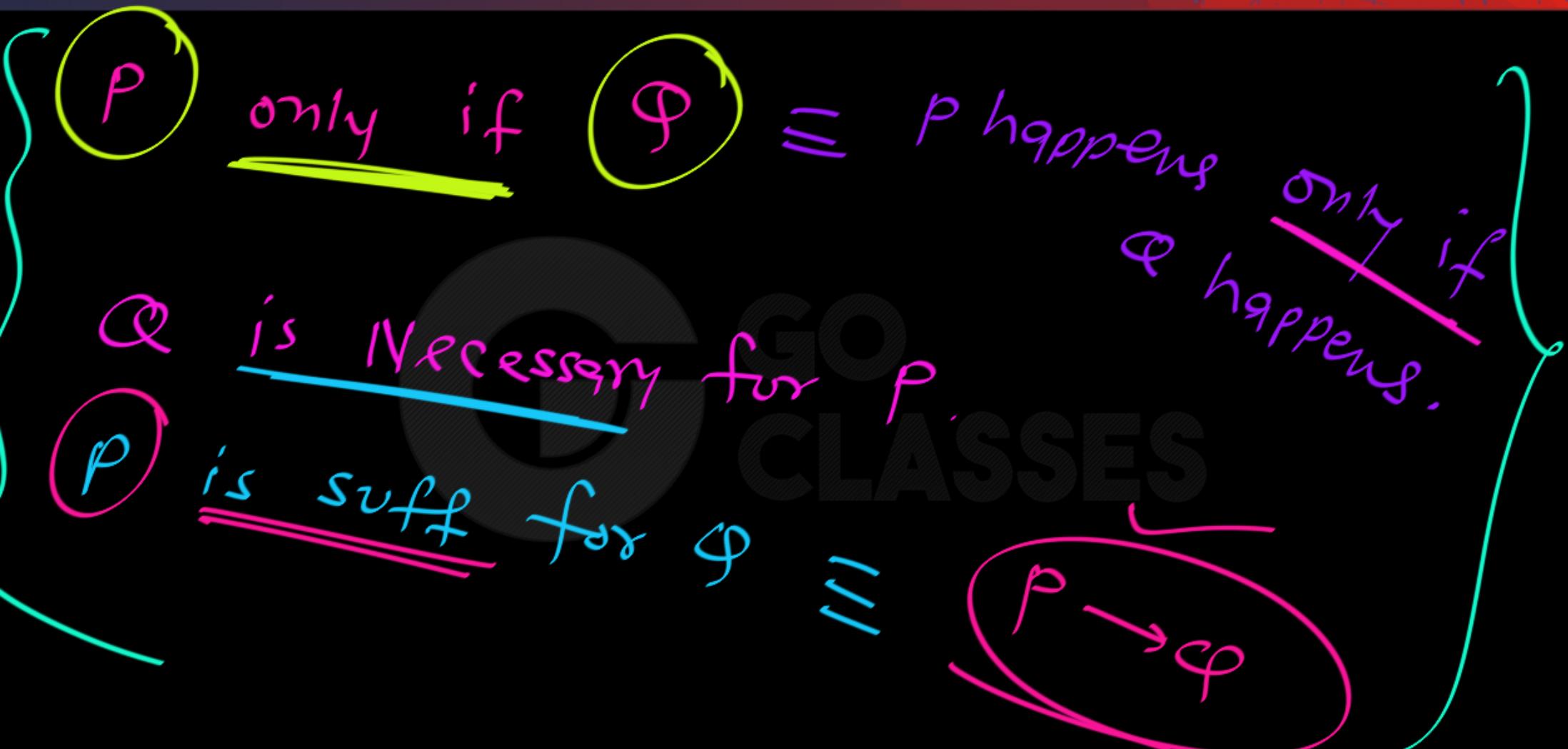
x's being less than 20 is a necessary condition for x's being less than 12. T

worry about the answer of other questions.



$P \rightarrow Q$ *only if* $\varphi \equiv P \rightarrow Q$

GO
CLASSES

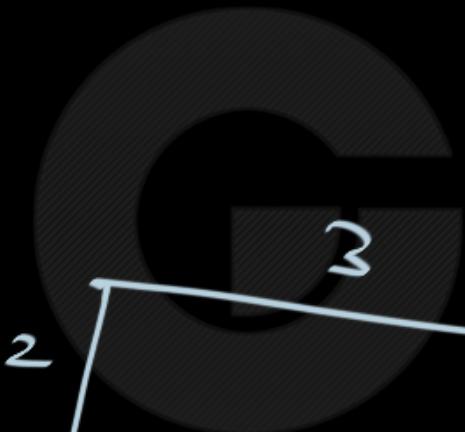


ΔC being Rectangle

is suff for

n being
square.

False.



→ Rectangle
↙ Square

HW 2 Question 2 :

6. Let proposition p be T and proposition q be F . Find the truth values for the following:

- (a) $p \vee q = T$
- (b) $q \wedge p = F$
- (c) $\neg p \vee q = F$
- (d) $p \wedge \neg q = T$
- (e) $q \rightarrow p = T$
- (f) $\neg p \rightarrow q = T$
- (g) $\neg q \rightarrow p = T$

$$\left. \begin{array}{l} T \vee F = T \\ T \wedge F = F \end{array} \right\}$$

**Discrete Mathematics
for Computer Science**

Gary Haggard

John Schlipf

Sue Whitesides

$\neg p \vee q$ $\neg(p \vee q)$ $(\neg p) \vee q$

Convention

$$\begin{array}{r} -2 + 5 \\ \hline -(-2+5) \\ (-2)+5 \end{array}$$

Priority order: (precedence order)

$\neg > \wedge > \vee > \rightarrow > \Leftrightarrow$

$$\neg p \rightarrow q \vee r \equiv (\neg p) \rightarrow (q \vee r)$$

HW 2 Question 3 :

7. Let proposition p be T , proposition q be F , and proposition r be T . Find the truth values for the following:

$$(a) p \vee q \vee r = T$$

$$(b) p \vee (\neg q \wedge \neg r) = T$$

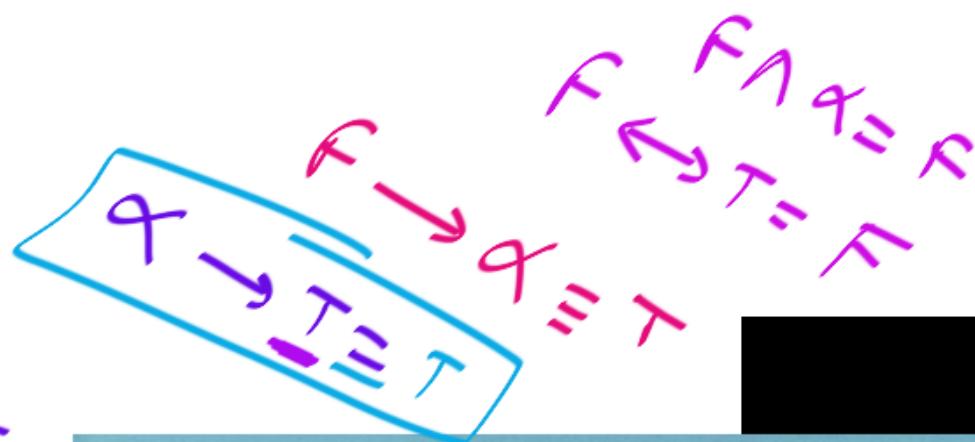
$$(c) p \rightarrow (q \vee r) = T$$

$$(d) (q \wedge \neg p) \leftrightarrow r = F$$

$$(e) \neg r \rightarrow (p \wedge q) = T$$

$$(f) (\neg p \rightarrow q) \rightarrow \neg r = T$$

$$(g) ((p \wedge r) \rightarrow (\neg q \vee p)) \rightarrow (q \vee r) = T$$

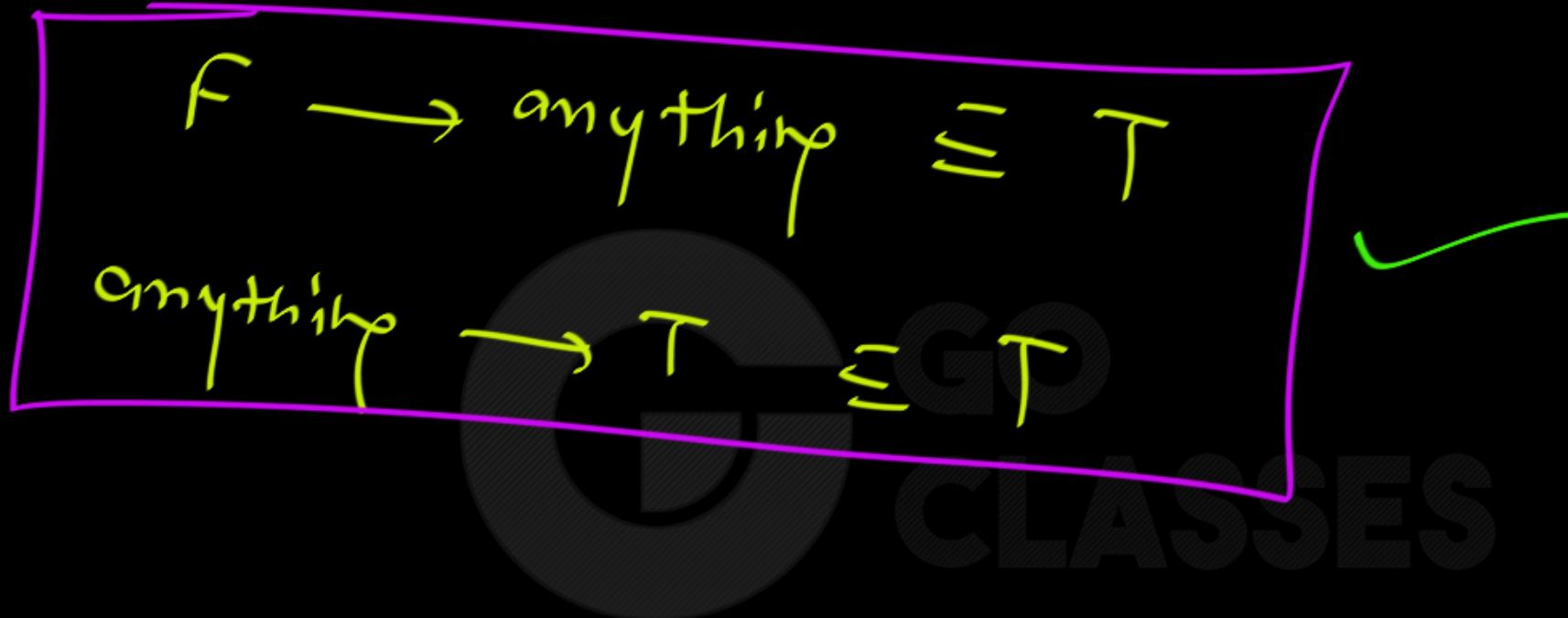


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3. Suppose P and Q are the statements: P: Jack passed math. Q: Jill passed math.

(a) Translate "Jack and Jill both passed math" into symbols. $P \wedge Q$

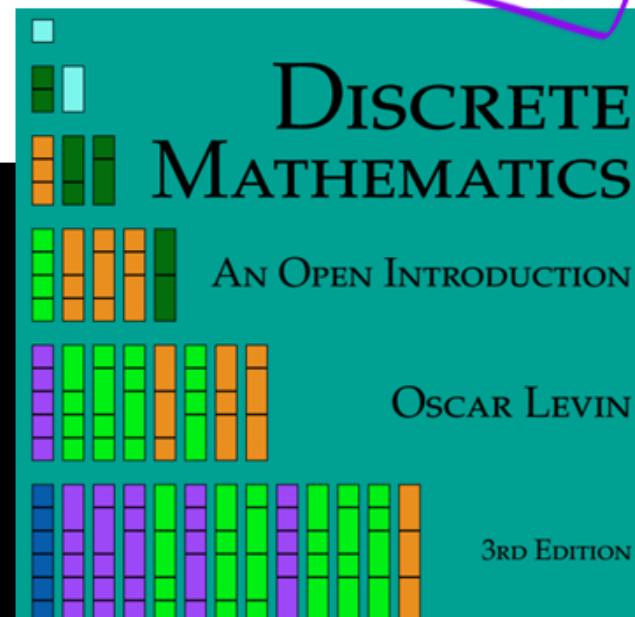
(b) Translate 'If Jack passed math, then Jill did not' into symbols.

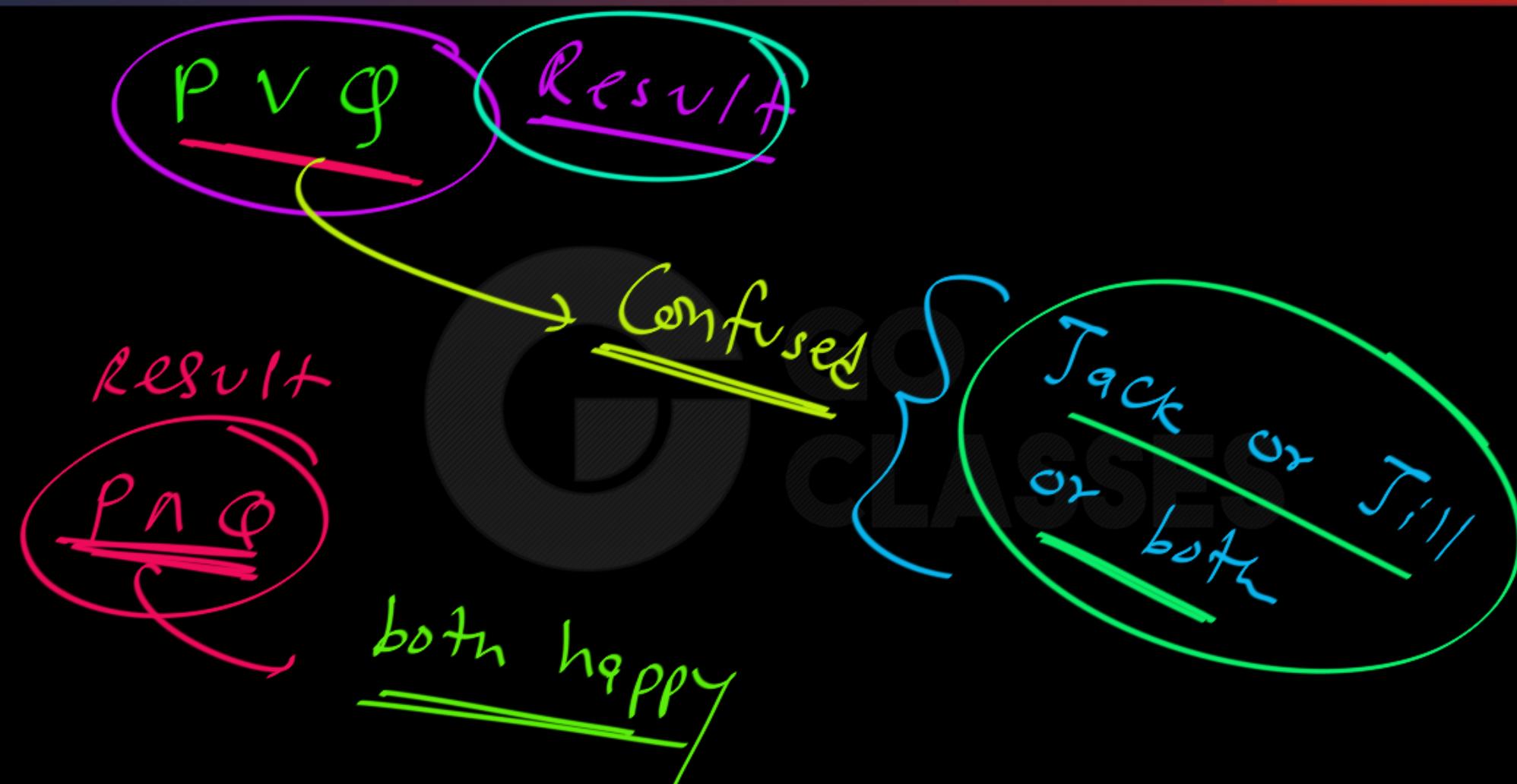
(c) Translate " $P \vee Q$ " into English. $\neg P \rightarrow \neg Q$

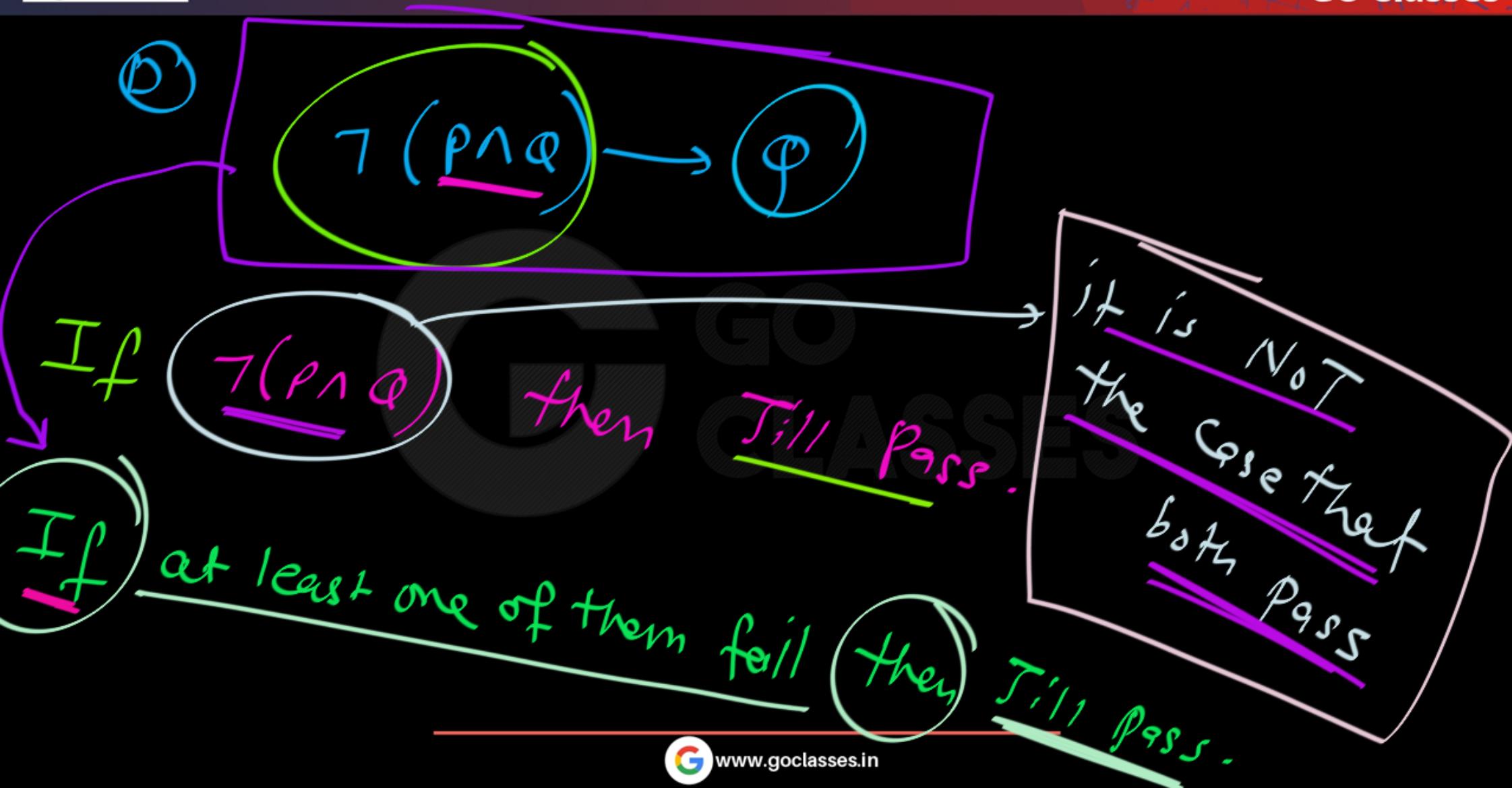
(d) Translate " $\neg(P \wedge Q) \rightarrow Q$ " into English.

$P \rightarrow \neg Q$

Jack or Jill or both



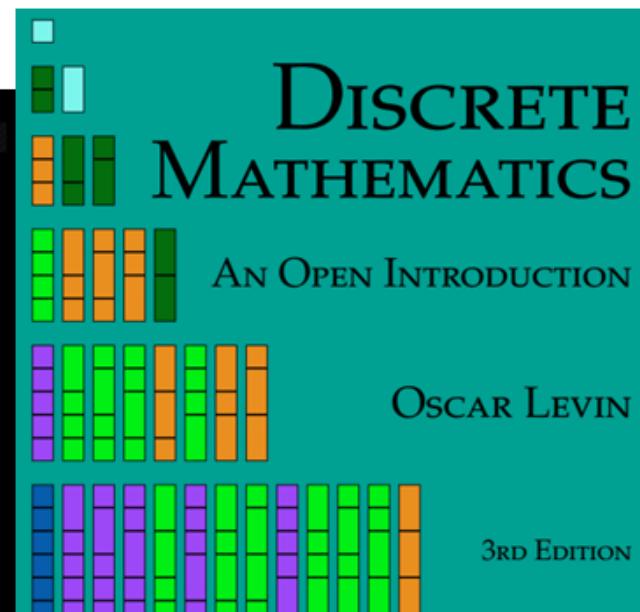






HW 2 Question 4 :

- (e) Suppose you know that if Jack passed math, then so did Jill.
What can you conclude if you know that:
- i. Jill passed math?
 - ii. Jill did not pass math?



HW 2 Question 4 :

(e) Suppose you know that if Jack passed math, then so did Jill.
What can you conclude if you know that:

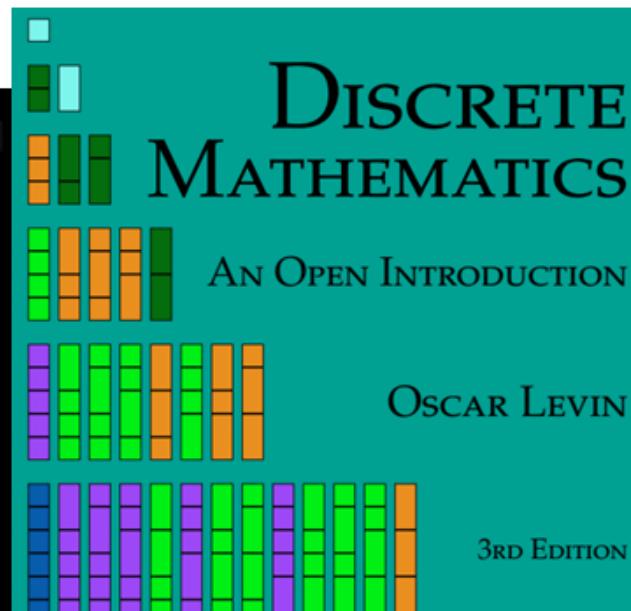
i. Jill passed math? \Rightarrow Can't Determine

ii. Jill did not pass math?



$P = \text{True/False}$

Tack fig/ls



Given:

$$\boxed{P \rightarrow Q} \equiv \text{True}$$

(a) $Q = \text{True}$ what can you say about P ?

$$P \rightarrow T \equiv \text{True}$$

$$\rho = f_9 / s_2$$

ok

$$P = T_{RQ}$$

Given:

$$P \rightarrow Q = \text{True}$$

a)

$Q = \text{false}$

what
can you say

P ??

$$P \rightarrow F = \text{True}$$

$P = \text{false}$

HW 2 Question 4 : Variation :

(e) Suppose you know that if Jack passed math, then so did Jill.

What can you conclude if you know that:

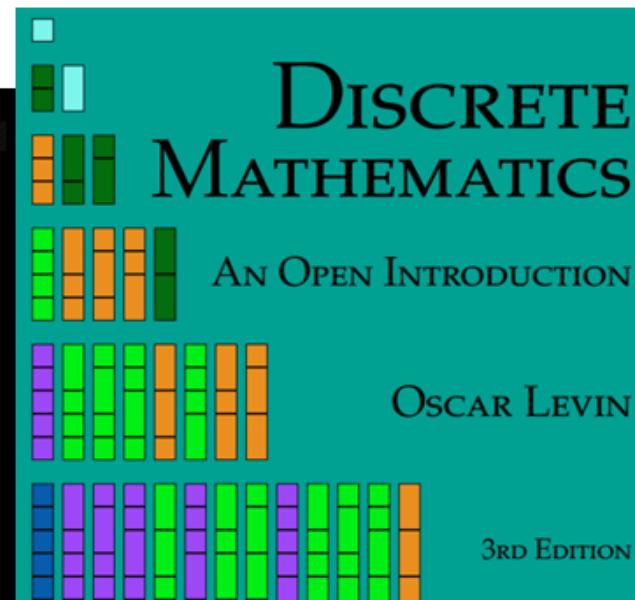
i. Jack passed math?

ii. Jack did not pass math?

$C_1 = \text{True}$

Jill Pass

Can not determine





Given:

$$P \rightarrow Q = \text{True}$$

(i)

$$P = \text{True}$$

Given in (i)

$$T \rightarrow Q = \text{True}$$

$$Q = \text{True}$$

~~True / False~~

Given:

$$P \rightarrow Q = \text{True}$$

(i)

$$P = \text{false}$$

Given in (i)

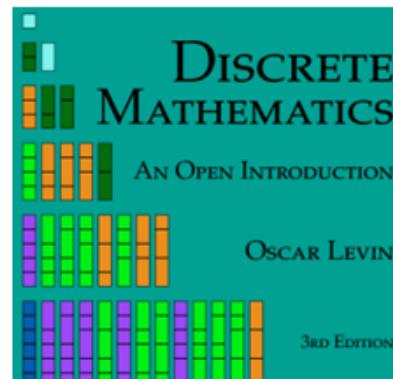
$$f \rightarrow Q = \text{True}$$

$Q = \text{True}$

or

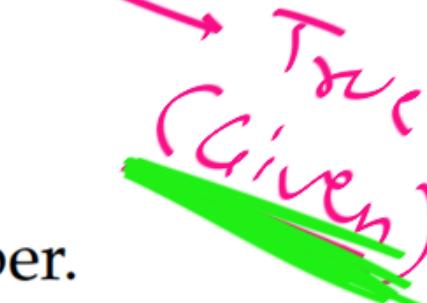
$Q = f_{\text{False}}$

4. Determine whether each molecular statement below is true or false, or whether it is impossible to determine. Assume you do not know what my favorite number is (but you do know that 13 is prime).
- (a) If 13 is prime, then 13 is my favorite number.
 - (b) If 13 is my favorite number, then 13 is prime.
 - (c) If 13 is not prime, then 13 is my favorite number.
 - (d) 13 is my favorite number or 13 is prime.
 - (e) 13 is my favorite number and 13 is prime.
 - (f) 7 is my favorite number and 13 is not prime.
 - (g) 13 is my favorite number or 13 is not my favorite number.



4. Determine whether each molecular statement below is true or false, or whether it is impossible to determine. Assume you do not know what my favorite number is (but you do know that 13 is prime).

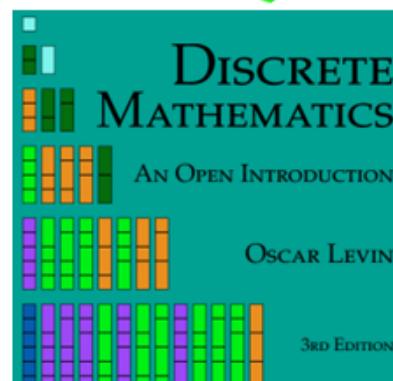
IDK (a) If 13 is prime, then 13 is my favorite number.



T (b) If 13 is my favorite number, then 13 is prime.

T (c) If 13 is not prime, then 13 is my favorite number.

T (d) 13 is my favorite number or 13 is prime.



IDK (e) 13 is my favorite number and 13 is prime.

F (f) 7 is my favorite number and 13 is not prime.



(g) 13 is my favorite number or 13 is not my favorite number.



$$T \rightarrow Q \equiv Q$$

$$T \rightarrow \boxed{\text{I Don't know}} \equiv \text{I don't know}$$

$$\boxed{\text{anything} \rightarrow T \equiv T}$$

$$\boxed{Fbk \rightarrow T \equiv T}$$



$f \rightarrow \text{anything} \equiv T$

$F \wedge \text{anything} \equiv F$

$\text{anything} \vee T \equiv T$

$P \wedge T \equiv P$

$IDk \wedge T = IDk$



(9)

$$(I_{DK}) \vee (\overline{I_{DK}})$$

$$P \vee \overline{P} = T$$

$$\begin{aligned} T \vee F &\longrightarrow T \\ F \vee T &\longrightarrow T \end{aligned}$$



GO Classes Enrolled Students' Doubt:

Should I solve Standard Resources/Books

Questions??



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solve... We Will Provide you in the HWs,
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So, NO Need. Just Follow Our Course.



GO Classes Enrolled Students' Doubt:

Should I study Standard Resources/Books??





NOTE:

We are Teaching you every subject from Several Standard Books/Resources.

We are putting Screenshots of Standard Books in the Slides, which you can read.

So, NO Need. Just Follow Our Course. ✓



3. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following.

- a) $p \wedge q$
- b) $\neg p \vee q$
- c) $q \rightarrow p$
- d) $\neg q \rightarrow \neg p$



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3. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following.

a) $p \wedge q$

b) $\neg p \vee q$

c) $q \rightarrow p$

d) $\neg q \rightarrow \neg p$

Prop. Variables

$P \rightarrow Q = \text{false}$
 $P = T, Q = F$

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So Far.. ANY Doubt or Confusion..??

Let's Make Sure “No Doubt Stays Unresolved”