Propositional Logic

Next Chapter:

Implication Continued...

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GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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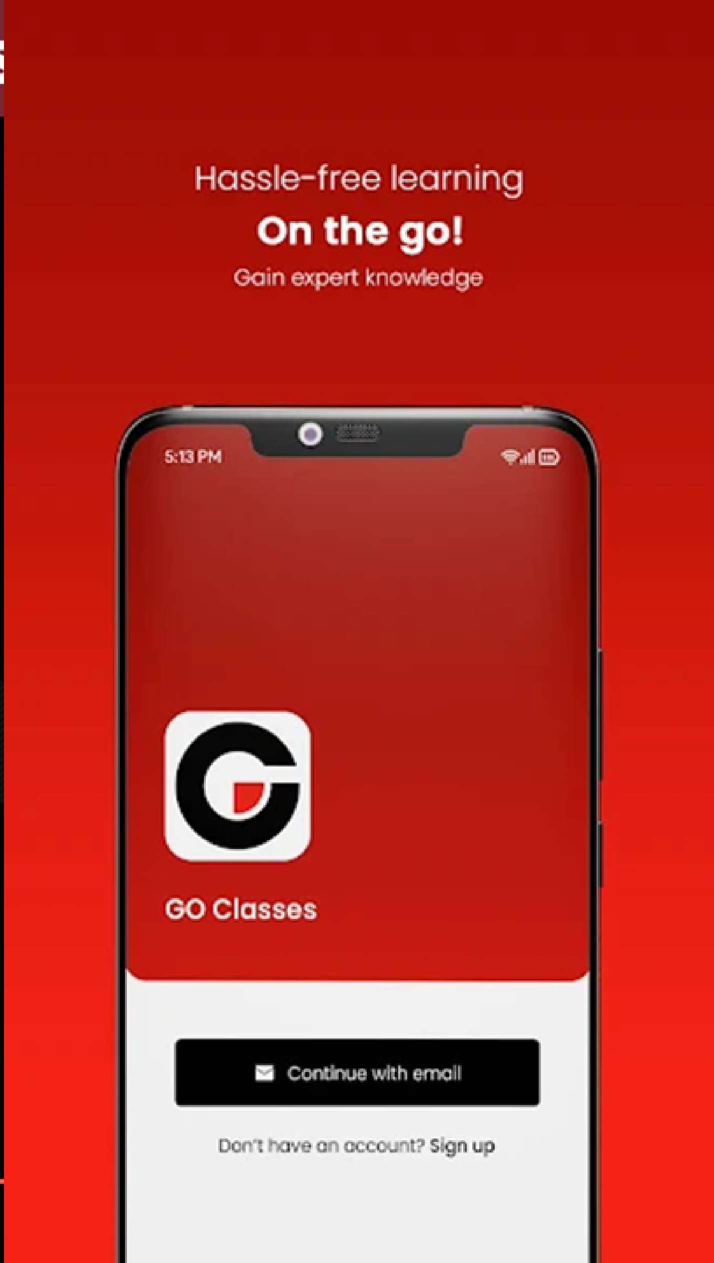

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Next Topic:

Implication Continued...

Few Very Important Points

about Implication

1.

$P \rightarrow Q$ is false **ONLY WHEN** (P True & Q False)

Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

The only way for $p \rightarrow q$ to be false is for p to be true and q to be false.

2.

$P \rightarrow Q$ is True in three cases:

- P= True, Q=True OR
- P= False, Q=True OR
- P= False, Q=False

2.

P → Q is True in three cases:

- P= True, Q=True
- P= False, Q=True
- P= False, Q=False

$\boxed{\text{P= false OR Q = True}}$

P = false, Q = false
P = True, Q = True
P = false; Q = True

3.

P → Q is True WHEN (P False OR Q True)

p=false

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

$P \rightarrow Q$ Implication Statement

① false when $\boxed{P = T \wedge Q = F}$

② True when $\boxed{P = F \text{ OR } Q = T}$

$a \vee b$ True

$$\left\{\begin{array}{ll} a = T, & b = F \\ a = F, & b = T \\ a = T, & b = T \end{array}\right\} \text{3 Cases Possible.}$$

4.

- $P \rightarrow Q$ == If P true then Q true
- $P \rightarrow Q$ =??= If Q false then P false

??

$P \rightarrow Q$: If p true then Q True.

If Q false then Can p be True??

$P \to Q$: If P true then Q True.

If Q false then Can P be True ??

NO.

If P True then Q True

$P \rightarrow Q$: If P true then Q True.

$\equiv$ $\equiv$

If Q false then P false

Same

$\equiv$

$P \rightarrow Q$

Important

Note:

$\boxed{P \text{ is false}} \equiv \neg P$

$\boxed{Q \text{ is false}} \equiv \neg Q$

If P True then Q True

$\equiv P \rightarrow Q$

If Q false then P false

$\equiv$ If $\neg Q$ then $\neg P$

$\equiv \neg Q \rightarrow \neg P$

Same

$T \rightarrow T$ True

$F \rightarrow F$ True

$F \rightarrow T$ True

$T \rightarrow F$ false

4.

- $P \to Q \;==\; \sim Q \to \sim P$

Condition

P	Q	$P \to Q$	$\equiv$ $\neg Q \to \neg P$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

$\boxed{\text{If } p \text{ then } q} \equiv p \to q$

$|||$

$\boxed{\text{If } \neg q \text{ then } \neg p} \equiv p \to q$

$\neg q \to \neg p \equiv p \to q$

VERY Important:

- $P \rightarrow Q$ == If P true then Q true
- $P \rightarrow Q$ == If Q false then P false

Same

Conclusion:

- $P \rightarrow Q$
- If P happens then Q happens
- If P true then Q true
- If Q false then P false
- $\sim Q \rightarrow \sim P$

Same

Conclusion:

- If P happens then Q happens

$P \to Q$

|||

- If Q does Not happen then P does Not happen

Next Topic:

Implication Continued...

Necessary & Sufficient Conditions

Example:

Filling the GATE exam application form is _______ for cracking the GATE exam.

Necessary

Sufficient

Example:

Filling the GATE exam application form is _______ for cracking the GATE exam.

Necessary

Sufficient

Enough

Example:

Filling the GATE exam application form is ______ for cracking the GATE exam.

P

Q

Necessary

P is Necessary for Q

≡ without P, Q can NoT happen.

If P false then Q false

$\equiv \neg P \rightarrow \neg q \equiv Q \rightarrow P$

If Q True then P True

$\equiv Q \rightarrow P$

$$\boxed{\alpha \to \beta \equiv \neg\beta \to \neg\alpha}$$

Last lecture

$$p \to q \equiv \neg q \to \neg p$$

$$q \to p \equiv \neg p \to \neg q$$

$\boxed{\begin{array}{l} P \text{ is Necessary for } Q \\ \equiv \quad Q \rightarrow P \end{array}}$

Example:

"Being Natural number" is ______ "Being Integer number"

Necessary

Sufficient

Example:

"Being Natural number" is ________ "Being Integer number"

~~Necessary~~

enough $\equiv$ Sufficient

-2

Integer ✓

Not Natural

Example:

"Being Natural number" is ______ "Being Integer number"

~~Necessary~~

Sufficient

Example:

"Being Natural number" is ______ "Being Integer number"

p sufficient q

If Natural then Integer

$\boxed{p \text{ is sufficient for } q}$

$\equiv$ If p True then Q True. $\equiv p \to q$

P is sufficient for Q

$\equiv$ If P True then Q True

$\equiv$ $P \to Q$

Sufficiency:

enough

"P is sufficient for Q" means

Happening of P is enough for Happening of Q.

If P happens then Q happens.

$\equiv$ $P \rightarrow Q$ ✓

Example:

For integers > 2;

Being Prime is ________ for Being Odd.

Example:

3, 4, 5, 6, - - - - - - - -

For integers > 2;

Being Prime is ________ for Being Odd.

necessary

9 → odd ✓
9 → prime ✗

Example:

For integers > 2;

sufficient

Being Prime is ________ for Being Odd.

P Q

P is sufficient for Q.

≡ If P happens then Q happens ≡ $P \to Q$

P is sufficient for Q

If P happens then Q happens.

$$\boxed{P \rightarrow Q}$$

Example:

Getting Top 10 rank in GATE exam is ________

for going to IITB MTech.

Example:

Getting Top 10 rank in GATE exam is ________

P

for going to IITB MTech.

Q

Necessary

Sufficient

If Top 10 then IITB.

Top 10

P is sufficient for Q

IITB

If P then Q

$P \rightarrow Q$

Example:

GO Classes course is __________ for cracking top rank in the GATE exam.

Example:

p

GO Classes course is __________ for cracking top rank in the GATE exam.

q

Sufficient

$$\boxed{p \rightarrow q}$$

Energy is Necessary for Driving Car.

$\neg$ Energy $\rightarrow$ $\neg$ Drive Car

Necessity:

"P is necessary for Q" means

Without P, Q cannot happen.

NOT Happening P Implies Not Happening of Q

$$\neg P \rightarrow \neg Q \equiv Q \rightarrow P$$

P is Necessary for Q

without P, Q can not happen.

If P false then Q false

$\neg P \rightarrow \neg Q \equiv Q \rightarrow P$

Example: 3, 4, 5, - - - - - -

For integers > 2;

Being Odd is ________ for Being Prime.

Example: 3, 4, 5, - - - - - -

For integers > 2;

Being Odd is ________ for Being Prime.

sufficient ✗

9 → odd ✓

9 → prime ✗

Example:

3, 4, 5, - - - - - - -

For integers > 2;

Being Odd is ________ for Being Prime.

Necessary

$\neg$ Odd $\rightarrow$ $\neg$ Prime $\equiv$ Prime $\rightarrow$ odd being odd is Necessary for being prime

p

q

P is Necessary for Q

Without P, Q can Not happen.

If P NoT happen then Q Not happen.

If P false then Q false

$$\neg P \rightarrow \neg Q \equiv Q \rightarrow P$$

P is Necessary for Q

Translation

Example:

Air is ________ for being alive.

Example:

Air is ________ for being alive.

sufficient

Air
water

Example:

Air is ________ for being alive.

Necessary

Without Air, You can Not be Alive.

$\boxed{\neg \text{Air} \rightarrow \neg \text{Life}}$

$\boxed{\text{Life} \rightarrow \text{Air}}$

Same

Air is Necessary for Life.

Life $\rightarrow$ Air

P is sufficient for Q

If P happens then Q happens.

$P \rightarrow Q$

P is Necessary for Q

Without P, Q can not happen.

$$\neg P \longrightarrow \neg Q \equiv Q \longrightarrow P$$

Suppose that A and B are statements.

We say that the statement A is sufficient for the statement B if:

- If A happens, then B happens.
- $A \rightarrow B$

Suppose that A and B are statements.

We say that the statement A is necessary for the statement B if:

- Without A, B cannot happen.
- If A does not happen then B does not happen.
- If B happens then A happens.
- $B \rightarrow A$

Conclusion:

1. p is sufficient for q

$\equiv p \to q \equiv \neg q \to \neg p$

$\equiv$ q is Necessary for p

$\equiv p \to q$

Same

Conclusion:

2) P is Necessary for Q

$\equiv \neg P \rightarrow \neg Q \equiv Q \rightarrow P$

Q is Sufficient for P.

$\equiv Q \rightarrow P$

Same

Important:

1. α is sufficient for β

$\alpha \rightarrow \beta$

2. α is Necessary for β

$\beta \rightarrow \alpha$

Important:

P is sufficient for Q $\equiv P \to Q$

$\equiv$ Q is Necessary for P $\equiv P \to Q$

Important:

α is sufficient for β

$\equiv$ β is Necessary for α

Same

Tips (Trick):

α $\boxed{\text{is sufficient for}}$ β $\longrightarrow$ $\alpha \to \beta$

α $\boxed{\text{is Necessary for}}$ β $\longleftarrow$ $\beta \to \alpha$

Note:

Tricks (shortcuts) are for Dumb people.

"Understanding" Concepts is for toppers

P is Sufficient for Q

$\equiv P \rightarrow Q$

P is Necessary for Q

$\equiv$ Q is sufficient for P $\equiv Q \rightarrow P$

Example:

Studying Chemistry is ________

for cracking GATE CSE exam.

Sufficient ✗

Necessary ✓✗

Example:

n being even is ___________ for n+2 being even.

Example:

n being even is ~~sufficient~~ for n+2 being even.

$\boxed{n \text{ even} \longrightarrow n+2 \text{ even}}$

Example:

n being even is __________ for n+2 being even.

necessary

$\boxed{\text{n NOT even}} \longrightarrow \boxed{\text{n+2 not even}}$

Example:

n being even is __________ for n+2 being even.

necessary and sufficient

Sufficient Condition :

Definition of "sufficient condition" :

Definition: A condition A is said to be *sufficient* for a condition B, if (and only if) the truth (/existence /occurrence) [as the case may be] of A guarantees (or brings about) the truth (/existence /occurrence) of B.

For example, while air is a necessary condition for human life, it is by no means a sufficient condition, i.e. it does not, by itself, i.e. alone, suffice for human life. While someone may have air to breathe, that person will still die if s/he lacks water (for a number of days), has taken poison, is exposed to extremes of cold or heat, etc.

Sufficient Condition :

Example :
For the whole numbers greater than two, being odd is necessary to being prime, since two is the only whole number that is both even and prime.

BUT
For the whole numbers greater than two, being odd is not sufficient to being prime, since there are many odd numbers which are not prime, example : 9.

Necessary Condition :

Definition of "necessary condition" :

Definition: A condition A is said to be *necessary* for a condition B, if (and only if) the falsity (/nonexistence /non-occurrence) [as the case may be] of A guarantees (or brings about) the falsity (/nonexistence /non-occurrence) of B.

•Here are a number of examples, all - more or less - saying the same thing:

"Air is necessary for human life."
•"Human beings must have air to live."
•"Without air, human beings die (i.e. do not live)."
•"If a human being is alive, then that human being has air (to breathe).“

•In an `if-then' statement (such as the last example immediately above), the clause that follows the "then" (i.e. the so-called `consequent') states the necessary condition for the antecedent (i.e. the clause immediately following the "if"). Thus that some *human being has air (to breathe)* is a **necessary condition for** *that human being's being alive*.

More Examples :

Example 1 :

For the whole numbers greater than two, being odd is necessary to being prime, since two is the only whole number that is both even and prime.

BUT

For the whole numbers greater than two, being odd is not sufficient to being prime, since there are many odd numbers which are not prime, example : 9.

Example 2 :

Being at least 18 years old is necessary for casting vote in India. If you are under 18 years old, then it is impossible for you to be a voter. That is, if you are a voter in India, it follows that you must be at least 18 years old.
NOTE that Being at least 18 years old is not sufficient for casting vote in India. Because prisoners cannot vote.

More Examples :

Example 3 :
A number's being divisible by 4 is sufficient (but not necessary) for it to be even, but being divisible by 2 is both sufficient and necessary.

Example 4 :

A number x is rational is sufficient but not necessary to x being a real number (since there are real numbers that are not rational).

Example 4 :

Reading Chemistry is NEITHER sufficient NOR necessary to crack GATE CSE exam.

NOTE :

Mathematically speaking, necessity and sufficiency are dual to one another.
For any statements S and N, the assertion that "N is necessary for S" is equivalent to the assertion that "S is sufficient for N".

If P is sufficient for Q, then Q is necessary for P.

If x is a necessary condition for y, then y is a sufficient condition for x.
And, equivalently,
If y is a sufficient condition for x, then x is a necessary condition for y.

- "Being a father is a sufficient condition for being male, and being male is a necessary condition for being a father." (But being a father is not a necessary condition for being a male; and being a male is not a sufficient condition for being a father.)

More Examples :

Air is *necessary* for (human) life. Without air, there is no (human) life. But air is NOT sufficient to keep us alive.

"Wanting to succeed is neither a necessary nor a sufficient condition for success."

"Being the consistent student in a class is neither a necessary nor a sufficient condition for achieving the highest grade in that class.“
(But you should be consistent as it improves chances of being successful. So, Be consistent in your preparation, be attentive in live classes, do homework on time, discuss with friends and teachers, and remember that we at GO Classes are always there to help you with your preparation. Keep preparing. You are doing well.)

"Today's being neither Saturday nor Sunday is both a necessary and a sufficient condition for today's being a weekday."

[TRICKY, but true.] "X's-being-a-necessary-condition-for-Y is both a necessary and a sufficient condition for Y's-being-a-sufficient-condition-for-X."

"A table's being square is a sufficient, but not a necessary condition, for its having four sides."

"A table's having four sides is a necessary, but not a sufficient, condition for its being square."

32

Suppose first that p implies q. Then knowing that p is true is **sufficient** (i.e., enough evidence) for you to conclude that q is true. It's possible that q could be true even if p weren't, but having p true ensures that q is also true.

Now suppose that not-p implies not-q. If you know that p is false, i.e., that not-p is true, then you know that not-q is true, i.e., that q is false. Thus, in order for q to be true, p **must** be true: without that, you automatically get that q is false. In other words, in order for q to be true, it's **necessary** that p be true; you can't have q true while p is false.

Conclusion:

p is sufficient for q : $p \to q$

p is Necessary for q : $q \to p$

If α is suff for β

then β is Necessary for α.

If α is Necessary for β

then β is sufficient for α.

Next Topic:

Implication Continued...

Various English Translations of

P->Q

$p \rightarrow q$: prop. logic Expression

1. p Implies q
2. If p then q
3. If p, q
4. whenever p, q
5. when p, q
6. Given that p, q
7. provided that p, q

NOTE : In Logic,

If == When == Whenever == Provided that == Given that

$\boxed{\text{Provided that}}$ p, q

If p, q $\equiv$ $p \to q$

$\boxed{\text{Given that}}$ p, q

$\boxed{\text{If } p, q}$: $p \to q$

q $\boxed{\text{provides that}}$ p

Same $\left\{ \begin{array}{l} q \text{ If } p \\ \text{If } p, q \end{array} \right\}$ $p \to q$

$\boxed{p \to q}$:

If p, q

q, if p

same

when p, q $\equiv$ q, when p

Equivalent Forms of "If p then q":

$p \to q$

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"

we will study Later.

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

$P \to Q$:

- P is sufficient for Q ✓
- Q is Necessary for P ✓

$P \to Q$:

- P is sufficient for Q
- A sufficient Condition for Q is P.

$P \to Q$

$P \rightarrow Q$:

- Q is necessary for P
- A necessary condition for P is Q.

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$$

$P \rightarrow Q$:

① If P then Q

② If P is true then Q is true.

③ If Q is false then P is false.

$\neg Q \rightarrow \neg P \equiv P \rightarrow Q$

$P \rightarrow Q$

① If P, Q

② $\boxed{Q \text{ follows from } P}$ ✓

③ Q if P

"Only when"

P $\boxed{\text{only when}}$ Q

necessity

P happens Only when Q happens

Q is necessary for P. $\equiv$ $P \to Q$

You can crack GATE only when

p

You fill the GATE form.

q

p only when q

q is necessary for p $\equiv$ p is sufficient for q

$\equiv (p \to q)$

p $\boxed{\text{only if}}$ q $\equiv$ $p \to q$

$\Updownarrow$

q is necessary for p $\equiv$ $p \to q$

Note:

α $\boxed{\text{only if}}$ β : $\alpha \rightarrow \beta$

α $\boxed{\text{if}}$ β : $\beta \rightarrow \alpha$

$\boxed{A}$ $\boxed{\text{only if}}$ $\boxed{B}$: $A \rightarrow B$

A $\boxed{\text{if}}$ B : $B \rightarrow A$

If you are in Delhi, You are in India. Correct

D I

$D \to I$

① being in Delhi is suff for being in India.

② being in India is Necessary for being in Delhi.

If you are in Delhi, You are in India. Correct

D I

$D \rightarrow I$

3 whenever D then I

4 D only if I

5 I if D

If n is natural number then n is integer.

N I Correct

$N \to I$

1. N is suff for I
2. I is Necessary for N
3. N only when I

$P \rightarrow Q$

~~Many English Translations~~

Done ✓

Next Topic:

Logical Connective

8. Bi-Implication Operator