Octal, Hexadecimal

Number Systems

Recap :

Number System

Binary, Decimal Number Systems

Number System:

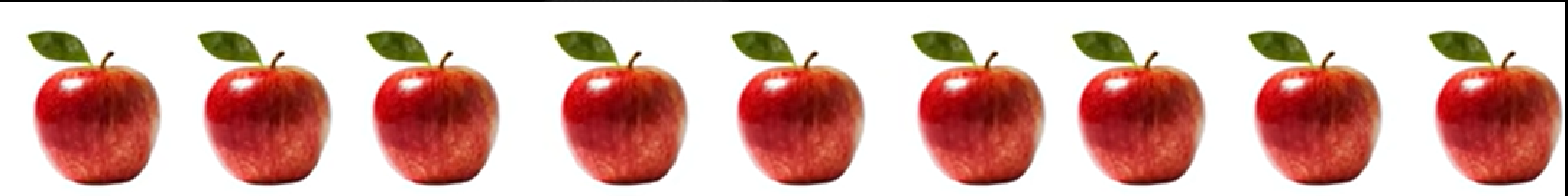
A System of Counting

Number systems are simply ways to count things.

Or ways to represent numbers.

Number System:

0. Roman System



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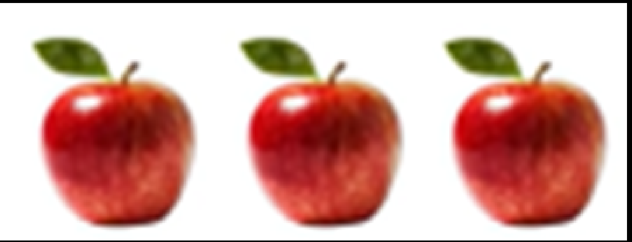

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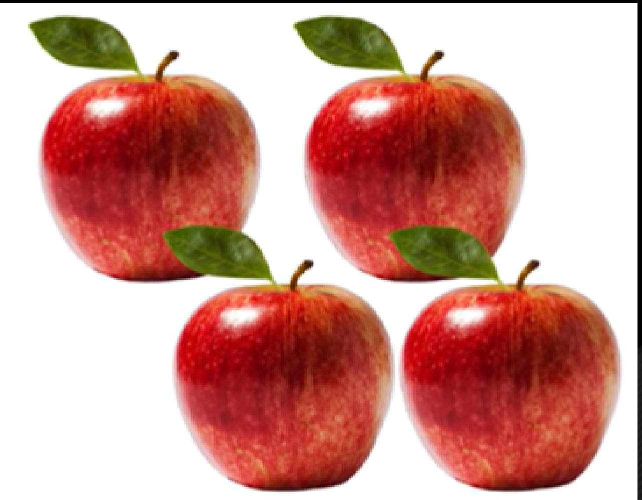
Number System:

1. Unary System

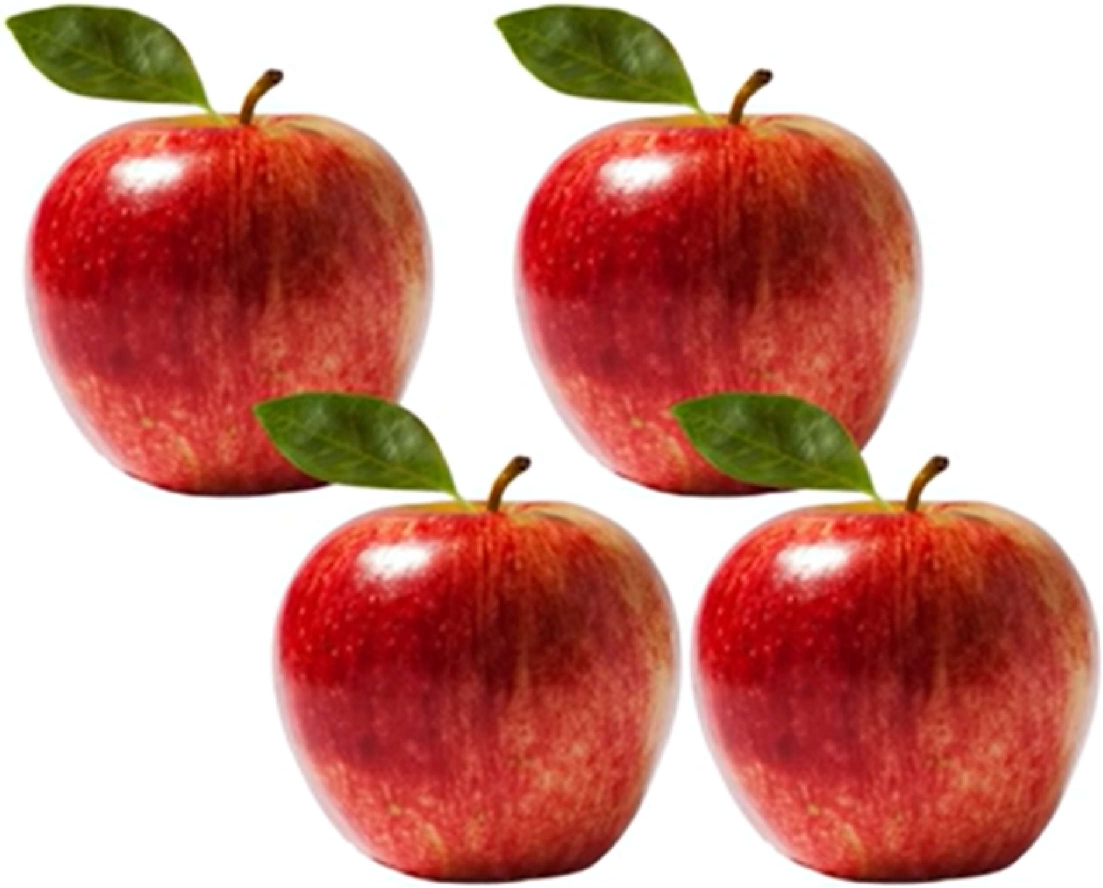
How small babies count?

One Symbol = {1}



= 1111

= 11



Number System:

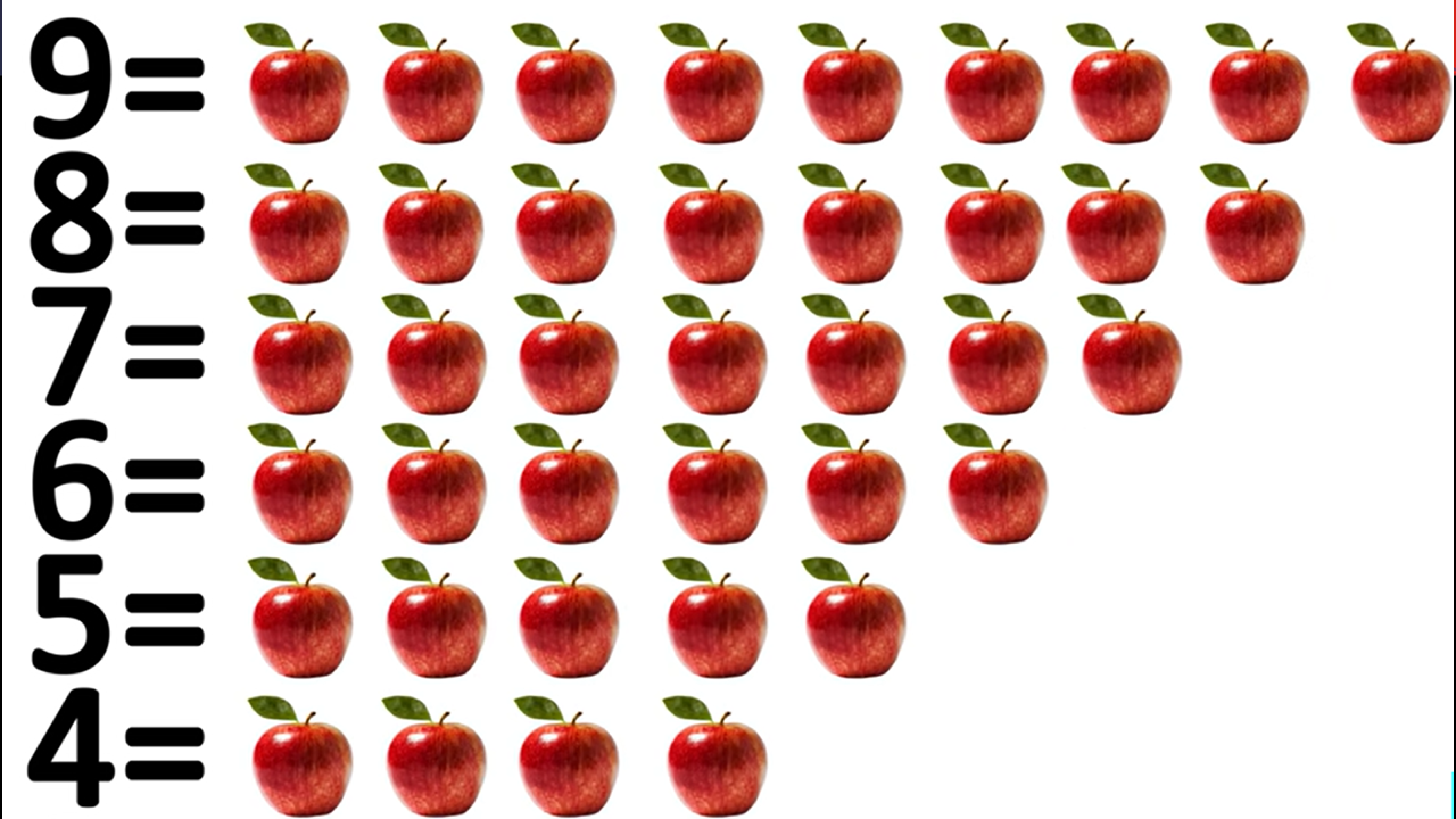
2. Decimal System — for Humans

How a normal human counts?

10 symbols (0,1,2,3,...,9)

- **Decimal system** uses 10 symbols ***(digits)***
  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- The ***decimal system*** is the number system that we use everyday


9=
8=
7=
6=
5=
4=

2	2	7	0	6
10^4	10^3	10^2	10^1	10^0

$$(6701)_{10} = \underbrace{6\times 10^3} + \underbrace{7\times 10^2} + \underbrace{0\times 10^1} + \underbrace{1\times 10^0}$$

Decimal System

$$\underbrace{6000 + 700 + 1}$$

$$= 6701$$

Number System:

3. Binary System: The Computer Number System

How a computers counts?

2 symbols (0,1) (bits)

Base Ten:

Greater than 9=New Digit

Binary:

Greater than 1=New Digit

Binary Number

0

Amount of Things

Binary Number

1

Amount of Things

Binary Number

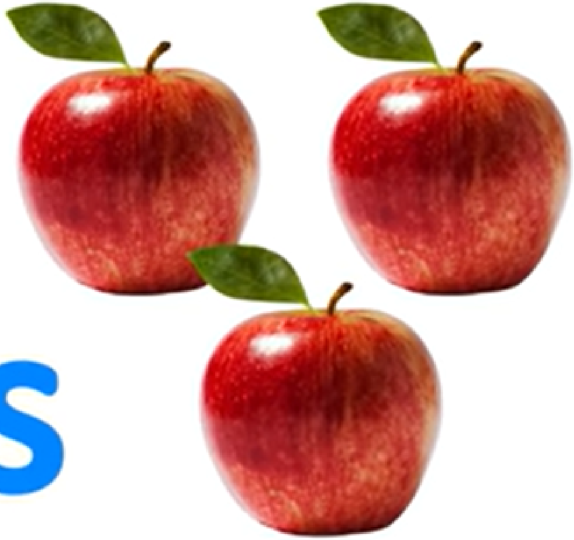
10

Amount of Things

Binary Number

11

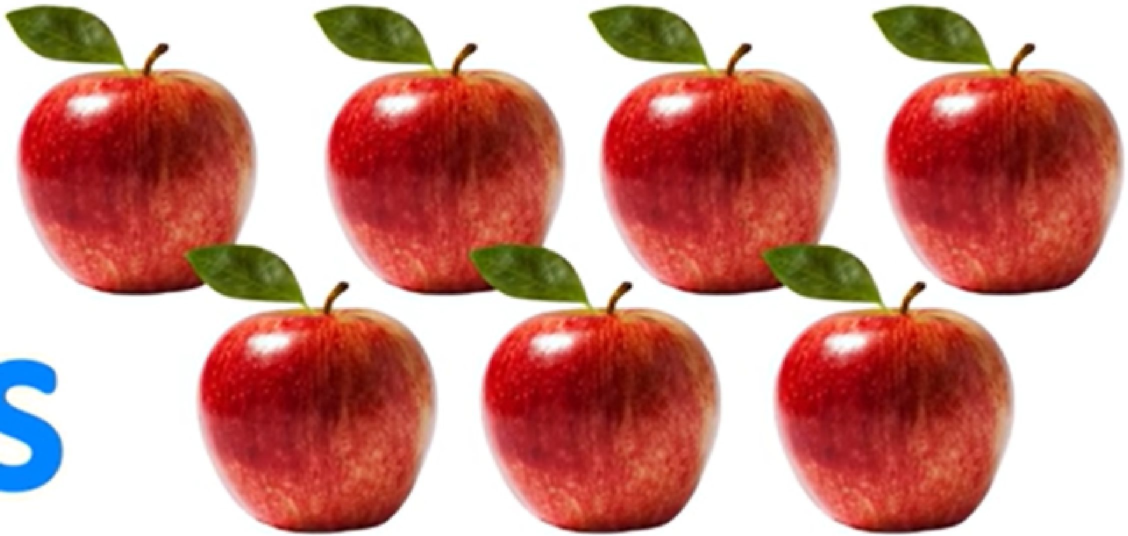
Amount of Things



Binary Number

111

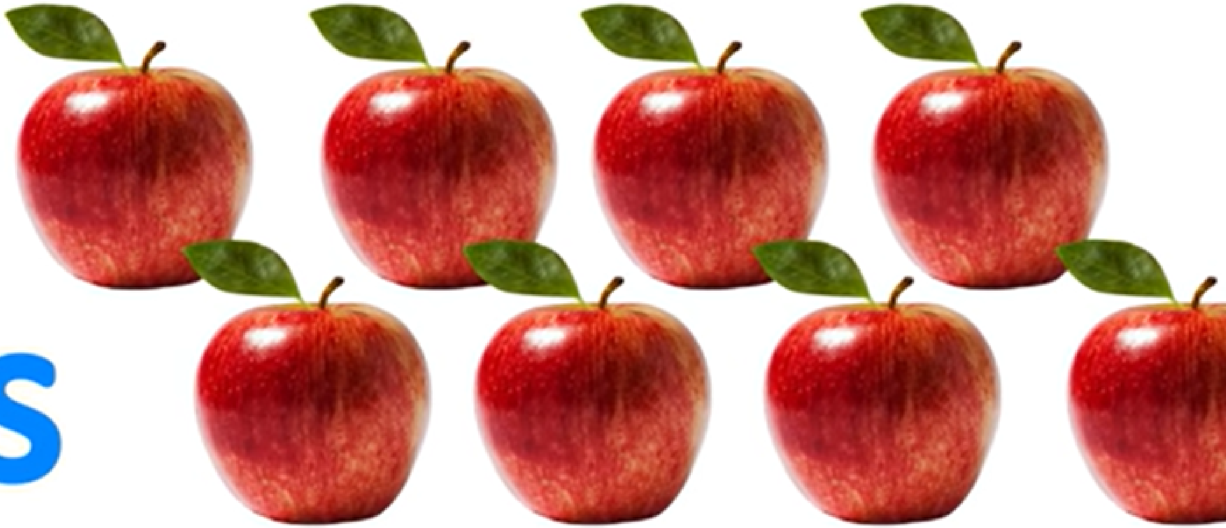
Amount of Things



Binary Number

1000

Amount of Things



0	1	1	0	0	1

2^5 2^4 2^3 2^2 2^1 2^0

$$\underbrace{(101100)}_{2} = \underline{1\times 2^{3}} + \underline{1\times 2^{2}} + \underline{0\times 2^{1}} + \underline{0\times 2^{0}} + \underline{0\times 2^{4}} + \underline{1\times 2^{5}}$$

Binary Number

$$= 32 + 4 + 8$$

$$= (44)_{10}$$

Number System:

4. Octal System:

8 symbols (0,1,2,3,4,5,6,7)

items	octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10

items	octal
9	11
10	12
11	13
12	14
13	15
14	16
15	17
16	20
17	21

Items	Octal
63	77

100
101
102
103
⋮
107

110
111
112
113
⋮
117

120

121
122
⋮
127
⋮

Octal

$$(77)_8 = 7 \times 8^1 + 7 \times 8^0$$

$$= 56 + 7$$

$$= 63$$

Octal System

Base 8

Octal Number System

Base of the number system

The base of the number system is defined as the total number of symbols(digits) available in the number system.

The base value of a number system is the number of different values the set has before repeating itself. For example, decimal has a base of ten values, 0 to 9.

Octal:

$\left\{ \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ 7 \end{matrix} \right\}$ 8 values (No Repetition any symbol)

1	0
1	1
⋮	

$$(abc)_8 = (\ ?\ )_{10}$$

$$8^2 \quad 8^1 \quad 8^0$$

$$= a \times 8^2 + b \times 8^1 + c \times 8^0$$

$$= (64a + 8b + c)_{10}$$

$$(100)_8 = (\quad ? \quad)_{10}$$

$8^2 \quad 8^1 \quad 8^0$

$$= 8^2 \times 1 + \underbrace{0 \times 8^1 + 0 \times 8^0}_{0}$$

$$64 + 0$$

$$= (64)_{10}$$

Number System:

4. Hexa-Decimal System:

16 symbols (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)

16 Symbols:

Hexadecimal	Decimal
0	0
1	1
2	2
⋮	
9	9
A	10
B	11
C	12
D	13
E	14
F	15

items	HD value
0	0
1	1
⋮	
9	9
10	A
11	B
⋮	
15	F
16	1 0

HD value

16 values { 0 ↓ F

1 0
↓
1F

20
↓
2F

30
↓
3F

FF

1 0 0
↓
1 0 F

1 1 0
↓
1 1 F

$$(10F)_{16} = 1 \times 16^2 + 0 \times 16^1 + \underbrace{F \times 16^0}$$

$16^2 \quad 16^1 \quad 16^0$

$$= 16^2 + 15$$

$$= (271)_{10}$$

$\underbrace{\text{Base 16}}$

HD System

Base of the number system

The base of the number system is defined as the total number of symbols(digits) available in the number system.

The base value of a number system is the number of different values the set has before repeating itself. For example, decimal has a base of ten values, 0 to 9.

Digital Logic :

Next Sub-Topic :

Base r System to Decimal Conversion

(Same as you read a decimal number - Easy Peasy)

Base r system: $\underbrace{r > 1}$; $\underbrace{r \in N}$

#symbols $= r$ → $\{0, 1, --, r-1\}$

Base $r = 18$ → Symbol 10 ✗ 15 ✗

Symbols: $0, 1, --9, A, B, C, D, E, F, G, H$

Octal NS:

$$\underbrace{r = 8}_{}$$

$$\{0, 1, --, 7\}$$

Binary NS: $r = 2$

$$\{0, 1\}$$

$$(\overset{r^3}{a}\ \overset{r^2}{b}\ \overset{r^1}{c}\ \overset{r^0}{d})_r = (\ ?\ )_{10}$$

Base r

$$= a \times r^3 + b \times r^2 + c \times r^1 + d \times r^0$$

$(11)_{16} = B$? ~~X~~

$(11)_{16} = (17)_{10}$

$\rightarrow$ 17 Apples

$(11)_{10} = (B)_{16}$ ✓

$(11)_{16} = 1 \times 16^1 + 1 \times 16^0 = (17)_{10}$ ✓

16^1 16^0 $\frac{16}{r}$

Number Systems and Conversion

When we write decimal (base 10) numbers, we use a positional notation; each digit is multiplied by an appropriate power of 10 depending on its position in the number. For example,

$$953.78_{10} = 9 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$$

Similarly, for binary (base 2) numbers, each binary digit is multiplied by the appropriate power of 2:

$$\begin{aligned} 1011.11_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} \\ &= 8 + 0 + 2 + 1 + \tfrac{1}{2} + \tfrac{1}{4} = 11\tfrac{3}{4} = 11.75_{10} \end{aligned}$$

$$(\overset{r^2}{a}\,\overset{r^1}{b}\,\overset{r^0}{c}\,.\,\overset{r^{-1}}{d}\,\overset{r^{-2}}{e})_r = (\,?\,)_{10}$$

Radix Point

Base r

$\{0, 1, --, r-1\}$

$$\boxed{a \times r^2 + b \times r^1 + c \times r^0 + d \times r^{-1} + e \times r^{-2}}$$

$$(101.01)_2 = (\ ?\ )_{10}$$

$$2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2}$$

$$= 2^2 \times 1 + 1 \times 2^0 + 1 \times 2^{-2}$$

$$= 4 + 1 + \frac{1}{4}$$

$$= (5.25)_{10}$$

$$(74001.26)_8 = (\ ?\ )_{10}$$

$$8^4 \quad 8^3 \quad 8^1 \quad 8^0 \quad 8^{-1} \quad 8^{-2}$$

$$= 7 \times 8^4 + 4 \times 8^3 + 1 + 2 \times 8^{-1} + 6 \times 8^{-2}$$

$$= (\qquad)_{10}$$

Radix r $\longrightarrow$ Decimal

$$(\underset{r^3}{d}\;\underset{r^2}{c}\;\underset{r^1}{b}\;\underset{r^0}{a}\;.\;\underset{r^{-1}}{e}\;\underset{r^{-2}}{f}\;\underset{r^{-3}}{g})_r \longrightarrow (d \times r^3 + c \times r^2 + b \times r^1 + a \times r^0 + e \times r^{-1} + f \times r^{-2} + g \times r^{-3})_{10}$$

Any positive integer R ($R > 1$) can be chosen as the *radix* or *base* of a number system. If the base is R, then R digits $(0, 1, \ldots, R - 1)$ are used. For example, if $R = 8$, then the required digits are 0,1,2,3,4,5,6, and 7. A number written in positional notation can be expanded in a power series in R. For example,

$$\begin{aligned} N &= (a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3})_R \\ &= a_4 \times R^4 + a_3 \times R^3 + a_2 \times R^2 + a_1 \times R^1 + a_0 \times R^0 \\ &\qquad + a_{-1} \times R^{-1} + a_{-2} \times R^{-2} + a_{-3} \times R^{-3} \end{aligned}$$

where a_i is the coefficient of R^i and $0 \le a_i \le R - 1$. If the arithmetic indicated in the power series expansion is done in base 10, then the result is the decimal equivalent of N. For example,

$$\begin{aligned} 147.3_8 &= 1 \times 8^2 + 4 \times 8^1 + 7 \times 8^0 + 3 \times 8^{-1} = 64 + 32 + 7 + \tfrac{3}{8} \\ &= 103.375_{10} \end{aligned}$$

Binary numbers to Decimal Number

$(N)_2 = (11100110)_2$

decimal value is given by,

$$(N)_2 = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 = (230)_{10}$$

Binary fractional number to Decimal number

A binary fractional number $(N)_2 = 101.101$

Its decimal value is given by

$$(N)2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$+ 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$
$$= 4 + 0 + 1 + \frac{1}{2} + 0 + \frac{1}{8}$$
$$= 5 + 0.5 + 0.125 = (5.625)_{10}$$

For bases greater than 10, more than 10 symbols are needed to represent the digits. In this case, letters are usually used to represent digits greater than 9. For example, in hexadecimal (base 16), A represents 10_{10}, B represents 11_{10}, C represents 12_{10}, D represents 13_{10}, E represents 14_{10}, and F represents 15_{10}. Thus,

$$A2F_{16} = 10 \times 16^2 + 2 \times 16^1 + 15 \times 16^0 = 2560 + 32 + 15 = 2607_{10}$$

Examples

$$(331)_8 = 3 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 = 192 + 24 + 1 = (217)_{10}$$

$$(D9)_{16} = 13 \times 16^1 + 9 \times 16^0 = 208 + 9 = (217)_{10}$$

$$(33.56)_8 = 3 \times 8^1 + 3 \times 8^0 + 5 \times 8^{-1} + 6 \times 8^{-2} = (27.69875)_{10}$$

$$(E5.A)_{16} = 14 \times 16^1 + 5 \times 16^0 + \underline{10 \times 16^{-1}} = (304.625)_{10}$$

$$\frac{10}{16}$$

Digital Logic :

Next Sub-Topic :

Decimal to Base r System Conversion

(Do you understand or have you by-hearted?)

$$N = (ar^2 + br + c)_{10}$$

Divide by r :

Quotient $= ar + b = Q_1$

Remainder $= c = R_1$

$$ar^2 + br + c = Q_1 r + R_1$$

$$ar^2 + br + c = (ar + b)r + c$$

$$Q_1 = (ar + b)$$

Divide by r : Quotient $= a = Q_2$

Remainder $= b = R_2$

$$ar + b = Q_2 r + R_2$$

$$\boxed{ar + b = ar + b}$$

$Q_2 = a$

Divide by r

$$\begin{cases} \text{Quotient} = 0 = Q_3 \\ \text{Remainder} = a = R_3 \end{cases}$$

$$a = Q_3 r + R_3$$

$$a = a$$

$$(abc)_r \Rightarrow \left(N = \underbrace{ar^2 + br + c}\right)_{10}$$

$$N = \left(ar^2 + br + c\right)_{10} \Rightarrow (abc)_r$$

$$c = \boxed{N \bmod r}$$

$$b = \left(\frac{N}{r}\right) \bmod r$$

$$a = Q_2 \bmod r$$

LSP, MSP;

most significant Position

least significant point

Ex: 9 2 1 0 7

MSP

LSP

Q:
$$(6079)_{10} = (\ ?\ )_8$$

$$(6079)_{10} = (p\,x\,y\,z\,w)_8$$

$$(8^3x + 8^2y + 8z + w) = 6079 = N$$

w : LSP $= N \bmod 8 = 6079 \bmod 8 = 7$

$R_1 = 7$; $Q_1 = 759 = 8^2 x + 8y + z$

$z = Q_1 \bmod 8 = z = 7$

$Q_2 = 94 = 8x + y$; $R_2 = 7$

$y = Q_2 \bmod 8 = 6 = y$

$Q_3 = x = 11$; $R_3 = 6$

$Q_3 = x = 11$

x :

$Q_3 \bmod 8 = 11 \bmod 8 = 3 = R_4$

$1 = R_5$

		Remainder
8	6079	7
8	759	7
8	94	6
8	11	3
8	1	1
	0	

LSP

MSP

(13677)

$(N)_{10} \longrightarrow (?)_r$?

r	N
r	Q_1
r	Q_2
	$\vdots$
	0

Remainder

R_1 — LSP

R_2

R_m — MSP

$$(900)_{10} \longrightarrow (\,?\,)_{12}$$

Remainder

12	900
12	75
12	6
	0

0 — LSP

3

6 — MSP

$(630)_{12}$ ✓

$$(11)_{10} \longrightarrow (\ ?\ )_{16} = \cancel{(11)_{16}}$$

$$\begin{array}{c|c} 16 & 11 \\ \hline & 0 \end{array}$$

Remainder

11 = B

$$= (B)_{16}$$

$$(31)_{10} \longrightarrow (\ ?\ )_{16}$$

Remainder

16	31
16	1
	0

15=F

1

$(\cancel{115})$

$(1F)_{16}$

$$(115)_{16} = 16^2 + 16 + 5$$

$$(11)_{10} = (B)_{16}$$

$$(11)_{16} = 16 + 1 = (17)_{10}$$

$$(17)_{10} = (11)_{16}$$

Rem

16	17	1
16	1	1
	0	

$$(29)_{10} = (\ ?\ )_{16}$$

Rem

16	29	13 = D	—— LSP
16	1	1	—— MSP
	0		

$\cancel{(113)}$

$(1D)_{16}$

Convert 53_{10} to binary.

LSP = LSB

MSP = MSB

$$
\begin{array}{rll}
2\,\underline{|53} & & \\
2\,\underline{|26} & \text{rem.} = 1 = a_0 = \text{LSB} & \\
2\,\underline{|13} & \text{rem.} = 0 = a_1 & \\
2\,\underline{|6} & \text{rem.} = 1 = a_2 & 53_{10} = 110101_2 \\
2\,\underline{|3} & \text{rem.} = 0 = a_3 & \\
2\,\underline{|1} & \text{rem.} = 1 = a_4 & \\
0 & \text{rem.} = 1 = a_5 = \text{MSB} &
\end{array}
$$

	Quotient	Remainder
$156 \div 2$	78	0 = LSB
$78 \div 2$	39	0
$39 \div 2$	19	1
$19 \div 2$	9	1
$9 \div 2$	4	1
$4 \div 2$	2	0
$2 \div 2$	1	0
$1 \div 2$	0	1 = MSB

$(156)_{10} = (10011100)_2$

$$\underbrace{(.ab)_{10}}_{N} \longrightarrow (\ ?\ )_r$$

$$\longrightarrow (.\underline{x}yz)_r$$

$$\frac{x}{r} + \frac{y}{r^2} + \frac{z}{r^3} = .ab = N$$

find x: $(N * r) \longrightarrow$ integer part $= x$

$$\left(\frac{x}{r}+\frac{y}{r^2}+\frac{z}{r^3}\right)r = x+\frac{y}{r}+\frac{z}{r^2}$$

integer

fractional

$$\underbrace{\left(\frac{y}{r}+\frac{z}{r^2}\right)}_{\text{fractional Part}} = M$$

find y

$(M \times r) \to$ integer Part $= y$

Eg:

$$(.152)_{10} \longrightarrow (.1156)_8$$

$8 = r$

$$\begin{array}{r} .152 \\ \times\ 8 \\ \hline 1.216 \end{array}$$

$$\begin{array}{r} .216 \\ \times\ 8 \\ \hline 1.728 \end{array}$$

$$\begin{array}{r} .728 \\ \times\ 8 \\ \hline 5.824 \end{array}$$

$$\begin{array}{r} .824 \\ \times\ 8 \\ \hline 6.592 \end{array}$$

$(.25)_{10} \longrightarrow (\ ?\ )_2$

$\underline{2} = r$

$.25$
$\times 2$

$\underline{0}.50$
2

$\underline{1}.\underbrace{00}_{\text{Stop}}$

$(.01)_2$

$(.3)_{10} \longrightarrow (?)_2$

$$\begin{array}{r} .3 \\ \times 2 \\ \hline 0.6 \\ \times 2 \\ \hline 1.2 \end{array} \qquad \begin{array}{r} .2 \\ \times 2 \\ \hline 0.4 \\ \times 2 \\ \hline 0.8 \end{array} \qquad \begin{array}{r} .8 \\ \times 2 \\ \hline 1.6 \end{array}$$

(0.0100110011001)

$$(0.3)_{10} \longrightarrow (.0100\,1\,\overline{1001})_2$$

keep repeting

Base Conversions

- From base-r >> decimal is easy
  - expand the number in power series and add all the terms
- Decimal >> base-r requires division
- Simple idea:
  - divide the decimal number successively by r
  - accumulate the remainders
- If there is a fraction, then integer part and fraction part are handled separately.

Next, we will discuss conversion of a decimal *integer* to base R using the division method. The base R equivalent of a decimal integer N can be represented as

$$N = (a_n a_{n-1} \cdots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R^1 + a_0$$

If we divide N by R, the remainder is a_0:

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R^1 + a_1 = Q_1, \text{ remainder } a_0$$

Then we divide the quotient Q_1 by R:

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_3 R^1 + a_2 = Q_2, \text{ remainder } a_1$$

Next we divide Q_2 by R:

$$\frac{Q_2}{R} = a_n R^{n-3} + a_{n-1} R^{n-4} + \cdots + a_3 = Q_3, \text{ remainder } a_2$$

This process is continued until we finally obtain a_n. Note that the remainder obtained at each division step is one of the desired digits and the least significant digit is obtained first.

	Quotient	Remainder
$678 \div 8$	84	6
$84 \div 8$	10	4
$10 \div 8$	1	2
$1 \div 8$	0	1

$(678)_{10} = (1246)_8$

	Quotient	Remainder
$678 \div 16$	42	6
$42 \div 16$	2	A
$2 \div 16$	0	2

$(678)_{10} = (2A6)_{16}$

Convert 0.625_{10} to binary.

$$
\begin{array}{ccc}
F = \ .625 & F_1 = \ .250 & F_2 = \ .500 \\
\times \quad 2 & \times \quad 2 & \times \quad 2 \\
\hline
1.250 & 0.500 & 1.000 \\
\hline
(a_{-1} = 1) & (a_{-2} = 0) & (a_{-3} = 1)
\end{array}
\qquad .625_{10} = .101_2
$$

$(\cdot 101)$

This process does not always terminate, but if it does not terminate, the result is a repeating fraction.

Convert 0.7_{10} to binary.

$$
\begin{array}{r}
.7 \\
2 \\
\hline
(1).4 \\
2 \\
\hline
(0).8 \\
2 \\
\hline
(1).6 \\
2 \\
\hline
(1).2 \\
2 \\
\hline
(0).4 \\
2 \\
\hline
(0).8
\end{array}
$$

$(.10110\ 0110)$

← process starts repeating here because 0.4 was previously obtained

$0.7_{10} = 0.1\underline{0110}\ \underline{0110}\ \underline{0110}\ \ldots_2$

$(31.4)_{10} \longrightarrow (\;?\;)_4 \quad 4 = r$

integer part

$(31)_{10} \longrightarrow (\;?\;)_4$

4	31	rem
4	7	3 - LSP
4	1	3
	0	1 - MSP

$(133)_4$

fraction part $(.121212\ldots)$

$.4$
$\times 4$
1.6
4
2.4

$.4$
2

$$(31.4)_{10} \longrightarrow (133.121212\ldots)_{4}$$

$$(23.31)_4 \longrightarrow (\ ?\ )_7$$

$$(23.31)_4 \longrightarrow (?)_7$$

$4^1 \quad 4^0 \quad 4^{-1} \quad 4^{-2}$

$$(?)_{10}$$

$$\left(2\times 4^1 + 3 + \frac{3}{4} + \frac{1}{16}\right)_{10}$$

$$= (\underline{11.8125})_{16}$$

$(11.8125)_{10} \longrightarrow (14.5454\ldots)_7$

integer part

$(11)_{10} \longrightarrow (?)_7$

$$\begin{array}{r|l} 7 & 11 \\ \hline 7 & 1 \\ \hline & 0 \end{array}$$

4 = LSP

1 = MSP

$(14)_7$

fraction part: $= (.5454545454\ldots)$

$$\begin{array}{r} .8125 \\ \times 7 \\ \hline 5.6875 \\ \hline 7 \\ \hline 4.8125 \\ \hline \end{array}$$

$.8125$

7

Convert 231.3_4 to base 7.

$$231.3_4 = 2 \times 16 + 3 \times 4 + 1 + \tfrac{3}{4} = 45.75_{10}$$

$$\begin{array}{rl} 7\,/\underline{45} & \\ 7\,/\underline{\ \ 6} & \text{rem. } 3 \\ 0 & \text{rem. } 6 \end{array}
\qquad
\begin{array}{r} .75 \\ \underline{7} \\ (5).25 \\ \underline{7} \\ (1).75 \\ \underline{7} \\ (5).25 \\ \underline{7} \\ (1).75 \end{array}$$

$$45.75_{10} = 63.5151\ldots_7$$

Digital Logic :

Next Sub-Topic :

Why Octal, Hexadecimal Systems?

(Why are they important??)

$$(1000)_{10} = (\underbrace{1111101000}_{\text{long sequence}})_2$$

$512 = 2^9$

1 1 1 1 1 0 1 0 0 0

512 256 128 64 32 8

Computer :

2	1000	0
2	500	0
2	250	:
		1
	⋮	⋮
	1	

Octal : $\boxed{\underbrace{0 \text{ — } 7}}$

Base $8 = 2^3$

$\underbrace{\text{---} \quad \text{---} \quad \text{---}}$

Binary	Octal
0 0 0	0
0 0 1	1
⋮	
1 1 1	7

$$(\underbrace{011}\ \underbrace{011}\ \underbrace{010}\ \underbrace{101}\ \underbrace{100}\ \underbrace{101})_2$$

$$(332545)_8$$

HexaDecimal:

$$\underbrace{0 \;-\;-\;-\; F}_{}$$

Base $= \underbrace{16 = 2^{4}}$

$(\; - - \;\; - - - - \;\; - - - - \;)_2$

Binary	HD
0 0 0 0	1
_ _ _ _	
_ _ _ _	
_ _ _ _	
1 1 1 1	F

$$00\,(0\,1\;1\,0\,1\,1\;0\,1\,0\,1\;0\,1\,1\,0\;0\,1\,0\,1)_2$$

$$(1\;\;B\;\;5\;\;6\;\;5)_{16}$$

How it is Beneficial for Computer ?

$$(1000)_{10} \longrightarrow (\;?\;)_2$$

$$(3E8)_{16}$$

$(1000)_{10} \longrightarrow (\quad ? \quad)_{16}$

		Rem
16	1000	8
16	62	14 = E
16	3	3
	0	

$\underline{(3E8)_{16}}$

~~(3 14 8)~~

$(3E8)_{16}$

$(11\ 1110\ 1000)_2$

Some features of Binary Numbers

- Require very long strings of 1s and 0s
- Some simplification can be done through grouping
- 3-bit groupings: Octal (radix 8) groups three binary digits

  Digits will have one of the eight values 0, 1, 2, 3, 4, 5, 6 and 7
- 4-digit groupings: Hexa-decimal (radix 16)

  Digits will have one of the sixteen values 0 through 15.

  Decimal values from 10 to 15 are designated as A (=10), B (=11), C (=12), D (=13), E (=14) and F (=15)

0000 abc.de000

Conversion to an octal number

- Group the binary digits into groups of three
- $(11011001)_2 = (011)\ (011)\ (001) = (331)_8$

- Conversion to an hexa-decimal number
  - Group the binary digits into groups of four
  - $(11011001)_2 = (1101)\ (1001) = (D9)_{16}$

D 9

$(\quad . \quad)_2$

$(001101.101100)_2$

$(15.54)_8$

$(\quad \cdot \quad)_2$

$(1101.1011)_2$

$(D \; . \; B)_{16}$

Conversion from binary to hexadecimal (and conversely) can be done by inspection because each hexadecimal digit corresponds to exactly four binary digits (bits). Starting at the binary point, the bits are divided into groups of four, and each group is replaced by a hexadecimal digit:

$$1001101.010111_2 = \underbrace{0100}_{4}\ \underbrace{1101}_{D}\ .\ \underbrace{0101}_{5}\ \underbrace{1100}_{C} = 4D.5C_{16} \tag{1-1}$$

As shown in Equation (1-1), extra 0's are added at each end of the bit string as needed to fill out the groups of four bits.

Digital Logic :

Next Topic :

(Straight) Binary Arithmetic

(Just like our school life arithmetic for decimals)

Decimal Addition: $\Rightarrow \underbrace{\text{Base} = 10}$

$$
\begin{array}{r}
1\ \ 1\ \ \ \ 1\ \ \ \ \\
2\ \ 3\ \ 0\ \ 4 \\
+\ 9\ \ 8\ \ 7\ \ 6 \\
\hline
12\ \ 1\ \ 8\ \ 0 \\
\hline
\end{array}
$$

$\underbrace{10 - \text{Base}} = 10 - 10 = 0$

$> \text{Base}$

$12 - \text{Base} = 2$

$8 + 3 = 11 - \text{Base} = 1$

Binary Addition: Base = 2

$$
\begin{array}{ccccccc}
 & 1 & 1 & 1 & 1 & 1 & \\
 & 1 & 0 & 1 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 1 & 1 & 1 \\
\hline
1 & 1 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

$1+1 = 2 - \text{Base} = 0$

$\geq \text{Base}$

$1+1+1 = 3 - \underset{2}{\text{Base}} = 1$

$\geq \text{Base}$

Binary Addition:

a	b	sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\begin{array}{r} 1 \\ +1 \\ \hline 2 = 10 \end{array}$$

$$\begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

1 → Carry, 0 → Sum

	sum	Carry
1+1+1	1	1

Carry

sum

$1+1+1 = 3 = 11$

$$\begin{array}{r} 1 \\ 1 \\ + \; 1 \\ \hline 1\;1 \end{array}$$

Carry

Sum

$$
\begin{array}{r}
1\;1\;1\;1\;1\;\; \\
1\,1\,0\,1\,1\,1 \\
+\;\;0\,1\,1\,1\,0\,1 \\
\hline
1\,0\,1\,0\,1\,0\,0 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
1\;1\;1\;\; \\
1\,1\,1\,1 \\
1\,1\,1\,1 \\
\hline
1\,1\,1\,1\,0 \\
\hline
\end{array}
$$

Binary Arithmetic

Arithmetic operations in digital systems are usually done in binary because design of logic circuits to perform binary arithmetic is much easier than for decimal. Binary arithmetic is carried out in much the same manner as decimal, except the addition and multiplication tables are much simpler.

The addition table for binary numbers is

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 0 \quad \text{and carry 1 to the next column}$$

Carrying 1 to a column is equivalent to adding 1 to that column.

Add 13_{10} and 11_{10} in binary.

$$
\begin{array}{rr}
 & 1111 \leftarrow \text{carries} \\
13_{10} = & 1101 \\
11_{10} = & \underline{1011} \\
 & 11000 = 24_{10}
\end{array}
$$