In this Lecture:

- Finding Dual of a function from Truth Table.
- Relation between Complement(f) & Dual(f)

- **Special Types of Boolean Functions:**

Neutral Function, Self Dual Function,

Functions whose complement is same as their dual.

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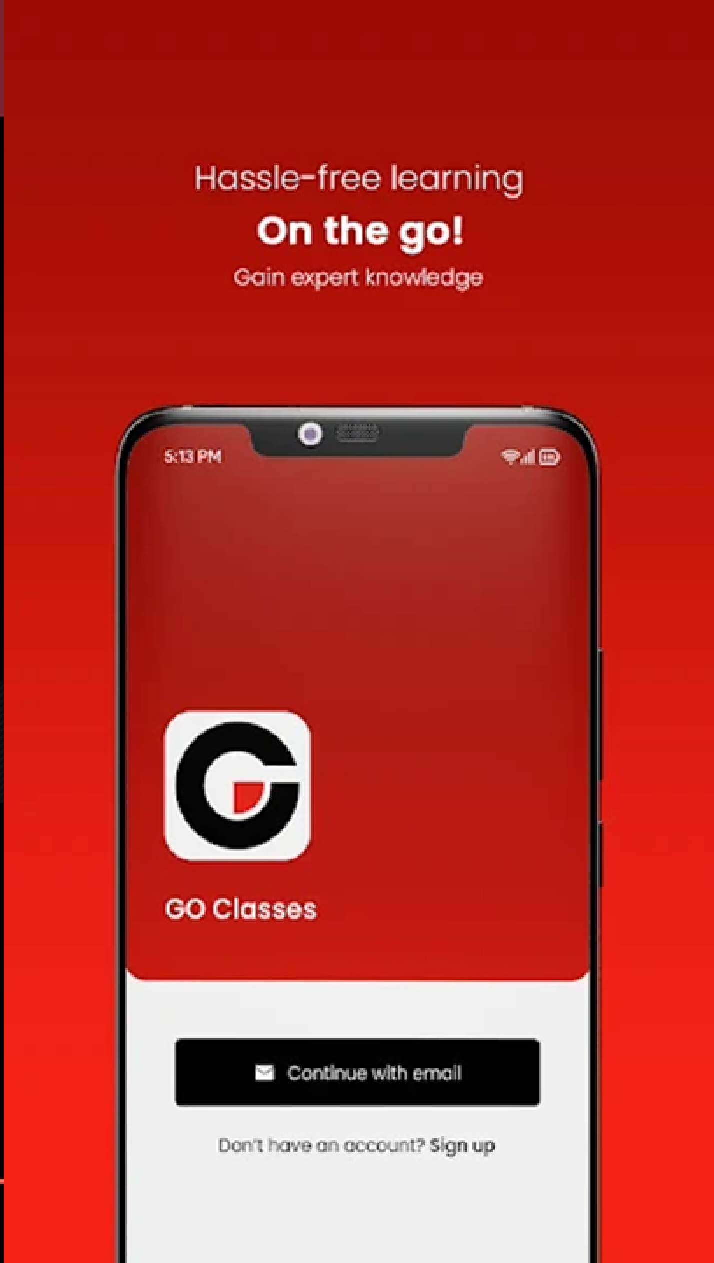

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In this Lecture:

- Finding Dual of a function from Truth Table.
- Relation between Complement(f) & Dual(f)

- Special Types of Boolean Functions:

Neutral Function, Self Dual Function,

Functions whose complement is same as their dual.

Recap:

Complement of a Boolean function

Table 2.2
Truth Tables for F_1 and F_2

x	y	z	F_1	F_2	$\overline{F_1}$	$\overline{F_2}$
0	0	0	0	0		
0	0	1	1	1	?	?
0	1	0	0	0		
0	1	1	0	1		
1	0	0	1	1		
1	0	1	1	1		
1	1	0	1	0		
1	1	1	1	0		

Table 2.2
Truth Tables for F_1 and F_2

	x	y	z	F_1	F_2	$\overline{F_1}$	$\overline{F_2}$
0	0	0	0	0	0	1	1
1	0	0	1	1	1	0	0
2	0	1	0	0	0	1	1
3	0	1	1	0	1	1	0
4	1	0	0	1	1	0	0
5	1	0	1	1	1	0	0
6	1	1	0	1	0	0	1
7	1	1	1	1	0	0	1

any Boolean function f

- variables	f	$\overline{f}$
	a_0	$\overline{a_0}$
	a_1	$\overline{a_1}$
	a_2	$\overline{a_2}$
	$\vdots$	
	a_m	$\overline{a_m}$

input Combinations

$a_i \to 0$ OR $a_i \to 1$

$\overline{0} = 1$

$\overline{1} = 0$

Recap:

De-Morgan's Law

Prove DeMorgan's Theorem

$$\overline{XY} = \overline{X} + \overline{Y}$$

X	Y	$\overline{XY}$
0	0	1
0	1	1
1	0	1
1	1	0

X	Y	$\overline{X} + \overline{Y}$
0	0	1
0	1	1
1	0	1
1	1	0

$$\overline{a + b} = \overline{a} \cdot \overline{b}$$

$$\overline{\alpha + \beta} = \overline{\alpha} \cdot \overline{\beta}$$

$$\overline{a \cdot b} = \overline{a} + \overline{b}$$

$$\overline{\alpha \cdot \beta} = \overline{\alpha} + \overline{\beta}$$

Finding Complement of a Boolean Expression:

If the Boolean Expression f consists of ONLY

Variables, Literals, AND, OR, NOT, 0, 1

Then we can use De-Morgan's Law to find

Complement of f.

DeMorgan's Theorem

- Procedure for complementing expressions
- Replace...
  - AND with OR, OR with AND
  - 1 with 0, 0 with 1
  - X with $\overline{X}$, $\overline{X}$ with X

$$\overline{XY} = \overline{X} + \overline{Y}$$

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$

$$f(a, \bar{a}, b, \bar{b}, +, \cdot, 0, 1)$$

$$\downarrow$$

$$\bar{f}(\bar{a}, a, \bar{b}, b, \cdot, +, 1, 0)$$

Literals

$$f(\boxed{a, \bar{a}, b, \bar{b}}, +, \cdot, 0, 1)$$

$$\bar{f}(\bar{a}, a, \bar{b}, b, \cdot, +, 1, 0)$$

$f = \overline{a+b}$

$\overline{f} = \overline{\overline{a+b}} = a+b$

$\overline{\overline{x}} = x$

DeMorgan's Practice

$$F = \overline{\overline{A\bar{B}C} + \overline{ACD} + B\bar{C}}$$

$$g = \left(\overline{A\bar{B}C}\right) + \left(\overline{ACD}\right) + \left(B\bar{C}\right)$$

$$\bar{g} = F = \left(\overline{\overline{A\bar{B}C}}\right) \cdot \left(\overline{\overline{ACD}}\right) \cdot \left(\overline{B\bar{C}}\right)$$

$$\bar{g} = F = \left(A\bar{B}C\right) \cdot \left(ACD\right)\left(\bar{B} + C\right)$$

$$\boxed{\bar{\bar{\alpha}} = \alpha}$$

$$\left.\begin{aligned} g &= \alpha + \beta + \gamma \\ \overline{g} &= \overline{\alpha} \cdot \overline{\beta} \cdot \overline{\gamma} \end{aligned}\right\}$$

$$f = (A \cdot \overline{B} \cdot C)(A \cdot C \cdot D)(\overline{B} + C)$$

$$= \underbrace{A\overline{B}CD}(\overline{B} + C)$$

$$= A\overline{B}CD\overline{B} + A\overline{B}CDC$$

$$= A\overline{B}CD + A\overline{B}CD = \underline{\underline{A\overline{B}CD}}$$

$$\left.\begin{array}{l} \alpha + \alpha = \alpha \\ \alpha \cdot \alpha = \alpha \\ \overline{\overline{\alpha}} = \alpha \end{array}\right\}$$

$$\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$$

DeMorgan's Practice

$$F = \overline{\overline{A\bar{B}C} + \overline{ACD} + B\bar{C}}$$

$$= (A\bar{B}C)(ACD)(\overline{B\bar{C}})$$

$$= (A\bar{B}CD)(\bar{B}+C)$$

$$= A\bar{B}CD + A\bar{B}CD$$

$$= A\bar{B}CD$$

$$\overline{\alpha\beta} = \overline{\alpha} + \overline{\beta}$$

$$\overline{\alpha+\beta+\gamma} = \overline{\alpha}\cdot\overline{\beta}\cdot\overline{\gamma}$$

$$\alpha + \cancel{\overline{\alpha}}\beta = \alpha + \beta \checkmark$$

Write the complement of the following Boolean Expression and then simplify using the laws of Boolean Algebra including DeMorgan's Law:

$$(A+\overline{B}+C)\,(\overline{A}+\overline{B})$$

- $A\,C+\overline{B}\,C$
- $A\,B+B\,\overline{C}$
- $A\,\overline{B}+A\,C$
- $\overline{B}+A\,\overline{C}$
- $B+A\,\overline{C}$

Write the complement of the following Boolean Expression and then simplify using the laws of Boolean Algebra including DeMorgan's Law:

$F = (A+\overline{B}+C)\,(\overline{A}+\overline{B})$

- $AC+\overline{B}C$
- $AB+B\overline{C}$ ✓
- $A\overline{B}+AC$
- $\overline{B}+A\overline{C}$
- $B+A\overline{C}$

$\overline{F} = \overline{(A+\overline{B}+C)\cdot(\overline{A}+\overline{B})}$

$= (\overline{A}\cdot B\cdot \overline{C}) + (A\cdot B)$

$= \overline{A}B\overline{C} + AB$

$= B(A+\cancel{\overline{A}}\,\overline{C}) = B(A+\overline{C})$

$= AB + B\overline{C}$

Complement of a Function

The complement of a function F is F' and is obtained from an interchange of 0's for 1's and 1's for 0's in the value of F. The complement of a function may be derived algebraically through DeMorgan's theorems, listed in Table 2.1 for two variables. DeMorgan's theorems can be extended to three or more variables. The three-variable form of the first

Complement of f :

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① from truth table ✓

② from expression ✓

Recap:

Dual of a Boolean function

Finding Dual of a Boolean Expression:

If the Boolean Expression f consists of ONLY $\boxed{\text{Variables, Literals, AND, OR, NOT, 0, 1}}$ Then we can easily find Dual of f.

- Duality
  - a dual of a Boolean expression is derived by replacing
    • by +, + by •, 0 by 1, and 1 by 0, and leaving variables unchanged

literals

$$f(\underbrace{a, \bar{a}, b, \bar{b}}, +, \cdot, 0, 1)$$

No change

$$f_d(a, \bar{a}, b, \bar{b}, \cdot, +, 1, 0)$$

EXAMPLE 1.6.3. *The dual of $x\overline{y} + \overline{x}z$ is $(x + \overline{y}) \cdot (\overline{x} + z)$.*

$$f = (x \cdot \bar{y}) + (\bar{x} \cdot z)$$

$$f_d = (x + \bar{y}) \cdot (\bar{x} + z)$$

$$f_d = (x + \bar{y})(\bar{x} + z)$$

$$f = x\bar{y} + \bar{x}z$$

$$f_d = (x+\bar{y})(\bar{x}+z)$$

$$\overline{f} = (\bar{x}+y)(x+\bar{z})$$

1.6. Dual.

Definition 1.6.1. *The* **dual** *of a Boolean expression is the expression one obtains by interchanging addition and multiplication and interchanging 0's and 1's. The dual of the function F is denoted F^d.*

- Duality
  - a dual of a Boolean expression is derived by replacing
    - • by +, + by •, 0 by 1, and 1 by 0, and leaving variables unchanged

Question 2: Finding the Dual of a Boolean Expression

Select the dual of the following Boolean expression:

$$\bar{B} + C + \bar{A}(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$

- ○ $B + \bar{C} + A(A + B) + \bar{B}(A + B)$
- ○ $\bar{A} + \bar{B} + C$
- ○ $\bar{B}C(\bar{A} + \bar{A}\bar{B})(B + \bar{A}\bar{B})$
- ○ $B\bar{C}(A + AB)(\bar{B} + AB)$
- ○ $\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + ABC$

Question 2: Finding the Dual of a Boolean Expression

Select the dual of the following Boolean expression:

$$F = (\bar{B}) + (C) + (\bar{A}(\bar{A} + \bar{B})) + (B(\bar{A} + \bar{B}))$$

- $B + \bar{C} + A(A + B) + \bar{B}(A + B)$
- $\bar{A} + \bar{B} + C$
- $\bar{B}C(\bar{A} + \bar{A}\bar{B})(B + \bar{A}\bar{B})$ ✓
- $B\bar{C}(A + AB)(\bar{B} + AB)$
- $\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + ABC$

$$F_d = (\ ) \cdot (\ ) \cdot (\ ) \cdot (\ )$$

$\bar{B}$ $\quad C \quad$ $\bar{A} + \bar{A}\bar{B}$ $\quad$ $B + \bar{A}\bar{B}$

Find the dual of the following Boolean expression:

$$(a * 1)' + (b' + 0)'$$

- $(a' + 0) + (b * 1)$
- $(a + 1)' * (b' * 0)'$
- $(a + 0) * (b' * 0)$
- $(a + 0)' * (b' * 1)'$
- None of the above.

Method 1 :

Find the dual of the following Boolean expression:

$$(a * 1)' + (b' + 0)'$$

$$(a+0)' \cdot (\bar{b} \cdot 1)'$$

- $(a' + 0) + (b * 1)$
- $(a + 1)' * (b' * 0)'$
- $(a + 0) * (b' * 0)$
- ✓ $(a + 0)' * (b' * 1)'$
- None of the above.

Find the dual of the following Boolean expression:

$$\underbrace{(a * 1)' + (b' + 0)'}$$

Simplify

method :

$x \cdot 1 = x$

$x + 0 = x$

- $(a' + 0) + (b * 1)$
- $(a + 1)' * (b' * 0)'$
- $(a + 0) * (b' * 0)$
- $(a + 0)' * (b' * 1)'$
- None of the above.

$a' + b$

Dual

$\bar{a} b$ → Answer

$$(x\bar{y}z)^D = x + \bar{y} + z$$

$$(x + y + \bar{z})^D = x \cdot y \cdot \bar{z}$$

Dual of a Boolean Function :

Given a Boolean algebra expression the dual of the expression is obtained by interchanging the constants 0 and 1 and interchanging the operations of AND and OR. Variables and Literals are left unchanged.

$$(XYZ\ldots)^D = X + Y + Z + \cdots \qquad (X + Y + Z + \cdots)^D = XYZ\ldots$$

Boolean Equation:

$$f = G$$

$$f \xrightarrow{\text{Dual}} F_d = G_d$$

$$f \xrightarrow{\text{Complement}} \overline{F} = \overline{G}$$

3. Write the dual of each Boolean ~~expression~~ Equations (1 point):

a. $a(a+b)=a$

b. $a+a'=1$

c. $(a+b)(a'+b')=ab'+a'b$

d. $a'1+a'0=a'$

3. Write the dual of each Boolean ~~expression~~ Equations (1 point):

a. $a(a+b) = a$

$$\boxed{a + ab = a}$$

↳ Dual of option a.

b. $a + a' = 1$

$$\boxed{a\bar{a} = 0}$$

Dual of option b.

c. $(a+b)(a'+b') = ab' + a'b$

$$\boxed{ab + a'\bar{b} = (a+b')(a'+b)}$$

↳ Dual of equation c.

d. $a'1 + a'0 = a'$

$$\boxed{(a'+0)\cdot(a'+1) = a'}$$

Dual of eq. d.

3. Write the dual of each Boolean ~~expression~~ (1 point):

Equations

a. $a(a+b)=a$

b. $a+a'=1$

c. $(a+b)(a'+b')=ab'+a'b$

d. $a'1+a'0=a'$

Dual

$$\boxed{(a'+0)\cdot(\bar{a}+1)=a'}$$

$$f(a, \bar{a}, 0, 1, \cdot, +, \text{NOT})$$

$$f_d(a, \bar{a}, 1, 0, +, \cdot, \text{NOT})$$

What is the dual of this expression?

$$\overline{X} + \overline{Y} = \overline{XY}$$

What is the dual of this ~~expression~~?

Equation

$$\overline{X} + \overline{Y} = \overline{XY} \rightarrow (x \cdot y)'$$

Dual

$$\boxed{\bar{x}\,\bar{y} = (x+y)'}$$

What is the dual of this expression?

$$\overline{X} + \overline{Y} = \overline{XY}$$

dual

$$\overline{X}\,\overline{Y} = \overline{X + Y}$$

Very Important:

Relationship between

Complement of f & Dual of f

for any boolean function f,

Relationship b/w $\underbrace{\overline{f} \,,\; f_d}$

$$f_d(a, \bar{a}, \cdot, +, 1, 0)$$

$$f(a, \bar{a}, +, \cdot, 0, 1)$$

$$\bar{f}(\bar{a}, a, \cdot, +, 1, 0)$$

look at $\bar{f}$ & f_d

$f_d(a, \bar{a}, \cdot, +, 1, 0)$

f

$\bar{f}(\bar{a}, a, \cdot, +, 1, 0)$

literals change

No change

look at $\bar{f}$ & f_d

for any function $f(a,b,c)$

If $f^d(a,b,c)$ is Given.

How to find $\overline{f}(a,b,c) = ?$

$$\overline{f} = f^d(\bar{a}, \bar{b}, \bar{c})$$

for any function $f(a,b,c)$

If $\overline{f}(a,b,c)$ is given then

How to find $f^d(a,b,c) = ?$

$$f^d = \overline{f}(\bar{a},\bar{b},\bar{c})$$

eg: $f_{(a,b)} = a + b$

$f^d = ab$ $\qquad$ $\bar{f} = \bar{a}\,\bar{b}$

Change literals

Change literals

eg: $f(a,b) = 0$

a	b	f
0	0	0
0	1	0
1	0	0
1	1	0

$f^d = 1$; $\overline{f} = 1$

Change literals

No literal

Eg: $f = ab$

$f_d = \underline{a+b}$ $\bar{f} = \underline{\bar{a}+\bar{b}}$

Change literals

Note:

$$f^{d}(a,b) = \overline{f}(\bar{a},\bar{b})$$

$$\overline{f}(a,b) = f^{D}(\bar{a},\bar{b})$$

for any function $f(a,b)$

$$f^d(\overset{a}{0},\overset{b}{0}) = \overline{f}(\overset{\bar{a}}{1},\overset{\bar{b}}{1})$$

$$f^d(0,1) = \overline{f}(1,0)$$

$f(a,b) = \bar{a} + b \rightarrow f_d = \bar{a}b$

$\bar{f} = a\bar{b}$

a	b	$f(a,b)$
0	0	1
0	1	1
1	0	0
1	1	1

$f^d(0,0) = 0 = \bar{f}(1,1) = 0$

$f^d(1,0) = 0 = \bar{f}(0,1) = 0$

$f^d(0,1) = 1 = \bar{f}(1,0) = 1$

$$f(a,b) = \bar{a} + b \xrightarrow{\text{Dual}} \bar{a}\,b$$

$$\xrightarrow{\text{Complement}} a\,\bar{b}$$

changing literals

a	b	$f(a,b)$	$\bar{f} = a\bar{b}$	$f_d = \bar{a}b$
0	0	1	0	0
0	1	1	0	1
1	0	0	1	0
1	1	1	0	0

$$f^{d}(0,1) = \bar{f}(1,0)$$

$f^d(a, \bar{a}, b, \bar{b}, \underbrace{+, \cdot, 0, 1}_{\text{same}})$

$\bar{f}(\bar{a}, a, \bar{b}, b, \overline{+, \cdot, 0, 1})$

$$f(\overset{a}{\underset{0}{d}}, \overset{b}{\underset{0}{d}}) = \bar{f}(\overset{\bar{a}}{1}, \overset{\bar{b}}{1})$$

$$\overset{d}{f}(\underset{a}{0}, \underset{b}{1}) = \bar{f}(\underset{\bar{a}}{1}, \underset{\bar{b}}{0})$$

$$f^{d}(1, 0) = \overline{f(0, 1)}$$

$a \quad b \qquad \bar{a} \quad \bar{b}$

$$f^{d}(1, 1) = \overline{f(0, 0)}$$

$a \quad b \qquad \bar{a} \quad \bar{b}$

Relationship b/w $\overline{f}$ & f^d :

a	b	f(a,b)	$\overline{f}$	f^d
0	0	a_0		
0	1	a_1	?	?
1	0	a_2		
1	1	a_3		

Relationship b/w $\overline{f}$ & f^d :

a	b	f(a,b)	$\overline{f}$	f^d
0	0	$a_0 \rightarrow$	$\overline{a_0}$	
0	1	$a_1 \rightarrow$	$\overline{a_1}$	?
1	0	$a_2 \rightarrow$	$\overline{a_2}$	
1	1	$a_3 \rightarrow$	$\overline{a_3}$	

Relationship b/w $\overline{f}$ & f^d:

a	b	f(a,b)	$\overline{f}$	f^d
0	0	a_0	$\overline{a_0}$	$\overline{a_3}$
0	1	a_1	$\overline{a_1}$	$\overline{a_2}$
1	0	a_2	$\overline{a_2}$	$\overline{a_1}$
1	1	a_3	$\overline{a_3}$	$\overline{a_0}$

Reverse of $\overline{f}$

Boolean Algebra: Example 2

Find the complement of F.

$$F = A\overline{B} + \overline{A}B$$

$$\overline{F} =$$

$$f(a,b) = a\bar{b} + \bar{a}b \longrightarrow f^d(a,b) = (a+\bar{b})(\bar{a}+b)$$

$$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$$

1

$$\bar{f} \longleftarrow \boxed{\text{Change Literals}} \longrightarrow f_d$$

$f(a,b) = a\bar{b} + \bar{a}b \longrightarrow f^d(a,b) = (a+\bar{b})(\bar{a}+b)$

$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$

a	b	f^d
0	0	1

a	b	$\bar{f}$
1	1	1

$$\boxed{f^d(0,0) = \bar{f}(1,1)}$$

$$f(a,b) = \underbrace{a\bar{b} + \bar{a}b} \rightarrow f^d(a,b) = \underbrace{(a+\bar{b})}(\bar{a}+b)$$

$$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$$

a	b	f^d
0	1	0

a	b	$\bar{f}$
1	0	0

$$\boxed{f^d(0,1) = \bar{f}(1,0)}$$

$$f(a,b) = \underbrace{a\bar{b} + \bar{a}b}_{} \rightarrow f^d(a,b) = (a+\bar{b})(\bar{a}+b)$$

$$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$$

a	b	f^d
1	0	0

a	b	$\bar{f}$
0	1	0

$$\boxed{f^d(1,0) = \bar{f}(0,1)}$$

$$f(a,b) = \underbrace{a\bar{b} + \bar{a}b}$$

$$f^d(a,b) = (a+\bar{b})(\bar{a}+b)$$

$$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$$

a	b	f^d
1	1	1

a	b	$\bar{f}$
0	0	1

$$\boxed{f^d(1,1) = \bar{f}(0,0)}$$

$$f(a,b) = \underbrace{a\bar{b} + \bar{a}b} \rightarrow f^{d}(a,b) = \underbrace{(a+\bar{b})(\bar{a}+b)}$$

$$\bar{f}(a,b) = (\bar{a}+b)(a+\bar{b})$$

$$f^{d}(0,0) = \bar{f}(1,1)$$

$$f^{d}(0,1) = \bar{f}(1,0) \qquad f^{d}(1,0) = \bar{f}(0,1)$$

$$f^{d}(1,1) = \bar{f}(0,0)$$

Boolean Algebra: Example 2

Find the complement of F.

$$F = A\overline{B} + \overline{A}B$$

$$\overline{F} = \overline{A\overline{B} + \overline{A}B}$$

$$\overline{(A\overline{B})}\,\overline{(\overline{A}B)} \qquad \text{(by DeMorgan's)}$$

$$(\overline{A} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{B}) \qquad \text{(by DeMorgan's)}$$

$$(\overline{A} + B)(A + \overline{B}) \qquad \text{(by involution)}$$

Example:

$$F = xy' + x'z$$

$$f = x\bar{y} + \bar{x}z$$

$$f_d = (x+\bar{y})(\bar{x}+z)$$

$$\bar{f} = (\bar{x}+y)(x+\bar{z})$$

$f = x\bar{y} + \bar{x}z$

$f_d = (x+\bar{y})(\bar{x}+z)$

$\bar{f} = (\bar{x}+y)(x+\bar{z})$

$\underbrace{f^d(0,1,0)}_{0} \stackrel{?}{=} \underbrace{\bar{f}(1,0,1)}_{0}$

$0 = 0$ ✓

$f^d(0,1,0) = \bar{f}(1,0,1)$

$f = x\bar{y} + \bar{x}z$

$f^d = (x+\bar{y})(\bar{x}+z)$

$\bar{f} = (\bar{x}+y)(x+\bar{z})$

$f^d(\underbrace{0,1,1}_{0}) \overset{?}{=} \bar{f}(\underbrace{1,0,0}_{0})$

$0 = 0$ ✓

$$f = x\bar{y} + \bar{x}z$$

$$f^d = (x+\bar{y})(\bar{x}+z)$$

$$\bar{f} = (\bar{x}+y)(x+\bar{z})$$

$$f^d(0,0,0) = \overline{f(1,1,1)}$$

$$f = x\bar{y} + \bar{x}z$$

$$f^d = (x+\bar{y})(\bar{x}+z)$$

$$\bar{f} = (\bar{x}+y)(x+\bar{z})$$

$$\underbrace{f^d(1,1,0)}_{0} \overset{?}{=} \overline{\underbrace{f(0,0,1)}_{1}} = 0$$

$f = x\bar{y} + \bar{x}z$

$f^d = (x+\bar{y})(\bar{x}+z)$

$\bar{f} = (\bar{x}+y)(x+\bar{z})$

$f^d(\underbrace{a,b,c}_{\text{some input combination}}) = \overline{f(a',b',c')}$

$$f = x\bar{y} + \bar{x}z$$

$$f^d = (x+\bar{y})(\bar{x}+z)$$

$$\bar{f} = (\bar{x}+y)(x+\bar{z})$$

	xyz	f	$\bar{f}$	f_d
0	0 0 0	0		
1	0 0 1	1	1	1
2	0 1 0	0		
3	0 1 1	1		
4	1 0 0	1		
5	1 0 1	1		
6	1 1 0	0		
7	1 1 1	0		

$$f = x\bar{y} + \bar{x}z$$

$$f^d = (x+\bar{y})(\bar{x}+z)$$

$$\bar{f} = (\bar{x}+y)(x+\bar{z})$$

	xyz	f	$\bar{f}$	$\rightarrow f_d$
0	0 0 0	0	1	?
1	0 0 1	1	0	
2	0 1 0	0	1	
3	0 1 1	1	0	
4	1 0 0	1	0	
5	1 0 1	1	0	
6	1 1 0	0	1	
7	1 1 1	0	1	

$f = x\bar{y} + \bar{x}z$

$f^d = (x+\bar{y})(\bar{x}+z)$

$\bar{f} = (\bar{x}+y)(x+\bar{z})$

	x y z	f	$\bar{f}$	f_d
0	0 0 0	0	1	1
1	0 0 1	1	0	1
2	0 1 0	0	1	0
3	0 1 1	1	0	0
4	1 0 0	1	0	0
5	1 0 1	1	0	1
6	1 1 0	0	1	0
7	1 1 1	0	1	1

Reverse $\bar{f}$ Table.

$$f^{d}(x, y, z) = \overline{f(\bar{x}, \bar{y}, \bar{z})}$$

$$\overline{f(0,0,0)} = f^{d}(1,1,1)$$

$$\overline{f(0,1,0)} = f^{d}(1,0,1)$$

$$\overline{f(1,1,0)} = f^{d}(0,0,1)$$

Relationship b/w

$\overline{f}, f^d$

$f_{(a,b)} = 1$ $\quad f^d = 0$; $\bar{f} = 0$

a	b	f	$\bar{f} \rightarrow f^d$	
0	0	1	0	0
0	1	1	0	0
1	0	1	0	0
1	1	1	0	0

Reverse of $\bar{f}$

Boolean algebra practice 3

Find the complement of F.

$F(V, W, X, Y, Z)$

$$F = (\overline{V}W + X)Y + \overline{Z}$$

find $\overline{F}$ ✓

find F^d ✓

Verify

$F^d(0,1,0,0,1) = \overline{F}(1,0,1,1,0)$

In previous Q:

Verify:

$$f^d(0,0,0,0,0) = \overline{f}(1,1,1,1,1)$$

$$f^d(1,1,0,0,0) = \overline{f}(0,0,1,1,1)$$

& Few more.

Boolean algebra practice 3

Find the complement of F.

$$F = (\overline{V}W + X)Y + \overline{Z}$$

$$\overline{F} = \overline{(\overline{V}W + X)Y + \overline{Z}}$$

$$\overline{((\overline{V}W + X)Y)}\,\overline{\overline{Z}} \qquad \text{(by DeMorgan's)}$$

$$(\overline{(\overline{V}W + X)} + \overline{Y})Z \qquad \text{(by DeMorgan's \& involution)}$$

$$(\overline{\overline{V}W}\,\overline{X} + \overline{Y})Z \qquad \text{(by DeMorgan's)}$$

$$((\overline{\overline{V}} + \overline{W})\overline{X} + \overline{Y})Z \qquad \text{(by DeMorgan's)}$$

$$((V + \overline{W})\overline{X} + \overline{Y})Z \qquad \text{(by null)}$$

Standard Order of Truth Table

$f(a,b) = ab$

a	b	f
1	0	0
1	1	1
0	0	0
0	1	0

→ Correct Truth Table.

→ NOT in Standard Order

$f(a,b) = ab$

T_1

a	b	f
1	0	0
1	1	1
0	0	0
0	1	0

"Standard order" of Truth Table:

Decimal value	a	b	f
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

In Standard order

$f = ab$

b	a	f
0	1	0
0	0	0
1	0	0
1	1	1

→ Correct Truth Table for f.

Truth Table :

Simply tells us, for every input Combinations, what is the output.

Standard order of a Truth Table:

Input Combinations are written in order of their Decimal Value from 0 to $2^n - 1$.

Which Truth Tables Are the Same?

1)

B	A	F
0	0	0
0	1	0
1	0	1
1	1	0

2)

A	B	F
0	0	0
0	1	0
1	0	1
1	1	0

3)

B	A	F
1	1	0
1	0	1
0	1	0
0	0	0

Which Truth Tables Are the Same?

1)

B	A	F
0	0	0
0	1	0
1	0	1
1	1	0

$F = B\overline{A}$

2)

A	B	F
0	0	0
0	1	0
1	0	1
1	1	0

$F = A\overline{B}$

$\neq$

3)

B	A	F
1	1	0
1	0	1
0	1	0
0	0	0

$F = B\overline{A}$

$1 = 3$

GO CLASSES

GO Classes

Table 2.2

Truth Tables for F_1 and F_2

Decimal value	x	y	z	F_1	F_2
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	1	1
5	1	0	1	1	1
6	1	1	0	1	0
7	1	1	1	1	0

Standard order of Truth Table.

Important Topic:

Finding Dual of a function from Truth Table

Finding Dual when Expression is Given:

Very Easy & Well Known.

Find Dual if Truth Table is Given:

for any function f :

If we have Truth Table in the "Standard Order" then To find f^d, Reverse the $\bar{f}$ Column.

Table 2.2

Truth Tables for F_1 and F_2

x	y	z	F_1	F_2
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

F_1^d F_2^d

? ?

Table 2.2
Truth Tables for F_1 and F_2

In Standard order

x	y	z	F_1	F_2	$\overline{F_1}$	F_1^d
0	0	0	0	0	1	0
0	0	1	1	1	0	0
0	1	0	0	0	1	0
0	1	1	0	1	1	0
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	1	0	0	0
1	1	1	1	0	0	1

Reverse of $\overline{F_1}$ Column

Table 2.2

Truth Tables for F_1 and F_2

x	y	z	F_1	F_2
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

$\overline{F_2}$	F_2^d
1	1
0	1
1	0
0	0
0	0
0	1
1	0
1	1

Reverse & $\overline{F_2}$ Column

Note:

n variables → 2^n Rows

let's call m.

input variables	f	$\bar{f}$	f^d
0	a_0 →	$\bar{a}_0$	$\bar{a}_m$
1	a_1 →	$\bar{a}_1$	$\bar{a}_{m-1}$
2	a_2 →	$\bar{a}_2$	⋮
3	a_3 →	$\bar{a}_3$	⋮
⋮	⋮	⋮	
$2^n = m$	a_m →	$\bar{a}_m$	$\bar{a}_0$

In standard order

Reverse of $\bar{f}$ column.

Note:

n variables $\longrightarrow$ 2^n Rows — let's call m.

input variables	f		f^d
0	a_0		$\overline{a_m}$
1	a_1		$\overline{a_{m-1}}$
2	a_2		$\vdots$
3	a_3		$\vdots$
$\vdots$	$\vdots$		
$2^n = m$	a_m		$\overline{a_0}$

In standard order

Reverse of Complement of f.

$f(a)$

$f_1^d = 1$

$f_1 = 0 \longrightarrow f_1^d = 1$

$a \longrightarrow f^d = a$

a

a	f_1	f_2	f_3	f_4	f_1^d	f_2^d	f_3^d	f_4^d
0	0	0	1	1	1	0	1	0
1	0	1	0	1	1	1	0	0

$f(a,b) \rightarrow$ Standard order Truth Table.

$f^d = a + b$

	a	b	$f = ab$	f^d
0	0	0	0	0
1	0	1	0	1
2	1	0	0	1
3	1	1	1	1

Reverse of f'

$f(a,b) \longrightarrow$ Standard order Truth Table.

	a	b	f = 0	f^d
0	0	0	0	1
1	0	1	0	1
2	1	0	0	1
3	1	1	0	1

$f^d = 1$

Reverse of $\bar{f}$

$$f = a \oplus b = \bar{a}b + a\bar{b}$$

Dual

$$f^d = (\bar{a}+b)(a+\bar{b}) = \bar{a}\bar{b} + ab$$

$$= a \odot b$$

$f(a,b) \rightarrow$ Standard order Truth Table.

$f^d = a \odot b$

	a	b	$f = a \oplus b = a\bar{b} + \bar{a}b$	f^d
0	0	0	0	1
1	0	1	1	0
2	1	0	1	0
3	1	1	0	1

Reverse of $\bar{f}$

$f(a,b) \longrightarrow$ Standard order Truth Table.

$f^d = a'b$

	a	b	$f = a' + b$	f_d
0	0	0	1	0
1	0	1	1	1
2	1	0	0	0
3	1	1	1	0

Reverse of f'

$f(a,b) \rightarrow$ Standard order Truth Table.

$f^d \rightarrow f^d = b$

	a	b	$f = b$	f^d
0	0	0	0	0
1	0	1	1	1
2	1	0	0	0
3	1	1	1	1

Reverse of f

$f(a,b) \rightarrow$ Standard order Truth Table.

$f^d = \bar{a}$

	a	b	$f = \bar{a}$	f^d
0	0	0	1	1
1	0	1	1	1
2	1	0	0	0
3	1	1	0	0

Note: for any f, if we have Truth Table in the $\boxed{\text{standard order}}$

f	$\bar{f}$	f^d
a_0	$\bar{a}_0$	$\bar{a}_m$
a_1	$\bar{a}_1$	$\bar{a}_{m-1}$
a_2	$\bar{a}_2$	$\vdots$
$\vdots$	$\vdots$	
a_m	$\bar{a}_m$	$\bar{a}_0$

Reverse of $\bar{f}$

Note:

$f(a, b, c, d)$

$f^d(0, 1, 1, 0) = \overline{f(1, 0, 0, 1)}$

$\overline{f}(1, 0, 0, 0) = f^d(0, 1, 1, 1)$

Note: f over 3 variables a, b, c

$f^d(0) = \bar{f}(7)$

Decimal value

$\bar{f}(5) = f^d(2)$

101 010

Note: f over 3 variables a, b, c

$$f^d(0) = \bar{f}(7)$$

Decimal value

$$\bar{f}(0) = f^d(7)$$

$$\bar{f}(x) = f^d(7-x)$$

Note: f over n variables → How many Rows $= 2^n$

$$f^d(\underbrace{0}_{\text{Decimal value}}) = \bar{f}(2^n - 1)$$

$0 \rightarrow$ $00\ldots0$ input Combination

$2^n - 1 \rightarrow$ $11\ldots1$ input Combination

Note: f over n variables → How many Rows $= 2^n$

$$f^d(t) = \overline{f}\big(\underbrace{(2^n - 1) - t}_{\text{Decimal value}}\big)$$

t → Decimal value

Note: f over n variables :

$$f^d(x) = \overline{f\Big(\underbrace{(2^n-1)-x}\Big)}$$

Decimal value

Relationship b/w

$f, \bar{f}$:

Note: f over n variables :

$f(x) = 1 \longrightarrow \bar{f}(x) = 0$

Decimal value

$f(x) = 0 \longrightarrow \bar{f}(x) = 1$

Relationship b/w

$f, \quad f^d$

Note: f over 3 variables :

$f(1) = 0 \longrightarrow f^d(\underline{\quad}) = ?$

Decimal value

for which combination, we can guarrantee something in f^d ?

Note: f over 3 variables :

$f(1) = 0 \longrightarrow f^{d}(6) = 1$

Decimal value

$\bar{f}(1) = 1$

Note: f over 3 variables:

$f(1) = 0 \longrightarrow f^d(6) = 1$

Decimal value

Note: f over 3 variables :

$f(3) = 0 \longrightarrow f^{d}(m) = ?$

Decimal value

Note: f over 3 variables :

$f(3) = 0 \longrightarrow f^d(4) = 1$

Decimal value

$\bar{f}(3) = 1$

Note: f over 3 variables :

$f(3) = 0 \longrightarrow f^d(4) = 1$

Decimal value

Note: f over 3 variables :

$f(5) = 1 \longrightarrow f^d(\sim) = ?$

Decimal value

Note: f over 3 variables :

$f(5) = 1 \longrightarrow f^d(2) = 0$

Decimal value

$\overline{f}(5) = 0$

Note: f over 3 variables:

$f(5) = 1 \longrightarrow f^d(2) = 0$

Decimal value

Note: f over 3 variables :

$$f(x) = 0 \longrightarrow f^{d}(7-x) = 1$$

Decimal Value

Note: f over 3 variables :

$$f(t) = 1 \longrightarrow f^{d}(7-t) = 0$$

Decimal value

Note: f over n variables :

$f(x) = 0 \longrightarrow f(2^n - 1 - x) = 1$

Decimal Value

Note: f over n variables :

$$f(x) = 1 \longrightarrow f(2^n - 1 - x) = 0$$

Decimal Value

Q: $f(a,b,c) = \sum(0, 2, 3, 4)$

$\overline{f} = ?$

$f^d = ?$

$$f(x) = 0 \longrightarrow f^{d}(2^{n}-1-x) = 1$$

Q: $f(a,b,c) = \sum(\underbrace{0, 2, 3, 4}_{})$

f = 1 for these combination

$\overline{f} = \sum(1, 5, 6, 7)$

$f^d = ? = \sum(\quad ? \quad)$

Q: $f(a,b,c) = \sum(0, 2, 3, 4)$

$f = 0$ for Combination 1, 5, 6, 7

So $f^d = 1$ for " 7-1, 7-5, 7-6, 7-7

$f^d = ? = \sum(0, 1, 2, 6)$ ✓

Special Type of Boolean Function:

Neutral Boolean Function

A boolean function f over n variables is <u>Neutral</u> iff

f = 1 for Half of input Combinations.

f = 0 " " " " " " .

f is Neutral :

Number of input Combinations for which f = 1

"

" " " " " " f = 0.

GATE CSE 1989 | Question: 4-x

asked in **Digital Logic** Nov 30, 2016 • edited Apr 16, 2021 by **Lakshman Bhaiya**

1,936 views

8

A switching function is said to be neutral if the number of input combinations for which its value is 1 is equal to the number of input combinations for which its value is 0. Compute the number of neutral switching functions of n variables (for a given n).

gate1989 descriptive digital-logic boolean-algebra

$n = 1$ $f(a)$

Neutral

a	f_1	f_2	f_3	f_4
0	0	0	1	1
1	0	1	0	1

$n = 2 \quad f(a, b)$

a	b	f
0	0	
0	1	
1	0	
1	1	

Neutral functions

for 2 input Combinations $f = 1$

for other 2, $f = 0$

$n = 2 \quad f(a, b)$

Neutral functions

a	b	f_1	f_2	f_3	f_4	f_5	f_6
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	0
1	0	1	0	0	1	1	0
1	1	1	1	0	0	0	1

$f \rightarrow$ Neutral function

a	b	f
0	0	
0	1	
1	0	
1	1	

$\rightarrow$ two 1's

two 0's

4 Rows ;

Select 2 Rows for 1's ;

Automatically Remaining two Rows 0's

$f \rightarrow$ Neutral function

a	b	f
0	0	
0	1	
1	0	
1	1	

two 1's

two 0's

4 Rows; select 2 Rows for 1's

$^4C_2 = 6$

n variables ;

$a_1, a_2 - - a_n$	f
2^n Rows	$\boxed{2^{n-1}\ 1's}$ $2^{n-1}\ 0's$

$\to$ Neutral function

Neutral functions = $\boxed{{}^{2^n}C_{2^{n-1}}}$

n Variables Neutral function:

2^n Rows; Select which Half of them Contain 1. The Remaining Half will Automatically Contain 0's.

n Variables Neutral function:

2^n Rows; Select which Half of them Contain 1.

$$\boxed{{}^{2^n}C_{2^{n-1}}}$$

Covers All possibilities.

Q: Is there any function f for which $f = \bar{f}$?

Q: Is there any function f for which $f = \bar{f}$? No

$f = \bar{f}$ Never Happen

f	$\bar{f}$
0	1
1	0
1	0
0	1
1	0
⋮	⋮

$f \neq \bar{f}$ Always

$f = \bar{f}$ Never

Q: for all functions f

$f = f^d$?

Q: for all functions f

$f = f^d$? → No

$f = a+b$

$f^d = ab$

NOT same

Q: Is there any function f for which $f = f^d$?

Q: Is there any function f for which $f = f^d$? Yes

$f = a \longrightarrow f^d = a$

$f = \overline{a} \longrightarrow f^d = \overline{a}$

$f = f^d$ → Happens for Some functions → Self Dual functions

$f = \overline{f}$ → Never Happens

Special Type of Boolean Function:

Self Dual Function

Special Type of Boolean Function:

Self Dual Function f

f is selfDual iff $f = f^d$

$f(a)$

a $\quad$ $\bar{a}$

self Dual

a	f_1	f_2	f_3	f_4
0	0	0	1	1
1	0	1	0	1

$f_1 = 0$

$f_1^d = 1$

$f_4 = 1$

$f_4^d = 0$

$f(a,b)$

In standard order

a	b	f	$\overline{f}$	f^d
0	0	a_0	$\overline{a_0}$	$\overline{a_3}$
0	1	a_1	$\overline{a_1}$	$\overline{a_2}$
1	0	a_2	$\overline{a_2}$	$\overline{a_1}$
1	1	a_3	$\overline{a_3}$	$\overline{a_0}$

$f(a, b)$

a	b	f	f^d
0	0	a_0	$\overline{a_3}$
0	1	a_1	$\overline{a_2}$
1	0	a_2	$\overline{a_1}$
1	1	a_3	$\overline{a_0}$

Self Dual f

$f = f^d$

$a_0 = \overline{a_3}$

$a_1 = \overline{a_2}$

$a_2 = \overline{a_1}$

$a_3 = \overline{a_0}$

$f(a, b)$

$a_i \rightarrow 0$, $a_i \rightarrow 1$

Self Dual $f \longrightarrow f = f^d$

$a_0 = \overline{a_3} \iff a_0 \neq a_3$

$a_1 = \overline{a_2} \iff a_1 \neq a_2$

$a_2 = \overline{a_1} \iff a_2 \neq a_1$

$a_3 = \overline{a_0} \iff a_3 \neq a_0$

Self Dual f:

a	b	$f = f^d$	f
0	0	$a_0 = \overline{a_3}$	a_0
0	1	$a_1 = \overline{a_2}$	a_1
1	0	$a_2 = \overline{a_1}$	a_2
1	1	$a_3 = \overline{a_0}$	a_3

Diff.

Different

Self Dual f:

a	b	f	f^d
0	0	a_0	$\overline{a_3}$
0	1	a_1	$\overline{a_2}$
1	0	a_2	$\overline{a_1}$
1	1	a_3	$\overline{a_0}$

f is self Dual

$a_0 \neq a_3$

$a_1 \neq a_2$ &

for 3 variables:

	a	b	c	$f = f_d$
0	0	0	0	a_0
1	0	0	1	a_1
2	0	1	0	a_2
3	0	1	1	a_3
4	1	0	0	a_4
5	1	0	1	a_5
6	1	1	0	a_6
7	1	1	1	a_7

$\neq$ $\neq$ $\neq$ Diff

Every self Dual function is Neutral ? $\longrightarrow$ YES

Every Neutral fun. is self Dual ? No.

a	b	f
0	0	1
0	1	0
1	0	0
1	1	1

Neutral ✓

~~NOT self Dual~~

In self Dual, they must be Different.

How many self Dual functions ?

a	b	f
0	0	0
0	1	1
1	0	0
1	1	1

Automatically

How many self Dual functions ?

a	b	f
0	0	a_0?
0	1	a_1?
1	0	$\overline{a_1}$
1	1	$\overline{a_0}$

$a_0 \rightarrow$ 2 choices

$a_1 \rightarrow$ 2 choices

$\overline{a_1} \rightarrow$ 1 choice

$\overline{a_0} \rightarrow$ 1 choice

$2 \times 2 = 4$ ✓

How many self Dual functions ? → $2 \times 2 \times \dots \times 2$ (2^{n-1} times)

a_1	a_2	a_3	---	a_n	$f = f_d$
0	0	0	- -	0	→ 2 choices
					→ 2 choices
1	0	0	- - -	0	→ 1 choice (No choice)
1	1	- - -		1	→ 1 choice (No choice)

$\rightarrow 2^{n-1}$ Rows

number of self Dual functions

on n-variables :

$$2^{\left(2^{n-1}\right)}$$

Reason: Each Row in first half of Truth Table has 2 choices.

Note: In Self Dual function f:

$$f(x) \neq f(2^n - 1 - x)$$

Decimal value

$\forall$ $0 \leq x \leq 2^n - 1$

$f(a,b,c) \longrightarrow$ Self Dual :

$f(0) \neq f(7)$

$f(1) \neq f(6)$

$f(2) \neq f(5)$

$f(3) \neq f(4)$

$f(x) \neq f(2^n - 1 - x)$

Q: $f = \sum(0, 2, 3, 4)$

Is f self Dual ?

Q: $f = \sum(0, 2, 3, 4)$

NOT a self Dual fun.

Is f self Dual ?

$f = 1$ for these.

$f(3) \neq f(7-3)$

$f(3) \neq f(4)$

Q: $f = \sum(0, 2, 3, 6)$

Is f self Dual ?

Q: $f = \Sigma(0, 2, 3, 6)$ → Yes.

Is f self Dual ? → Yes ✓

for these, f = 1

$f(0) = 1 \quad f(7) = 0$

$f(2) = 1 \quad f(5) = 0$

$f(3) = 1 \quad f(4) = 0$

$f(6) = 1 \quad f(1) = 0$

Note:

f is Self Dual iff

$f(x) \neq f(2^n - 1 - x) \quad \forall\, 0 \le x \le 2^n - 1$

Decimal value

Special Type of Boolean Function:

Functions whose Complement is Same as their Dual functions f for which

$$\boxed{\bar{f} = f^d}$$

→ No Standard Name for these functions.

Functions whose Complement is Same as their Dual:

$\bar{f} = f^d$

Let's first discuss the **Misconceptions** Around Such Function functions f which $\overline{f} = f^d$:

Misconceptions for these functions:

(1) such functions are called Orthogonal.

Not a standard word.

(2) These functions are Neutral.

(3) # such functions $= {}^{2^{n-1}}C_{2^{n-2}}$

functions f which $\bar{f} = f^d$:

for these functions :

(1) such functions are called Orthogonal

(2) These functions are Neutral.

(3) # such functions = $^{2^{n-1}}C_{2^{n-2}}$

WRONG

Part 2.17 - Orthogonal function

[redacted]

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$\bar{f} = f^d$

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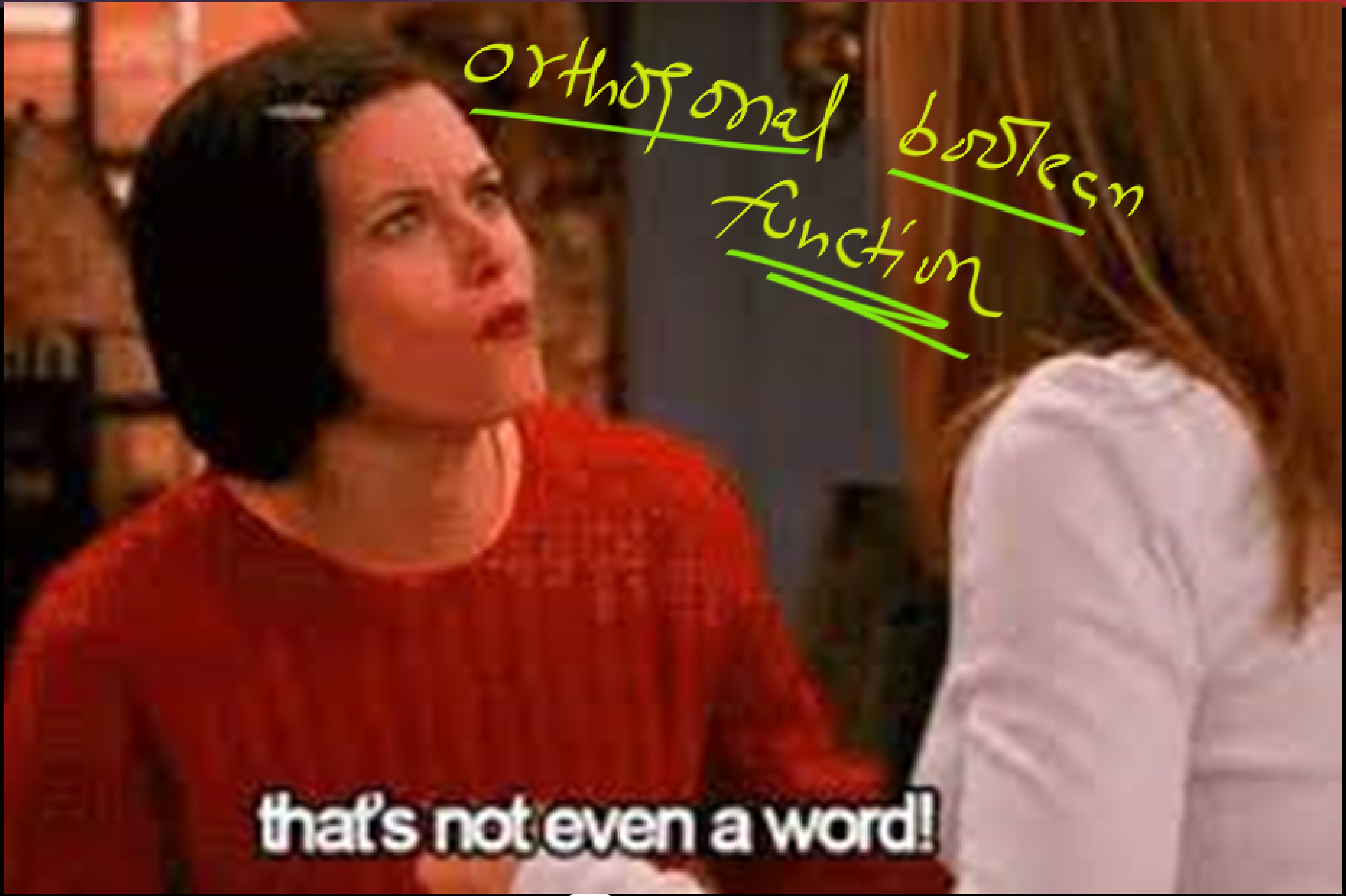
Entire Video

" Analysis is WRONG.


orthogonal boolean function
that's not even a word!

Functions whose Complement is Same as their Dual:

Analysis with Proof

functions f for which $\overline{f} = f^d$

No standard Name for such functions

GO function

functions f for which $\bar{f} = f^d$

$f \rightarrow a \oplus b$

a	b	f	$\bar{f}$	f^d
0	0	0	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	1	1

$\bar{f} = f^d$

So $\oplus$ is GO function.

functions f for which $\bar{f} = f^d$

Q: Such function is Neutral ?

functions f for which $\bar{f} = f^d$

Q: Such function is Neutral? No

a	b	f	$\bar{f}$	f^d
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	0	1	1

$\bar{f} = f^d$

but f is NOT Neutral.

How many GO functions over a, b ?

a	b	f	$\bar{f} = f^d$
0	0	a_0	$\bar{a}_0 = \bar{a}_3$
0	1	a_1	$\bar{a}_1 = \bar{a}_2$
1	0	a_2	$\bar{a}_2 = \bar{a}_1$
1	1	a_3	$\bar{a}_3 = \bar{a}_0$

$\bar{a}_0 = \bar{a}_3$

$a_0 = a_3$

$\bar{a}_1 = \bar{a}_2$

$a_1 = a_2$

How many GO functions over a, b ?

a	b	f	$\overline{f} = f^d$	f
0	0	a_0	$\overline{a_0} = \overline{a_3}$	a_0
0	1	a_1	$\overline{a_1} = \overline{a_2}$	a_1
1	0	a_2	$\overline{a_2} = \overline{a_1}$	a_2
1	1	a_3	$\overline{a_3} = \overline{a_0}$	a_3

=

=

How many GO functions over a, b ?

a	b	Go fun f	f_1	f_2	f_3	f_4
0	0	a_0	0	0	1	1
0	1	a_1	0	1	0	1
1	0	a_1	0	1	0	1
1	1	a_0	0	0	1	1

4 Go functions

GO functions on 2 variables: 4

formula: $^{2^{n-1}}C_{2^{n-2}} = {}^{2^1}C_{2^0} = {}^{2}C_{1} = 2$

WRONG

Go functions over 3 Variables :

a	b	c	Go fun f	$\bar{f} = f^d$
0	0	0	a_0	$\overline{a_0} = \overline{a_7}$
0	0	1	a_1	$\overline{a_1} = \overline{a_6}$
0	1	0	a_2	$\overline{a_2} = \overline{a_5}$
0	1	1	a_3	$\overline{a_3} = \overline{a_4}$
1	0	0	a_4	$\vdots \quad \vdots$
1	0	1	a_5	
1	1	0	a_6	
1	1	1	a_7	$\overline{a_7} = \overline{a_0}$

Go functions over 3 Variables :

a	b	c	Go fun f	$\bar{f} = f^d$
0	0	0	a_0	$\overline{a_0} = \overline{a_7}$
0	0	1	a_1	$\overline{a_1} = \overline{a_6}$
0	1	0	a_2	$\overline{a_2} = \overline{a_5}$
0	1	1	a_3	$\overline{a_3} = \overline{a_4}$
1	0	0	a_4	⋮
1	0	1	a_5	⋮
1	1	0	a_6	
1	1	1	a_7	$\overline{a_7} = \overline{a_0}$

$a_0 = a_7$

$a_1 = a_6$

$a_2 = a_5$

$a_3 = a_4$

Go functions over 3 Variables :

a	b	c	Go fun f
0	0	0	a_0
0	0	1	a_1
0	1	0	a_2
0	1	1	a_3
1	0	0	a_4
1	0	1	a_5
1	1	0	a_6
1	1	1	a_7

=

=

=

same

GO functions over 3 Variables :

GO fun = Palindrom function

a	b	c	f
0	0	0	a_0
0	0	1	a_1
0	1	0	a_2
0	1	1	a_3
1	0	0	a_3
1	0	1	a_2
1	1	0	a_1
1	1	1	a_0

a_0 to a_3 → 2-2 choices each

a_3 to a_0 (lower rows) → No choice

Go functions over 3 Variables :

8 length palindrom bit strings

$= 2^4$ ✓

Go functions over n variables :

$a_1 \; a_2 \; \cdots \; a_n$	f
0	a_0
1	a_1
2	$\vdots$
$\vdots$	$\vdots$
$\vdots$	a_1
$2^n - 1$	a_0

same

same

Go functions over n variables :

$$= 2^{\left(2^{n-1}\right)}$$ ✓

number of 2^n length palindrome bit strings

$$= 2^{\left(2^{n-1}\right)}$$ ✓

Go functions $\equiv$ Palindrom function

$$f(x) = f(\underbrace{2^n - 1 - x}) $$

Decimal value

Go functions $\equiv$ Palindrom function

$$f(a,b,c) = f(\bar{a},\bar{b},\bar{c})$$

$$f(0,0,0) = f(1,1,1)$$

$$f(0,1,0) = f(1,0,1)$$

$$f(1,1,0) = f(0,0,1)$$

Go functions $\equiv$ Palindrom function

$$\equiv f(a_1, a_2, \ldots, a_n) = f(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n)$$

#Go functions $= 2^{\left(2^{n-1}\right)}$ ✓