Addition, Subtraction of Signed Binary Numbers

Digital Logic :

Next Topic :

Addition of two numbers in the "signed-magnitude system"

Signed

$$(+11) + (-13) = -(13 - 11)$$

$$(-11) + (-17) = -(11 + 17)$$

$$(+11) + (+17) = +(11 + 17)$$

$$(-11) + (+17) = +(17 - 11)$$

Sign-magnitude Addition of signed No.

= Decimal Addition of signed Numbers

$(+70) + (-78) = -(78-70) = -8$

$(-70) + (+78) = +(78-70) = +8$

$(+70) + (+78) = +(78+70) = +148$

$$(-70) \overset{\checkmark}{+} (-78) = -(70+78) = -148$$

$$(+m) + (+N) = +(m+N)$$

$$(-m) + (-N) = -(m+N)$$

$$(+m) + (-N) = +(m-N) \quad N<m$$

$$(+m) + (-N) = -(N-m) \quad N>m$$

Arithmetic Addition

The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude. For example, $(+25) + (-37) = -(37 - 25) = -12$ is done by subtracting the smaller magnitude, 25, from the larger magnitude, 37, and appending the sign of 37 to the result. This is a process that requires a comparison of the signs and magnitudes and then performing either addition or subtraction. The same procedure applies to binary numbers

In sign-mag;

$M = \underline{\underline{1}}\,\underbrace{011}$ $\quad (-3)$

$N = \underline{\underline{0}}\,\underbrace{111}$ $\quad + (+7)$

$\underline{\underline{1}}\,\underbrace{0\ 1\ 1}_{\text{magnitude}} + \underline{\underline{0}}\,\underbrace{1\ 1\ 1}$ → bigger magnitude

Result: $\underset{\text{sign}}{\underline{\underline{0}}}\ \underline{1}\ \underline{0}\ \underline{0} \Rightarrow +4$

① Comparison of sign bit

② Comparison of magnitude

$$\begin{array}{r} 1\,1\,1 \\ -\,0\,1\,1 \\ \hline 1\,0\,0 \\ \hline \end{array}$$

111 — bigger mag

011 — smaller mag

M 1 0 1 1 — (−3)

\+ +

N 1 1 1 1 — (−7)

Result : 1 1010 ⇒ −10

Sign

① Compare Sign bit

both Negative

$$\begin{array}{r} 011 \\ +\ 111 \\ \hline 1010 \end{array}$$

M 0 0 1 1 —— (+3)
+ +
N 0 1 1 1 —— (+7)

Result: 0 1010 ⇒ +10

sign

① Compare sign bit

both positive

$$\begin{array}{r} 011 \\ +\ 111 \\ \hline 1010 \end{array}$$

m 1 1 1 1 → −7

N 0 1 0 1 → +5

Result: 1 010 → −2

sign

① Comparison of sign bit

one +ve, one −ve

② Comparison of magnitude

7, 5

$$\begin{array}{r} 111 \\ -\ 101 \\ \hline 010 \end{array}$$

In Sign magnitude System :

All Signed numbers ⇒ Represented in Sign-mag.

Arithmatic of Signed Numbers :

Input ⟶ Signed magnitude
output ⟶

Subtraction of Signed numbers in SM System:

$(A) - (B)$

$\Downarrow$

$A + (-B)$

$(+10) - (+5) = (+10) + (-5)$

$(-10) - (-5) = (-10) + (+5)$

In Sign-magnitude System:

for Addition of two Signed Numbers:

Cost: 1 Comparison of sign bits
2 If sign bits Different then Comparison of magnitude

Cost:

3 If signs are same then we do Addition

Adder Circuit

Else if signs are Different then we do Subtraction of magnitudes

Subtracter Circuit

Cost : Comparison $O(n)$ time

Hardware for Addition

" Subtraction

Too much

Sign-mag

$$(\underline{0}\,1010) + (\underline{1}\,1011)$$

magnitude

Decimal

$$(\underline{+}\,10) + (\underline{-}\,11)$$

Digital Logic :

Next Topic :

Addition of two numbers in the

$\boxed{\text{2's Complement system}}$

2's Complement System :

① All Signed numbers in 2's Comp Rep.

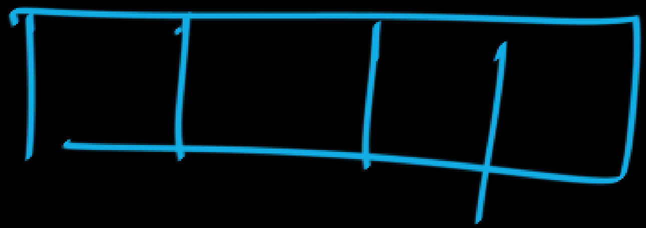
Ex: Signed int a = −6

1	0	1	0

Unsigned int a = 10

1	0	1	0

$\boxed{\text{Signed int } \underline{a = 10}}$



Invalid

using 4-bits:

$$\boxed{-8 \longrightarrow +7}$$

(2) Signed Arithmatic in 2's Comp System:

input, output in 2's Comp Rep.

Eg: 5 bits:

input

$(+5) + (-4)$

$00101 + 11100$

$$\begin{array}{r} 00101 \\ 11100 \\ \hline 1\,\boxed{00001} \\ \hline \end{array}$$

Dis-card

$\rightarrow 1$ ✓

$(+5) + (-6) = -1$

I   II

00101 + 11010

$$
\begin{array}{r}
00101 \\
11010 \\
\hline
11111 \rightarrow -1 \\
\hline
\end{array}
$$

$(-5) + (-4) = -9$

$\overline{\overline{\pi}}$ $\overline{\overline{\pi}}$

11011 11100

$$
\begin{array}{r}
11011 \\
11100 \\
\hline
1\,\boxed{10111}
\end{array}
\rightarrow -9
$$

Discard

[redacted] the rule for adding numbers in the signed-complement system does not require a comparison or subtraction, but only addition. The procedure is very simple and can be stated as follows for binary numbers:

The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits. A carry out of the sign-bit position is discarded.

$(+5) + (-4)$ using 5-bits

$00101 + 11100$

$$\begin{array}{r} 00101 \\ 11100 \\ \hline 1\ \boxed{00001} \end{array}$$

Discarded ← 1

$00001 \rightarrow 1$ ✓

in 2's comp

2's Complement System for Signed numbers:

$$\underbrace{(\pm m)}_{\substack{\downarrow \\ \text{in 2's Comp Rep}}} \pm \underbrace{(\pm N)}_{\substack{\downarrow \\ \text{in 2's Comp Rep}}} = R \Rightarrow \text{also in 2's Comp Rep}$$

Numerical examples for addition follow:

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+19	00010011	+ 7	00000111

+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	−19	11101101

Note that negative numbers must be initially in 2's-complement form and that if the sum obtained after the addition is negative, it is in 2's-complement form. For example, −7 is represented as 11111001, which is the 2s complement of +7.

In each of the four cases, the operation performed is addition with the sign bit included. Any carry out of the sign-bit position is discarded, and negative results are automatically in 2's-complement form.

Eg: $(+9) + (-12)$ using 6 bits

$001001 + 110100$

$$\begin{array}{r} 001001 \\ 110100 \\ \hline \boxed{111101} \end{array}$$

in 2's comp Rep

$\to -3$ ✓

The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits. A carry out of the sign-bit position is discarded.

Numerical examples for addition follow:

$$
\begin{array}{rr}
+\ 6 & 00000110 \\
+13 & 00001101 \\
\hline
+19 & 00010011
\end{array}
\qquad
\begin{array}{rr}
-\ 6 & 11111010 \\
+13 & 00001101 \\
\hline
+\ 7 & 00000111
\end{array}
$$

$$
\begin{array}{rr}
+\ 6 & 00000110 \\
-13 & 11110011 \\
\hline
-\ 7 & 11111001
\end{array}
\qquad
\begin{array}{rr}
-\ 6 & 11111010 \\
-13 & 11110011 \\
\hline
-19 & 11101101
\end{array}
$$

Digital Logic :

Next Topic :

Addition of two numbers in the

1's Complement system

input
output
1's Comp Representation

1's Comp Addition ⇒ we Add End Carry to Result to get final Result.

5 bits:

$$(+5) + (+4) = 9$$

$$\text{II} \qquad \text{II}$$

00101 00100

$$\begin{array}{r} 00101 \\ 00100 \\ \hline 01001 \end{array} \rightarrow 9$$

5 bits:

$(+5) + (-4)$

$+4 = 00100$

00101 11011

00101
11011

1 | 00000
→ +1

00001

5 bits:

$(-5) + (+4)$

$+5 = 00101$

11010 00100

is in 1's comp Rep.

11010
00100

11110 $\rightarrow -(1)$

5 bits:

$(-5) + (-4)$

11010 11011

$$
\begin{array}{r}
11010 \\
11011 \\
\hline
1\ \boxed{10101} \\
+1 \\
\hline
\end{array}
$$

$\rightarrow$ 10110 $\rightarrow$ -9