Set Theory

Functions

Very Special types of relations

Website : https://www.goclasses.in/

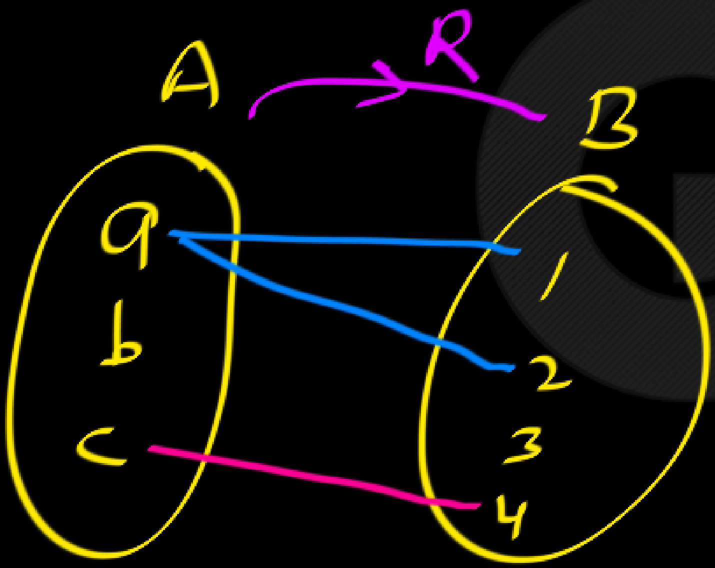
Functions

Objectives:

1. Determine whether a relationship is a function or not
2. Determine the domain, co-domain, range of a function, and the inverse image of x
3. Prove or disprove whether a function is one-to-one or not
4. Determine whether a function is onto or not
5. Determine the inverse of a one-to-one correspondence
6. Determine the composition of two functions
7. Show that if two functions are one-to-one (onto) so too is their composition

Relation from set A to set B ;

$R ; A \longrightarrow B$



$\boxed{R \subseteq A \times B}$

$a \longrightarrow \{1, 2\}$

$b \not\longrightarrow$

$c \longrightarrow 4$

To How many "b" is Related ?

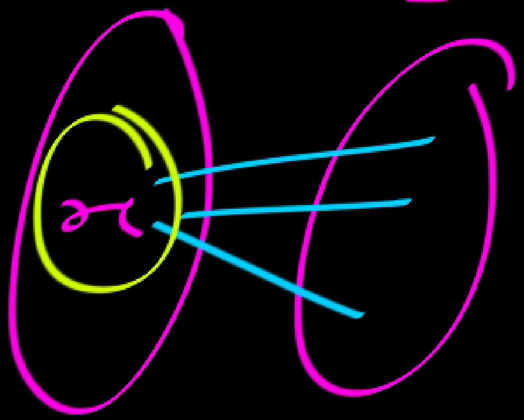
0

$|A| = 5, \quad |B| = 10$

Relation $R : S \to B$

Q: $x \in A$ then $\left|\underbrace{\{y \mid \underline{xRy}, y \in B\}}_{\text{set of those elements of B to which } x \text{ is Related.}}\right| = ?$ $\Rightarrow$ 0 to 10

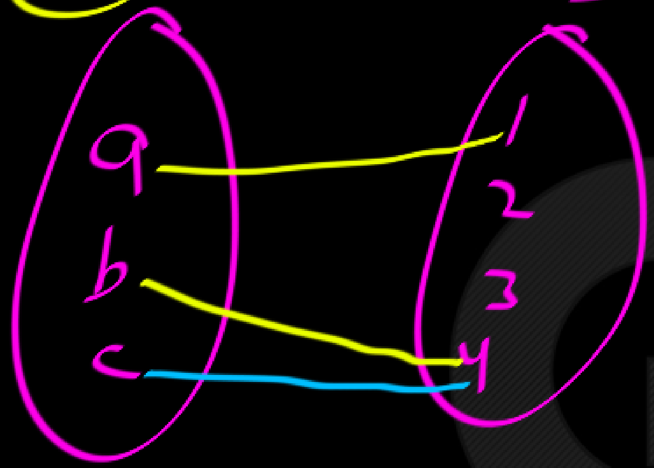
$A \xrightarrow{R} B$



A binary relation R from A to B is said to be a *function* if for every element a in A, there is a unique element b in B so that (a, b) is in R. For a function R from A to B, instead of writing $(a, b) \in R$, we also use the notation $R(a) = b$, where b is called the *image* of a. The set A is called the *domain* of the function R, and the set B is called the *range* of the function R. [redacted]

function from A to B is a Relation in which "EVERY" element of A (individually) Related to "EXACTLY one element" of B.

$A \xrightarrow{f_1} B$ — function

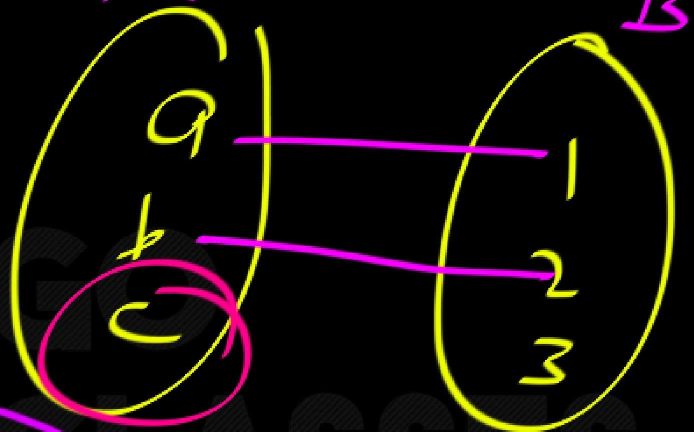


$a \longrightarrow 1$

$b \longrightarrow 4$

$c \longrightarrow 4$

$A \xrightarrow{f_2} B$ — Relation But not function

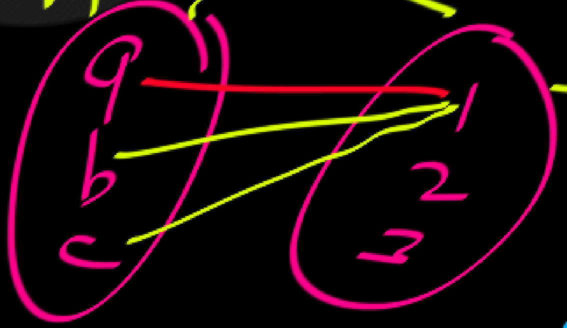


$a \longrightarrow 1$

$b \longrightarrow 2$

$c \not\longrightarrow$

$A \xrightarrow{f} B$ — function



$a \rightarrow 1$

$b \rightarrow 1$

$c \rightarrow 1$

$f : A \rightarrow B$

Domain

Co-Domain

function / mapping / Transformation

Relation:

$R : A \rightarrow B$

Domain of Relation R

CoDomain of R

Relation on set A : $R : A \rightarrow A$

Definitions

Let A and B be sets.

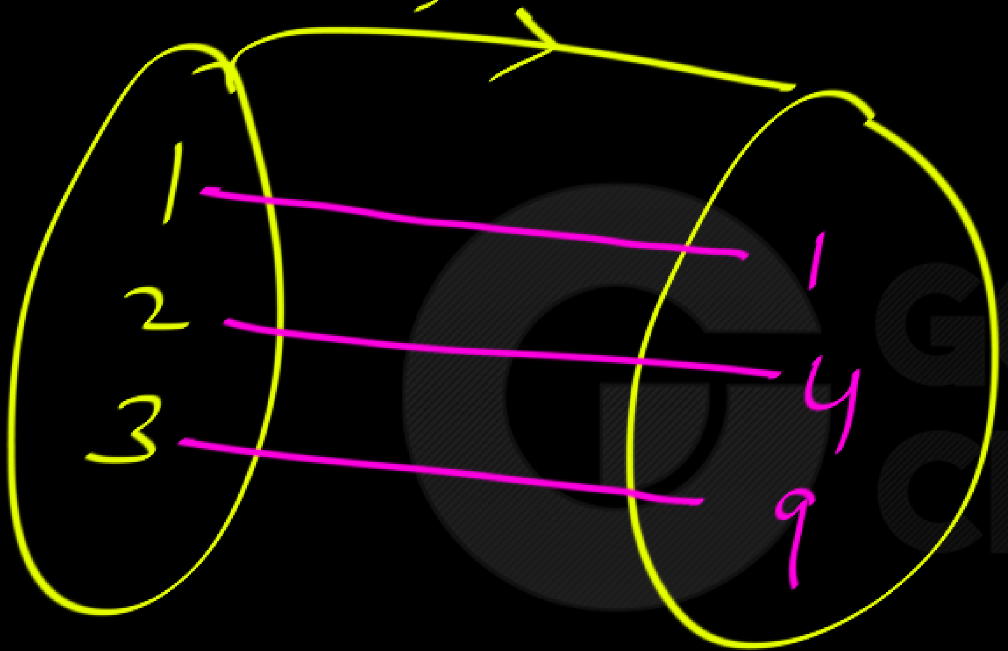
- A **function** f from A to B is an assigment of exactly one element of B to each element of A. *Note: exactly one element of* B. We can write this as $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A. We write $f : A \to B$.

  Given an input element a in A, there is a unique output element b in B that is related to a by f.

- A is the **domain** of f and B is the **co-domain** of f.
- If $f(a) = b$, we say that b is the **image** of a and a is a **pre-image**, or **inverse image**, of b.
- The **range** of f is the set of all images of elements of A.

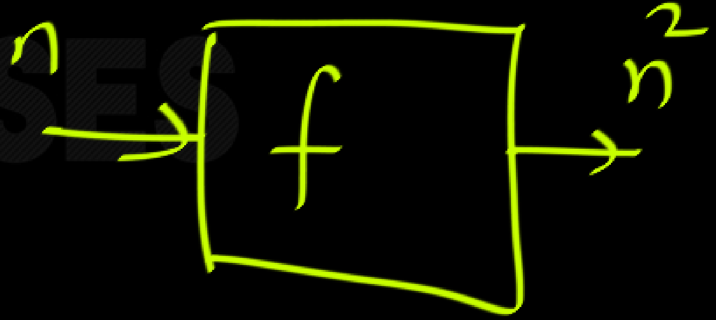
We say that a function "maps" one set to another.

$f(n) = n^2$



$f(n) = n^2$

Transformation



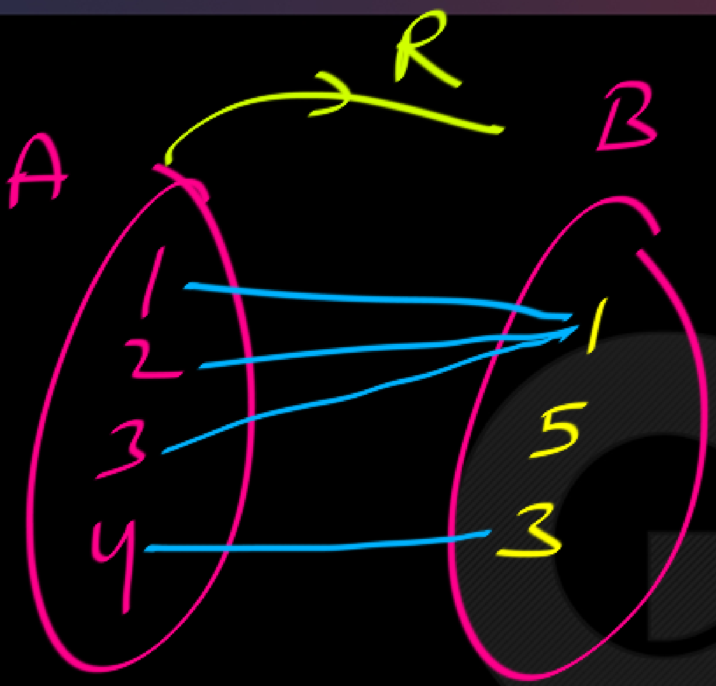
$f(1) = 1^2 = 1$; $f(2) = 2^2 = 4$; $f(3) = 3^2 = 9$

function f is like a Transformation.

input $\quad$ output

$$a \longrightarrow \boxed{f} \longrightarrow b$$

$$f(a) = b$$



$R : A \rightarrow B$

A → Domain

B → CoDomain

$1 \rightarrow \boxed{R} \rightarrow 1$

$4 \rightarrow \boxed{R} \rightarrow 3$

$R_{(4)} = 3$

$R_{(2)} = 1$

$R_{(1)} = 1 ; \ R_{(3)} = 1$

R : function

image of 4 = 3

$a \rightarrow \boxed{\text{function } f} \rightarrow b$

$a \in$ Domain

$b \in$ Co-Domain

Pre-image of b = a

$\boxed{f(a) = b}$

f of a is 1

Image of a is 1 (under f)

Image of "a" (under f) is b.

$f(a) = b$

Pre-image : $f: A \to B$

Pre-image of x (under f) = _ _ _ _

$x \in$ Co-Domain

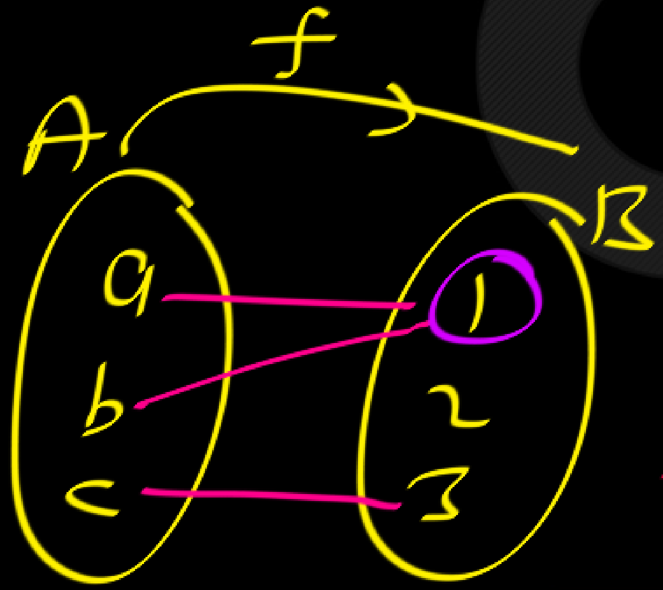


image of $a = 1$ ✓

Preimage of $1 = \{a, b\}$

$\boxed{\text{image of } 3 = \text{Nonsense}}$

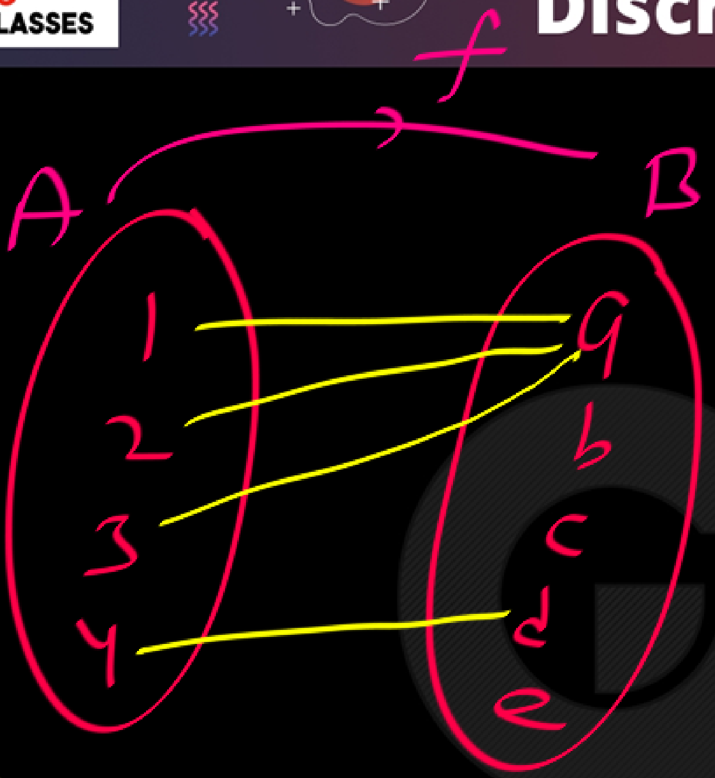


image of 1 = a

image of 4 = d

Preimage(a) = {1, 2, 3}

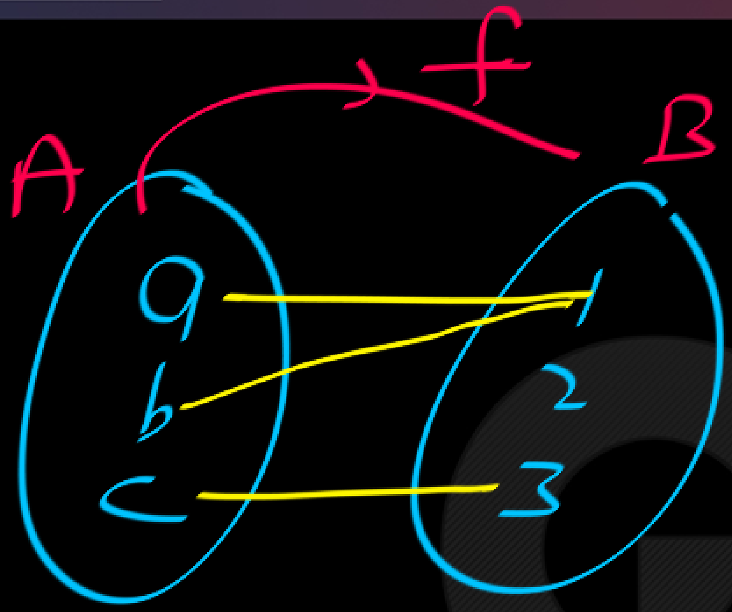
Preimage(b) = ϕ = None

Preimage(d) = {4}

Preimage(e) = ϕ = None

$f(1) = a$ $\quad f(4) = d$

$f(2) = a$



~~image of a = {1}~~

image of a = 1 ✓

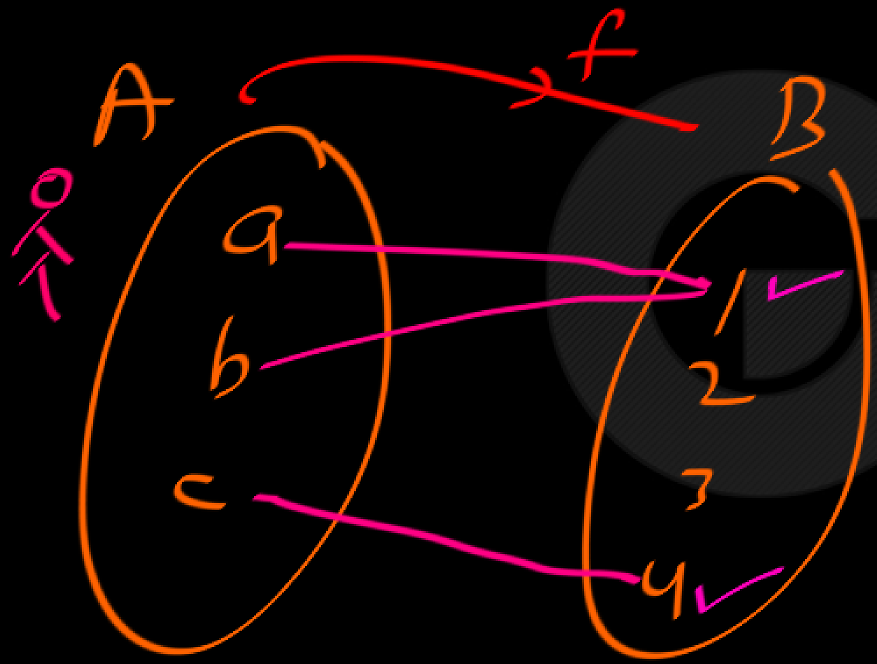
$f(c) = 3$

$f(b) = 2$

Pre-image of 1 = $\{a, b\}$

" " 2 = ϕ

"Range" of a function :



Range ≡ Reach

Which elements of Co-Domain are Reachable from Domain.

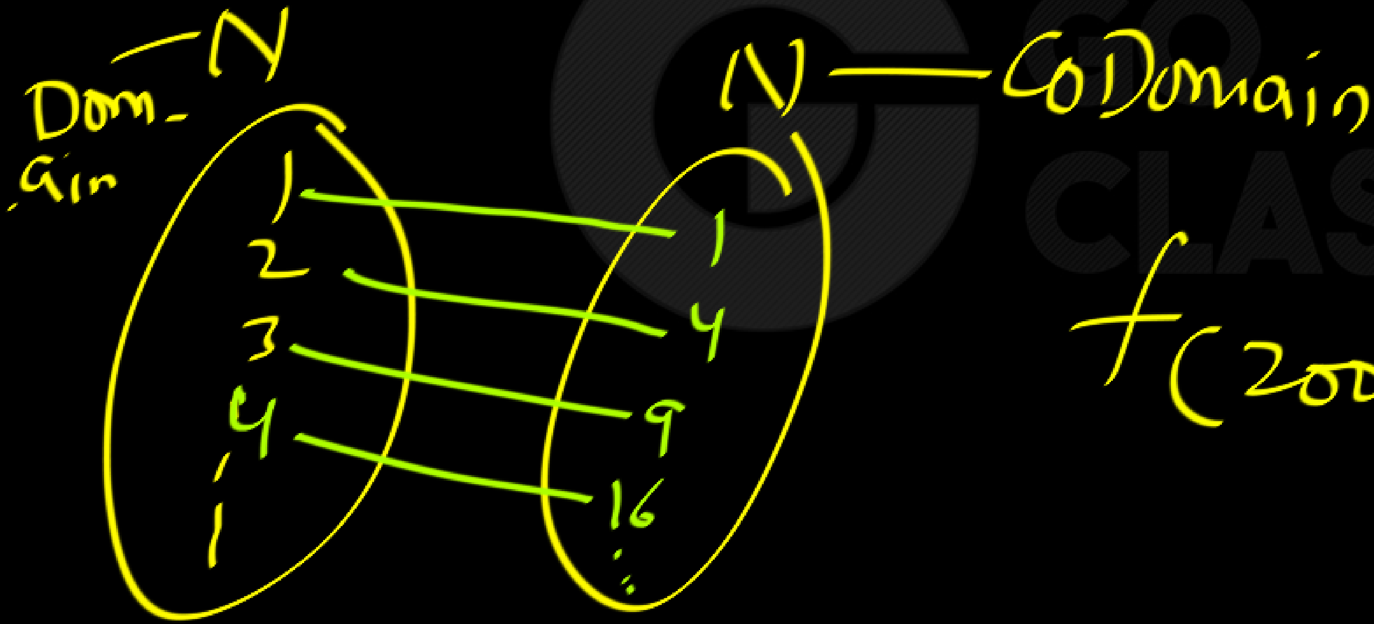
$\text{Range}(f) = \{1, 4\}$

$$\text{Range} = \left\{ y \;\middle|\; \begin{array}{l} y \in \text{Co-Domain}; \\ \text{Pre-image}(y) \text{ is Non-Empty} \end{array} \right\}$$

$$\text{Range}(f) = \left\{ y \;\middle|\; \begin{array}{l} y \in \text{Co-Domain}; \\ \exists_{x \in \text{Domain}},\ f(x) = y \end{array} \right\}$$

Q: $f : N \rightarrow N$; $f(x) = x^2$

Q: Is f a function? Yes.



$f(200) = 40000$

$Range(f) = \{1, 2^2, 3^2, 4^2, 5^2, 6^2, ---\}$

$7 \notin Range(f)$ ✓

$9 \in Range(f)$ ✓

$8 \in Range(f)$ ✗

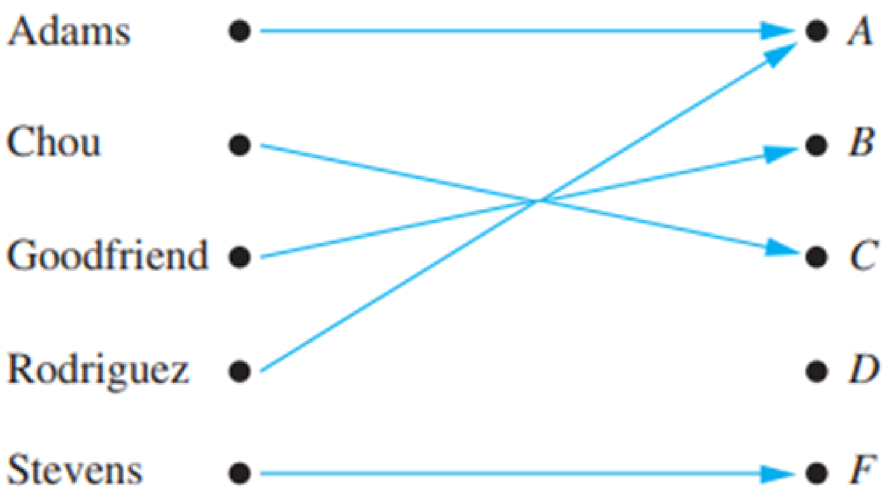


FIGURE 1 **Assignment of Grades in a Discrete Mathematics Class.**

Remark: Functions are sometimes also called **mappings** or **transformations**.

Function Requirements

There are rules for functions to be well defined, or correct.

- No element of the domain must be left unmapped.
- No element of the domain may map to more than one element of the co-domain.

It is allowable, however, for elements of the co-domain to be unmapped or to have multiple elements from the domain map to single elements in the co-domain.

So basically what we are saying is that every element in the domain has to be mapped to an output, and there can be only one output for every input. No input can have to possible outputs.

When we define a function we specify its domain, its codomain, and the mapping of elements of the domain to elements in the codomain. Two functions are **equal** when they have the same domain, have the same codomain, and map each element of their common domain to the same element in their common codomain. Note that if we change either the domain or the codomain

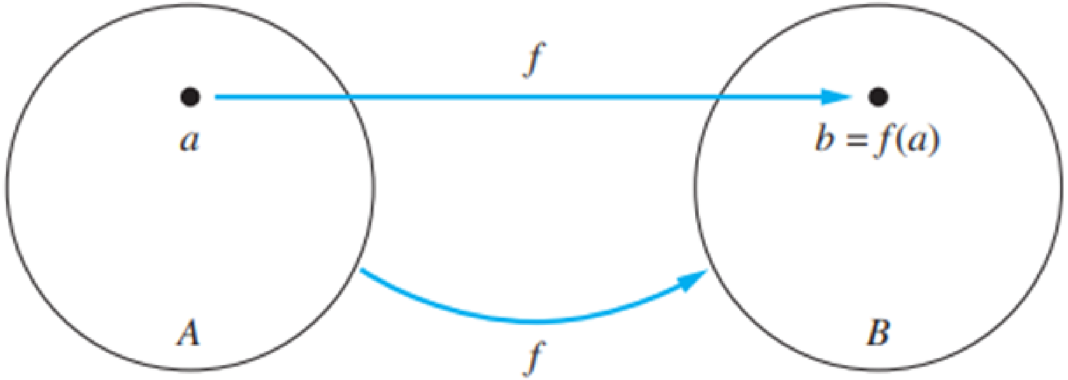


FIGURE 2 The Function f Maps A to B.

of a function, then we obtain a different function. If we change the mapping of elements, then we also obtain a different function.

Let f be the function that assigns the last two bits of a bit string of length 2 or greater to that string. For example, $f(11010) = 10$. Then, the domain of f is the set of all bit strings of length 2 or greater, and both the codomain and range are the set $\{00, 01, 10, 11\}$.

f: set of strings of length ≥ 2 → Co-Domain

Domain

Range $= \{00, 01, 10, 11\}$

$f(1000101) = 01$

$f(0001000) = 00$

$f(x) = x^2$

1 $\mathbb{Z} \longrightarrow \mathbb{Z}$ ✓

$f(0) = 0$; $f(-4) = 16$; $f(4) = 16$

Range $= \{0, 1, 2^2, 3^2, 4^2, ---\}$

$f(x) = x^2$

$N = \{1, 2, 3, \ldots\}$ given

① $Z \longrightarrow N$ ✗

Not a function

"0" has No image.

Let $f: \mathbf{Z} \rightarrow \mathbf{Z}$ assign the square of an integer to this integer. Then, $f(x) = x^2$, where the domain of f is the set of all integers, the codomain of f is the set of all integers, and the range of f is the set of all integers that are perfect squares, namely, $\{0, 1, 4, 9, \ldots\}$. ◀

Keneth Rosen:

1. Why is f not a function from $\mathbf{R}$ to $\mathbf{R}$ if

a) $f(x) = 1/x$?

b) $f(x) = \sqrt{x}$?

c) $f(x) = \pm\sqrt{(x^2 + 1)}$?

Keneth Rosen:

1. Why is f not a function from $\mathbf{R}$ to $\mathbf{R}$ if

a) $f(x) = 1/x$? — No image for 0

b) $f(x) = \sqrt{x}$? — No image for Negative numbers

c) $f(x) = \pm\sqrt{(x^2+1)}$? — every element has two images.

image of $-1 = \sqrt{-1} \notin R$

$\sqrt{-1} = i$

$f : \mathbb{R} \rightarrow \mathbb{R}$; $f(x) = \frac{1}{x}$

f is not a function because

1. there is No mapping for ∞
2. " " " " " $-\infty$
3. " " " " " π
4. " " " " " 0

2. Determine whether f is a function from $\mathbf{Z}$ to $\mathbf{R}$ if

a) $f(n) = \pm n$.

b) $f(n) = \sqrt{n^2 + 1}$.

c) $f(n) = 1/(n^2 - 4)$.

2. Determine whether f is a function from **Z** to **R** if

× **a)** $f(n) = \pm n$. $f(1) = -1, 1$

✓ **b)** $f(n) = \sqrt{n^2 + 1}$. — $f(-5) = \sqrt{26}$

× **c)** $f(n) = 1/(n^2 - 4)$. No image of 2

$\sqrt{25} = 5$ ✓

$= \pm 5$ ×

Note: $\sqrt{}$ is a function from $\mathbb{N}$ to $\mathbb{R}$.

$\sqrt{25} = 5$

$x^2 = a$ then $x = \pm\sqrt{a}$

$x^2 = 25$ then $x = \pm 5$

$x^2 = a^2$ then $x = \pm a$

Write down all the functions from the two-element set $\{1, 2\}$ to the two-element set $\{a, b\}$.

$2^2 = 4$

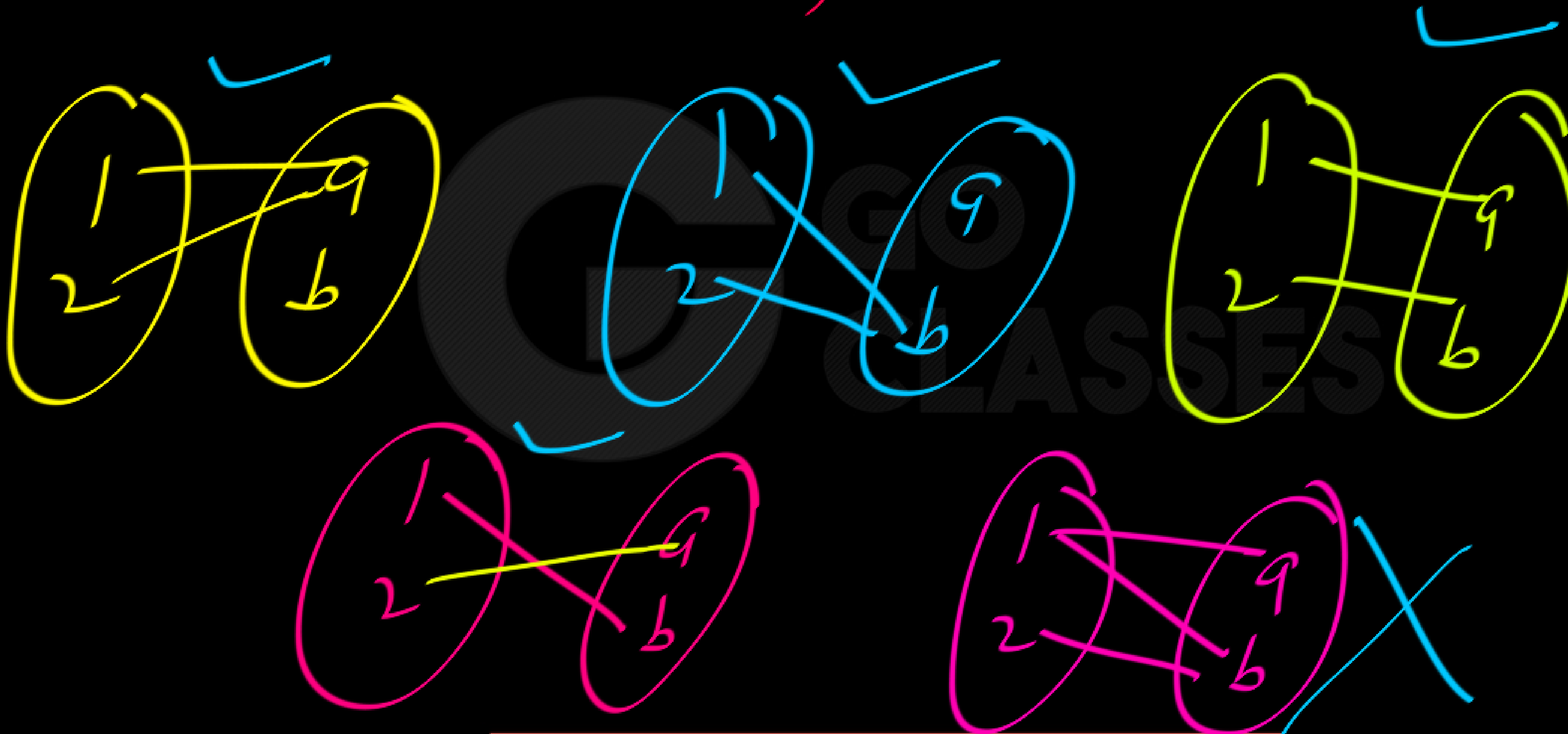
How many functions are there from a two-element set to a three-element set?

$3^2 = 9$

How many functions are there from a three-element set to a two-element set?

$2^3 = 8$

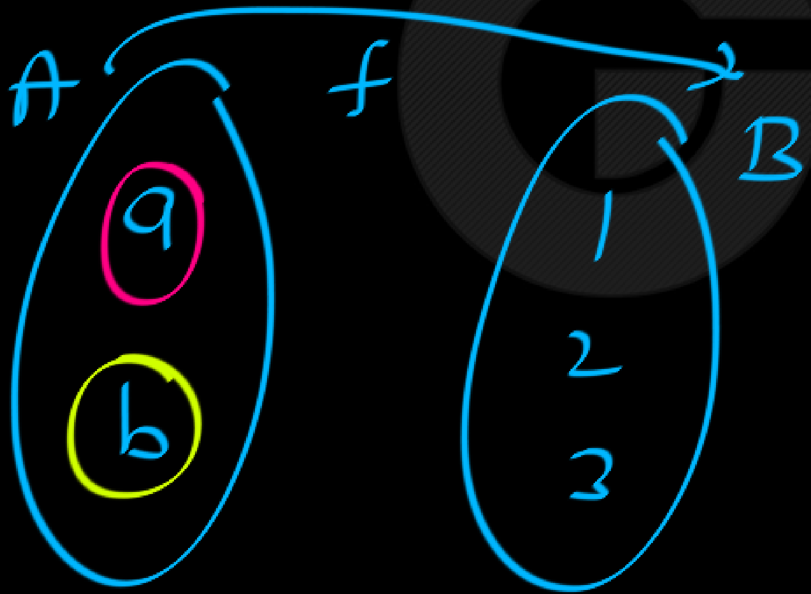
$f : \{1,2\} \rightarrow \{a,b\}$ # functions = 4



$A = \{a, b\}$; $B = \{1, 2, 3\}$

$f : A \longrightarrow B$

#functions possible?



$\underset{\downarrow}{a}$ $\quad$ $\underset{\downarrow}{b}$

3 choices × 3 choices

$= 3^2 = 9$ function

$a - 1$ $\begin{cases} b - 1 \\ b - 2 \\ b - 3 \end{cases}$

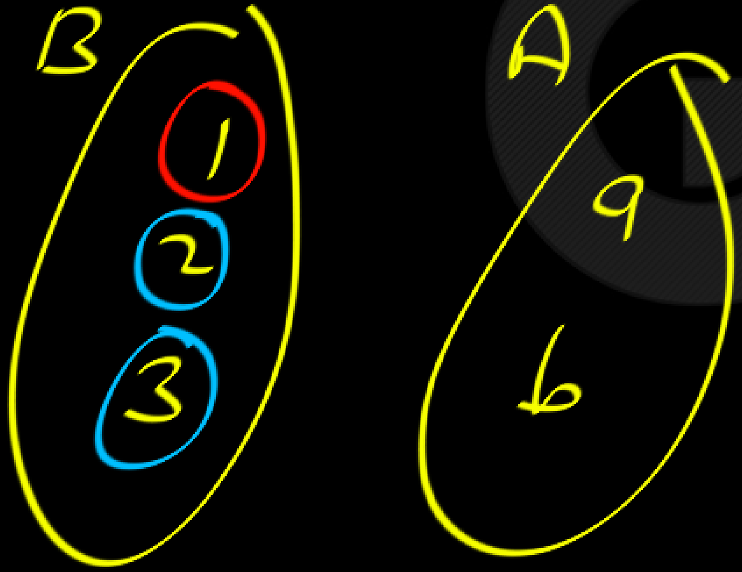
$a - 2$ $\begin{cases} b - 1 \\ b - 2 \\ b - 3 \end{cases}$

$a - 3$ $\begin{cases} b - 1 \\ b - 2 \\ b - 3 \end{cases}$

Total 9 functions

$A = \{a, b\}$; $B = \{1, 2, 3\}$

$f : B \longrightarrow A$; #function = ?

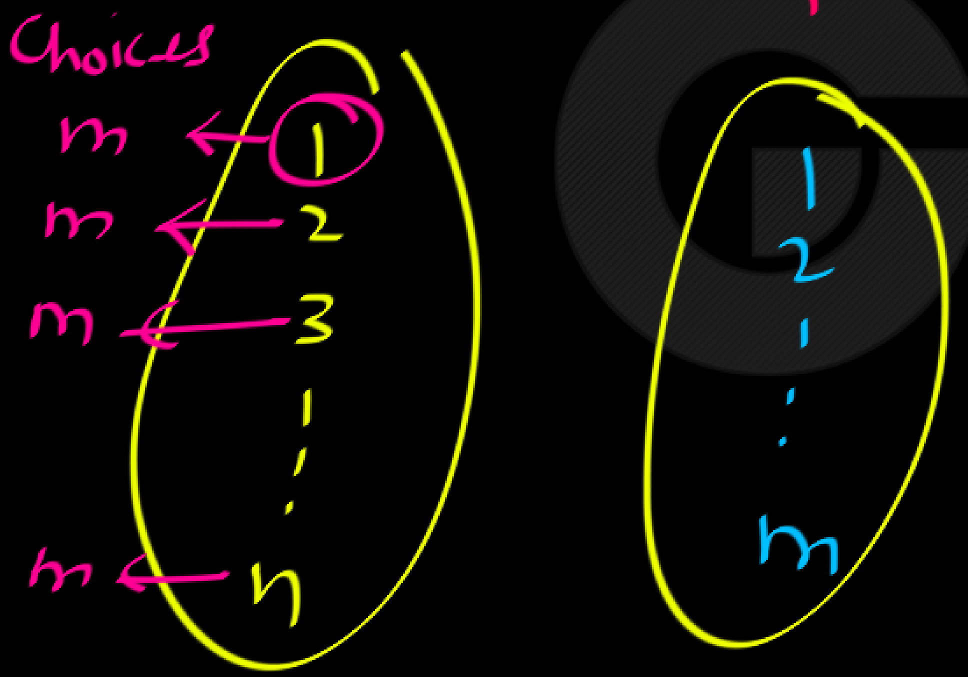


1 ↓ 2 choices × 2 ↓ 2 choices × 3 ↓ 2 choices

$= 2^3 = \underline{8 \text{ functions}}$

$|\text{Domain}| = n$; $|\text{Co-Domain}| = m$

#functions = ?



#functions :

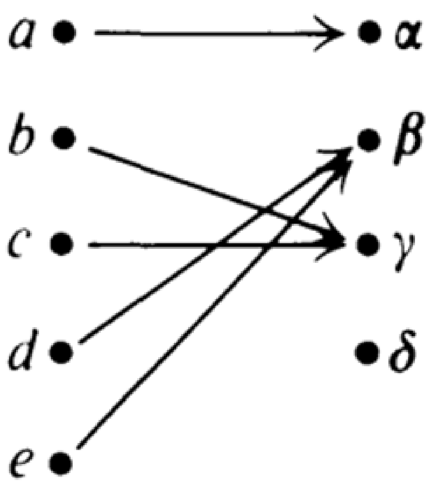
$$\underbrace{m \times m \times m \,--\, \times m}_{n \text{ times}}$$

$= m^n$

Note: #functions possible

$$= \boxed{|\text{Co-Domain}|^{|\text{Domain}|}}$$

Representations of functions:



most Popular

(a)

	α	β	γ	δ
a	√			
b			√	
c			√	
d		√		
e		√		

(b)

	R
a	α
b	γ
c	γ
d	β
e	β

(c)

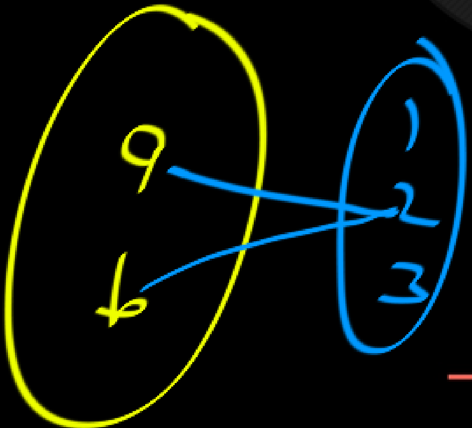
$f : \{a, b\} \to \{1, 2, 3\}$

$f(a) = 2, \quad f(b) = 2$

every function is Relation.

1. Set Representation of f:

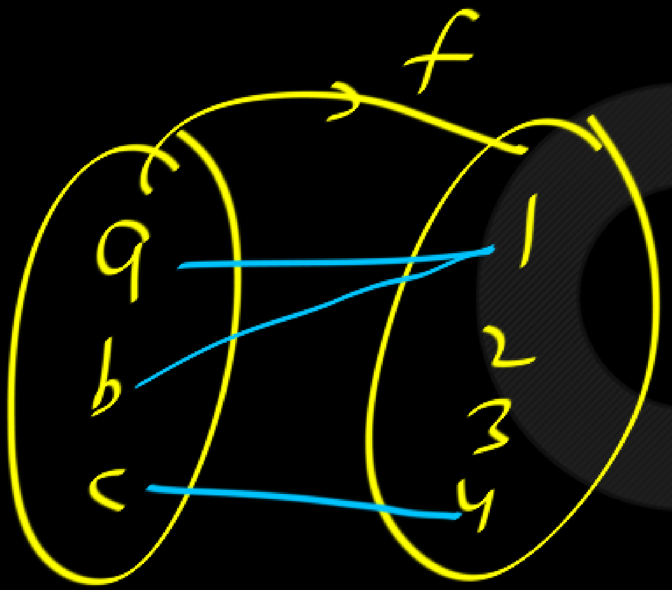
$$\{(a,2), (b,2)\}$$

2. 

Domain \ CoDomain	1	2	3
a		✓	
b		✓	

f	
a	2
b	2

Image for a Subset of Domain:



$f(a) = 1$

$f(\text{Domain}) = \text{Range} = \{1, 4\}$

$f(\{a, b\}) = \{1\}$

$f(\{a, c\}) = \{1, 4\}$

Image for a Subset of Domain:

$f : D \to C$

let $S \subseteq D$

$$f(S) = \{ y \mid y \in C;\ \exists x \in S\ f(x) = y \}$$

$$f(\{a,b,c\}) = \left\{ y \;\middle|\; \begin{array}{l} f(a) = y \ \underline{\text{OR}} \\ f(b) = y \ \underline{\text{OR}} \\ f(c) = y \end{array} \right\}$$

<u>Note:</u>

$$f(\text{Domain}) = \text{Range}$$

Set A, B

$A \cup B$ = merge all elements of A and B

$A \cup B = \{ y \mid y \in A \text{ OR } y \in B \}$

$f : Z \rightarrow Z \quad ; \quad f(x) = x^2 + 2$

$S \subseteq Z \quad ; \quad S = \{-10, 0, 5\}$

$f(S) = \{102, 2, 27\}$ ✓

Let f be a function from A to B and let S be a subset of A. The *image* of S under the function f is the subset of B that consists of the images of the elements of S. We denote the image of S by $f(S)$, so

$$f(S) = \{t \mid \exists s \in S\ (t = f(s))\}.$$

We also use the shorthand $\{f(s) \mid s \in S\}$ to denote this set.

Remark: The notation $f(S)$ for the image of the set S under the function f is potentially ambiguous. Here, $f(S)$ denotes a set, and not the value of the function f for the set S.

Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. The image of the subset $S = \{b, c, d\}$ is the set $f(S) = \{1, 4\}$.

Real-Valued function : Co-Domain = $\mathbb{R}$

Integer-Valued function : Co-Domain = $\mathbb{Z}$

Natural- " " : Co-Domain = $\mathbb{N}$

Boolean-Valued " : " = $\{0,1\}$

A function is called **real-valued** if its codomain is the set of real numbers, and it is called **integer-valued** if its codomain is the set of integers. Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.

$f : \{1,2,3\} \longrightarrow R \qquad f(x) = x^2$

$g : \{1,2,3\} \longrightarrow R \qquad g(x) = x+1$

$f+g : \{1,2,3\} \longrightarrow R$

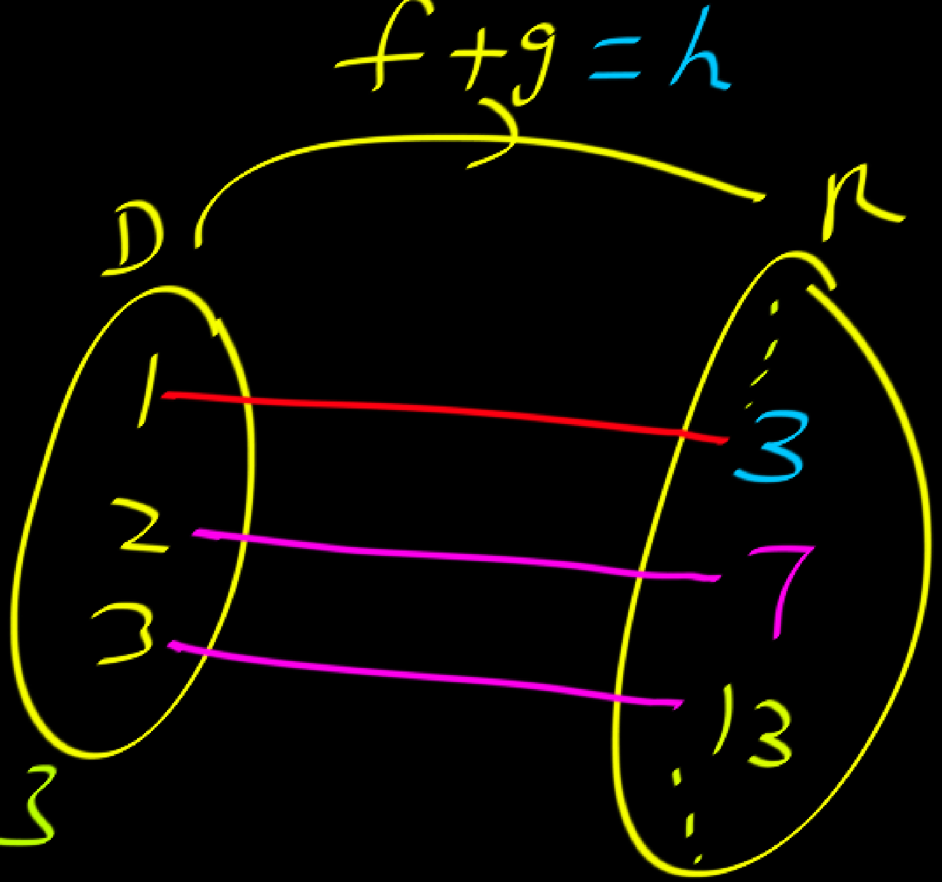
$(f+g)(x) = f(x) + g(x)$

$f(x) : x^2$

$g(x) = x+1$

Range(f) = $\{1, 4, 9\}$

Range(g) = $\{2, 3, 4\}$



Range$(f+g)$ = $\{3, 7, 13\}$

$h(2) = f(2) + g(2)$
$= 4+3$

$h(1) = f(1) + g(1) = 1+2 = 3$

$f : \{1,2,3\} \rightarrow R \qquad f(x) = x^2$

$g : \{1,2,3\} \rightarrow R \qquad g(x) = x+1$

$fg : \{1,2,3\} \rightarrow R$

$(fg)(x) = f(x) * g(x)$

$f(x) = x^2$

$g(x) = x+1$

$h = f \times g$

$h(1) = f(1) \times g(1) = 2$

$h(2) = f(2) \times g(2) = 12$

$h(3) = f(3) \times g(3) = 36$

A function is called **real-valued** if its codomain is the set of real numbers, and it is called **integer-valued** if its codomain is the set of integers. Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.

DEFINITION 3 Let f_1 and f_2 be functions from A to $\mathbf{R}$. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to $\mathbf{R}$ defined for all $x \in A$ by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x),$$
$$(f_1 f_2)(x) = f_1(x) f_2(x).$$

Note that the functions $f_1 + f_2$ and $f_1 f_2$ have been defined by specifying their values at x in terms of the values of f_1 and f_2 at x.

Let f_1 and f_2 be functions from $\mathbf{R}$ to $\mathbf{R}$ such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$?

$f_1 : R \to R \quad f_1(x) = x^2$

$f_2 : R \to R \quad f_2(x) = x - x^2$

$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + x - x^2$

$= x$

$h_1 = f_1 + f_2 \quad ; \quad \underline{h_1(x) = x}$

$$h_2 = f_1 \times f_2$$

$$(f_1 \times f_2)(x) = f_1(x) \times f_2(x)$$

$$= (x^2)(x - x^2)$$

$$h_2(x) = x^3 - x^4$$

Let f_1 and f_2 be functions from $\mathbf{R}$ to $\mathbf{R}$ such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$?

Solution: From the definition of the sum and product of functions, it follows that

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + (x - x^2) = x$$

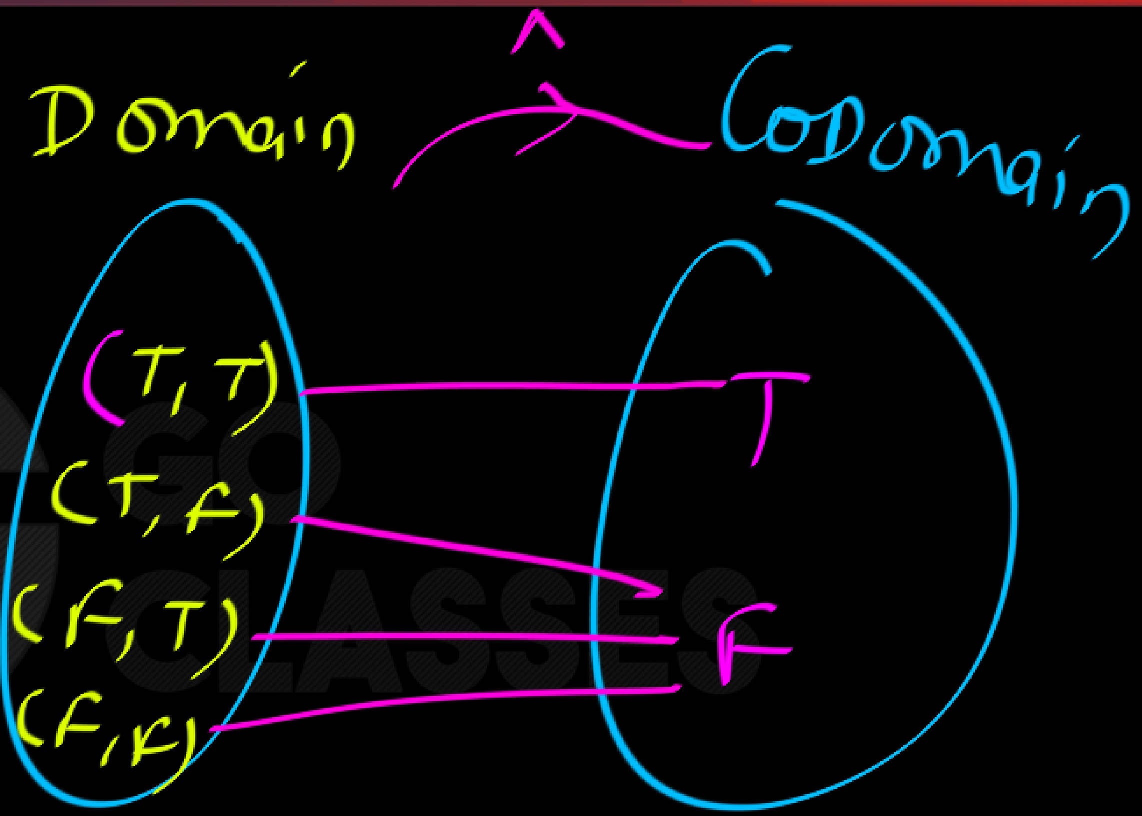
and

$$(f_1 f_2)(x) = x^2(x - x^2) = x^3 - x^4.$$

◀

Conjunction :

P	Q	P ∧ Q
T	T	T
F	T	F
F	F	F
T	F	F

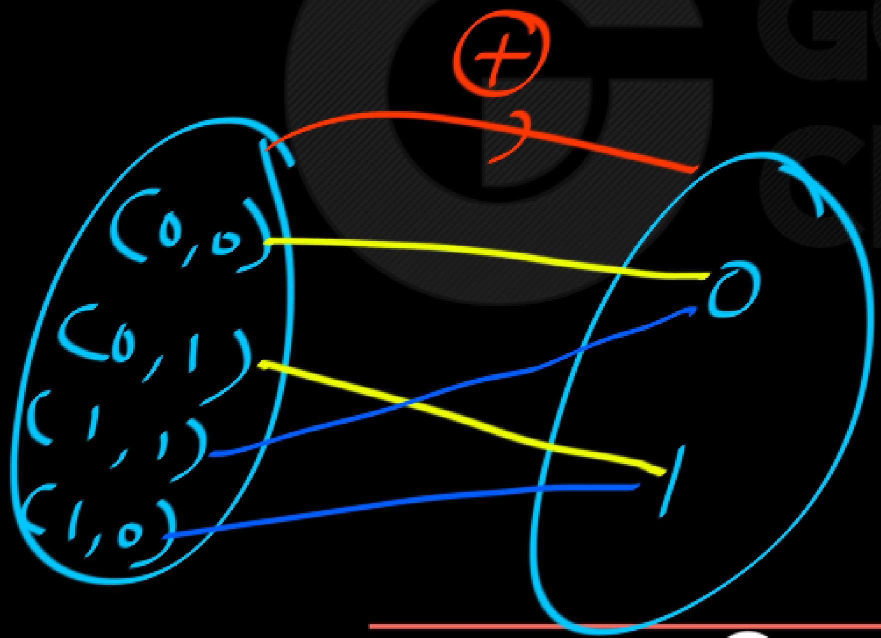


Boolean Functions

A **boolean function** is a function whose domain (input) is the set of all ordered n-tuples of 0's and 1's and whose co-domain (output) is the set $\{0, 1\}$.

In Digital logic :

Boolean Expression ≡ Boolean function

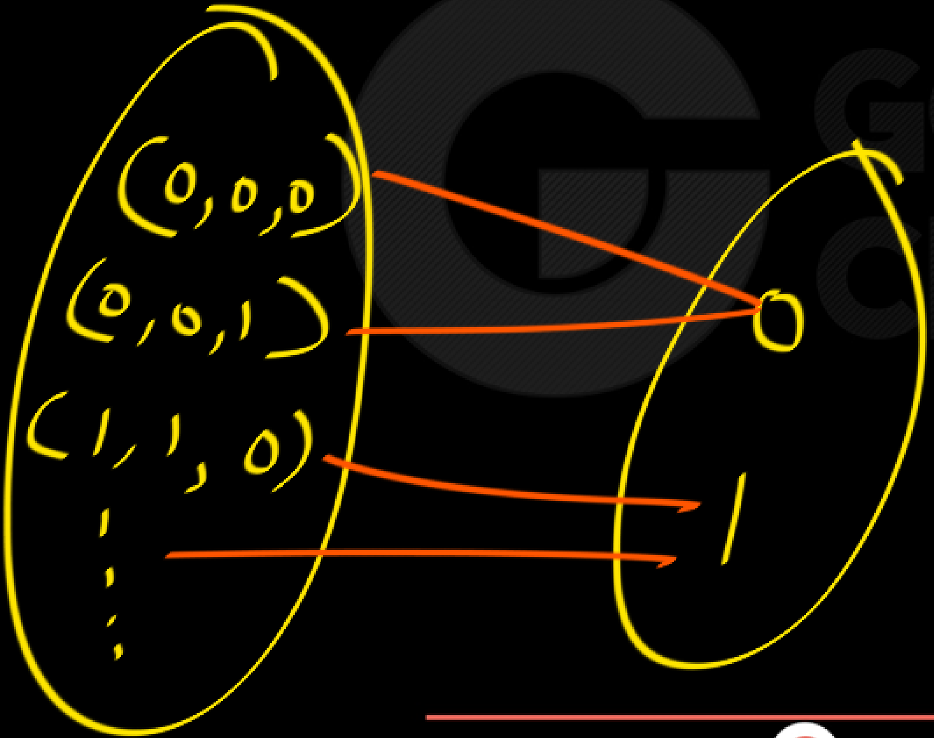


Ex-OR = function

AND = function

Q: 3 boolean variable a, b, c

Domain → 8 elements



Special Type of functions :

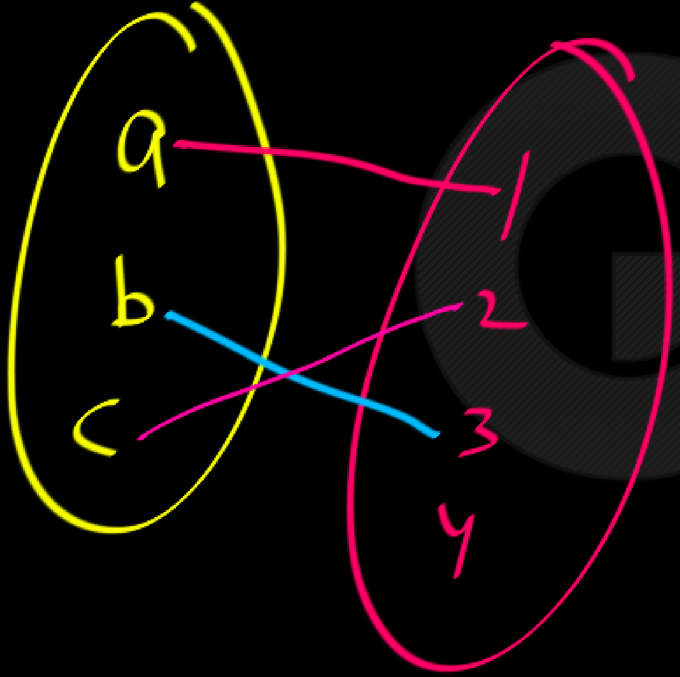
① one-one function

② onto "

③ bijection

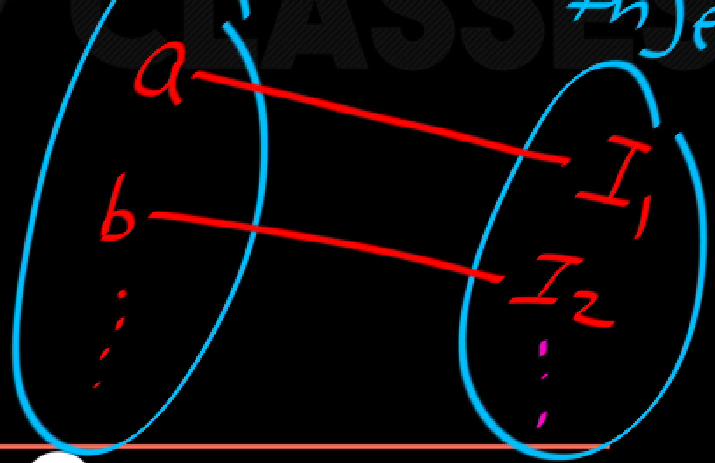
One-one function / Injection / Injective function

Injunction :



people

Injections / Syringe



Onto function: (आच्छादित)

Covering the Complete Co-Domain

Co-Domain = Range

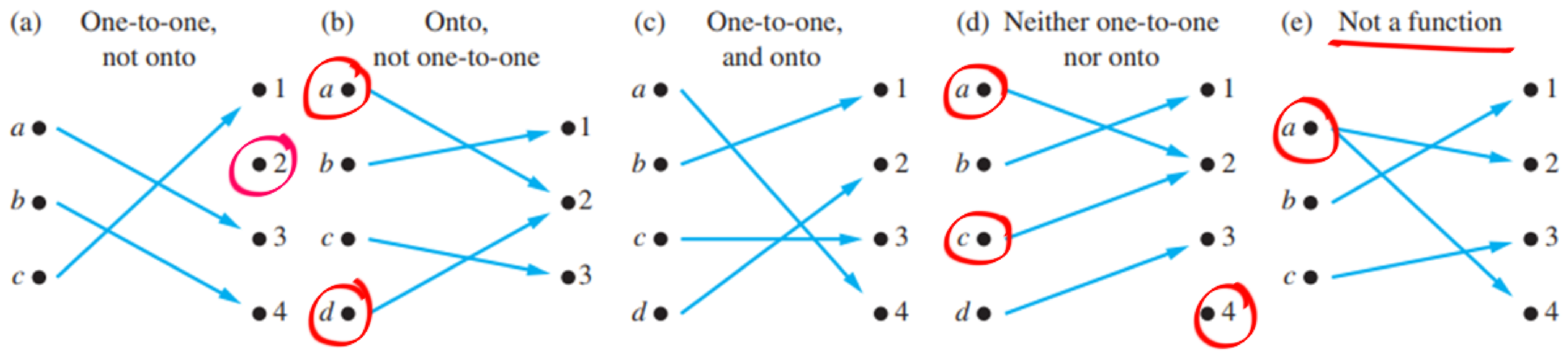


FIGURE 5 **Examples of Different Types of Correspondences.**

$a \neq b$

but

$f(a) = f(b)$

One-to-One Functions / Injection / Injective / Injunction

Let F be a function from a set X to a set Y. F is **one-to-one** if, and only if, for all elements x_1 and x_2 in X,

$$\text{if } F(x_1) = F(x_2), \text{ then } x_1 = x_2$$

or

$$\forall x_1, x_2 \in X, \text{ if } F(x_1) = F(x_2) \text{ then } x_1 = x_2.$$

The function,

$$f = (1, b), (2, a), (3, c)$$

from $X = \{1, 2, 3\}$ to $Y = \{a, b, c, d\}$ is one-to-one. {*from Johnsonbaugh, p. 119*}

One-one function $f : D \to C$

1. If $a \neq b \in D$ then $f(a) \neq f(b)$

OR

** 2. $\boxed{\text{If } f(a) = f(b) \text{ then } a = b}$

Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

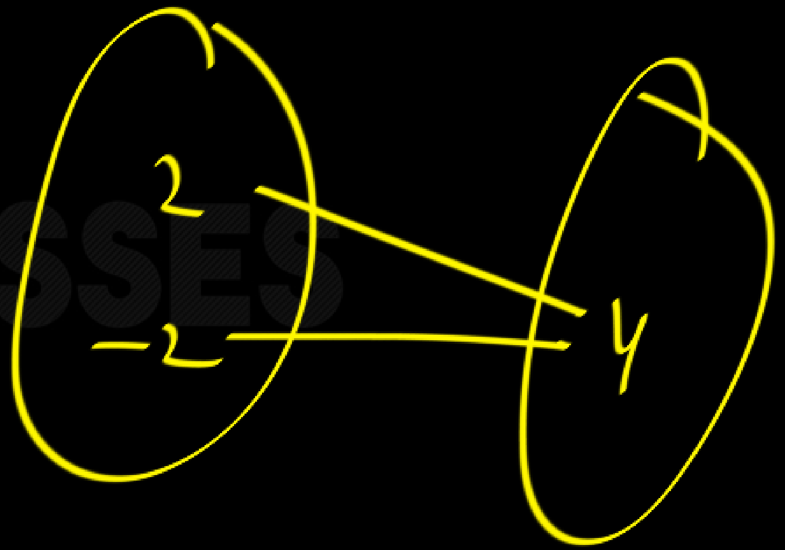
Solution: The function $f(x) = x^2$ is not one-to-one because, for instance, $f(1) = f(-1) = 1$, but $1 \neq -1$.

Note that the function $f(x) = x^2$ with its domain restricted to $\mathbf{Z}^+$ is one-to-one. (Technically, when we restrict the domain of a function, we obtain a new function whose values agree with those of the original function for the elements of the restricted domain. The restricted function is not defined for elements of the original domain outside of the restricted domain.) ◀

$f : \mathbb{Z} \to \mathbb{Z}$; $f(x) = x^2$

Not one-one.

$f(-2) = f(2) = 4$



$f : N \rightarrow N \; ; \; f(x) = x^2$

one-one ✓

$f : N \rightarrow Z \; ; \; f(x) = x^2$

one-one ✓

Consider the function $f : R \rightarrow R$ where $f(x) = 3x + 7$ for all $x \in R$. Then for all $x_1, x_2 \in R$, we find that

$$f(x_1) = f(x_2) \Rightarrow 3x_2 + 7 = 3x_2 + 7 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2,$$

so the given function is one-to-one.

On the other hand, suppose that $g : R \rightarrow R$ is the function defined by $g(x) = x^4 - x$ for each real number x. Then

$$g(0) = (0)^4 - 0 = 0 \text{ and } (1)^4 - 1 = 1 - 1 = 0.$$

Consequently, g is **not** one-to-one since $g(0) = g(1)$ but $0 \neq 1$. { from Grimaldi, p 255}

$f : R \to R \quad ; \; f(x) = 3x + 7$

Is f one-one ? — Yes

Ans: Assume $f(a) = f(b)$

$3a + 7 = 3b + 7$

$a = b$

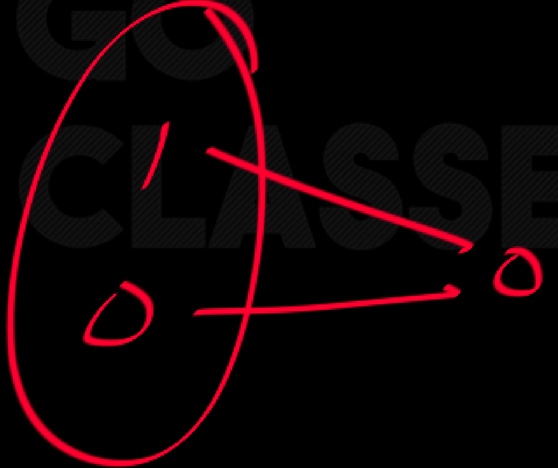
$\boxed{a = b}$

$f : R \rightarrow R \; ; \; f(x) = x^4 - x$

Is f one-one ? — No.

$f(1) = 0$

$f(0) = 0$



Onto / Surjective / Surjection :

$f : D \to C$

$\forall y \in C \; ; \; \exists x \in D \quad f(x) = y$

onto / Surjective Functions / surjection

- A function $f : A \to B$ is called **surjective** (or **onto**) if each element of the codomain has at least one element of the domain associated with it.
  - A function with this property is called a **surjection**.
- Formally:

  For any $b \in B$, there exists at least one $a \in A$ such that $f(a) = b$.

- An intuition: surjective functions cover every element of B with at least one element of A.

Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

Solution: The function f is not onto because there is no integer x with $x^2 = -1$, for instance. ◀

Is the function $f(x) = x + 1$ from the set of integers to the set of integers onto?

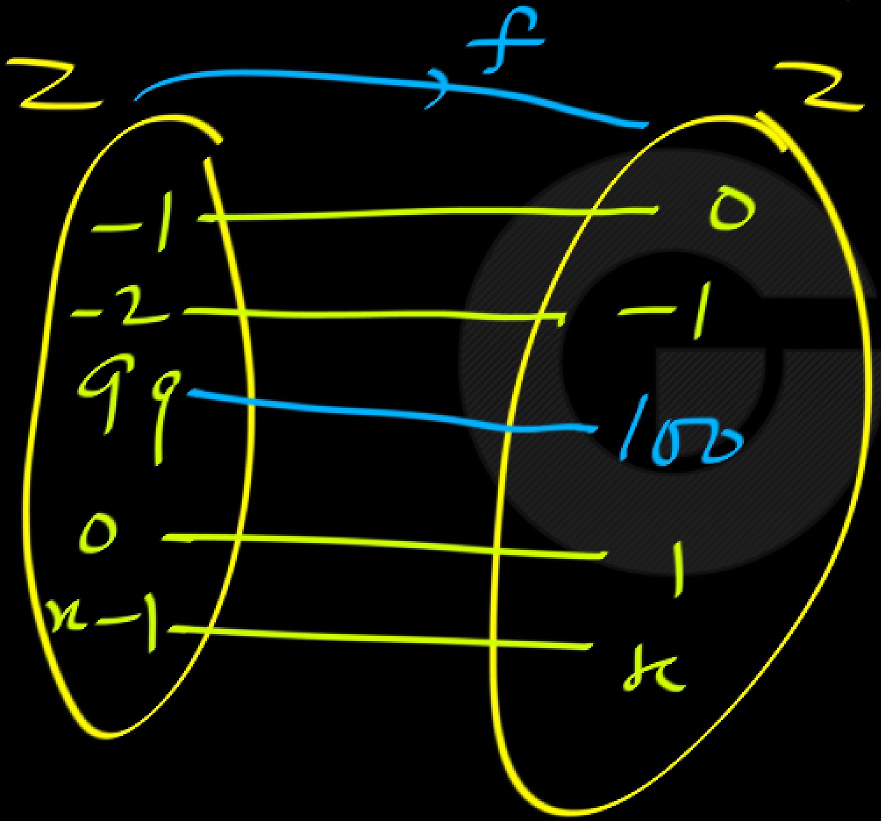
$f : N \to N$

$f(x) = x^2$

Not onto.

No Pre-image of 3.

$f : \mathbb{Z} \to \mathbb{Z}$; $f(x) = x + 1$



$f(99) = 100$

$\text{Pre-image}(y) = y - 1$

From Co-Domain Point of View:

① Injective:

Def: Every element of Co-Domain has at most one Pre-image.

From Co-Domain Point of View:

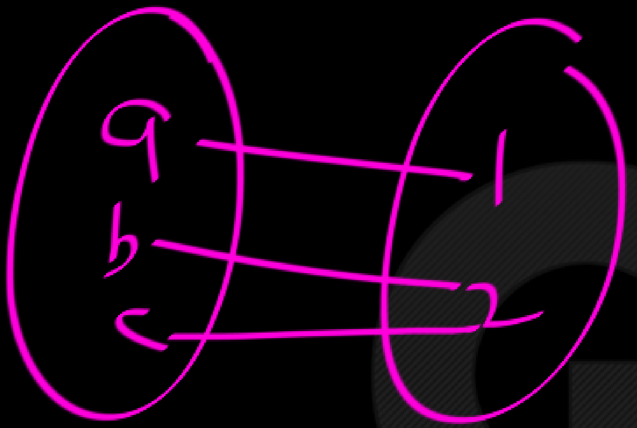
② **Surjective:**

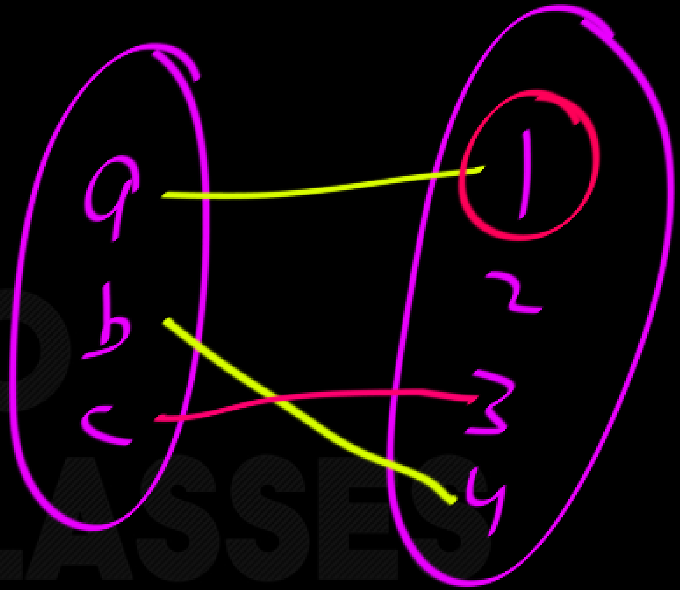
Def: Every element of "Co-Domain" has at least one Pre-image.

From Co-Domain Point of View:

③ $\boxed{\text{Surjective and Injective}}$: Bijection :

Def: "Every element" of "Co-Domain" has Exactly one Pre-image.





onto ✓

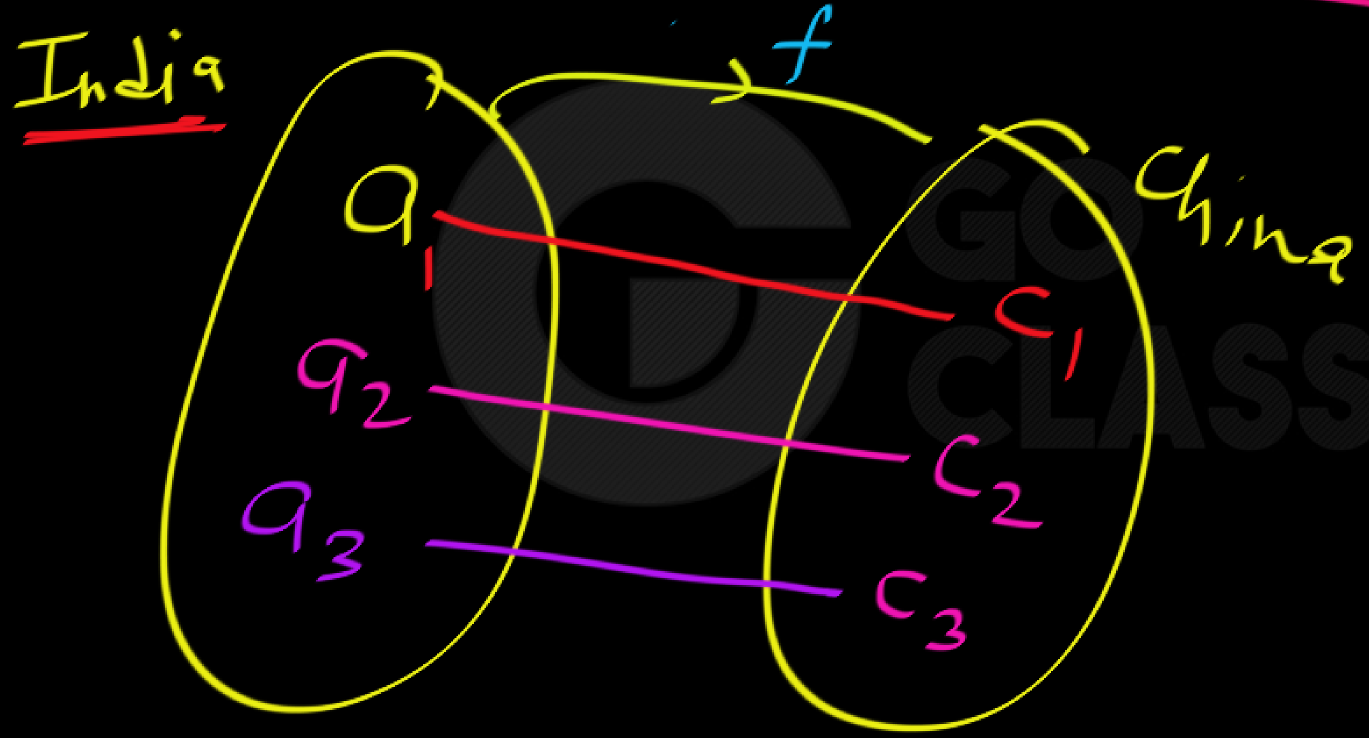
Pre-image(1) = {a}

'' '' (2) = {b, c}

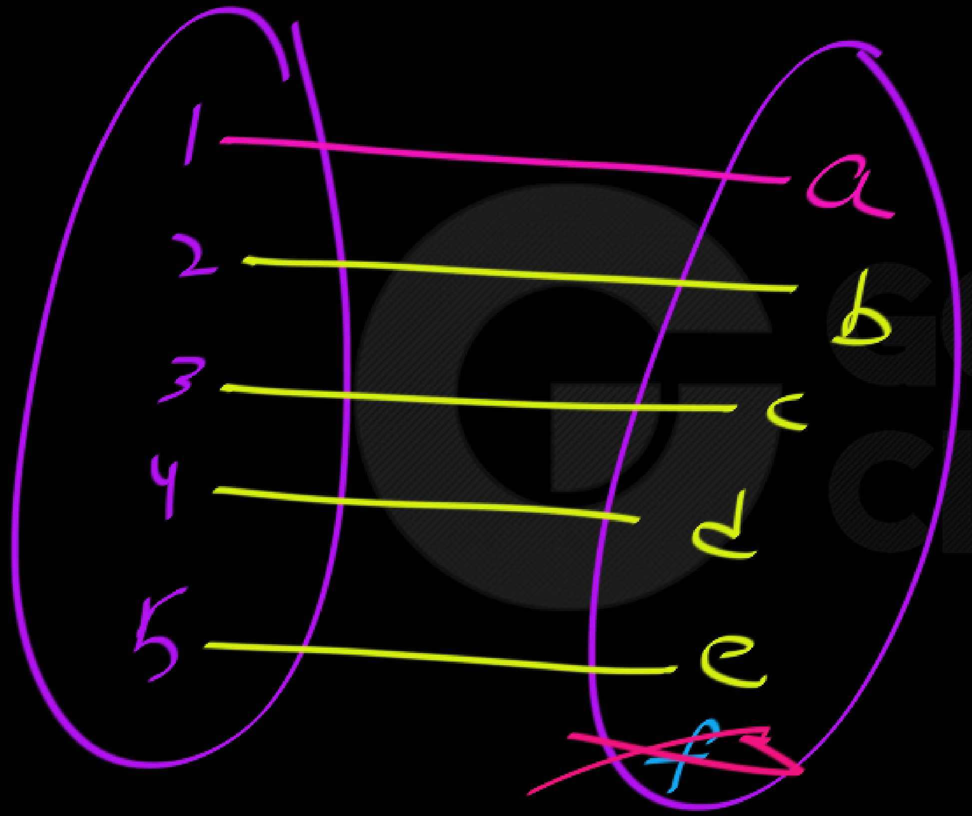
Injections and Surjections

- An injective function associates **at most** one element of the domain with each element of the codomain.
- A surjective function associates **at least** one element of the domain with each element of the codomain.
- What about functions that associate **exactly one** element of the domain with each element of the codomain?

Bijective function : $\boxed{\text{1-1 and onto}}$



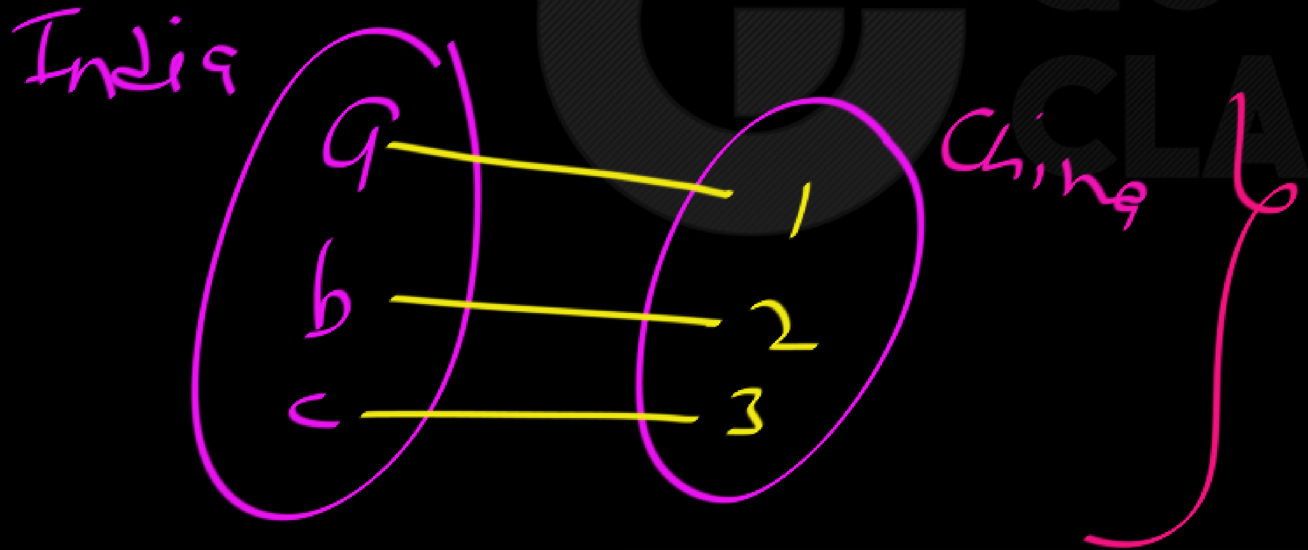
Bijective :— 1-1 and onto



Bijective ≡ One-one Correspondance

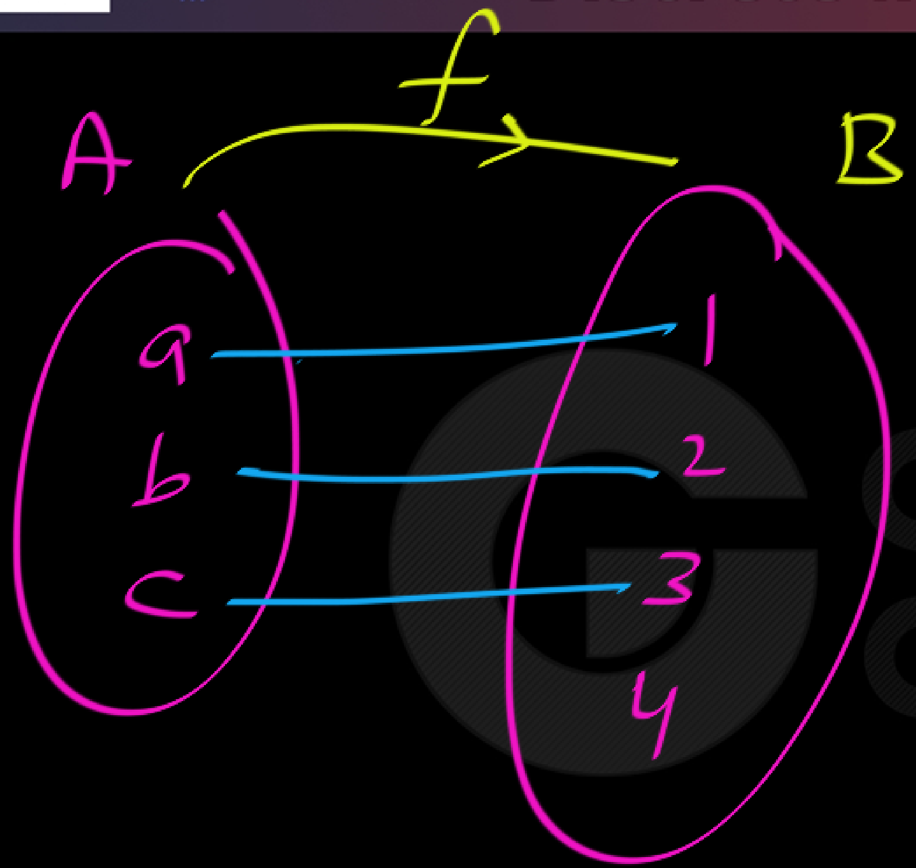
f is bijective function $A \longrightarrow B$

then $\boxed{|A| = |B|}$



Bijections

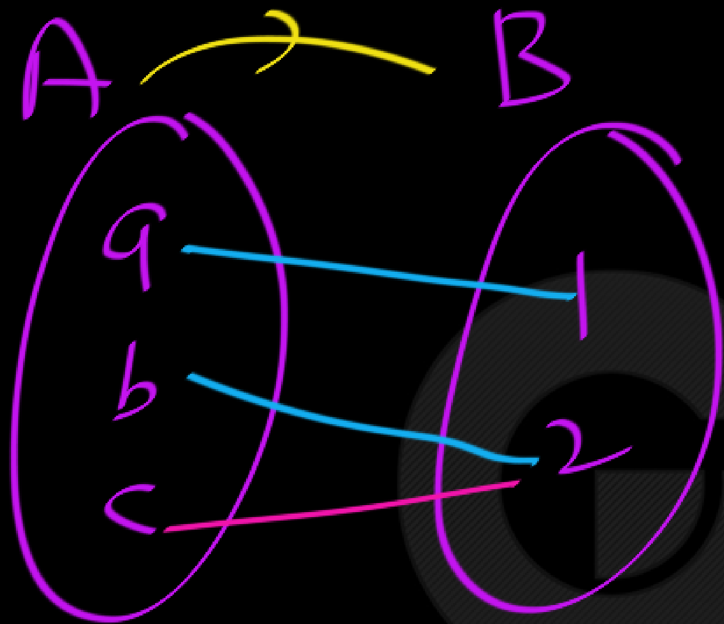
- A function that associates each element of the codomain with a unique element of the domain is called **bijective**.
  - Such a function is a **bijection**.
- Formally, a bijection is a function that is both **injective** and **surjective**.
- A bijection is a one-to-one correspondence between two sets.



Is onto function Possible?

No.

$|A| < |B|$ then onto Not Possible.



Is 1-1 function possible?

NO

$|A| > |B|$ then 1-1 function NOT possible.

|Domain| > |Co-Domain| : 1-1 Not possible.

|Domain| < |Co-Domain| : onto NOT possible.

Note: $f : A \longrightarrow B$

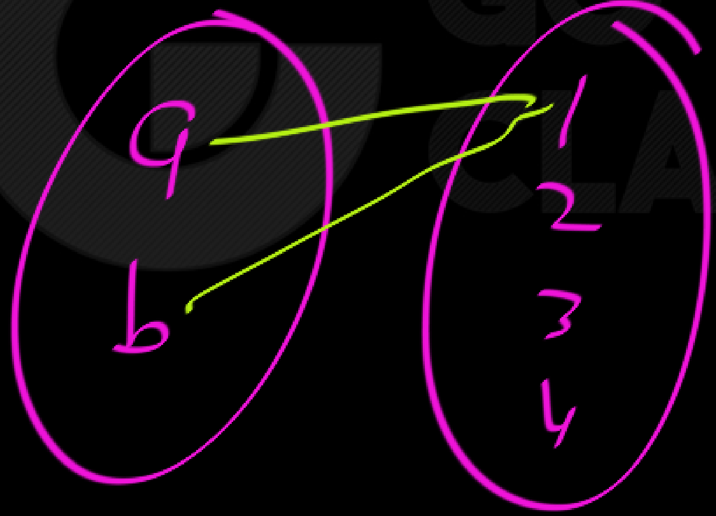
These exists at least one onto function then $|A| \geq |B|$ onto function possible.

One-one function possible : $|A| \leq |B|$

Q: $|\text{Domain}| \le |\text{Co-Domain}|$

So Every function will be 1-1 fun ?

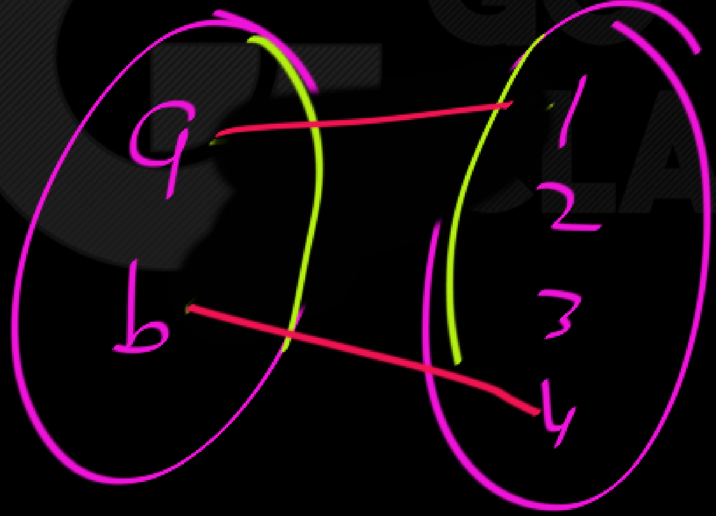
No.



Q: |Domain| ≤ |Co-Domain|

So some function will be 1-1 fun?

Yes:



Note:

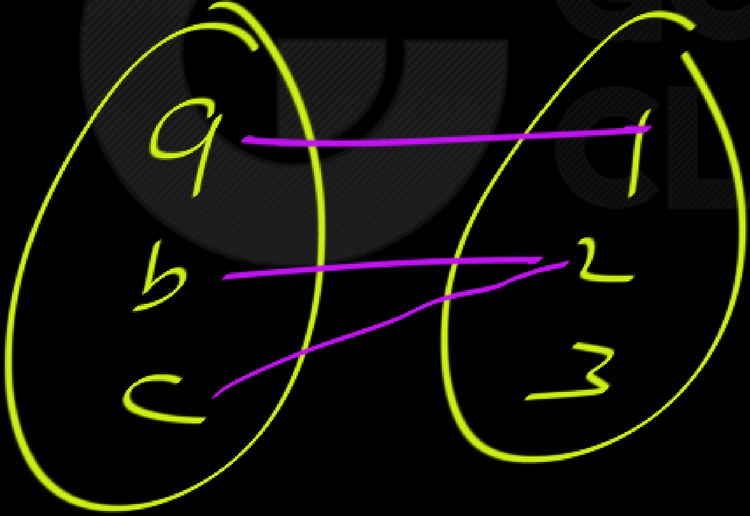
Bijective function exists

iff $\boxed{|\text{Domain}| = |\text{Co-Domain}|}$

Q: |Domain| = |Co-Domain|

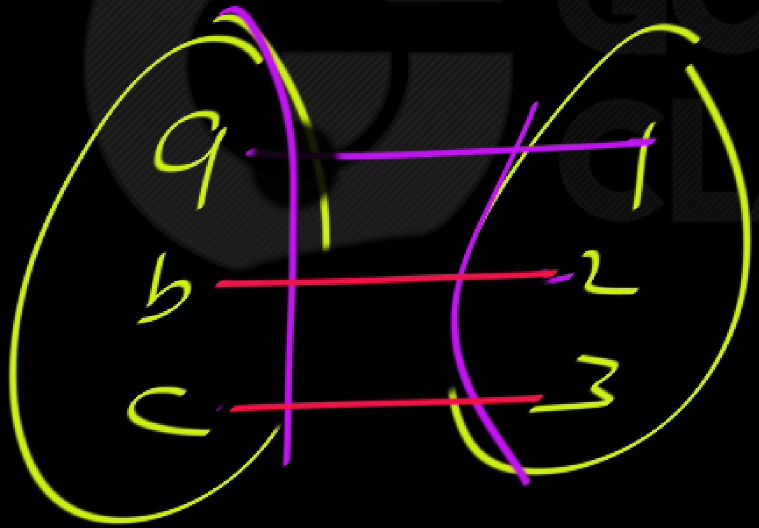
Every function is Bijective ?

No.



Q: |Domain| = |Co-Domain|
Some function is Bijective ?

Yes

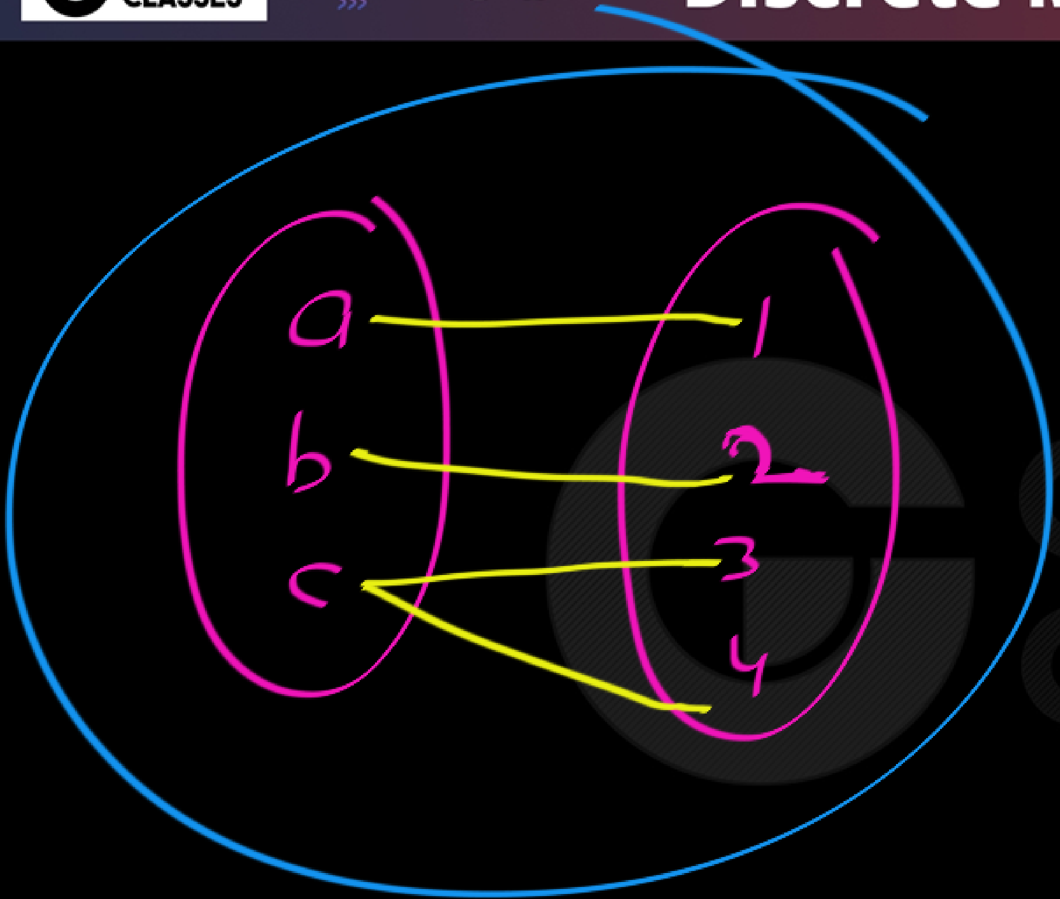


Q: |Domain| < |Co-Domain|

Some function is Bijective?

No.

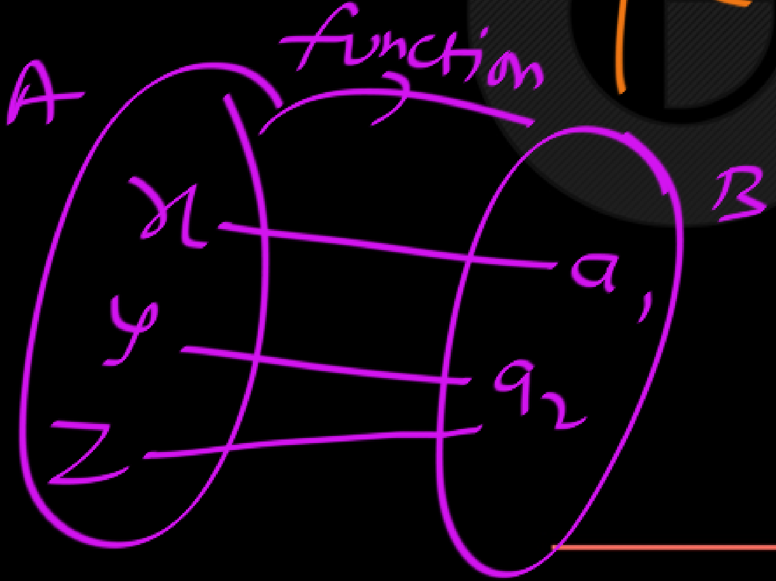
Onto Not possible



Not a function

Q : function $f : A \to B$

$x \in A$; $|\{ y \mid f(x) = y \}| = 1$



Q : Relation R : $A \rightarrow B$

$\underline{x \in A}$: $\left| \{ y \mid xRy \} \right| = 0$ to $|B|$