Set Theory

Next Chapter:

Special Types of Relations

(Symmetric, Transitive etc)

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GATE CSE AIR 53; AIR 67;
AIR 107; AIR 206; AIR 256

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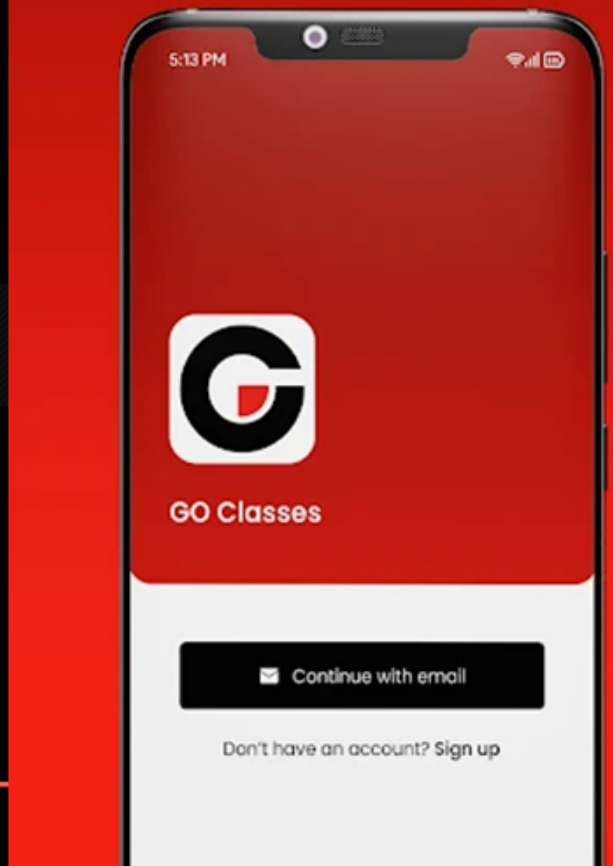

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Next Topic:

Symmetric Relation

(Symmetric, Anti-Symmetric, Asymmetric)

Symmetric Relation :

If $a R b$ then $b R a$

Symmetric property

Symmetric Relation :

Base set : A

Relation R on A $\equiv$ $R : A \longrightarrow A$

Relation R is Symmetric iff

$$\forall_{a,b \in A} \left(aRb \longrightarrow bRa \right)$$

Assume, Base Set A ; $R : A \rightarrow A$

For any x, y ∈ A (Assume x,y are different elements):

Possibilities

- x R y & y R x
- x R y & y R x
- x R y & y R x
- x R y & y R x
- x R x
- y R y

OR OR OR

Assume, Base Set A ; $R : A \rightarrow A$

For any x, y ∈ A (Assume x,y are different elements):

If R is Symmetric ⇒

- x R y & y R x — Allowed
- x R y & y R x — NOT Allowed
- x R y & y R x — " "
- x R y & y R x — Allowed
- x R x — "
- y R y — "

Symmetric :

$R : A \longrightarrow A$

$$\forall_{x,y \in A} \left(xRy \longrightarrow yRx \right)$$

$$\forall_{x,y \in A} \left(yRx \longrightarrow xRy \right)$$

Consider the following relations on {1, 2, 3, 4}: → Base set

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\},$

$R_2 = \{(1, 1), (1, 2), (2, 1)\},$

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\},$

Which Relation is Symmetric?

Consider the following relations on {1, 2, 3, 4}:

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,

$R_2 = \{(1, 1), (1, 2), (2, 1)\}$,

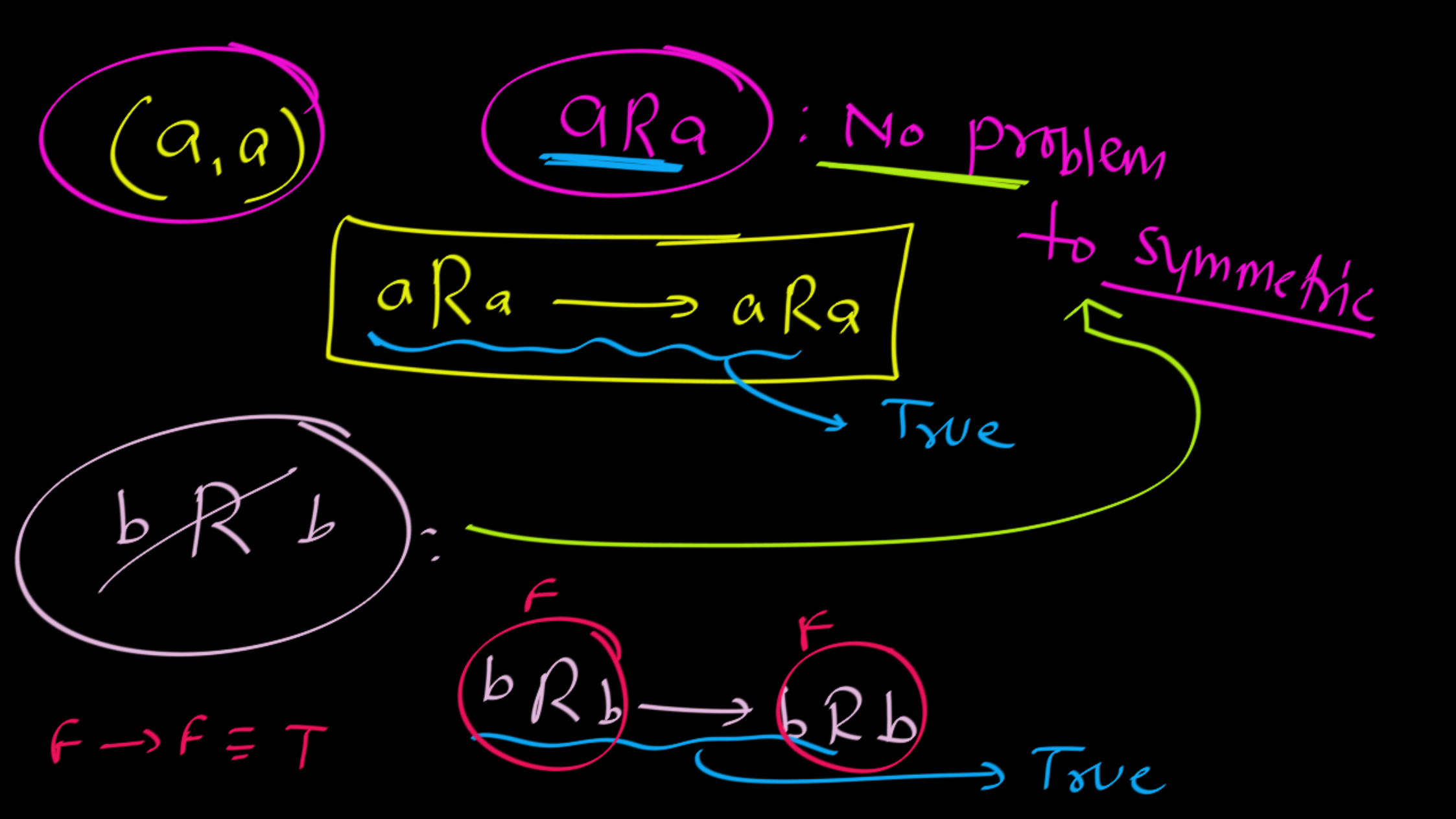
$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

R_1: 3 R_1,4 But 4 R_1,3

→ Not Symmetric

R_2: Symm.

R_3: Symm


(a, a)
aRa : No problem to symmetric
aRa ⟶ aRa
True
bRb :
F
F
bRb ⟶ bRb
True
F → F ≡ T

Example 1:

Equality Relation on Z is Symmetric??

$R : Z \rightarrow Z$; $a R b$ iff $a = b$

Yes

$x, y \in Z$

if xRy then yRx

$x = y$ $\quad$ $y = x$

Example 2:

Let Set A = {1,2,3}

Subset Relation on P(A) is Symmetric??

Example 2:

Let Set A = {1,2,3}

Subset Relation on P(A) is Symmetric??

No

Base set : P(A)

$R : P(A) \longrightarrow P(A)$; $X R Y$ iff $X \subseteq Y$

$A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Basic set

on

xRy iff $x \subseteq y$

$\{1\}\ R\ \{1,2\}$

$\{1,2\}\ \not R\ \{1\}$

Symmetry

- In some relations, the relative order of the objects doesn't matter.
- Examples:
  - If $x = y$, then $y = x$.
  - If $u \leftrightarrow v$, then $v \leftrightarrow u$.
  - If $x \equiv_k y$, then $y \equiv_k x$.
- These relations are called ***symmetric***.
- Formally: a binary relation R over a set A is called *symmetric* if

$$\forall a \in A.\ \forall b \in A.\ (aRb \rightarrow bRa)$$

("If a is related to b, then b is related to a.")

Matrix Representation of Symmetric Relation:

Base set $A = \{1, 2, 3\}$

Sym. Rel (R)	1	2	3
1	✗	✓	✓
2	✓	✓	✗
3	✓	✗	✗

matrix M

a R a fine

a $\not R$ a fine

✓ Symmetric matrix

$M = M^T$

Matrix Representation of Symmetric Relation:

Base set $A = \{1, 2, 3\}$

Sym. Rel (R)	1	2	3
1	✓	✗	✗
2	✗	✗	✓
3	✗	✓	✗

Matrix M

Symmetric Matrix

$M = M^T$

Matrix Representation of Symmetric Relation:

Base set $A = \{1, 2, 3\}$

Sym. Rel R	1	2	3
1	X	X	✓
2	X	X	✓
3	✓	✓	X

Symm. matrix

Consider the following relations on $\{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\},$$

$$R_2 = \{(1, 1), (1, 2), (2, 1)\},$$

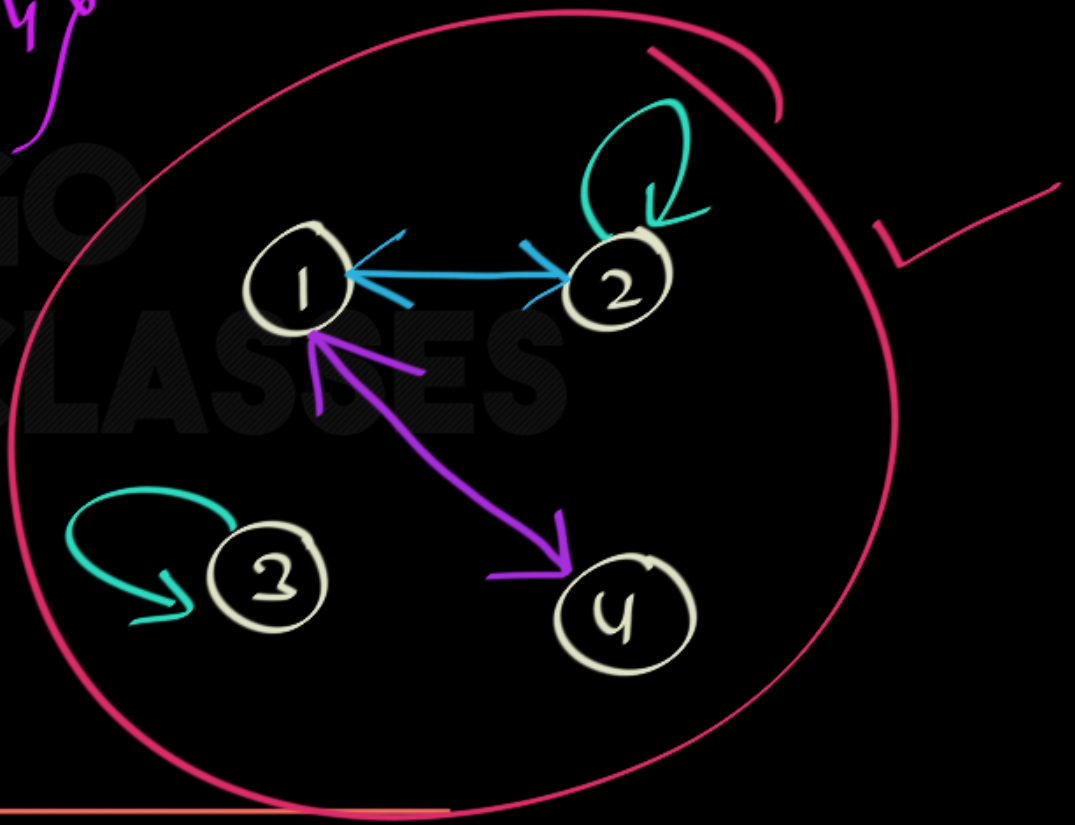
$$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\},$$

Graph Representation of Symmetric Relation:

Base set : $A = \{1, 2, 3, 4\}$

Sym. Rel R on A :

Graph Rep of R :



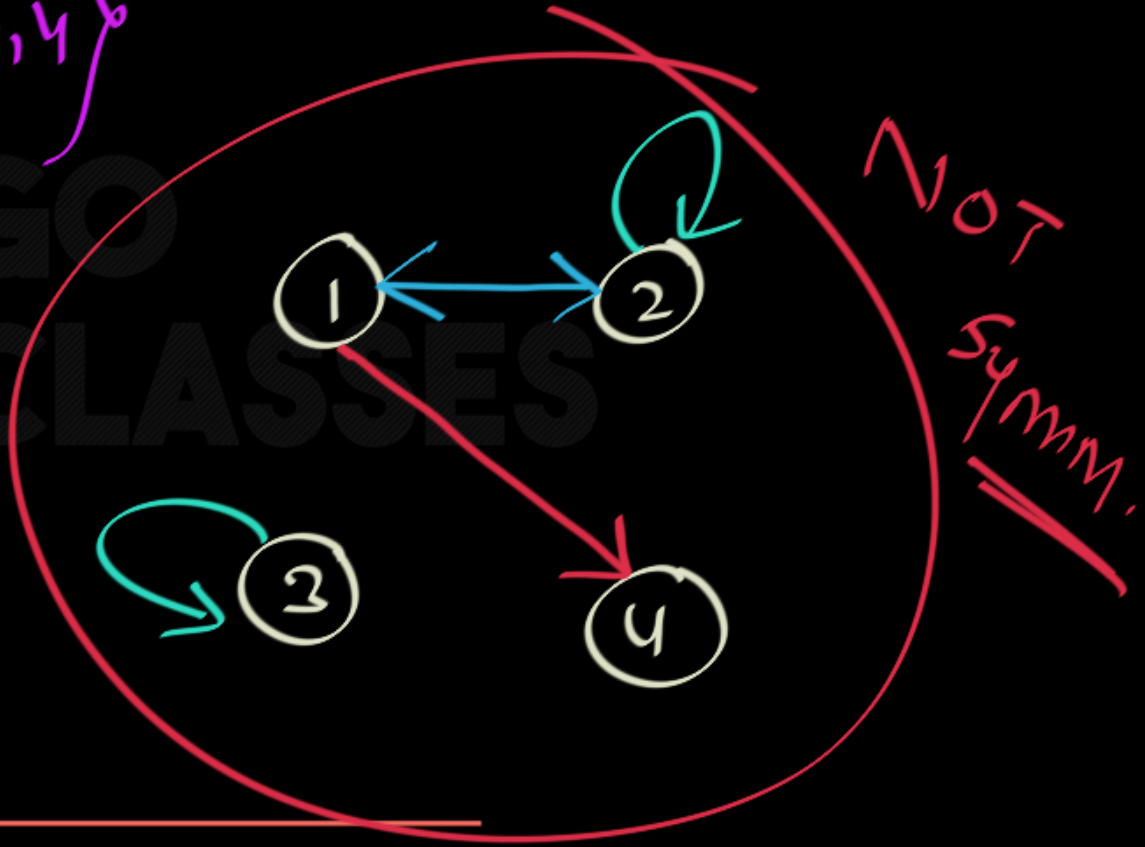
2 R 2 line ; 2 R 2 line

Graph Representation of Symmetric Relation:

Base set : $A = \{1, 2, 3, 4\}$

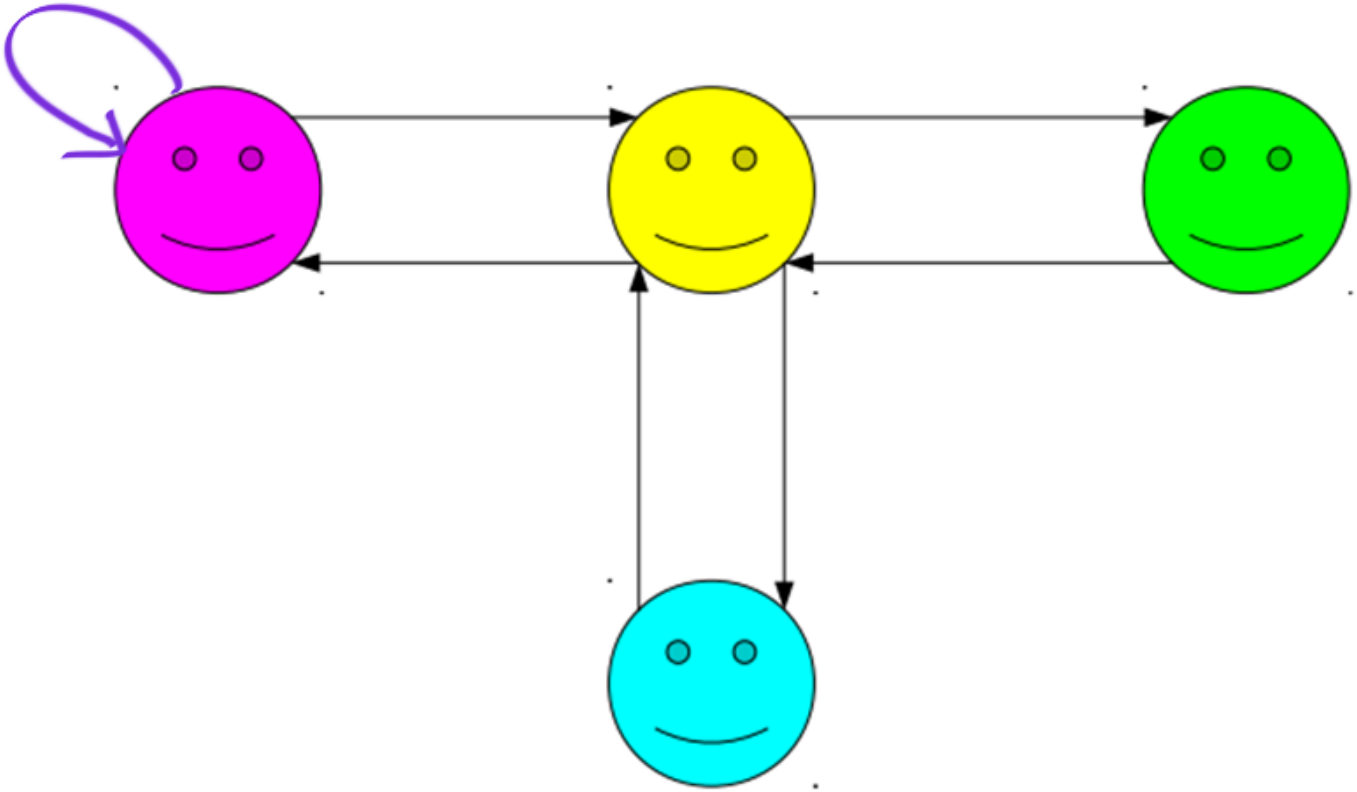
Sym. Rel R on A :

Graph Rep of R :



2 R 2 line ; 2 $\not R$ 2 line

An Intuition for Symmetry

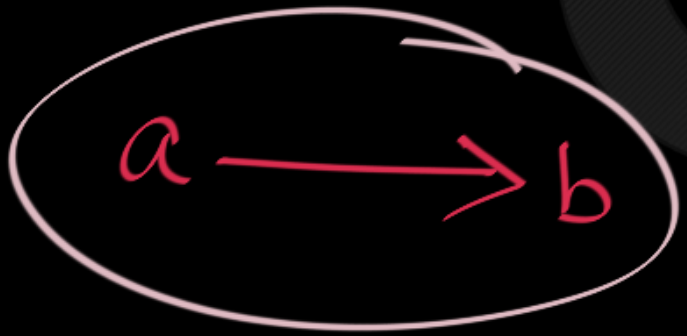


$$\forall a \in A.\ \forall b \in A.\ (aRb \rightarrow bRa)$$

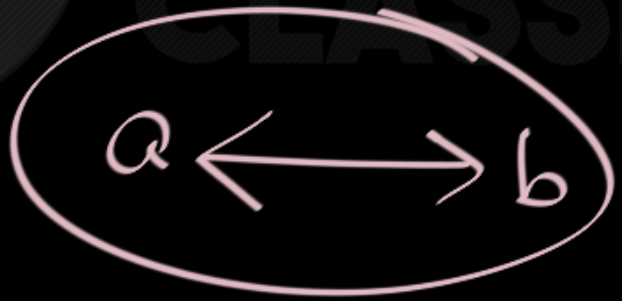
("If a is related to b, then b is related to a.")

Graph Rep of Sym Rel:

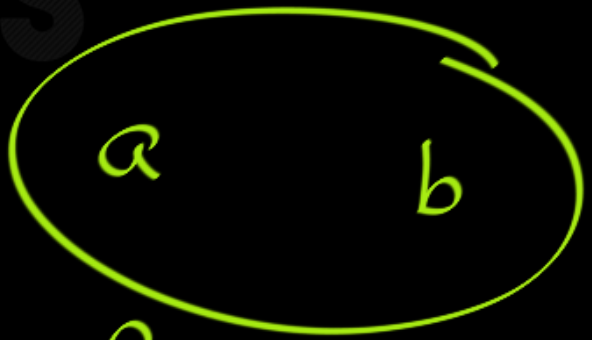
No Unidirectional Edges.



Not Allowed

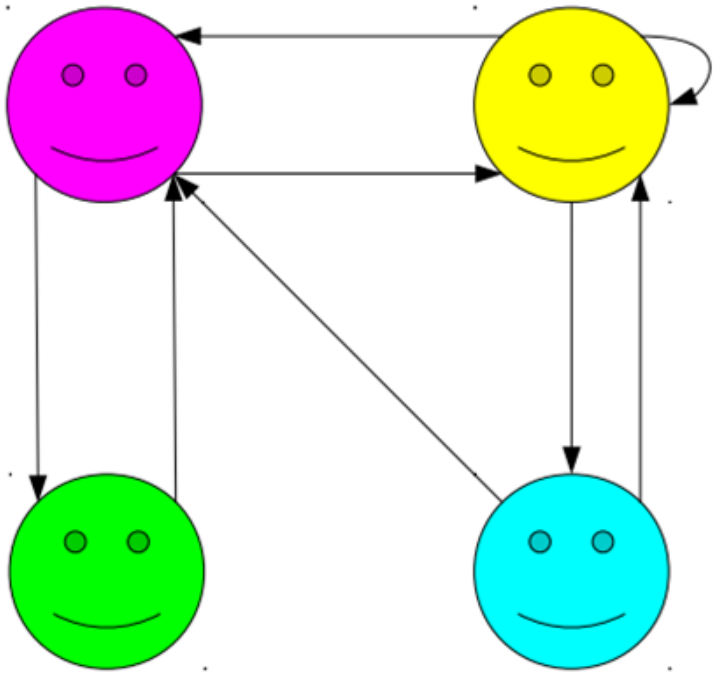


fine



fine

Is This Relation Symmetric?

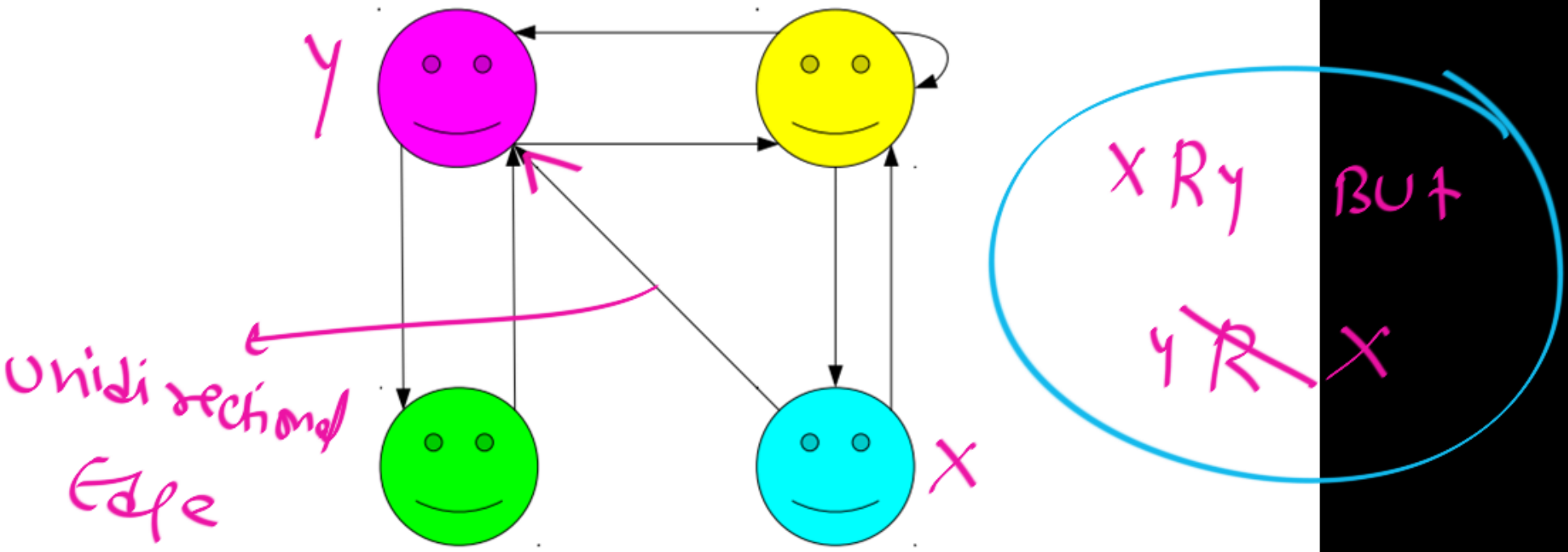


$$\forall a \in A.\ \forall b \in A.\ (aRb \rightarrow bRa)$$

("If a is related to b, then b is related to a.")

Is This Relation Symmetric?

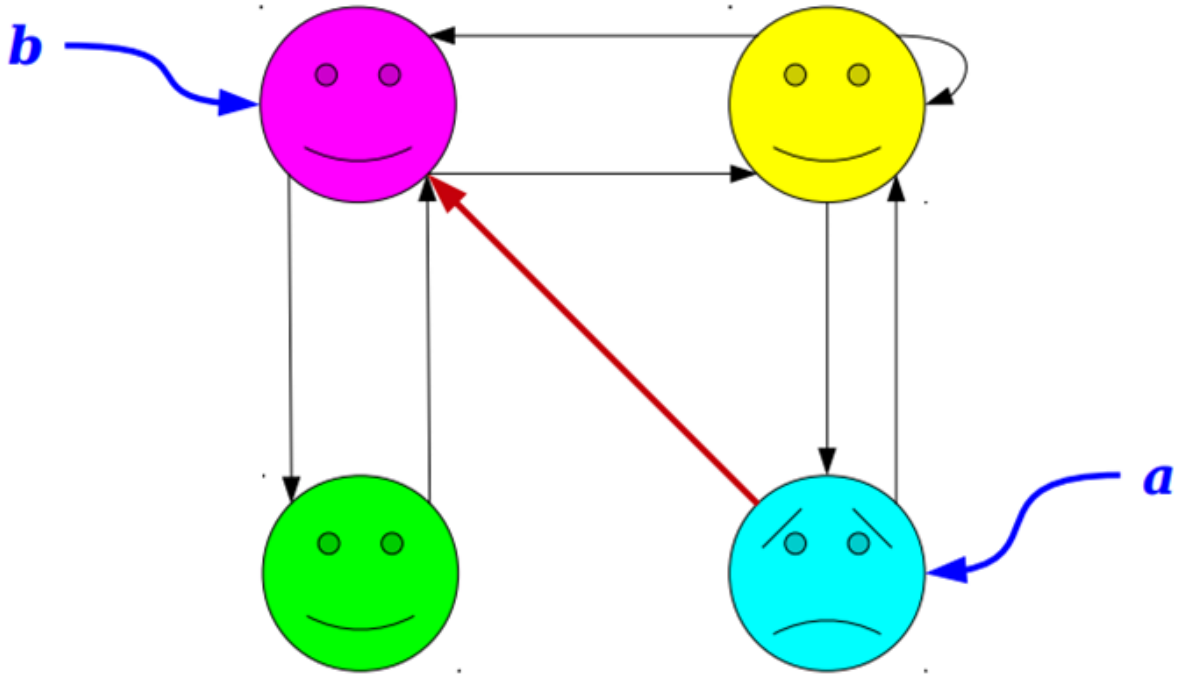
No



$$\forall a \in A.\ \forall b \in A.\ (aRb \rightarrow bRa)$$

("If a is related to b, then b is related to a.")

Is This Relation Symmetric?



$$\forall a \in A.\ \forall b \in A.\ (aRb \rightarrow bRa)$$

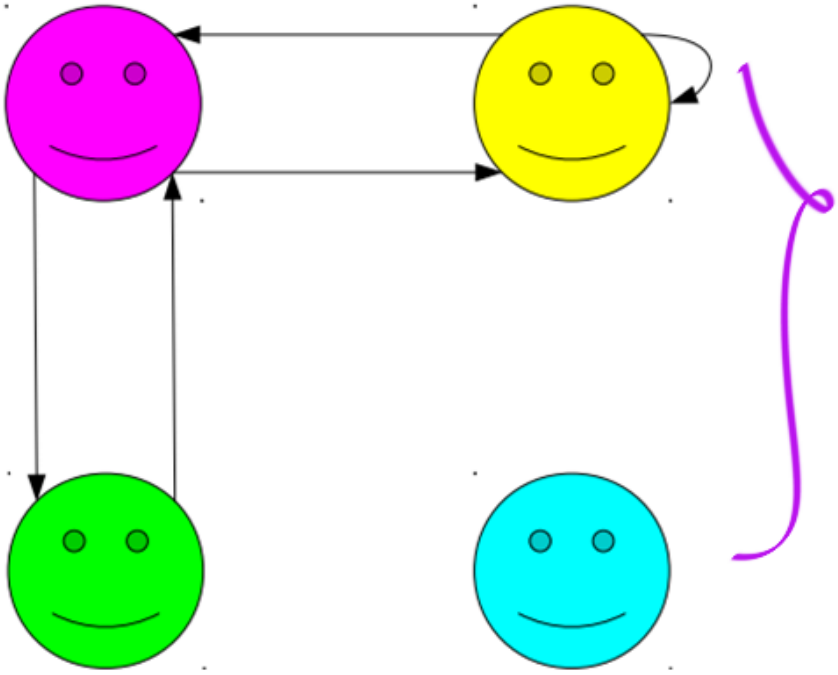
("If a is related to b, then b is related to a.")

"Not Symmetric" Relation:

A Rel R on set A, is NOT Symmetric iff

$$\exists_{a,b} \left(aRb \;\&\; bRa \right)$$

Graph Rep of Rel R



R is symm.

$$\exists a \in A.\ \exists b \in A.\ (aRb \wedge bRa)$$

("Some a relates to some b, but not vice-versa")

"Anti-Symmetric" Relation:

"Anti Symmetric" Relation:

for two different elements a, b

$aRb \;\&\; bRa$ should NOT happen.

"Anti Symmetric" Relation:

$R : A \longrightarrow A$

$x, y \in A$ where $x \neq y$

(1) $x \not R y$ and $y \not R x$: Allowed

(2) $x \not R y$ and $y R z$; "

(3) $x R y$ and $y \not R x$: "

(4) $x R y$ and $y R x$: → NOT Allowed

$x R x$ fine

$x \not R x$ fine

Assume, Base Set A ;

For any x, y ∈ A (Assume x,y are different elements):

- x R y & y R x — NOT Allowed
- x R y & y R x
- x R y & y R x
- x R y & y R x — Allowed
- x R x
- y R y

$x \neq y$

Antisym Rel.