Propositional Logic

Next Chapter:

Logical Connectives

"Implication"

- By Deepak Poonia (IISc Bangalore)

Instructor:

Deepak Poonia

IISc Bangalore

GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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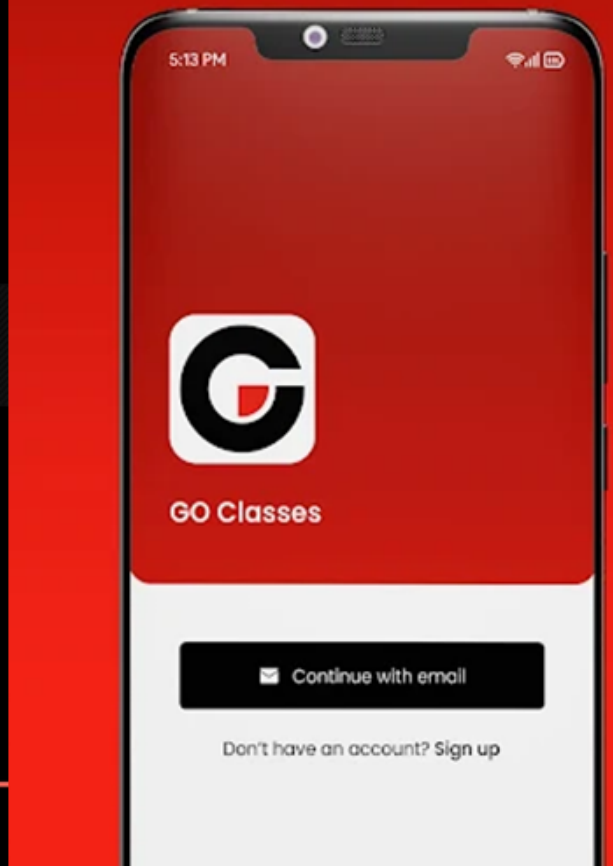

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Propositional Connectives

- **Logical NOT: $\neg p$**
  - Read "**not** p"
  - $\neg p$ is true if and only if p is false.
  - Also called **logical negation**.
- **Logical AND: $p \wedge q$**
  - Read "p **and** q."
  - $p \wedge q$ is true if both p and q are true.
  - Also called **logical conjunction**.
- **Logical OR: $p \vee q$**
  - Read "p **or** q."
  - $p \vee q$ is true if at least one of p or q are true (inclusive OR)
  - Also called **logical disjunction**.

Propositional Logic

Next Topic:

Logical Connectives

"Implication"

English Statement:

Today is Sunday and I am happy.

S H

Propositional Logic Expression:

$S \wedge H$

AND ≡ Conjunction

English Statement:

Today is Sunday OR I am happy.

S V H

Propositional Logic Expression:

S V H

English Statement:

Today is Sunday OR I am happy But Not Both.

S H

Propositional Logic Expression:

$S \oplus H$

Exclusive OR

English Statement:

Conditional Statement

If Today is Sunday then I am happy.

Condition

Consequence / Conclusion

Propositional Logic Expression:

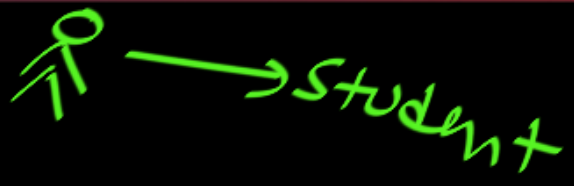
??

English Statement: → Conditional Statement (Implication Statement)

If Today is Sunday then I am happy.

Propositional Logic Expression: → ??

English Statement:

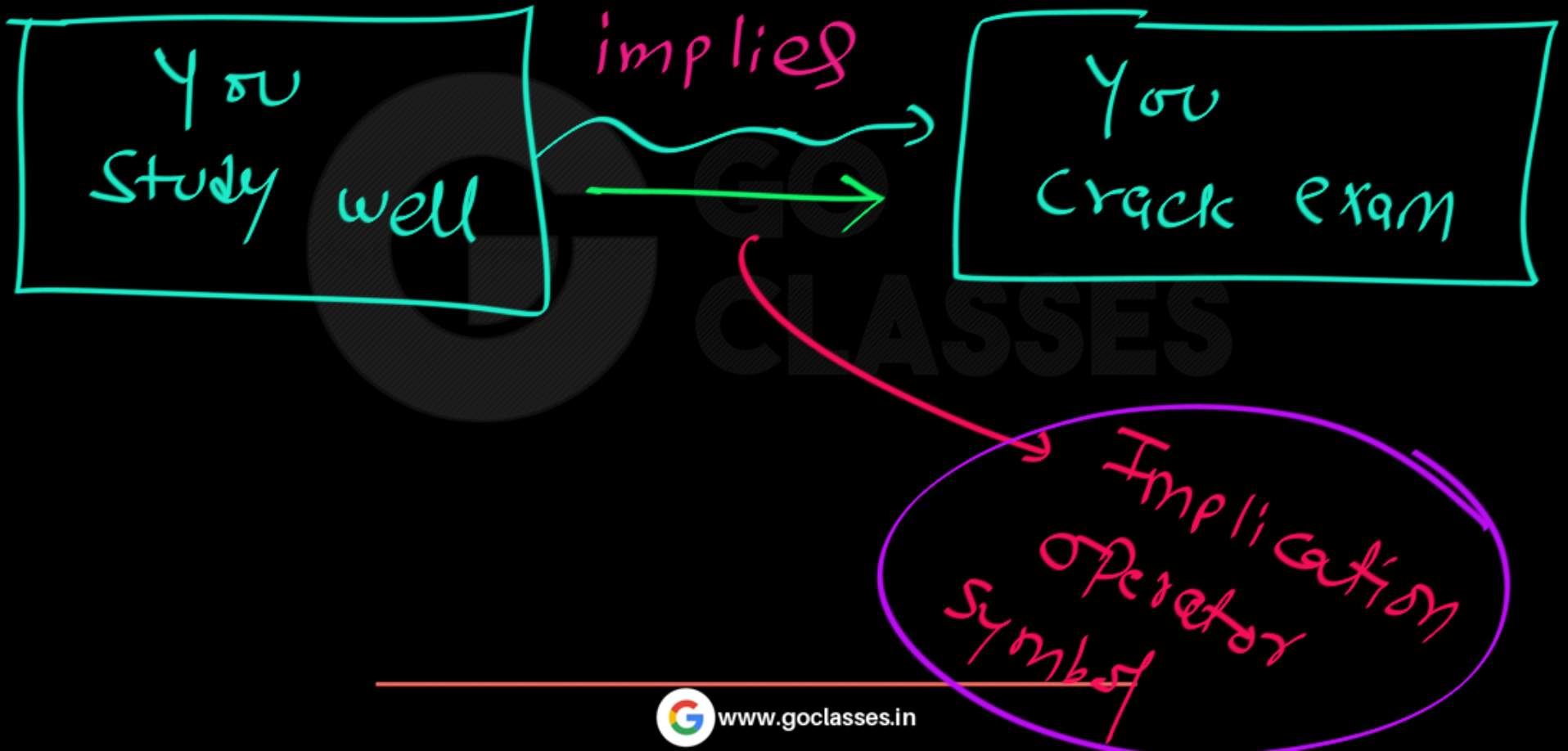


If you study well then you will crack exam.

Condition / Antecedent / Hypothesis / Premise

Conclusion / Consequence

Conditional Statement (Implication)


You Study well
implies
You Crack exam
Implication operator symbol

English Statement:

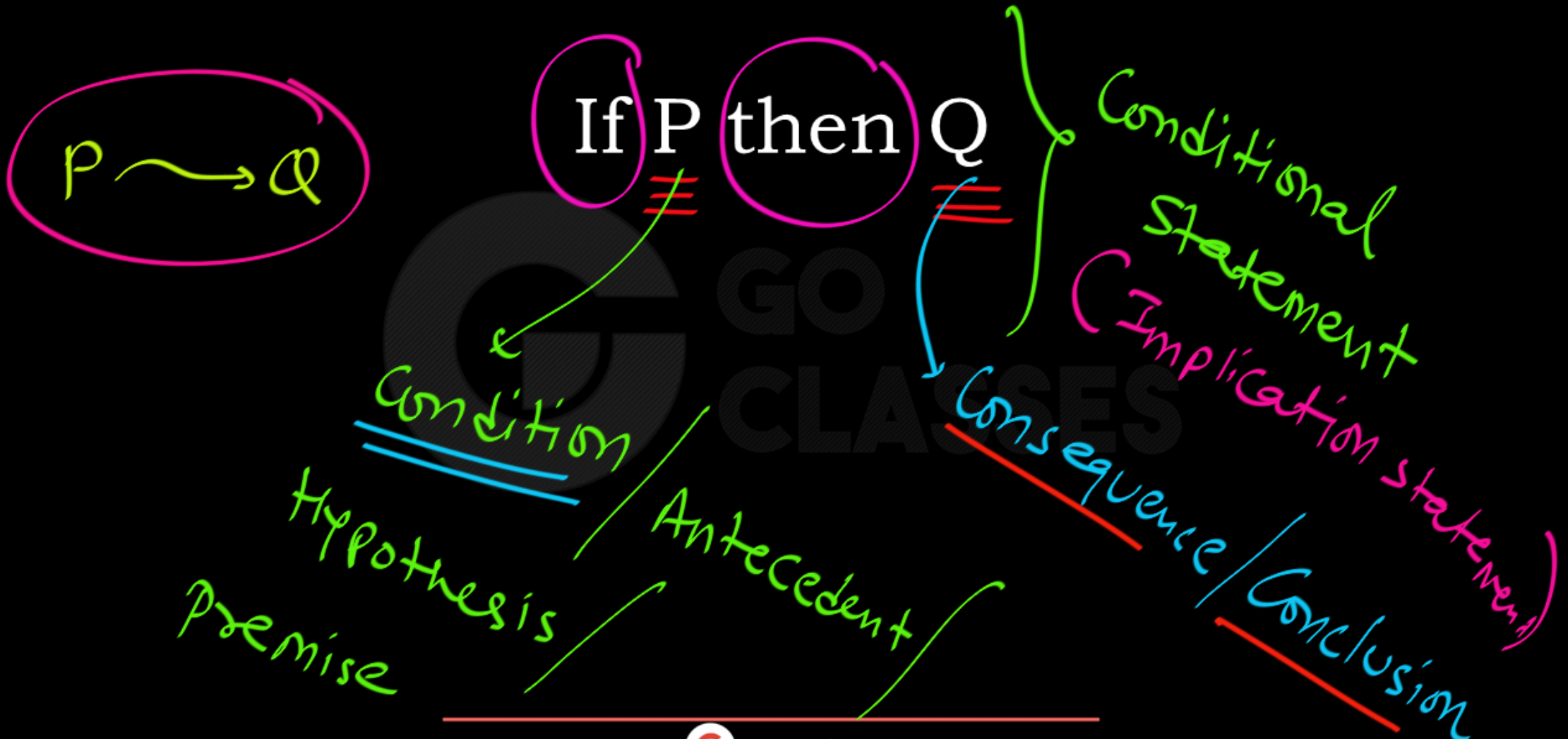
If You work for me then I'll give you money.

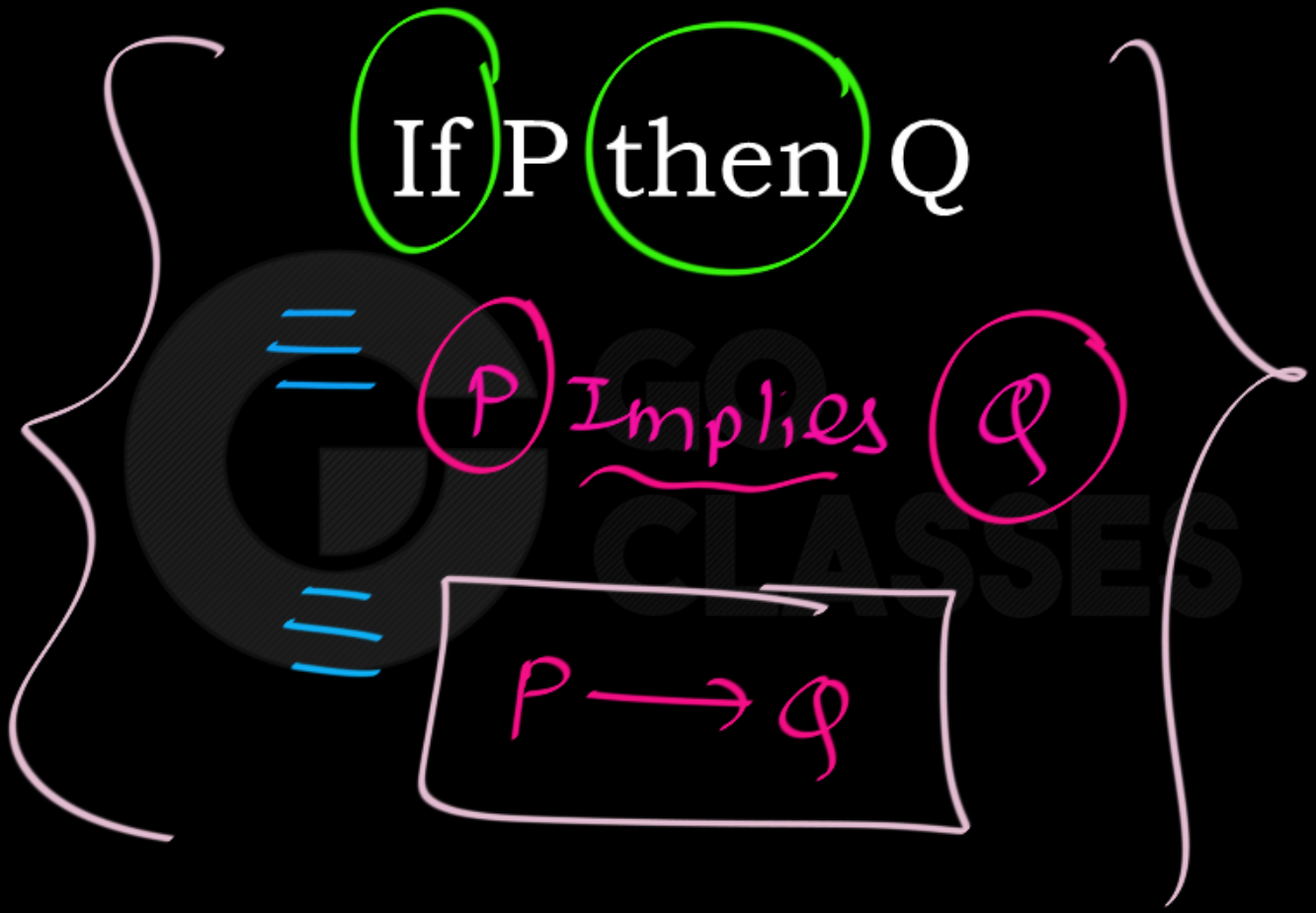
w m

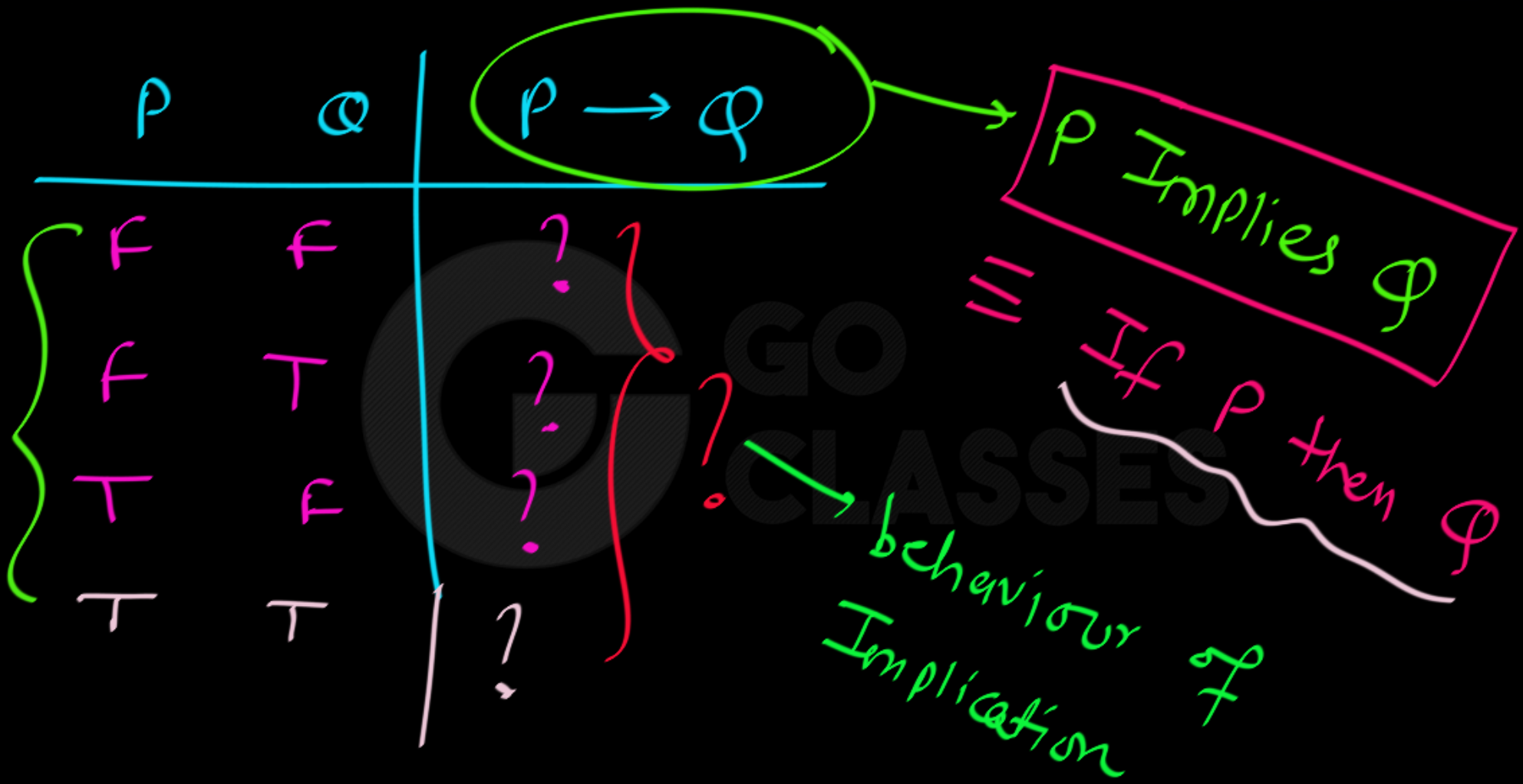
Propositional Logic Expression:

$w \rightarrow m$

Implication Symbol : $\rightarrow$


P → Q
If P then Q
Conditional statement
Implication statement
Condition
Hypothesis
Premise
Antecedent
Consequence / Conclusion


If P then Q
P Implies Q
P → Q


P
Q
P → Q
F F ?
F T ?
T F ?
T T ?
? → behaviour of Implication
P Implies Q
≡ If P then Q

Assume an Airline makes this Claim:

If You fly with us then You'll get free food.

When will this Claim be FALSE??

Assume an Airline makes this Claim:

If You fly with us then You'll get free food.

p: You fly with us
q: You'll get free food.

p	q	Claim
T	T	T
T	F	F
F	F/T	T

Assume an Airline makes this Claim:

If You fly with us then You'll get free food.

p

q

p	q	Claim
T	T	T
T	F	F
F	T	T
F	F	T

Assume an Airline makes this Claim:

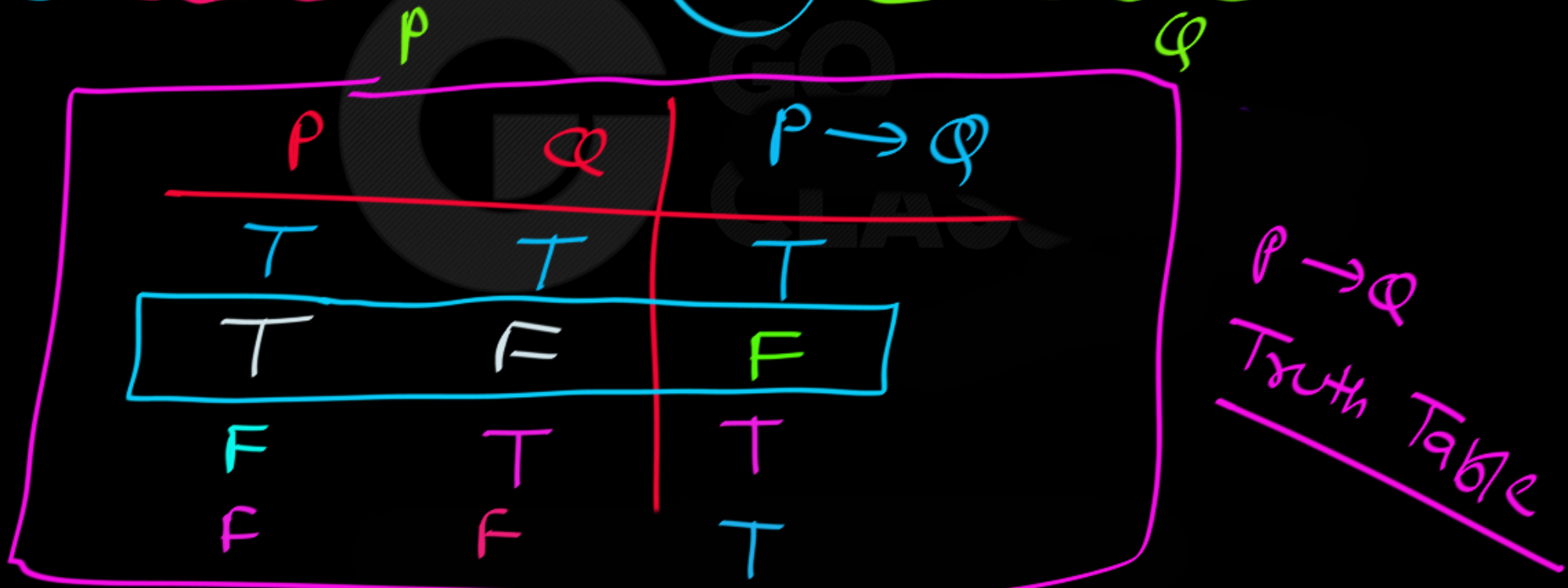
If You fly with us then You'll get free food.

p: You fly with us
q: You'll get free food.

p	q	Claim $(p \rightarrow q)$
T	T	T
T	F	F
F	T	T
F	F	T

Assume an Airline makes this Claim:

If You fly with us then You'll get free food.



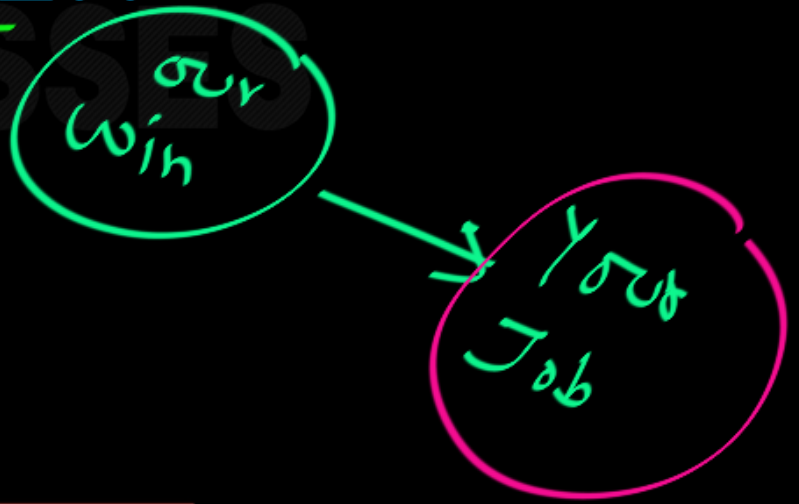
Assume a Political Party makes this Claim:

If we win then You'll get job.

W J

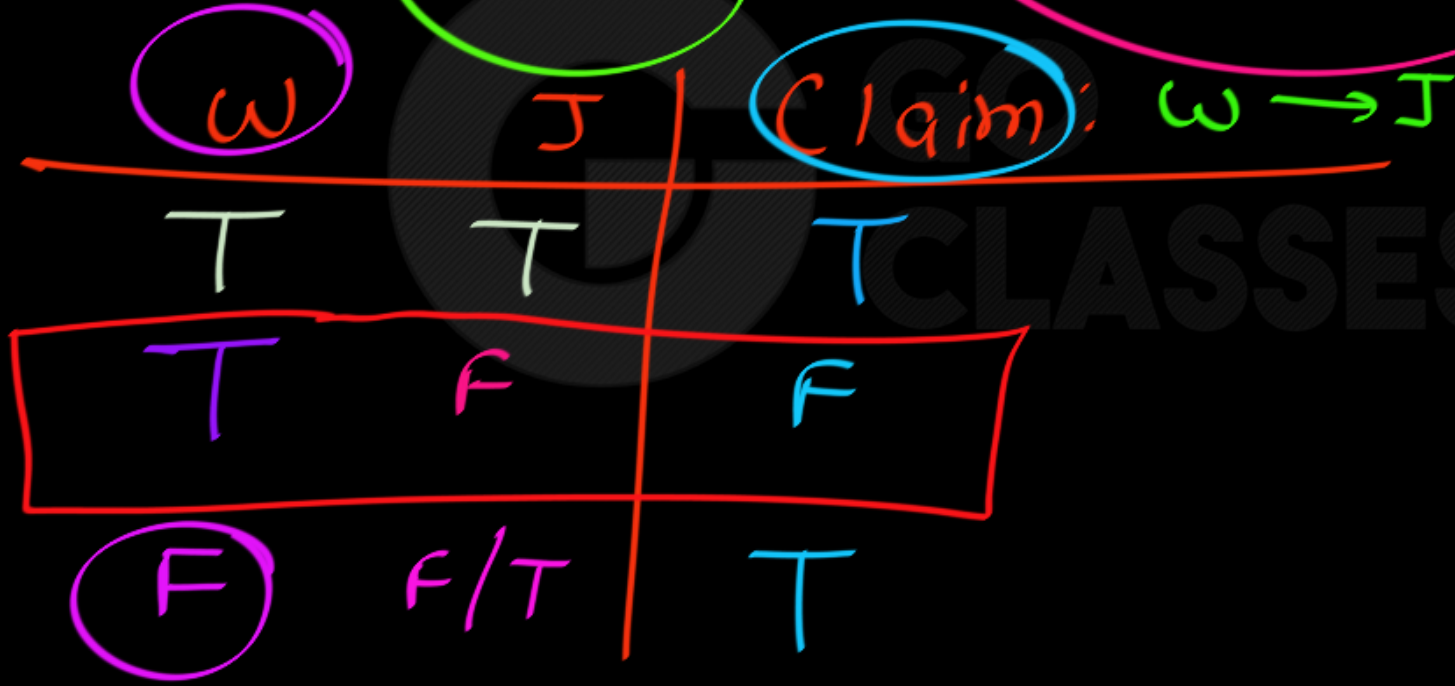
When will this Claim be FALSE??

Claim: $W \rightarrow J$



Assume a Political Party makes this Claim:

If we win then You'll get job.



Assume a Political Party makes this Claim:

If we win then You'll get job.

Assume Someone makes you a promise:

If You work for me then I'll give you money.

w m

When will this Promise be FALSE??

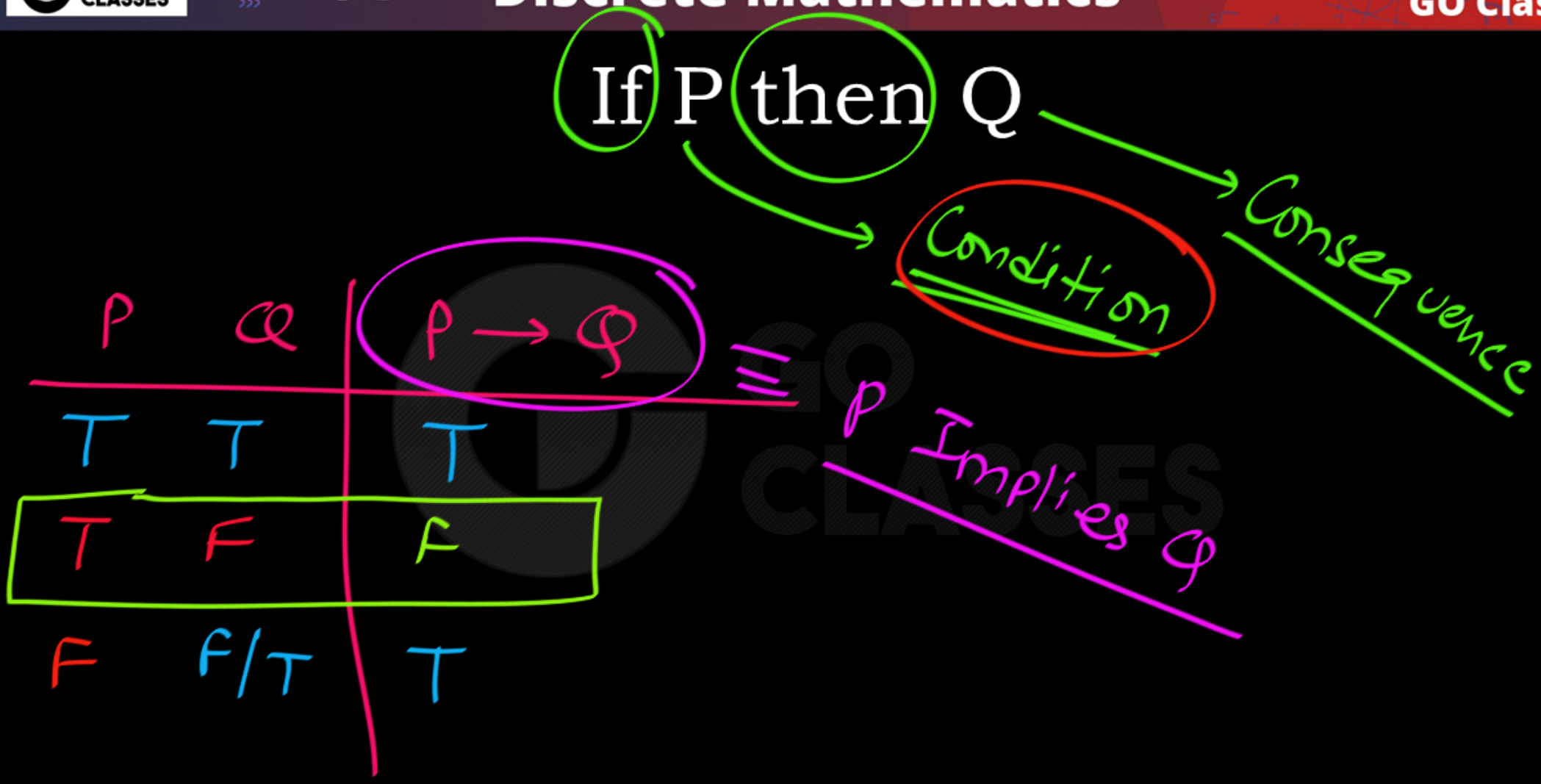
Promise : $w \rightarrow m$

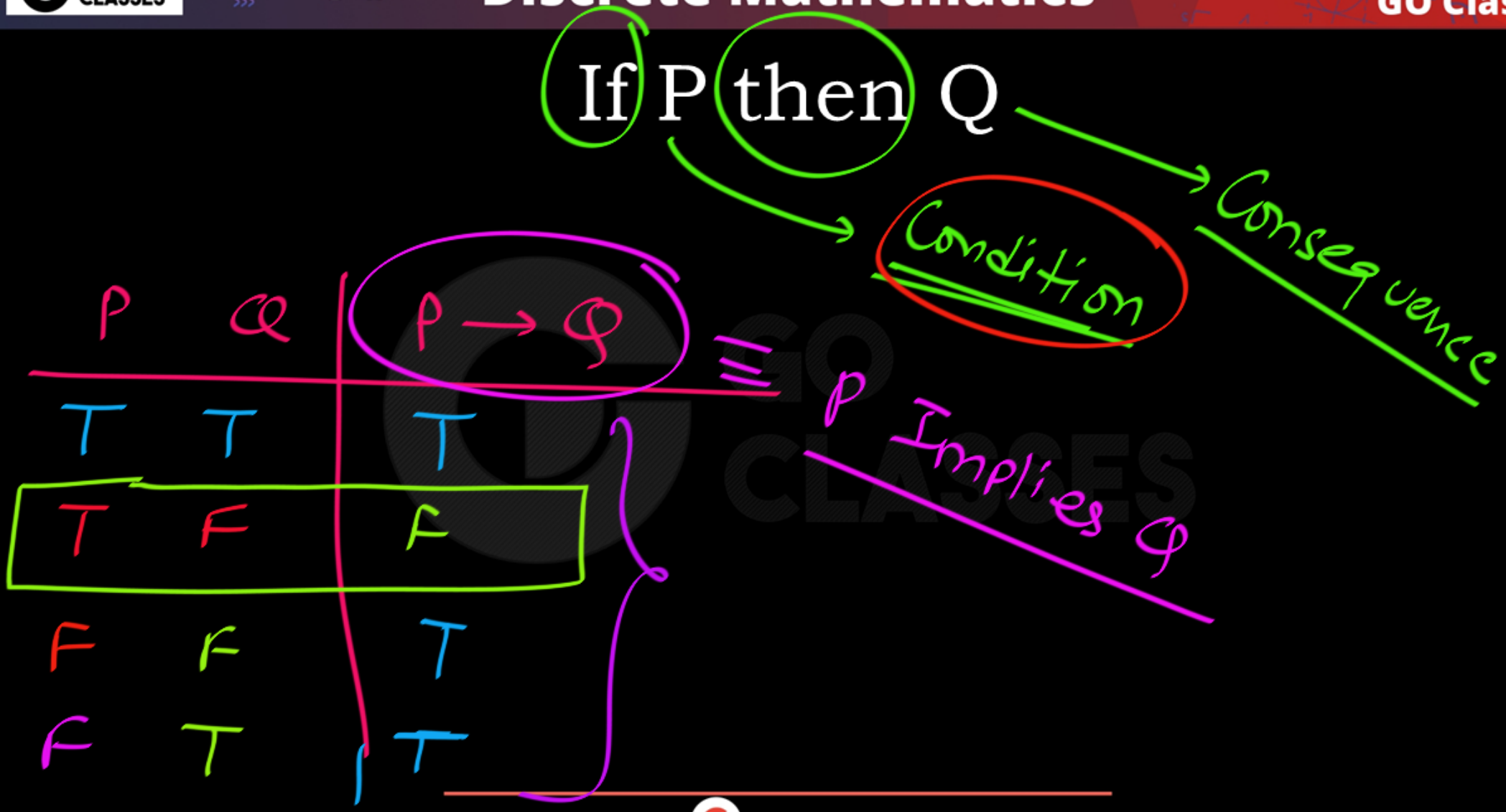
Assume Someone makes you a promise: $w \rightarrow m$

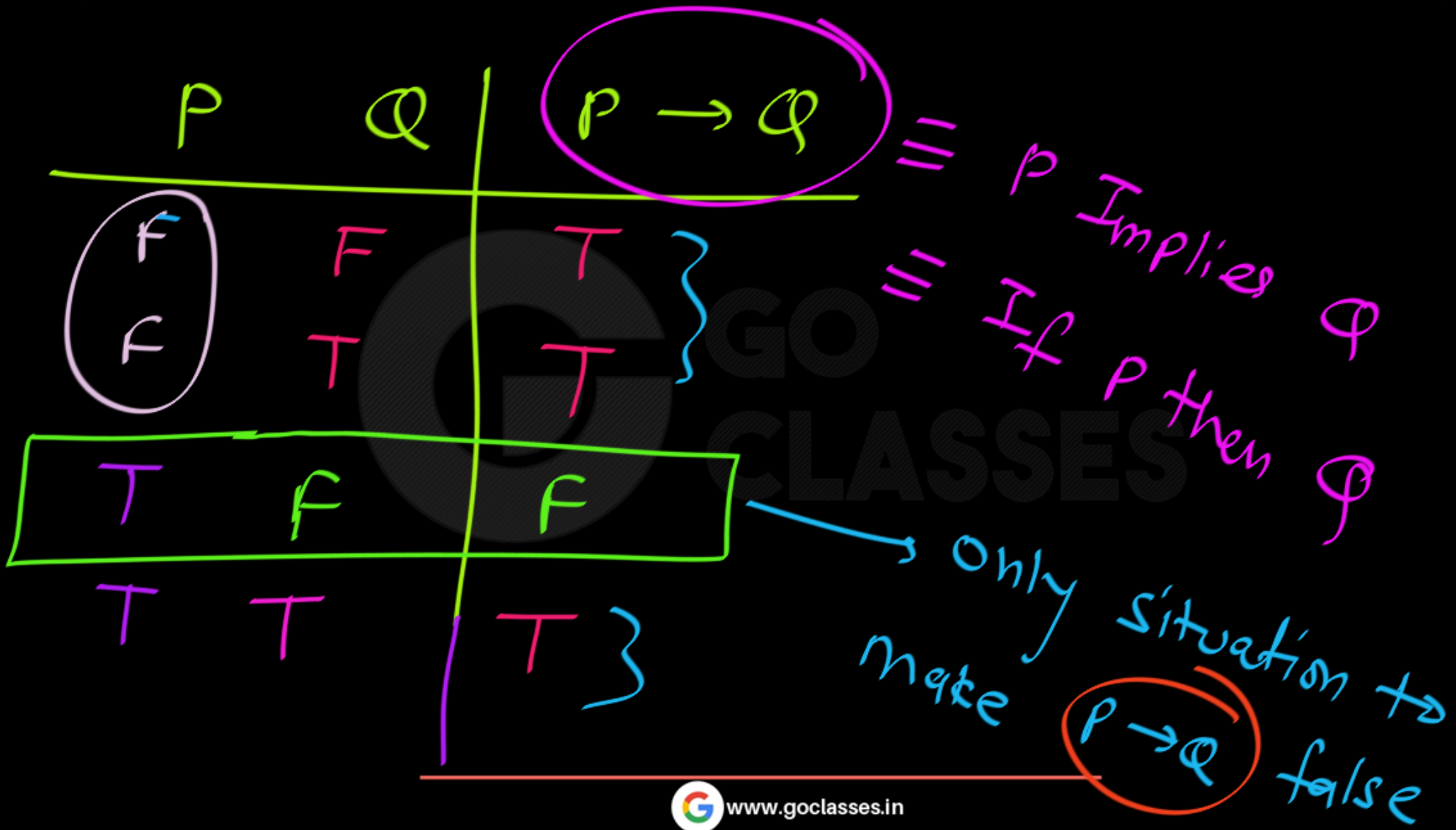
If You work for me then I'll give you money.

w m

w	m	$w \rightarrow m$
T	T	T
T	F	F
F	F	T
F	T	T


If P then Q
Condition
Consequence
P
Q
P → Q
T T T
T F F
F F/T T
≡ P Implies Q


If P then Q
Condition
Consequence
P
Q
P → Q
T T T
T F F
F F T
F T T
≡ P Implies Q


P
Q
P → Q
F
F
T
F
T
T
T
f
F
T
T
T
≡ P Implies Q
≡ If P then Q
only situation to make P → Q false

$p \longrightarrow q$ is false

only when

$p = T$ and $q = f$

Assume a Father Promises his child:

If You study today then I'll let you watch TV.

S ω

When will this Promise be FALSE??

Assume a Father Promises his child:

If You study today then I'll let you watch TV.

S ω

S	ω	S → ω
T	T	T
T	F	F
F	F	T
F	T	T

S → ω promise

T T T

T F F → promise Broken

F F T

F T T

$P \rightarrow Q$

false only when

P = True
and
Q = False

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Here is the truth table for an implication:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

You can think of an implication as a conditional promise. If you keep the promise, it's true. If you break the promise, it's false.

If I tell my kids, "I'll give you a cookie if you clean up." Then they clean up. I better give them a cookie. If I don't, I've lied. However, if they don't clean up, I can either give them a cookie or not. I didn't promise either if they didn't keep up their end of the bargain.

So in other words, an implication is false only if the hypothesis is true and conclusion is false.

Conditional Statements

Implications. Implication Operator, "if...then...", has symbol $\rightarrow$.

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

p: This book is interesting. q: I am staying at home.

p → q: If this book is interesting, then I am staying at home.

Implication operator

Implication ≡ Conditional Promise

$P \rightarrow Q$ means :

- P = True → Q = True
- P = False → Q = F, Q = T

Implication ≡ Conditioned Promise

$P \rightarrow Q$ is True

1. $P = T$, $Q = T$
2. OR $P = F$, $Q = F$
3. OR $P = F$, $Q = T$

3 situations

Implication

- An important connective is logical implication: $p \rightarrow q$.
- Recall: $p \rightarrow q$ means "if p is true, q is true as well."
- Recall: $p \rightarrow q$ says **nothing** about what happens if p is false.
- Recall: $p \rightarrow q$ says **nothing** about causality; it just says that if p is true, q will be true as well.

Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

p → q means that if we ever find that p is true, we'll find that q is true as well.

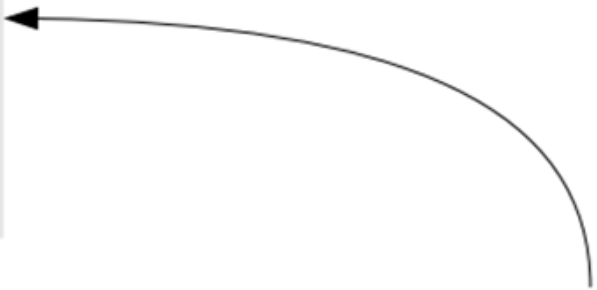
Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	

p → q should mean when p is true, q is true as well. But here p is true and q is false!

Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	
T	T	

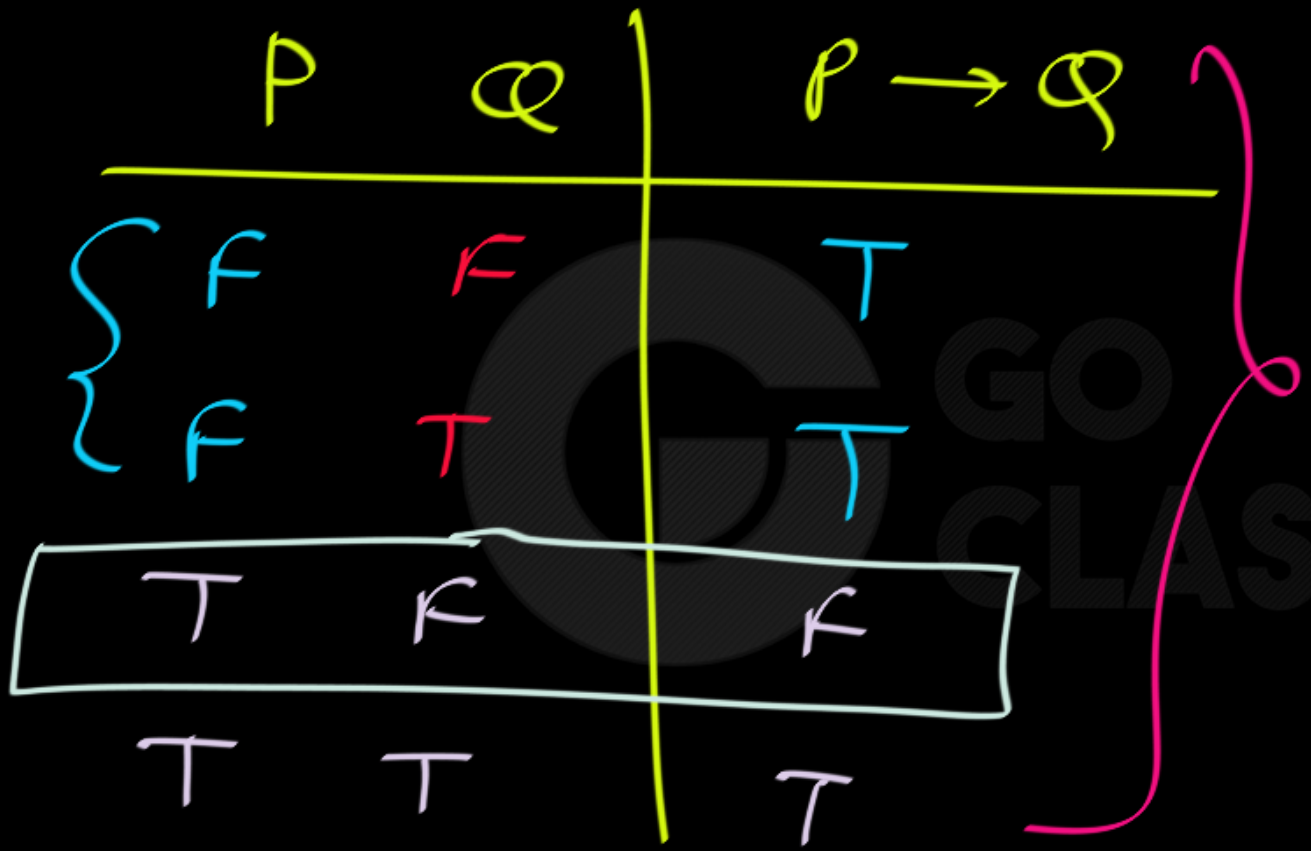


In both of these cases, p is false, so the statement "if p, then q" is vacuously true.

Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

The only way for $p \rightarrow q$ to be false is for p to be true and q to be false.


P
Q
P → Q
F
F
T
F
T
T
T
F
F
T
T
T

Truth Table for Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

The implication is only false if p is true and q isn't. It's true otherwise.

You will need to commit this table to memory. We're going to be using it *a lot* over the rest of the week.

The implication p → q is the proposition that is often read "if p then q."

"If p then q" is false precisely when p is true but q is false.

One way to think of the meaning of p → q is to consider it a contract that says if the first condition is satisfied, then the second will also be satisfied. If the first condition, p, is not satisfied, then the condition of the contract is null and void. In this case, it does not matter if the second condition is satisfied or not, the contract is still upheld.

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When p Does Not Imply q

- $p \rightarrow q$ means “if p is true, q is true as well.”
- Recall: The **only way** for $p \rightarrow q$ to be false is if we know that p is true but q is false.
- Rationale:
  - If p is false, $p \rightarrow q$ doesn't guarantee anything. It's true, but it's not **meaningful**.
  - If p is true and q is true, then the statement “if p is true, then q is also true” is itself true.
  - If p is true and q is false, then the statement “if p is true, q is also true” is false.

Conditional Statements are One-Way:

$P \rightarrow Q$ Means :

P = True $\longrightarrow$ Q = True

P = false $\longrightarrow$ Q = True or false

$P \rightarrow Q$ means:

$P = True$ ⟶ $Q = True$ ✓

$P = True$ ⟵ $Q = True$ ✗

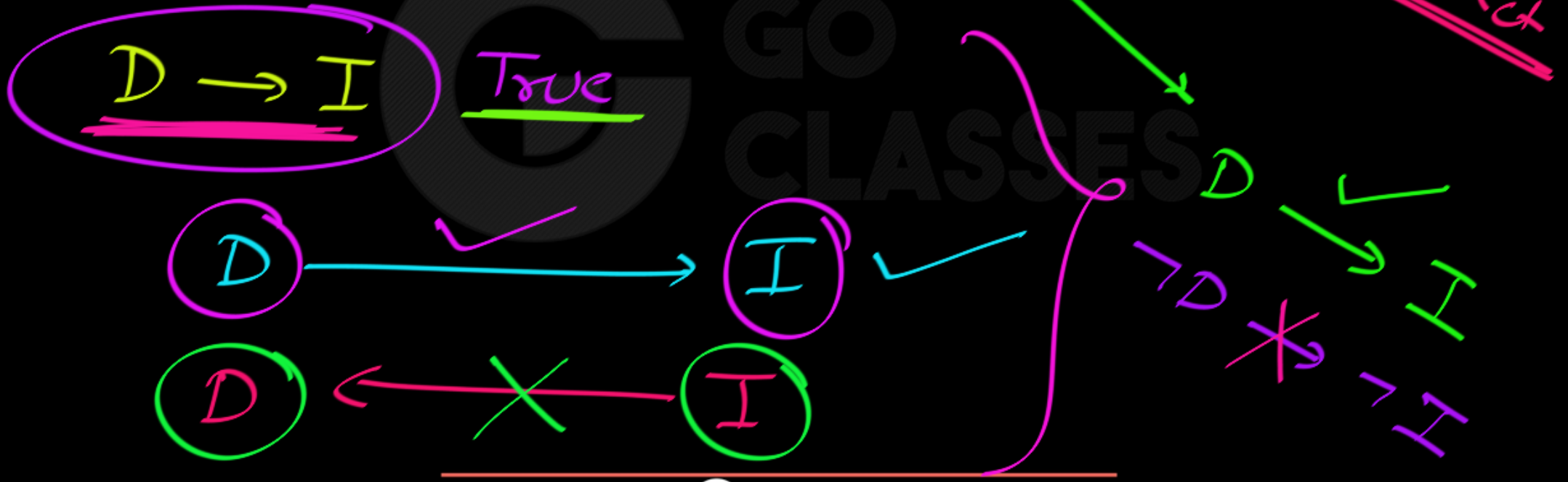
If you are in Delhi, then You are in India.

D

I

Correct

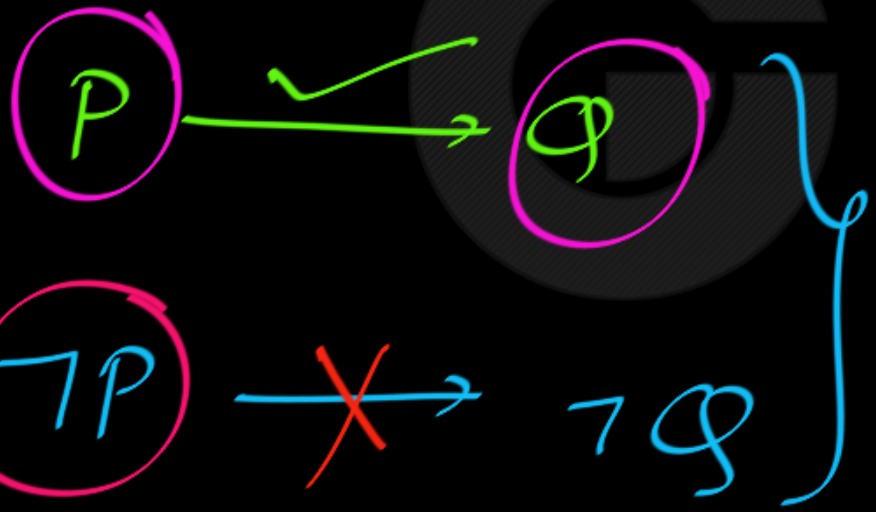
$D \rightarrow I$ True



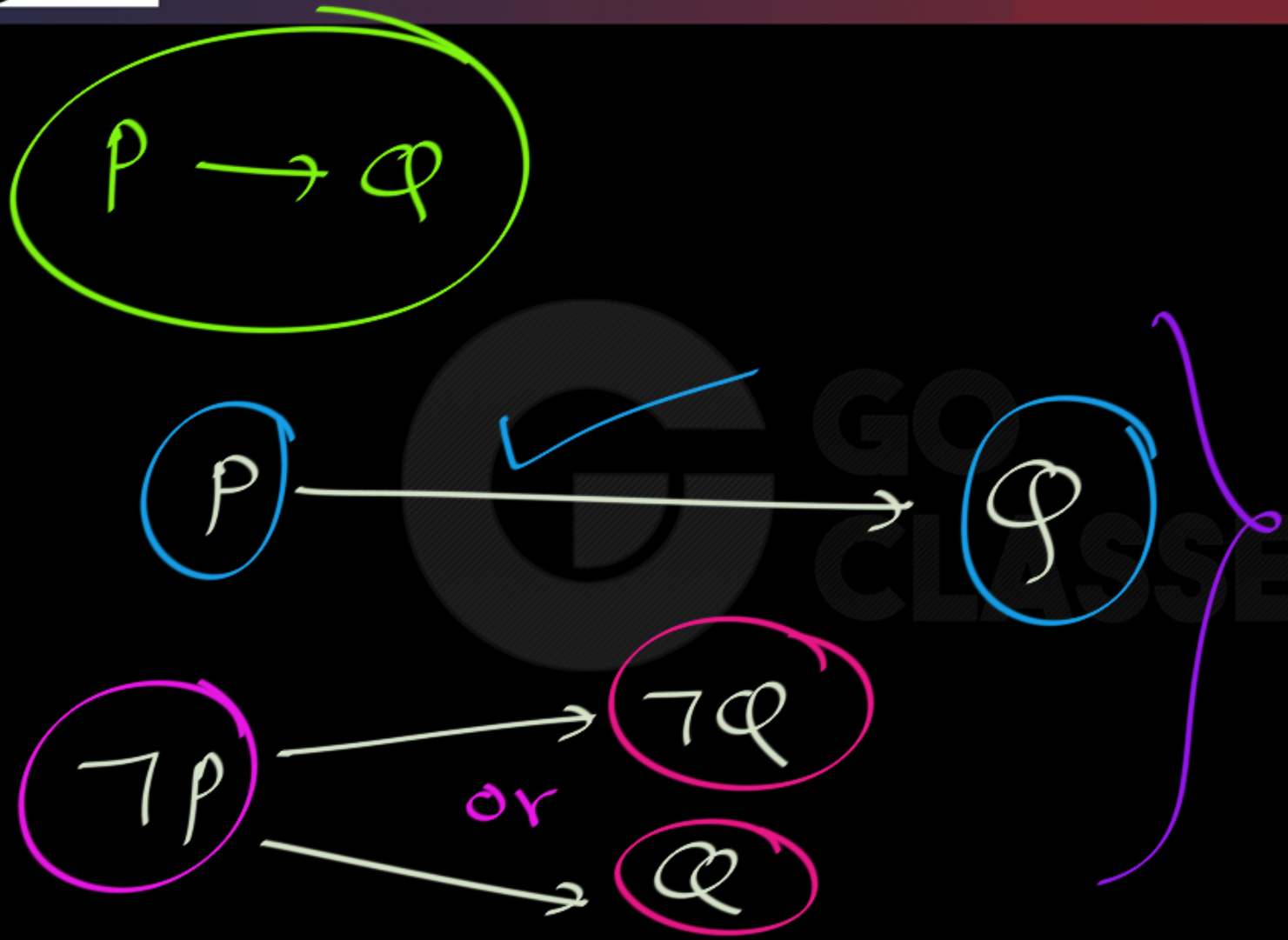
If Score is 2, then Score is even.

p q

Correct



Score : 4, 5


P → Q
P
Q
¬P
¬Q
or
Q

$$P \rightarrow Q \equiv Q \leftarrow P$$

Only ONE WAY to make

$(P \rightarrow Q)$ False

Three WAYs to make $(P \rightarrow Q)$ True

Conditional Connective : (Implication)

$\rightarrow$

$p \rightarrow q$

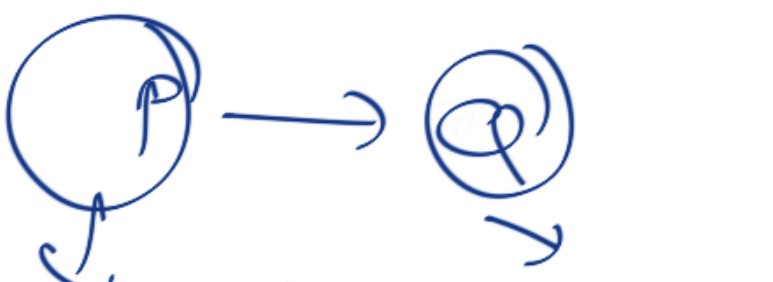
$T \rightarrow T$	True
$T \rightarrow F$	false
$F \rightarrow T$	True
$F \rightarrow F$	True

The ONLY situation In which Implication Statement is false :

Condition = True
and
Conclusion = false

$P \rightarrow Q$ is True when P is false or Q is True.

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T



3 ways in which Implication Statement $\underbrace{P \longrightarrow Q}$ is True:

$$\left\{ \begin{array}{l} P = T, \; Q = T \\ P = F, \; Q = T \\ P = F, \; Q = F \end{array} \right\}$$

Conditional Statement:

If p then q $\equiv$ $p \rightarrow q$

when p True ⟶ q True

But when p false, q True or q false

So, Conditional statements are one way. $p \rightarrow q$

Propositional Variables : p, q

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

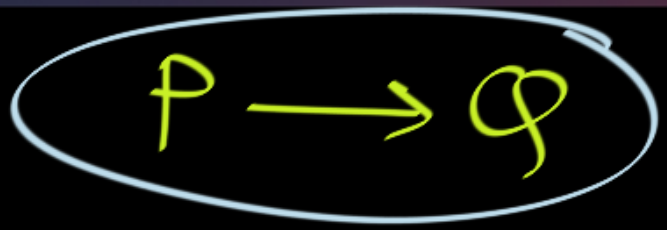
$\equiv$ p Implies q

$\equiv$ If p then q

Next Topic:

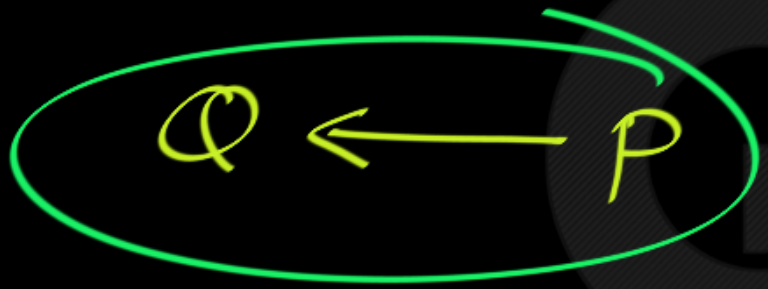
Implication Continued...

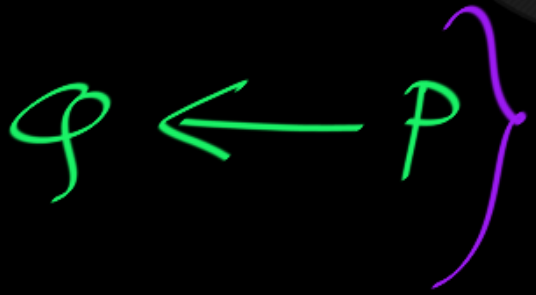
P->Q Vs Q->P



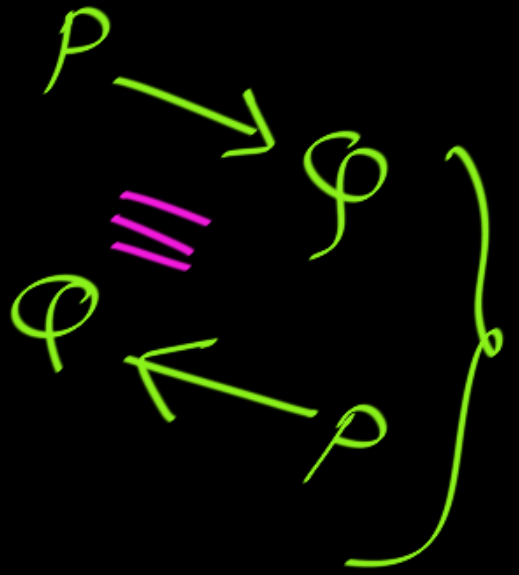
P Implies Q

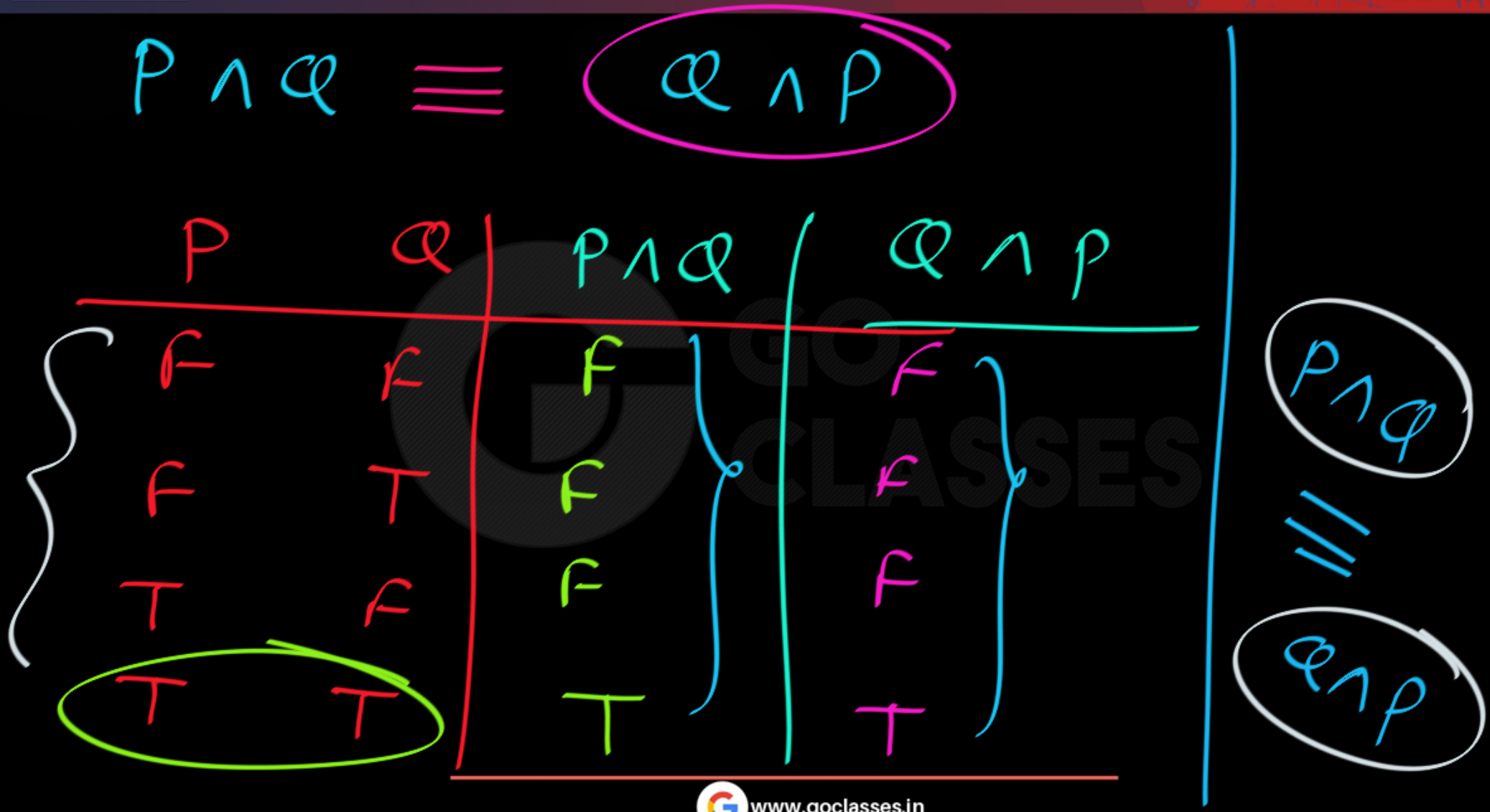
|||

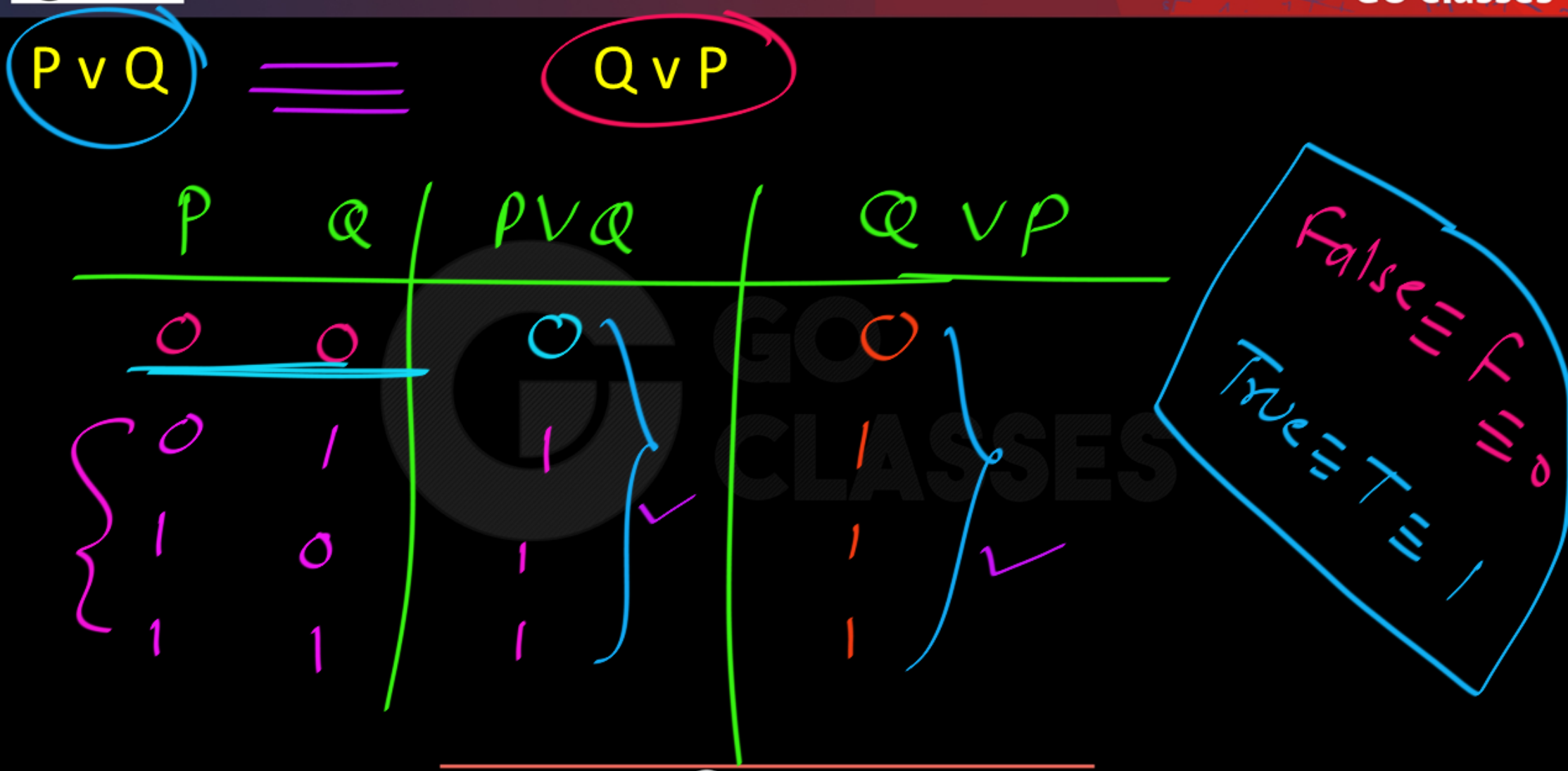




P Implies Q




P ∧ Q ≡ Q ∧ P
P
Q
P ∧ Q
Q ∧ P
F F F F
F T F F
T F F F
T T T T
P ∧ Q
≡
Q ∧ P

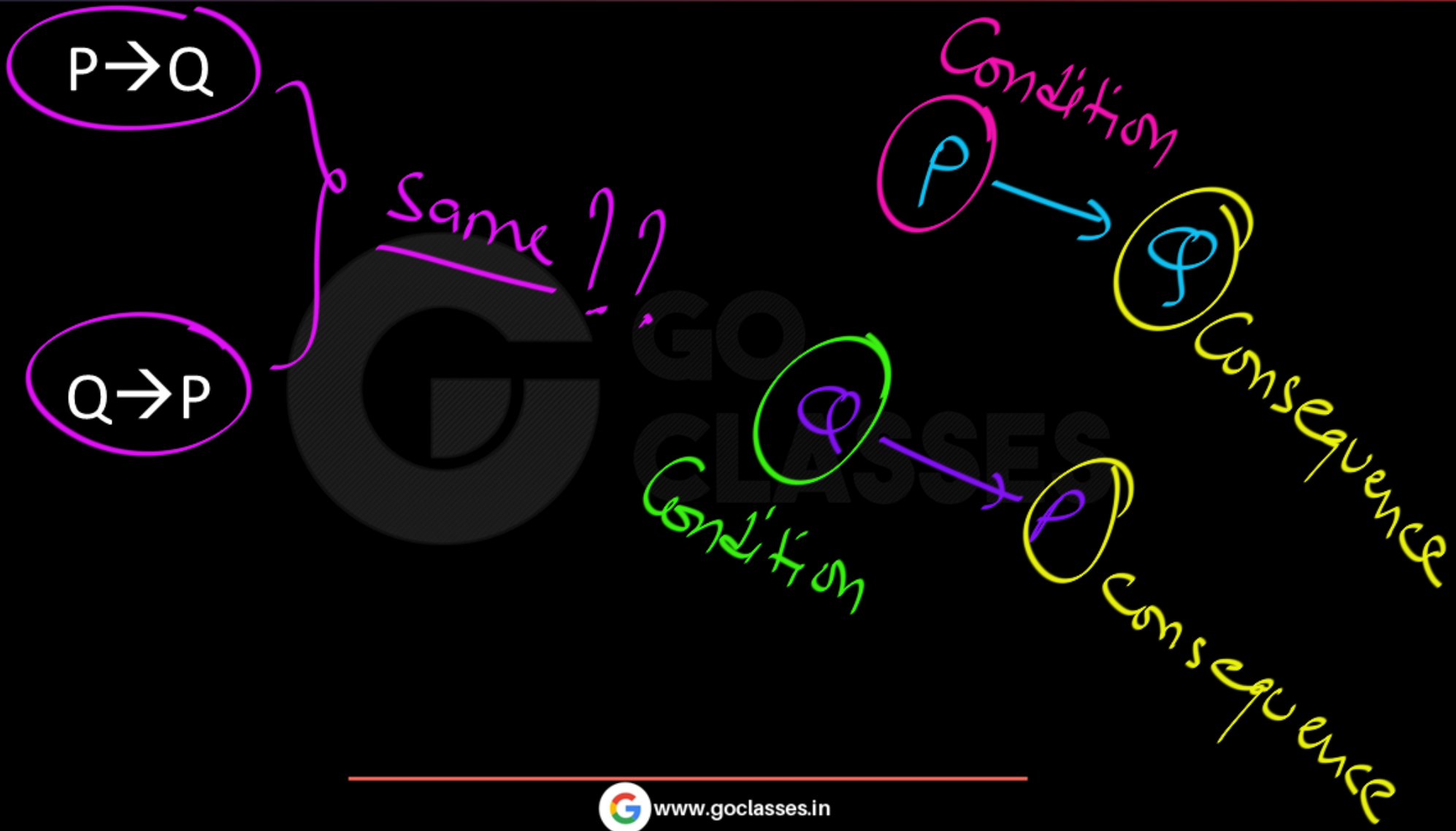

P v Q
Q v P
P Q P∨Q Q∨P
0 0 0 0
0 1 1 1
1 0 1 1
1 1 1 1
False = F = 0
True = T = 1

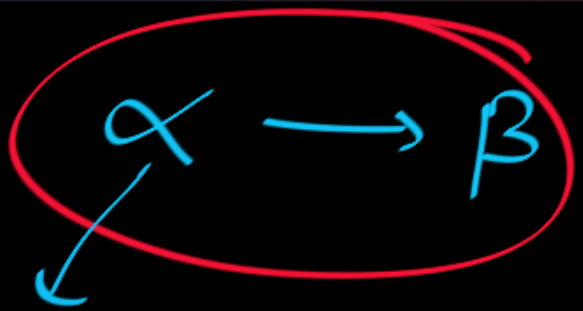
$P \oplus Q \equiv Q \oplus P$

Ex-or

P	Q	$P \oplus Q$	$Q \oplus P$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

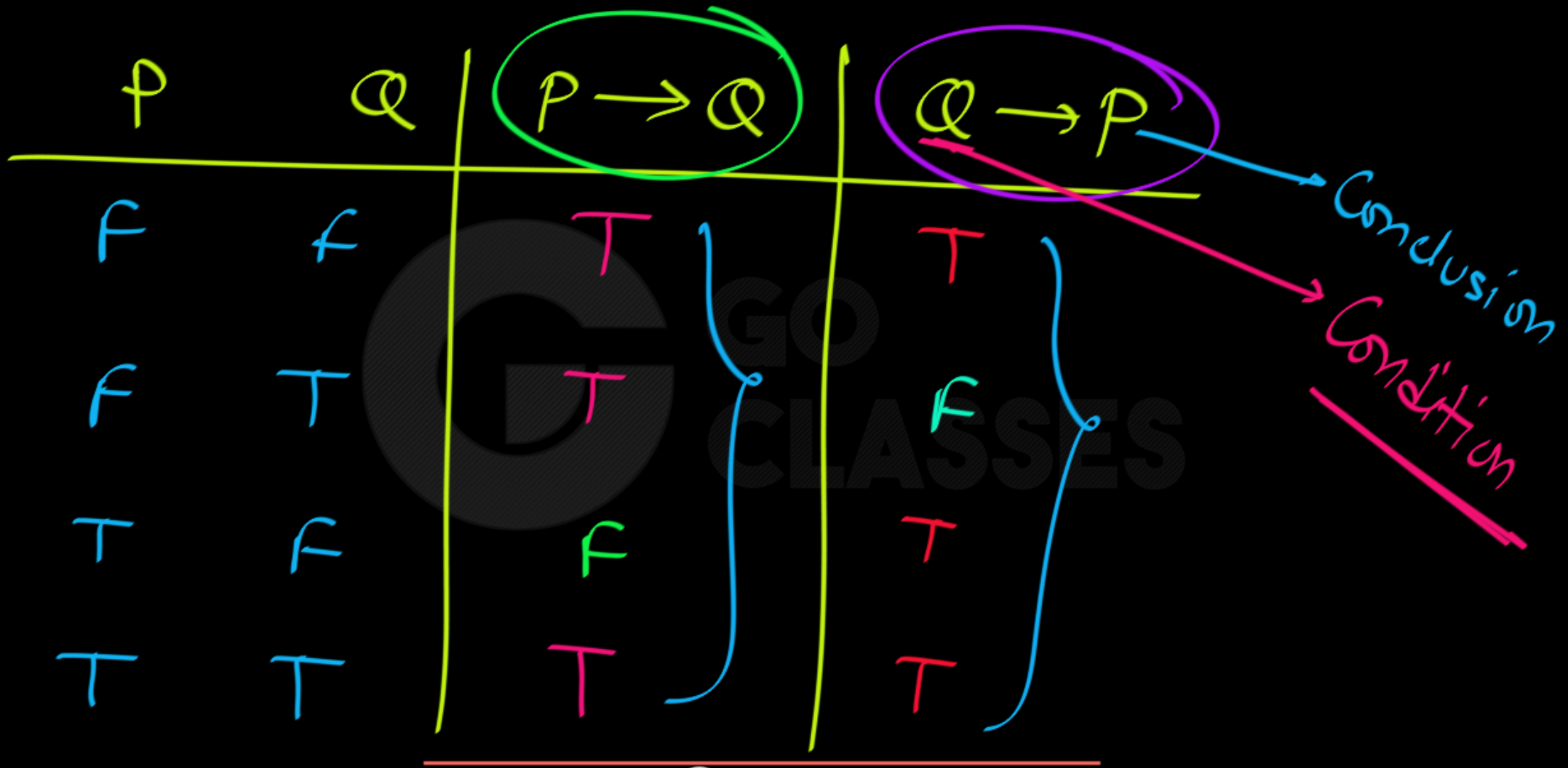
$P \oplus Q \equiv Q \oplus P$

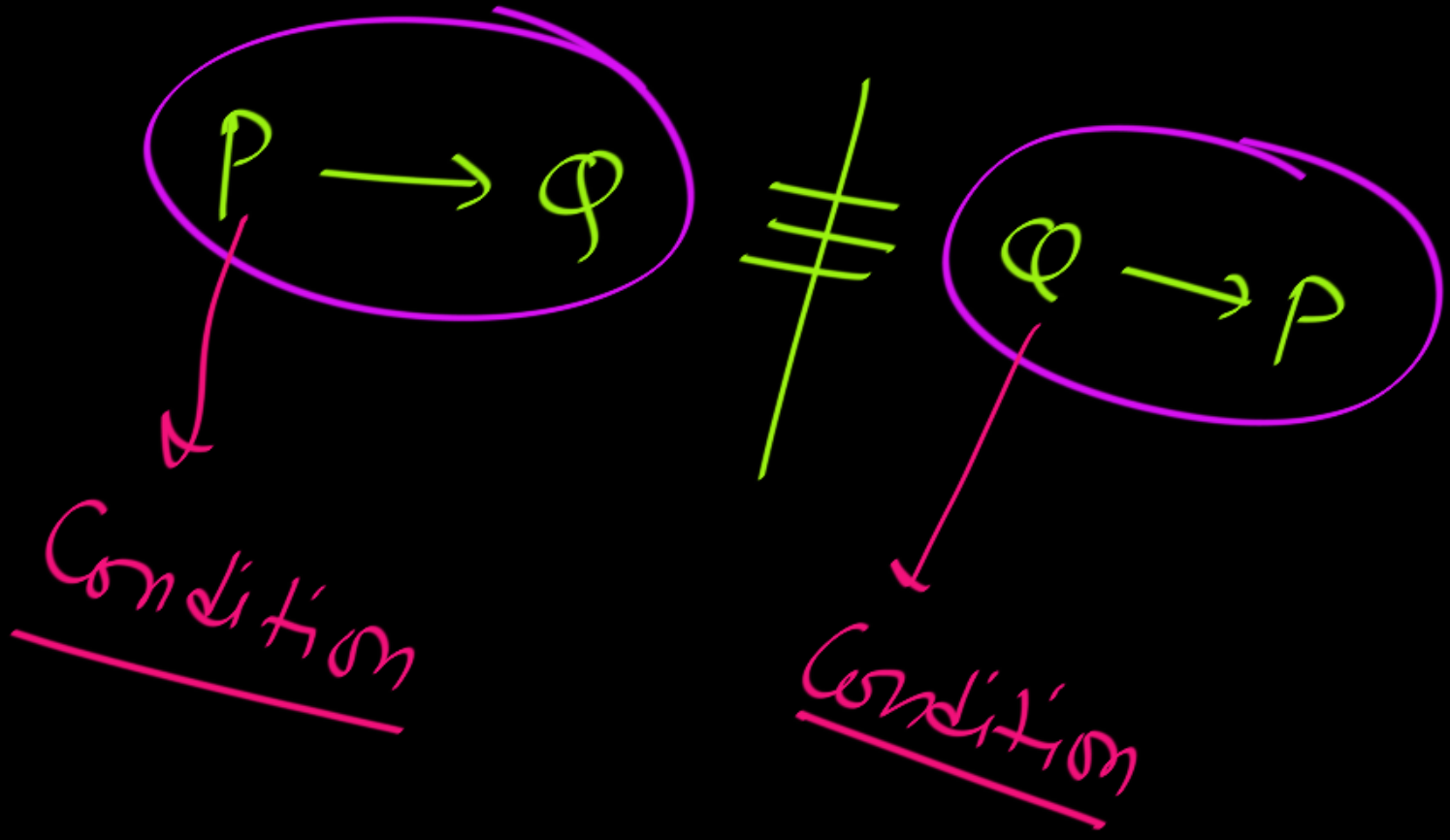

P→Q
Q→P
Same ??
Condition
P
Consequence
Q
Condition
Q
Consequence
P



Condition

α	β	α → β
F	F	T
F	T	T
T	F	F
T	T	T


p
q
p → q
q → p
T
F
T
F
T
F
T
F
T
T
T
T
Conclusion
Condition


$p \rightarrow q$
$\not\equiv$
$q \rightarrow p$
Condition
Condition

P : You are PM of India

Q : You are Indian.

$P \rightarrow Q$ ∫ ??

$Q \rightarrow P$

P : You are PM of India

Q : You are Indian.

$P \rightarrow Q$ True

$Q \rightarrow P$ False

$P \rightarrow Q \neq Q \rightarrow P$

If Q then P

A : It is a natural number

B : It is an integer.

$A \longrightarrow B$ True

If A then B

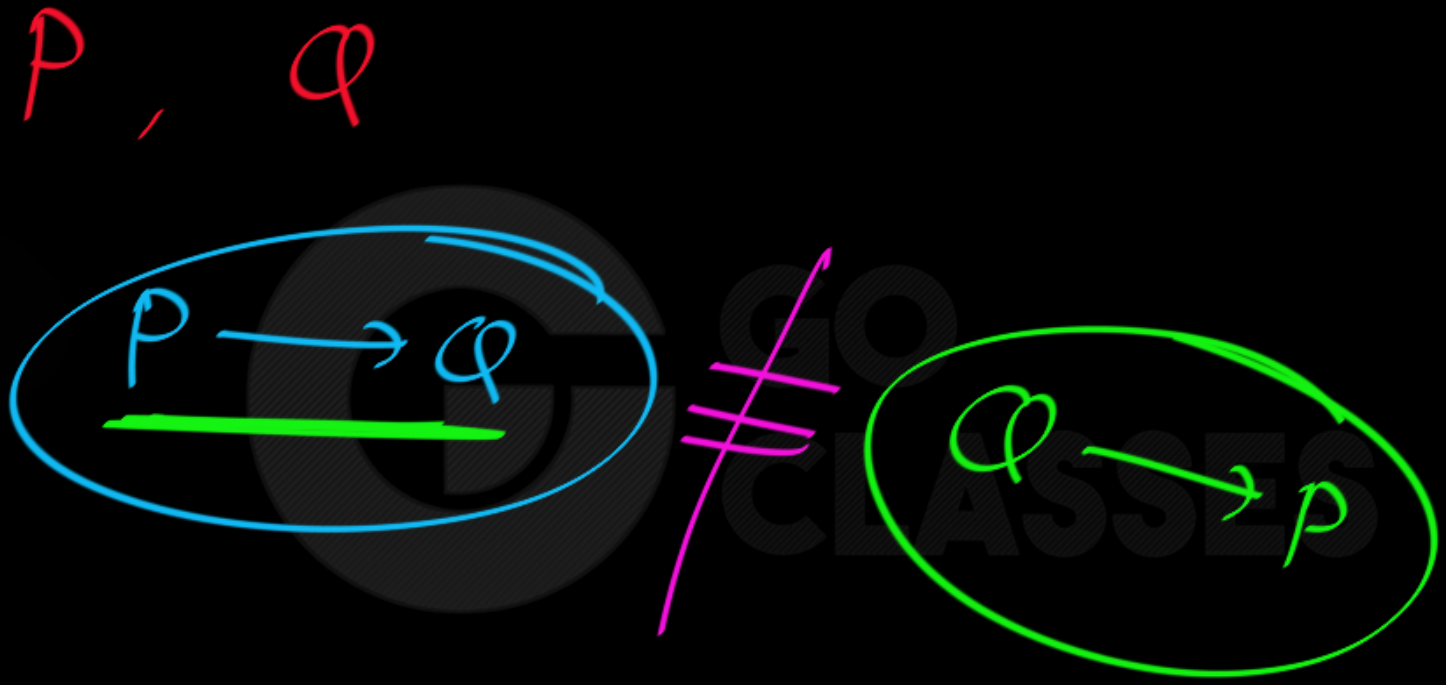
$B \longrightarrow A$ False

If B then A

-2

So, $(p \rightarrow q)$ doesn't mean that $(q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T


P , Q
P → Q
≠
Q → P

Next Topic:

Implication Continued...

Necessary & Sufficient Conditions