Sachin Mittal

Co-founder and Instructor at GO Classes
MTech IISc Bangalore
Ex Amazon Applied Scientist
GATE AIR 33

Logarithms

....are another way of thinking about a^x

Logarithms

- Definition
- Log Properties and their proofs
- Exercise questions
- Log Graph
- Importance of Log in Computer Science

$2^3 = 8$

$2^5 = 32$

$3^4 = 81$

$\log_2 8 = 3$

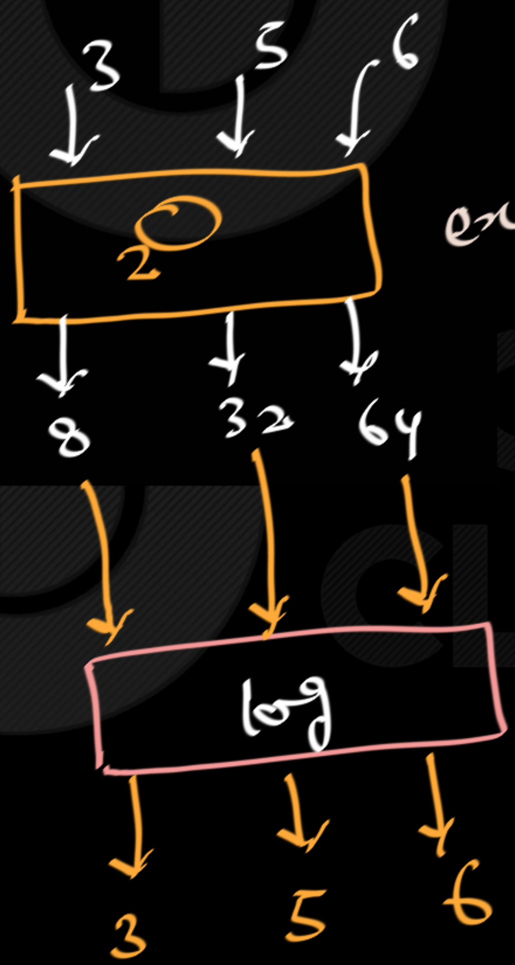
$\log_2 32 = 5$

$\log_3 81 = 4$

$\log_2 8 = 3$

$\log_2 32 = 5$

$\log_3 81 = 4$



exponent m/c

log m/c

$$\boxed{\log_b x = y \iff x = b^y}$$

$x = b^y$

$\log_b x = \log_b b^y$

$y \log_b b$

$= y$

or

$\log_b x = y$

Evaluate the logarithmic expressions.

a. $\log_{10} 10{,}000$ **b.** $\log_5\left(\frac{1}{125}\right)$ **c.** $\log_{1/2}\left(\frac{1}{8}\right)$

$x = b^y \iff \log_b x = y$

$\log_{10} 10{,}000 = 4$

$\log_{10} 10{,}000 = y$

$10{,}000 = 10^y$

$\Rightarrow y = 4$

https://people.ucsc.edu/~miglior/chapter%20pdf/Ch10_SE.pdf

Evaluate the logarithmic expressions.

a. $\log_{10} 10{,}000$ **b.** $\log_5\left(\frac{1}{125}\right)$ **c.** $\log_{1/2}\left(\frac{1}{8}\right)$

$\log_5\left(\frac{1}{125}\right) = y$

$\Rightarrow 5^y = \frac{1}{125} = \frac{1}{(5)^3} = 5^{-3}$

$\Rightarrow \underline{y = -3}$

https://people.ucsc.edu/~miglior/chapter%20pdf/Ch10_SE.pdf

Evaluate the logarithmic expressions.

a. $\log_{10} 10{,}000$ **b.** $\log_5\left(\frac{1}{125}\right)$ **c.** $\log_{1/2}\left(\frac{1}{8}\right)$

$\log_{1/2} 1/8 = y$

$(1/2)^y = \frac{1}{8} \Rightarrow \left(\frac{1}{2}\right)^y = (1/2)^3$

$\Rightarrow \underline{y = 3}$

https://people.ucsc.edu/~miglior/chapter%20pdf/Ch10_SE.pdf

Find the value of the following expression

$\log_4 1 = y$

$1 = 4^y$

$\Rightarrow y = 0$

$\log_b 1 = ?$

$\log_b 1 = y$

$\Rightarrow 1 = b^y$

$\boxed{y = 0}$

$$\log_4 1 = y$$

$$\Rightarrow \quad 4^y = 1$$

$$\left(\begin{array}{c}\text{Any Non-Zero} \\ \text{number}\end{array}\right)^0 = 1$$

$$\underline{\underline{y = 0}}$$

$$\boxed{\log 1 = 0}$$

Observations...

(a) $\log_a a = 1$

(b) $\log_a 1 = 0$

$\log_a a = y$

$a = a^y$

$\Rightarrow \quad y = 1$

Find the value of the following expressions

Undefined

$\log_2 0$

$\log_3 -1$

$\log_2 0 = y$

$\Rightarrow 0 = 2^y$

$2^2 = 4$

$y = -2 \quad 2^{-2} = 1/4 \;,\; y = 2$

$2^3 = 8$

$y = -10 \quad 2^{-10} \quad 1/2^{10} \;,\; y = 3$

Find the value of the following expressions

Undefined

$\log_2 0$

$\log_3 -1 = y$

$-1 = 3^y$

there is no such y.

Find the value of the following expressions

$\log_2 0$

Undefined

$\log_3 -1$

Undefined

$log(10) = 1$

$log(100) = 2$

$log(1000) = 3$

$log(10000) = 4$

$log(100000) = 5$

$log(1000000) = 6$

$log(10000000) = 7$

$log(100000000) = 8$

log with base 10

n $\quad$ $\log n$

Formal Definition of log

Definition of a Logarithm Function

If x and b are positive real numbers such that $b \neq 1$, then $y = \log_b x$ is called the **logarithmic function** with base b and

$$y = \log_b x \quad \text{is equivalent to} \quad b^y = x$$

https://people.ucsc.edu/~miglior/chapter%20pdf/Ch10_SE.pdf

Common bases to be used.

$$\log \rightarrow \begin{cases} \log_{10} & (\text{common log}) \\ \log_e & (\text{natural log}) \\ \log_2 & (\text{for computer science}) \end{cases}$$

Few Properties

$$\log_b a^n = n log_b a$$

$$\log 5^3 = 3 \log 5$$

$$\log 2^{10} = 10 \log 2$$

$$\log_b mn = log_b m + log_b n$$

$$\log 3 \cdot 5 = \log 3 + \log 5$$

$$\log_b \frac{m}{n} = log_b m \; - \; log_b n$$

$$\log \frac{3}{5} = \log 3 - \log 5$$

$$\log_b mn = \underbrace{log_b m}_{x} + \underbrace{log_b n}_{y}$$

$x = \log_b m \Rightarrow m = b^x$

$y = \log_b n \Rightarrow n = b^y$

RHS : $x+y$

LHS. $\log_b b^x \cdot b^y = \log_b b^{x+y} = x+y \log_b b$

$= x+y$

$$\log_b \frac{m}{n} = \underbrace{\log_b m}_{x} - \underbrace{\log_b n}_{y}$$

$x = \log_b m \Rightarrow m = b^x$

$y = \log_b n \Rightarrow n = b^y$

RHS : $x - y$

LHS : $\log_b \frac{b^x}{b^y} = \log_b b^{x-y} = x-y \log_b b$

$= x-y$

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to be proved

$$\log_b a^n = n\log_b a$$

$$\boxed{x = ny}$$

$\log_b a^n \to x$, $\quad \log_b a \to y$

$x = \log_b a^n$

$\Rightarrow a^n = b^x$

$\log_b a = y$

$a = b^y$

$(b^y)^n = b^x = b^{yn} = b^x$

$\Rightarrow x = ny$

$$(b^y)^n = b^{ny}$$

$$\downarrow$$

$$b^y \cdot b^y \cdot b^y \cdots b^y$$

$$= b^{y+y+\cdots y}$$

$$= b^{ny}$$

$$(2^3)^2 = 2^{3\times 2} = 2^6 = 64$$

$$(2^3)^2 \neq 2^{(3^2)} = 2^9$$

$$= 512$$

$$2^{3^{2}} = 2^{\left(3^{2}\right)} = 2^{9} = 512 \checkmark \text{ (Right)}$$

$$= 2^{3\times2} = 2^{6} = 64 \quad \times \text{(wrong)}$$

$$\left(2^{3}\right)^{2} = 2^{3\times2} = 2^{6} = \underline{\underline{64}} \quad \text{Right}$$

$$\rightarrow 2^{\left(3^{2}\right)} = 2^{9} = 512 \quad \text{wrong}$$

You don't need to prove these properties again in your entire life.

Theorem 2.5 $\log_a(x^r) = r\log_a(x)$

Proof:
Set $b = \log_a(x^r), c = \log_a(x)$, so that $a^b = x^r, a^c = x$. We then get

$$a^b = x^r$$
$$a^b = (a^c)^r$$
$$b = rc$$
$$\log_a(x^r) = r\log_a(x)$$

This final result gives us a method for converting between bases:

https://math.berkeley.edu/~ehallman/fall-2014/exp_log.pdf

Solve following expressions -

1. $\log_5(5\sqrt{5})$ $\Rightarrow$

$\log_5 5 + \log_5 \sqrt{5}$

$= 1 + \log_5 5^{1/2}$

$1 + \frac{1}{2} \log_5 5$

$\Rightarrow 1 + \frac{1}{2} = 3/2$

2. $\log_2(\frac{1}{8})$

$$\log_2 \frac{1}{8} = \log_2 \left(\frac{1}{2}\right)^3 = \log_2 2^{-3}$$

$$= -3 \log_2 2$$

$$= -3$$

1. $\log_5(5\sqrt{5})$

$$\Rightarrow \quad \log_5(5\sqrt{5}) = \log_5(5^1 \cdot 5^{\frac{1}{2}}) = \log_5(5^{\frac{3}{2}}) = \frac{3}{2}$$

2. $\log_2(\frac{1}{8})$

$$\Rightarrow \quad \log_2(\frac{1}{8}) = \log_2(\frac{1}{2^3}) = \log_2(2^{-3}) = -3$$

Few Properties (Contd..)

$$a = b^{\log_b a}$$

$$b^{\log_b a} = a$$

e^n n^2

$e^n \leftrightarrow e^{\log_e n^2}$

$n \leftrightarrow 2 \log_e n$

$$a = b^{\log_b a}$$

$\log_b$ both side

$$\log_b a = \log_b b^{\log_b a}$$

$$= \underline{\log_b a} \times \underline{\underline{\log_b b}}$$

$\underline{\underline{\text{H.P.}}}$

$$10^{\log_{10} \pi} = \pi$$

$$10^{\log_{10}(x^2+y^2)} = x^2+y^2$$

$$10^{\log_{10} 10^{3x^3}} = 10^{3x^3}$$

$$
\begin{aligned}
10^{\log_{10}\pi} &= \pi \\
10^{\log_{10}(x^2+y^2)} &= x^2+y^2 \\
10^{\log_{10}10^{3x^3}} &= 10^{3x^3}
\end{aligned}
$$

Fun Exercise:

Prove product and division property of logarithm using

$$a = b^{\log_b a}$$

$$\log_b mn = log_b m + log_b n$$

$m = b^{\log_b m}$

$n = b^{\log_b n}$

$$\log_b \frac{m}{n} = log_b m - log_b n$$

$\log_b mn = log_b m + log_b n$

$m = b^{\log_b m}$

$n = b^{\log_b n}$

LHS

$\log_b b^{\log_b m + \log_b n}$

$= \underline{\log_b m + \log_b n} \cdot \log_b b$

$$\begin{aligned} xy &= 10^{\log_{10} x} \times 10^{\log_{10} y} \\ &= 10^{\log_{10} x + \log_{10} y} \end{aligned}$$

$$\begin{aligned} \frac{x}{y} &= \frac{10^{\log_{10} x}}{10^{\log_{10} y}} \\ &= 10^{\log_{10} x - \log_{10} y} \end{aligned}$$

Few Properties (Contd..)

$$\log_a x = \frac{\log_b x}{\log_b a}$$

change of base property

Proof :

$\log_a x = y$

$\Rightarrow x = a^y$

LHS : y

RHS : $\dfrac{\log_b a^y}{\log_b a} = \dfrac{y \cdot \cancel{\log_b a}}{\cancel{\log_b a}} = y$

Theorem 2.6 *For any bases a and b, we have* $\log_a(x) = \log_b(x)/\log_b(a)$

Proof: Let $c = \log_a(x)$, so that $a^c = x$. We then get

$$\begin{aligned} a^c &= x \\ \log_b(a^c) &= \log_b(x) \\ c\log_b(a) &= \log_b(x) \\ c &= \log_b(x)/\log_b(a) \\ \log_a(x) &= \log_b(x)/\log_b(a) \end{aligned}$$

If we take $b = e$ in particular, this gives us $\log_a(x) = \ln(x)/\ln(a)$.

https://math.berkeley.edu/~ehallman/fall-2014/exp_log.pdf

Fun Exercise:

Prove change of base property of logarithm using

$$x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

L.H.S $\Rightarrow \log_a a^{\log_a x} \Rightarrow \log_a x$

RHS

$$\frac{\log_b a^{\log_a x}}{\log_b a} = \frac{\log_a x \cancel{\bigcirc}}{\cancel{\bigcirc}}$$

H.P.

Proof. Take the base-b logarithm of both sides of the equation $x = a^{\log_a x}$, obtaining

$$\log_b x = \log_b a^{\log_a x} = (\log_a x) \cdot (\log_b a),$$

which is equivalent to the desired result.

Question: True / False

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_2 4 = \frac{1}{\log_4 2}$$

$$\frac{\log_a a}{\log_a b} = \frac{1}{\log_a b}$$

Question: True / False

$$\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = \log_b x$$

$$\log_b \frac{1}{\sqrt{b}} = \log_b \frac{1}{b^{1/2}}$$

$$= \log_b b^{-1/2}$$

$$\frac{\log_b \sqrt{x}}{\log_b \frac{1}{\sqrt{b}}} = \frac{\frac{1}{2}\log_b x}{-\frac{1}{2}\log_b b} = -\log_b x$$

$$\log_b b \rightarrow 1$$

Find the value of $\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$

$\Rightarrow 2$

$$\frac{\log_{10} 4}{\log_{10} 3} \times \frac{\log_{10} 5}{\log_{10} 4} \times \frac{\log_{10} 6}{\log_{10} 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}$$

$$= \frac{\log_{10} 9}{\log_{10} 3} = \log_3 9 = 2$$

Prove that: $\dfrac{\log_a n}{\log_{ab} n} = 1 + \log_a b$

LHS

$$\Rightarrow \frac{\log_a n}{\left(\dfrac{\log_a n}{\log_a ab}\right)} = \frac{\cancel{\log_a n} \times \log_a ab}{\cancel{\log_a n}}$$

$$= \log_a ab = \log_a a + \log_a b$$

$$= 1 + \log_a b$$

Few Properties (Contd..)

$$a^{\log_b c} = c^{\log_b a}$$

RHS $\log_b c^{\log_b a}$

$= \log_b a \cdot \log_b c$

$\log_b$ at both side

$\log_b a^{\log_b c}$

LHS. $\log_b c \cdot \log_b a$

Proof

If you apply the logarithm with base a to both sides you obtain,

$$\log_a a^{\log_b c} = \log_a c^{\log_b a}$$

$$\log_b c = \log_b a \log_a c$$

$$\frac{\log_b c}{\log_b a} = \log_a c$$

All Properties of log at once

$$\log_b a^n = n log_b a$$

$$\log_b mn = log_b m + log_b n$$

$$\log_b \frac{m}{n} = log_b m \; - \; log_b n$$

$$a = b^{\log_b a}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^{\log_b c} = c^{\log_b a}$$

Question

If $(\log_2 3)\cdot(\log_3 4)\cdot(\log_4 5)\cdots\cdots(\log_n(n+1)) = 10.$

Find 'n'.

Sol^n

Question

If $(\log_2 3) \cdot (\log_3 4) \cdot (\log_4 5) \cdots\cdots (\log_n(n+1)) = 10.$

Find 'n'.

Sol^n $\Rightarrow$

$$\frac{\cancel{\log 3}}{\boxed{\log 2}} \times \frac{\cancel{\log 4}}{\cancel{\log 3}} \times \frac{\log 5}{\cancel{\log 4}} \cdots \frac{\boxed{\log n+1}}{\cancel{\log n}}$$

$$\frac{\log n+1}{\log 2} = 10$$

$$\frac{\log_{30} n+1}{\log_{30} 2} = 10$$

change of base property

$$\log_2 n+1 = 10$$

$$n+1 = 2^{10}$$

$$\Rightarrow n+1 = 1024$$

$$\Rightarrow n = \underline{\underline{1023}}$$

Question

MCQ

$$\frac{1}{\log_{xy}(xyz)}+\frac{1}{\log_{yz}(xyz)}+\frac{1}{\log_{zx}(xyz)}=$$

(a) 1 (b) 2

(c) 3 (d) 4

$$\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$$

$$\log_{xyz}(xy \cdot yz \cdot zx)$$

$$= \log_{xyz}(xyz)^2 = 2$$

MSQ

(More than one option could be correct)

Which of the following is/are CORRECT ?

A. $\log_b a^n = n log_b a$

B. $\log_{b^m} a = m\ log_b a$

C. $\log_{b^m} a = \frac{1}{m}\ log_b a$

D. $\log_{b^m} a^n = \frac{n}{m}\ log_b a$

MSQ

(More than one option could be correct)

Which of the following is/are CORRECT ?

A. $\log_b a^n = n log_b a$

B. $\log_{b^m} a = m\, log_b a$

C. $\log_{b^m} a = \frac{1}{m}\, log_b a$

D. $\log_{b^m} a^n = \frac{n}{m}\, log_b a$

LHS for option B

$$\log_{b^m} a = \frac{\log_b a}{\log_b b^m}$$

$$= \frac{1}{m} \log_b a$$

$$\log_b a^n = n \log_b a$$

$$\log_{b^m} a = \frac{1}{m} \log_b a$$

Question

$\log a^n/b^n + \log b^n/c^n + \log c^n/a^n$

(a) 1 (b) n

(c) 0 (d) 2

Question

$\log a^n/b^n + \log b^n/c^n + \log c^n/a^n$

(a) 1 (b) n

(c) 0 (d) 2

$\rightarrow \log \cancel{\frac{a^n}{b^n}} \times \cancel{\frac{b^n}{c^n}} \times \cancel{\frac{c^n}{a^n}}$

$\Rightarrow \log 1$

$= 0$

Solve for the value of x -

$$\log_9(x-5)+\log_9(x+3)=1$$

Solve for the value of x -

$$\log_9(x-5) + \log_9(x+3) = 1$$

$\Rightarrow \quad \log_9 (x-5)(x+3) = 1$

$(x-5)(x+3) = 9$

$\Rightarrow \quad x^2 - 2x - 15 = 9 \quad \Rightarrow \quad x^2 - 2x - 24 = 0$

$x^2 - 2x - 24 = 0$

$\Rightarrow \quad x^2 - 6x + 4x - 24 = 0$

$x(x-6) + 4(x-6) = 0$

$(x+4)(x-6) = 0 \quad \Rightarrow \quad x = -4 \; \times$

or

$x = +6 \; \checkmark$

Question- MCQ

Given, $\log_a x = \dfrac{1}{\alpha}, \log_b x = \dfrac{1}{\beta}, \log_c x = \dfrac{1}{\gamma}$, then $\log_{abc} x$ equals :

(a) $\alpha\beta\gamma$

(b) $\dfrac{1}{\alpha\beta\gamma}$

(c) $\alpha + \beta + \gamma$

(d) $\dfrac{1}{\alpha + \beta + \gamma}$

Question- MCQ

Given, $\log_a x = \dfrac{1}{\alpha}, \log_b x = \dfrac{1}{\beta}, \log_c x = \dfrac{1}{\gamma}$, then $\log_{abc} x$ equals :

(a) $\alpha\beta\gamma$

(b) $\dfrac{1}{\alpha\beta\gamma}$

(c) $\alpha + \beta + \gamma$

(d) $\dfrac{1}{\alpha+\beta+\gamma}$ ✓

$\hookrightarrow \dfrac{1}{\log_x abc}$

$\log_x abc = \log_x a + \log_x b + \log_x c$

$= \alpha + \beta + \gamma$

$\alpha = \log_x a$

$\beta = \log_x b$

$\gamma = \log_x c$

Question:-

If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is

(*a*) $\log_a (\log_b a)$

(*b*) $\log_a (\log_e a) + \log_a(\log_e b)$

(*c*) $-\log_a (\log_a b)$

(*d*) $\log_a (\log_e b) - \log_a (\log_e a)$

Question:-

If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is

(*a*) $\log_a (\log_b a)$

(*b*) $\log_a (\log_e a) + \log_a (\log_e b)$

(*c*) $-\log_a (\log_a b)$

(*d*) $\log_a (\log_e b) - \log_a (\log_e a)$

a, b, c in GP then $b^2 = ac$

$$\left(a^{x/2}\right)^2 = \left(\log_x a\right)\left(\log_b x\right)$$

$$\left(a^{x/2}\right)^2 = \left(\log_x a\right)\left(\log_b x\right)$$

$$a^x = \log_x a \cdot \log_b x$$

$$\log_b a$$

$$= \log_b x \cdot \log_x a$$

$$= \frac{\cancel{\log x}}{\log b} \cdot \frac{\log a}{\cancel{\log x}} = \frac{\log a}{\log b}$$

$$\left(a^{x/2}\right)^2 = (\log_x a)(\log_b x)$$

$$a^x = \log_x a \cdot \log_b x$$

$$= \log_b x \cdot \log_x a$$

$$= \log_b a$$

$$\Rightarrow x = \underline{\log_a(\log_b a)}$$

Sol. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then $\left(a^{x/2}\right)^2 = (\log_b x) \times (\log_x a)$

$\Rightarrow a^x = \log_b a \Rightarrow \log a^x = \log(\log_b a) \Rightarrow x \log a = \log(\log_b a) \Rightarrow x \log_a a = \log_a(\log_b a)$

$\Rightarrow \boldsymbol{x = \log_a(\log_b a)}$.

Question:-

If $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$, which of the following option is CORRECT?

(a) $xyz = 1$ (b) $x^a y^b z^c = 1$

(c) $x^{b+c} y^{c+a} z^{a+b} = 1$ (d) All the options are correct.

$$\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b} = K$$

$$\frac{\log x}{b-c} = K \Rightarrow \log x = K(b-c)$$

$$\Rightarrow x = 10^{K(b-c)}$$

$$\frac{\log y}{c-a} = K \Rightarrow y = 10^{K(c-a)}$$

$$z = 10^{K(a-b)}$$

(a) $xyz = 1$ (b) $x^a y^b z^c = 1$

(c) $x^{b+c} y^{c+a} z^{a+b} = 1$ (d) All the options are correct.

option A

$x = 10^{k(b-c)}$

$y = 10^{k(c-a)}$

$z = 10^{k(a-b)}$

$xyz = 10^{k(b-c)} \cdot 10^{k(c-a)} \cdot 10^{k(a-b)}$

$= 10^{k(\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a}+\cancel{a}-\cancel{b})}$

$= 10^0 = 1$

(a) ~~$xyz = 1$~~ (b) ~~$x^a y^b z^c = 1$~~

(c) $\underbrace{x^{b+c} y^{c+a} z^{a+b} = 1}_{\text{H.W.}}$ (d) All the options are correct.

Option B

$x = 10^{k(b-c)}$

$y = 10^{k(c-a)}$

$z = 10^{k(a-b)}$

$x^a y^b z^c = \left(10^{k(b-c)}\right)^a \left(10^{k(c-a)}\right)^b \left(10^{k(a-b)}\right)^c$

$= 10^{ka(b-c) + kb(c-a) + kc(a-b)}$

$= 10^0 = 1$

$$\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b}=k$$

$\Rightarrow x = 10^{k(b-c)}, y = 10^{k(c-a)}, z = 10^{k(a-b)}$

$\therefore\ xyz = 10^{k(b-c+c-a+a-b)} = 10^0 = 1$

Therefore option (a) is correct.

$x^a\ y^b\ z^c = 10^{k[a(b-c)+b(c-a)+c(a-b)]}$

$= 10^{k(ab-ac+bc-ab+ca-bc)}$

$= 10^{K.0} = 1$

Therefore option (b) is correct.

$x^{b+c}\ y^{c+a} z^{a+b} = 10^{k[(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)]}$

$= 10^{k.0} = 1$ ← option c

Therefore option (c) is also correct.

Since all the first three options are correct, we choose option (d) as the correct answer.

Question:-

The number of meaningful solutions of

$$\log_4(x-1) = \log_2(x-3) \text{ is}$$

(*a*) zero (*b*) 1 (*c*) 2 (*d*) 3

(IIT 2001)

$$\log_4(x-1) = \log_2(x-3) \text{ is}$$

Question:-

The number of meaningful solutions of

$$\log_4(x-1) = \log_2(x-3) \text{ is}$$

(*a*) zero (*b*) 1 (*c*) 2 (*d*) 3

(IIT 2001)

$$\log_4(x-1) = \log_2(x-3)$$

$$\log_{2^2}(x-1) = \frac{1}{2}\log_2(x-1) = \log_2(x-3)$$

$$\frac{1}{2}\log_3(x-1) = \log_2(x-3)$$

$$\log_2(x-1) = \log_3(x-3)^2$$

$$\Rightarrow \quad x-1 = (x-3)^2$$

one - one function

if

$f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2$

$$\Rightarrow \quad x-1 = (x-3)^2$$

$$\Rightarrow \quad x-1 = x^2+9-6x$$

$$\Rightarrow \quad x^2-7x+10 = 0$$

$$\Rightarrow \quad x^2-5x-2x+10 = 0$$

$$x(x-5)-2(x-5) = 0$$

$$\Rightarrow \quad (x-2)(x-5) = 0$$

$$x = 2 \text{ or } 5$$

$$\cancel{=} \qquad =$$

$$\boxed{\log_b a \qquad \begin{matrix} a > 0 \\ b > 0, \ b \neq 1 \end{matrix}}$$

9. $\log_4(x-1) = \log_2(x-3) \Rightarrow \log_{2^2}(x-1) = \log_2(x-3)$

$$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3) \Rightarrow \log_2(x-1) = 2\log_2(x-3)$$

$$\left[\text{Using } \log_{a^m}(b^n) = \frac{n}{m}\log_a b\right]$$

$\Rightarrow \log_2(x-1) = \log_2(x-3)^2$

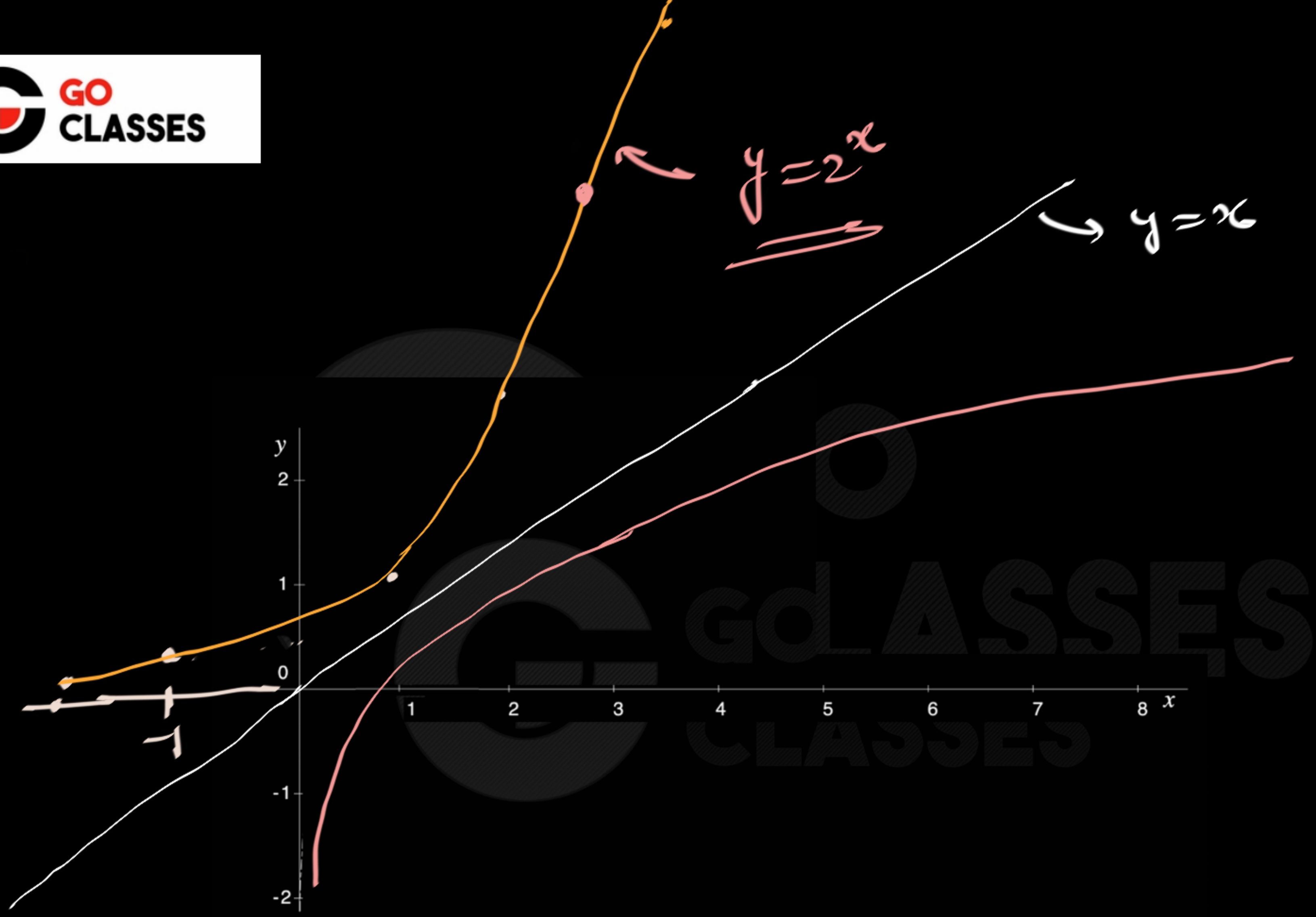
$\Rightarrow (x-1) = (x-3)^2 \Rightarrow x - 1 = x^2 - 6x + 9$

$\Rightarrow x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2 \text{ or } 5$

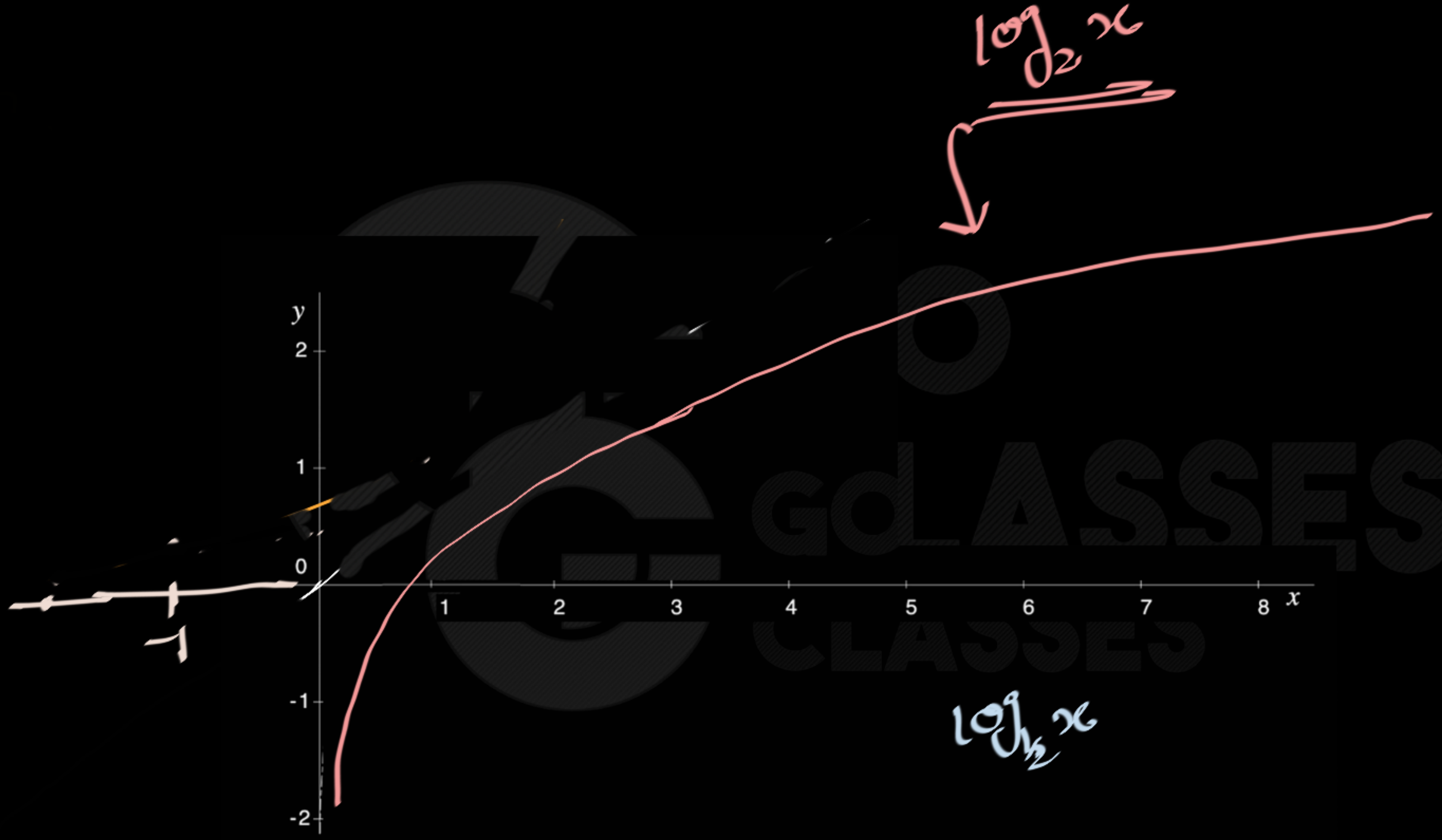
Neglecting $x = 2$ as $\log_2(x-3)$ is defined when $x > 2$.

$\Rightarrow$ There is only one meaningful solution of the given equation.

Graphs of Logarithmic Functions

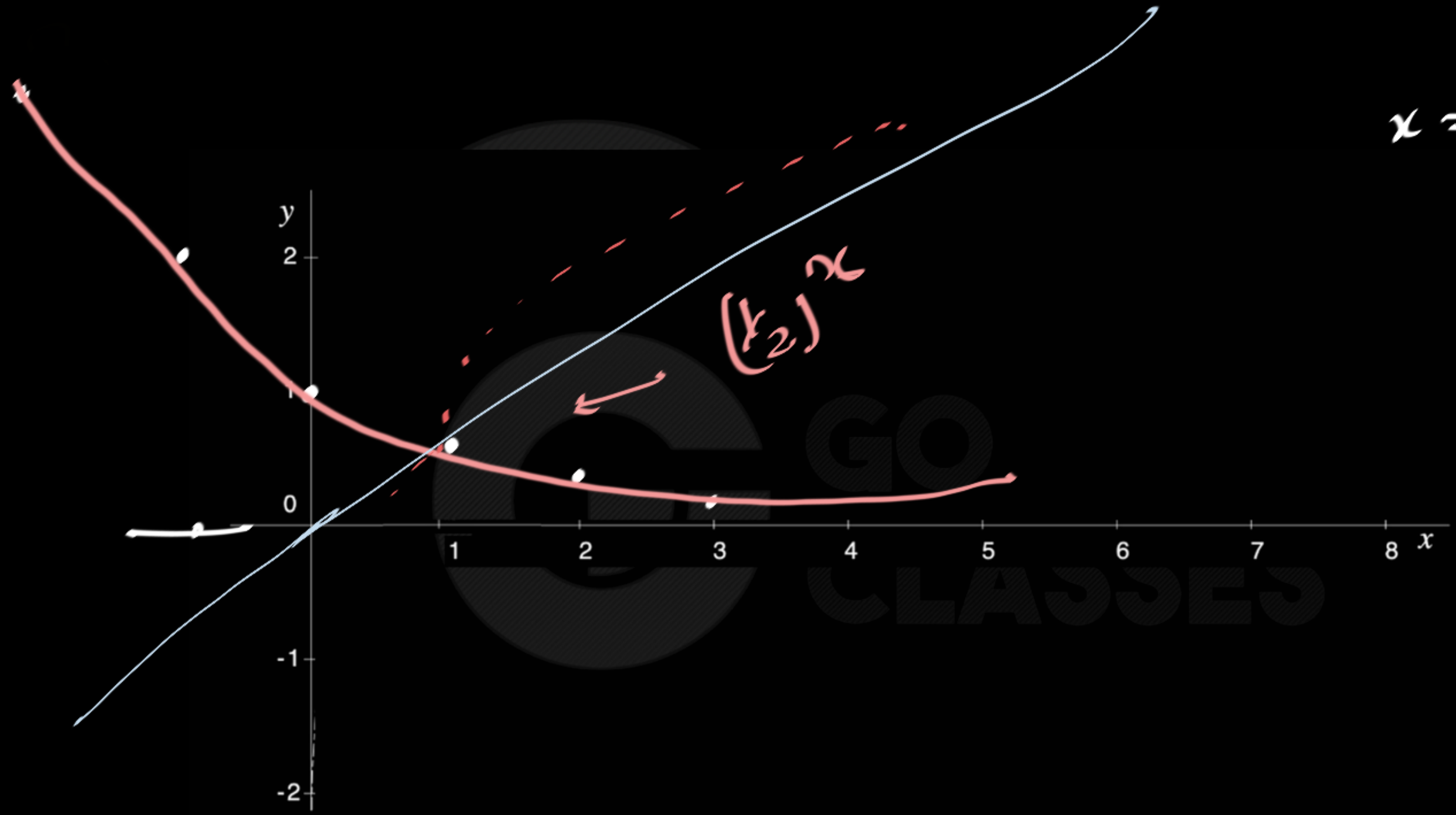


$y = 2^x$
$y = x$
y
2
1
0
-1
-2
-1
1
2
3
4
5
6
7
8
x



$b>0$

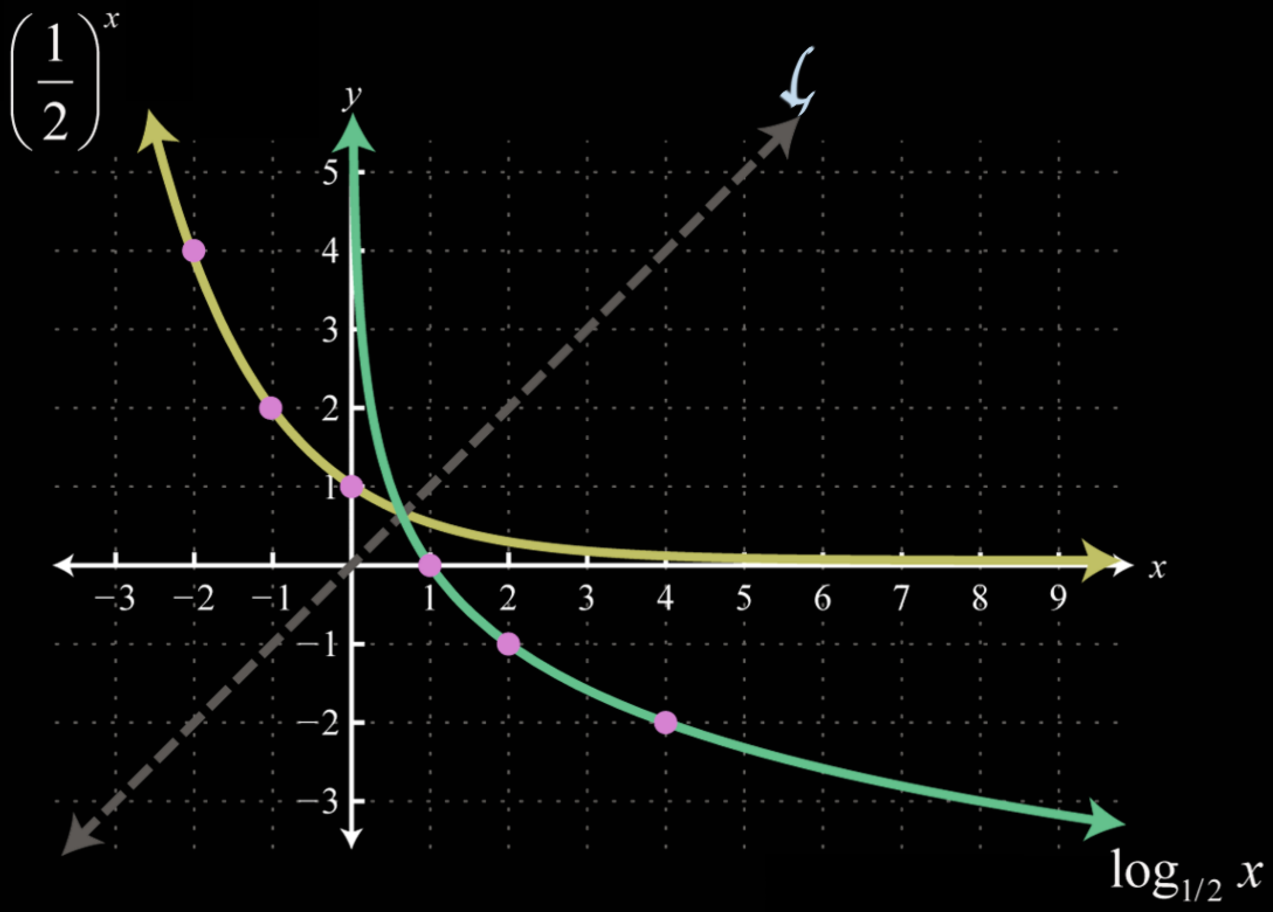
$b \neq 1$



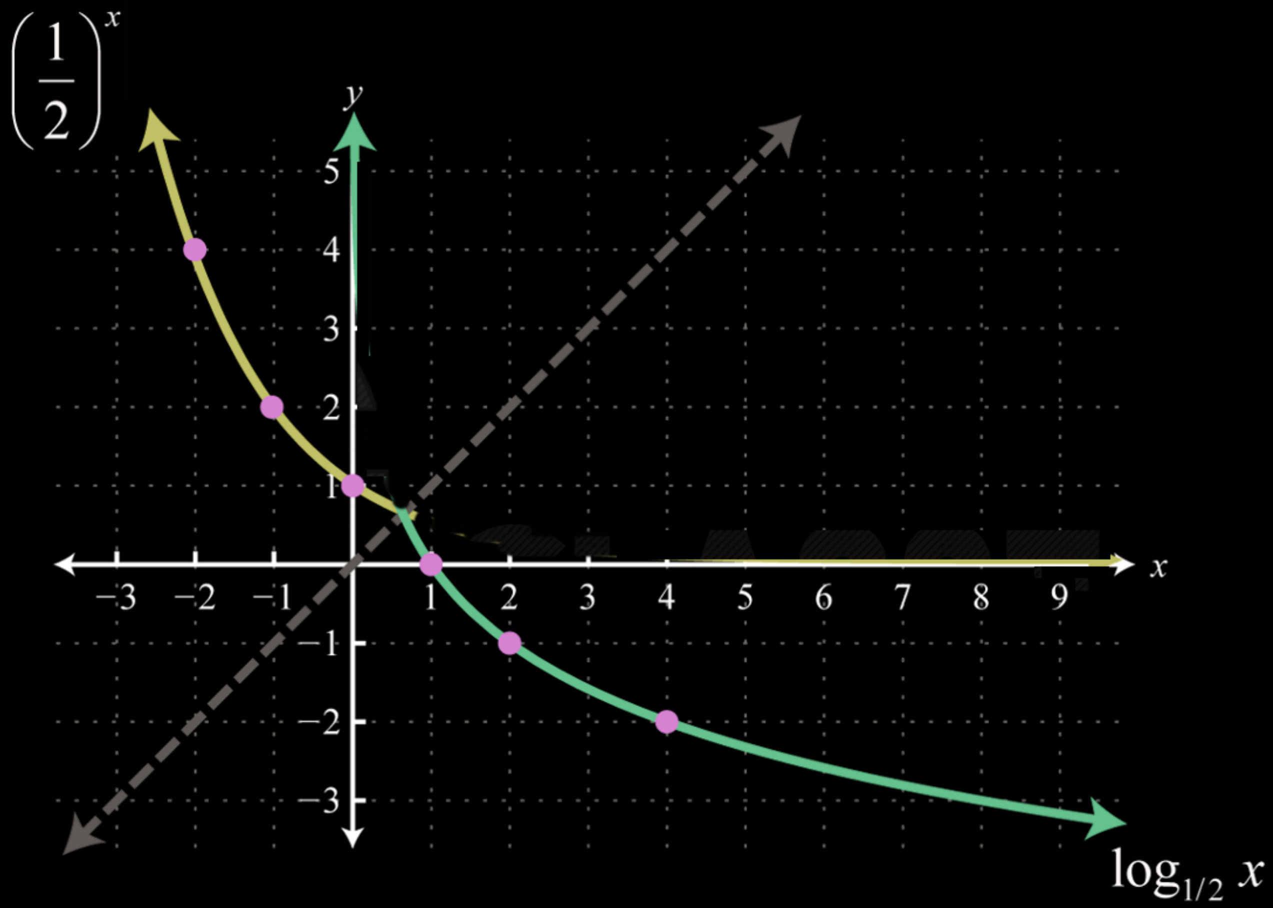
$x = -1 \quad (1/2)^{-1} = 2$

$(1/2)^x$

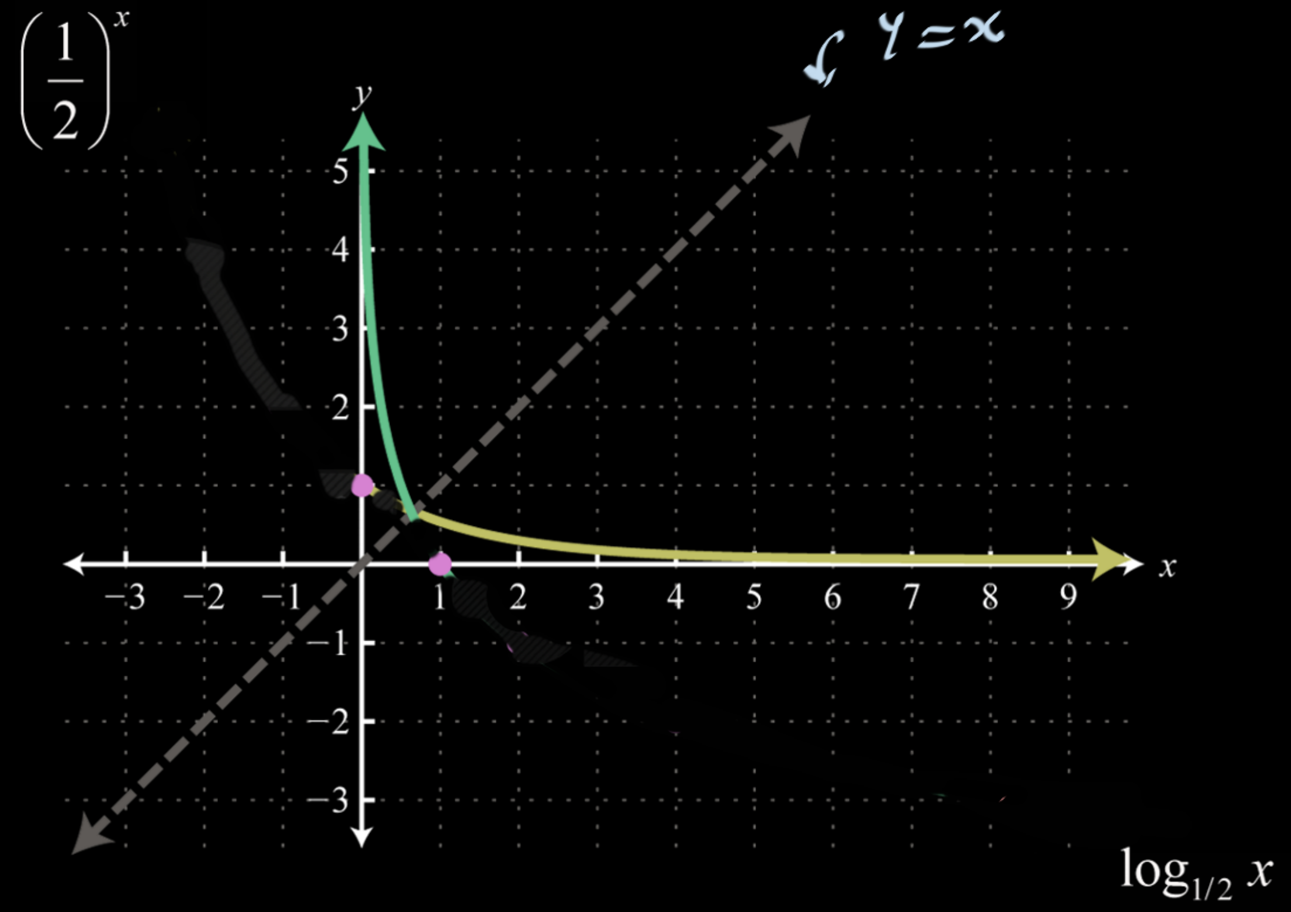
$\log_{1/2} x$



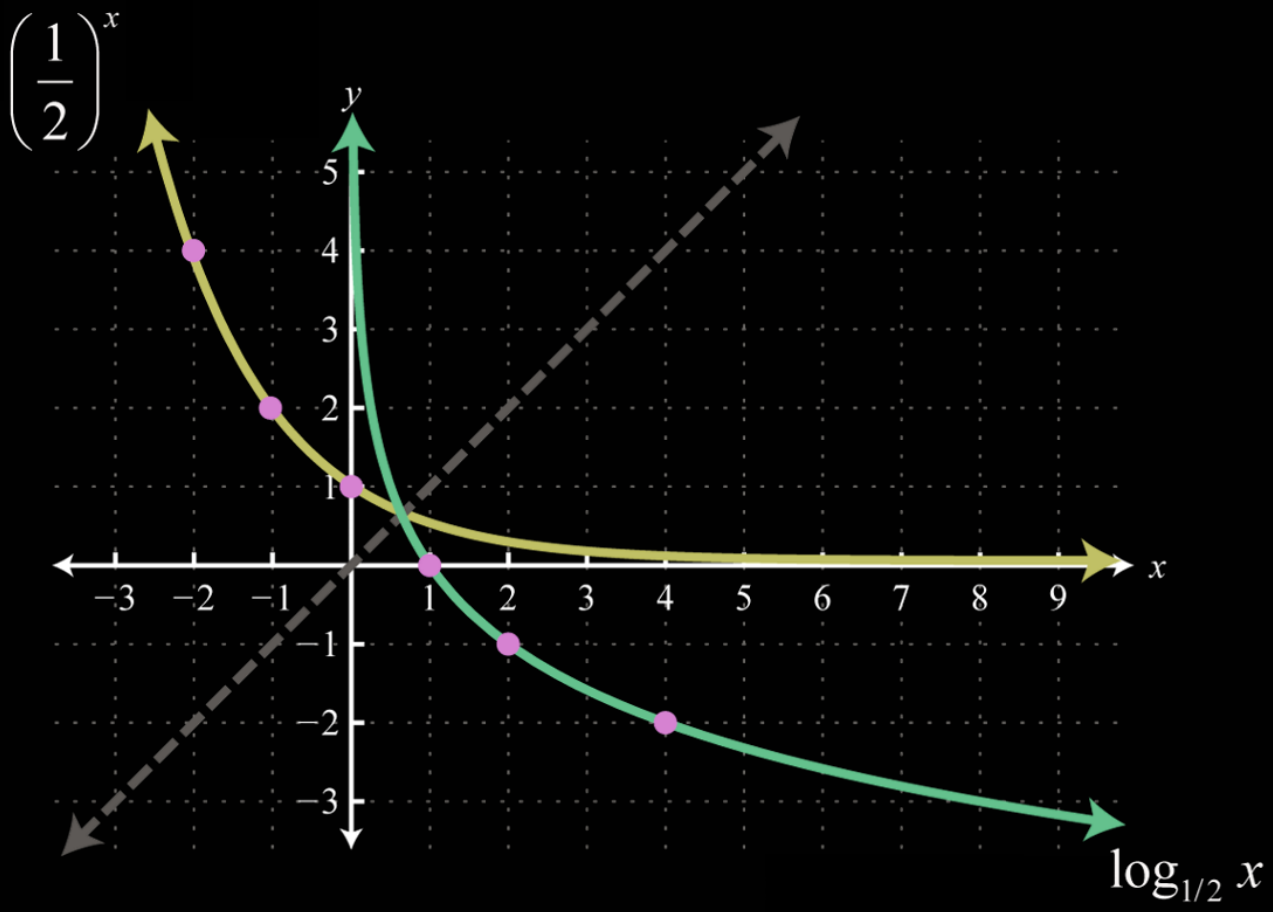
$\left(\frac{1}{2}\right)^x$
y
x
$\log_{1/2} x$



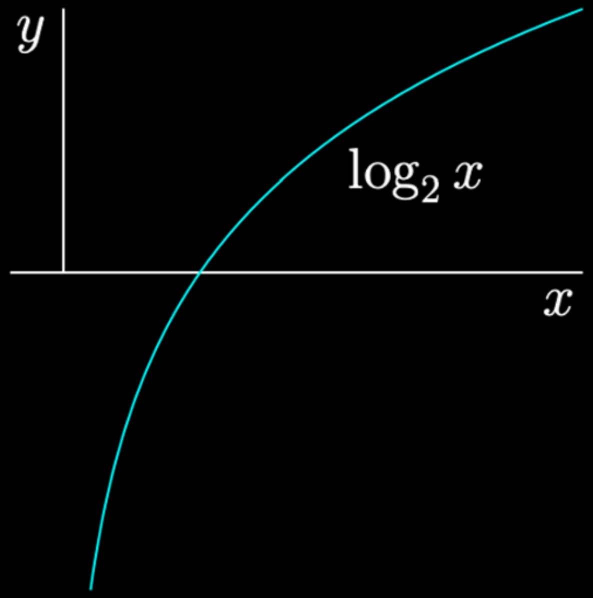
$\left(\frac{1}{2}\right)^x$
y
5
4
3
2
1
−3
−2
−1
1
2
3
4
5
6
7
8
9
x
−1
−2
−3
$\log_{1/2} x$



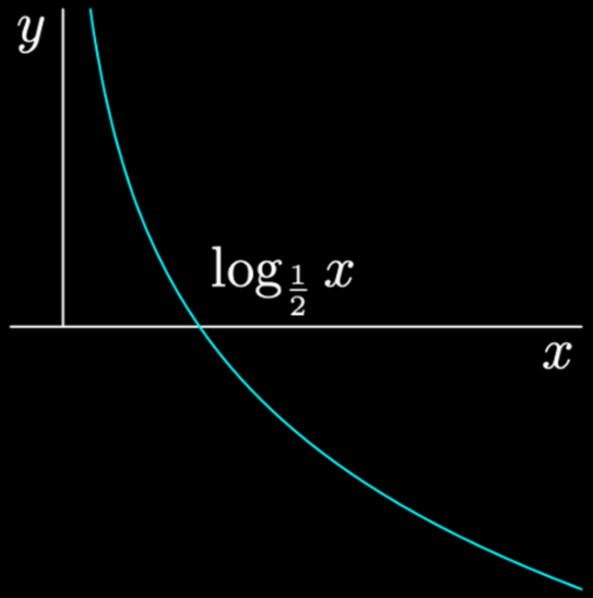
$\left(\frac{1}{2}\right)^x$
$y = x$
y
x
$\log_{1/2} x$



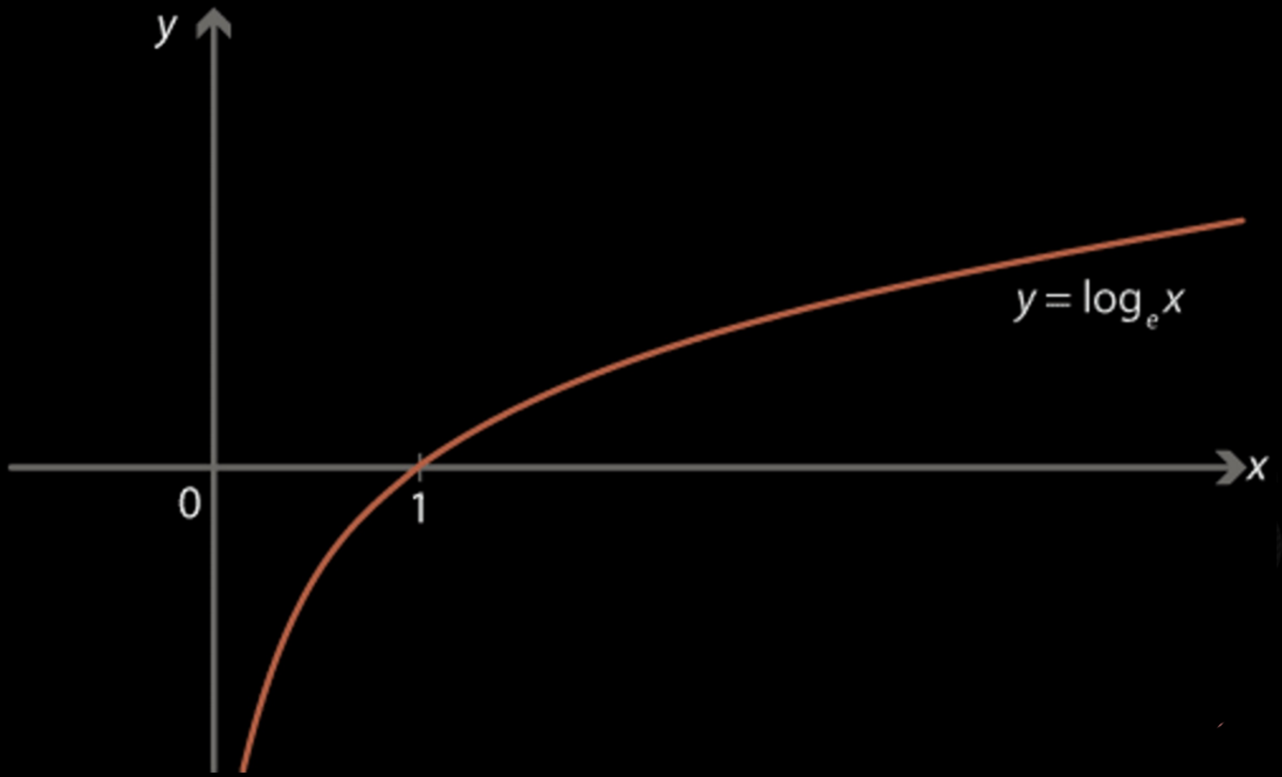
$\left(\frac{1}{2}\right)^x$
y
5
4
3
2
1
−1
−2
−3
−3
−2
−1
1
2
3
4
5
6
7
8
9
x
$\log_{1/2} x$



y
$\log_2 x$
x



y
$\log_{\frac{1}{2}} x$
x



y
x
0
1
$y = \log_e x$

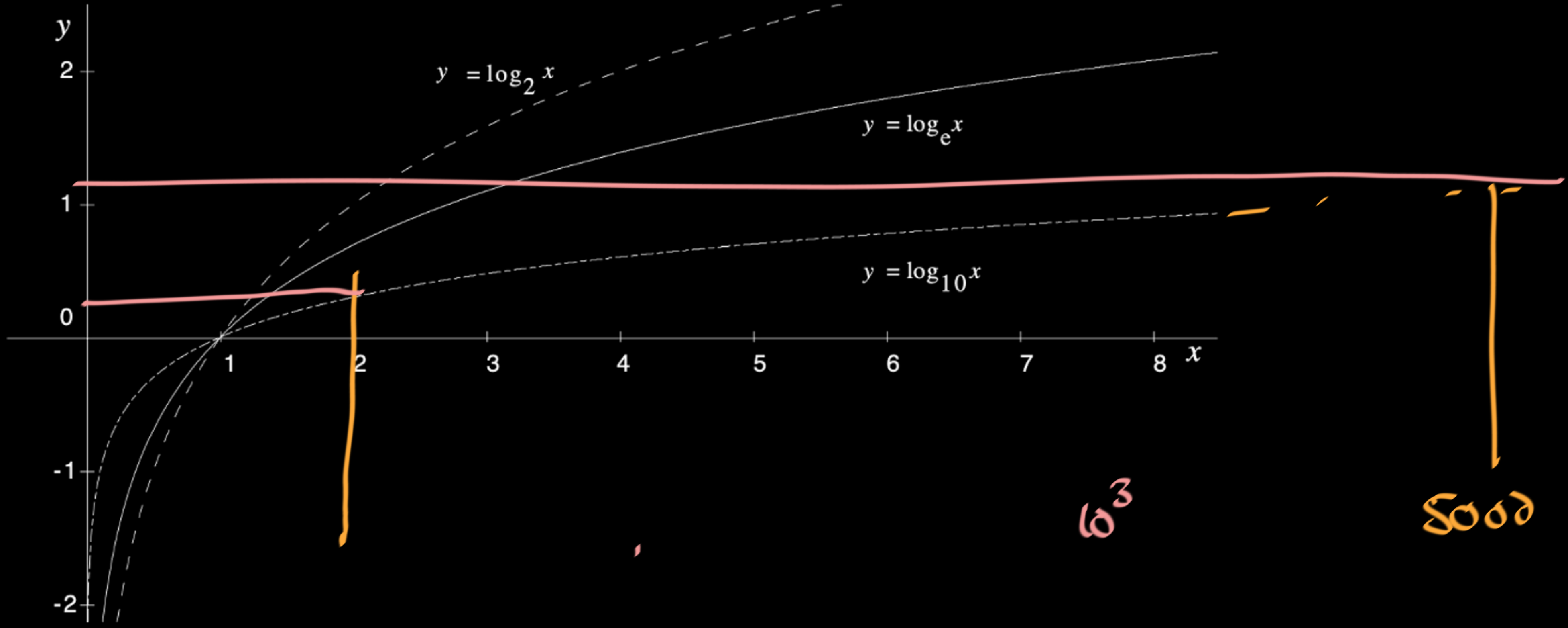


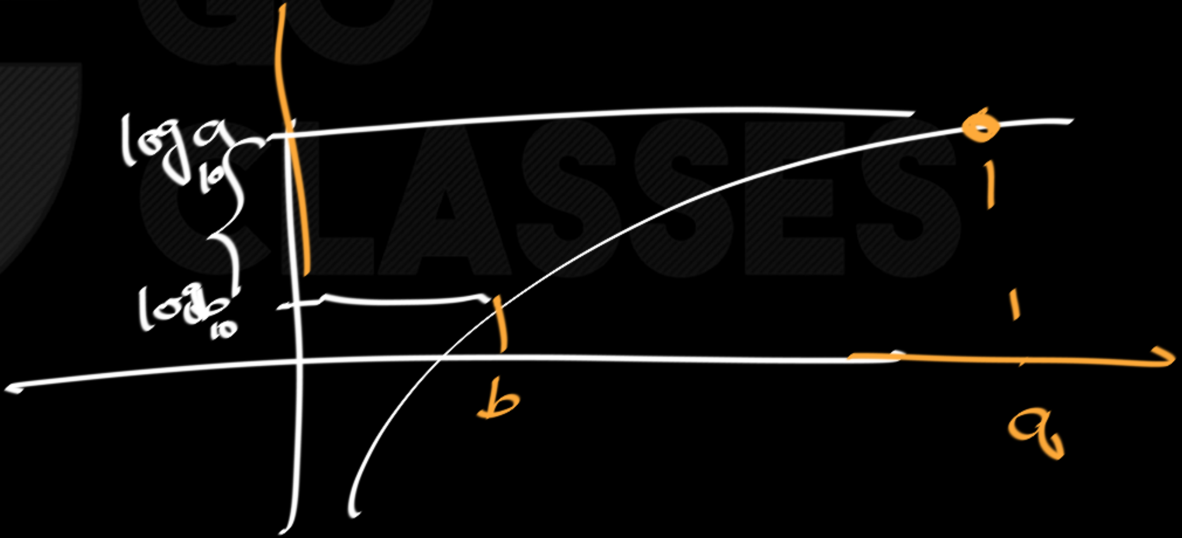
Figure 3: Graph of $f(x) = \log_b x$ for various values of b. Different function

$$R^+ \xrightarrow{\log_2 x} R$$

$$R^+ \xrightarrow{\log_{10} x} R$$

True/False

$$a > b \Rightarrow \log_{10} a > \log_{10} b$$

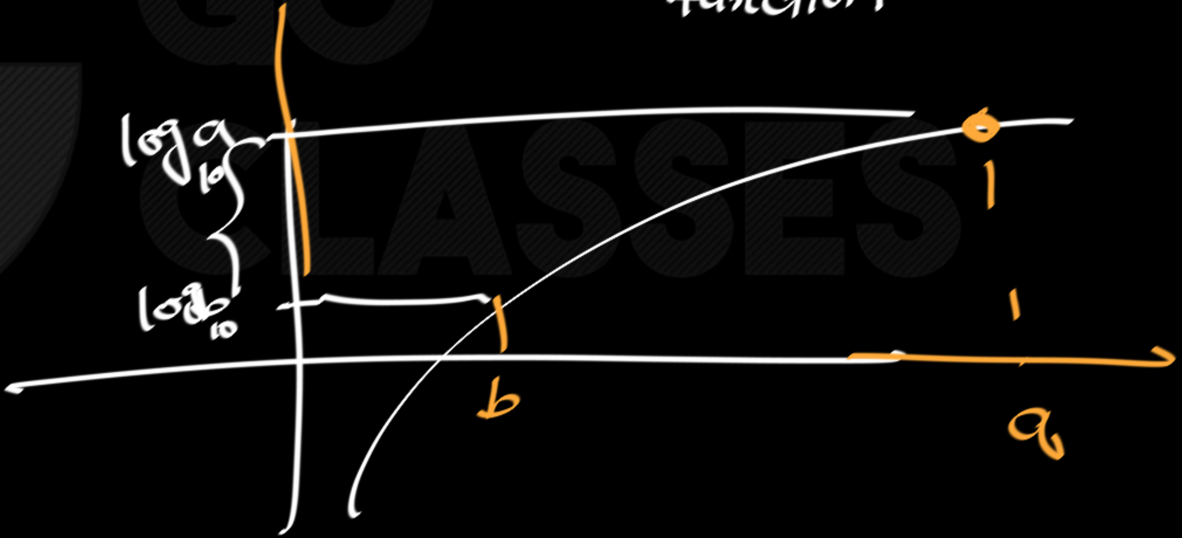


True/False

$a > b \Rightarrow \log_{10} a > \log_{10} b$

TRUE

bcoz $\log_{10}$ is increasing function



$$\log_1 x = \frac{\log_a x}{\log_a 1}$$ (That is why we do not define log with base 1)

undefined

$\log_1 5 = ?$

$\log_1 5 = a$ find a

$5 = 1^a$ find a

$\log_b a$

defined only for

$a > 0$

$b > 0$ and $b \neq 1$

Logarithm in Computer Science

1.2.2 Logarithms

In computer science, all logarithms are to the base 2 unless specified otherwise.

Definition 1.1
$X^A = B$ if and only if $\log_X B = A$

Several convenient equalities follow from this definition.

Source: MARK ALLEN WEISS

Is it a coincidence that

logarithm is an anagram of algorithm ?? !!

anagram

race	care
part	trap
heart	earth
knee	keen

How many times must we double 1 before we get to $n = 2^K$

$1 \longrightarrow 2 \longrightarrow 2^2 \longrightarrow 2^3 \longrightarrow \cdots\cdots \quad \underset{n}{2^K}$

K times we need to double up.

$n = 2^K \Rightarrow \boxed{K = \log_2 n}$

for ($i=1$ $i = 2*i$)

{

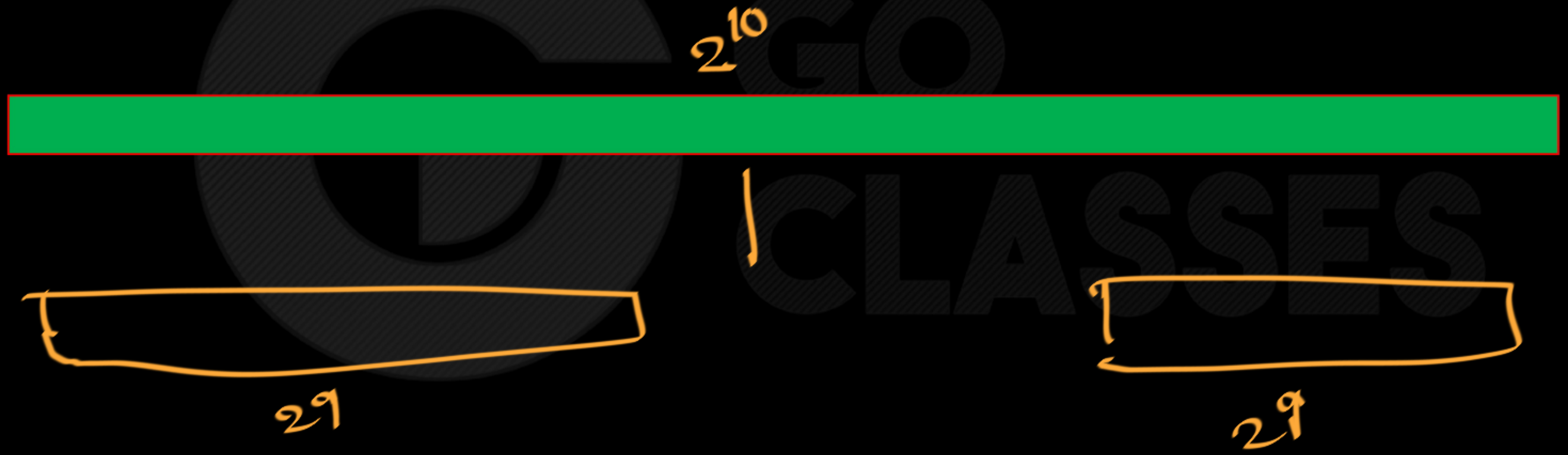
}

$\leftarrow \log_2 n$

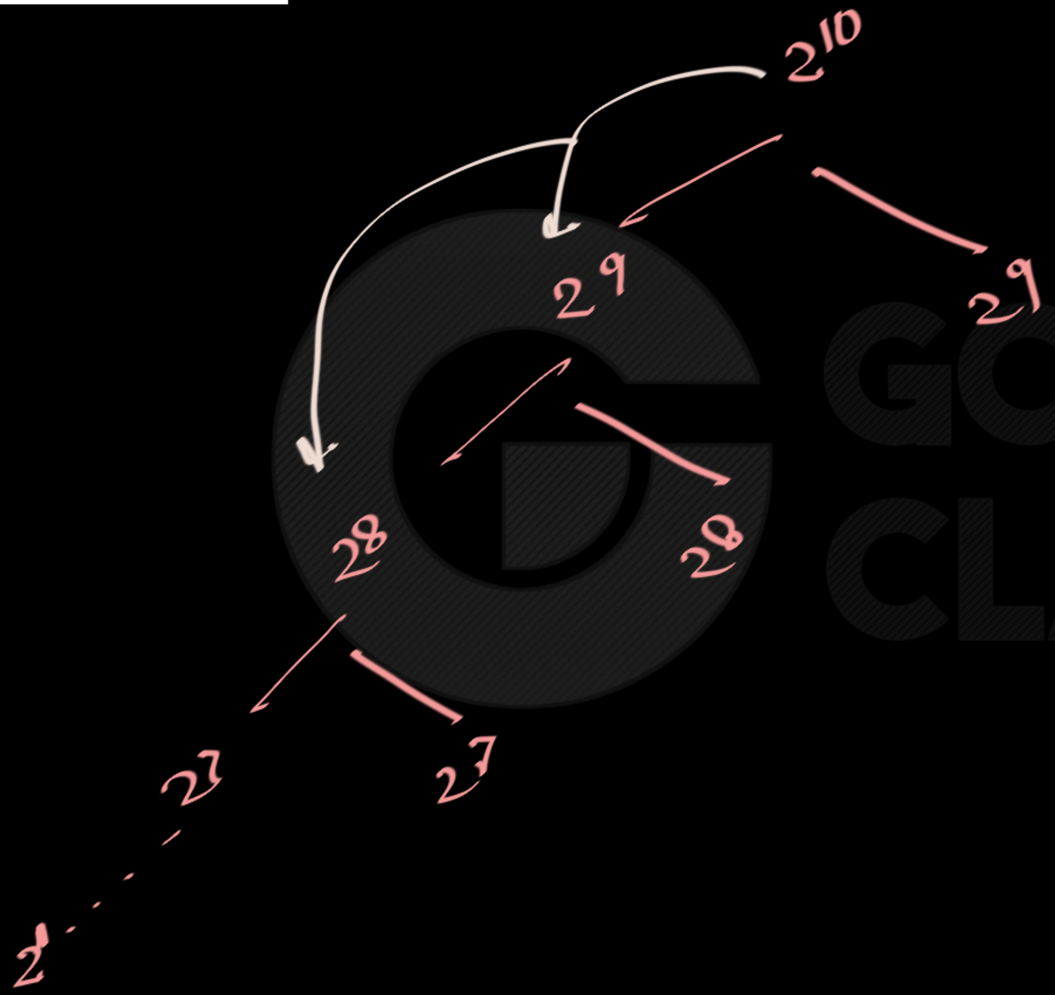
Howmany times you can divide following rod in halves?.

Rod length = 2^{10}

$$\frac{2^{10}}{2} = 2^9$$



merge sort



10 times

$\log_2 2^{10}$ times

Logarithms in binary search

Logarithms in sorting

Logarithms in binary trees (AVL trees)

Many algorithms has $\log$ in their time complexities

merge sort

quick sort

log log

- Double logarithms occasionally crop up in CS, and when they do it can be great news!

$log(10) = 1$

$log(100) = 2$

$log(1000) = 3$

$log(10000) = 4$

$log(100000)$ 5

$log(1000000)$ 6

$log(10000000)$ 7

$log(100000000) =$ 8

$$\log_{10}\log_{10} 10 = \log_{10} 1 = 0$$

$$\log_{10}\log_{10} 100 = \log_{10} 2 \approx 0.30103$$

$$\log_{10}\log_{10} 1,000 = \log_{10} 3 \approx 0.47712$$

$$\log_{10}\log_{10} 1,000,000 = \log_{10} 6 \approx 0.77815$$

$$\log_{10}\log_{10} 1,000,000,000 = \log_{10} 9 \approx 0.95424$$

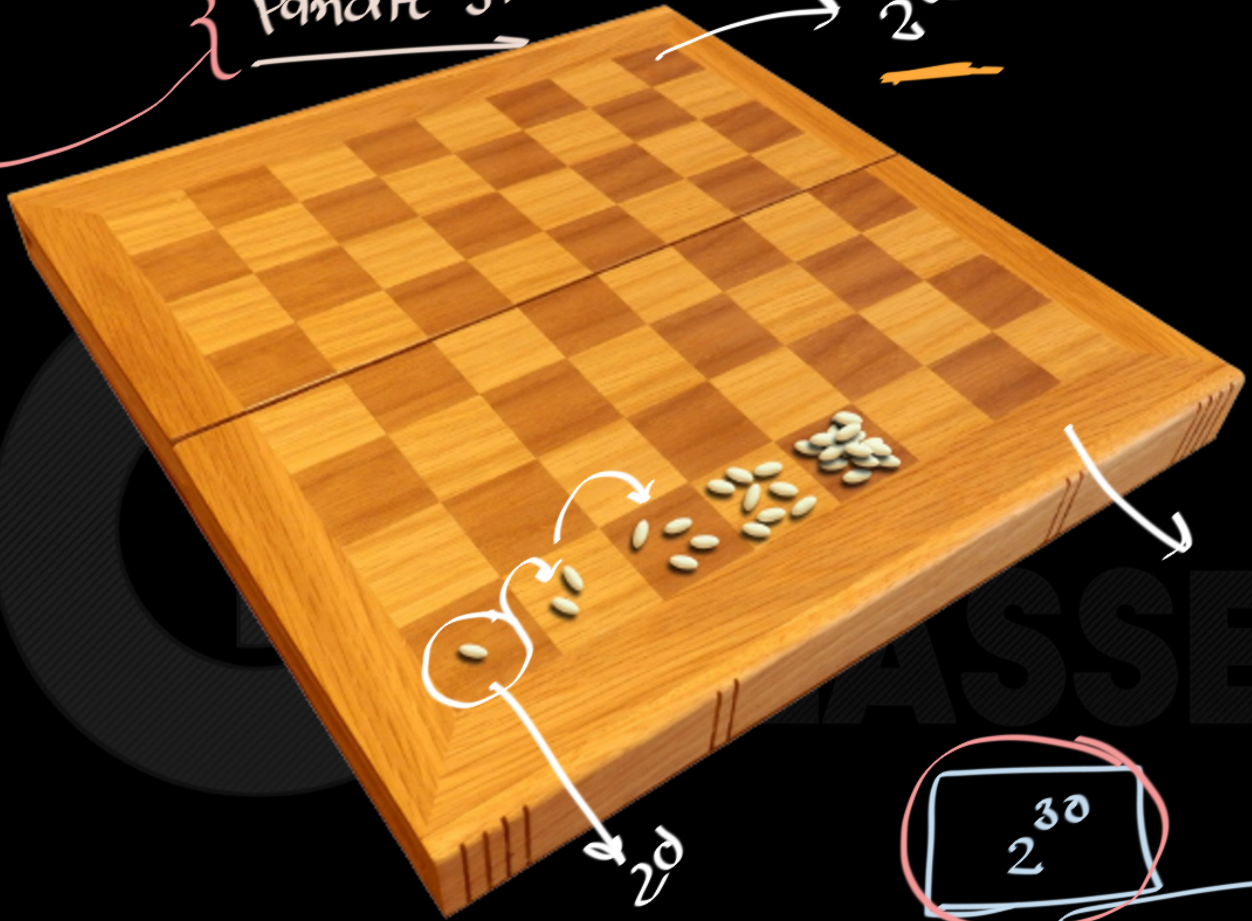
$$\log_{10}\log_{10} 10,000,000,000 = \log_{10} 10 = 1$$

$$\log_{10}\log_{10} \underbrace{100,000,000,000,000,000,000}_{10^{20}} = \log_{10} 20 \approx \underline{1.30103}$$

King

Pandit Ji

2^{63}



64 Squares

2^{0}

$\boxed{2^{30}}$

$2^{0} + 2^{1} + \dots 2^{29}$

Rice Grain Story

The problem may be solved using simple addition. With 64 squares on a chessboard, if the number of grains doubles on successive squares, then the sum of grains on all 64 squares is: 1 + 2 + 4 + 8 + ... and so forth for the 64 squares. The total number of grains can be shown to be $2^{64}-1$ or 18,446,744,073,709,551,615 (eighteen quintillion, four hundred forty-six quadrillion, seven hundred forty-four trillion, seventy-three billion, seven hundred nine million, five hundred fifty-one thousand, six hundred and fifteen, over 1.4 trillion metric tons), which is over 2,000 times the annual world production of wheat.[1]

Source: wikipedia

GO CLASSES