Graph Theory

Rooted Tree

Graph Theory :

Recap :

Questions on Trees, Forests

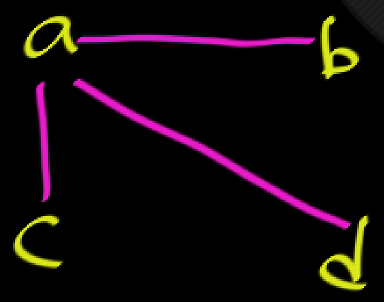
Website : https://www.goclasses.in/

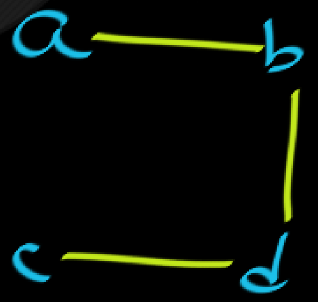
Q:

What is the maximum number of edges in a Forest on n vertices?

Q:

What is the maximum number of edges in a Forest on n vertices? = n - 1





NOTE:

Acyclic Graph

forest with maximum number of Edges

≡ Tree ≡ maximally Acyclic

≡ maximally forest

Q:

What is the maximum number of edges in a Forest which is NOT Tree, on n vertices?

Q:

What is the maximum number of edges in a Forest which is NOT Tree, on n vertices?

$= n-2$

If $(n-1)$ Edges then forest will be a Tree.

Q:

What is the maximum number of edges in a Forest which is NOT Tree, on n vertices?

$= n-2$

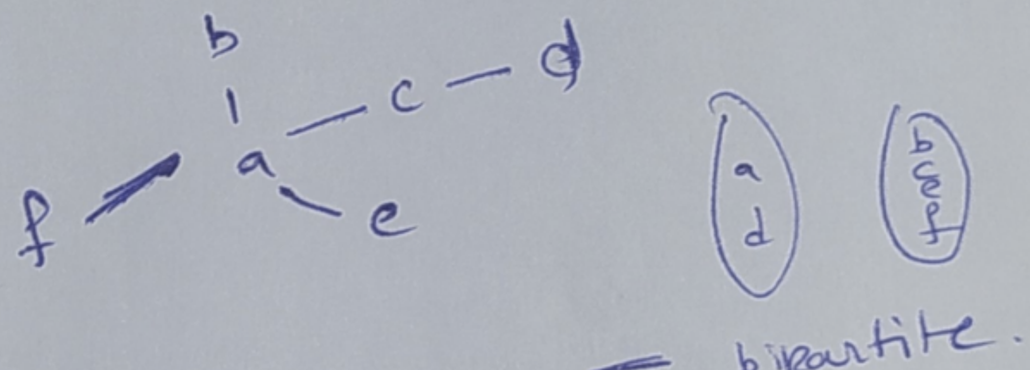
If $(n-1)$ Edges then forest will be a Tree.

Q:

What is the minimum number of edges in a Connected Graph on n vertices?

But total deg = $2n-2$)

Q) Consider a graph G such that atleast one vertex v is connected to all other vertices, and there is some other vertex of degree more than 1. Such G is not bepartite.



YES, your question's statement is true, in a graph

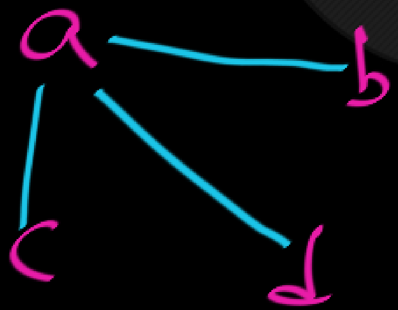
1. if at least one(assume exactly one) vertex is connected to all other vertices(n-1) then it will automatically pushes all other other vertices to 2nd partitions.

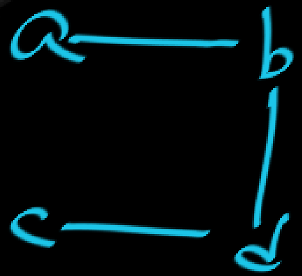
2. and there is some vertex(assume exactly one) have more then 1 degree then it will definitely creates an edge from one vertex to some other in same partition.

So, G is not a bipartite graph.

Q:

What is the minimum number of edges in a Connected Graph on n vertices? $= n-1$





So Tree.

Tree $\equiv$ Connected Graph with minimum number of edges

$\equiv$ minimally connected

Q:

What is the minimum number of edges in a Connected Cyclic Graph on n vertices?

Q:

What is the minimum number of edges in a Connected Cyclic Graph on n vertices? $= n$

Connected with $n-1$ Edges $\equiv$ Tree

Q:

What is the minimum number of edges in a Connected Acyclic Graph on n vertices?

Q:

What is the minimum number of edges in a Connected Acyclic Graph on n vertices?

Tree

$= n-1$

Q: Is it possible?

A tree with 9 vertices and the sum of the degrees of all the vertices is 18.

Q: Is it possible? — No

A tree with 9 vertices and the sum of the degrees of all the vertices is 18. $\Rightarrow$ No

Tree with n vertices $\Rightarrow |E| = n-1$

Total Degree $= 2(n-1)$

for $|V|=9$;
Total Degree $= 2(8) = 16$

Q: Is it possible?

A graph with 9 vertices, 9 edges, and no cycles.

Q: Is it possible? — No

A graph with 9 vertices, 9 edges, and no cycles. ⇒ forest

In a forest, max. $|E| = n-1 = 9-1 = 8$

In a Tree on n vertices ;

min $|E| = n-1$

max $|E| = n-1$

n-1 Edges

In a forest on n vertices ;

$$\min |E| = 0 \quad (E_n)$$

$$\max |E| = n-1 \quad (\text{Tree})$$

Q: Is it possible?

Graph has 8 vertices, 8 edges, and no cycles.

Q: Is it possible? $\Rightarrow$ NO

Graph has 8 vertices, 8 edges, and no cycles.

forest

Max $|E| = n-1$

Q: Is it necessarily a Tree??

Graph G with n vertices, one vertex of degree n–1 and rest of the vertices of degree 1.

Q: Is it necessarily a Tree?? → Yes.

a_1

Graph G with n vertices, one vertex of degree n–1 and rest of the vertices of degree 1.

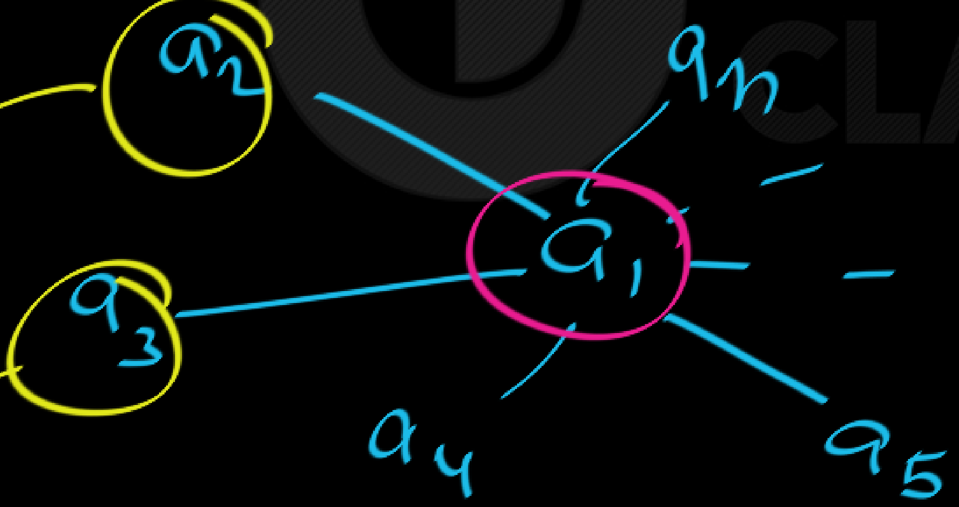
Graph, n vertices

$a_1, a_2, a_3, ---, a_n$

$Deg = n-1$ means a_1 is Adjacent to all others.

Deg = 1

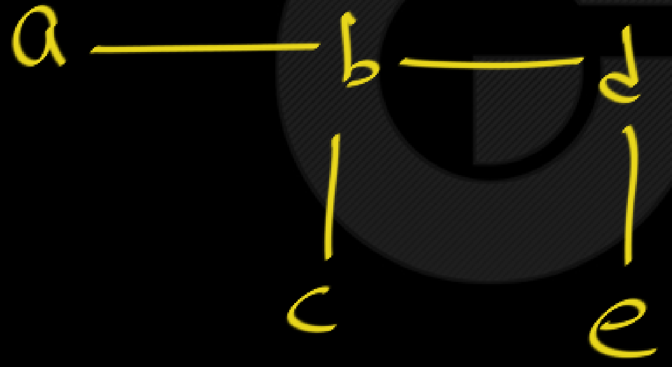
Deg = 1



Star Graph;

Tree

Star Graph $\longrightarrow$ Tree



Tree ✓

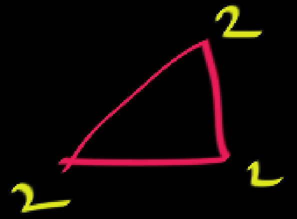
NOT Star Graph

Q:

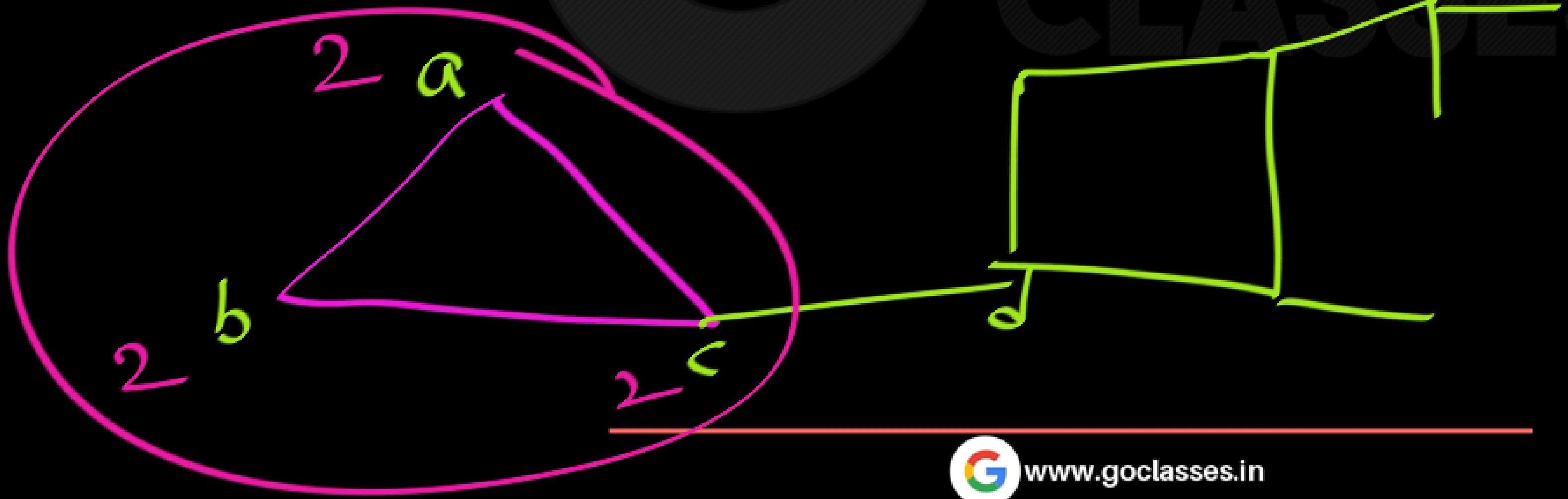
A cyclic graph must have at least 3 vertices of degree at least 2? True or False?

Q:

Smallest Cycle Possible



A cyclic graph must have at least 3 vertices of degree at least 2? True or False?



Ans:

In any cycle of a cyclic graph, every vertex in it has a degree greater or equal to 2 (since we have to both "enter" and "exit" the vertex).

Q: True or False?

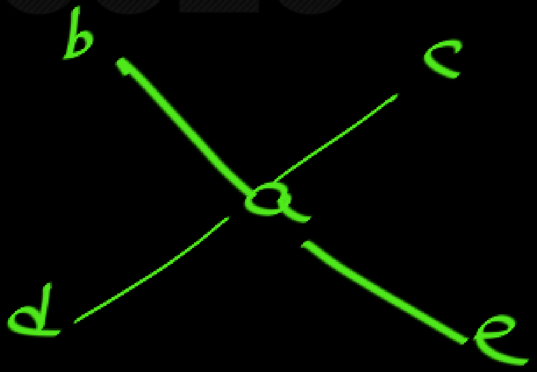
Consider a graph G such that at least one vertex v is connected to all other vertices.

Such G is not bipartite.

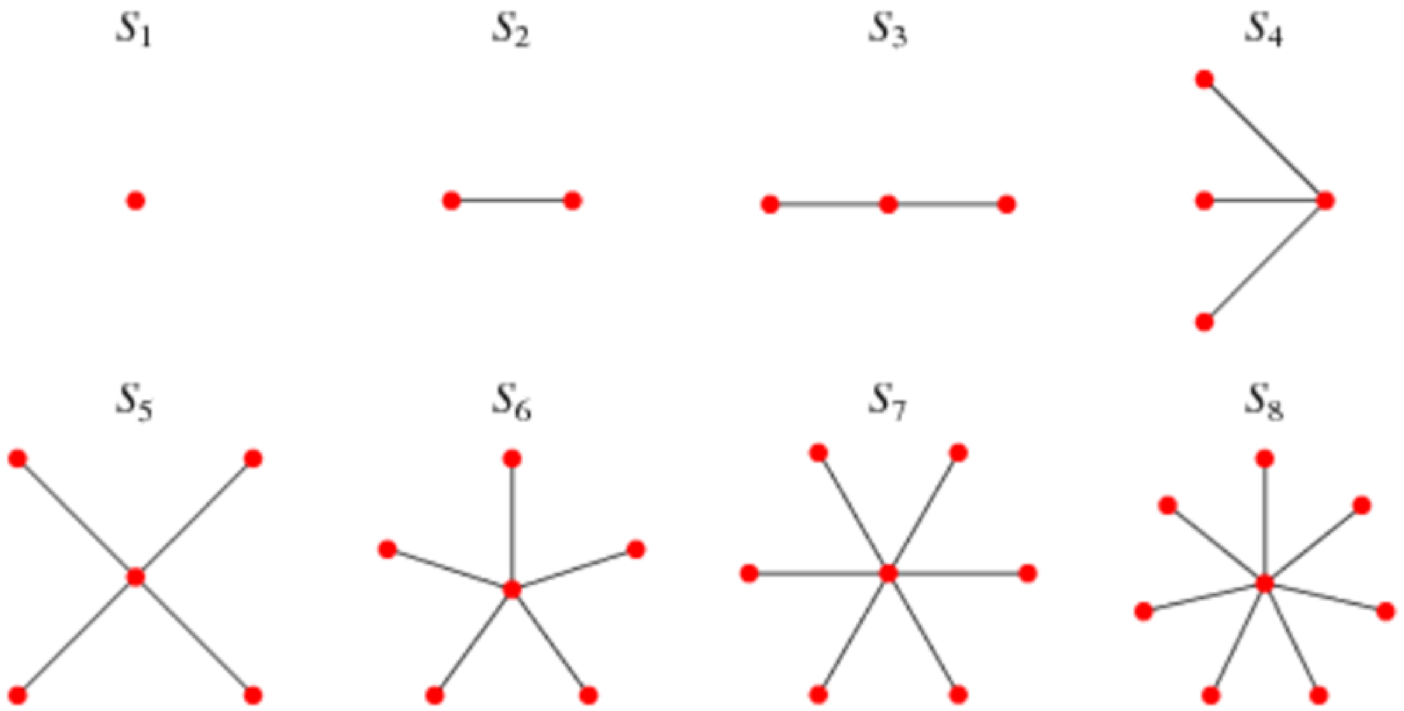
Q: True or False?

Consider a graph G such that at least one vertex v is connected to all other vertices.

Such G is not bipartite.



There are infinitely-many counterexamples to the claim as stated. Consider the graph on $n+1$ vertices where v is connected to each of the other n vertices but no other edges are included (this is sometimes called the star graph S_{n+1}). The graph is bipartite: put v in one set and the remaining n vertices in another.



Q: True or False?

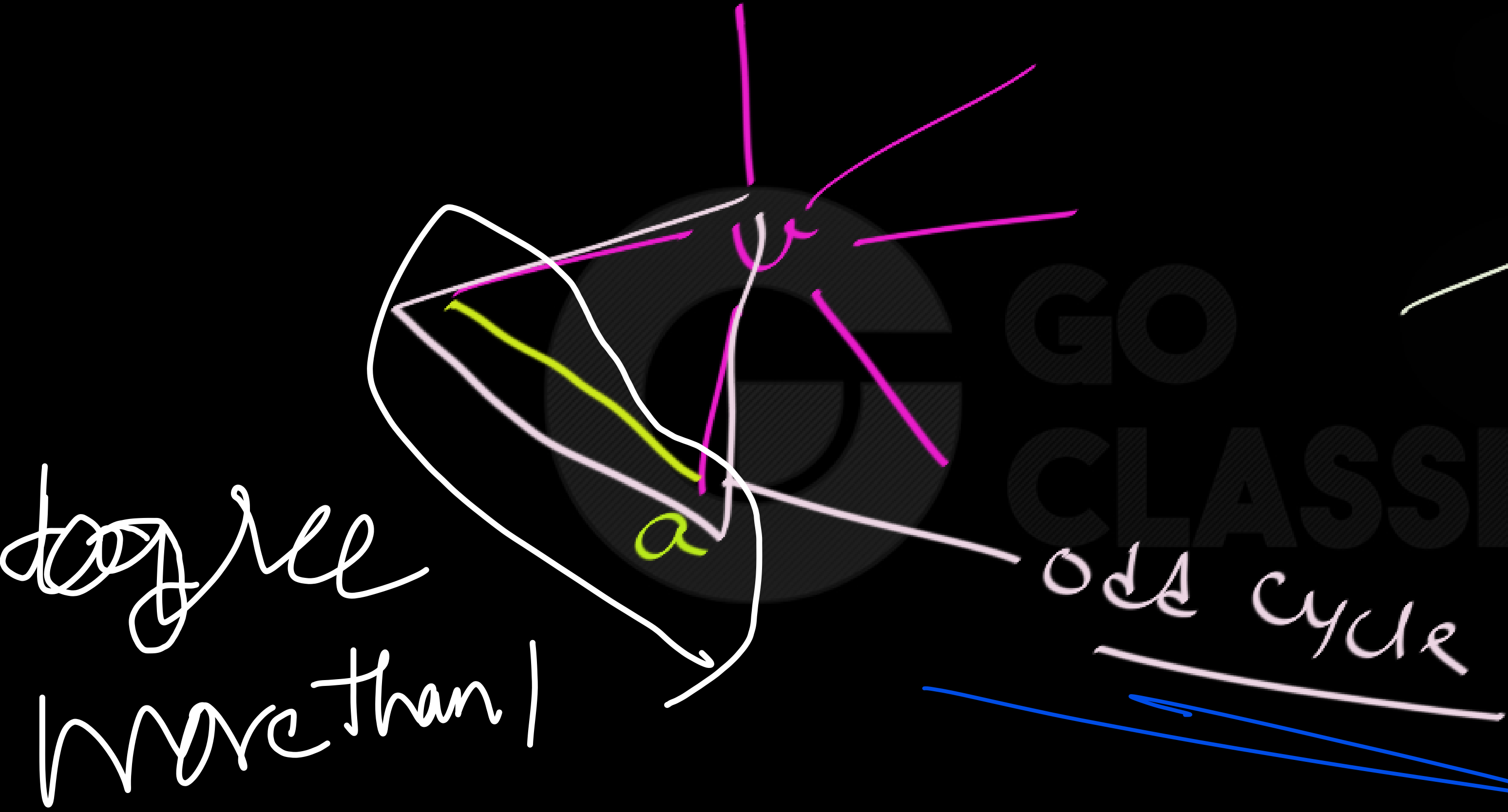
Consider a graph G such that at least one vertex v is connected to all other vertices, and there is some other vertex of degree more than 1. Such G is not bipartite.

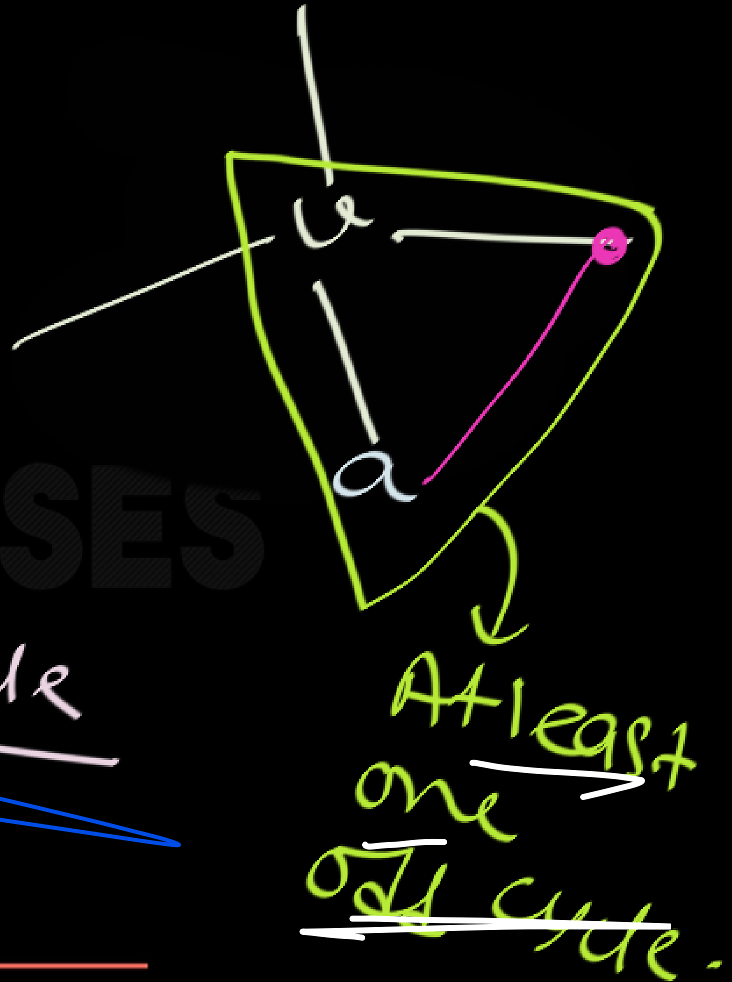
Q: True or False?

True

Consider a graph G such that at least one vertex v is connected to all other vertices, and there is some "other" vertex of degree more than 1. Such G is not bipartite.

a


u
a
degree
more than 1
odd cycle


u
a
Atleast one odd cycle.

Bipartite iff NO odd cycle.

Bipartite iff All cycles are even.

Q: True or False?

Let G be a graph with at least two vertices,

with NO vertex of degree 1.

G cannot be a tree.

Q: True or False?

Let G be a graph with at least two vertices,

with NO vertex of degree 1.

G cannot be a tree.

Assume G is Tree : $Deg(v) \geq 1 \ \forall v$

G :

	a_1	a_2	a_3	---	a_n
Deg =	≥ 2	≥ 2	≥ 2	---	≥ 2

Total Degree $\geq 2n$

But in Tree, Total Deg $= 2n - 2$

Contradiction

G NOT Tree.

Q: True or False?

Let G be a graph with at least two vertices,

with exactly one vertex of degree 1.

G cannot be a tree.

Q: True or False?

Let G be a graph with at least two vertices, with exactly one vertex of degree 1.

G cannot be a tree.

Assume G = Tree $\rightarrow$ Connected $\rightarrow$ $\text{Deg}(v) \geq 1$ $\forall v$

G: a_1 a_2 a_3 - - - a_n

Deg: 1 ≥ 2 ≥ 2 ≥ 2

Total Deg $\geq 2(n-1) + 1 = 2n-1$

But in Tree, Total Deg $= 2n-2$

Contradiction

So G NOT Tree

NOTE: Every Tree with $n \geq 2$ has at least two leaves.

leaf: Deg 1 vertex.

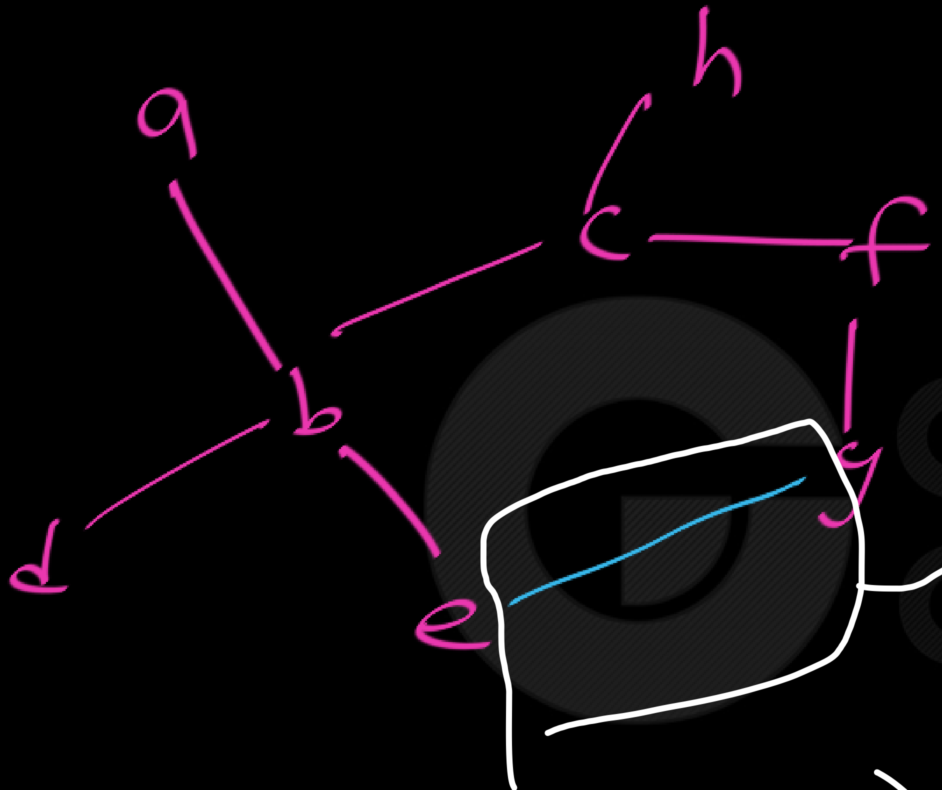
Pendent vertex.

Q: True or False?

To a tree if we add an edge between two existing vertices of T, it creates exactly one cycle.

Q: True or False?

To a tree if we add an edge between two existing vertices of T, it creates exactly one cycle.



any edge (add)

Tree + one Edge

Cyclic Graph with EXACTLY one cycle.

Q: True or False?

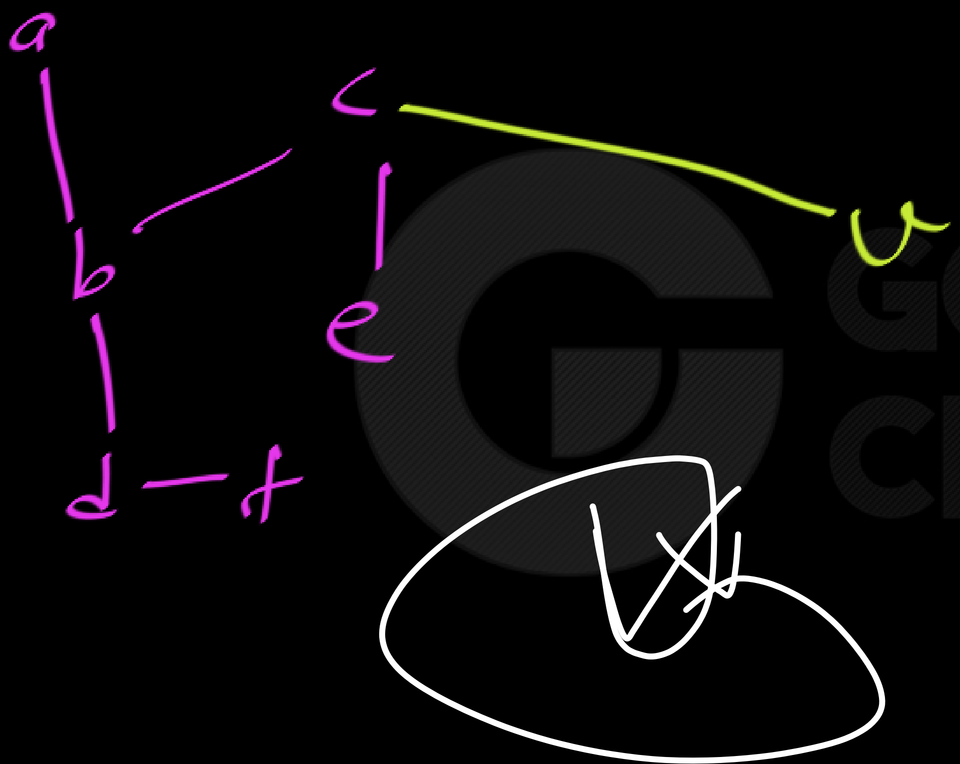
To a tree if we add a new vertex with one edge, it creates a cycle. not Possible

$e + v$

Q: True or False?

To a tree if we add a new vertex with one edge, it creates a cycle.

can



Tree + one new vertex of Degree 1

Tree

Algorithms :

MST

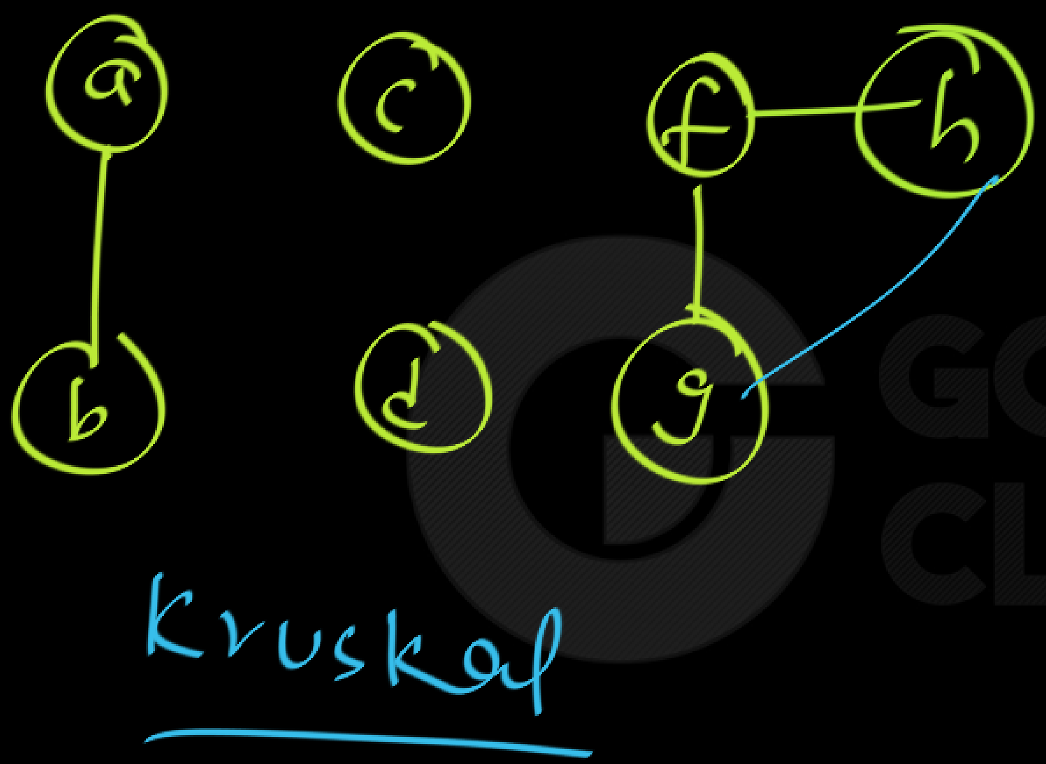
no cycle

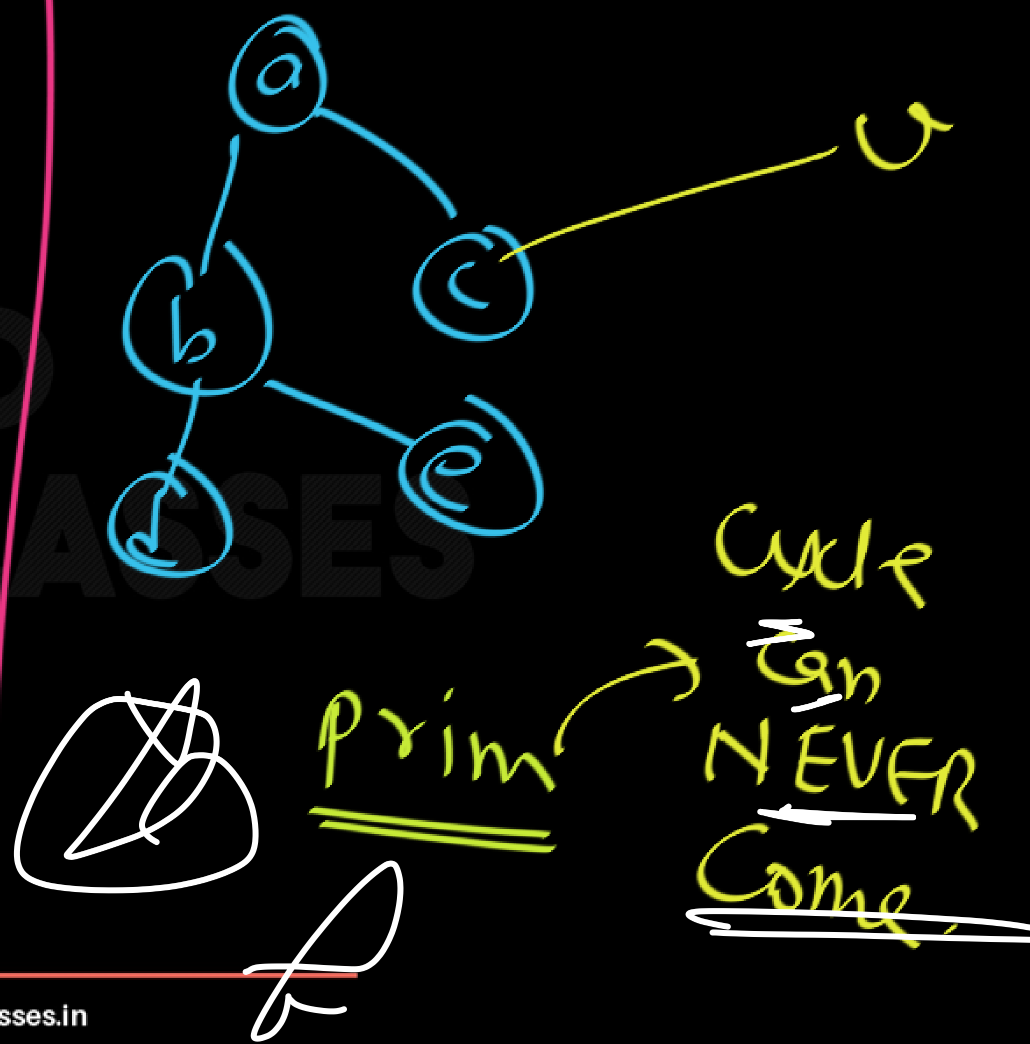
Prim's ⇒ intermediate Tree + one new vertex with one Edge

Tree+

Kruskal's → Cycle can occur.

Intermediate forest + Add one more Edge


a
b
c
d
f
h
g
Kruskal


a
b
c
d
e
v
Prim
Cycle can NEVER come

Q: True or False?

If G has no cycles but by adding one edge between any two vertices will create a cycle then G is a tree.

Q: True or False?

forest = no cycle

If G has no cycles but by adding one edge between "any two vertices" will create a cycle then G is a tree.

True

Q: True or False?

Let G be a connected graph, then G is a tree iff G has no cycles

Q: True or False?

Let G be a connected graph, then G is a tree iff G has no cycles

def of tree

Q: True or False?

Let T be a tree. G is a new graph by adding vertex x and connect it to one vertex through an edge in T. Then G is a Tree.

Q: True or False?

Let T be a tree. G is a new graph by adding vertex x and connect it to one vertex through an edge in T. Then G is a Tree.

Tree + new vertex with one Edge ⟹ Tree

Q: True or False?

Adding one edge to a tree creates exactly one cycle.

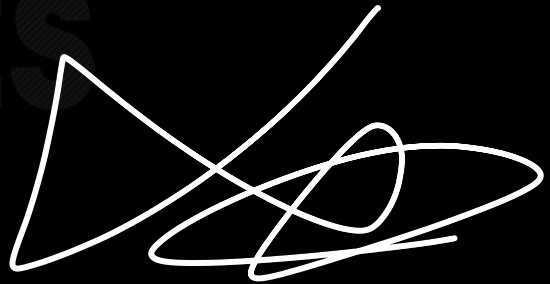
T

Q: True or False?

Every tree with $n \geq 2$ vertices has at least two vertices of degree one.

Q: True or False?

A finite graph in which every vertex has degree ≥ 2 has a cycle



$\underbrace{\text{Tree with } n \geq 2}$ $\Longrightarrow$ At least two vertices have Degree 1.

Q:

Removing any one edge from a tree results in

___2___ number of trees.

Tree ka koi bhi one edge

remove karne se 2 Tree form honge.

Q:

Removing any one edge from a tree results in

___2___ number of trees.

2.11.2 Trees: TIFR2011-B-33

Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

A. G is connected and has $n-1$ edges.

B. G is acyclic and connected.

C. G is acyclic and has $n-1$ edges.

D. G is acyclic, connected and has $n-1$ edges.

E. G has $n-1$ edges.

2.11.2 Trees: TIFR2011-B-33

Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

A. G is connected and has $n-1$ edges.

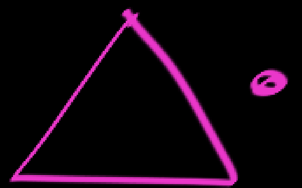
B. G is acyclic and connected.

C. G is acyclic and has $n-1$ edges.

D. G is acyclic, connected and has $n-1$ edges.

E. G has $n-1$ edges.

Definition (iff)



$n=4, |E|=3$

Necessary for Tree, NOT sufficient

2.4.22 Graph Connectivity: GATE2014-3-51

If G is the forest with n vertices and k connected components, how many edges does G have?

A. $\lfloor \frac{n}{k} \rfloor$

B. $\lceil \frac{n}{k} \rceil$

C. $n-k$

D. $n-k+1$

k Trees

Conn components = tree

forest G:

	Tree		Tree		Tree
	G_1	G_2	G_3	$- -$	G_k
$\lvert V \rvert =$	n_1	n_2	n_3	$- - -$	n_k
$\lvert E \rvert =$	$n_1 - 1$	$n_2 - 1$	$n_3 - 1$	$- - -$	$n_k - 1$

Total $\lvert E \rvert$ in forest $G = \underbrace{(n_1 + n_2 + \cdots + n_k)} - k$

$= n - k$

In forest, Every Component is a Tree.

If $K = 1$ then $\boxed{\text{forest} \equiv \text{Tree}}$

$|E| = n-1$

K component

No of edge $= n-K$

2.4.7 Graph Connectivity: GATE1995-1.25

The minimum number of edges in a connected cyclic graph on n vertices is:

A. $n-1$ B. n C. $n+1$ D. None of the above

2.4.5 Graph Connectivity: GATE1993-8.1

Consider a simple connected graph G with n vertices and n edges $(n > 2)$. Then, which of the following statements are true?

A. G has no cycles
B. The graph obtained by removing any edge from G is not connected
C. G has at least one cycle
D. The graph obtained by removing any two edges from G is not connected
E. None of the above

def hai cycle

2.4.5 Graph Connectivity: GATE1993-8.1

Consider a simple connected graph G with n vertices and n edges $(n > 2)$. Then, which of the following statements are true?

A. G has no cycles
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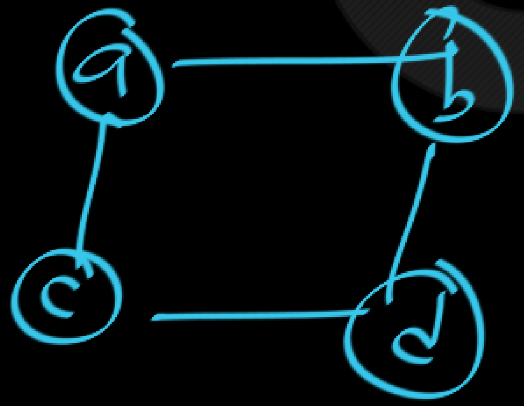
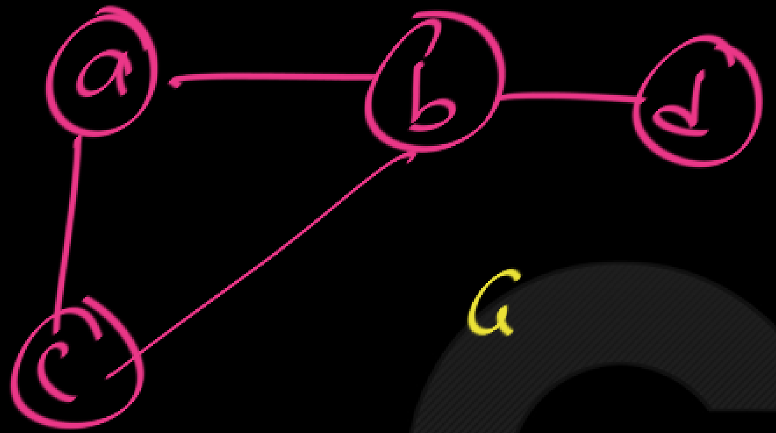
E. None of the above

(b)



Remove any one Edge

Connected



G has at least one cycle? Yes.

Exactly one cycle $\rightarrow$ At least one cycle

At least one cycle $\not\rightarrow$ Exactly one cycle

Connected
n vertices
n edges

Remove 2 Edges →

n vertices
n-2 Edge

Can it be Connected?

No

minimally Connected Graph has n-1 Edges

2.4.13 Graph Connectivity: GATE2004-IT-5

What is the maximum number of edges in an acyclic undirected graph with n vertices?

A. $n-1$ B. n C. $n+1$ D. $2n-1$

2.4.13 Graph Connectivity: GATE2004-IT-5

What is the maximum number of edges in an acyclic undirected graph with n vertices?

A. $n-1$ B. n C. $n+1$ D. $2n-1$

Theorem 30. *The following statements are equivalent for a graph T:*

(1) T *is a tree.*

(2) Any two vertices in T *are connected by a unique path.*

(3) T *is minimally connected, i.e.* T *is connected but* $T - e$ *is disconnected for any edge* e *of* T.

(4) T *is maximally acyclic, i.e.* T *is acyclic but* $T + uv$ *contains a cycle for any two non-adjacent vertices* u, v *of* T.

(5) T *is connected and* $|E(T)| = |V(T)| - 1$.

(6) T *is acyclic and* $|E(T)| = |V(T)| - 1$.

Graph Theory :

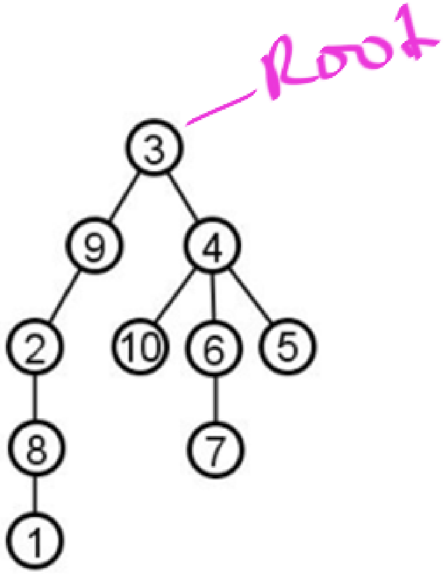
Next Topic :

Rooted Trees

(Special Types of DAGs) Binary Trees

Website : https://www.goclasses.in/

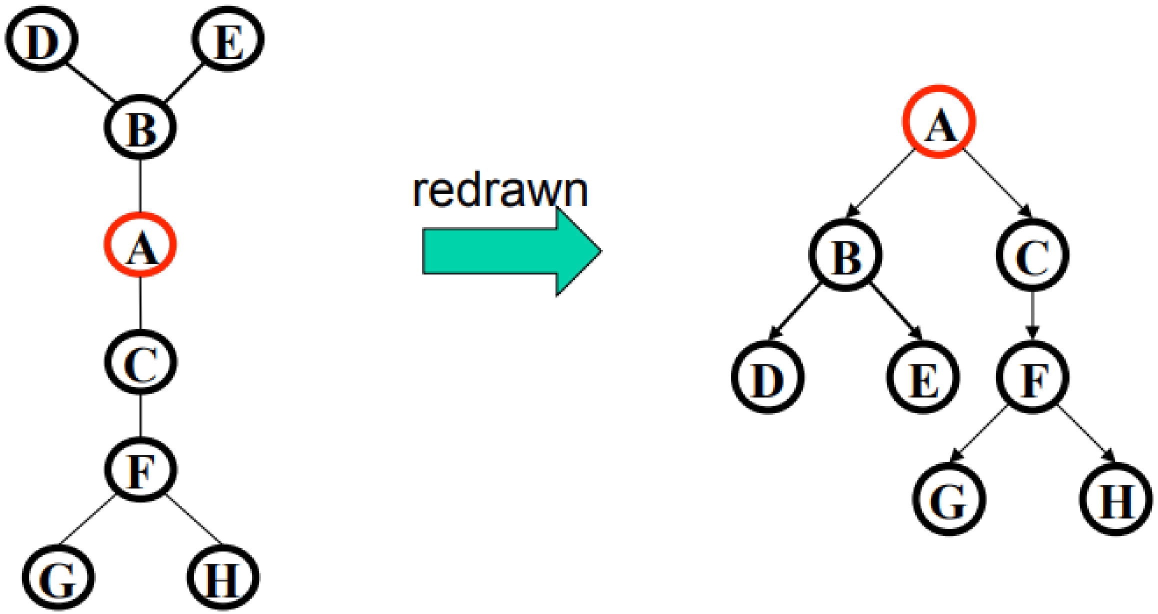
Rooted Trees



- A rooted tree has a designated root vertex and every edge is directed away from the root.
- Vertex v is a parent of vertex u if there is an edge from v to u; and u is called a child of v
- Vertices with the same parent are called siblings
- Vertex v is an ancestor of u if v is u's parent or an ancestor of u's parent.
- Vertex v is a descendant of u if u is v's ancestor

Rooted Trees

- We are more accustomed to rooted trees where:
  - We identify a unique root
  - We think of edges as directed: parent to children
- Given a tree, picking a root gives a unique rooted tree
  - The tree is just drawn differently



Rooted Trees

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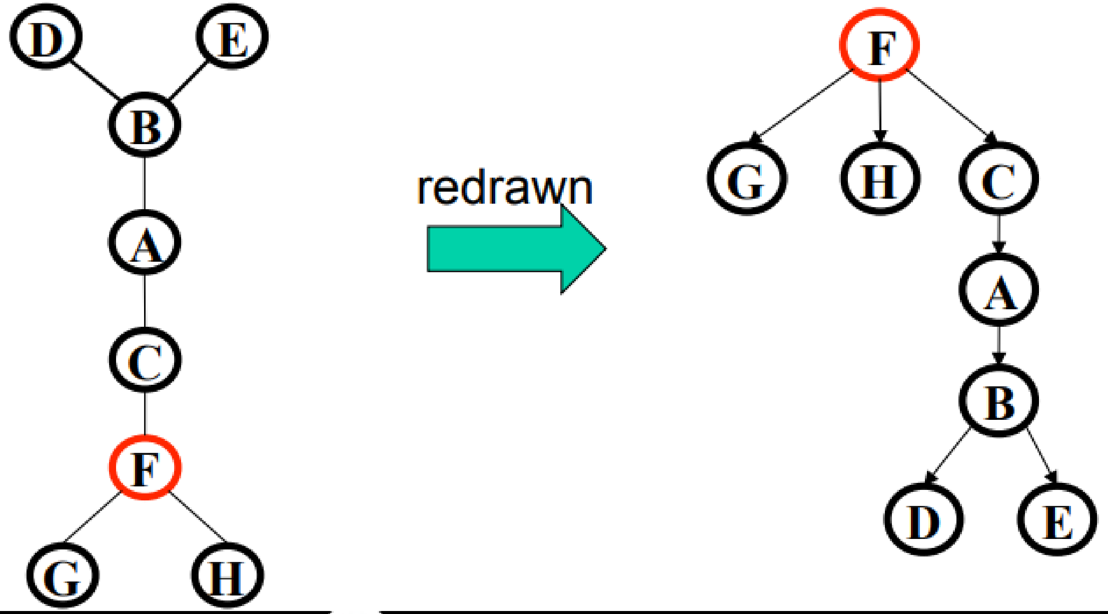
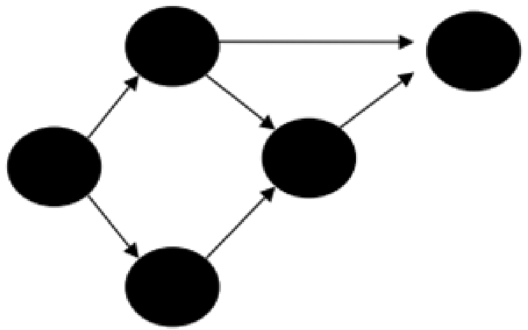
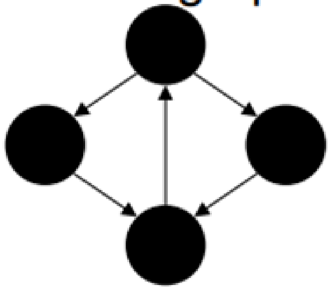


Directed Acyclic Graphs (DAGs)

- A DAG is a directed graph with no (directed) cycles
  - Every rooted directed tree is a DAG
  - But not every DAG is a rooted directed tree

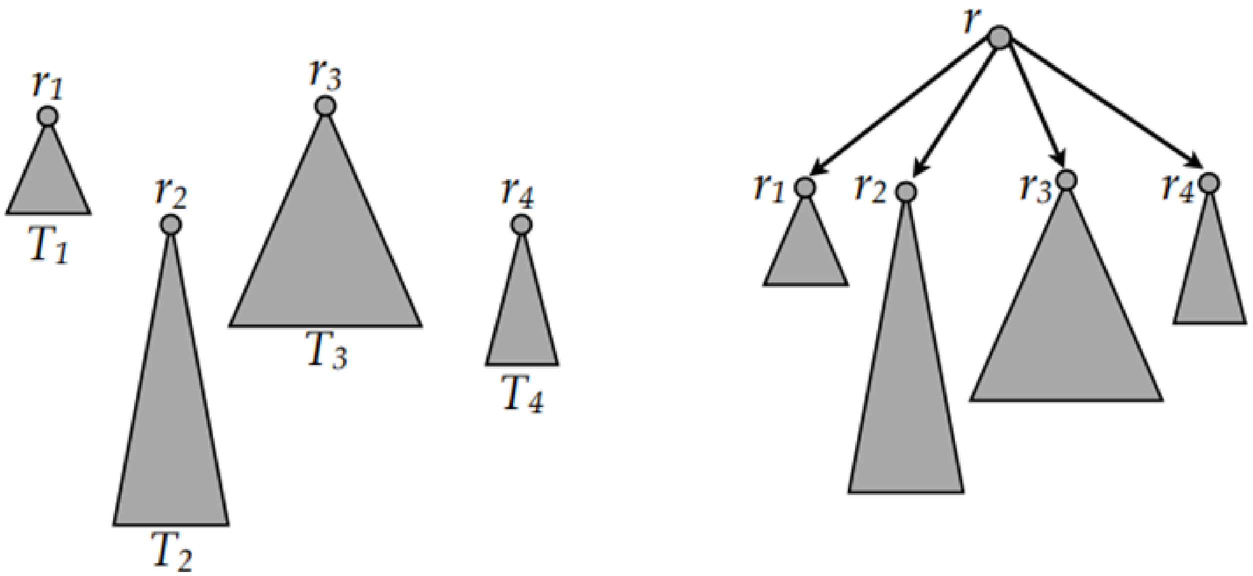


  - Every DAG is a directed graph
  - But not every directed graph is a DAG



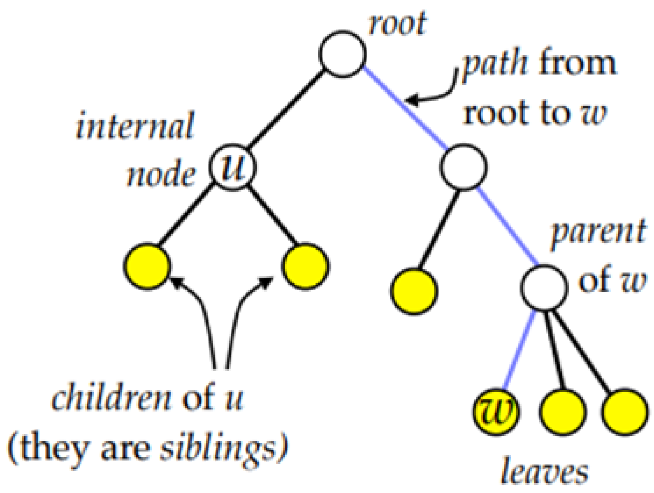
Definition – Rooted Tree

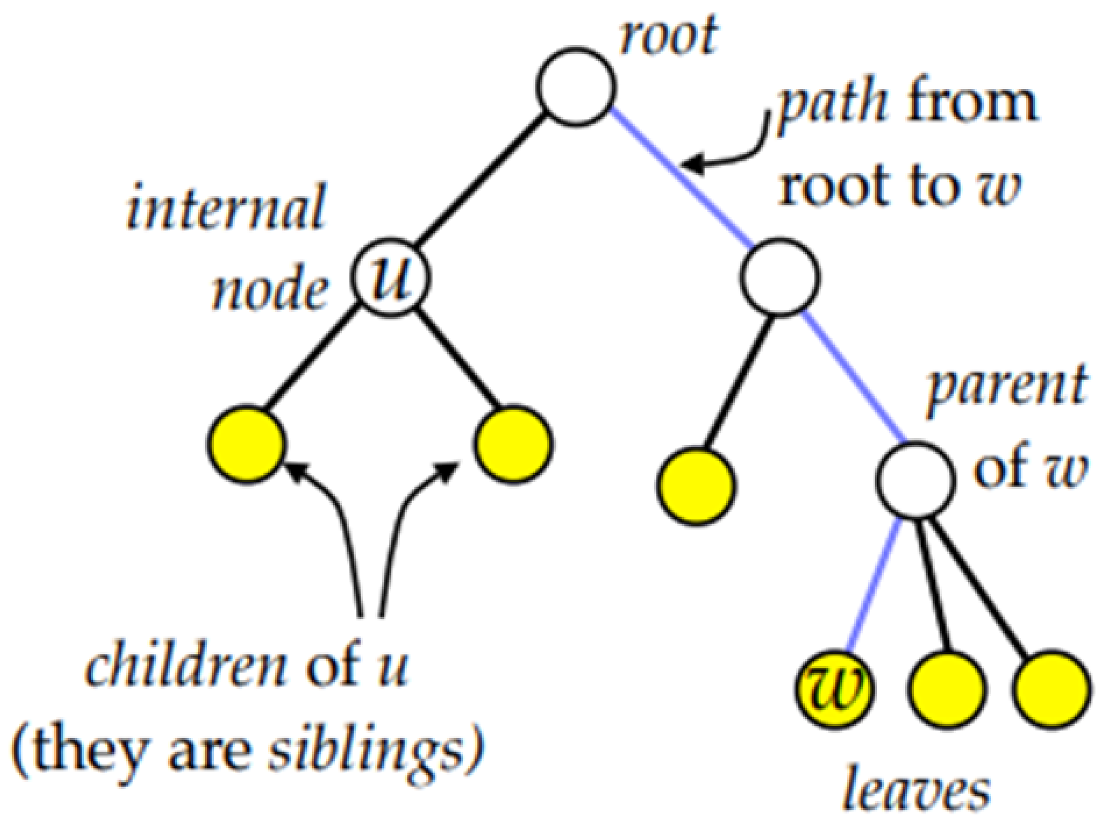
- Λ is a tree
- If $T_1, T_2, \ldots, T_k$ are trees with roots $r_1, r_2, \ldots, r_k$ and r is a node $\notin$ any T_i, then the structure that consists of the T_i, node r, and edges (r, r_i) is also a tree.



Terminology

- r is the <u>*parent*</u> of its <u>*children*</u> $r_1, r_2, \ldots, r_k$.
- $r_1, r_2, \ldots, r_k$ are <u>*siblings*</u>.
- <u>*root*</u> = distinguished node, usually drawn at top. Has no parent.
- If all children of a node are Λ, the node is a <u>*leaf*</u>. Otherwise, the node is a <u>*internal node*</u>.
- A <u>*path*</u> in the tree is a sequence of nodes $u_1, u_2, \ldots, u_m$ such that each of the edges (u, u_{i+1}) exists.
- A node u is an <u>*ancestor*</u> of v if there is a path from u to v.
- A node u is a <u>*descendant*</u> of v if there is a path from v to u.




root
path from
root to w
internal
node
u
parent
of w
w
children of u
(they are *siblings*)
leaves

Rooted trees

- A *rooted tree* is a tree with a designated vertex v_0 called the *root*. According to Theorem 30, each vertex v_i is connected to the root by a unique path $v_i, v_{i-1}, \ldots, v_1, v_0$, in which case v_{i-1} is called *father* (or *parent*) of v_i, and v_i is a *child* (or *son*) of v_{i-1}.

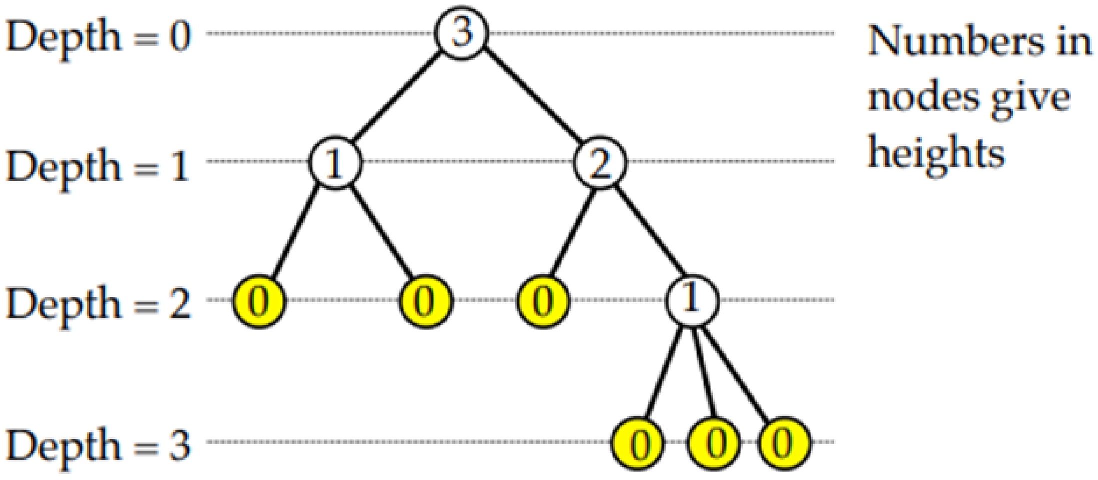
- The *height* of a rooted tree is the distance from the root to the farthest leaf.

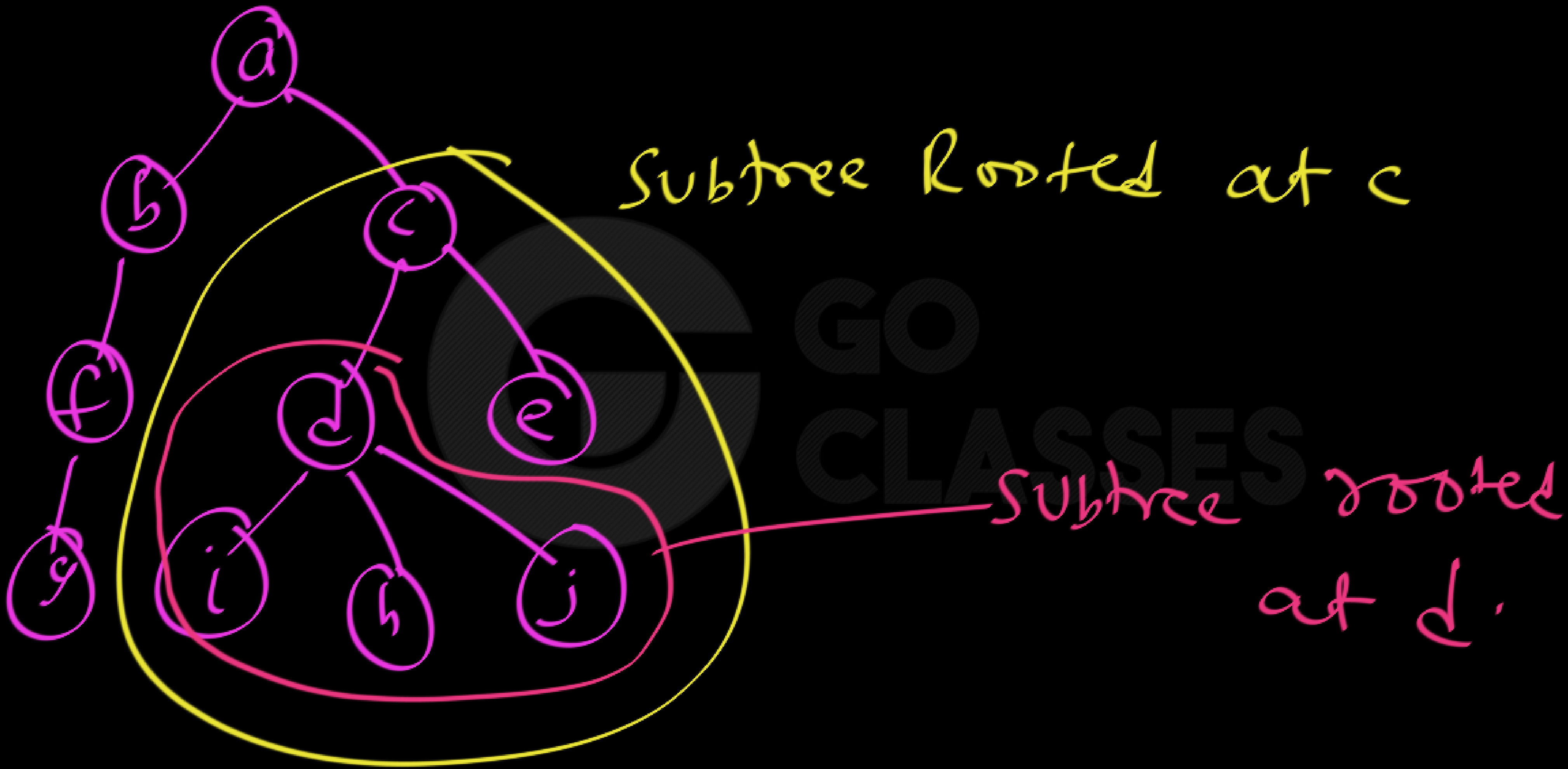
- A *binary* tree is a rooted tree in which every vertex has at most two children.

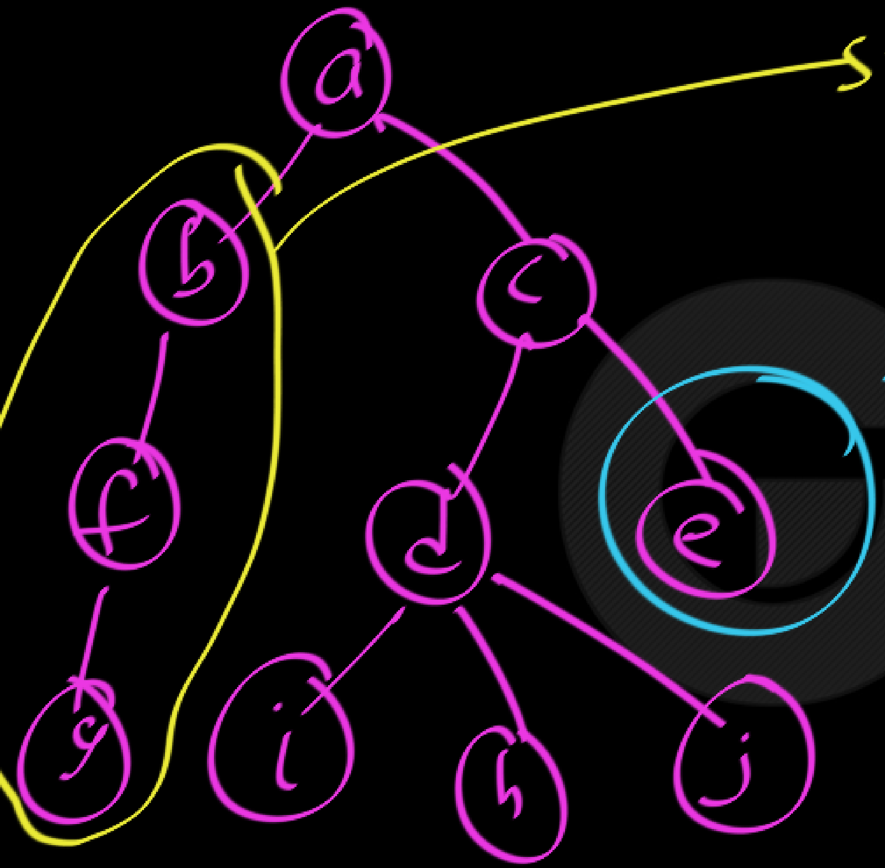
- A *full binary tree* is a binary tree in which every non-leaf vertex has exactly two children.

Height & Depth

- The *height* of node u is the length of the longest path from u to a leaf.
- The *depth* of node u is the length of the path from the root to u.
- Height of the tree = maximum depth of its nodes.
- A *level* is the set of all nodes at the same depth.




a
b
c
f
d
e
g
i
h
j
Subtree Rooted at c
Subtree rooted at d.

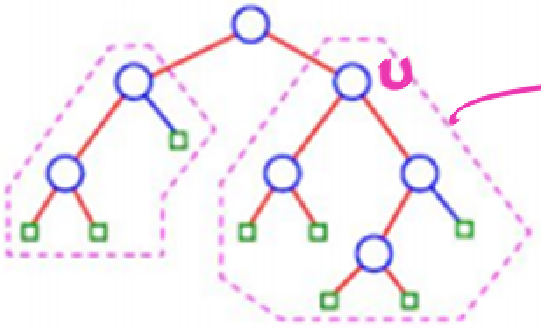


subtree Rooted at b.

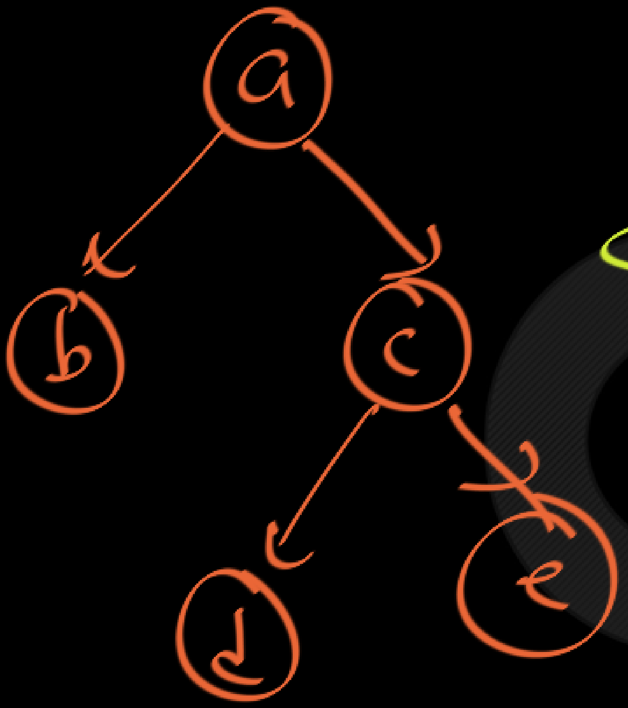
subtree Rooted at e

Subtrees

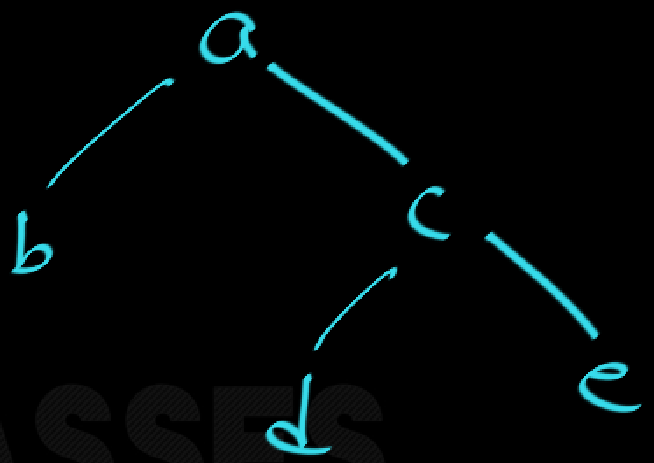
- Given a rooted tree and a node v, the subtree rooted at v includes v and its descendants.



Subtree Rooted at u.



underlying structure



Rooted Tree

Tree

Basic Properties

- Every node except the root has exactly one parent.
- A tree with n nodes has n-1 edges (every node except the root has an edge to its parent).
- There is exactly one path from the root to each node.

"Rooted Tree" → Directed Graph

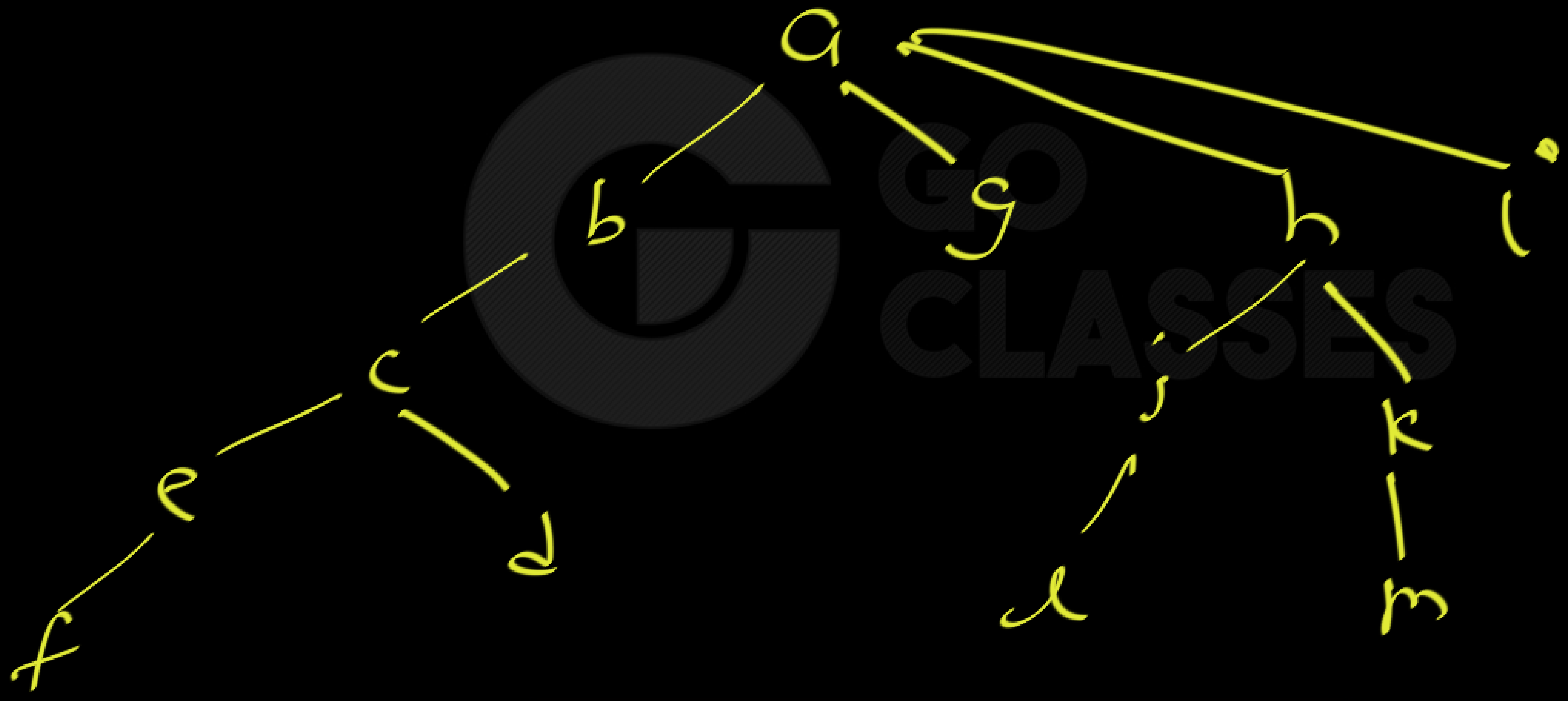
→ very special

→ Term "Degree" is Defined (but in another way)

Degree of a node in a Rooted Tree:

$\equiv$ number of children.

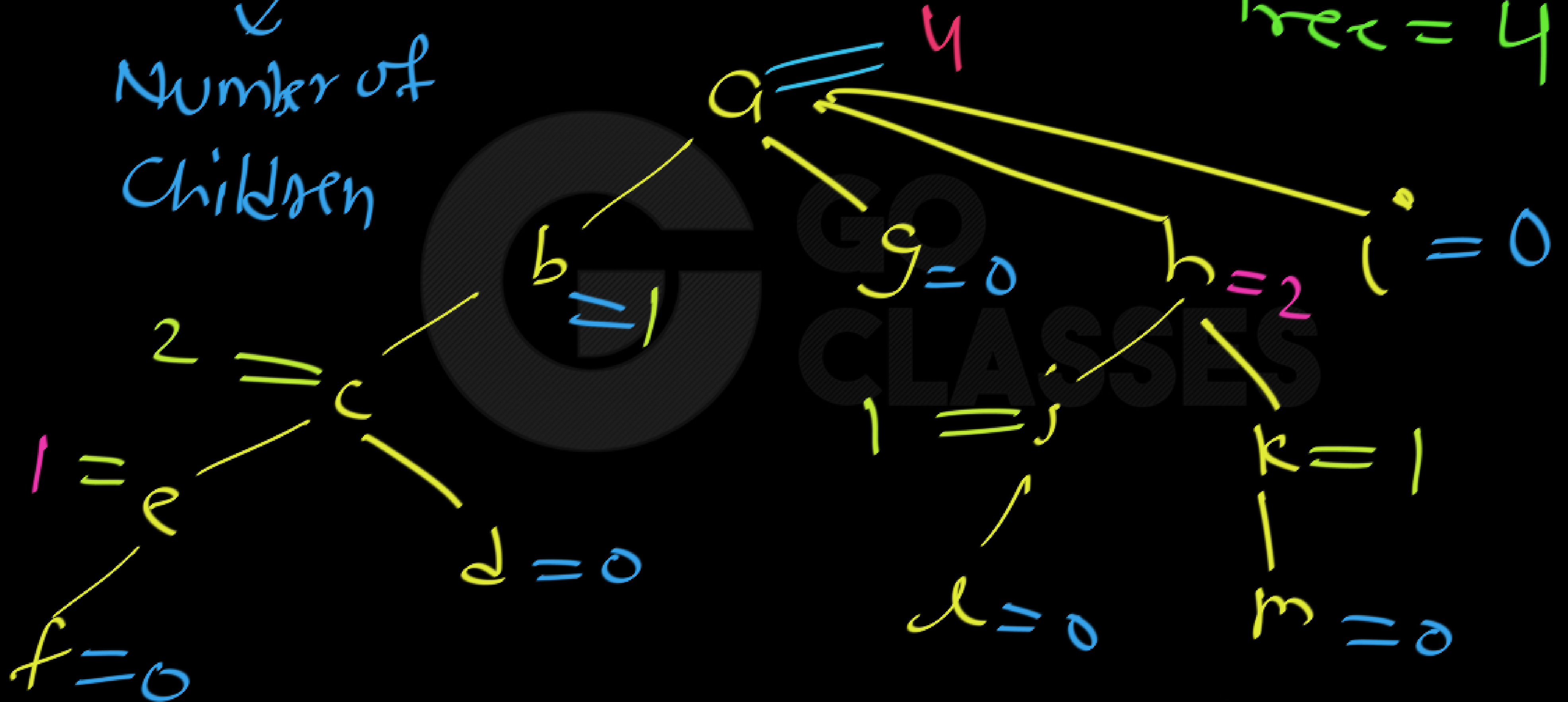
find Degree of all nodes;

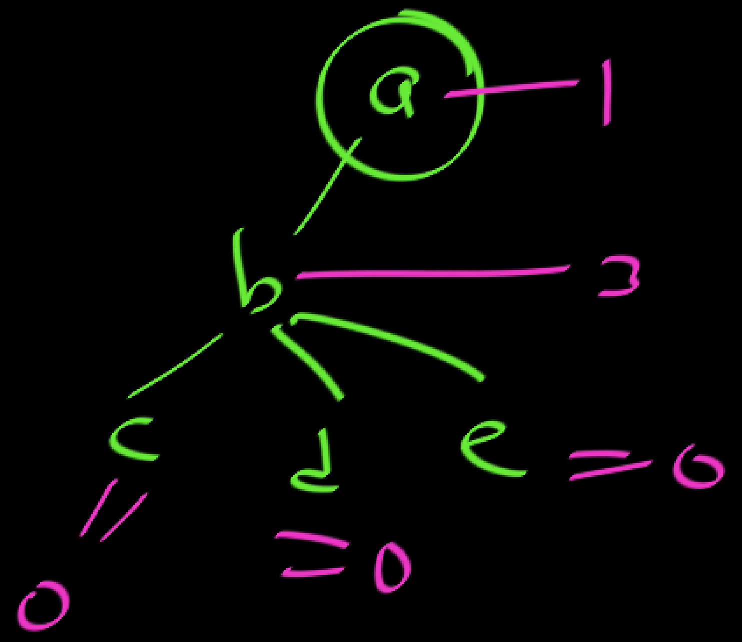


find Degree of all nodes;

Degree of Tree = 4

Number of Children





Degree of Tree = 3

- **DEFINITION:** Nodes with no children are called *leaf* nodes. All other nodes are called *interior* nodes.

- **DEFINITION:** The *degree* of a node is the number of its children. The degree of a tree is the maximum degree of any of its nodes.

- **DEFINITION:** Nodes with the same parent are called *siblings*.

- Note that the edges are **directed** edges. This means there is a direction associated with them. [redacted]
Nobody draws the edges as directed edges, it's just understood.

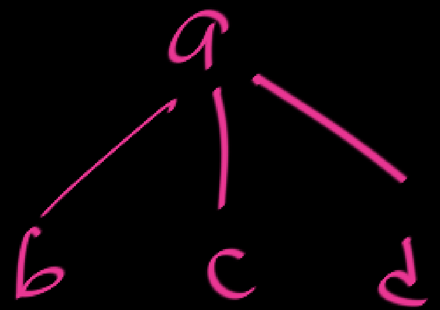
Graph Theory :

Next Topic :

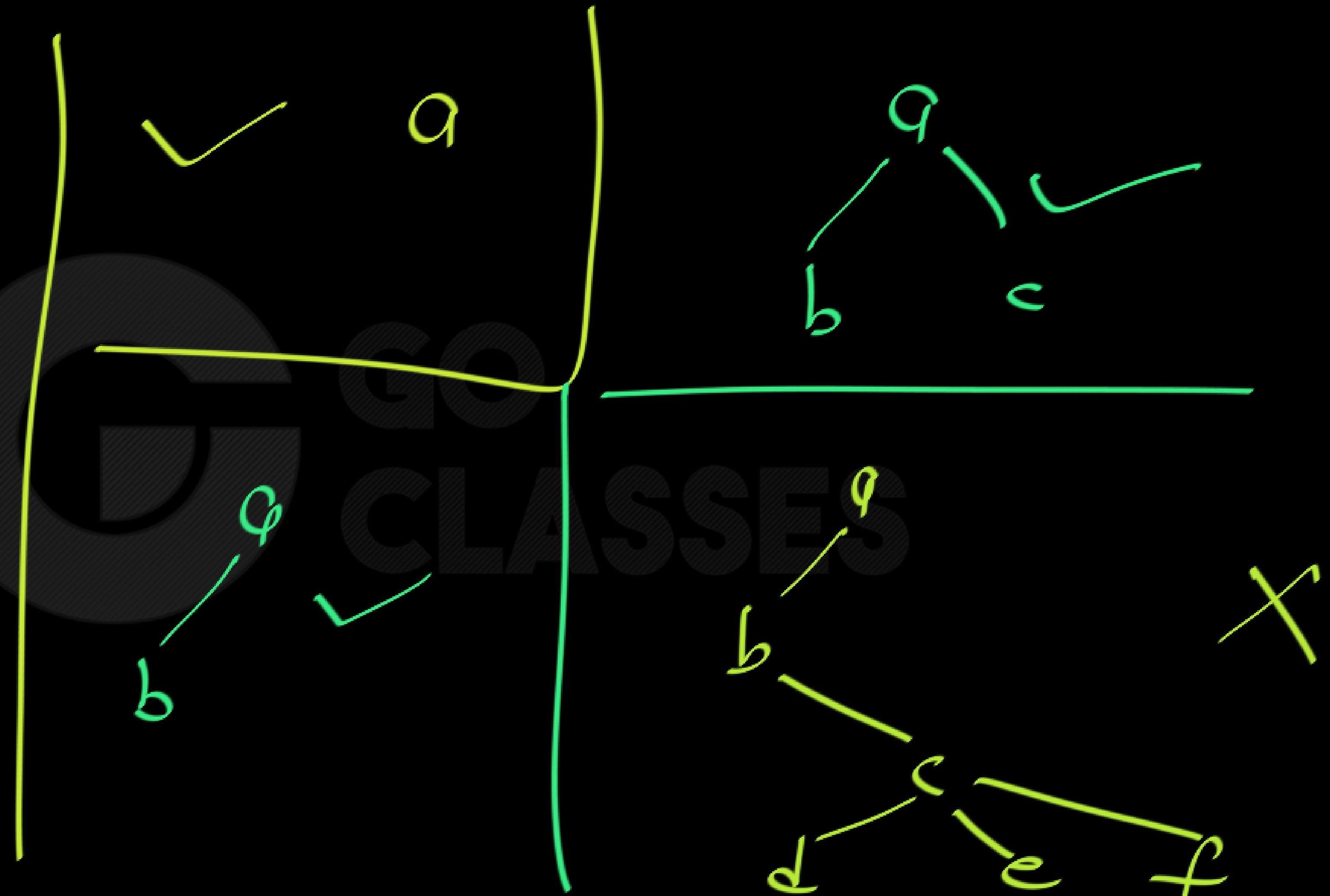
Binary Trees

Website : https://www.goclasses.in/

- The *height* of a rooted tree is the distance from the root to the farthest leaf.
- A *binary* tree is a rooted tree in which every vertex has at most two children.
- A *full binary tree* is a binary tree in which every non-leaf vertex has exactly two children.



NOT binary Tree



Leaf: node without children.

Deg 0 node.

Degree 2 node: exactly 2 child node

Q: L = no. of Leaves (Deg 0 nodes)

D'' = no. of Degree 2 nodes.

find Relation b/w them.

Proof 1: Binary Tree → Rootes Tree → Directed Graph

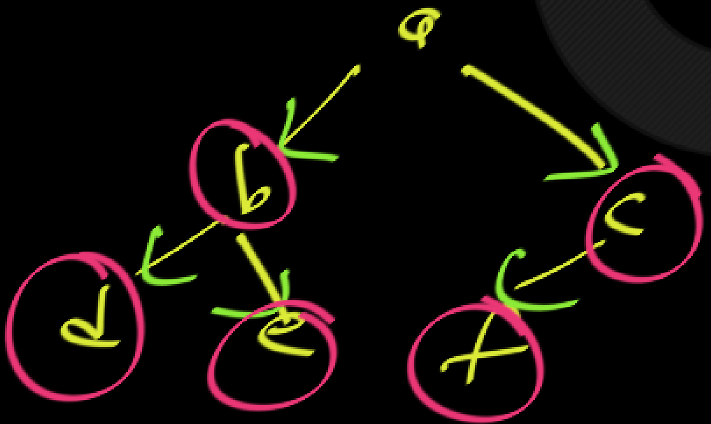
Total InDeg = Total outDeg
= |E| = n-1

Total InDeg = Total outDeg = |E| = n-1

Total In Deg :

In Deg of every node Except Root = 1

In Deg (Root) = 0



$\boxed{\text{Total In Deg} = n-1}$

In a Binary Tree:

How many type of Nodes?

Deg 0 nodes ≡ Leaves

Deg 1 "

Deg 2 "

L = Number of Leaves (Dep o node)

D' = " " Dep-1 nodes

D'' = " " Dep-2 "

Total OutDeg :

OutDeg of leaves $= 0$
OutDeg of Deg-1 node $= 1$
" " " Deg-2 " $= 2$

Total OutDegree $= L(0) + D'(1) + 2D''$

So,

$$\boxed{n = L + D' + D''}$$

$$D' + 2D'' = n - 1 = L + D' + D'' - 1$$

$$= \boxed{D'' = L - 1}$$

$$\boxed{L = D'' + 1}$$

In Binary Tree :

$$\boxed{L = D'' + 1}$$

#Leaves = number of 2-Deg nodes + 1

Proof 2 :

Binary Tree → Rooted Tree

own Degree Definition

as a

underlying Graph

Tree

own $\boxed{\text{Degree}}$ Definition ≡ let's use Order

Let's use order word for Tree instead of Degree.

$\boxed{n \geq 2}$

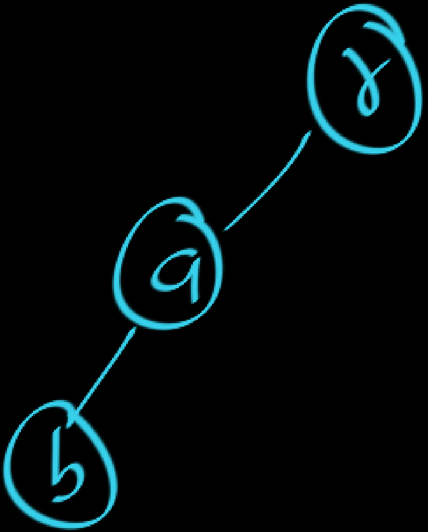
Order of Leaf = 1

Order of Deg-1 Node = 2

" " Deg-2 " = 3

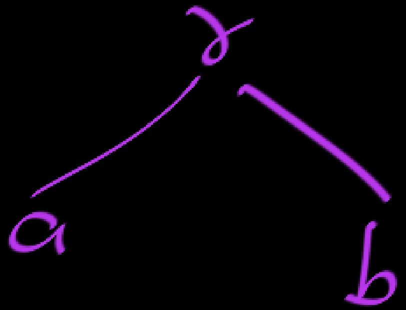
If Root is Deg-1 Node

order(Root) = 1

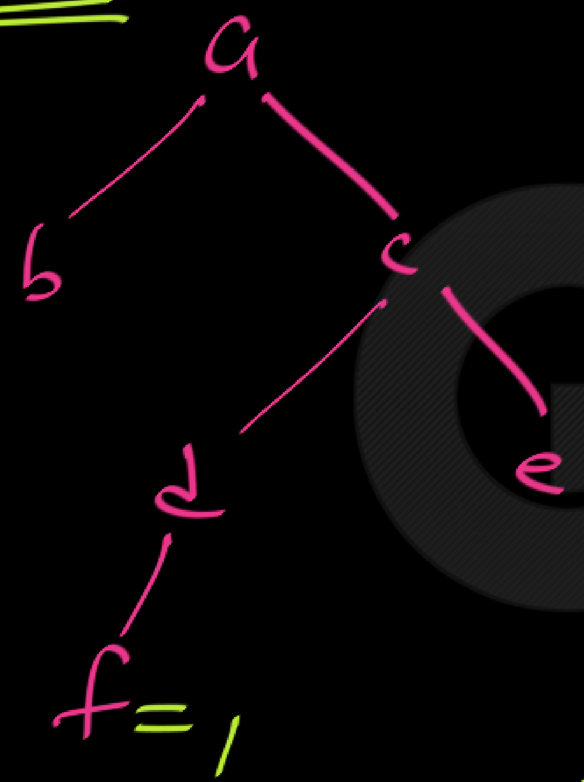


If Root is Deg-2 Node:

order(Root) = 2



Order:



Total order $= 2(n-1)$

$$L(1) + D'(2) + D''(3) - 1$$

$$= 2(n-1)$$

$$L + \cancel{2D'} + 3D'' - 1 = 2(L + \cancel{D'} + D'' - 1)$$

$$\boxed{D'' = L - 1}$$

$$\boxed{L = D'' + 1}$$

In A binary tree if there are N leaf nodes then the number of nodes having 2 children will be N-1

GATE CSE 2015 Set 2

A binary tree T has 20 leaves. The number of nodes in T having two children is ______.

GATE CSE 2015 Set 2

A binary tree T has 20 leaves. The number of nodes in T having two children is ______.

Rooted Tree

19

GATE CSE 2015 Set 3

Consider a binary tree T that has 200 leaf nodes. Then the number of nodes in T that have exactly two children are _____.

199

Graph Theory :

Next Topic :

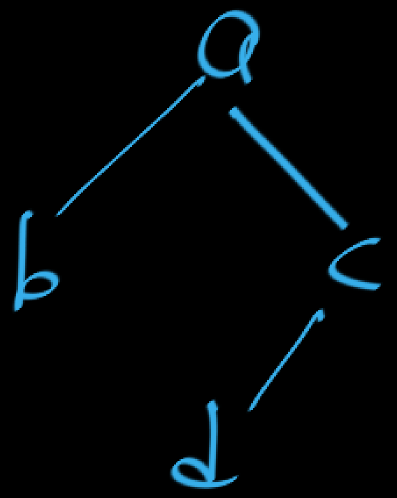
Full Binary Trees

Website : https://www.goclasses.in/

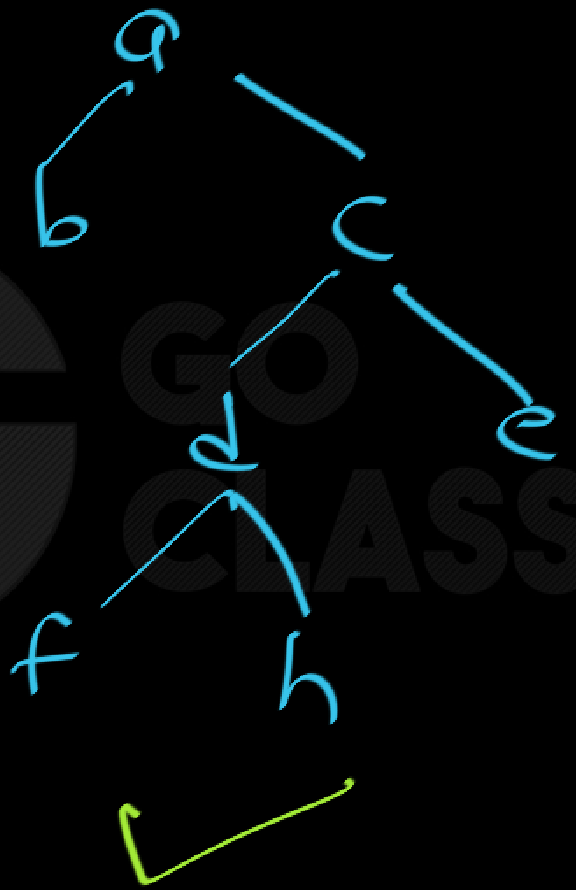
full Binary Tree : (FBT)

No node has Exactly one child.

Every internal node has 2 children.



FBT ✗



In FBT,

$$\boxed{D' = 0}$$

$$\boxed{n = L + D''}$$

In FBT,

$D' =$ Number of 1-child nodes $= 0$

$L =$ " " 0 - " " $= L$

$D'' =$ " " 2 - " " $= D''$

$$\boxed{n = L + D''}$$

In FBT, If L is known then

$D'' = ? = L - 1$

$n = ? = D'' + L = (L-1) + L = 2L - 1$

$\boxed{n = 2L - 1}$ ✓

In Binary Tree, If L is known then

$D'' = ? = L - 1$

$n = ? = ? = L + D'' + D'$?

We can not find n.

In FBT, If n is known then

$$D'' = ? = L - 1 = \frac{n+1}{2} - 1 = \frac{n-1}{2}$$

$$L = ? = \frac{n+1}{2}$$

$$n = 2L - 1$$

In FBT, If D'' is known then

$$n = ? = 2L - 1 = 2(D'' + 1) - 1$$
$$= 2D'' + 1$$

$$L = ? = D'' + 1$$

$L =$

In FBT,

Number of internal nodes $= D''$

$= L - 1$

In BT,

Number of internal nodes $= D'' + D'$

- A *full binary tree* is a binary tree in which every non-leaf vertex has exactly two children.

<u>Theorem</u>: Let T be a nonempty, full binary tree Then:

I = D''

(a) If T has I internal nodes, the number of leaves is $L = I + 1$.

(b) If T has I internal nodes, the total number of nodes is $N = 2I + 1$.

(c) If T has a total of N nodes, the number of internal nodes is $I = (N - 1)/2$.

(d) If T has a total of N nodes, the number of leaves is $L = (N + 1)/2$.

(e) If T has L leaves, the total number of nodes is $N = 2L - 1$.

(f) If T has L leaves, the number of internal nodes is $I = L - 1$.

Basically, this theorem says that the number of nodes N, the number of leaves L, and the number of internal nodes I are related in such a way that if you know any one of them, you can determine the other two.

GATE IT 2006

In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

A. 10
B. 11
C. 12
D. 15

GATE IT 2006

In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

A. 10
B. 11
C. 12
D. 15

$$\boxed{L = D'' + 1}$$

$$= 10 + 1 = 11$$

$$n = L + D' + D'' = 11 + 5 + 10 = 26$$

2.11.1 Trees: GATE2010-1

Let $G = (V, E)$ be a graph. Define $\xi(G) = \sum_d i_d * d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

A. $|S| = 2|T|$
B. $|S| = |T| - 1$
C. $|S| = |T|$
D. $|S| = |T| + 1$

2.11.1 Trees: GATE2010-1

Total Degree

Let $G = (V, E)$ be a graph. Define $\boxed{\xi(G) = \sum_d i_d * d}$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then [redacted]

A. $|S| = 2|T|$

B. $|S| = |T| - 1$

C. $|S| = |T|$

D. $|S| = |T| + 1$

Tree $S \Rightarrow n_1$ vertices

Tree $T \Rightarrow n_2$ vertices

Total Deg $= 2(n_1 - 1) = 2(n_2 - 1) =$ Total Deg

$n_1 = n_2$; $|E(S)| = |E(T)|$

$|S| =$ number of vertices
$|T| =$ " " "

$|S| = |T|$

$|S| =$ number of Edges
$|T| =$ " " "

$|S| = |T|$

Graph Theory :

Next Topic :

Questions related to Components

Website : https://www.goclasses.in/

Give the minimum and the maximum number of edges in an undirected connected graph of n vertices?

Connected Graph

minimally connected

min $|E| = n-1$ (Tree)

max $|E| = {}^{n}C_{2} = \frac{n(n-1)}{2}$ (K_n)

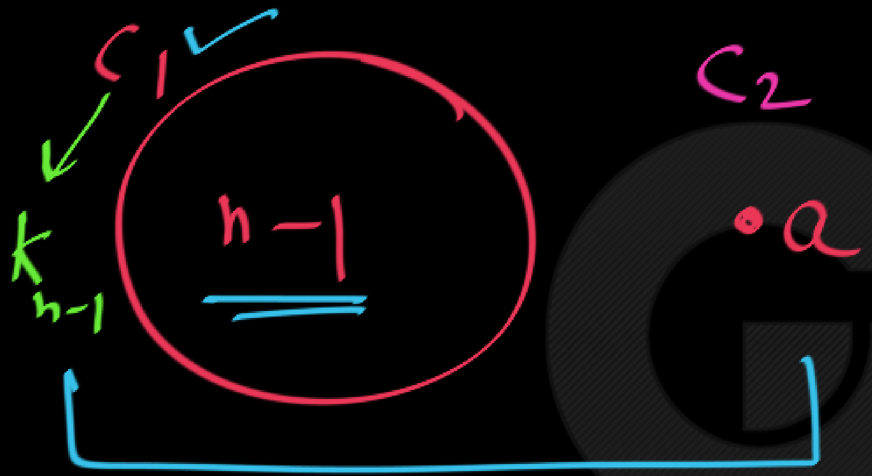
Maximum number of edges in unconnected graph

Maximum number of edges in unconnected graph

Disconnected Graph $\Rightarrow$ At least two Components.

two Components. to get max. number of Edges.

2 Components $\longrightarrow$ want max $|E|$?



$$\max |E| = {}^{n-1}C_2 = \frac{(n-1)(n-2)}{2}$$

$$\max |E| = {}^{n/2}C_2 + {}^{n/2}C_2$$

$$= 2\,\frac{\left(\frac{n}{2}\right)\left(\frac{n}{2}-1\right)}{2} = \frac{n(n-2)}{4}$$

Disconnected Graph : $\Rightarrow$ #Components ≥ 2

we want "max $|E|$" $\Rightarrow$ #Components = 2

To get max $|E|$:

$|E| = \frac{(n-1)(n-2)}{2} \Leftarrow K_{n-1}$

Minimum number of Edges in a Connected Graph??

$n-1$

minimum

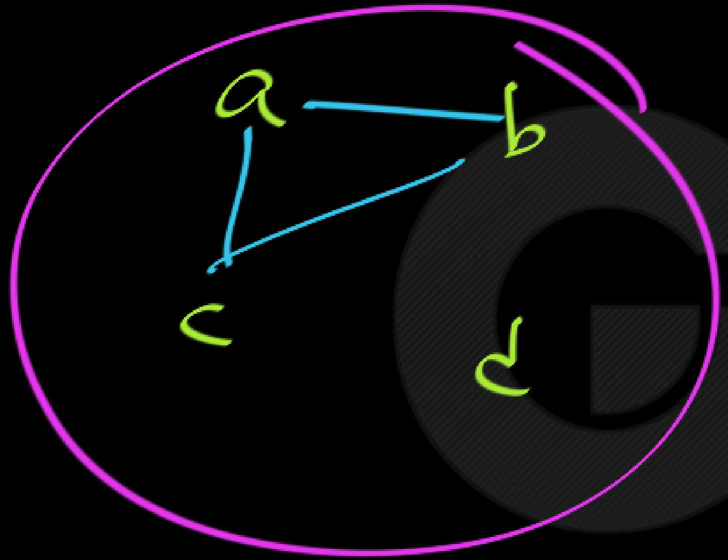
How many edges must a graph with N vertices have in order to guarantee that it is connected?

minimum

How many edges must a graph with N vertices have in order to guarantee that it is connected?

$$= {}^{(n-1)}C_2 + 1$$

4 vertices $\longleftrightarrow$ will 3 Edges guarrantee that it will be Connected.



"Disconnected Graph" $\rightarrow$ $\max |E| = {}^{n-1}C_2$

$\underbrace{G \text{ with } > {}^{n-1}C_2 \text{ Edges}}$

$\downarrow$

Connected ✓

How many edges must a graph with N vertices have in order to guarantee that it is connected?

$= {}^{n-1}C_2 + 1$

Every simple undirected graph with more than $(n-1)(n-2)/2$ edges is connected

1. There's a **connected** graph with 40 vertices, what is the minimum amount of edges there could be?
2. There's a graph with 40 vertices, what is the minimum amount of edges that **guarantees** a connected graph?

1. There's a **connected** graph with 40 vertices, what is the minimum amount of edges there could be?

$\Rightarrow 40 - 1 = 39$

2. There's a graph with 40 vertices, what is the minimum amount of edges that **guarantees** a connected graph?

$\Rightarrow {}^{39}C_2 + 1 = \frac{39 \times 38}{2} + 1 =$

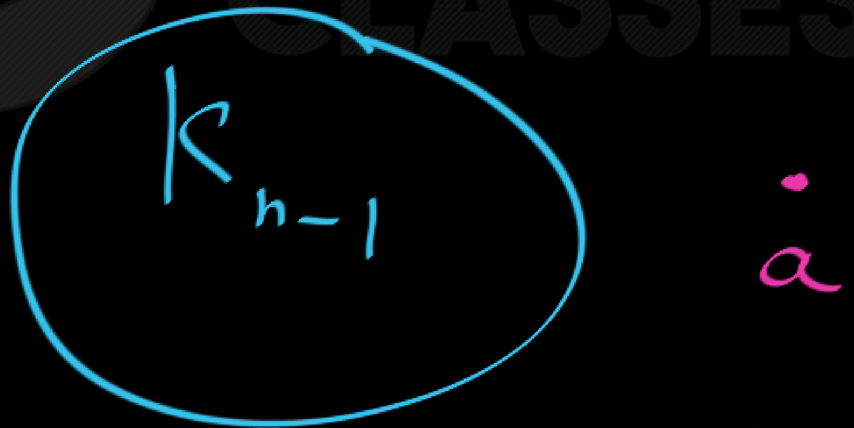
① Already have : Connected Graph

② Already have : 40 vertices

Graph with 2 Components $\Rightarrow$

max $|E| = {}^{n-1}C_2$

To get this:



Graph with 3 Components $\Rightarrow$

max $|E| = {}^{n-2}C_2$

To get this:

K_{n-2} a b

Graph with 4 Components $\Rightarrow$

max $|E| = {}^{n-3}C_2$

To get this:

K_{n-3}

a b c

Graph with k Components $\Rightarrow$

$\max |E| = {}^{n-k+1}C_2$

To get this:

K_{n-k+1} $\quad a_1 \quad a_2 \quad a_3 \quad \cdots \quad a_{k-1}$

Graph is Connected iff one Component.

2.4.4 Graph Connectivity: GATE1991-01,xv

The maximum number of possible edges in an undirected graph with n vertices and k components is ______.

2.4.4 Graph Connectivity: GATE1991-01,xv

The maximum number of possible edges in an undirected graph with n vertices and k components is ______.

$$^{n-k+1}C_2 = \frac{(n-k+1)(n-k)}{2}$$

- Show that if a graph with n vertices has more than $\binom{n-1}{2}$ edges, then it is connected.

2.4.11 Graph Connectivity: GATE2003-8, ISRO2009-53

HW

Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie down between

A. k and n B. $k-1$ and $k+1$ C. $k-1$ and $n-1$ D. $k+1$ and $n-k$

2.4.4 Graph Connectivity: GATE1991-01,xv

The maximum number of possible edges in an undirected graph with n vertices and k components is ______.

2.4.4 Graph Connectivity: GATE1991-01,xv

The maximum number of possible edges in an undirected graph with n vertices and k components is ______.

$$^{n-k+1}C_2 = \frac{(n-k+1)(n-k)}{2}$$

- Show that if a graph with n vertices has more than $\binom{n-1}{2}$ edges, then it is connected.

HW

- Prove that a graph is connected if and only if for every partition of its vertex set into two non-empty sets A and B there is an edge $ab \in E(G)$ such that $a \in A$ and $b \in B$.