Graph Theory :

Recap :

(For Simple Graph) Walk, Trail, Path

Cycle, Circuit

[†] Since the terminology of graph theory is not standard, the reader may find some differences between terms used here and in other texts.

DISCRETE AND COMBINATORIAL MATHEMATICS

An Applied Introduction

FIFTH EDITION

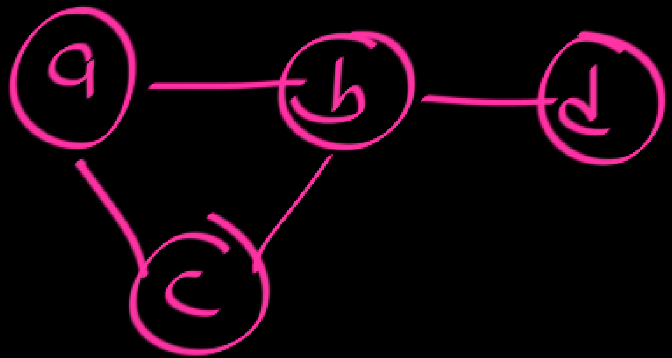
RALPH P. GRIMALDI

Rose-Hulman Institute of Technology

Definition: A walk in a graph is an alternating sequence of vertices and edges, beginning and ending with a vertex, in which each edge is incident with the vertex immediately preceding it and the vertex immediately following it. The length of a walk is the number of edges in it. A walk is closed if the first vertex is the same as the last and otherwise it is called open.

Definition: A trail is a walk in which all edges are distinct; a path is a walk in which "all" vertices are distinct. A closed trail is called a circuit. A circuit in which the first vertex appears exactly twice (at the beginning and at the end) and in which no other vertex appears more that once is called a cycle. An n-cycle is a cycle with n vertices. It is even is n is even and odd if n is odd.

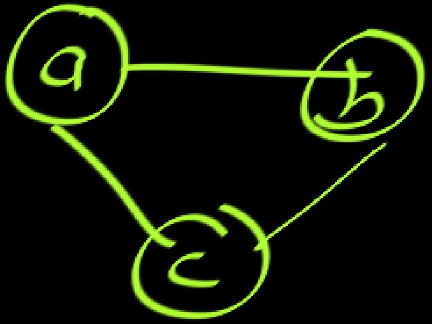
Cycle:



Cycle:

3 length cycle:

Point of view 1

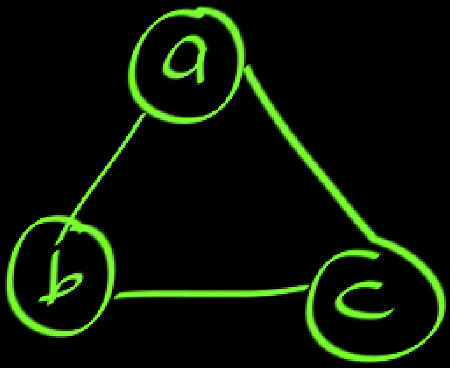


No vertex Repeats

Point of view 2

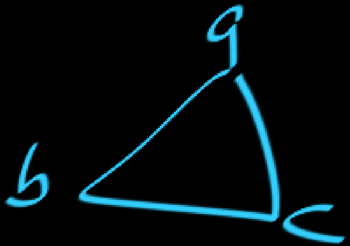
a — b — c — a

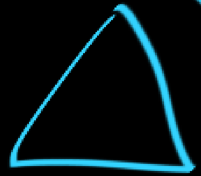
No vertex Repeats Except first, Last.

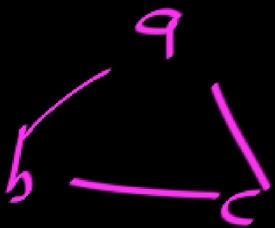


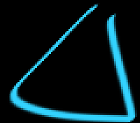
How many 3-length cycles ?

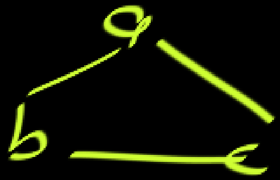
Think of this Graph as a "City"

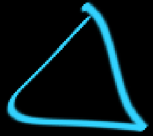
a — b — c — a ≡ 

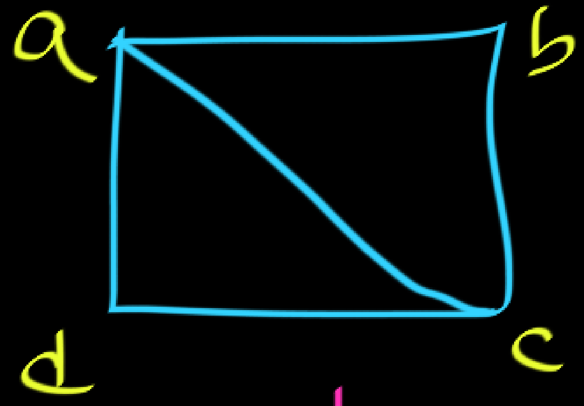
≡ 

b — a — c — b ≡ 

≡ 

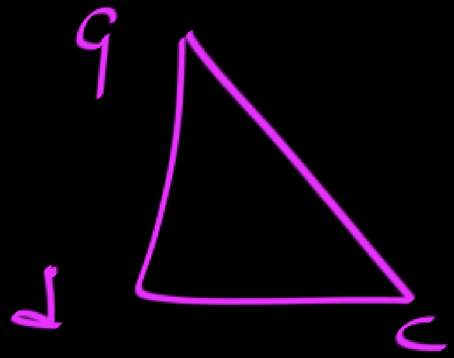
c — a — b — c ≡ 

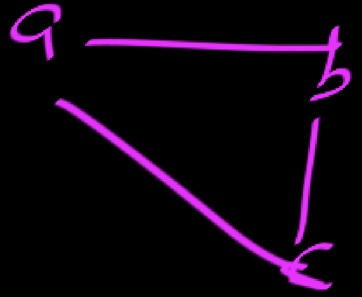
≡ 



3-length cycles = 2

Think of this Graph as a "City"





We can make a small observation here:

In a path, no vertex appears twice. This means that also no edge can repeat. Because if an edge repeats, then that edge is incident on a vertex that must also repeat, which means that it is not a path.

Walks, trails, paths, and cycles

A walk is an alternating list $v_0, e_1, v_1, e_2, \ldots, e_k, v_k$ of vertices and edges such that for $1 \leq i \leq k$, the edge e_i has endpoints v_{i-1} and v_i.

The **length** of the walk is the number of edges in the walk. A walk of length zero is a **trivial walk**.

Definition

A **trail** is a walk with no repeated edges. A **path** is a walk with no repeated vertices. A **circuit** is a closed trail and a **trivial circuit** has a single vertex and no edges.

Definition

A **cycle** is a nontrivial circuit in which the only repeated vertex is the first/last one.

Note that a walk may repeat both vertices and edges.

A trail is a walk with no repeated edge.

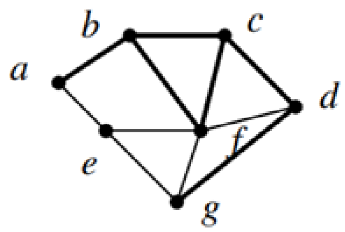
A path is a walk with no repeated vertex.

A u, v-walk, u, v-trail, u, v-path is a walk, trail, path, respectively, with first vertex u and last vertex v.

If $u = v$ then the u, v-walk and u, v-trail is closed. A closed trail (without specifying the first vertex) is a circuit. A circuit with no repeated vertex is called a cycle.

The length of a walk trail, path or cycle is its number of edges.

draw a picture illustrating each concept. We will therefore draw eight graphs, the first four will illustrate a walk which is not a path or trail, then we have a closed walk which is not a path or trail, then we will have a trail which is not a path and lastly a path. Here are the first four figures:



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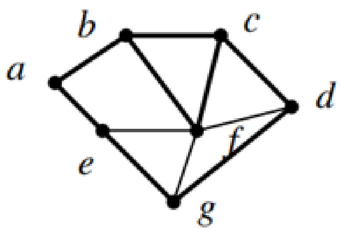

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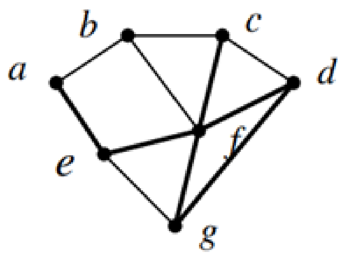

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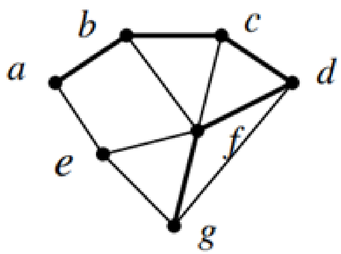

abcdfg

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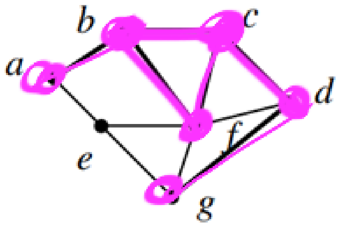

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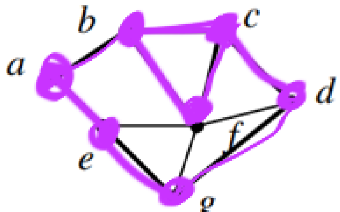

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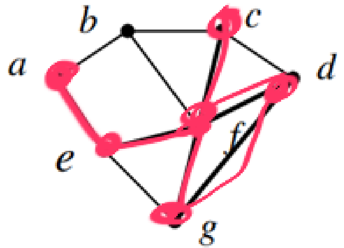

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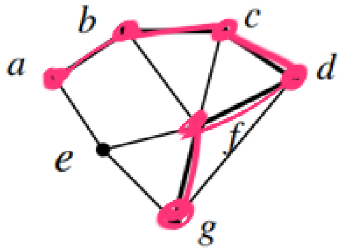


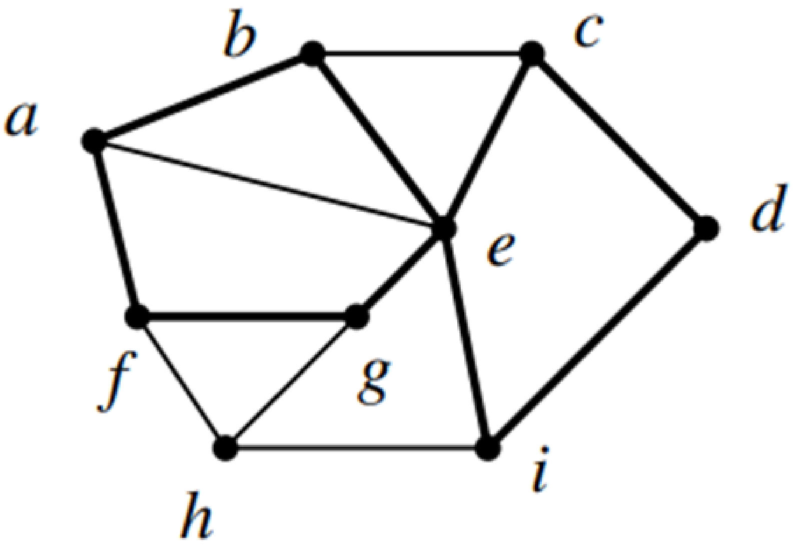
abcdfg

walk ✓
open walk ✓
Trail X
Path X

walk ✓
Closed walk ✓
Trail X
Path X

walk ✓
Open walk ✓
Trail ✓
Circuit X
Path X

walk ✓
open walk ✓
Trail ✓
Path ✓
cycle X



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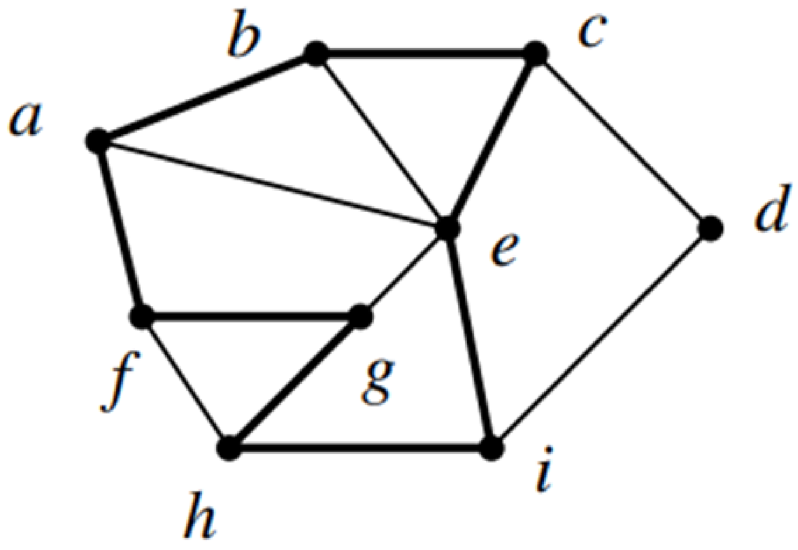

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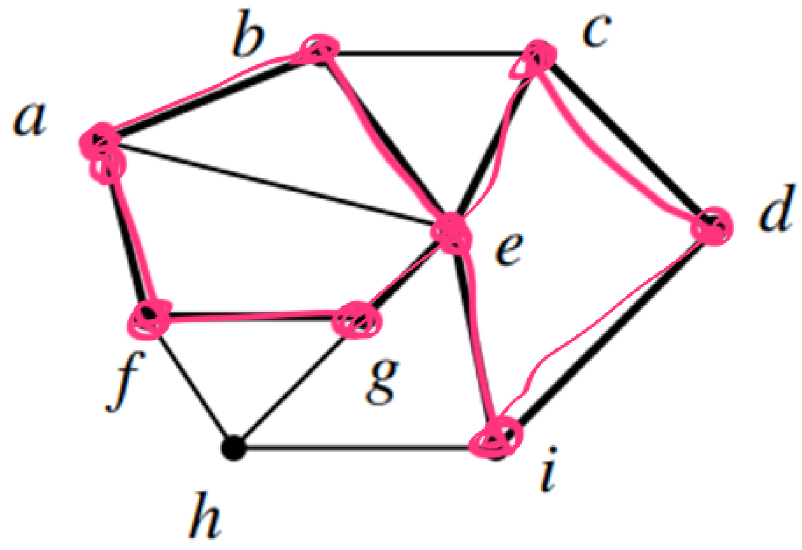

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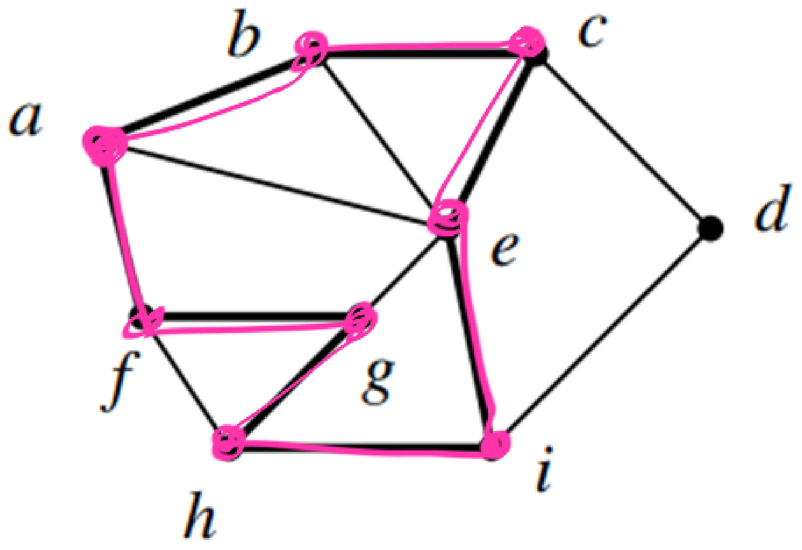

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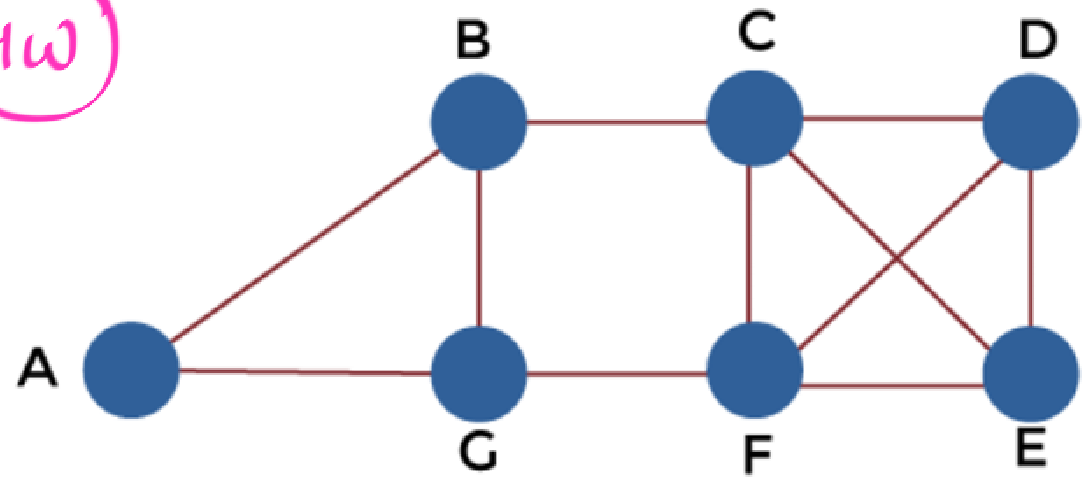
Circuit ✓ Closed walk ✓
Trail ✓ Path ✗
walk ✓ Cycle ✗

Cycle ✓
Path ✗ Circuit ✓
Closed walk ✓
Trail ✓

GO CLASSES

Example 1: In this example, we will consider a graph.

Hw



Now we have to find out which sequence of the vertices determines walks. The sequence is described below:

1. A, B, G, F, C, D
2. B, G, F, C, B, G, A
3. C, E, F, C
4. C, E, F, C, E
5. A, B, F, A
6. F, D, E, C, B

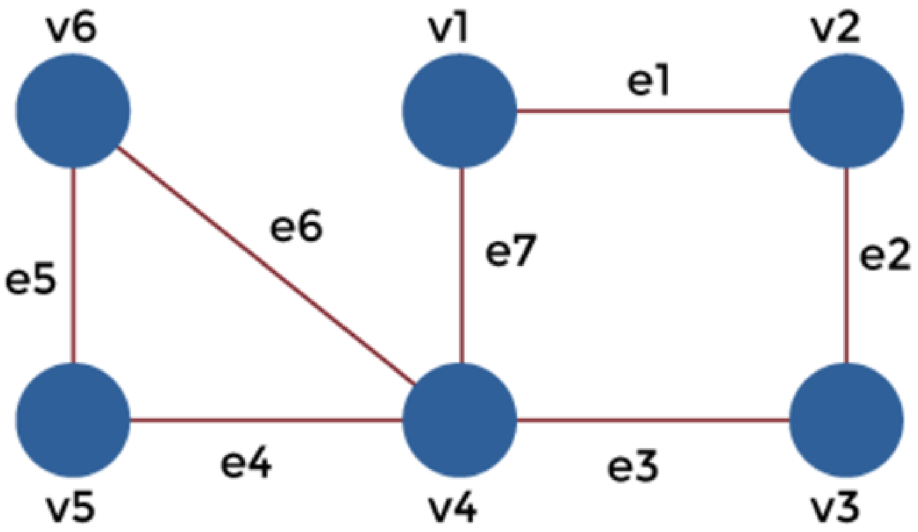
For those sequences which are walk, we have to also determine whether it is a cycle, path, circuit, or trail.

Solution: In the above graph, A, B, C, D, E, F, and G are the vertices, and the line between two vertices is the edges, i.e., the line between A and B is an edge. To solve this, we will first determine sequence no 1. After that, we will proceed to the next.

1. Sequence no 1 is a **Trail** because there is no repeated edge in the sequence ABGFCB.
2. Sequence no 2 is a **Walk** because the sequence BGFCBGA contains the repeated edges and vertices.
3. Sequence 3 is a **Cycle** because the sequence CEFC does not contain any repeated vertex or edge except the starting vertex C.
4. Sequence no 4 is a **Walk** because the sequence CEFCE contains the repeated edges and vertices.
5. Sequence no 5 is **Not a Walk** because there is no direct path to go from B to F. That's why we can say that the sequence ABFA is not a Walk.
6. Sequence no 6 is a **Path** because the sequence FDECB does not contain any repeated edges and vertices.

Example 2: In this example, we will consider a graph.

HW



With the help of below sequences, we have to determine the nature of walk in each case:

1. v1, e1, v2, e2, v3, e2, v2
2. v4, e7, v1, e1, v2, e2, v3, e3, v4, e4, v5
3. v1, e1, v2, e2, v3, e3, v4, e4, v5
4. v1, e1, v2, e2, v3, e3, v4, e7, v1
5. v6, e5, v5, e4, v4, e3, v3, e2, v2, e1, v1, e7, v4, e6, v6

Solution: In the above graph, v1, v2, v3, v4, v5, v6 are the vertices and e1, e2, e3, e4, e5, e6, e7 are the edges. To solve this, we will first determine sequence no 1. After that, we will proceed to the next.

1. Sequence no 1 is an **Open Walk** because the starting vertex and the last vertex are not the same. The starting vertex is v1, and the last vertex is v2.
2. Sequence no 2 does not have a path. It is a **trail** because the trail can contain the repeated edges and vertices, and the sequence v4v1v2v3v4v5 contains the repeated vertex v4.
3. Sequence no 3 is a **Path** because the sequence v1e1, v2e2, v3e3, v4e4, and v5 does not contain any repeated edges and vertices.
4. Sequence no 4 is a **Cycle** because the sequence v1e1, v2e2, v3e3, v4e7, v1 does not contain any repeated vertex or edge except the starting vertex v1.
5. Sequence no 5 does not have a cycle. It is a **Circuit** because a circuit can contain the repeated vertex, but it cannot contain repeated edges except the starting vertex. The sequence v6e5, v5e4, v4e3, v3e2, v2e1, v1e7, v4e6, v6 contains the repeated vertex v4.

Now let us examine special types of walks.

Consider any x-y walk in an undirected graph $G = (V, E)$.

a) If no edge in the x-y walk is repeated, then the walk is called an x-y *trail*. A closed x-x trail is called a *circuit*.

b) If no vertex of the x-y walk occurs more than once, then the walk is called an x-y *path*. When $x = y$, the term *cycle* is used to describe such a closed path.

The term *cycle* will always imply the presence of at least three distinct edges (from the graph).

What is REALLY Important to understand:

Cycle:

The term *cycle* will always imply the presence of at least three distinct edges (from the graph).

Path:

a ; $v_1 \sim\sim\sim v_n$ $v_i \neq v_j$; $v_1 \neq v_n$

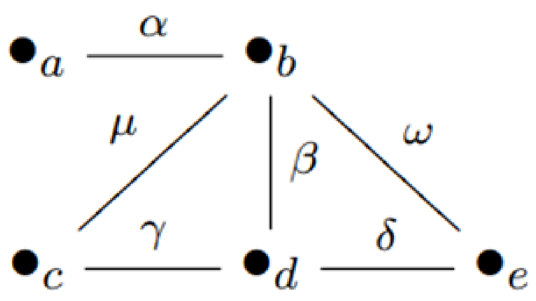
Walk:

v_1 v_n $\underbrace{v_1 - v_n \text{ walk}}$

Definition 2. A *path* is a walk that does not repeat an vertex. A *trail* is a walk that does not repeat any edge.

A walk of legth zero or one (in a graph!) will always be a path. A path is always a trail, but not vice-versa. For example, given

$G:$



the walk

$$(a, \alpha, b, \beta, d, \gamma, c, \mu, b, \omega, e)$$

is a trail but not a path.

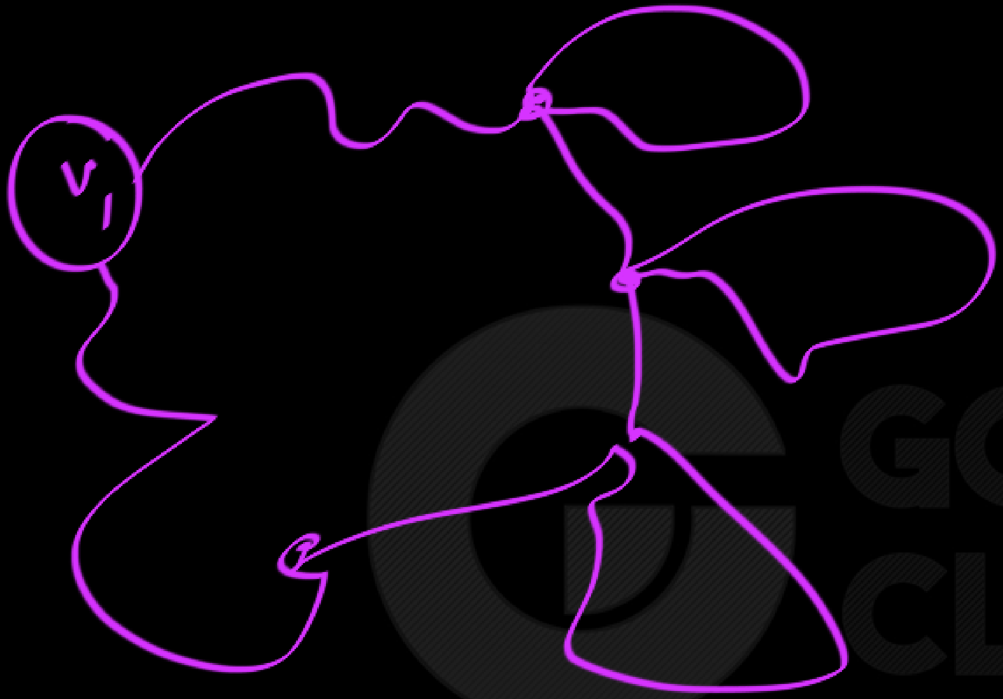
Path $\longrightarrow$ Trail $\longrightarrow$ walk

Cycle $\longrightarrow$ circuit

Lemma 1. *In a graph, if there is a walk from v to w then there is a path from v to w.*

Every Path is a walk.

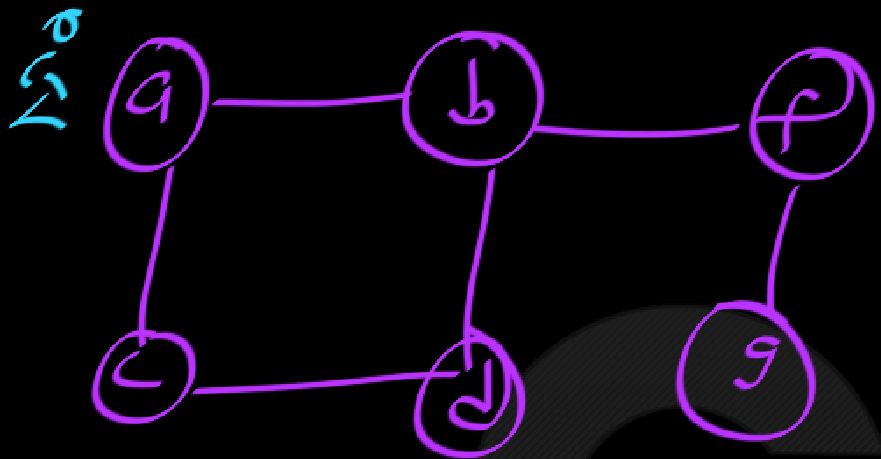
Every walk is a Path $\Longrightarrow$ Wrong



v_1

$v_1 - v_1$ walk

$v_1 - v_1$ path



a – g walk? Yes

a – b – d – c – a – b – f – b – f – g

walk

a – b – f – g Path ✓

Path   a – g path

Lemma 2. *There is always a walk from v to v.* Trivial wak.

3 If there is a walk from v to w then there is a walk from w to v.

4 If there is a walk from v to w and a walk from w to q then there is a walk from v to q.

Graph Theory :

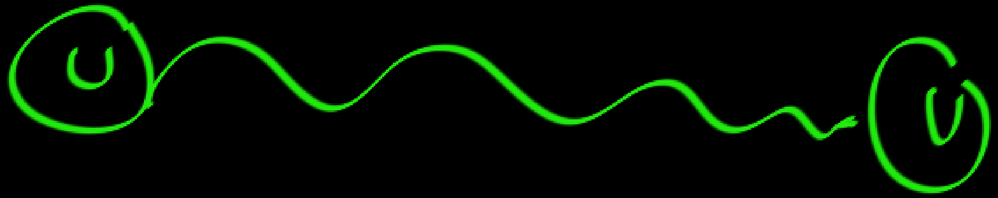
Next Topic : (For Simple Graph)

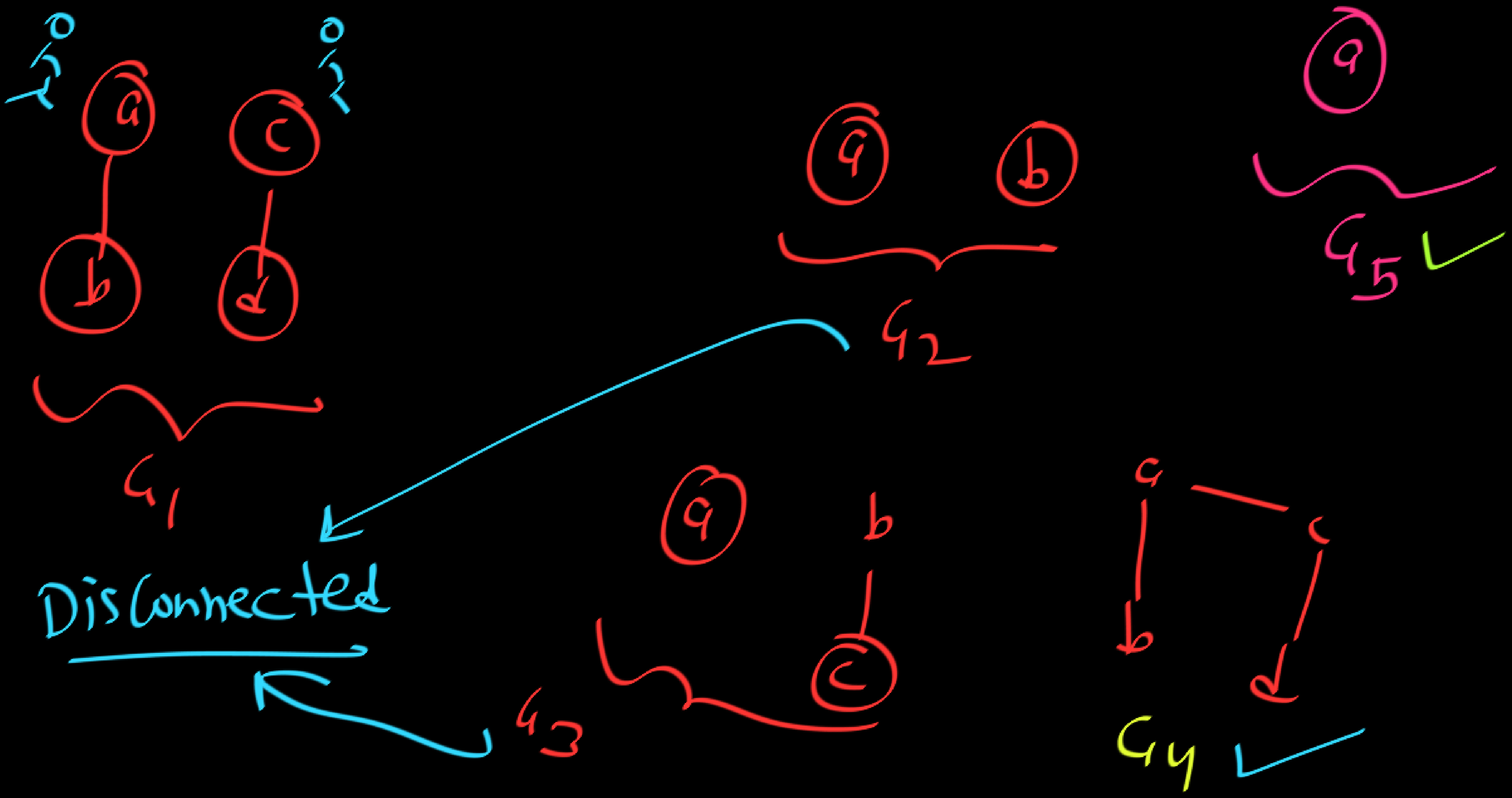
Distance, Diameter

Website : https://www.goclasses.in/

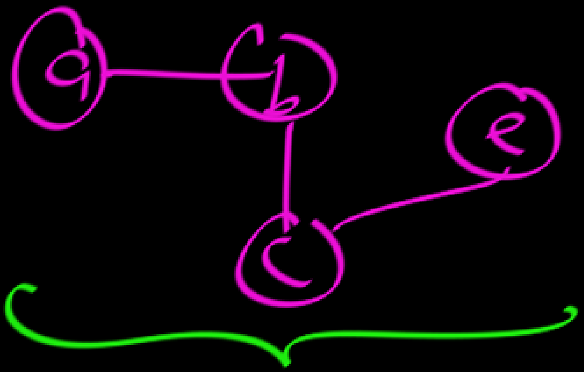
Connected Graph:

A Graph G is Connected iff there is path b/w every two vertices.

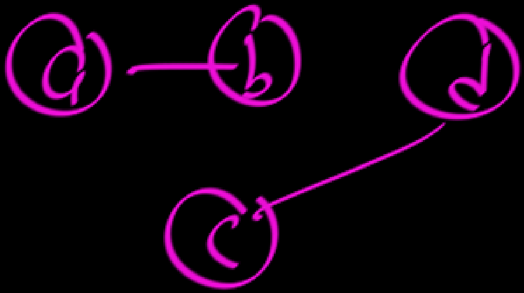




a
b
c
d
G_1
a
b
G_2
a
G_5
Disconnected
a
b
c
G_3
a
b
c
d
G_4

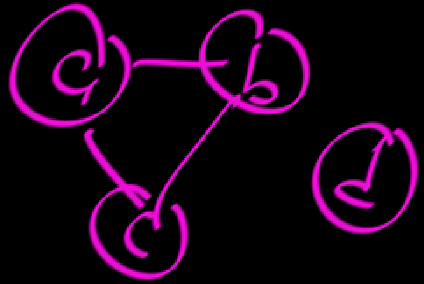


G_1



G_2

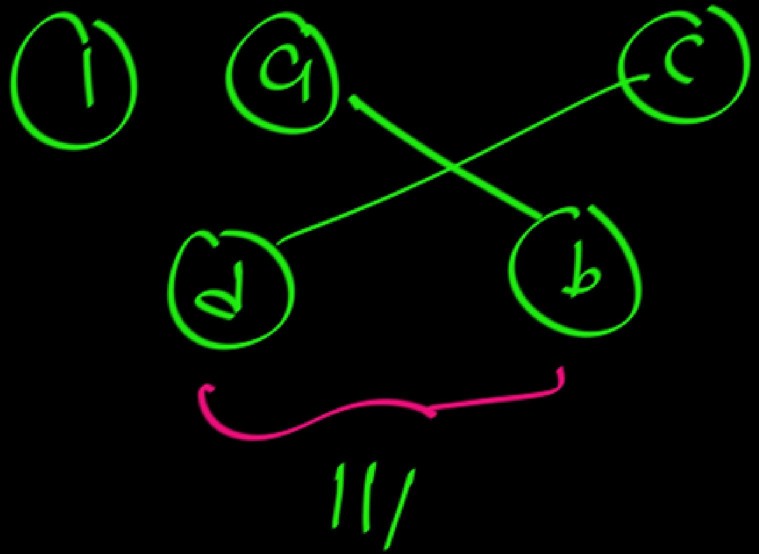
there is No path from b to d.



G_3

there is No path from b to d.

Which Graph is Connected ?



② Intersection Graph :

$V = \{A_1, A_2, A_3, A_4\}$

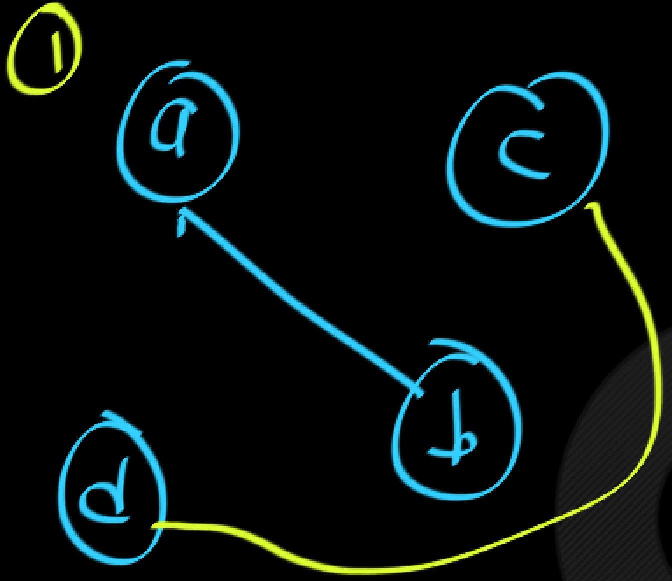
$A_1 = \{1, 2, 3\}$

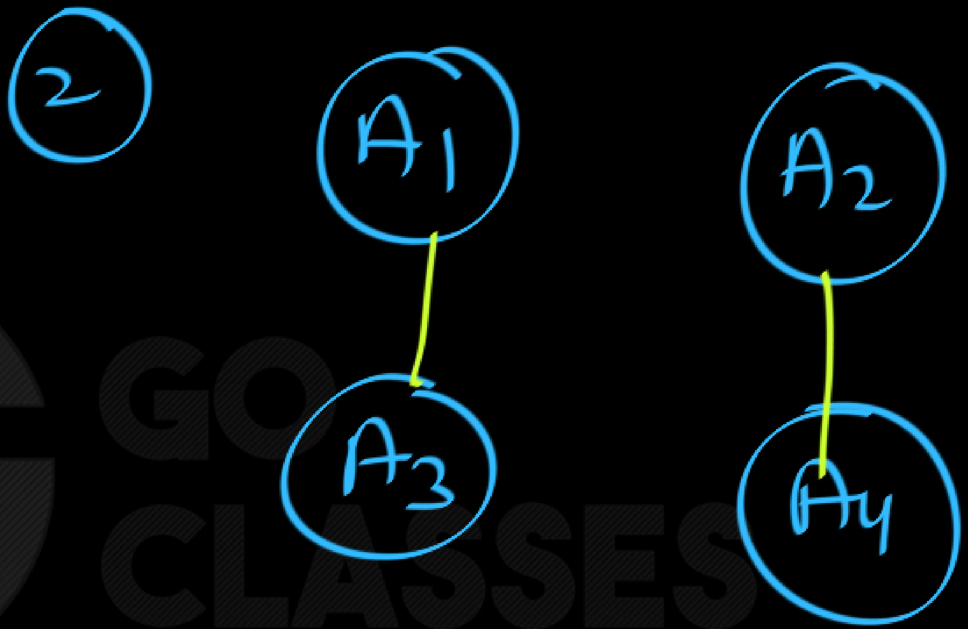
$A_2 = \{4, 5\}$

$A_3 = \{2, 6\}$

$A_4 = \{5, 7\}$

$uv \in E$ iff $u \cap v \neq \phi$



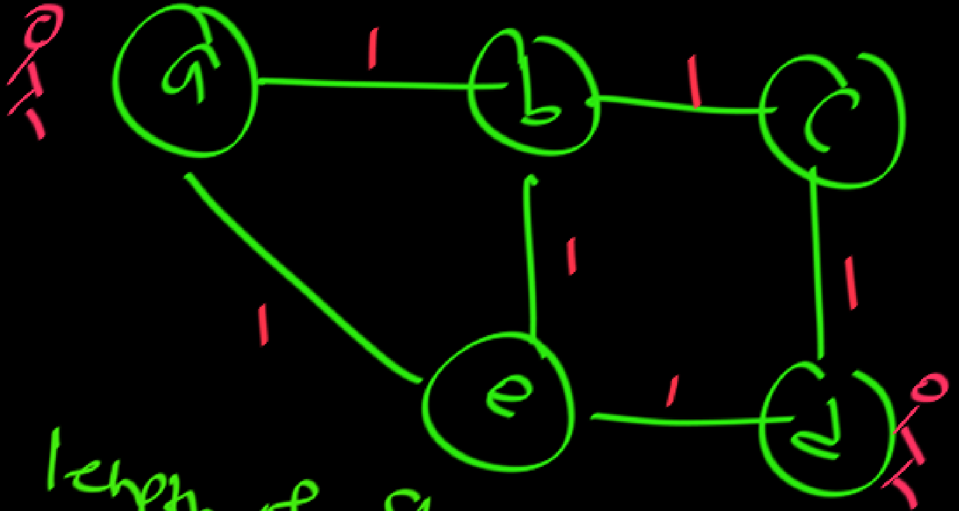


Disconnected

Disconnected

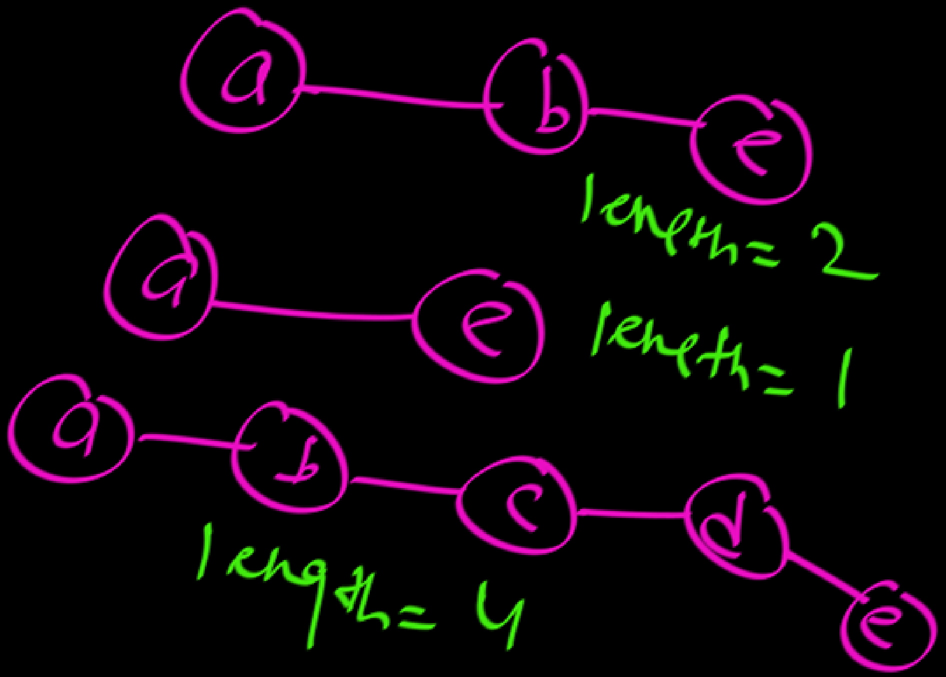
length of Path:

Number of Edges in the path.



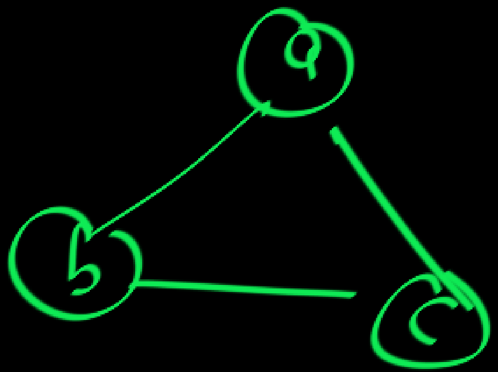
length of Shortest path from a to e = 1 ✓

many paths from a to e:



Distance :

Distance b/w "a" , "b" is the "length of Shortest Path" b/w a, b.



Dist(a,b) = 1
Dist(a,c) = 1
Dist(b,c) = 1

Paths a — b
{ a — c — b ✓
a — b ✓

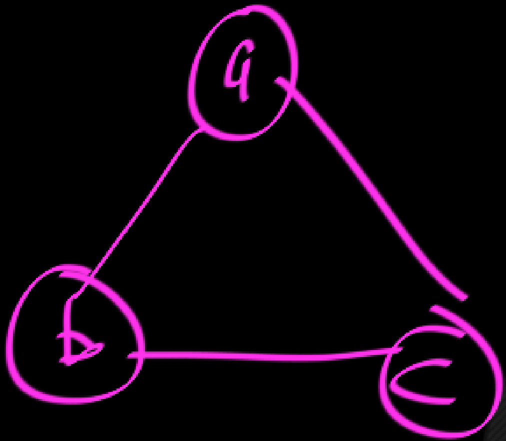
[PATH, CONNECTEDNESS, DISTANCE, DIAMETER]

- A graph G is *connected* if every pair of distinct vertices is joined by a path. Otherwise it is *disconnected*.
- The *distance* between two vertices a and b, denoted $dist(a, b)$, is the length of a shortest path joining them.
- The *diameter* of a connected graph, denoted $diam(G)$, is $\max_{a,b \in V(G)} dist(a, b)$.

Diameter:

Longest Among All shortest Distances.

Longest "Shortest path".



Dist(a, a) = 0

" (a, b) = 1

" (a, c) = 1

" (b, b) = 0

" (b, c) = 1

Diameter = 1

"lonest path" from a to b = a — c — b ✓

$$\text{Diam}(G) = \max_{\forall a, b} \left\{ \text{Dist}(a,b) \right\}$$

Diameter(G) tell us that from any vertex to any vertex the Distance is $\leq$ Diam(G)

[PATH, CONNECTEDNESS, DISTANCE, DIAMETER]

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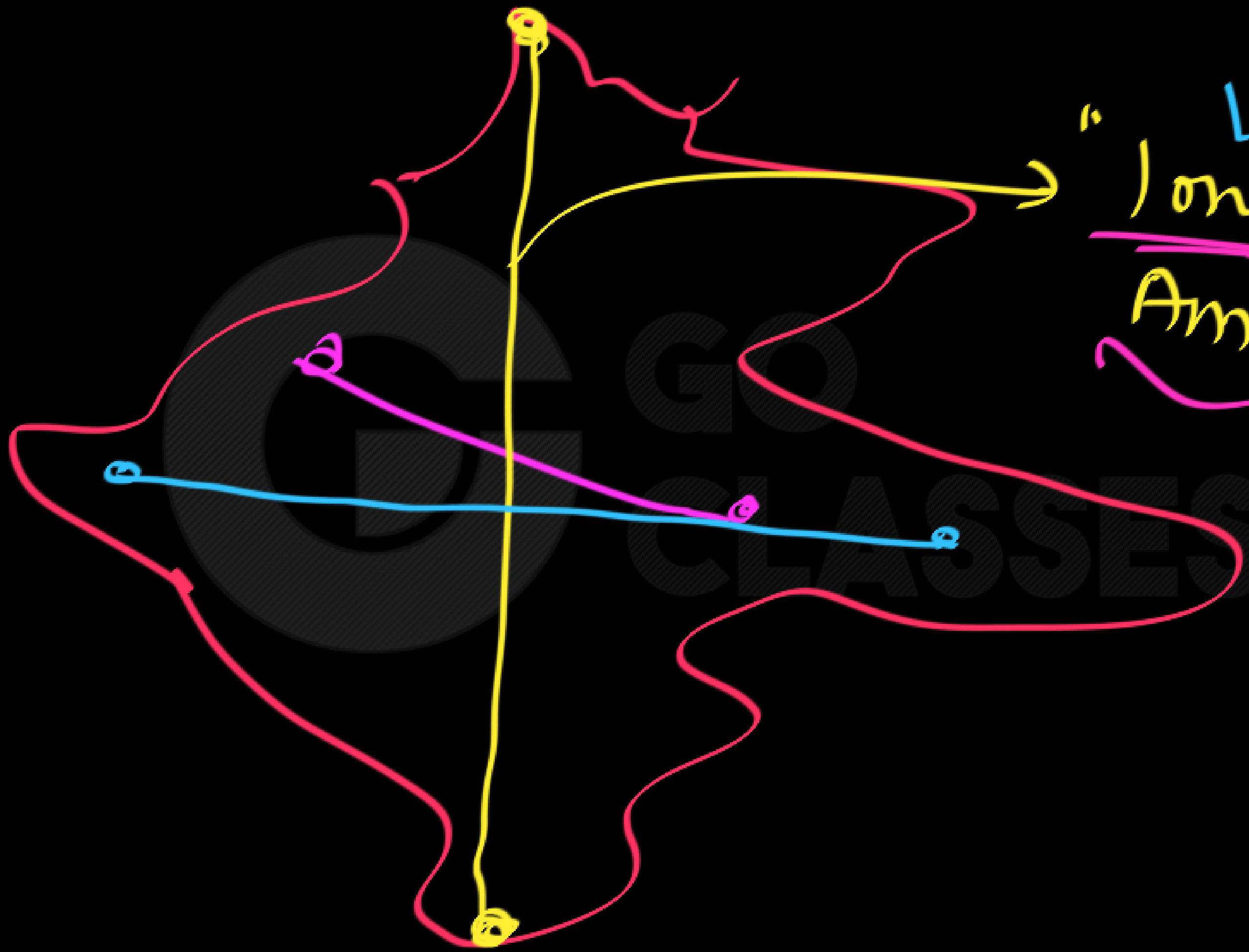
India:



"longest" Among all shortest Distances

Google

diameter of india

All Images News Shopping Maps More Tools

About 58,80,00,000 results (0.69 seconds)

India measures **3,214 km (1,997 mi) from north to south and 2,933 km (1,822 mi) from east to west**. It has a land frontier of 15,200 km (9,445 mi) and a coastline of 7,516.6 km (4,671 mi).

Borders: Total land borders: 15,200 km (9,400 ...

Largest lake: Loktak Lake (freshwater); 287 km2 ...

Coastline: 7,516.6 km (4,670.6 mi)

Region: South Asia; (Indian subcontinent)

https://en.wikipedia.org › wiki › Geography_of_India

Geography of India - Wikipedia

Graph Theory :

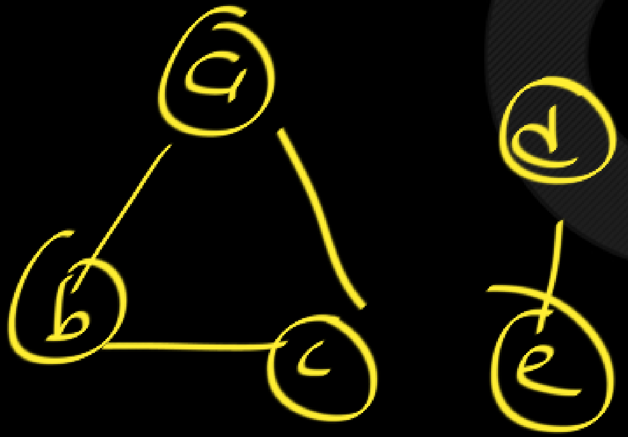
Next Topic : (For Simple Graph)

Special Type of Graphs

Website : https://www.goclasses.in/

Diameter of Disconnected Graph:

∞ ✓



Graph:

Diameter = 0

Graph Theory :

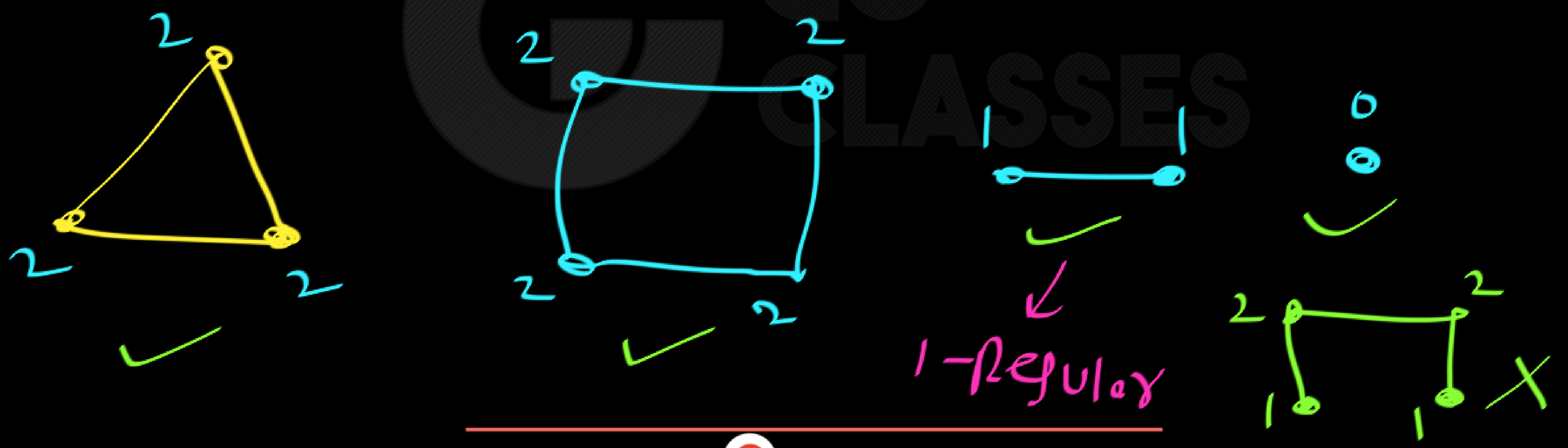
Next Topic : (For Simple Graph)

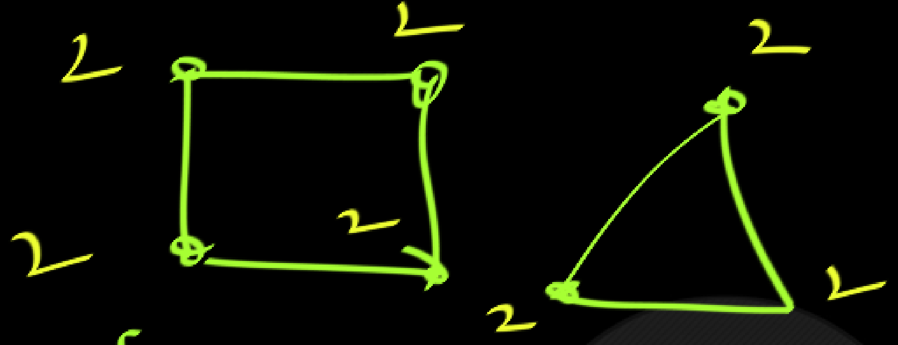
Regular Graph

Website : https://www.goclasses.in/

Regular graph:

Every vertex has Same Degree.





G

Regular ✓

2-Regular

G — Regular ✓

0-Regular

Complete Analysis of d-Regular Graph:

Assume $|V| = n$

① Degree Sequence: $\underbrace{d, d, d, d, \ldots, d}_{n \text{ times}}$

② $\Delta(G) = d$
$\delta(G) = d$

Avg Deg $= d = \frac{nd}{n}$

Total Deg $= nd$

(3) #Edges = $\quad nd = \underbrace{2|E|}$

$|E| = \frac{nd}{2}$

(4) Connected or Disconnected

(5) Diameter: finite or ∞

Definition 11. [Degree sequence, regular graph]

- If $V(G) = \{v_1, v_2, \ldots, v_n\}$, then the sequence $(deg(v_1), deg(v_2), \ldots, deg(v_n))$ is called the *degree sequence* of G.
- If $deg(v_1) = deg(v_2) = \ldots = deg(v_n)$, then G is a *regular* graph.
- If the degree of each vertex is d, then the graph is d-regular.
- 3-regular graphs are called *cubic*.

Graph Theory :

Next Topic : (For Simple Graph)

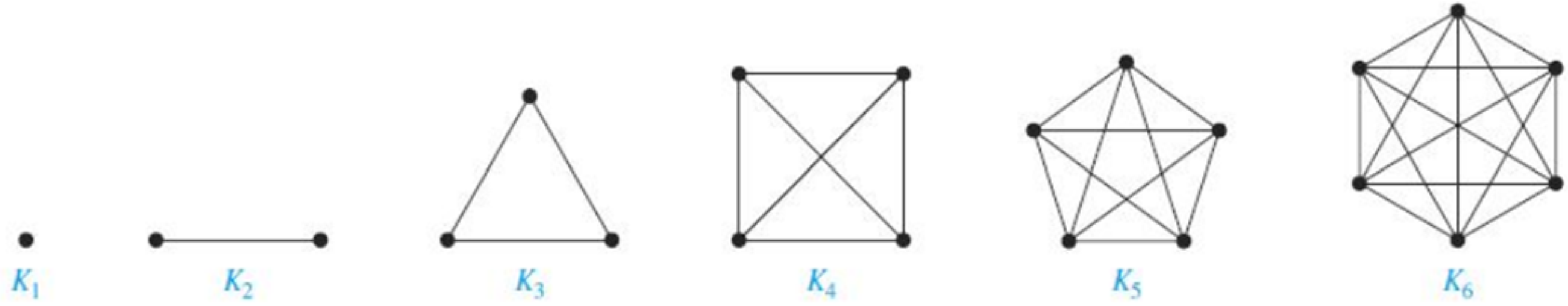
Complete Graph

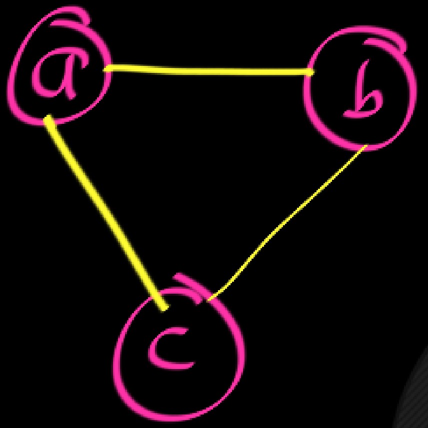
Website : https://www.goclasses.in/

Special Graphs

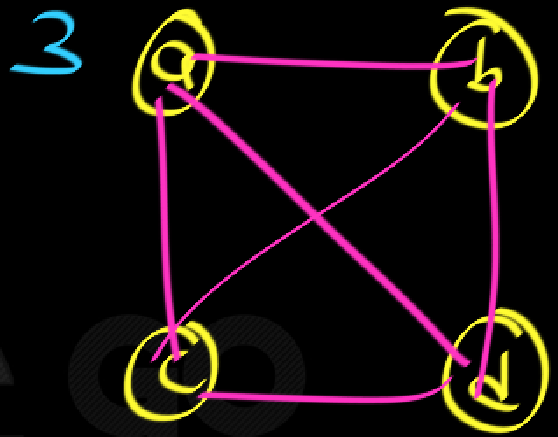
Complete Graphs

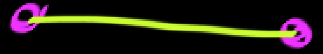
A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices. Has $\dfrac{n(n-1)}{2}$ edges.



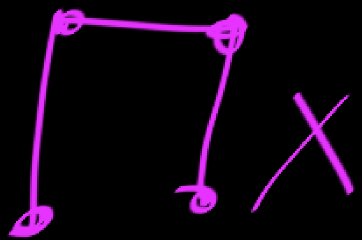


K_3





K_2



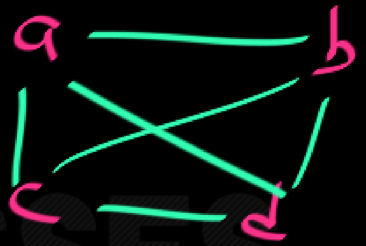
K_4

K_n : $\rightarrow$ $|V| = n$

1. Degree of every vertex $= n-1$
2. $\Delta = \delta = \text{Avg Deg} = n-1$
3. $(n-1)$ regular ✓ $d = (n-1)$
4. Deg seq : $\underbrace{(n-1), (n-1), --, (n-1)}_{n \text{ times}}$

(5) #Edges $= \frac{n\,d}{2} = \frac{n(n-1)}{2}$ ✓

$\to {}^{n}C_{2} = \frac{n(n-1)}{2}$



(6) Diameter : 1

(7) Connected ✓

Graph Theory :

Next Topic : (For Simple Graph)

Empty/Null/EdgeLess Graph

Website : https://www.goclasses.in/

Edgeless Graph on n-vertices (E_n):

Graph without Edges.

"Every Graph" must have at least one vertex.

$E_n \longrightarrow |V| = n$

① Degree of each vertex = 0

② 0 - Regular ✓

③ $\delta = \Delta =$ Avg Deg = 0

Total Deg = 0

#Edges = 0

Connected iff $n=1$

Else Disconnected

Diameter =
0 — $n=1$
∞ — otherwise

Graph Theory :

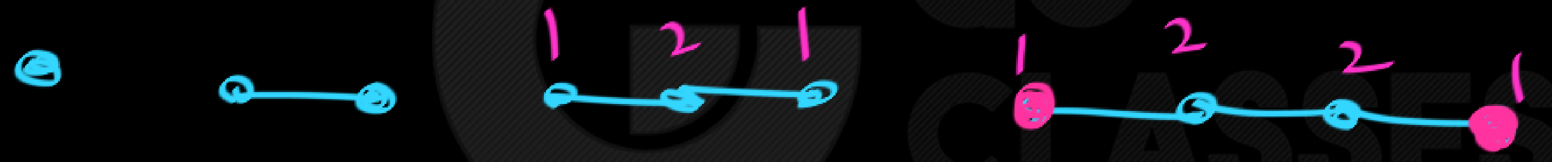
Next Topic : (For Simple Graph)

Path Graph

Website : https://www.goclasses.in/

Path Graph: (P_n) : — Straight line

$a_1 — a_2 — a_3 — - - - — a_n$



P_1 P_2 P_3 P_4

Deg seq = 2, 2, 1, 1

P_n: $\rightarrow$ $|V| = n$

Diameter $= n-1$ ✓

$P_1 \Rightarrow$ Diam $= 0$

Degree Sequence: $\begin{cases} 0 & P_1 \\ 1,1 & P_2 \\ \underbrace{2,2,\ldots,2}_{n-2 \text{ times}}, \underbrace{1,1}_{2 \text{ times}} & P_n,\ n \geq 3 \end{cases}$

$\left.\begin{array}{l} \Delta = 2 \\ \delta = 1 \end{array}\right\}$

Edges $= n-1$

Total Deg $= 2(n-1)$

Connected ✓

Graph Theory :

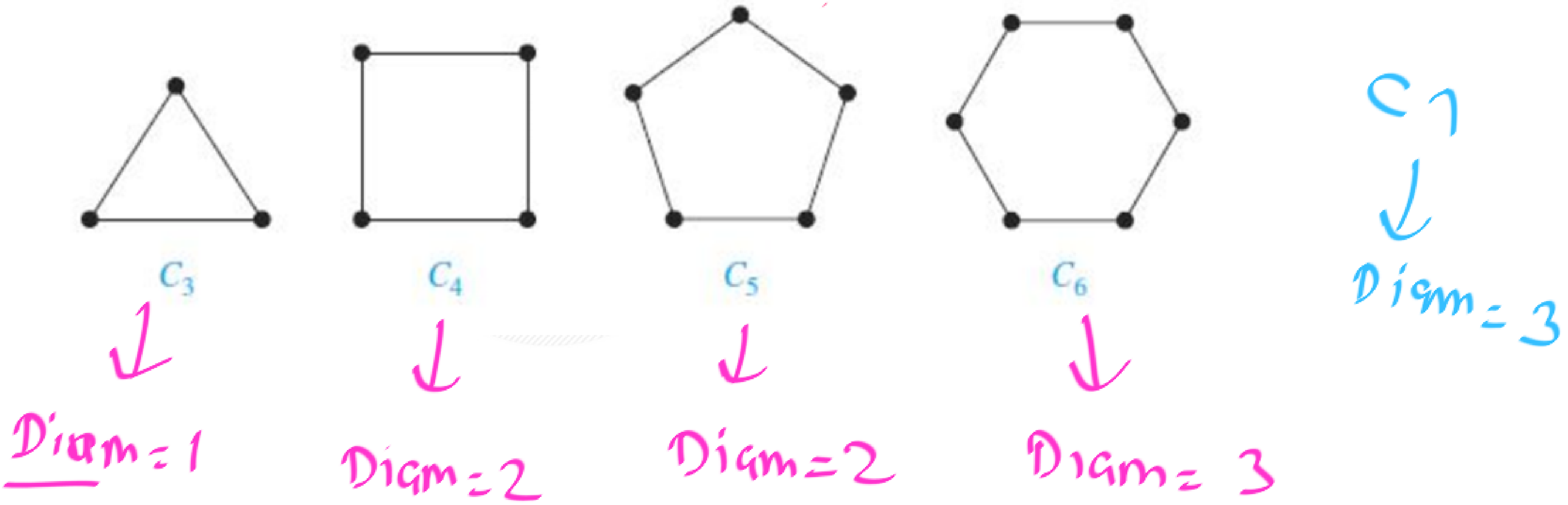
Next Topic : (For Simple Graph)

Cycle Graph

Website : https://www.goclasses.in/

Cycles

A *cycle* $C_n, n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$. Has n edges.



C_n ; $n \geq 3$

$\rightarrow |V| = n$

④ #Edges $= \frac{2n}{2} = n$

① Degree of each vertex = 2

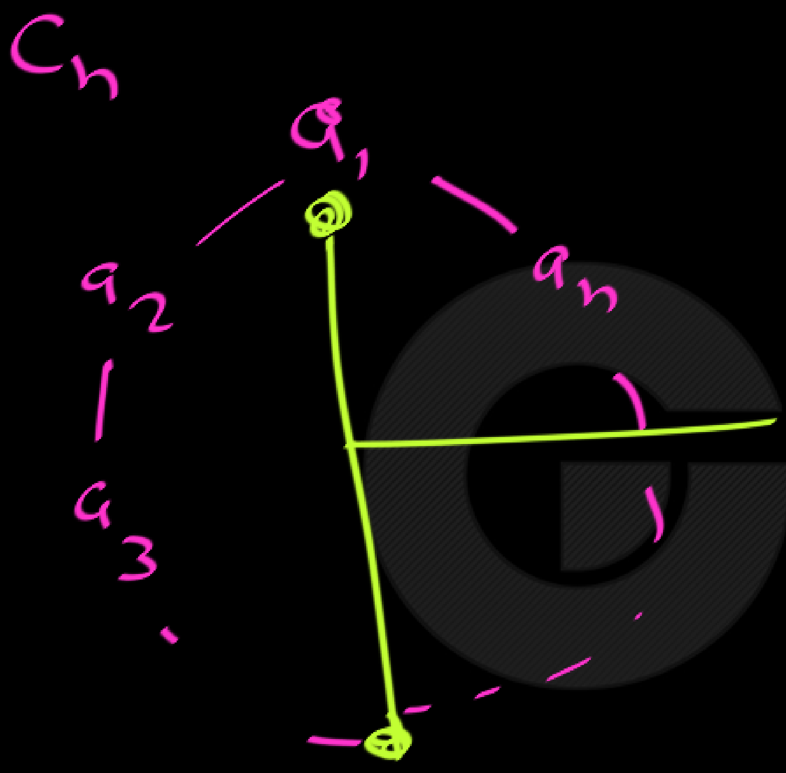
② 2-Regular (d=2)

⑤ Diameter = ? $= \left\lfloor \frac{n}{2} \right\rfloor$

③ $\delta = \Delta =$ Avg Deg $= 2$

Total Deg $= 2n$

⑥ Connected



Diameter

$$\left\lfloor \frac{n}{2} \right\rfloor$$

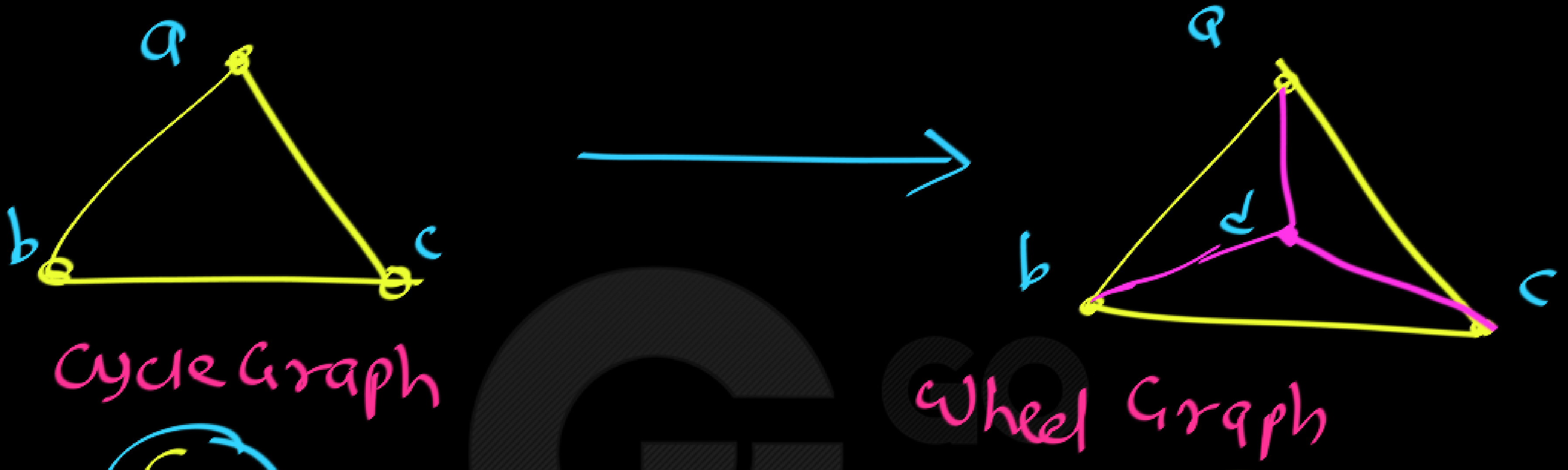
$$\begin{cases} \dfrac{n}{2} & n \text{ even} \\[2ex] \dfrac{n-1}{2} & n \text{ odd} \end{cases}$$

Graph Theory :

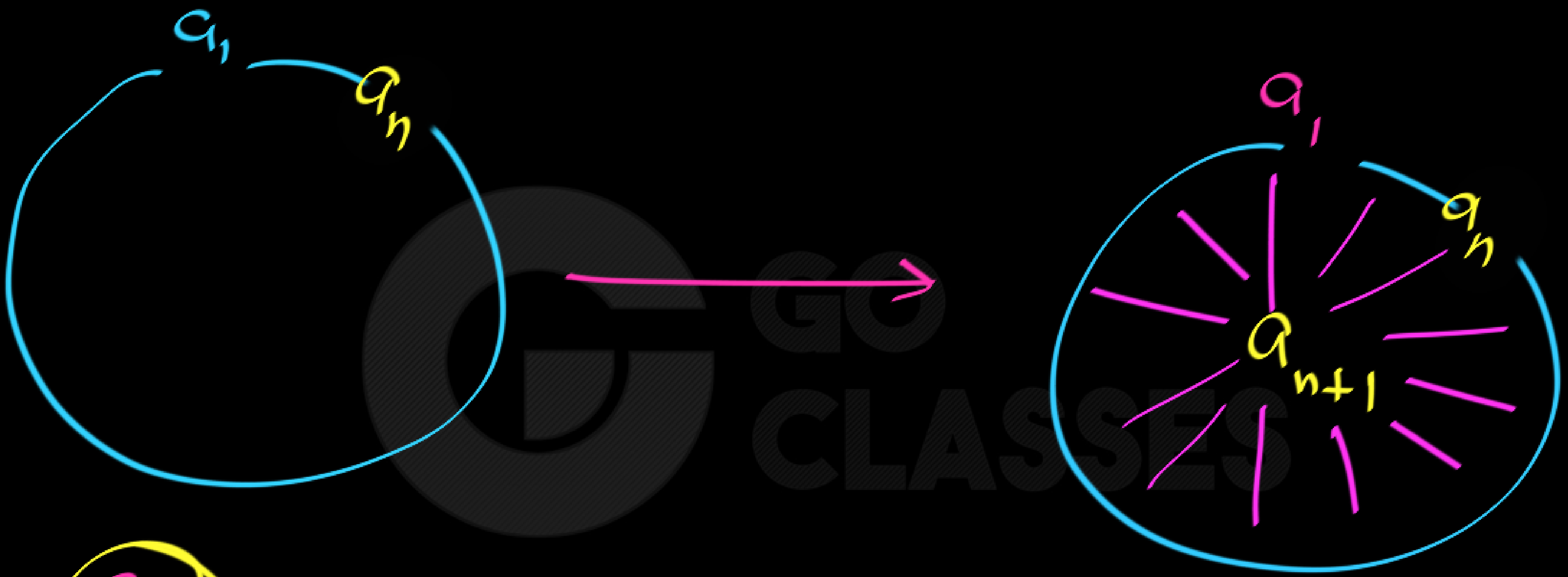
Next Topic : (For Simple Graph)

Wheel Graph

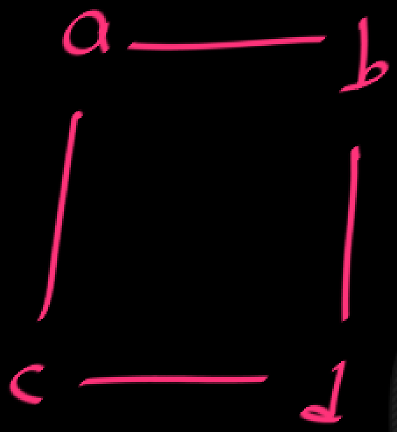
Website : https://www.goclasses.in/

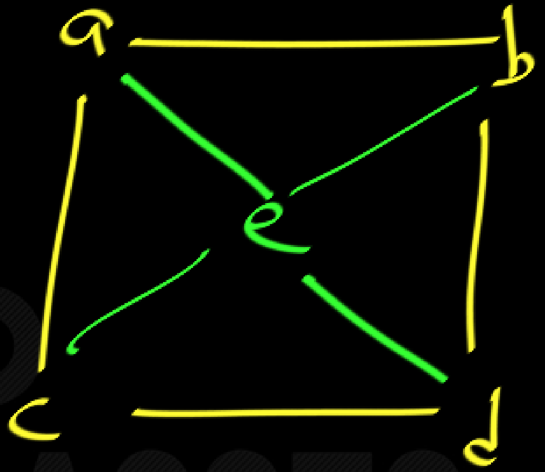


C_3 → W_3 = W_3 is a Wheel Graph on 4-vertices.



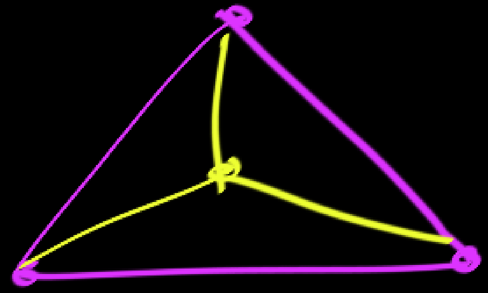
$$C_n \longrightarrow W_n \implies |V| = n+1$$





$C_4 \longrightarrow W_4 \longrightarrow |V| = 5$

$W_3 \rightarrow |V| = 3 + 1 = \underline{\underline{4}}$



Degree seq = $\underbrace{3, 3, 3, 3}$

W_3 = 3-Regular ($d = 3$)

$\delta = \Delta =$ Avg Deg $= 3$

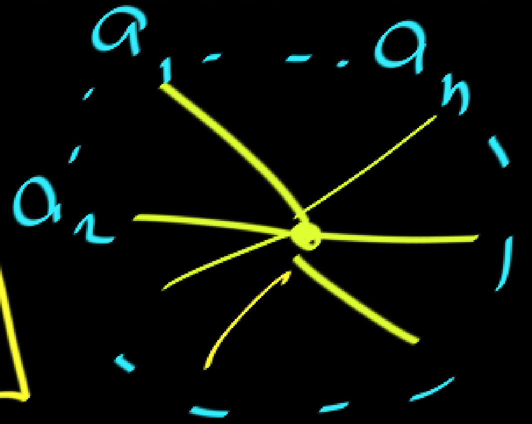
#Edges $= \frac{nd}{2} = \frac{4 \times 3}{2} = 6$

$\boxed{W_3 = K_4}$

Diam(W_3) = 1

Connected ✓

W_n ; $n \geq 4$



$|V| = \text{order}(W_n) = \boxed{n+1 \text{ vertices}}$

Deg seq : $n, \underbrace{3, 3, 3, \ldots, 3}_{n \text{ times}}$

Connected ✓

NOT Regular Graph.

Diameter = 2

$\delta = 3$; $\Delta = n$; Total Deg = $n + 3(n) = 4n$

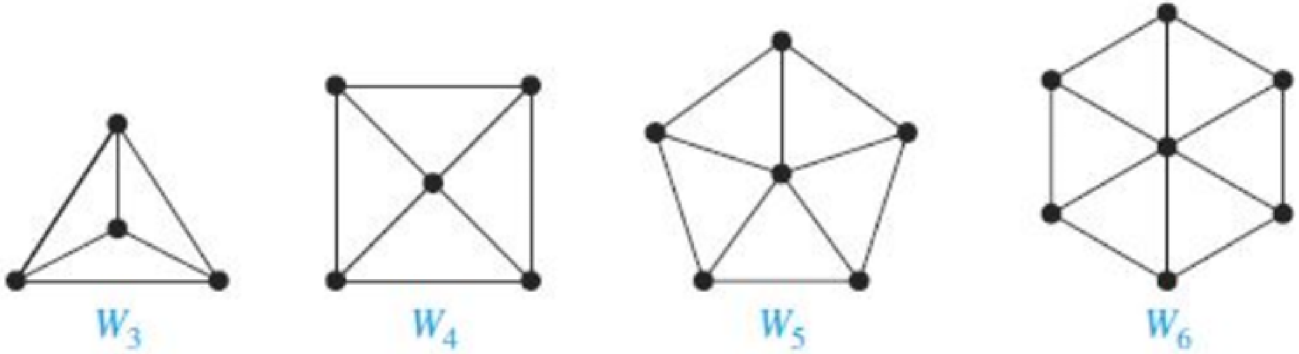
$$\text{Avg Deg} = \frac{4n}{n+1}$$

$$|\text{Edges}| = \frac{4n}{2} = 2n$$

$$\boxed{|\text{Edges}| = \frac{\text{Total Deg}}{2}}$$

Wheels

We obtain a *wheel* W_n when we add an additional vertex to a cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n, by new edges. Has $2n$ edges and $n+1$ vertices.



Graph Theory :

Next Topic : (For Simple Graph)

HyperCube Graph

Website : https://www.goclasses.in/

Hamming distance :

Hamming Distance between two strings of equal length is the number of positions at which the corresponding symbols are different. In other words, it measures the minimum number of substitutions required to change one string into the other, or the minimum number of errors that could have transformed one string into the other.

a b c
a b d
HD = 1

a b c
a c b
HD = 2

0 0 1
1 0 1
HD = 1

1 1 1
0 0 0
HD = 3

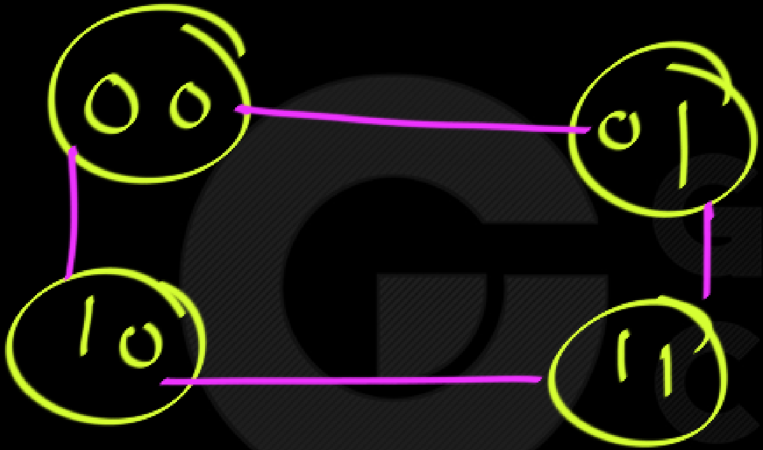
1 0 1
0 1 0
HD = 3

Hypercube Graph :

$Q_2(V, E)$

$|V| = 4$

Q_2 :



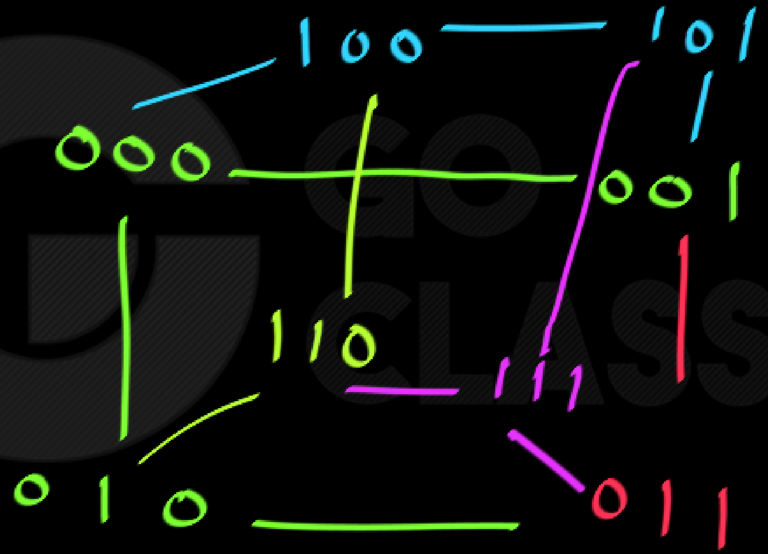
$\boxed{uv \in E \text{ iff } HD(u,v) = 1}$

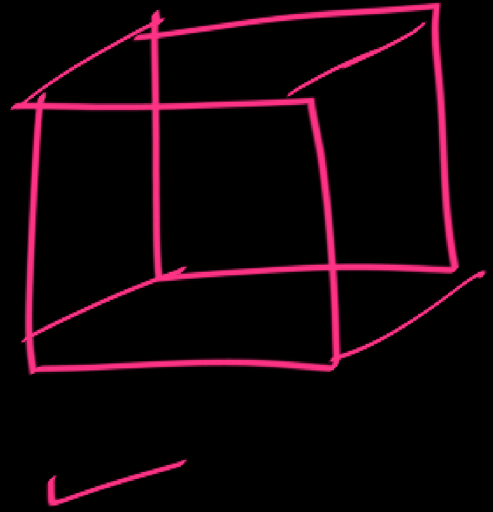
$Q_{2} \longrightarrow$ 2 length bit strings

Q_3 : → $\boxed{\text{3-length bit strings}}$ Vertices

$|V| = 8$

$Q_3 =$





Definition [edit]

The Hamming distance between two equal-length strings of symbols is the number of positions at which the corresponding symbols are different.[1]

Examples [edit]

The symbols may be letters, bits, or decimal digits, among other possibilities. For example, the Hamming distance between:

- "**karolin**" and "**kathrin**" is 3.
- "**karolin**" and "**kerstin**" is 3.
- "**kathrin**" and "**kerstin**" is 4.
- **0000** and **1111** is 4.
- **2173896** and **2233796** is 3.

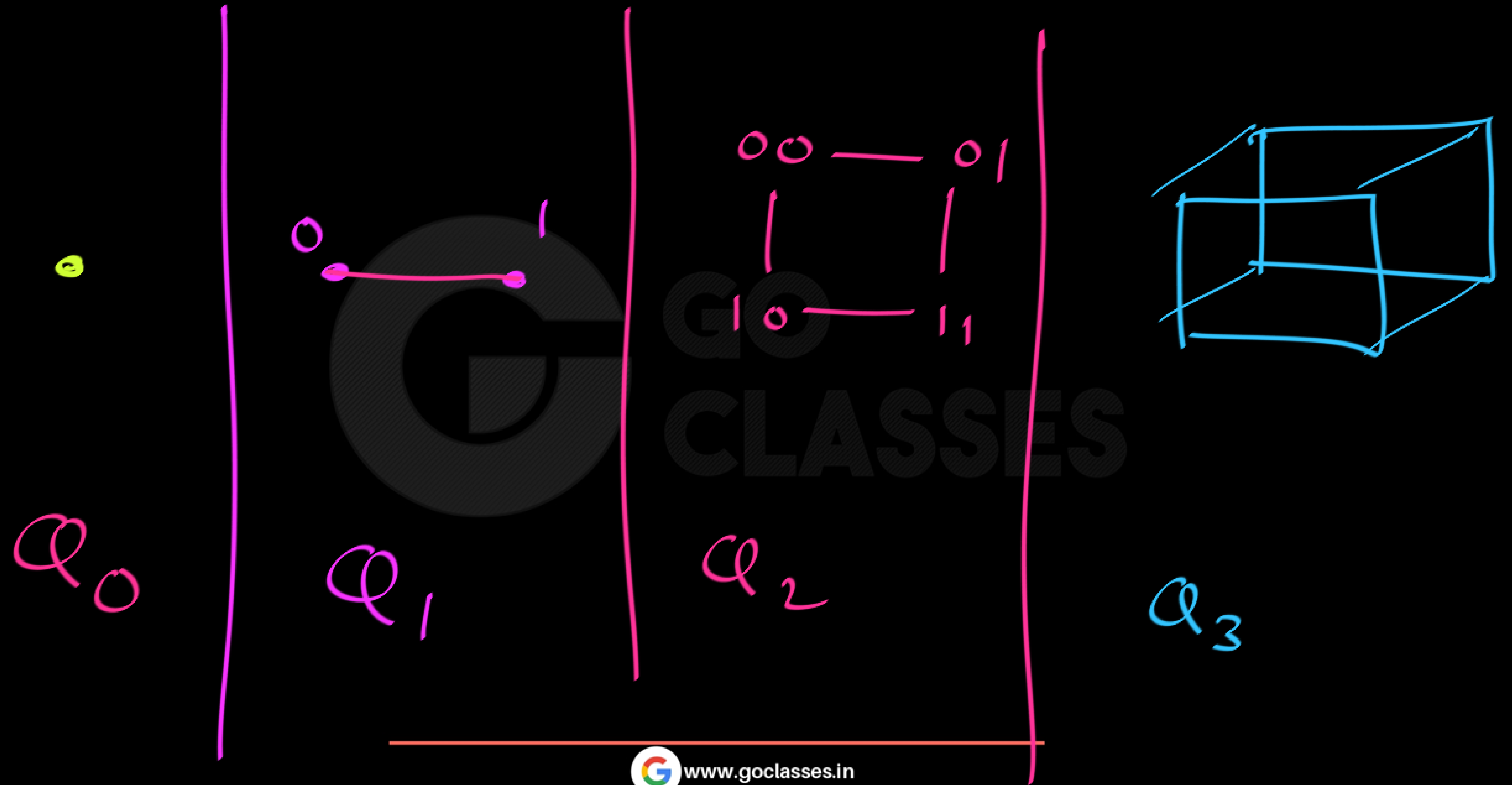
Hyper Cube Graph
n-Cube Graph
n-Dimensional Hypercube

$\Big\}$ Same Q_n

$Q_n(V, E)$

Vertices = bit-strings of length n

Two vertices are Adjacent iff they Differ in Exactly one Position.


Q0
0
1
Q1
00
01
10
11
Q2
Q3

$Q_n \longrightarrow$ n-length bit strings

Diam = n

① Order$(Q_n) = |V| = 2^n$ ✓

② Degree of each vertex $= n_{C_1} = n$

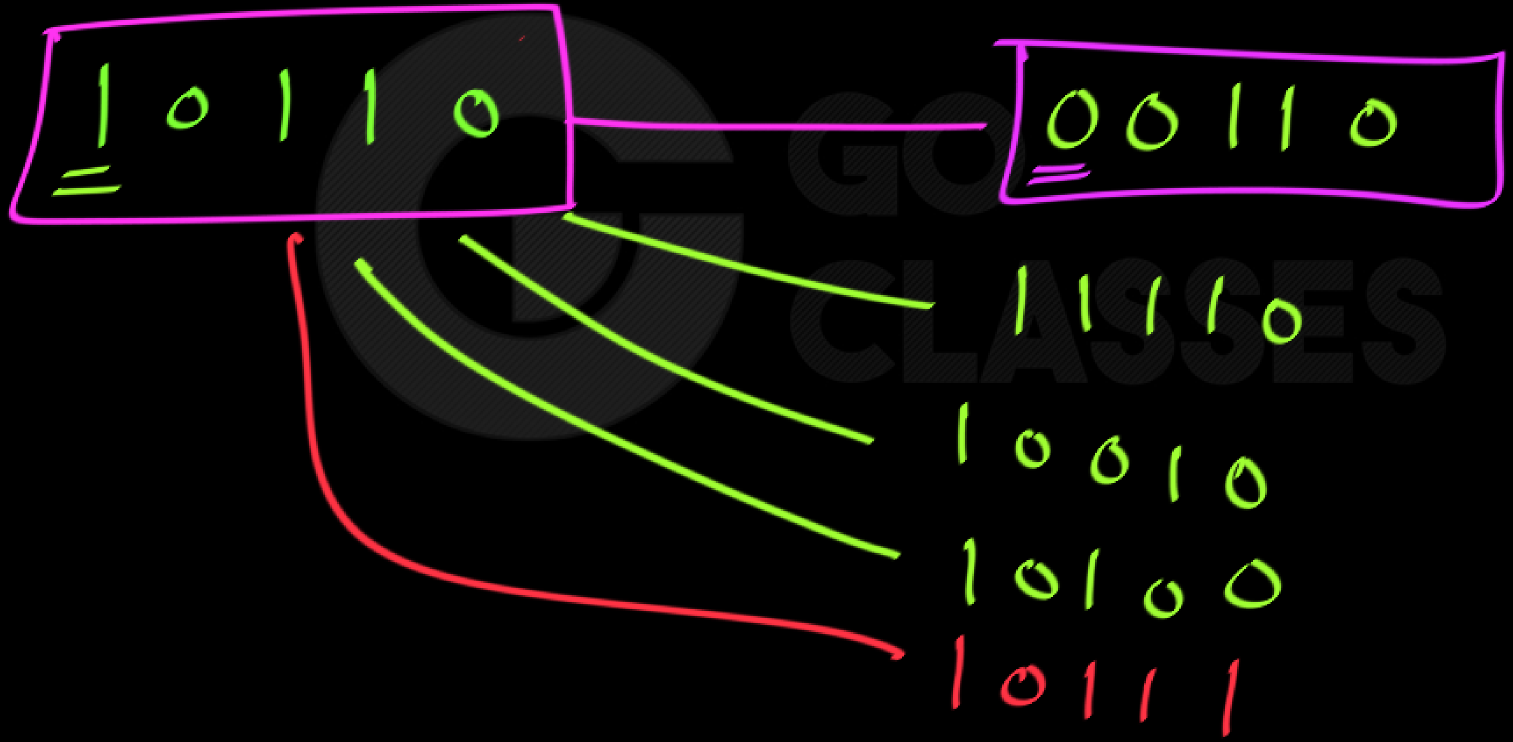
③ n-Regular $(d = n)$

$\delta = \Delta = \text{Avg Deg} = n$

Total Deg $= 2^n \times n$

$|E| = \frac{2^n \times n}{2} = n \times 2^{n-1}$

$Q_5 \Longrightarrow$ Degree of a vertex



$Q_n \Longrightarrow$ Degree of a vertex

$\underline{\underline{n}}$

$\boxed{a_1\, a_2\, a_3 - - a_n}$

$a_1'\, a_2\, a_3 - - a_n$

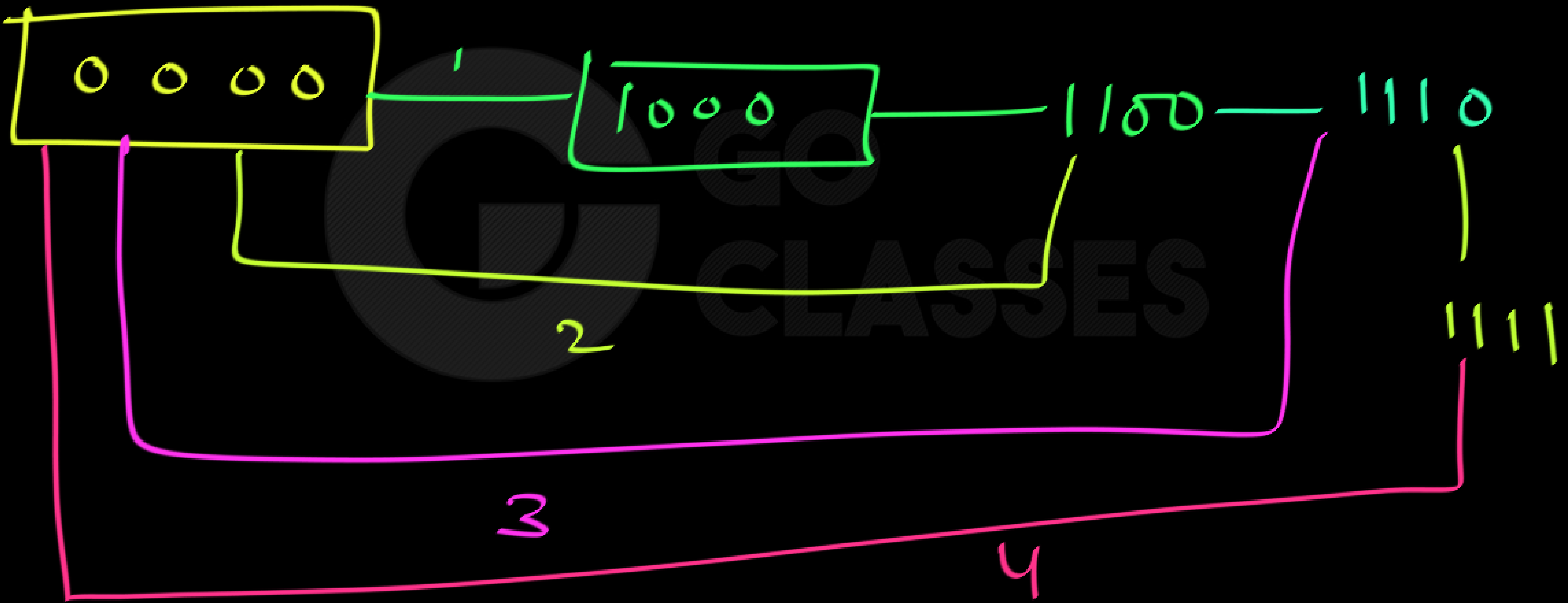
$a_1\, a_2'\, a_3 - - a_n$

$a_1\, a_2\, a_3' - - a_n$

$- - - - -$

n Neighbours

Diameter of Q_4 : $= 4$



Diameter of Q_n:

$a_1 a_2 \ldots a_n$

Distance $= n$

$a_1' a_2' \ldots a_n'$

- Q_n is a *hypercube*, i.e. the graph whose vertex set is the set of all binary strings $a_1a_2\ldots a_n$ of length n (i.e. $a_i \in \{0,1\}$ for each i), where two vertices $a_1a_2\ldots a_n$ and $b_1b_2\ldots b_n$ are adjacent if and only if there exists an i such that $a_i \neq b_i$ and $a_j = b_j$ for all $j \neq i$ (see Figure 3 for Q_3.)

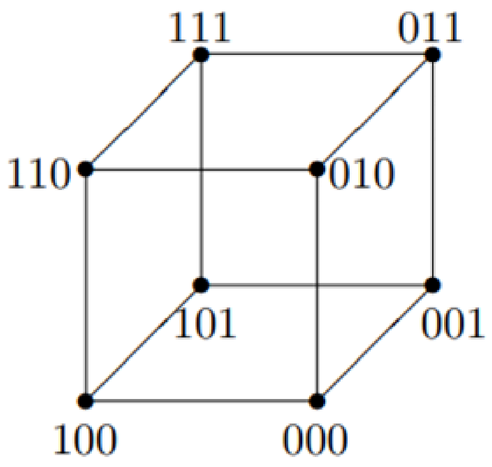
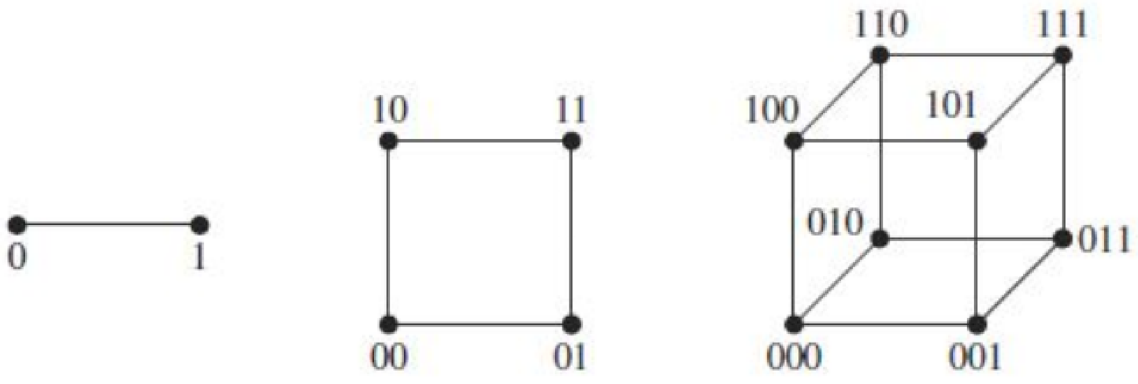


Figure 3: Q_3

n-Cubes

An n-*dimensional hypercube*, or n-*cube*, denoted by Q_n, is a graph that has vertices representing the 2^n bit strings of length n. Two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position. Has 2^n vertices and $n2^{n-1}$ edges (note that there are 0 edges in Q_0).

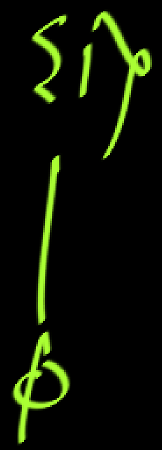


Q_1 Q_2 Q_3

Boolean Lattice; (Boolean Algebra)

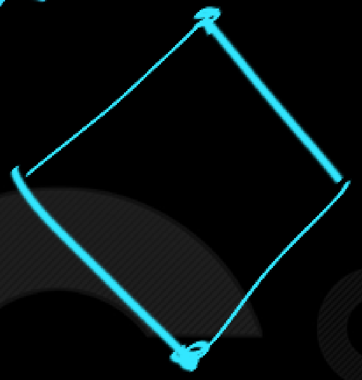
Every boolean lattice $\equiv (P(A), \subseteq)$ for some set A.

Q_1



$(P\{1\}, \subseteq)$

Q_2



$(P\{1,2\}, \subseteq)$

Q_3

$(P\{1,2,3\}, \subseteq)$

Boolean Lattice $\equiv \underbrace{(P(A), \subseteq)}_{\text{Powerset Lattice}}$ for some set A

$\equiv$ Powerset Lattice

$\equiv Q_n, \; n \geq 0$

Q: How many Edges in Hasse Diagram of $(P(A), \subseteq)$ where $|A| = 10$?

Q: How many Edges in Hasse Diagram of $(P(A), \subseteq)$ where $|A| = 10$?

$\underbrace{(P(A), \subseteq)}$ → Boolean Lattice

|||

Q_n, for some n → $n = 10$

$Q_{10} \Rightarrow$

$|V| = 2^{10}$ ✓

$|E| = 10 \times 2^{9}$ ✓

Q: How many Edges in Hasse Diagram of $(\underline{\underline{P(A)}}, \subseteq)$ where $\underline{\underline{|A| = n}}$?

$|P(A)| = 2^n$

$\underline{\underline{Q_n}}$

$|V| = 2^n$ ✓

$|E| = n \times 2^{n-1}$ ✓

Hypercube Graph Q_n :

$|V| = 2^n$

$|E| = n \times 2^{n-1}$

Degree $= n$

Diameter $= n$

Connected

n-regular

$|E| = \frac{\text{Total Deg}}{2} = \frac{2^n \times n}{2}$

EXERCISES

- Find the diameter of K_n, P_n, C_n, Q_n

Exercises

$n \geq 1$

- Find the diameter of K_n, P_n, C_n, Q_n, W_n

K_n: 1

P_n: $n-1$

C_n: $\left\lfloor \frac{n}{2} \right\rfloor$

Q_n: n

W_n: $2, n \geq 4$

$1, n = 3$

2.2.1 Degree Of Graph: GATE1987-9c

Show that the number of odd-degree vertices in a finite graph is even.

2.2.3 Degree Of Graph: GATE1995-24

Prove that in finite graph, the number of vertices of odd degree is always even.

2.2.9 Degree Of Graph: GATE2013-25

Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.

A. P only
B. Q only
C. Both P and Q
D. Neither P nor Q

2.2.1 Degree Of Graph: GATE1987-9c

Show that the number of odd-degree vertices in a finite graph is even.

2.2.3 Degree Of Graph: GATE1995-24

Prove that in finite graph, the number of vertices of odd degree is always even.

2.2.9 Degree Of Graph: GATE2013-25

Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.

Total Deg $= 2|E|$ even

A. P only
B. Q only
C. Both P and Q
D. Neither P nor Q

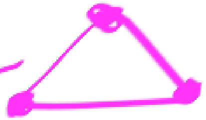
2.2.13 Degree Of Graph: TIFR2012-B-2

In a graph, the degree of a vertex is the number of edges incident (connected) on it. Which of the following is true for every graph G?

A. There are even number of vertices of even degree.
B. There are odd number of vertices of even degree.
C. There are even number of vertices of odd degree.
D. There are odd number of vertices of odd degree.
E. All the vertices are of even degree.

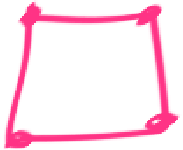
2.2.13 Degree Of Graph: TIFR2012-B-2

In a graph, the degree of a vertex is the number of edges incident (connected) on it. Which of the following is true for every graph G?



A. There are even number of vertices of even degree.

B. There are odd number of vertices of even degree.



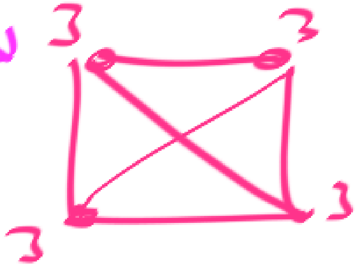
C. There are even number of vertices of odd degree.

Always

D. There are odd number of vertices of odd degree.

NEVER

E. All the vertices are of even degree.



(may or may not)

NOTE:

$a + b + c + d = t$ (Given)

let $k \le a$

$k + b + c + d \le t$

let $m \ge a$

$m + b + c + d \ge t$

$$a + b + c + d = t$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$Avg + Avg + Avg + Avg = t$$

$$\boxed{Avg = \frac{a+b+c+d}{4}}$$

Degree Summation formula:

$$\sum_{\forall v \in V} deg(v) = 2|E|$$

Avg Deg $= \Omega \implies \boxed{n \times \Omega = 2|E|}$ ✓

min Deg $= \delta \implies n\delta \leq 2|E|$

max Deg $= \Delta \implies n\Delta \geq 2|E|$

$$\boxed{\min \leq \text{Avg} \leq \max}$$

Avg Deg $= r$

Deg Summation in terms of

① Avg Deg: $\boxed{2|E| = r \cdot n}$

② δ : $n\delta \leq 2|E|$ ✓

③ Δ : $n\Delta \geq 2|E|$ ✓

2.2.11 Degree Of Graph: GATE2017-2-23

G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is ________ .

2.2.11 Degree Of Graph: GATE2017-2-23

G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is ________ . 16

$\delta = 3$

$$\boxed{n\delta \le 2|E|}$$

$$3n \le 2 \times 25$$

$$3n \le 50$$

$$\boxed{n \le \frac{50}{3}}$$

$$n = \left\lfloor \frac{50}{3} \right\rfloor = 16$$

$$\underbrace{\text{Deg sum}} = 2|E|$$

$$\text{Deg}_1 + \text{Deg}_2 + \text{Deg}_3 + \cdots + \text{Deg}_n = 2|E|$$

$$\text{Avg} + \text{Avg} + \text{Avg} + \cdots \text{Avg} = 2|E|$$

$$\delta + \delta + \delta + \cdots + \delta \leq 2|E|$$

2.2.4 Degree Of Graph: GATE2003-40

A graph $G = (V, E)$ satisfies $|E| \leq 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V}\{\text{degree }(v)\}$. Therefore, min-degree of G cannot be

A. 3
B. 4
C. 5
D. 6

2.2.4 Degree Of Graph: GATE2003-40

$|E| = e \; ; \; |V| = n$

A graph $G = (V, E)$ satisfies $|E| \leq 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V}\{\text{degree}(v)\}$. Therefore, min-degree of G cannot be

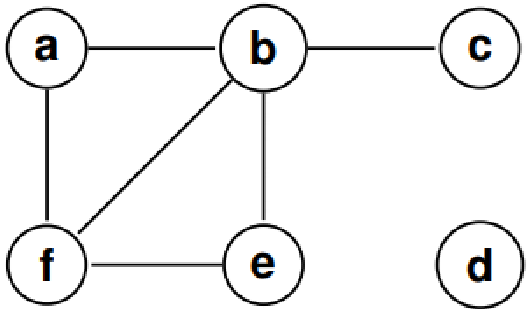
A. 3 B. 4 C. 5 D. 6

$e \leq 3n - 6 \; ; \; n\delta \leq 2e$

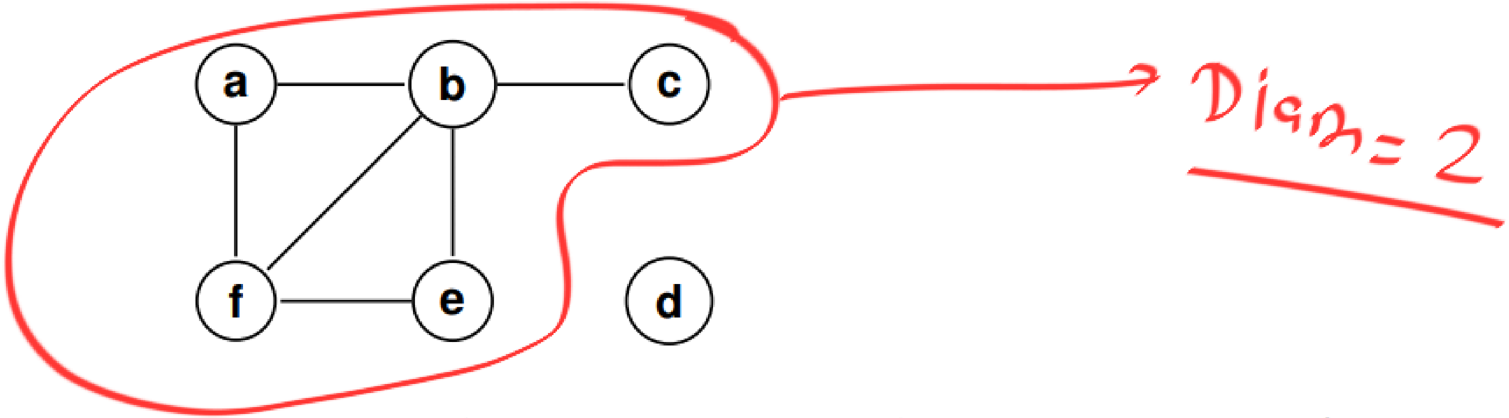
$3n-6 \geq e \geq \frac{n\delta}{2}$

$\frac{n\delta}{2} \leq 3n - 6 \Rightarrow \boxed{\delta \leq \frac{6n - 12}{n}} \Rightarrow \delta \neq 6$

Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



We have 6 vertices and 6 edges. deg(a) = 2, deg(b) = 4, deg(c) = 1, deg(d) = 0, deg(e) = 2, deg(f) = 3. c is a pendant and d is isolated.

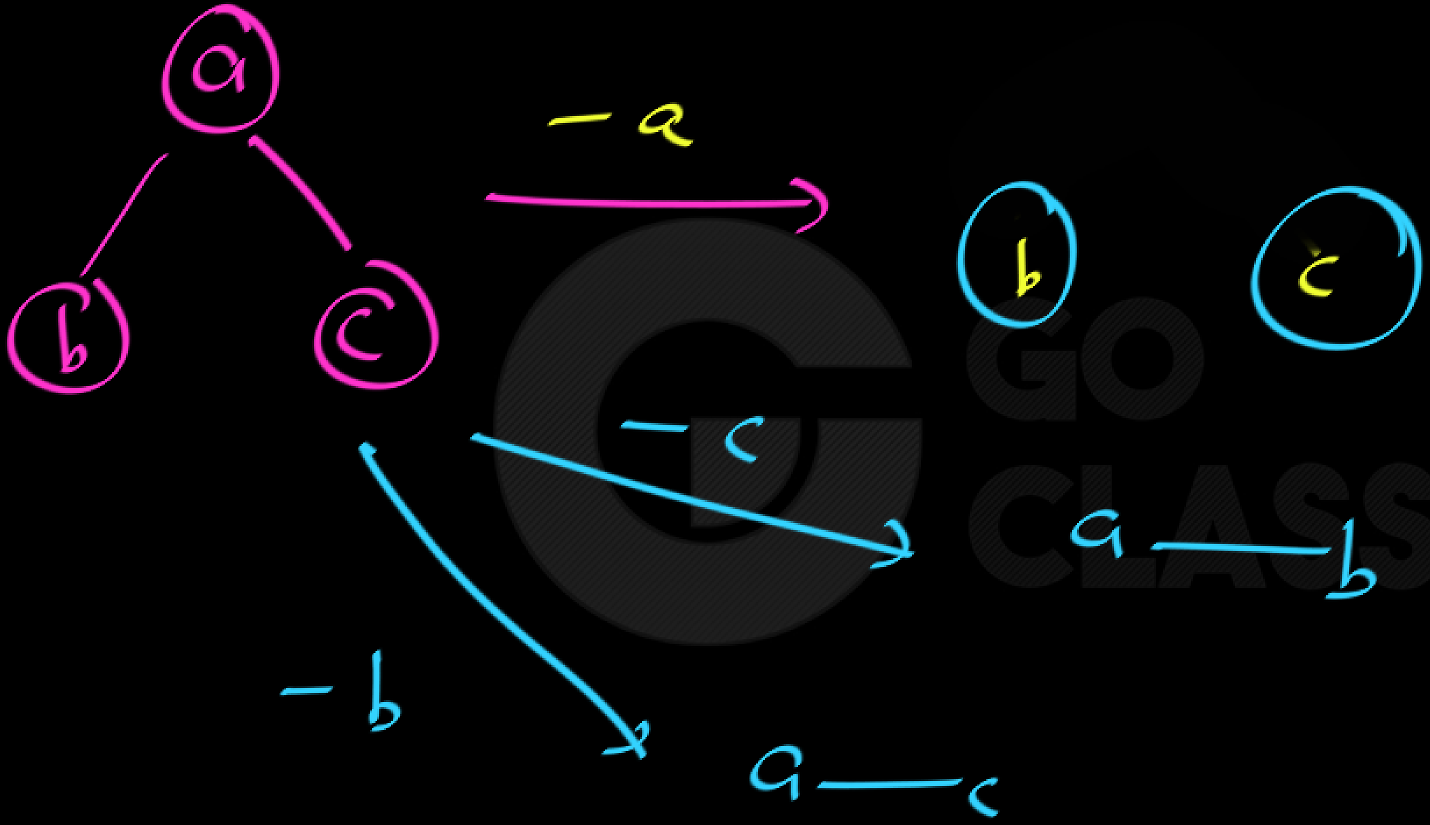
Diameter = ∞

Graph Theory :

Next Topic :

Deletion of Vertex, Edge

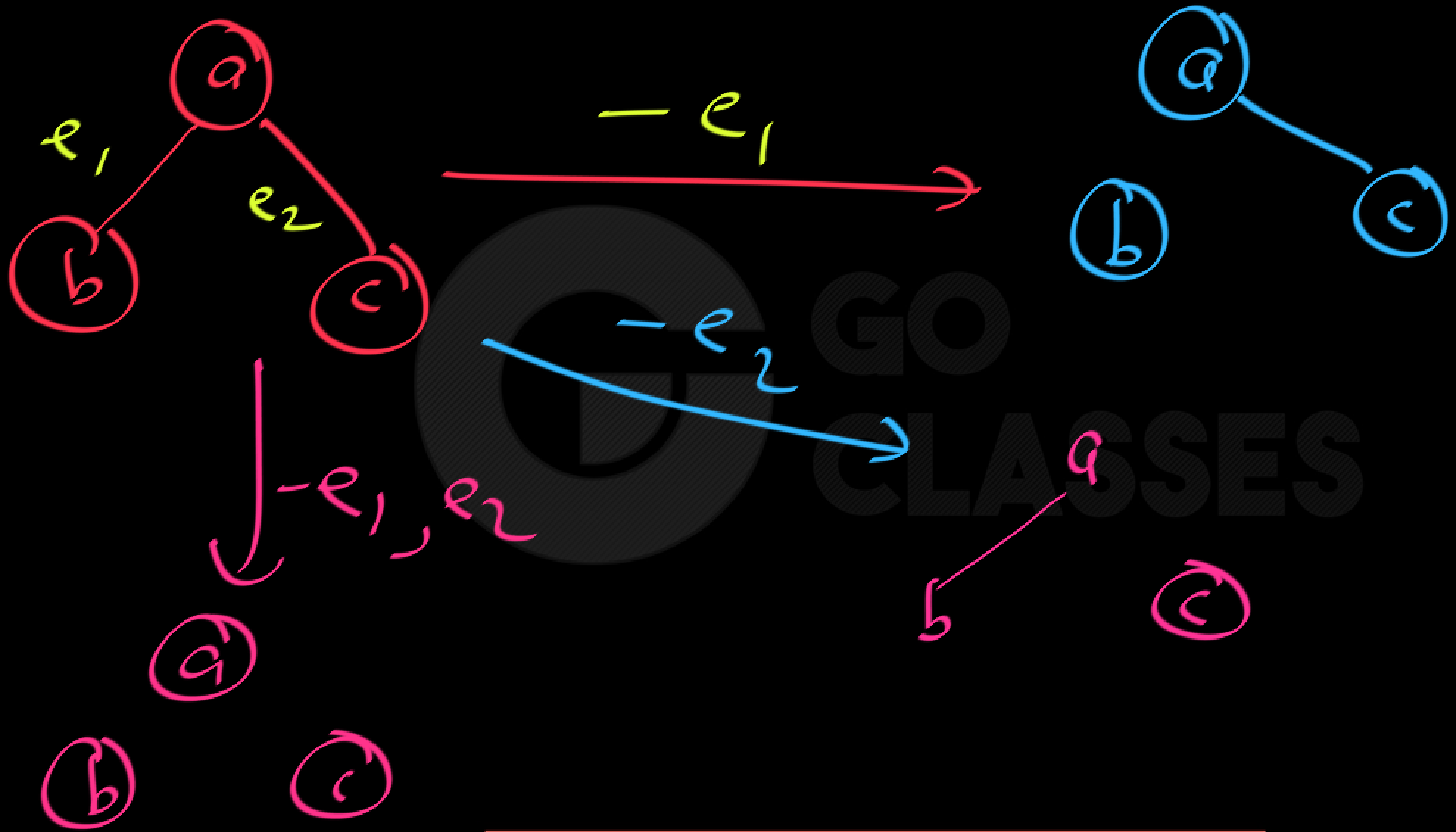
Website : https://www.goclasses.in/


a
b
c
− a
b
c
− c
a — b
− b
a — c

NOTE:

When we delete a vertex v, all Edges incident on v, are Automatically gone.

Deletion of an Edge $\Rightarrow$ only that Edge is Deleted.


a
b
c
e_1
e_2
$-e_1$
a
b
c
$-e_2$
a
b
c
$-e_1, e_2$
a
b
c

Graph Theory :

Next Topic :

Subgraph

Induced, Spanning Subgraphs

Website : https://www.goclasses.in/

Graph G $\xrightarrow{\text{Delete 0 or more vertices}}$

or

Delete 0 or more Edges

or

both

Graph H

is subgraph of G.

$H \leq G$ (H is subgraph of G)

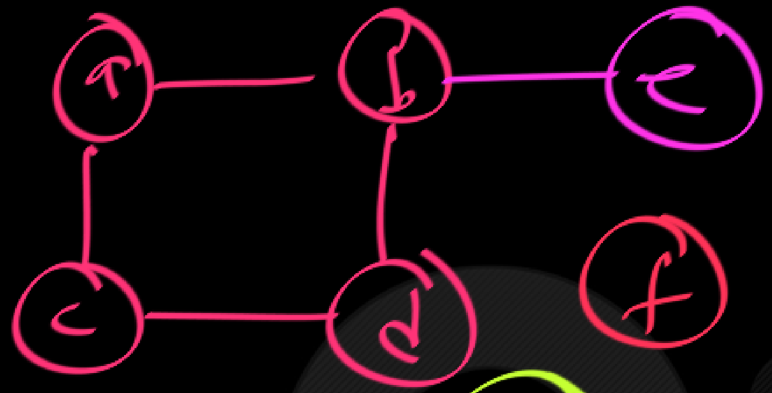
iff $\left[\begin{array}{c} V(H) \subseteq V(G) \\ \text{and} \\ E(H) \subseteq E(G) \end{array} \right]$ H is subgraph of G.

Definition 8. [GRAPH CONTAINMENT RELATIONS] Given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, the graph G_1 is said to be

- a *subgraph* of $G_2 = (V_2, E_2)$ if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$, i.e. G_1 can be obtained from G_2 by deleting some vertices and some edges;
- a *spanning subgraph* of G_2 if $V_1 = V_2$ and $E_1 \subseteq E_2$, i.e. G_1 can be obtained from G_2 by deleting some edges but not vertices;
- an *induced subgraph* of G_2 if G_1 is a subgraph of G_2 and every edge of G_2 with both endpoints in V_1 is also an edge of G_1, i.e. G_1 can be obtained from G_2 by deleting some vertices but not edges.

- What are the subgraphs, induced subgraphs and spanning subgraphs of K_n?

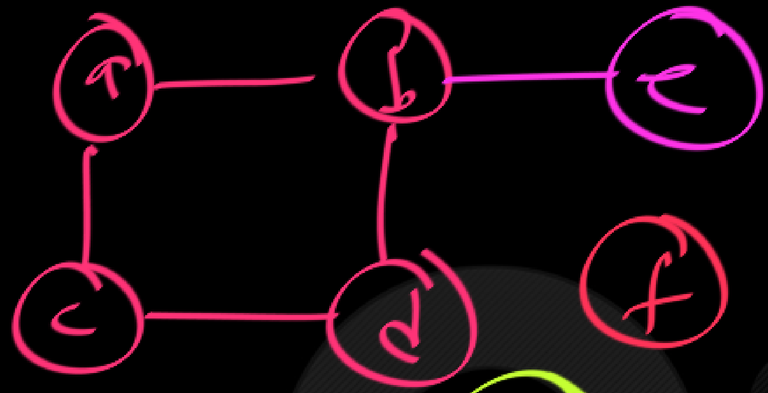
G :



which is a subgraph of G?

1. G
2. f
3. a — d
4. a h
5. a — b — e
6. a e

G :



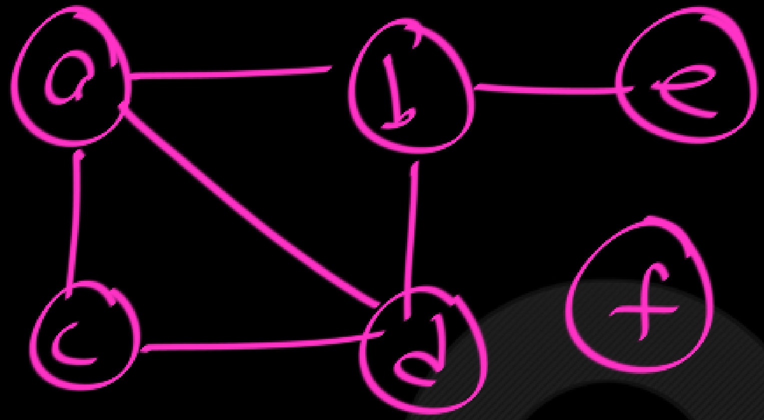
$\boxed{G \le G}$

which is a subgraph of G ?

1. G
2. f
3. a — d
4. a h
5. a — b — e
6. a e
7. a b c d

Subgraph

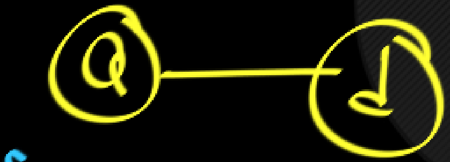
- Spanning Subgraph : Vertex Deletion NOT Allowed.
- Induced Subgraph : (Extra) Edge Deletion NOT Allowed.
  - vertex and its incident edges Can be deleted.



Not Induced

Spanning

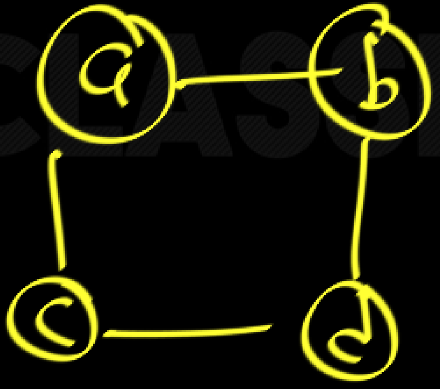
1



Induced ✓

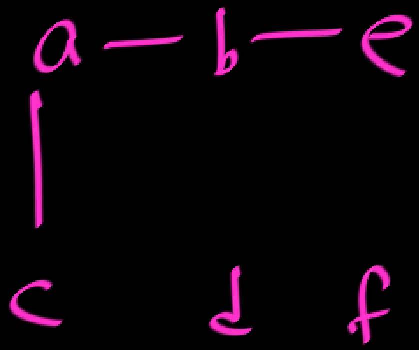
Not spanning

2



Neither

3



$G \leq G$

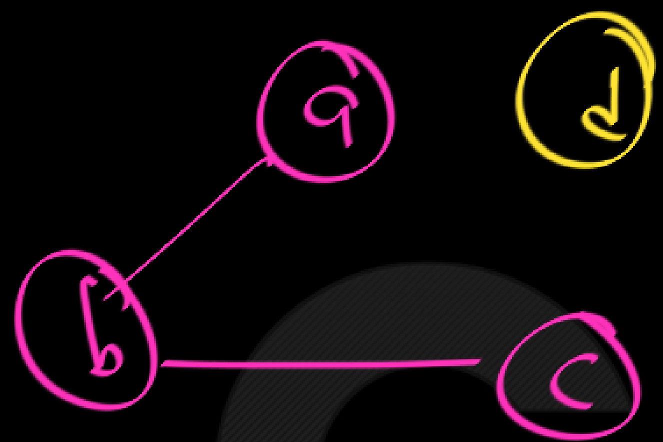
Spanning Subgraph ✓

Induced Subgraph ✓

Q: for any G,
How many Spanning AND Induced Subgraphs are there?

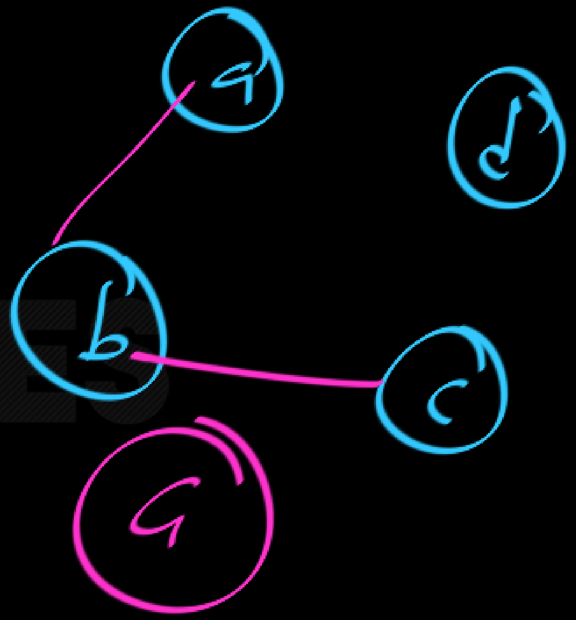
$= 1$; G itself

G :



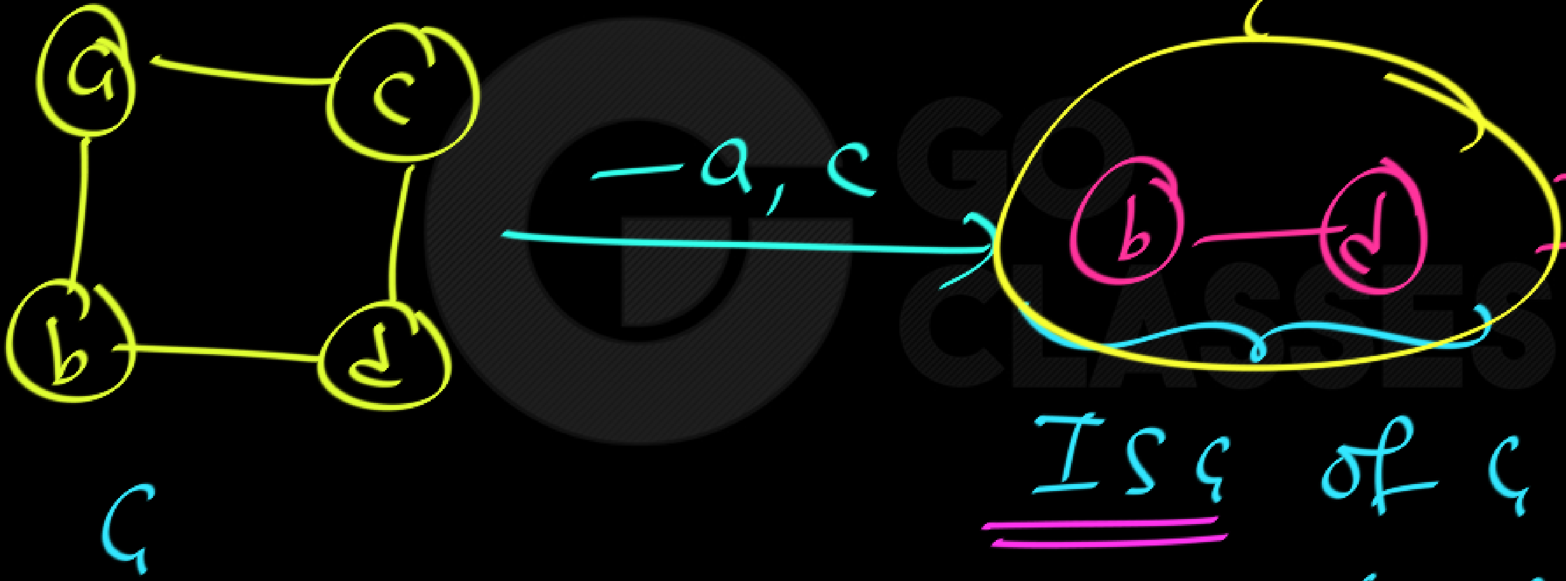
Spanning

Induced



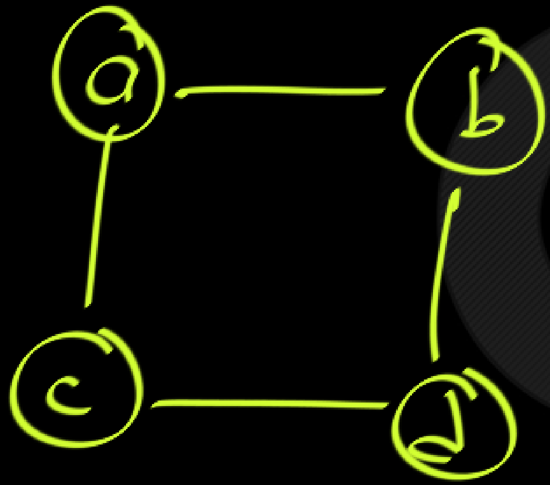
Induced SubGraph: (ISG)

Induced by b, d



ISG of G by {b, d}

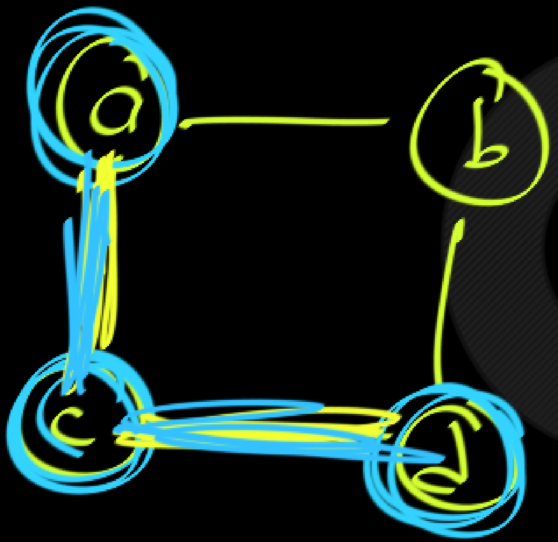
Induced SubGraph:

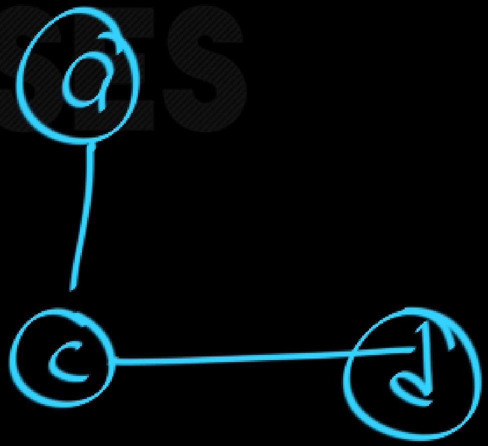


Graph Induced by a, d;

Induced SubGraph:

Graph Induced by a, d, c :





NOTE: $G(V,E)$; $S \subset V$

$G \xrightarrow{-S}$ Graph Induces by $V-S$

$G \xrightarrow{?}$ Graph Induces by S

$-(V-S)$

Definition 9. [Induced subgraph relation]

Given a graph G and a subset $U \subseteq V(G)$, we denote by

- $G[U]$ the subgraph of G induced by U, i.e. the graph with vertex set U whose vertices are adjacent if and only if they are adjacent in G,
- $G - U$ the subgraph of G induced by $V(G) - U$, i.e. the graph obtained from G by deleting the vertices of U.

We say that G contains a graph H as an induced subgraph if H is isomorphic to an induced subgraph of G, in which case we also say that H is contained in G as an induced subgraph, or simply, H is an induced subgraph of G. We denote by

- $H < G$ the fact that H is an induced subgraph of G.

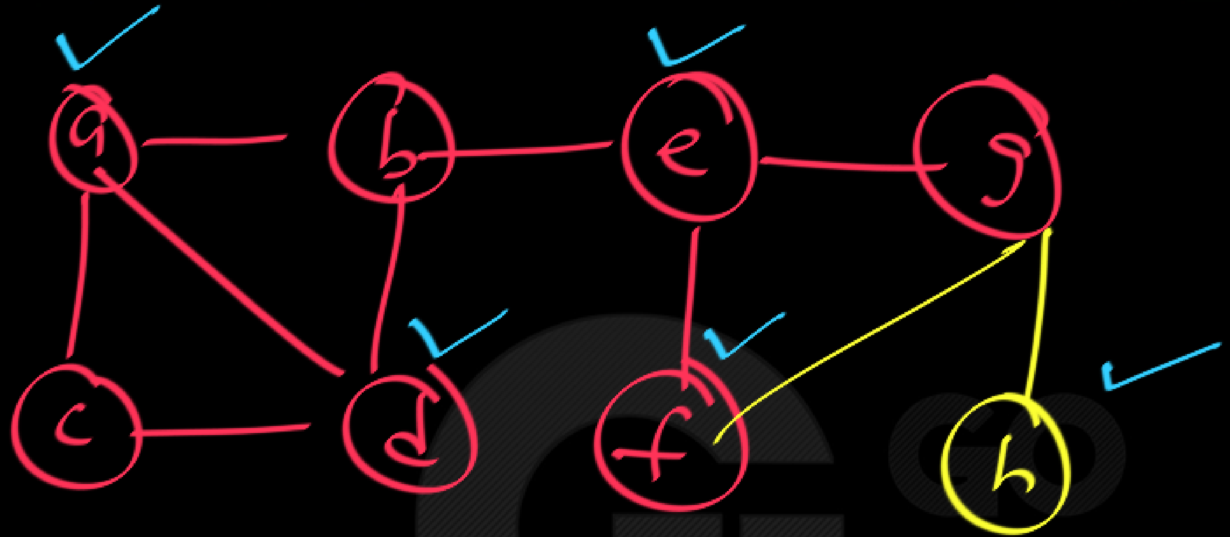
$G(V, E)$; $S \subseteq V$

Graph Induced by S :

$H(S, E')$

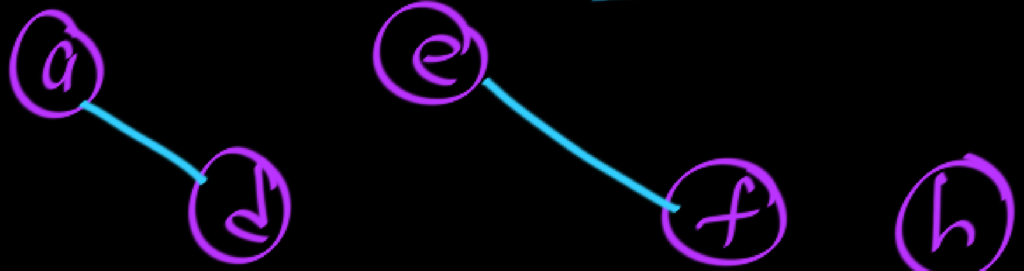
Vertex set

$E' = \{ e \mid e \in E$ and both end points of e are in $S \}$

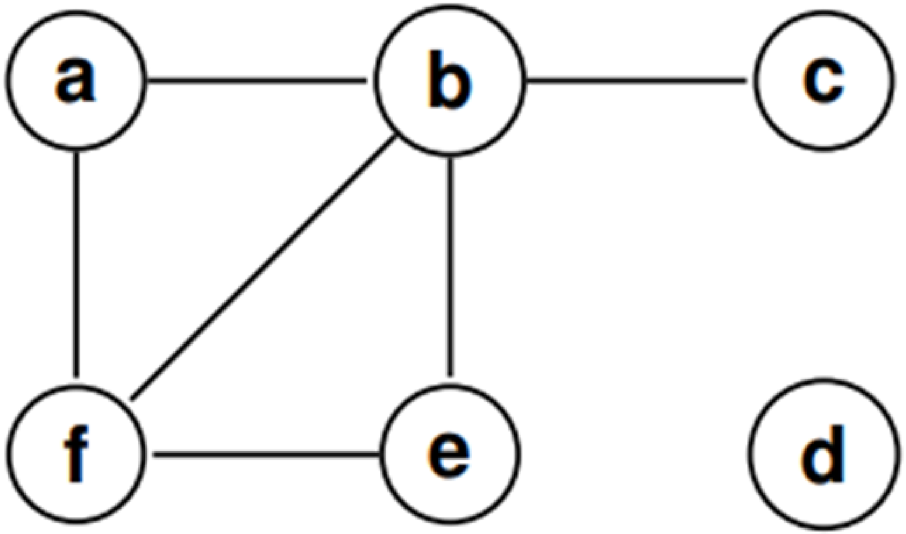


Graph Induced by $S = \{a, e, f, h, d\}$

$V = S$



For the following graph G find

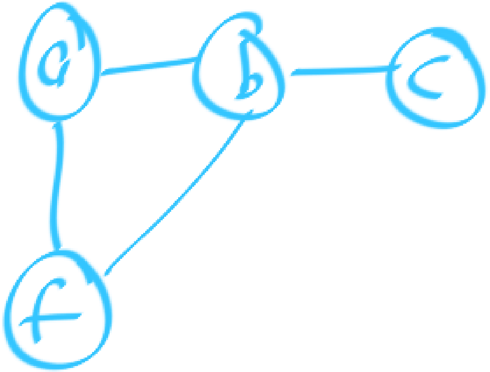


a) a subgraph induced by the vertices a, b, c, and f.

For the following graph G find

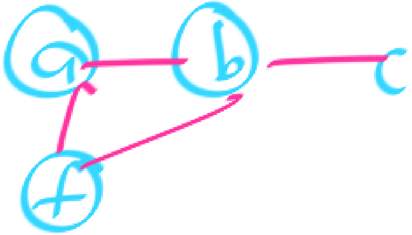
way1:

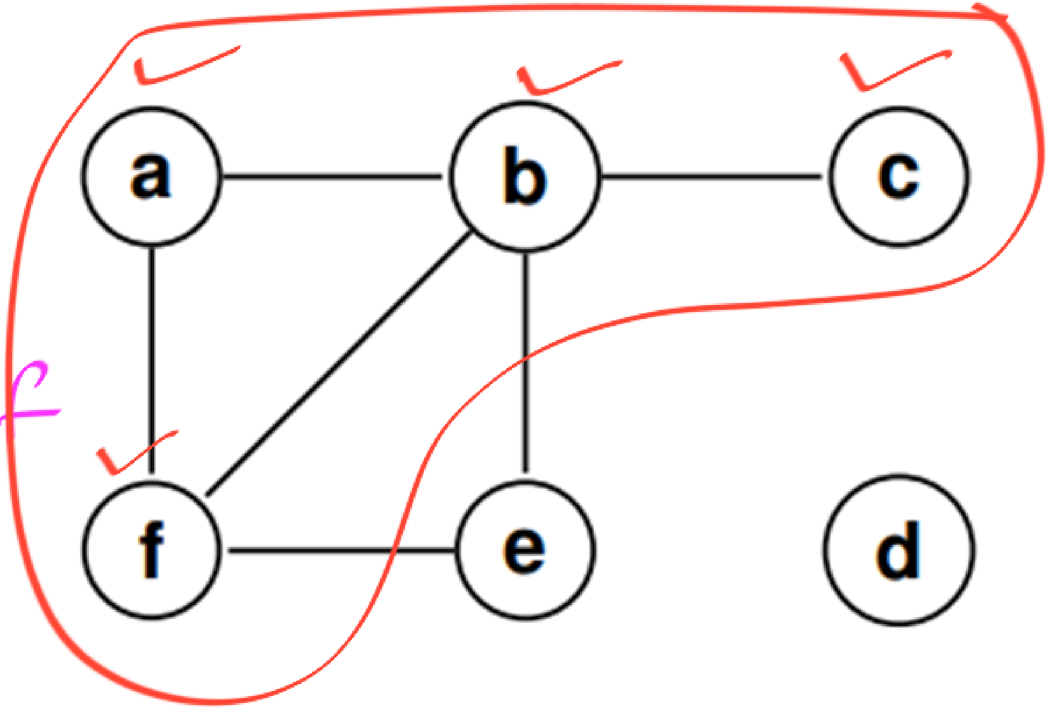
Delete e, d



way2

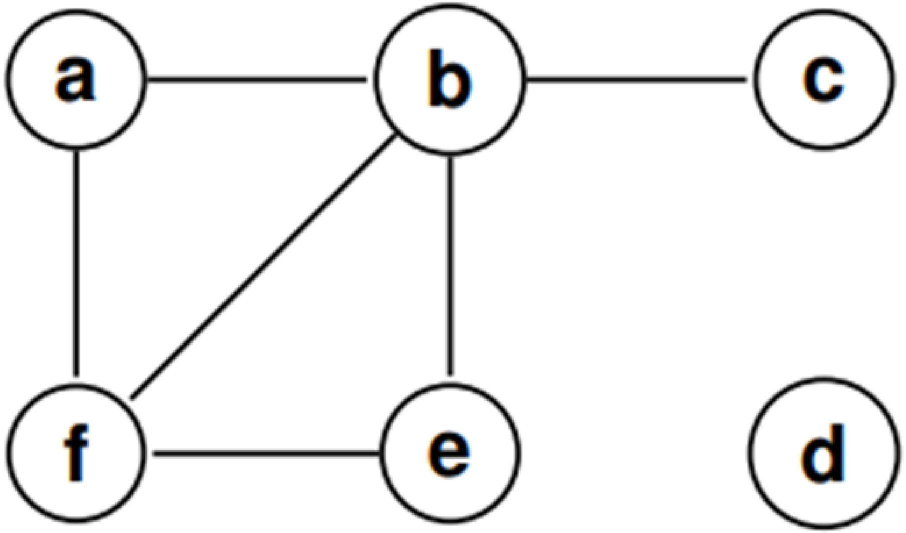
Just look at a,b,c,f





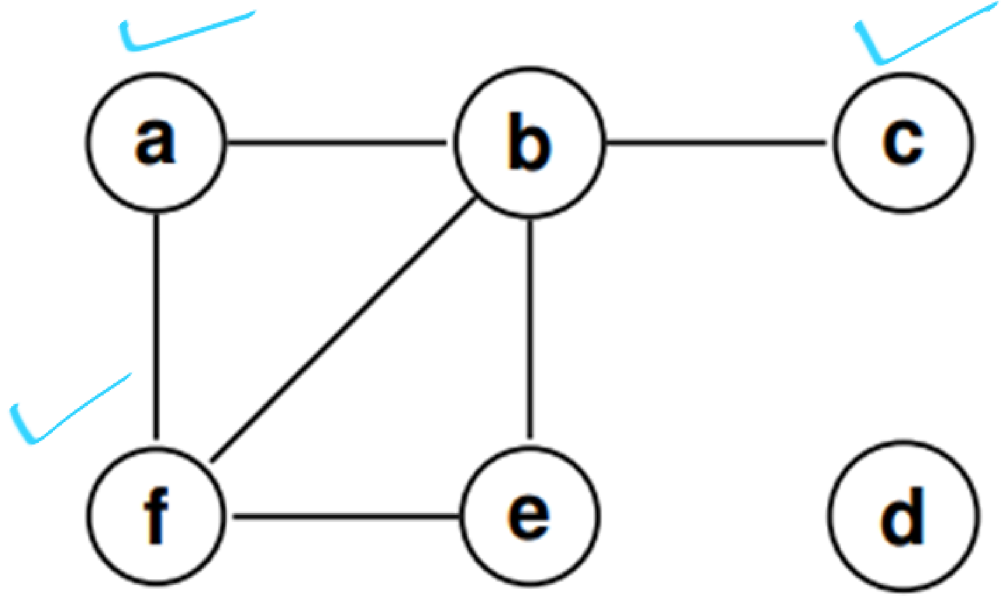
a) a subgraph induced by the vertices a, b, c, and f.

For the following graph G find



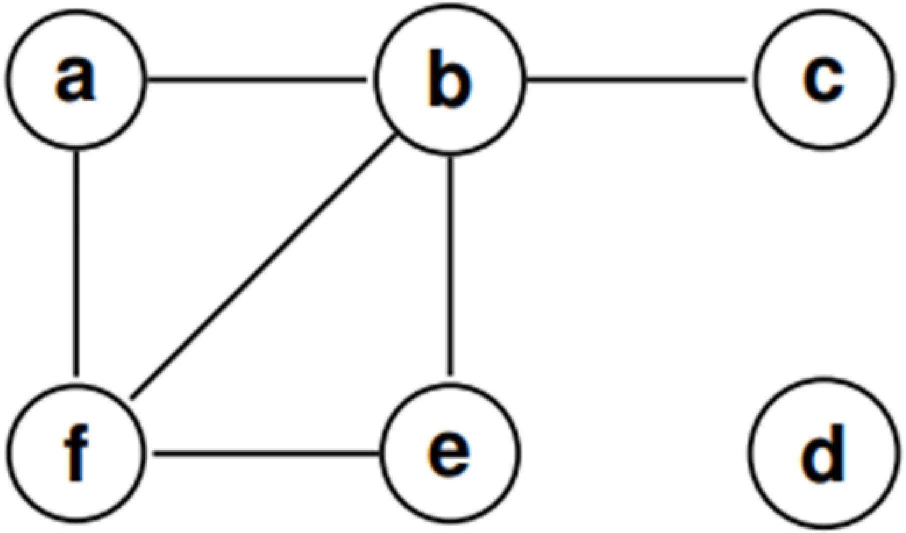
a) a subgraph induced by the vertices a, [illegible] c, and f.

For the following graph G find



a) a subgraph induced by the vertices a, c, and f.

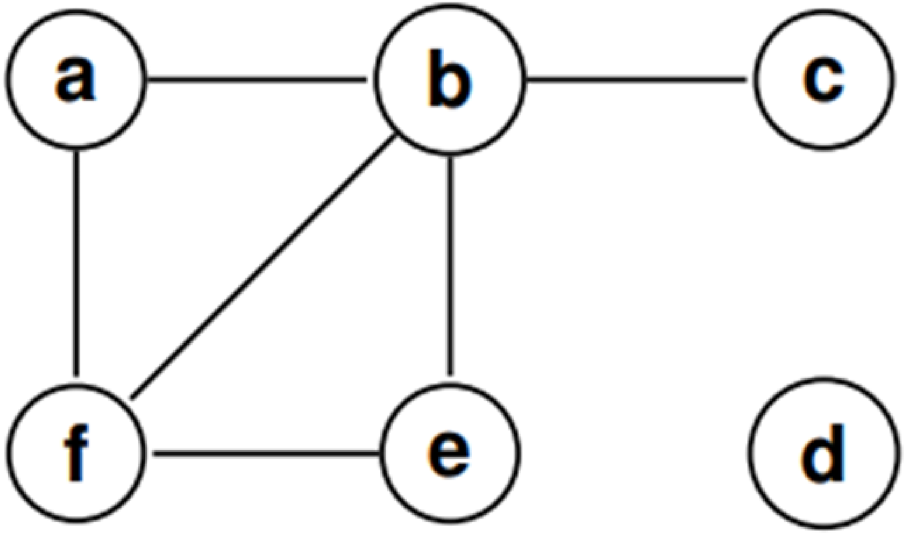
For the following graph G find



a) a subgraph induced by the vertices $a, b, c,$ [redacted] f. , d, e

For the following graph G find

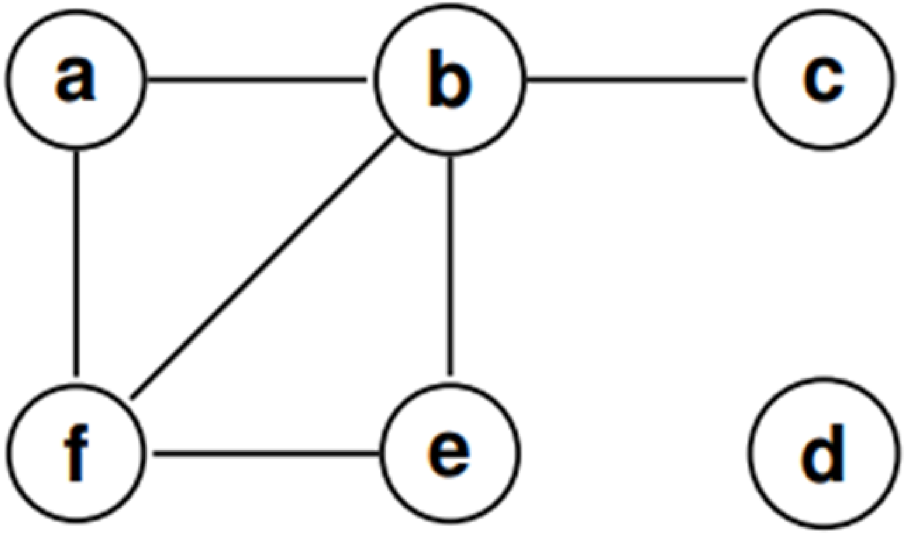
G itself



a) a subgraph induced by the vertices $a, b, c,$ [redacted] f. , d, e

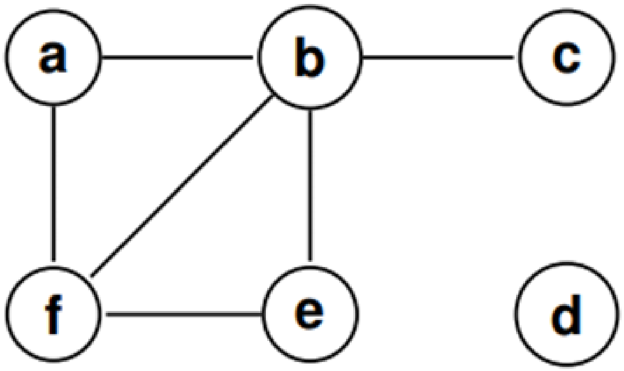
For the following graph G find

a c

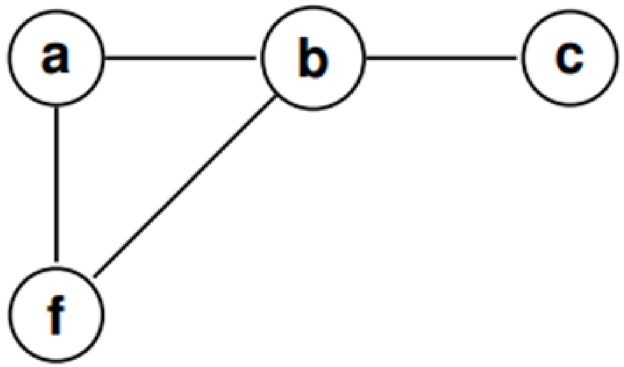


a) a subgraph induced by the vertices a[redacted] c,[redacted]

For the following graph G find



a) a subgraph induced by the vertices a, b, c, and f.



Every subgraph of a complete graph is also a complete graph. True/False ?

Every induced subgraph of a complete graph is also a complete graph. True/False ?

Every spanning subgraph of a complete graph is also a complete graph. True/False ?

Every graph of n vertices is subgraph of n vertices complete graph. True/False ?

A subgraph of G can be both induced and spanning subgraph. True/False?

There is exactly one subgraph of G which is both induced and spanning subgraph. True/Fase?

Every subgraph of a complete graph is also a complete graph. True/False ?

Every induced subgraph of a complete graph is also a complete graph. True/False ?

Every spanning subgraph of a complete graph is also a complete graph. True/False ?

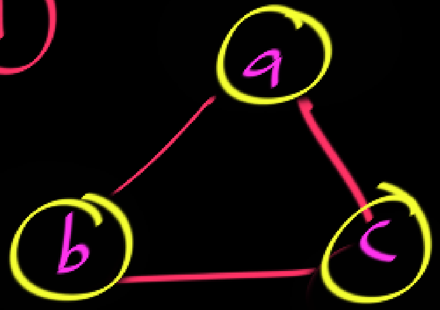
Every graph of n vertices is subgraph of n vertices complete graph. True/False ?

A subgraph of G can be both induced and spanning subgraph. True/False?

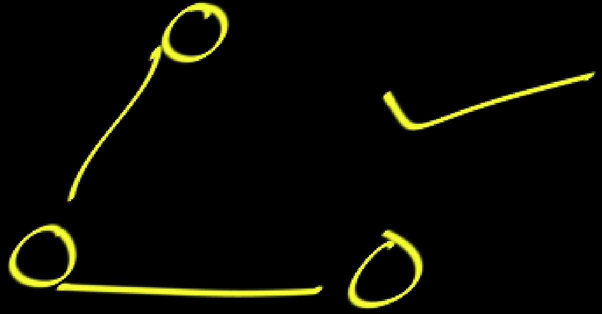
There is exactly one subgraph of G which is both induced and spanning subgraph. True/Fase?

G itself

1



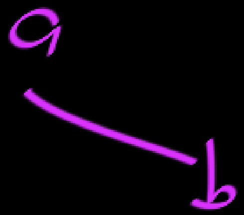
Subgraph

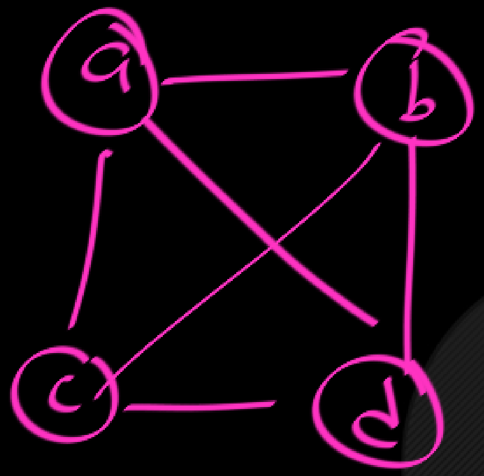


K_3

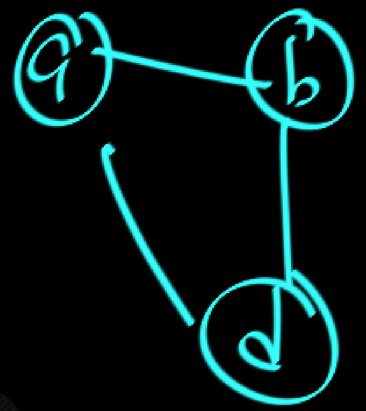
induced by a, b

NOT Complete



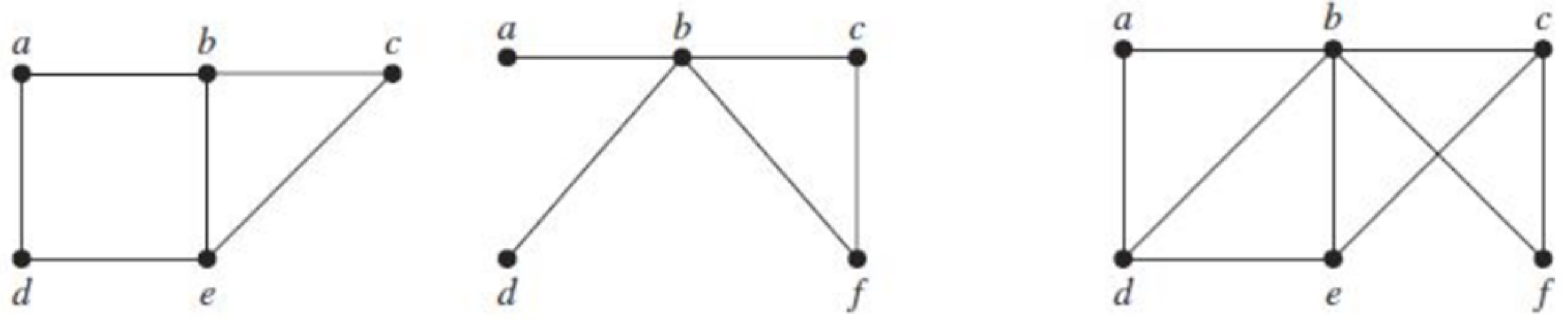


induced by a, b, d



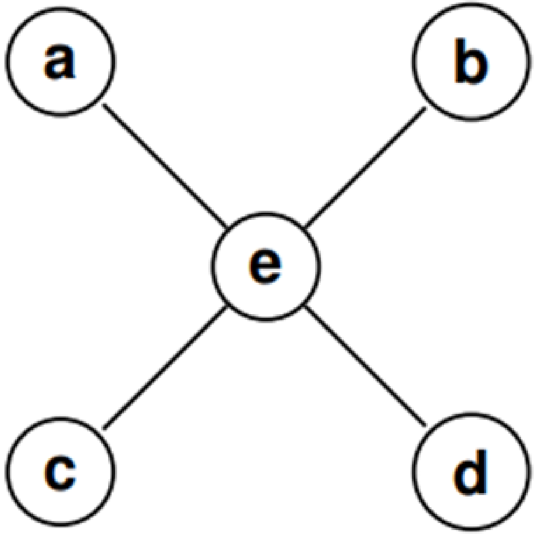
Graph Union

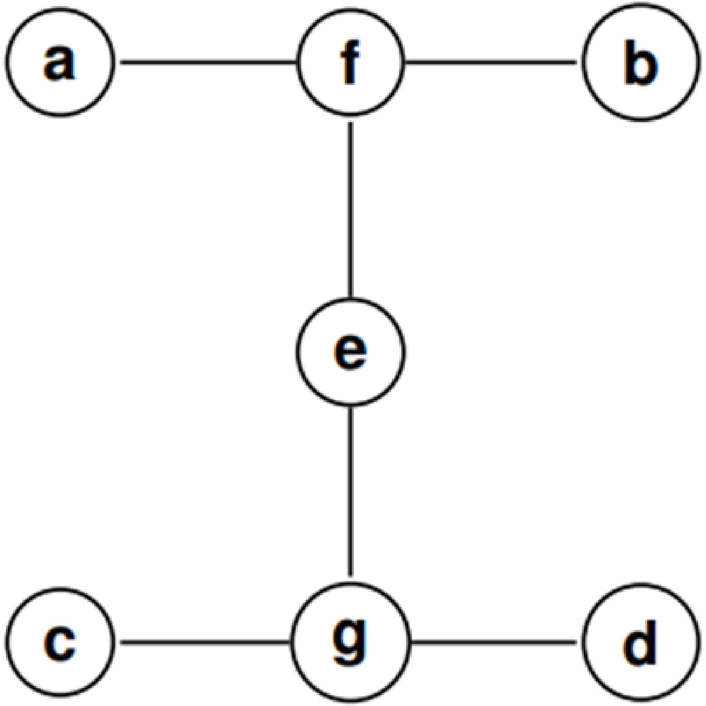
The union of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_1 \cup G_2$.



G_1 G_2 $G_1 \cup G_2$

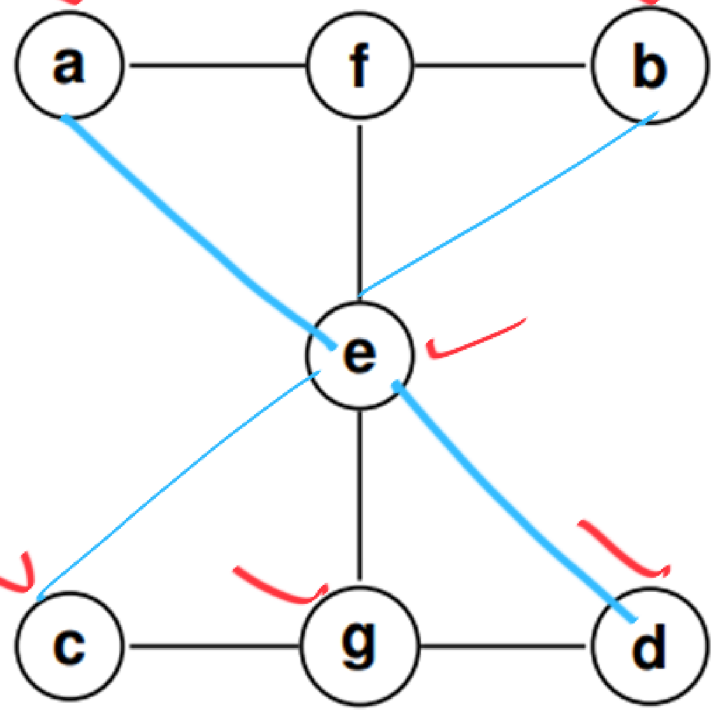
Find the union of the given pair of simple graphs.

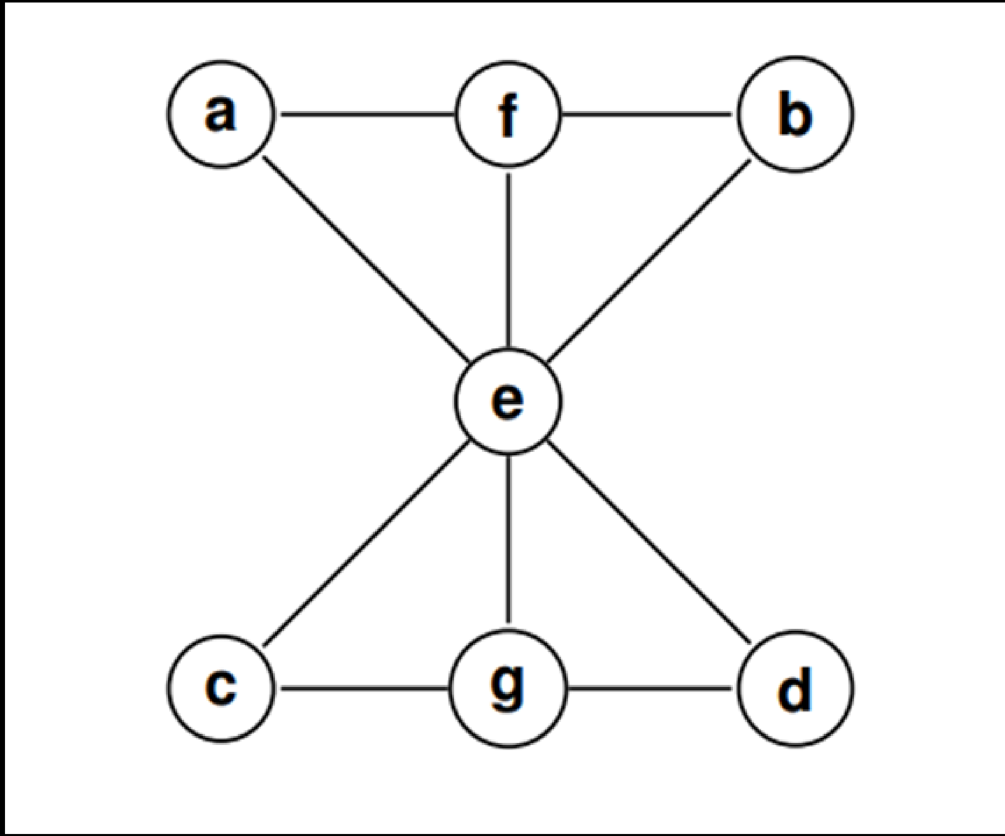




Find the union of the given pair of simple graphs.

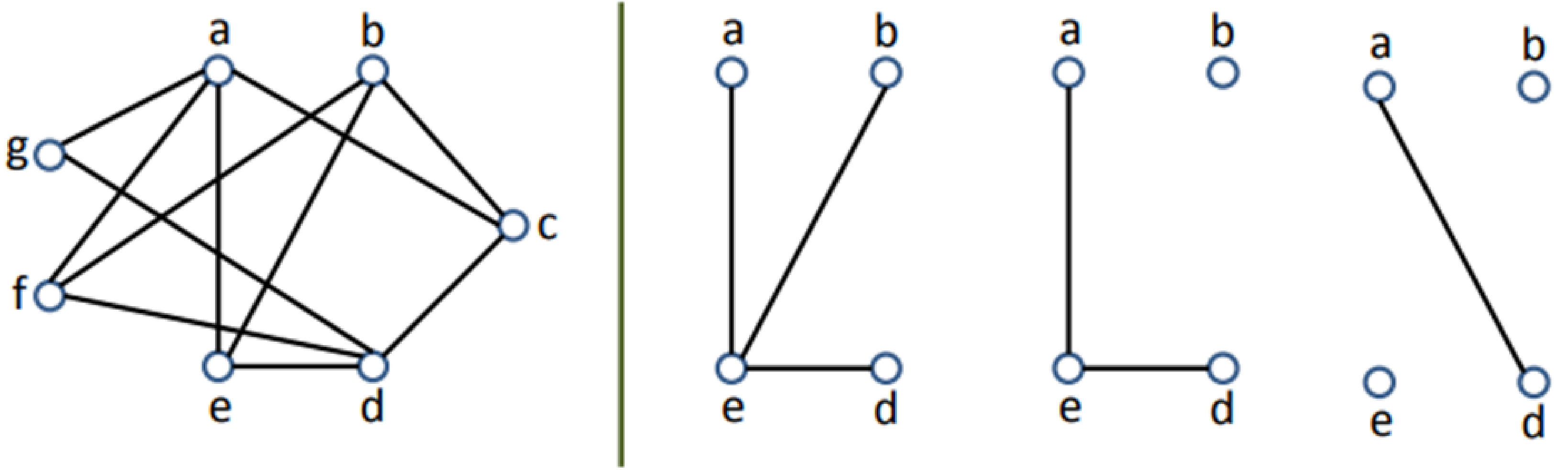
$\Rightarrow$




a
f
b
e
c
g
d

If G = (V, E) is a graph, then G' = (V', E') is called a subgraph of G if $V' \subseteq V$ and $E' \subseteq E$.

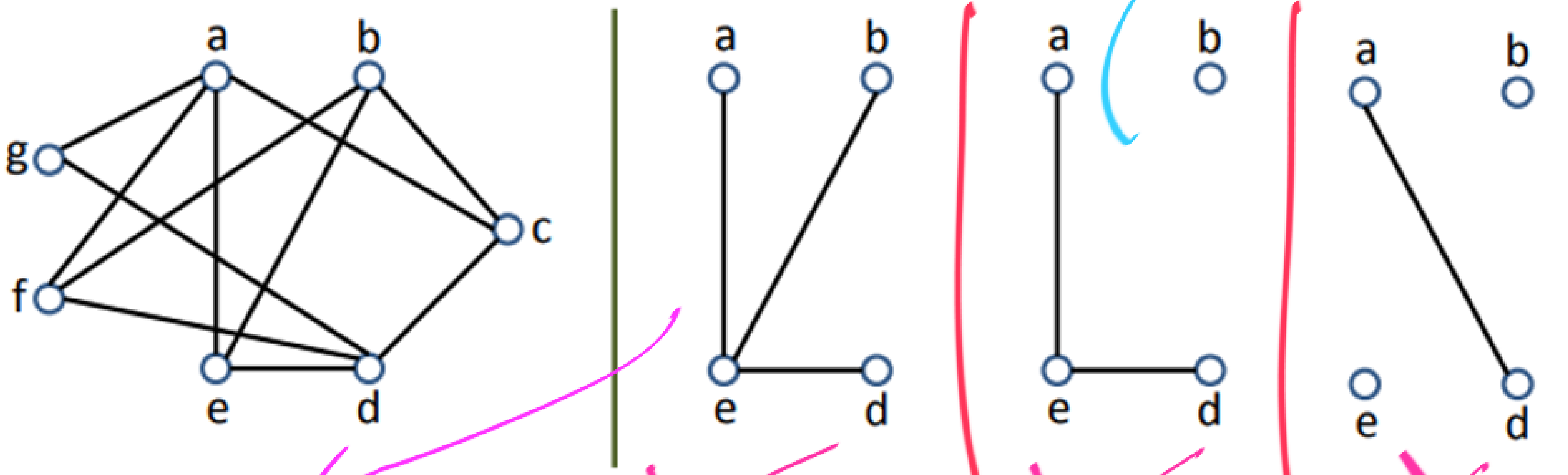
- Which one is a subgraph of the leftmost graph G ?



If G = (V, E) is a graph, then G' = (V', E') is called a **subgraph** of G if $V' \subseteq V$ and $E' \subseteq E$.

NOT Induced

- Which one is a subgraph of the leftmost graph G ?

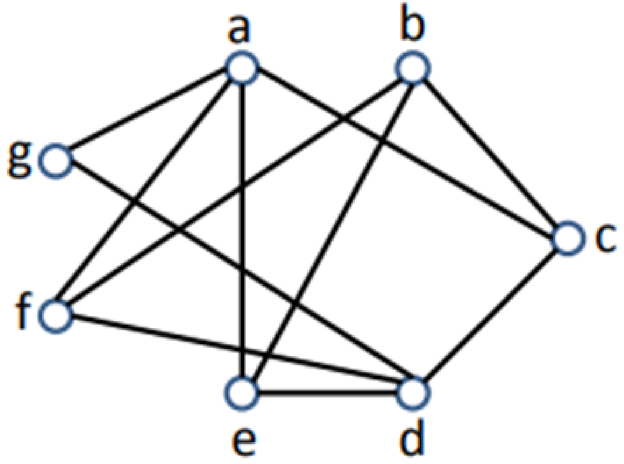


induced by a,b,e,d

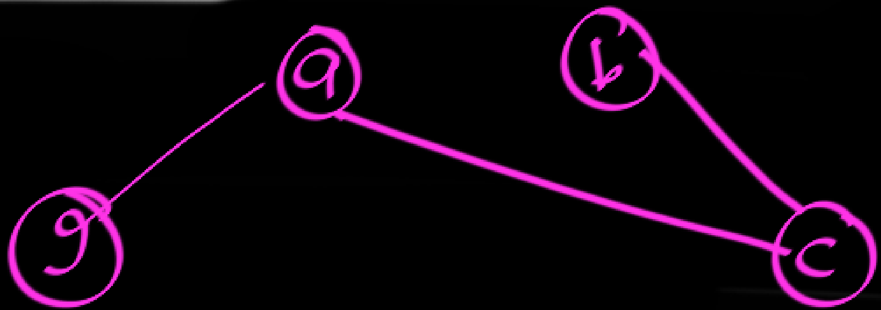
If G = (V, E) is a graph, then the subgraph of G induced by $U \subseteq V$ is a graph with the vertex set U and contains exactly those edges from G with both endpoints from U

Ex : Consider the graph on the right side

What is its subgraph induced by the vertex set { a, b, c, g } ?

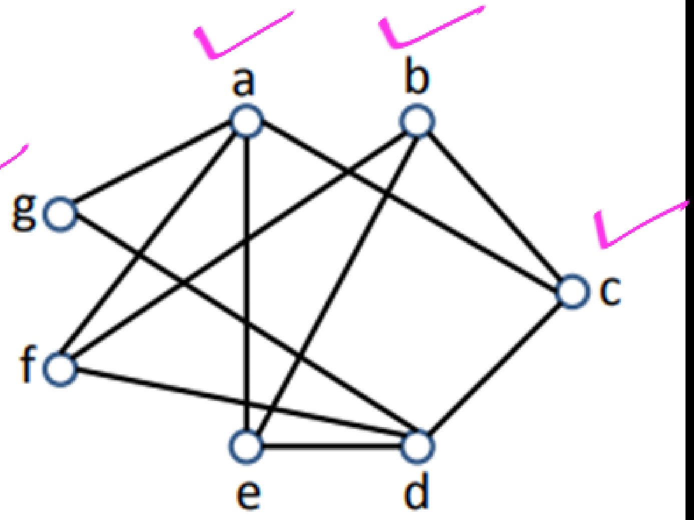


[redacted]



Ex : Consider the graph on the right side

What is its subgraph induced by the vertex set { a, b, c, g } ?



2.4.6 Graph Connectivity: GATE1994-2.5

The number of edges in a regular graph of degree d and n vertices is ____________

2.4.6 Graph Connectivity: GATE1994-2.5

The number of edges in a regular graph of degree d and n vertices is ______________

$$|E| = \frac{nd}{2}$$

2.4.12 Graph Connectivity: GATE2004-IT-37

What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree $2, 3$ vertices of degree 4 and remaining of degree 3?

A. 10 B. 11 C. 18 D. 19

2.4.12 Graph Connectivity: GATE2004-IT-37

What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

A. 10 B. 11 C. 18 D. 19

$n ? = 6 + 3 + t = 19$

t

Total Degree $= t(3) + 3(4) + 6(2) = 24 + 3t$

$24 + 3t = 2 \times 27 = 54$

$3t = 30 \Rightarrow t = 10$

2.4.29 Graph Connectivity: TIFR2019-B-3

A graph is d – regular if every vertex has degree d. For a d – regular graph on n vertices, which of the following must be TRUE?

A. d divides n
B. Both d and n are even
C. Both d and n are odd
D. At least one of d and n is odd
E. At least one of d and n is even

2.4.29 Graph Connectivity: TIFR2019-B-3

A graph is d – regular if every vertex has degree d. For a d – regular graph on n vertices, which of the following must be TRUE?

A. d divides n — may or may not

B. Both d and n are even

C. Both d and n are odd — NEVER

D. At least one of d and n is odd

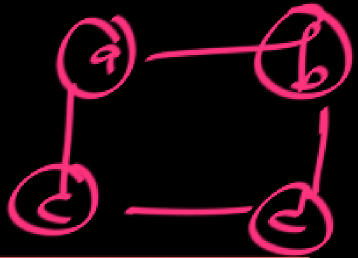
E. At least one of d and n is even

$$nd = 2|E| = \text{even}$$

$$\boxed{n \times d = \text{even}}$$

2 Regular

$d = 2, n = 3$



$d = 2$

$n = 4$