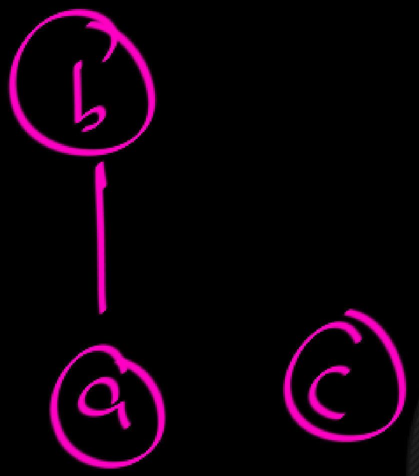
Partial Order Relations

Next Topic:

Finding Cardinality of a POR from Hasse Diagram

Website : https://www.goclasses.in/



$|\text{Base set}| = 3$

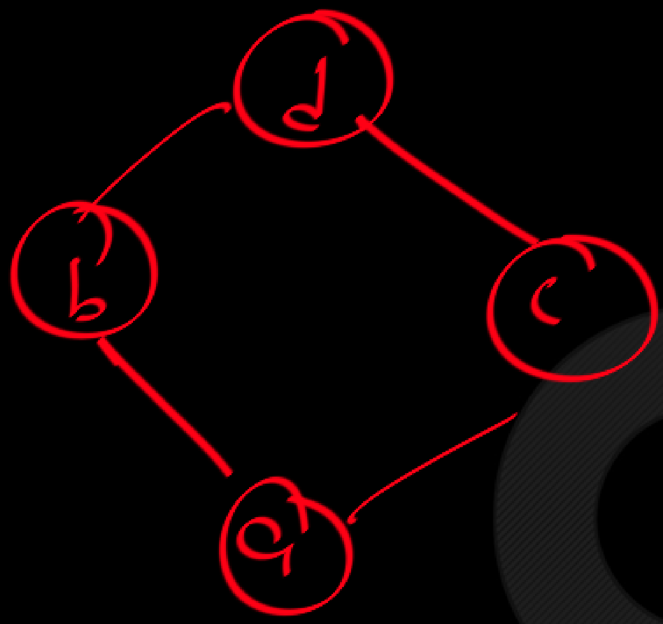
$|\text{PoR}| = 4$ ✓

$\text{PoR} = \{(a,a),(b,b),(c,c),(a,b)\}$

HD

Representation

#Edges = 1



HD of

POR R

$$|R| = ? = \underbrace{(a,x)}_{4} + \underbrace{(b,x)}_{2} + \underbrace{(c,x)}_{2} + 1$$

$$= \{(a,a), (b,b), (c,c), (d,d), (a,b), (a,c), (a,d), (b,d), (c,d)\}$$

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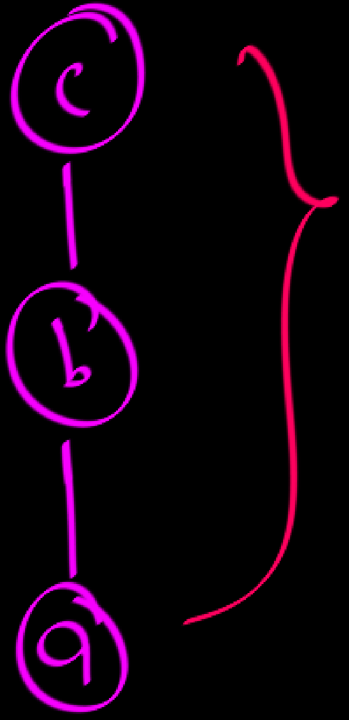


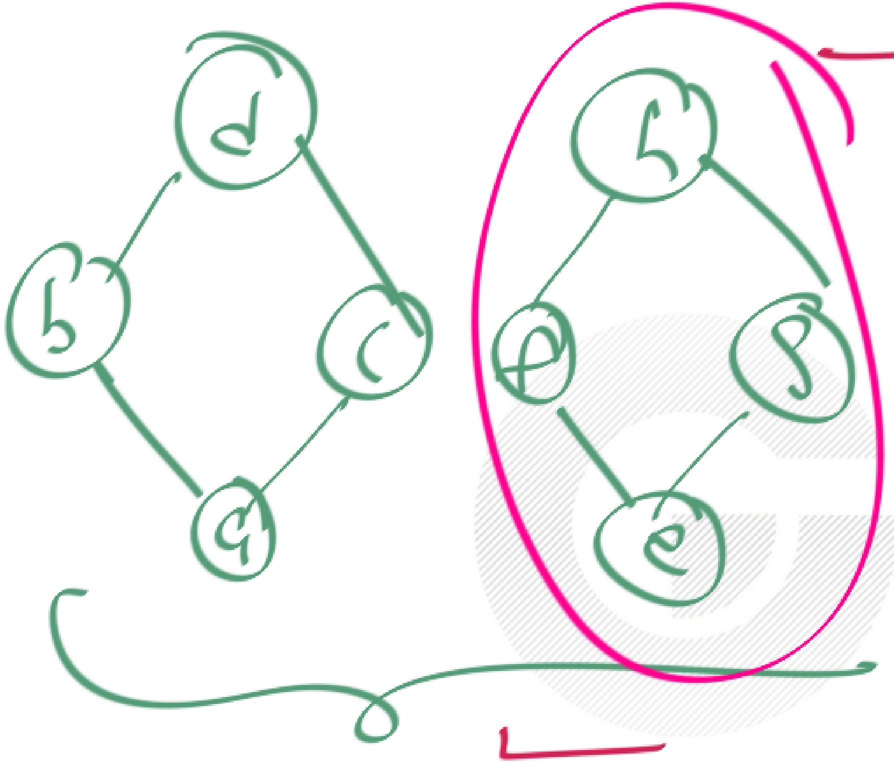
HD of R

$|R| = ? = \underset{3}{(a,x)} , \underset{2}{(b,x)} , \underset{1}{(c,x)}$

$3 + 2 + 1$

$\underline{6}$

$R = \{(a,a), (a,b), (a,c), (b,b), (b,c), (c,c)\}$



poset

Base set = {a, b, c, d, e, f, g, h}

$|POR| = ? = 8 + 3 + 1 + 1$

18

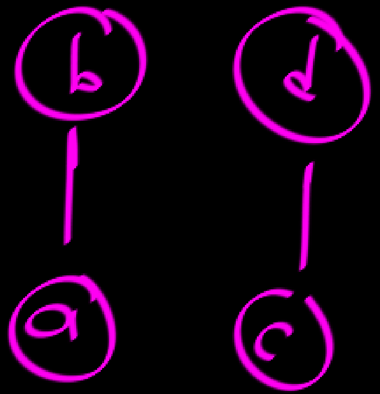
Ref Pairs

(a,x) (b,x) (c,x)

$+ 5$

Not lattice

$d \vee h = DNE$



HD of R_1

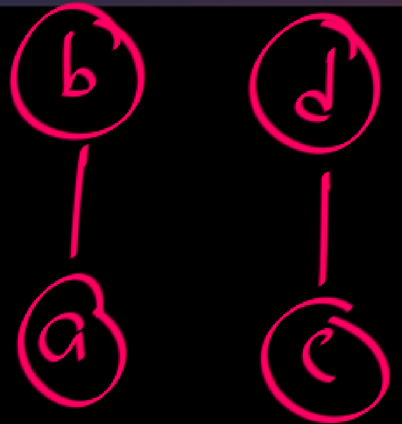
POR

$$R_1 = \{(a,a), (b,b), (a,b), (c,c), (d,d), (c,d)\}$$

Base set: $\{a, b, c, d\} = A$

$$\underbrace{(A, R_1) = R_1(A)}_{\text{Poset}}$$

$$|R_1| = 6$$



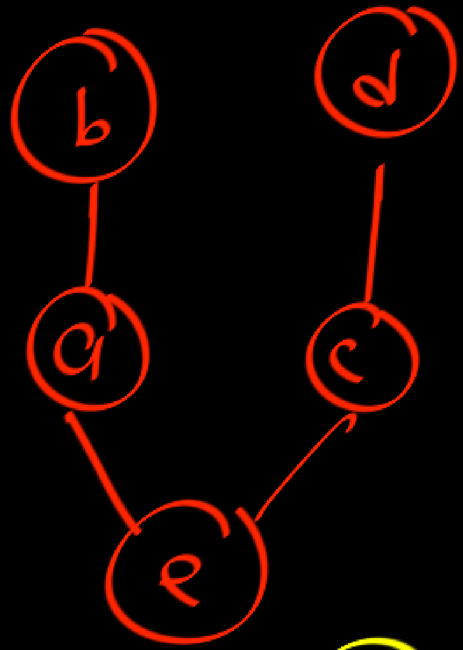
HD of R_1

$|R_1| = \underbrace{(a,x)}_{2}, \underbrace{(b,x)}_{1}, \underbrace{(c,x)}_{2}, \underbrace{(d,x)}_{1}$

(a,a) (b,b) (c,c) (d,d)

(a,b) (c,d)

$|R_1| = 6$



Base set of $R = \{a, b, c, d, e\}$

$|\text{Base set}| = 5$

#Edges in HD = 4

$\forall x_i \in R_x$

least

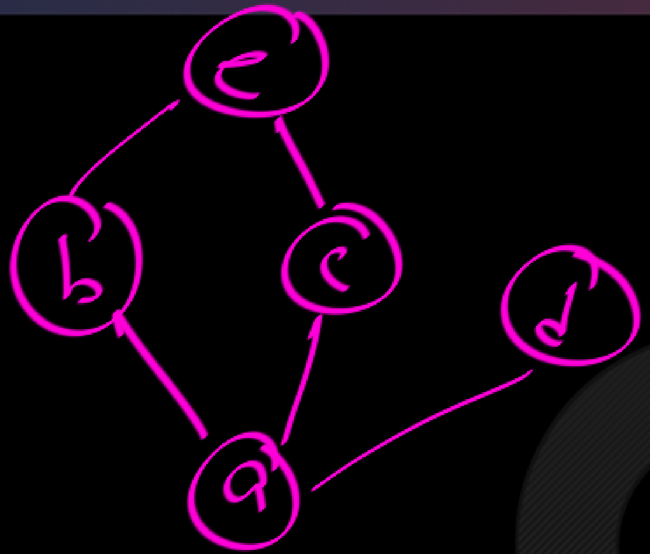
HD of R

POR

$(e, x), (a, x), (b, x), (c, x), (d, x)$

$|R| = 5 + 2 + 1 + 2 + 1$

$= 11$



#Edges in HD = 5

$|R_2| = \underset{5}{(a,x)} + \underset{2}{(b,x)} + \underset{2}{(c,x)} + \underset{1}{(e,x)} + \underset{1}{(d,x)}$

$|R_2| = 11$

HD of R_2

POR ← Ref, Transitive, Antisym

(a, x), ∀x → least

Q: 3 element poset such that cardinality of POR is maximum possible; so $|POR| = ?$

Base set $A = \{a, b, c\}$

POR on A; maximum possible cardinality of POR?

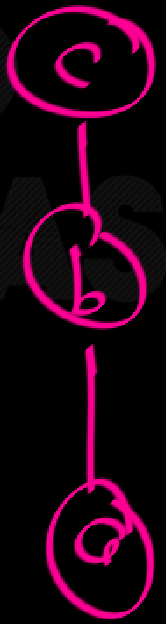
Q: 3 element poset such that cardinality of POR is maximum possible; so

$|POR| = ?$ $\Rightarrow$ 6

$|POR| =$ (a, x), (b, x)

3 + 2 + 1

= 6

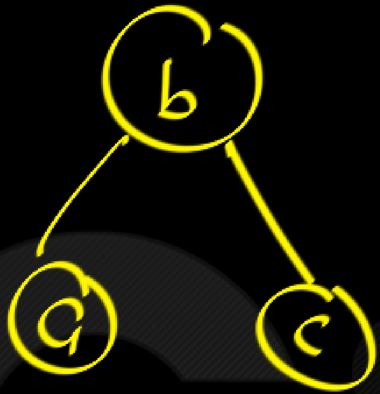


Chain

Every pair of elements is comparable.

HD of R_1

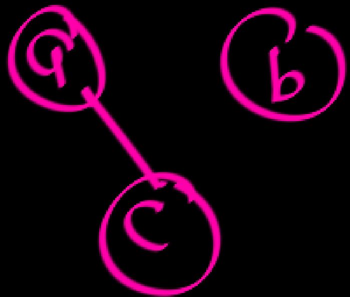
$|R_1| = 3$



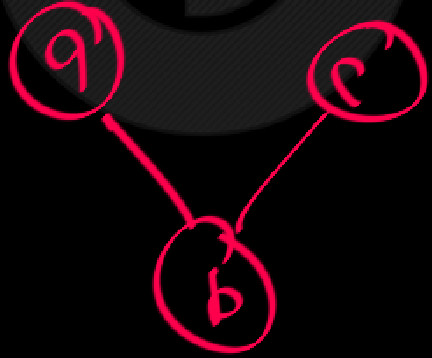
$|R_3| = 5$

$|R_4| = 3+2+1 = 6$

HD of R_2

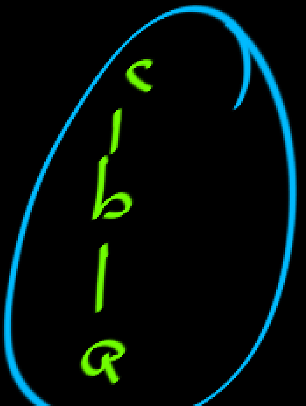


$|R_2| = 4$



$|R_5| = 5$

Conclusion: on "n" elements, the Largest possible POR is a Chain (TOR).



Q: On a set A of n elements, is Full Relation a POR?

$R = \boxed{A \times A}$ on A

full Relation

$|\text{Base set}| = n$

Q: On a set A of n elements, is Full Relation a POR?

$R = \boxed{A \times A}$ on A

full Relation

No.

POR ✗

Eq. Rel

$A = \{1, 2\}$

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$ – NOT Antisym

If set A, $|A| \geq 2$ then

$A \times A$ is NEVER POR

$A \times A$ is Always Eq. Rel, for A.

#Eq. Classes $= 1$

Q: set A, $|A| = n$, $A \times A \Rightarrow$ POR

then $|A|$? $\Rightarrow$ 0 or 1

$A = \{a\}$; $A \times A = \{(a,a)\}$ — Ref — Antisym — Trans

POR ✓

If $|A| = 1$ then $A \times A \Rightarrow$ POR ✓

If $|A| = 0$ then $A \times A \Rightarrow$ POR ✓

Note: If Base set is Empty; $A = \phi$

on ϕ, Define R.

$R = \phi$

on ϕ,

Every Relation $= \phi$

Every Relation on ϕ:

1. Ref ✓
2. Sym ✓
3. Antisym ✓

Trans ✓

Asym ✓

Irref ✓

If Domain is Empty :

$$\forall_x \boxed{\alpha} \equiv \text{True}$$

$$\exists_x \boxed{\alpha} = \text{false}$$

When Base set Empty

Ref: $\forall_x (xRx)$ — True

Sym: $\forall_{x,y} (xRy \to yRx)$ — True

Antisym: $\forall_{x,y} ((xRy \wedge yRx) \Rightarrow x=y)$ — True

Irreflexive: $\forall_x (x \not R x)$ — True

When Base set Empty

Ref: $\forall_x (xRx)$ — True

Trans: $\forall_{x,y,z} \big((xRy) \wedge (yRz) \rightarrow (xRz) \big)$ — True

Asym: $\forall_{x,y} \big(xRy \longrightarrow y \cancel{R} x \big)$ — True

Q: On a non-empty set A, Assume Full Relation a POR, then what is $|A|$? $\Rightarrow$ 1 ✓

$|A| = 1$

$A = \{a\}$

$A \times A = \{(a,a)\}$

Refl ✓

Sym ✓

Antisym ✓

Trans ✓

Eq Rel ✓

POR ✓

Q: Empty Relation on Non-Empty set

Ref	Irref
Sym	
Antisym	
Asym	
Trans	

Q: Empty Relation on Non-Empty set

- Ref ✗
- Sym ✓
- Antisym ✓
- Asym ✓
- Trans ✓

- Irref ✓

Q: On a set A of n elements, what is the largest and smallest partial order relations that can be defined?

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$$|\text{largest possible POR}| = \frac{n(n+1)}{2}$$

$$|\text{smallest possible POR}| = n$$ ✓

Smallest Possible POR on a set A:

Identity Relation

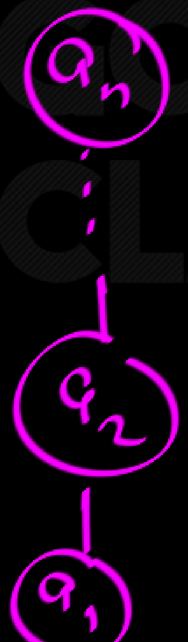
$R = \{(a,a) \mid a \in A\}$ — Smallest possible POR

POR
- Ref — All Ref pairs in POR
- Antisym
- Trans

$A = \{a_1, a_2, a_3, ---, a_n\}$ — Base set

Hasse Diagram of Largest possible POR :

= Chain = TOR



$\Rightarrow |POR| = ?$

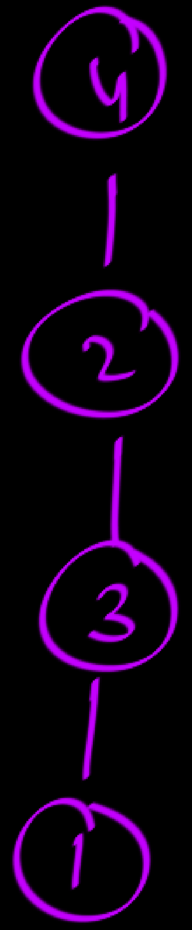
HD of Relation R ; $|R| = ?$

$$(a_1, x) \quad (a_2, x) \quad (a_n, x)$$

$$|R| = n + n-1 + \dots + 1$$

$$|R| = \frac{n(n+1)}{2}$$

Cardinality of Largest possible POR on n elements.



HD of R

$$|R| = \frac{n(n+1)}{2} = \frac{4(5)}{2} = 10$$

$$|R| = \underset{4}{\underset{\downarrow}{(1,x)}} + \underset{3}{\underset{\downarrow}{(3,x)}} + \underset{2}{\underset{\downarrow}{(2,x)}} + \underset{1}{\underset{\downarrow}{(4,x)}}$$

$$= 10$$

Cardinality of largest POR on 4 elements.