Complement of a Number

Subtraction using r's Complement

Recap :

Number System

Binary, Decimal Number Systems

Base b representation

- Interpreting a string of digits in base b
- Every number can be written uniquely in any base
- Converting to base b

Working in base b

Base b representation is a way to write numbers using the digits $\{0, 1, \ldots, (b-1)\}$.

Common bases:

- you use base 10 (**decimal**) every day (digits are $\{0, 1, \ldots, 9\}$)
- base 2 (**binary**) uses digits $\{0, 1\}$. It is convenient for digital logic, a digit (called a **bit**) can be represented using a single wire: the wire has high voltage for 1, low for 0. Binary numbers are often designated by a trailing b: for example 1101b.
- base 16 (**hexadecimal**) uses the digits $\{0, 1, 2, \ldots, 9, A, B, C, D, E, F\}$. it is useful becausee a single digit can be represented using 4 bits. Hex numbers are often written with a prefix of "0x": for example 0xFC39.
- base 8 (**octal**) uses the digits $\{0, 1, 2, \ldots, 7\}$, and is occasionally used when 3-bit numbers are useful.

A string of digits in base b, written $(a_n a_{n-1} \ldots a_3 a_2 a_1 a_0)_b$, represents the number $a_0 b^0 + a_1 b^1 + a_2 b^2 + \cdots + a_n b^n$.

Next Topic :

Subtraction

Life would be so good if we did not have to borrow.

Math Review: Subtraction

$$
\begin{array}{r}
39 \\
-\ 15 \\
\hline
??
\end{array}
$$

24

Math Review: Subtraction

No need to borrow

$$\begin{array}{r} 82 \\ -\ 61 \\ \hline 21 \end{array} \qquad \begin{array}{r} 48 \\ -\ 26 \\ \hline 22 \end{array} \qquad \begin{array}{r} 32 \\ -\ 11 \\ \hline 21 \end{array}$$

Math Review: Subtraction

Need to borrow

$$\begin{array}{r} 82 \\ -\ 64 \\ \hline 18 \end{array} \qquad \begin{array}{r} 48 \\ -\ 29 \\ \hline 19 \end{array} \qquad \begin{array}{r} 32 \\ -\ 13 \\ \hline 19 \end{array}$$

When can you Guarantee that you don't need to Borrow while subtraction?

When can you Guarantee that you don’t need to Borrow while subtraction?

When we subtract a number from ALL 9’s.

Another Way to Do Subtraction

$$82 - 64 =$$

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

Another Way to Do Subtraction

$$82 - 64 =$$

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + \underbrace{(99 - 64)}_{\text{No Need to Borrow}} - 100 + 1$$

$$82 - 64 = \underbrace{82 + 35 + 1}\ - 100$$

$$= (118 - 100) \equiv \left(\text{Discarding end Carry from 118}\right)$$

End Carry

$$= 18$$

When can you Guarantee that you don't need to Borrow while subtraction? **When we subtract a number from ALL 9's.**

Complement Operation that we will study now, will help us with Subtraction by using this same idea.

Digital Logic :

Next Topic :

Complement of a number in Base r

r's, (r-1)'s Complement

Radix / Base r $(r \geq 2)$

$(r-1)$'s Complement

(Diminished Radix Complement)

r's Complement

(Radix Complement)

Radix / Base 10

9's Complement
(Diminished Radix Complement)

10's Complement
(Radix Complement)

Radix / Base 2

1's Complement
(Diminished Radix Complement)

2's Complement
(Radix Complement)

for N in Base r $(r \geq 2)$

n Digits

$(r-1)$'s Comp

$\underbrace{(r^n - 1)}$ $- N$

r's Comp

$\underbrace{r^n - N}$

Base $\underline{10}$: r

$N = \underline{\underline{82}} \rightarrow n = 2$

9's Comp of N : $(10^2 - 1) - N = 99 - 82$
$= \underline{\underline{17}}$

10's Comp of N :

$\rightarrow 10^2 - N = 100 - 82 = \underline{\underline{18}}$

Base 10 :

$N = 2301 \rightarrow n = 4$

9's Comp of N : $(10^4 - 1) - N = 9999 - 2301$
$= 7698$

10's Comp of N : (9's Comp of N) + 1
$= 7699$

Base 10 :

$N = 082 \rightarrow n = 3$

9's Comp of N : $(10^3 - 1) - N = 999 - 082$
$= 917$

10's Comp of N : $917 + 1 = \underline{\underline{918}}$

Base r :

Number = N

Number of Digits in N

Two "operations" on N :

① $(r-1)$'s Comp op^n of $N = (r^n - 1) - N$

② r's " " of $N = r^n - N$

Base 10 :

n Digits

9's Complement of N :

$$\underbrace{999\cdots9}_{n\text{ times}} - N$$

10' Complement of N : (9's Comp of N) + 1

Base 2 :

n Digits

1's Complement of N :

$$\underbrace{11111\ldots1}_{n \text{ times}} - N$$

2' Complement of N : (1's Comp of N) + 1

1's Comp of (01011101) =

$$
\begin{array}{ccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
\end{array}
$$

Base 2:

n Digits

1's Complement of N:

Complement every bit

2' Complement of N: (1's Comp of N) + 1

2's Comp of $\overbrace{0\,1\,1\,0\,1\,0}\boxed{1\,0\,0}$

1's Comp

$$\begin{array}{r} 1\,0\,0\,1\,0\,1\,0\,1\,1 \\ +1 \\ \hline 1\,0\,0\,1\,0\,1\,\boxed{1\,0\,0} \\ \hline \end{array}$$

2' Comp of N :

N = 1011101110

N = 101111011

2's Comp

0100010010

010000011

10's Comp of 8201129600

9's Comp

1798870399

+1

1798870400

10' Comp:

$$N = \underbrace{e\ d\ c\ b}\ a\ \underbrace{0\ 0\ 0}$$

$$(9-e)(9-d)(9-c)(9-b)(10-a)000$$

COMPLEMENTS OF NUMBERS

Complements are used in digital computers to **simplify the subtraction operation** and for logical manipulation. Simplifying operations leads to simpler, less expensive circuits to implement the operations. There are two types of complements for each base-r system: the radix complement and the diminished radix complement. The first is referred to as the r's complement and the second as the $(r - 1)$'s complement. When the value of the base r is substituted in the name, the two types are referred to as the 2's complement and 1's complement for binary numbers and the 10's complement and 9's complement for decimal numbers.

Diminished Radix Complement

Given a number N in base r having n digits, the $(r - 1)$'s complement of N, i.e., its diminished radix complement, is defined as $(r^n - 1) - N$. For decimal numbers, $r = 10$ and $r - 1 = 9$, so the 9's complement of N is $(10^n - 1) - N$. In this case, 10^n represents a number that consists of a single 1 followed by n 0's. $10^n - 1$ is a number represented by n 9's. For example, if $n = 4$, we have $10^4 = 10{,}000$ and $10^4 - 1 = 9999$. It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9. Here are some numerical examples:

The 9's complement of 546700 is $999999 - 546700 = 453299$.

The 9's complement of 012398 is $999999 - 012398 = 987601$.

For binary numbers, $r = 2$ and $r - 1 = 1$, so the 1's complement of N is $(2^n - 1) - N$. Again, 2^n is represented by a binary number that consists of a 1 followed by n 0's. $2^n - 1$ is a binary number represented by n 1's. For example, if $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$. Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1. However, when subtracting binary digits from 1, we can

have either $1 - 0 = 1$ or $1 - 1 = 0$, which causes the bit to change from 0 to 1 or from 1 to 0, respectively. Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.** The following are some numerical examples:

The 1's complement of 1011000 is 0100111.

The 1's complement of 0101101 is 1010010.

The $(r - 1)$'s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

Radix Complement

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Comparing with the $(r - 1)$'s complement, we note that the r's complement is obtained by adding 1 to the $(r - 1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$. Thus, the 10's complement of decimal 2389 is $7610 + 1 = 7611$ and is obtained by adding 1 to the 9's complement value. The 2's complement of binary 101100 is $010011 + 1 = 010100$ and is obtained by adding 1 to the 1's-complement value.

Since 10 is a number represented by a 1 followed by n 0's, $10^n - N$, which is the 10's complement of N, can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9. Thus,

the 10's complement of 012398 is 987602

and

the 10's complement of 246700 is 753300

The 10's complement of the first number is obtained by subtracting 8 from 10 in the least significant position and subtracting all other digits from 9. The 10's complement of the second number is obtained by leaving the two least significant 0's unchanged, subtracting 7 from 10, and subtracting the other three digits from 9.

Similarly, the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits. For example,

the 2's complement of 1101100 is 0010100

and

the 2's complement of 0110111 is 1001001

The 2's complement of the first number is obtained by leaving the two least significant 0's and the first 1 unchanged and then replacing 1's with 0's and 0's with 1's in the other four most significant digits. The 2's complement of the second number is obtained by leaving the least significant 1 unchanged and complementing all other digits.

Complement

- Complementing is an operation on base-r numbers
- Goal: To simplify subtraction operation
  – Rather turn the subtraction operation into an addition operation
- Two types
  1) Radix complement (r's complement)
  2) Diminished complement ((r-1)'s complement)
- When r = 2
  1) 2's complement
  2) 1's complement

How to Complement?

- A number M in base-r (n-digit)
  - $r^n - M$ r's complement
  - $(r^n-1) - M$ (r-1)'s complement

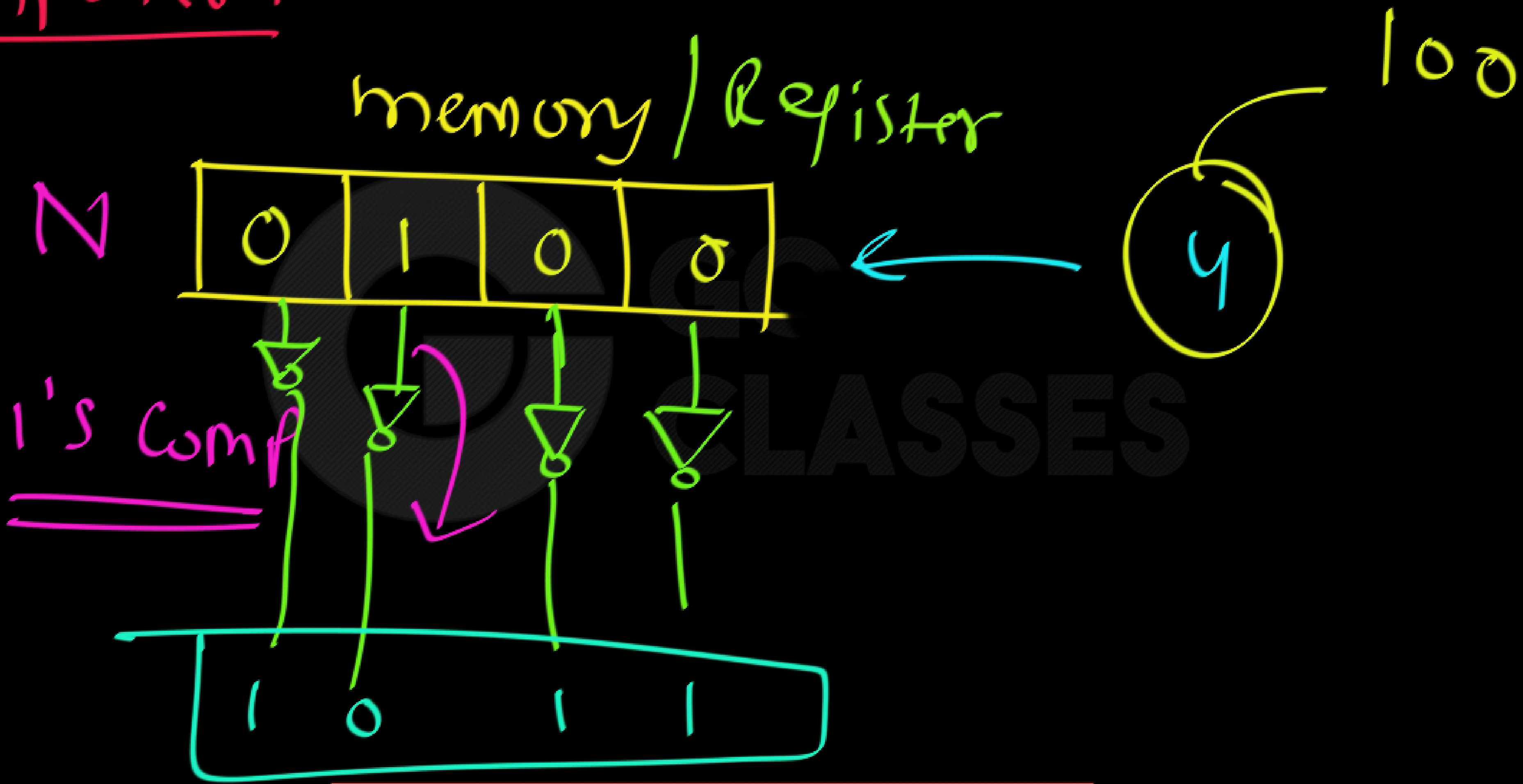
  where n is the number of digits we use
- Example: Base r = 2, #Digits n = 4, Given M = 7
  - $r^n = 2^4 = 16$, $r^n - 1 = 15$.
  - 2's complement of 7 → 9
  - 1's complement of 7 → 8
- **Easier way** to compute 1's and 2's complements
  - Use binary expansions
  - 1's complement: negate
  - 2's complement: negate + increment

Better expressed this way

1's complement: flip

2's complement: flip + 1

Computer:



How to Complement?

- 10’s complement of 9 is 0+1=1
- 10’s complement of 09 is 90+1=91
- 10’s complement of 009 is 990+1=991
- 9’s complement of 9 is 0
- 9’s complement of 09 is 90
- 9’s complement of 009 is 990

- 2’s complement of 100 is 011+1=100
- 2’s complement of 111 is 000+1=001
- 2’s complement of 000 is 000
- 1’s complement of 11110001 is 00001110

Digital Logic :

Next Topic :

Subtraction of unsigned numbers using r's Complement

M – N

unsigned

Case 1: M >= N

Case 2: M < N

M – N

Case 1: M >= N

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

Another Way to Do Subtraction

$$82 – 64 = 82 + 100 – 100 - 64$$

$$= 82 + (100 – 64) - 100$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

$$= 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

Does not require borrows

$$= 82 + (99 - 64) + 1 - 100$$

9’s Complement
(subtract each digit from 9)

$$\begin{array}{r} 99 \\ -\ 64 \\ \hline 35 \end{array}$$

10’s Complement

(subtract each digit from 9 and add 1 to the result)

$$\begin{array}{r} 99 \\ -\ 64 \\ \hline 35 + 1 = 36 \end{array}$$

Another Way to Do Subtraction

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

$$= 82 + 35 + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

$$= \boxed{82 + 36} - 100$$

// add the first two

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

$$= 82 + 36 - 100$$ // add the first two

$$= 118 - 100$$ // delete the leading 1

$$= 18$$

End Carry

Discard the end Carry

$$\underbrace{\overset{M}{82} - \overset{N}{64}}$$

$$\underbrace{M \geq N}$$

using 10's Comp :

$$82 + \boxed{(100 - 64)} \underline{\underline{-100}}$$

Discard

$$= \left(\underline{\underline{82 + r\text{'s Comp of } 64}}\right)$$

End Carry

M − N

Case 1: M >= N

$$\left.\begin{array}{r} M \\ -\ N \\ \hline \\ \hline \end{array}\right\} \rightarrow \begin{array}{r} M \\ +\ (r\text{'s Comp of } N) \\ \hline \boxed{\text{Result}} \\ \hline \end{array}$$

end Carry → Discard

$\overset{M}{28} - \overset{N}{19}$

$\underbrace{M \geq N}$

Using 10's Comp:

$28 + (10\text{'s Comp of } 19)$

$= 28 + (81) = 109$

Discard

$= 09$

$M - N$

unsigned

Case 1: $M \geq N$

End Carry

$$\begin{array}{r} M \\ - N \\ \hline ? \end{array}$$

$$\begin{array}{r} M \\ + (r\text{'s Comp of } N) \\ \hline 1 \ \boxed{\text{Result}} \end{array}$$

End Carry Discarded

$M - N$

Case 2: $M < N$

$$\overset{M}{20} - \overset{N}{78} = 20 + 100 - 100 - 78$$

$$= 20 + (100 - 78) - 100$$

$$= \boxed{20 + 22} - 100$$

$$= \boxed{42 - 100} = -(100 - 42) = -\left(\begin{array}{c}\text{10's} \\ \text{Comp} \\ \text{of} \\ 42\end{array}\right)$$

$$\begin{array}{r} 20 \\ -78 \\ \hline -58 \\ \hline \end{array} \longrightarrow \begin{array}{r} 20 \\ +\,(10\text{'s Comp of } 78) \\ \hline \hline \end{array}$$

$$\begin{array}{r} 20 \\ +\ 22 \\ \hline 42 \\ \hline \end{array} \longrightarrow -\,(10\text{'s Comp of } 42) = -(58)$$

final Result

Case 2: $\underbrace{M < N}$ → No End Carry

$$\left.\begin{array}{r} M \\ -N \\ \hline \\ \hline \end{array}\right\} \rightarrow \begin{array}{r} M \\ +(r\text{'s Comp of } N) \\ \hline \text{Result} \\ \hline \end{array}$$

→ Correct Answer: $-$(r's Comp of Result)

$M - N$

Case 2: $M < N$

$$\begin{array}{r} 34 \\ -\ 98 \\ \hline -\ 64 \\ \hline \end{array}$$

34

$+$ (10's Comp of 98)

$$\begin{array}{r} 34 \\ +\ 02 \\ \hline 36 \\ \hline \end{array}$$

Correct Answer

$-$ (10's Comp of 36)

Subtraction using r's Comp :
of Unsigned numbers (M, N)

$\boxed{M - N}$ Target

1) $M + (\text{r's Comp of } N) = \underbrace{M + (r^n - N)}$

① $\underbrace{(M-N)+r^n}$

Case 1: End Carry exist $\Longleftrightarrow$ $M \geq N$

Correct Answer: Just Discard Carry

You have your correct Answer.

① $(M - N) + r^n$

Case2 : End Carry Does not exist $\iff M < N$

Correct Answer : $-(r\text{'s Comp of result})$

Subtraction with Complements

The direct method of subtraction taught in elementary schools uses the borrow concept. In this method, we borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit. The method works well when people perform subtraction with paper and pencil. However, when subtraction is implemented with digital hardware, the method is less efficient than the method that uses complements.

The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Eg: $(\underset{M}{239}) - (\underset{N}{82})$ (using 10's Comp)

$$\begin{array}{r} 239 \\ -\ \ 82 \\ \hline ? \\ \hline \end{array} \quad \begin{array}{r} 239 \\ -\ 082 \\ \hline \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 239 \\ +\ 918 \\ \hline 1157 \\ \hline \end{array}$$

$M \geq N$ ⟷ end Carry

Correct Answer 157 ✓

EXAMPLE 1.5

Using 10's complement, subtract $72532 - 3250$.

$$\begin{aligned} M &= \quad 72532 \\ \text{10's complement of } N &= + \underline{\ 96750} \\ \text{Sum} &= \quad 169282 \\ \text{Discard end carry } 10^5 &= - \underline{100000} \\ Answer &= \quad 69282 \end{aligned}$$

Note that M has five digits and N has only four digits. Both numbers must have the same number of digits, so we write N as 03250. Taking the 10's complement of N produces a 9 in the most significant position. The occurrence of the end carry signifies that $M \geq N$ and that the result is therefore positive.

Binary Subtraction (using 2's Comp)

Unsigned Numbers:

$M = 1111010$

$N = 1101011$

M − N using 2's Comp:

$$
\begin{array}{r}
1111010 \\
-\;1101011 \\
\hline
\\
\hline
\end{array}
\qquad
\begin{array}{r}
1111010 \\
+\;0010101 \\
\hline
1\;\boxed{0001111}
\end{array}
$$

End Carry

Discard

Correct Answer

$M \geq N \longleftrightarrow$ End Carry

M−M using 2's Comp:

$$\begin{array}{rcccccccc} N & 1 & 1 & 0 & 1 & 0 & 1 & 1 & \\ - & 1 & 1 & 1 & 1 & 0 & 1 & 0 & M \\ \hline \end{array}$$

$$\begin{array}{rccccccc} & & 1 & & 1 & 1 & & \\ & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ + & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ \hline \end{array}$$

N<M ⟺ No end Carry

Correct Answer:

−(0 0 0 1 1 1 1)

EXAMPLE 1.7

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

(a)

$$
\begin{aligned}
X &= 1010100 \\
\text{2's complement of } Y &= + \underline{0111101} \\
\text{Sum} &= 10010001 \\
\text{Discard end carry } 2^7 &= - \underline{10000000} \\
\textit{Answer}: X - Y &= 0010001
\end{aligned}
$$

(b)

$$
\begin{aligned}
Y &= 1000011 \\
\text{2's complement of } X &= + \underline{0101100} \\
\text{Sum} &= 1101111
\end{aligned}
$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{2's complement of } 1101111) = -0010001$.

Digital Logic :

Next Topic :

Subtraction of unsigned numbers using (r-1)'s Complement

M − N using 9's Complement

$$82 - 24 = 82 + (100 - 24) - 100$$

$$= 82 + (99 - 24) + 1 - 100$$

$$= \boxed{82 + (9\text{'s Comp of } 24)} + 1 - 100$$

Result

$$\begin{array}{r} 82 \\ -24 \\ \hline \end{array}$$

$$\begin{array}{r} 82 \\ +75 \\ \hline 157 \\ +1 \\ \hline 58 \\ \hline \end{array}$$

End Carry

→ Correct Answer

M − N using (r − 1)'s Comp :

$$\left.\begin{matrix} M \\ -N \end{matrix}\right\} \Rightarrow \begin{array}{l} \quad M \\ + (r-1)\text{'s Comp of } N \\ \hline \text{Result} \\ \hline \end{array}$$

If

(1) $m \geq N \Longleftrightarrow$ End Carry

Correct Answer : $\boxed{\text{Discard Carry, Add +1 to the Result}}$

$\Updownarrow$

$\boxed{\text{Add the end Carry to Result}}$

(2) If $\underbrace{M < N}$: $\Longleftrightarrow$ No end Carry

Correct Answer:

$-\left(\underline{(r-1)}\text{'s Comp of Result}\right)$

Eg: 24 − 82 using 9's Comp

M N

$$\begin{array}{r} 24 \\ -82 \\ \hline ? = -58 \\ \hline \end{array} \rightarrow \begin{array}{r} 24 \\ +17 \\ \hline 41 \\ \hline \end{array}$$

No End Carry $\Longleftrightarrow$ $M < N$

Correct Answer: −(9's comp of 41)

= −(58)

Understand:

$$24 - 82 = 24 + (100 - 82) - 100$$

$$= \boxed{24 + (99 - 82)} \underbrace{+ 1 - 100}$$

$$= \underline{\text{Result}} - 99 = \boxed{-(\underbrace{99 - \text{Result}})}$$

Correct Answer

<u>Correct Answer</u> = $-(99 - \text{Result})$

$= -(9\text{'s comp of Result})$

Subtraction of unsigned numbers can also be done by means of the $(r - 1)$'s complement. Remember that the $(r - 1)$'s complement is one less than the r's complement. Because of this, the result of adding the minuend to the complement of the subtrahend produces a sum that is one less than the correct difference when an end carry occurs. Removing the end carry and adding 1 to the sum is referred to as an *end-around carry.*

Binary subtraction: Using 1's Comp:

$$\overset{M}{1011} - \overset{N}{100}$$

$$= \begin{array}{r} 1011 \\ -\ 0100 \\ \hline \end{array}$$

$$\begin{array}{r} 1011 \\ +\ 1011 \\ \hline 1\,0110 \\ \hline \end{array}$$

$M \geq N \Leftrightarrow$ End Carry

Correct Ans:

$$\begin{array}{r} 0110 \\ +1 \\ \hline 0111 \\ \hline \end{array}$$

M − N

$4 \leftarrow 100 - 1101 \rightarrow 13$

$$
\begin{array}{r}
01001 \\
-\ 1101 \\
\hline
\end{array}
\qquad
\begin{array}{r}
0100 \\
+\ 0010 \\
\hline
0110
\end{array}
$$

M < N ⟺ No end Carry

Correct Ans:

$-(\text{1's comp of } 0110)$

$= -1001$

$= -9$ ✓

(a) $X - Y = 1010100 - 1000011$

$$\begin{array}{rr} X = & 1010100 \\ \text{1's complement of } Y = & +\ \underline{0111100} \\ \text{Sum} = & 10010000 \\ \text{End-around carry} = & +\ \underline{\qquad\quad 1} \\ \textit{Answer}\text{: } X - Y = & 0010001 \end{array}$$

(b) $Y - X = 1000011 - 1010100$

$$\begin{array}{rr} Y = & 1000011 \\ \text{1's complement of } X = & +\ \underline{0101011} \\ \text{Sum} = & 1101110 \end{array}$$

There is no end carry. Therefore, the answer is $Y - X = -$(1's complement of 1101110) $= -0010001$.

Don't by-heart ;

Understand ;

If you forget, you

can Derive.

Eg: Assume you forgot $(r-1)$'s Subtraction Algorithm:

Eg: $\overset{M}{31} - \overset{N}{78}$ using 9's comp.

$31 - 100 + 100 - 78$

$= 31 + (100 - 78) - 100$

$$= 31 + (99 - 78) + 1 - 100$$

$$= 31 + (9\text{'s comp of } 78)$$

Result

$$= \boxed{M + (r-1\text{'s comp of } N)} - 99 = -(99 - \text{Result})$$

$$\left.\begin{array}{r} M \\ -N \end{array}\right\} \longrightarrow \begin{array}{l} \quad M \\ + (r-1)\text{'s Comp of } N \\ \hline \quad \text{Result} \\ \hline \end{array}$$

Correct Ans: $-$ (9's Comp of Result)

r's, $(r-1)$'s Complement

for Radix point :

$r = 10$

$$39.502 \xrightarrow{9\text{'s Comp}} 60.497$$

$$39.502 \xrightarrow{10\text{'s Comp}} 60.498$$

$\underbrace{r = 2}$

$10100.1011 \xrightarrow{\text{1's comp}} 01011.0100$

$10100.1011 \xrightarrow{\text{2's comp}} 01011.0101$

In the previous definitions, it was assumed that the numbers did not have a radix point. If the original number N contains a radix point, the point should be removed temporarily in order to form the r's or $(r - 1)$'s complement. The radix point is then restored to the complemented number in the same relative position. It is also worth mentioning that **the complement of the complement restores the number to its original value.** To see this relationship, note that the r's complement of N is $r^n - N$, so that the complement of the complement is $r^n - (r^n - N) = N$ and is equal to the original number.

$$N \xrightarrow{r\text{'s Comp}} \underline{(r^n - N)} = M$$

$$\Big\downarrow r\text{'s Comp}$$

$$r^n - r^n + N = N$$

$$N \xrightarrow{(r-1)'s \text{ Comp}} \underline{(r^n - 1)} - N$$

$$\big\downarrow (r-1)'s \text{ Comp}$$

$$(r^n - 1) - r^n + 1 + N = N$$

Q: Why Define "Complement operation" ?

Subtraction $\xrightarrow{\text{Converted}}$ Addition

Hard

for Humans

easy

Q: Why Define "Complement operation"?

Subtraction $\xrightarrow{\text{Converted}}$ Addition

for Digital Circuits $\Rightarrow$ Same H/w for $+, -$

Digital Logic :

Next Topic :

Representations of Signed Binary Numbers

Decimal: +99

−99

Computer: → binary

−4 →

−	1	0	0

??

Signed Binary Number

- Sign-magnitude System
- Sign-Complement System
  - 1's Comp System
  - 2's Comp System

What about negative numbers?

- Change the encoding.
  - Sign and magnitude
  - Ones compliment
  - Twos compliment

Digital Logic :

Next Topic :

Representations of Signed Binary Numbers

$\underbrace{\text{Signed-Magnitude Number System}}$

+4 0100

-4 1100

SM

Number system

Sign-Magnitude representation

- "+" sign before a number indicates it as a positive number
- "-" sign before a number indicates it as a negative number
- Not very convenient on computers

• Replace "+" sign by "0" and "-" by "1"

$(+1100101)_2 \rightarrow (01100101)_2$

$(+101.001)_2 \rightarrow (0101.001)_2$

$(-10010)_2 \rightarrow (110010)_2$

$(-110.101)_2$ --. $(1110.101)_2$

SM system:

Signed Number:

Actual Number

$\underline{1}\,\underline{0\,0} \longrightarrow -0$

$\underline{0}\,\underline{0\,0} \longrightarrow +0$

$\left.\begin{matrix}-0\\+0\end{matrix}\right\}\ 0$

9 in 4 bits ? (in SM system)

$\cancel{\underline{\underline{1}}\underbrace{001}} \Longrightarrow -1$

Sign and magnitude

- Most significant bit is sign

- Rest of bits are magnitude

$$0110 = (6)_{10} \qquad 1110 = (-6)_{10}$$

- Two representations of zero

$$0000 = (0)_{10} \qquad 1000 = (-0)_{10}$$

In SM system :

$\underbrace{1 1 1}_{\text{Signed Number}} \Rightarrow \cancel{7}$

-3

Digital Logic :

Next Topic :

Representations of Signed Binary Numbers

1's Complement Number System

for $\underbrace{N \geq 0}$ $\Longrightarrow$ SM system
||
1's Comp system

Ex: 6

0	1	1	0

SM system

0	1	1	0

1's Comp system

for $\boxed{N < 0}$ then

in 1's Comp System, N is

Stored as (1's Comp of +N)

$\underbrace{\text{1's Comp of } +N}_{\text{Operation}}$

5 bits ← 1's Comp system

+9 → | 0 | 1 | 0 | 0 | 1 |

−9 → | 1 | 0 | 1 | 1 | 0 |

+9 = 01001 $\xrightarrow{\text{1's comp}}$ 10110

1's Comp system:

Signed

4 → +4 → | 0 | 1 | 0 | 0 |

−4 → | 1 | 0 | 1 | 1 |

$+4 = 0100 \xrightarrow{\text{1's comp}} 1011$

-8 in 4 bits (in 1's comp system)

0	1	1	1

mistake

mistake:

$8 = \underbrace{1000} \xrightarrow{\text{1's comp}} \underbrace{0111}$

-8 in 4 bits (in 1's comp system)

→ Not possible :

mistake :

$+8 = \underbrace{01000}$ $\xrightarrow{\text{1's comp}}$ $\boxed{\underbrace{10111}}$

$N < 0$, in 1's Comp sys:

find $\pm N$

1's comp operation

operation

N in 1's system

MSB

1001 $\Rightarrow$ $-(0110)$ $= -6$

$-ve$

Ones compliment

e

- Compliment bits in positive value to create negative value

-ve

- Most significant bit still a sign bit

+ve

$0110 = (6)_{10}$ $\quad$ $1001 = (-6)_{10}$

- Two representations of zero

-ve

+ve

$0000 = (0)_{10}$ $\quad$ $1111 = (-0)_{10}$

Q: In Computer;

m | 1 | 0 | 0 | 1 |

then $(m)_{10}$?

Q: In Computer;

m

1	0	0	1

then $(m)_{10}$?

Depends on Your Interpretation

M

1	0	0	1

If unsigned

9

If signed

If SM system

-1

If 1's Comp system

$-(6)$

Digital Logic :

Next Topic :

Representations of Signed Binary Numbers

2's Complement Number System

$$\boxed{N \geq 0}$$

SM system $\equiv$ 1's Comp System $\equiv$ 2's Comp Sys for $\underbrace{N < 0}$

N will be stored as 2' Comp

of +N

$N = 8$ → 5 bits

0	1	0	0	0

$N = -5$ →

1	1	1	0	1	1

$+5 = 00101 \xrightarrow{\text{2's comp}} 11011$

2's Complement Numbers

In the 2's complement number system, a positive number, N, is represented by a 0 followed by the magnitude of N as in the sign and magnitude system; however,

a negative number, $-N$, is represented by its 2's complement, N^*. If the word length is n bits, the 2's complement of a positive integer N is defined as

N^* is obtained by complementing N bit-by-bit and then adding 1. An alternative way to form the 2's complement of N is to start at the right and leave any 0's on the right end and the first 1 unchanged, then complement all bits to the left of the first 1.

GO CLASSES

Twos compliment

0111

- Compliment bits in positive value and add 1 to create negative value
- Most significant bit still a sign bit

-ve

+ve 0110 = $(6)_{10}$ 1001 [redacted] = -7

- One representation of zero

-ve -ve

+ve 0000 = $(0)_{10}$ 1000 = $(-8)_{10}$ 1111 = $(-1)_{10}$

- One more negative number than positive

MIN: 1000 = $(-8)_{10}$ MAX: 0111 = $(7)_{10}$

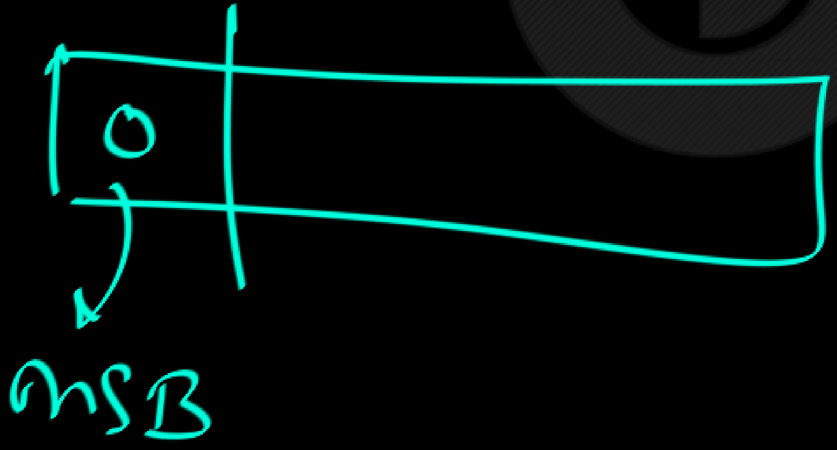
In all Systems

Positive Integers are Stored Exactly Same.

In all system:

+ve Number

-ve number



Signed

1	0	1	1

$\xrightarrow{\text{in SM}} -3$

$-ve$

in 1's system $= -(0100) = -4$

in 2's system $-(0101) = -5$

Signed

0	1	1	0

in SM → $+6 = 6$

in 1's → $+6 = 6$

in 2's → $+6 = 6$

Unsigned

1	0	1	1

in SM → 11

in 1's → 11

in 2's → 11

for Signed Binary Numbers

SIGNED BINARY NUMBERS

Positive integers (including zero) can be represented as unsigned numbers. However, to represent negative integers, we need a notation for negative values. In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign. Because of hardware limitations, computers must represent everything with binary digits. It is customary to represent the sign with a bit placed in the leftmost position of the number. The convention is to make the sign bit 0 for positive and 1 for negative.

It is important to realize that both signed and unsigned binary numbers consist of a string of bits when represented in a computer. The user determines whether the number is signed or unsigned. If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number. If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number. For example, the string of bits 01001 can be considered as 9 (unsigned binary) or as +9 (signed binary) because the leftmost bit is 0. The string of bits 11001 represents the binary equivalent of 25 when considered as an unsigned number and the binary equivalent of −9 when considered as a signed number. This is because the 1 that is in the leftmost position designates a negative and the other four bits represent binary 9. Usually, there is no confusion in interpreting the bits if the type of representation for the number is known in advance.

As an example, consider the number 9, represented in binary with eight bits. +9 is represented with a sign bit of 0 in the leftmost position, followed by the binary equivalent of 9, which gives 00001001. Note that all eight bits must have a value; therefore, 0's are inserted following the sign bit up to the first 1. Although there is only one way to represent +9, there are three different ways to represent −9 with eight bits:

signed-magnitude representation:	10001001
signed-1's-complement representation:	11110110
signed-2's-complement representation:	11110111

In signed-magnitude, −9 is obtained from +9 by changing only the sign bit in the leftmost position from 0 to 1. In signed-1's-complement, −9 is obtained by complementing all the bits of +9, including the sign bit. The signed-2's-complement representation of −9 is obtained by taking the 2's complement of the positive number, including the sign bit.

1's Comp opn Vs 1's Comp system

Operation on any Number

Storage system to store

Number system

Computer can use Signed Numbers

00101 $\xrightarrow{\text{1's Complement}}$ 11010

In 1's system what this represents → +5 = 5

$10101 \xrightarrow{\text{1's Complement}} 01010$

In 1's system what this representes

$\rightarrow -(01010)$

$= -10$

10101 $\xrightarrow{\text{2's Complement}}$ 01011

In 2's system what this represents

$\rightarrow -(01011)$

$= -11$

980102 $\xrightarrow{\text{9's Comp}}$ 019897

In 9's Comp system what this represents

meaningless

nonsense

Complement opⁿ :

any base r

(r−1)'s Comp
r's Comp opⁿ

Signed-Complement System

Number systems for signed binary numbers.