



## Propositional Logic

### Homework 3

# 5 Types of Questions in Propositional Logic

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**Detailed Solutions** of this Homework

is discussed in the “Live Doubts Session-3” of

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## 1.11 Propositional Logic Questions

1. Suppose that the statement  $p \rightarrow \neg q$  is false. Find all combinations of truth values of  $r$  and  $s$  for which  $(\neg q \rightarrow r) \wedge (\neg p \vee s)$  is true.

**Detailed Solutions** of this Homework is discussed in the  
“Live Doubts Session-3” of Goclasses Discrete Mathematics Course-2024”  
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2. Find all combinations of truth values for  $p, q$  and  $r$  for which the statement  $\neg p \leftrightarrow (q \wedge \neg(p \rightarrow r))$  is true.

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9. You have discovered an old paper on graph theory that discusses the *viscosity* of a graph (which for all you know, is something completely made up by the author). A theorem in the paper claims that “if a graph satisfies *condition (V)*, then the graph is *viscous*.” Which of the following are equivalent ways of stating this claim?

- (a) A graph is viscous only if it satisfies condition (V).
- (b) A graph is viscous if it satisfies condition (V).
- (c) For a graph to be viscous, it is necessary that it satisfies condition (V).
- (d) For a graph to be viscous, it is sufficient for it to satisfy condition (V).





- (e) Satisfying condition (V) is a sufficient condition for a graph to be viscous.
- (f) Satisfying condition (V) is a necessary condition for a graph to be viscous.
- (g) Every viscous graph satisfies condition (V).
- (h) Only viscous graphs satisfy condition (V).

**Detailed Solutions** of this Homework is discussed in the

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**EXAMPLE 7**

In [Sm78] Smullyan posed many puzzles about an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people  $A$  and  $B$ . What are  $A$  and  $B$  if  $A$  says “ $B$  is a knight” and  $B$  says “The two of us are opposite types?”

Source: Discrete Mathematics and Its Applications 7<sup>th</sup> Edition Kenneth H. Rosen

Detailed Solutions of this Homework is discussed in the

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## EXAMPLE 7



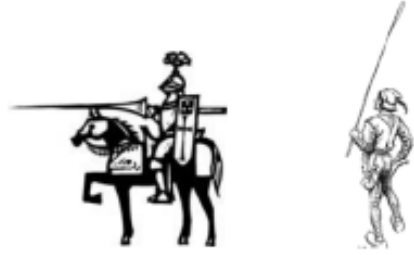
In [Sm78] Smullyan posed many puzzles about an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people  $A$  and  $B$ . What are  $A$  and  $B$  if  $A$  says “ $B$  is a knight” and  $B$  says “The two of us are opposite types?”

**Solution:** Let  $p$  and  $q$  be the statements that  $A$  is a knight and  $B$  is a knight, respectively, so that  $\neg p$  and  $\neg q$  are the statements that  $A$  is a knave and  $B$  is a knave, respectively.

We first consider the possibility that  $A$  is a knight; this is the statement that  $p$  is true. If  $A$  is a knight, then he is telling the truth when he says that  $B$  is a knight, so that  $q$  is true, and  $A$  and  $B$  are the same type. However, if  $B$  is a knight, then  $B$ 's statement that  $A$  and  $B$  are of opposite types, the statement  $(p \wedge \neg q) \vee (\neg p \wedge q)$ , would have to be true, which it is not, because  $A$  and  $B$  are both knights. Consequently, we can conclude that  $A$  is not a knight, that is, that  $p$  is false.

If  $A$  is a knave, then because everything a knave says is false,  $A$ 's statement that  $B$  is a knight, that is, that  $q$  is true, is a lie. This means that  $q$  is false and  $B$  is also a knave. Furthermore, if  $B$  is a knave, then  $B$ 's statement that  $A$  and  $B$  are opposite types is a lie, which is consistent with both  $A$  and  $B$  being knaves. We can conclude that both  $A$  and  $B$  are knaves. 





# Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.
- Puzzle 1: You meet two people on the island, Arnold and Bob. Arnold says “Either I am a knave, or Bob is a knight”. Is Arnold a knight or a knave? What about Bob?