Homework 3
Questions and Solutions

Please join our public telegram group by clicking here –

https://t.me/GATECSE_Goclasses

If you are an Enrolled Student, Please drop us a message or mail to join Private Telegram groups.

Question 1

Find the summation of following GP

$$\sum_{r=1}^{15}(3 \times 2^r)$$

$$\sum_{r=1}^{15}(3\times 2^r) = (3\times 2^1)+(3\times 2^2)+(3\times 2^3)+(3\times 2^4)+\dots+(3\times 2^{15})$$

$$= \underbrace{6+12+24+48+\dots}_{\text{This is a G.P.}}$$

$a = 6$

$r = 2$

$n = 15$

USING $S_n = \dfrac{a(r^n-1)}{r-1}$

$$S_{15} = \frac{6(2^{15}-1)}{2-1} = 196602$$

Question 2

The sum to infinity of a geometric progression of positive terms is 270 and the sum of its first two terms is 240 .

Find a and r ?

$S_\infty = 270$

$\frac{a}{1-r} = 270$

$\boxed{a = 270(1-r)}$

$S_2 = 240$

$a + ar = 240$

$\boxed{a(1+r) = 240}$

$270(1-r)(1+r) = 240$

$(1-r)(1+r) = \frac{8}{9}$

$1 - r^2 = \frac{8}{9}$

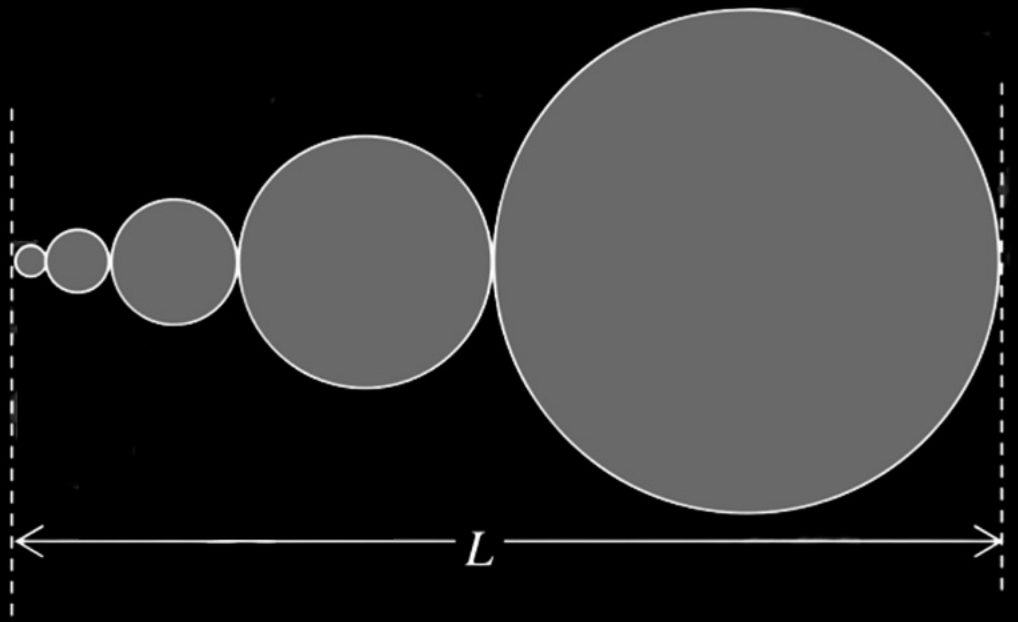
$\frac{1}{9} = r^2$

$r = +\frac{1}{3}$ (POSITIVE TERMS)

& $a = 270(1-r)$

$a = 270\left(1 - \frac{1}{3}\right)$

$a = 180$



Question 3

The figure shows a pattern of 5 circles, touching externally, whose centres lie on a straight line of length L units.

The radii of these circles form a geometric progression, where the radius of the smaller circle is 3 units and that of the fifth (larger) circle is 48 units.

a) Find the common ratio of the geometric progression.

b) The pattern is extended by 5 more circles to 10 circles. Determine the new value of L .

Question Source: https://madasmaths.com/archive/maths_booklets/basic_topics/various/geometric_series.pdf

a) USING $u_n = ar^{n-1}$

$$\Longrightarrow \quad 48 = 3 \times r^4$$

$$\Longrightarrow \quad 16 = r^4$$

$$\Longrightarrow \quad r = +2$$ (ALL TERMS ARE POSITIVE)

b) USING THE SUM FORMULA FOR $n=10$ & SUM, NOTING THAT IT NEEDS TO BE DOUBLED (DIAMETER SUM IS NEEDED)

$$L_{NEW} = 2 \times \frac{a(r^n - 1)}{r-1} \qquad r=2,\ a=3,\ n=10$$

$$L_{NEW} = 2 \times \frac{3(2^{10}-1)}{2-1}$$

$$L_{NEW} = 6138$$

AGP : Question 4:

Find the sum of the series $1 . 2 + 2 . 2^2 + 3 . 2^3 + \ldots . + 100 . 2^{100}$

Solution

$$S = 1.2 + 2.2^2 + 3.2^3 + \ldots. + 100.2^{100}$$

$$\Rightarrow 2S = \quad 1.2^2 + 2.2^3 + \ldots. + 99.2^{100} + 100.2^{101}$$

On subtracting ;

$$\Rightarrow -S = 1.2 + 1.2^2 + 1.2^3 + \ldots. + 1.2^{100} - 100.2^{101}$$

$$\Rightarrow -S = 1.2\left(\frac{2^{100}-1}{2-1}\right) - 100.2^{101}$$

$$\Rightarrow S = -2^{101} + 2 + 100.2^{101} = 99.2^{101} + 2$$

AGP :

Question 5: Compute the sum

$$\sum_{k=1}^{\infty} \frac{k}{4^k}$$

Solution. Let the sum be S. Then

$$S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \dots,$$

and if we divide by 4,

$$\frac{S}{4} = \frac{1}{4^2} + \frac{2}{4^3} + \dots.$$

Note that if we subtract these in a special way (by subtracting the ones with common denominators), we get

$$\frac{3S}{4} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3},$$

so $S = \frac{4}{9}$. $\square$

Bonus Question

If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to

A $\frac{121}{10}$

B $\frac{441}{100}$

C 100

D 110

Solution

$$S = 10^9 + 2.11^1.10^8 + \ldots + 10.11^9$$

$$\frac{11}{10} \cdot S = \qquad 11^1 \cdot 10^8 + \ldots + 9 \cdot 11^9 + 11^{10}$$

$$\Rightarrow -\frac{1}{10}S = 10^9 + 11^1 \cdot 10^8 + 11^2 \cdot 10^7 + \ldots + 11^9 - 11^{10}$$

$$\Rightarrow -\frac{1}{10}S = \quad 10^9\left(\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right) - 11^{10} \Rightarrow -\frac{1}{10}S = 11^{10} - 10^{10} - 11^{10}$$

$$S = 10^{11}$$

$$S = 100 \cdot 10^9$$

$$\Rightarrow k = 100.$$