



Next Chapter:

Propositional Logic

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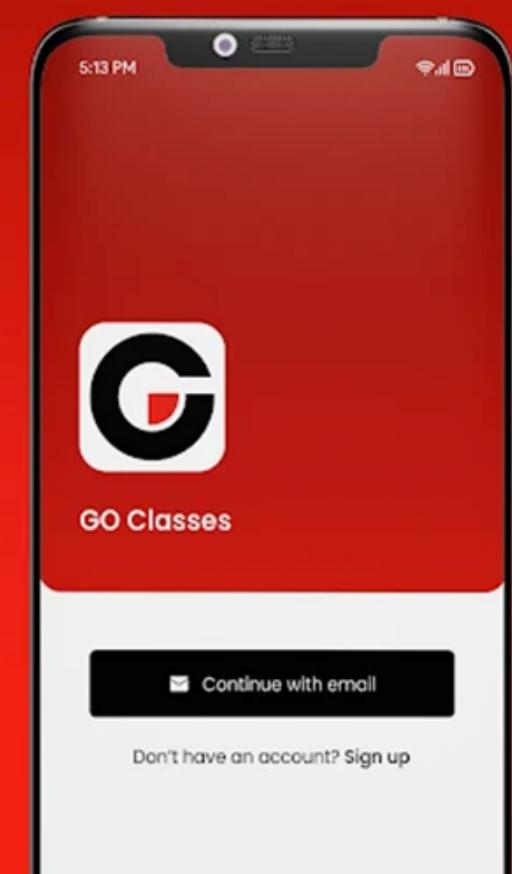
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Next Chapter:

Propositional Logic



Mathematical Logic:

- Propositional Logic
 - First Order Logic
- 



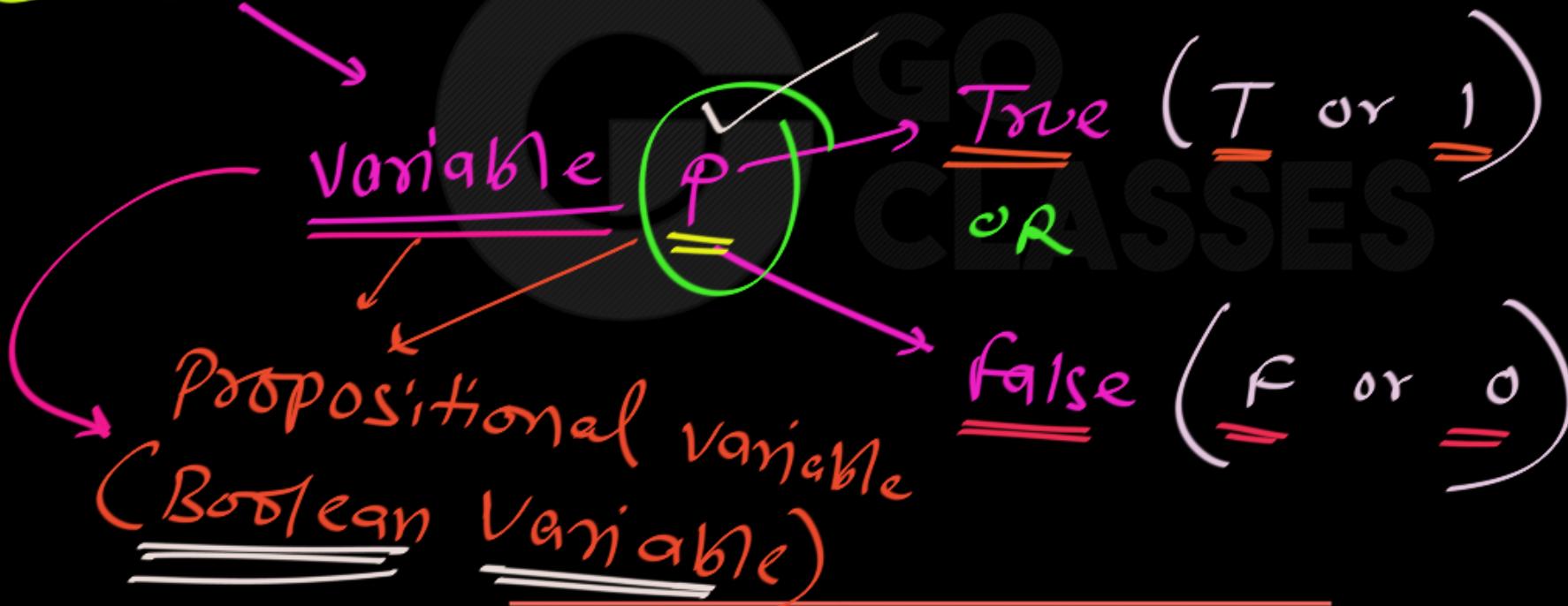
Mathematical Logic:

Propositional Logic:

The Simplest type of Logic.

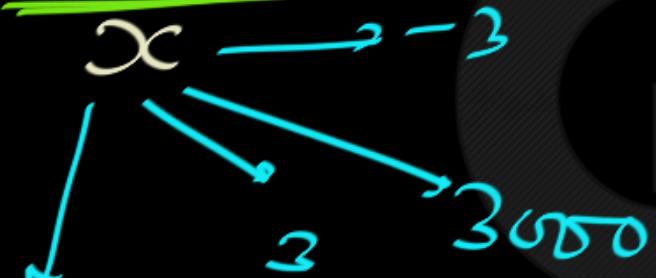
Propositional Logic:

A world of True, False.



{ Normal Mathematics }

Integer Variable



Real Variable

$y \rightarrow \pi$
 $y \rightarrow 2.3$
 $y \rightarrow 0$

Prop. logic

Very Simple World

$p \rightarrow T$ or
 $q \rightarrow F$
 $q \rightarrow T$
 $q \rightarrow F$ or



Propositional Logic:

1. What is Proposition??



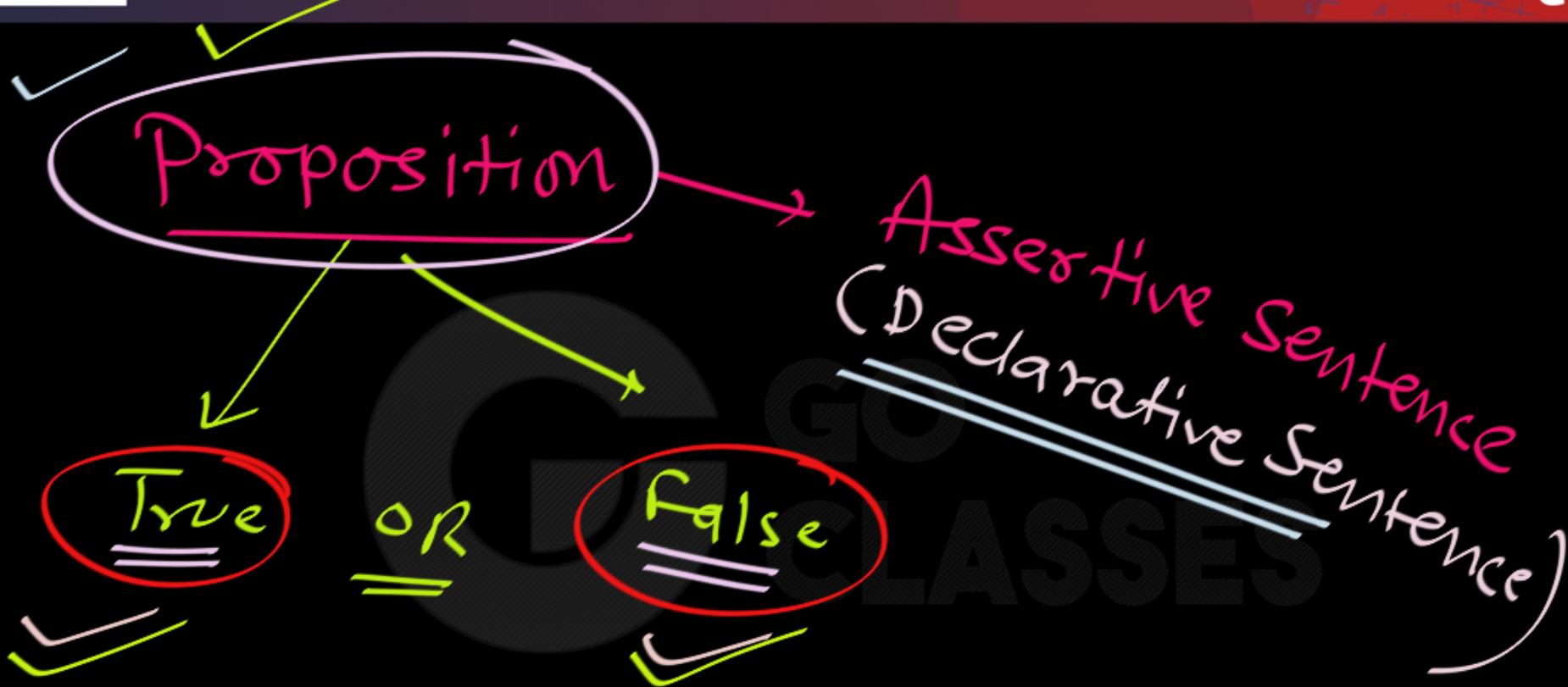


Propositional Logic:

1. Proposition: Statement :

A proposition is a declarative sentence(that is, a sentence that declares a fact) that can be either true or false; it must be one or the other, and it cannot be both.

A proposition/statement is a declarative sentence to which it is possible to assign a value of either true or false.



Examples of Propositions:

“Jaipur is the capital of the India.” is a proposition.

All cows are brown. ✓

False

The Earth is further from the sun than Venus. ✓

$2 \times 2 = 5$. ✓

2 × 2 is 5 ✓

Examples of Propositions :

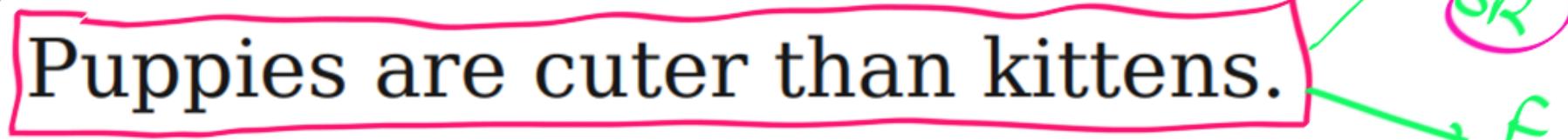
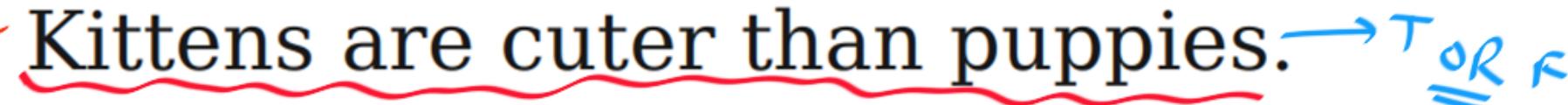
T or F

“There are 1000 stones on the Mars.” is a proposition.

“In 7th century, 90000 people were born”.

True or False

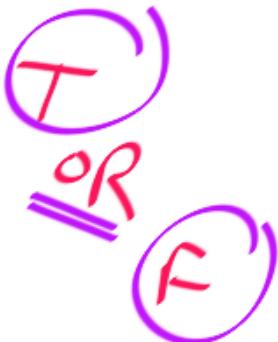
Some Sample Propositions

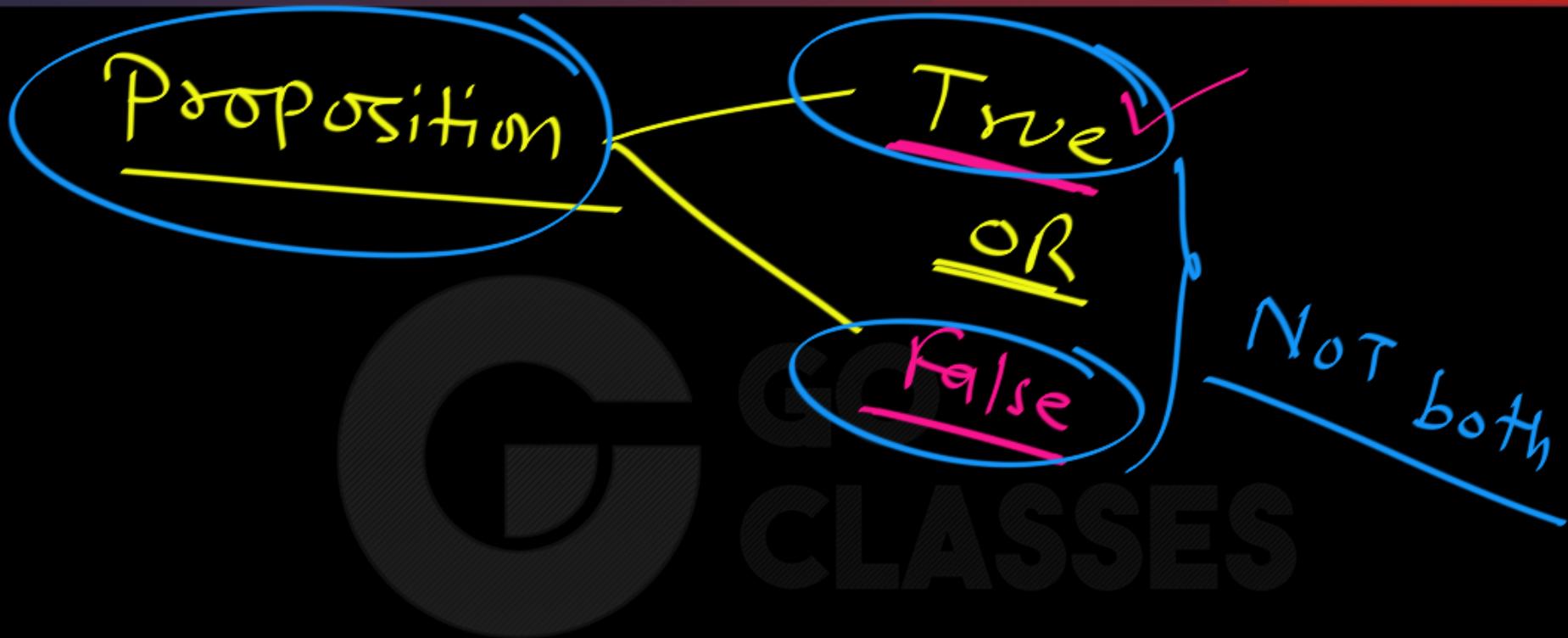
- ✓ • Puppies are cuter than kittens.
- ✓ • Kittens are cuter than puppies.
- ✓ • Usain Bolt can outrun everyone in this room.
- ✓ • CS103 is useful for cocktail parties.
- ✓ • This is the last entry on this list.



More Propositions

- ✓ • I came in like a wrecking ball.
- ✓ • I am a champion.
- ✓ • You're going to hear me roar.
- ✓ • We all just entertainers.







We got the Idea of “what is a proposition/statement”..

But What is NOT a Proposition then??



Things That Aren't Propositions



Things That Aren't Propositions





Sentences That Aren't Propositions:

"Look out!" is not a proposition.

Imperative sentence

"How far is it to the next town?" is not a proposition.

Question

" $x + 2 = 2x$ " is not a proposition.

Assertive sentence

$x : \text{integer}$

free Variable



Sentences That Aren't Propositions :

“Do you want to go to the movies?”

Since a question is not a declarative sentence, it fails to be a proposition.



“Clean up your room.” Likewise, an imperative is not a declarative sentence; hence, fails to be a proposition.



Sentences That Aren't Propositions :

“ $2x = 2 + x$.” This is a declarative sentence, but unless x is assigned a value or is otherwise prescribed, the sentence neither true nor false, hence, not a proposition.

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Which of the following is a Propositions??

$$4n = 2 + 3n$$

For all integers n, $2n$ is an even number.



Which of the following is a Proposition??

$$4n = 2 + 3n \quad \times$$

free variable n

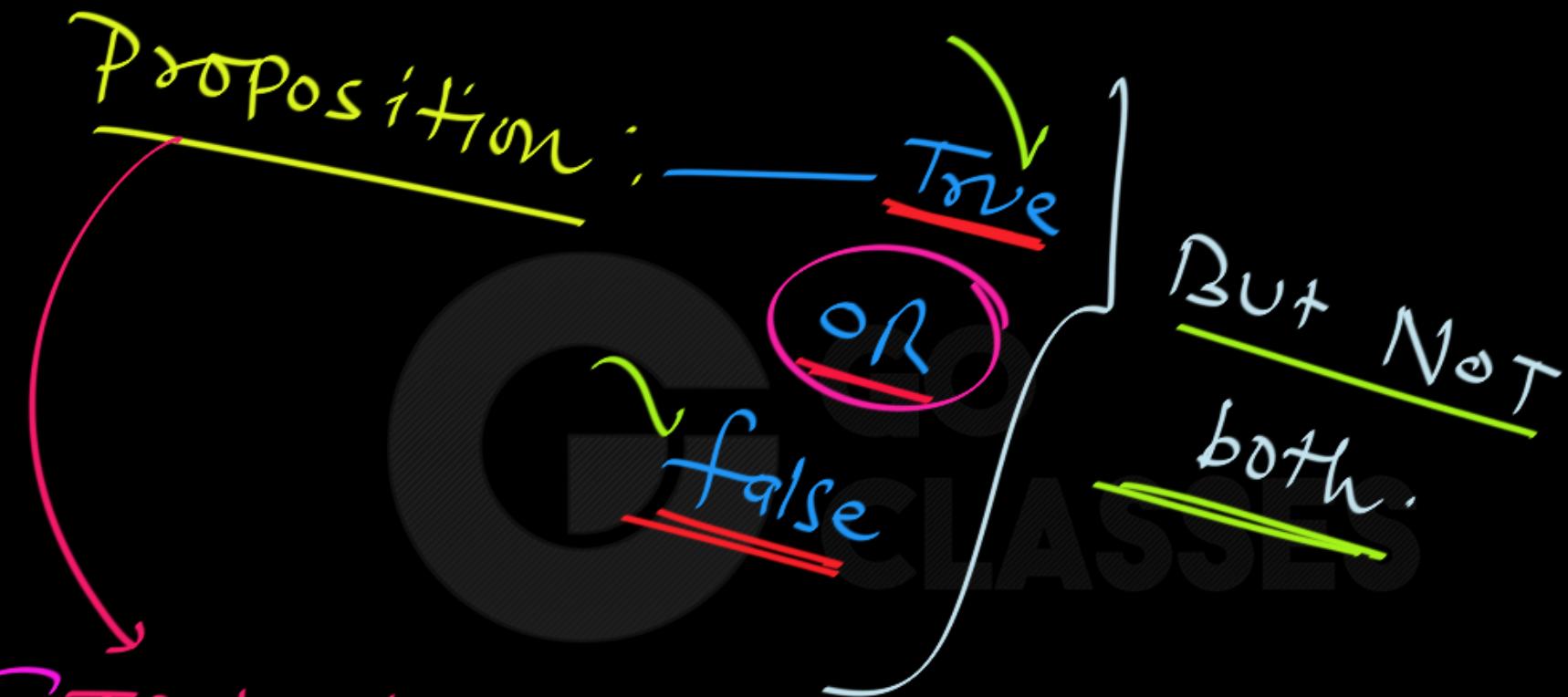
integer

✓
For all integers n, 2n is an even number.

True ✓

Not
free Variable

Proposition ✓



{ If it is True then it cannot be false .
If it is false " " " " " True .



Sentences That Aren't Propositions :

“This sentence is false.” : is a Paradox, hence, Not a proposition.

What happens if you assume this statement is true? false? This example is called a paradox and is not a proposition, because it is neither true nor false.

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$s :$

"s is false"

→ sentence

Case 1:

$s = \text{True}$

This is True

If

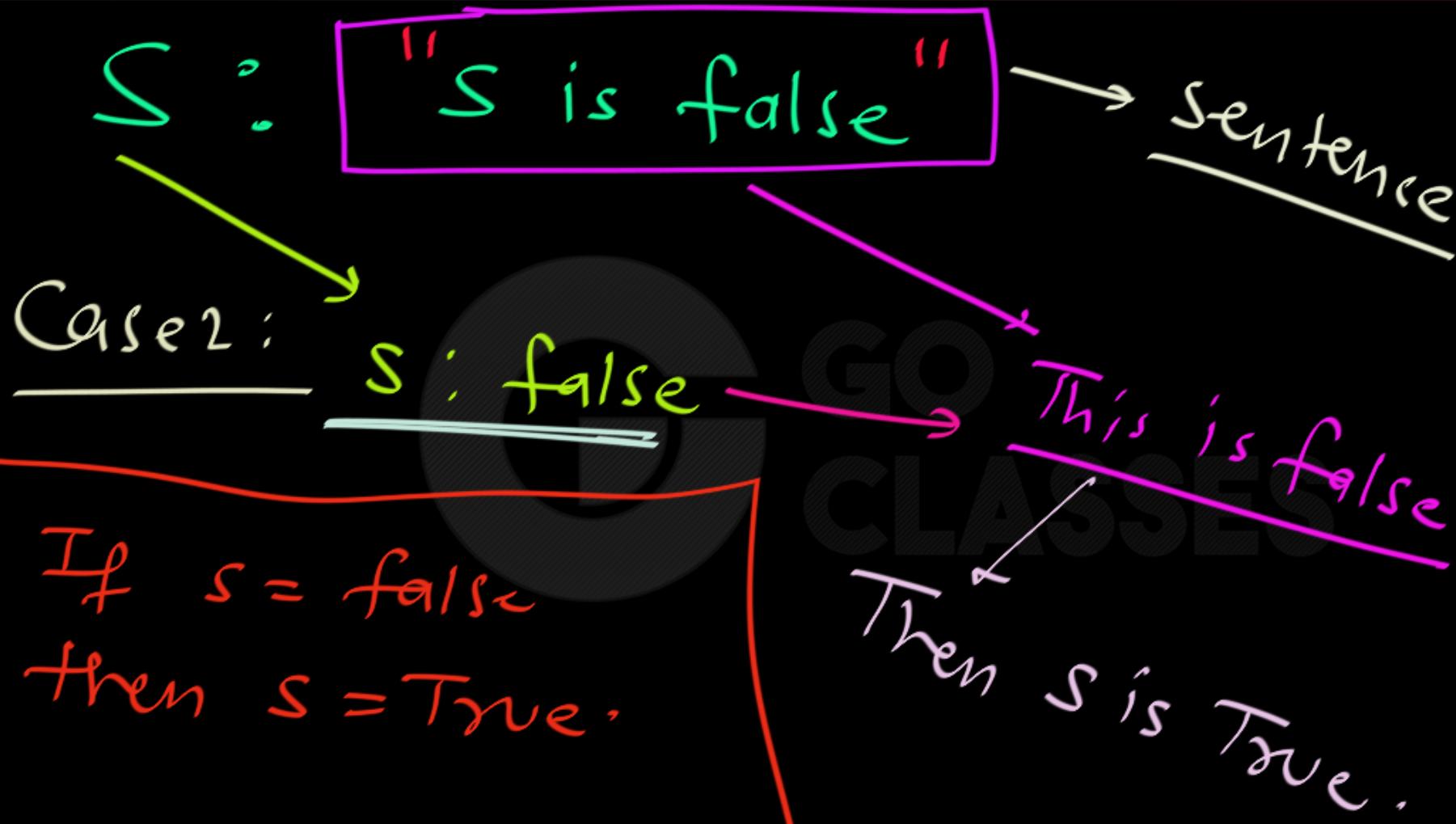
$s = \text{True}$

then

$s = \text{false}$

s is false

Paradox



$s :$

" s is false"

→ sentence

a Paradox

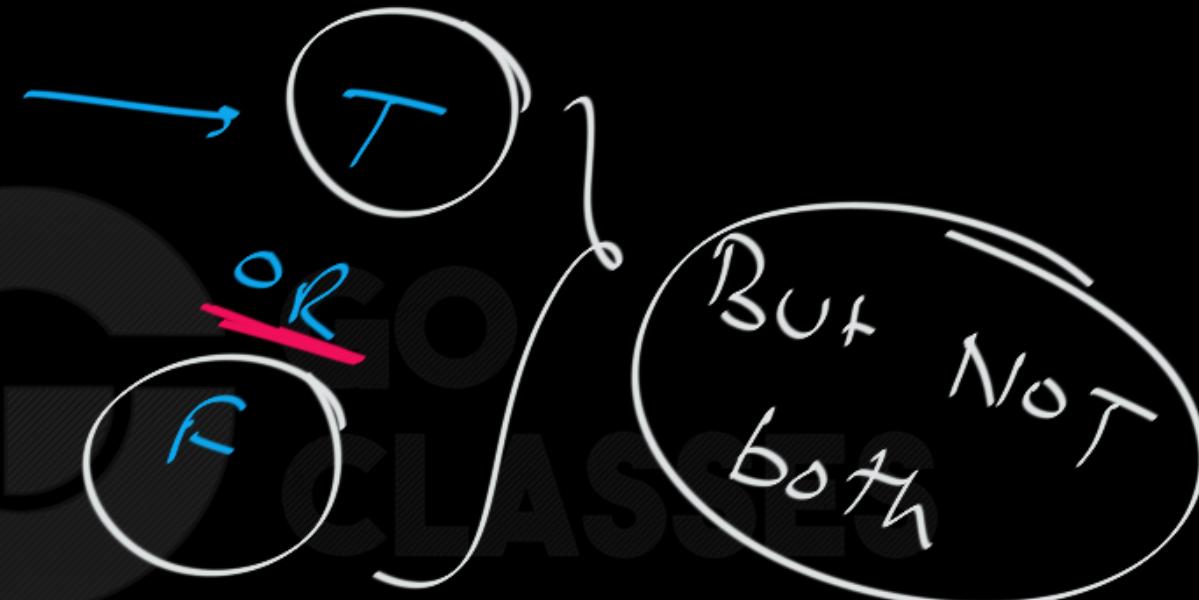
If $s = \text{True}$ then $s = \text{false}$

If $s = \text{false}$ then $s = \text{True}$

a NOT
proposition



Proposition:





A **proposition** is a statement that is either true or false.

In other words, English sentences can be propositions, but not all are (for example, commands and questions can't be propositions).



Not a proposition:

- ① Question
- ② Imperative Sentence, Command
- ③ Sentences with free Variables
- ④ Paradox



Propositional logic

World
of
True, false

Proposition → True
on
False

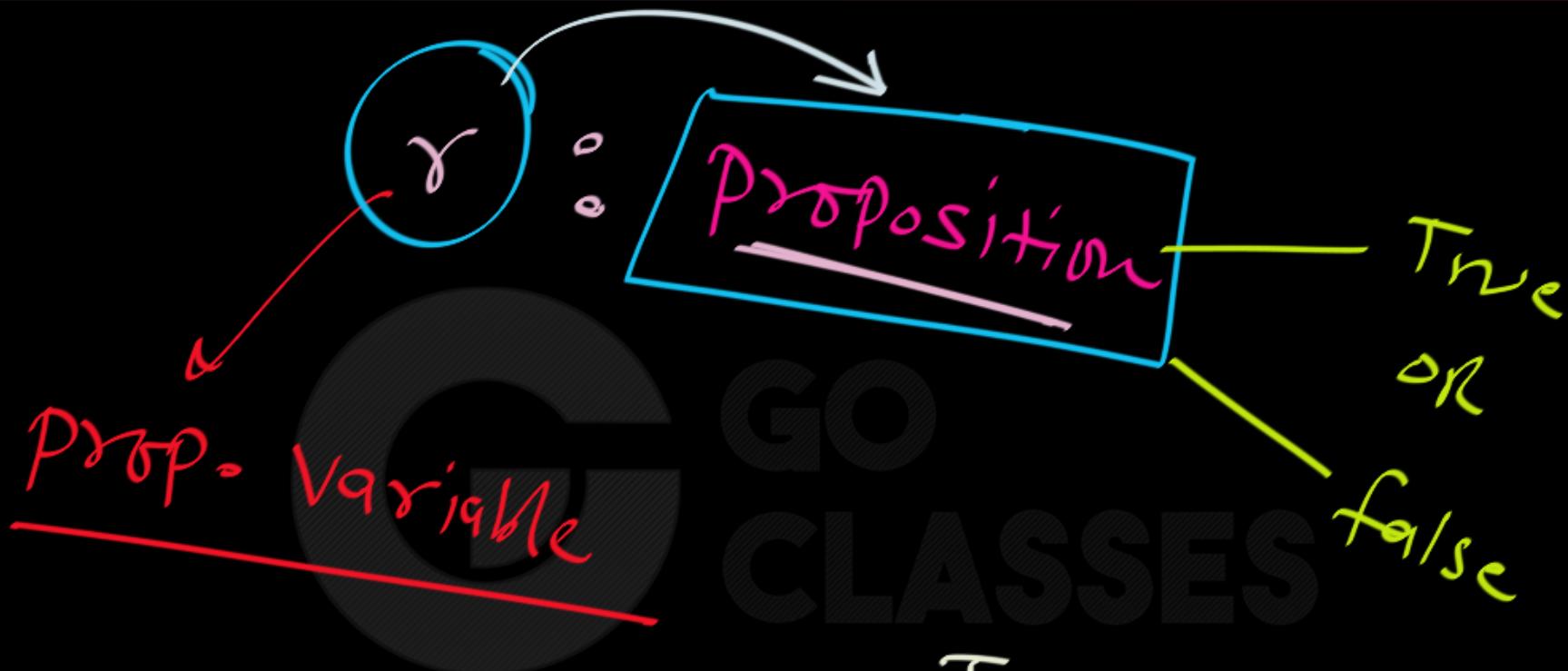
"Not True" ≡ false

"Not false" ≡ True



Propositional Logic:

1. Proposition
2. Propositional Variables



Propositional Variables

- Each proposition will be represented by a propositional variable.
- Propositional variables are usually represented as lower-case letters, such as p, q, r, s , etc.
- Each variable can take one of two values: true or false.



The sun is shining.

I feel happy.

Cats are furry.

Elephants are heavy.

True

on

false

P

H

C

Propo. Variable

R



example, we can define some propositional variables to stand for the following English statements:

- A = The sun is shining.
- B = I feel happy.
- C = Cats are furry.
- D = Elephants are heavy.

prop. variable $\begin{array}{c} T \\ \diagup \quad \diagdown \\ \text{on} \\ \diagdown \quad \diagup \\ F \end{array}$

Propositional Variables

- **Propositional Variables** - variables that represent propositions: p, q, r, s
 - E.g. Proposition p - "Today is Friday."
- Truth values - $\textcircled{T, F}$



Prop - Variable

P True
OR
false

R → True → Truth value of R
is "T".



Prop - Variable

P True
OR
false

S → False



Propositional Variables :

Each proposition will be represented by a propositional variable.

We use letters to denote propositional variables (or statement variables), that is, variables that represent propositions.

Each variable represents some proposition.

Truth Value :

The truth value of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

Each (propositional)variable can take one of two values: true or false.



{ True $\equiv T \equiv 1$
false $\equiv F \equiv 0$ }

Truth Values
of propositions



Next Lecture:

Atomic & Compound Propositions