Propositional Logic

Next Chapter:

Bi-implication

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GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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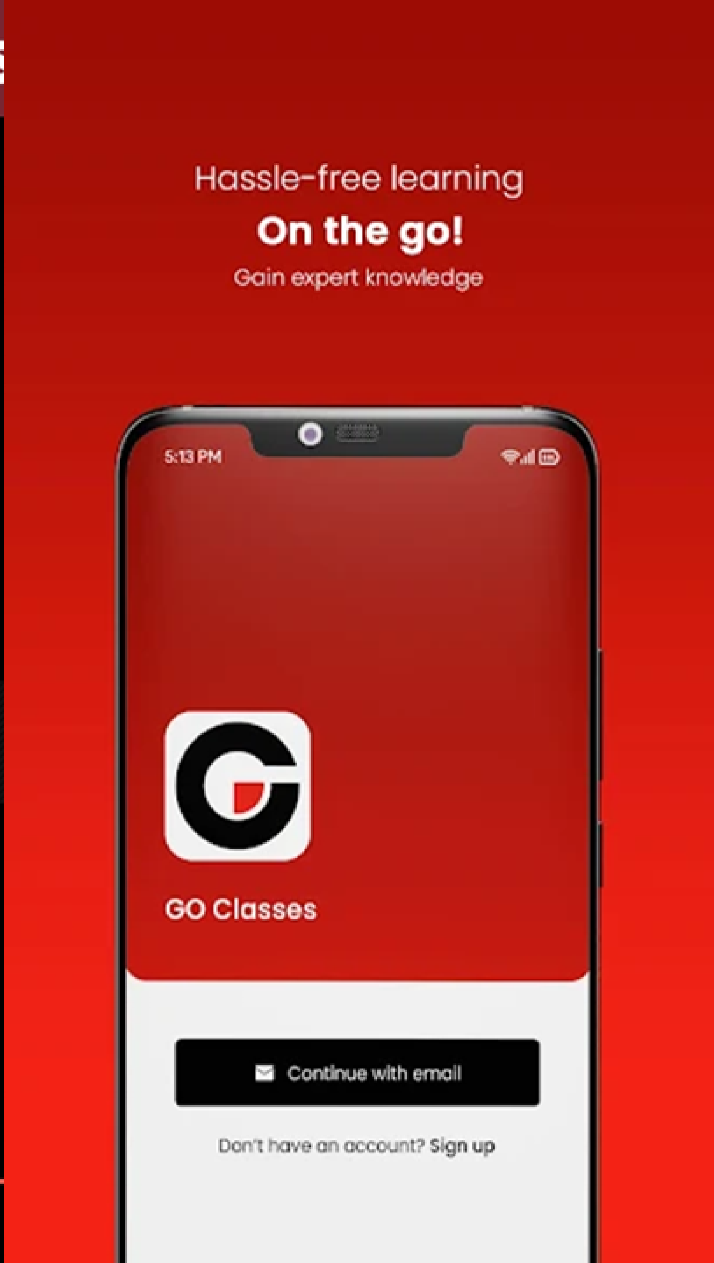

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Next Topic:

Logical Connective

Bi-implication

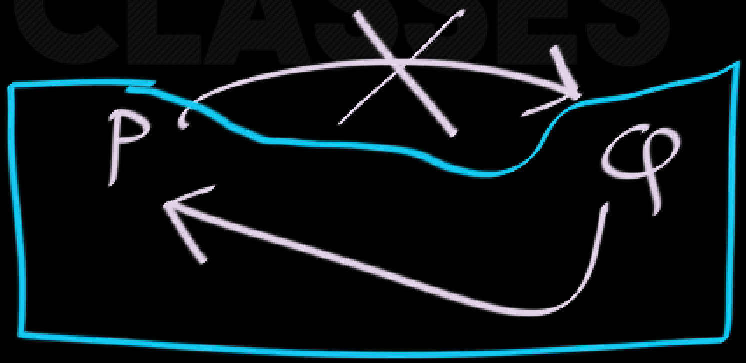
"Implication" is used to represent One-Way statements.

P: You appear in the GATE exam.

Q: You top the GATE exam.

$P \to Q$ ✗

$Q \to P$ ✓



$Q \to P$

"Implication" is used to represent One-Way statements.

But sometimes, statements are Two-ways.

P: Value is 2.
Q: Value is an Even Prime number.

"Implication" is used to represent One-Way statements.

But sometimes, statements are Two-ways.

$P \rightleftarrows Q$

P: Value is 2.
Q: Value is an Even Prime number.

$P \rightleftarrows Q$

"Implication" is used to represent One-Way statements.

But sometimes, statements are Two-ways.

P: n is even
Q: n+2 is even.

$P \rightleftarrows Q$

$$A \rightleftarrows B$$

Two way Relationship

$A \longrightarrow B$ and $B \longrightarrow A$

"Implication" is used to represent One-Way statements.

$$A \rightleftarrows B$$

But sometimes, statements are Two-ways.

To represent Two-Way statements, we have a new logical connective, "Bi-Implication"

Statements: A , B

2-way Relationship b/w A, B

$A \rightleftarrows B$

$A \to B$ and $B \to A$ $\equiv$ $A \longleftrightarrow B$

$$(A \longleftrightarrow B) = (A \to B) \wedge (B \to A)$$

Symbol : $\longleftrightarrow$ bi-implication (biconditional)

two way Implication

BiConditional Operator : $\longleftrightarrow$

$$\boxed{(p \longleftrightarrow q)} = \boxed{(p \to q) \land (q \to p)}$$

Same

Truth Table:

$P \leftrightarrow Q$

$(P \rightarrow Q) \wedge (Q \rightarrow P)$

$\equiv$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$P \leftrightarrow Q$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

To Represent "two way Conditional Statements" we use the following logical Connective :

$\longleftrightarrow$ (biconditional / biimplication)

$P \longleftrightarrow Q$ means $P \longrightarrow Q$ (P implies Q)

and

$P \longleftarrow Q$ (Q implies P)

So,

$$(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

The biconditional statement is equivalent to $(p \to q) \land (q \to p)$.

In other words, for $p \leftrightarrow q$ to be true we must have both p and q true or both false.

$P \longleftrightarrow Q$: P biimplication Q

P	Q	$P \longleftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

$P \leftrightarrow Q$: P biimplication Q

P	Q	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

$\equiv$

Similar to Equality

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The Biconditional

- The **biconditional** connective $p \leftrightarrow q$ is read "p if and only if q."
- Intuitively, either both p and q are true, or neither of them are.

p	q	$p \leftrightarrow q$
F	F	
F	T	F
T	F	F
T	T	

One of p or q is true without the other.

The Biconditional

- The **biconditional** connective $p \leftrightarrow q$ is read "p if and only if q."
- Intuitively, either both p and q are true, or neither of them are.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Both p and q are false here, so the statement "p if and only if q" is true.

The Biconditional

- The **biconditional** connective $p \leftrightarrow q$ is read "p if and only if q."
- Intuitively, either both p and q are true, or neither of them are.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

One interpretation of $\leftrightarrow$ is to think of it as equality: the two propositions must have equal truth values.

Note:

$$(P \leftrightarrow Q) \equiv \overline{(P \oplus Q)}$$

$$P \oplus Q = \overline{(P \leftrightarrow Q)}$$

EXOR

P	Q	$P \oplus Q$	$P \leftrightarrow Q$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	F	T

Various English Translations

of

Bi-implication

$P \longleftrightarrow Q$

- P biimplication Q
- P implies Q and Q implies P.
- P implies Q and vice versa.

$P \leftrightarrow Q \equiv (P \rightarrow Q)$ and $(Q \rightarrow P)$

(Q if P) and (Q only if P)

$\equiv$ Q if and only if P

$P \longleftrightarrow Q \equiv (P \rightarrow Q)$ and $(Q \rightarrow P)$

(P only if Q) and (P if Q)

$\boxed{\text{P if and only if Q}}$

$$p \leftrightarrow q \equiv (p \rightarrow q) \text{ and } (q \rightarrow p)$$

$p \leftrightarrow q \equiv p$ $\boxed{\text{if and only if}}$ q

$\equiv$ p iff q

iff

$P \longleftrightarrow Q \equiv (P \rightarrow Q)$ and $(Q \rightarrow P)$

(P is sufficient for Q) and (P is Necessary for Q)

$\equiv$ $\boxed{\text{P is sufficient and Necessary for Q}}$

$$P \longleftrightarrow Q \equiv (P \to Q) \text{ and } (Q \to P)$$

$P \longleftrightarrow Q \equiv$ P is Necessary and Sufficient for Q

$$P \leftrightarrow Q \equiv (P \to Q) \text{ and } (Q \to P)$$

1. P biimplication Q
2. P if and only if Q
3. P iff Q
4. P is Necessary and sufficient for Q.

There are some other common ways to express $p \leftrightarrow q$:

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q."

Next Topic:

Propositional Formula

Vs

Propositional Variable