Next Topic (VERY Important):

Minimization of Expression

Using *K-Map*

A Karnaugh map (K-map) is a pictorial method used to minimize Boolean expressions without having to use Boolean algebra theorems and equation manipulations.

A K-map can be thought of as a special version of a truth table, designed **to ease the minimization**.

Using a K-map, expressions with upto four variables are easily minimized.

K-map is used to get :

minimum sum-of-products expression for a function.

as well as

minimum product-of-sums expression for a function.

First we study how to use K-map to get :

minimum sum-of-products expression for a function.

So, from now onwards, we will be focusing on getting Minimum SOP expression.

Karnaugh Maps (K-map)

- Alternate representation of a truth table

Karnaugh maps

- All functions can be expressed with a map

- There is one square in the map for each minterm in a function's truth table

X	Y	F
0	0	m0
0	1	m1
1	0	m2
1	1	m3

X \ Y	0	1
0	m0 $\overline{X}\overline{Y}$	m1 $\overline{X}Y$
1	m1 $X\overline{Y}$	m3 XY

Next Topic:

Idea Behind K-map

Idea behind K-map method :

when only one-variable changes in two minterms; then we can combine;

$$Ax + Ax' = A(x+x')$$
$$= A$$

And the variable which changes can be removed.

Minimization

- Minimization can be done using
  - Boolean algebra
    - To combine terms

$$B\,\overline{C} + B\,C = B(\,\overline{C} + C) = B$$

- Or equivalently
  - Karnaugh maps
    - Visual identification of terms that can be combined

Idea behind k-map:

Truth Table of any f is represented in such a way (in matrix format/in a map format) that Adjacent cells/minterms differ by only one variable.

Eg: Truth Table Representation of f

a	b	f
0	0	
0	1	
1	0	
1	1	

Adjacent minterms
Do not differ
by only one
Variable

K-map Rep of f:

a \ b	0	1
0	00	01
1	10	11

Three Variables: $f(a,b,c)$ ~ K-map

Truth Table

a	b	c	minterms
0	0	0	$a'b'c'$
0	0	1	$a'b'c$
0	1	0	$a'bc'$
0	1	1	$a'bc$
1	0	0	⋮
1	0	1	⋮
1	1	0	⋮
1	1	1	

a \ bc	00	01	11	10
0	0	1	3	2
1	4	5	7	6

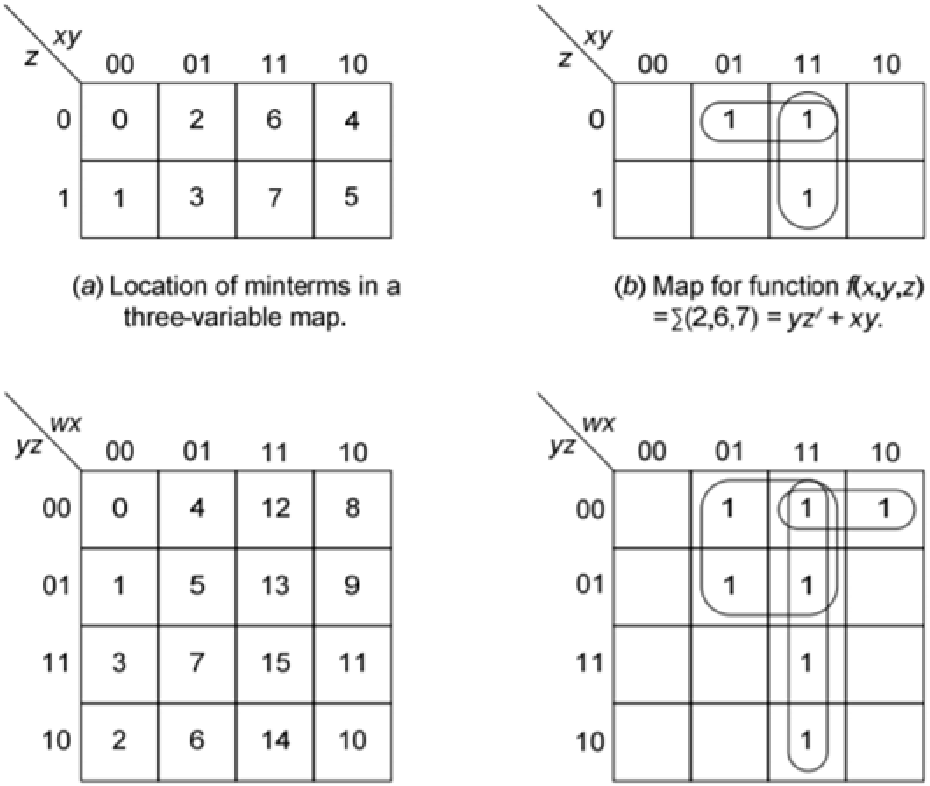
$a'b'c$

abc

The Map Method

Algebraic procedure to combine terms using the *Aa* + *Aa'* = *A* rule

Karnaugh map: modified form of truth table



(*a*) Location of minterms in a three-variable map.

(*b*) Map for function *f*(*x*,*y*,*z*) $=\sum(2,6,7) = yz' + xy$.

(*c*) Location of minterms in a four-variable map.

(*d*) Map for function *f*(*w*,*x*,*y*,*z*) $=\sum(4,5,8,12,13,14,15) = wx + xy' + wy'z'$.

Intro to K-map

K-maps consist of one square for each possible minterm in a function

- 2 variable - 4 squares (2^2)
- 3 variable - 8 squares (2^3)
- 4 variable - 16 squares (2^4)
- n variable - 2^n squares

Re-arranging the Truth Table

- A two-variable function has four possible minterms. We can re-arrange these minterms into a Karnaugh map

x	y	minterm
0	0	$x'y'$
0	1	$x'y$
1	0	xy'
1	1	xy

X \ Y	0	1
0	$x'y'$	$x'y$
1	xy'	xy

- Now we can easily see which minterms contain common literals
  - Minterms on the left and right sides contain y' and y respectively
  - Minterms in the top and bottom rows contain x' and x respectively

X \ Y	0	1
0	$x'y'$	$x'y$
1	xy'	xy

	y'	y
x'	$x'y'$	$x'y$
x	xy'	xy

$f(A, B)$

Map 3.1 Two-variable Karnaugh maps.

B \ A		
	$A'B'$	AB'
	$A'B$	AB

B \ A		A
	m_0	m_2
B	m_1	m_3

B \ A	0	1
0	0 (00)	2 (10)
1	1 (01)	3 (11)

3 variable K-map

Map 3.3 Three-variable maps.

C \ AB	A′ B′ 00	A′ B 01	A B 11	A B′ 10
C′ 0	A′ B′ C′	A′ B C′	A B C′	A B′ C′
C 1	A′ B′ C	A′ B C	A B C	A B′ C

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

Next Topic:

Adjacent Cells & Cubes in K-map

Adjacent Cell / min terms (Neighbour Cells)

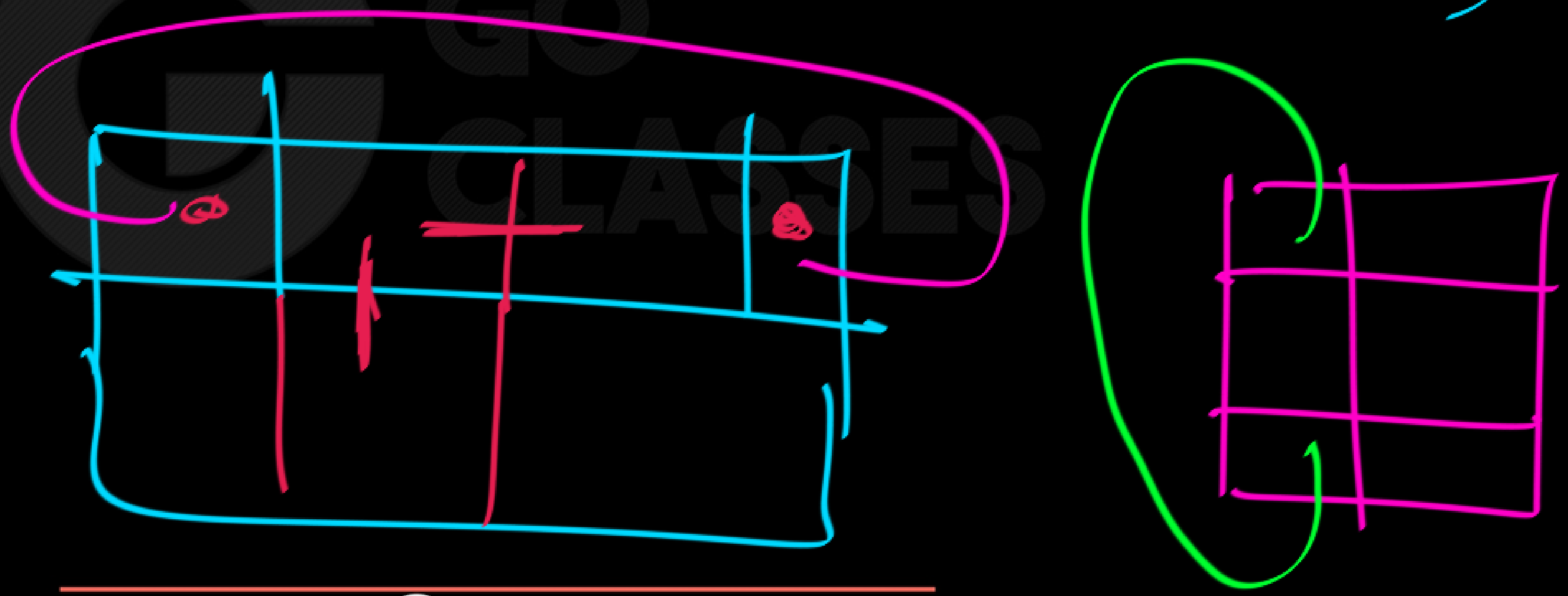
Any two cells which differ by only one variable.

$(ab'c, ab'c')$ ✓ $(ab'c, abc')$ ✗

Conclusion: In K map, Adjacent cells:

(1) Side by side (L-R or Up-Down)

(2)



Cube in a kmap;

any collection of 2^m cells such that every cell is adjacent to m cells in the cube.

Next Topic:

WHY 2^m Cells in a Cube in K-map ??

Reason:

Because we want (minimum) SOP.

$$f = \underbrace{P_1}_{\text{Product term}} + \underbrace{P_2}_{\text{Product term}} + \underbrace{P_3}$$

Q: one product term Covers How many minterm ?

also product term (PT)

Q: one product term Covers How many minterm ?

→ also product term

Eg: $f(a,b,c)$ → Product term: ab

$\begin{Bmatrix} abc \\ ab\bar{c} \end{Bmatrix}$

PT ab Covers 2 minterms.

$$\underbrace{abc + ab\bar{c}}_{} = ab$$

Eg: $f(a,b,c) \Rightarrow PT = a'$

$$\left.\begin{array}{l} \bar{a}\,\bar{b}\,\bar{c} \\ \bar{a}\,\bar{b}\,c \\ \bar{a}\,b\,\bar{c} \\ \bar{a}\,b\,c \end{array}\right\}$$

PT a' Covers 2^2 minterms.

$f(a,b,c)$

PT	Covered minterms	# minterms Covered
$ab'c$	$ab'c$	1
$a'b$	$a'bc'$, $a'bc$	2
a	100, 101, 110, 111	2^2

Q: a Sum term Covers how many Maxterms?

also a Sum term

$f(a,b,c)$

sum term	#maxterms Covered	maxterms Covered
$a'+b+c$	1	$a'+b+c$
$a'+b$	2^1	$a'+b+c'$; $a'+b+c$
a'	2^2	$1+0+0$; $1+0+1$; $1+1+0$ $1+1+1$

from k-map → we want

minimum POS then we Combine 2^m cells.

$(S_1)(S_2)(S_3)$

One more Interesting thing:

Two minterms:

$ab'c' + a'b'c$

Cannot Combine / Reduce.

$ab'c + ab'c'$

ab'

$ab'c + ab'c'$

ab'

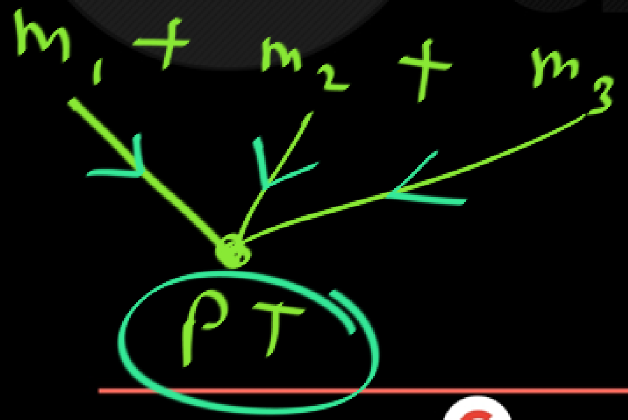
Note: Target from k-map = minimum SOP (product terms)

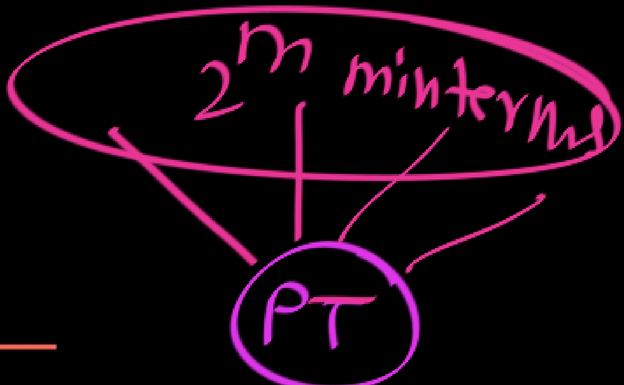
2 minterms can be reduced to single product term iff they change by single variable.

Q: Can we Reduce 3 minterms to a simple product term?

Q: Can we Reduce 3 minterms to a Simple Product term? **No**

Proof: think in Reverse Direction.





Q: Can we Reduce 4 minterms to a Simple Product term?

Q: Can we Reduce 4 minterms to a simple product term? Yes.

Iff every minterm is Adjacent to minterms.

$ab'c + abc + ab'c' + abc'$

$ac \qquad ac'$

a

$$ab'c + abc + ab'c' + abc'$$

$$a$$

Note: In K-map, we create Cubes of size 2^m so that we can write product term for that cube.

Writing Product terms

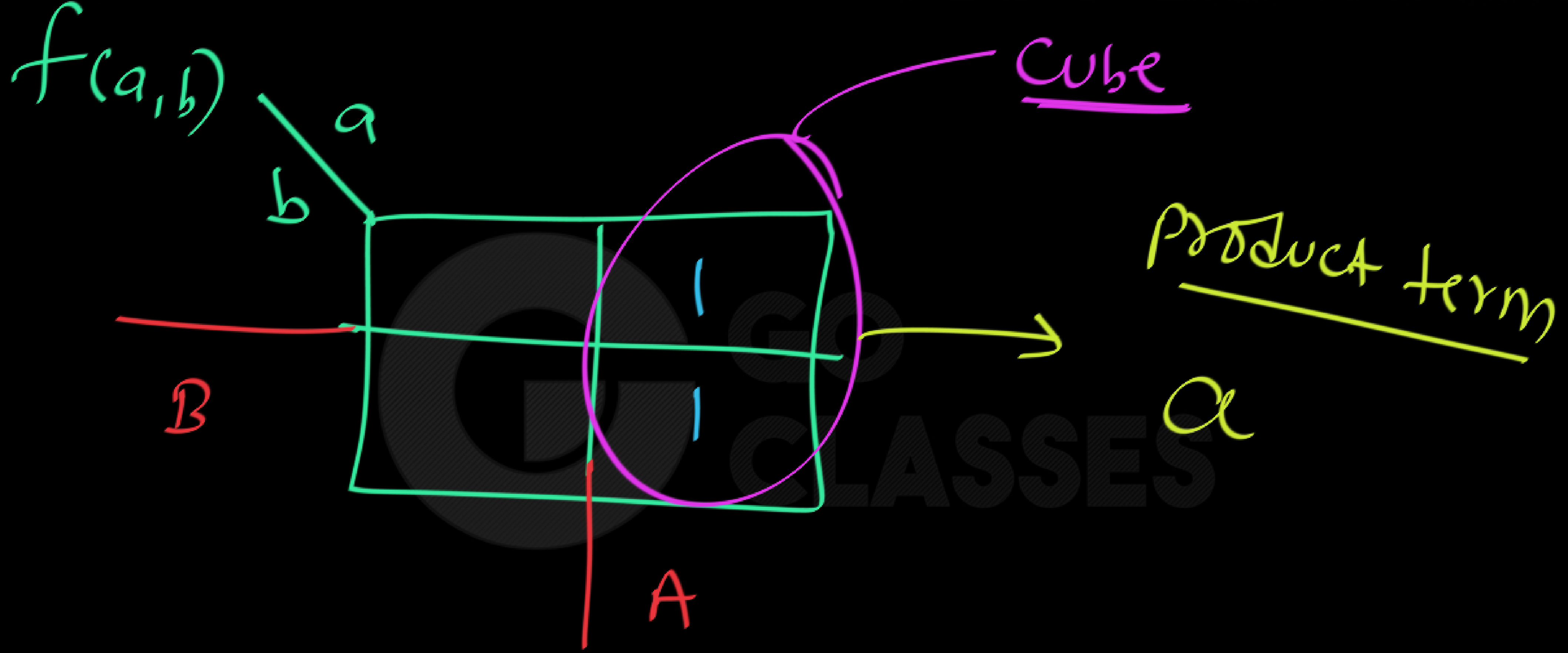
from Cubes :

To find

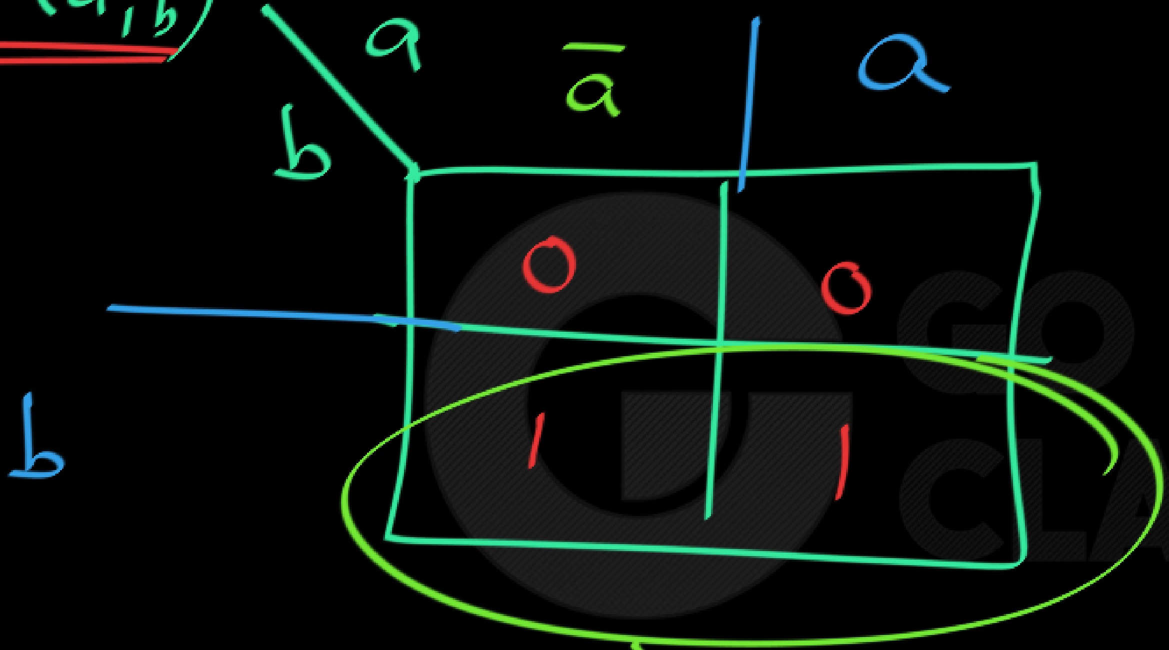
SOP

only Cover 1's

$b \backslash a$	$\overline{A}$ 0	A 1
$\overline{B}$ 0		
B 1		


f(a,b)
a
b
B
A
1
1
Cube
Product term
a

f(a, b)

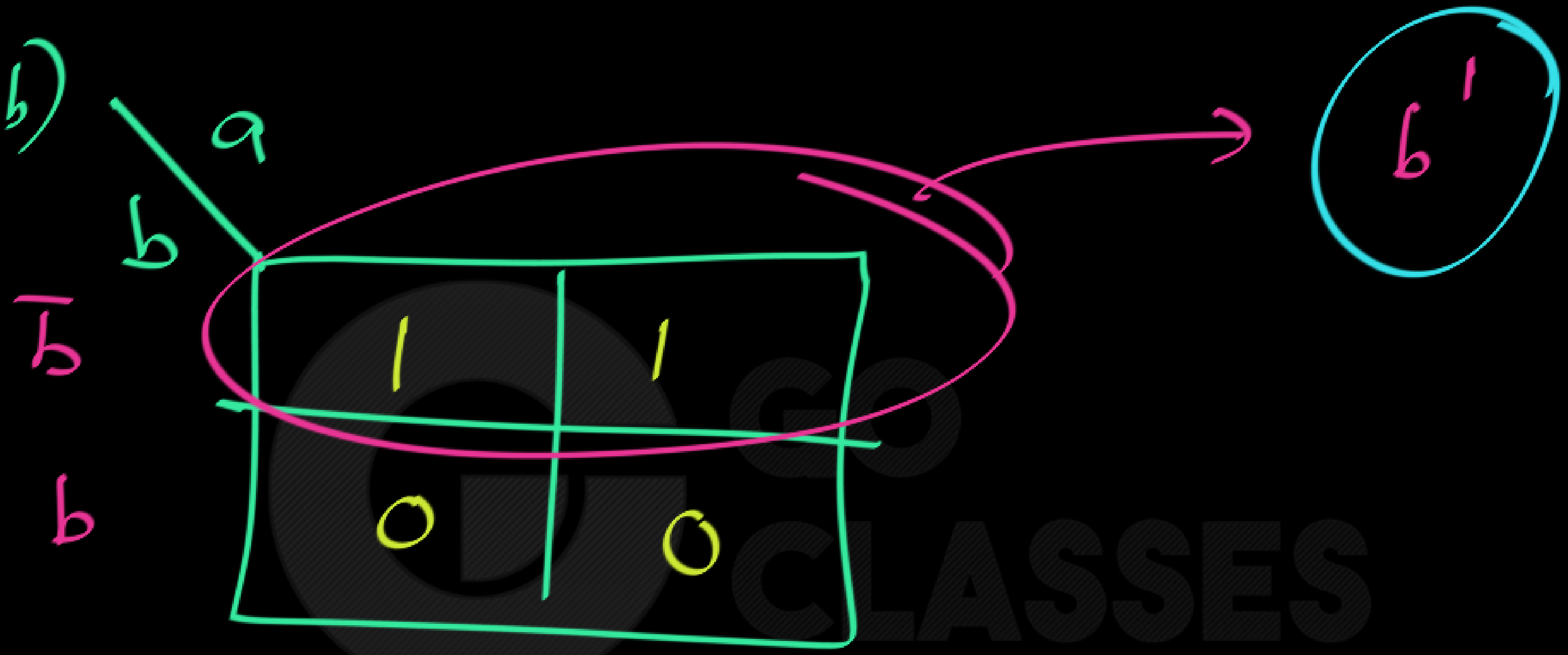


b ✓

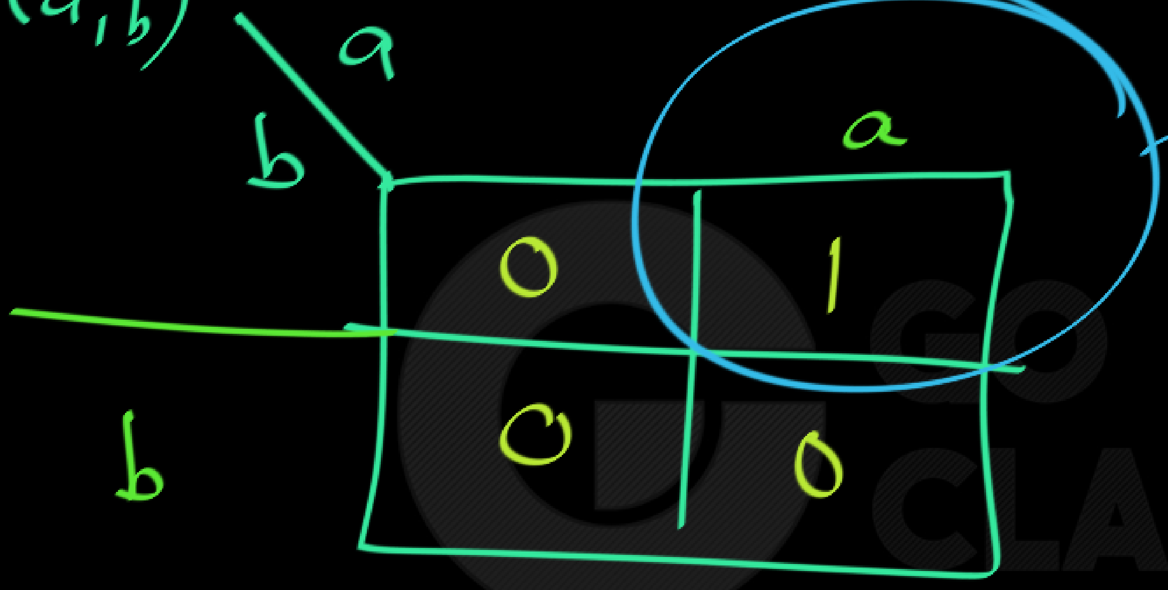
mSOP :

Cover only 1's
(No 0 should be covered)

$f(a,b)$

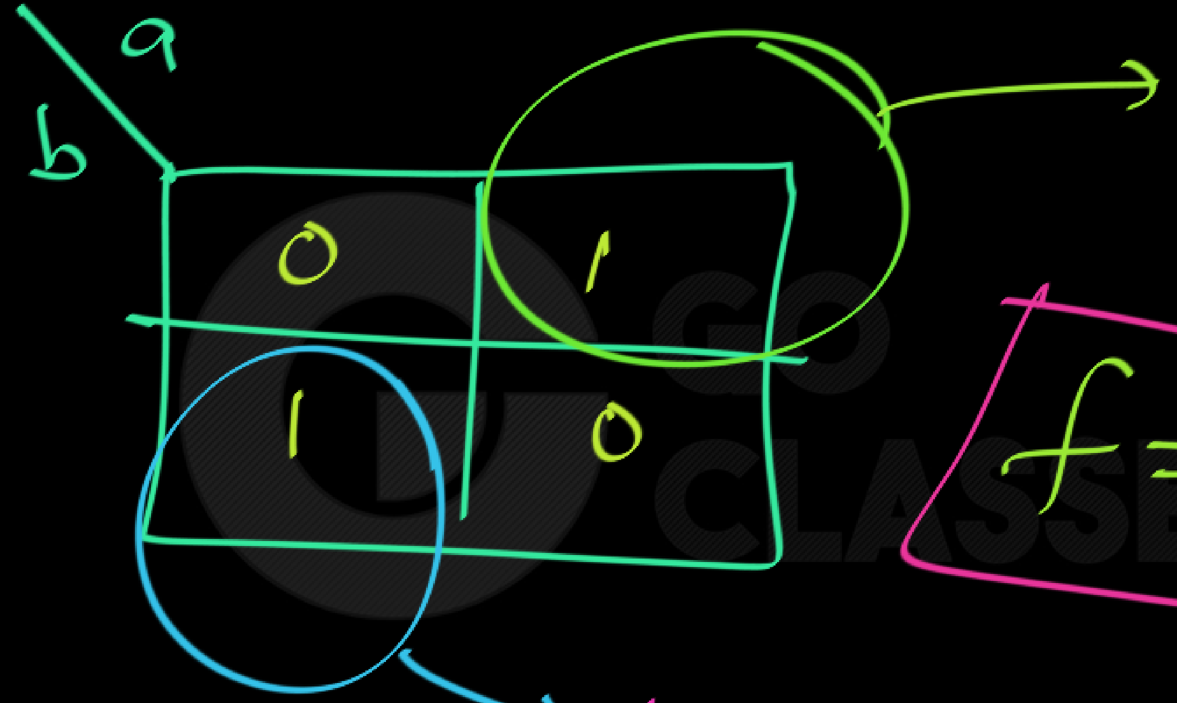


$f(a,b)$



$a\bar{b}$

$f(a,b)$

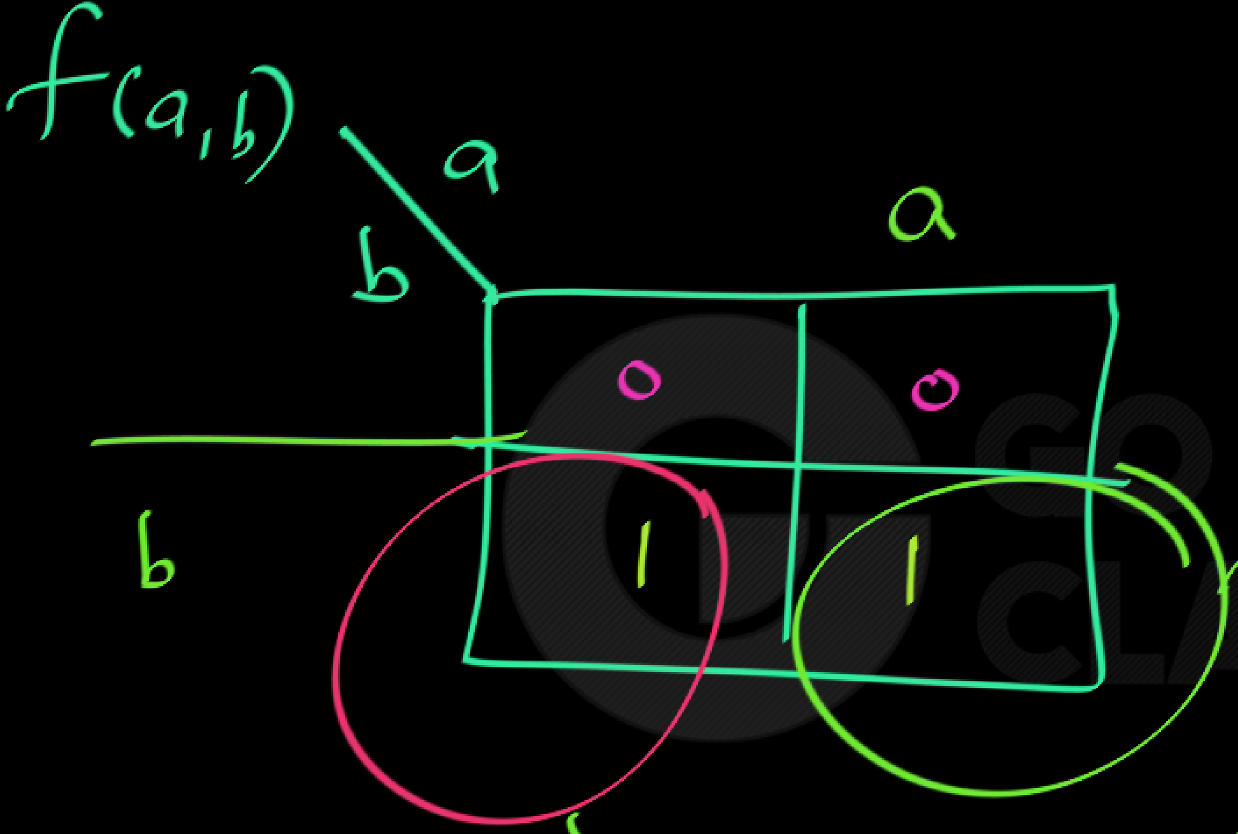


$\rightarrow a\bar{b}$

$\rightarrow b\bar{a}$

$$\boxed{f = a\bar{b} + b\bar{a}}$$

$\rightarrow$ msop



$\rightarrow ba$

$\rightarrow b\bar{a}$

Not minimum

$$\boxed{f = ba + b\bar{a}}$$

$f(a,b)$

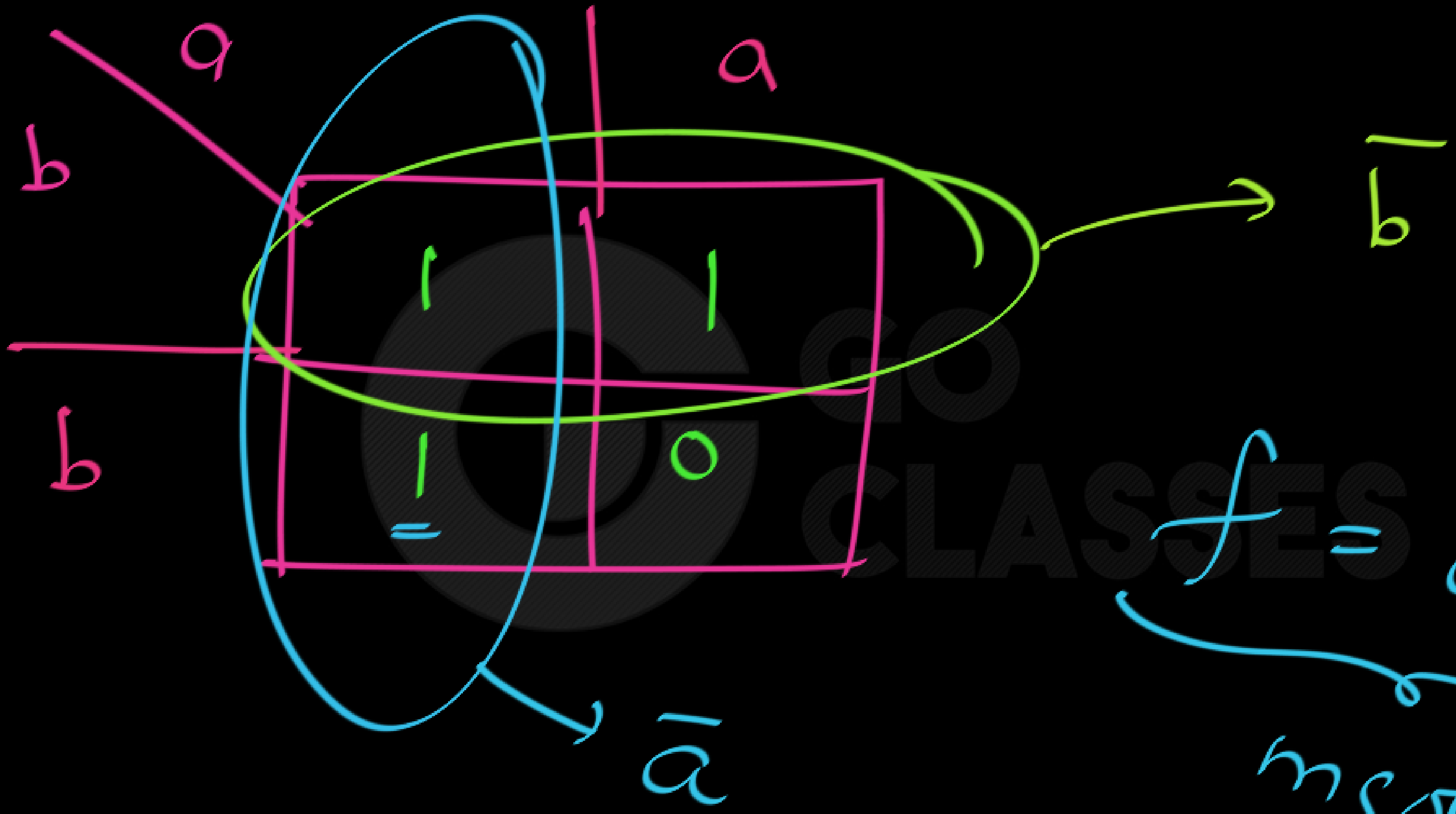
$b \backslash a$		
	0	0
	1	1

minimum SOP

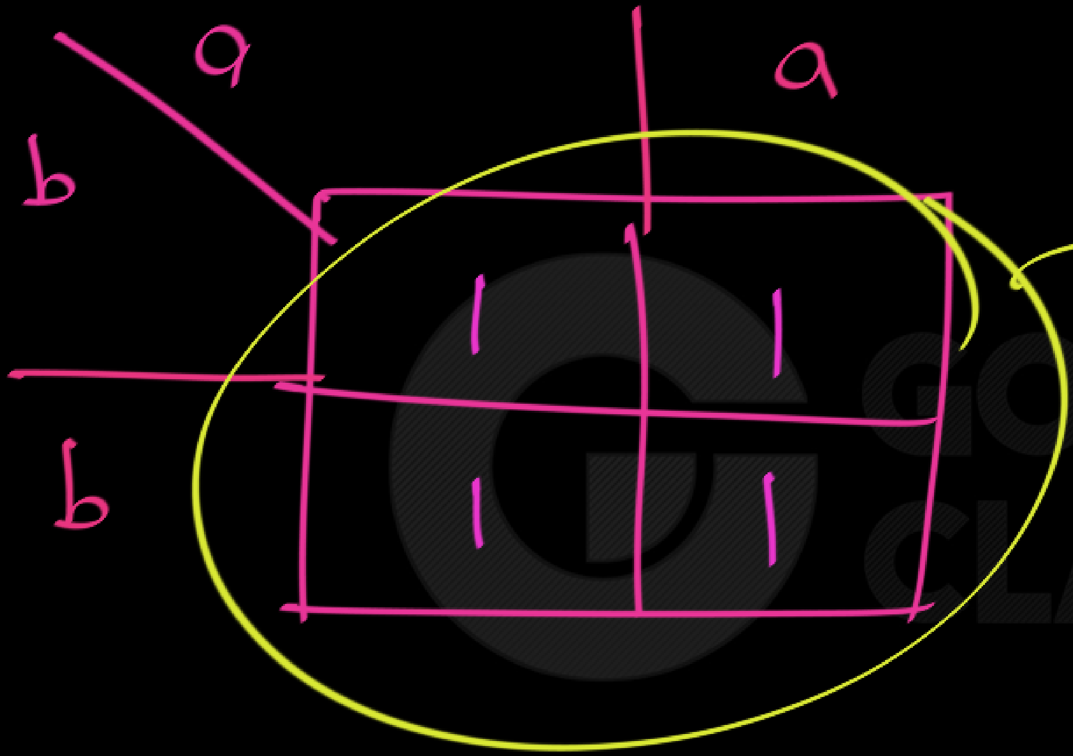
b ✓

To get minimum sop

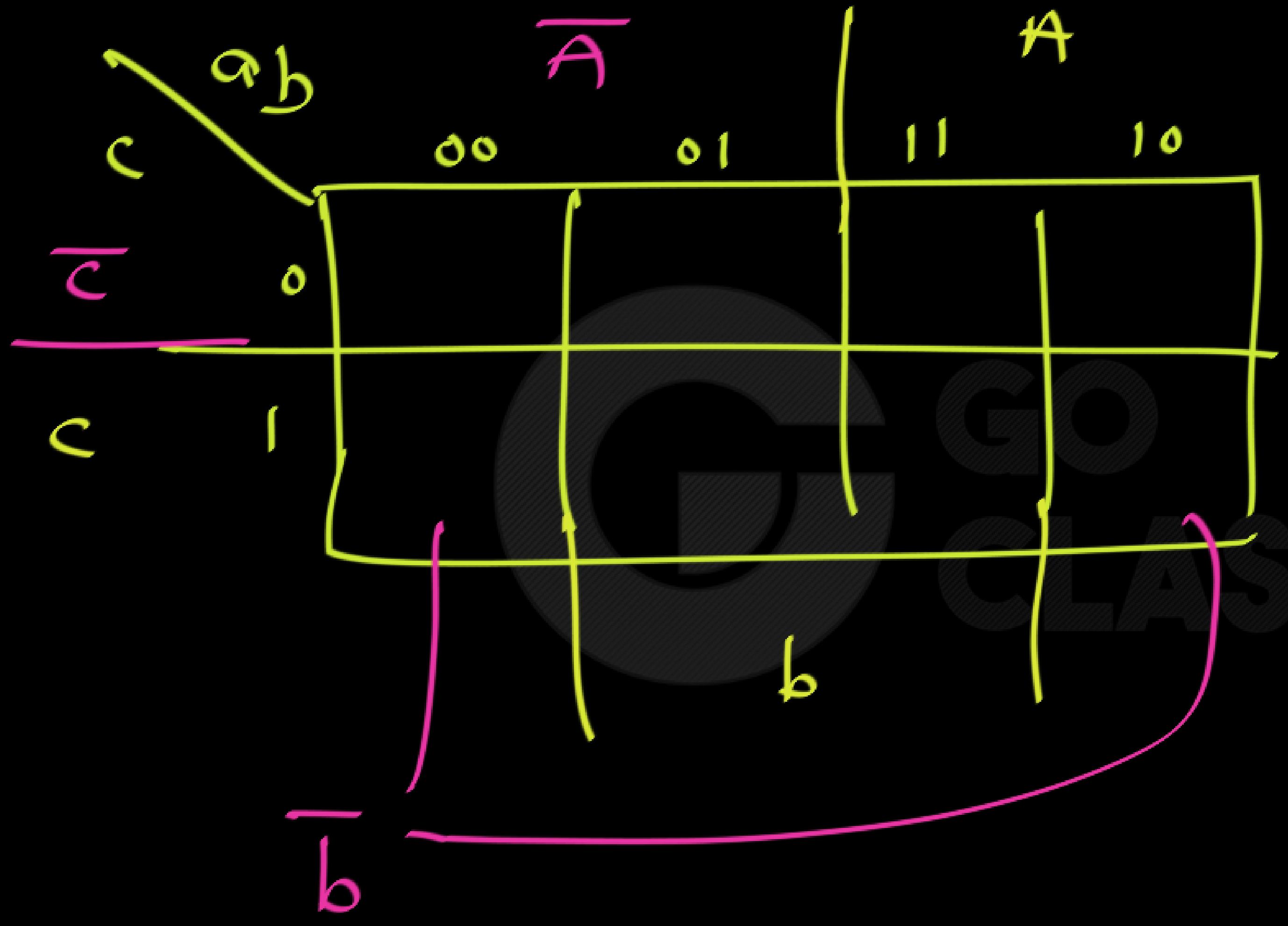
make "every cube" as big as possible.



$$\underbrace{f = \bar{a} + \bar{b}}_{\text{msop}}$$



$f = 1$


ab
c
$\bar{A}$
A
00
01
11
10
$\bar{c}$
0
c
1
b
$\bar{b}$

c \ ab	00	01	11	10
0	0	0	1	1
1	0	0	1	1

A

b

$\bar{c}$

c

2 size → ab

2^1 size → $a\bar{b}$

$$\boxed{f = ab + a\bar{b}}$$

not minimum

A

c \ ab	00	01	11	10
0	0	0	1	1
c 1	0	0	1	1

b

2^2 size

a

Note: f on n variables;

$\boxed{\text{Cube with } 2^{m} \text{ cells}}$

- #variables that change $= m$ ✓
- #variables that do not change $= (n-m)$ ✓
- Product term contains $(n-m)$ literals.

A

c \ ab	00	01	11	10
0	1	1	0	0
c 1	0	1	0	1

b

find msop?

c \ ab	00	01	11	10
0	1	1	0	0
1	0	1	0	1

A

b

SOP → Cover 1's NOT 0's.

$\bar{a}\bar{c}$

$\bar{a}b$

$ac\bar{b}$

2^0 Size

$f = \bar{a}\bar{c} + \bar{a}b + ac\bar{b}$

A

c \ ab	00	01	11	10
0	1	0	0	1
c 1	1	0	1	1

b

find mSOP?

c \ ab	00	01	11	10
0	1	0	0	1
c 1	1	0	1	1

A

b

$\bar{b}$

ac

$$\boxed{f = \bar{b} + ac}$$ ✓

A

c \ ab	00	01	11	10
$\bar{c}$ 0	1			1
c 1	1			1

b

$\bar{b}$

Size : 2

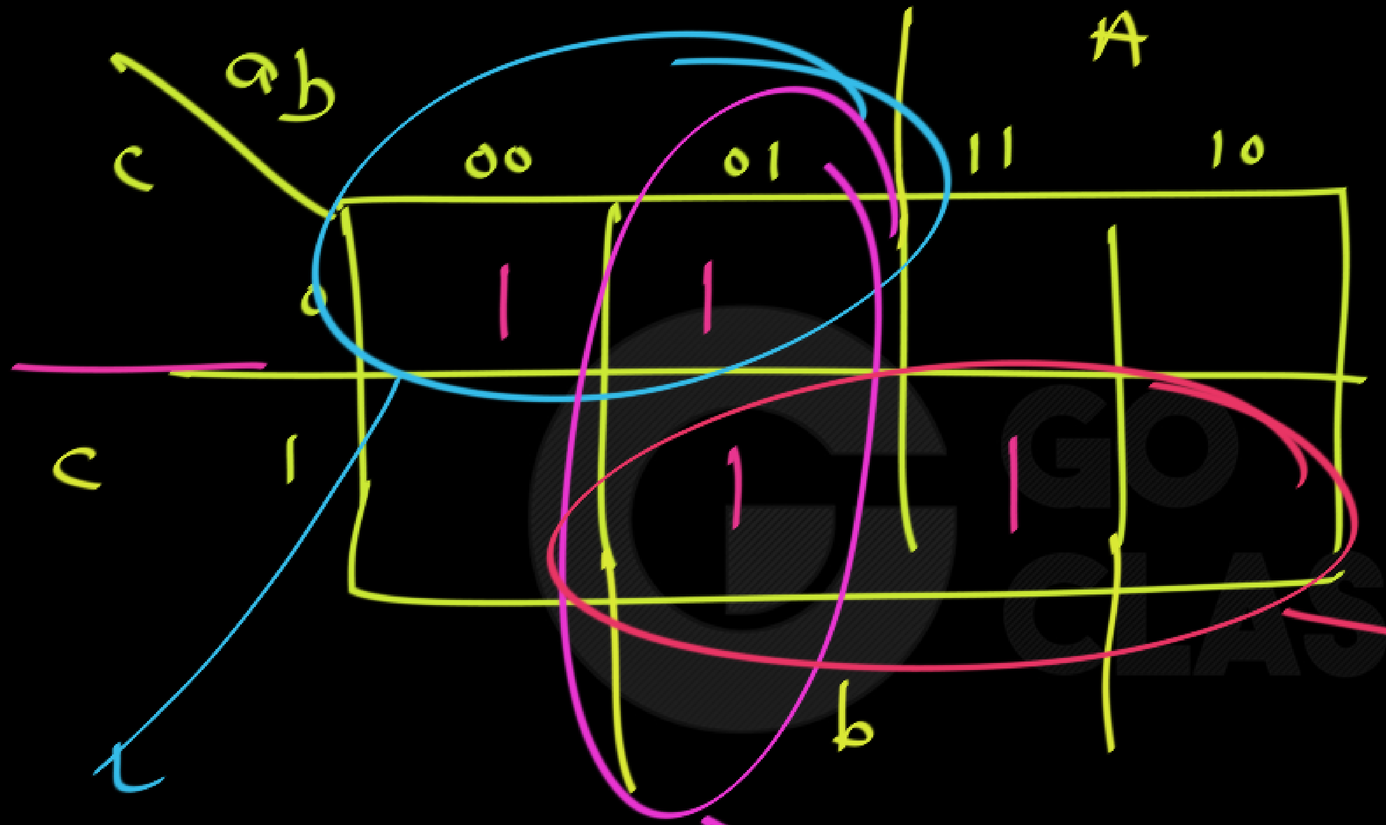
Changing variables = a, c

A

c \ ab	00	01	11	10
c' 0	1	1		
c 1		1	1	

b

find msop ?



$\bar{a}\bar{c}$

$\bar{a}b$

bc

$$f = \bar{a}\bar{c} + \bar{a}b + bc$$

minimum?

No.

A

c \ ab	00	01	11	10
0	1	1		
1		1	1	

b

$\bar{a}\bar{c}$

bc

$$\boxed{f = \bar{a}\bar{c} + bc}$$

msop

A

c \ ab	00	01	11	10
0	0	1	0	1
c 1	0	1	0	1

b

A

	ab $\to$	00	01	11	10
$\bar{c}$	0	0	1	0	1
c	1	0	1	0	1

b

$\to a\bar{b}$

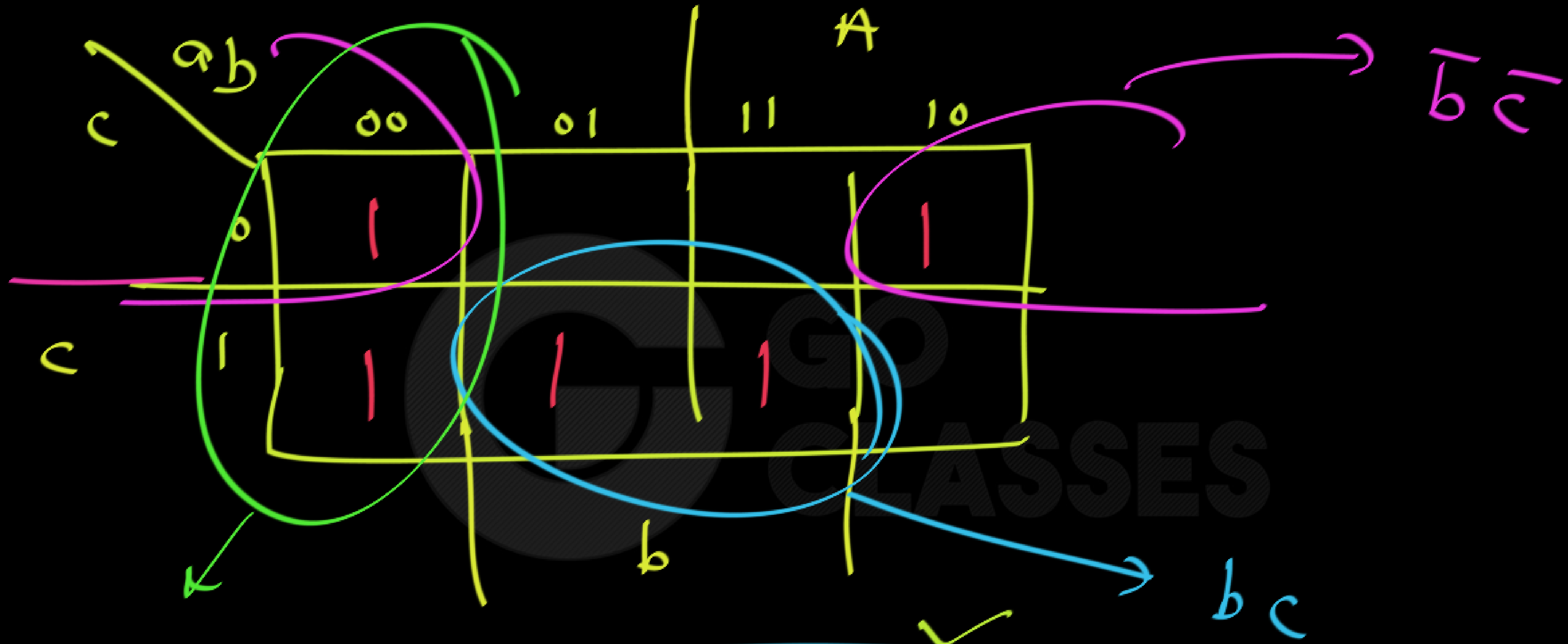
$\to \bar{a}b$

$$\boxed{f = a\bar{b} + \bar{a}b}$$

A

c \ ab	00	01	11	10
0	1			1
c 1	1	1	1	

b



$\bar{a}\bar{b}$

$$f = \boxed{\bar{a}\bar{b} + bc + \bar{b}\bar{c}}$$

3 terms

6 literals

A

c \ ab	00	01	11	10
0	1			1
1	1	1	1	

b

$\bar{b}\,\bar{c}$

$b\,c$

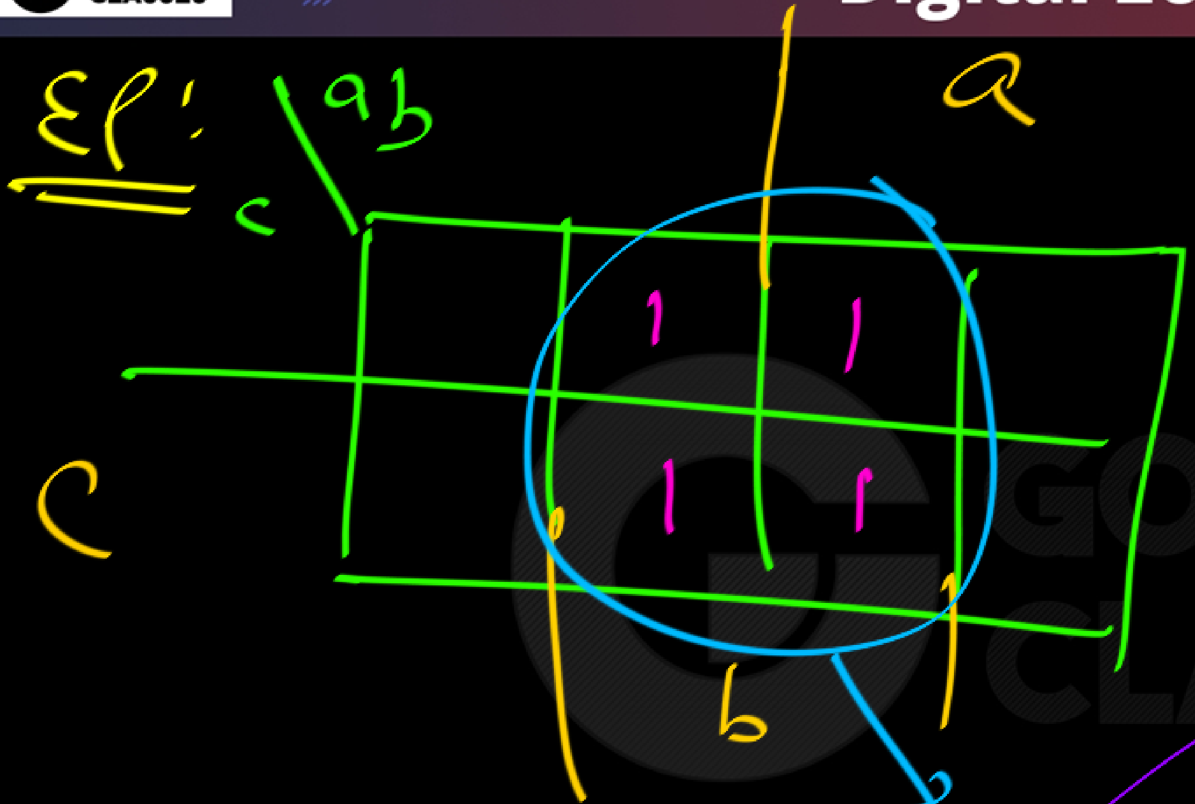
$c\,\bar{a}$

$$f = \boxed{\bar{a}c + bc + \bar{b}\,\bar{c}}$$

3 terms
6 literals

So minimum SOP may not be unique.

Eg:



minimum SOP:

b

a'bc'

abc'

a'bc

abc

0

1

Variable a, c changing

$a'bc'$

abc'

$a'bc$

abc

→ 2 variables change → a, c

4 minterms in cube c_1

$$a'bc' + abc' + a'bc + abc$$

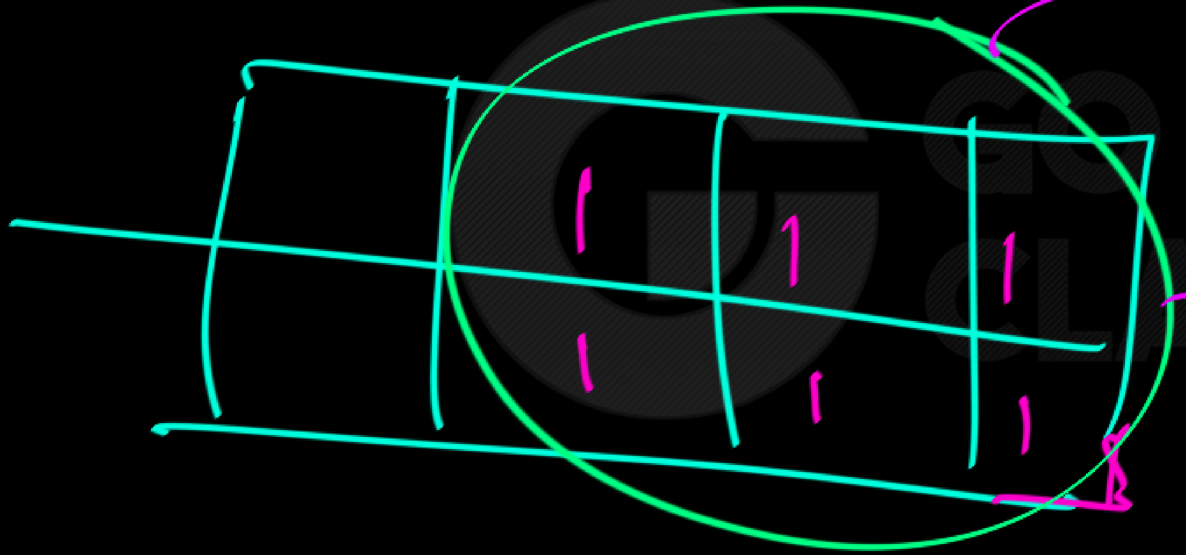
$$bc' + bc$$

$$b$$

larger cube ≡ small product term.

small cube ≡ large product term.

why $\underline{2^m}$ cells in a cube?



Not a cube

We cannot create a product term from these 6 cells.

Next Topic (VERY Important):

Using ***K-Map***

To get Minimized SOP Expression

"minimized"

Every cube as big as possible

SOP

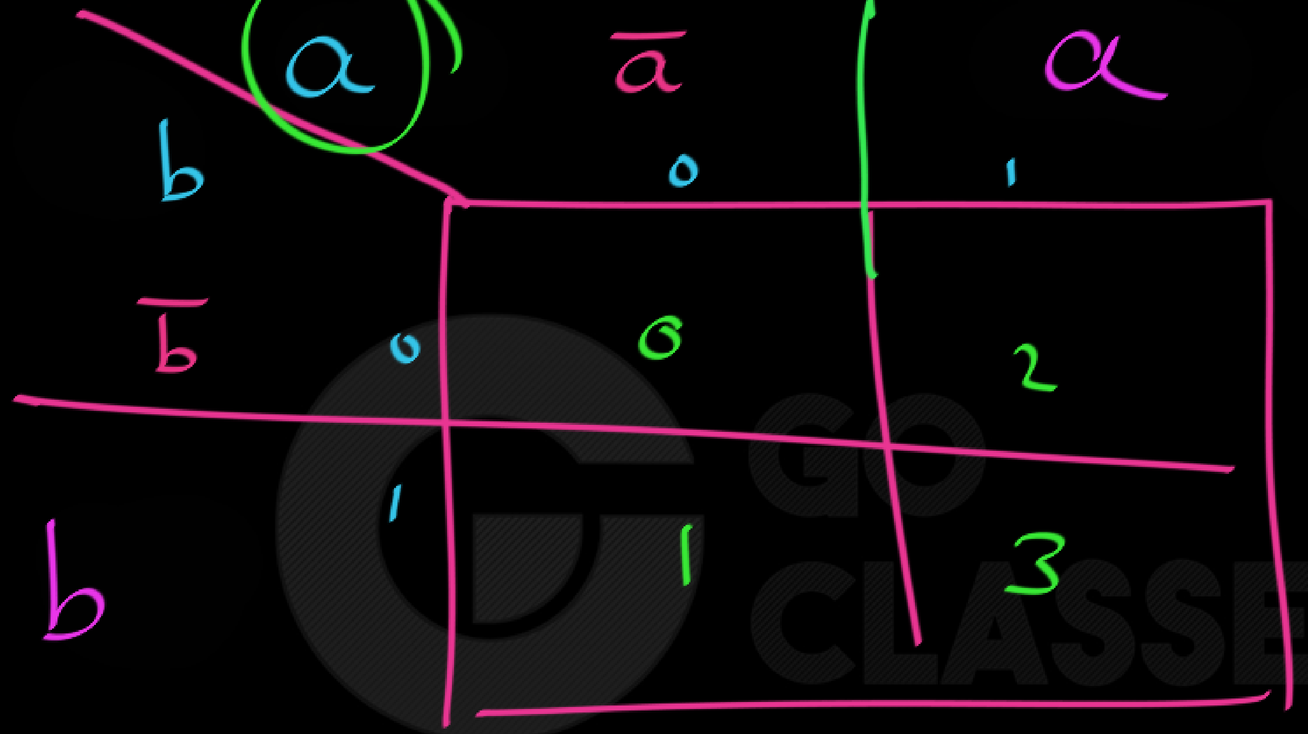
Expression:

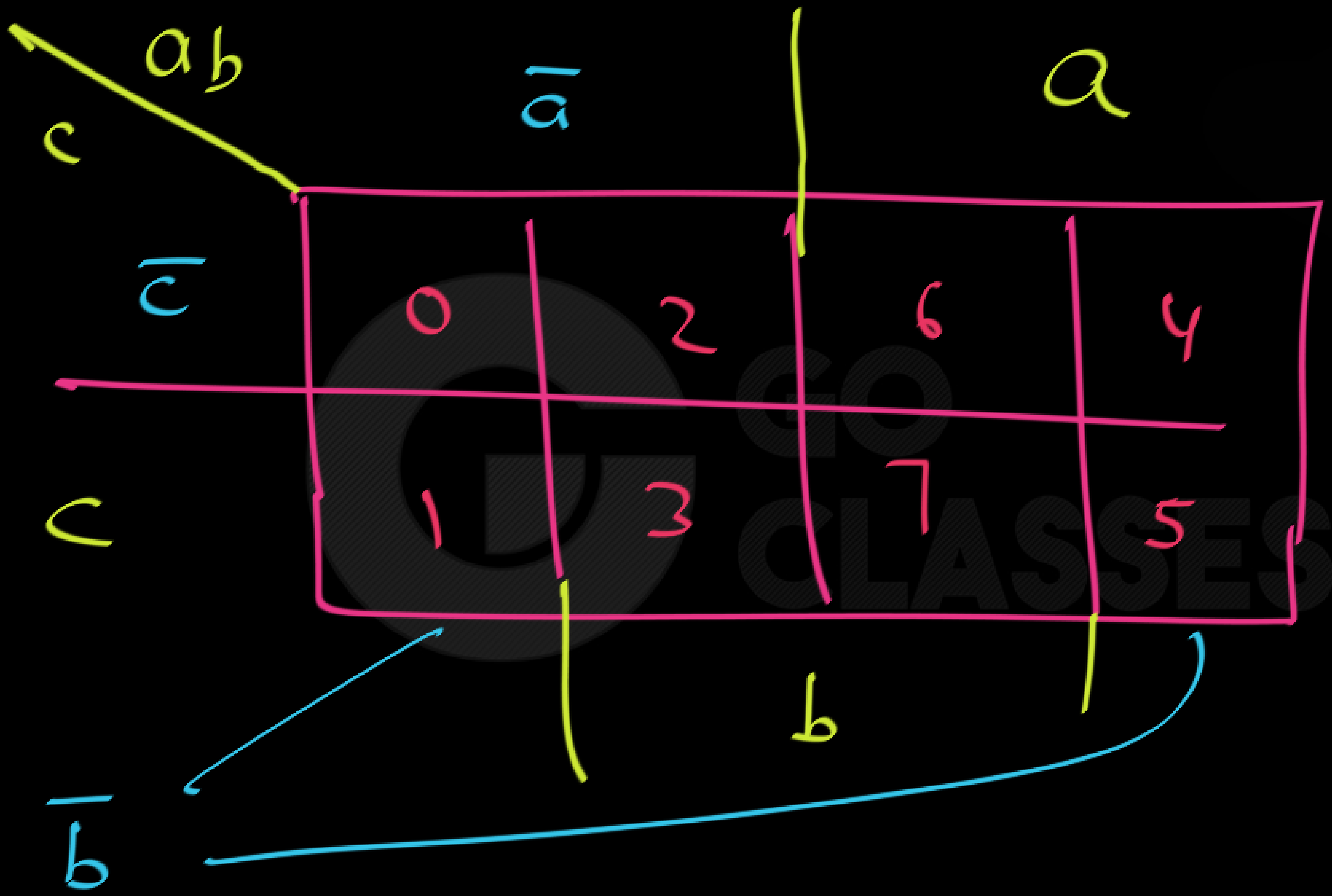
Covering all 1's and writing product terms

$a \backslash b$	0	1
0	0	1
1	2	3

b

a


a
b
$\bar{a}$
a
0
1
$\bar{b}$
0
0
2
b
1
1
3


ab
c
$\bar{a}$
a
$\bar{c}$
c
0
2
6
4
1
3
7
5
b
$\bar{b}$

A

cd \ ab	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

C

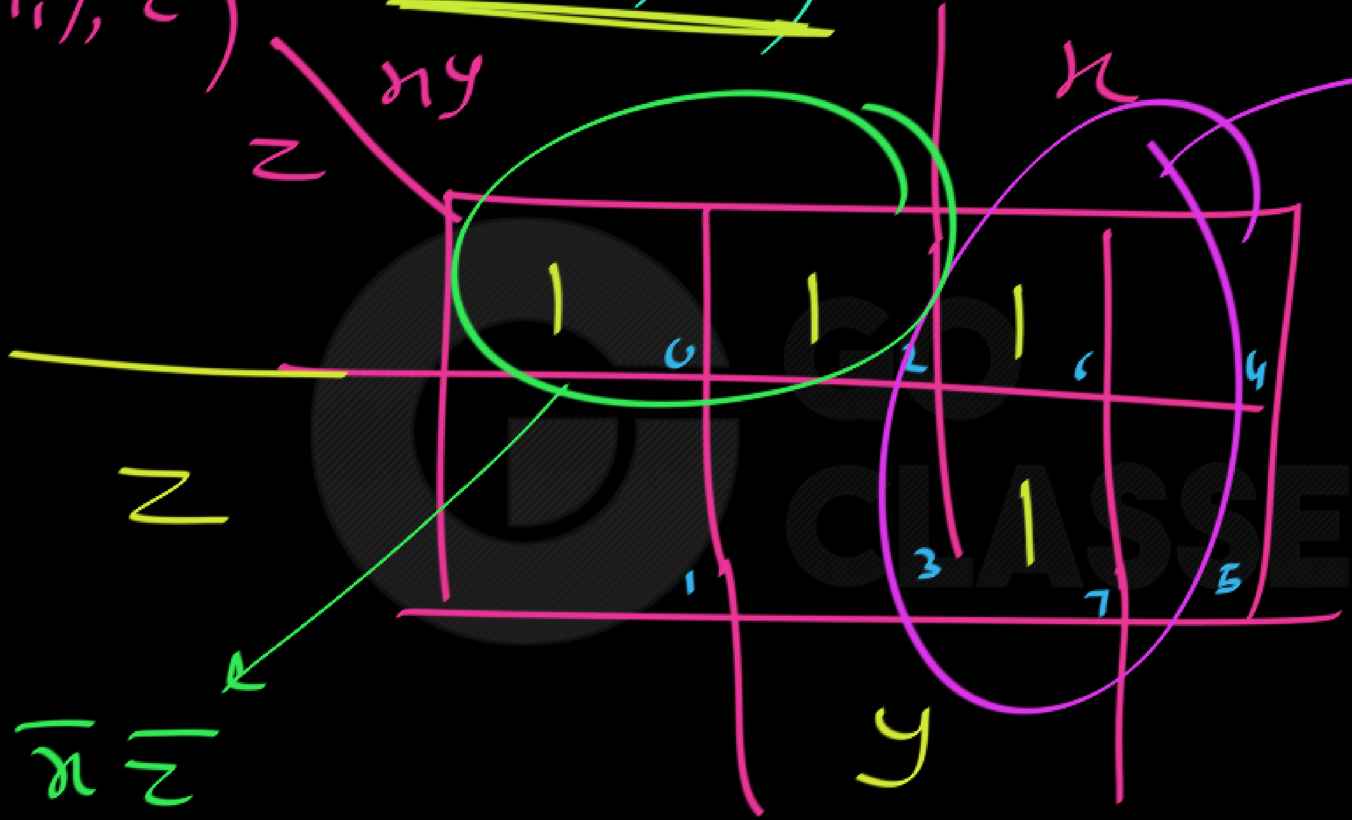
D

B

5. Simplify the following Boolean function , using three- variable maps:

a) $F(x,y,z) = \Sigma\ (0,2,6,7)$
b) $F(A,B,C\ ) = \Sigma\ (0,2,3,4,6)$

$f(x,y,z) = \sum(0,2,6,7)$



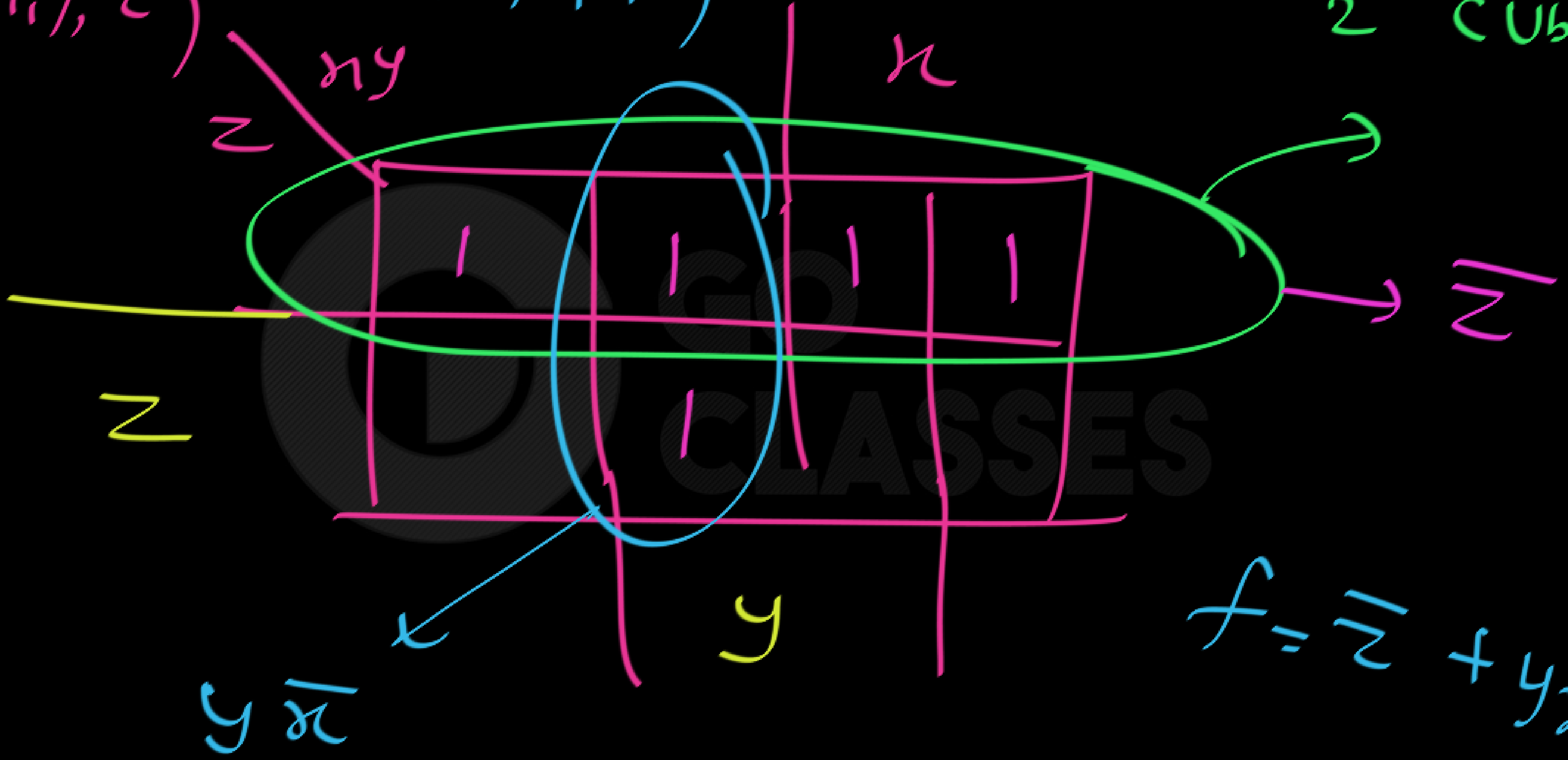
$\bar{x}\bar{z}$

xy

$f = xy + \bar{x}\bar{z}$

$f(x,y,z) = \Sigma(0,2,3,4,6)$

2^2 cube



$f = \bar{z} + y\bar{x}$

5. Simplify the following Boolean function , using three- variable maps:

[redacted]

Hw

c) $F(a,b,c) = \Sigma\,(0,1,2,3,7)$
d) $F(x,y,z) = \Sigma\,(3,5,6,7)$

A Three-Variable Karnaugh Map

- For a three-variable expression with inputs x, y, z, the arrangement of minterms is more tricky:

X \ YZ	00	01	11	10
0	x'y'z'	x'y'z	x'yz	x'yz'
1	xy'z'	xy'z	xyz	xyz'

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- Another way to label the K-map (use whichever you like):

			y	y
	x'y'z'	x'y'z	x'yz	x'yz'
X	xy'z'	xy'z	xyz	xyz'
		z	z	

			y	y
	m_0	m_1	m_3	m_2
X	m_4	m_5	m_7	m_6
		z	z	

Why the funny ordering?

- With this ordering, any group of 2, 4 or 8 adjacent squares on the map contains common literals that can be factored out

			y	y
	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
X	$xy'z'$	$xy'z$	xyz	xyz'
		Z	Z	

$$
\begin{aligned}
& x'y'z + x'yz \\
= \; & x'z(y' + y) \\
= \; & x'z \bullet 1 \\
= \; & x'z
\end{aligned}
$$

- "Adjacency" includes wrapping around the left and right sides:

			y	y
	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
X	$xy'z'$	$xy'z$	xyz	xyz'
		Z	Z	

$$
\begin{aligned}
& x'y'z' + xy'z' + x'yz' + xyz' \\
= \; & z'(x'y' + xy' + x'y + xy) \\
= \; & z'(y'(x' + x) + y(x' + x)) \\
= \; & z'(y'+y) \\
= \; & z'
\end{aligned}
$$

- We'll use this property of adjacent squares to do our simplifications.

GO CLASSES

Karnaugh Map Simplifications

- Imagine a two-variable sum of minterms:

$$x'y' + x'y$$

- Both of these minterms appear in the top row of a Karnaugh map, which means that they both contain the literal x'

		y
	$x'y'$	$x'y$
X	xy'	xy

- What happens if you simplify this expression using Boolean algebra?

$$\begin{aligned} x'y' + x'y &= x'(y' + y) && [\text{ Distributive }] \\ &= x' \bullet 1 && [\ y + y' = 1\ ] \\ &= x' && [\ x \bullet 1 = x\ ] \end{aligned}$$

More Two-Variable Examples

- Another example expression is x'y + xy
  - Both minterms appear in the right side, where y is uncomplemented
  - Thus, we can reduce x'y + xy to just y

		y
	x'y'	x'y
X	xy'	xy

- How about x'y' + x'y + xy?
  - We have x'y' + x'y in the top row, corresponding to x'
  - There's also x'y + xy in the right side, corresponding to y
  - This whole expression can be reduced to x' + y

		y
	x'y'	x'y
X	xy'	xy

Example K-map Simplification

- Let's consider simplifying $f(x,y,z) = xy + y'z + xz$
- First, you should convert the expression into a sum of minterms form, if it's not already
  - The easiest way to do this is to make a truth table for the function, and then read off the minterms
  - You can either write out the literals or use the minterm shorthand
- Here is the truth table and sum of minterms for our example:

x	y	z	f(x,y,z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned} f(x,y,z) &= x'y'z + xy'z + xyz' + xyz \\ &= m_1 + m_5 + m_6 + m_7 \end{aligned}$$

GO CLASSES

Unsimplifying Expressions

- You can also convert the expression to a sum of minterms with Boolean algebra
  - Apply the distributive law in reverse to add in missing variables.
  - Very few people actually do this, but it's occasionally useful.

$$
\begin{aligned}
xy + y'z + xz &= (xy \bullet 1) + (y'z \bullet 1) + (xz \bullet 1) \\
&= (xy \bullet (z' + z)) + (y'z \bullet (x' + x)) + (xz \bullet (y' + y)) \\
&= (xyz' + xyz) + (x'y'z + xy'z) + (xy'z + xyz) \\
&= xyz' + xyz + x'y'z + xy'z
\end{aligned}
$$

- In both cases, we're actually "unsimplifying" our example expression
  - The resulting expression is larger than the original one!
  - But having all the individual minterms makes it easy to combine them together with the K-map

Making the Example K-map

- Next up is drawing and filling in the K-map
  - Put 1s in the map for each minterm, and 0s in the other squares
  - You can use either the minterm products or the shorthand to show you where the 1s and 0s belong
- In our example, we can write f(x,y,z) in two equivalent ways

$$f(x,y,z) = x'y'z + xy'z + xyz' + xyz$$

			y	y
	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x	$xy'z'$	$xy'z$	xyz	xyz'
		z	z	

$$f(x,y,z) = m_1 + m_5 + m_6 + m_7$$

			y	y
	m_0	m_1	m_3	m_2
x	m_4	m_5	m_7	m_6
		z	z	

- In either case, the resulting K-map is shown below

			y	y
	0	1	0	0
x	0	1	1	1
		z	z	

Grouping the Minterms Together

- The most difficult step is grouping together all the 1s in the K-map
  - Make rectangles around groups of one, two, four or eight 1s
  - All of the 1s in the map should be included in at least one rectangle
  - Do *not* include any of the 0s

			y	
	0	1	0	0
X	0	1	1	1
		z	z	

- Each group corresponds to one product term. For the simplest result:
  - Make as few rectangles as possible, to minimize the number of products in the final expression.
  - Make each rectangle as large as possible, to minimize the number of literals in each term.
  - It's all right for rectangles to overlap, if that makes them larger.

Reading the MSP from the K-map

- Finally, you can find the minimal SoP expression
  - Each rectangle corresponds to one product term
  - The product is determined by finding the common literals in that rectangle

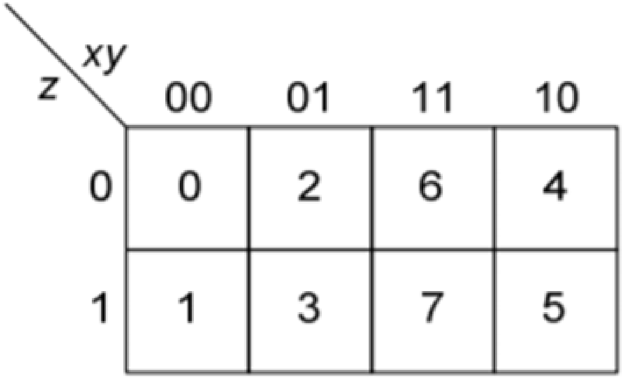
			y	y
	0	1	0	0
X	0	1	1	1
		Z	Z	

			y	y
	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
X	$xy'z'$	$xy'z$	xyz	xyz'
		Z	Z	

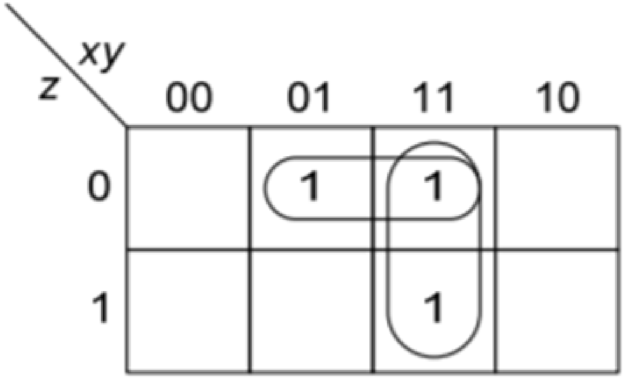
- For our example, we find that $xy + y'z + xz = y'z + xy$. (This is one of the additional algebraic laws from last time.)

Example: $f = yz' + xy$

- Use of cell 6 in forming both cubes justified by idempotent law



(a) Location of minterms in a three-variable map.



(b) Map for function $f(x,y,z) = \sum(2,6,7) = yz' + xy$.

Corresponding algebraic manipulations:

$$f = x'yz' + xyz' + xyz$$
$$= x'yz' + xyz' + xyz' + xyz$$
$$= yz'(x' + x) + xy(z' + z)$$
$$= yz' + xy$$

Practice K-map 1

- Simplify the sum of minterms $m_1 + m_3 + m_5 + m_6$

			y	y
x				
		z	z	

			y	y
	m_0	m_1	m_3	m_2
x	m_4	m_5	m_7	m_6
		z	z	

Solutions for Practice K-map 1

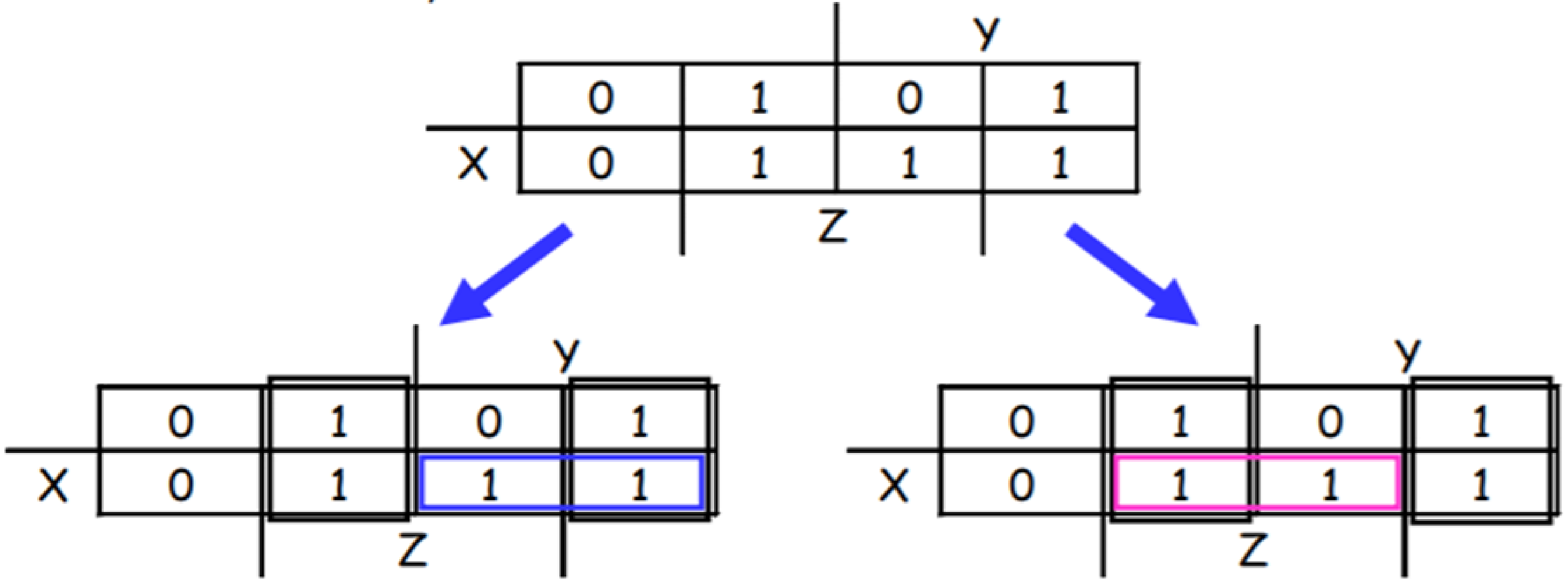
- Here is the filled in K-map, with all groups shown
  - The magenta and green groups overlap, which makes each of them as large as possible
  - Minterm m_6 is in a group all by its lonesome

			y	y
	0	1	1	0
X	0	1	0	1
		Z	Z	

- The final MSP here is $x'z + y'z + xyz'$

K-maps can be tricky!

- There may not necessarily be a *unique* MSP. The K-map below yields two valid and equivalent MSPs, because there are two possible ways to include minterm m_7

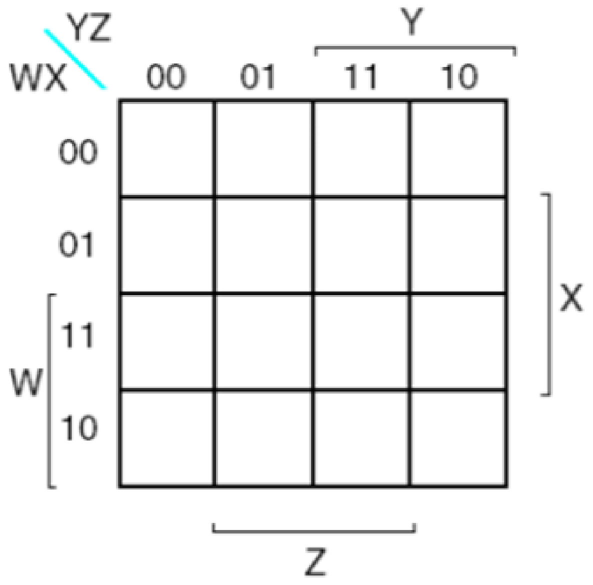


$y'z + yz' + xy$ $\qquad$ $y'z + yz' + xz$

- Remember that overlapping groups is possible, as shown above

Four-variable K-maps

- We can do four-variable expressions too!
  - The minterms in the third and fourth columns, *and* in the third and fourth rows, are switched around.
  - Again, this ensures that adjacent squares have common literals



- Grouping minterms is similar to the three-variable case, but:
  - You can have rectangular groups of 1, 2, 4, 8 or 16 minterms
  - You can wrap around *all four* sides

4 variable!

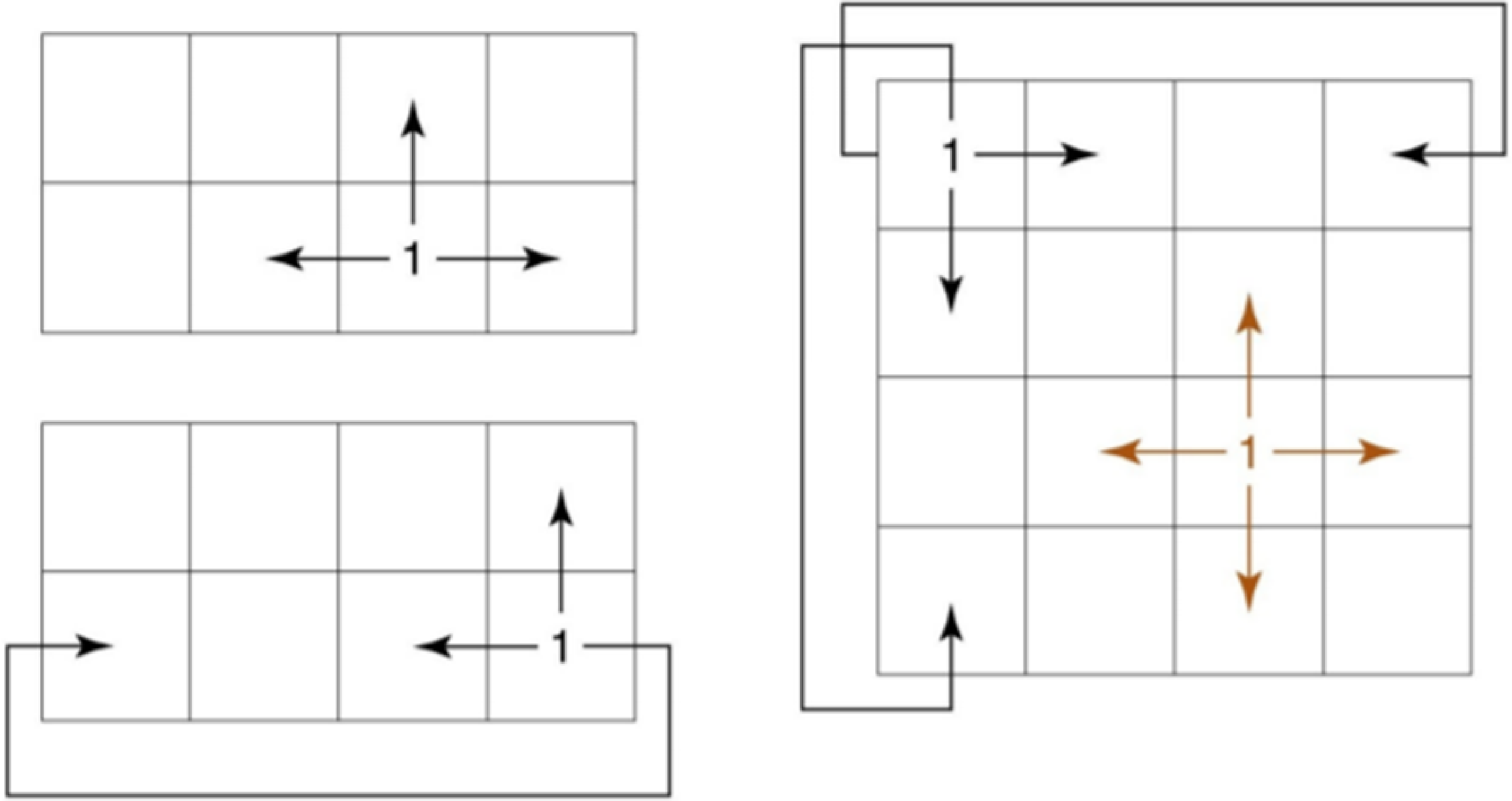
Map 3.8 The four-variable map.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

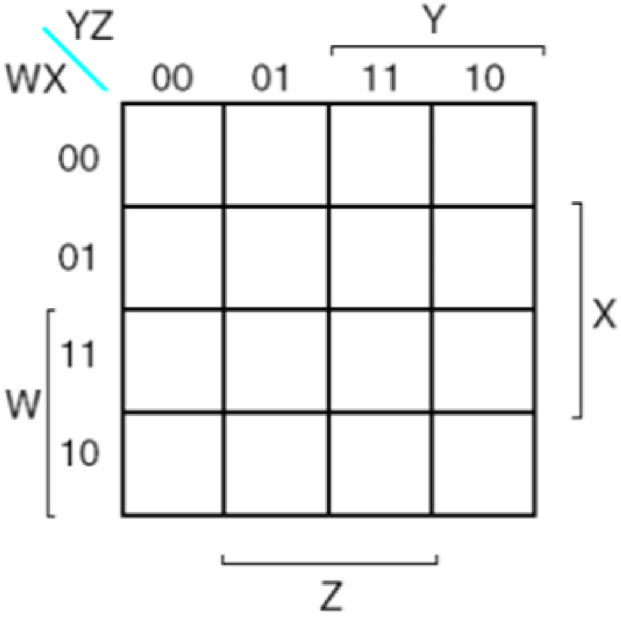
CD \ AB	00	01	11	10
00	$A'B'C'D'$	$A'BC'D'$	$ABC'D'$	$AB'C'D'$
01	$A'B'C'D$	$A'BC'D$	$ABC'D$	$AB'C'D$
11	$A'B'CD$	$A'BCD$	$ABCD$	$AB'CD$
10	$A'B'CD'$	$A'BCD'$	$ABCD'$	$AB'CD'$

Finding Minimum SOP Using K-Map

Map 3.14 Adjacencies on three- and four-variable maps.



Four-variable K-maps



			y	y	
	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'	
	w'xy'z'	w'xy'z	w'xyz	w'xyz'	x
w	wxy'z'	wxy'z	wxyz	wxyz'	x
w	wx'y'z'	wx'y'z	wx'yz	wx'yz'	
		z	z		

			y	y	
	m_0	m_1	m_3	m_2	
	m_4	m_5	m_7	m_6	x
w	m_{12}	m_{13}	m_{15}	m_{14}	x
w	m_8	m_9	m_{11}	m_{10}	
		z	z		

Example: Simplify $m_0+m_2+m_5+m_8+m_{10}+m_{13}$

- The expression is already a sum of minterms, so here's the K-map:

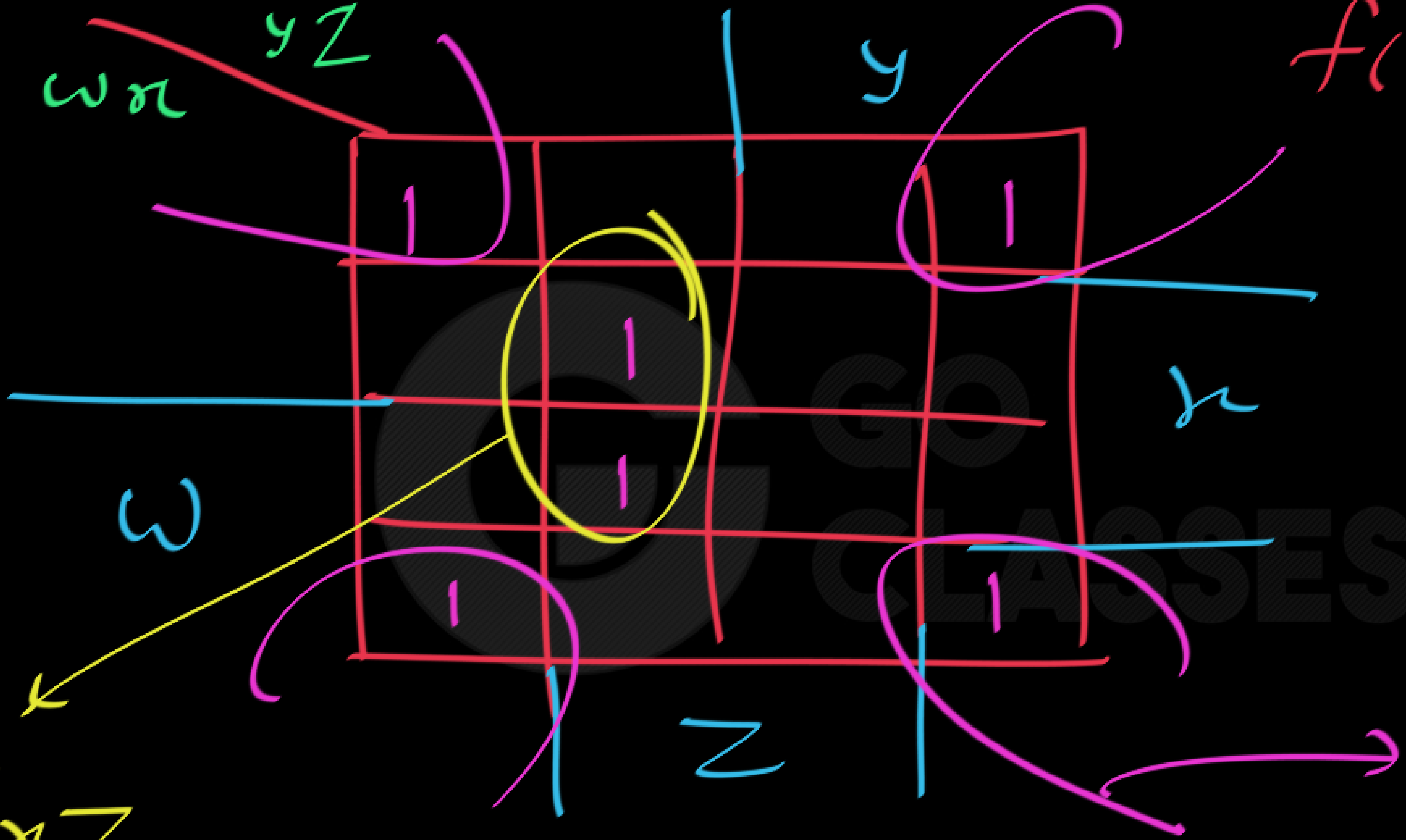
			y		
	1	0	0	1	
	0	1	0	0	x
w	0	1	0	0	
	1	0	0	1	
		z			

			y		
	m_0	m_1	m_3	m_2	
	m_4	m_5	m_7	m_6	x
w	m_{12}	m_{13}	m_{15}	m_{14}	
	m_8	m_9	m_{11}	m_{10}	
		z			

- We can make the following groups, resulting in the MSP $x'z' + xy'z$

			y		
	1	0	0	1	
	0	$\boxed{1}$	0	0	x
w	0	$\boxed{1}$	0	0	
	1	0	0	1	
		z			

			y		
	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'	
	w'xy'z'	w'xy'z	w'xyz	w'xyz'	x
w	wxy'z'	wxy'z	wxyz	wxyz'	
	wx'y'z'	wx'y'z	wx'yz	wx'yz'	
		z			



$$f(wxyz) = \underbrace{\bar{y}xz + \bar{z}\bar{x}}$$

$$\bar{y}xz$$

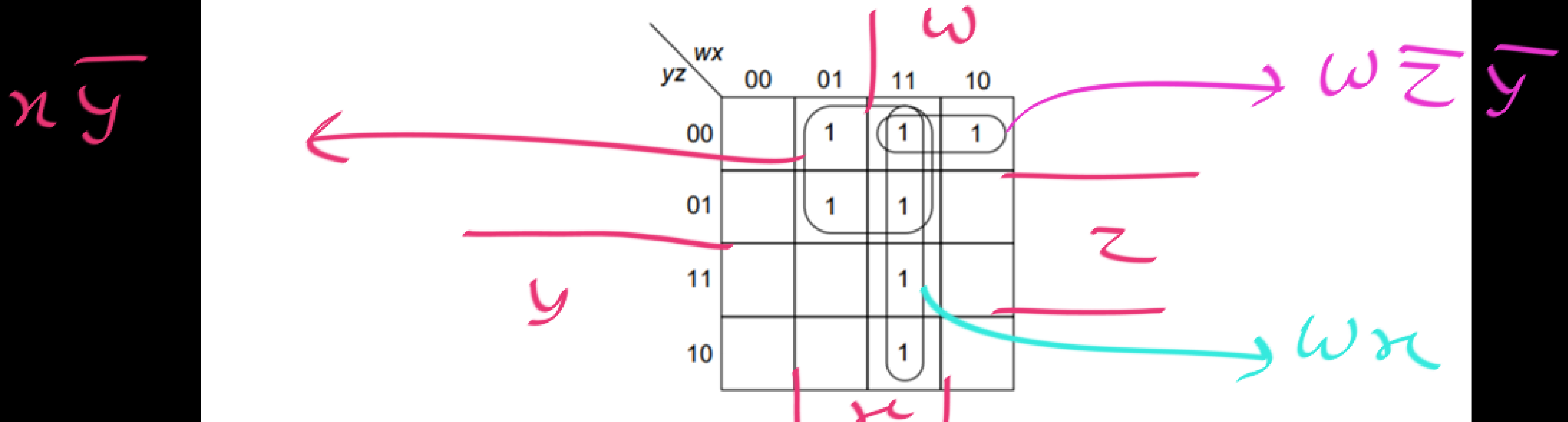
$$\rightarrow \bar{z}\bar{x}$$

Simplification and Minimization of Functions

Cube: collection of 2^m cells, each adjacent to *m* cells of the collection

- Cube is said to cover these cells
- Cube expressed by a product of *n-m* literals for a function containing *n* variables
- *m* literals not in the product said to be eliminated

Example: $w'xy'z' + w'xy'z + wxy'z' + wxy'z = xy'(w'z' + w'z + wz' + wz) = xy'$



(*d*) Map for function $f(w,x,y,z) = \sum(4,5,8,12,13,14,15) = wx + xy' + wy'z'$.

Minimal expression: cover all the 1 cells with the smallest number of cubes such that each cube is as large as possible

- A cube contained in a larger cube must never be selected
- If there is more than one way of covering the map with a minimal number of cubes, select the cover with larger cubes
- A cube contained in any combination of other cubes already selected in the cover is redundant by virtue of the consensus theorem

Rules for minimization:

1. First, cover those 1 cells by cubes that cannot be combined with other 1 cells; continue to 1 cells that have a single adjacent 1 cell (thus can form cubes of only two cells)
2. Next, combine 1 cells that yield cubes of four cells, but are not part of any cube of eight cells, and so on
3. Minimal expression: collection of cubes that are as large and as few in number as possible, such that each 1 cell is covered by at least one cube

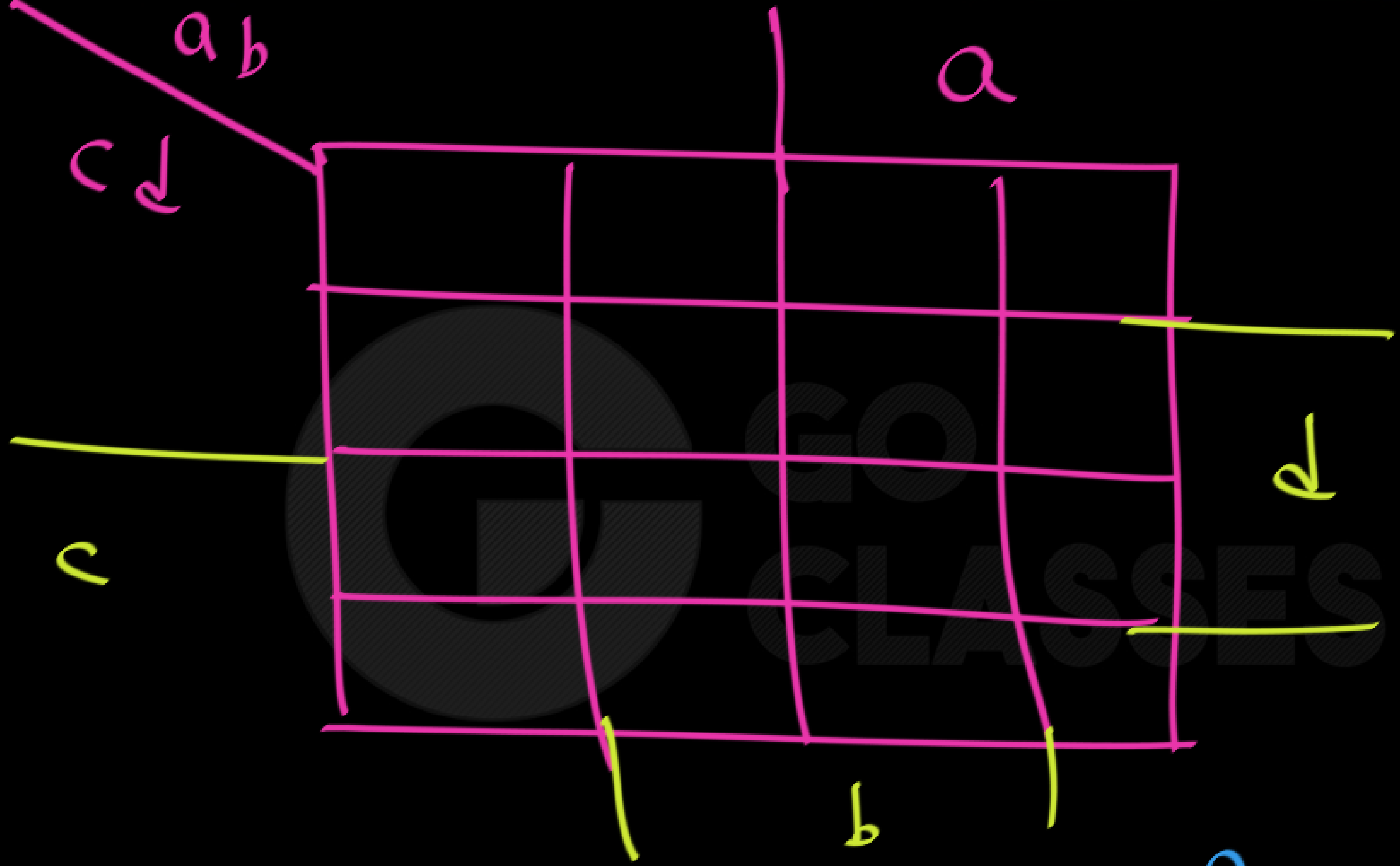
ab　　　　　a　　　　　$f(a,b,c,d)$

cd

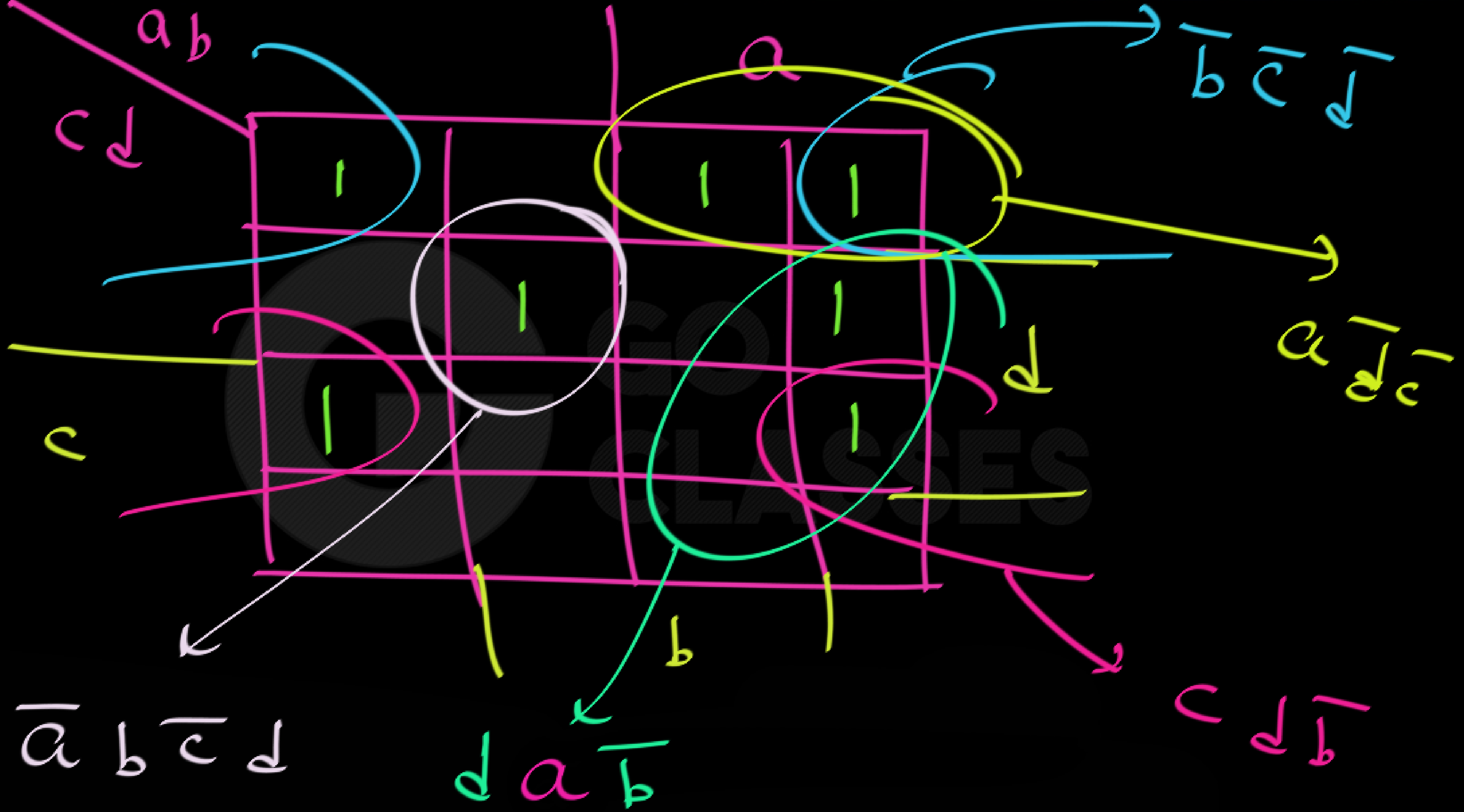
0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

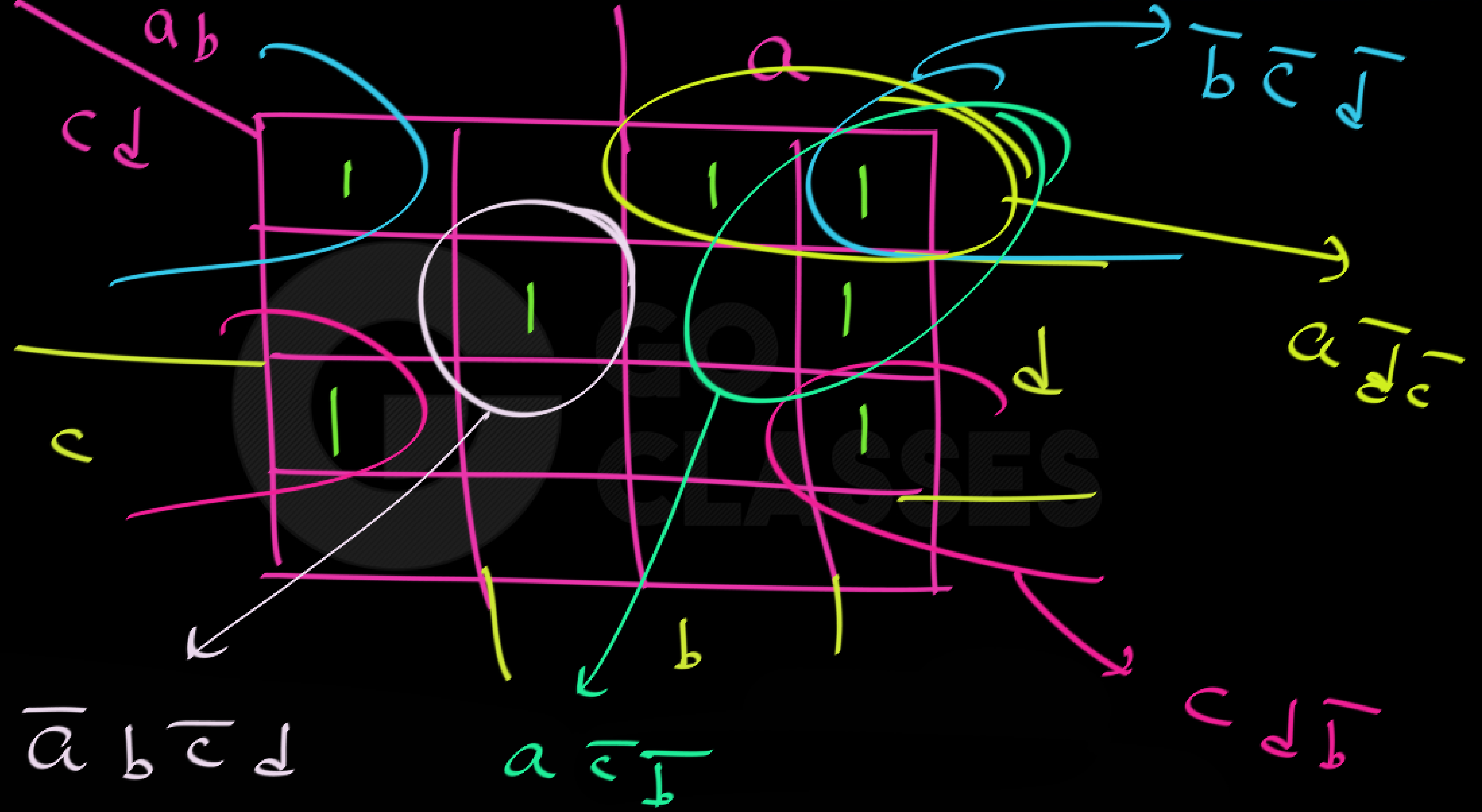
c　　　　　d

b



$f = \sum(0, 3, 5, 8, 9, 11, 12)$ min. SOP ? → Not be Unique.


ab
cd
a
$\bar{b}\bar{c}\bar{d}$
1
1
1
1
1
1
1
$a\bar{d}\bar{c}$
d
c
b
$\bar{a}b\bar{c}d$
$da\bar{b}$
$cd\bar{b}$


ab
cd
a
$\bar{b}\bar{c}\bar{d}$
1
1
1
1
1
1
1
$a\bar{d}\bar{c}$
d
c
b
$\bar{a}b\bar{c}d$
$a\bar{c}\bar{b}$
$cd\bar{b}$

ab

a

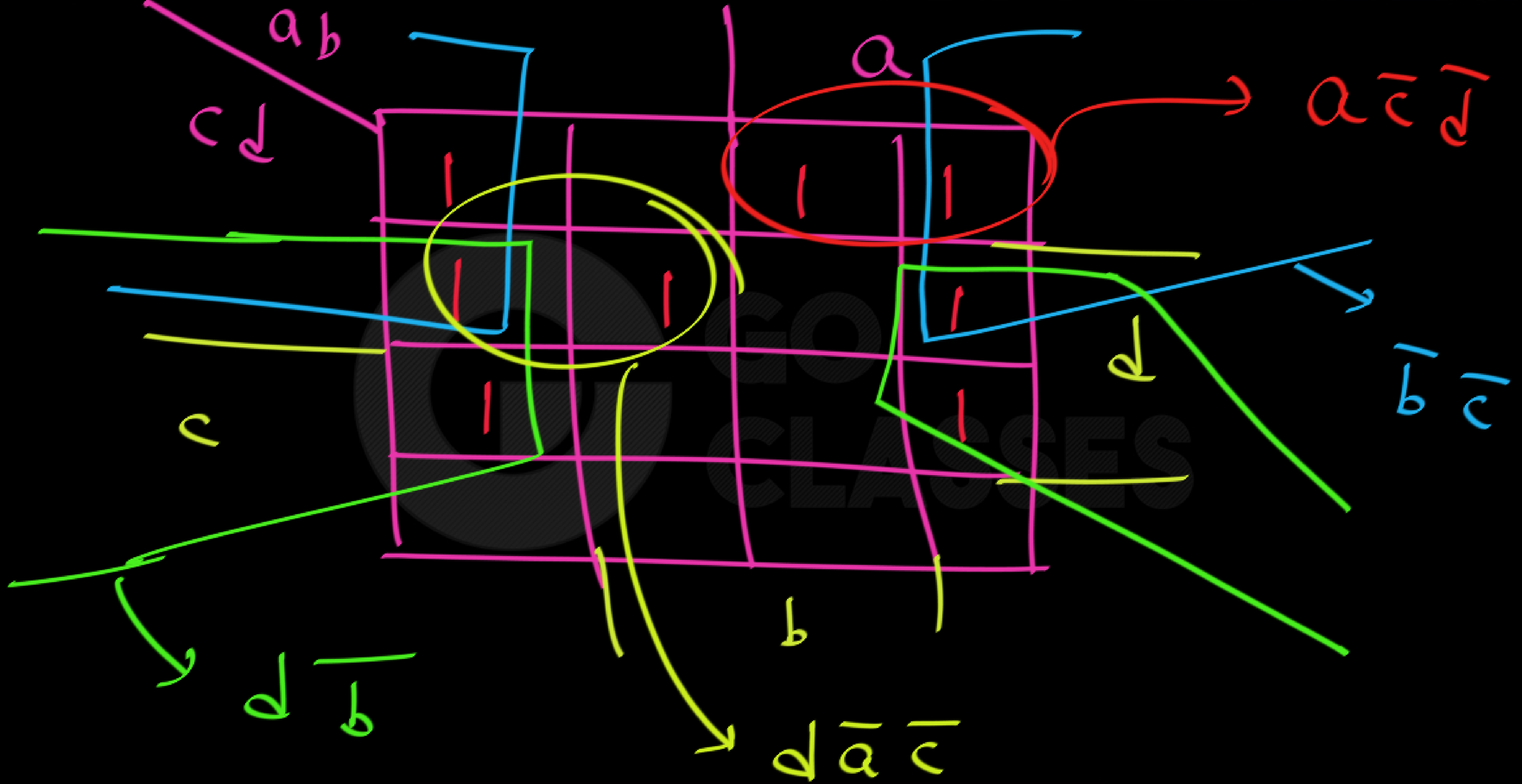
cd

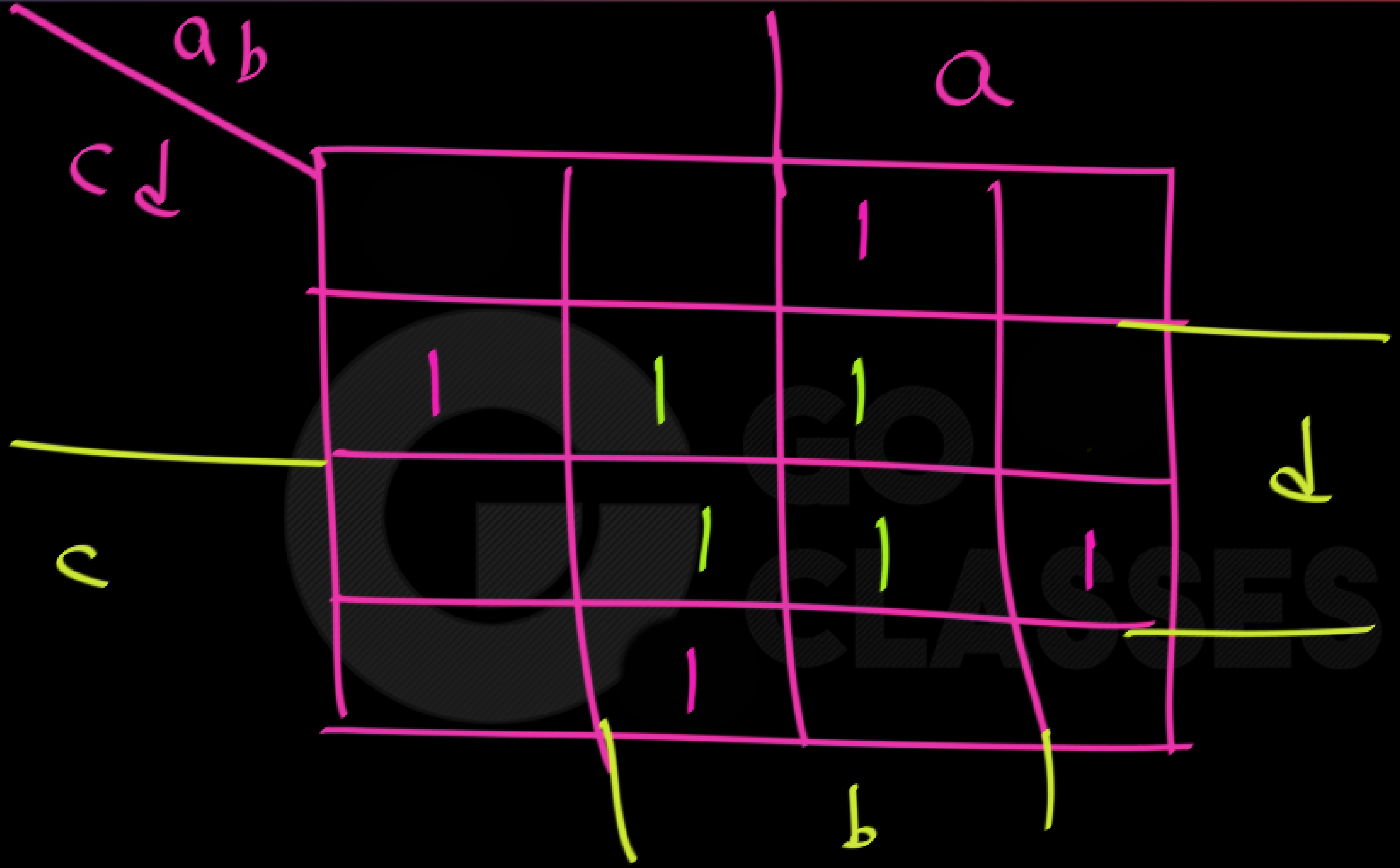
1		1	1
1	1		1
1			1

c

d

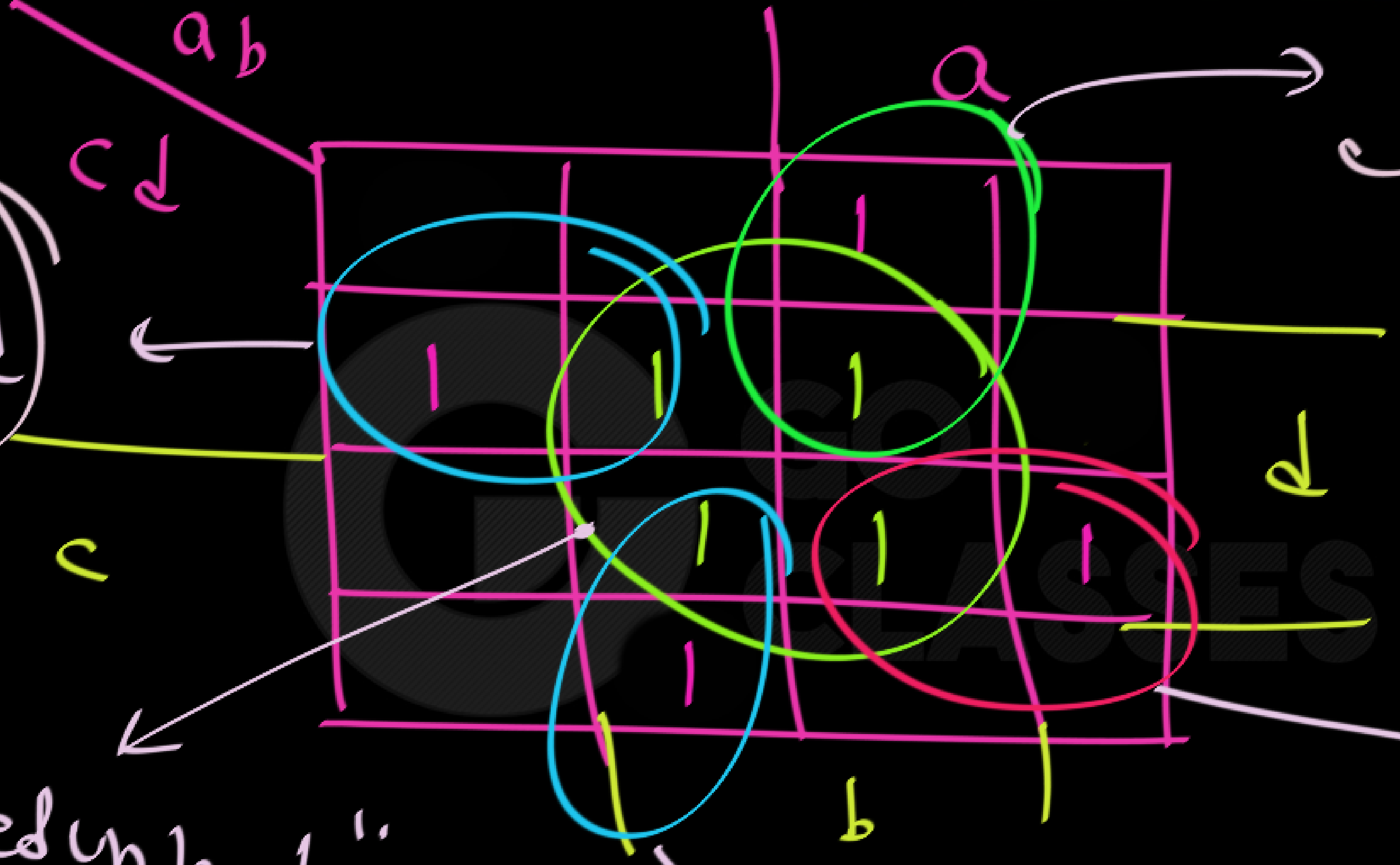
b


ab
cd
a
$a\bar{c}\bar{d}$
1
1
1
1
1
1
1
1
c
d
$\bar{b}\bar{c}$
b
$d\bar{b}$
$d\bar{a}\bar{c}$


ab
a
cd
c
d
b

$ab\bar{c}$

$\bar{a}\bar{c}d$



"Redundant"

cda

$\bar{a}bc$

ab, cd

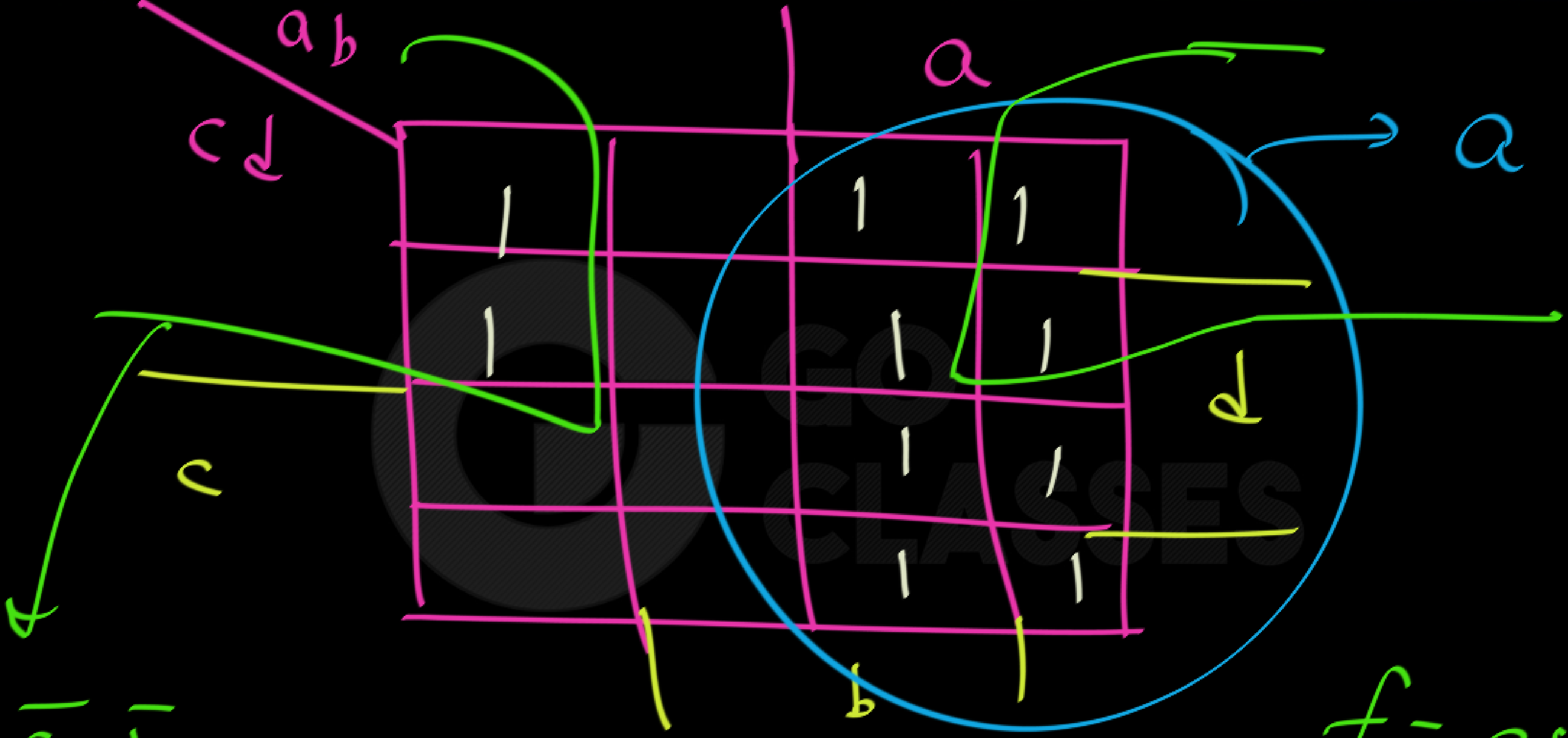
a

1		1	1
1		1	1
		1	1
		1	1

c

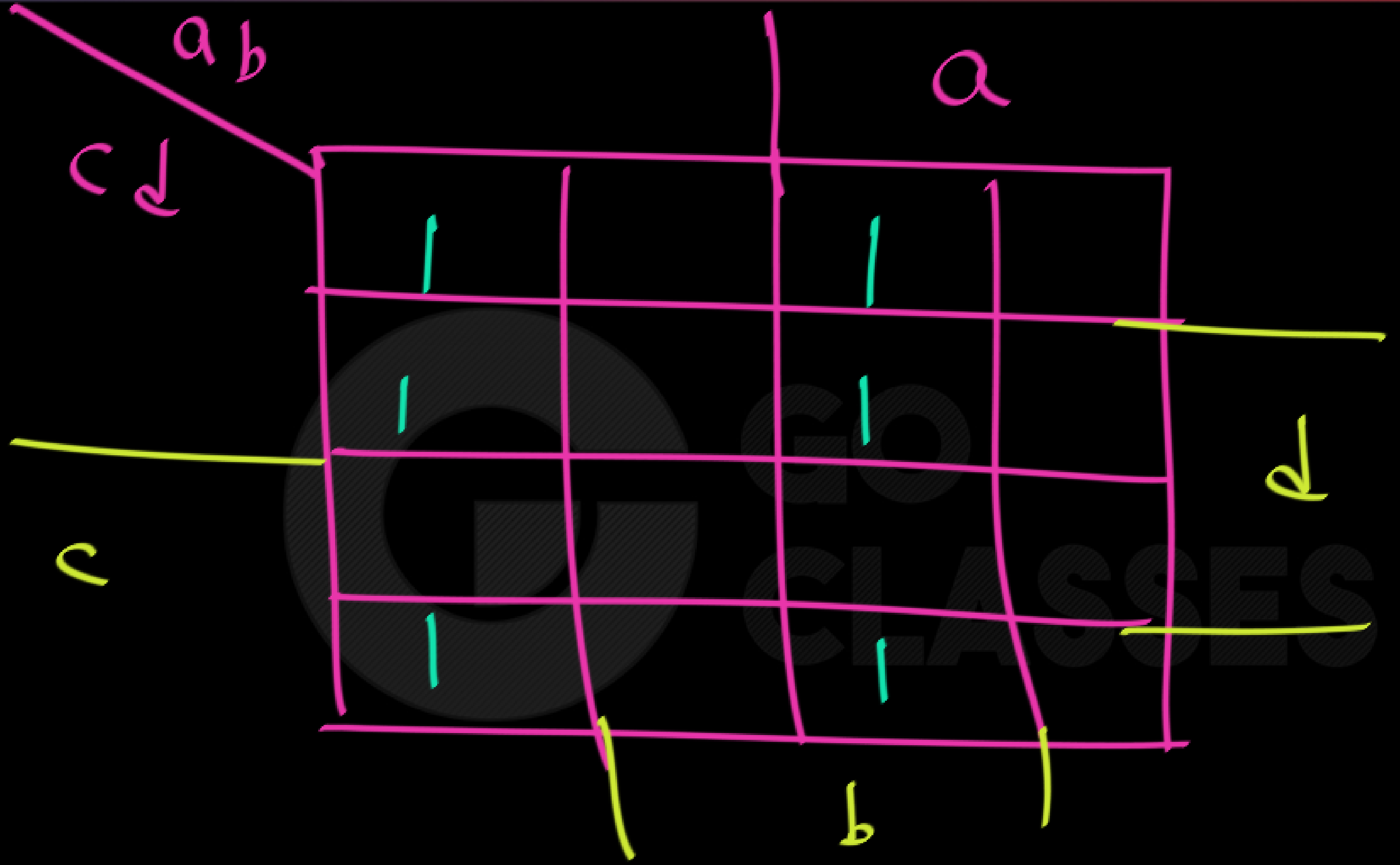
d

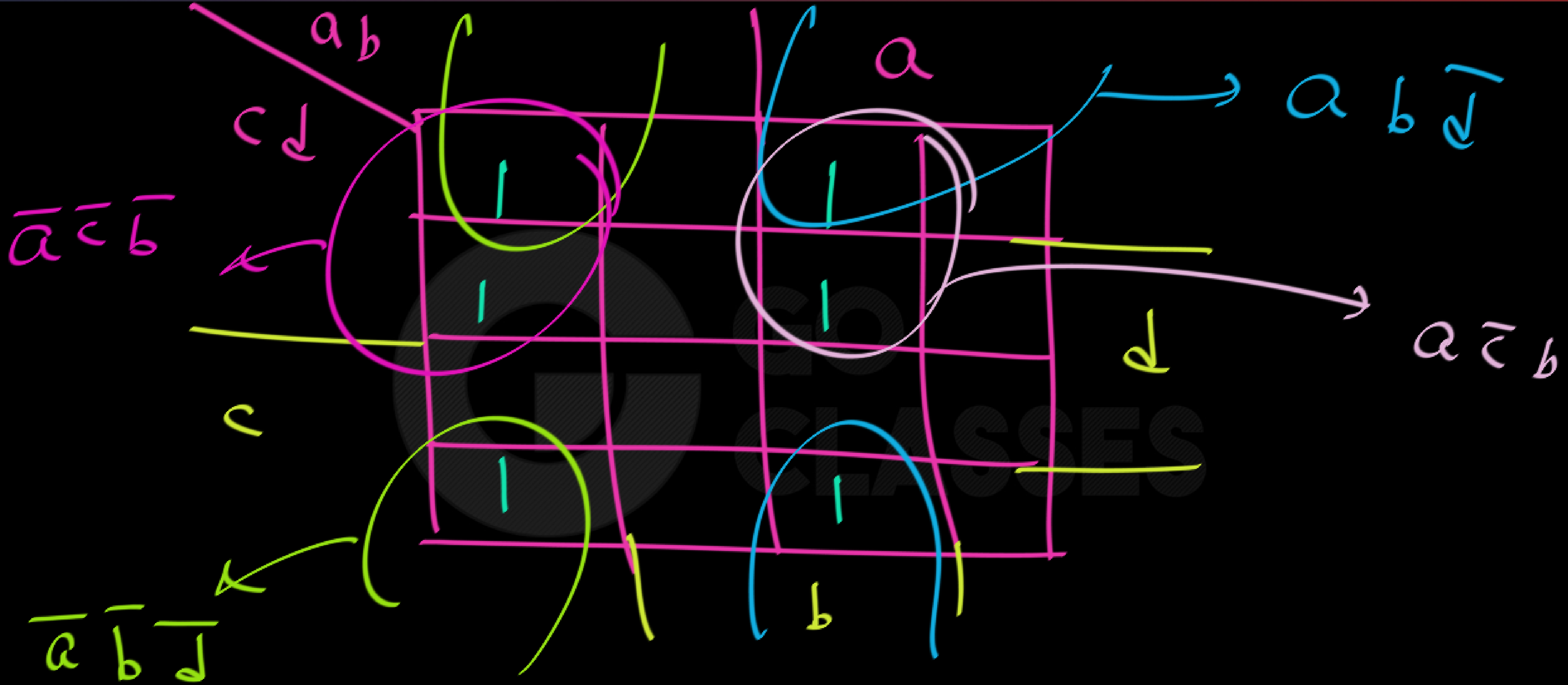
b



$\underbrace{\bar{c}\,\bar{b}}$

$\underbrace{f = a + \bar{c}\bar{b}}$


ab
cd
a
c
d
b
1
1
1
1
1
1


ab
cd
a
$ab\bar{d}$
$\bar{a}\bar{c}\bar{b}$
1
1
1
1
1
1
d
$a\bar{c}b$
c
b
$\bar{a}\bar{b}\bar{d}$

ab

cd

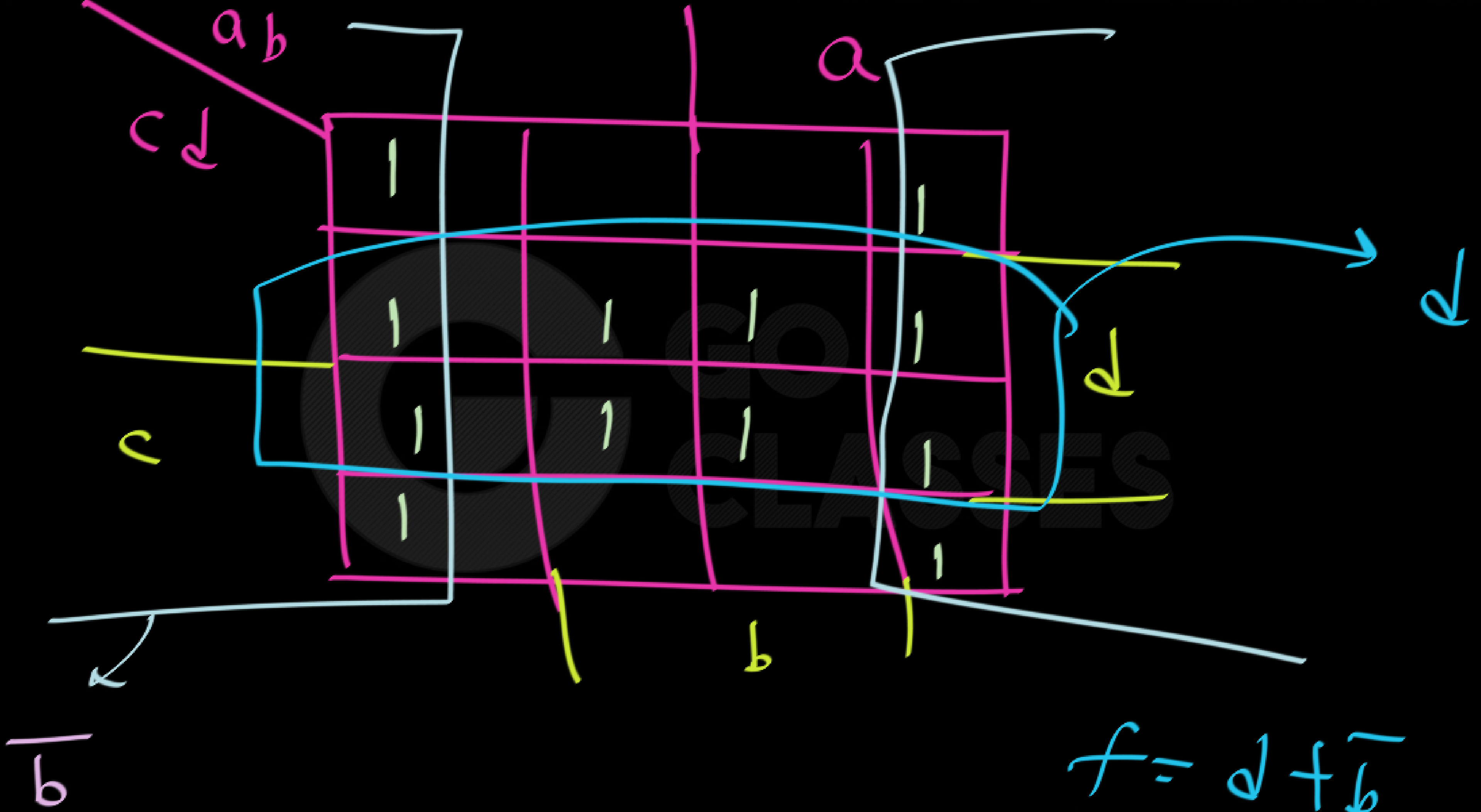
a

1			1
1	1	1	1
1	1	1	1
1			1

d

c

b



$$f = d + \bar{b}$$

Next Topic (VERY Important):

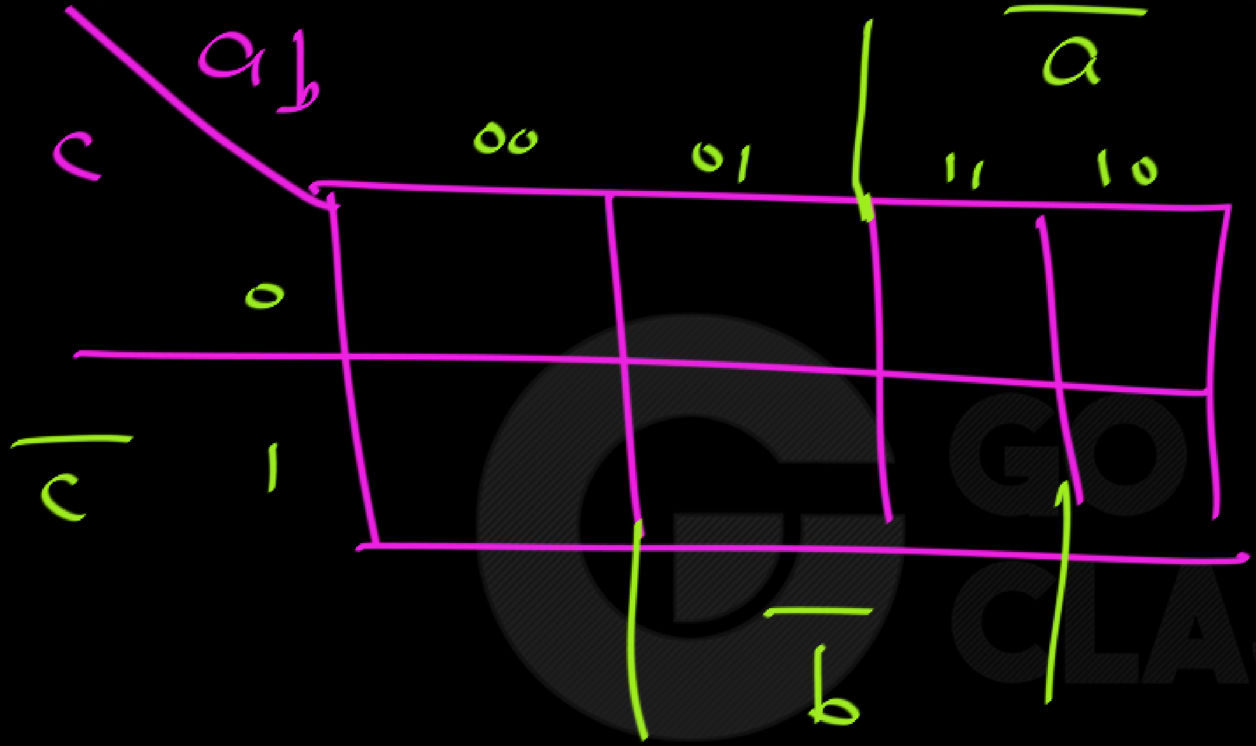
Using *K-Map*

To get Minimized **POS** Expression

minimized POS Expression

Every cube as big as possible.

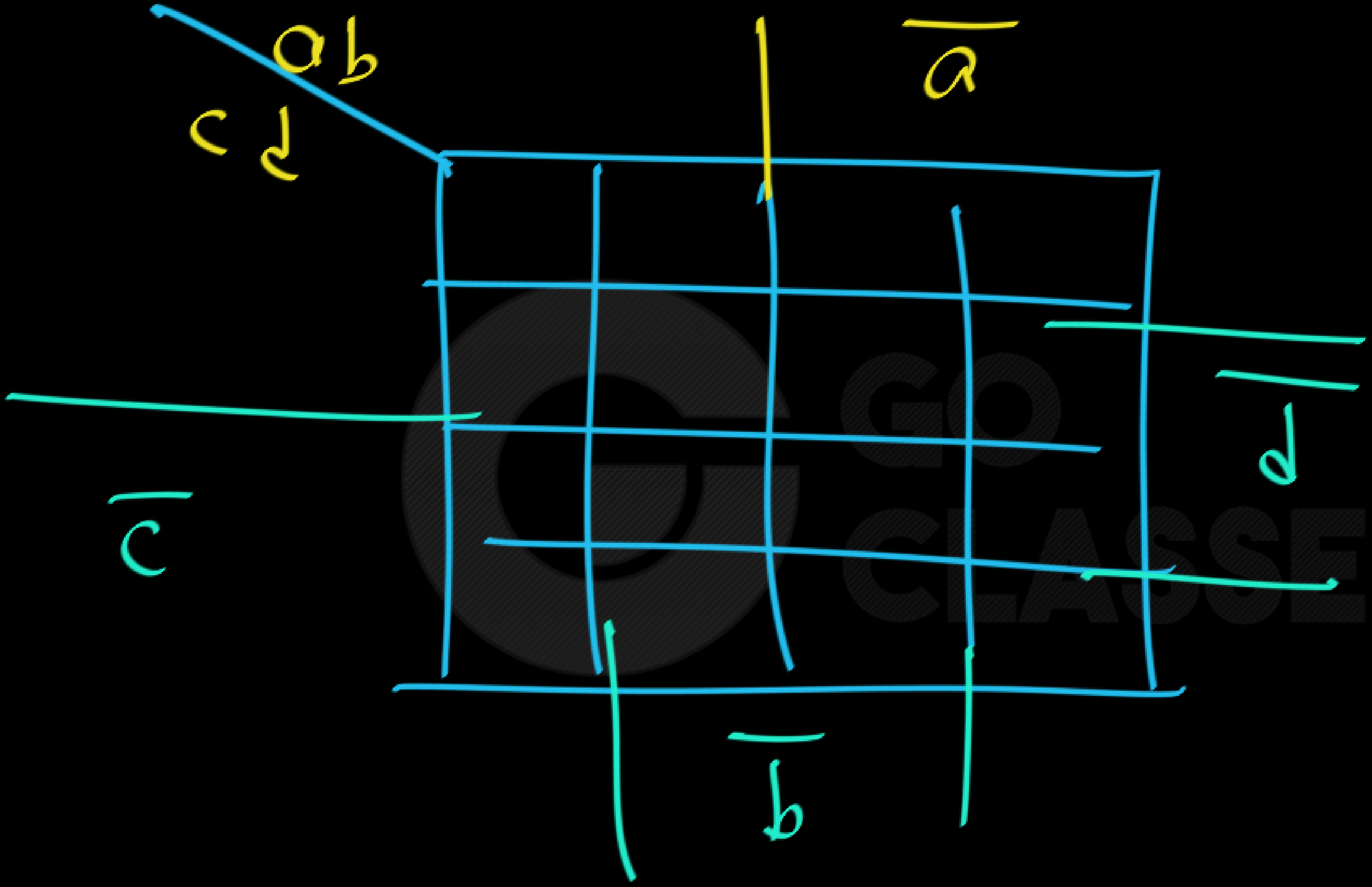
Cover 0's and write sum terms



Aim: POS

a=0 $\quad a$

a=1 $\quad \bar{a}$



Aim:
POS

Minimal Product-of-sums

Dual procedure: product of a minimum number of sum factors, provided there is no other such product with the same number of factors and fewer literals

- Variable corresponding to a 1 (0) is complemented (uncomplemented)
- Cubes are formed of 0 cells

Example: either one of minimal sum-of-products or minimal product-of-sums can be better than the other in literal count

yz \ wx	00	01	11	10
00				
01		1		1
11				
10		1		1

(*a*) Map of $f(x,y,z) = \sum(5,6,9,10)$
$= w'xy'z + wx'y'z + w'xyz' + wx'yz'$.

yz \ wx	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	0	0	0	0
10	0	1	0	1

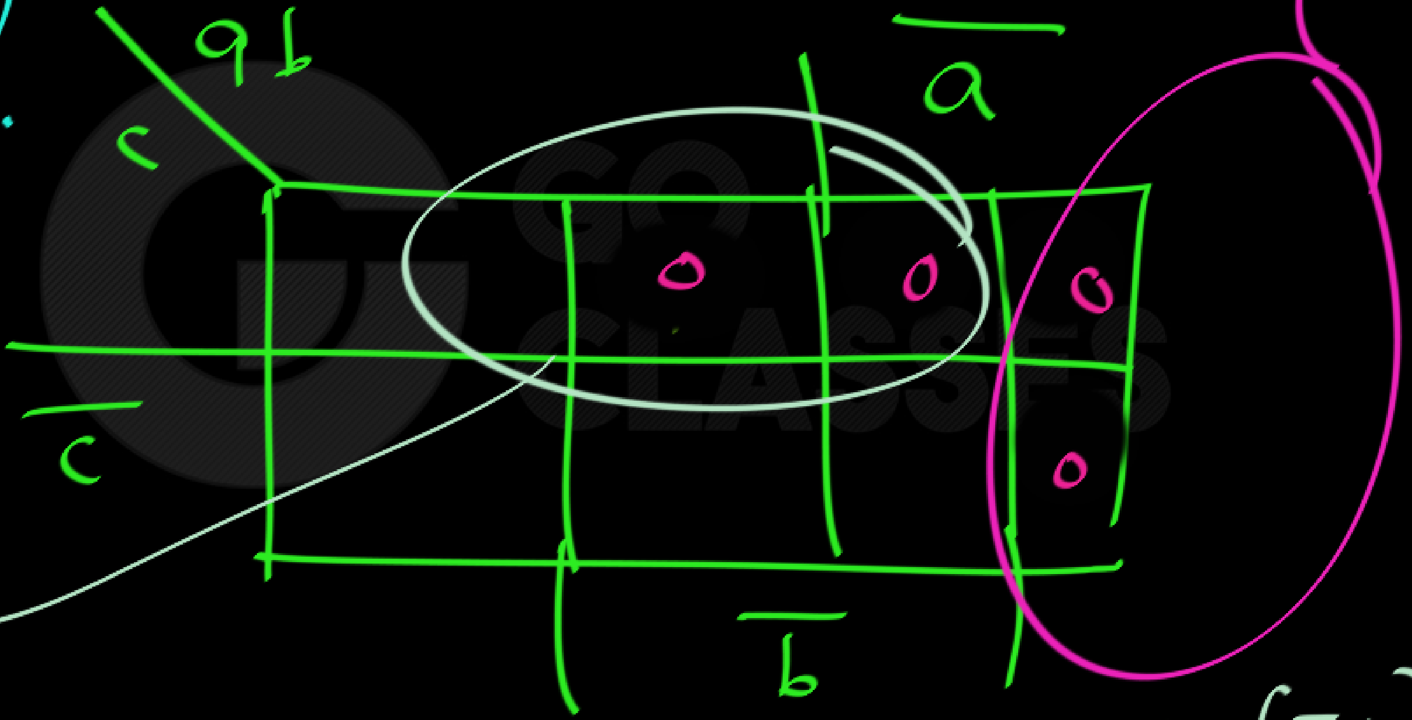
(*b*) Map of $f(x,y,z)$
$=\prod(0,1,2,3,4,7,8,11,12,13,14,15)$
$= (y + z)(y' + z')(w + x)(w' + x')$.

Eg: $f(a,b,c) = \sum(0, 1, 3, 7)$

mpos?

Eg: $f(a,b,c) = \sum(0, 1, 3, 7)$

mPOS?



$\bar{a} + b$

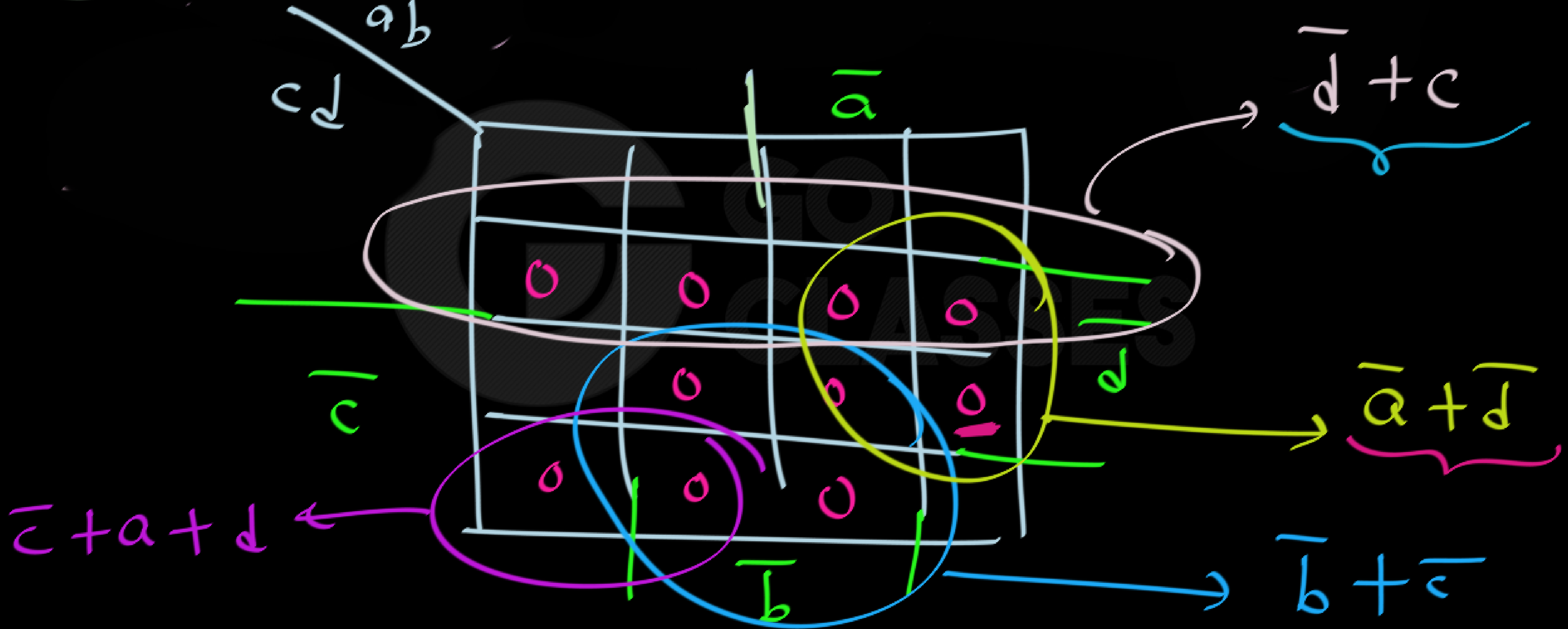
$\bar{b} + c$

$f = (\bar{a}+b)(\bar{b}+c)$

Q: $f(a,b,c,d) = \sum(0, 3, 4, 8, 10, 12)$

mpos?

POS $f = (\bar{d}+c)(\bar{c}+a+d)(\bar{a}+\bar{d})(\bar{b}+\bar{c})$



a

b

c \ ab	00	01	11	10
0	0	1	1	1
1	1	0	0	1

c

$a=1$
$b=0$

$a=1$
$b=0$

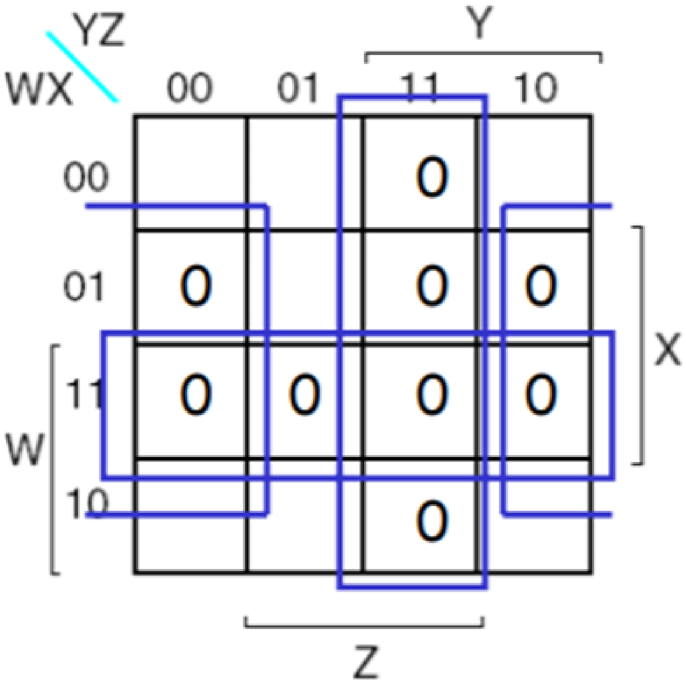
$\underbrace{c\bar{b}}$ $\quad \bar{c}b \quad a\bar{b}$

$f(a,b,c) = c\bar{b} + \bar{c}b + a\bar{b}$

PoS Optimization from SoP

$F(W,X,Y,Z) = \Sigma m(0,1,2,5,8,9,10)$

$= \Pi\, M(3,4,6,7,11,12,13,14,15)$

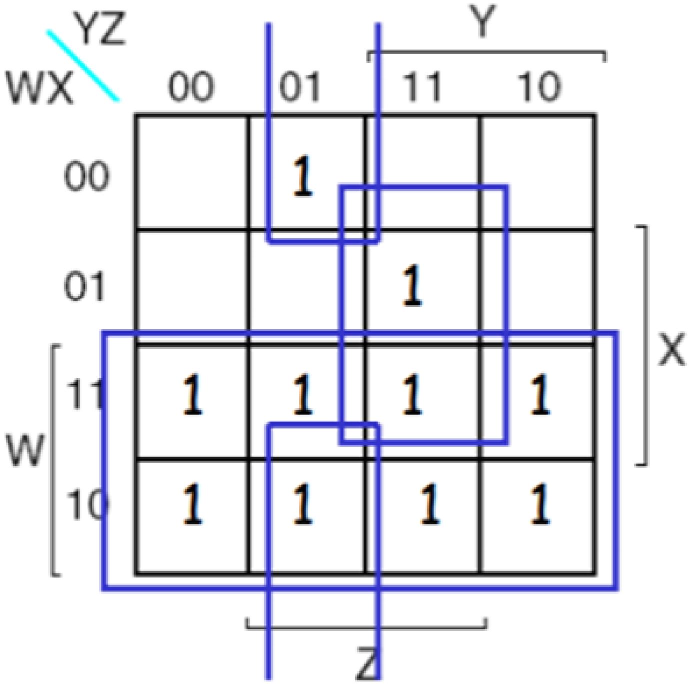


$F(W,X,Y,Z) = (W' + X')(Y' + Z')(X' + Z)$

SoP Optimization from PoS

$$F(W,X,Y,Z) = \prod M(0,2,3,4,5,6)$$

$$= \sum m(1,7,8,9,10,11,12,13,14,15)$$



$$F(W,X,Y,Z) = W + XYZ + X'Y'Z$$