Homework 2
Questions and Solutions

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Question 1

If $(x + 1)$, $3x$ and $(4x + 2)$ are first three terms of an A.P. then find its 5^{th} term

Solution

Solution : $(x + 1), 3x, (4x + 2)$ are in AP

$\Rightarrow \quad 3x - (x + 1) = (4x + 2) - 3x$

$\Rightarrow \quad x = 3$

$\therefore \quad a = 4, d = 9 - 4 = 5$

$\Rightarrow \quad T_5 = 4 + (4)5 = 24$

Question 2

Example 1 (AMC 10B/12B 2004). *A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains* 100 *cans, how many rows does it contain?*

https://www.andrew.cmu.edu/user/daltizio/Sequences%20and%20Series.pdf

Solution

Solution. Note that the number of cans on each of the n rows forms an arithmetic sequence with first term 1 and common difference 2. The total number of cans on n rows is thus

$$\frac{1}{2}n[2 + (n-1) \cdot 2] = \frac{1}{2}n[2n] = n^2.$$

Setting this equal to 100 gives $n = \boxed{10}$. ■

Question 3

Find the 18^{th}, 23^{rd} and n^{th} terms of the arithmetic progression.

$-11, -9, -7, -5\ldots\ldots\ldots\ldots\ldots\ldots$

Solution

Here $a = -11$, and $d = -9 + 11 = 2$

Thus,

$$a_{18} = a + (18 - 1)\, d$$
$$= -11 + (17)\,(2)$$
$$= -11 + 34 = 23;$$
$$a_{23} = a + (23 - 1)\mathrm{d}$$
$$= -11 + (22)\,(2)$$
$$= -11 + 44 = 33; \text{ and}$$
$$a_n = a + (n - 1)d$$
$$= -11(n - 1)\,(2)$$
$$= -11 + 2n - 2 = 2n - 13$$

Question 4

Example 4 : If pth term of an A.P. is q and its qth term is p, show that its rth term is $p+q-r$. What is its $(p+q)th$ term ?

Solution

Solution : If d is the common difference of the A.P., then

$$a_p - a_q = (p - q)\, d$$

$$\Rightarrow \quad q - p = (p - q)\, d$$

$$\Rightarrow d = \frac{q - p}{p - q} = -1$$

Now,

$$a_r - a_p = (r - p)\, d = (r - p)(-1)$$

$$\Rightarrow a_r = a_p - r + p$$

$$= q - r + p = p + q - r$$

$$\therefore \; a_{p+q} = p + q - (p + q) = 0 \qquad [\text{ put } r = p + q]$$

Question 5

Find the sum of 23 terms and n terms of the A.P.

16, 11, 6, 1.....

Solution

Here $a = 16,\ a + d = 11$ Therefore $d = -5$

Since $S_n = \frac{n}{2}\ [2a + a + (n-1)d]$ we get

$$S_{23} = \frac{23}{2}\ [2(16) + (23-1)(-5)] = \frac{23}{2}\ (32 - 110)$$

$$= \frac{23}{2}\ (-78) = -23 \times 39 = -\ 897.$$

Bonus Question

Find the sum of all integers between 100 and 1000 which are divisible by 9.

The first integer greater than 100 and divisible by 9 is 108 and the integer just smaller than 1000 and divisible by 9 is 999. Thus, we have to find the sum of the series.

$108 + 117 + 126 + \ldots\ldots\ldots\ldots + 999.$

Here $t_1 = a = 108$, $d = 9$ and $l = 999$

Let n be the total number of terms in the series be n. Then

$999 = 108 + 9\,(n - 1) \Rightarrow 111 = 12 + (n - 1) \Rightarrow n = 100$

Hence, the required sum $= \frac{n}{2}(a + l) = \frac{100}{2}(108 + 999)$

$= 50\,(1107) = 55350.$