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MTech IISc Bangalore

Ex Amazon Applied Scientist

GATE AIR 33

in 2017

older than IBM

NLP

Research eng.

9 months

Xerox labs

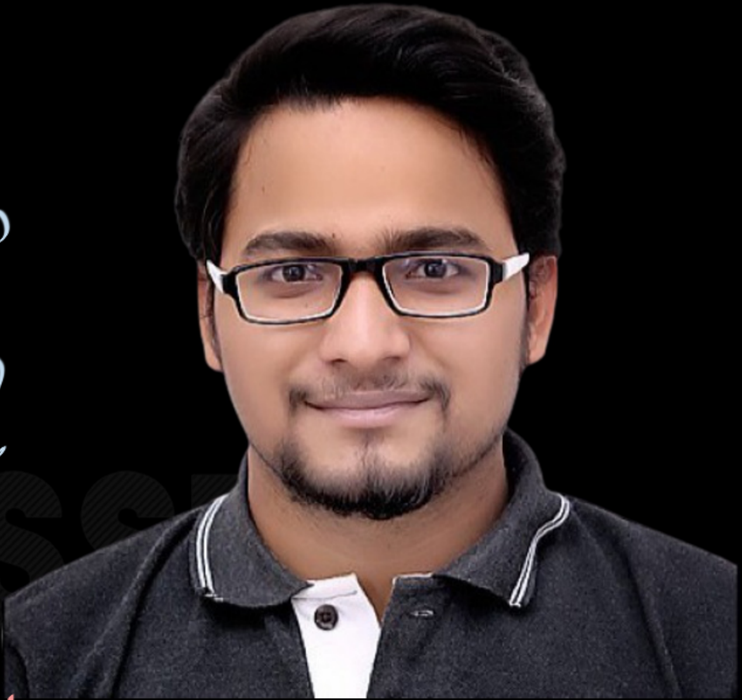
Amazon

1 year 2 months

time series

2017 - 2019

IISc CSA



Welcome to the first course...

The Fundamental Course

Why Fundamental course ?

There are many students from interdisciplinary branches

We do not assume any prerequisite whatsoever

These concepts get often use in all subjects

The Fundamental Course

A course to get you started for computer science..

Course content

- Sequence and Series
- Modular Arithmetic
- Logarithmic (log properties, graph and exercises)
- Importance of logarithmic in computer science
- Finding roots of polynomials (quadratic and cubic)
- Proof Techniques

Topic 1: Sequence and Series

Summation

Sigma Notation

Last value of *i*

$$\sum_{i=1}^{n} x_i$$

Formula for the terms

First value of *i*

unrolling

$$\sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5)$$

$i = 1, \quad 2, \quad 3, \quad 4, \quad 5$

(Correct)

$$\sum_{i=1}^{10} i^2 \overset{A.}{=} 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots 10^2$$

$$\overset{B.}{=} (1 + 2 + 3 + 4 + \cdots 10)^2$$

(incorrect)

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + (n-1) + n$$

Example 1: Write out explicitly the following sums:

a) $\displaystyle\sum_{i=3}^{6} i$, b) $\displaystyle\sum_{i=1}^{3} (2i+1)$, c) $\displaystyle\sum_{i=1}^{4} 2^i$.

$3+4+5+6$

$\underline{2^1 + 2^2 + 2^3 + 2^4}$

b) $\sum_{i=1}^{3}(2i+1)$,

$i=1$ $i=2$ $i=3$

$(2(1)+1) + (2(2)+1) + (2(3)+1)$

$(2+1) + (4+1) + (6+1)$

$= 3 + 5 + 7$

The above sums when written out are:

a) $\displaystyle\sum_{i=3}^{6} i = 3 + 4 + 5 + 6\,,$

b) $\displaystyle\sum_{i=1}^{3}(2i+1) = (2\times 1+1) + (2\times 2+1) + (2\times 3+1) = 3 + 5 + 7\,,$

c) $\displaystyle\sum_{i=1}^{4} 2^i = 2^1 + 2^2 + 2^3 + 2^4\,.$

It is important to realise that the choice of symbol for the variable we are summing over is arbitrary, e.g., the following two sums are identical:

$$\sum_{i=1}^{4} i^3 = \sum_{j=1}^{4} j^3 = 1^3 + 2^3 + 3^3 + 4^3 .$$

The variable that is summed over is called a **dummy variable**.

$$\sum_{j=1}^{4} j^3 = \sum_{i=1}^{4} i^3 = \sum_{j=1}^{4} j^3 = \sum_{k=1}^{4} k^3 \cancel{=} \sum_{t=1}^{4} i^3$$

$$x = x$$

Shifting indices of Summation

$$\sum_{j=0}^{3} 2j$$

Shift summation to 1 as start

$$\sum_{j=1}^{b}$$

a

$a = ?$

$b = ?$

given

$$\sum_{j=0}^{3} 2j$$

Asked

$$\sum_{i=1}^{4}$$

$j=0$	1	2	3
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
$i=1$	2	3	4

given

$j=0$ 1 2 3

$$\sum_{j=0}^{3} 2j$$

Asked

$$\sum_{i=1}^{4} 2(i-1)$$

$i = j+1$

relationship

$i=1$

$j = i-1$

4

$$\sum_{j=0}^{3} 2j = 2(0) + 2(1) + 2(2) + 2(3)$$

$$0 + 2 + 4 + 6$$

$$\sum_{i=1}^{4} 2(i-1) = 2(1-1) + 2(2-1) + 2(3-1) + 2(4-1)$$

$$0 + 2 + 4 + 6$$

Shifting the index of summation (example)

Consider the following summation. Change the index of summation to run between 1 and 4.

$$\sum_{j=0}^{3} 2j$$

Solution:

$$\sum_{j=0}^{3} 2j$$

j=0,1,2,3 k=1,2,3,4 k = j+1

$$\sum_{j=0}^{3} 2j = \sum_{k=1}^{4} 2(k-1)$$

$$\sum_{i=2}^{19} i(i-2)$$

Start summation from 1

Step 1: find out the relationship b/w i & j

$j = i - 1$

$i = 2, 3, \ldots 19$

$j = 1, 2, 3$

$$\sum_{j=1}^{18} (j+1)(j+1-2)$$

$$= \sum_{j=1}^{18} (j+1)(j-1)$$

$$\sum_{j=1}^{18} j^2 - 1$$

A. $\sum_{i=1}^{18}(i-1)(i-3)$

B. $\sum_{i=1}^{18}(i^2-1)$

$$\sum_{i=2}^{19} i(i-2)$$

Start summation from 1

$j = i - 1 \Rightarrow \quad i = j + 1$

A. $\sum_{i=1}^{18}(i-1)(i-3)$

B. $\sum_{i=1}^{18}(i^2-1)$

$$\sum_{j=1}^{18} \underbrace{(j+1)(j-1)}_{= (j^2 - 1)}$$

$$\sum_{i=2}^{19} i(i-2)$$

Start summation from 1

$i = \quad 2 \quad 3 \quad 4 \quad 5 \;\text{----}\; 19$

$j = \quad 1 \quad 2 \quad 3 \quad 4 \quad\quad 18$

$\boxed{j = i-1} \rightarrow i = j+1$

A. $\sum_{\cancel{i}=1}^{18} (\cancel{i}-1)(\cancel{i}-3)$

B. $\sum_{\cancel{i}=1}^{18} (\cancel{i}^2-1)$

$$\sum_{j=1}^{18} (j+1)(j-1) = \sum_{j=1}^{18} j^2-1$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a(a-b) + b(a-b)$$

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$= \underline{a^2 - b^2}$$

formula

$$\sum_{i=1}^{n} \frac{i-1}{m}$$

MSQ

Start summation from 0 or 2

A) $\sum_{i=0}^{n} \frac{i-2}{m}$

B) $\sum_{i=0}^{n-1} \frac{i}{m}$

c) $\sum_{i=0}^{n-1} \frac{i-2}{m}$

d) $\sum_{i=2}^{n+1} \frac{i-2}{m}$

$$\sum_{i=1}^{n} \frac{i-1}{m}$$

$i = 1, 2, \ldots n$

$j = 0, 1$

$j = i-1$

$i = j+1$

$$\sum_{j=0}^{n-1} \frac{j+\cancel{1} \; \cancel{-1}}{m}$$

$=$

if they ask to start from 2

$i = 1, 2, \ldots n$

$$\sum_{i=1}^{n} \frac{i-1}{m}$$

$j = 2, 3, 4 \ldots$

$i = j - 1 \}$ ← crucial step

$$\sum_{j=2}^{n+1} \frac{j-2}{m}$$

↖ j कि दुनिया

Exercise 2. Express the following in summation notation.

(a) $1 + 20 + 400 + 8,000$, (b) $-3 - 1 + 1 + 3 + 5 + 7$.

$$\sum_{i=0}^{3} 20^i$$

AP

$$\cancel{\sum_{k=-3}^{5} k+2}$$

$$\sum_{t=-2}^{3} 2t+1 = -3 + \cdots$$

Exercise 2(a) To write $1 + 20 + 400 + 8,000$ as a sum, note that

$$\begin{aligned} 1 &= 20^0\,, \\ 20 &= 20^1\,, \\ 400 &= 20^2\,, \\ 8,000 &= 20^3\,. \end{aligned}$$

This shows that we may write

$$1 + 20 + 400 + 8,000 = \sum_{j=0}^{3} 20^j\,.$$

or, alternatively,

$$1 + 20 + 400 + 8,000 = \sum_{i=1}^{4} 20^{i-1}\,.$$

Exercise 2(b) The sum $-3-1+1+3+5+7$ is a sum over a consecutive range of odd integers. It may be written in many different ways. Here are three possibilities.

$$-3-1+1+3+5+7=\sum_{j=-1}^{4}(2j-1),$$

or

$$\sum_{i=1}^{6}[(2j-1)-4].$$

or

$$\sum_{j=-4}^{3}$$

our Answer

$$\sum_{t=-2}^{3}(2t+1)$$

There are many other possibilities.

Summation identities

$$\sum_i a x_i = a \sum_i x_i$$

$$\sum_i (x_i + y_i) = \sum_i x_i + \sum_i y_i$$

$$\sum_{i=a}^{b} f(x) = \sum_{i=0}^{b} f(x) - \sum_{i=0}^{a-1} f(x)$$

$$\sum_i 2i = 2\sum i$$

$$\sum_i ti = \sum_i it$$

$$= t\sum_i i$$

$$\sum_i (it)^2 = t^2 \sum_i i^2$$

$$\sum_i (x_i + y_i) = \sum_i x_i + \sum_i y_i$$

Rearrange

$y_1 + y_2 + y_3$

$x_1 + x_2 + x_3$

$(x_1 + y_1) + (x_2 + y_2) + (x_3 + y_3)$

$$\sum_i i + i^2 = \sum_i i + \sum_i i^2$$

$1+2+3$

$1^2+2^2+3^2$

$1+1^2$

$2+2^2$

$1+2+3$

$3+2+1$

$$\sum (i+3) = \sum i + \sum 3$$

$$= \sum_{i=1}^{5} i + 15$$

$$\sum_{i=1}^{5} 3 = 3 \sum_{i=1}^{5} i^0 = 3 \times 5 = 15$$

$$= 3\left(\underbrace{1^0 + 2^0 + 3^0 + 4^0 + 5^0}_{1+1+1+1+1} \right) \rightarrow 5$$

$$\sum_{i=1}^{5} (i+3) = \sum i + 3 \quad \text{incorrect}$$

$$= \sum i + 15 \quad \checkmark$$

Will this help me in GATE ?

GATE CSE 2003 | Question: 64

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Let **S** be a stack of size $n \geq 1$. Starting with the empty stack, suppose we push the first n natural numbers in sequence, and then perform n pop operations. Assume that Push and Pop operations take X seconds each, and Y seconds elapse between the end of one such stack operation and the start of the next operation. For $m \geq 1$, define the stack-life of m as the time elapsed from the end of $Push(m)$ to the start of the pop operation that removes m from **S**. The average stack-life of an element of this stack is

A. $n(X+Y)$
B. $3Y+2X$
C. $n(X+Y)-X$
D. $Y+2X$

https://gateoverflow.in/951/gate-cse-2003-question-64

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Let us represent stack-life of i^{th} element as $S(i)$. The i^{th} element will be in stack till $(n-i)$ elements are pushed and popped. Plus one more Y for the time interval between the push of i^{th} element and the $i+1^{th}$ element. So,

$$S(i) = Y + 2.(n-i)(Y+X) = Y + 2.(n-i)Z$$

Best answer

average stack-life will, $A = \Sigma \frac{S(i)}{n}$

$$\Sigma\left(Y + 2(n-i)Z\right)$$

$$\sum_{i=1}^{n} \left(y + 2(n-i) z \right)$$

$$\sum_{i=1}^{n} (n-i)$$

$$n-1 + n-2 + \cdots + 1 + 0$$

$$1+2+3+ \cdots n-1$$

$$= \sum_{i=1}^{n} y + \sum_{i=1}^{n} 2(n-i) z$$

$$\sum_{i=1}^{n-1} i$$

$$ny + 2z \sum_{i=1}^{n} (n-i)$$

$$\sum_{i=1}^{n-1} i$$

$$\Rightarrow \sum_{i=0}^{n-1} i \Rightarrow 0 + 1 + 2 + \cdots$$

Yes $\sum (n-1) = \underline{\sum n + \sum (-1)} = \sum n - (\sum 1)$

$$\sum (n-i) = \sum n + \sum (-i)$$

$$= \sum n - (\sum i)$$

AP $\frac{n(n+1)}{2}$

$1+2+3+\cdots n$

$$\sum_{i=1}^{n} 3i =$$

$1+1+1=3$

$1+1=2$

$1+1+1+1=4$

$3\cdot 1 + 3\cdot 2 + 3\cdot 3 + \cdots$

$$\sum_{i=1}^{n} 3 = \sum_{i=1}^{n} 3i^0 = 3\sum_{i=1}^{n} i^0$$

$$= 3(\underbrace{1^0 + 2^0 + 3^0 + \cdots n^0}_{n}) = \underline{\underline{3n}}$$

1 1 1 1