# Inverse of a function

# Instructor:

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GATE CSE AIR 53; AIR 67; AIR 107; AIR 206

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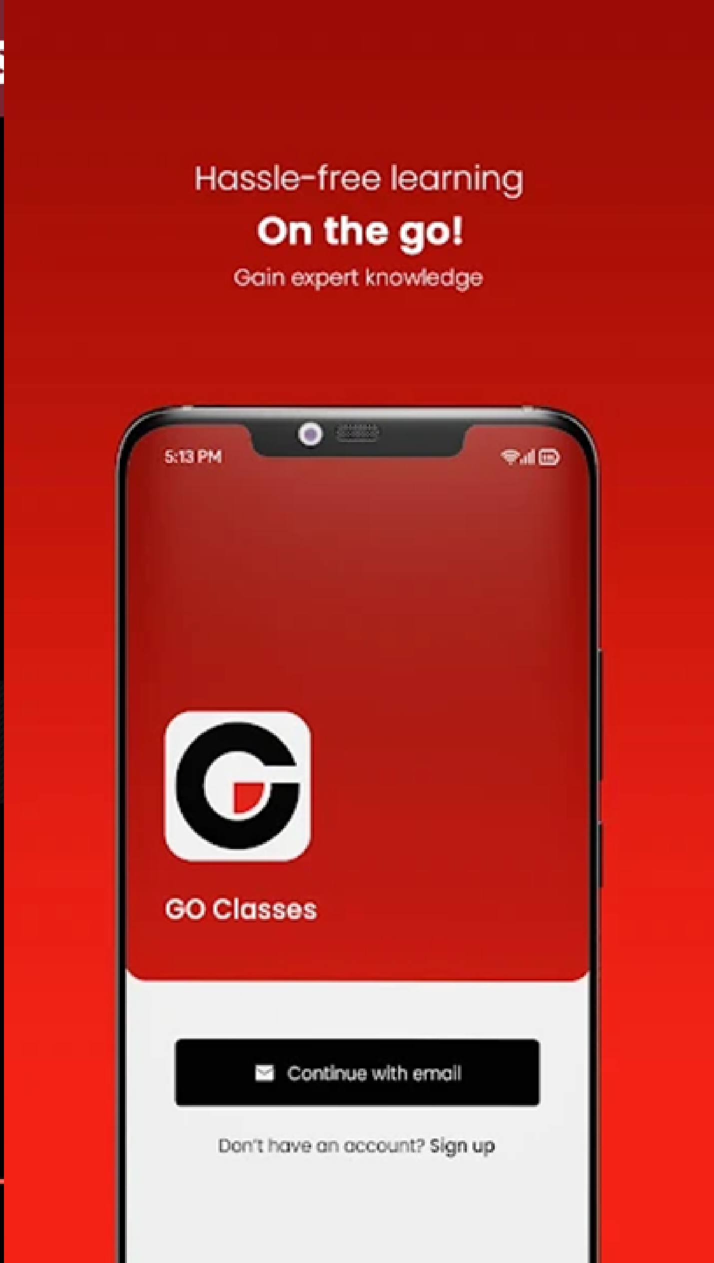

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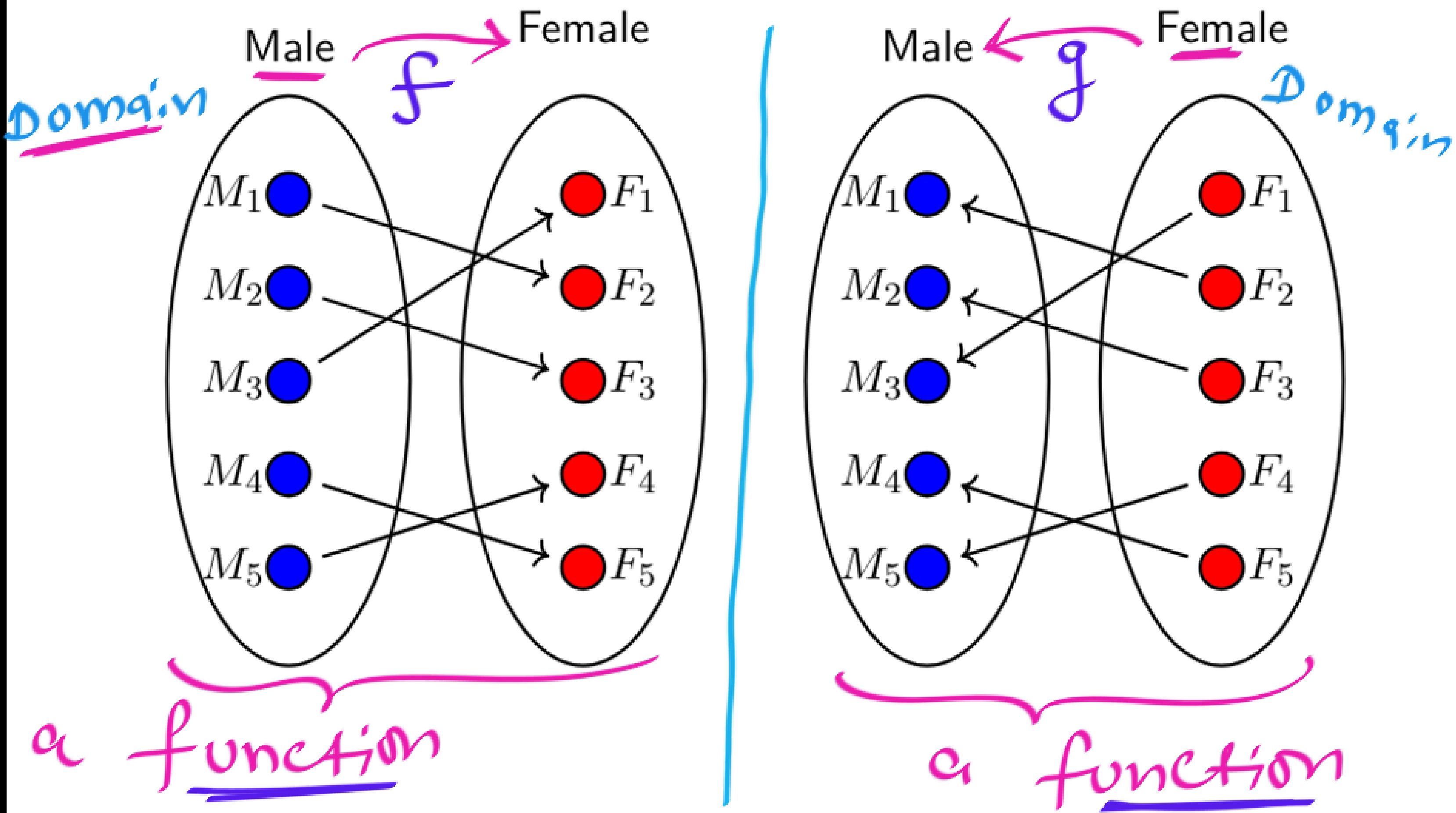
# Inverse of a function

# Inverse functions

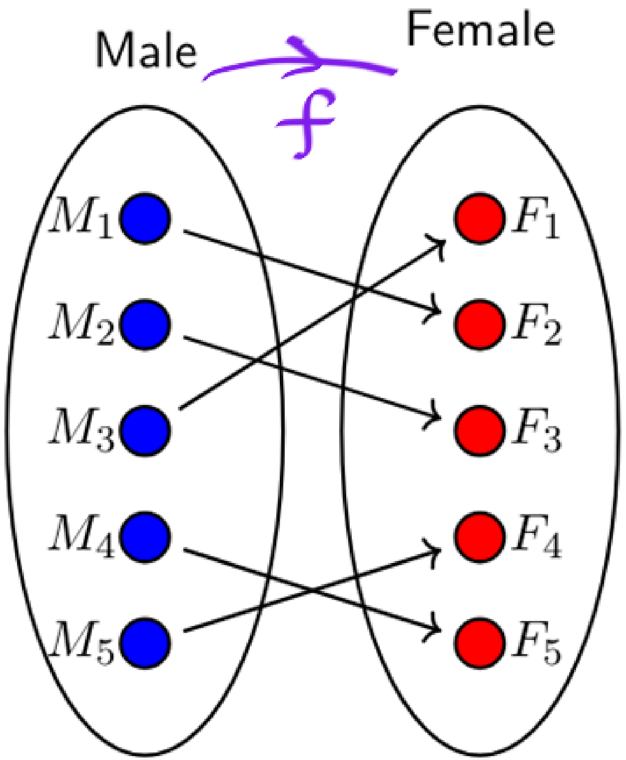


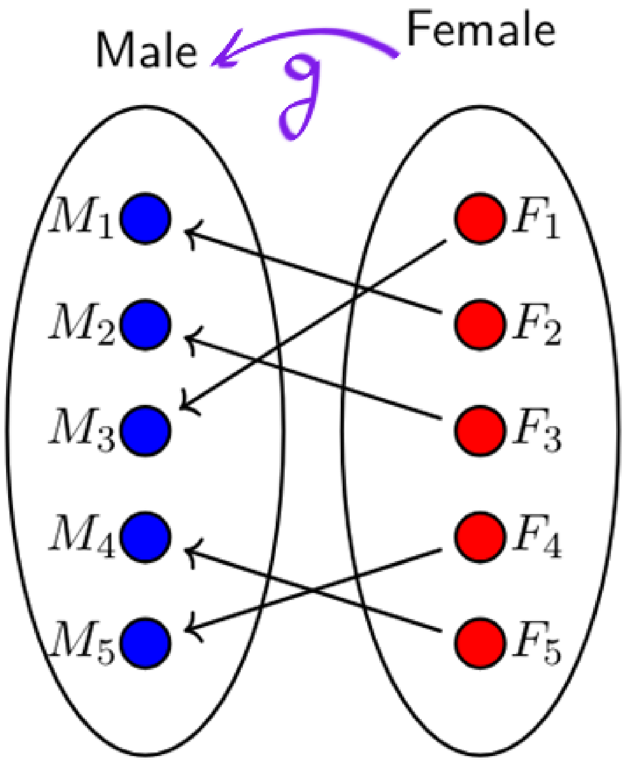
- What is the difference between the two marriage functions?



# Inverse functions

- What is the difference between the two marriage functions? Reversal of mapping




$m_1 \xrightarrow{f} f_2$

$F_2 \xrightarrow{g} m_1$

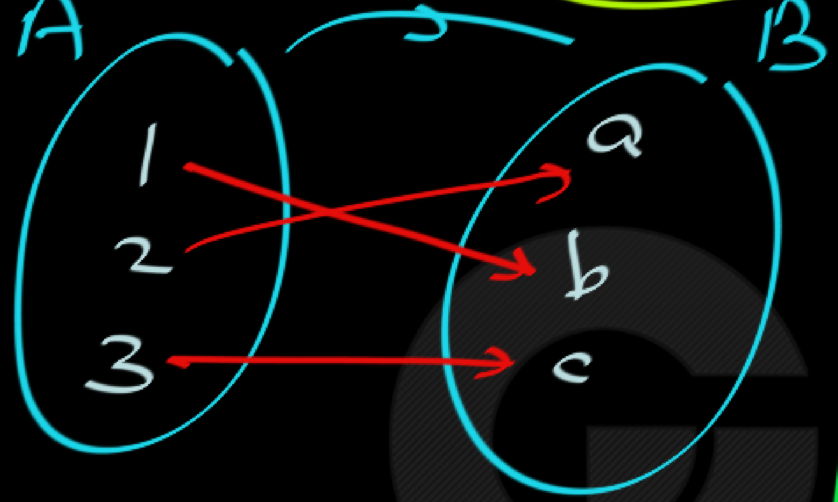
# Function g "Reverses the mapping" given by function f .

We will say :

g is Inverse of f .

Notation: $g = f^{-1}$

Domain $\qquad f$ function



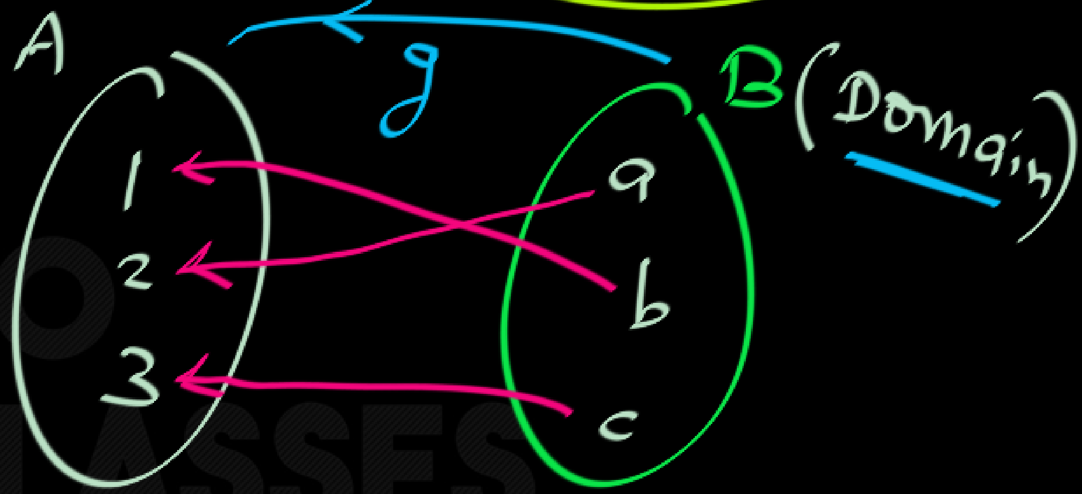

$1 \xrightarrow{f} b$

$2 \xrightarrow{f} a$

$3 \xrightarrow{f} c$

$f: A \to B$

another function $g$




$g: B \to A$

$a \xrightarrow{g} 2$

$b \xrightarrow{g} 1$

$c \xrightarrow{g} 3$

$g = f^{-1}$

$f^{-1}$ : Inverse of f

Single thing

another function g

Just a Notation

$f^{-1}$

NOT "power -1"

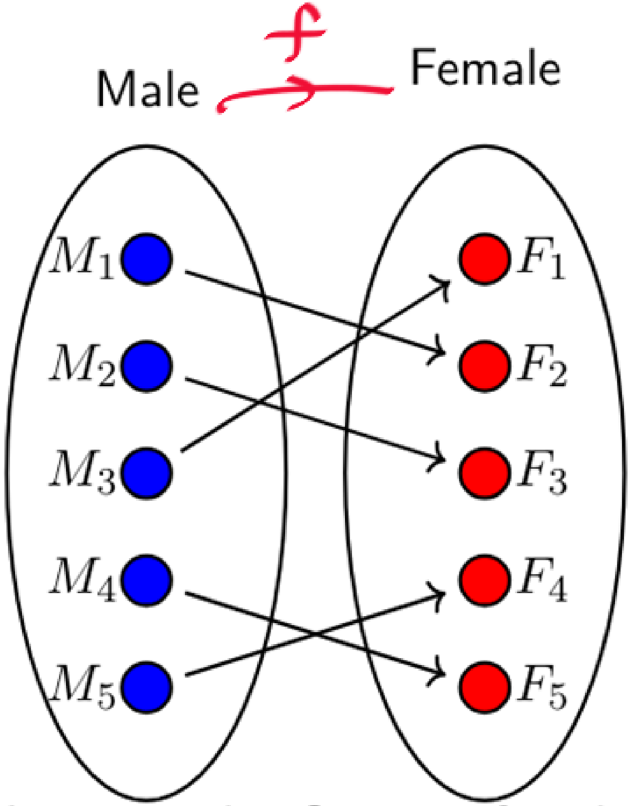
$f^{-1}$ : Just a Notation for Inverse of $f$.

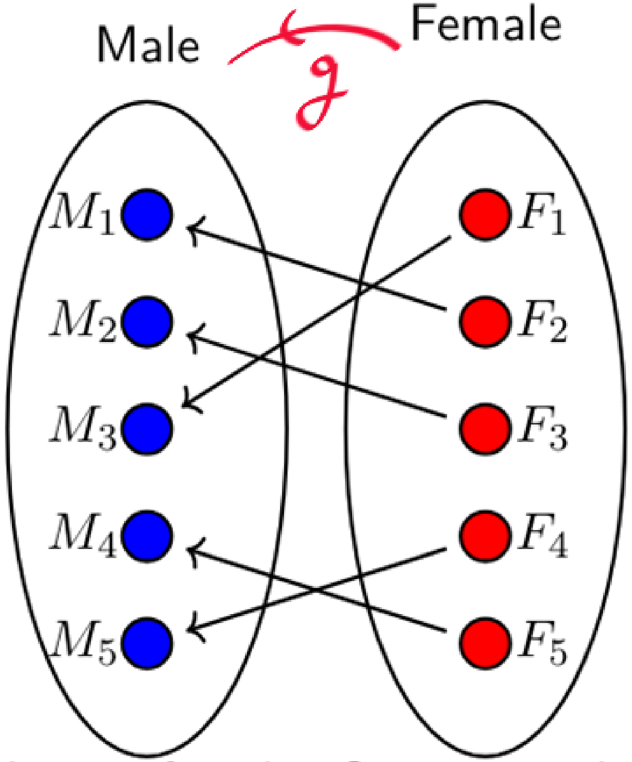
Notation

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

# Inverse functions

- What is the difference between the two marriage functions?




- Input: male. Output: female.
- $F$

- Input: female. Output: male.
- $F^{-1}$

$g = F^{-1}$

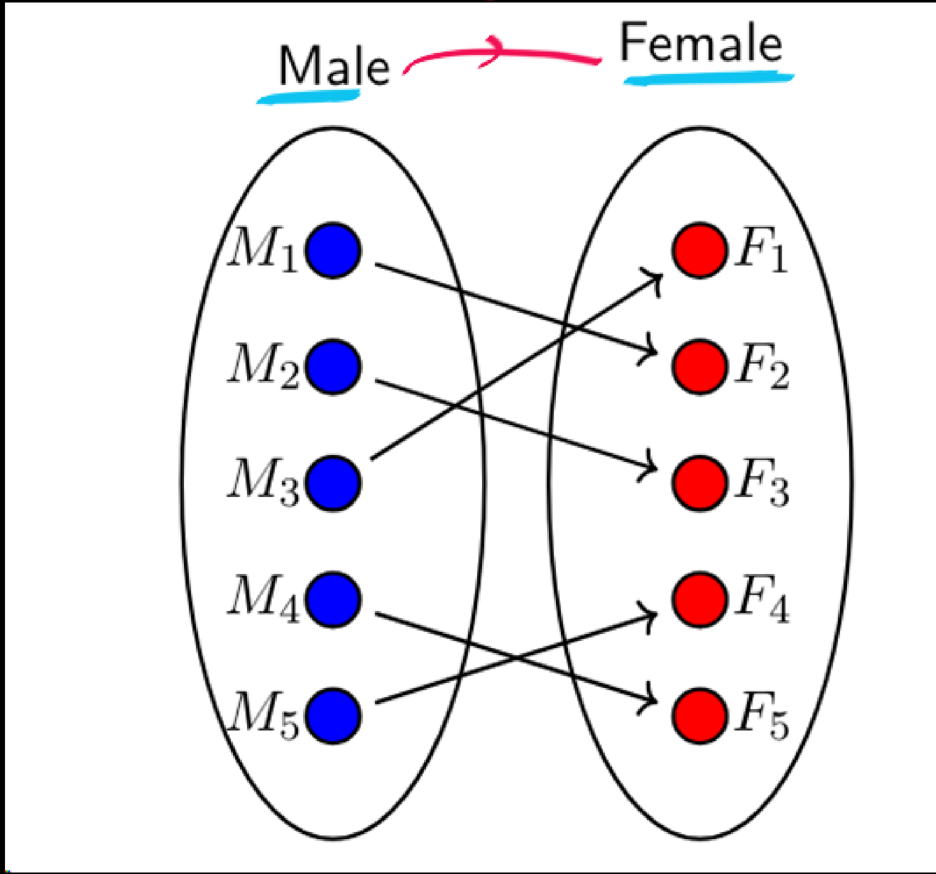
# Informal Definition (for Understanding Purpose Only):

Function f is Invertible

iff

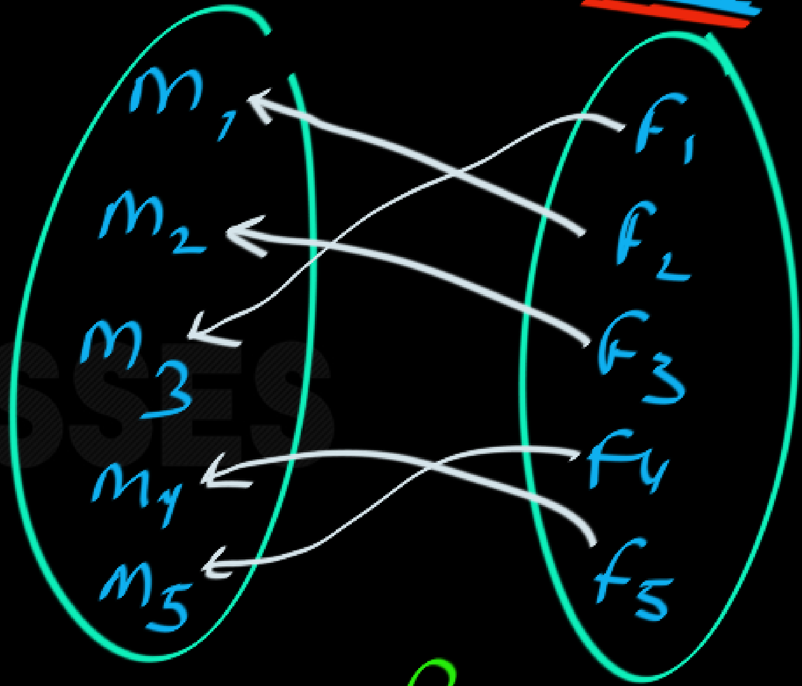
when we look at function f in the Reverse Direction, that also is a function.

$f$ : male $\longrightarrow$ female $f$




$g$

male $\longleftarrow$ female

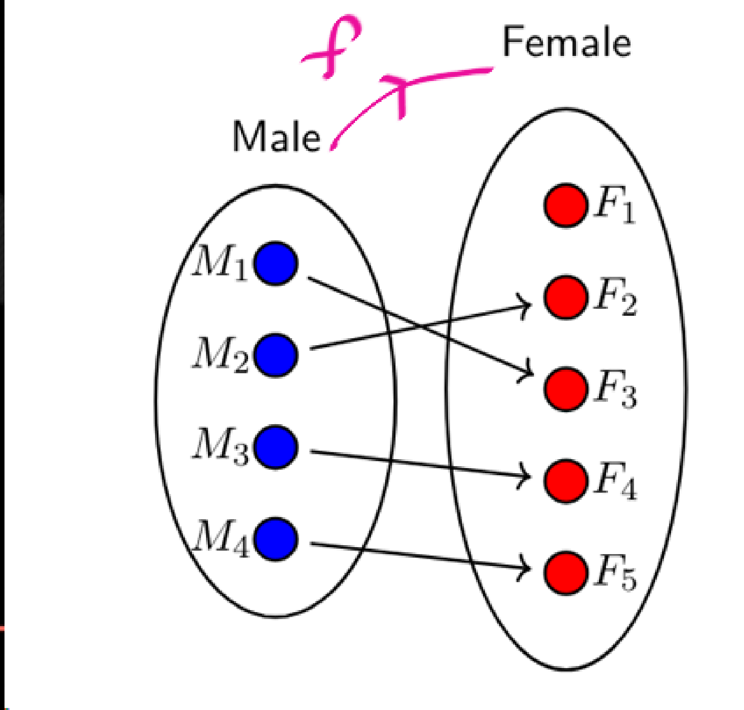



$g$ function

$f$ is Invertible. $g = f^{-1}$

Consider the following function f.

In the Reverse Direction, do we get a function??

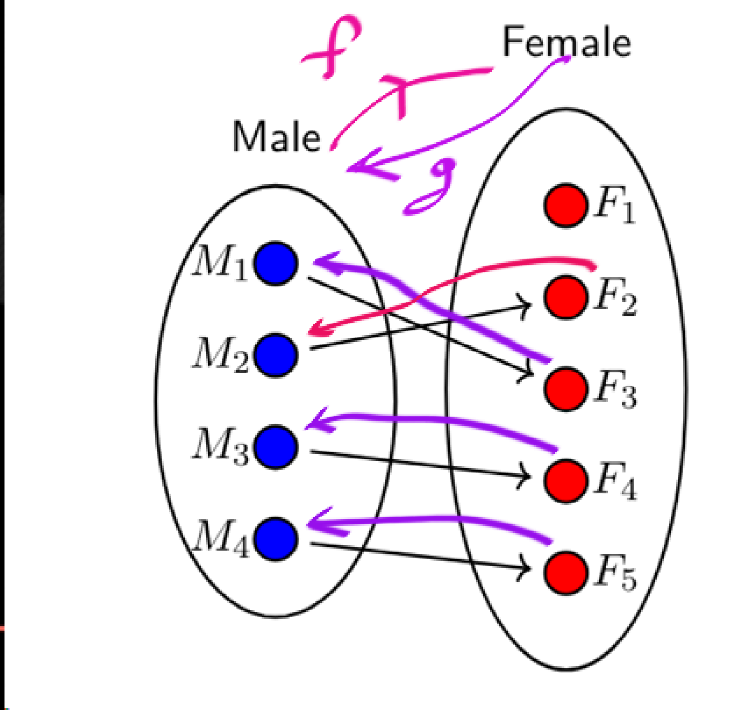


Consider the following function f.

In the Reverse Direction, do we get a function?? → No.

No. ✓

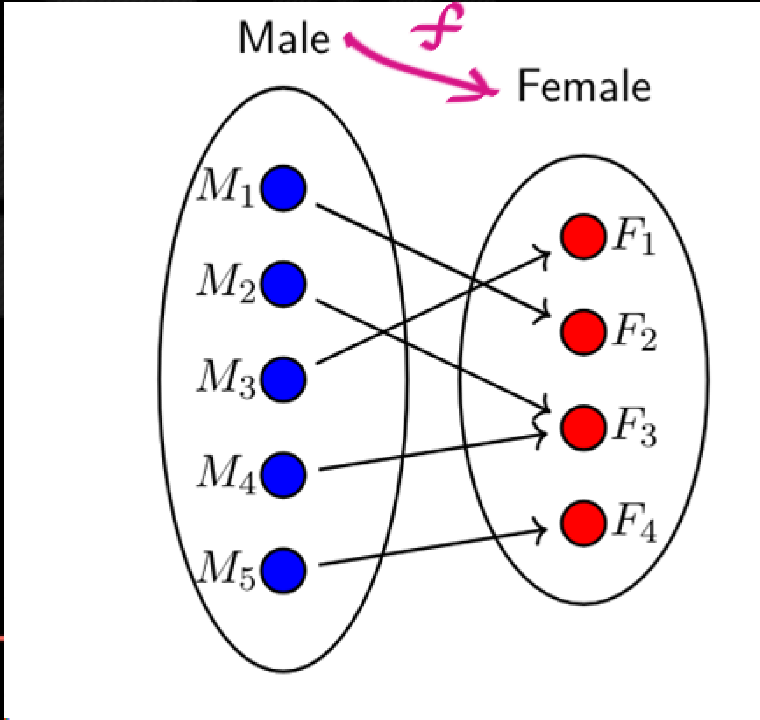
because function f is NOT onto.




because $F_1$ has no image (in g)

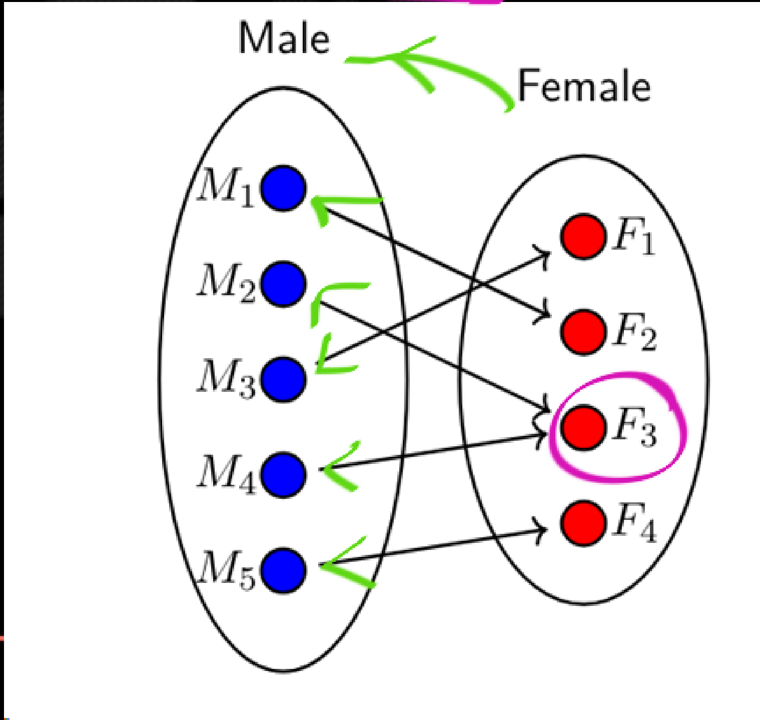
Consider the following function f.

In the Reverse Direction, do we get a function??



Consider the following function f.

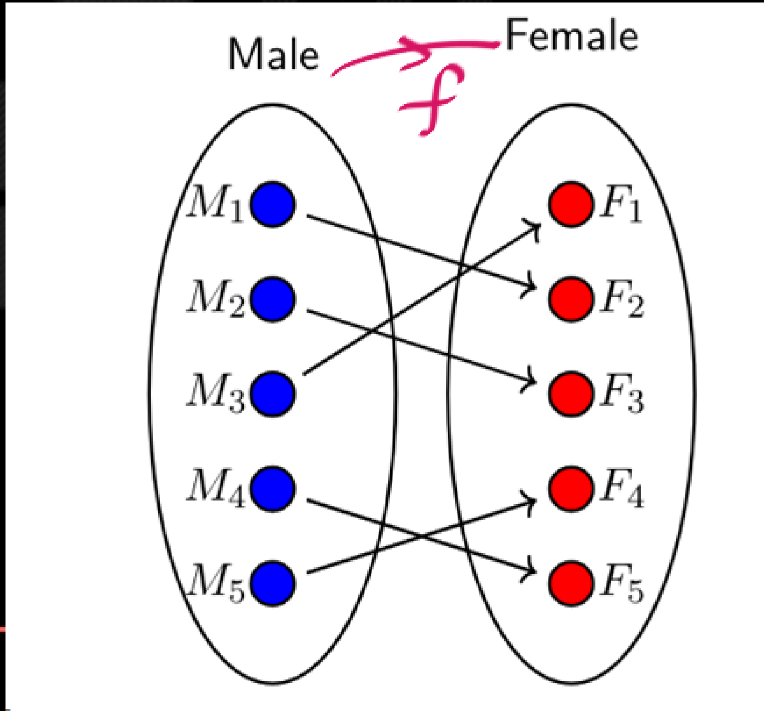
In the Reverse Direction, do we get a function?? → NO




function f is NOT one-one.

# Consider the following function f.

## In the Reverse Direction, do we get a function?? → Yes.



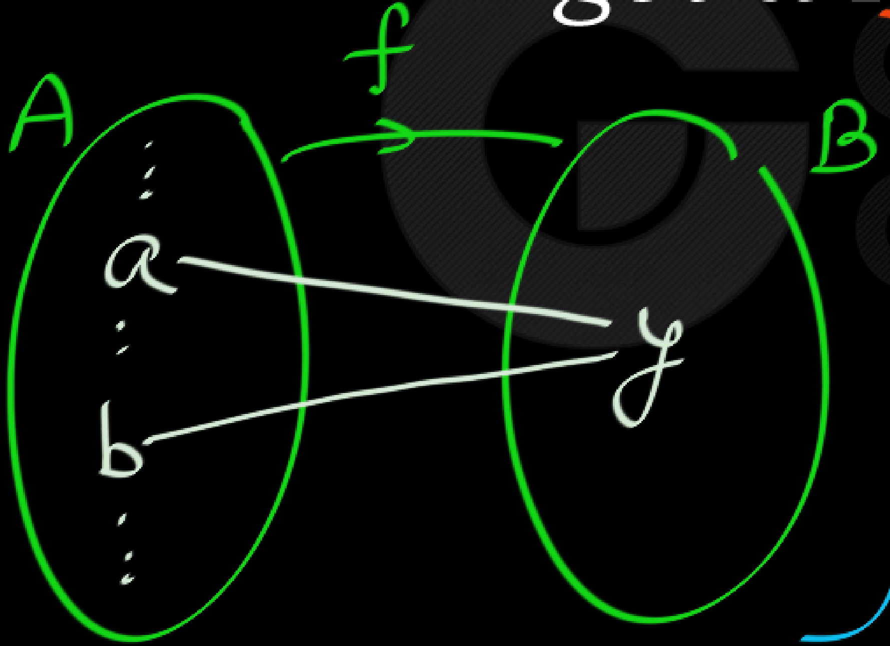

Why?

f is a bijection.

# Note:

For any Function f,
when we look at function f in the Reverse Direction, we get a function iff f is bijective.

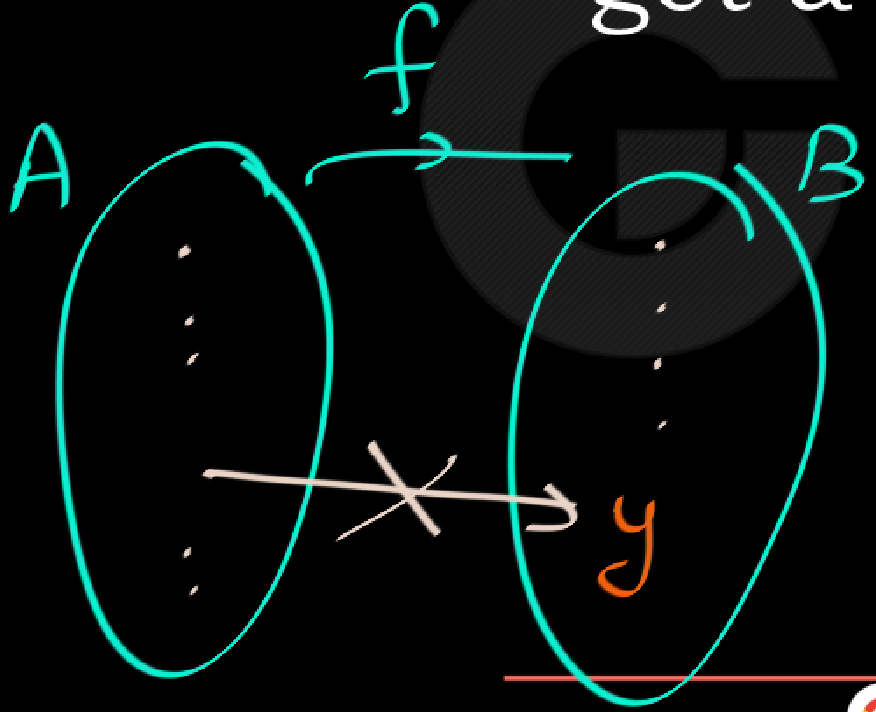
If Function f is NOT One-One, then in the Reverse Direction, we don't get a function.




In Reverse Direction, we are NOT getting a function.

If Function f is NOT Onto,

then in the Reverse Direction, we don't get a function.



Note:

For any Function f,

when we look at function f in the Reverse Direction, we get a function iff f is bijective.

# Note:

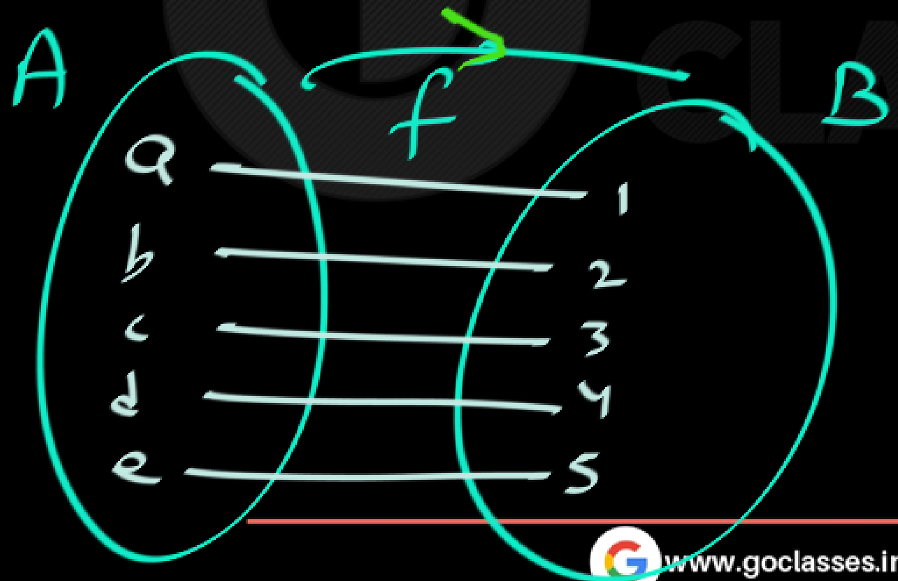
Function f is Invertible iff f is bijective.

Invertible function == Bijective function

Invertible function $\longrightarrow$ one-one & onto

$\longleftarrow$

bijective function f $\longrightarrow$ invertible function




bijective function f, in the Reverse Direction, is also a function.

# Note:

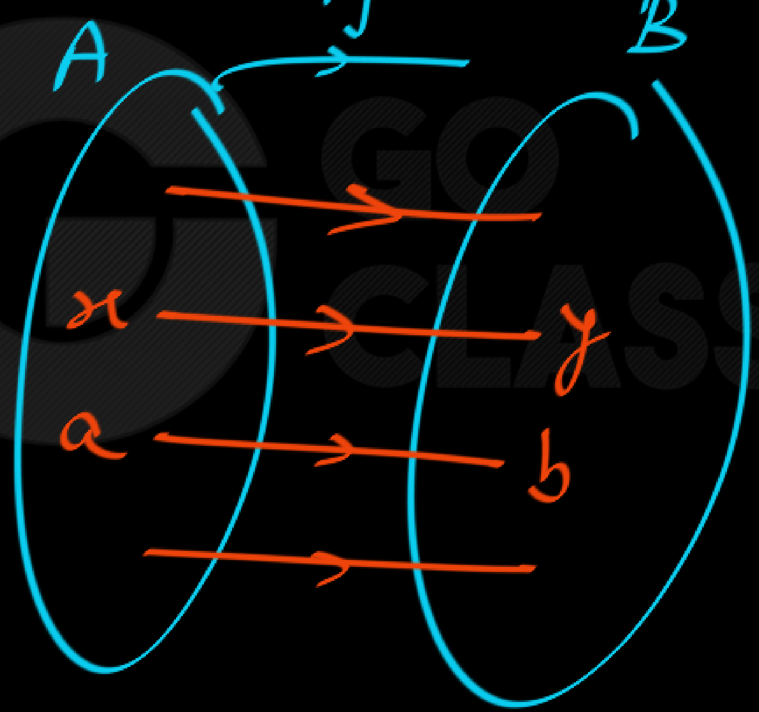
bijective

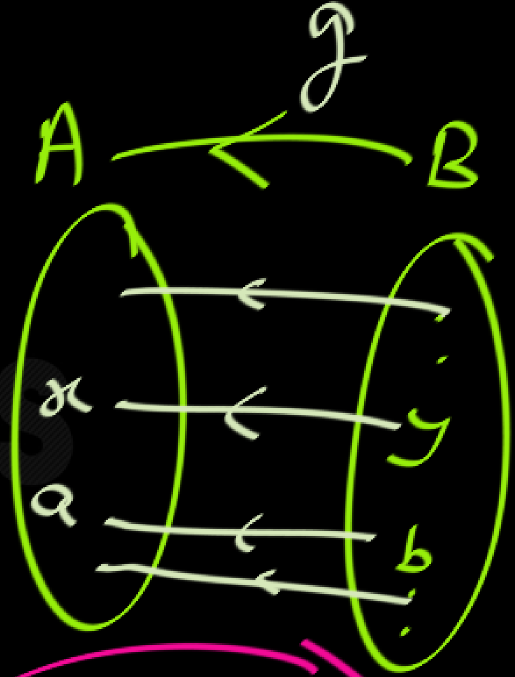
If function f is a Invertible function, then in the Reverse Direction the function we get is called Inverse of f.

If $f$ is Invertible $\longleftrightarrow$ bijective

$f : A \longrightarrow B$

invertible



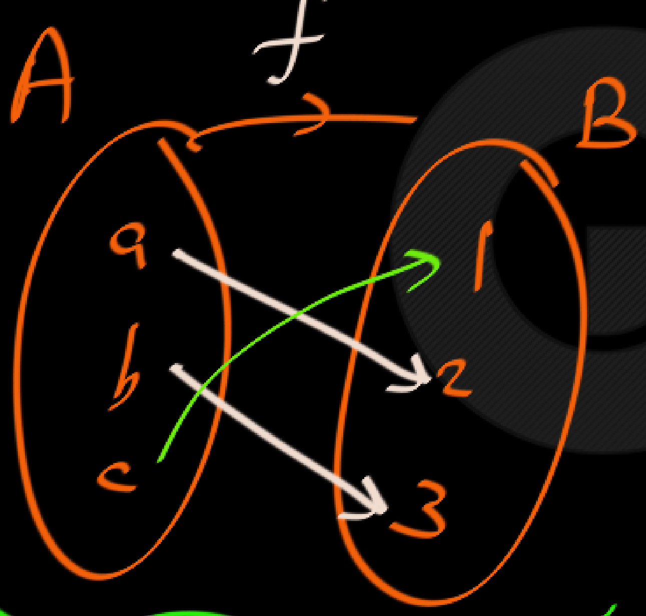

$g = f^{-1}$

**EXAMPLE 18** Let $f$ be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is $f$ invertible, and if it is, what is its inverse?

**EXAMPLE 18** Let $f$ be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is $f$ invertible, and if it is, what is its inverse?

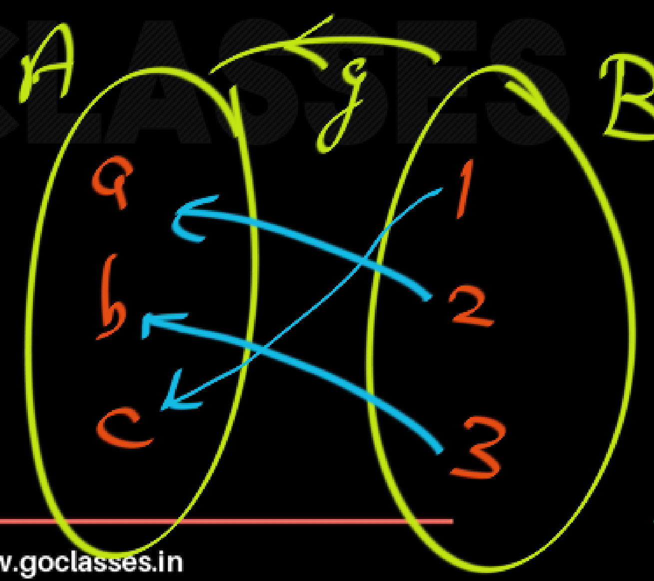
$f : A \to B$

bijective $\longleftrightarrow$ one-one & onto




f : Invertible

Inverse of $f = f^{-1} = g$




$g : B \to A$

$g(3) = b$

$g(1) = c = f^{-1}(1)$

$f^{-1}(2) = g(2) = a$

Note:

1. Invertible ≡ bijective ≡ one-one & onto.

2. If $f : A \to B$ is Invertible then we can find $\boxed{\text{Inverse of } f} = g = f^{-1}$

$g : B \to A$
$f^{-1} : B \to A$

Just "Reverse the mapping done by $f$".

**EXAMPLE 18** Let $f$ be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is $f$ invertible, and if it is, what is its inverse?

*Solution:* The function $f$ is invertible because it is a one-to-one correspondence. The inverse function $f^{-1}$ reverses the correspondence given by $f$, so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$. ◀

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is $f$ invertible, and if it is, what is its inverse?

**EXAMPLE 19** Let $f : \mathbf{Z} \rightarrow \mathbf{Z}$ be such that $f(x) = x + 1$. Is $f$ invertible. [redacted]

$f : Z \rightarrow Z$

$f(x) = x + 1$

one-one & onto

Checking one-one? → Yes.

Assume $f(a) = f(b)$ → Prove that $a = b$

$a + 1 = b + 1$

$a = b$ ✓

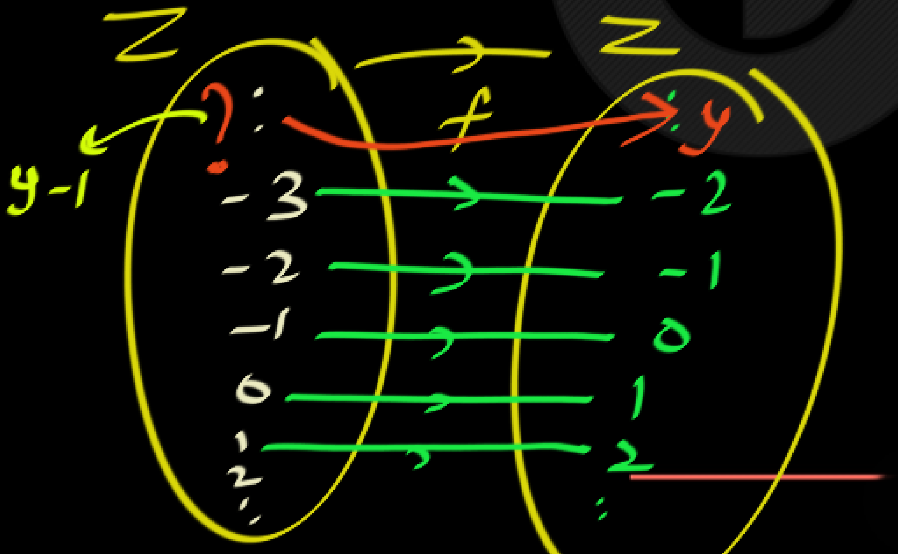
**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is $f$ invertible [redacted]

one-one & onto

$f : Z \to Z$

$f(x) = x + 1$

Checking onto? → Yes.




Take $\boxed{y \in \text{Co-Domain}}$ &

find who maps to $y$.

Assume "$a$" maps to $y$

$f(a) = y \Rightarrow a + 1 = y \Rightarrow a = y - 1$

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is $f$ invertible. [redacted] → Yes ✓

one-one & onto

$$\left.\begin{array}{l} f : Z \to Z \\ f(x) = x + 1 \end{array}\right\} \text{bijective} \equiv \text{Invertible}$$

→ bijective ✓

→ Invertible ✓

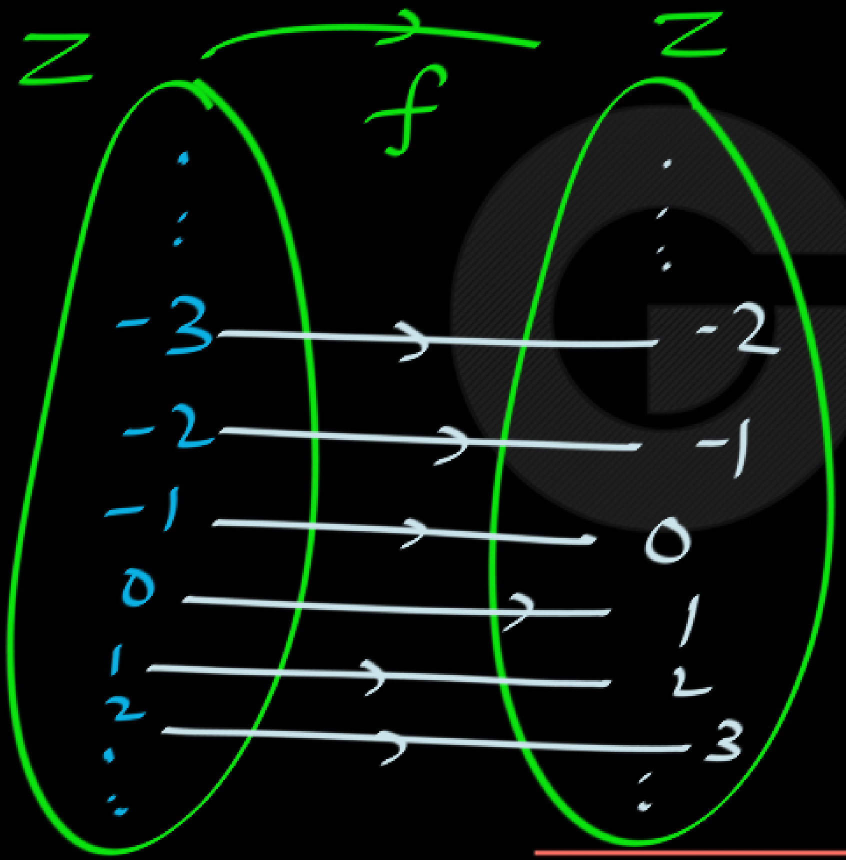
**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. [redacted], what is its inverse?

To find Inverse of f,

Please Apply the Definition/Idea

of Inverse.

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. [redacted], what is its inverse?

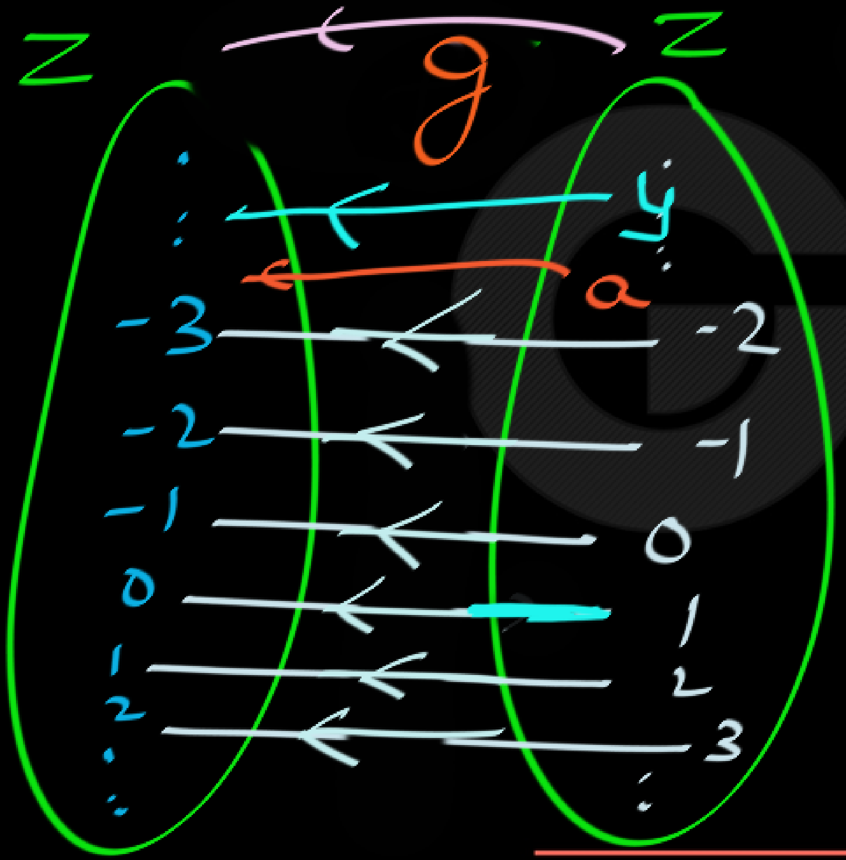



$f(x) = x+1$

$f^{-1} = g$ → Inverse of f

g : Look at f in Reverse Direction.

**EXAMPLE 19** Let $f : \mathbf{Z} \rightarrow \mathbf{Z}$ be such that $f(x) = x + 1$. [redacted], what is its inverse?




$g(0) = -1$

$g(1) = 0$

$g(-2) = -3$

$g(y) = y - 1$

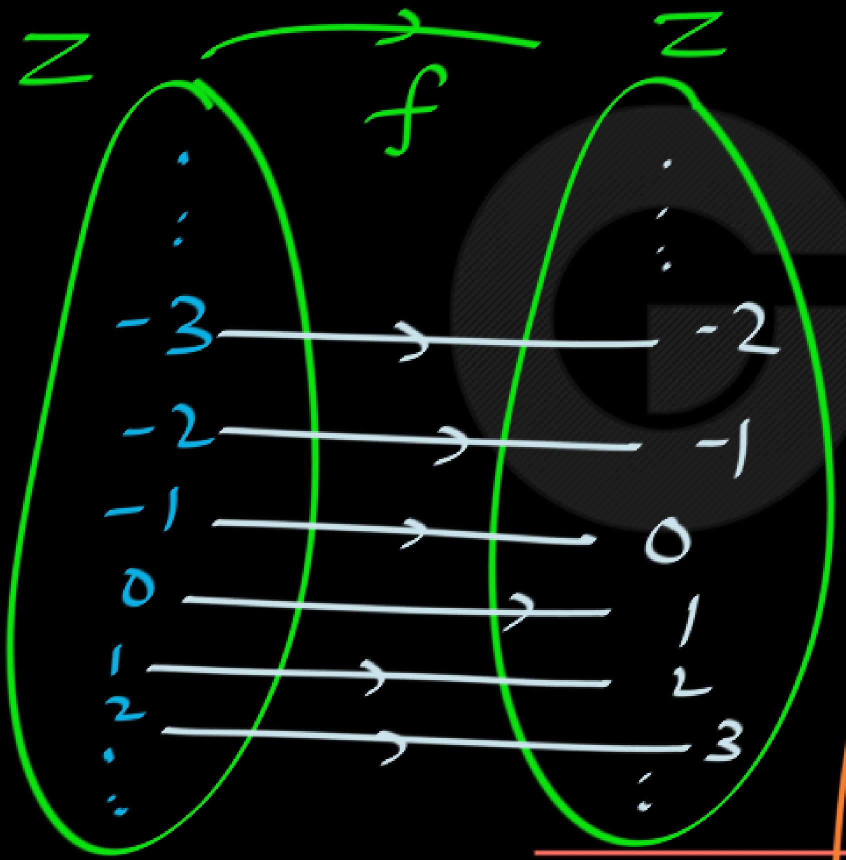
$g(a) = a - 1$

$g(x) = x - 1$

$g = f^{-1}$

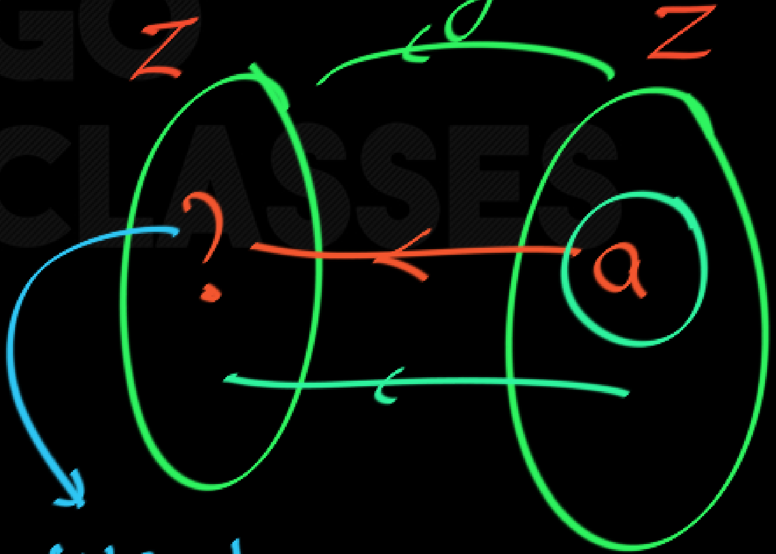
$f^{-1}(x) = x - 1$

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. [redacted], what is its inverse?




$f(x) = x + 1$

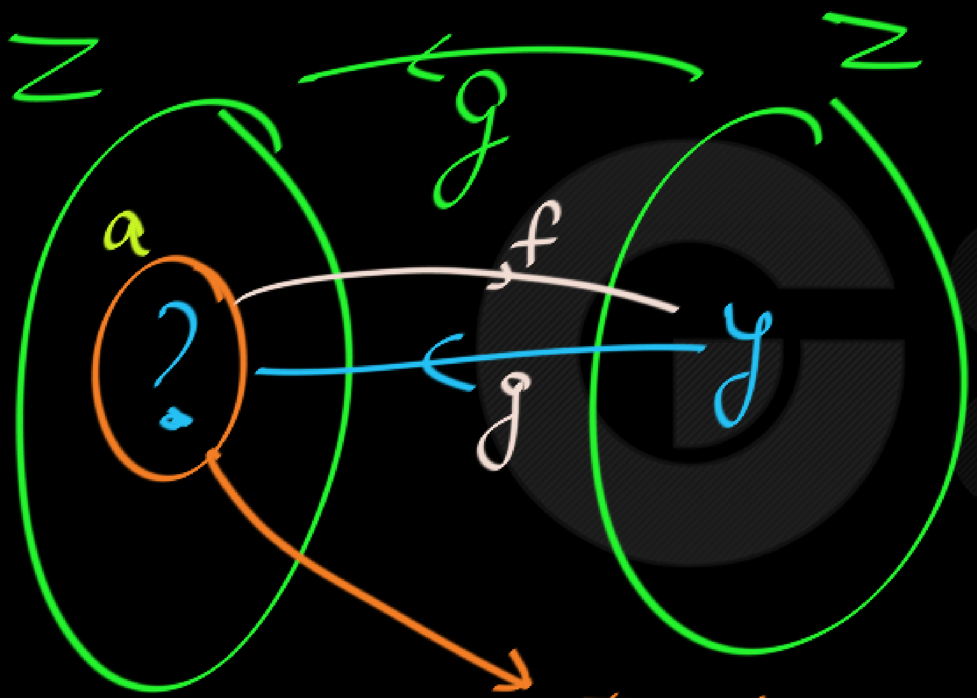
$g = f^{-1}$




$g(a) = ?$

we have to find.

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. [redacted] what is its inverse?




$f(a) = y$

$a + 1 = y$

$\boxed{a = y - 1}$

$a = g(y)$

$g(y) = y - 1$

$g(x) = x - 1$

$f^{-1}(x) = x - 1$

To find $\rightarrow$ assume 'a'

find a.

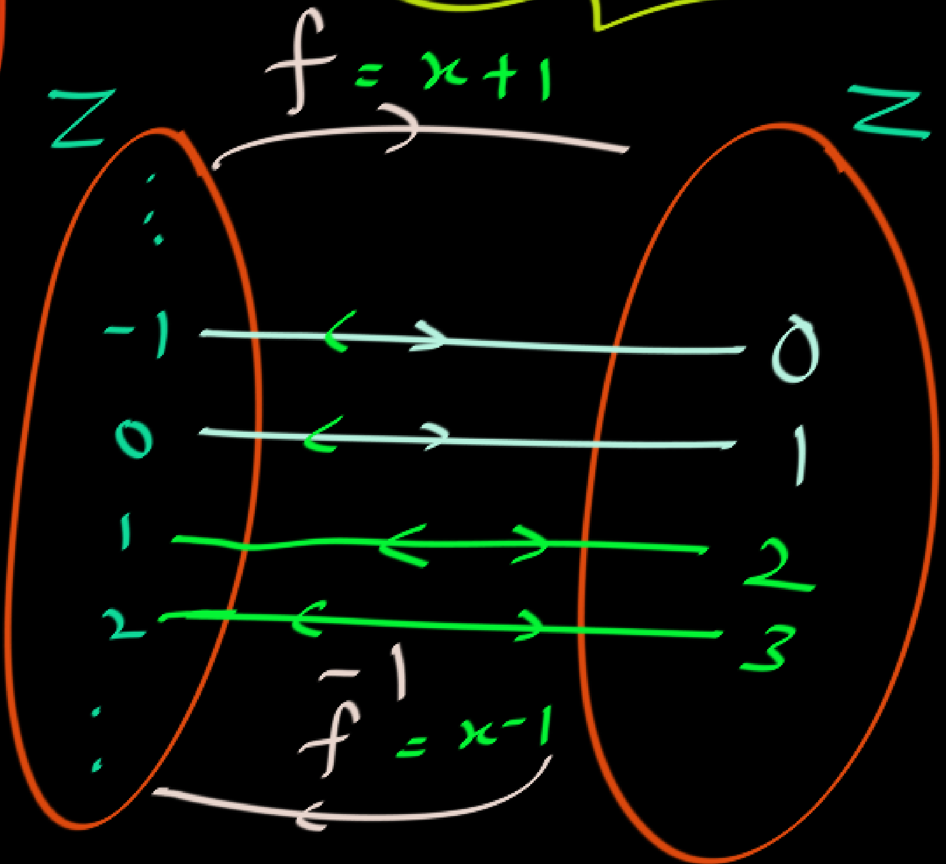
$g = f^{-1}$

$f^{-1}$ : Just a Notation $= g$

$$f : \mathbb{Z} \to \mathbb{Z}$$

$$f(x) = x + 1$$

$$f^{-1}(x) = x - 1$$




$$f(-1) = 0$$

$$f^{-1}(2) = 1$$

$$f^{-1}(0) = 1$$

**EXAMPLE 19** Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is $f$ invertible, and if it is, what is its inverse?

*Solution:* The function $f$ has an inverse because it is a one-to-one correspondence, as follows from Examples 10 and 14. To reverse the correspondence, suppose that $y$ is the image of $x$, so that $y = x + 1$. Then $x = y - 1$. This means that $y - 1$ is the unique element of $\mathbf{Z}$ that is sent to $y$ by $f$. Consequently, $f^{-1}(y) = y - 1$. ◀

**EXAMPLE 20** Let $f$ be the function from $\mathbf{R}$ to $\mathbf{R}$ with $f(x) = x^2$. Is $f$ invertible?

**EXAMPLE 20** Let $f$ be the function from **R** to **R** with $f(x) = x^2$. Is $f$ invertible? No.

$f : R \to R$

$f(x) = x^2$

NOT one-one.

$f(2) = f(-2)$
$= 4$

NOT onto because Negative numbers have No pre-image.

Pre-image of 3: $\sqrt{3}, -\sqrt{3}$

**EXAMPLE 20** Let $f$ be the function from **R** to **R** with $f(x) = x^2$. Is $f$ invertible?

*Solution:* Because $f(-2) = f(2) = 4$, $f$ is not one-to-one. If an inverse function were defined, it would have to assign two elements to 4. Hence, $f$ is not invertible. (Note we can also show that $f$ is not invertible because it is not onto.) ◀

## Problem

- Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Find its inverse function.

## Problem

- Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Find its inverse function.

$f : R \to R$

$f(x) = 4x - 1$

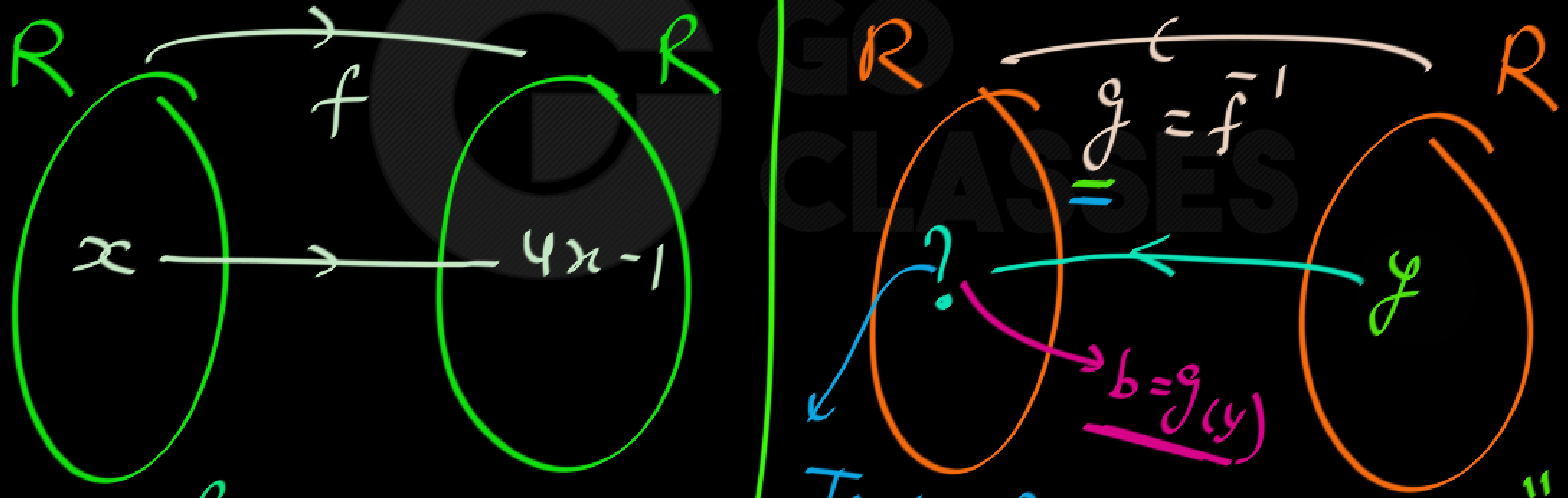
→ one-one & onto ✓

→ invertible ✓

→ bijective ✓

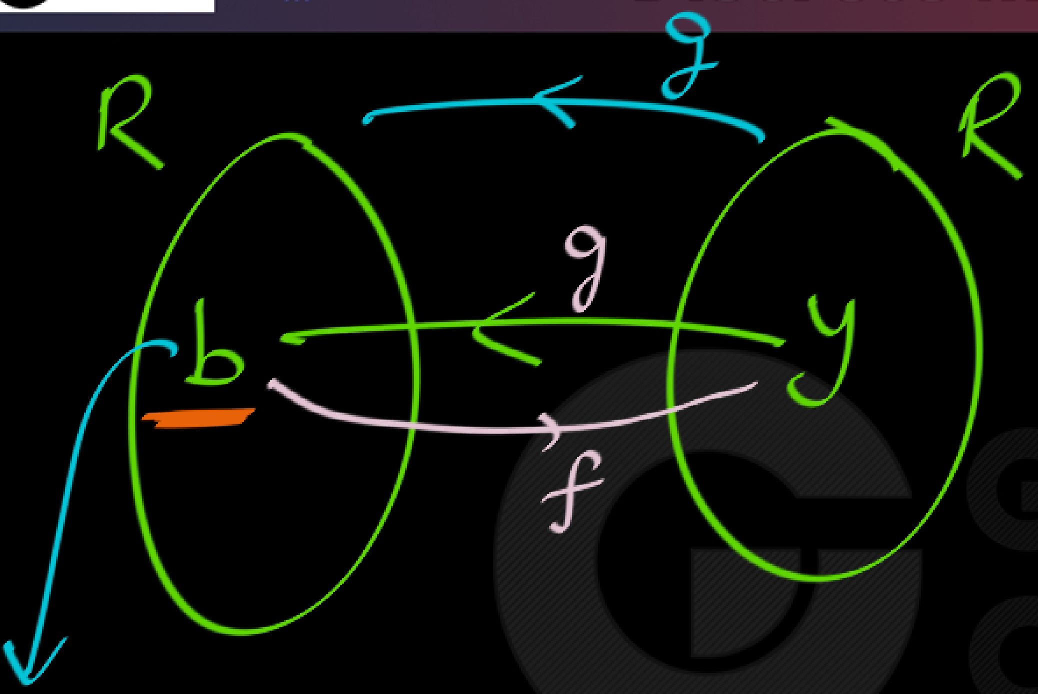
## Problem

- Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Find its inverse function.




$f(x) = 4x - 1$

To be found ⇒ Assume "b" = g(y)




We need to find.

$b = g(y)$

$\underline{f}(b) = y$

$4b - 1 = y$

$b = \dfrac{y+1}{4}$

$$b = \boxed{g(y) = \frac{y+1}{4}}$$

$$g(x) = \frac{x+1}{4}$$

$g = f^{-1}$

$g(x) = \dfrac{x+1}{4}$

$$\boxed{f^{-1}(x) = \frac{x+1}{4}}$$

## Problem

- Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Find its inverse function.

## Proof

For any $y$ in $R$, by definition of $f^{-1}$

- $f^{-1} =$ unique number $x$ such that $f(x) = y$

  Consider $f(x) = y$

  $\implies 4x - 1 = y \qquad (\because$ Defn. of $f)$

  $\implies x = \frac{y+1}{4} \qquad (\because$ Simplify$)$

- Hence, $f^{-1}(y) = \frac{y+1}{4}$ is the inverse function.

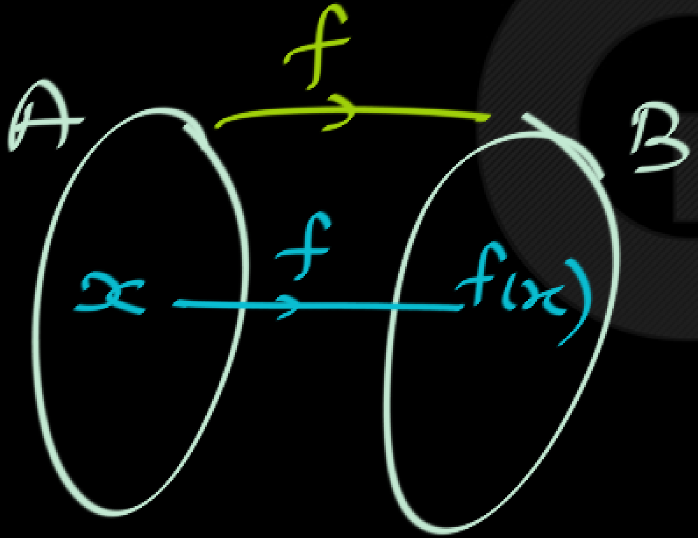
Now consider a one-to-one correspondence $f$ from the set $A$ to the set $B$. Because $f$ is an onto function, every element of $B$ is the image of some element in $A$. Furthermore, because $f$ is also a one-to-one function, every element of $B$ is the image of a *unique* element of $A$. Consequently, we can define a new function from $B$ to $A$ that reverses the correspondence given by $f$. This leads to Definition 9.

**DEFINITION 9**

Let $f$ be a one-to-one correspondence from the set $A$ to the set $B$. The *inverse function* of $f$ is the function that assigns to an element $b$ belonging to $B$ the unique element $a$ in $A$ such that $f(a) = b$. The inverse function of $f$ is denoted by $f^{-1}$. Hence, $f^{-1}(b) = a$ when $f(a) = b$.

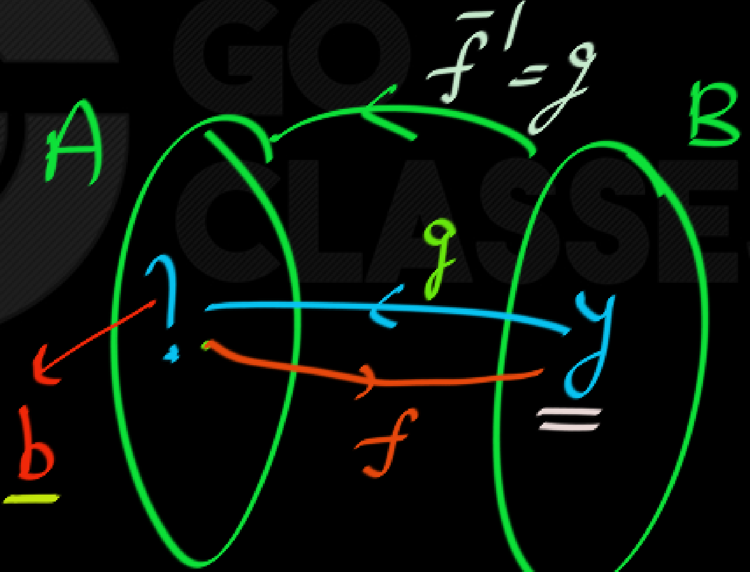
Summary: If "f" is Invertible

$f : A \to B$




$f(x)$ : Given

Question: find $f^{-1}$.




$g : B \to A$

To find: "b"

$\boxed{b = g(y)}$ ✓

$f(b) = y$

A one-to-one correspondence is called **invertible** because we can define an inverse of this function. A function is **not invertible** if it is not a one-to-one correspondence, because the inverse of such a function does not exist.

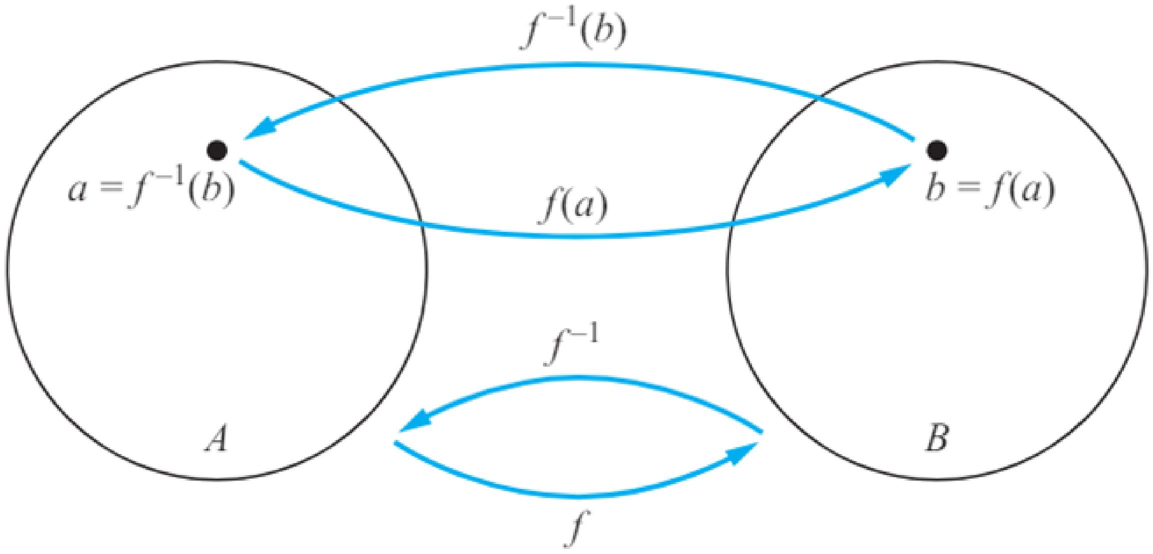



**FIGURE 6** **The Function $f^{-1}$ Is the Inverse of Function $f$.**

# Question:

## True or False??

If $X$ and $Y$ are sets and $F : X \to Y$ is a one-to-one correspondence, then $F^{-1} : Y \to X$ is also a one-to-one correspondence.

# Question:

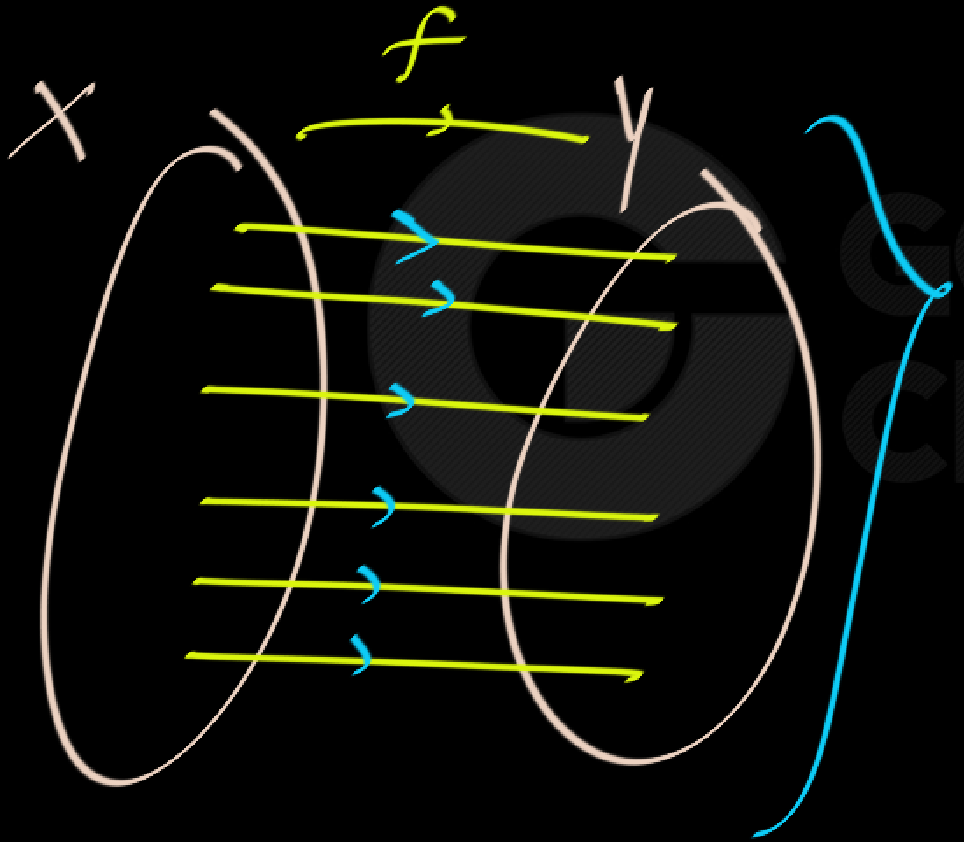
True or False??

If $X$ and $Y$ are sets and $F : X \to Y$ is a one-to-one correspondence, then $F^{-1} : Y \to X$ is also a one-to-one correspondence.

bijection ≡ Invertible

bijective

Proof : Given: $f : X \to Y$ bijective ≡ Invertible




In the Reverse Direction of f, we have 1-1 Correspondance.

# Inverse functions

**Theorem**

- If $X$ and $Y$ are sets and $F : X \to Y$ is a one-to-one correspondence, then $F^{-1} : Y \to X$ is also a one-to-one correspondence.

# Question:

If function f is invertible,

What is $(f^{-1})^{-1} = ??$

# Question:

If function f is invertible,

What is $(f^{-1})^{-1}$ = ?? $= f$

**Theorem 7.** *Let $A$ and $B$ be nonempty sets, and suppose $f : A \to B$ is invertible. Then $f^{-1} : B \to A$ is also invertible, and $(f^{-1})^{-1} = f$.*

## 4.3.6 Functions: GATE CSE 1996 | Question: 2.1 top

https://gateoverflow.in/2730

Let $R$ denote the set of real numbers. Let $f: R \times R \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of $f$ is given by

A. $f^{-1}(x, y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

B. $f^{-1}(x, y) = (x - y, x + y)$

C. $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

D. $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

## 4.3.6 Functions: GATE CSE 1996 | Question: 2.1 top

https://gateoverflow.in/2730

Let $R$ denote the set of real numbers. Let $f : R \times R \to R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of $f$ is given by

≡ Invertible

Given: f is Invertible

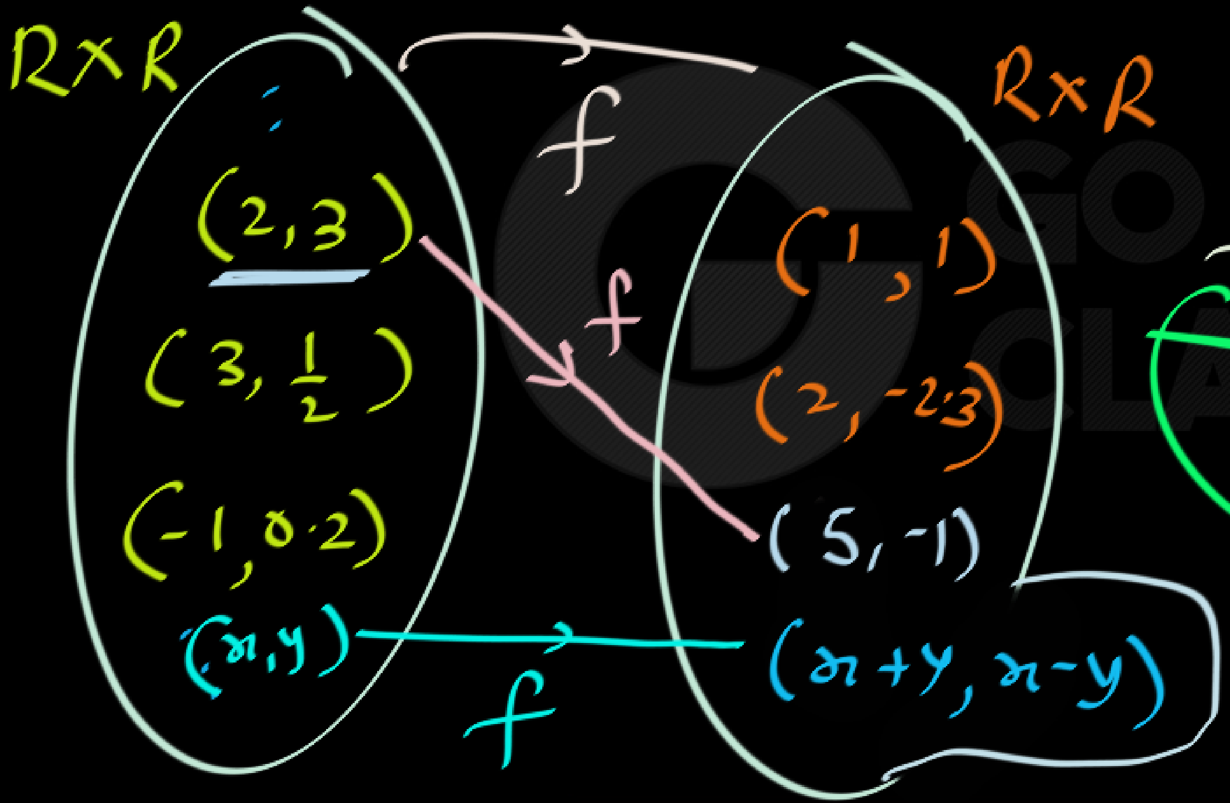
A. $f^{-1}(x, y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

B. $f^{-1}(x, y) = (x - y, x + y)$

C. $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

D. $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

Understand $f$ : $f : R \times R \longrightarrow R \times R$

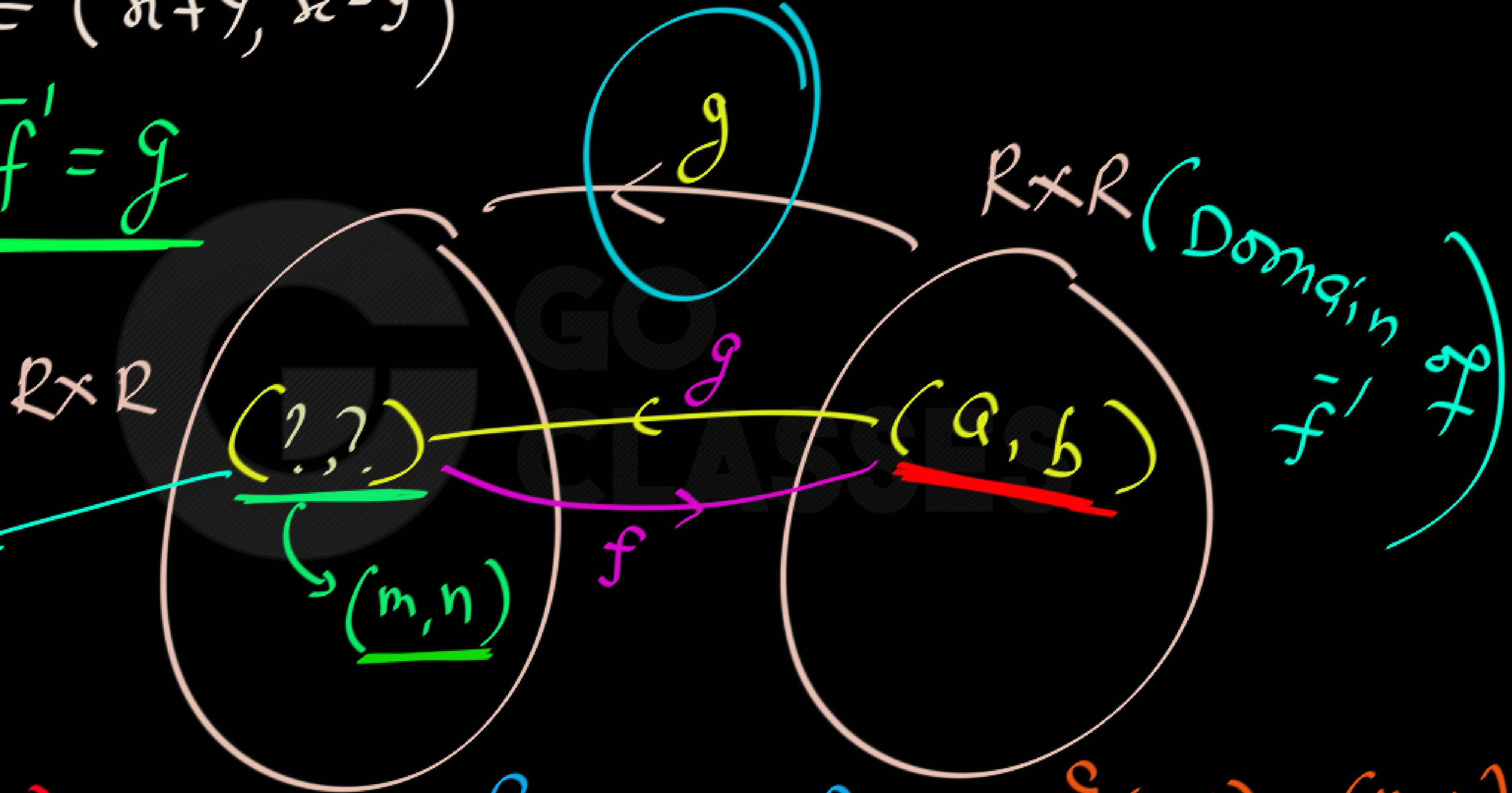



$f(x,y) = (x+y, x-y)$

bijective $\equiv$ Invertible

$f(x, y) = (x+y,\ x-y)$

Let: $\underline{f^{-1} = g}$




To find

Assume $(m, n)$

find $m, n$?

$g(a, b) = (m, n)$

$$f(m,n) = (a,b)$$

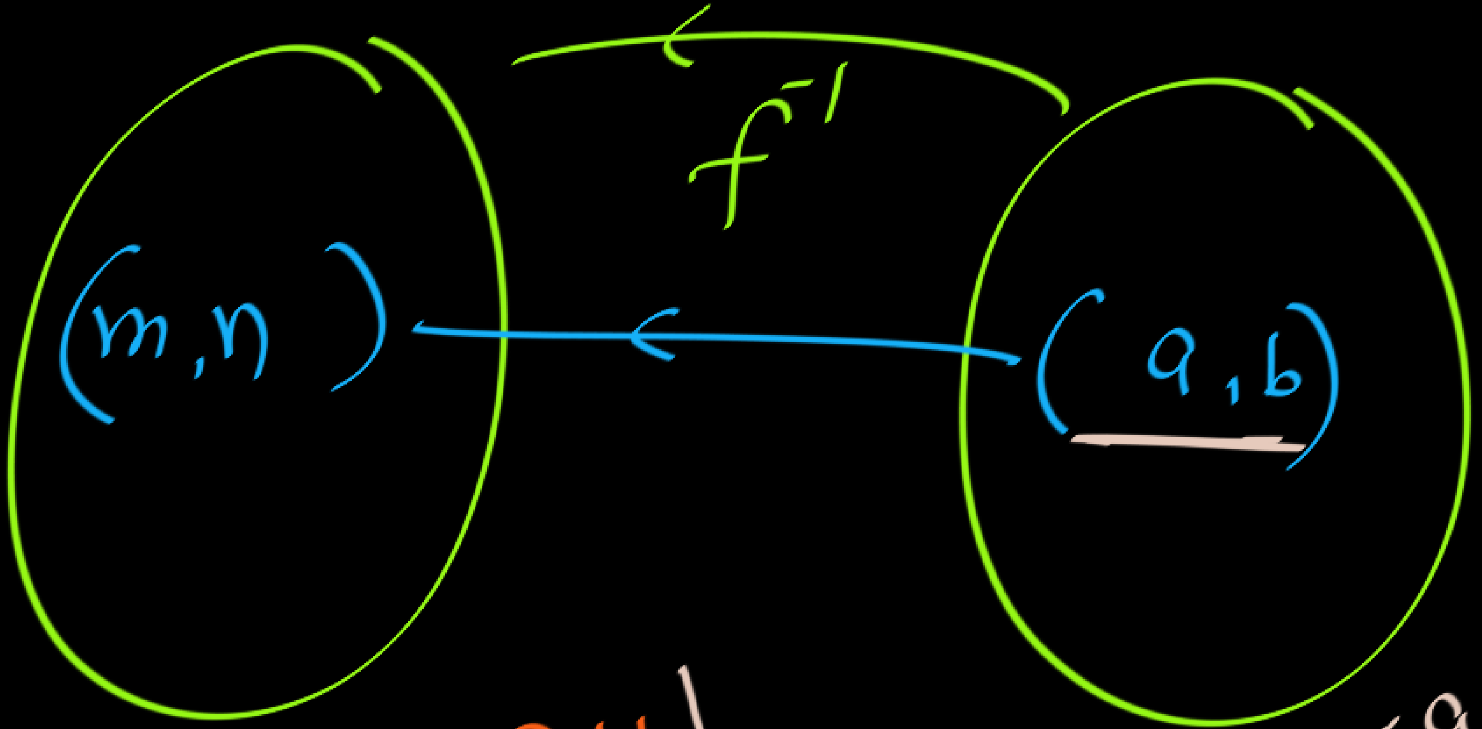
$$(m+n,\ m-n) = (a,b)$$

$$\begin{cases} a = m+n \\ b = m-n \end{cases} \rightarrow \underline{\text{find } m, n.}$$

$$a+b = 2m$$

$$m = \frac{a+b}{2} \checkmark$$

$$n = \frac{a-b}{2} \checkmark$$




$$m = \frac{a+b}{2}$$

$$n = \frac{a-b}{2}$$

$$g(a,b) = \left(\frac{a+b}{2}, \frac{a-b}{2}\right)$$

$$f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$$

$$f^{-1} = g$$

**Note:**

Ordered Pairs Equality:

$(x, y) = (p, q)$

iff

$x = p$, & $y = q$

Quickly: Take some Random ordered pair & check.

### 4.3.6 Functions: GATE CSE 1996 | Question: 2.1 top

https://gateoverflow.in/2730

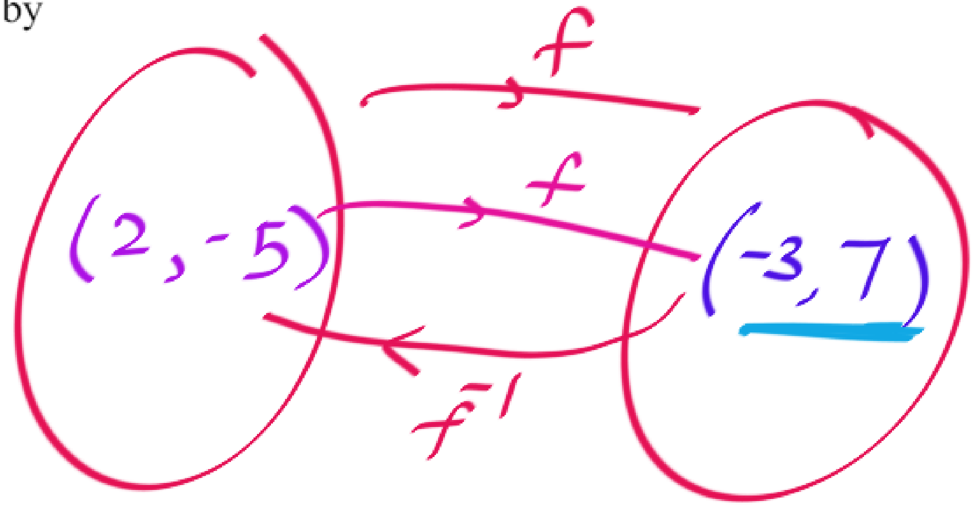
Let $R$ denote the set of real numbers. Let $f : R \times R \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of $f$ is given by

A. $f^{-1}(x, y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$ ✗

B. $f^{-1}(x, y) = (x - y, x + y)$ ✗

C. $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$ ✓

D. $f^{-1}(x, y) = [2(x - y), 2(x + y)]$ ✗




check all options for $f^{-1}(-3, 7) \Rightarrow$ must be $(2, -5)$

Now we will see the Quick Method to find Inverse of a function...

Algorithm

BUT

The Proper Idea (Definition) that we have applied so far in all questions, is More Intuitive & Best way..

# Algorithm to find Inverse of a function:

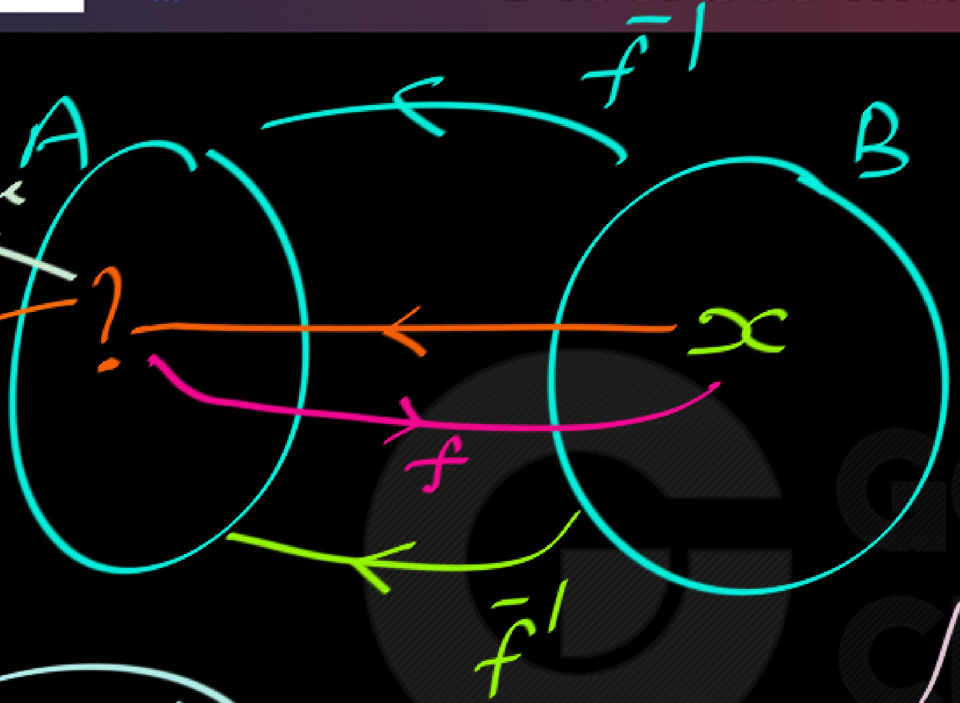
① Note: Given: Invertible function $f$

$\boxed{f(x) = \cdots}$ Given

② To find: $f^{-1}(x) = ?$

Assume $y$

To find




**To find:** $f^{-1}(x)$

**To find:**

$f^{-1}(x) = y$ (Assumed)

$\boxed{\text{Apply: } f(y) = x}$

get: $y = \dots = f^{-1}(x)$ ✓

$\underbrace{f: A \to B}$

$f(x) = --$

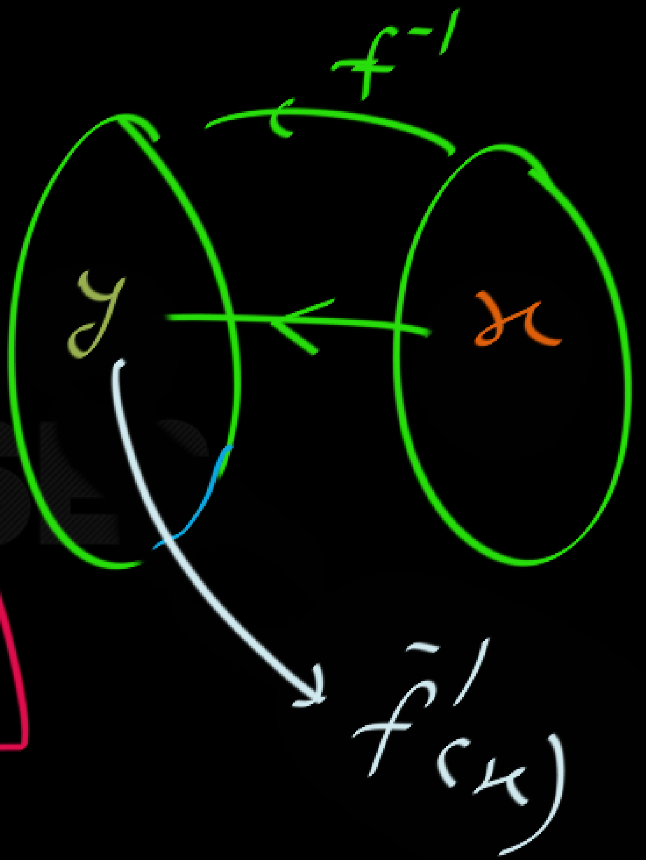
Given

# Algorithm to find Inverse of a function:

① Given: $f(x) = \text{- - -}$

② Apply $\underbrace{f(y) = x}$

get $\boxed{y = \text{- - - - -} = f^{-1}(x)}$



**6.7.** The function $f(n) = 2n$ is a bijection from $\mathbb{Z}$ to the even integers and the function $g(n) = 2n + 1$ is a bijection from $\mathbb{Z}$ to the odd integers. What are $f^{-1}$, $g^{-1}$

**6.7.** The function $f(n) = 2n$ is a bijection from $\mathbb{Z}$ to the even integers and the function $g(n) = 2n + 1$ is a bijection from $\mathbb{Z}$ to the odd integers. What are $f^{-1}$, $g^{-1}$

Find $f^{-1}(x) = ?$

Algo: Given: $f(x) = 2x$

Apply: $\boxed{f(y) = x}$ find $y$

$y = f^{-1}(x) = \frac{x}{2}$

$2y = x \Rightarrow y = x/2$

**6.7.** The function $f(n) = 2n$ is a bijection from $\mathbb{Z}$ to the even integers and the function $g(n) = 2n + 1$ is a bijection from $\mathbb{Z}$ to the odd integers. What are $f^{-1}$, $g^{-1}$

find $g^{-1} = ?$

Given: $g(x) = 2n + 1$

Apply: $\boxed{g(y) = x}$

find $y$

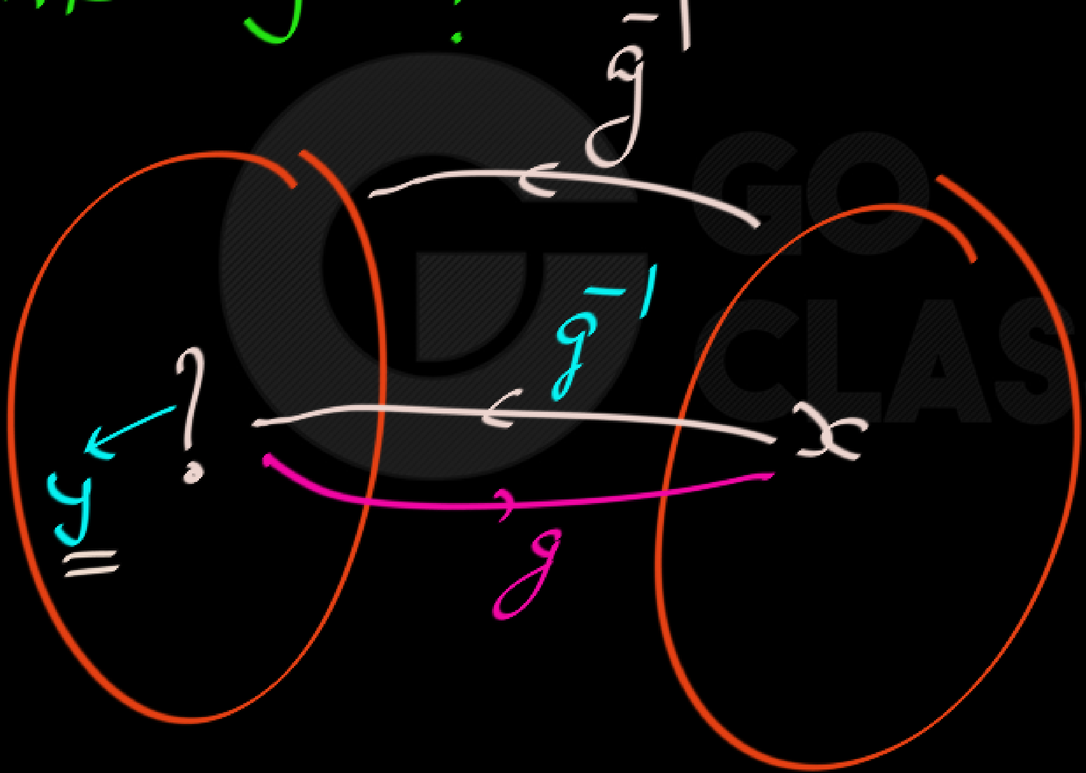
$2y + 1 = x$

$y = x - 1/2$

$y = f^{-1}(x) = \frac{x-1}{2}$

**Given:** $g(n) = 2n + 1$ (Given)

To find $g^{-1} = ?$




To find

$\underline{g^{-1}(x) = y}$

$\underbrace{g(y)} = x$

$2y + 1 = x$

$y = \dfrac{x-1}{2} = g^{-1}(x)$

# Next:

## Questions based on Inverse & Composition Combo

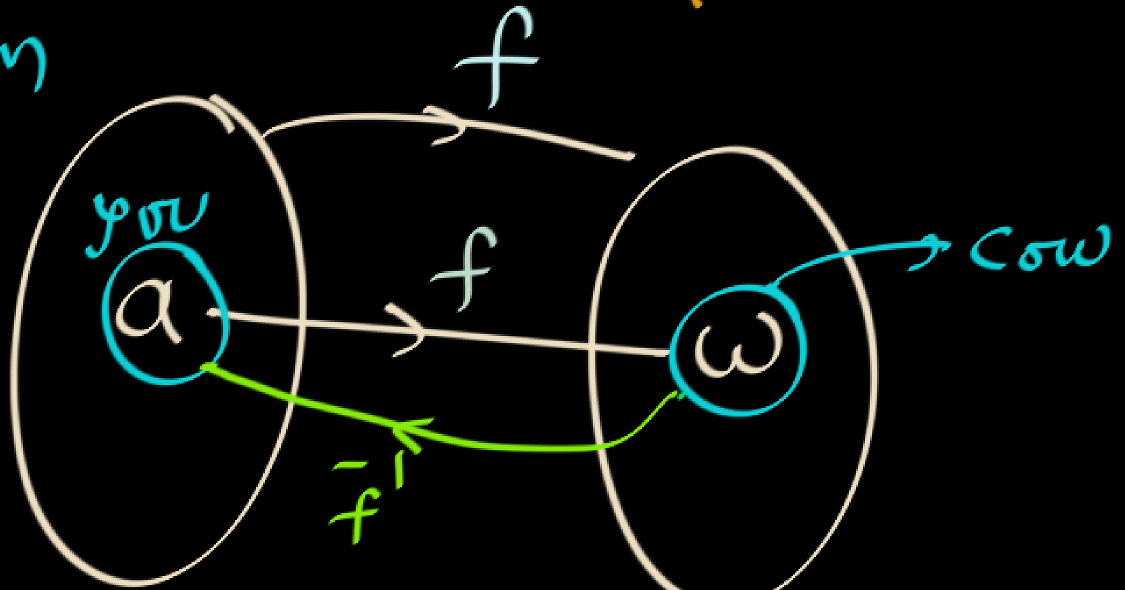
# What does Inverse Function Signify??

# What does it ACTUALLY DO??

$f$ : Invertible function

function $\equiv$ mapping $\equiv$ Transformation

magician



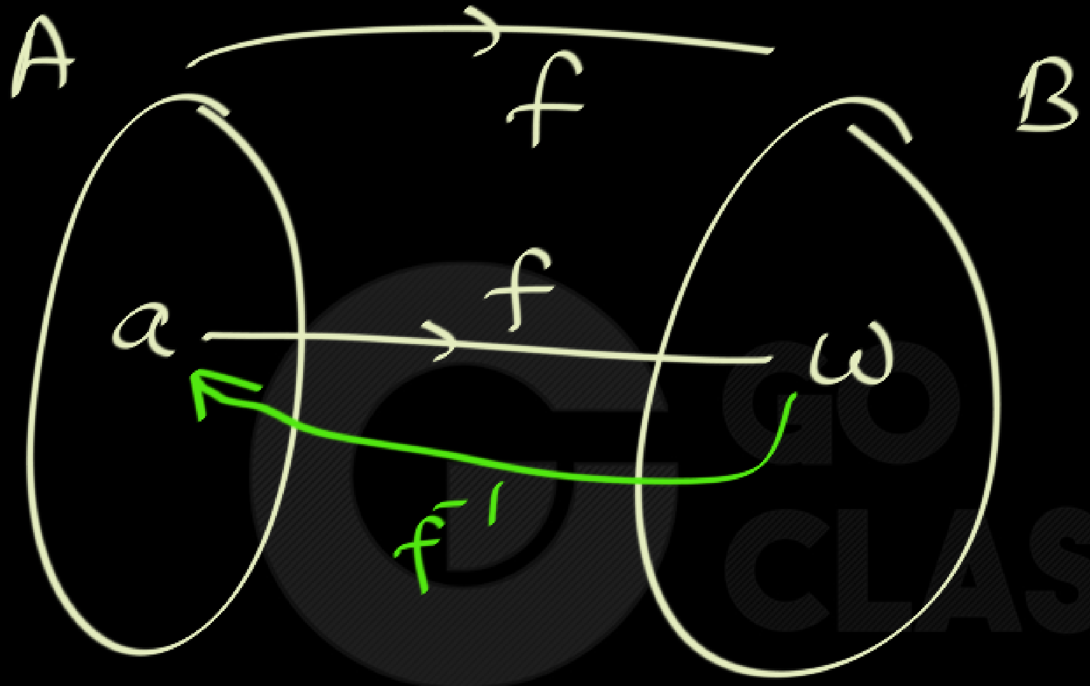

Inverse function Does "Undo"

# What does Inverse Function Signify??

# What does it ACTUALLY DO??

Answer:

If f is invertible,
Inverse of f "Undo" the transformation done by f.

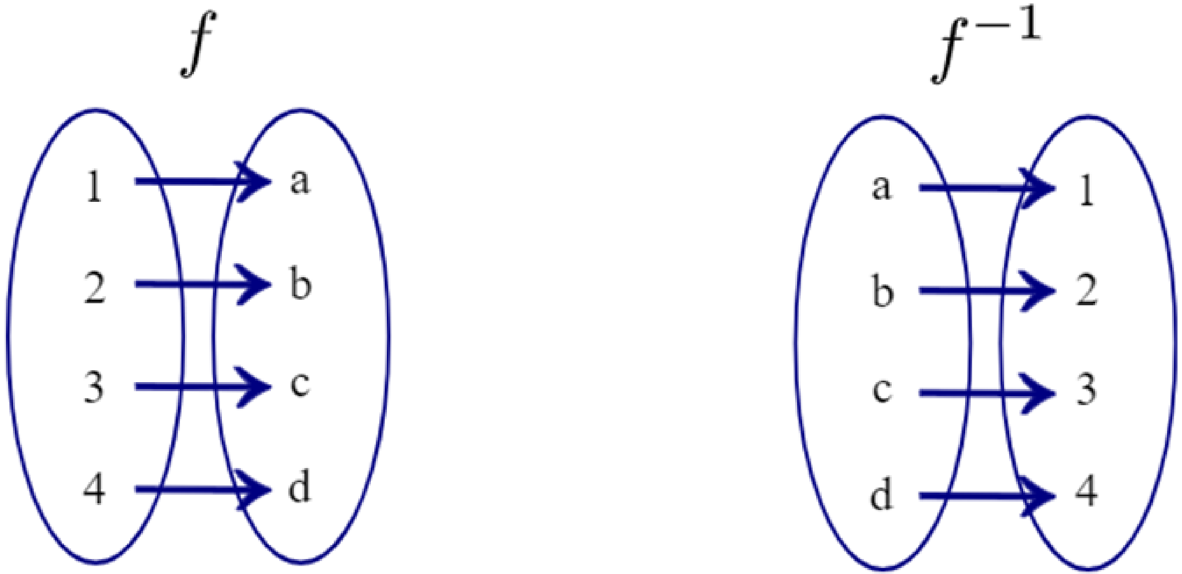



Note:

f must be Invertible to Define $f^{-1}$.

# 2.4 Inverse Functions ¶

In mathematics, an *inverse* is a function that serves to "undo" another function. That is, if $f(x)$ produces $y$, then putting $y$ into the inverse of $f$ produces the output $x$. A function $f$ that has an inverse is called invertible and the inverse is denoted by $f^{-1}$. It is best to illustrate inverses using an arrow diagram:




Notice how $f$ maps 1 to $a$, and $f^{-1}$ undoes this, that is, $f^{-1}$ maps $a$ back to 1. Don't confuse $f^{-1}(x)$ with exponentiation: the inverse $f^{-1}$ is *different* from $\frac{1}{f(x)}$.

# Identity Function:

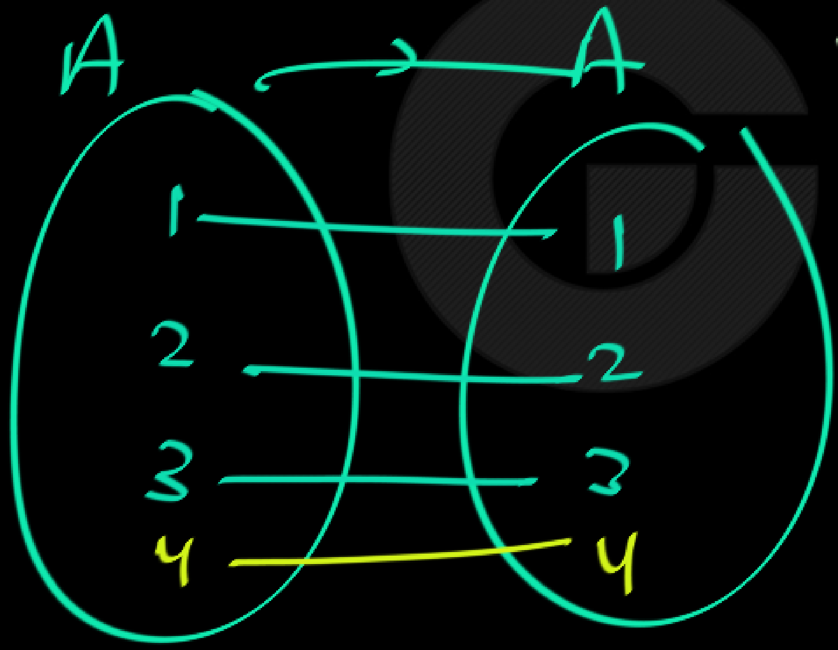
Let $A$ be a set. The *identity function* on $A$ is the function $\iota_A : A \to A$, where

$$\iota_A(x) = x$$

for all $x \in A$. In other words, the identity function $\iota_A$ is the function that assigns each element to itself. The function $\iota_A$ is one-to-one and onto, so it is a bijection. (Note that $\iota$ is the Greek letter iota.)

Identity function

$\underbrace{f : A \to A}$




$f(x) = x$ ✓

Identity function

$I_A : A \to A$

$I_B \rightarrow$ Identity function

$\rightarrow I_B : B \rightarrow B$

$I_B(x) = x$ ✓

# Question:

Let f: A → B be an Invertible Function.

What is $f \circ f^{-1}$ & $f^{-1} \circ f$ ??

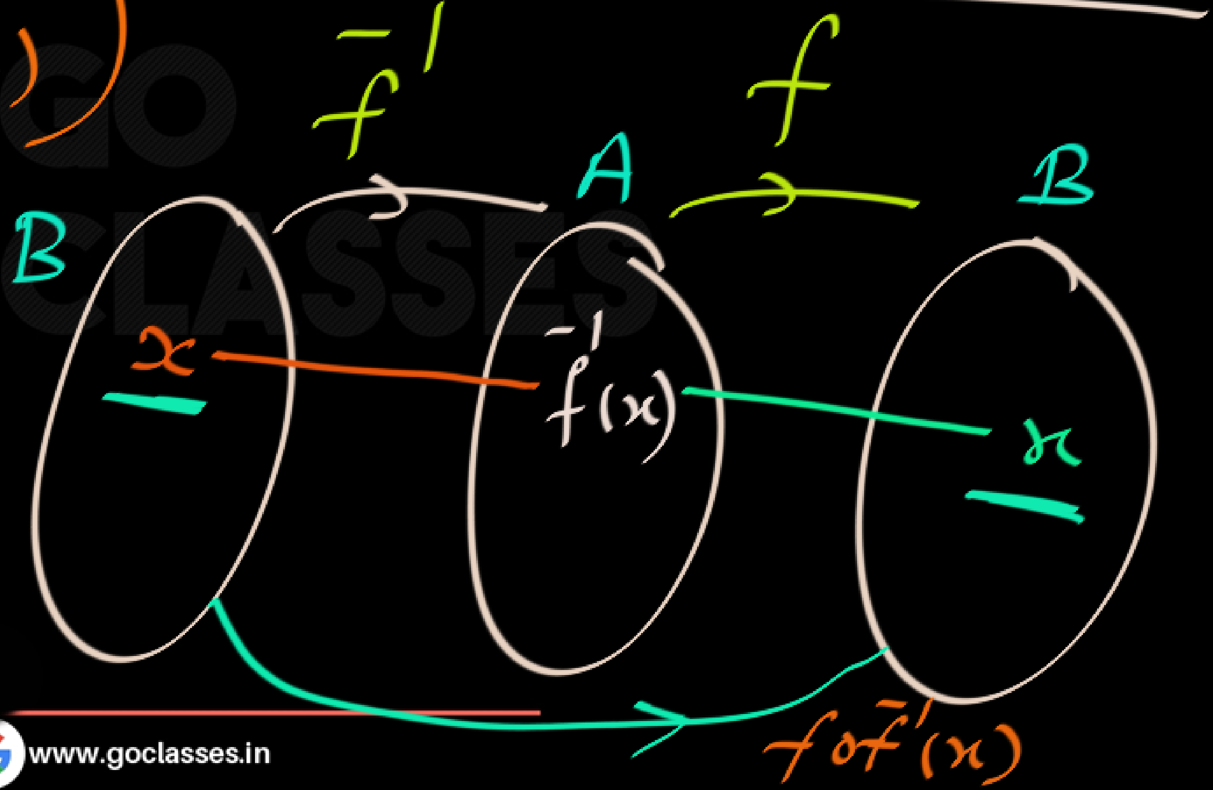
$f : A \rightarrow B$ → Invertible

So $f \circ f^{-1}$ is Identity function $B \rightarrow B$

$f \circ f^{-1}(x) = f\left(f^{-1}(x)\right)$

Composition function

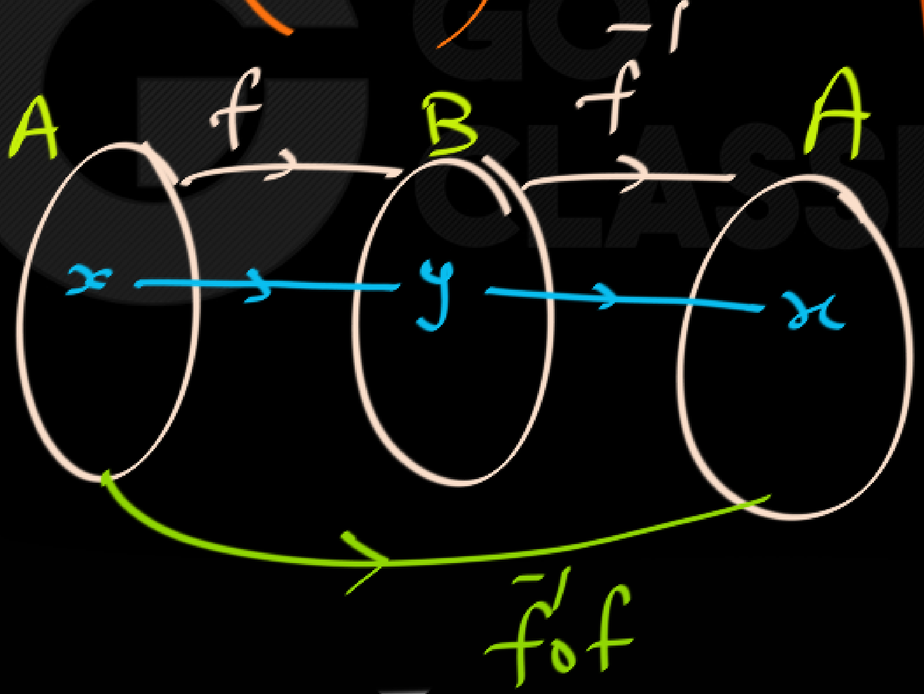
$$\boxed{f \circ f^{-1} = I_B}$$



$f : A \to B$ Invertible

$f^{-1} \circ f(x) = ? = f^{-1}(f(x))$

Composition function




$f^{-1} \circ f$ is Identity function from A to A.

$f^{-1} \circ f = I_A$

# Conclusion:

Invertible function $f : A \rightarrow B$

$$\left\{ \begin{aligned} f \circ f^{-1}(x) &= I_B(x) \\ f^{-1} \circ f(x) &= I_A(x) \end{aligned} \right\}$$

Q: $f : Z \to Z$ ; $\underbrace{f(x) = x+1}$ → Invertible

find $\underbrace{f \circ f^{-1}(x) = ?}$

Q: $f : Z \to Z$ ; $\underbrace{f(x) = x+1}$ → Invertible

find $\underbrace{f \circ f^{-1}(x) = ?}$ $= I_Z$ → Identity function

$$\boxed{f \circ f^{-1}(x) = x}$$ ✓

$I_Z(x) = x$

# Question:

Let $f: A \rightarrow B$ be an Invertible Function.

What is $\underbrace{f \circ f^{-1}}_{I_B}$ & $\underbrace{f^{-1} \circ f}_{I_A}$ ??

When the composition of a function and its inverse is formed, in either order, an identity function is obtained. To see this, suppose that $f$ is a one-to-one correspondence from the set $A$ to the set $B$. Then the inverse function $f^{-1}$ exists and is a one-to-one correspondence from $B$ to $A$. The inverse function reverses the correspondence of the original function, so $f^{-1}(b) = a$ when $f(a) = b$, and $f(a) = b$ when $f^{-1}(b) = a$. Hence,

$$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a,$$

and

$$(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b.$$

Consequently $f^{-1} \circ f = \iota_A$ and $f \circ f^{-1} = \iota_B$, where $\iota_A$ and $\iota_B$ are the identity functions on the sets $A$ and $B$, respectively. That is, $(f^{-1})^{-1} = f$.

# Next:

## More Questions based on Inverse & Composition Combo

# Question:

If f & g are Invertible functions such that

$f: A \rightarrow B$, $g: B \rightarrow C$

Then $g \circ f$ is Invertible?

# Question:

If f & g are Invertible functions such that

$$\boxed{f: A \rightarrow B,\ g: B \rightarrow C} \rightarrow g \circ f$$

Defined

Then $g \circ f$ is Invertible? → Yes.

Remember: $f: A \to B$, $g: B \to C$

1. If $f, g$ both 1-1, then $g \circ f$ 1-1.
2. If $f, g$ both onto, then $g \circ f$ onto.
3. If $f, g$ both bijective, then $g \circ f$ bijective.

invertible

Invertible

# Question:

If f & g are Invertible functions such that

$f: A \to B,\ g: B \to C$

What is $(g_o f)^{-1} = ??$

$g^{-1} o f^{-1}$?

$f^{-1} o g^{-1}$? or

# Question:

If f & g are Invertible functions such that

$f: A \to B$, $g: B \to C$

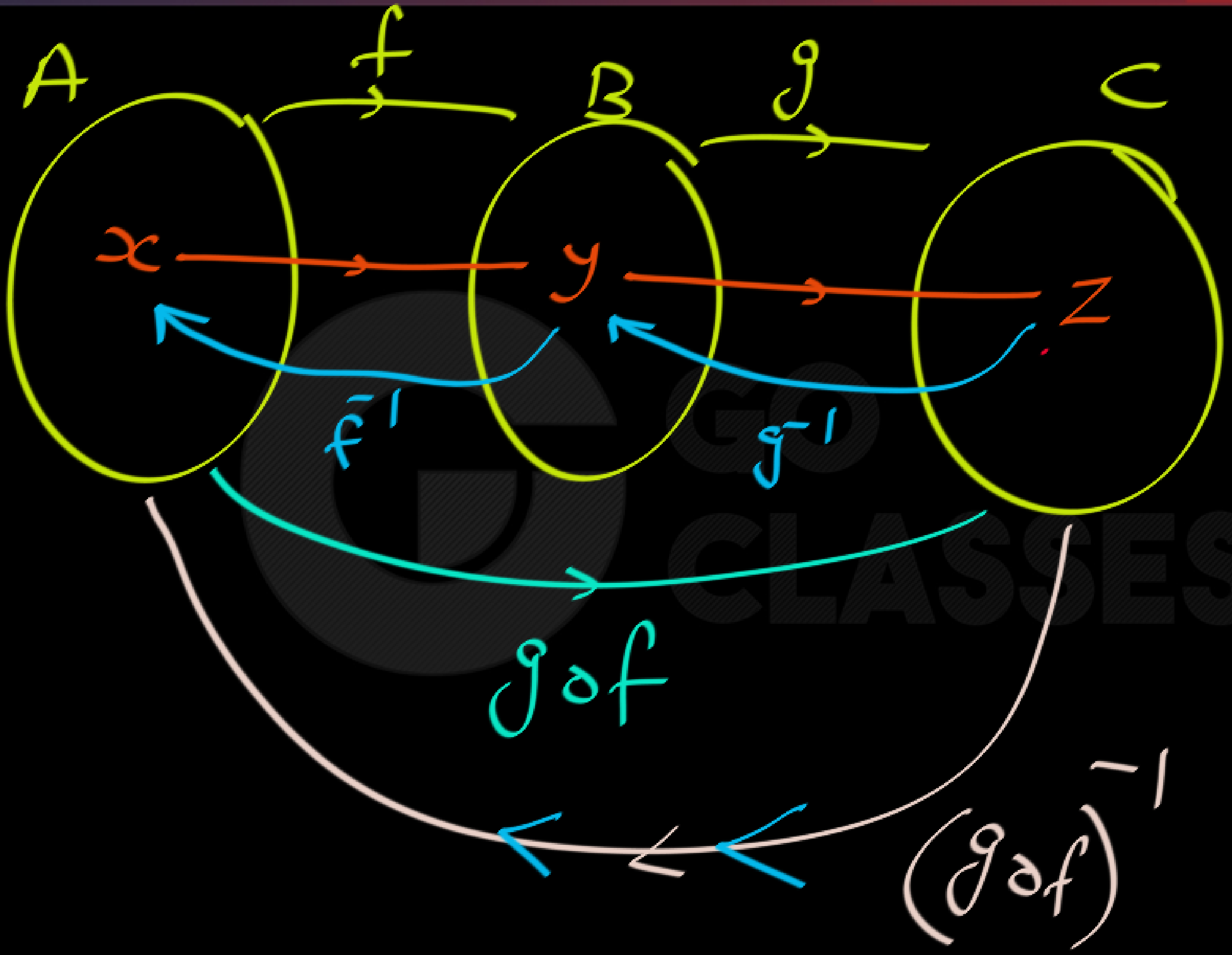
What is $(g \circ f)^{-1} = ??$

$f^{-1} \circ g^{-1}$

$g^{-1} \circ f^{-1}$?

or

$f^{-1} \circ g^{-1}$?




$$f^{-1}(g^{-1}(z)) = f^{-1} \circ g^{-1}$$

__Note:__ Invertible functions $f, g$

$f : A \to A$ ; $g : A \to A$

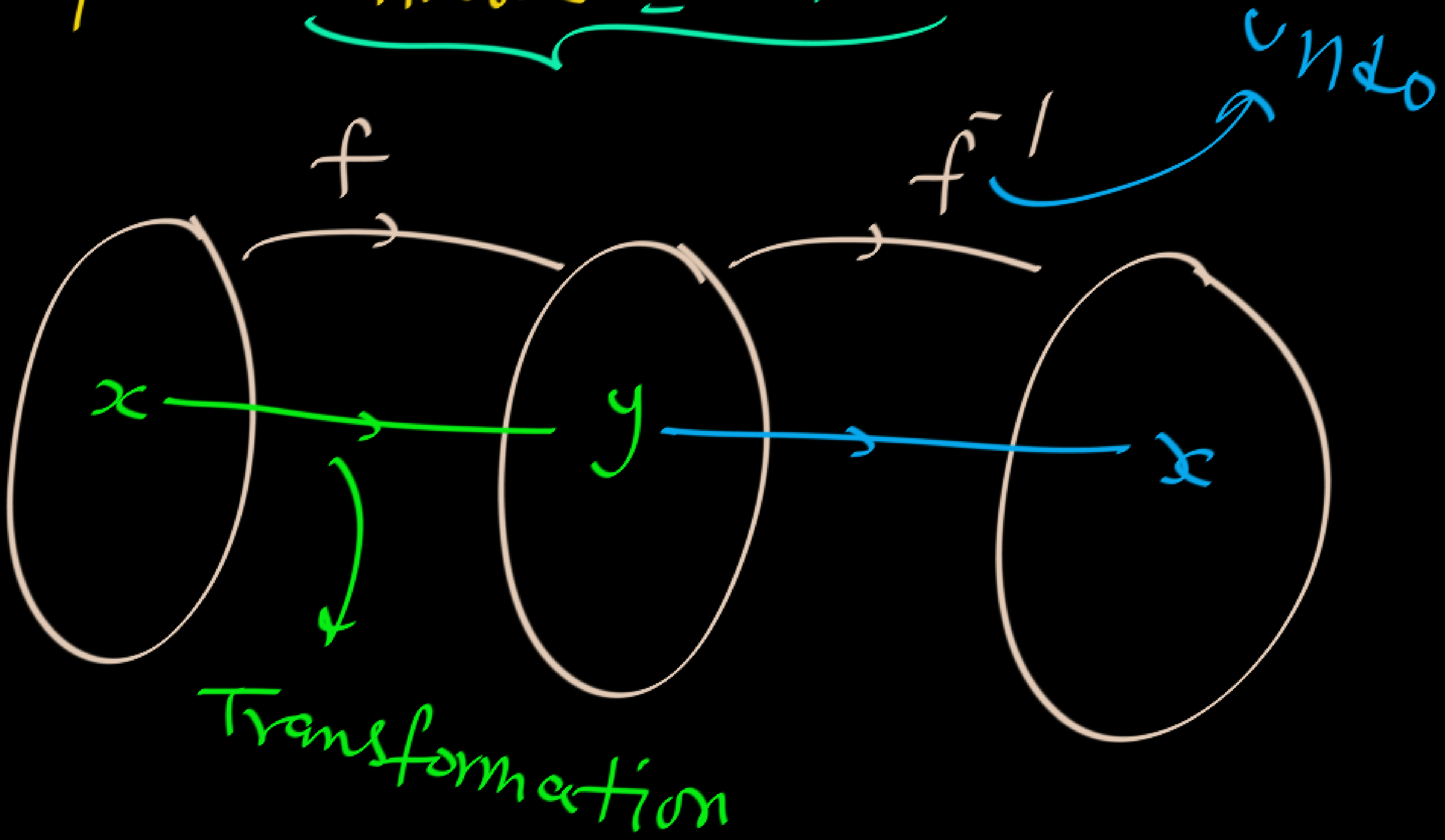
$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

Easily proven

## Summary:

Inverse $\equiv$ undo

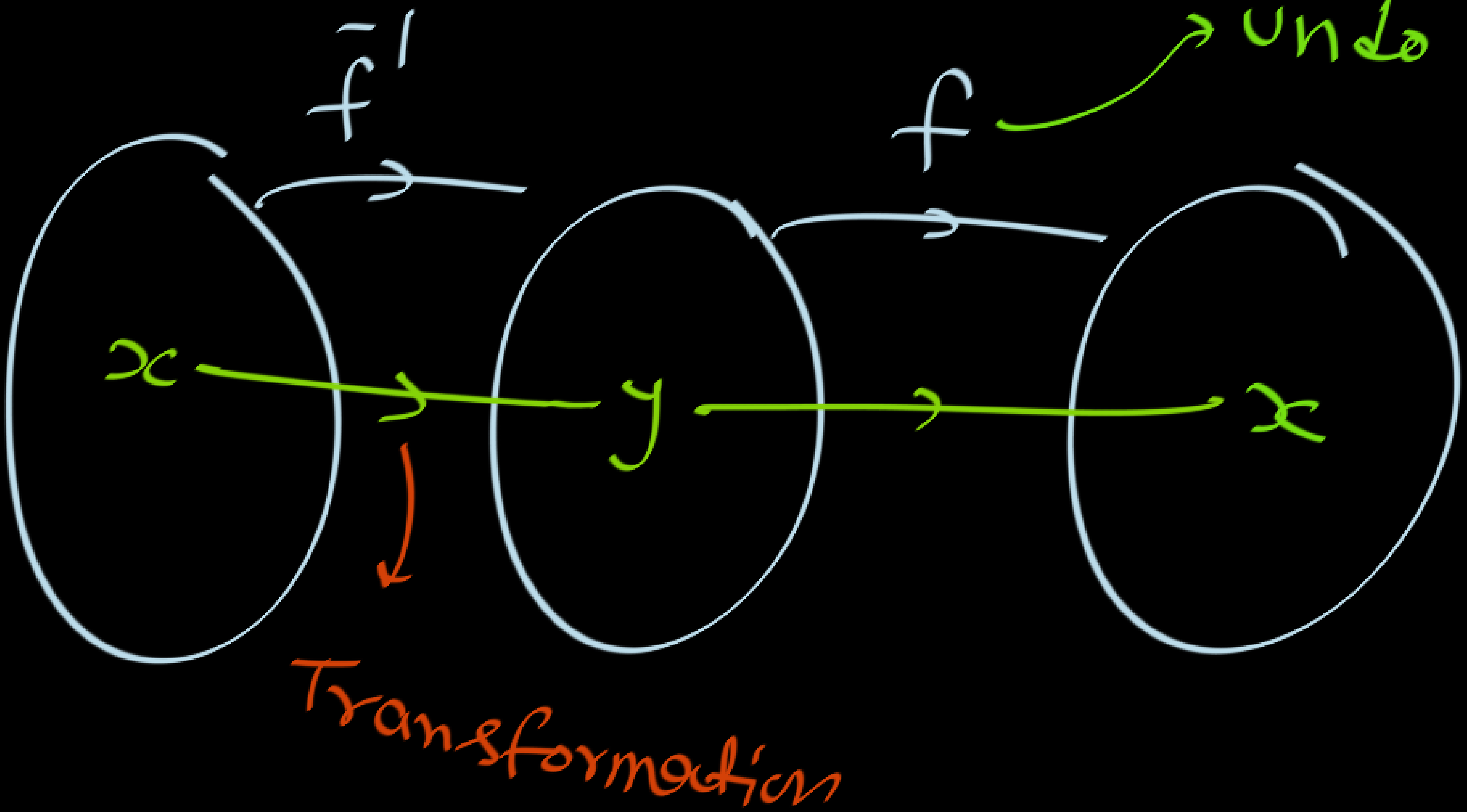


Summary: $\underbrace{\text{Inverse} \equiv \text{undo}}$

for $f$, inverse is $f^{-1}$.

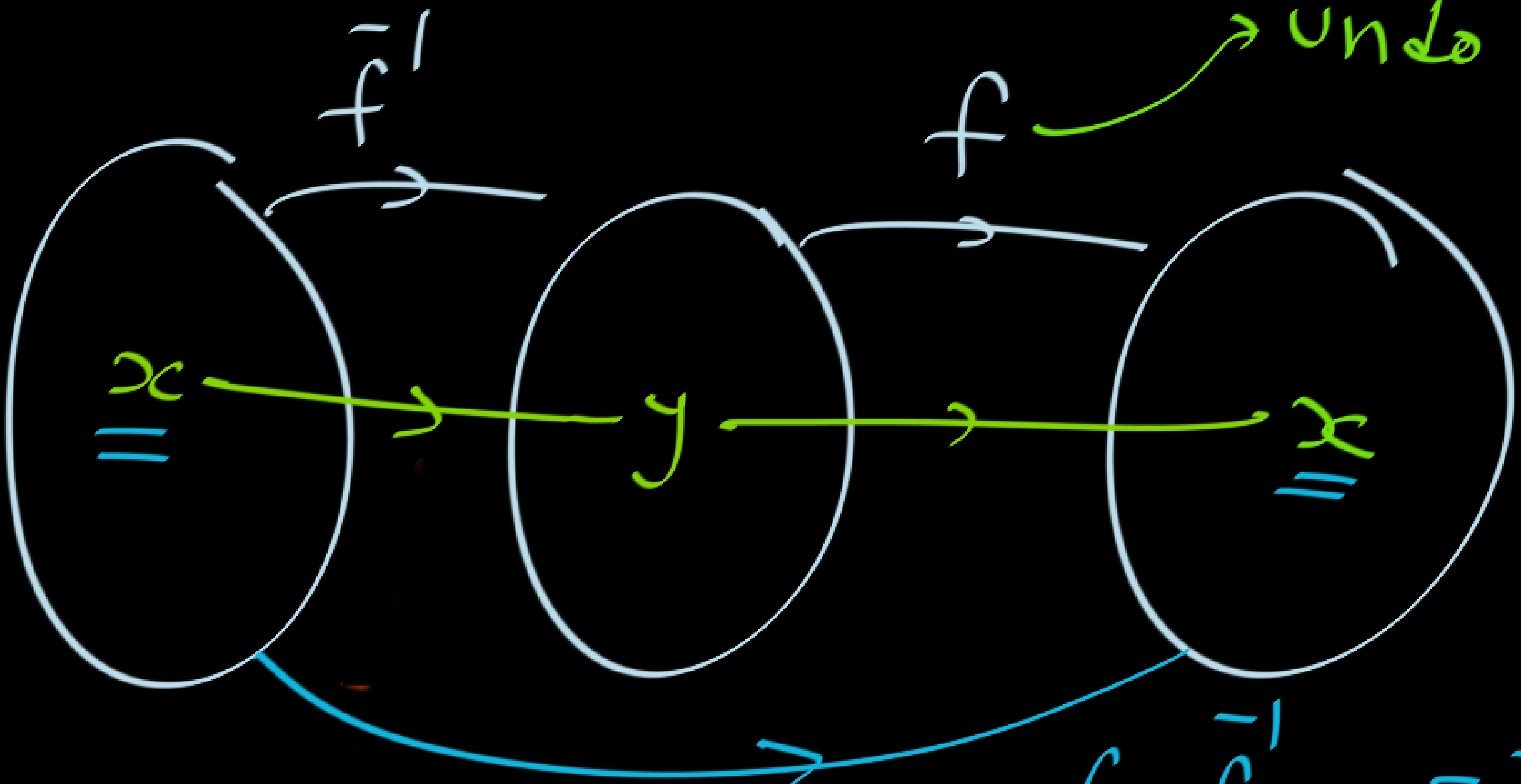
for $f^{-1}$, inverse is $f$.

Summary: $\underbrace{\text{Inverse} \equiv \text{undo}}$



# Summary:

Inverse $\equiv$ undo




$f \circ f^{-1}(x) = x$ ✓