Propositional Logic:

Propositional Logic:

Proposition
Propositional Variable
Logical Connectives (Negation, AND, OR, Implication, Bi-implication, Exclusive OR, NAND, NOR etc)

What is Propositional Logic?

1. What is Proposition?:

In Simple Words,
A proposition(or statement) is just a sentence that can be either true or false.

More formally,
A proposition is a sentence that declares a fact that either true or false but not both.

Proposition → True OR false

Not True ≡ false

Not false ≡ True

Examples of Propositions :

Examples:

Sentence	Is it a proposition?	Truth value
$1 + 2 = 4$	Yes	F
$2 \times 3 = 6$	Yes	T

We refer to T (true) or F (false) as the truth value of the proposition.

Example of Proposition:

Grass is green : T

2 is an odd prime AND 3 is an even number. : F

Truth value

1+2=3 OR 2+2=5. : T

"Grass is green and snow is white" : T

Example of "Not a Proposition":

Grass

Hello!!

1+2

Is grass green?

2. What is Atomic and Compound Proposition?

Grass is green.

Snow is white.

Atomic Proposition

Propositions

"Grass is green and snow is white"

Compound Proposition

prop

prop

2. What is Atomic Proposition?

Atomic Proposition :

"No part is a proposition".

An atomic proposition is one whose truth or falsity does not depend on the truth or falsity of any other proposition.

So, Atomic Proposition is, by itself, True or False.

Grass is green.

Snow is white.

"Grass is green and snow is white“

Consider

"Grass is green and snow is white"

This is a proposition, since it makes a claim that is either true or false (in our world, it is true, but in other logically possible worlds, it might be false).

However, this statement is not an atomic proposition, since it has a part (e.g. "Grass is green") that is a proposition

"Grass is green" itself is an atomic proposition, since there is no smaller part that is still a proposition (e.g. "Grass" is not a proposition)

In the same way,
in natural language(like English) we may call a proposition atomic if we cannot find a part of it that is again a proposition :
"Socrates is a man", — Atomic proposition

while we can call it compound if we can analyze it in parts that are themselves propositions (glued together by connectives) :

"Socrates is a man and Plato is his pupil". — Compound Prop.

Prop Prop


Proposition
Atomic
Compound

3. Propositional Variable:

Grass is green. : p — Prop. Variable

Snow is white. : q — Prop. Variable

"Grass is green and snow is white" : p and q ≡ $p \wedge q$

p

q

Not a Prop. Variable

In normal mathematics;

$x + 2y = xy$

expression

x — Variable

y — Variable

xy — Not variable

We represent Atomic Propositions by Variables, called Propositional Variables.

Atomic Propositions are represented by Propositional Variables,

Compound propositions are Not.

Example of Proposition:

Grass is green : p prop. variable

"2 is an odd prime AND 3 is an even number." : a ∧ b
a b

"1+2=3 OR 2+2=5." : p ∨ q → prop. Expression
p q

Grass is green. — p

Snow is white. — q

"Grass is green and snow is white"

p ∧ q

expression

Not a variable

"Grass is not green"

Grass is green. — p

Snow is white.

"Grass is green and snow is white"

"Grass is not green" == "It is Not the case that grass is green"

¬ p

We'll, generally, use symbols such as p, q, and r as propositional variables—that is, variables that represent atomic propositions,

Propositional variables may also be called atomic propositions,

as opposed to compound propositions, which are composed from other propositions using "and," "or,", "not" etc.

A proposition is a sentence like $p \rightarrow (p \rightarrow q)$.

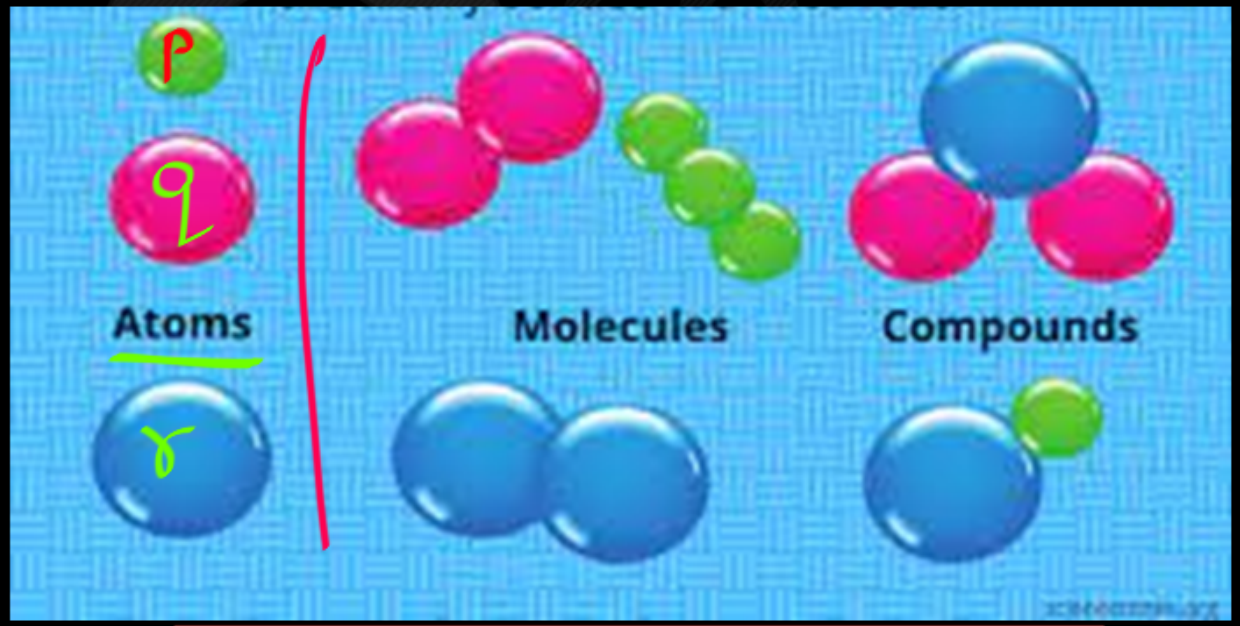
An atomic proposition is like p.

Like with atoms, an atomic proposition is the fundamental block from which more complex structures can be built.

A proposition is a sentence like $p \rightarrow (p \rightarrow q)$.

An atomic proposition is like p.

Like with atoms, an atomic proposition is the fundamental block from which more complex structures can be built.



4. Propositional Formula/Expression/Form

well-formed formula

Prop. Variable : p, q, r

$p \vee q$; $q \to r$; $p \wedge \overline{p}$; $\overline{p}$

PE PE PE PE

PE = Prop. Expression (PE)

Prop. formula

① Every propositional Variable (Atomic Proposition) is a prop. formula,

eg: p, q, r

Prop. formula/Expression :

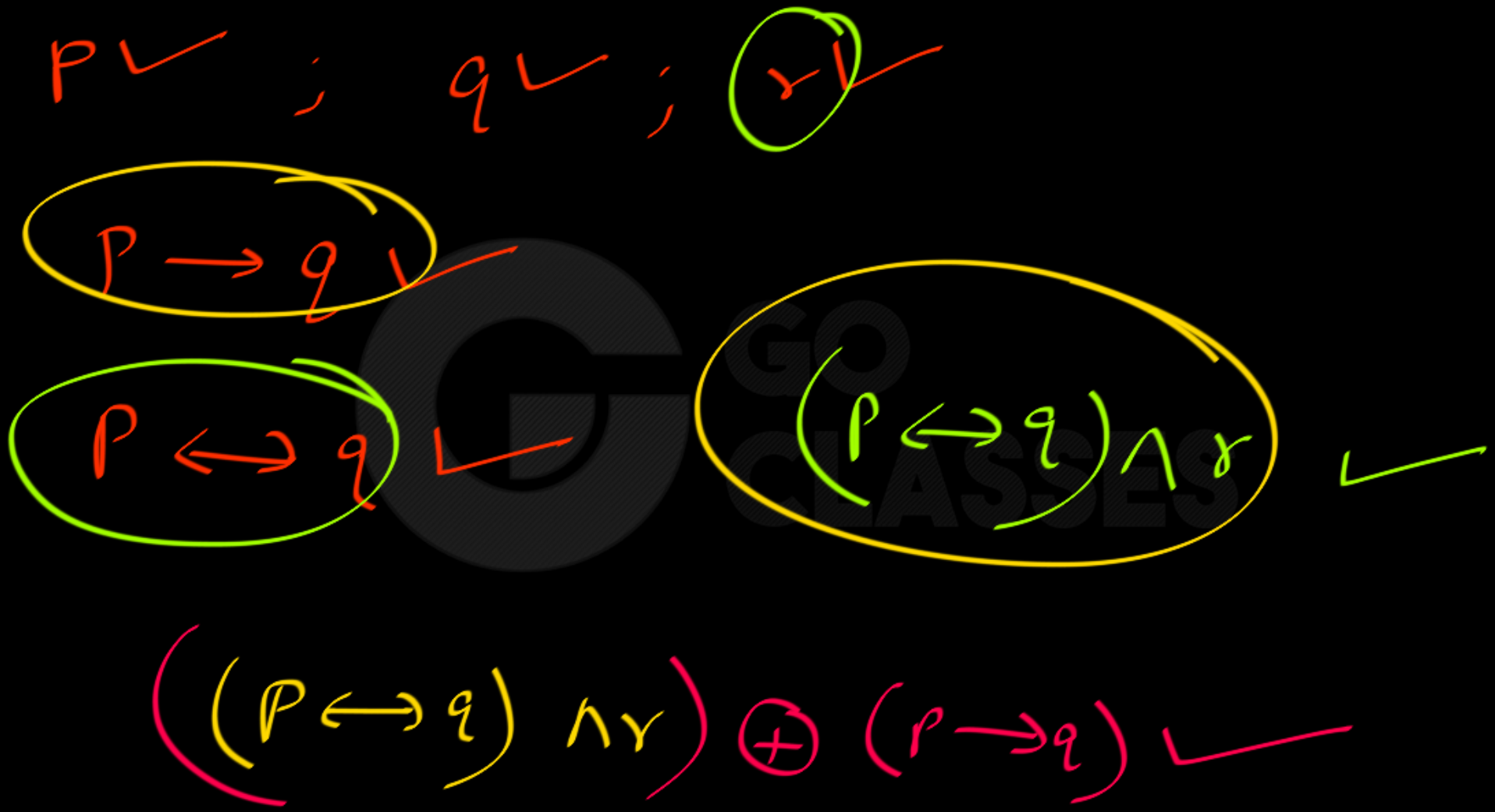
(2) T, F are prop. formula.

(3) If G, H are prop. formula then

$\neg G$, $G \vee H$, (G) , $G \uparrow H$,

$G \wedge H$, $G \rightarrow H$, $G \leftrightarrow H$, $G \downarrow H$,

$G \oplus H$


p ; q ; r
p → q
p ↔ q
(p ↔ q) ∧ r
((p ↔ q) ∧ r) ⊕ (p → q)

Not prop. formula (Not well formed formula)

p q 7

p 7

p →

p q ∧

p ∧

Formulas in logic are formed analogously to arithmetic expressions in mathematics, with parentheses, variables (atomic propositions) and binary connectives analogous to the binary arithmetic operations like addition and multiplication.

Definition 2.1 (Propositional Formulas) *Formulas are recursively defined as follows.*

- *Atomic propositions* $x \in X$ *are formulas.*
- *Symbols* $\top$ *and* $\bot$ *are formulas (respectively representing the constant* true *and* false*).*
- *If* α *and* β *are formulas, then so are*
  1. $\neg\alpha$,
  2. $(\alpha \wedge \beta)$,
  3. $(\alpha \vee \beta)$,
  4. $(\alpha \rightarrow \beta)$, *and*
  5. $(\alpha \leftrightarrow \beta)$.

$(P \rightarrow q) \rightarrow r$

$P \wedge \bar{P}$

$T \rightarrow P$

$P \vee \bar{P}$

$P \rightarrow F$

Atom: P, Q, r, T, F

$P \wedge Q$

$(Q \rightarrow r) \wedge T$

Propositional Logic(PL)

PL Syntax

Atom truth symbols $\top$ ("true") and $\bot$ ("false")
propositional variables $P, Q, R, P_1, Q_1, R_1, \cdots$

Literal atom α or its negation $\neg\alpha$

Formula literal or application of a
logical connective to formulae F, F_1, F_2

$\neg F$	"not"	(negation)
$F_1 \wedge F_2$	"and"	(conjunction)
$F_1 \vee F_2$	"or"	(disjunction)
$F_1 \rightarrow F_2$	"implies"	(implication)
$F_1 \leftrightarrow F_2$	"if and only if"	(iff)

Literal

$p, \neg p$

prop. variable

$\neg p \vee (p \rightarrow q)$

$p \vee (\neg p \vee q)$

Example:

formula F : $(P \wedge Q) \rightarrow (\top \vee \neg Q)$

atoms: $P, Q, \top$

literal: $\neg Q$, Q , P, T

subformulas: $P \wedge Q$, $\top \vee \neg Q$

Propositional Variable : $P, \bar{Q}, R$

→ Atomic proposition

Vs

Propositional formula :

$\bar{P} \wedge (P \rightarrow Q)$

$P \wedge \bar{P}$

$P \wedge Q$

$P \leftrightarrow \bar{P}$

Atomic propositions and logical connectives

An *atomic proposition* is a statement or assertion that must be true or false.

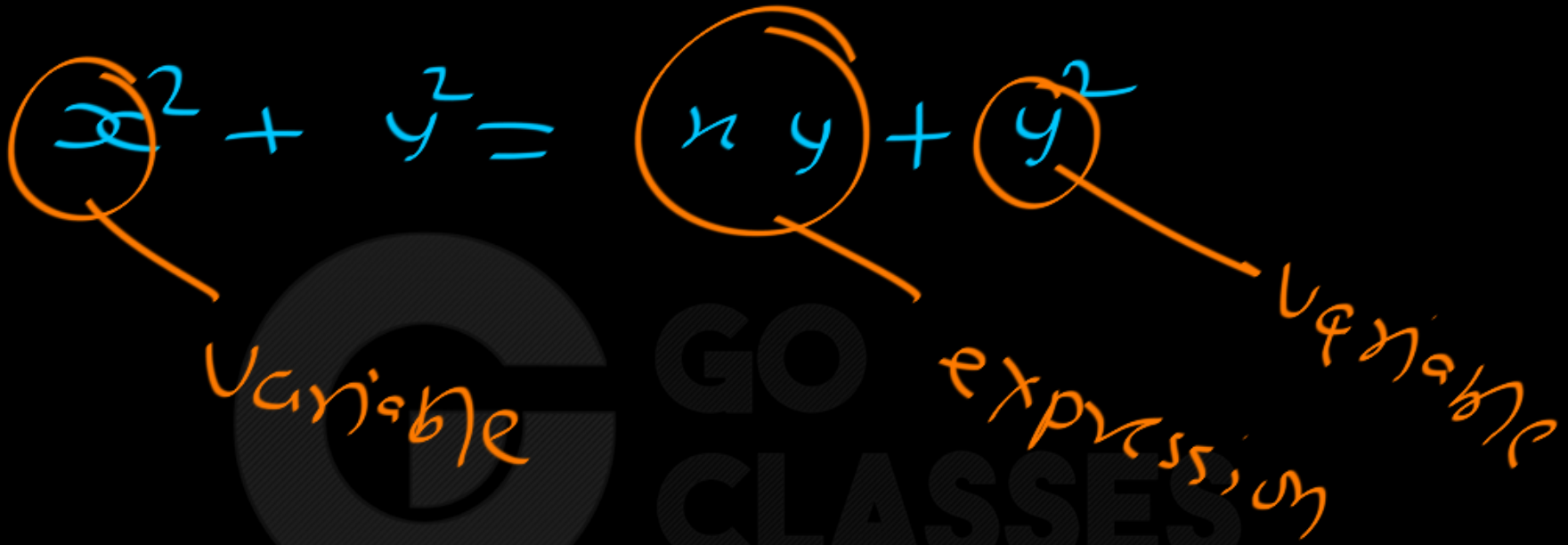
Examples of atomic propositions are: "5 is a prime" and "program P terminates".

Propositional formulas are constructed from atomic propositions by using *logical connectives*.

Connectives	
0	false
1	true
$\neg$	not
$\wedge$	and
$\vee$	or
$\rightarrow$	conditional (implies)
$\Leftrightarrow$	biconditional (equivalent)

A typical propositional formula is $A \wedge (B \vee C) \rightarrow B$.

The *truth value* of a propositional formula can be calculated from the truth values of the atomic propositions it contains.


$x^2 + y^2 = xy + y^2$
variable
expression
variable

5. What is Propositional Logic?

A World of True or False.

Every Variable is either true or false. Every variable is called Propostional Variable as it represents some atomic propostion.

Every Variable is also called a Boolean Variable as it can take only one of the two possible values.

Propositional logic is Collection/set of all propositional formulas.

Prop. logic = { $p, q, r, s, \ldots$
T, F
$\neg p, \neg q, r \wedge s, r \rightarrow s, \ldots$ }