Propositional Logic

Next Chapter:

- Logical Arguments
- Rules of Inference

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GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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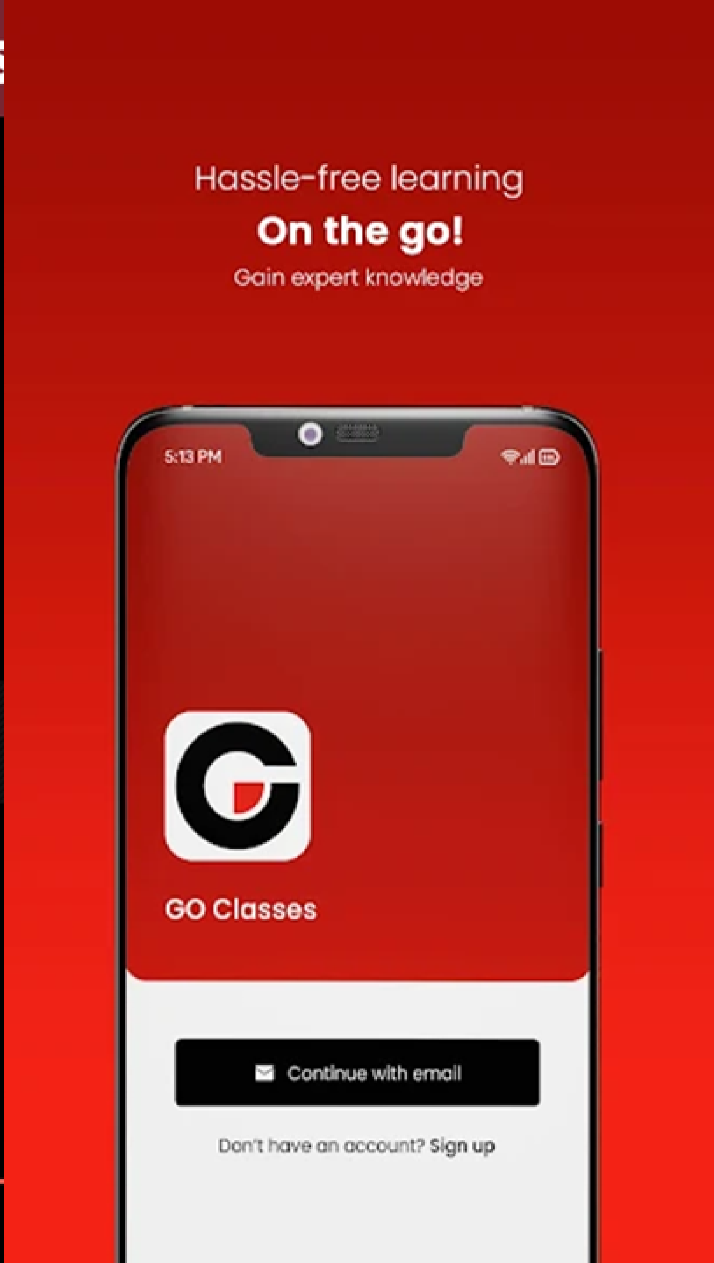

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Next Topic:

Logical Arguments

"Analysis of Implication" is VERY Important to solve MANY Question Very Quickly.

Now we study "Logical Arguments" in which we use the "Analysis of Implication" that we studied in previous lecture.

Let's Build Intuition first.

First build Intuition, then we'll study formal methods.

If it is raining, He'll take umbrella.

It is raining.

Can we infer "He'll take umbrella" ?

YES

If it is raining, He'll take umbrella.

It is not raining.

Can we infer "He'll not take umbrella" ? No

If it is raining, He'll take umbrella.

It is not raining.

Can we infer "He'll take umbrella" ? No

"If you have access to the network, then you can change your grade."

"You have access to the network."

Can we infer "You can change your grade." ?

YES

If I work all night on this homework, then I can answer all the exercises.

If I answer all the exercises, I will understand the material.

I work all night on this homework.

Can we infer “I will understand the material.” ?

If I work all night on this homework, then I can answer all the exercises.

If I answer all the exercises, I will understand the material.

I work all night on this homework.

Can we infer "I will understand the material." ?

YES

Conclusion

Propositional Logic

Next Topic:

(Logical) Arguments

Formal Definition & Method

"If you have a current password, then you can log onto the network."

"You have a current password."

premises
(Knowledge base)
(Antecedents)
(Hypothesis)

Argument

Therefore,
"You can log onto the network."

Conclusion / Consequence

Valid Argument

If it is raining, He'll take umbrella.

It is not raining.

Hence,
He'll not take umbrella.

premises

logical
Argument

Invalid
Argument

Conclusion

Argument:

P_1

P_2

P_3

P_4

$\therefore$ C

Sequence of Statements

Argument

Argument :

Argu-
-ment

P_1
P_2
P_3
P_4 } → Premises / Knowledge Base (KB)
Hypothesis / Antecedents

$\therefore$ C → Conclusion / Consequence

---

therefore / Hence / so

Argument:

Premises

P_1

P_2

$\therefore C$

Conclusion

Valid iff

if we Assume all premises to be True, then Conclusion must be true.

Argument:

P_1

P_2

$\therefore C$

Invalid iff

it is possible that All premises are true, still Conclusion is false.

Arguments

Definition: An argument has the form

$$\begin{array}{l} A_1 \\ A_2 \\ \vdots \\ A_n \\ \hline B \end{array}$$

$A_1, \ldots, A_n$ are called the *premises* of the argument; B is called the *conclusion*. An argument is *valid* if, whenever the premises are true, then the conclusion is true.

Checking if a given argument is Valid or Not?

An Argument is Invalid if it is Possible to make

"ALL Premises True & Conclusion False"

Checking if a given argument is Valid or Not?

An Argument is Valid if it is NEVER Possible to make

"ALL Premises True & Conclusion False"

Argument:

$$\left.\begin{array}{c} A \\ B \\ C \\ \hline \therefore D \\ \hline \end{array}\right\} \text{Valid iff}$$

$$\boxed{A \wedge B \wedge C \longrightarrow D \text{ is Tautology}}$$

Argument:

$$\begin{array}{l} A \\ B \\ C \\ \hline \therefore D \\ \hline \end{array}$$

Invalid iff

$$\boxed{A \wedge B \wedge C \longrightarrow D}$$

is NOT a Tautology.

Formal Definition of Valid Argument :

Argument :

P_1
P_2
P_3
$\therefore C$

Valid iff

$\boxed{P_1 \land P_2 \land P_3 \rightarrow C}$ is Tautology.

Checking if a given argument is Valid or Not?

Method:

Make the Conclusion False… NOW, TRY to make ALL premises True…

If Possible, then Argument is Invalid.

If Not Possible, then Argument is Valid.

Argument:

$$\left\{\begin{array}{l} p_1 \\ p_2 \\ p_3 \end{array}\right.$$

$\therefore$ c (F)

Method:

(1) make c = false

(2) Try to make ALL premises (p_1, p_2, p_3) True

(3) Possible: Arg. Invalid

NOT possible: Arg. valid.

By an argument, we mean a sequence of statements that end with a conclusion.

By valid, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises, of the argument.

That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false.

Argument

premises

$$\left.\begin{array}{l} p \rightarrow q \\ p \end{array}\right.$$

$$\therefore q$$

therefore

Conclusion

Valid Arg or Not ?

① $\overset{T}{p} \to \overset{F}{q}$ → Can NoT make True.

② p → p = True

$\therefore q$ = false

It is NOT possible that All premises True & conclusion false ∴ Valid Arg.

Intuitively:

$\overset{T}{p} \to \overset{T}{q}$

p T

$\therefore q$

Valid

"If you have access to the network, then you can change your grade."
"You have access to the network."

∴ "You can change your grade."

N G N G

English Description

$N \rightarrow G$

N

$\therefore G$

Argument form

Valid Argument form

$A \to B$

A

$\therefore B$

Valid Arg

Valid Argument form

Modus Ponens

"If you have access to the network, then you can change your grade."
"You have access to the network."

$\therefore$ "You can change your grade."

Argument

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Argument form

φ :

$$\begin{array}{l} p \lor \varphi \\ p \\ \hline \therefore \varphi \\ \hline \end{array}$$

Valid or Not?

q :

$$\begin{cases} (1) & \overset{T}{p} \vee \overset{F}{q} = \text{True} \\ (2) & p = \text{True} \end{cases}$$

$\therefore$ $q = F$

All premises True

&

Conclusion false

Invalid Arg.

φ :

$p \vee \varphi$

$\neg p$

$\therefore \varphi$

Valid or Not?

φ :

① $\overset{F}{p} \vee \overset{F}{\varphi}$ = false

② $\neg p$ = True ⟶ $p = f$

$\therefore$ $\varphi = F$

Conclusion false & All premises True

NOT possible

Valid Arg

Q: Intuitively:

$p \vee q$ → At least one of P, Q is True

$\neg p$ → Q is false.

$\therefore q$ → Q is True.

Correct Inference

Valid Arg

Q : Intuition:

$P \vee Q$ → At least one of P, Q is True

P : P is True

$\therefore$ Q : $\boxed{Q \text{ is True}}$ → NOT Correct Conclusion

Invalid Arg

Q:

$$P \rightarrow Q$$

$$Q \rightarrow R$$

$\therefore$ $P \rightarrow R$

Valid Arg or Not?

φ:

① $\overset{T}{P} \rightarrow \varphi \equiv$ True $\varphi = T$

② $\overset{T}{\varphi} \rightarrow \overset{F}{R} \equiv$ false

---

$\therefore P \rightarrow R$ false $\Rightarrow$ $P = T$, $R = F$

Valid Arg

formal method:

Intuition:

Q:

$P \rightarrow Q$

$Q \rightarrow R$

$\therefore P \rightarrow R$

Correct Conclusion

P happens → Q → R happens

if P happens then R happens.

Q : Is this argument valid?

If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow.
Therefore, if it rains today, then we will have a barbecue tomorrow.

Q : Is this argument valid?

R ¬B

If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. E ¬B

Therefore, if it rains today, then we will have a barbecue tomorrow.

R E

English Description → Argument form

B: Barbecue today.
E: Barbecue Tom.

Argument form:

$R \rightarrow \neg B$

$\neg B \rightarrow E$

$\therefore R \rightarrow E$

Valid Arg. form

Argument form:

$R \rightarrow \neg B$

$\neg B \rightarrow E$

$\therefore R \rightarrow E$

Correct Conclusion

If R then E

$R \rightarrow \neg B \rightarrow E$

Valid Arg.

Argument form:

$\overset{T}{R} \rightarrow \neg B$ ① = True → $\neg B$ = True

$\overset{T}{\neg B} \rightarrow \overset{F}{E}$ ② = false

$\therefore R \rightarrow E$ false ⇒ $R = T; E = F$

Valid Arg.

Arg. form:

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ \hline \therefore A \rightarrow \underline{\underline{C}} \\ \hline \end{array}$$

Valid Arg form ✓

An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is *valid* if the truth of all its premises implies that the conclusion is true.

An *argument form* in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is *valid* no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

Propositional Logic

Next Sub-Topic:

Rules of Inference

(some popular, frequently used Valid Arguments)

Rules of Inference

Some popular, frequently used Valid Argument forms...

Rules of Inference

Some popular, frequently used Valid Argument forms...

Basically, they tell us "How to correctly Infer some information from a given set of statements"

① $p \to q$: If p happens, q happens

p : p happens.

$\therefore q$: q happens

Correct Inference

(1)

$P \rightarrow Q$

P

$\therefore Q$

Valid Argument form

$A \rightarrow B$

A

$\therefore B$

---

(1)

$$\left.\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \\ \hline \end{array}\right\}$$

Modus ponens

(2)

$$p \rightarrow q$$

$$\neg q$$

$$\therefore \neg p$$

Valid or Not ?

(2) formal method:

$\overset{T}{p} \rightarrow \overset{F}{\varphi}$ (1) = false

$\neg\varphi$ (2) = True $\Rightarrow$ $\varphi = F$

$\therefore$ $\neg p$ = false $\Rightarrow$ p = True

Valid Arg.

② Intuitively :

$P \rightarrow Q$: if P happens then Q happens.

$\neg Q$: Q does not happen.

$\therefore \neg P$: P Does not happen.

Valid inference

Valid Arg.

2

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

→ Valid Argument form

→ Modus Tollens

Rules of Inference

Some popular, frequently used Valid Arguments forms have special names.

(These names are Not important, But can be remembered for some exams)

①

$$\left.\begin{array}{l} A \to B \\ A \\ \hline \therefore B \\ \hline \end{array}\right\} \text{Modus Ponens}$$

②

$$\left.\begin{array}{l} A \to B \\ \neg B \\ \hline \therefore \neg A \\ \hline \end{array}\right\} \text{Modus Tollens}$$

3

p

q

$\therefore p$

Valid or Not ?

3

p ✓

q ✓

$\therefore p$

valid ✓

Standard Rules of Inference

Each of the following is based on a tautology.

- Modus Ponens
$$\begin{array}{rl} & p \\ & p \implies q \\ \hline \therefore & q \end{array}$$

- Modus Tollens
$$\begin{array}{rl} & \neg q \\ & p \implies q \\ \hline \therefore & \neg p \end{array}$$

- Conjunctive Simplification
$$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \end{array}$$

- Disjunctive Syllogism
$$\begin{array}{rl} & p \vee q \\ & \neg p \\ \hline \therefore & q \end{array}$$

(4)

$$\left.\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \\ \hline \end{array}\right\}$$ valid Arg form

5

$A \rightarrow B$

$B \rightarrow C$

$\therefore A \rightarrow C$

Valid Arg.

A happens $\rightarrow$ B happens $\rightarrow$ C happens

$$
\begin{tikzcd}
A \arrow[r] \arrow[rr, bend right, squiggly] & B \arrow[r] & C
\end{tikzcd}
$$

Transitivity of Implication

If $A \rightarrow B$ and $B \rightarrow C$

then $A \rightarrow C$

$\Rightarrow$ Note: Implication is Transitive.

5

$A \rightarrow B$

$B \rightarrow C$

$\therefore A \rightarrow C$

Transitivity of Implication

Hypothetical Syllogism

- Hypothetical Syllogism

$$\begin{array}{rl} & p \implies q \\ & q \implies r \\ \hline \therefore & p \implies r \end{array}$$

7

p

$\therefore \; p \vee q$

Valid

8

$p \wedge q$

$\therefore \; p$

Valid

$p \wedge q$

$\therefore \; q$

Valid

9

$P \vee Q$

$\neg P \vee R$

$\therefore Q \vee R$

Valid or Not ?

formal:

(9) $\neg P \lor Q^{F}$ ① = True → $P = T$

$F\ \neg P \lor R^{F}$ ② = false

$\therefore\ Q \lor R$ = false ⟹ $Q = F;\ R = F$

Valid Arg

Intuition:

(9)

$P \vee Q$

$\neg P \vee R$

$\therefore Q \vee R$

Correct Conclusion

if P is false then $Q \vee R = \text{True}$

If P is True then $Q \vee R = \text{True}$

9

$P \vee Q$

$\neg P \vee R$

$\therefore Q \vee R$

Valid Arg form

Resolution

- Addition $\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$

- Conjunctive Simplification (alternate version) $\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$

- Resolution $\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$

Example 1. Identify the rules of inference used in each of the following arguments.

(a) Alice is a math major. Therefore, Alice is either a math major or a c.s. major.

(b) If it snows today, the college will close. The college is not closed today. Therefore it did not snow today.

(c) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will get sunburn.

Example 2. Use rule of inference to show that the premises “Henry works hard”, “If Henry works hard then he is a dull boy”, and “If Henry is a dull boy then he will not get the job” imply the conclusion “Henry will not get the job.”