Combinatorics

Recap

Basic Counting Principles

Sum, Product Rules

Website : https://www.goclasses.in/

1.1.1 Sum Rule

First though, we will count baby outfits. Let's say that a baby outfit consists of *either* a top *or* a bottom (but not both), and we have 3 tops and 4 bottoms. How many baby outfits are possible? We simply add $3 + 4 = 7$, and we have found the answer using the sum rule.

Theorem 1.1.1: Sum Rule

If an experiment can either end up being one of N outcomes, or one of M outcomes (where there is no overlap), then the number of possible outcomes of the experiment is:

$$N + M$$

More formally, if A and B are sets with no overlap ($A \cap B = \emptyset$), then $|A \cup B| = |A| + |B|$.

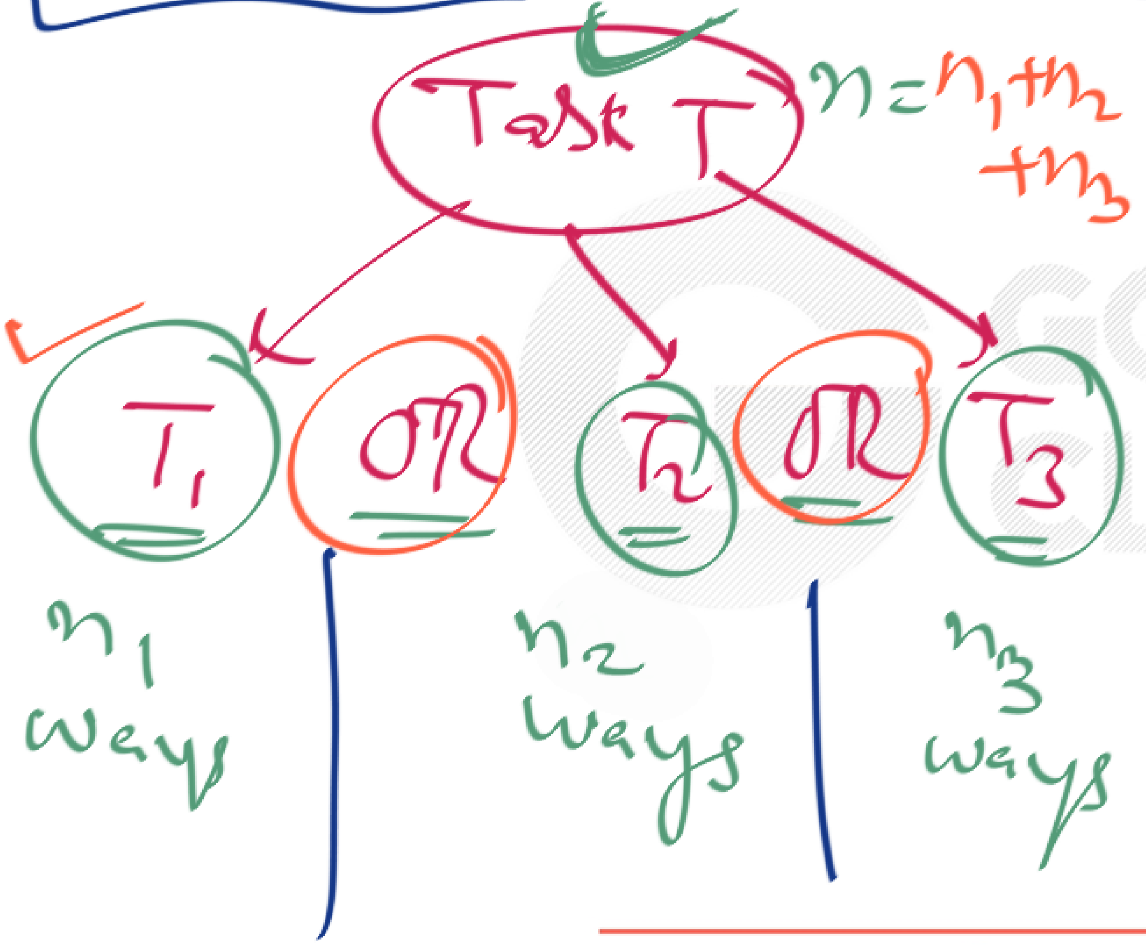
Example(s)

Suppose you must take a natural science class this year to graduate at any of the three UW campuses: Seattle, Bothell, and Tacoma. Seattle offers 4 different courses, Bothell offers 7, and Tacoma only 2. How many choices of class do you have in total?

Solution By the sum rule, it is simply $4 + 7 + 2 = 13$ different courses (since there is no overlap)! □

We'll see some examples of the Sum Rule combined with the Product Rule (next), so that they can be a bit more complex!

Sum Rule:



Very Imp: T_1, T_2, T_3 Should Not have any way Common.

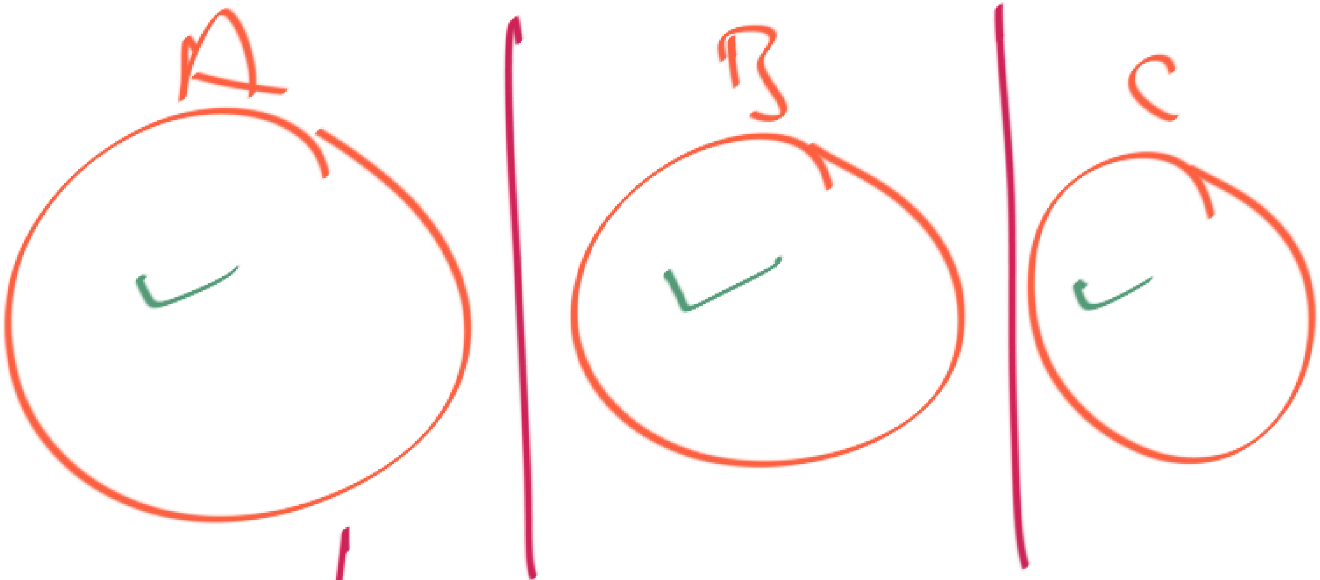
T_1, T_2, T_3 are Completely Disjoint.

Sum Rule:

Set A, B, C

Cond: All sets must be mutually (Pairwise) Disjoint.

$\underbrace{\text{Disjoint}}_{\text{Nothing common}}$



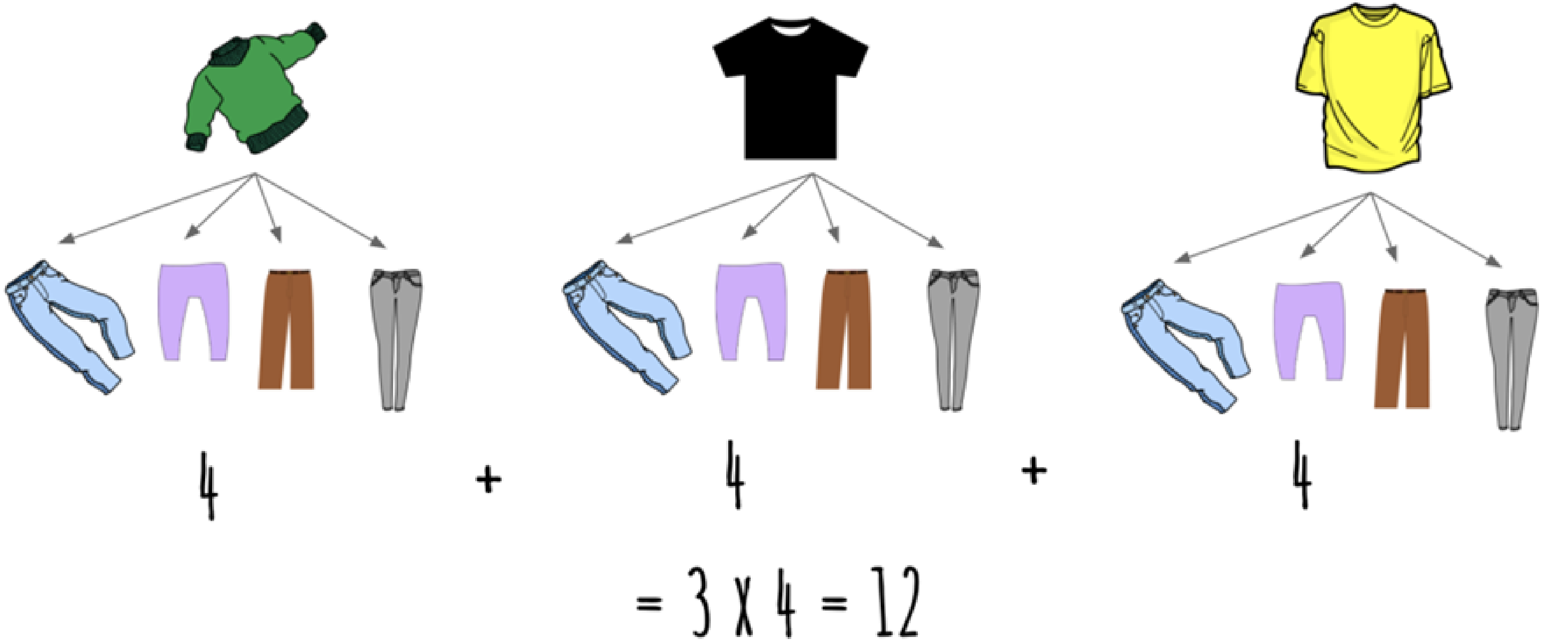
$$|A \cup B \cup C| = |A| + |B| + |C|$$

1.1.2 Product Rule

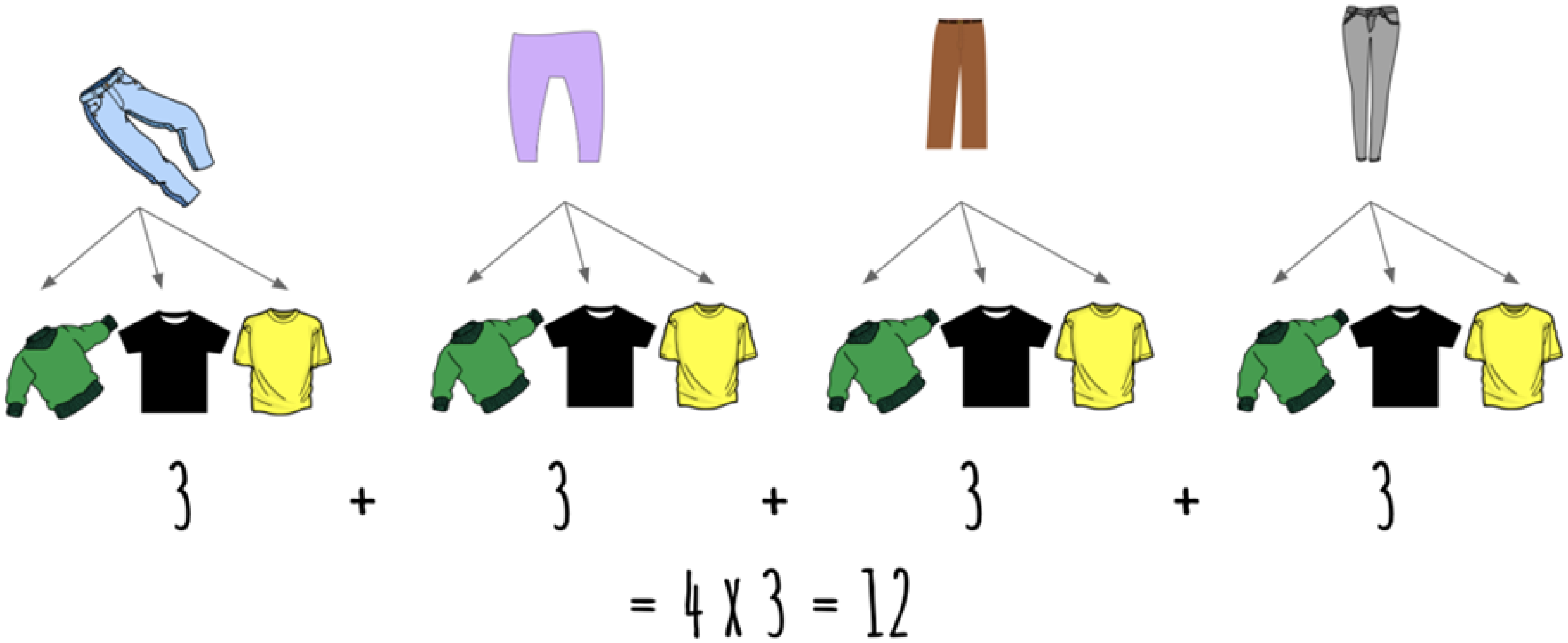
Now we will count real outfits. Let's say that a real outfit consists of *both* a top *and* a bottom, and again, we still have 3 tops and 4 bottoms. then how many outfits are possible?

Well, we can consider this from first picking out a top. Once we have our top, we have 4 choices for our bottom. This means we have 4 choices of bottom for each top, which we have 3 of. So, we have a total of

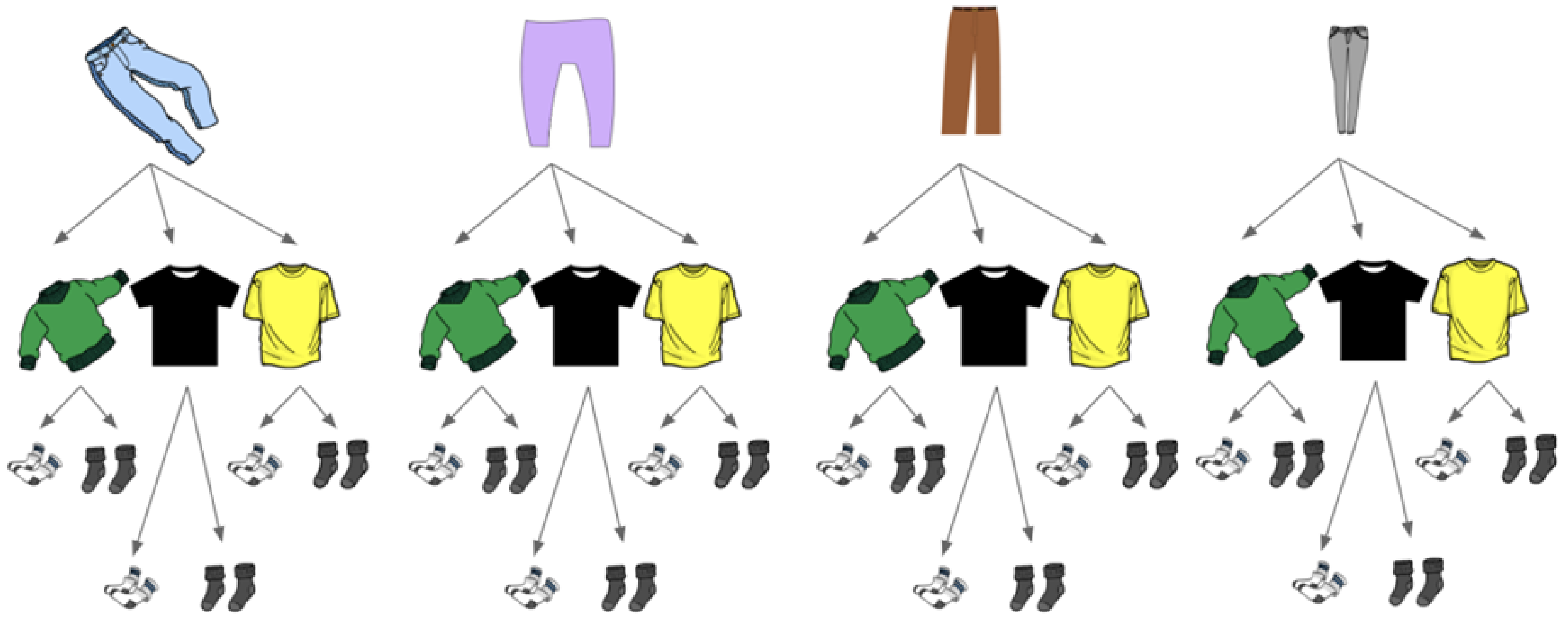
$4 + 4 + 4 = 3 \cdot 4 = 12$ outfit choices.



We could also do this in reverse and first pick out a bottom. Once we have our bottom, we have 3 choices for our top. This means we have 3 choices of top for each bottom, which we have 4 of. So, we still have a total of $3 + 3 + 3 + 3 = 4 \cdot 3 = 12$ outfit choices. (This makes sense - the number of outfits should be the same no matter how I count!)



What if we also wanted to add socks to the outfit, and we had 2 different pairs of socks? Then, for each of the 12 choices outlined above, we now have 2 choices of sock. This brings us to a total of 24 possible outfits.



$4 \times 3 \times 2 = 24$

This could be calculated more directly rather than drawing out each of these unique outfits, by multiplying our choices: 3 tops $\cdot$ 4 bottoms $\cdot$ 2 socks $=$ 24 outfits.

Theorem 1.1.2: Product Rule

If an experiment has N_1 outcomes for the first stage, N_2 outcomes for the second stage, $\ldots$, and N_m outcomes for the m^{th} stage, then the total number of outcomes of the experiment is $N_1 \cdot N_2 \cdots N_m$.

More formally, if A and B are sets, then $|A \times B| = |A| \cdot |B|$ where $A \times B = \{(a, b) : a \in A, b \in B\}$ is the Cartesian product of sets A and B.

Product Rule

Sequence of Tasks

T_1 AND T_2 AND T_3

Sum Rule

we have Alternatives

T_1 OR T_2 OR T_3

To motivate us for Combinatorics(also known as Counting Principles):

Let's consider how easy or difficult it is for a robber to randomly guess your PIN code. Every debit card has a PIN code that their owners use to withdraw cash from ATMs or to complete transactions.
How secure are these PINs, and how safe can we feel?

ATM $\longrightarrow$ 4 Digit PIN Code

How many PIN Codes possible for Debit Card?

T_1 and T_2 and T_3 and T_4

0	0	0	

$$10 \times 10 \times 10 \times 10 = 10^4$$

$$= 10{,}000$$

Example 1: In New Hampshire, license plates consisted of two letters followed by 3 digits. How many possible license plates are there?

(a) $26^2 \times 10^3$?

(b) $26 \times 25 \times 10 \times 9 \times 8$?

(c) No idea.

L L D D D

$26 \times 26 \times 10 \times 10 \times 10$

Example 1: In New Hampshire, license plates consisted of two letters followed by 3 digits. How many possible license plates are there? (No Repetition of letters, Digits)

(a) $26^2 \times 10^3$?

(b) $26 \times 25 \times 10 \times 9 \times 8$?

(c) No idea.

L L D D D

26 25 10 9 8

Example 1: In New Hampshire, license plates consisted of two letters followed by 3 digits. How many possible license plates are there?

Answer: 26 choices for the first letter, 26 for the second, 10 choices for the first number, the second number, and the third number:

$$26^2 \times 10^3 = 676,000$$

Example 2: A traveling salesman wants to do a tour of all 50 state capitals. How many ways can he do this?

Answer: 50 choices for the first place to visit, 49 for the second, $\ldots$: 50! altogether.

ISRO2014-19

asked in Combinatory Sep 2, 2015 • retagged Jun 27, 2017 by **Arjun**

7

The number of bit strings of length 8 that will either start with 1 or end with 00 is?

A. 32
B. 128
C. 160
D. 192

8 length – Task T ⇒ $2^7 + 2^6 - 2^5$

$128 + 64 - 32 = 128 + 32 =$

T_1: 1 _ _ _ _ ⇒ 2^7

OR

T_2: _ _ _ 0 0 ⇒ 2^6

Common

1 _ _ _ 0 0 ⇒ 2^5

Bit strings Starting with 1 : $\Rightarrow 2^7$

8-length

1							
1 choice	2	2	2	2	2	2	2

ending with 00

			- -		0	0
2	2	2	2			

$= 2^6$

1						0	1	0
	2	2	2	2	2			

2^5

Example: How many bit strings of length 8 either start with a 1 bit or end with the two bits 00?

Solution:

- Number of bit strings of length 8 that start with 1: $2^7 = 128$.
- Number of bit strings of length 8 that end with 00: $2^6 = 64$.
- Number of bit strings of length 8 that start with 1 and end with 00: $2^5 = 32$.

Applying the subtraction rule, the number is $128 + 64 - 32 = 160$. $\square$

Example 1: Suppose variable names in a programming language can be either a single uppercase letter or an uppercase letter followed by a digit. Find the number of possible variable names.

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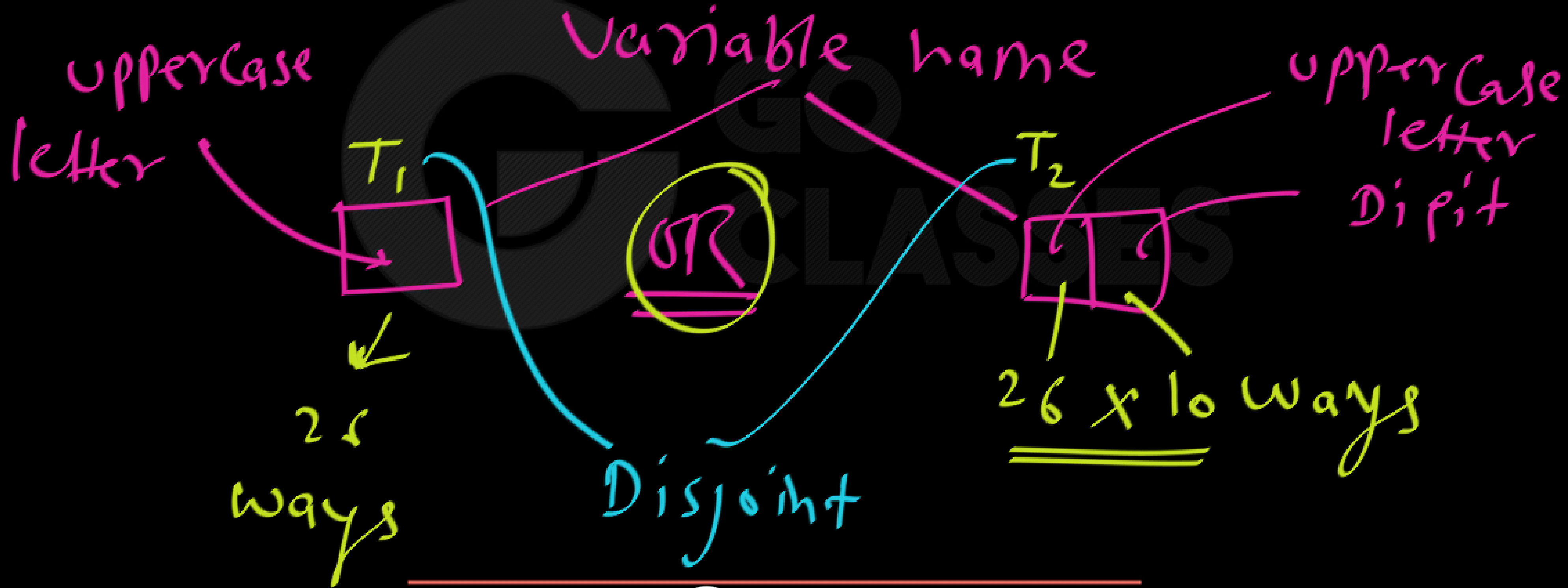


Example 1: Suppose variable names in a programming language can be either a single uppercase letter or an uppercase letter followed by a digit. Find the number of possible variable names.

Solution: Use the sum and product rules: $26 + 26 \cdot 10 = 286$. □

6 length bitstring start with 101

OR end with 001 ? [redacted]

6 length bitstring start with 101 OR end with 001 ? [redacted]

T_1: 101_ _ _ OR _ _ _001 :T_2

2^3 $\qquad$ 2^3

Common

101001 = 1 common string

Ans:

$2^3 + 2^3$

-1

$= 15$

4 length bitstring start with 101

OR end with 001 ? [redacted]

4 length bitstring Start with 101

OR end with 001 ? [redacted]

T_1: 101_ → 2 ways

OR

T_2: _001 → 2 ways

Disjoint

Ans:

$2+2=4$

4 length bitstring start with 101

OR end with 001 ?

1	0	1	

Start with 101 $\Rightarrow$ 2 strings

end " 001 $\Rightarrow$ 2 strings

$2 + 2 = 4$

Can they happen simultaneously?? No

Coin Flip

Head or Tail?

Example(s)

1. How many outcomes are possible when flipping a coin n times? For example, when $n = 2$ there are four possibilties: HH, HT, TH, TT.

F_1 f_2 f_3 - - - - f_n

$2 \times 2 \times 2$ - - $\times 2$ $= 2^n$

outcomes

Possible outcomes

{Head or Tail}

2. How many subsets of the set $[n] = \{1, 2, \ldots, n\}$ are there?

Subset :

$\{1, 3, 4\}$ — 1 — Present
2 — Absent
3 — P
4 — P
5 — A
6 — A
⋮
n — A

$\{\ \}$ — 1 — A
2 — A
3 — A
⋮
n — A

$S = \{1, 2, \ldots, n\}$

Subset of S : $\{\ \ ,\ \ ,\ \ ,\ \ \}$

$S = \{1, 2, \ldots, n\}$

P or A   P or A   P or A

subsets $= 2^n$

Example(s)

1. How many outcomes are possible when flipping a coin n times? For example, when $n = 2$ there are four possibilties: HH, HT, TH, TT.

2. How many subsets of the set $[n] = \{1, 2, \ldots, n\}$ are there?

Solution

1. The answer is 2^n: for the first flip, there are two choices: H or T. Same for the second flip, the third, and so on. Multiply 2 together n times to get 2^n.

2. This may be hard to think about at first. But think of the subset $\{2, 4, 5\}$ of the set $\{1, 2, 3, 4, 5, 6, 7\}$ as follows: for each number in the set, either it is in the subset or not. So there are two choices for the first element (in or out), and for each of them. This gives 2^n as well!

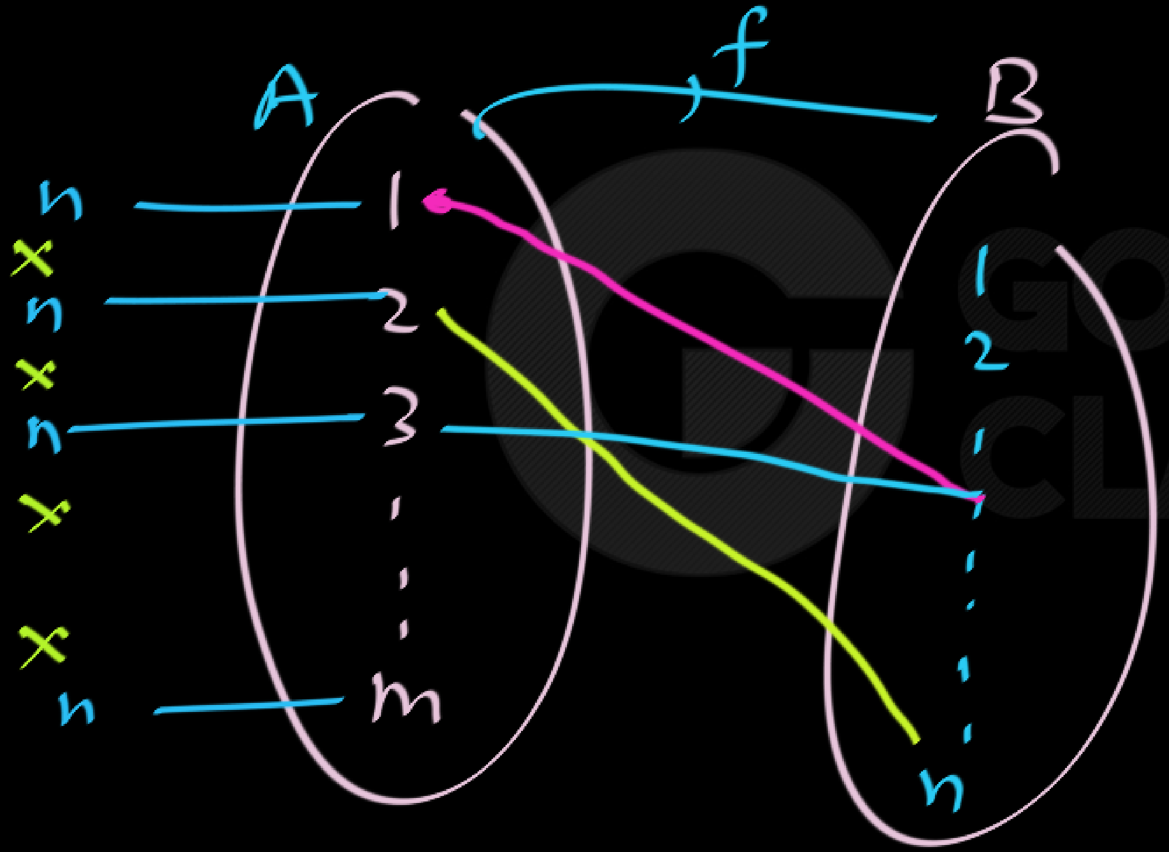
$\square$

Counting Functions How many functions are there from a set with m elements to a set with n elements?

Solution: A function corresponds to a choice of one of the n elements in the codomain for each of the m elements in the domain. Hence, by the product rule there are $n \cdot n \cdot \cdots \cdot n = n^m$ functions from a set with m elements to one with n elements. For example, there are $5^3 = 125$ different functions from a set with three elements to a set with five elements. ◀

$f : A \to B$; $|A| = m$; $|B| = n$

functions $= n^m$



Task:

map1 AND map 2
↓ n choices
↓ n choices

AND map3 - - -
→ n choices

AND map m → n ways

Q: Number of functions possible from Set A to Set B?

1. A^B
2. B^A

Q: Number of functions possible from Set A to Set B?

1. A^B
2. B^A

Non sense

$|B|^{|A|}$

technicalities

$A = \{a, b\}$

$B = \{1, 2\}$

$\{a, b\}^{\{1, 2\}}$

Non sense

Q :

Set A, B. |A| = 3, |B|=5, How many 1-1 functions from A to B possible?

Q :

Set A, B. |A| = 3, |B|=5, How many 1-1 functions from A to B possible?

Different elements have Different image.

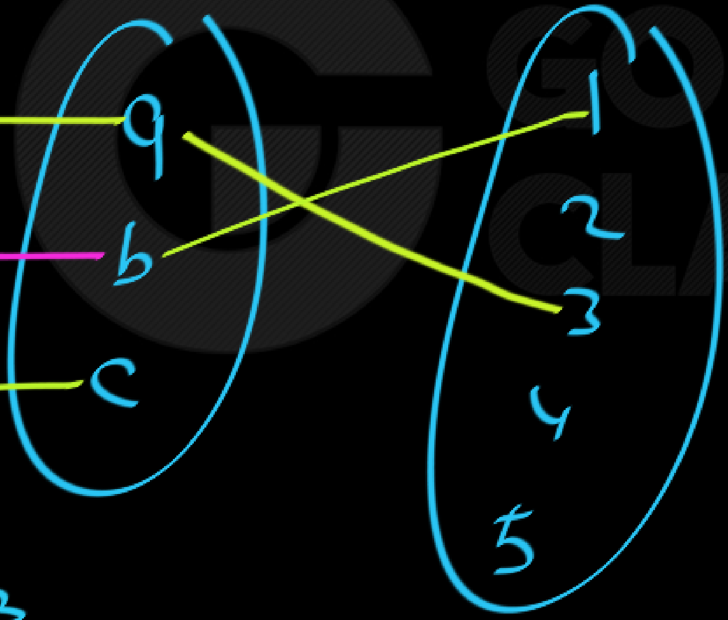
5 choices

×

4 choices

×

3 choices



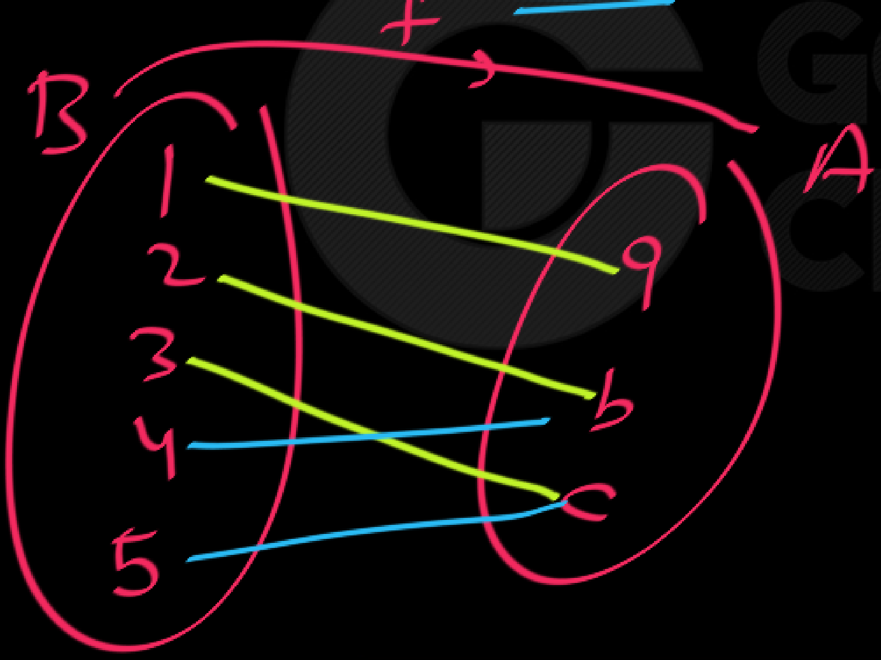
Ans: $5 \times 4 \times 3$

Q :

Set A, B. |A| = 3, |B|=5, How many 1-1 functions from B to A possible?

Q :

Set A, B. |A| = 3, |B|=5, How many 1-1 functions from B to A possible? → 0 ✓



injective function exists iff

|Domain| ≤ |Co-Domain|

A student ID is made up of 3 letters followed by two digits.

a. How many possible ID's exist? $\Rightarrow (26)^3 \times (10)^2$

b. How many ID's are possible if duplicate letters or numbers aren't allowed?

$\Rightarrow 26 \times 25 \times 24 \times 10 \times 9$

[redacted]

Combinatorics

Next Sub-Topic:

Counting by Cases

Mutually Exclusive, Exhaustive Cases

Website : https://www.goclasses.in/

Mutually Exclusive $\equiv$ Disjoint

Mutually Exclusive Cases $\Rightarrow$ NOT Counting same thing more than one time. ✓

$\rightarrow$ Can Apply Sum Rule. ✓

Exhaustive Cases:

All Desired possibilities Covered.

Eg: 8 length bit strings — Target

Starting with 101
Starting with 001

} mut. Exclusive

→ NOT Exhaustive.

Target – 8 length bit strings

Case 1 : Starting with 0

Case 2 : Starting with 1

ME & Exh.

Target – 8 length bit strings – Desired

Starting with 000
" " 001
" " 010
" " 011
" " .
" " .
" " 111

→ 8 Cases

ME & Exh

ME & Exh :

Count Everything Exactly once. Desired

Don't Count Same thing more than one time

Count Everything in at least one Case.

A student ID is made up of 3 letters followed by two digits.

c. How many student ID's are possible with an even number of "A"'s?

A student ID is made up of 3 letters followed by two digits.

c. How many student ID's are possible with an even number of "A"'s?

Case 1: #A's = 0

L L L D D

25 25 25 10 10

$= (25)^3 \times (10)^2$

Case 2: $\boxed{\#A's = 2}$ ⇒ Exactly 2 A's

mutually Exclusive

Case (i)

A	A	25	10	10

+

Case (ii)

25	A	A	10	10

+

Case (iii)

A	25	A	10	10

Ans:

$$\boxed{\#A' = 0} + \boxed{\#A's = 2}$$

$$(25)^3 \times (10)^2 + 3 \times (25) \times (10)^2$$

$$(25) \times (10)^2 \left[625 + 3 \right]$$

Example(s)

Flamingos Fanny and Freddy have three offspring: Happy, Glee, and Joy. These five flamingos are to be distributed to seven different zoos so that no zoo gets both a parent and a child :(. It is not required that every zoo gets a flamingo. In how many different ways can this be done?

F Fr

H, G, J

offspring

/ˈɒfsprɪŋ/

noun

a person's child or children.
"the offspring of middle-class parents"

Similar: children | sons and daughters | progeny | family | youngsters | babies

- an animal's young.
- the product or result of something.
  "child hunger is the offspring of political indfference, unaccountable governance, and economic mismanagement"

Example(s) — stanford

Flamingos Fanny and Freddy have three offspring: Happy, Glee, and Joy. These five flamingos are to be distributed to seven different zoos so that no zoo gets both a parent and a child :(. It is not required that every zoo gets a flamingo. In how many different ways can this be done?

Case 1: both Parents Together

$\boxed{F + Fr}$ H G J

7 × 6 × 6 × 6

Choices → zoos   Choices

7×6^3

Case 2: Parents NOT Together

Baghban

F | Fr   H G J

7 × 6 × 5 5 5

Choices   Choices

$7 \times 6 \times 5^3$

Example(s)

Flamingos Fanny and Freddy have three offspring: Happy, Glee, and Joy. These five flamingos are to be distributed to seven different zoos so that no zoo gets both a parent and a child :(. It is not required that every zoo gets a flamingo. In how many different ways can this be done?

Solution There are two disjoint (mutually exclusive) cases we can consider that cover every possibility. We can use the sum rule to add them up since they don't overlap!

1. **Case 1: The parents end up in the same zoo.** There are 7 choices of zoo they could end up at. Then, the three offspring can go to any of the 6 other zoos, for a total of $7 \cdot 6 \cdot 6 \cdot 6 = 7 \cdot 6^3$ possibilities (by the product rule).

2. **Case 2: The parents end up in different zoos.** There are 7 choices for Fanny and 6 for Freddy. Then, the three offspring can go to any of the 5 other zoos, for a total of $7 \cdot 6 \cdot 5^3$ possibilities.

The result, by the sum rule, is $7 \cdot 6^3 + 7 \cdot 6 \cdot 5^3$. (Note: This may not be the only way to solve this problem. Often, counting problems have two or more approaches, and it is instructive to try different methods to get the same answer. If they differ, at least one of them is wrong, so try to find out which one and why!) $\square$

Combinatorics

Next Topic

The Complement Rule

Desired = Total - Undesired

Website : https://www.goclasses.in/

The Complement Rule :

It is sometimes easier to calculate the undesired cases than it is to calculate the desired cases itself. So, find undesired cases, and total cases, then once this is done, we have :

Desired cases = Total Cases - Undesired cases

Q :

Set A, B. |A| = 3, |B|=5, How many functions from A to B are Not injective?

Desires = Not Injective functions

$$\#\text{Non 1-1 fun} = \underbrace{\#\text{Total fun}}_{\downarrow} - \underbrace{\#\text{1-1 fun}}_{\downarrow}$$

$$\#\text{NOT 1-1 fun} = 5^3 - (5 \times 4 \times 3)$$

$$= 125 - 60$$

$$= \underline{65}$$

Q.

Of all 4-letter words in the English alphabet, how many do not begin with an ‘L’ ?

Q.

Of all 4-letter words in the English alphabet, how many do not begin with an 'L' ?

method 1: $\Rightarrow 25 \times (26)^3$

25	26	26	26

choices

method 2 $\Rightarrow (26)^4 - (26)^3$

NOT begin with L = Total − Start with L

Total = $(26)^4$

$\boxed{L \mid \;\; \mid \;\; \mid \;\;} = (26)^3$

A student ID is made up of 3 letters followed by two digits.

d. How many students ID's have some repetition?

A student ID is made up of 3 letters followed by two digits.

d. How many students ID's have some repetition?

Some Repetition = Total − No Repetition

" " $= (26)^3 \times (10)^2 - 26 \times 25 \times 24 \times 10 \times 9$

A student ID is made up of 3 letters followed by two digits.

d. How many students ID's have some repetition of letters.

A student ID is made up of 3 letters followed by two digits.

d. How many students ID's have some repetition of letters.

Some Repetition of letter =

Total − No Repetition of letters

$$(26)^3 \times (10)^2 - 26 \times 25 \times 24 \times (10)^2$$

A student ID is made up of 3 letters followed by two digits.

d. How many students ID's have some repetition?

Instead, we can find the number of possibilities with no repeats and subtract from the total.

$$\text{ID's with repetition} = \textit{Sample space} - \textit{no repeats}$$
$$= \left(26^3 \cdot 10^2\right) - (26 \cdot 25 \cdot 24 \cdot 10 \cdot 9)$$
$$= 353{,}600$$

Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

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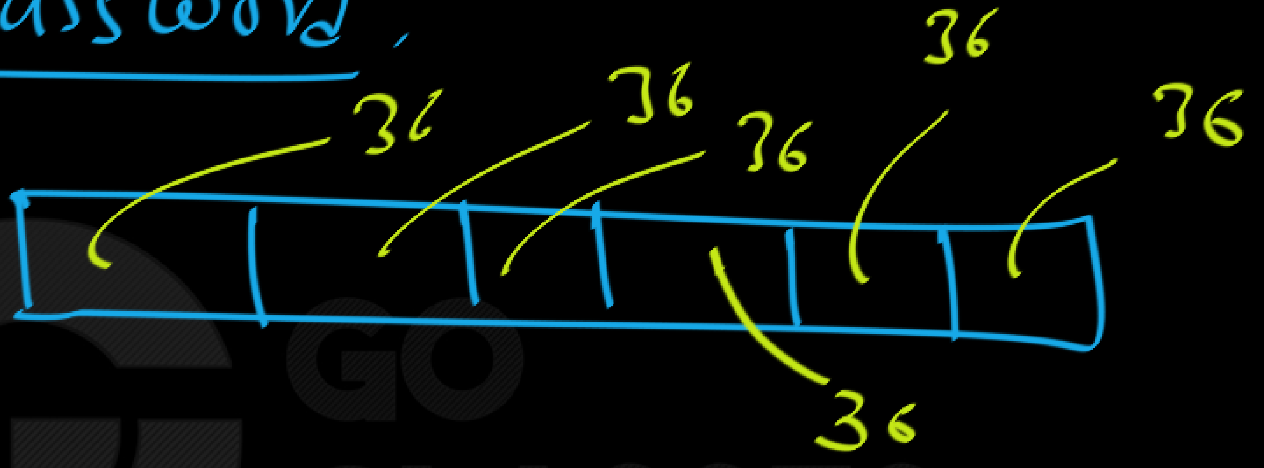
Password: $P_6 + P_7 + P_8$

$(36)^6 - (26)^6 + (36)^7 - (26)^7 + (36)^8 - (26)^8$

P_6: → At least one Digit = Total − No Digit

$= (36)^6 - (26)^6$

6 length Password:

Total = 

$(36)^6$

No Digit: $(26)^6$

Combinatorics

Next Sub-Topic:

Know about Cards

4 Suits/types; 13 of same type/suit

Website : https://www.goclasses.in/

Types

Deck of Cards = 52 Cards

Standard

for every type : 13 Card Value

2, 3, - - , 10, J, Q, K, A

Q:

Consider the number of ways of picking two cards of which there is at least one ace from a deck of cards when drawing a pair of cards one after other (order matters)

Q:

Consider the number of ways of picking two cards of which there is at least one ace from a deck of cards when drawing a pair of cards. one after one (order matters)

At least one Ace = Total − No Ace

$52 \times 51 - 48 \times 47$

Q:

Consider the number of ways of picking two cards of which there is at least one ace from a deck of cards when drawing a pair of cards one after other (order matters)

Can you Directly solve using product Rule !

52 | 4

$= 52 \times 4$

wrong

52

?

So

52×4
is wrong.

J ◇

52 ways

4 now

A ◇

52 ways

51 now

Product Rule:

T

T_1 — m ways

T_2 — n ways

However you do T_1

In "whatever way" we do T_1, there should be n ways for T_2.

2 letters without Repetition:

T_1	T_2
26 ways	25 ways

$= 26 \times 25$ ✓

T_1		T_2
a	—	25 ways
b	—	25 "
c	—	" "
⋮		
z	—	25 ways

NOTE :

The applicability of the product rule hinges upon the independence of the number of outcomes for each of the first and second stages. For example, consider the number of ways of picking two cards of which there is at least one ace from a deck of cards when drawing a pair of cards.

If you happen to pick an ace first, then there are a possible 51 cards you can pick for your second choice. BUT, if you do not pick an ace first, you must pick one of four cards instead. In any event, what we see is that the number of possible ways to execute the second choice is NOT always the same after first stage is done. Thus, we can NOT apply the produce rule in such a simple manner.

Q: Drawing 2 cards (one after one) (order matters), containing at least one Ace.

Case 1: $\boxed{A \mid \ }$

Case 2: $\boxed{\ \mid A}$

$\Rightarrow$ NOT mut. Exclusive

Q: Drawing 2 cards (one after one) (order matters), containing at least one Ace.

Case 1:

Ace	Non Ace

4×48

Case 2:

Non Ace	Ace

48×4

Case 3:

Ace	Ace

4×3

Q: Drawing 2 Cards (one after one) (order matters), Containing at least one Ace.

Case 1: | Ace | Anything |

Case 2: | Anything | Ace |

Common = | Ace | Ace |

$(4 \times 51) + (4 \times 51) - (4 \times 3)$

method 1: $52 \times 51 - 48 \times 47 = 396$

method 2: $(4 \times 48) \times 2 + 4 \times 3 = 396$

method 3: $(4 \times 51) \times 2 - (4 \times 3) = 396$

NOTE :

Basically, While applying the Product Rule, MAKE SURE that HOWEVER the first task is done, the second task has same “m” number of ways.

i.e. For EVERY way of first task, there should be “m” ways for second task.

Even though product rule can be applied in many situations BUT Product Rule is particularly BEST when :

We have to do n tasks, and these tasks are completely independent of each other i.e. happening of any task has no effect on others.

GATE CSE 2021

Consider the following sets, where $n \geq 2$:

S_1: Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$
S_2: Set of all functions from the set $\{0, 1, 2, ..., n^2 - 1\}$ to the set $\{0, 1, 2\}$

$|S_1| = ?$ $|S_2| = ?$

S_1:

$\{a, b, c\}$

$$\begin{bmatrix} a & a & b & \\ & & & \\ b & - & - & c \end{bmatrix}_{n \times n} = (3)^{(n^2)}$$

3 choices

$= \#\text{matrices}$

S_2: $A \longrightarrow B$

$|A| = n^2$; $|B| = 3$

$\#\text{functions} = (3)^{(n^2)}$

Q.

How many 4 digit pin-code are there, not starting with 0 ?

Q.

How many 4 digit pin-code are there, not starting with 0 ?

9 10 10 10

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be an Odd number ?

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be an Odd number ?

9	9	8	5

Choices

WRONG Answer

5	0	3	
9	9	8	3

Choices

Guideline 1: While Applying Product Rule

Start with the most Restricted Place.

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be an Odd number ?

T_2			T_1
6	5	0	3

8 × 8 × 7 × 5 choices

Choices choices choices

= 8 × 8 × 7 × 5

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be even number ?

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be even number ?

T_2 T_1

?

9 choices — 0

8 choices — 2

5 choices

T_2 T_1

$\underbrace{9 \times 8 \times 7 \times 5}$

wrong

Q.

How many 4 digit numbers are there, not starting with 0, without repetition, also must be even number ?

Case 1:

			0

9 choices, 8, 7

$9 \times 8 \times 7$

Case 2:

3			2

8 choices, 8 choices, 7 choices, 4 choices

Non zero Even

$9 \times 8 \times 7 + 8 \times 8 \times 4 \times 7$

HW Determine the number of 6-digit integers (no leading zeros) in which:

a. No digit may be repeated

b. Digits may be repeated

c. No digit may be repeated, even number

d. Digits may be repeated, even number

e. No digit may be repeated, divisible by 5

GATE2001

How many 4-digit even numbers have all 4 digits distinct

A. 2240

B. 2296

C. 2620

D. 4536

GATE2001

How many 4-digit even numbers have all 4 digits distinct

A. 2240 B. 2296 C. 2620 D. 4536

Ex :

How many elements in the A*B (Cartesian product of set A,B) ?

set A; $|A| = m$; set B; $|B| = n$

$A * B = \{ (x, y), \mid x \in A, y \in B \}$

we need to create ordered pairs.

(T_1, T_2)

$m \times n = mn$

The total number of relations that can be formed between two sets is the number of subsets of their Cartesian product.

For example:

$$n(A) = p$$
$$n(B) = q$$
$$\implies n(AXB) = pq$$
$$Number\ of\ relations\ between\ A\ and\ B = 2^{pq}$$

Remember that if $n(T) = m$, then the number of subsets of set T will be 2^m

Q : You are doing web-check-in for your flight to book seat. Among the available seats, there are 30 "Free" Middle Seats, 20 "Free" Windows Sets, "10" Paid seats.

You do not have money. How many ways to book one seat for you?

Ans: $30 + 20 = 50$

Q : You are doing web-check-in for your flight to book seat. Among the available seats, there are 30 “Free” Middle Seats, 20 “Free” Windows Sets, “10” Paid seats.

You do not have money. How many ways to book one seat for you if two free seats are already booked by your bother and sister before you do the booking for yourself?

Ans: $30 + 20 - 2 = 48$

Q : You are doing web-check-in for your flight to book seat. Among the available seats, there are 30 “Free” Middle Seats, 20 “Free” Windows Sets, “10” Paid seats.

You do not have money. How many ways to book one seat for you if two paid seats are already booked by your bother and sister before you do the booking for yourself?

$30 + 20 = 50$ ✓

Guideline 2: Just because some information is given, Doesn't mean it is useful.

Read Carefully and Solve Accordingly.

Q : You are doing web-check-in for your flight to book seat. Among the available seats, there are 30 “Free” Middle Seats, 20 “Free” Windows Sets, “10” Paid seats.

You do not have money. How many ways to book 2 seats for you and your sister?

You	sis

50 × 49

Q :

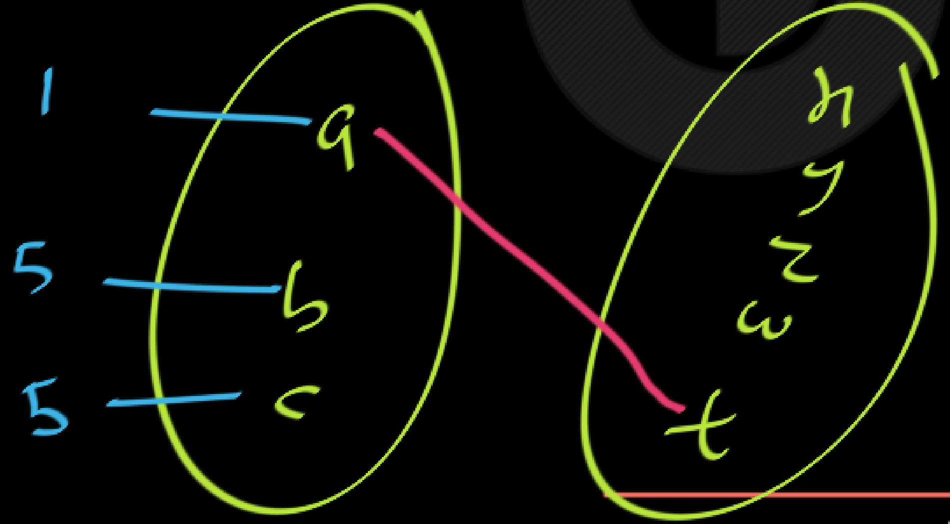
Set A, B. A = {a,b,c} ; B={x,y,z,w,t}

|A| = 3, |B|=5, How many functions from A to B are there such that “a” maps to “t”?

Q :

Set A, B. A = {a,b,c} ; B={x,y,z,w,t}

|A| = 3, |B|=5, How many functions from A to B are there such that “a” maps to “t”?



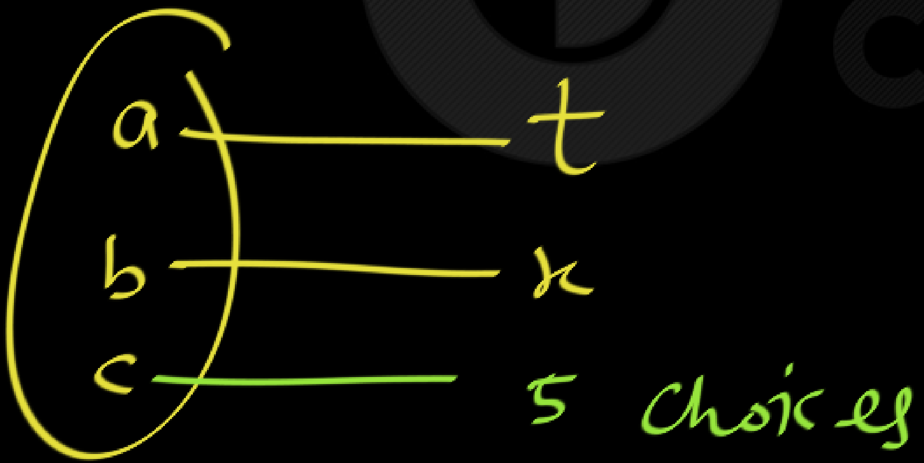
$= 5 \times 5 = 25$

Q :

Set A, B. A = {a,b,c} ; B={x,y,z,w,t}

|A| = 3, |B|=5, How many functions from A to B are there such that “a” maps to “t”, “b” maps to “x”?

= 5



Q :

Set A, B. A = {a,b,c} ; B={x,y,z,w,t}

|A| = 3, |B|=5, How many one-one functions from A to B are there such that "a" maps to "t", "b" maps to "x"?

a — t

b — x

c — 3 choices

3

GATE CSE 2021

There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

The number of ways in which this can be done is ________.

$J_1 < J_2 < J_3 < J_4 < J_5 < J_6$

$\underline{c_1} < c_2 < \underline{c_3}$

At least one Job to c_2 = Total − No Job to c_2

$= 3^4 - 2^4$

J_2 to J_4
2 choices each

Combinatorics

Next Topic:

Division Rule

Life is so much better if we ignore small differences.

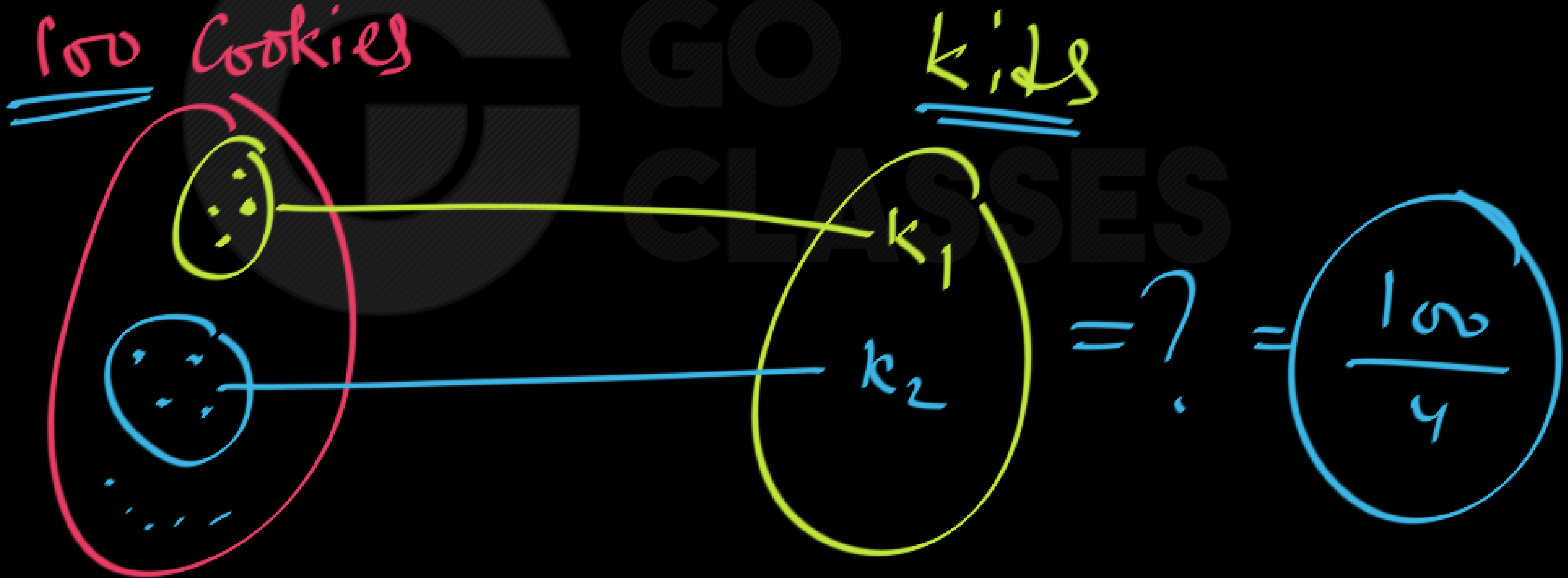
Website : https://www.goclasses.in/

Idea of Division Rule :

To count the number of cows in your field, first count the number of *legs* and then divide by four.

Q :

Suppose 100 cookies are distributed equally to a group of kids. Each kid receives 4 cookies. How many kids are there?



Division Rule:

easy to Calculate   askes

Q. How many ways can the full committee of n people be split into two sides(Yes, No) on an issue?

Q. How many ways can the full committee of n people be split into two sides(Yes, No) on an issue?

Yes | No

P_1 | P_2

P_2 | P_1

Different

Ans: $= 2^n$

$P_1 \rightarrow$ 2 Choices

$P_2 \rightarrow$ 2 Choices

⋮

$P_n \rightarrow$ 2 Choices

Q. How many ways can the full committee of n people be split into two team1

$$\left(\frac{2^n}{2}\right)$$

$P_1, P_3 \mid P_2, P_4$

$P_2, P_4 \mid P_1, P_3$

Same

easy $\Rightarrow$ Teams labdles

Askes: teams NOT labelled

2^n

(T_1, T_2) $\to$ $\{\ \}$

(T_2, T_1)

(T_3, T_4) $\to$ $\{\ \}$

(T_4, T_3)

2 $\to$ 1

2 $\to$ 1

2 $\to$ 1

$\Rightarrow \dfrac{2^n}{2}$

$n = 4$; a, b, c, d — People

Teams labelled

(ab, cd)
(cd, ab)
(a, bcd)
(bcd, a)
(, abcd)
(abcd,)

2^n

teams NOT labelled

{ab, cd}
{a, bcd}
{abcd, }

$\Rightarrow \frac{2^n}{2}$