Graph Theory :

Lecture 2 :

Degree Summation, Path, Cycle, Walk, Special Type of Graphs

Website : ~~https://www.goclasses.in/~~

Graph Theory :

Recap :

Basic of Graph Theory

Website : https://www.goclasses.in/

1 Basic Definitions and Concepts in Graph Theory

A **graph** $G(V, E)$ is a set V of **vertices** and a set E of **edges**. In an **undirected** graph, an edge is an unordered pair of vertices. An ordered pair of vertices is called a **directed** edge. If we allow **multi-sets** of edges, i.e. multiple edges between two vertices, we obtain a **multigraph**. A self-loop or **loop** is an edge between a vertex and itself. An undirected graph without loops or multiple edges is known as a **simple** graph. In this class we will assume graphs to be simple unless otherwise stated.

If vertices a and b are endpoints of an edge, we say that they are **adjacent** and write $a \sim b$. If vertex a is one of edge e's endpoints, a is **incident** to e and we write $a \in e$. The **degree** of a vertex is the number of edges incident to it.

1.1.1 Graphs and Their Relatives

A *graph* consists of two finite sets, V and E. Each element of V is called a *vertex* (plural *vertices*). The elements of E, called *edges*, are unordered pairs of vertices. For instance, the set V might be $\{a, b, c, d, e, f, g, h\}$, and E might be $\{\{a, d\}$, $\{a, e\}$, $\{b, c\}$, $\{b, e\}$, $\{b, g\}$, $\{c, f\}$, $\{d, f\}$, $\{d, g\}$, $\{g, h\}\}$. Together, V and E are a graph G.

Graphs have natural visual representations. Look at the diagram in Figure 1.2. Notice that each element of V is represented by a small circle and that each element of E is represented by a line drawn between the corresponding two elements of V.

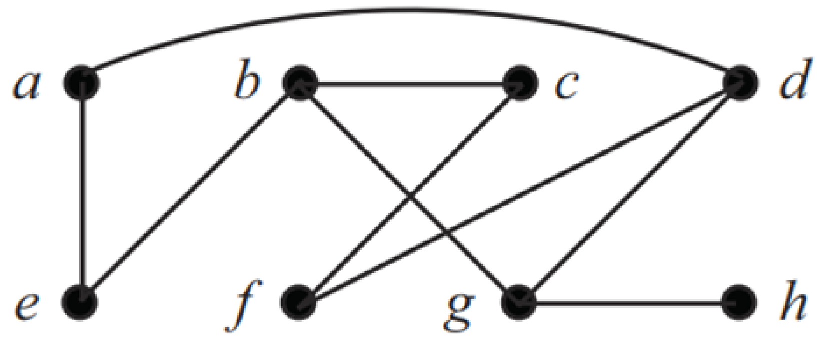


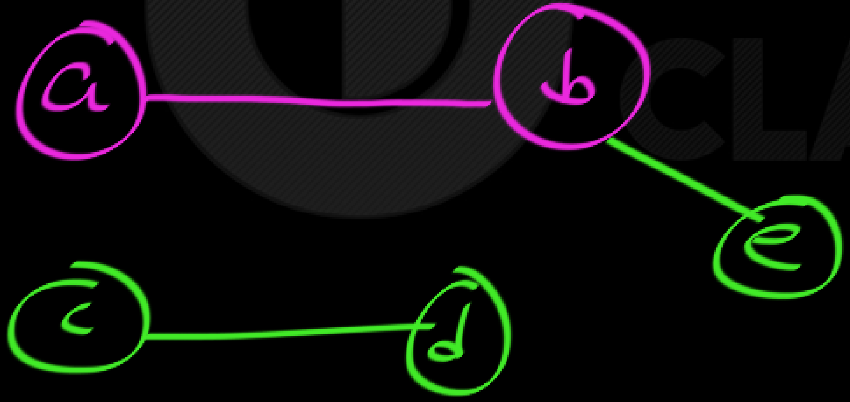
FIGURE 1.2. A visual representation of the graph G.

Find Type of Graph?

Facebook Friendship: →

Undirected Edges ✓
No self loop ✓
No multi Edges ✓

Simple Graph



Find Type of Graph?

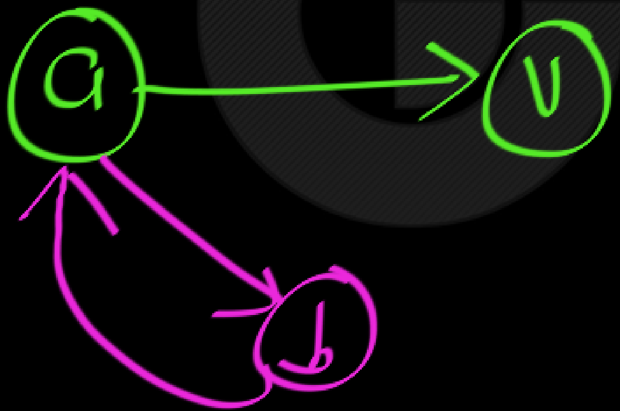
Instagram Following:

→ Directed Edges ✓

No self loops

No multi Edges

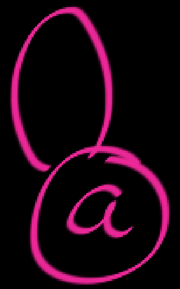
Simple Di-graph



We can write $\{a, a\}$ to denote a loop in an undirected graph, but $\{a, a\}$ is considered the same as (a, a).

In general, if a graph G is not specified as directed or undirected, it is assumed to be undirected. When it contains no loops it is called *loop-free*.

self-loop



$\equiv \{a, a\}$

In undirected Graph

$\equiv (a, a)$

In Di-graph

Order, Size of a Graph:

The order of a graph G is the cardinality of its vertex set, and the size of a graph is the cardinality of its edge set.

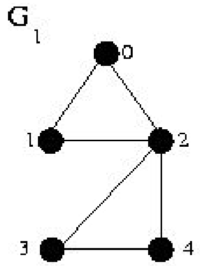
$$\text{order}(G) = |V| = n(G)$$

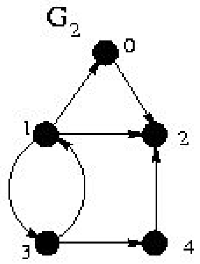
$$\text{size}(G) = |E|$$

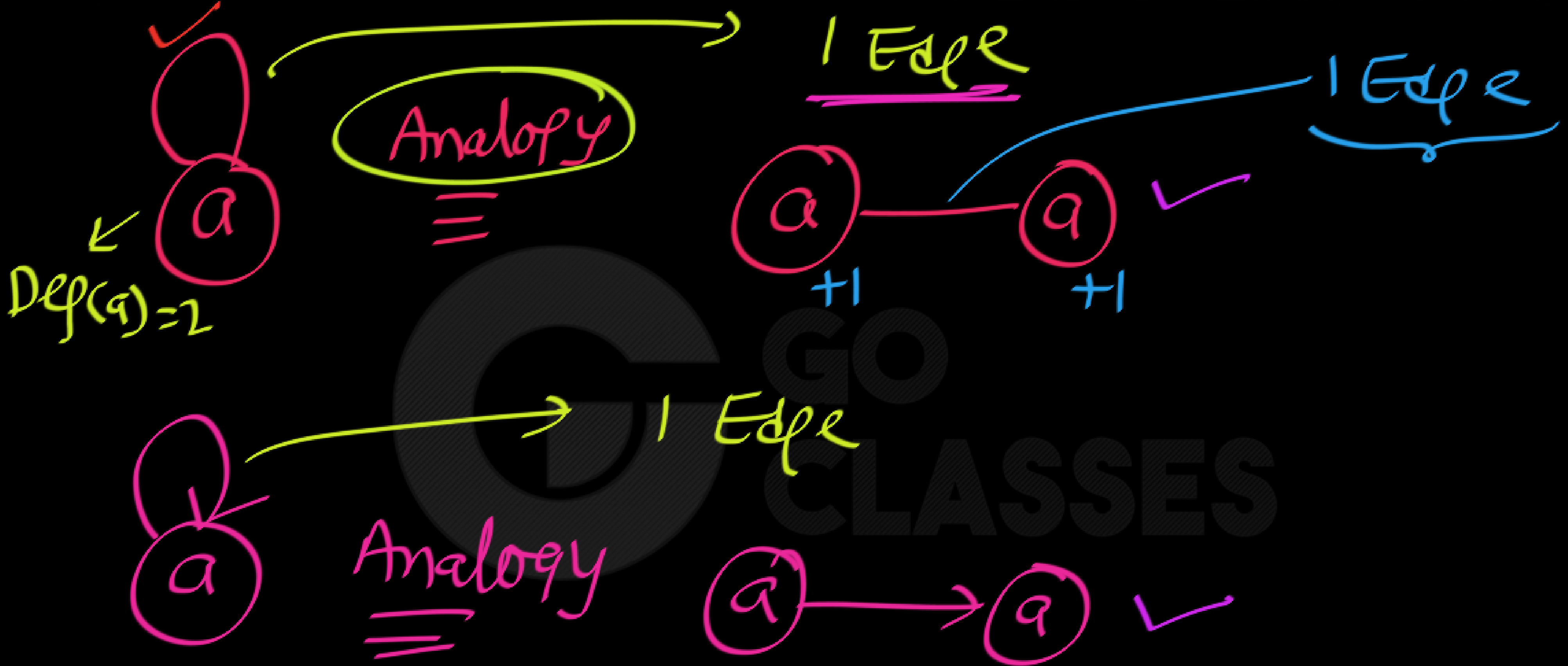
Example

- G1 is a graph of order 5
- G2 is a digraph of order 5
- The size of G1 is 6 where E(G1) =
  - {(0, 1), (0, 2), (1, 2), (2, 3), (2, 4), (3, 4)}
- The size of the digraph G2 is 7 where E(G2) =
  - {(0, 2), (1, 0), (1, 2), (1, 3), (3, 1), (3, 4), (4, 2)}.

unDirected






1 Edge
Analogy
a
Deg(a)=2
1 Edge
a
a
+1
+1
1 Edge
Analogy
a
a
a

Let G be an undirected graph or multigraph. For each vertex v of G, the *degree of* v, written $\deg(v)$, is the number of edges in G that are incident with v. Here a loop at a vertex v is considered as two incident edges for v.

For the graph in Fig. 11.32, $\deg(b) = \deg(d) = \deg(f) = \deg(g) = 2$, $\deg(c) = 4$, $\deg(e) = 0$, and $\deg(h) = 1$. For vertex a we have $\deg(a) = 3$ because we count a loop twice. Since h has degree 1, it is called a *pendant* vertex.

Size(G) = 8

Order(G) = 8

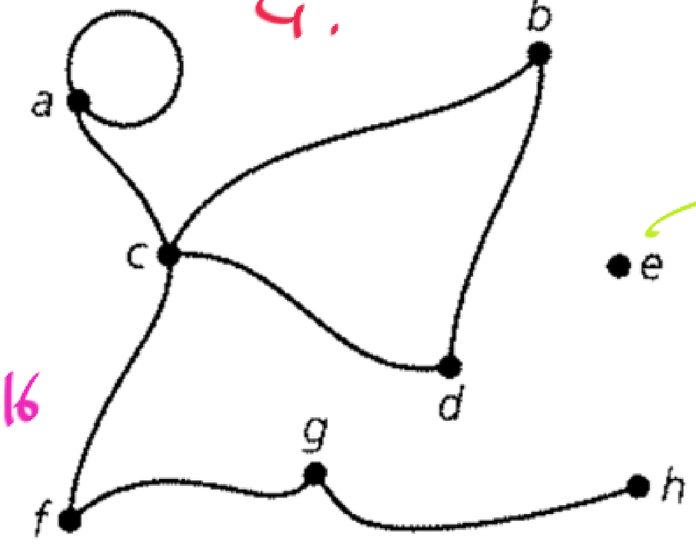
δ

min-Deg(G) = 0

max-Deg(G) = 4

Δ

G:



Isolated

Figure 11.32

Total Deg = $\sum \mathrm{Deg}(v) = 16$

Avg-Deg = $\frac{\text{Total Deg}}{\#\text{vertice}}$ = 2 order(G) = 8

size(G) = 9

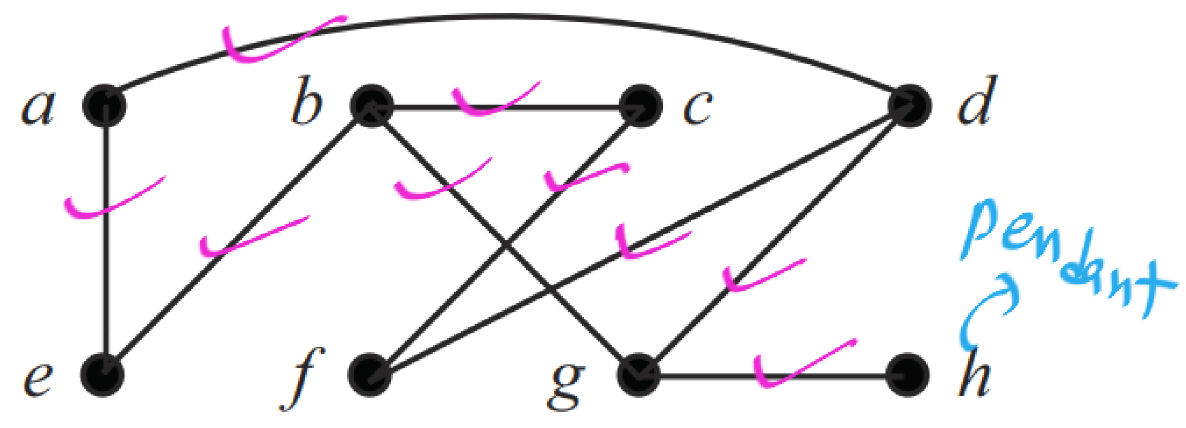


FIGURE 1.2. A visual representation of the graph G.

Degree:

a = 2
b = 3
c = 2
d = 3
e = 2
f = 2
g = 3
h = 1

$\delta(G) = 1$

$\Delta(G) = 3$

Total Deg = $\sum_{\forall v} \text{Deg}(v) = 18$

Avg-Deg(G) = $\frac{18}{8} = 2.25$

#Even-Deg vertices = 4

#Odd-Deg vertices = 4

The *degree* of v, denoted by $\deg(v)$, is the number of edges incident with v. In simple graphs, this is the same as the cardinality of the (open) neighborhood of v. The *maximum degree* of a graph G, denoted by $\Delta(G)$, is defined to be

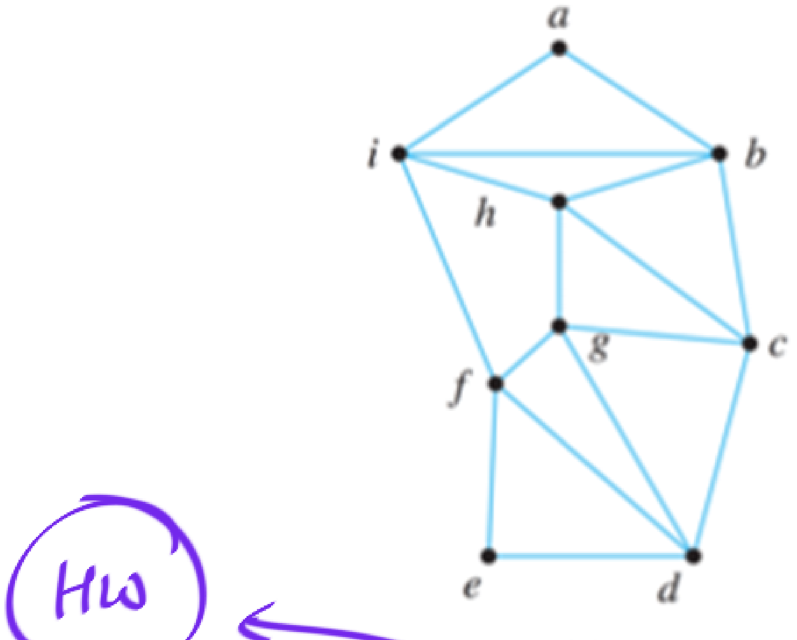
$$\Delta(G) = \max\{\deg(v) \mid v \in V(G)\}.$$

Similarly, the *minimum degree* of a graph G, denoted by $\delta(G)$, is defined to be

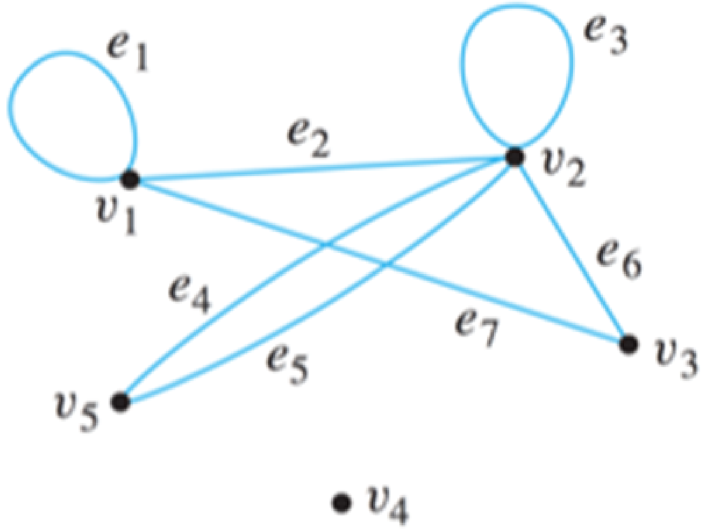
$$\delta(G) = \min\{\deg(v) \mid v \in V(G)\}.$$

The *degree sequence* of a graph of order n is the n-term sequence (usually written in descending order) of the vertex degrees.

Concept 8.1 (Graph Descriptors). For each graph below, determine: the vertex set, the edge set, the order, the size, the minimum degree, the maximum degree, the total degree, the average degree, and the degree sequence.



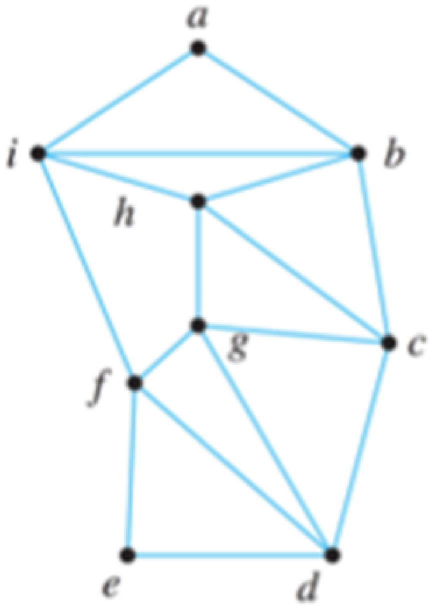
(a)



(b)

Concept 8.1 (Graph Descriptors). For each graph below, determine: the vertex set, the edge set, the order, the size, the minimum degree, the maximum degree, the total degree, the average degree, and the degree sequence.

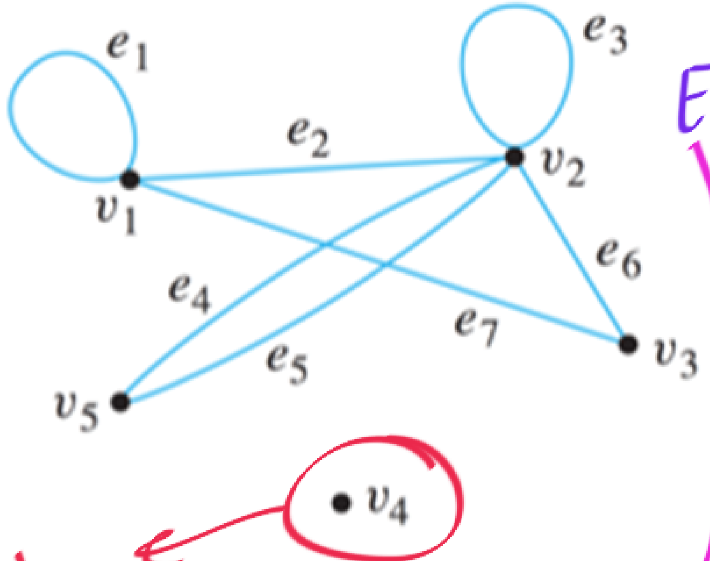
$V = \{v_1, v_2, v_3, v_4, v_5\}$



HW

(a)

Simple Graph



Isolated

(b)

✓ Psudograph

$E = \{\{v_1, v_1\}, \{v_1, v_2\}, \{v_2, v_5\}, \{v_2, v_5\}, \text{- - - -}\}$

multiset

(b) order(G) = 5 ; size(G) = 7 ;

Deg(v_2) = 6 ; Deg(V_1) = 4 ; Deg(V_5) = 2 ; Deg(V_4) = 0
Deg(V_3) = 2

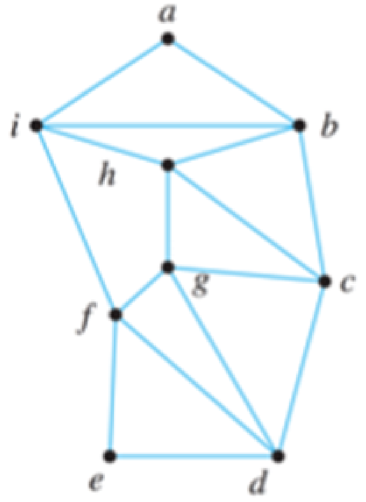
Total Deg = $\sum$ Deg(v) = 14

$\delta(G) = 0$
$\Delta(G) = 6$

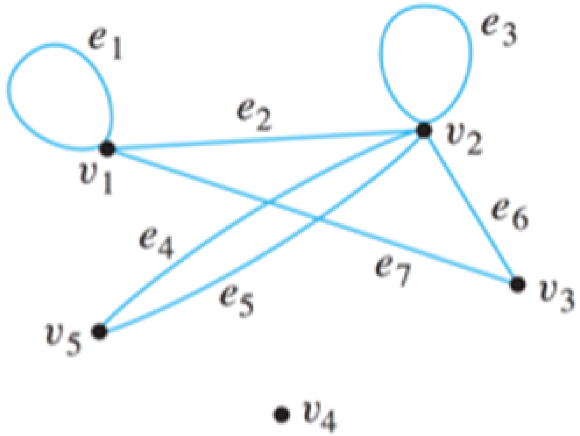
Avg Deg = $\frac{14}{5}$

Deg Sequence :
6, 4, 2, 2, 0

#odd-Deg vertices = 0



(a)



(b)

Concept 8.2 (Adjacency & Incidency). Answer the following questions about the graph (b) show on the right in the previous problem.

(a) Find all edges that are incident on v_1.

(b) Find all vertices that are adjacent to v_2.

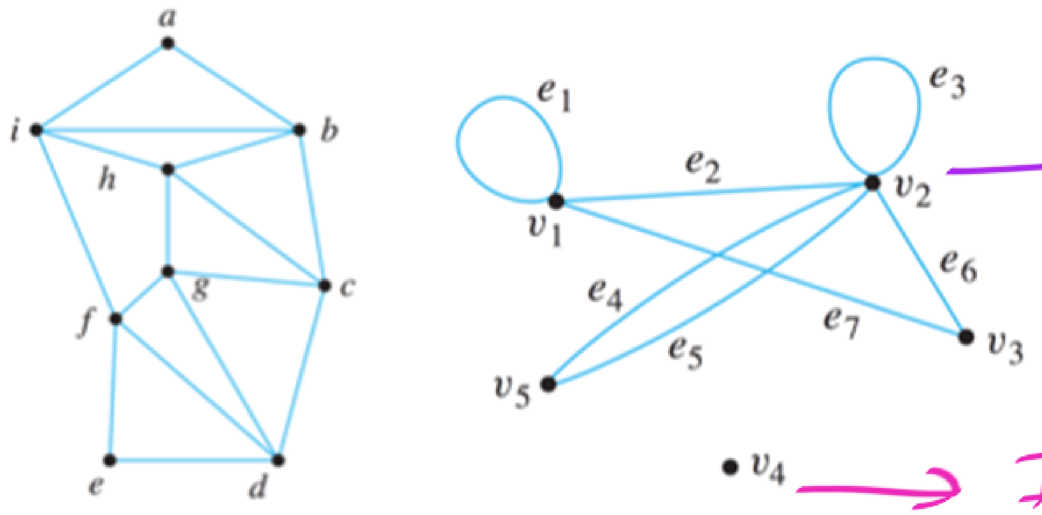
(c) Find all edges that are adjacent to e_2.

(d) Find all loops.

(e) Find all parallel edges.

(f) Find all isolated vertices.

(g) Find the degree of v_2.



(a) (b)

→ Deg = 6

→ Isolated

Concept 8.2 (Adjacency & Incidency). Answer the following questions about the graph (b) show on the right in the previous problem.

(a) Find all edges that are incident on v_1. → e_1, e_2, e_7

$N(V_3) = \{V_2, V_1\}$

(b) Find all vertices that are adjacent to v_2. → V_2, V_3, V_1, V_5

(c) Find all edges that are adjacent to e_2. → $e_1, e_7, e_3, e_6, e_4, e_5$

(d) Find all loops. → $\{v_1, v_1\}, \{v_2, v_2\}$

(e) Find all parallel edges. → e_4, e_5

(f) Find all isolated vertices.

(g) Find the degree of v_2.

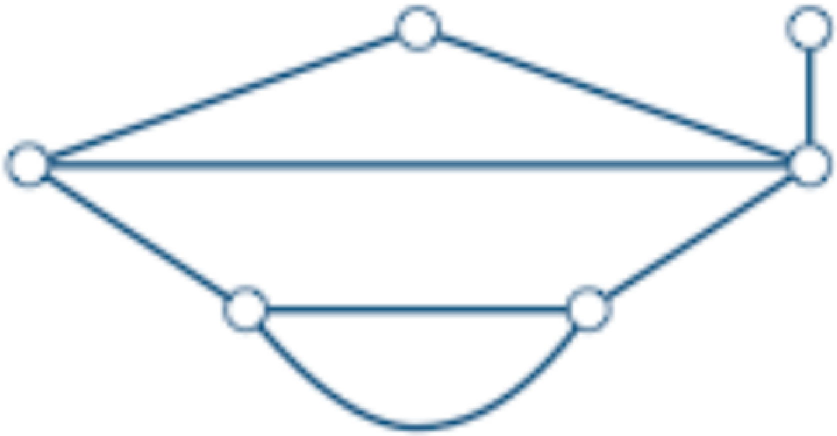
Graph Theory :

Next Topic :

Degree Summation

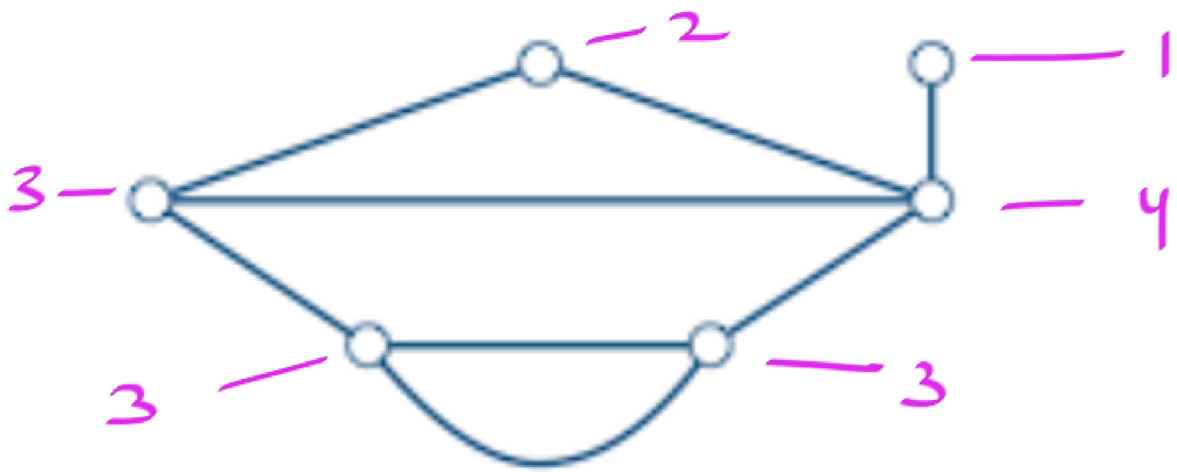
Website : https://www.goclasses.in/

Q: The sum of the degrees of all the vertices in the network below is:



A 6 B 7 C 8 D 15 E 16

Q: The sum of the degrees of all the vertices in the network below is:



A 6 B 7 C 8 D 15 E 16

Total Degree = Degree Summation

$$= \sum_{\forall v \in V} Deg(v)$$

$$\text{Avg. Deg} = \frac{\text{Total Deg}}{|V|}$$

① min $\leq$ Avg $\leq$ max

② $\delta \leq$ Avg Deg $\leq \Delta$ max Deg

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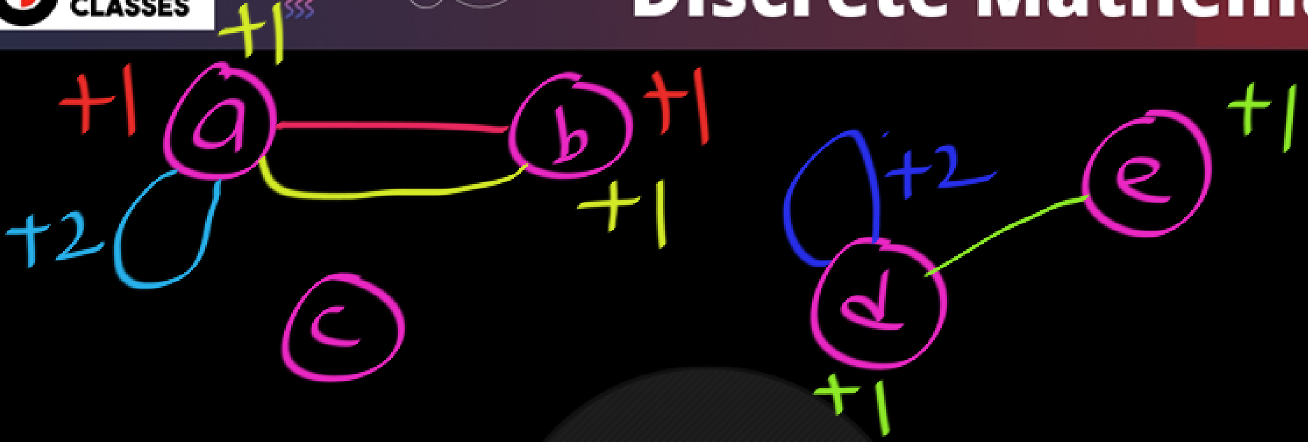
for Graph with n vertices:

$n\delta \leq$ Total Deg $\leq n\Delta$



#Edges =	0	1	2	3	4	5
Total Deg =	0	2	4	6	8	10

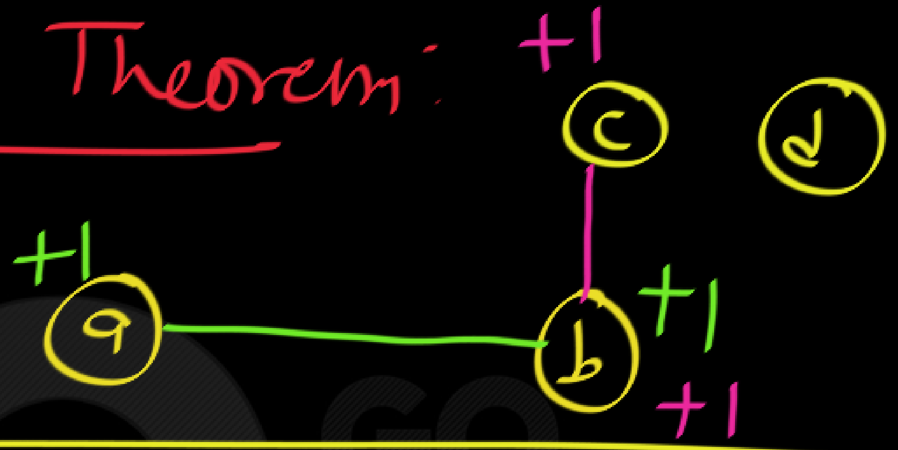
$$\underbrace{\text{Total Deg}} = 2 * |E|$$

Why? Every Edge contributes a value of 2 in $\underbrace{\text{Total Deg}}$.

$$\sum \text{Deg}(v) = 2|E|$$

Handshaking Theorem:

People:



#Handshakes =	1	2	- -
Total Deg $= \sum_{\forall \text{ persons } p} \text{Handshakes}(p)$	2	4	- -

THE HANDSHAKING THEOREM Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} \deg(v).$$

(Note that this applies even if multiple edges and loops are present.)

Proof. In counting the sum $\sum_{v \in V} deg(v)$, we count each edge of the graph twice, because each edge is incident to exactly two vertices. $\square$

EXAMPLE 3 How many edges are there in a graph with 10 vertices each of degree six?

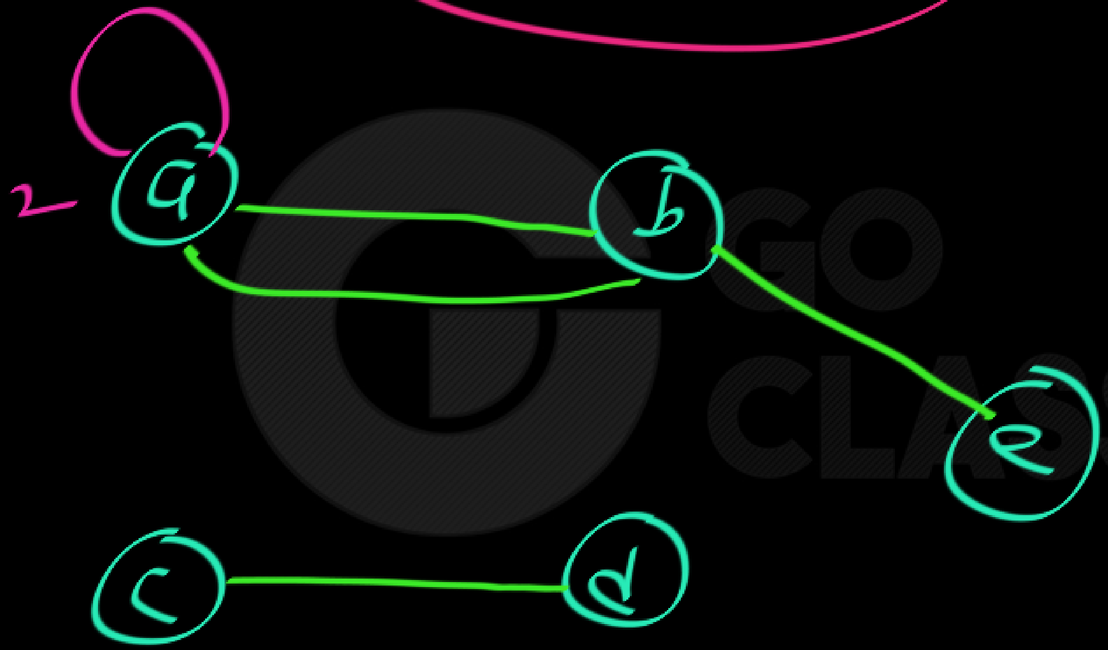
EXAMPLE 3 How many edges are there in a graph with 10 vertices each of degree six?

Solution: Because the sum of the degrees of the vertices is $6 \cdot 10 = 60$, it follows that $2m = 60$ where m is the number of edges. Therefore, $m = 30$. ◀

$2|E| =$ Total Deg $= 10 \times 6 = 60$

$|E| = 30$

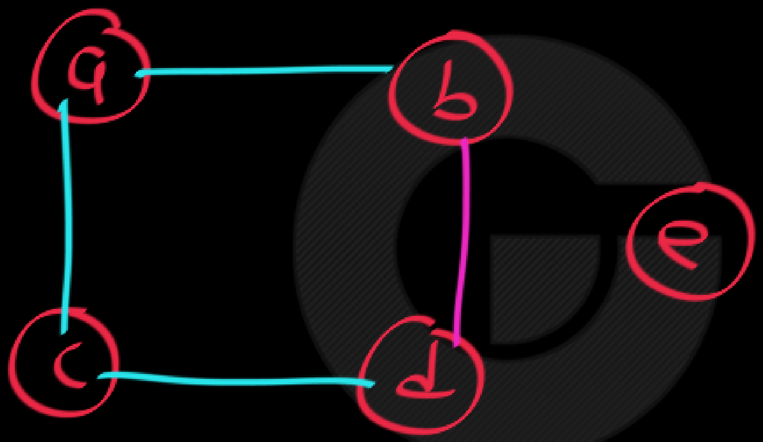
Try to create Odd number of Odd-Deg Vertices:



#Odd-Deg Vertices

$= \cancel{0}\ \cancel{0}\ \cancel{2}\ \cancel{0}\ \cancel{2}\ 4$

Try to create Odd number of Even-Deg vertices:



#Even-Dep vertices

= ~~5~~

~~3~~

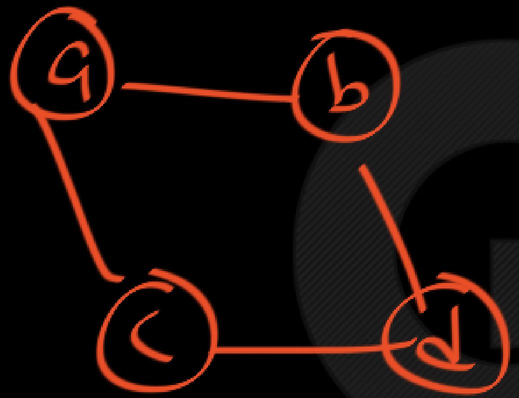
=

+

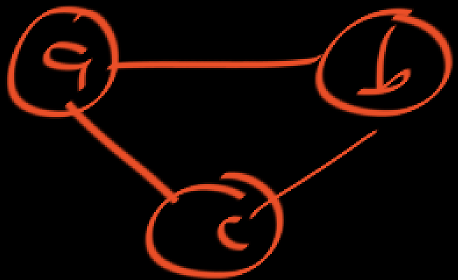
~~3~~

5

Try to create Even number of Even-Deg vertices:



#Even-Deg vertices = 4



#Even-Deg vertices = 3

Even + Even = Even

Odd + Even = Odd

Odd + Odd = Even

$$\underbrace{\text{Odd} + \text{Odd} + \text{Odd} + \text{Odd} + \cdots + \text{Odd}}_{\text{Even Number of Odd Values}} = \underline{\underline{\text{Even}}}$$

Theorem: In any Graph, number of odd-Deg vertices is Always Even.

Proof:

Total Deg $= 2|E| =$ Even

Total Deg of Even-Deg vertices + Total Deg of odd-Deg vertices = Even

Total Deg of Even-Deg vertices + Total Deg of Odd-Deg vertices = Even

Even

must be Even

(Odd + Odd + ... + Odd) = Even

\# Odd-Deg vertices = Even

An undirected graph has an even number of vertices of odd degree.

Proof: Let V_1 and V_2 be the set of vertices of even degree and the set of vertices of odd degree, respectively, in an undirected graph $G = (V, E)$ with m edges. Then

$$2m = \sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v).$$

Because $\deg(v)$ is even for $v \in V_1$, the first term in the right-hand side of the last equality is even. Furthermore, the sum of the two terms on the right-hand side of the last equality is even, because this sum is $2m$. Hence, the second term in the sum is also even. Because all the terms in this sum are odd, there must be an even number of such terms. Thus, there are an even number of vertices of odd degree. ◁

If $G = (V, E)$ is an undirected graph or multigraph, then $\sum_{v \in V} \deg(v) = 2|E|$.

Proof: As we consider each edge $\{a, b\}$ in graph G, we find that the edge contributes a count of 1 to each of $\deg(a)$, $\deg(b)$, and consequently a count of 2 to $\sum_{v \in V} \deg(v)$. Thus $2|E|$ accounts for $\deg(v)$, for all $v \in V$, and $\sum_{v \in V} \deg(v) = 2|E|$.

This theorem provides some insight into the number of odd-degree vertices that can exist in a graph.

For any undirected graph or multigraph, the number of vertices of odd degree must be even.

Theorem 1.1. *In a graph G, the sum of the degrees of the vertices is equal to twice the number of edges. Consequently, the number of vertices with odd degree is even.*

Proof. Let $S = \sum_{v \in V} \deg(v)$. Notice that in counting S, we count each edge exactly twice. Thus, $S = 2|E|$ (the sum of the degrees is twice the number of edges). Since S is even, it must be that the number of vertices with odd degree is even. $\square$

Graph Theory :

Next Topic : Directed Graphs

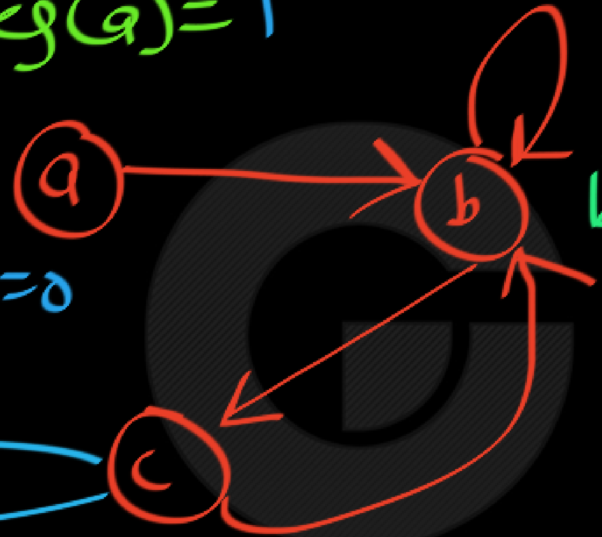
In-Degree, Out-Degree of a vertex

Website : https://www.goclasses.in/

Directed Graph:

$a^{+} = \text{out Deg}(a) = 1$

$a^{-} = \text{In-Deg}(a) = 0$



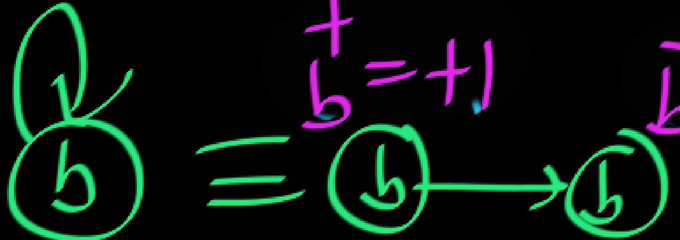
out-Deg(b)

$b^{+} = 2$

$b^{-} = 3$

$\text{In}(c) = 1$

$\text{out}(c) = 1$



$b^{+} = +1$

$b^{-} = +1$

Undirected Graph → Concept of Degree of Vertex

Directed Graph → ~~Degree~~ In-Degree, out-Degree

Terminology for graphs with directed edges reflects the fact that edges in directed graphs have directions.

DEFINITION 4

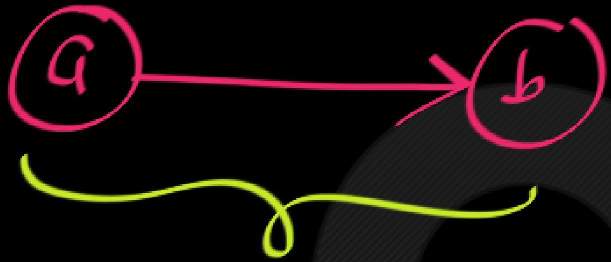
When (u, v) is an edge of the graph G with directed edges, u is said to be *adjacent to* v and v is said to be *adjacent from* u. The vertex u is called the *initial vertex* of (u, v), and v is called the *terminal* or *end vertex* of (u, v). The initial vertex and terminal vertex of a loop are the same.

Because the edges in graphs with directed edges are ordered pairs, the definition of the degree of a vertex can be refined to reflect the number of edges with this vertex as the initial vertex and as the terminal vertex.

DEFINITION 5

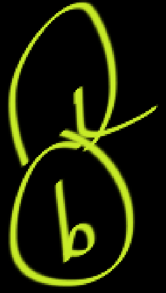
In a graph with directed edges the *in-degree of a vertex* v, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex. The *out-degree of* v, denoted by $\deg^+(v)$, is the number of edges with v as their initial vertex. (Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.)

Directed: (a, b) Edge



means a is adj to b

b is adj from a



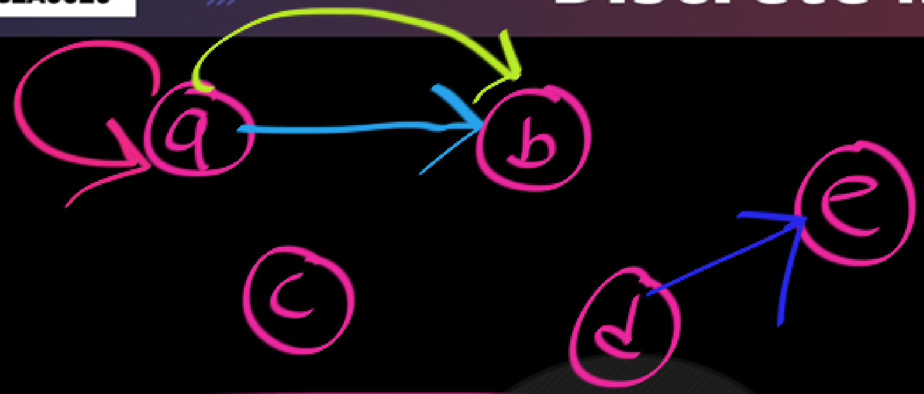
(b, b) Edge

b is Adj to b.

X a, b are adjacent

$$\text{Total-In Deg} = \sum_{\forall v \in V} \text{In-Deg}(v)$$

$$\text{Total-out Deg} = \sum_{\forall v \in V} \text{out-Deg}(v)$$



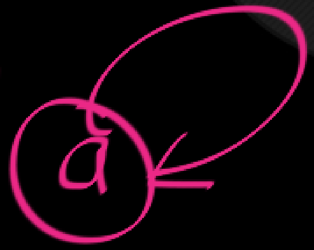
#Edges	0	1	2	3	4
Total In-Deg	0	1	2	3	4
Total out-Deg	0	1	2	3	4

Total InDeg = Total outDeg = $|E|$

Why?

$a^{+} = +1$ $\quad$ $b^{-} = +1$

$a^{-} = +1$



$a^{+} = +1$

Find the in-degree and out-degree of each vertex in the graph G with directed edges shown in Figure 2.

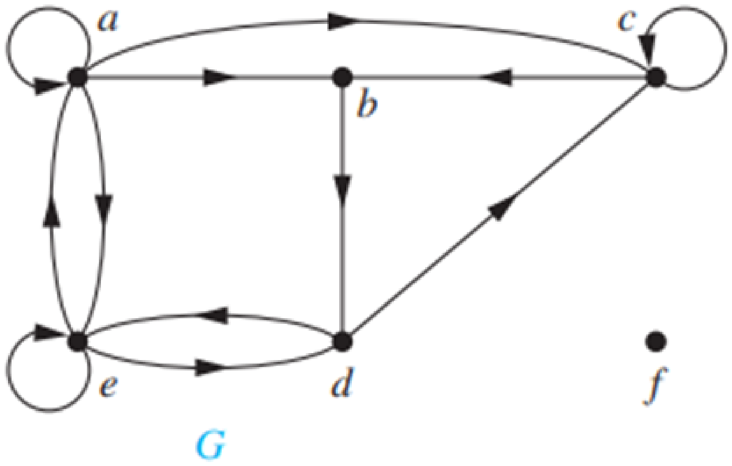


FIGURE 2 **The Directed Graph G.**

Solution: The in-degrees in G are $\deg^-(a) = 2$, $\deg^-(b) = 2$, $\deg^-(c) = 3$, $\deg^-(d) = 2$, $\deg^-(e) = 3$, and $\deg^-(f) = 0$. The out-degrees are $\deg^+(a) = 4$, $\deg^+(b) = 1$, $\deg^+(c) = 2$, $\deg^+(d) = 2$, $\deg^+(e) = 3$, and $\deg^+(f) = 0$. ◀

Because each edge has an initial vertex and a terminal vertex, the sum of the in-degrees and the sum of the out-degrees of all vertices in a graph with directed edges are the same. Both of these sums are the number of edges in the graph. This result is stated as Theorem 3.

THEOREM 3 Let $G = (V, E)$ be a graph with directed edges. Then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|.$$

Now, Back to "Simple Graphs"

Undirected ✓

No self loop ✓

No multiEdges ✓

Graph Theory :

Next Topic :

(For Simple Graph) Walk, Trail, Path

Cycle, Circuit

† Since the terminology of graph theory is not standard, the reader may find some differences between terms used here and in other texts.

DISCRETE AND COMBINATORIAL MATHEMATICS

An Applied Introduction

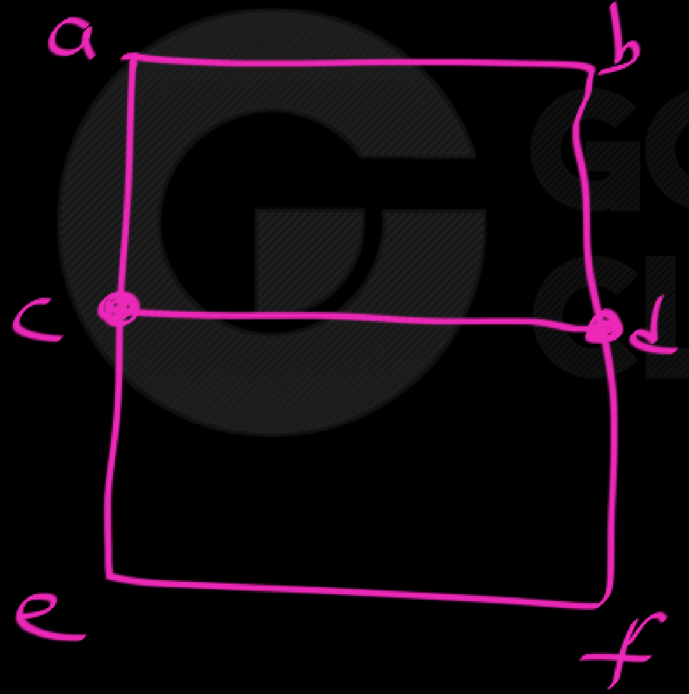
FIFTH EDITION

RALPH P. GRIMALDI

Rose-Hulman Institute of Technology

Walk ; Trail ; Path :

Park :



walk :

a – b walk :

a — b ✓

a — c — d — b ✓

a — b — d — b ✓

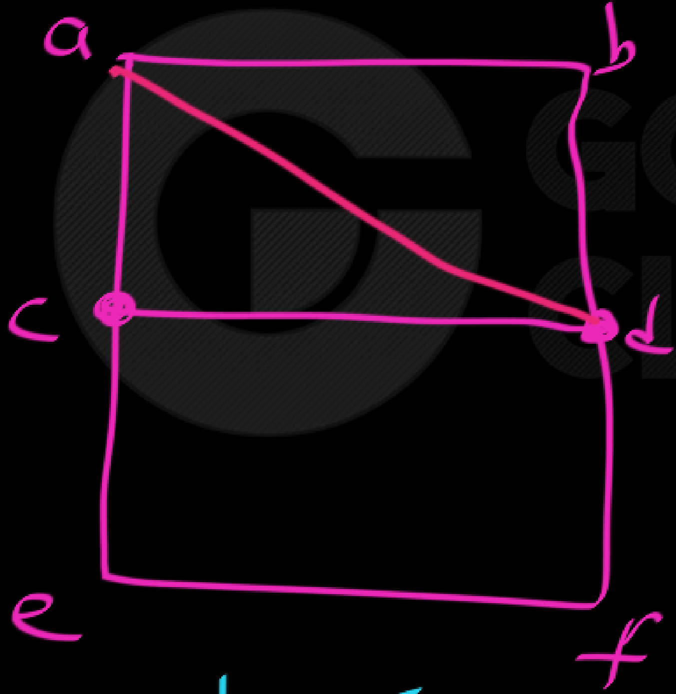
a — c — d — b — a — c — d — b

Walk : ; Trail ; Path :

Trail ⟹ No Edge Repetition

a — b Trail

Park :



a — b ✓

a — c — d — b ✓

a — b — d — b ✗

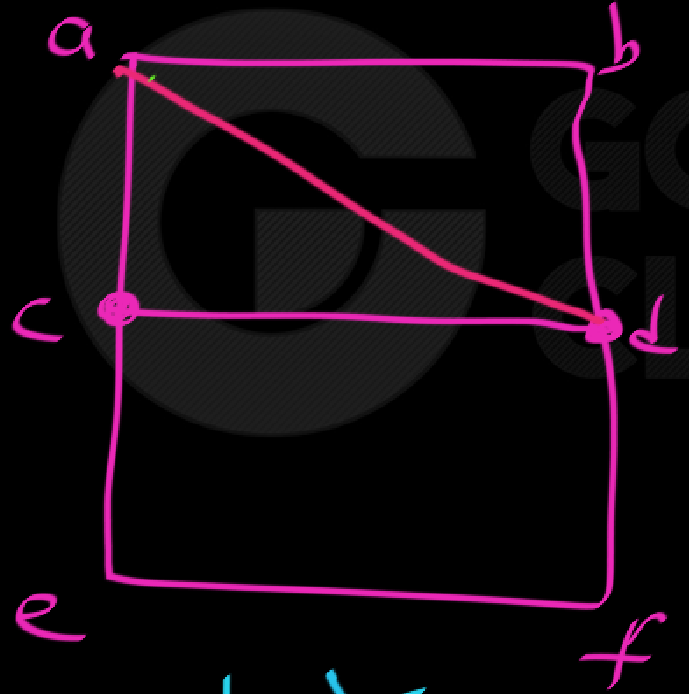
a — c — d — b — a — c — d — b ✗

a — c — d — a — b ✓

Walk ; Trail ; Path :

Path $\Rightarrow$ NO Repetition

Park :



a — b Path

a — b ✓

a — c — d — b ✓

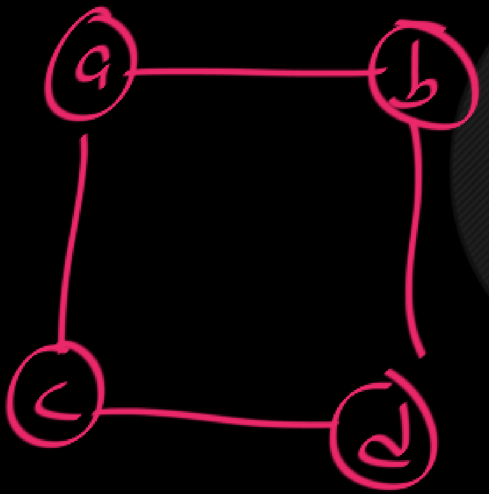
a — b — d — b ✗

a — c — d — b — a — c — d — b ✗

a — c — d — a — b ✗

walk:

$v_1 \underset{\text{Edge}}{—} v_2 \underset{\text{Edge}}{—} v_3 — \cdots — v_n$



a — b walk:

a — d — b ✗

No such Edge

a — c — b ✗

Walk:

open walk:

$v_1 \neq v_n$

Closed walk:

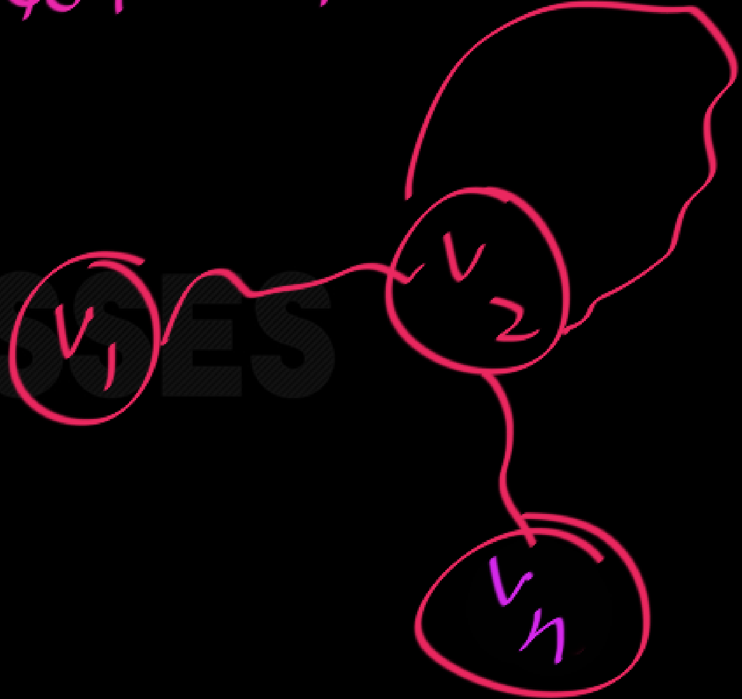
$\underbrace{v_1 = v_n}$

Trail → Vertex Repetition Allowed

Edge " NOT "

Closed Trail = Circuit

$v_1 = v_n$



Path: → No Repetition at all.

→ 

$v_1 \neq v_n$

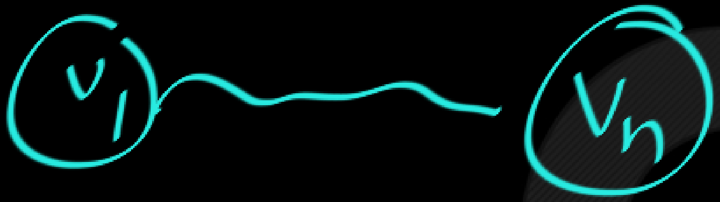
Cycle: → At least 3 vertices

Path But $v_1 = v_n$

like a

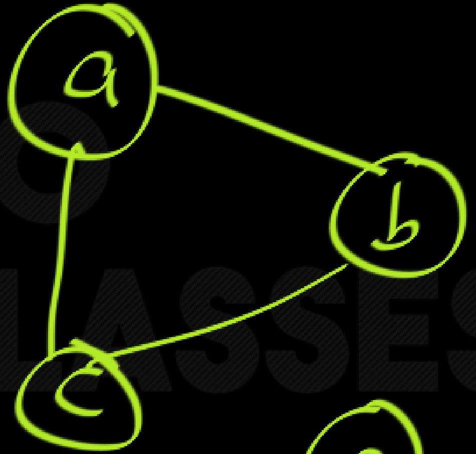
→ No Edge Repetition; No Vertex Except 1st, last

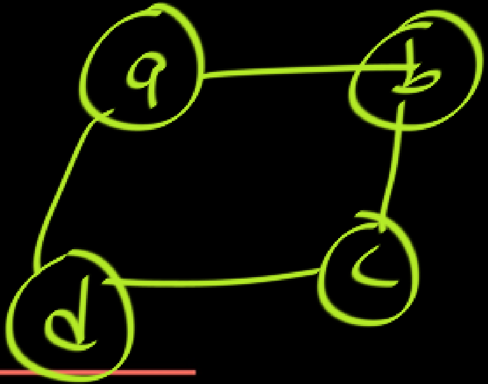
Path



$v_1 \neq v_n$

Cycle ≥ 3 vertices





Path $\not\longrightarrow$ cycle

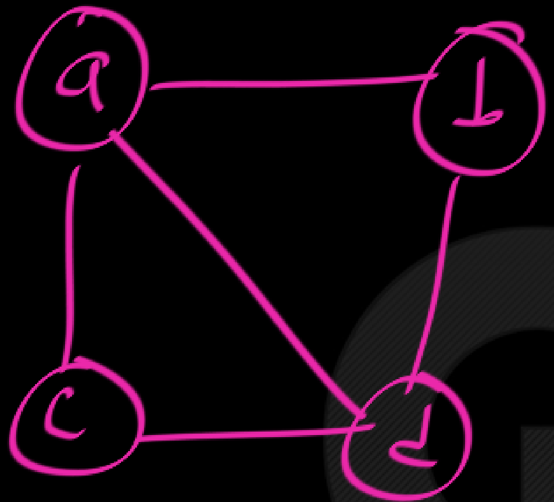
Path

cycle

Walks, trails and paths

Definition

A **walk** in a graph is a sequence of alternating vertices and edges $v_1 e_1 v_2 e_2 \ldots v_n e_n v_{n+1}$ with $n \geq 0$. If $v_1 = v_{n+1}$ then the walk is **closed**. The **length** of the walk is the number of edges in the walk. A walk of length zero is a **trivial walk**.

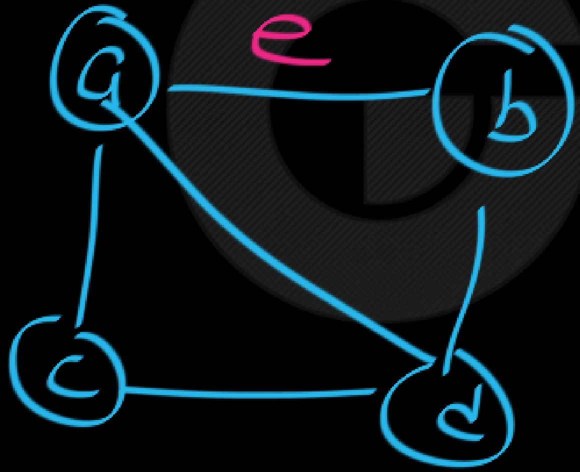


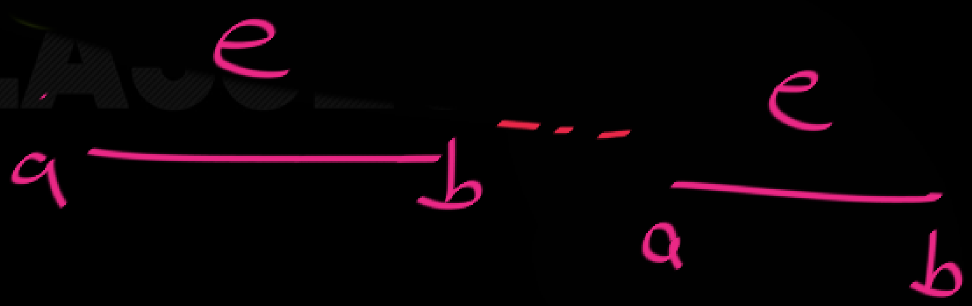
a — a walk

① a — Trivial walk

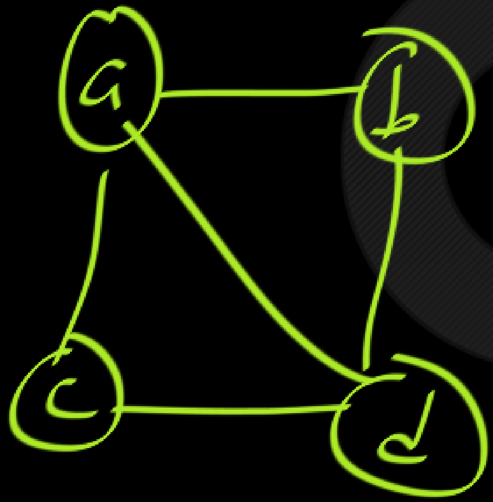
② a — c — a — b — d — a ✓

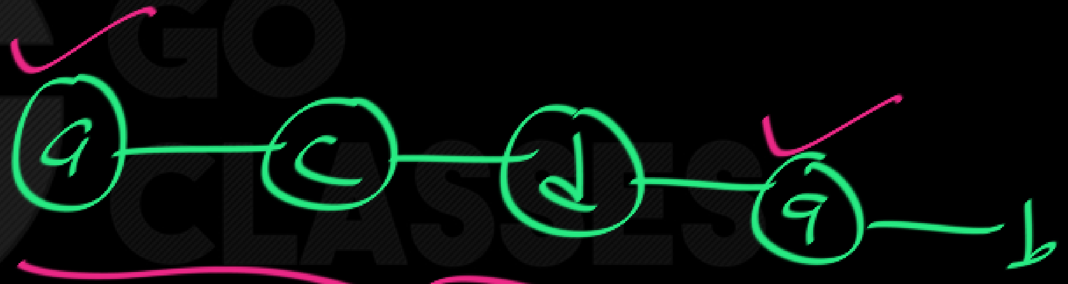
No Vertex Repetition $\Longrightarrow$ No Edge Repetition





No Vertex Repetition $\not\Longleftarrow$ No Edge Repetition





No Edge Repetition But Vertex Repeating

Definition

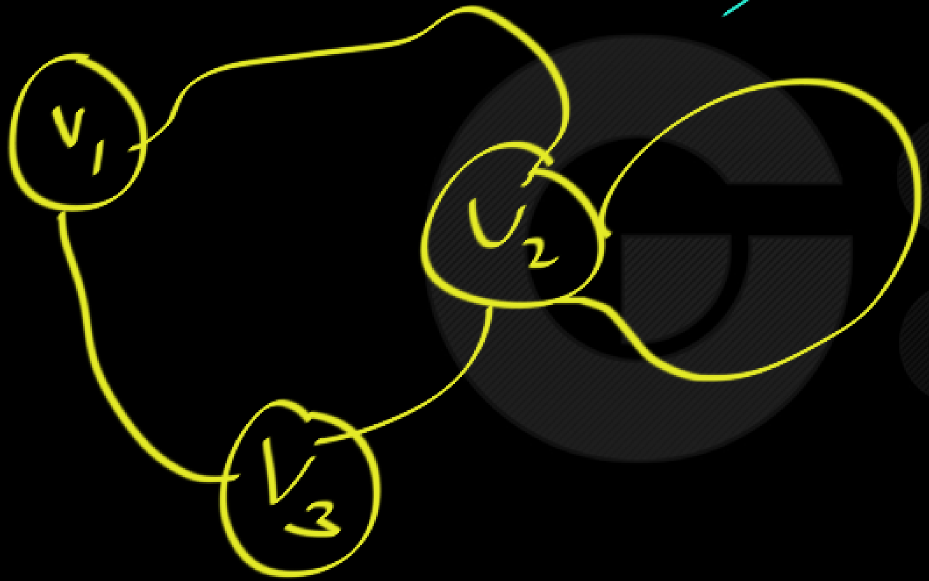
A **trail** is a walk with no repeated edges. A **path** is a walk with no repeated vertices. A **circuit** is a closed trail and a **trivial circuit** has a single vertex and no edges.

at least three vertices

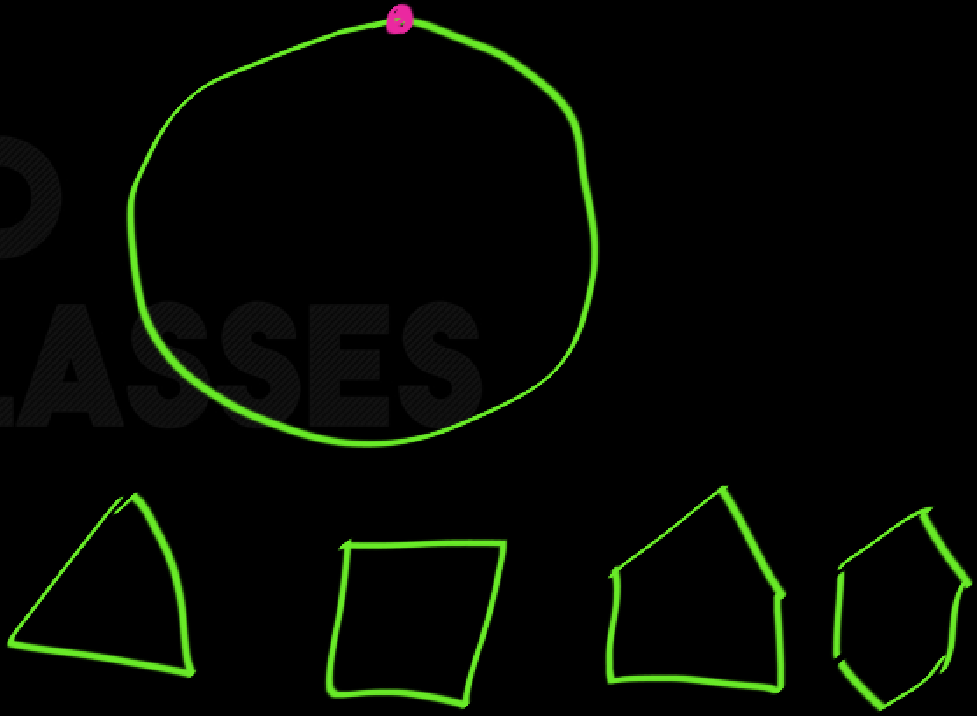
Definition

A **cycle** is a nontrivial circuit in which the only repeated vertex is the first/last one.

Circuit (Closed Trail)



Cycle



A **walk** is a sequence of vertices $v_1, v_2, \ldots, v_k$ such that $\forall i \in 1, 2, \ldots, k-1$, $v_i \sim v_{i+1}$. A **path** is a walk where $v_i \neq v_j, \forall i \neq j$. In other words, a path is a walk that visits each vertex at most once. A **closed walk** is a walk where $v_1 = v_k$. A **cycle** is a closed path, i.e. a path combined with the edge (v_k, v_1). A graph is **connected** if there exists a path between each pair of vertices.

Let x, y be (not necessarily distinct) vertices in an undirected graph $G = (V, E)$. An x-y *walk* in G is a (loop-free) finite alternating sequence

$$x = x_0, e_1, x_1, e_2, x_2, e_3, \ldots, e_{n-1}, x_{n-1}, e_n, x_n = y$$

of vertices and edges from G, starting at vertex x and ending at vertex y and involving the n edges $e_i = \{x_{i-1}, x_i\}$, where $1 \leq i \leq n$.

The *length* of this walk is n, the number of edges in the walk. (When $n = 0$, there are no edges, $x = y$, and the walk is called *trivial*. These walks are not considered very much in our work.)

Any x-y walk where $x = y$ (and $n > 1$) is called a *closed walk*. Otherwise the walk is called *open*.

Note that a walk may repeat both vertices and edges.

For the graph in Fig. 11.4 we find, for example, the following three open walks. We can list the edges only or the vertices only (if the other is clearly implied).

1) $\{a, b\}, \{b, d\}, \{d, c\}, \{c, e\}, \{e, d\}, \{d, b\}$: This is an a-b walk of length 6 in which we find the vertices d and b repeated, as well as the edge $\{b, d\}$ $(= \{d, b\})$.
2) $b \to c \to d \to e \to c \to f$: Here we have a b-f walk where the length is 5 and the vertex c is repeated, but no edge appears more than once.
3) $\{f, c\}, \{c, e\}, \{e, d\}, \{d, a\}$: In this case the given f-a walk has length 4 with no repetition of either vertices or edges.

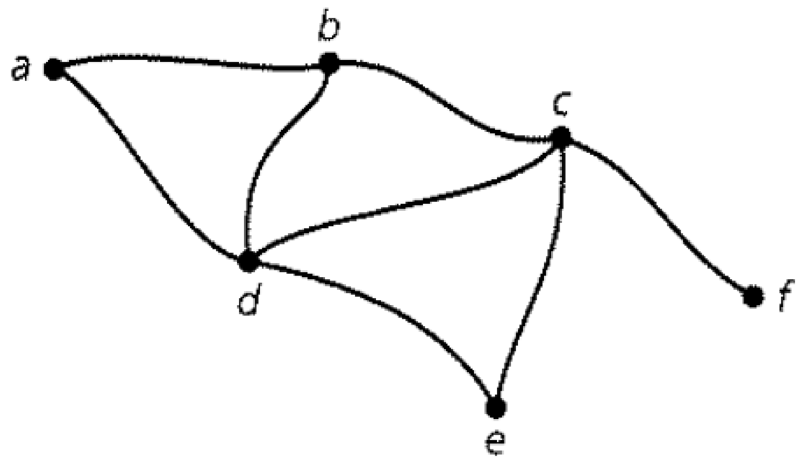


Figure 11.4

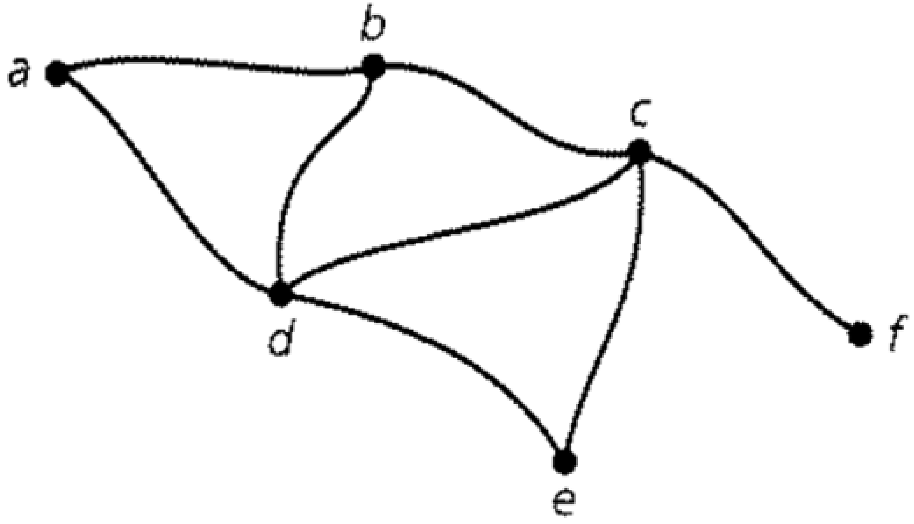
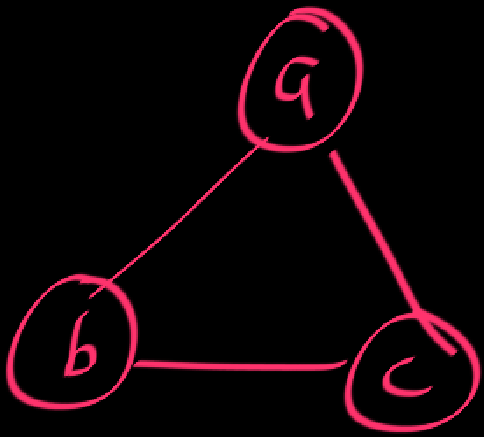


Figure 11.4

Since the graph of Fig. 11.4 is undirected, the a-b walk in part (1) is also a b-a walk (we read the edges, if necessary, as $\{b, d\}$, $\{d, e\}$, $\{e, c\}$, $\{c, d\}$, $\{d, b\}$, and $\{b, a\}$). Similar remarks hold for the walks in parts (2) and (3).

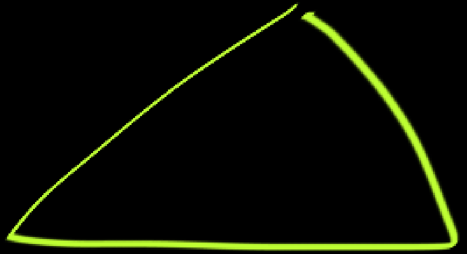
Finally, the edges $\{b, c\}$, $\{c, d\}$, and $\{d, b\}$ provide a b-b (closed) walk. These edges (ordered appropriately) also define (closed) c-c and d-d walks.



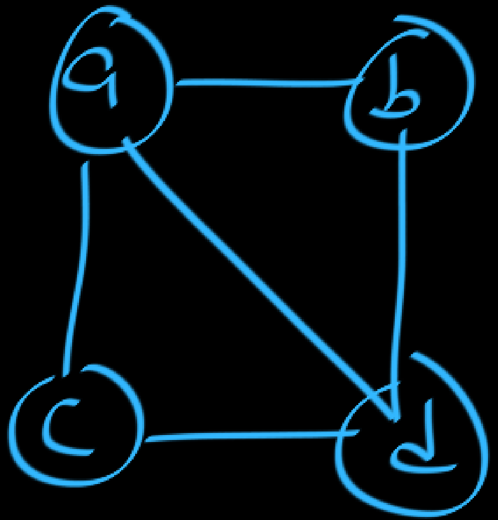
How many 3-length cycles?

= 1 ✓

3-Edge

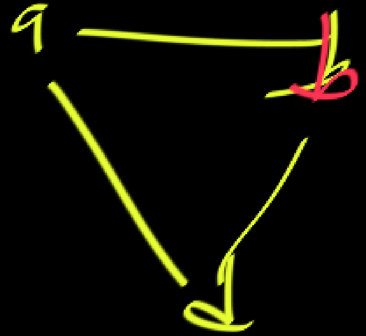


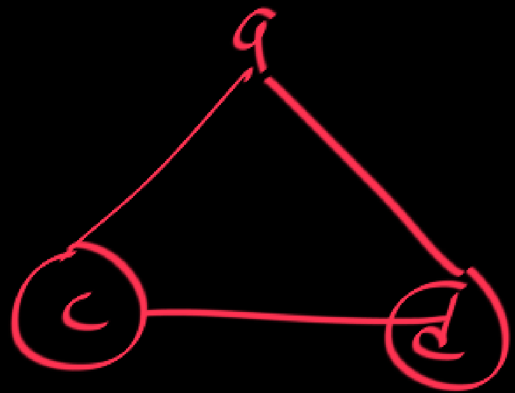
How many 6-length cycles $\Rightarrow$ 0



How many 3-length cycles!

2





a) The b-f walk in part (2) of Example 11.1 is a b-f trail, but it is not a b-f path because of the repetition of vertex c. However, the f-a walk in part (3) of that example is both an f-a trail (of length 4) and an f-a path (of length 4).

b) In Fig. 11.4, the edges $\{a, b\}$, $\{b, d\}$, $\{d, c\}$, $\{c, e\}$, $\{e, d\}$, and $\{d, a\}$ provide an a-a circuit. The vertex d is repeated, so the edges do *not* give us an a-a cycle.

c) The edges $\{a, b\}$, $\{b, c\}$, $\{c, d\}$, and $\{d, a\}$ provide an a-a cycle (of length 4) in Fig. 11.4. When ordered appropriately these same edges may also define a b-b, c-c, or d-d cycle. Each of these cycles is also a circuit.

Important:

$\boxed{\text{Cycle}}$ → Most Imp and Universal

Path
Walk
Trail

Path, Walk → NOT universally Defines

Trail → NOT at all Imp.

Path: $v_1 \neq v_n$

Now let us examine special types of walks.

Consider any x-y walk in an undirected graph $G = (V, E)$.

a) If no edge in the x-y walk is repeated, then the walk is called an x-y *trail*. A closed x-x trail is called a *circuit*.

b) If no vertex of the x-y walk occurs more than once, then the walk is called an x-y *path*. When $x = y$, the term *cycle* is used to describe such a closed path.

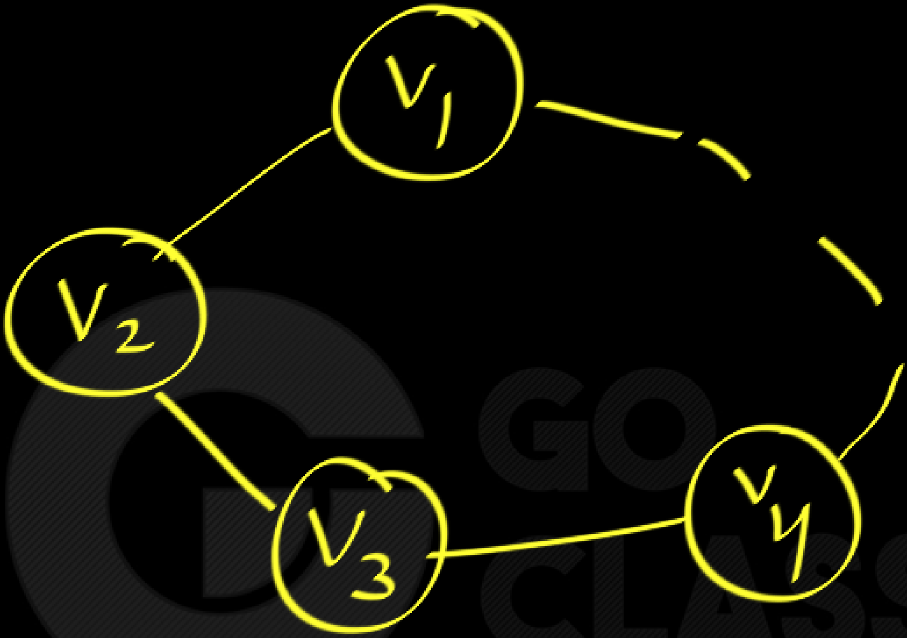
The term *cycle* will always imply the presence of at least three distinct edges (from the graph).

Before continuing, we summarize (in Table 11.1) for future reference the results of Definitions 11.2 and 11.3. Each occurrence of "Yes" in the first two columns here should be interpreted as "Yes, possibly." Table 11.1 reflects the fact that a path is a trail, which in turn is an open walk. Furthermore, every cycle is a circuit, and every circuit (with at least two edges) is a closed walk.

Table 11.1

Repeated Vertex (Vertices)	Repeated Edge(s)	Open	Closed	Name
Yes	Yes	Yes		Walk (open)
Yes	Yes		Yes	Walk (closed)
Yes	No	Yes		Trail
Yes	No		Yes	Circuit
No	No	Yes		Path
No	No		Yes	Cycle

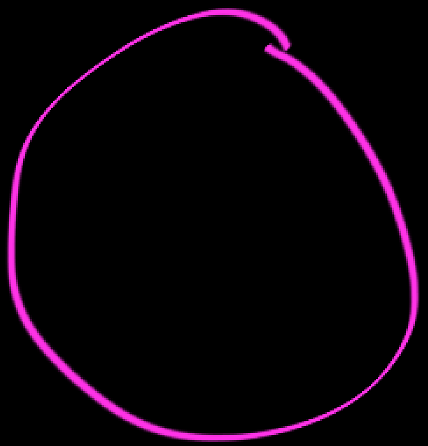
Cycle :



$\boxed{v_i \neq v_j}$

must have ≥ 3 vertices ✓

Cycle



Path