



Set Theory

Recap

Sub-lattices, Bounded Lattice

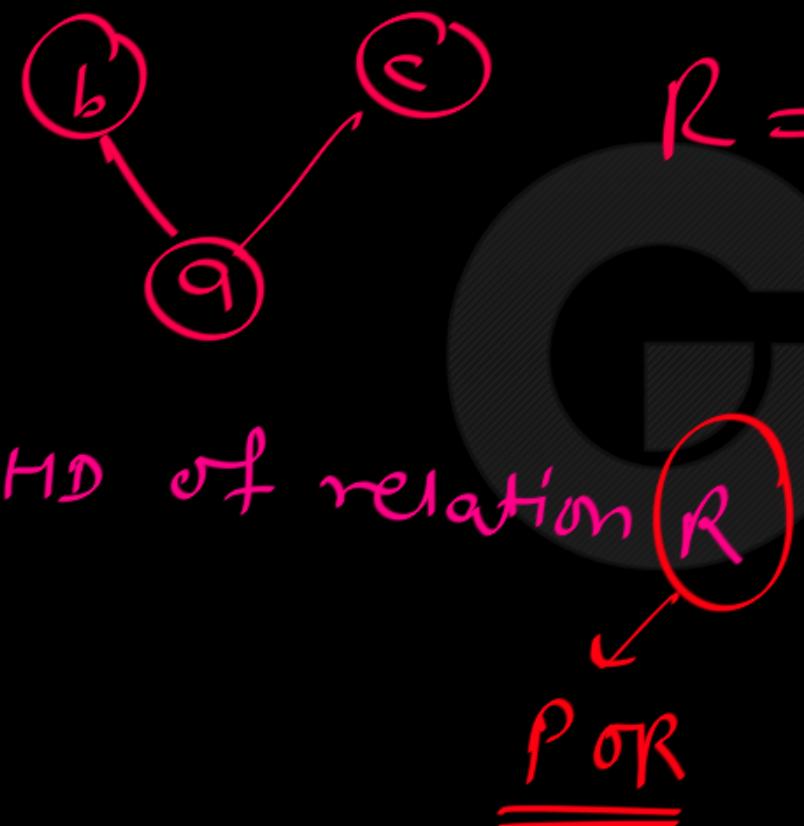
Website : <https://www.goclasses.in/>



NOTE:

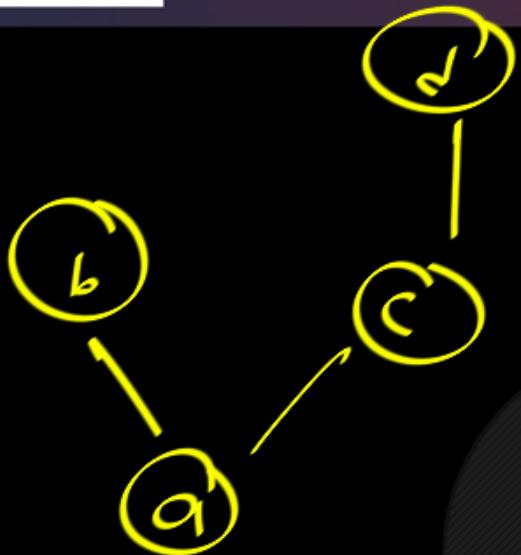
In POSET, When we are discussing same Relation R on a base set A, then we can directly say that A is a poset.

Basically, When (A,R) is a poset, and we are considering same relation R, then we can simply say that A is a poset.



Base set = {a, b, c}

$$R = \{(a,a), (a,b), (a,c), (b,b), (c,c)\}$$



Hasse Diagram
of Relation \mathcal{S}

Base set: $\{a, b, c, d\}$

$$\mathcal{S} = \{(a, a), (a, b), (a, c), (a, d), (b, b), (c, c), (c, d)\}$$

$$a \mathcal{S} a$$
$$a \mathcal{S} b$$

$$b \not\mathcal{S} a$$
$$c \not\mathcal{S} a$$



If (A, R) is a poset;

If Relation " R " is fixed then

we can say that "A" is poset

(under relation R)



Sublattices [\[edit\]](#)

A *sublattice* of a lattice L is a subset of L that is a lattice with the same meet and join operations as L . That is, if L is a lattice and M is a subset of L such that for every pair of elements a, b in M both $a \wedge b$ and $a \vee b$ are in M , then M is a sublattice of L .^[2]





A sublattice of a lattice L is a nonempty subset of L that is a lattice with the same meet and join operations as L .



The Intuition behind Sublattice :

Lattice L ; You want to take
L and want to Create Sublattice ;
Take those elements so take
their GLB, LUB also



Note: When we say that L is a lattice, we mean some relation R is there (which is fixed) such that (L, R) is lattice.



Q: True/False?

(L, R) is lattice

Let L be a lattice. If S is subset of L and S is a lattice

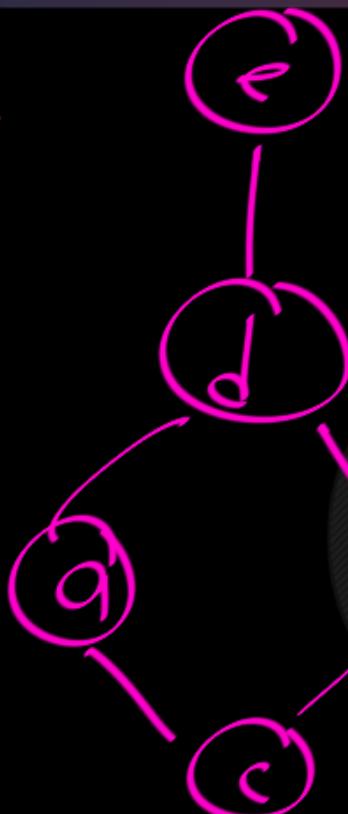
then S is a sublattice of L.

= False

Under some R relation

EP:

are
like

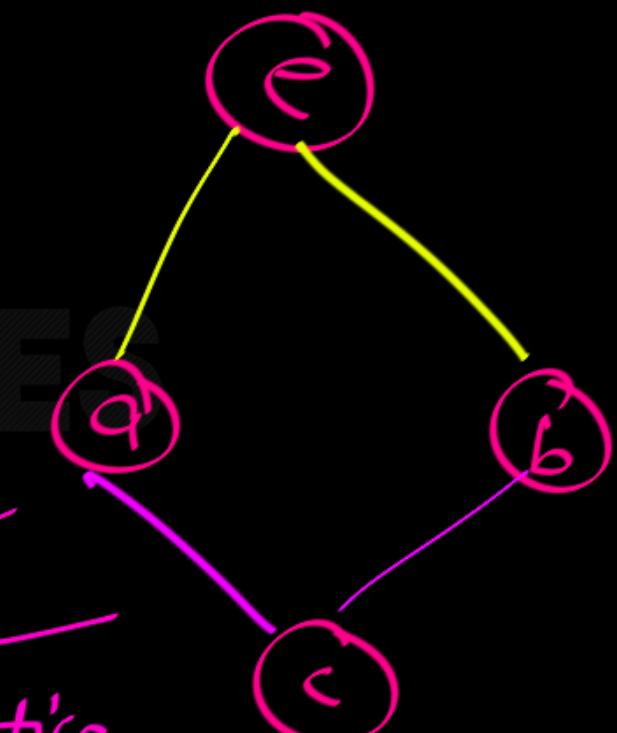


Lattice (L, R)

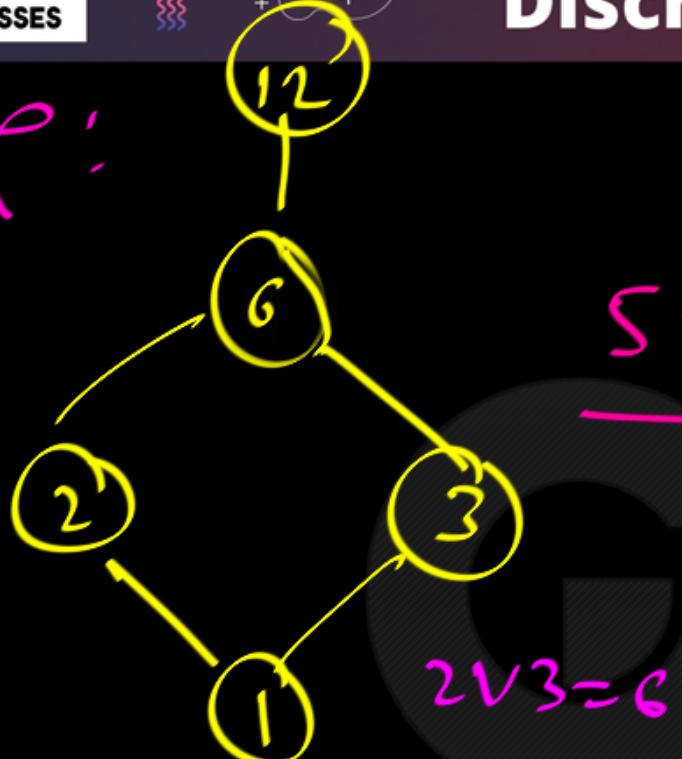
$L = \{a, b, c, d, e\}$ (under R)

$S = \{a, b, c, e\}$

Lattice (S, R)
Not sublattice
of L



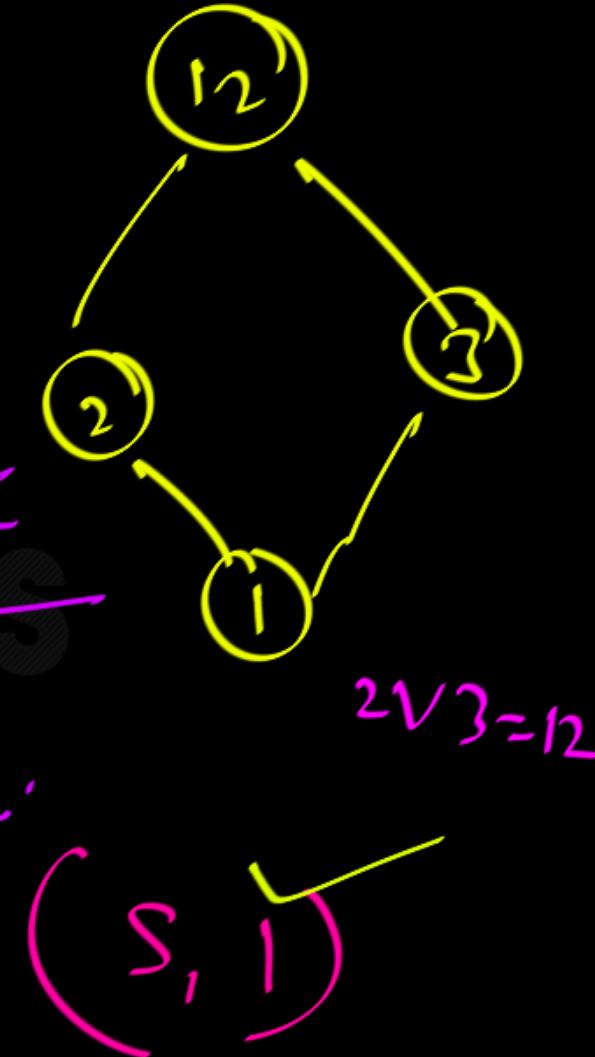
Ex:



$$S = \{1, 2, 3, n\}$$

Lattice (L, \leq)
Base set \sqcup Relation

lattice
But not
sublattice of L





$(\{1, 2, 3, 12\}, |)$ — Lattice S

$$2 \vee 3 = 12$$

$(L, |) = (\{1, 2, 3, 4, 12\}, |)$ — Lattice L

$$2 \vee 3 = 4$$

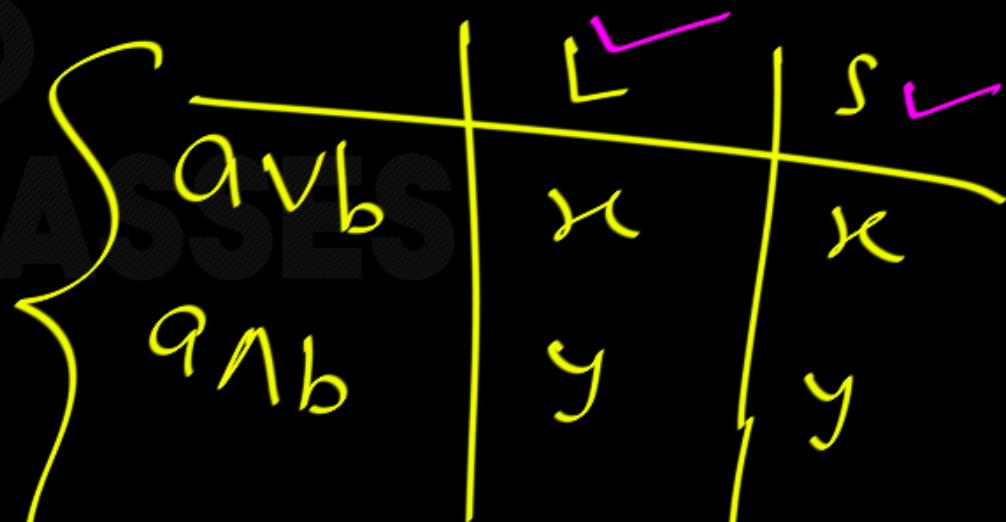
so S is
not sublattice
of L.

Lattice L (under relation R) then

$S \subseteq L$, S is Sublattice of L (R under)

iff

$\forall a, b \in S$





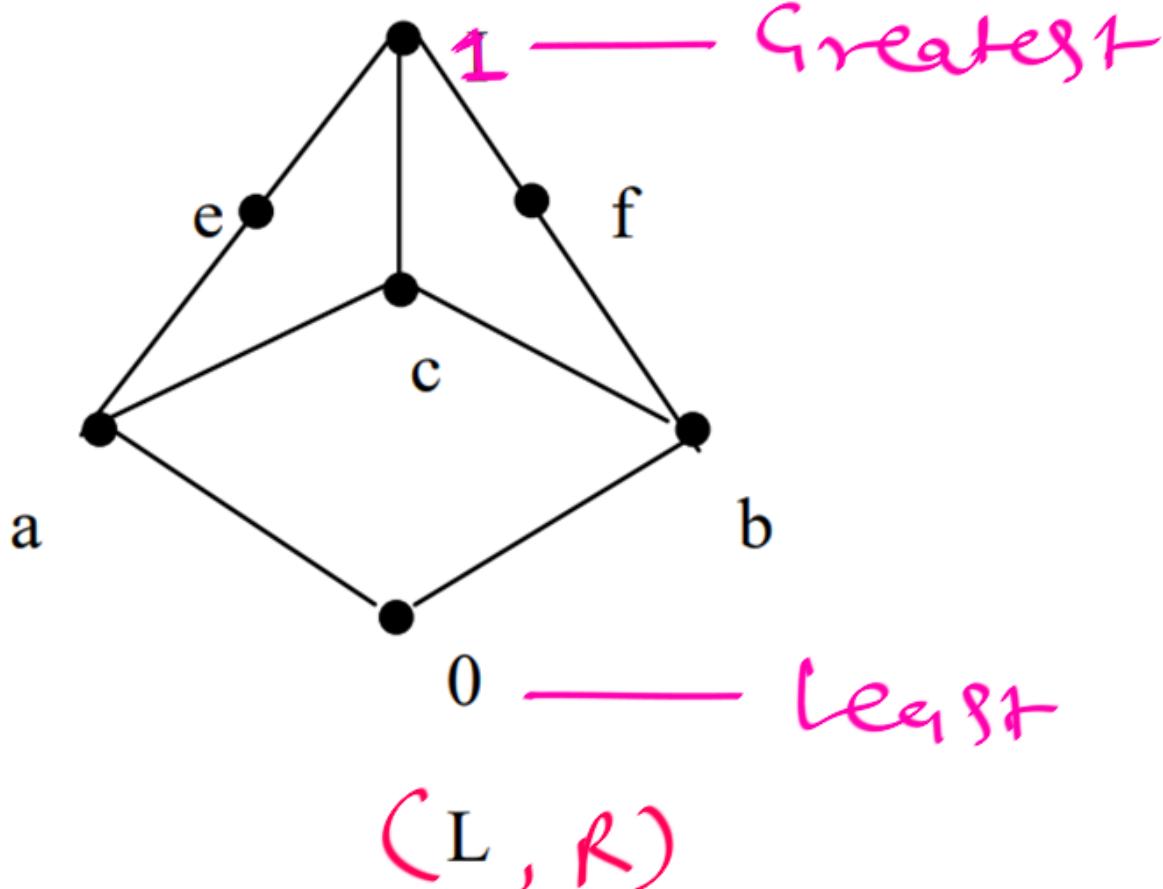
So, Sublattice itself is a lattice. ✓

However, a subset of L which is a lattice need not be a sublattice.





For example, in the diagram:



$|R|=?$

Q1: Is L a lattice?

$$a \vee b = c$$

$$a \wedge b = o$$

$$a \vee f = 1$$

$$a \wedge f = o$$

$$b \vee e = 1$$

$$b \wedge e = o$$

$$c \vee e = 1$$

$$c \wedge e = a$$

$$c \vee f = 1$$

$$c \wedge f = b$$

$$e \vee f = 1$$

$$e \wedge f = o$$

$|R| =$

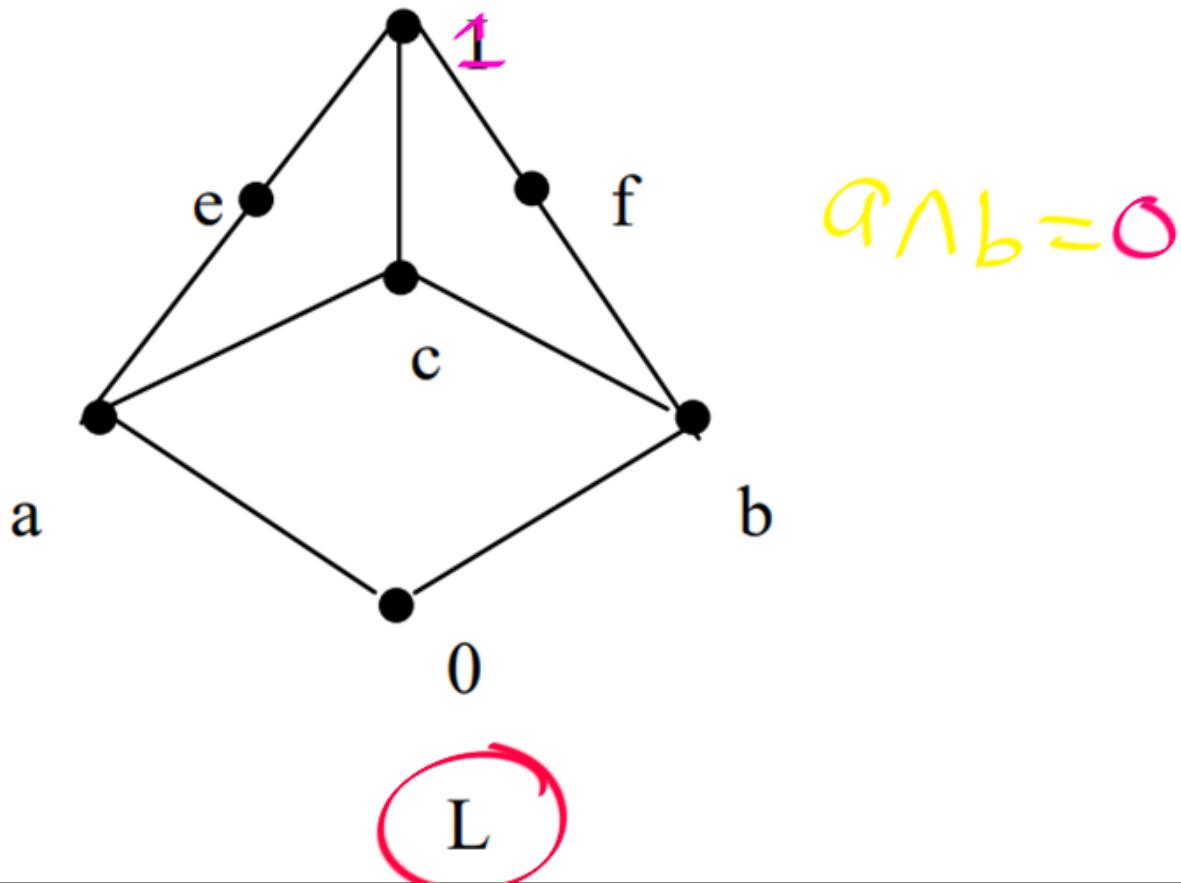
$$\begin{array}{cccc} (0, \kappa) & (a, \kappa) & (b, \kappa) & (c, \kappa) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & + & 4 & f \\ \end{array}$$

$$\begin{array}{ccc} (e, \kappa) & (f, \kappa) & (s, \kappa) \\ \downarrow & \downarrow & \downarrow \\ 2 & + & 2 & + & 1 \\ \end{array}$$

$$|R| = 22$$



For example, consider the lattice shown in the diagram:



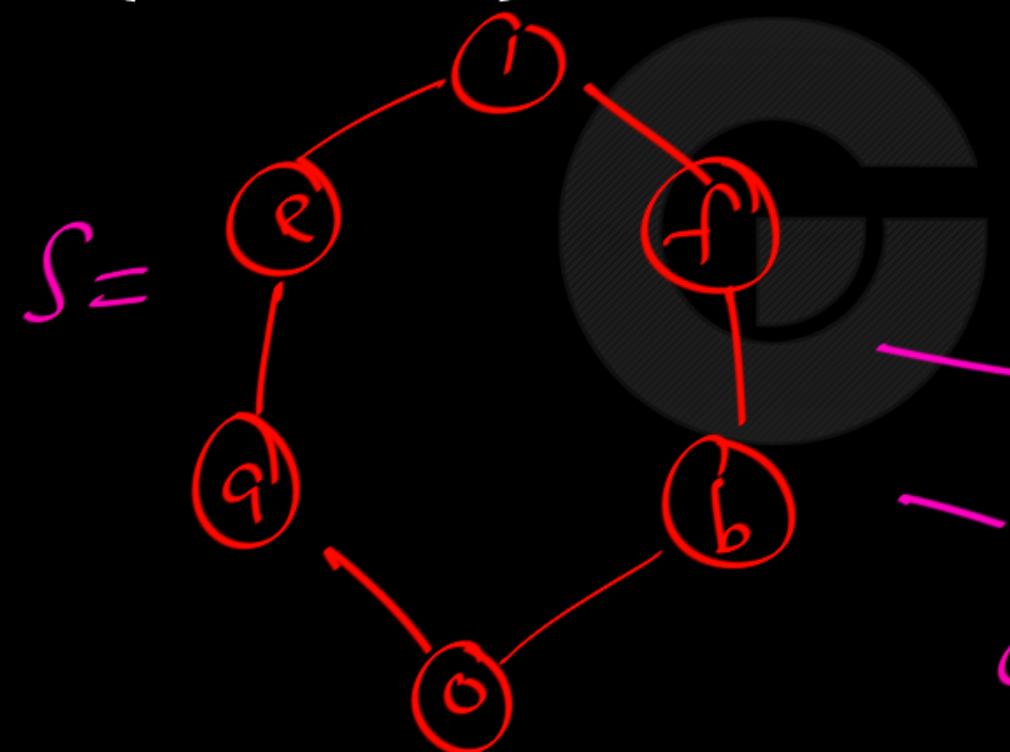


Q:

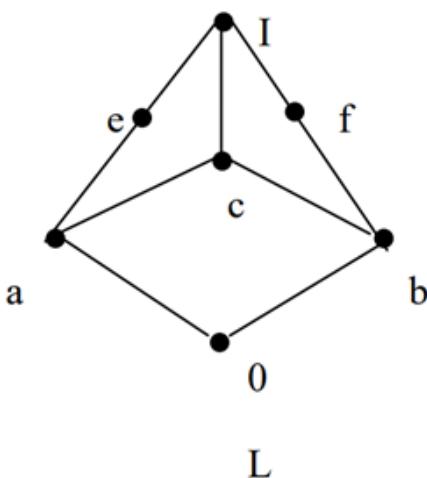
1. $\{a, e, f, b, 1\}$ is a sublattice of L ? (Under Relation is same as L)
-
- ```
graph TD; 1((1)) --- e((e)); 1 --- f((f)); a((a)) --- e; f --- b((b));
```
- $a \wedge b = DNE$
- not even a lattice
- not a sublattice of  $L$



Q:

2.  $\{a, e, f, b, 1, 0\}$  is a sublattice?

For example, consider the lattice shown in the diagram:



$$a \vee b = c$$

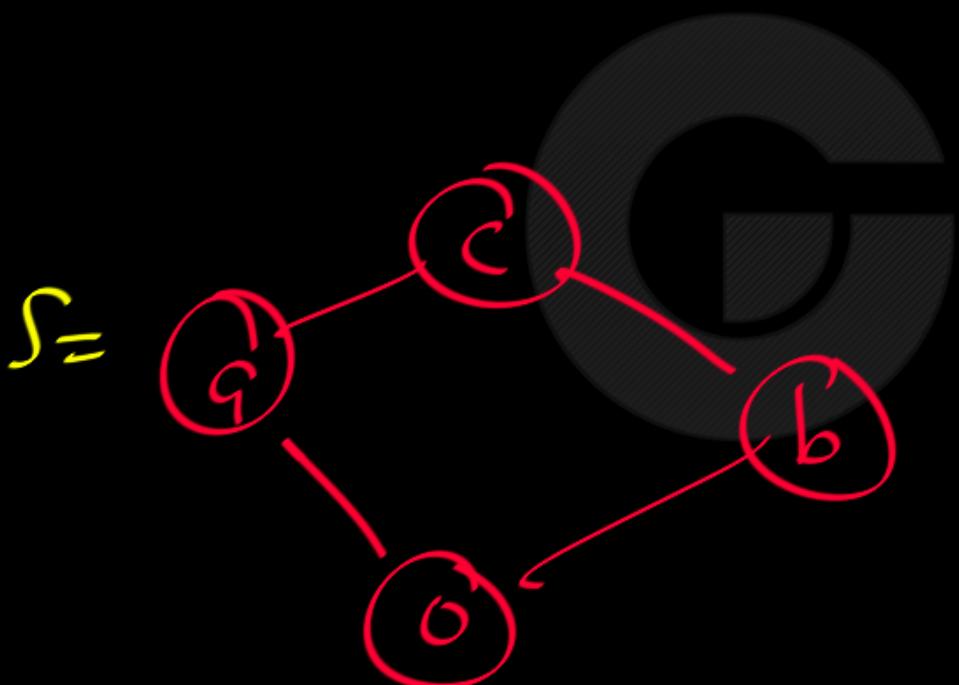
lattice  $L$

not sublattice of  $L$

$a \vee b$  in  $S = 1$

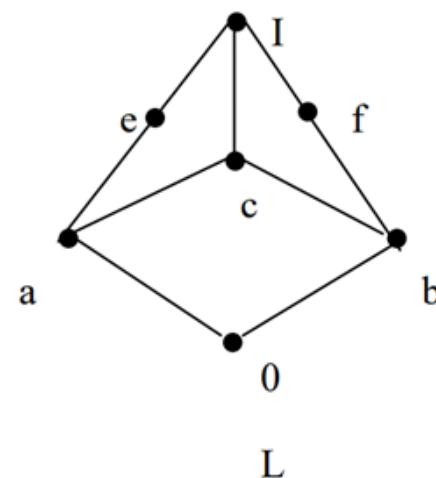


Q:

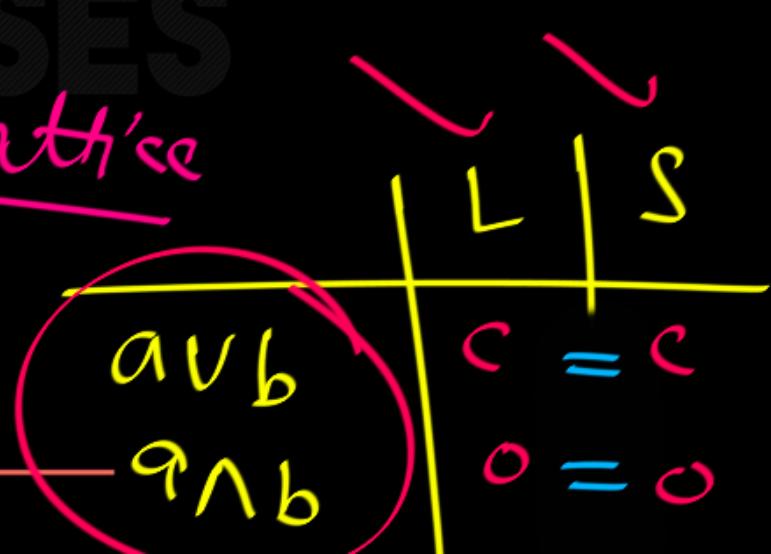
3.  $\{a, c, b, o\}$  is a sublattice?

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For example, consider the lattice shown in the diagram:

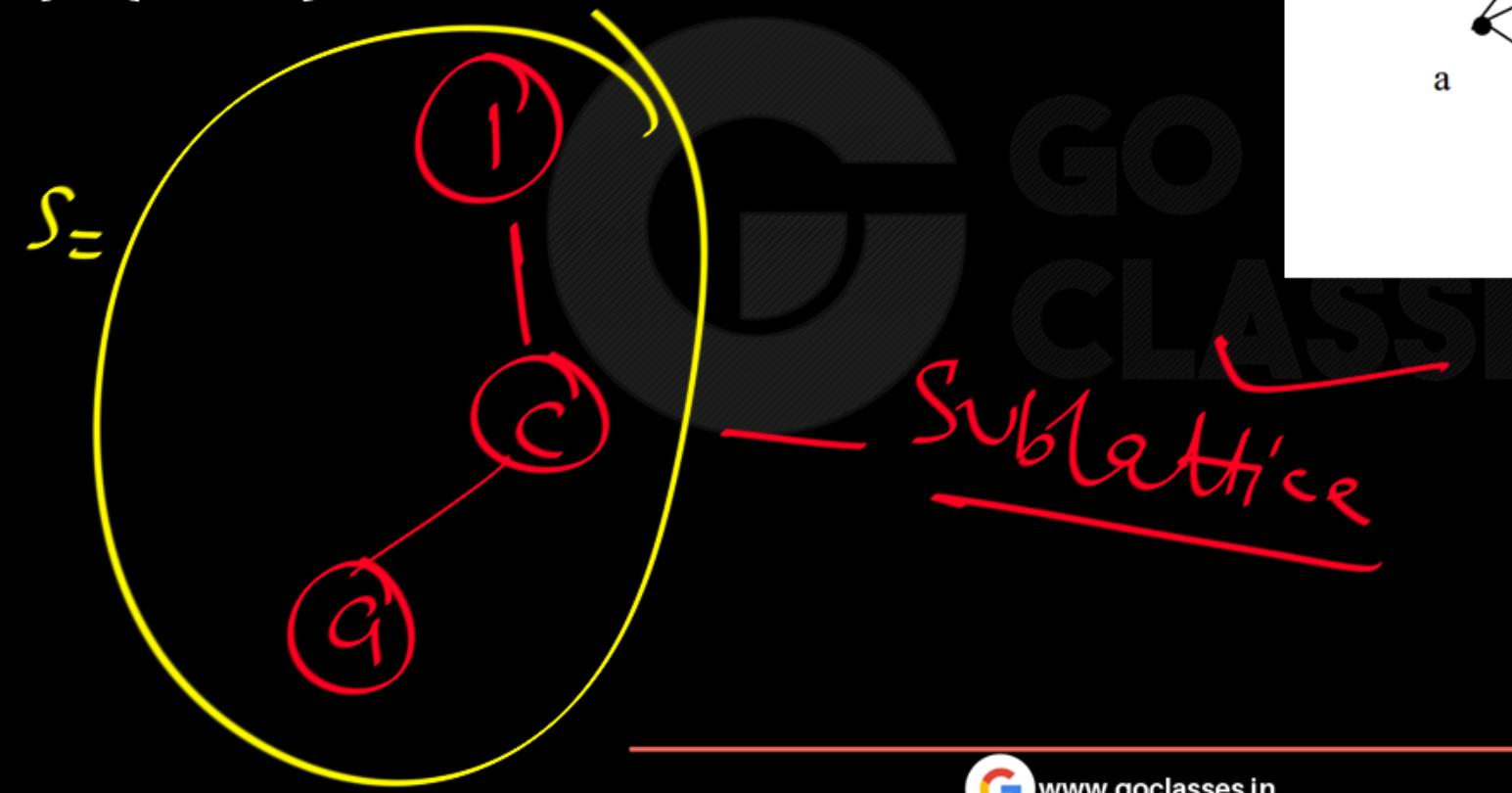


Sublattice

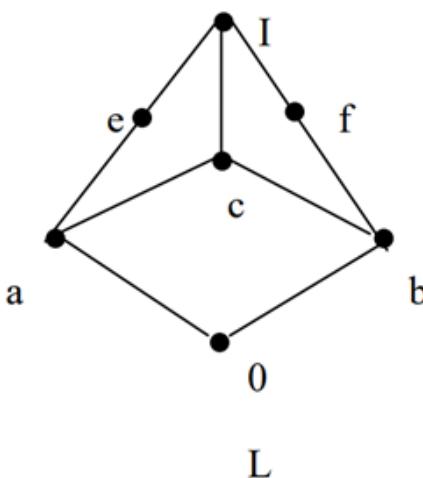




Q:

4.  $\{a, c, 1\}$  is a sublattice?

For example, consider the lattice shown in the diagram:





Q:

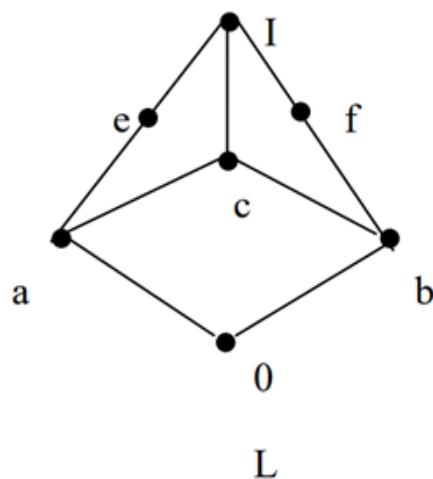
5.  $\{e, f, c\}$  is a sublattice?

S:

$e$      $c$      $f$

$e \vee c = f$      $e \vee c = \text{DNE}$     *not even lattice*  
*not sublattice of L,*

For example, consider the lattice shown in the diagram:



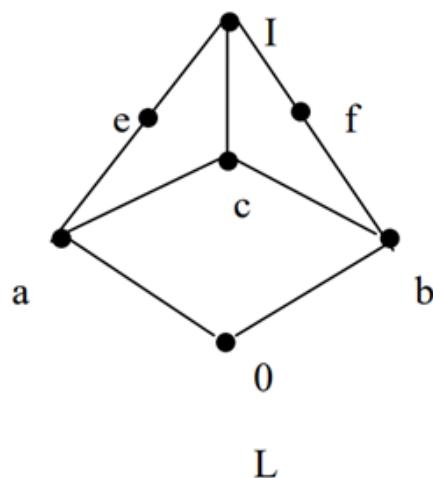
$$e \vee c = 1$$



Q:

6.  $\{0, a, c, 1\}$  is a sublattice?Sublattice

For example, consider the lattice shown in the diagram:





Note: let  $L$  be a lattice and  $S \subseteq L$ .

To check if  $S$  is sublattice of  $L$ : Just check  $\vee, \wedge$  of Non-Comparable Pairs of  $S$ .