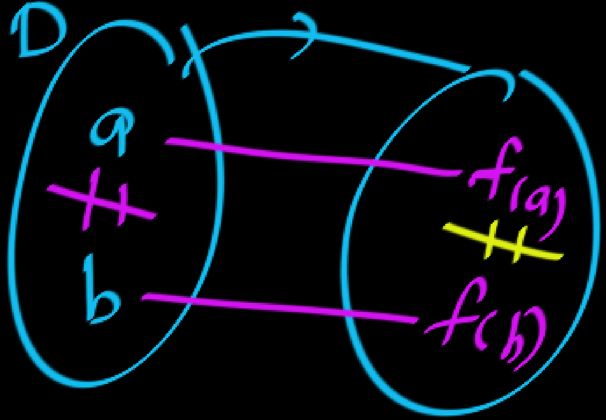
Injections and Surjections

- An injective function associates **at most** one element of the domain with each element of the codomain.
- A surjective function associates **at least** one element of the domain with each element of the codomain.
- What about functions that associate **exactly one** element of the domain with each element of the codomain?

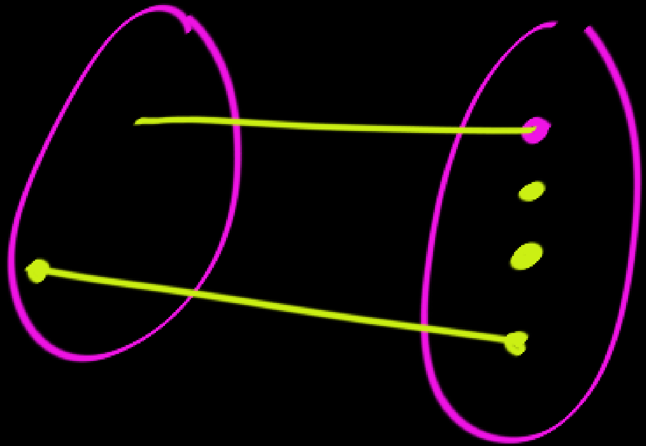
Injective :

from Domain point of view



If $a \neq b$ then $f(a) \neq f(b)$

from Co-Domain point of view



Bijections

- A function that associates each element of the codomain with a unique element of the domain is called **bijective**.
  - Such a function is a **bijection**.
- Formally, a bijection is a function that is both **injective** and **surjective**.
- A bijection is a one-to-one correspondence between two sets.

Question:

Every relations are functions, but some functions are not relations.

True False

Relation → function — false

Converse: function → Relation — True

Inverse: Not Relation → Not function — True

$$\left.\begin{aligned} p \rightarrow q &\equiv \neg q \rightarrow \neg p \\ q \rightarrow p &\equiv \neg p \rightarrow \neg q \end{aligned}\right\}$$

Conditional statement $\equiv$ Contrapositive

Converse of $p \rightarrow q$ $\equiv$ Inverse of $p \rightarrow q$

Converse of $p \rightarrow q$ is Contrapositive of Inverse.

Let $f(n) = 2$ for $n \leq 2$ and $f(n) = f(n-1) + f(n-2) + 1$. Compute the values of $f(n)$ for $n \leq 6$.

Let $f(n) = 1$ for $n \leq 2$ and $f(n) = 2f(n-1) - 1$. Compute the values of $f(n)$ for $n \leq 5$.

Let $\boxed{f(n) = 2 \text{ for } n \leq 2}$ and $f(n) = f(n-1) + f(n-2) + 1$. Compute the values of $f(n)$ for $n \leq 6$.

$f_1 = 2$; $f_2 = 2$; $f_3 = f_2 + f_1 + 1 = 5$; $f_4 = f_3 + f_2 + 1 = 8$;
$f_5 = 14$; $f_6 = 23$

Let $f(n) = 1$ for $n \leq 2$ and $f(n) = 2f(n-1) - 1$. Compute the values of $f(n)$ for $n \leq 5$. — $f(n) = 1$; $\forall n \geq 1$

$f(n) = 1$ when $n \leq 2$
$f(1) = 1$; $f(2) = 1$

$f_3 = 2f_2 - 1 = 2(1) - 1$
$= 1$

$f_4 = 2f_3 - 1 = 1$; $f_5 = 2f_4 - 1$
$= 1$

Suppose that $f : A \to B$.

To show that f is injective Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$.

To show that f is not injective Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

To show that f is surjective Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

To show that f is not surjective Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$.

$f : A \to B$

① To prove f is Injective:

$$f(a) = f(b) \longrightarrow \boxed{a = b}$$

② To prove f is NOT Injective:

Some a, b ; $a \neq b$ but $f(a) = f(b)$

(C) To prove f is surjective :

Arbitrary $\boxed{y \in \text{Co-Domain}}$

then find $x \in$ Domain such that

$f(x) = y$

(D) To prove f is Not onto :

find some $y \in$ Co-Domain ; y Doesn't have Pre-image.

In each of the following, assume that $f : Z \to Z$. Then identify whether each is a function, onto function, one-to-one function, bijection.

18. $f(x) = x^2 - 1$

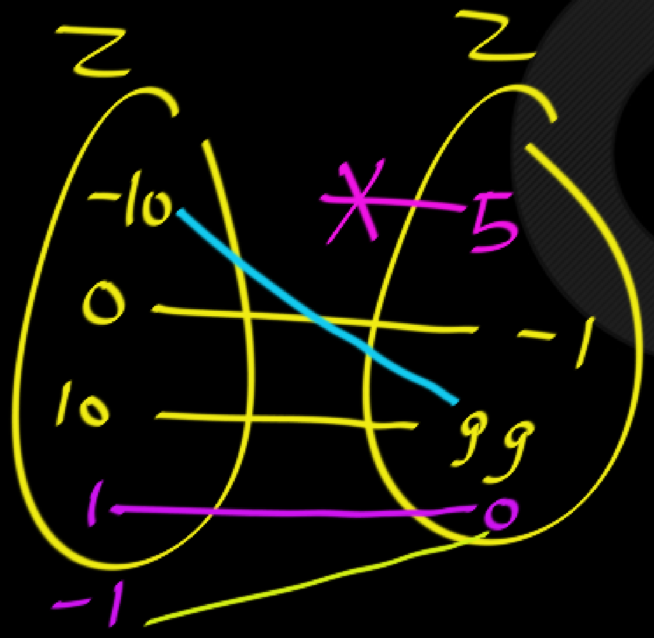
19. $f(x) = x^2 + 1$

20. $f(x) = \sqrt{x}$

$f : \mathbb{Z} \to \mathbb{Z}$

Pre-image of $8 = \{-3, 3\}$

① $f(x) = x^2 - 1$



① function? ✓

$x \in \mathbb{Z}$; $\underline{x^2 - 1} \in \mathbb{Z}$

② one-one? ✗

$f(1) = f(-1) = 0$

③ onto: There is no $x \in \mathbb{Z}$ such that $f(x) = 5$

No

NOT bijection

$f: Z \to Z$

② $f(x) = x^2 + 1$

NOT bijection

$f(1) = f(-1) = 2$

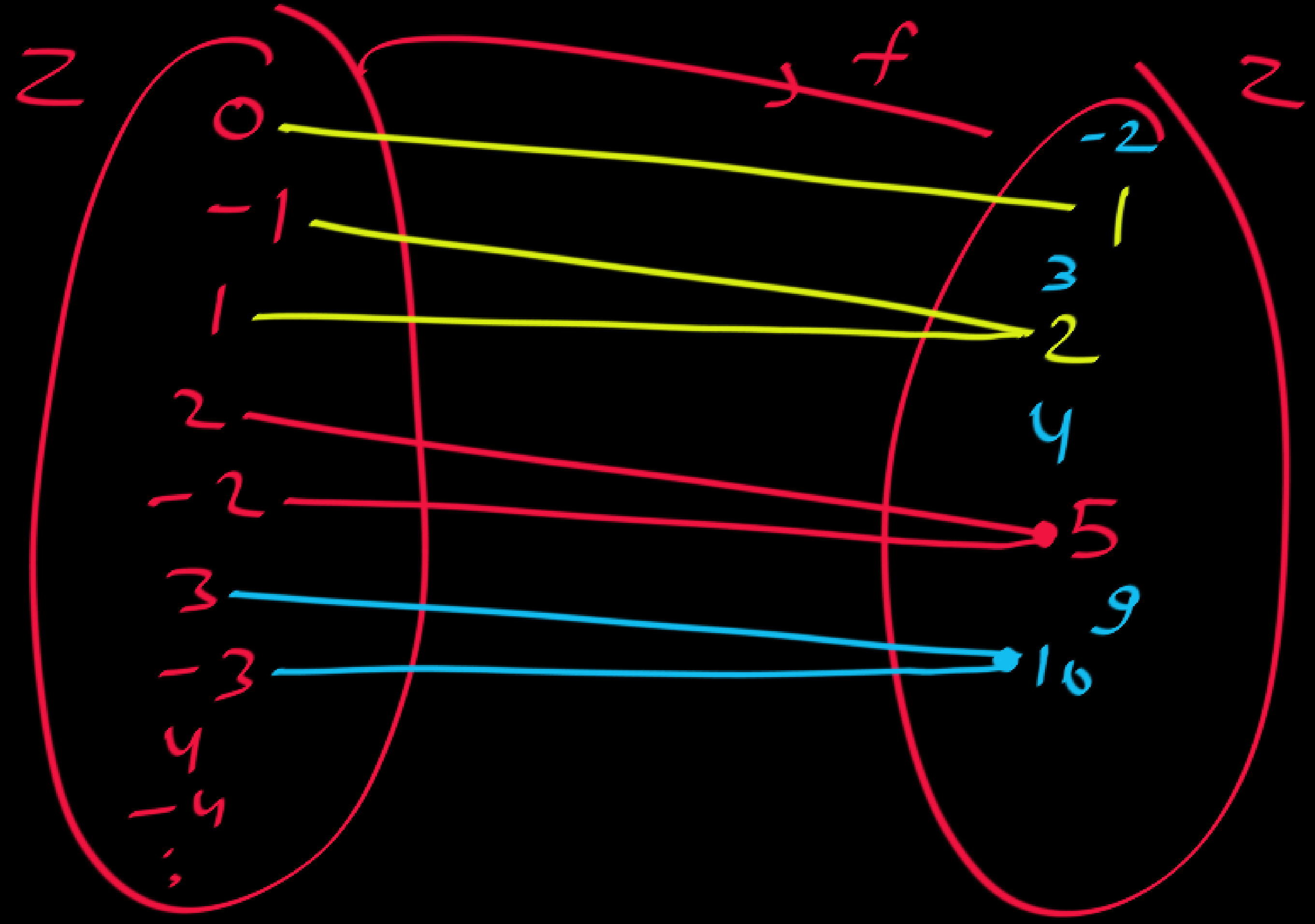
One-one ✗

Not onto ✗

No $x \in Z$

$f(x) = 11$

$f: Z \to Z \;;\; f(x) = x^2 + 1$



No pre-image of

-4, 4,

9, 6, - -

- - .

$f : \mathbb{Z} \to \mathbb{Z}$

③ $f(x) = \sqrt{x}$ Not even a function

No image of -1.

$f(-1) = \sqrt{-1} \notin$ Co-Domain

not integer

$f : \mathbb{N} \to \mathbb{Z}$

$f(x) = \sqrt{x}$ — function ✗ $\quad f(2) = \sqrt{2} \notin \mathbb{Z}$

$f(25) = \sqrt{25} = \cancel{\pm 5}$

$\sqrt{25} = 5$ ✓ $\qquad \sqrt{25} \neq \pm 5$

Note:

$x^2 = 25$

then $x = \pm 5$

$x^2 = a^2$

then $x = \pm a$

$x = \sqrt{25}$

$x = 5$ ✓

$x = \pm 5$ ✗

$x = \sqrt{a^2} \Rightarrow x = |a|$

$$x = \sqrt{(-5)^2} \Rightarrow x = 5$$

$$x = \sqrt{(-5)^2} = \sqrt{25} = 5$$

$f : N \rightarrow R$

$f(x) = \sqrt{x}$ —— function ✓

① one-one ✓

② onto ✗ No Pre-image of $-5, 0, 5, \ldots$

In each of the following, assume that $f : Z \to Z$. Then identify whether each is a function, onto function, one-to-one function, bijection.

22. $f(x) = 5$

23. $f(x) = 2^x$

24. $f(x) = (i)\frac{x}{2}$ if x is even; $(ii) 2x - 1$ if x is odd

$f : Z \to Z$

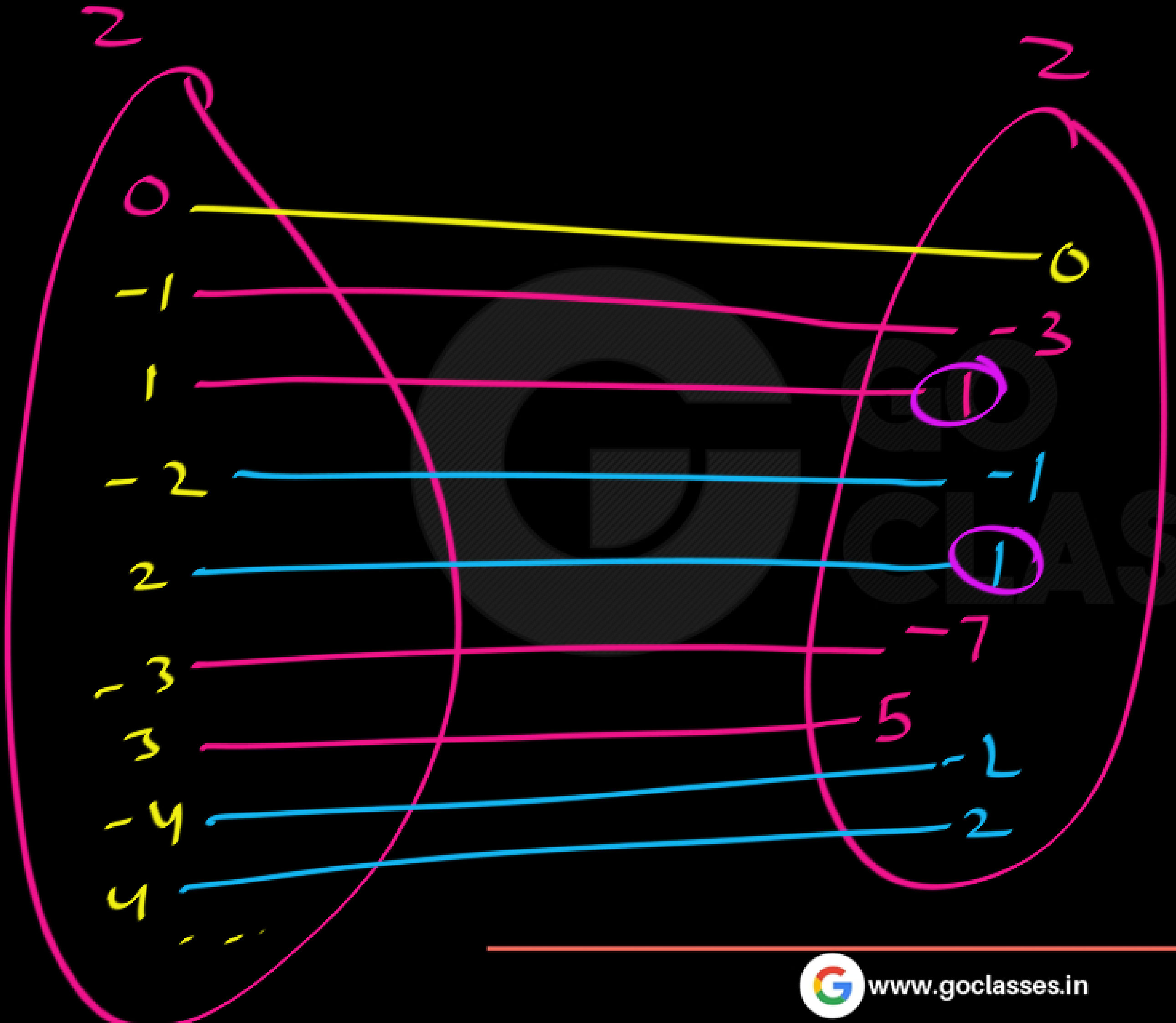
(iii) $f(x) = \begin{cases} x/2 & \boxed{x = \text{even}} \\ 2x - 1 & x = \underline{\text{odd}} \end{cases}$

$f(-1) = 2(-1) - 1 = -3$; $f(1) = 1$

$f : Z \to Z$

(iii) $f(x) = \begin{cases} x/2 & \boxed{x = \text{even}} \\ 2x - 1 & x = \text{odd} \end{cases}$

$$f(\overset{\text{even}}{-10}) = \frac{-10}{2} = -5 \; ; \; f(\overset{\text{odd}}{-15}) = 2(-15) - 1 = -31$$

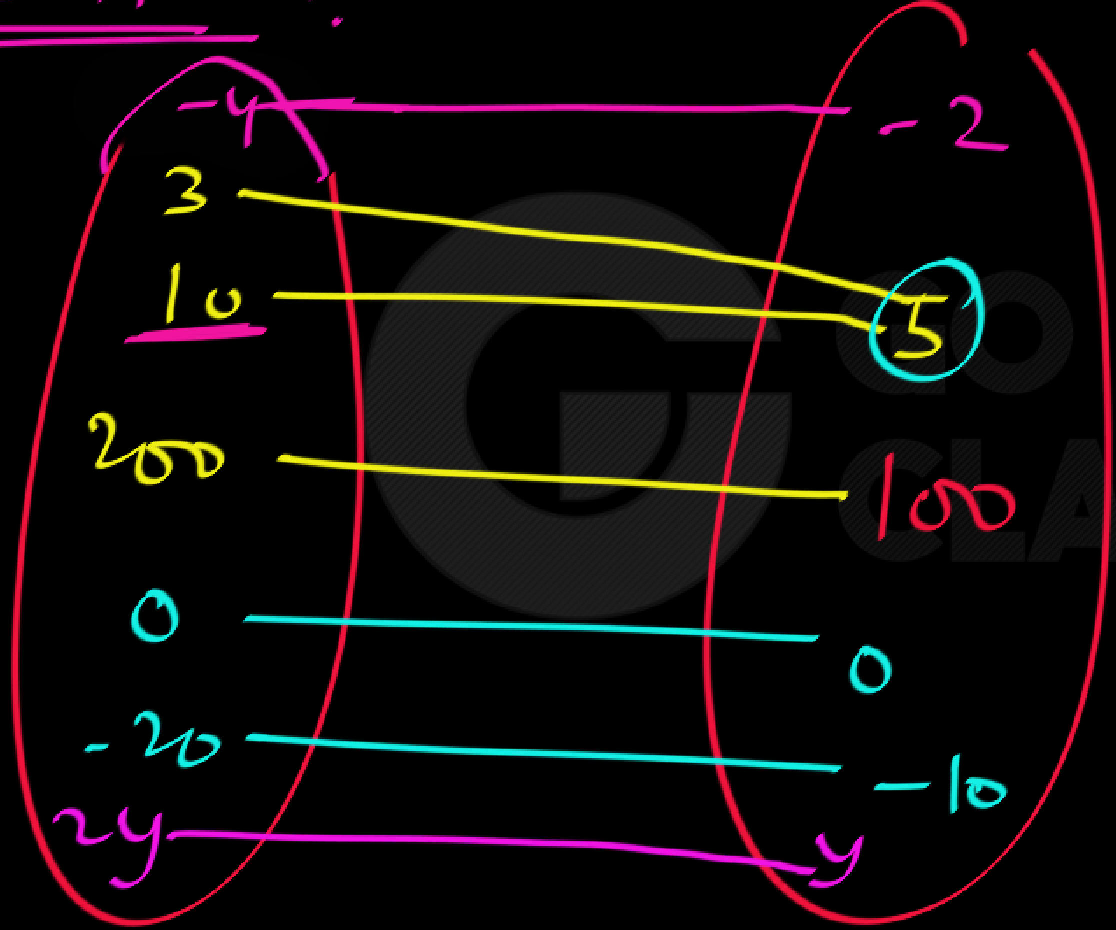


$f(1) = 1$
$f(2) = 1$

$f(3) = 5$
$f(10) = 5$

Not one-one.

Onto?



for all $y \in$ Co-Domain

at least one Pre-image is

2y.

$f : Z \to Z$

(1) $f(x) = 5$ — function

$f(0) = 5$; $f(-2) = 5$; $f(10) = 5$

one-one ✗

onto ✗

for which Co-Domain, onto ?

$\{5\}$

$f : Z \rightarrow Z$

$f(x) = 2^{x}$ — function ✗

$f(-1) = 2^{-1} = \frac{1}{2} = 0.5 \notin$ Co-Domain

$f: R \to R$

$f(x) = 2^x$ — function ✓

one-one? Yes. ✓

Proof: Assume $f(x) = f(y)$

$2^x = 2^y \Rightarrow x = y$

$f: R \rightarrow R$

$f(x) = 2^x$

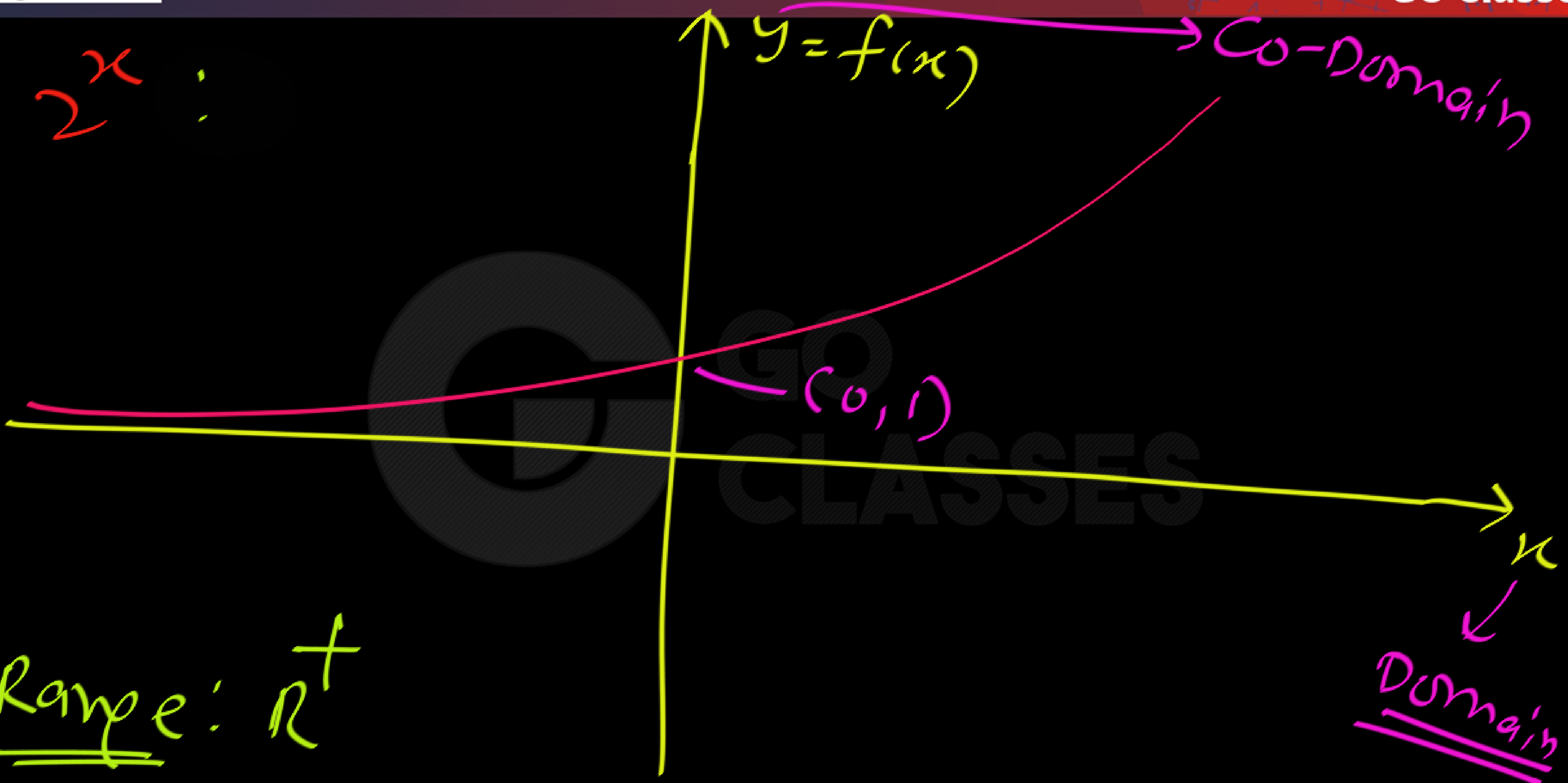
Onto? ✗

Range: R^+

No pre-image of 0, −1, −0.5, - - - -

↳ Non-positive numbers.

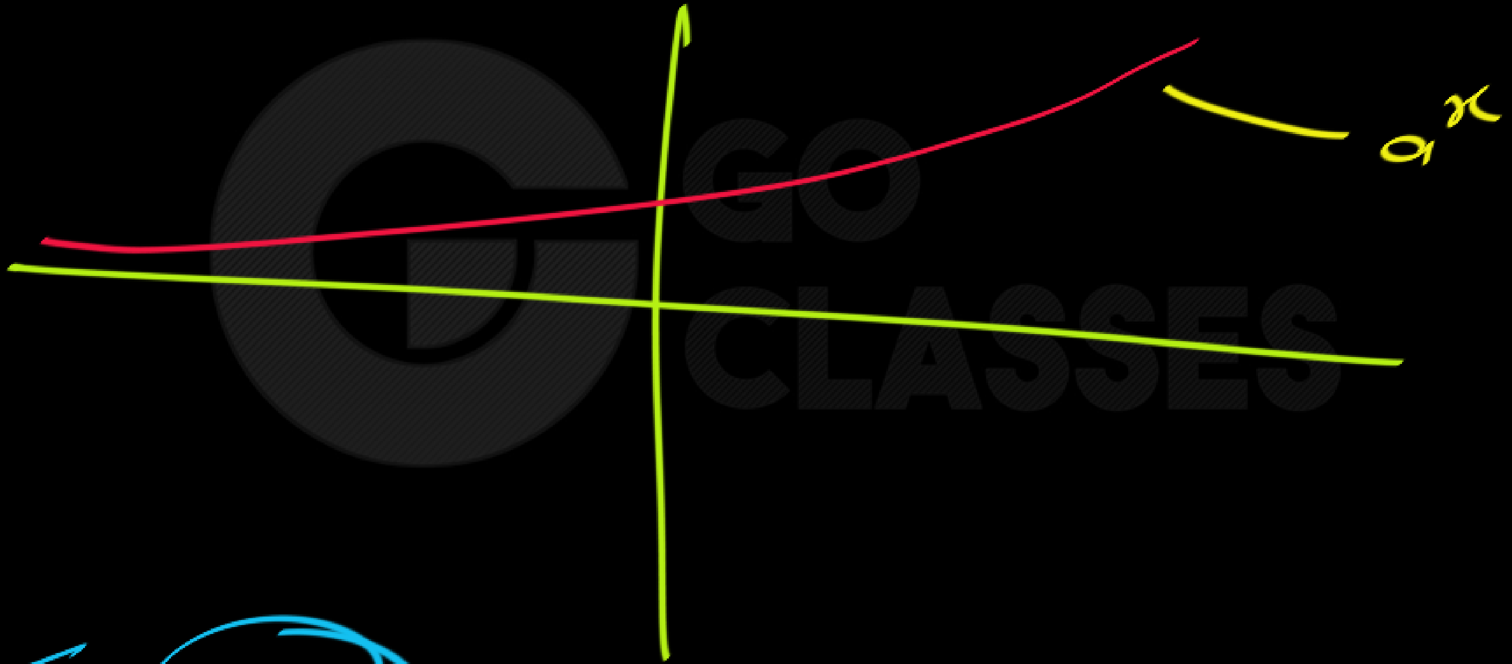
2^x :



Range: R^+

$a > 1$

a^x :



e^x ✓ $(e > 1)$

$f : R \rightarrow R^+$

$f(x) = 2^x ; e^x ; 5^x ; (5.5)^x \ldots$

$= a^x , a > 1$

onto ✓ ; one-one ✓ ; bijection

Set Theory

Next Topic: Combining Relations

Operations on Relations

Website : https://www.goclasses.in/

Combining Relations

Because relations from A to B are subsets of A × B, two relations from A to B can be combined in any way two sets can be combined.

$R : A \longrightarrow B$

$R \subseteq A \times B$ → a set. → can apply set operations

Relation $\longrightarrow$ set

On Relations, set operations can be applied.

$\cup$

$\cap$

$\oplus$

$-$

$\times$

EXAMPLE 17 Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. The relations $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ and $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$ can be combined to obtain

$$R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\},$$
$$R_1 \cap R_2 = \{(1, 1)\},$$
$$R_1 - R_2 = \{(2, 2), (3, 3)\},$$
$$R_2 - R_1 = \{(1, 2), (1, 3), (1, 4)\}.$$

$R_1 \oplus R_2 = R_1 \Delta R_2 = (R_1 - R_2) \cup (R_2 - R_1)$

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$

EXAMPLE 18 Let A and B be the set of all students and the set of all courses at a school, respectively. Suppose that R_1 consists of all ordered pairs (a, b), where a is a student who has taken course b, and R_2 consists of all ordered pairs (a, b), where a is a student who requires course b to graduate. What are the relations $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \oplus R_2$, $R_1 - R_2$, and $R_2 - R_1$?

Solution: The relation $R_1 \cup R_2$ consists of all ordered pairs (a, b), where a is a student who either has taken course b or needs course b to graduate, and $R_1 \cap R_2$ is the set of all ordered pairs (a, b), where a is a student who has taken course b and needs this course to graduate. Also, $R_1 \oplus R_2$ consists of all ordered pairs (a, b), where student a has taken course b but does not need it to graduate or needs course b to graduate but has not taken it. $R_1 - R_2$ is the set of ordered pairs (a, b), where a has taken course b but does not need it to graduate; that is, b is an elective course that a has taken. $R_2 - R_1$ is the set of all ordered pairs (a, b), where b is a course that a needs to graduate but has not taken. ◀

A = Set of Students

B = " " Courses

$R_1 = \{(a, b) \mid$ Student a has taken course b $\}$

$R_1 : A \rightarrow B$

$R_2 = \{(x, y) \mid$ Student x needs course y to pass $\}$

$R_2 : A \rightarrow B$

$$R_1 \cup R_2 = \{ (a,b) \mid \text{student } a \text{ has taken course } b \text{ OR needs } b \}$$

$$R_1 \cap R_2 = \{ (a,b) \mid \text{student } a \text{ has taken course } b \text{ and needs } b \text{ to pass} \}$$

$R_1 \oplus R_2 = \{ (a,b) \mid$ Student a has taken Course b but not needs b OR needs b but not taken $\}$

$R_1 - R_2 = \{ (a,b) \mid$ Student a has taken Course b But not needs b to pass $\}$

taken but not needed

$$R_2 - R_1 = \{(a, b) \mid \text{Studen a needs b but not taken b}\}$$

$$\underline{R_1 \times R_2} = \{((\underline{a, b}), (\underline{x, y})) \mid \text{Stud a takes b, Stud x needs y}\}$$

EXAMPLE 19 Let R_1 be the "less than" relation on the set of real numbers and let R_2 be the "greater than" relation on the set of real numbers, that is, $R_1 = \{(x, y) \mid x < y\}$ and $R_2 = \{(x, y) \mid x > y\}$. What are $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, $R_2 - R_1$, and $R_1 \oplus R_2$?

$R_1 \cup R_2 = \{(x,y) \mid x \neq y\} = \{(x,y) \mid x<y \text{ OR } x>y\}$

$R_1 \cap R_2 = \{\ \}$ R_1, R_2 are Disjoint

$R_1 \oplus R_2 = R_1 \cup R_2 = \{(x,y) \mid x \neq y\}$

$R_1 - R_2 = R_1$; $R_2 - R_1 = R_2$

$$R_1 \times R_2 = \left\{ \big((a,b), (x,y) \big) \;\middle|\; a < b,\ x > y \right\}$$

EXAMPLE 19 Let R_1 be the "less than" relation on the set of real numbers and let R_2 be the "greater than" relation on the set of real numbers, that is, $R_1 = \{(x, y) \mid x < y\}$ and $R_2 = \{(x, y) \mid x > y\}$. What are $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, $R_2 - R_1$, and $R_1 \oplus R_2$?

Solution: We note that $(x, y) \in R_1 \cup R_2$ if and only if $(x, y) \in R_1$ or $(x, y) \in R_2$. Hence, $(x, y) \in R_1 \cup R_2$ if and only if $x < y$ or $x > y$. Because the condition $x < y$ or $x > y$ is the same as the condition $x \neq y$, it follows that $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$. In other words, the union of the "less than" relation and the "greater than" relation is the "not equals" relation.

Next, note that it is impossible for a pair (x, y) to belong to both R_1 and R_2 because it is impossible that $x < y$ and $x > y$. It follows that $R_1 \cap R_2 = \emptyset$. We also see that $R_1 - R_2 = R_1$, $R_2 - R_1 = R_2$, and $R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2 = \{(x, y) \mid x \neq y\}$. ◀

Relation $\longrightarrow$ Set

$\not\longleftarrow$

Special type of sets ✓

So we have special operations only for Relations $\longrightarrow$ { Composition ✓, Inverse ✓

There is another way that relations are combined that is the composition of relations.

Set Theory

Next Topic: Composition of Relations

Composition of Functions

Website : https://www.goclasses.in/

Composition — Relation operation

Create new Relations

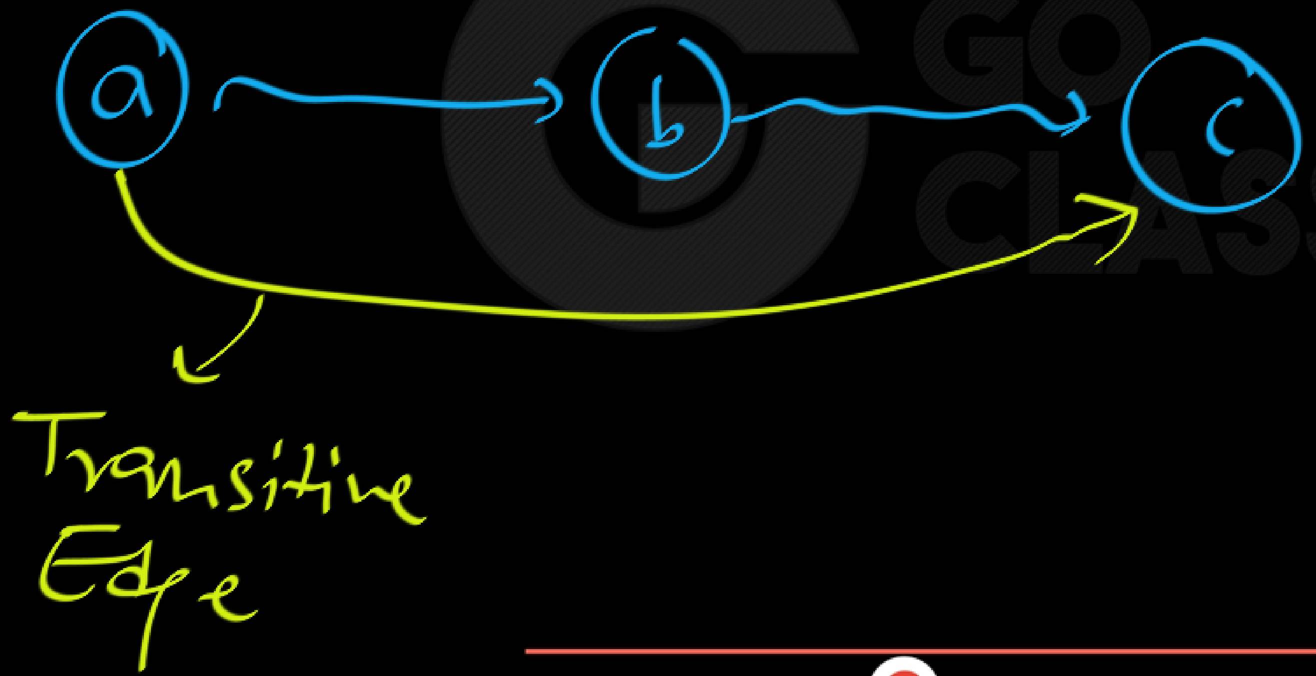
$R_1 = \{(1,1),(1,2)\}$

$R_2 = \{(2,3)\}$

Union — operation name

$\underbrace{R_1 \cup R_2}_{\text{symbol}} = S = \{(1,1),(1,2),(2,3)\}$ — new Relation

Composition : An operation that creates new Relation Transitively,



$$R_1 = \{ (1,1), (1,2) \}$$

$$\underbrace{1 \to 1}_{R_1} \underbrace{\to 4}_{R_2}$$

$$R_2 = \{ (2,3), (2,4), (1,4) \}$$

$$\underbrace{1 \to 2}_{R_1} \underbrace{\to 3}_{R_2}$$

$$R_2 \cdot R_1 = \{ (1,4), (1,3) \}$$

new Relation

Compositions


www.microsoft.com
www.apple.com
www.google.com
zune

Set Theory

Next Topic:

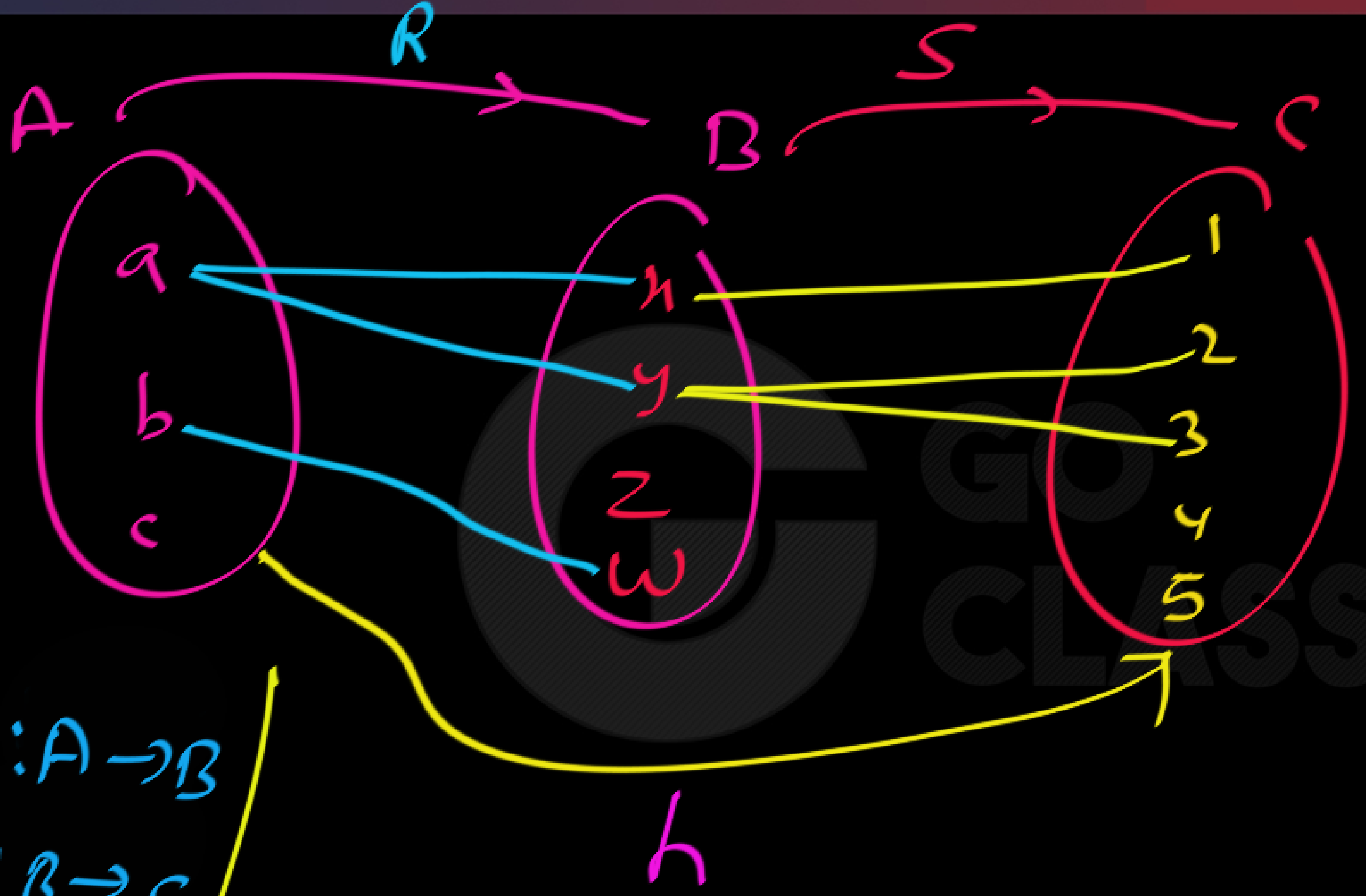
Composition of Relations

Website : https://www.goclasses.in/

1 Composition of Relations

In this section we will study what is meant by composition of relations and how it can be obtained.

Suppose that we have three sets A, B and C; a relation R defined from A to B, and a relation S defined from B to C. We can now define a new relation known as the *composition of* R *and* S, written as $S \circ R$. This new relation is defined as follows. If a is an element in A and c is an element in C, then $a(S \circ R)c$ if and only if there exists some element b in B, such that aRb and bSc. This means that we have a relation $S \circ R$ from a to c, if and only if we can reach from a to c in two steps; i.e. from a to b related by R and from b to c related by S. In this manner relation $S \circ R$ can be interpreted as R followed by S, since this is the order in which the two relations need to be considered, first R then S.



$R : A \to B$

$S : B \to C$

$h : A \to C$

$a \to 1$

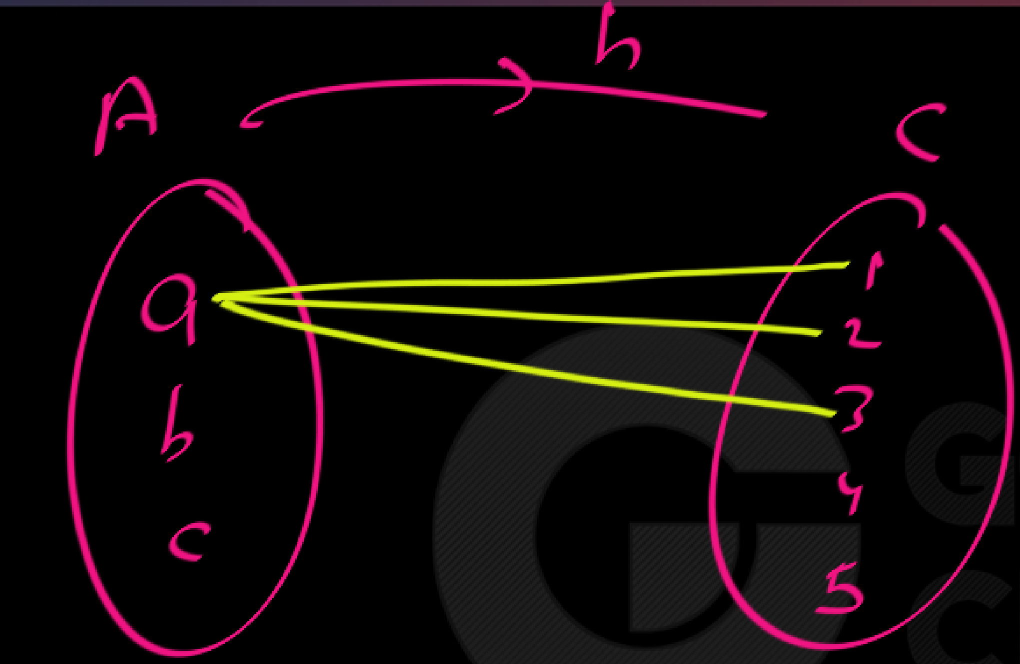
$a \to 2$

$a \to 3$

$b \cancel{\to}$

$c \cancel{\to}$

$h(a) = S(R(a))$



Composition of R and S

$$h = \{(a,1),(a,2),(a,3)\}$$

$$h(\alpha) = S(R(\alpha))$$

$$\boxed{h = S \circ R}$$

second first

† Example 1:

$R: A \to A$; $S: A \to A$

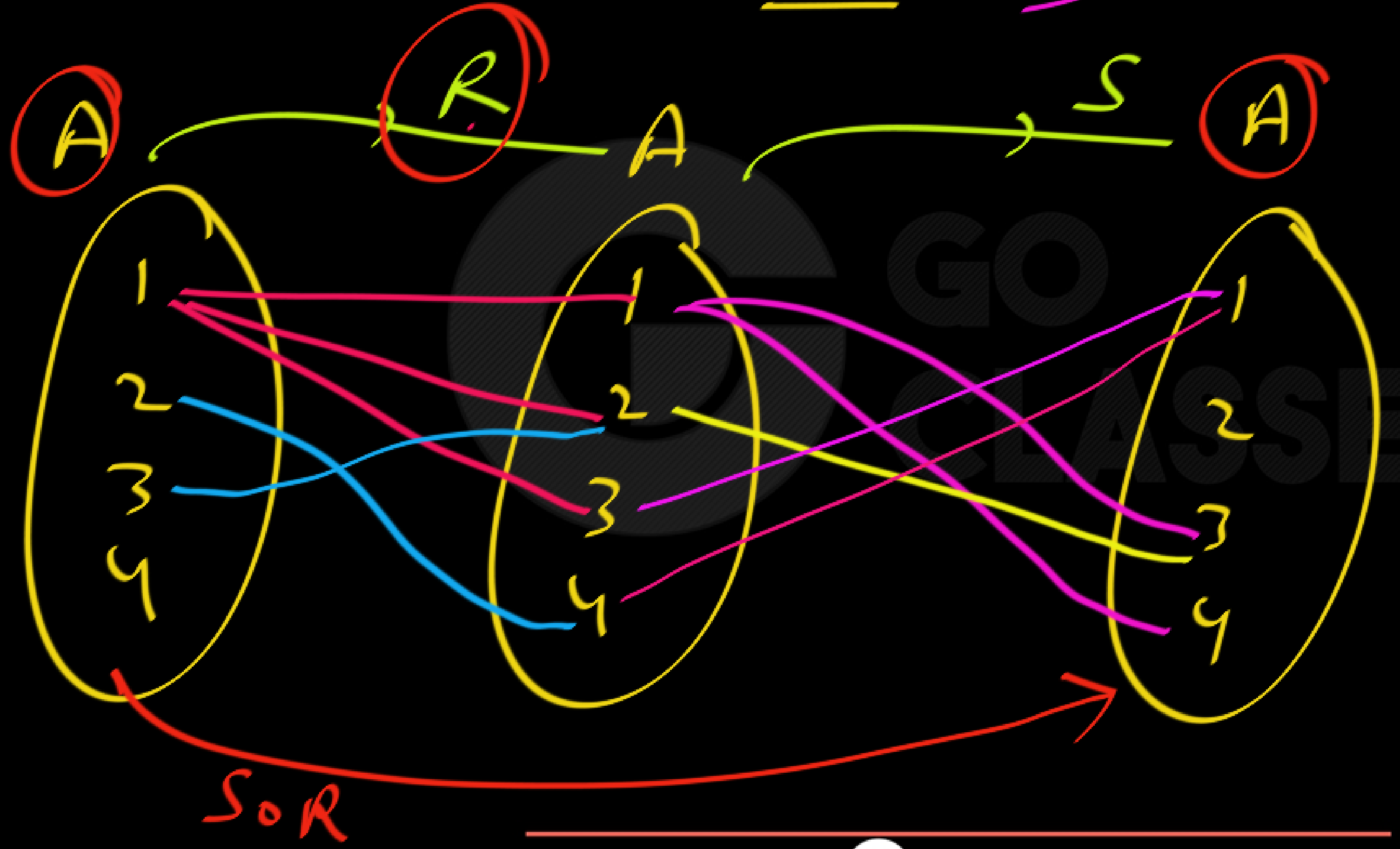
Let us try to understand this better through an example.
Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (1, 1), (1, 3), (2, 4), (3, 2)\}$,
and $S = \{(1, 4), (1, 3), (2, 3), (3, 1), (4, 1)\}$.
Find $S \circ R$.

$|S \circ R| = ? = 5$

$S \circ R = \{(1,3), (1,1), (1,4), (2,1), (3,3)\}$

$S \circ R = S(R)$

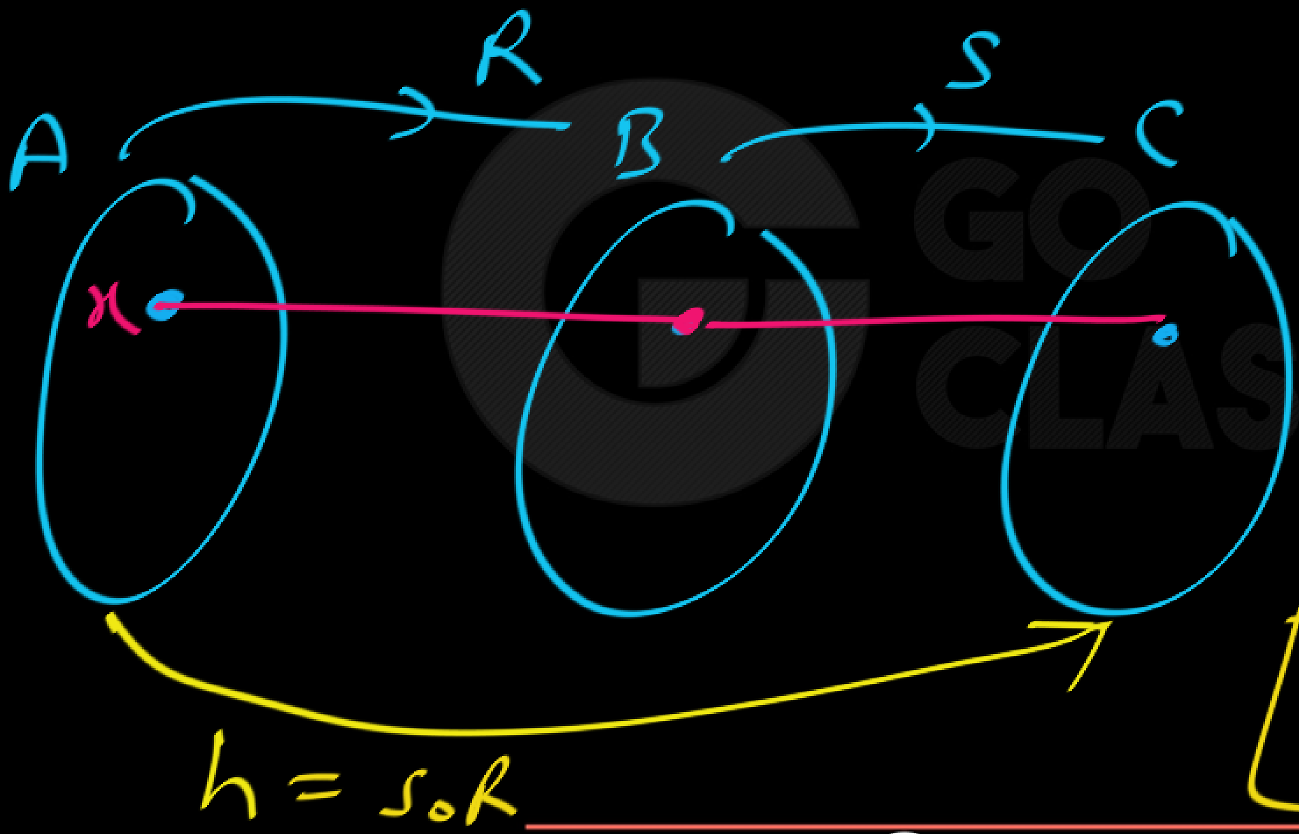
$S \circ R$



$S \circ R : A \to A$

$1 \to 3$
$1 \to 4$
$1 \to 1$
$2 \to 1$
$3 \to 3$

Composition of Relations: (Transitively Relate)



$R : A \to B$

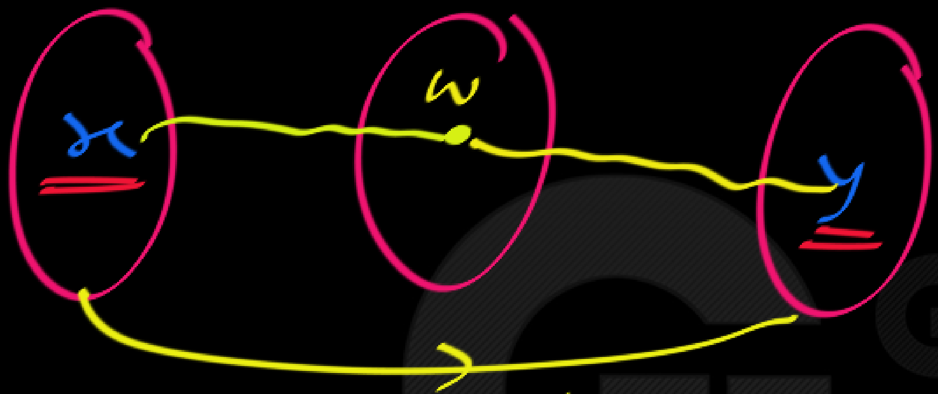
$S : B \to C$

$h : A \to C$

$h = S(R(x))$

$\boxed{h = S \circ R}$

$A \xrightarrow{R} B \xrightarrow{S} C$



$S \circ R = h$

$x \, (S \circ R) \, y$ iff

A C

$\exists w \, ; \; \left.\begin{array}{l} xRw \\ wSy \end{array}\right\}$

B

$\underline{\underline{S \circ R}}$:

$\underline{\underline{x}} \, (S \circ R) \, \underline{\underline{y}} = x h y$ iff $\exists w \in B$

$x \in A ; \; y \in C$

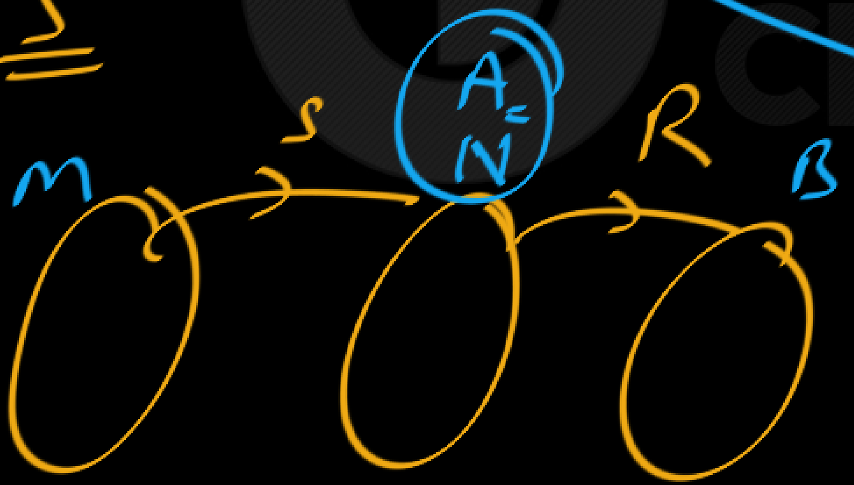
$(x, w) \in R ; \; (w, y) \in S$

Composition of Relations : (Transitively Relate)

To Define $R \circ S$: $R : A \to B$, $S : M \to N$

$A = N$

$R \circ S$



$R_1 : N \longrightarrow R$

$R_2 : R \longrightarrow N$

$R_1 \circ R_2$

$R_2 \cdot R_1$

Let us try to understand this better through an example.
Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (1, 1), (1, 3), (2, 4), (3, 2)\}$,
and $S = \{(1, 4), (1, 3), (2, 3), (3, 1), (4, 1)\}$.
Find $S \circ R$.

Solution:
Here we see that $(1,\ 2) \in R$ and $(2,\ 3) \in S$. This gives us $(1,\ 3) \in S \circ R$.
Similarly we can proceed with the others:

- $(1,\ 1) \in R$ and $(1,\ 4) \in S \Rightarrow (1,\ 4) \in S \circ R$
- $(1,\ 1) \in R$ and $(1,\ 3) \in S \Rightarrow (1,\ 3) \in S \circ R$
- $(1,\ 3) \in R$ and $(3,\ 1) \in S \Rightarrow (1,\ 1) \in S \circ R$
- $(2,\ 4) \in R$ and $(4,\ 1) \in S \Rightarrow (2,\ 1) \in S \circ R$
- $(3,\ 2) \in R$ and $(2,\ 3) \in S \Rightarrow (3,\ 3) \in S \circ R$

$\Rightarrow S \circ R = \{(1, 3), (1, 4), (1, 1), (2, 1), (3, 3)\}$.

What is the composite of the relations R and S, where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$?

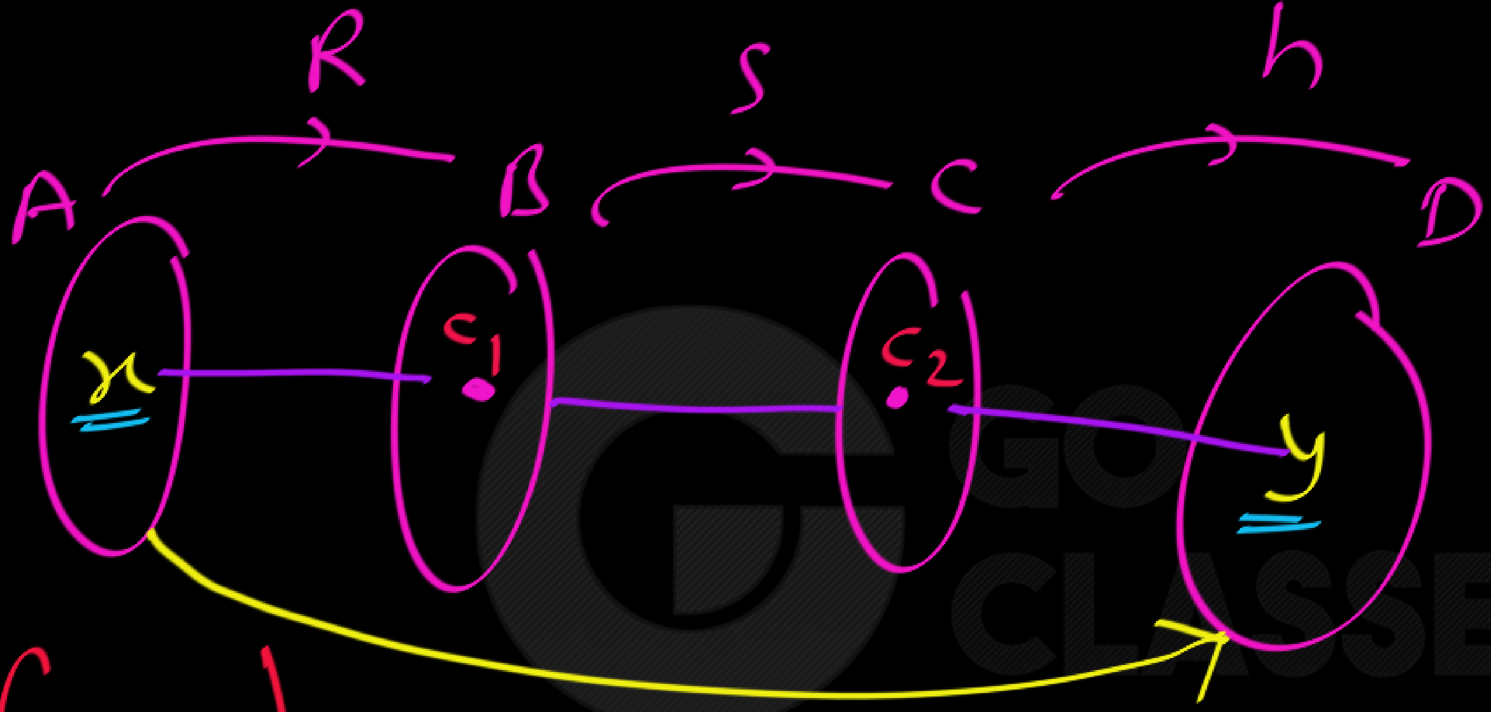
Solution: $S \circ R$ is constructed using all ordered pairs in R and ordered pairs in S, where the second element of the ordered pair in R agrees with the first element of the ordered pair in S. For example, the ordered pairs $(2, 3)$ in R and $(3, 1)$ in S produce the ordered pair $(2, 1)$ in $S \circ R$. Computing all the ordered pairs in the composite, we find

$$S \circ R = \{(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)\}.$$

◀

Note: Composition of Relations is Associative:

$$(R \circ h) \circ S = R \circ (h \circ S)$$

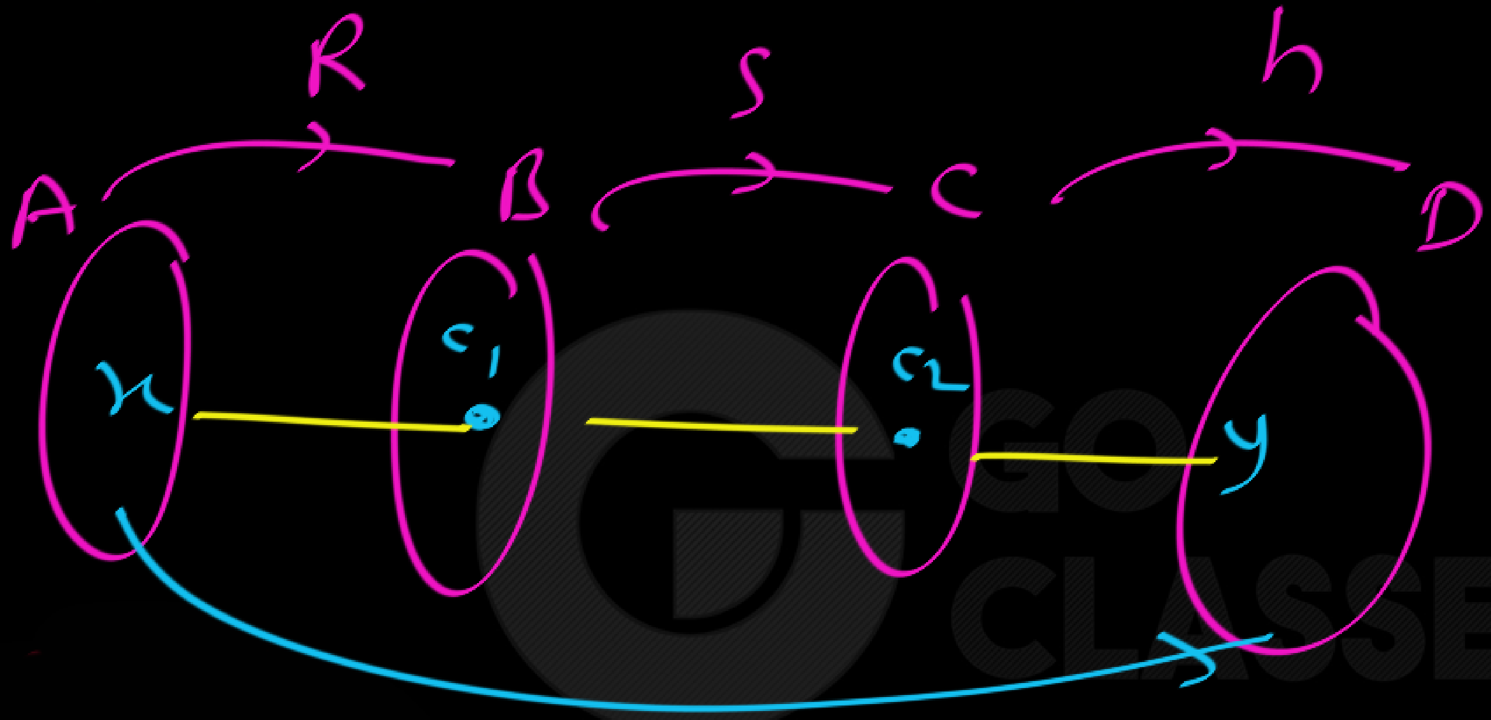


$(h \circ S) \circ R : A \to D$

$g = (h \circ S) \circ R$

$x \, g \, y$

$x R c_1$
$c_1 S c_2$
$c_2 h y$



$h \circ (R \circ S)$

§ Theorem

Let A, B, C and D be sets, R a relation from A to B, S a relation from B to C and T a relation from C to D. Then

$$T \circ (S \circ R) = (T \circ S) \circ R$$

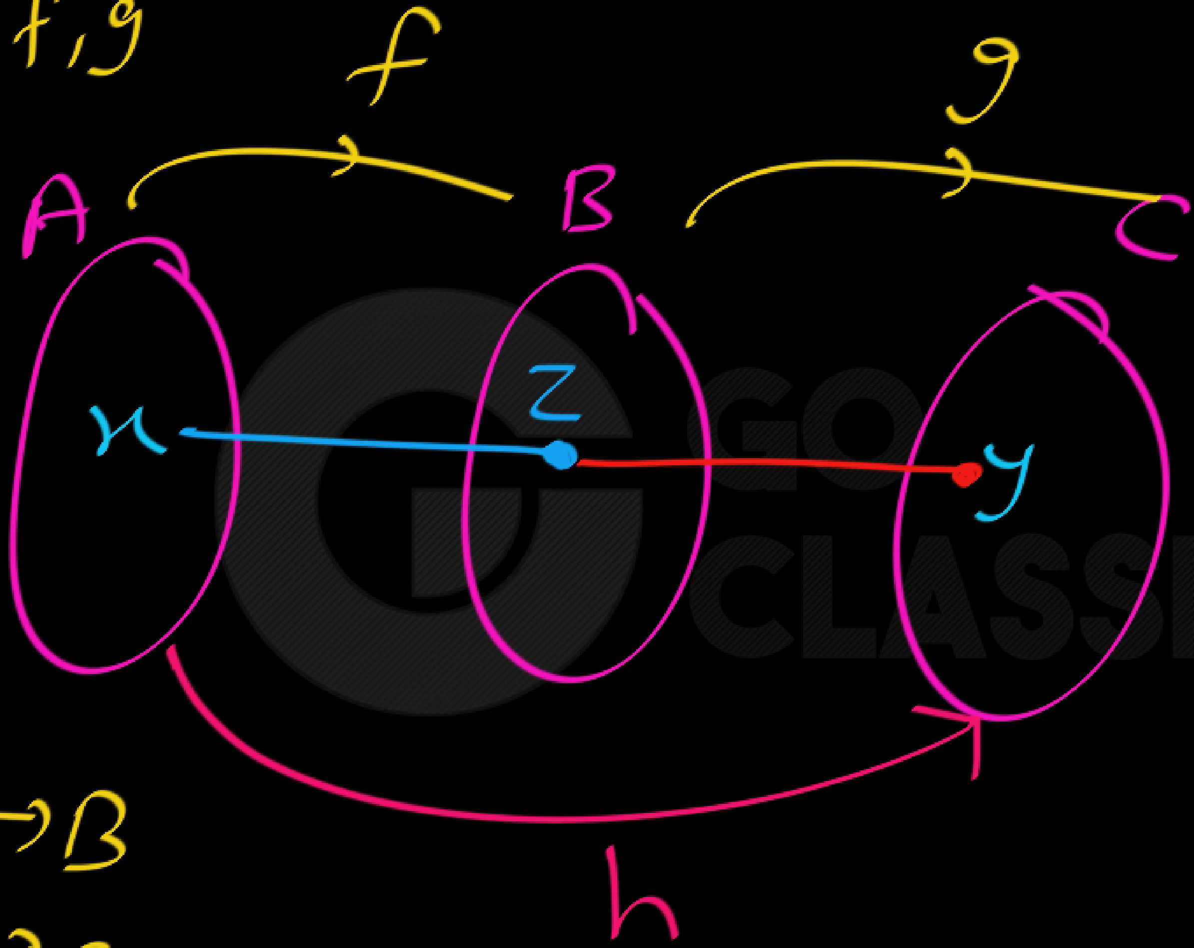
Set Theory

Next Topic: (Important)

Composition of Functions

Website : https://www.goclasses.in/

function f, g



$h : A \to C$

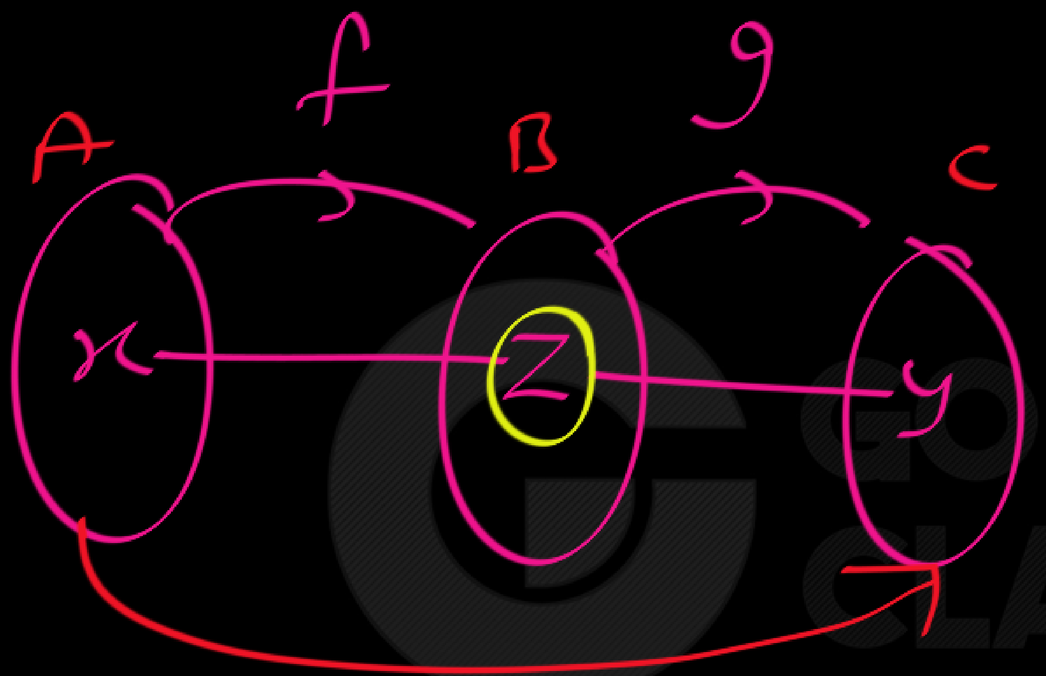
$h(x) = y$

iff

$\left.\begin{array}{l} f(x) = z \\ g(z) = y \end{array}\right\}$

$f : A \to B$

$g : B \to C$



$$f : A \to B$$

$$g : B \to C$$

$$h : A \to C$$

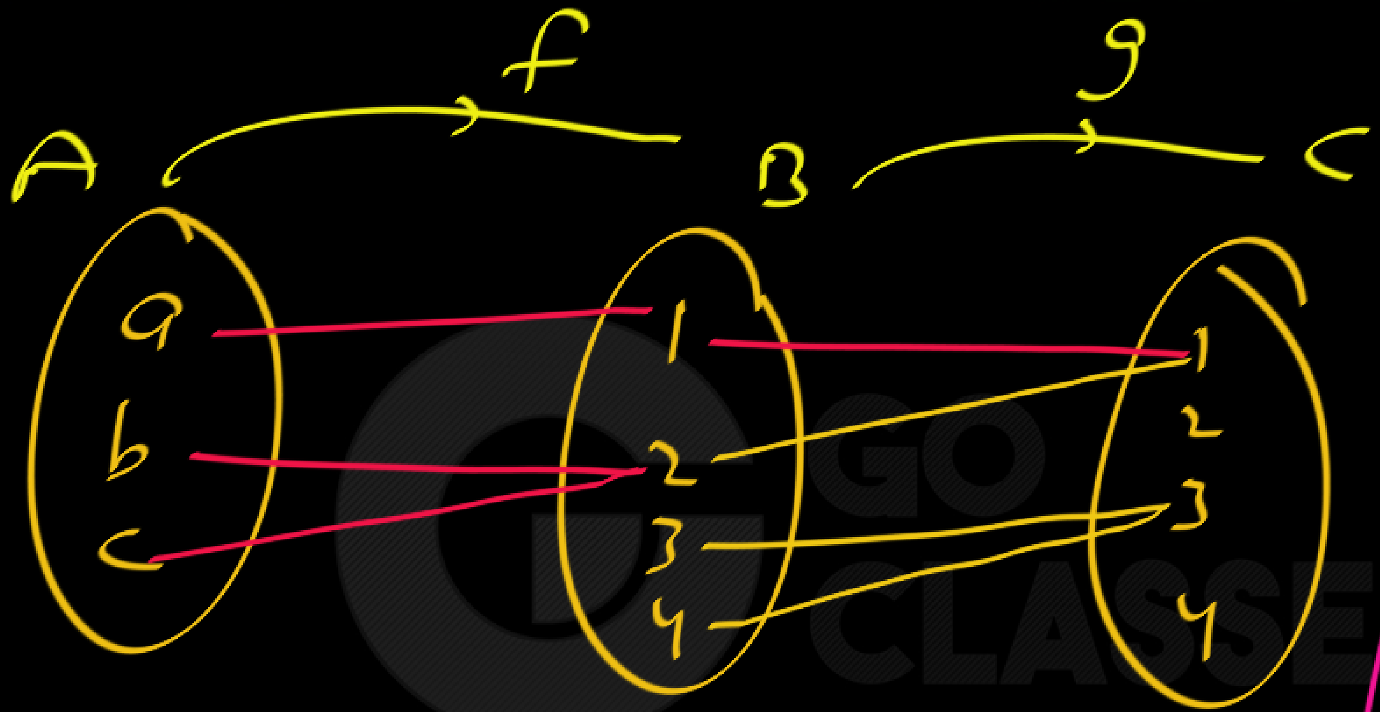
$$h(x) = g(f(x))$$

$$h = g \circ f \checkmark$$

$$h(x) = g(f(x))$$

$$g \circ f(x) = g(f(x))$$

$$f \circ g(x) = f(g(x))$$



$g \circ f(a) = g(f(a))$

$= g(1)$

$= 1$

$\underline{g \circ f(a) = 1}$

$f \circ g$ ✗

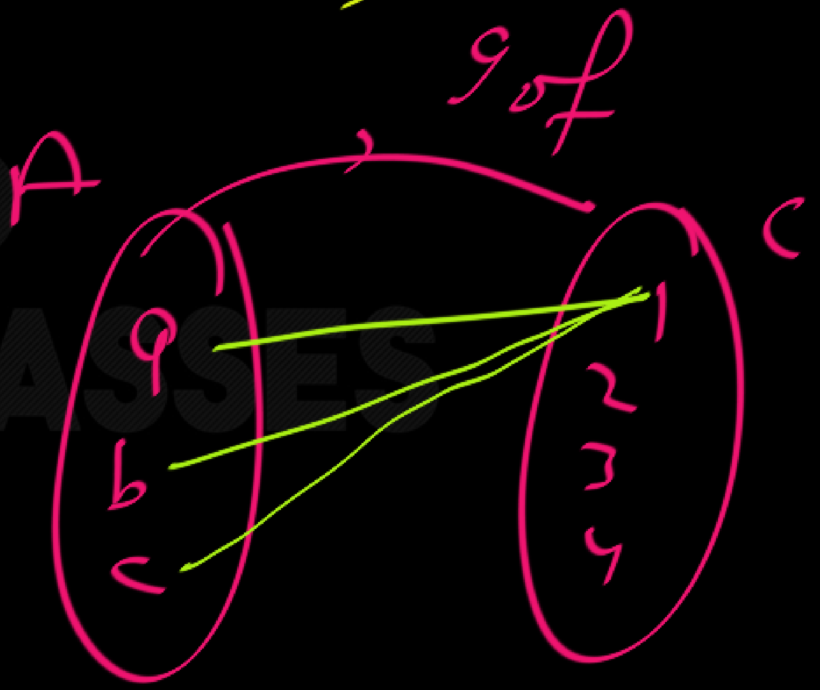
$g \circ f$ ✓

$g \circ f : A \to C$

$$g\circ f(b) = g(f(b)) = g(2) = 1$$

$$g\circ f(b) = 1$$

$$g\circ f(c) = 1$$



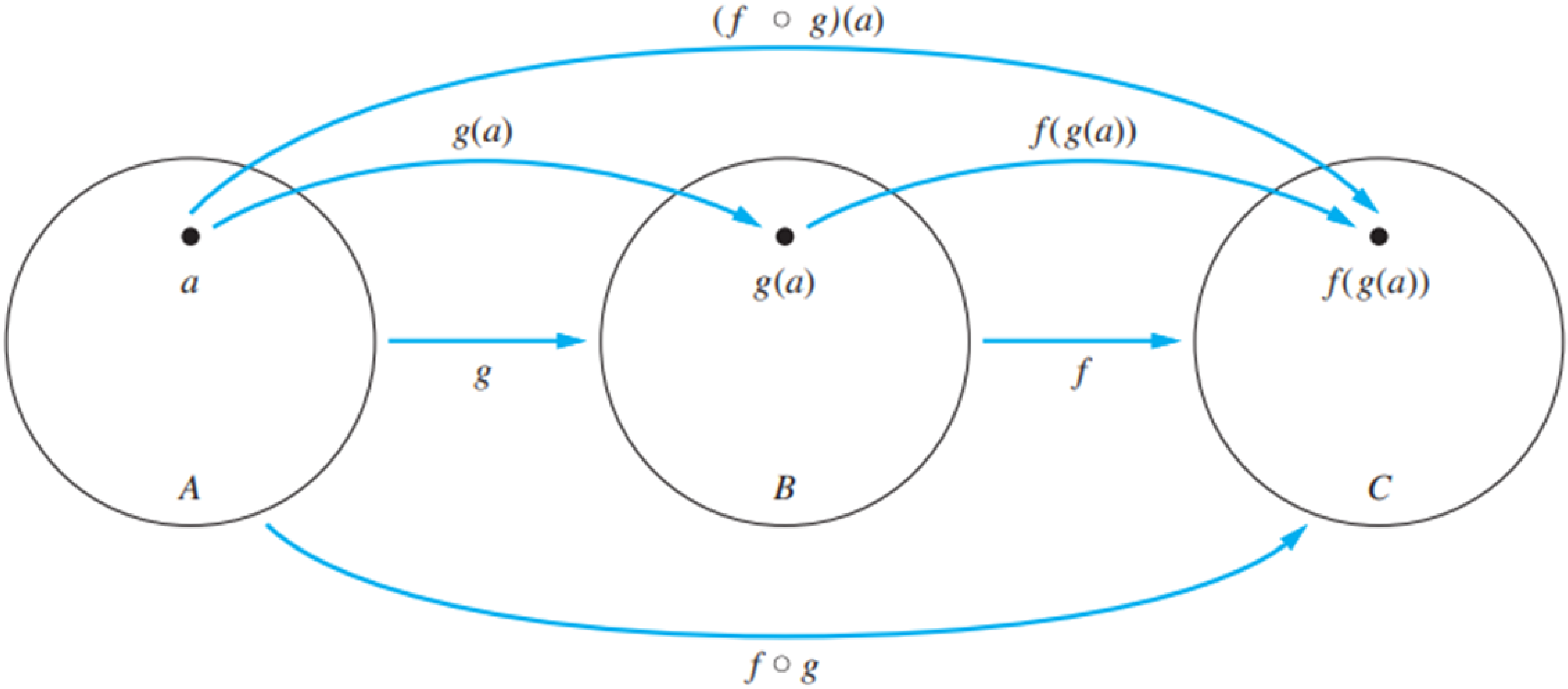


FIGURE 7 **The Composition of the Functions f and g.**

Q: $f : A \to B$

$g : M \to N$

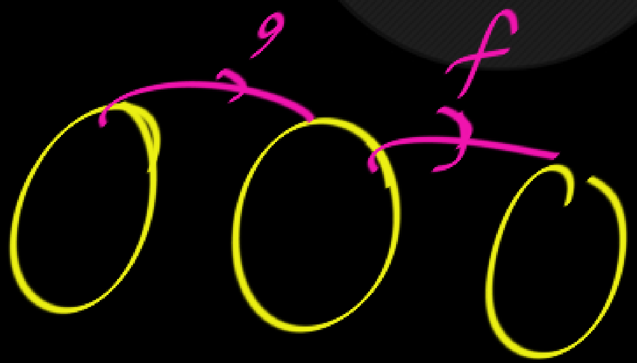
$f \circ g = f(g)$

To Defined $f \circ g$:

CoDomain of g

= Domain of f

$N = A$

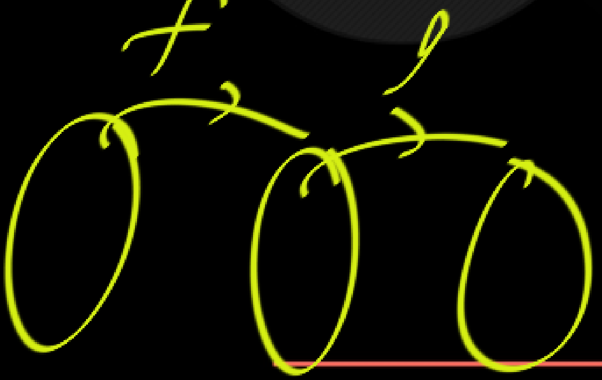


Q: $f : A \to B$

$g : M \to N$

$g \circ f = g(f)$

To Defined $g \circ f$: CoDomain of f = Domain of g



$B = M$

Function Composition

- Let $f : A \to B$ and $g : B \to C$.
- The **composition of *f* and *g*** (denoted $\boldsymbol{g \circ f}$) is the function $g \circ f : A \to C$ defined as

$$(g \circ f)(x) = g(f(x))$$

- Note that f is applied first, but f is on the right side!
- Function composition is **associative**:

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Function Composition

- Suppose $f : A \to A$ and $g : A \to A$.
- Then both $g \circ f$ and $f \circ g$ are defined.
- Does $g \circ f = f \circ g$?
- **In general, no:**
  - Let $f(x) = 2x$
  - Let $g(x) = x + 1$
  - $(g \circ f)(x) = g(f(x)) = g(2x) = 2x + 1$
  - $(f \circ g)(x) = f(g(x)) = f(x + 1) = 2x + 2$

$f(x) = 2x$

$g(x) = x+1$

$f \circ g(x) = f(g(x)) = f(x+1)$

$= 2(x+1) = 2x+2$

$g \circ f(x) = g(f(x)) = g(2x) = 2x+1$

Q: $f : N \to Z \qquad f(x) = x^2$

$g : N \to N \qquad g(x) = x + 1$

$g \circ f$ can not be Defined.

$$N \xrightarrow{f} \overset{N}{Z} \xrightarrow{g} N$$

$f \circ g$ ✓

$$N \xrightarrow{g} N \xrightarrow{f} Z$$

$f \circ g : N \rightarrow Z$

$f(x) = x^2$

$g(x) = x+1$

$f \circ g(1) = f(g(1)) = f(2) = 4$

$f \circ g(2) = f(3) = 9$

$f \circ g(3) = 16$

$f \circ g(4) = 25$

$f \circ g(6) = 49$

$f \circ g(5) = 36$

$f \circ g(x) = f(g(x))$

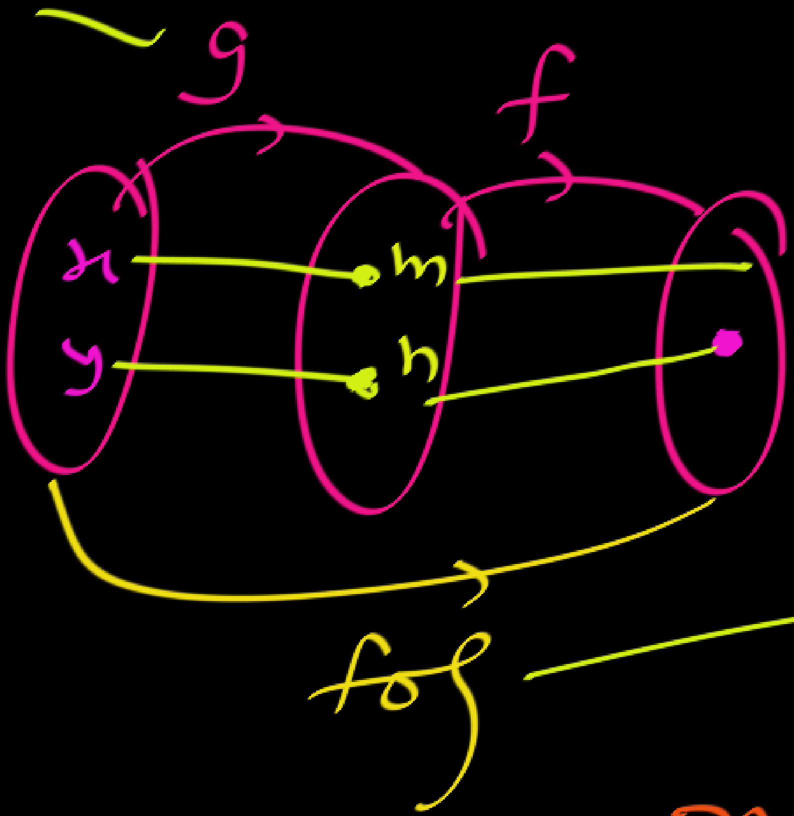
$= f(x+1)$

$= (x+1)^2$

Q: $f: A \to A$; $g: A \to A$

f, g = one-one (Given)

$f \circ g$ = one-one ?



1-1 ✓

$x \neq y \Rightarrow \underline{g(x) \neq g(y)}$ one-one

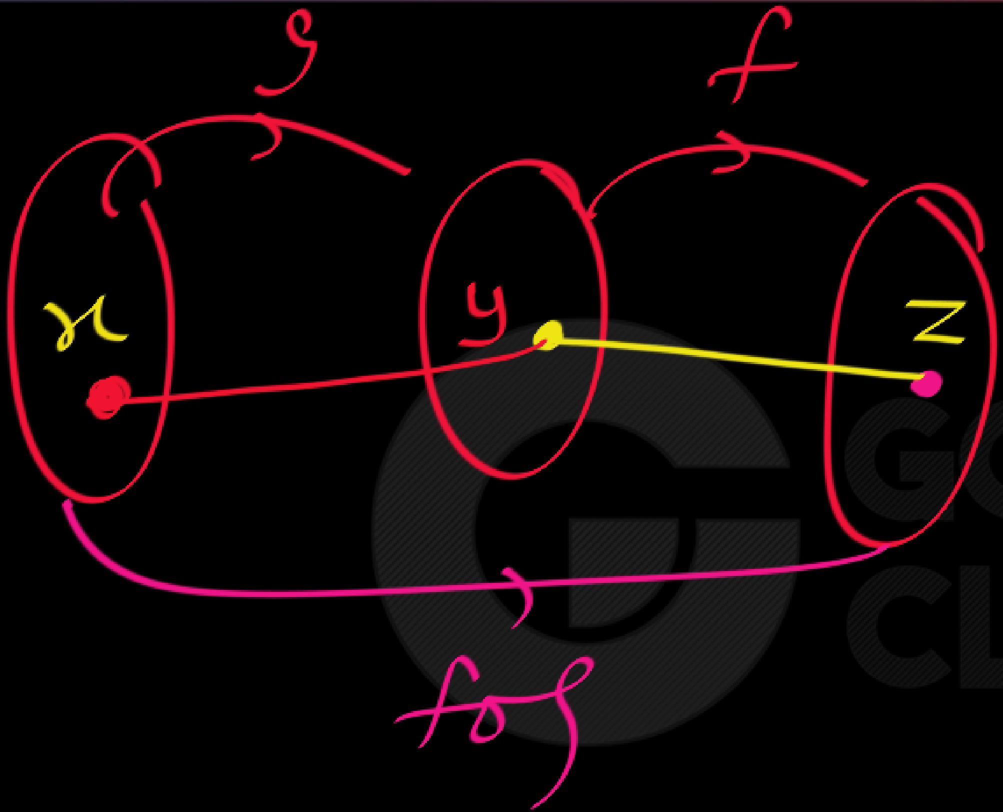
$m \neq n$ so

$f(m) \neq f(n)$ 1-1

Q: $f : A \to A$; $g : A \to A$

f, g = onto (Given)

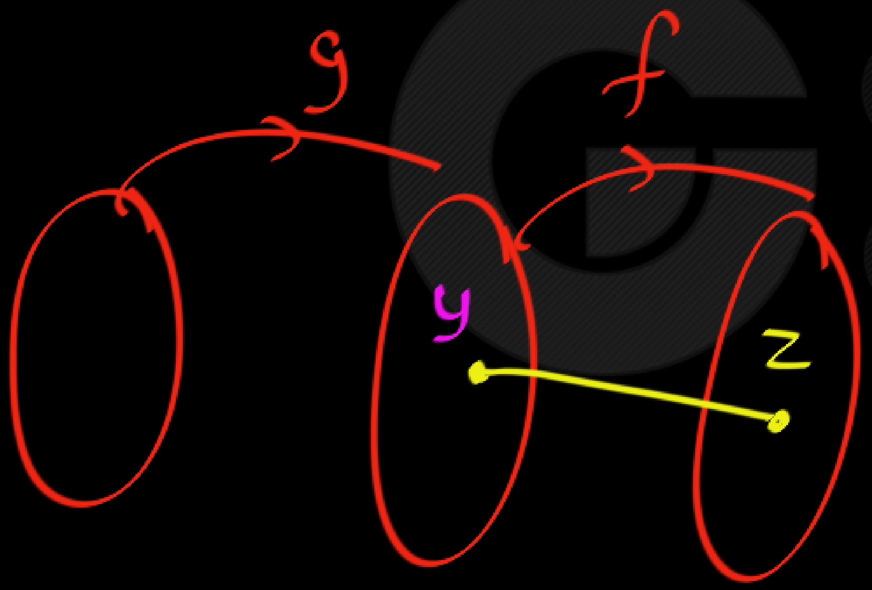
$f \circ g$ = onto ?

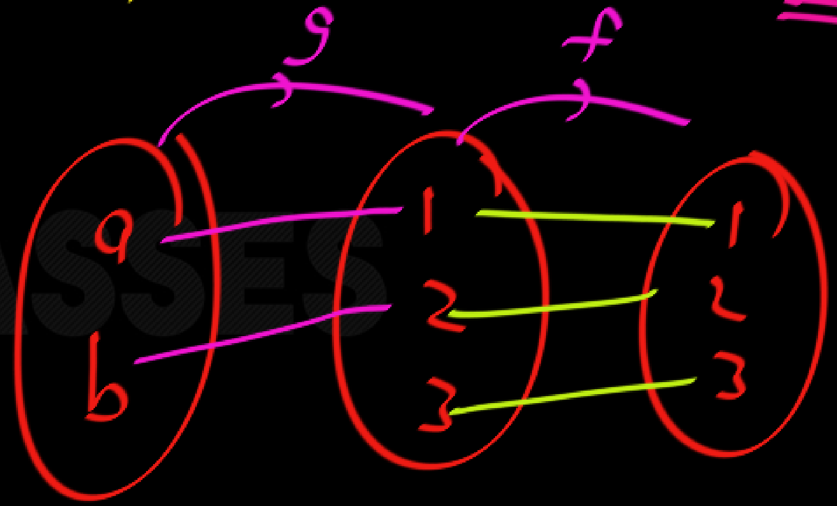


So fog = onto

Q : $f, g : A \longrightarrow A$

f = onto then $f \circ f$ = onto ? **No**





$f \circ f$ is Not onto.

Q : $f, g : A \to A$

f = onto then $g \circ f$ = onto ?

No