LOGICAL CONNECTIVES

Propositional logic

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Propositional Logic

Homework 1

Proposition, Propositional Variable

Conjunction, Disjunction, Negation

A Proposition/Statement is something which is either true or false, but not both simultaneously.

Note: A sentence Can NOT be called a Proposition/Statement if

(i) It is an exclamation or imperative sentence

(ii) It is an order or request or advice or command

(iii) It is a question

(iv) It is a paradox(self-contradictory)

(v) It involves some variable and the truth value depends on this variable.

Example of propositions:

Example of propositions:
John loves CSE 191.
2+3=5.
2+3=8.
Sun rises from West.

Example of non-propositions:
Does John love CSE 191?
$2 + 3$.
Solve the equation $2 + x = 3$.
$2 + x > 8$.

NOTE :

This much understanding of “what is a proposition” is enough as the definition of “proposition” in English/Natural languages is vague and in mathematical world, it is simply a True or False Fact.

Some authors will consider “Today is Monday” as a proposition, whereas, other authors will consider same sentence as Non-proposition.

So, we will not worry about it too much and we just need to know that every proposition is either true or false but not both.

Question 1 :

UGCNET-DEC2018-II: 1

1

In mathematical logic, which of the following are statements?

2.3k views

i. There will be snow in January.
ii. What is the time now?
iii. Today is Sunday.
iv. You must study Discrete mathematics

Choose the correct answer from the code given below:

A. i and iii
B. i and ii
C. ii and iv
D. iii and iv

Source : https://gateoverflow.in/288376/ugcnet-dec2018-ii-1

Solution 1: https://gateoverflow.in/288376/Ugcnet-dec2018-ii-1?show=372580#c372580

"Today is Sunday" is a Proposition ??

Some authors would consider it as a proposition (by fixing certain time, date, place etc), Other authors would not consider it as a proposition.

I suggest NOT to waste too much time on "which sentence is proposition?" type of question.

A Good exam will not ask such questions. If you find such questions in some test series, leave such test series.

In the (good)question, propositions will be given , not asked.

Example 1. Let p denote "Henry eats halibut" and q denote "Catherine eats kippers."

(a) The proposition $\neg p$ is read "Henry does not eat halibut."
(b) The proposition $p \wedge q$ is read "Henry eats halibut, and Catherine eats kippers."
(c) The proposition $p \rightarrow q$ is read "If Henry eats halibut, then Catherine eats kippers."
(d) The proposition $p \leftrightarrow q$ is read "Henry eats halibut if and only if Catherine eats kippers."
(e) The proposition $(\neg p) \vee (\neg q)$ is read "Henry does not eat halibut, or Catherine does not eat kippers."
(f) The proposition $p \leftrightarrow (\neg q)$ is read "Henry eats halibut if and only if Catherine does not eat kippers."

NOTES :

The negation of a statement p in symbolic form is written as "~ p".

Example :

Write the negation of the statement p :

p : New Delhi is a city.

Solution :

The negation of p is given by

~ p : New Delhi is not a city

Or ~ p : It is not the case that New Delhi is a city.

Or ~ p : It is false that New Delhi is a city.

Or ~ p : It is not true that New Delhi is a city.

Question 2:

We have seen that for two propositional variables, we have 4 rows in truth table(because with two propositional variables, total 4 combinations of truth values are possible).

For three propositional variables, in the truth table, how many rows we have ?

For Four propositional variables, in the truth table, how many rows we have ?

For n propositional variables, in the truth table, how many rows we have ?

Question :

We have seen that for two propositional variables, we have 4 rows in truth table.

For three propositional variables, in the truth table, how many rows we have ?

For Four propositional variables, in the truth table, how many rows we have ?

For n propositional variables, in the truth table, how many rows we have ?

Q.: for 3 variables, In Truth Table, we have 2^3 rows as every variable has 2 choices, True or false.

for n variables, no. of Rows in Truth Table $= 2^n$

Question 3:

It is known that p ∧ q is false, also p is true.

What can we say about q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p ∧ q is false, also p is true.

What can we say about q ?

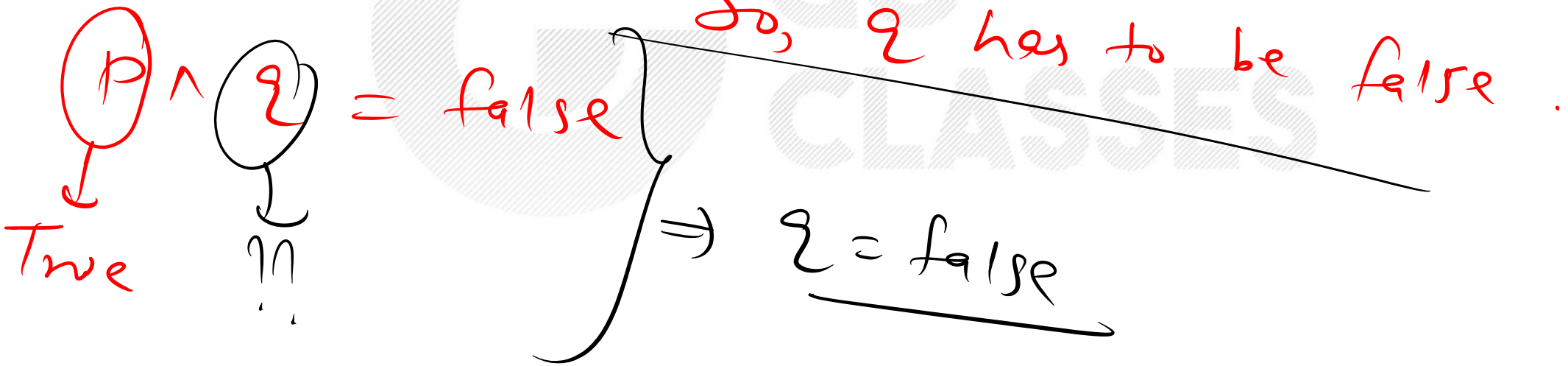
A. True

B. False.

C. Nothing can be said

$p \wedge q \equiv$ false Given

$p =$ True Given



Question 4:

It is known that p v q is false, also p is false.

What can we say about q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p v q is false, also p is false.

What can we say about q ?

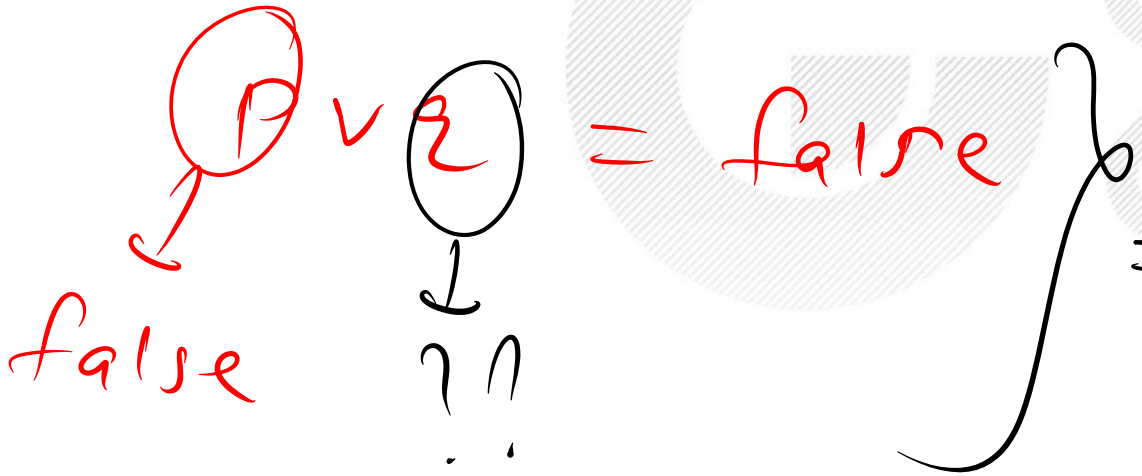
A. True

B. False.

C. Nothing can be said

False ∨ True = True

false ∨ false = false



Question 5:

It is known that p ∧ q is true, also p is true.

What can we say about q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

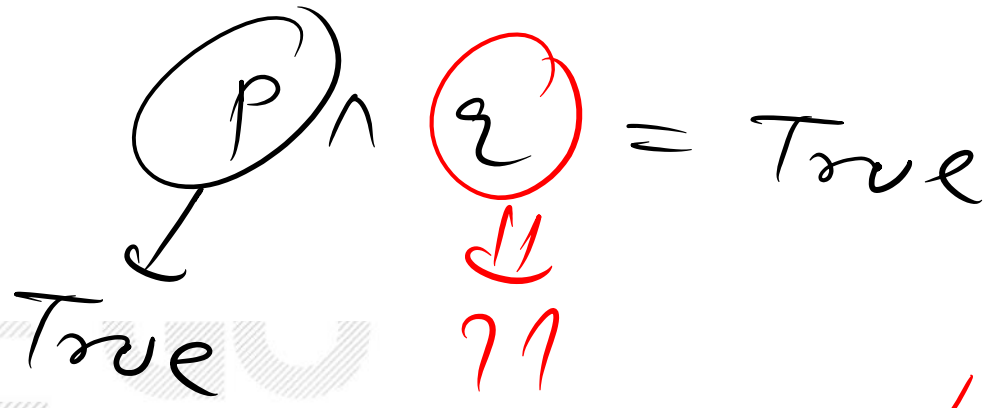
It is known that p ∧ q is true, also p is true.

What can we say about q ?

A. True

B. False.

C. Nothing can be said



True ∧ false = false

True ∧ True = True

q = True

Question 6:

It is known that p v q is true, also p is true.

What can we say about q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

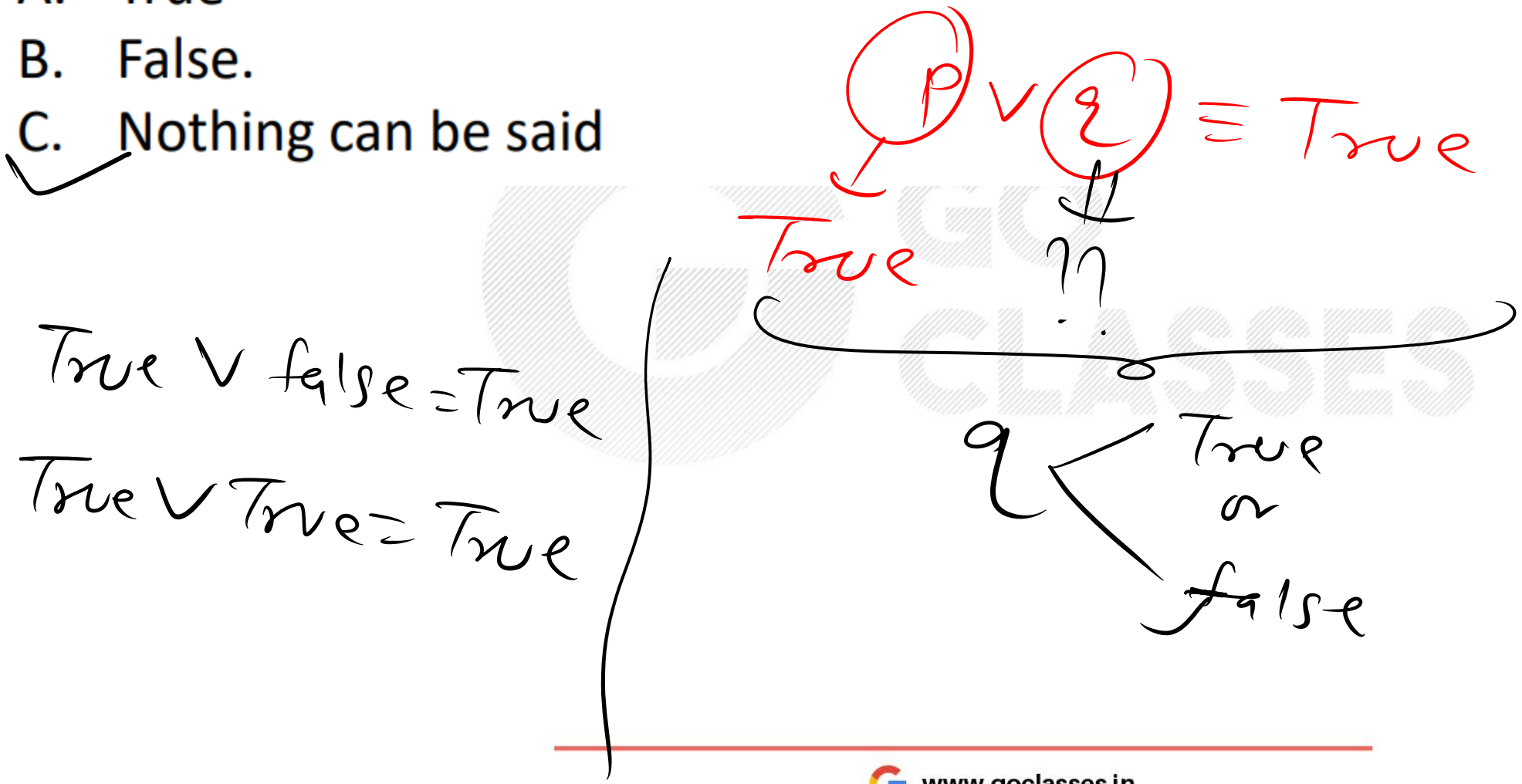
It is known that p v q is true, also p is true.

What can we say about q ?

A. True

B. False.

C. Nothing can be said



NOTE:

By the definition of a Proposition (or Propositional Variable), a **proposition can NEVER have multiple truth values.**

A proposition is Either True or False But Not Both.

For the previous question, answer will be “Truth Value can’t be determined”.. “Multiple Values” is a Wrong Answer.

Question 7:

It is known that p is true.

What can we say about $p \wedge q$?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

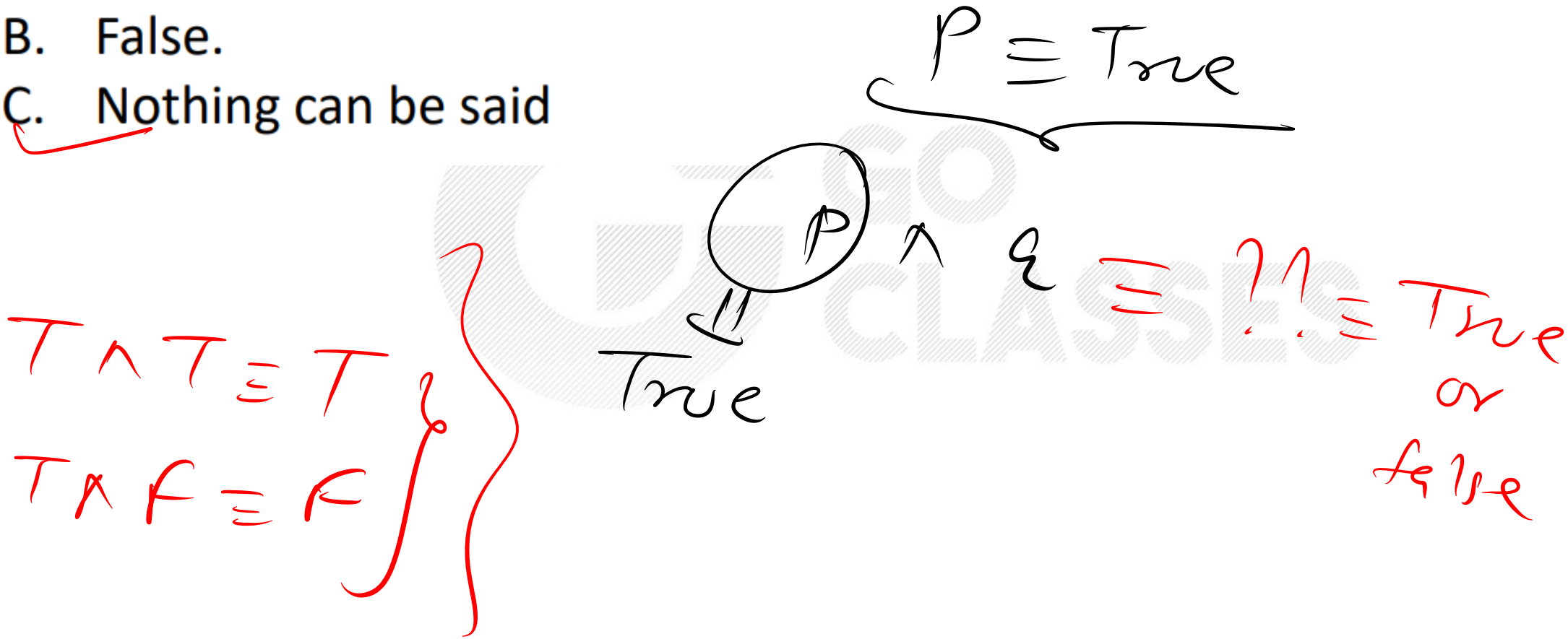
It is known that p is true.

What can we say about $p \wedge q$?

A. True

B. False.

C. Nothing can be said



Question 8:

It is known that p is true.

What can we say about $(\neg p) \wedge q$?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p is true.

What can we say about $(\neg p) \wedge q$?

A. True

B. False.

C. Nothing can be said

$p \equiv True$; $p' \equiv false$

$(p') \wedge q$

false

$\equiv ?? \equiv false$

false $\wedge$ True = false

false $\wedge$ false = false

Question 9:

It is known that p is true.

What can we say about ¬ (p ∧ q) ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

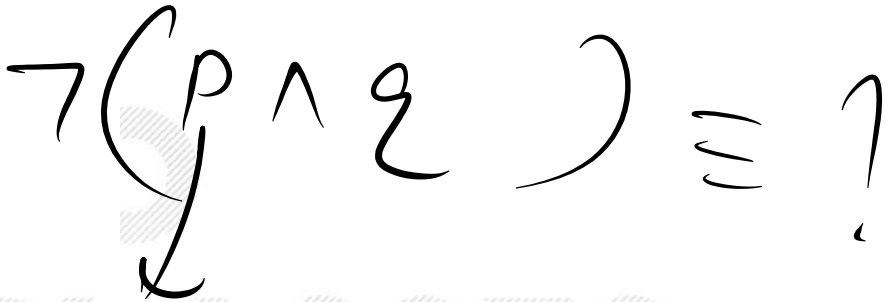
It is known that p is true.

What can we say about ¬ (p ∧ q) ?

A. True

B. False.

C. Nothing can be said



Question 10:

It is known that p is true.

What can we say about p v q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p is true.

What can we say about p v q ?

A. True

B. False.

C. Nothing can be said

T v F ≡ T

T v T ≡ T

True v q ≡ True

Question 11:

It is known that p is true.

What can we say about (¬ p) v q ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p is true.

What can we say about (¬ p) v q ?

A. True

B. False.

C. ~~Nothing can be said~~

$P \equiv$ True, so $p' \equiv$ false

$(p') \vee q$

F

$F \vee q \equiv q <$ True or false

Question 12:

It is known that p is true.

What can we say about (p) v (¬ q) ?

A. True

B. False.

C. Nothing can be said

D. Multiple Values

Question :

It is known that p is true.

What can we say about (p) v (¬ q) ?

A. True

B. False.

C. Nothing can be said

$P \equiv True$

$(P) \vee (q')$

$T \vee \beta \equiv T$

$True \vee q' \equiv True$

NOTE :

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