



Definition :

Definitions are "If and only if's"

Always

statements.

(Even if they are written as "If")



## Definition of Even number :

a number is even number

if it is

multiple  
of 2.

integers



## Definition of Even number :

a number is even number if it is integer multiple of 2.

$m_2 \rightarrow$  even



$m_2 \rightarrow \text{even}$

even  $\rightarrow \underline{\underline{m_2}}$  ?? Yes.

So the correct statement is :

Even if and only if  $m_2$



Definition is Always a

"If and only if "

Statement.



# Implication( one way statement)

tells you about "Property"

# Bi Implication

tells you the

"Definition".



Definition of a even number:

D1

integer multiple of 2

D2

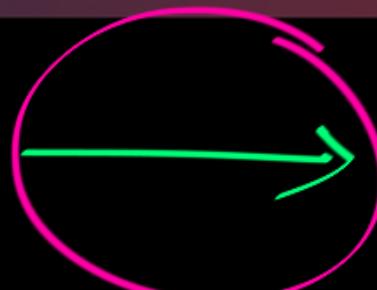
integer divisible by 2

D3

$$\underbrace{n \bmod 2}_{=} 0$$



Prime  $> 2$



odd



Is "being odd" Definition of  
"prime  $> 2$ " ?? No.



But "being odd" is  
Property of "Prime >2".



Boy → Human

"being Human" is Property of  
"being boy"



Define a boy :

boy is a Human.

No      Correct Definition



$$P \rightarrow Q$$

P is Property of Q X



Every P has Property Q.

# Properties of Natural numbers

- ① integer
- ②  $> 0$
- ③ rational
- ④ real
- ⑤ odd or even

$$\mathbb{N} \rightarrow \mathbb{I}$$

$$\mathbb{N} \rightarrow \text{real}$$

$$\mathbb{N} \rightarrow \text{rational}$$

Implication tells us Property.

Bi Implication tells us Definition.

Eg: Prime minister of India → Indian

So, PM has Property that He/She is Indian,

But Being Indian is Not the Definition of  
PM of Indian.

Eg: Prime number greater than 2  $\rightarrow$  Odd Number.

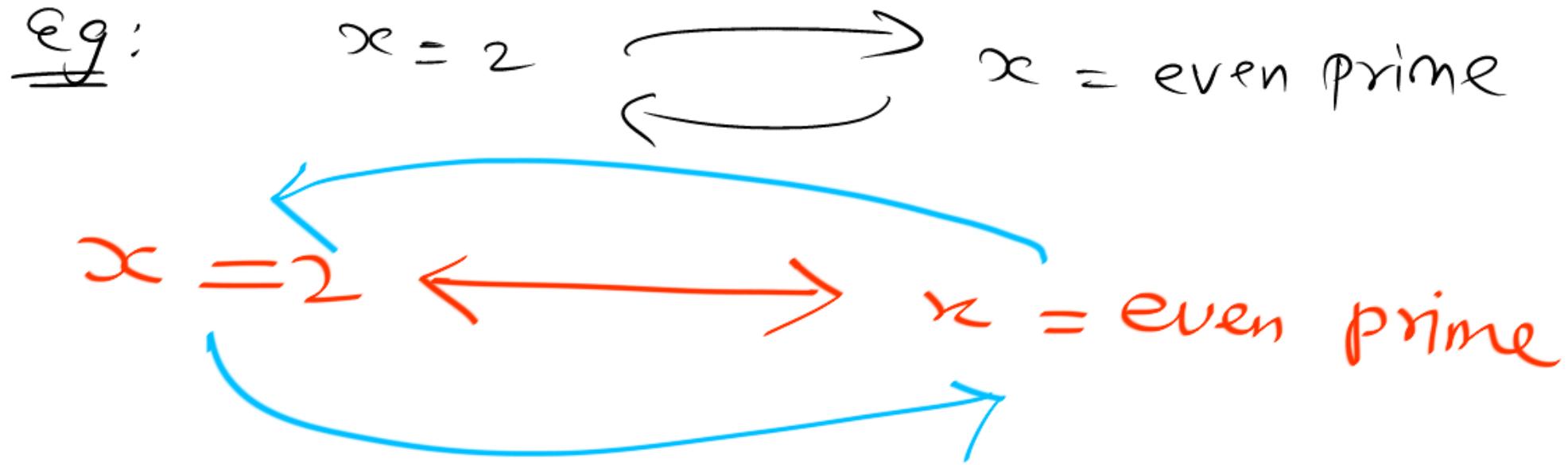
$\varphi$  is property of  $p$ .



$P \leftrightarrow Q$

$P$  is Definition of  $Q$ .  
 $Q$  " " Definition of  $P$ .

$P, Q$  are Definition of each other.



So "being 2" is Definition of "even prime"  
 "being prime" is || " " "being 2"  
 even

$P \rightarrow Q$

$Q$  is property of  $P$ .

$P$  satisfies  
Property  $Q$

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$P \leftrightarrow Q$

GO CLASSES

$P, Q$  are Definitions of each other.



## Definitions of even prime :

- (D1) is 2.
- (D2) Prime  $\Leftrightarrow$  Prime  $< 3$
- (D3) is prime and divisible by 2

So,

$$\underbrace{P \rightarrow Q}$$

Says that  $P$  satisfy property  $Q$ .

$$P \leftrightarrow Q$$

Says that  $P$ , and only  $P$  satisfies property  $Q$ .



# Recap So Far

- A **propositional variable** is a variable that is either true or false.
- The **logical connectives** are
  - Negation:  $\neg p$
  - Conjunction:  $p \wedge q$
  - Disjunction:  $p \vee q$
  - Implication:  $p \rightarrow q$
  - Biconditional:  $p \leftrightarrow q$



$$\begin{array}{l} P \rightarrow Q \\ P \rightarrow R \end{array} \quad \left. \begin{array}{l} P \rightarrow Q \\ P \rightarrow R \end{array} \right\}$$

Q, R are

Properties of P

If P is True, Q True, R True, P satisfies Q, R

# Logical Connectives

- **Logical NOT:**  $\neg p$

- Read “**not**  $p$ ”
- $\neg p$  is true if and only if  $p$  is false.
- Also called **logical negation**.

- **Logical AND:**  $p \wedge q$

- Read “ $p$  **and**  $q$ .”
- $p \wedge q$  is true if both  $p$  and  $q$  are true.
- Also called **logical conjunction**.

- **Logical OR:**  $p \vee q$

- Read “ $p$  **or**  $q$ .”
- $p \vee q$  is true if at least one of  $p$  or  $q$  are true (inclusive OR)
- Also called **logical disjunction**.



# Some Popular Boolean Operators

<u>Formal Name</u>	<u>Nickname</u>	<u>Arity</u>	<u>Symbol</u>
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$