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Summation identities

$$\sum_i a x_i = a \sum_i x_i$$

$$\sum_i (x_i + y_i) = \sum_i x_i + \sum_i y_i$$

$$\sum_{i=a}^{b} f(x) = \sum_{i=0}^{b} f(x) - \sum_{i=0}^{a-1} f(x)$$

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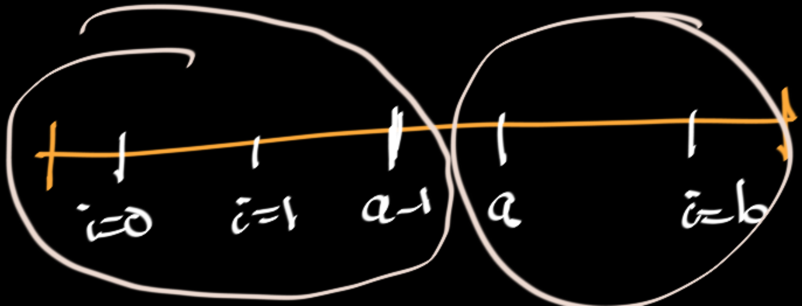
$$\sum_{i=a}^{b} f(x) = \sum_{i=0}^{b} f(x) - \sum_{i=0}^{a-1} f(x)$$

Rearrange

$$\sum_{i=0}^{b} f(x) = \sum_{i=0}^{a-1} f(x) + \sum_{i=a}^{b} f(x)$$



$$1+2+3+4+5+6 = \underline{(1+2+3+4)} + \underline{(5+6)}$$

$$1+2+3+\cdots a-1+a+\cdots b$$

$$= (1+2+3+\cdots a-1) + (a+a+1+\cdots b)$$

$$\sum_{x=1}^{b} f(x)$$

$$f(1) + f(2) + \cdots f(a-1) + f(a) + \cdots f(b)$$

$$\left(f(1) + f(2) + \cdots f(a-1)\right) + \left(f(a) + \cdots f(b)\right)$$

$$\sum_{x=1}^{a-1} f(x) + \sum_{x=a}^{b} f(x)$$

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Q. T/F

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x)$$

Q. T/F

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x)$$

$f(0) + f(1) + f(a) f(b)$

$$\underbrace{f(0) + f(1) + \cdots f(a)}_{} + \underbrace{f(a) + f(a+1) + \cdots f(b)}_{}$$

Q. T/F

false

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x)$$

Q. T/F

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x) - \underbrace{f(a)}_{\text{removing one time}}$$

f(a) one time

every term is just one time

f(a) is 2 times

Q. T/F

TRUE

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x) - f(a)$$

Q. is this be true given that

$f(a) = 0$

$$\sum_{x=0}^{b} f(x) = \sum_{x=0}^{a} f(x) + \sum_{x=a}^{b} f(x)$$

Basics of maths

Summation

Sequence and Series

Sequence

The idea of sequence is straightforward

$a1, a2, a3, \ldots$

Sequence

3, 5, 7

7 5, 3

Series

if $a_1, a_2, \ldots, a_n$ is a sequence of n terms,

then the corresponding series is $a_1 + a_2 + \cdots + a_n$.

Series

people.math.harvard.edu/~engelwar/MathS305/Seq...

Another way to think about s_n is that it is given by the sum of the first n terms of the sequence $\{a_n\}$, namely

$$s_n = a_1 + a_2 + \ldots + a_n.$$

$$\{a_n\} = \{a_1, a_2, \ldots a_n\}$$

$1+8$

$2+3+4 = 4+3+2$

Arithmetic Progression

2, 4, 6, 8, 10, 12, 14, . . .

Series

AP

GP

HP

2, 4, 6 is following some

common difference. which is "2"

AP.

AP is a sequence of terms which follows a specific common difference.

$$2 \xrightarrow{+2} 4 \xrightarrow{+2} 6$$

people.math.osu.edu/fowler.291/sequences-and-series.pdf

Definition An **arithmetic progression** (sometimes called an arithmetic sequence) is a sequence where each term differs from the next by the same, fixed quantity.

common difference

Example 1 : Find the first term and the common difference of each of the following arithmetic progressions.

		first	common diff
(i)	7, 11, 15, 19, 23 ………………	7	4
(ii)	$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ 1 … … … … … … … … … ….	$1/6$	$1/6$
(iii)	$a+2b, a+b, a, a-b, a-2b, \ldots \ldots \ldots \ldots \ldots \ldots$	$a+2b$	$-b$

Solution :

	First term	Common difference
(i)	7	4
(ii)	$\frac{1}{6}$	$\frac{1}{6}$
(iii)	$a+2b$	$-b$

Example:

If the first 3 terms in an arithmetic progression are 8,5,2 then what is the 16th term? In this progression a = 8 and d = −3

$8, \ 5, \ 2, \ -1, \ -4, \ -7, \ \ldots$

$a = 8$

$d = -3$

$a_{16} = a_1 + 15d$

$= 8 + 15(-3) = \underline{\underline{-37}}$

$$a_1 \xrightarrow{d} a_2 \xrightarrow{d} a_3 \xrightarrow{d} a_4 \; \cdots \; a_n$$

Drive a formula for n^{th} term

(given first term a_1 and common diff d)

$a_2 = a_1 + d$

$a_3 = a_1 + 2d$

$a_4 = a_1 + 3d$

$a_n = a_1 + (n-1)d$

formula for n^{th} term in AP

$$a_n = a_1 + (n-1)d$$

Recursive formula : $a_n = a_{n-1} + d$

Example:

Given that 2x, 5 and 6 – x are the first three terms in an arithmetic progression , what is d?

$$2x \xrightarrow{+d} 5, \quad 6-x$$

$$8, \quad 5, \quad 2$$

$$d = -3$$

another way

$$2x + d = 5$$

$$5 + d = 6 - x$$

$$5 - 2x = 6 - x - 5$$

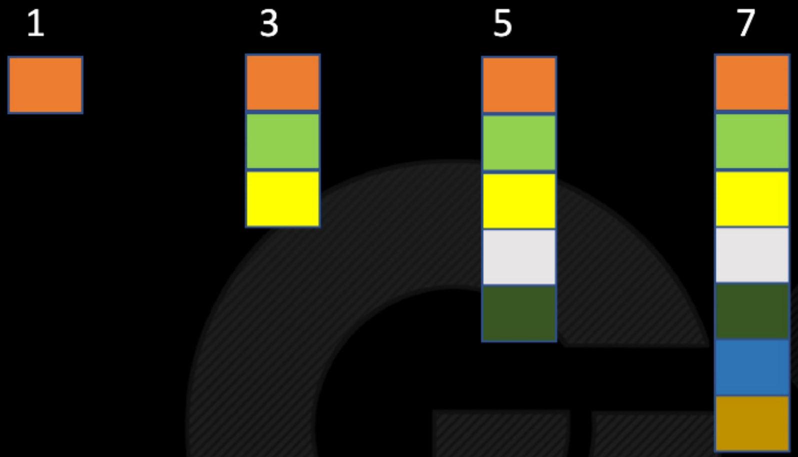
$$\Rightarrow 5 - 2x = 1 - x$$

$$x = 4$$

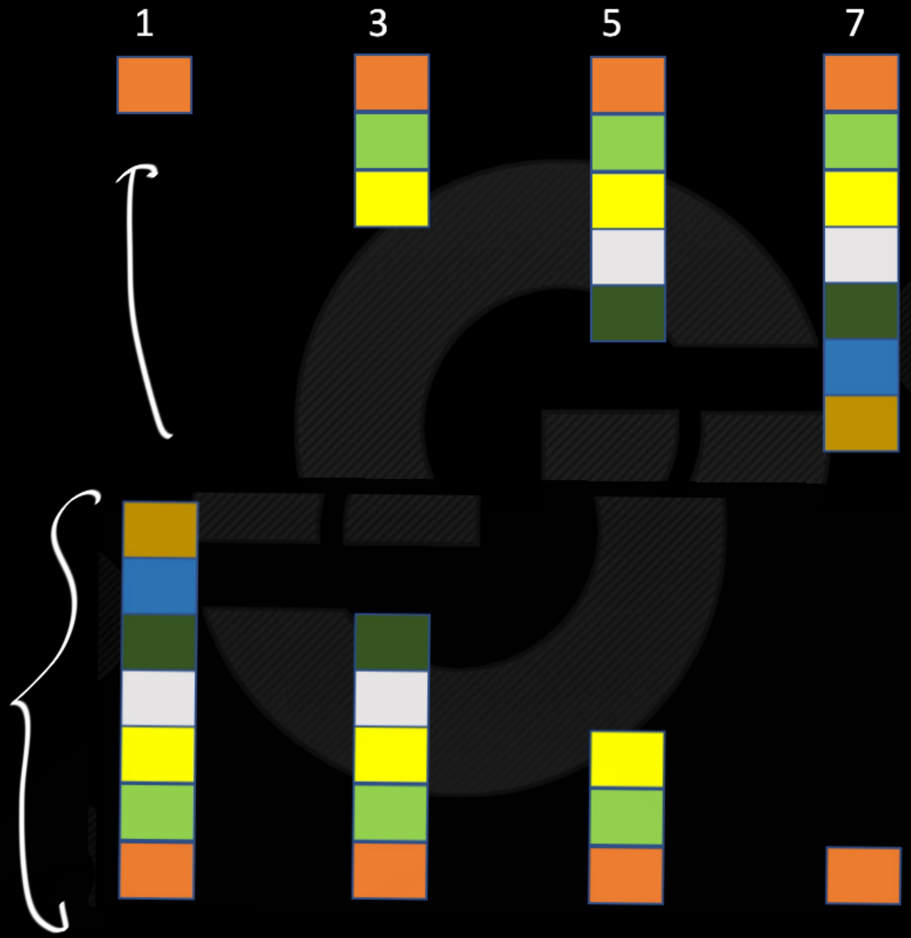
Sum of first n terms in AP

$a_1, \quad a_2, \quad a_3 \quad \ldots \quad a_n$

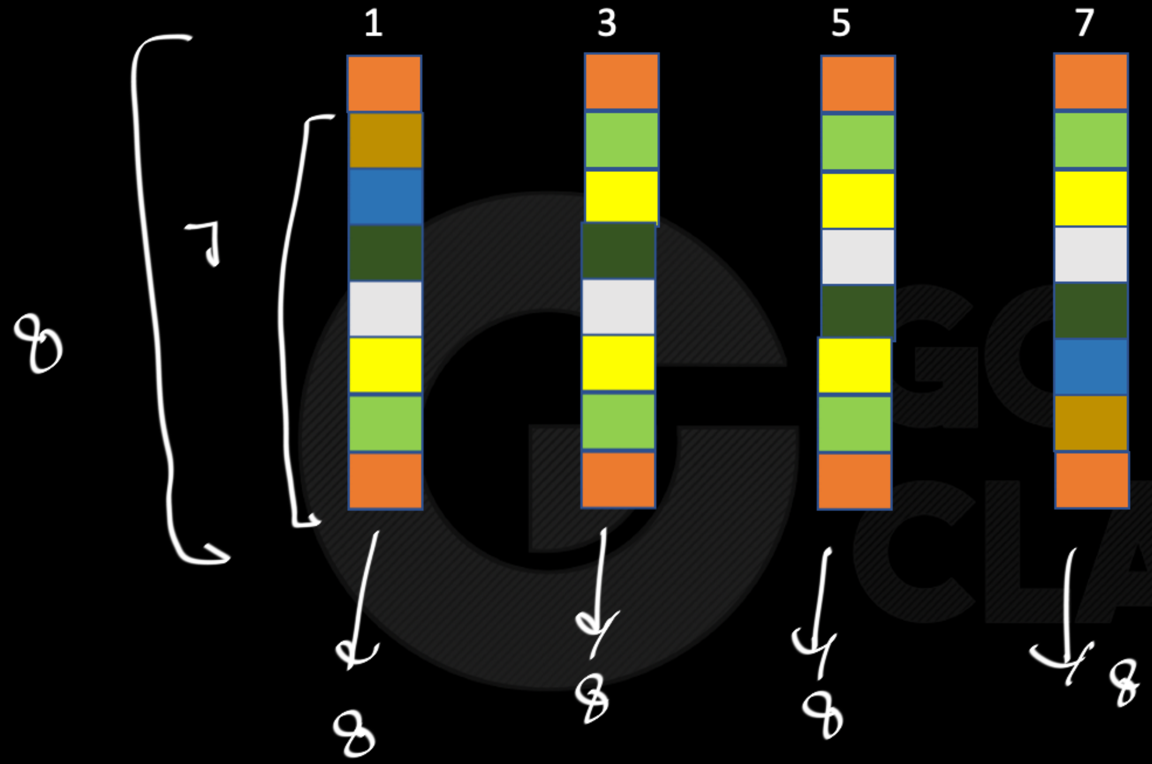
$a_1 + a_2 + a_3 + \ldots \ldots a_n$



$1+3+5+7=?$



$1 + 3 + 5 + 7 = ?$



$$2(1+3+5+7) = x$$

$$1+3+5+7 = \frac{x}{2}$$

$$\frac{8 \times 4}{2}$$

Gauss was just 9 year old

$$S = 1 + 2 + 3 + \cdots + 100$$

$$S = 100 + 99 + 98 + \cdots + 1$$

$$2S = 101 + 101 + 101 + \cdots 101 \Rightarrow 2S = 100 \times 101$$

$$S = \frac{100 \times 101}{2} = 50 \times 101$$

$a \qquad a+d \qquad \ldots \qquad l-d \qquad l$

Summation of first & last term $= a + l$

$$a+l = a+\cancel{d} + l-\cancel{d}$$

$$= a+l$$

1 5 9 13 17, 21

1+21 = 5+17 = 9+13

22 22 22

$a \qquad a+d \qquad \qquad l-d \qquad l$

if there are n terms then there are

$n/2$ pairs, each pair sum $= a+l$

$$a + a+d + \cdots l-d + l = \frac{n}{2}(a+l)$$

$a \qquad a+d \qquad\qquad l-d \qquad l$

$a + a+d + \cdots l-d + l \qquad \frac{n}{2}(a+l)$

$a+d + l-d$

$= a+l + a+l + a+l$

l

$a_1 + a_2 + \cdots\ a_n$

$l = a + (n-1)d$

$= \frac{n}{2}(a + l)$

$= \frac{n}{2}(a + a + (n-1)d) = \frac{n}{2}(2a + (n-1)d)$

$a \qquad b \qquad c$ (in AP)

$b - a = c - b$

$\Rightarrow \quad 2b = a + c \qquad \Rightarrow \quad b = \dfrac{a+c}{2}$

$$a \xrightarrow{d} b \xrightarrow{d} c \xrightarrow{d} d \xrightarrow{d} e$$

$$a \xrightarrow{2d} c \xrightarrow{2d} e$$

(in AP)

$$c = \frac{a+e}{2}$$

1, 5, 9, 13, 17

$\frac{n-1}{2}(a+l)$ + middle term

5 terms.

$\frac{5-1}{2}$ pairs

$= 2$ pairs

$\frac{a+l}{2}$

1, 5, 9, 13, 17

$$9 = \frac{1+17}{2}$$

$$\frac{n-1}{2}(a+l) + \frac{a+l}{2}$$

$$= \frac{n}{2}(a+l)$$

$1 + 3 + 5$

$$\frac{n}{2}\left(a + a + (n-1)d\right)$$

$$\frac{n}{2}\left(a + l\right)$$

$$= \frac{3}{2}\left(1 + 5\right)$$

$$= \frac{3}{2} \times 6 = 9$$

formula to sum n numbers –

$$S_n = \frac{n}{2}(a + l)$$

$$= \frac{n}{2}\left(2a + (n-1)d\right)$$

odd

$1 + 3 + 5 + \cdots\cdots$ (AP)

No. of terms $= 15$
(n)

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

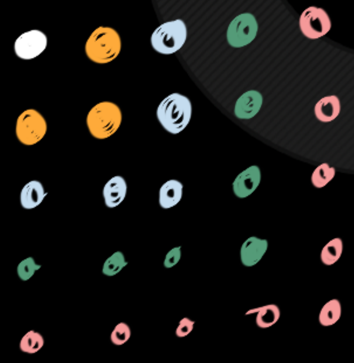
$= \frac{15}{2}\left(2 + 14 \times 2\right)$

$= \underline{\underline{225}}$

$$\underbrace{1 + 3 + 5 + 7}_{4^2 = 16} + 9$$

$$\underbrace{7 + 9}_{}$$

$$5^2 = 25$$



[first n consecutively odd numbers is n^2]

$$1 + 2 + 3 + 4 + \dots \qquad \dots 97 + 98 + 99 + 100$$

The Story of Gauss

There are several stories of his early genius. One story has it that in primary school after the young Gauss misbehaved, his teacher, J.G. Büttner, gave him a task: add a list of integers in arithmetic progression; as the story is most often told, these were the numbers from 1 to 100. The young Gauss reputedly produced the correct answer within seconds, to the astonishment of his teacher and his assistant Martin Bartels. Gauss's presumed method was to realize that pairwise addition of terms from opposite ends of the list yielded identical intermediate sums: $1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$, and so on, for a total sum of $50 \times 101 = 5050$.

Source- wikipedia

- the normal distribution, also known as the Gaussian distribution, the most common bell curve in statistics;
- the Gauss Prize, one of the highest honors in mathematics;
- gauss, CGS unit for magnetic field.
- Gaussian elimination
- If we except the great name of Newton it is probable that no mathematicians of any age or country have ever surpassed Gauss - The British mathematician Henry John Stephen Smith

Question

- Find the sum of following AP –

$$1,3,\ldots\ldots 15$$

Few Formulas

$$1 + 2 + 3 + \cdots n = \frac{n(n+1)}{2}$$

using AP formula

$$1^2 + 2^2 + 3^2 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Remember

Example 1 (AMC 10B/12B 2004). *A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?*

H.W.

https://www.andrew.cmu.edu/user/daltizio/Sequences%20and%20Series.pdf

Solution. Note that the number of cans on each of the n rows forms an arithmetic sequence with first term 1 and common difference 2. The total number of cans on n rows is thus

$$\frac{1}{2}n[2+(n-1)\cdot 2]=\frac{1}{2}n[2n]=n^2.$$

Setting this equal to 100 gives $n=\boxed{10}$. ■

Question

If $(x + 1)$, $3x$ and $(4x + 2)$ are first three terms of an A.P. then find its 5^{th} term

H.W.

Solution : $(x + 1), 3x, (4x + 2)$ are in AP

$\Rightarrow \quad 3x - (x + 1) = (4x + 2) - 3x$

$\Rightarrow \quad x = 3$

$\therefore \quad a = 4, d = 9 - 4 = 5$

$\Rightarrow \quad T_5 = 4 + (4)5 = 24$

$$\sum_{k=2}^{\infty} k = -1 + \sum_{k=1}^{\infty} k$$

$j = k - 1$

$$\sum_{j=1}^{\infty} (j+1)$$

$k = 2, \quad 3, \quad \ldots \quad \infty$

$j = 1 \quad 2 \quad \ldots \quad \infty$

Tharun kumar reddy Ganga to Everyone 8:52 PM

T

Sir actually 1+2+3+4.... infinity is -1/12 according to ramanujan

$$\sum_{k=2}^{n} k = -1 + \sum_{k=1}^{n} k$$

$$= \sum_{j=1}^{n-1} (j+1)$$

$j = k-1$

$$\sum_{j=1}^{n-1} j + \sum_{j=1}^{n-1} 1$$

$$\sum_{j=1}^{n} j$$

$k = 2, 3, \ldots \infty$

$j = 1 \quad 2 \quad \ldots \infty$

$n-1$