Digital Logic :

Lecture 5 :

Standard Forms of Boolean Expressions

Minterm, Maxterm, SOP, POS

Be Careful with Bars

$\overline{XY} = \overline{X} + \overline{Y}$

$$\overline{X}\,\overline{Y} \neq \overline{XY}$$

Let's check all the combinations of X and Y:

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \cdot \overline{Y}$	XY	$\overline{XY}$
0	0	1	1	1	0	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	0	1	0

Boolean Algebra :

Next Topic :

Standard Forms of Boolean Expressions

Standard Boolean Expression Formats

Standard Forms

- There are many ways to express a boolean expression

$$F = XYZ + XYZ + XZ$$
$$= XY(Z + Z) + XZ$$
$$= XY + XZ$$

- It is useful to have a standard or canonical way

- Derived from truth table

- Generally not the simplest form

$F = a + b$

$F = \bar{a}b + a\bar{b} + ab$

a	b	$f(a,b) = a+b$
0	0	0
0	1	1
1	0	1
1	1	1

$F = a + b$ is 1 iff

$a=0, b=1$ $\bar{a}b$

OR — $+$

$a=1, b=0$ $a\bar{b}$

OR — $+$

$a=1, b=1$ — ab

Over 2 Boolean variables, we can only have 16 different Boolean expressions/functions.

$2^{\binom{2}{2}}$ $\binom{2}{2}$ $= 2$

Same Boolean expression/function can be written in many ways if some standard representation is not followed.

Truth Tables for the 16 Functions of Two Binary Variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Truth Tables for the 16 Functions of Two Binary Variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

$F_0 = 0$ $F_1 = xy$ $F_3 = x$ $F_5 = y$ $F_6 = x \oplus y$ $F_{14} = \overline{xy}$ $F_{15} = 1$

Boolean Expressions for the 16 Functions of Two Variables

Boolean Functions	Operator Symbol	Name	Comments
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_3 = x$		Transfer	x
$F_4 = x'y$	y/x	Inhibition	y, but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$(x \oplus y)'$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y, then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x, then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1

Two principle standard forms

- Sum-of-products (SOP)
- Product-of-sums (POS)

Product term: AND of literals OR a single Literal

a ✓

$\bar{a}$ ✓

aa ✓

$a\bar{a}$ ✓

abc ✓

$a\bar{b}c$ ✓

$aa\bar{b}\bar{a}$ ✓

$a+b$ ✗

$\overline{a+b+c}$ ✗

$\overline{ab}$ ✗

$\bar{a}\bar{b}$ ✓

$\bar{a}\overline{bc}$ ✗

$\bar{a}\bar{b}\bar{c}$ ✓

$\bar{a}\bar{b}\bar{c}$ ✓

$\bar{a}\bar{b}\bar{b}a$ ✓

Sum term: OR of literals OR a single Literal

a ✓

$a+a$ ✓

$a+b$ ✓

$a+\bar{a}$ ✓

ab ✗

$a+ab$ ✗

$\overline{a+b}$ ✗

$\bar{a}+\bar{b}$ ✓

$a+\overline{b+c}$ ✗

$a+\bar{b}+\bar{c}$ ✓

$\overline{ab}$ ✗

$\bar{a}+\bar{b}$ ✓

Some Definitions

- A ***literal*** is a complemented or uncomplemented boolean variable.
  - *Examples:* a and $\bar{a}$ are distinct literals. $\bar{a}+cd$ is not.

- A ***product term*** is a single literal or a logical product (AND) of two or more literals.
  - *Examples:* $a, \bar{a}, ac, \bar{a}cd, aa\bar{a}b$ are product terms; $\bar{a}+cd$ is not a product term.
- A ***sum term*** is a single literal or a logical sum (OR) of two or more literals.
  - *Examples:* $a, \bar{a}, a+c, \bar{a}+c+d$ are sum terms; $\bar{a}+cd$ is not a sum term.

Sum Term ?

$A + BB + C$ ✗

$A + B + C$ ✓

$A + B\overline{B}$ ✗

$A(B+B)C$ — NoT a product term

Product and sum terms

- Product term: logical AND of literals (e.g., $X\overline{Y}Z$)
- Sum term: logical OR of literals (e.g., $A + \overline{B} + C$)

SOP : Sum(OR) of Product terms.

SOP form $= P_1 + P_2 + P_3$

POS : Product(AND) of Sum terms.

POS form $= (S_1)(S_2)(S_3)$

POS ?

$a+b+c$ ✓

$a+\bar{b}$ ✓

$(a+b+c)$

$(a+\bar{b})$

We are talking about forms of Same Expression ;

$$\left.\begin{array}{l} aa \\ a+a \end{array}\right\}$$ Same But Different forms.

Expression = a

$(a) + (aa)$

Two forms

$a + aa$ — In SOP form

In POS form ~~X~~

$(a + aa)$

NOT a sum term

aa

In POS $(a)(a)$

In SOP ✓ (aa) ✓

$\underbrace{a + b + c}$

Expression

In SOP? ✓ $(a)+(b)+(c)$

In POS? ✓ $(a+b+c)$

Some Definitions

- A ***sum of products (SOP)*** expressions is a set of product (AND) terms connected with logical sum (OR) operators.
  - *Examples:* $a, \bar{a}, ab+c, \bar{a}c+bde, a+b$ are SOP expressions.

- A ***product of sum (POS)*** expressions is a set of sum (OR) terms connected with logical product (OR) operators.
  - *Examples:* $a, \bar{a}, a+b+c, (\bar{a}+c)(b+d)$ are POS expressions.

Sum of Product (SOP) form of an Expression: Sum (OR) of Product terms.

Eg: $E: \underbrace{a + bc}$ $\Rightarrow$ in SOP form

$E: a + bc$ — Not In POS form

$E: a+b$ $\begin{cases} \text{In POS} ✓ & (a+b) \\ \text{Is SOP} & (a)+(b) \end{cases}$

$E : a + bc$ — This form NOT in POS

$E : (a+b)(a+c)$ — Now this form In POS ✓

$\underbrace{a + aaa}$ — NOT in POS form

$(a + a)$ — Now it is in POS form.

Product of Sum (POS) form of an Expression: Product (AND) of sum terms.

Eg: $E = (a+b)(b+c) \Rightarrow$ in POS form

Which Expression is in SOP form?

~~1~~ $(ab+c)(c+a)$

~~2~~ $a+b$

~~3~~ a — SOP, POS

~~4~~ $ab+c$

~~4~~ $\bar{a}$

~~5~~ $(a+\bar{a})(b)$

~~6~~ $a+\bar{a}+b$

$E = (ab + c)(c + a)$ — This form is NOT SOP

$E = abc + aba + cc + ca$ — This form is in SOP.

Which Expression is in POS form?

~~(1)~~ $(ab+c)(c+a)$

Not a sum term

(2) $(a+b)$

(3) a

~~(7)~~ $ab+c$

(4) $\bar{a}$

(5) $(a+\bar{a})(b)$

(6) $(a+\bar{a}+b)$

Which Expression is in SOP, POS form?

(1) $a+b$ — SOP ✓ $(a)+(b)$; POS ✓ $(a+b)$

(2) $a(b+c)$ — POS ✓ SOP ✗

(3) $a+\bar{a}+b+bc$ — SOP ✓ ; POS ✗

(4) $ab+ac$ — SOP ✓ POS ✗

(5) $bc+ab$ — SOP ✓ ; POS ✗

(6) $(a+bc)(c)$ — SOP ✗ POS ✗

(7) $a+(b)(b+c)$ — POS ✗ SOP ✗

Sum-of-Products

A sum-of-products (SOP) expression is a Boolean expression in a specific format. The term sum-of-products comes from the expression's form:

a sum (OR) of one or more products (AND).

Below is an example of an SOP expression:

$$\underbrace{\bar{A}BC\bar{D}} + \underbrace{A\bar{B}D} + \underbrace{C\bar{D}} + \underbrace{A\bar{D}}$$

Q:

$$\overline{abc} + bc \Rightarrow \text{Not SOP}$$

product term

Not a product term

Sum of product

$E = \overline{abc} + bc$ — NOT In SOP form

NOT product term

$E = \bar{a} + \bar{b} + \bar{c} + bc$ — In SOP form

Sum-of-Products

An SOP expression cannot have more than one variable combined in a term with an inversion bar.

The following is Not an SOP expression:

Not product term

$$(\overline{AB})\overline{CD} + A\overline{B}D + C\overline{D} + A\overline{D}$$

Q : Convert this expression into SOP form.

NOT SOP — $(\overline{\overline{AB}})C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$

NOT SOP — $(\overline{A} + \overline{B})C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$

SOP — $\overline{A}C\overline{D} + \overline{B}C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$

This expression is now considered to be in SOP format.

E: $\overline{abc}$ $\longrightarrow$ Not in SOP form

$\downarrow$

E: $\underbrace{\bar{a}+\bar{b}+\bar{c}}$ $\Longrightarrow$ Now in SOP form

$\overline{abc}$ ⇒ Not a product term

Product term : AND of literals

$\overline{abc}$ ⇒ Not a literal

$\overline{abc}$ ⇒ Not product of literals

$\bar{a}\,\bar{b}\,\bar{c}$ ⇒ Product of literals

PoS & SoP

- Sum of products (SoP): OR of ANDs

$$\text{e.g., } F = \overline{Y} + \overline{X}Y\overline{Z} + XY$$

- Product of sums (PoS): AND of ORs

$$\text{e.g., } G = X(\overline{Y} + Z)(X + Y + \overline{Z})$$

Do it yourself

- Put the right ticks in the following table.

Expression	*SOP?*	*POS?*
$X' \cdot Y + X \cdot Y' + X \cdot Y \cdot Z$		
$(X+Y') \cdot (X'+Y) \cdot (X'+Z')$		
$X' + Y + Z$		
$X \cdot (W' + Y \cdot Z)$		
$X \cdot Y \cdot Z'$		
$W \cdot X' \cdot Y + V \cdot (X \cdot Z + W')$		

Do it yourself

- Put the right ticks in the following table.

Expression	*SOP?*	*POS?*
$X' \cdot Y + X \cdot Y' + X \cdot Y \cdot Z$	✓	✗
$(X+Y') \cdot (X'+Y) \cdot (X'+Z')$	✗	✓
$X' + Y + Z$	✓	✓
$X \cdot (W' + Y \cdot Z)$	✗	✗
$X \cdot Y \cdot Z'$	✓	✓
$W \cdot X' \cdot Y + V \cdot (X \cdot Z + W')$	✗	✗

$x \cdot (y + \bar{w}z)$ — Not In SoP

literal ✓

$\underline{\underline{x}} \cdot (y \underline{\underline{+}} \underline{\underline{\bar{w}z}})$ — ~~Not POS~~

Not a literal

x → sum term ✓

$(y + \bar{w}z)$ → sum term?

Sum term → OR of literals.

Boolean Algebra :

Next Topic :

Minterm, Maxterm

Misconceptions Alert

Definitions

- Literals x_i or x_i'
- Product Term $x_2x_1'x_0$
- Sum Term $x_2 + x_1' + x_0$
- Minterm of n variables: A product of n literals in which every variable appears exactly once.
- Maxterm of n variables: A sum of n literals in which every variable appears exactly once.

Minterm :

a product term containing "EACH" variable "EXACTLY ONCE" in Complemented OR UnComplement form But not both.

Maxterm :

a Sum term Containing "EACH" variable "EXACTLY ONCE" in Complemented OR Uncomplement form But not both.

$f(a,b) \longrightarrow \#\text{variables} = 2 \begin{cases} a \\ b \end{cases}$

minterms of f :

a ✗

$ab\bar{b}$ ✗

abb ✗

$\bar{a}\bar{b}$ ✓

ab ✓

$\bar{a}b$ ✓

$a\bar{b}$ ✓

$a+\bar{b}$ ✗

b ✗

$\bar{b}$ ✗

$a+b$ ✗

$f(a, b)$

sum term

maxterms :

a ✗ $\quad ab$ ✗ $\quad \bar{a}a + b$ ✗

$a + b$ ✓ $\quad \bar{a} + \bar{b}$ ✓

$\bar{a} + b$ ✓

$a + \bar{b}$ ✓ $\quad \bar{a} + a + b$ ✗

- Minterm
  - An product of literals in which each variable is represented once and only once in either its complemented or uncomplemented form.

- Maxterm
  - An sum of literals in which each variable is represented once and only once in either its complemented or uncomplemented form.

for 1 variable a:

minterms = 2

max terms = 2

minterms: a, $\bar{a}$

max terms: a, $\bar{a}$

for 3 variables a, b, c :

minterms : abc, $\bar{a}bc$, $a\bar{b}c$, $\bar{a}\bar{b}\bar{c}$, ----

Product term

$\_\ \_\ \_ = 2 \times 2 \times 2 = 2^3$

$\underbrace{a, \bar{a}}_{2}$ $\underbrace{b, \bar{b}}_{2}$ $\underbrace{c, \bar{c}}_{2}$

minterms

for 3 variables a, b, c :

maxterms $\Rightarrow$ $a+b+c, \ \bar{a}+b+c, \ \bar{a}+\bar{b}+\bar{c}, \ \ldots$

Sum term

$\_ + \_ + \_ = 2 \times 2 \times 2 = 2^3 = 8$

$\underbrace{a, \bar{a}}_{2} \quad \underbrace{b, \bar{b}}_{2} \quad \underbrace{c, \bar{c}}_{2}$

maxterms

$a+b+c = a+c+b$

Note: In all Definitions

(Sum term, product term, minterm, maxterm, SOP, POS)

Order of terms/literals Does not matter.

for n variables : $\{a_1, a_2, \ldots, a_n\}$

$$\left.\begin{array}{l}\#\text{minterms} = 2^n \\ \#\text{maxterms} = 2^n\end{array}\right\}$$

$\_\ \_\ \_\ \_\ \_ \ldots \_ = 2^n$ ✓

$\_ + \_ + \_ + \_ + \cdots + \_$
$= 2^n$

Minterms

- **Minterms** are AND terms with every variable present in either true or complemented form.
- Given that each binary variable may appear normal (e.g., x) or complemented (e.g.,), there are 2^n minterms for *n* variables.
- **Example:** Two variables (X and Y)produce 2 x 2 = 4 combinations:
  - (both normal)
  - (X normal, Y complemented)
  - (X complemented, Y normal)
  - (both complemented)
- Thus there are **four minterms** of two variables.

Minterm

– A minterm of n variables = product of n literals in which each variable appears exactly once either in T or F form, but not in both. (Also known as a standard product term)

Minterm and Maxterm

- For a switching function, a ***literal*** is defined as a variable in uncomplemented or complemented form.
  - Example: x, x′, y, y′, etc.
- Consider an n-variable switching function $f(x_1, x_2, \ldots, x_n)$.
  - A product term (that is, an AND operation) of all the n literals is called a ***minterm***.
  - A sum term (that is, an OR operation) of all the n literals is called a ***maxterm***.
- Consider a 3-variable function f (A, B, C).
  - Examples of minterm: A′.B′.C′, A.B′.C, A.B.C, etc.
  - Examples of maxterm: (A + B′ + C′), (A′ + B′ + C′), (A + B + C), etc.

Minterms and Maxterms (1/2)

- A minterm of n variables is a product term that contains n literals from all the variables.

  Example: On 2 variables x and y, the minterms are:

  $x' \cdot y'$, $x' \cdot y$, $x \cdot y'$ and $x \cdot y$

- A maxterm of n variables is a sum term that contains n literals from all the variables.

  Example: On 2 variables x and y, the maxterms are:

  $x'+y'$, $x'+y$, $x+y'$ and $x+y$

- In general, with n variables we have 2^n minterms and 2^n maxterms.

Analysis of minterm :

$f(a,b) \rightarrow$ minterm : $\bar{a}b$

let's consider it as a function

a	b	$\bar{a}b$	$\bar{a}\bar{b}$	ab	$a\bar{b}$
0	0	0	1	0	0
0	1	1	0	0	0
1	0	0	0	0	1
1	1	0	0	1	0

So, Every minterm is 1 for Exactly one Row (input Combination)

Eg: $\bar{a}\,\bar{b}$ is 1 for $a=0, b=0$ Row;

So, we say that $\bar{a}\,\bar{b}$ is the minterm associated with Row $a=0, b=0$

Similarly,

minterm $\bar{a}b$ is Associated with $a=0, b=1$ Row.

" ab " " " $a=1, b=1$

" $a\bar{b}$ " " " $a=1, b=0$

Associates minterm with each Row

a	b	
0	0	$\bar{a}\,\bar{b}$
0	1	$\bar{a}\,b$
1	0	$a\,\bar{b}$
1	1	$a\,b$

Observe:

$\underbrace{a=0, b=1}$

minterm $\bar{a}\,b$

$\underbrace{a=0, b=0}$

minterm $= \bar{a}\,\bar{b}$

$f(a,b,c)$

a	b	c	(Associates) minterm → Product term
0	0	0	$\bar{a}\,\bar{b}\,\bar{c}$
0	0	1	$\bar{a}\,\bar{b}\,c$
0	1	0	$\bar{a}\,b\,\bar{c}$
0	1	1	$\bar{a}\,b\,c$
1	0	0	$a\,\bar{b}\,\bar{c}$
1	0	1	$a\,\bar{b}\,c$
1	1	0	$a\,b\,\bar{c}$
1	1	1	$a\,b\,c$

Analysis of maxterm :

sum term

$f(a,b) \rightarrow \underbrace{\bar{a}+b}$ maxterm

let's Consider it as a function

a	b	$\bar{a}+b$	$\bar{a}+\bar{b}$	$a+\bar{b}$	$a+b$
0	0	1	1	1	0
0	1	1	1	0	1
1	0	0	1	1	1
1	1	1	0	1	1

Every maxterm is 0 for Exactly one Row (input Combination)

Eg: maxterm $\bar{a}+b$ is 0 for $a=1, b=0$

So, we say that maxterm $\bar{a}+b$ is Associated with $a=1, b=0$ Row.

Similarly,

maxterm $a+b$ is Associated with $a=0, b=0$

" $a+\bar{b}$ " " " $a=0, b=1$

" $\bar{a}+\bar{b}$ " " " $a=1, b=1$

$f(a,b)$

(Associates) maxterms

a	b	maxterms
0	0	$a+b$
0	1	$a+\bar{b}$
1	0	$\bar{a}+b$
1	1	$\bar{a}+\bar{b}$

$\underbrace{a=0,\ b=1}$

maxterm $= a + \bar{b}$

$a=1,\ b=1$

$\begin{cases} \text{minterm} = ab \\ \text{maxterm} = \bar{a}+\bar{b} \end{cases}$

$f(a,b,c)$;

	product term — minterm	sum term — maxterm
$a=0,\ b=1,\ c=0$	$\bar{a}\,b\,\bar{c}$	$a+\bar{b}+c$
$a=1,\ b=0,\ c=0$	$a\,\bar{b}\,\bar{c}$	$\bar{a}+b+c$
$a=1,\ b=1,\ c=1$	$a\,b\,c$	$\bar{a}+\bar{b}+\bar{c}$
$a=0,\ b=0,\ c=0$	$\bar{a}\,\bar{b}\,\bar{c}$	$a+b+c$

Short Notation for minterm

	a	b	minterms
0	0	0	$\bar{a}\,\bar{b} = m_0$
1	0	1	$\bar{a}\,b = m_1$
2	1	0	$a\,\bar{b} = m_2$
3	1	1	$a\,b = m_3$

	a	b	Maxterm	Short Notation
0	0	0	$a+b$	M_0
1	0	1	$a+\bar{b}$	M_1
2	1	0	$\bar{a}+b$	M_2
3	1	1	$\bar{a}+\bar{b}$	M_3

$m_2 = a\bar{b}$

$b\bar{a}$

$m_2 \rightarrow 10$

we fix order

M_3 — maxterm for binary value 3.

Note:

for short notation of minterm, maxterm (m_i, M_i), one fixes order of variables is considered.

Eg: $f(a, b, c) \Rightarrow$

a b c

0 1 0

0 1 1

a b c

$m_0 = \bar{a}\,\bar{b}\,\bar{c}$

$m_2 = \bar{a}\,b\,\bar{c}$

$m_3 = \bar{a}\,b\,c$

Eg:

$f(c, a, b)$

order

$m_3 = \bar{c}ab$

011

c a b

$m_2 = \bar{c}a\bar{b}$

010

c a b

Note: by default, the order is "**Alphabatical**".

Ex:

abc	wxyz	xy
xyz	xy	pqrst
pqr	ab	

Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
4 1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterms and Maxterms (2/2)

- The minterms and maxterms on 2 variables are denoted by m0 to m3 and M0 to M3 respectively.

x	y	Minterms		Maxterms	
		Term	Notation	Term	Notation
0	0	$x' \cdot y'$	m0	$x+y$	M0
0	1	$x' \cdot y$	m1	$x+y'$	M1
1	0	$x \cdot y'$	m2	$x'+y$	M2
1	1	$x \cdot y$	m3	$x'+y'$	M3

- Each minterm is the complement of the corresponding maxterm
  - Example: $m2 = x \cdot y'$

    $m2' = (x \cdot y')' = x' + (y')' = x' + y = M2$

Standard Order

- Minterms and maxterms are designated with a subscript
- The subscript is a number, corresponding to a binary pattern
- The bits in the pattern represent the complemented or normal state of each variable listed in a standard order.
- All variables will be present in a minterm or maxterm and will be listed in the same order (usually alphabetically)
- Example: For variables a, b, c:
  - Maxterms: $(a + b + \overline{c}), (a + b + c)$
  - Terms: $(b + a + c), a\overline{c}$b, and $(c + b + a)$ are NOT in standard order.
  - Minterms: $a\overline{b}c, abc, a\overline{b}\overline{c}$
  - Terms: $(a + c), \overline{b}c$, and $(\overline{a} + b)$ do not contain all variables

Purpose of the Index

- The index for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.
- For Minterms:
  - "1" means the variable is "Not Complemented" and
  - "0" means the variable is "Complemented".
- For Maxterms:
  - "0" means the variable is "Not Complemented" and
  - "1" means the variable is "Complemented".

a	b	c	d	minterm	maxterm
0	1	0	0	$\bar{a} b \bar{c} \bar{d}$	$a + \bar{b} + c + d$
1	0	1	0	$a \bar{b} c \bar{d}$	$\bar{a} + b + \bar{c} + d$

Index Example in Three Variables

- Example: (for three variables)
- Assume the variables are called X, Y, and Z.
- The standard order is X, then Y, then Z.
- The Index 0 (base 10) = 000 (base 2) for three variables). All three variables are complemented for minterm 0 $(\overline{X}, \overline{Y}, \overline{Z})$ and no variables are complemented for Maxterm 0 (X, Y, Z).
  - Minterm 0, called m_0 is $\overline{X}.\overline{Y}.\overline{Z}$
  - Maxterm 0, called M_0 is $(X + Y + Z)$
  - Minterm 6 ? $= XY\overline{Z}$
  - Maxterm 6 ? $= \overline{X} + \overline{Y} + Z$

110

HW

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\bar{b}\bar{c}\bar{d}$	$a+b+c+d$
1	0001	$\bar{a}\bar{b}\bar{c}d$	?
3	0011	?	$a+b+\bar{c}+\bar{d}$
5	0101	$\bar{a}b\bar{c}d$	$a+\bar{b}+c+\bar{d}$
7	0111	?	$a+\bar{b}+\bar{c}+\bar{d}$
10	1010	$a\bar{b}c\bar{d}$	$\bar{a}+b+\bar{c}+d$
13	1101	$ab\bar{c}d$	?
15	1111	$abcd$	$\bar{a}+\bar{b}+\bar{c}+\bar{d}$

Function Tables for Both

- Minterms of 2 variables

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

- Each column in the maxterm function table is the complement of the column in the minterm function table since M_i is the complement of m_i.

- Properties of minterms and maxterms:
  - A minterm assumes value 1 for exactly one combination of variables.
  - A maxterm assumes the value 0 for exactly one combination of variables.
- Example:

Minterms

Each row has a unique minterm

X	Y	**Minterm**	$\overline{X}\,\overline{Y}$	$\overline{X}Y$	$X\overline{Y}$	XY
0	0	$\overline{X}\,\overline{Y}$	1	0	0	0
0	1	$\overline{X}Y$	0	1	0	0
1	0	$X\overline{Y}$	0	0	1	0
1	1	XY	0	0	0	1

The minterm is the product term that is 1 for only its row

Maxterms

Each row has a unique maxterm

X	Y	**Maxterm**	$X+Y$	$X+\overline{Y}$	$\overline{X}+Y$	$\overline{X}+\overline{Y}$
0	0	$X+Y$	0	1	1	1
0	1	$X+\overline{Y}$	1	0	1	1
1	0	$\overline{X}+Y$	1	1	0	1
1	1	$\overline{X}+\overline{Y}$	1	1	1	0

The maxterm is the sum term that is 1 for only its row

Boolean Algebra :

Next Topic :

Minterm, Maxterm Relationship

Minterm and Maxterm Relationship

- Review: DeMorgan's Theorem
  $\overline{x.y} = \overline{x} + \overline{y}$ and $\overline{x+y} = \overline{x}.\overline{y}$
- Two-variable example:
  $M_2 = \overline{x} + y$ and $m_2 = x.\overline{y}$
  Thus M_2 is the complement of m_2 and vice-versa.
- Since DeMorgan's Theorem holds for *n* variables, the above holds for terms of *n* variables
- giving:
  $M_i = \overline{m_i}$ and $m_i = \overline{M_i}$
  Thus M_i is the complement of m_i.

$f(a,b,c)$

$m_6 = ab\bar{c}$

$M_6 = \bar{a}+\bar{b}+c$ ← 110

$$\overline{m_6} = \overline{ab\bar{c}} = \underbrace{\bar{a}+\bar{b}+c}_{M_6}$$