# Translating English to FOL

English statement : P: All crows are black

P $\longrightarrow$ $\psi$

English statement

FOL Expression

$$P \equiv \psi$$

P True $\longrightarrow$ $\psi$ True

P false $\longrightarrow$ $\psi$ false

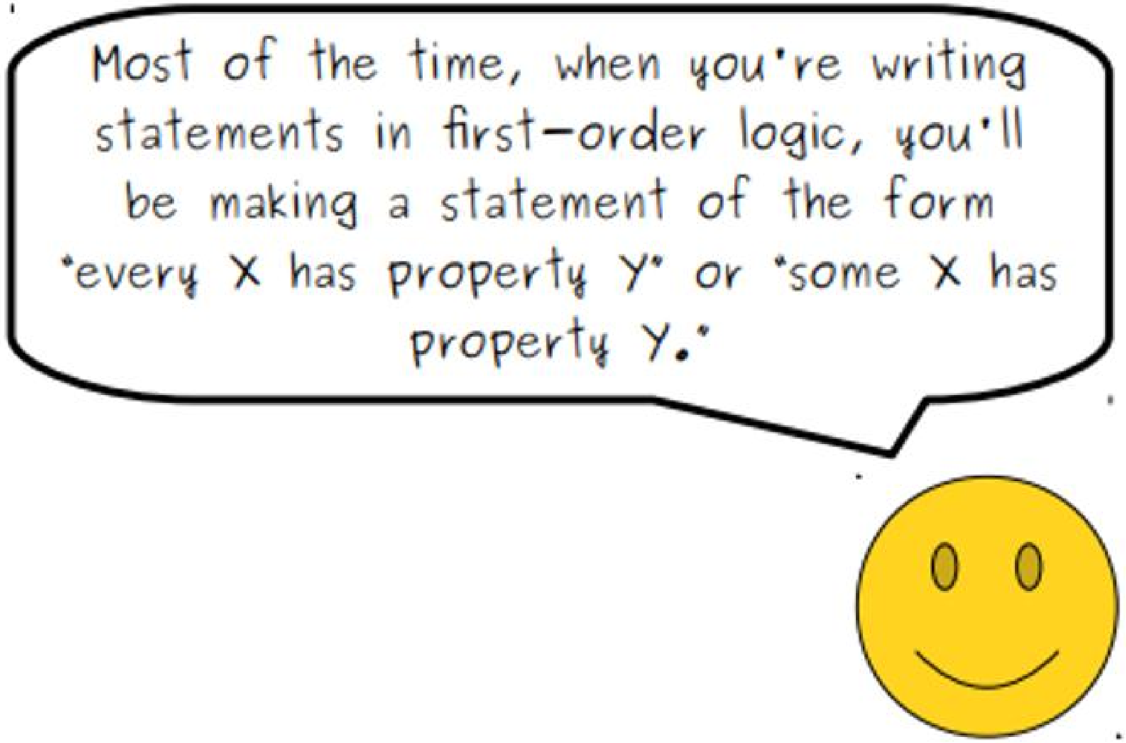
$P \equiv \psi$

English Statement

FOL Expression

Whenever P True $\longrightarrow$ $\psi$ True

Whenever p false $\longrightarrow$ $\psi$ false


Most of the time, when you're writing statements in first-order logic, you'll be making a statement of the form "every X has property Y" or "some X has property Y."

Four fundamental types of statements :

"All *P*s are *Q*s."

"Some *P*s are *Q*s."

"No *P*s are *Q*s."

"Some *P*s aren't *Q*s."

“All $A$'s are $B$'s”

“All crows are black”

All Natural numbers are integers.

All indians are asians.

(1) When Domain is Not specified, then Domain Contains Everything.

$\boxed{\text{All crows are Black}}$

≡ for every element in the Domain, If it is crow then it is black

$$\equiv \forall_x \left( Crow(x) \longrightarrow Black(x) \right)$$

P : All crows are black.

Q: $\forall_x \big( \text{Crow}(x) \wedge \text{Black}(x) \big)$

Everything in the Domain is crow and black.

P : All crows are black.

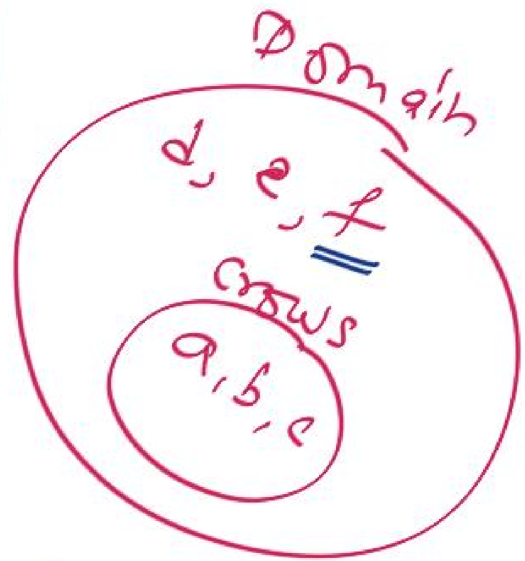
①: $\forall_x \left( Crow(x) \wedge Black(x) \right)$

Everything in the Domain is crow and black.

P : T

① : false

Scenario:




$Crow(f) = false$

1: All crows are Black.

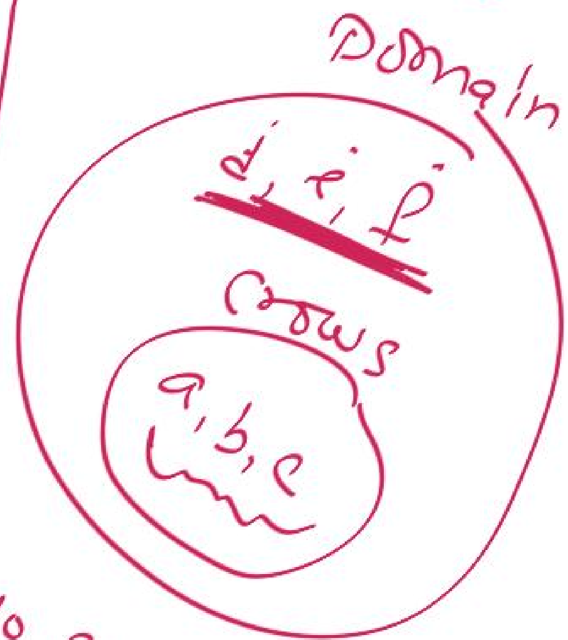
2: $\forall_x \boxed{(Crow(x) \lor Black(x))}$

Everything in the Domain is crow or Black.

P: F

2: T

Scenario




No crow is Black.

All Non-crow is Black.

P: All crows are Black

(3): $\forall_x \left( \text{crow}(x) \longleftrightarrow \text{Black}(x) \right)$

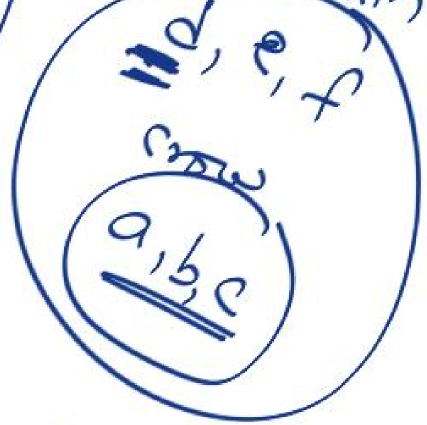
Everything in the Domain is Black iff it is crow.

p: True

3: false

Scenario

Domain




d ⇒ Not crow

d ⇒ Black

All crows are Black.

P: All crows are black.

Everything in the Domain, If it is crow then it is black.

Y: $\forall x \left( Crow(x) \longrightarrow Black(x) \right)$

P : All crows are Black

$\psi : \forall x\,(\text{Crow}(x) \rightarrow \text{Black})$

Domain: c, d crows, a, b

When P is True

$\psi$ cannot be false.

$\boxed{\text{Crow} \rightarrow \text{Black} \equiv \text{False}}$

T F

then $\psi$ is True

When P is false.

there is some crow which is Not Black.

$\psi$ will false.

All crows are Black

$$\equiv \forall_x \left( Crow(x) \rightarrow Black(x) \right)$$

Why??

Imp:

Template in FOL

**"All *A*'s are *B*'s"**

$\forall x\,(A(x) \rightarrow B(x))$

translates as

$$\forall x.\ (A(x) \rightarrow B(x))$$

“Some $A$ is a $B$”

Some Crows are Black.

At least one crow is Black. $\equiv$ There exist an element in the Domain such that it is a crow and it is Black.

$$\underbrace{\exists_x \left( Crow(x) \wedge Black(x) \right)}$$

P : Some crows are Black.

1 : $\exists x \left( \boxed{\text{Crow}(x) \rightarrow \text{Black}(x)} \right)$

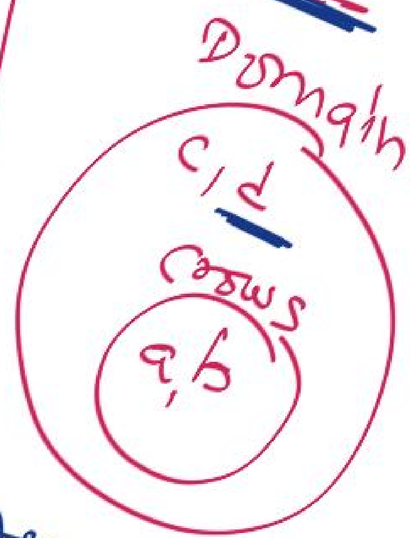
$\equiv \exists x \left( \neg \text{Crow}(x) \vee \text{Black}(x) \right)$

$\equiv$ there is some element in Domain which is Black or not crow.

P : false

1 : True

Scenario




Assume No Crow is Black.

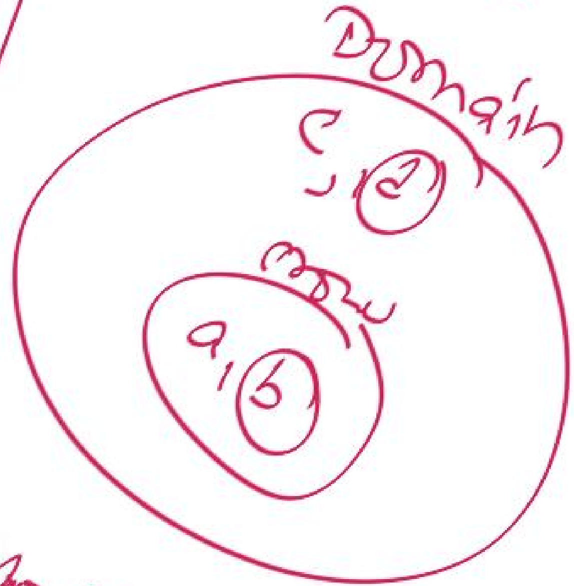
②: $\exists x\,(\boxed{\text{Crow}(x) \lor \text{Black}(x)})$

P: Some crows are Black

P: False

②: True

Scenario




No crow is Black

P: Some Crows are Black

$\psi$: $\exists x\,(\boxed{Crow(x) \wedge Black(x)})$

When P = True

$\psi$ = True

When P = false

No crow is Black

$\psi$ cannot be True

$\psi$ false

Some Crows are Black

$$\exists_x \left( Crow(x) \wedge Black(x) \right)$$

$$\boxed{P \rightarrow Q \equiv \neg P \vee Q}$$

Imp- Template in FOL

**"Some $A$ is a $B$"**

translates as $\exists_x (A(x) \land B(x))$

$$\exists x.\ (A(x) \land B(x))$$

"All $A$s are $B$s"

$\forall x. (A(x) \to B(x))$

"Some $A$s are $B$s"

$\exists x. (A(x) \land B(x))$

"No $A$s are $B$s"

$\forall x. (A(x) \to \neg B(x))$

"Some $A$s aren't $B$s"

$\exists x. (A(x) \land \neg B(x))$

It is worth committing these patterns to memory. We'll be using them throughout the day and they form the backbone of many first-order logic translations.

Some A's are Not B's

$$\exists_x \left( A(x) \wedge \neg B(x) \right)$$

Some crows are Not Black,

$$\exists x \left( Crow(x) \wedge \neg Black(x) \right)$$

No A's are B's

$$\forall_x \left( A(x) \rightarrow \neg B(x) \right)$$

$$\neg \exists_x \left( A(x) \wedge B(x) \right)$$

No crows are Black.

There Doesnot exist an element which is crow and Black.

$\equiv \neg \exists_x (Crow(x) \land Black(x))$

for every element, If it is crow then It is Not Black.

$\forall_x (Crow(x) \rightarrow \neg Black(x))$

$\exists_x$ : There exists $x$

$\neg \exists_x$ : There Does not exist $x$

"All $P$s are $Q$s."

$$\forall x. (P(x) \rightarrow Q(x))$$

"Some $P$s are $Q$s."

$$\exists x. (P(x) \land Q(x))$$

"No $P$s are $Q$s."

$$\forall x. (P(x) \rightarrow \neg Q(x))$$

"Some $P$s aren't $Q$s."

$$\exists x. (P(x) \land \neg Q(x))$$

All A's are B's : $\forall_x \left( A(x) \rightarrow B(x) \right)$

Some A's are B's : $\exists_x \left( A(x) \wedge B(x) \right)$

No A's are B's : $\forall_x \left( A(x) \rightarrow \neg B(x) \right)$

$$\equiv \neg \exists_x \left( A(x) \wedge B(x) \right)$$

Some A's are Not B's $\equiv \exists_x \left( A(x) \wedge \neg B(x) \right)$

*Available Predicates:*

$Orange(x)$
$Cat(x)$
$Fluffy(x)$

Imagine that we have these predicates available to us to use...

GO CLASSES

Every orange cat is fluffy.

$\forall_x$ ( orange cat $\longrightarrow$ fluffy )

$\forall_x$ ( [ (orange(x)) $\wedge$ (cat(x)) ] $\rightarrow$ fluffy(x) )

No orange cat is fluffy

$$\forall_x \left( \text{orange cat} \longrightarrow \neg \text{fluffy} \right)$$

$$\forall_x \left( \left[ (\text{orange}(x)) \land (\text{cat}(x)) \right) \longrightarrow \neg \text{fluffy}(x) \right)$$

$$\neg \exists_x \left( \text{orange}(x) \land \text{cat}(x) \land \text{fluff}(x) \right)$$

Some orange cats are fluffy

$\exists x \left( orange(x) \wedge cat(x) \wedge fluffy(x) \right)$

Some orange cats are not fluffy

$$\exists x\,(\text{orage}(x) \land \text{Cat}(x) \land \neg\text{fluffy}(x))$$

Not all crows are Black.

$\equiv$ Some crows are Not Black

$\equiv \exists_x \left( Crow(x) \wedge \neg Black(x) \right)$

Not all orange cats are fluffy

Some orange cats are Not fluffy.

$\exists x\,(\,\text{orange}(x) \wedge \text{Cat}(x) \wedge \neg\,\text{fluffy}(x)\,)$

All and only crows are Black.

Crows $\rightleftarrows$ Black

$\equiv$

$$\forall_x \left( Crow(x) \longleftrightarrow Black(x) \right)$$

All and only A's are B's

$$\forall_x \big( A(x) \longleftrightarrow B(x) \big)$$

$$A \rightleftarrows B$$

Only crows are Black.

$\equiv$ If it is Not crow then it Not Black

$\equiv$ If it is Black then it is crow

$$\equiv \forall_x \left( Black(x) \longrightarrow crow(x) \right)$$

$$\equiv \forall_x \left( \neg crow(x) \longrightarrow \neg Black(x) \right)$$

Only A's are B's

$$\forall_x \left( B(x) \longrightarrow A(x) \right)$$

$$\equiv \forall_x \left( \neg A(x) \longrightarrow \neg B(x) \right)$$

Not A $\longrightarrow$ not B

Contrapositive

B $\longrightarrow$ A

All and only orange cats are fluffy

$$\forall_x \left( \text{orange cat}(x) \longleftrightarrow \text{fluffy}(x) \right)$$

$$\equiv \forall_x \left( [\text{orange}(x) \wedge \text{cat}(x)] \longleftrightarrow \text{fluffy}(x) \right)$$

1. All A's are B's : $\forall_x \big(A(x) \rightarrow B(x)\big)$
2. Some A's are B's : $\exists_x \big(A(x) \wedge B(x)\big)$
3. No A's are B's :

$$\forall_x \big(A(x) \rightarrow \neg B(x)\big) \equiv \neg \exists_x \big(A(x) \wedge B(x)\big)$$

4. Some A's are Not B's :

$$\exists_x (A(x) \wedge \neg B(x))$$

5. Not All A's are B's :

Some A's are not B's

$$\equiv \exists_x (A(x) \wedge \neg B(x))$$

Not "all A's are B's":

$$\neg \forall_x \left( A(x) \rightarrow B(x) \right)$$

$$\equiv \exists_x \left( A(x) \wedge \neg B(x) \right)$$

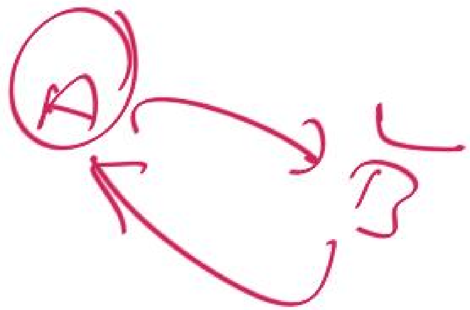
(6) Only A's are B's

$$\forall_x \left( \neg A(x) \longrightarrow \neg B(x) \right)$$

$$\equiv \forall_x \left( B(x) \longrightarrow A(x) \right)$$

7. All and only A's are B's

$$\forall_x \left( A(x) \longleftrightarrow B(x) \right)$$



$\underbrace{\underline{\text{English}} \longrightarrow \underline{\text{FOL}}}$

"All $P$s are $Q$s."

$$\forall x.\ (P(x) \to Q(x))$$

"Some $P$s are $Q$s."

$$\exists x.\ (P(x) \land Q(x))$$

$$\exists_x \left( P(x) \land Q(x) \right)$$

"No *P*s are *Q*s."

$$\forall x.\ (P(x) \to \neg Q(x))$$

$$\equiv \quad \neg \exists_x \left( P(x) \land Q(x) \right)$$

$\forall_x \left( P(x) \to \neg Q(x) \right)$

$\equiv$

$\neg \exists_x \left( P(x) \land Q(x) \right)$

Note: There exists $x$ : $\exists x$

There Does not exist $x$ : $\neg \exists x$

"Some *P*s aren't *Q*s."

$$\exists x.\ (P(x) \land \neg Q(x))$$

It is Not the case that All P's are Q's

$\neg \quad \forall x\,(P(x) \to Q(x))$

$\equiv \boxed{\neg \forall x\,\big(P(x) \to Q(x)\big)}$

$\equiv$

All P's are Q's

Some P's are Q's

Only P's are Q's $\equiv \forall x \, (Q(x) \rightarrow P(x))$

Only A's are B's $\equiv$ All B's are A's

$\equiv$ If Not A then Not B

$\equiv$ All B's are A's

$$\equiv \forall_x \left( B(x) \to A(x) \right) \equiv \forall_x \left( \neg A(x) \to \neg B(x) \right)$$

only Vaccinated people are in the flight.

$\equiv$ If you are in flight then you are Vaccinated.

$\equiv$ If you are Not Vaccinated then you are Not in flight.

$$\equiv \forall x \left( \text{In-flight}(x) \rightarrow \text{Vaccinated}(x) \right)$$

All and only A's are B's

$\equiv$ A iff B

$A \rightleftarrows B$

$$\equiv \forall_x \left( A(x) \leftrightarrow B(x) \right)$$

All A's are B's : $A(x) \rightarrow B(x)$

only A's are B's : $B(x) \rightarrow A(x)$

Not all A's are B's $\equiv$ It is Not the case that All A's are B's

$\equiv$

$\equiv \exists_x \left( A(x) \wedge \neg B(x) \right)$

$\equiv \neg \forall_x \left( A(x) \rightarrow B(x) \right)$

$\exists_x \left( A(x) \wedge \neg B(x) \right)$

All students are smart.

$\forall_x \left( Student(x) \rightarrow Smart(x) \right)$

There exists a student.

$$\exists_x \left( student(x) \right)$$

There exists a smart student.

$\exists x\,(\text{Student}(x) \wedge \text{Smart}(x))$

Every student loves someone.

Love(x, y) : x loves y

$\forall_x (\text{Student}(x) \rightarrow \boxed{\text{x loves someone}})$

$\forall_x (\text{Student}(x) \rightarrow \exists_y (\text{Love}(x, y)))$

Every student loves some student.

$\forall_x \left( \text{student}(x) \rightarrow \boxed{x \text{ loves some student}} \right)$

$\forall_x \left( \text{student}(x) \rightarrow \exists_y \left( \text{student}(y) \wedge \text{Loves}(x, y) \right) \right)$

Every student loves some other student.

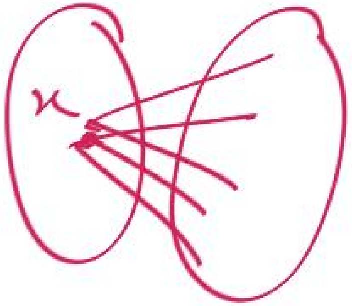
$\forall x \,( stu(x) \rightarrow x \text{ loves some other student} )$

$\forall x \,( Stu(x) \rightarrow \exists y \,( student(y) \wedge (x \neq y) \wedge loves(x,y) ))$

There is a student who is loved by every other student.

$x$

$$\exists_x \left( \text{Student}(x) \wedge \forall_y \left[ \text{Stu}(y) \wedge (x \neq y) \rightarrow \text{loves}(y, x) \right] \right)$$



$\exists_x$ ( Stud(x) $\wedge$

Bill is a student.

$\equiv$ Student ( Bill )

Student(x) : x is a student.

predicate.

Bill takes either Analysis or Geometry (but not both).

$\boxed{\text{Take}(x, y) : x \text{ takes } y}$

Given predicate

$\text{Take}(\text{Bill}, \text{Analysis}) \oplus \text{Take}(\text{Bill}, \text{Geometry})$

Bill takes Analysis or Geometry (or both).

Take(Bill, Analysis) $\vee$ Take(Bill, Geometry)

Bill takes Analysis and Geometry.

Take(Bill, Analysis) $\wedge$ Take(Bill, Geo)

Bill does not take Analysis

$\neg \text{Take}(\text{Bill}, \text{Analysis})$

No student loves Bill.

$\neg\exists_x$ : There Does not exist

$\neg\exists_x(\text{Student}(x) \wedge \text{Loves}(x, \text{Bill}))$

$\forall_x(\text{Student}(x) \rightarrow \neg\text{loves}(x, \text{Bill}))$

Bill has no sister.

Given Predicate

$\text{Sisterof}(x,y) \equiv$
$x$ is sister of $y$.

$\neg \exists_x (\text{Sisterof}(x, \text{Bill}))$

$\forall_x (\neg \text{Sisterof}(x, \text{Bill}))$

Bill has at least one sister.

$\exists_x ( \text{SisterOf}(x, \text{Bill}) )$

All dogs howl at night.

$$\forall_x \left( \underline{Dog(x)} \longrightarrow \underline{Howls(x)} \right)$$

All purple mushrooms are poisonous.

$\forall x\,(\,\text{Purplemush}(x) \rightarrow \text{Poison}(x)\,)$

$\forall x\,(\,[\text{Purple}(x) \wedge \text{mush}(x)] \rightarrow \text{Poison}(x)\,)$

Every gardener likes the sun.

$$\forall_x \left( \text{Gardener}(x) \rightarrow \text{like}(x, \text{sun}) \right)$$

No gardener likes the sun.

$$\neg \exists_x \left( Gardener(x) \wedge like(x, sun) \right)$$

$$\forall_x \left( Gardener(x) \rightarrow \neg like(x, sun) \right)$$

Not all gardeners like the sun.

$$\exists_x \left( \text{Gardener}(x) \land \neg \text{like}(x, \text{sun}) \right)$$

$$= \neg \forall_x \left( \text{Gardener}(x) \rightarrow \text{like}(x, \text{sun}) \right)$$

Every student walks or talks.

$$\forall x \left( \text{Stud}(x) \rightarrow (\text{walk}(x) \vee \text{talk}(x)) \right)$$

Some student walks or talks.

$\exists_x \left( stud(x) \wedge \left( walk(x) \vee talk(x) \right) \right)$

No student walks or talks.

$$\neg \exists_x \left( \text{student}(x) \wedge (\text{walk}(x) \vee \text{talk}(x)) \right)$$

$$\forall_x \left( \text{Stud}(x) \longrightarrow \neg (\text{walk}(x) \vee \text{talk}(x)) \right)$$

Only students walk or talk.

$$\forall_x \left( (\text{walk}(x) \vee \text{talk}(x)) \rightarrow \text{Student}(x) \right)$$

$\boxed{\text{Only A's are B's} \equiv \text{All B's are A's}}$

All and Only students walk or talk.

$$\forall x\,(\text{student}(x) \longleftrightarrow (\text{walk}(x) \lor \text{talk}(x)))$$

# Numerical quantification :

Now we will learn how to use FOL to express numerical quantifiers as the following:

at least two,
at most one,
exactly one,
at least three,
at most two,
exactly two, etc.

"Every cube is large."

$\forall_x \left( \text{Cube}(x) \rightarrow \text{Large}(x) \right)$

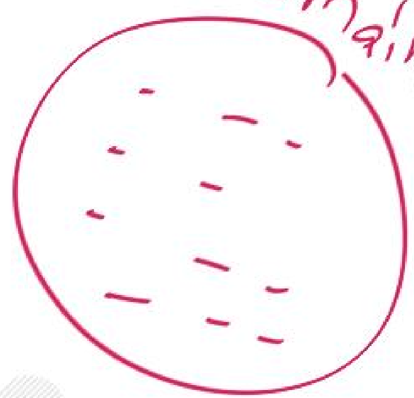
No cube is large.

$$\neg \exists_x \left( Cube(x) \wedge large(x) \right)$$

$$\forall_x \left( Cube(x) \rightarrow \neg large(x) \right)$$

"Nothing is large."

Domain



$$\neg \exists_x (\text{large}(x))$$

$$\forall_x (\neg \text{large}(x))$$

$\exists_x$ : there exists ; $\neg \exists_x$ : There Does not exist.

Some Cube is large.

$$\exists_x \left( Cube(x) \wedge large(x) \right)$$

All large cubes are nice.

$\forall x \left( [\text{large}(x) \land \text{cube}(x)] \rightarrow \text{nice}(x) \right)$

At least one:

There is at least one cube.

$\exists_x \; \text{Cube}(x)$

"There are at least two cubes."

$x$ $y$

$$\exists_x \exists_y \left( Cube(x) \land Cube(y) \land (x \neq y) \right)$$

$c_1$ $c_2$

$$\exists_x \exists_y \left( Cube(x) \land Cube(y) \land (x \neq y) \right)$$

There are at least two cubes.

False

"$\exists x \exists y \left( \text{Cube}(x) \wedge \text{Cube}(y) \right)$"

True

$x = c_1$

$y = c_1$

Domain

$x$

$y$

$c_1$

"There are at least 2 elements in the Domain"

$$\underbrace{\exists_x \exists_y \left( x \neq y \right)}$$

If we want to say simply that there are at least two things

$$\exists x \exists y \; x \neq y$$

There are at least three cubes.

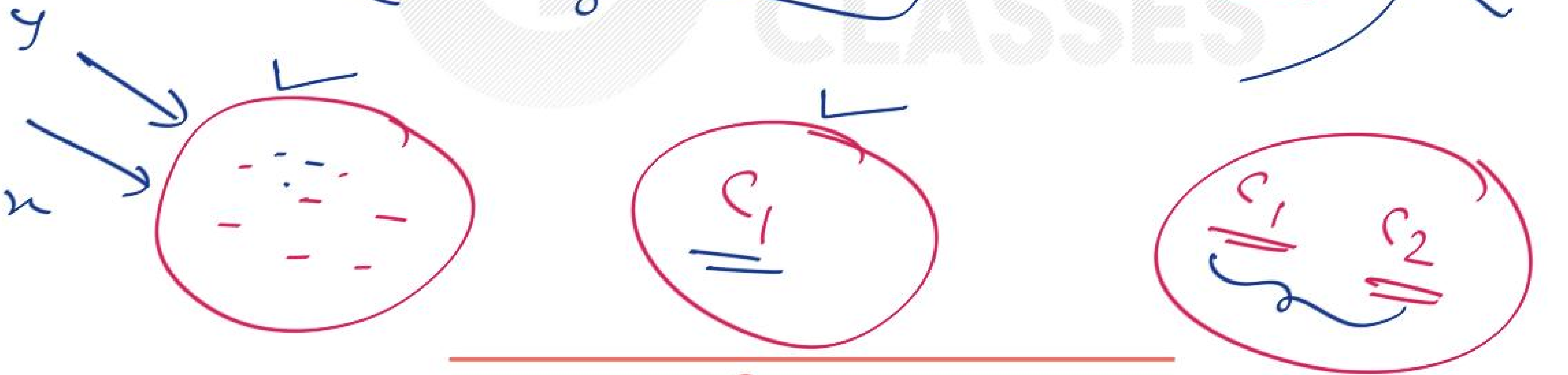
$$\exists x \exists y \exists z \left( Cube(x) \wedge Cube(y) \wedge Cube(z) \wedge (x \neq y) \wedge (y \neq z) \wedge (x \neq z) \right)$$

we can say simply that "there are at least three things"

$\exists x \exists y \exists z\, (x \neq y \land y \neq z \land x \neq z).$

# There is at most one cube.

$$\forall x \, \forall y \left( \underbrace{[\text{Cube}(x) \land \text{Cube}(y)]} \rightarrow (x = y) \right)$$



There is at most one cube.

No cube OR "Exactly one cube"

$\neg \exists_x \; Cube(x) \quad \vee \quad$ Exactly one cube

There is at most one element in the Domain.

$$\forall x \, \forall y \, (x = y)$$

no element

one element

a

a, b

If we want to say simply that "there is at most one thing"

$$\forall x \forall y \; x = y$$

There is at most one bird.

$$\forall_x \forall_y \left( bird(x) \wedge bird(y) \longrightarrow x = y \right)$$

There is exactly one cube.

At least one cube $\wedge$ at most one cube

$\exists_x \; Cube(x) \wedge \forall_x \forall_y \left( Cube(x) \wedge Cube(y) \rightarrow (x = y) \right)$

There is exactly one cube.

$$\exists_x \left( Cube(x) \wedge \forall_y \left( Cube(y) \rightarrow x = y \right) \right)$$

$$\exists_x \forall_y \left( Cube(x) \wedge \left( Cube(y) \rightarrow x = y \right) \right)$$

There is exactly one cube.

$x \rightarrow \text{cube}(x)$

$(x=y) \leftarrow \text{cube}(y)$

$\exists_x \forall_y \left( \text{cube}(y) \leftrightarrow x=y \right)$

$c_1$

$c_1$

There is Exactly one bird.

$$\left\{ \exists_x \; x \rightarrow bird(x) \quad \forall_y \; bird(y) \rightarrow x = y \right\}$$

$$\exists_x \forall_y \left( bird(y) \longleftrightarrow x = y \right)$$

There is exactly one cube.

$\exists x\, Cube(x) \wedge \forall x \forall y\, ((Cube(x) \wedge Cube(y)) \rightarrow x = y)$

$\exists x\, (Cube(x) \wedge \forall y\, (Cube(y) \rightarrow y = x))$

$\exists x \forall y\, (Cube(y) \leftrightarrow y = x)$

iff

There is at most one cube.

$$\forall_x \forall_y \left( Cube(x) \wedge Cube(y) \longrightarrow x = y \right)$$

there are at most two cubes.

$$\forall x \forall y \forall z \left( \text{cube}(x) \wedge \text{cube}(y) \wedge \text{cube}(z) \rightarrow \left( (x=y) \vee (y=z) \vee (x=z) \right) \right)$$

there are at most two cubes.

$$\forall x \forall y \forall z\, ((Cube(x) \wedge Cube(y) \wedge Cube(z)) \rightarrow (x = y \vee y = z \vee x = z)).$$

Exactly one cube!

$$\exists_x \left( \underline{\underline{cube(x)}} \wedge \forall_y \left( cube(y) \rightarrow y = x \right) \right)$$

There are exactly two cubes.

$$\exists_x \exists_y \Big( (x \neq y) \wedge Cube(x) \wedge Cube(y) \wedge \forall_z \big( Cube(z) \rightarrow (x = z) \vee (z = y) \big) \Big)$$

There are exactly two cubes.

$$\exists x\, \exists y\, (x \neq y \land \forall z\, (Cube(z) \leftrightarrow (z = x \lor z = y))).$$

"For every number there is a larger number."
Assume domain is set of numbers.

$$\underbrace{\forall_x \exists_y (y > x)}$$

$x$

There is a number that is larger than every "other" number.
Assume domain is set of numbers.

$$\exists x \forall y \left( (x \neq y) \rightarrow (x > y) \right)$$

Every positive number is a square.
Assume domain is set of numbers.

$x = y^2$

$4 = 2^2$
$9 = 3^2$

$$\forall_x \left( (x > 0) \longrightarrow \exists_z (z * z = x) \right)$$

$x < z < y$

If one number is less than another, then there is a number properly between the two.
Assume domain is set of numbers.

$$\forall_x \forall_y \left( (x < y) \longrightarrow \exists_z \left( (x < z) \wedge (z < y) \right) \right)$$

There are at least two numbers which are prime.
Assume domain is set of numbers.

$$\exists x \exists y \left( (x \neq y) \land prime(x) \land prime(y) \right)$$

There are at least three numbers which are prime.
Assume domain is set of numbers.

$$\exists_x \exists_y \exists_z \left( \text{Prime}(x) \wedge \text{Prime}(y) \wedge \text{prime}(z) \wedge (x \neq y) \wedge (y \neq z) \wedge (x \neq z) \right)$$

There are infinitely many numbers which are prime.
Assume domain is set of numbers.

$$\forall_x \exists y \left( (y > x) \wedge (\text{Prime}(y)) \right)$$

There is at most one number which is prime.
Assume domain is set of numbers.

$$\forall x \forall y \left( (\text{prime}(x) \land \text{prime}(y)) \rightarrow (x = y) \right)$$

There is exactly one number which is prime.
Assume domain is set of numbers.

$$\exists_x \left( Prime(x) \wedge \forall_y \left( Prime(y) \rightarrow y = x \right) \right)$$

Divides $a \mid b$ : $a\cancel{x} = b$

a divides b

Assume domain is set of integers.

$2 \mid 4$ ; $2(2)=4$

$$\exists_x \left( a \cdot x = b \right)$$

x is an even number

Assume domain is set of integers.

$x = 2(y)$

$4 = 2(2)$

$6 = 2(3)$

$10 = 2(5)$

$9 \neq 2(\ )$

$\exists_y (x = 2y)$

$y \mid x \rightarrow y=1$ or $y=x$

x is prime

Assume domain is set of integers.

$$\underbrace{(x>1) \wedge \left(\forall y \left(y \mid x \rightarrow (y=1) \vee (y=x)\right)\right)}$$

x is prime

Assume domain is set of integers.

$$(1 < x) \land \forall y \left( (y|x) \to \left( (y \approx 1) \lor (y \approx x) \right) \right)$$

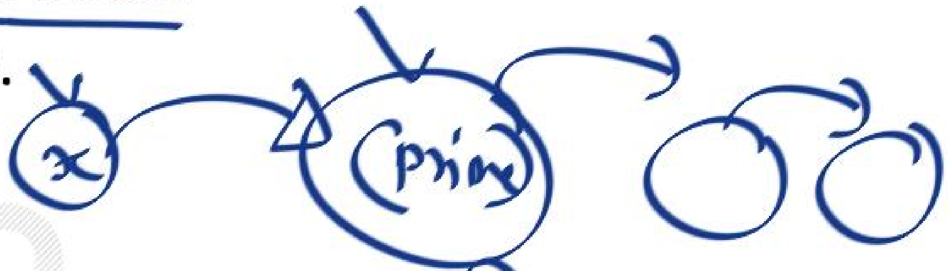
"The relation "divides" is transitive."

Assume domain is set of integers.

$a|b$ , $b|c$ then $a|c$

$$\underbrace{\forall_a \forall_b \forall_c \left( \left[ (a|b) \land (b|c) \right] \longrightarrow (a|c) \right)}$$

There are an infinite number of primes.
Assume domain is set of integers.




$$\forall_x \exists_y \left( (y > x) \wedge \text{Prime}(y) \right)$$

"There are an infinite number of pairs of primes that differ by the number 2." (Twin Prime Conjecture)
Assume domain is set of integers.

$x \rightarrow$ prime

(3, 5) (11, 13)

$$\forall_x \exists_y \left( (y > x) \land Prime(y) \land Prime(y+2) \right)$$

There are an infinite number of pairs of primes that differ by the number 2. (Twin Prime Conjecture)
Assume domain is set of integers.

$$\boxed{\forall x \exists y \Big( (x < y) \land prime^{\star}(y) \land prime^{\star}(y+2) \Big)}$$

$x$

"

All even numbers greater than two are the sum of two primes.
(Goldbach's Conjecture)
Assume domain is set of integers.

$$\forall_x \left( \boxed{even(x) \wedge (x>2)} \longrightarrow \exists_y \exists_z \left( Prime(y) \wedge Prime(z) \wedge x = y+z \right) \right)$$

All even numbers greater than two are the sum of two primes. (Goldbach's Conjecture)
Assume domain is set of integers.

$$\boxed{\forall x \left( \left( (2|x) \wedge (2 < x) \right) \rightarrow \right.}$$

$$\boxed{\left. \exists y \exists z \left( prime^{\star}(y) \wedge prime(z) \wedge (x \approx y + z) \right) \right)}$$