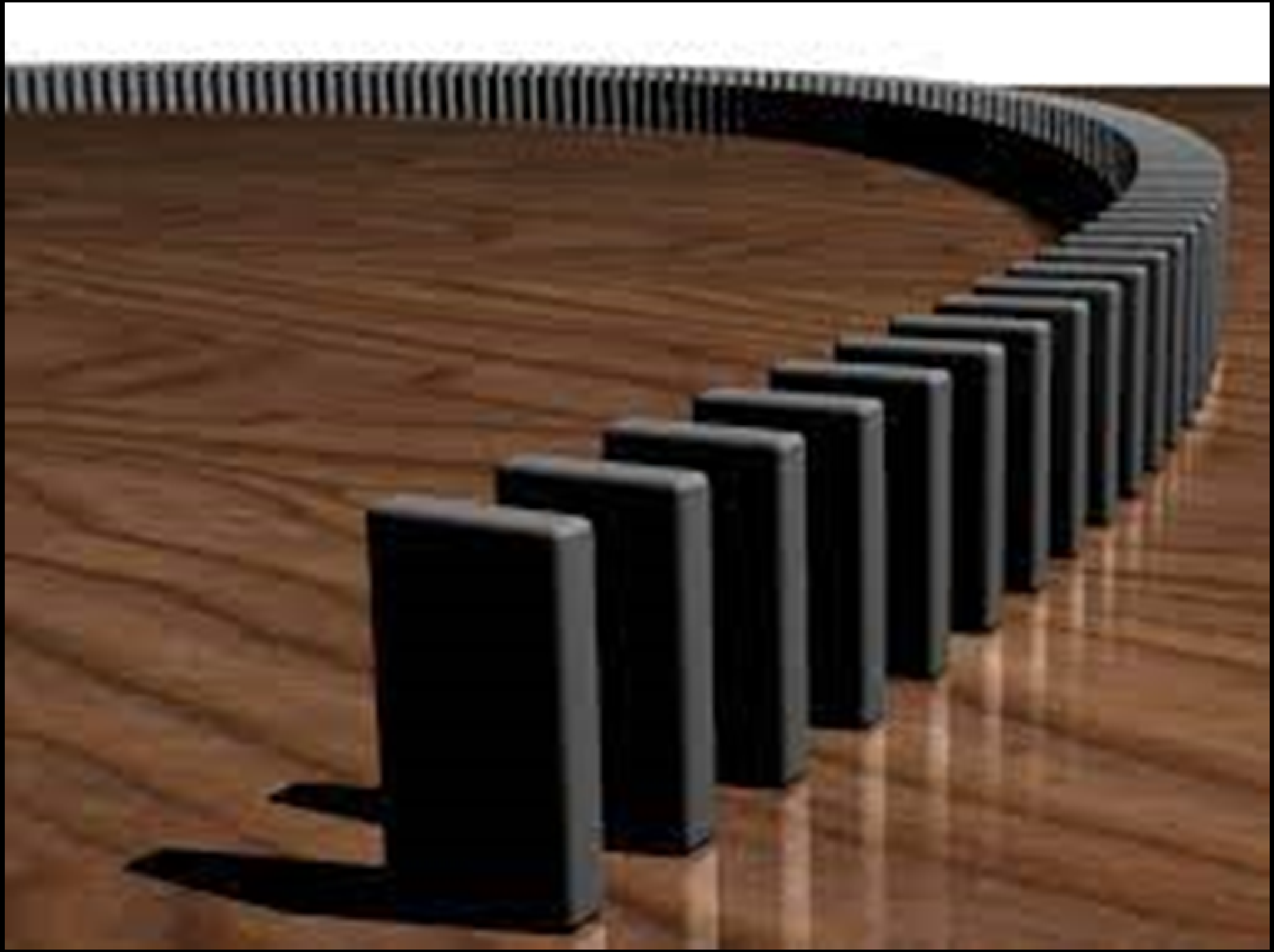
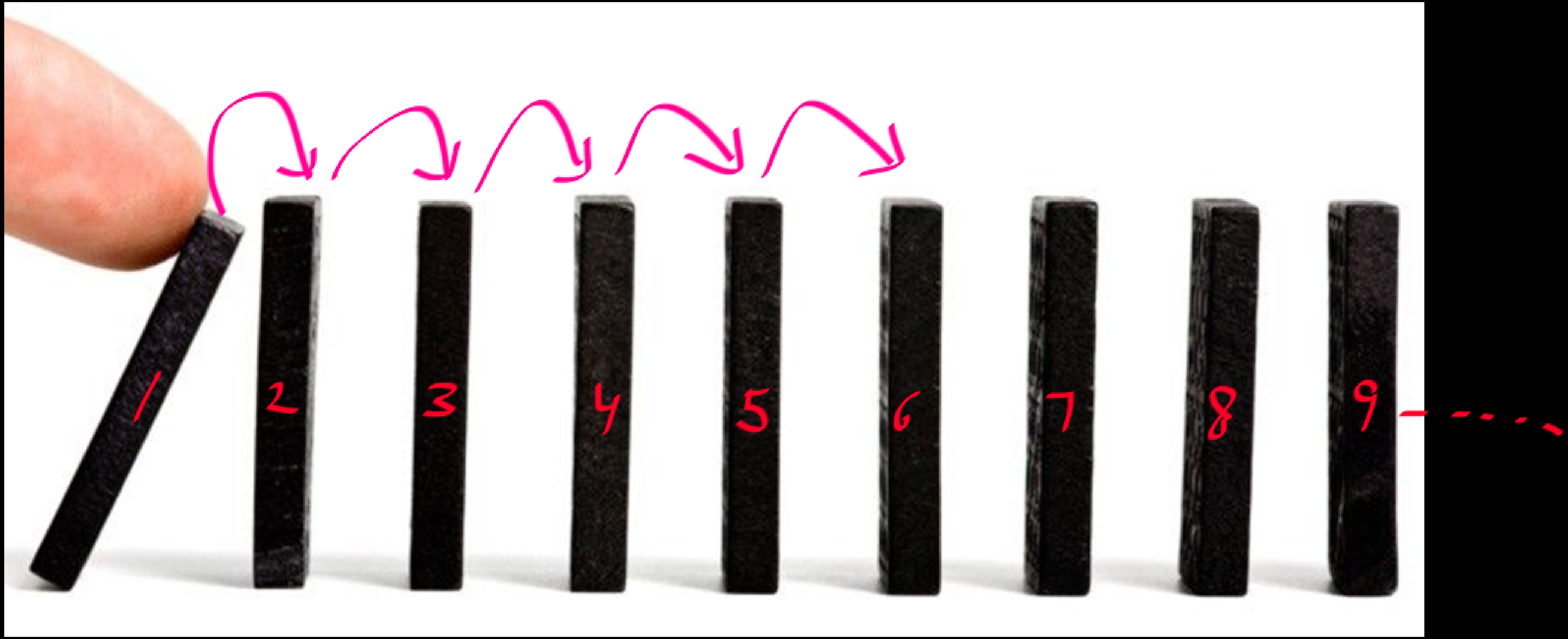
Proof Using Mathematical Induction:




1
2
3
4
5
6
7
8
9

Domino Effect

why all Domions are falling?

① Pushed first Domino } Starting Point

② k^{th} Domino is pushing $(k+1)^{th}$ Domino. } Induction

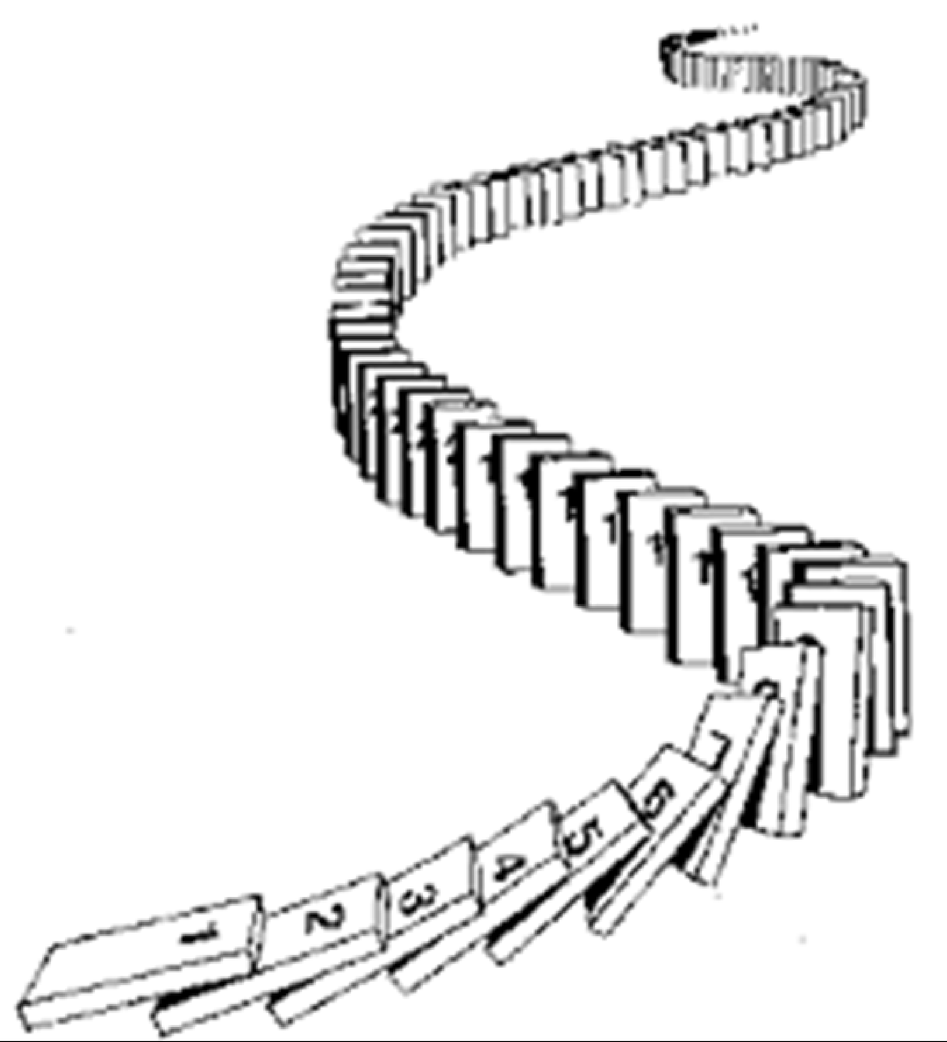
for every $k \geq 1$

$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to \cdots$

You are pushing

So all **Dominos** fall.


Step 1: Prove true for n=1
Step 2: Assume true for n=k
Step 3: Prove true for n=k+1
Step 4: Therefore, true for all positive integers by Mathematical Induction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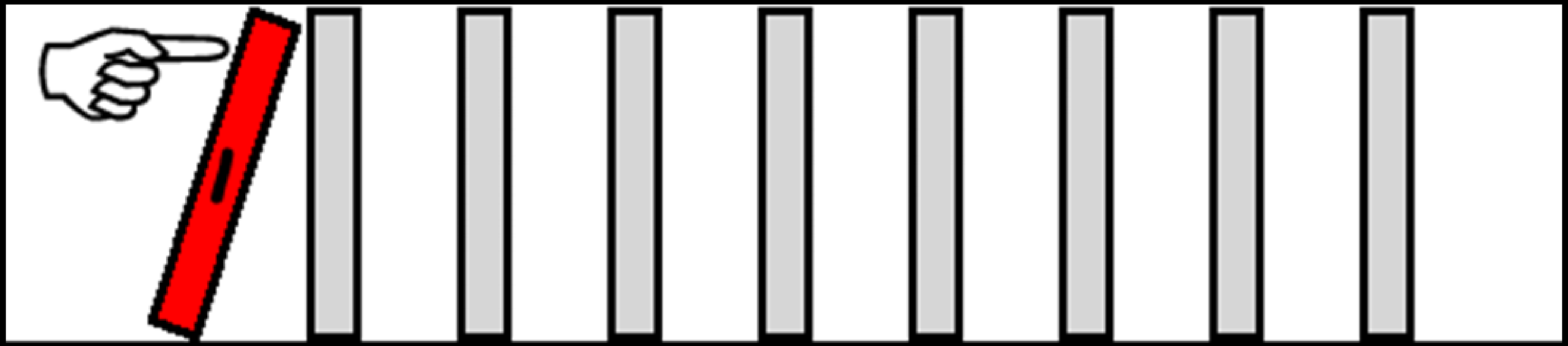
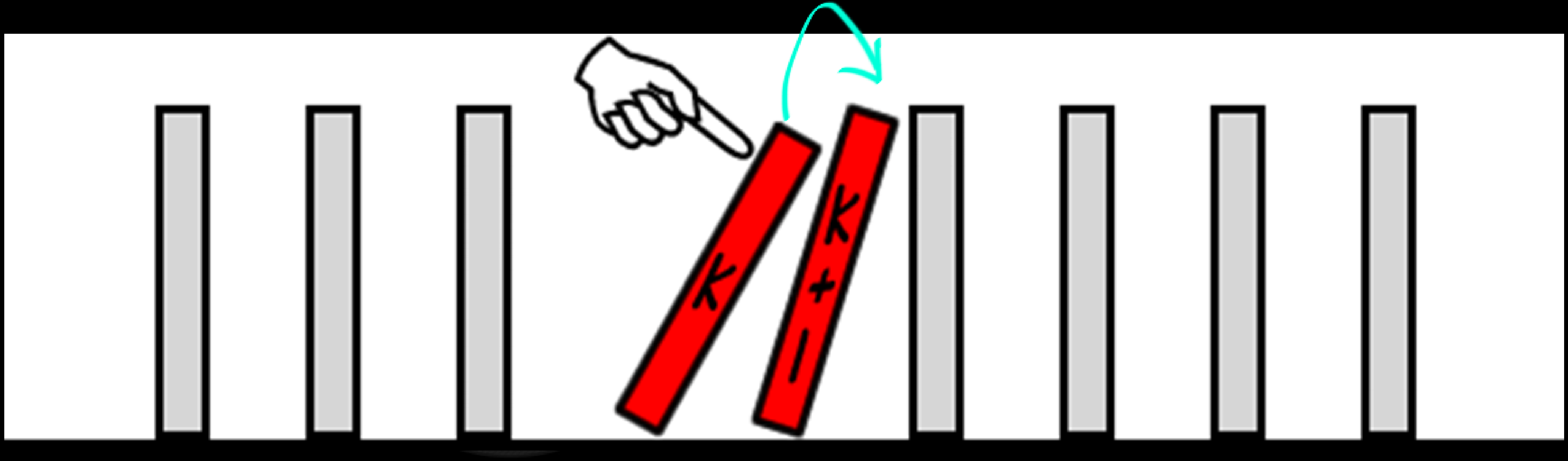


Starting
Point


K
K+1

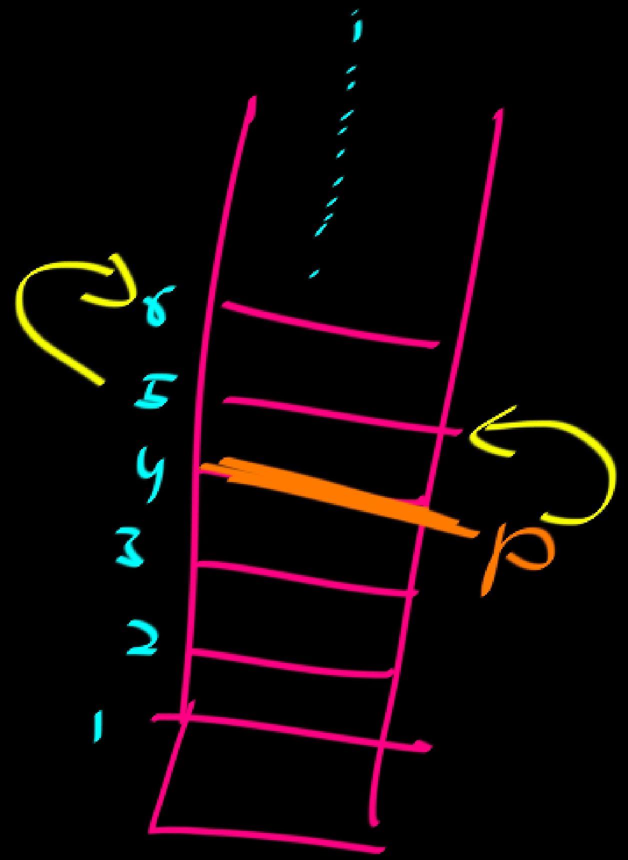

Step 1: Prove true for n=1
Step 2: Assume true for n=k
Step 3: Prove true for n=k+1
Step 4: Therefore, true for all positive integers by Mathematical Induction

 P

1. Can climb on 1^{st} step

2. If P is on k^{th} step

   P can go to $k+1^{th}$ step, } for every k



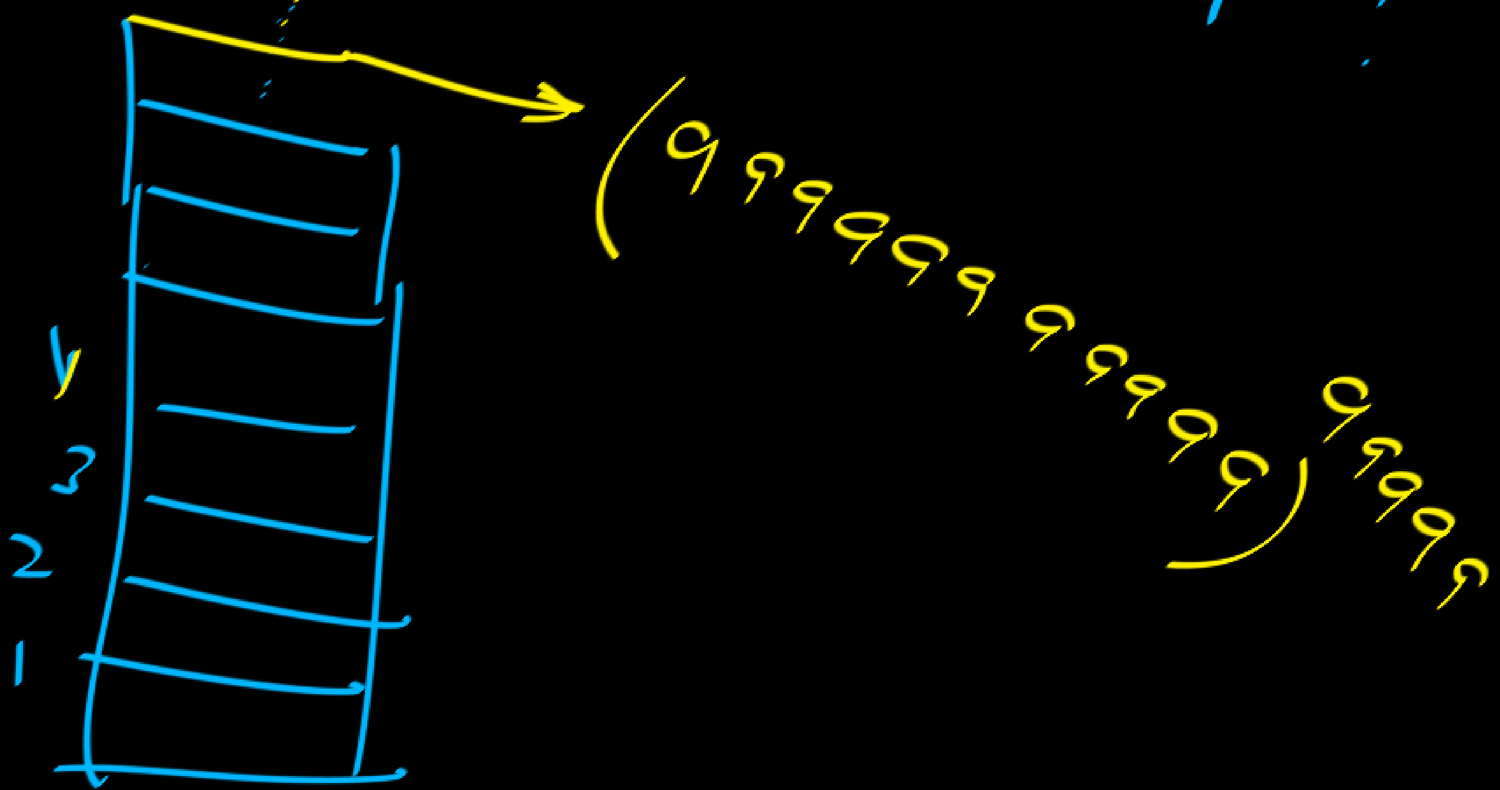
Q: Can P go to 100000^{th} step?

Yes.

Q: Can P go to $(9999999999999)^{th}$ step?

Yes

Q: Can P go to "∞th" step ?



So P can go any Step n

Where n is natural number.

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow \cdots (99)^{99} \rightarrow (99)^{99}+1 \rightarrow \cdots\cdots$

"∞" is NOT a number.

∞ is " a Natural number.

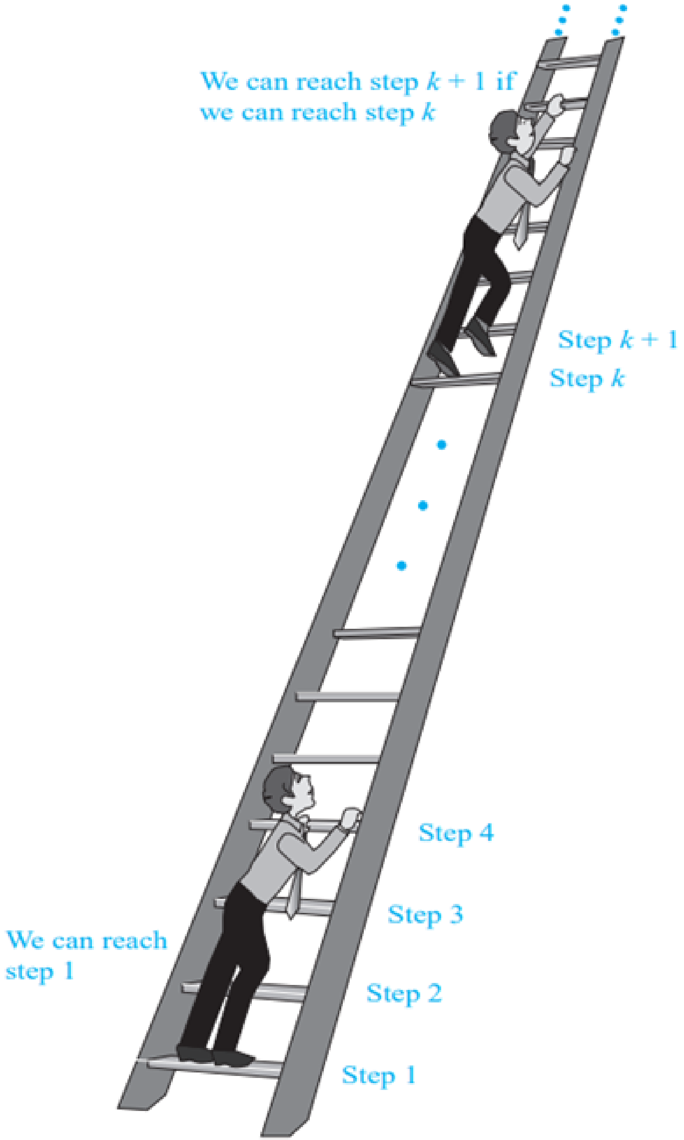


FIGURE 1 Climbing an Infinite Ladder.

Mathematical Induction

Introduction

Suppose that we have an infinite ladder, as shown in Figure 1, and we want to know whether we can reach every step on this ladder. We know two things:

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Can we conclude that we can reach every rung? By (1), we know that we can reach the first rung of the ladder. Moreover, because we can reach the first rung, by (2), we can also reach the second rung; it is the next rung after the first rung. Applying (2) again, because we can reach the second rung, we can also reach the third rung. Continuing in this way, we can show that we can reach the fourth rung, the fifth rung, and so on. For example, after 100 uses of (2), we know that we can reach the 101st rung. But can we conclude that we are able to reach every rung of this infinite ladder? The answer is yes, something we can verify using an important proof technique called **mathematical induction**. That is, we can show that $P(n)$ is true for every positive integer n, where $P(n)$ is the statement that we can reach the nth rung of the ladder.

To prove: P(n) for every $n \in \mathbb{N}$

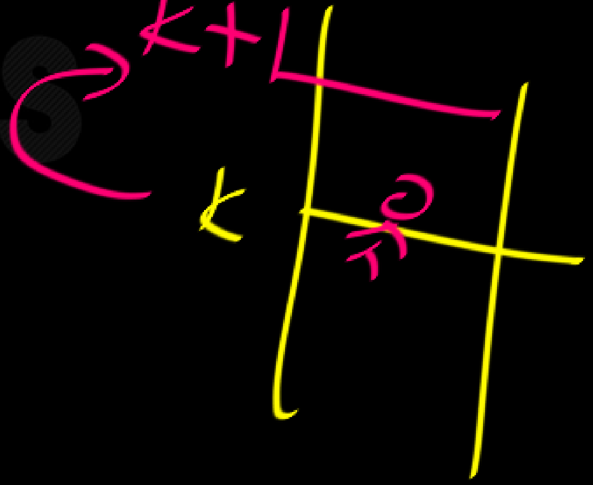
Proof by Induction:

① Starting Point (Base Case) : P(1) manually check

(2) Assume $P(k)$ is True

(3) Now Prove that

$P(k+1)$



If we can do this then

$$P(1) \to P(2) \to P(3) \to P(4) \text{ - - - - -}$$

$\downarrow$

manually Proven

So for every Natural number n ; $P(n)$ is True.

Mathematical Induction

The **principle of mathematical induction** states that if for some property P(n), we have that

P(0) is true

and

For any natural number n, P(n) → P(n + 1)

Then

For any natural number n, P(n) is true.

The **principle of mathematical induction** states that if for some property P(n), we have that

P(0) is true

If it starts...

and

For any natural number n, P(n) → P(n + 1)

Then

For any natural number n, P(n) is true.

The **principle of mathematical induction** states that if for some property P(n), we have that

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If it starts...

and it keeps going...

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The **principle of mathematical induction** states that if for some property P(*n*), we have that

P(0) is true

If it starts...

and it keeps going...

and

For any natural number *n*, P(*n*) → P(*n* + 1)

...then it's always true

Then

For any natural number *n*, P(*n*) is true.

Induction, Intuitively

- It's true for 0.
- Since it's true for 0, it's true for 1.
- Since it's true for 1, it's true for 2.
- Since it's true for 2, it's true for 3.
- Since it's true for 3, it's true for 4.
- Since it's true for 4, it's true for 5.
- Since it's true for 5, it's true for 6.
- …

Proof by Induction

- Suppose that you want to prove that some property P(n) holds of all natural numbers. To do so:
- Prove that P(0) is true.
  - This is called the **basis** or the **base case**.
- Prove that for any natural number n, if P(n) is true, then P(n + 1) is true as well.
  - This is called the **inductive step**.
  - P(n) is called the **inductive hypothesis**.
- Conclude by induction that P(n) holds for all n.

Mathematical Induction

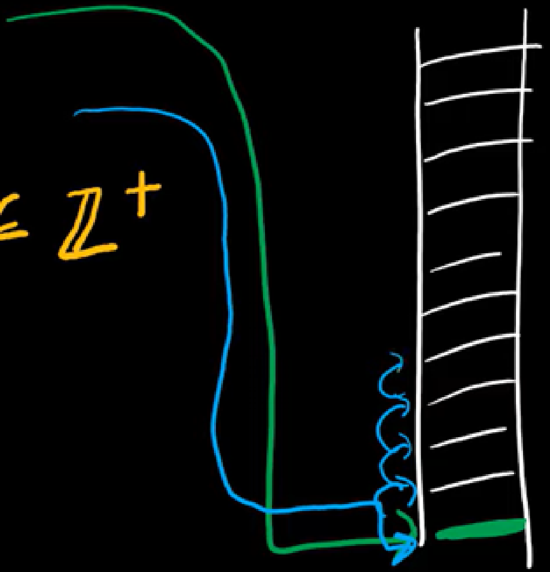
To prove $P(x)$ is true for $x \in \mathbb{Z}^+$, where $P(x)$ is a propositional function, we complete two steps:

1) Basis step - verify $P(1)$ is true

2) Inductive step - verify if $P(k)$ is true, then $P(k+1)$ is true $\forall k \in \mathbb{Z}^+$

   Inductive hypothesis: $P(k)$ is true

   Must show: $P(k) \rightarrow P(k+1)$



Conclusion: $P(x)$ is true $\forall\ k \in \mathbb{Z}^+$

Proving a Summation Formula

Show $1+2+3+\dots+n = \frac{n(n+1)}{2}$

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Show $1+2+3+\dots+n = \frac{n(n+1)}{2}$

$P(n)$: What you want to prove.

$P(n)$: $\boxed{1+2+3+\dots+n = \frac{n(n+1)}{2}}$

To prove : $\boxed{P(n)}$ for any $\underbrace{n \in N}$

Base Case : for $n = 1$

$P(1) : \quad 1 = \frac{1(1+1)}{2}$ ✓

$P(n): \underbrace{1+2+3+4+\cdots+n}_{n \text{ terms}} = \frac{n(n+1)}{2}$

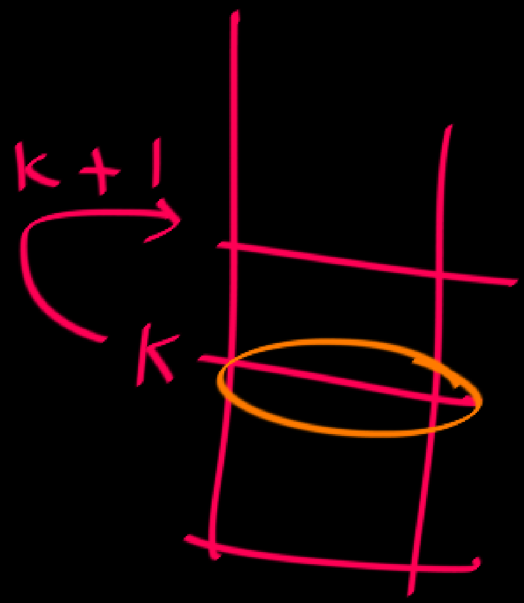
$P(1):\ 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$P(2):\ \underset{3}{1+2} = \frac{2(2+1)}{2}\ \ 3$

Assume P(k) is True.

So

$$\boxed{1+2+\cdots+k = \frac{k(k+1)}{2}}$$



Aim: $\underbrace{P(k)}_{\text{Assumed to be True}} \longrightarrow \underbrace{P(k+1)}_{\text{prove this}}$

$\boxed{\text{To prove:}}$

$P(k+1)$: $1+2+\cdots+k+k+1 = \frac{(k+1)(k+1+1)}{2}$

LHS:

$$\boxed{1+2+3+\cdots+k} + k+1$$

$$\frac{k(k+1)}{2} + k+1$$

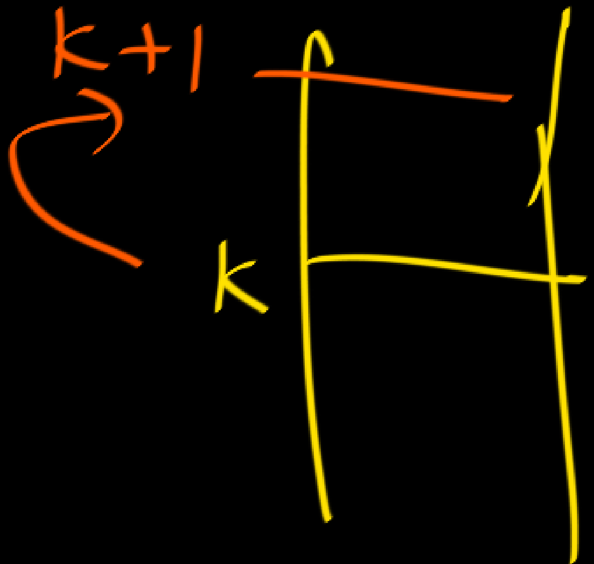
RHS

$$= \frac{k^2+k+2k+2}{2} = \frac{(k+1)(k+2)}{2}$$

So we proved

$$P(k) \rightsquigarrow P(k+1)$$

$$\begin{tikzcd} P(1) \arrow[r, bend left] & P(2) \arrow[r, bend left] & P(3) \arrow[r, bend left] & P(4) \arrow[d, bend left] \\ & - - - - - & & P(5) \arrow[ll] \end{tikzcd}$$



So for any $n \in \mathbb{N}$;

$P(n): 1+2+\cdots+n = \frac{n(n+1)}{2}$

is True.

To prove $\underbrace{P(n)}$

Starting point: P(1) ✓

Now prove that If P(k) is True then P(k+1) is True.

① $P(k) \rightarrow P(k+1)$

② $P(1)$

$P(n)$ for any $n \in \mathbb{N}$

Use mathematical induction to show that for all non-neg integers n,
$1+2+2^2+\dots+2^n = 2^{n+1}-1$

for all non-negative integers;

$P(n)$: $\boxed{1+2+2^2+\dots+2^n = 2^{n+1}-1}$

what you want to prove

Starting Point : (base Case)

$\underbrace{n=0}$

$P(0)$ ✓

$$P(n): \underbrace{\underset{\underbrace{}_{}}{1}\xrightarrow{} 2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^n}_{\underline{n+1} \text{ terms}} = 2^{n+1} - 1$$

$P(0)$: $\underline{LHS} = 1$; $RHS = 2^{0+1} - 1 = 1$

$\hookrightarrow$ 0+1 term

P(0) ✓

Assume P(k) is True

$$\boxed{1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1}$$

Prove that $\underbrace{P(k+1)}$ is True.

$P(k+1)$:

$$\boxed{1+2+2^2+\dots+2^k+2^{k+1} = \underbrace{2^{k+1+1}-1}_{\text{RHS}}}$$

To Prove

LHS:

$$\boxed{1+2+2^2+2^3+\cdots+2^k}+2^{k+1}$$

$$\underbrace{2^{k+1}-1}+2^{k+1}$$

RHS

$$=2\cdot 2^{k+1}-1=\boxed{2^{k+2}-1}$$

So we showed ;

① P(0) ✓

② If P(k) is True then

P(k+1) is True .

So

$$P(0) \longrightarrow P(1) \longrightarrow P(2) \longrightarrow P(3) \longrightarrow \ldots$$

So, for every whole number n

$P(n)$ is True.

Conjecture and prove a summation formula for the sum of the first n positive odd integers.

Proving an Inequality :

Prove $n < 2^n \ \forall n \in \mathbb{Z}^+$ using mathematical induction.

Proving an Inequality :

Prove $\boxed{n < 2^n}$ $\forall n \in \mathbb{Z}^+$ using mathematical induction.

let $P(n) : n < 2^n \ \forall n \in \mathbb{Z}^+$

① Basis : $P(1)$

$P(1) : 1 < 2^1$

$1 < 2$ ✓

② Inductive $P(k) \to P(k+1)$

IH : $k < 2^k$

show : $k+1 < 2^{k+1}$

$1 < 2^k \quad \forall k \in \mathbb{Z}^+$

$k+1 < 2^k + 1 < 2^k + 2^k = 2(2^k)$

$\boxed{k+1 < 2^{k+1}}$

$\square$

Prove $2^n < n!$ $\forall n \in \mathbb{Z}^+$ and $n \geq 4$.

Q :

Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.