Propositional Logic

Next Chapter:

Logical Connectives

(Continued)

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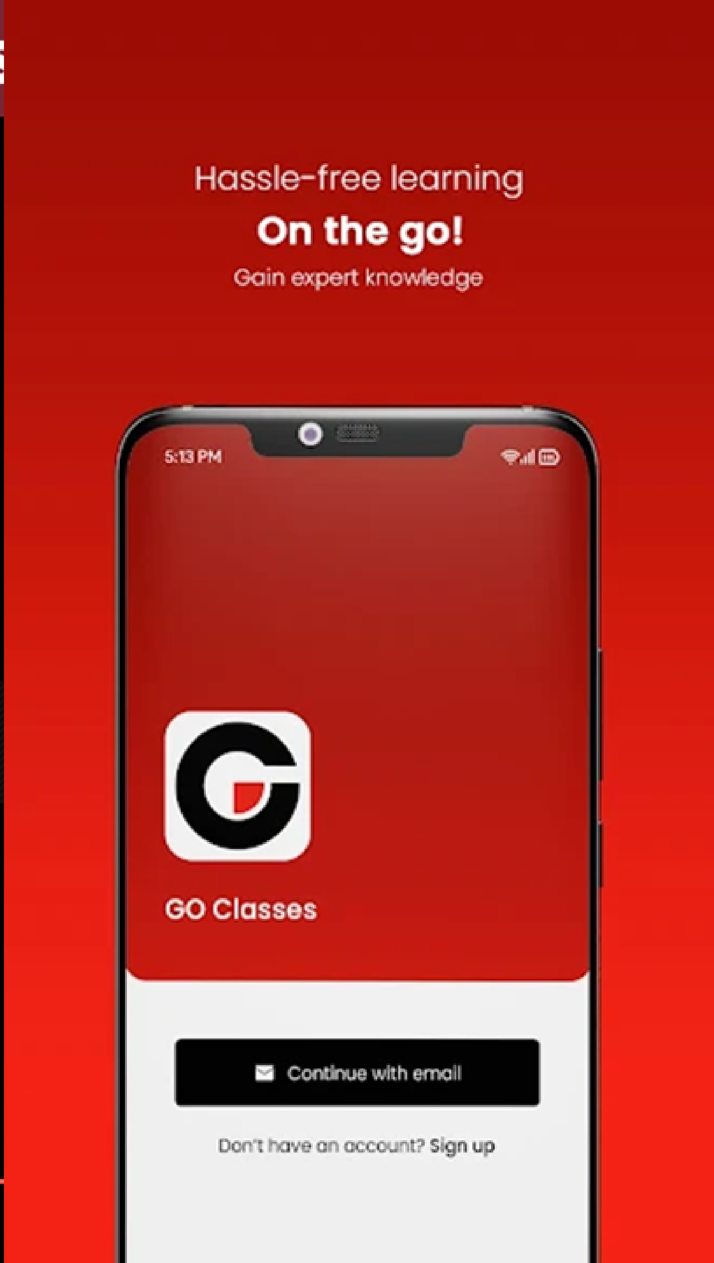

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Propositional Logic

Next Topic:

Logical Connectives

Continued...

Next Topic:

Logical Connectives –

4. Exclusive OR (xor)

Why “OR” is also called “Inclusive OR” ??

P, Q :

P ∨ Q

Disjunction (OR)

P	Q	P ∨ Q
T	F	T
F	T	T
T	T	T
F	F	F

OR ≡ Inclusive-OR ≡ Disjunction

A disjunction is true when at least one of the two propositions is true.

For instance, Consider the statement:

"Students who have taken calculus or computer science can take this class."

Here, we mean that students who have taken both calculus and computer science can take the class, as well as the students who have taken only one of the two subjects.

In a hotel:

E : You must have Soup or Salad.

Soup	Salad	E
T	F	T
F	T	T
T	T	T
F	F	F

Try you

Prop. logic

You can take Soup or Salad. $\Rightarrow$ Soup $\vee$ Salad

You can take Soup or Salad But not both.

or . . . but not both $\equiv$ Soup $\oplus$ Salad

In prop. logic:

$p \vee q$

Inclusive OR $\equiv$ OR

$\equiv$ Disjunction

$p \oplus q$

Exclusive OR

only one of p, q is True

M : You can come on Monday

S : You can come on Sunday

या

You can come on Monday OR on Sunday. $\equiv$ $M \vee S$

M	S
✓	✓

M : You can come on Monday

S : You can come on Sunday

You can come on Monday OR on Sunday. But not both.

$\boxed{M \oplus S}$

Ex-OR

P OR Q $\equiv$ $P \vee Q$

Inclusive OR

$\boxed{\text{P or Q But not both}}$ $\equiv$ $P \oplus Q$

Ex-OR

P	Q	P ∨ Q	P ⊕ Q
F	F	F	F
F	T	T	T
T	F	T	T
T	T	T	F

P ⊕ Q → Exclusive-OR

Exclusively one of P, Q must be True.

M : You can come on Monday
S : You can come on Sunday

You can come on Monday OR on Sunday. $M \vee Q$

You can come on Monday OR on Sunday BUT NOT BOTH.

$M \oplus S$

Exclusive News: → only one New channel has this News.

AT	NDTV	$A \oplus N$
F	F	F
T	F	T
F	T	T
T	T	F

$\oplus$ → Exclusive-OR

Exclusive OR $\oplus$

$\equiv$ ExOR

$\equiv$ XOR

Exclusive Or. Exclusive Or Operator, "xor", has symbol $\oplus$.

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

p: This book is interesting. q: I am staying at home.

$p \oplus q$: Either this book is interesting, or I am staying at home, but not both.

Ex-OR

Ex OR operator $\equiv \oplus$

$p \oplus q \equiv$ p or q but not both.

p	q	$p \oplus q$
F	F	F
T	F	T
F	T	T
T	T	F

or ---- but not both $\equiv$ $\oplus$

Truth Table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

The "exclusive or" is the binary operator which, when applied to two propositions p and q yields the proposition "p xor q", denoted $p \oplus q$, which is true if exactly one of p or q is true, but not both. It is false if both are true or if both are false.

Examples:

p: "The butler did it"

q: "The cook did it"

$p \vee q$: "Either the butler or the cook did it"

OR ≡ Disjunction ≡ Inclusive OR ≡ $\vee$

Exclusive OR ≡ $\oplus$ ≡ OR ... but not both

Examples:

p: "The butler did it"
q: "The cook did it"
$p \oplus q$: "Either the butler or the cook did it, but they did not do it together!"

p OR q but not both $\equiv$ $p \oplus q$

Next Topic:

Natural Languages(Like English) are

Ambiguous...

But Mathematical Logic Unambiguous..

English Statement:

Mary is poet Or singer.

No Ambiguity

$p \vee s$

$p \oplus s$

Prop. logic Expression

English Statement:

Mary is poet Or singer.

Translate in Logic:

$P \vee S$ ✓

$P \oplus S$

English Statement:

Mary is poet Or singer but not both

Translate in Logic:

$P \vee S$

$P \oplus S$

English

p or q

p or q but not both

prop. logic:

$p \vee q$

$p \oplus q$

Next Topic:

Logical Connectives –

5. NAND

6. NOR

NAND , NOR :

NAND $\equiv$ Negation of AND

NOR $\equiv$ " " OR

NAND $\equiv$ $\uparrow$

NOR $\equiv$ $\downarrow$

NAND , NOR :

The NAND, or "not-and," operator applies AND to its operands and then complements the result by applying NOT. We write p NAND q to denote NOT (p AND q).

Similarly, the NOR, or "not-or," operator takes the OR of its operands and complements the result; p NOR q denotes NOT (p OR q). The truth tables for NAND and NOR are shown in Fig. 12.4.

p	q	p NAND q
0	0	1
0	1	1
1	0	1
1	1	0

p	q	p NOR q
0	0	1
0	1	0
1	0	0
1	1	0

NAND operator :

N AND

Symbol : $\uparrow$

$P \uparrow Q \equiv \overline{(P \wedge Q)}$

P NAND Q

P	Q	$P \wedge Q$	$P \uparrow Q$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

P	Q	$P \uparrow Q \equiv \overline{(P \wedge Q)}$
F	F	T
F	T	T
T	F	T
T	T	F

NAND Truth Table

NoR operator :

NOR

Symbol: $\downarrow$

$$P \downarrow Q \equiv \overline{(P \vee Q)}$$

P NoR Q

P	Q	$P \vee Q$	$P \downarrow Q$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

P	Q	$P \downarrow Q$
F	F	T
F	T	F
T	F	F
T	T	F

NOR Truth Table

Next Topic:

Logical Connectives –

7. Implication