



Group Theory

Next Topic:

Practice Questions

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

*	e	a	b	c	d
e	e	a	b	c	d
a	a		e	b	
b	b				
c	c				
d	d				

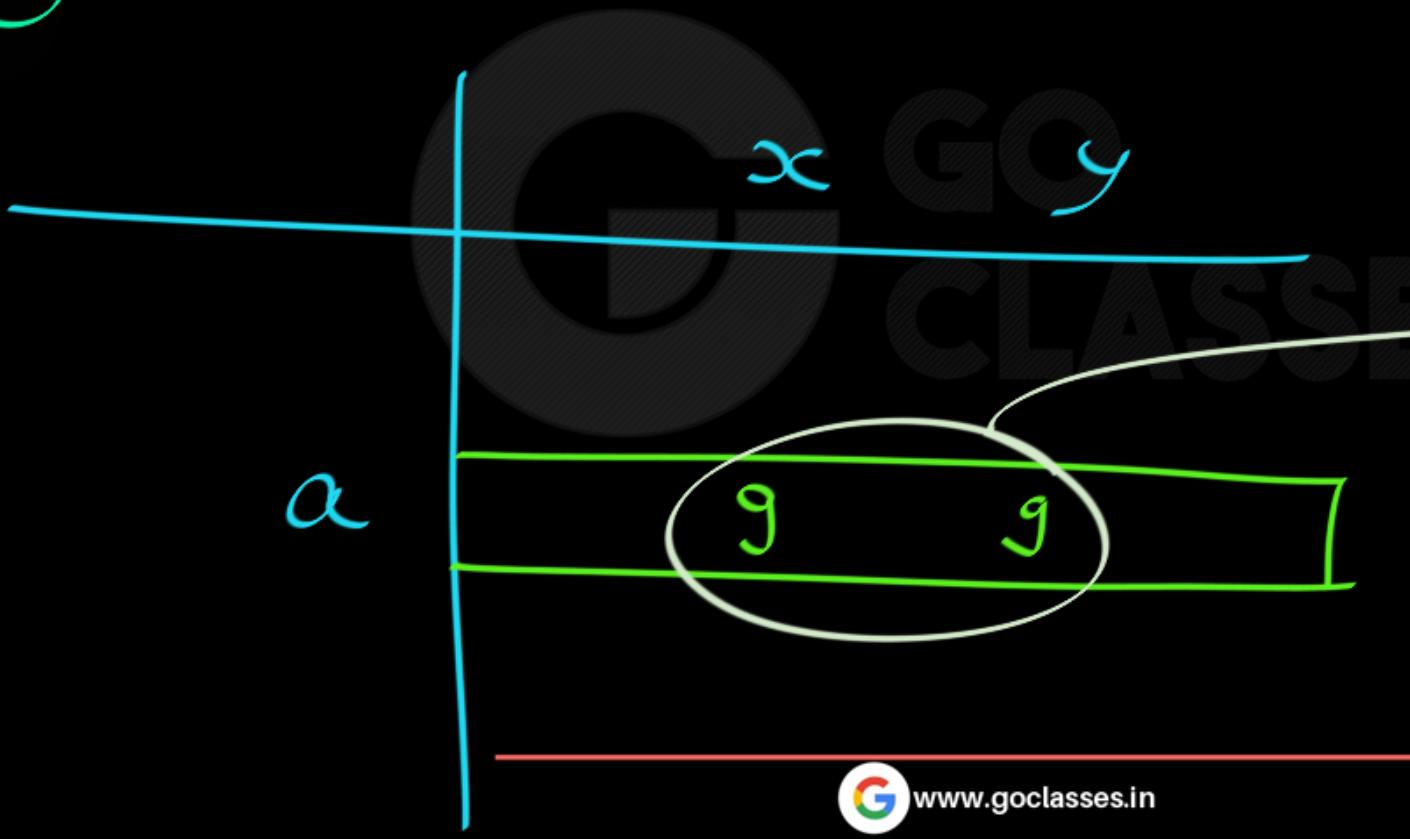
- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
- (e) Is G abelian? (2 marks)

Some Group Properties:

- ① In Cayley Table of a **Group**,
no element can repeat in any Row
No " ",
no " ",
repeat in any Column.
i.e. Every element appears Exactly Once
in each Row & in each Column.

Some Group properties:

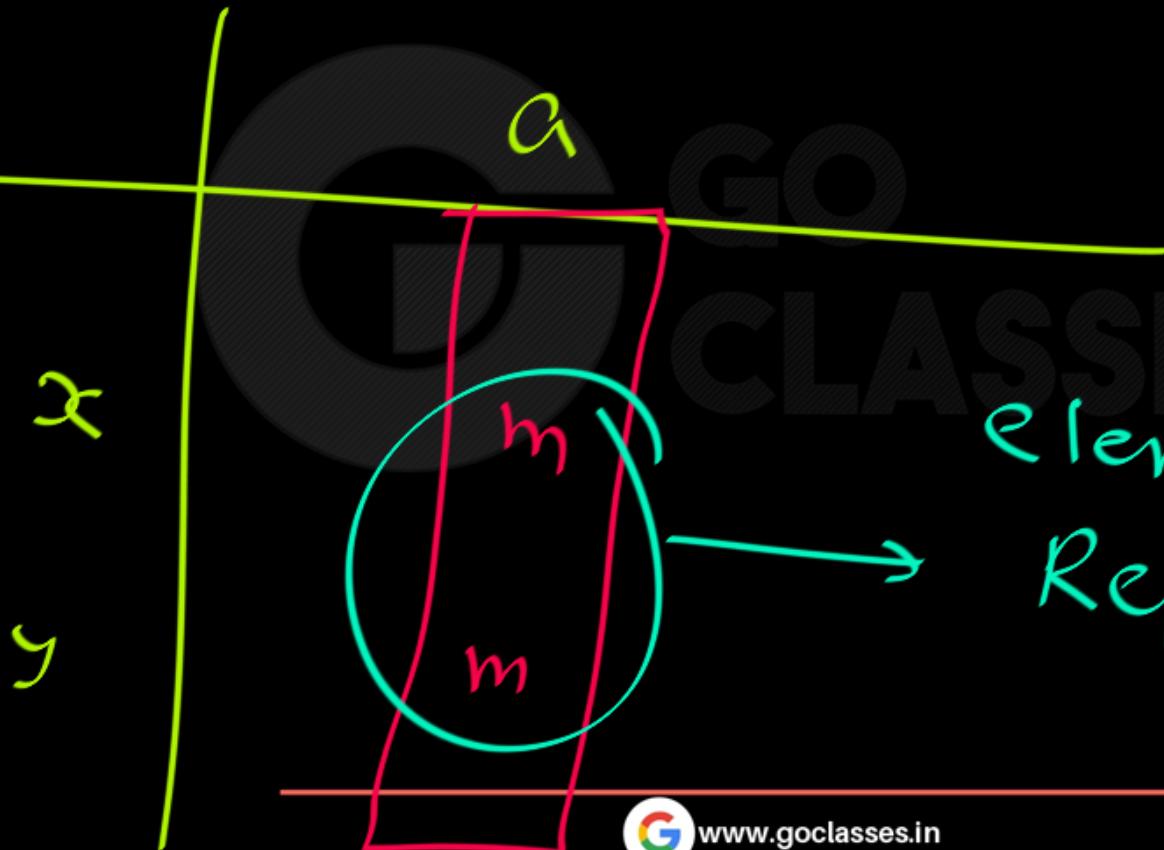
②



This Repetition
Can Not
happen.

Some Group Properties:

②



Element Can Not
Repeat in any
Column.

Some Group Properties:

(2)

If

$$\boxed{xy = e}$$

identity element

$$\begin{cases} x^{-1} = y \\ y^{-1} = x \end{cases}$$

$$\begin{aligned} & xy = e \\ & x^{-1} y = x^{-1} e \\ & \cancel{e} y = \cancel{x^{-1}} e \\ & y = x^{-1} \end{aligned}$$

$$\begin{aligned} & xy = e \\ & x(y^{-1}) = e y^{-1} \\ & x = y^{-1} \end{aligned}$$



Group $(\mathcal{Q}, *)$ binary opⁿ

If $\underline{ab = e}$ identity element

$\bar{a}^{-1} = b$; $\bar{b}^{-1} = a$

Sudoku

Group
Cayley Table

q	z	9	1	j
z	1	0	8	6
9	8	6	4	2
1	0	4	2	3

Group :

	a	b	c	d
a	a	b	c	d
b	b			
c	c			
d	d			

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

① identity:
e

*	e	a	b	c	d
e	e	a	b	c	d
a	a		e	b	
b	b				
c	c				
d	d				

- (a) Demonstrate $a * d = c$ and $a * a = d$. aktb (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. e X
b X
a X (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
- (e) Is G abelian? (2 marks)

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

① identity:
 e

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b				
c	c				
d	d				

$$\tilde{a}^{-1} = b$$

$$\tilde{b}^{-1} = a$$

$$a * \underline{?} = e$$

(4 marks)

(4 marks)

(5 marks)

 $\Rightarrow b$

$$\tilde{a}^{-1} = \infty$$

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
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- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
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- (e) Is G abelian? (2 marks)

$$a * x = e$$

$$a * \bar{b} = e$$



$$\begin{array}{l} \overset{-1}{a} = b \\ \overset{-1}{b} = a \end{array}$$

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

① identity:
e

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	C
b	b	e			
c	c	b			
d	d	C			

b, c

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
- (e) Is G abelian? (2 marks)

$b * a = e$

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e c	d	a	
c	c	b			
d	d	c			

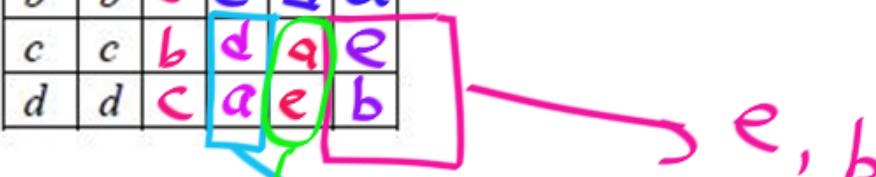
α
 $d \times c \times$
 $b \times e \times$

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
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*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
- (e) Is G abelian? (2 marks)

 e, b

(4 marks)

(4 marks)

(5 marks)

(5 marks)

(2 marks)

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

$$a * d = c$$

$$a * a = d$$

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

$$a * e$$

$$b * d$$

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
- (c) Implement group axiom, for each element of G , find its inverse. (5 marks)
- (d) Complete the above Cayley table of G . (5 marks)
- (e) Is G abelian? (2 marks)

Let $G = \langle \{e, a, b, c, d\}, *\rangle$ be a group with the following Cayley table partially completed.

$$\begin{aligned} e^{-1} &= e \\ a^{-1} &= b \\ b^{-1} &= a \end{aligned}$$

*	e	a	b	c	d
e	e	a	b	c	d
a	a	d	e	b	c
b	b	c	d	a	e
c	c	b	d	a	e
d	d	c	a	e	b

$$c^{-1} = d; d^{-1} = c$$

$$c * d = e$$

(4 marks)

identity

$$c^{-1} = d;$$

(4 marks)

$$d^{-1} = c$$

(5 marks)

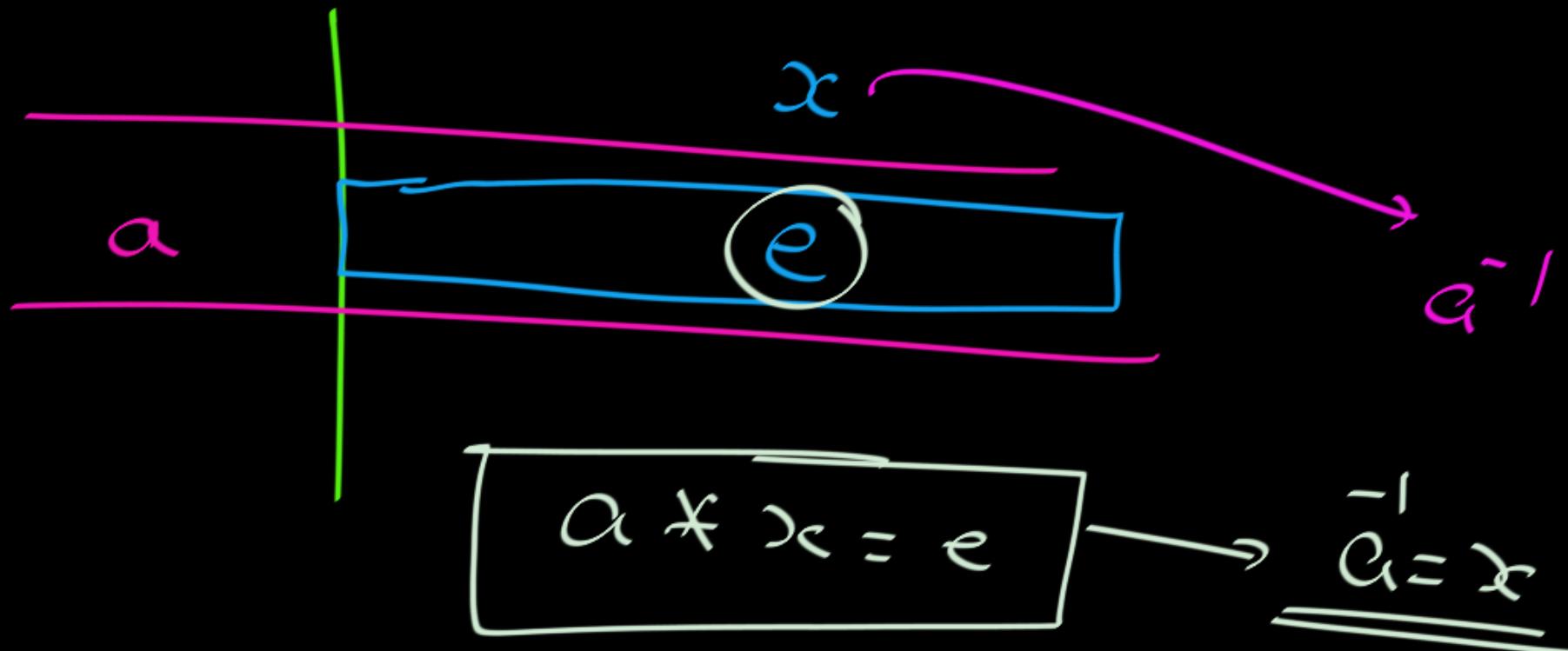
(a) Demonstrate $a * d = c$ and $a * a = d$.(b) Demonstrate $b * d = a$ and $b * c = d$.(c) Implement group axiom, for each element of G , find its inverse.(d) Complete the above Cayley table of G .(e) Is G abelian?

(5 marks)

(2 marks)

Group Cayley Table:

e : Identity element find \bar{a}^{-1}



Group Cayley Tables:

Abelian iff Table is symmetric

$\forall i$

i^{th} Row = i^{th} Column

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e	e	a	b	c	d
a	a	d	e	b	c
b	b	e	c	d	a
c	c	b	d	a	e
d	d	c	a	e	b

- (a) Demonstrate $a * d = c$ and $a * a = d$. (4 marks)
- (b) Demonstrate $b * d = a$ and $b * c = d$. (4 marks)
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