

Combinatorics

Discrete Mathematics

Questions:

Combinatorial Arguments

Website: https://www.goclasses.in/





Hockey-Stick Identity

For
$$n, r \in \mathbb{N}, n > r, \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$$
.

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n-1}{k} + \binom{n-2}{k} + \dots + \binom{k+1}{k} + \binom{k}{k}.$$





a combinatorial proof of $k\binom{n}{k} = n\binom{n-1}{k-1}$



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Story = n people --> Make a team of k, with one of them as captain.

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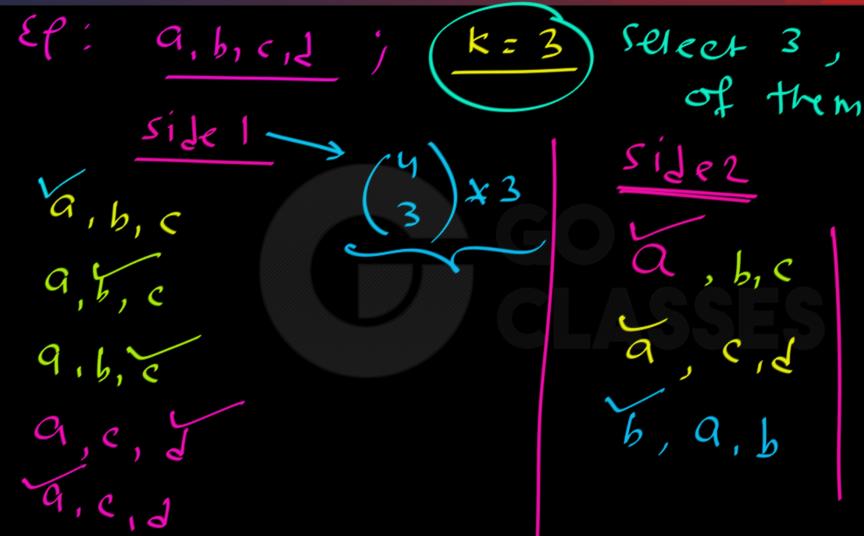
Side 1 = Select k people, then select one captain. $\binom{n}{k}$

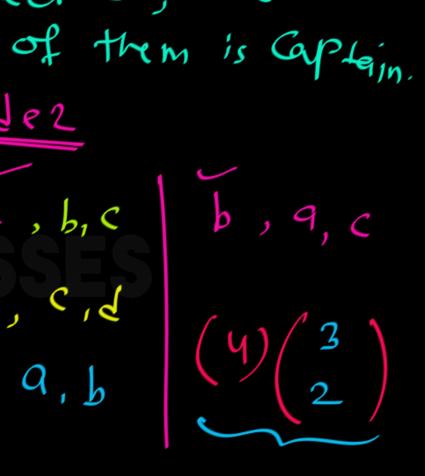
Side 2 = Select a captain first, then select k-1 people

from remaining n-1 people. =
$$\binom{h}{k-1}$$











a combinatorial proof of $k\binom{n}{k} = n\binom{n-1}{k-1}$

To select a committee of k people with a president from a group of n people, you can

- 1. select the k committee members, and then select the president from the members, or
- 2. select the president from the n people, and the k-1 remaining committee members from the remaining n-1 people.

You could also choose k-1 ordinary committee members from the group of n, and then a president from the remaining n-k+1, so $(n-k+1)\binom{n}{k-1}$ should also give the same value – Henry Oct 5, 2014 at 0:09 🥕





Combinatorial Proof -
$$\binom{n}{r} = \frac{n}{r} \binom{n-1}{r-1}$$





$$\begin{cases}
h \\
y
\end{cases} = (h) \begin{pmatrix} h-1 \\
r-1
\end{pmatrix} \qquad \begin{pmatrix} 50 \\
30
\end{pmatrix} = \frac{50}{30} \begin{pmatrix} 49 \\
29
\end{pmatrix}$$

$$= \begin{pmatrix} h \\
y
\end{cases} = \begin{pmatrix} h-1 \\
y-1
\end{pmatrix} \qquad \begin{pmatrix} 48 \\
21
\end{pmatrix} = \frac{48}{21} \begin{pmatrix} 47 \\
20
\end{pmatrix}$$

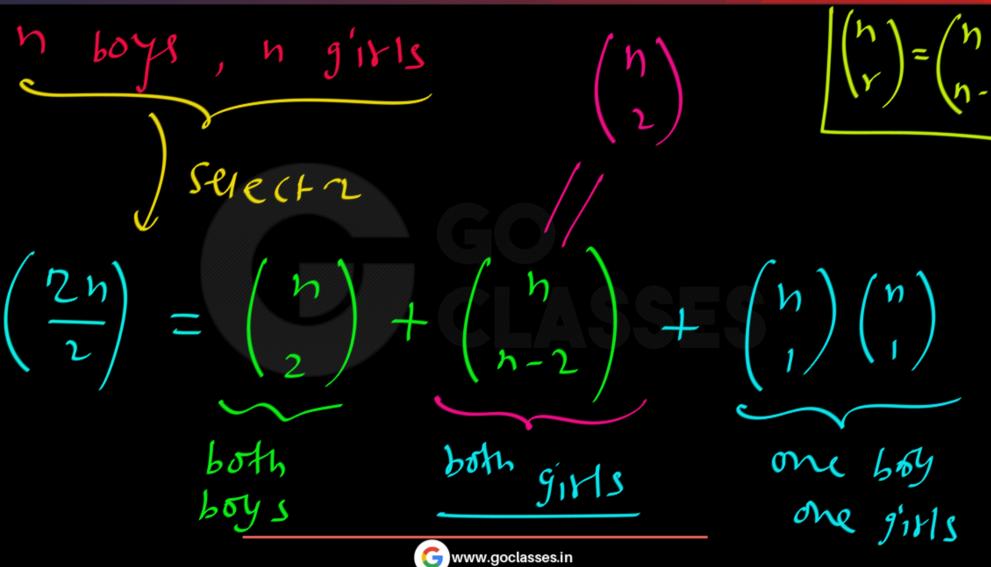
$$\frac{2}{30} \begin{pmatrix} 49 \\
29 \end{pmatrix}$$



Prove the identity $\binom{k}{2} + \binom{k}{k-2} + k^2 = \binom{2k}{2}$, where $k \geq 2$ using a combinatorial proof.

$$\binom{2k}{2} = k^2 + 2\binom{k}{2} = k^2 + \binom{k}{2} + \binom{k}{k-2}$$









$$\operatorname{Sum} \operatorname{of} k \binom{n}{k} \text{ is } n2^{n-1}$$

Proof that $\sum_{k=1}^{n} k \binom{n}{k}$ for $n \in \mathbb{N}$ is equal to $n2^{n-1}$.

Story: Make a Committee, with one of them as president.





$$\sum_{k=1}^{n} k \binom{n}{k} = \binom{n}{2}^{n-1}$$
a subset of n people, one of them is President



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■ https://gateoverflow.in/302843



Let $U = \{1, 2, ..., n\}$ Let $A = \{(x, X) \mid x \in X, X \subseteq U\}$. Consider the following two statements on |A|.

I.
$$|A| = n2^{n-1}$$

II.
$$|A| = \sum_{k=1}^n k \binom{n}{k}$$

Which of the above statements is/are TRUE?

- A. Only I
- B. Only II
- C. Both I and II
- Neither I nor II

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Let
$$U = \{1, 2, ..., n\}$$
 Let $A = \{(x, X) \mid x \in X, X \subseteq U\}$. Consider the following two statements on $|A|$.

I.
$$|A| = n2^{n-1}$$

I.
$$|A|=n2^{n-1}$$
II. $|A|=\sum_{k=1}^n k \binom{n}{k}$

Which of the above statements is/are TRUE?

- A. Only I
- B. Only II
- Both I and II
- Neither I nor II

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