Next Topic:

Checking Associative Property

in a Cayley Table

Structure : $(S, \#)$ binary operation

$(S, \#)$ is Associative iff
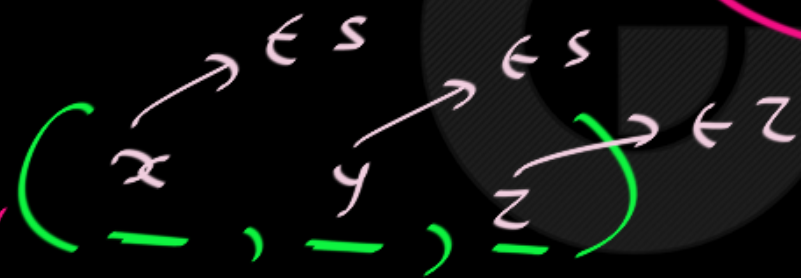
$\forall a, b, c \in S$

$$(a \# b) \# c = a \# (b \# c)$$

$$\left(S = \{a, b, c, d\} , \# \right) \quad \text{binary op}^n$$

To check for (Associative) prop:

$$\left(\underset{x}{\underbrace{}}^{\in S}, \underset{y}{\underbrace{}}^{\in S}, \underset{z}{\underbrace{}}^{\in z} \right)$$

Triple

$$(x \# y) \# z \overset{?}{=} x \# (y \# z)$$

no. of Triples:

$4 \times 4 \times 4 = 64$

(x, y, z)

$\in S$
$\in S$
$\in S$

4 choices

4 choices

$|S| = 4$

$$\left(S = \{a_1, a_2, \cdots, a_n\}, \ast \right) \quad \rightarrow \text{binary op}^n$$

$$|S| = n$$
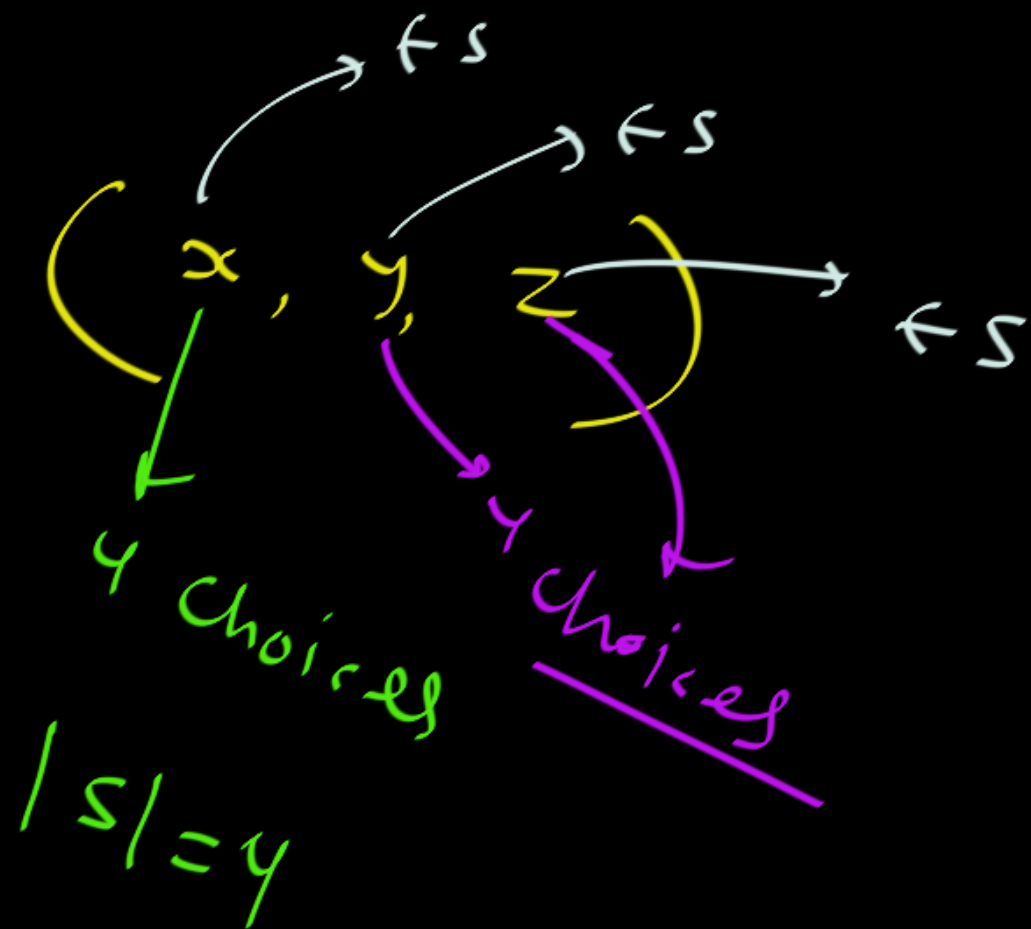
To check for (Associative) prop:

Time Complexity:

Check $(x \ast y) \ast z \overset{?}{=} x \ast (y \ast z)$ for every Triple (x, y, z)

$\leftarrow S$

n choices

$$\left(S = \{a_1, a_2, \text{--}, a_n\}, * \right)$$

To check for $\boxed{Associative}$ prop:

Time Complexity: no. of Triples $= \boxed{n^3}$

$$O(n^3) \checkmark$$

Is this structure Associative?

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

Is this structure Associative?

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

$(a * b) * c$

$b * c = a$

$a * (b * c)$

$a * a = a$

Is this structure Associative?

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

$S = \{ a, b, c, d, e \}$

$|S| = 5$

for 5^3 Triples we need to check Asso.

Is this structure Associative?

(c, e, e)

$(c * e) * e$

$\neq$

$c * (e * e)$

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

No

$(c * e) * e$

$a * e = $

$c * (e * e)$

$c * $

$= b$

I have the following table and I don't know how to determine if an operation is associative based on the table. Is there an easy way to do it? Or it's just brute force

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

In general, it is not possible to check for associativity simply by glancing at the Cayley table. This is, in part, because associativity is determined from a three termed equation $a(bc) = (ab)c$ whilst the Cayley table shows two-term products only.

$$c * (e * e) = c * c = b$$

$$(c * e) * e = a * e = a$$

and so

$$c * (e * e) \neq (c * e) * e$$

So $*$ is not associative.

NOTE:

In the GATE or any objective exam, they will not ask to check for Associativity by giving a Cayley table..

So, Don't Worry About It.