First Order Logic
Next Chapter:

Validity of FOL Expression

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GATE CSE AIR 53; AIR 67;
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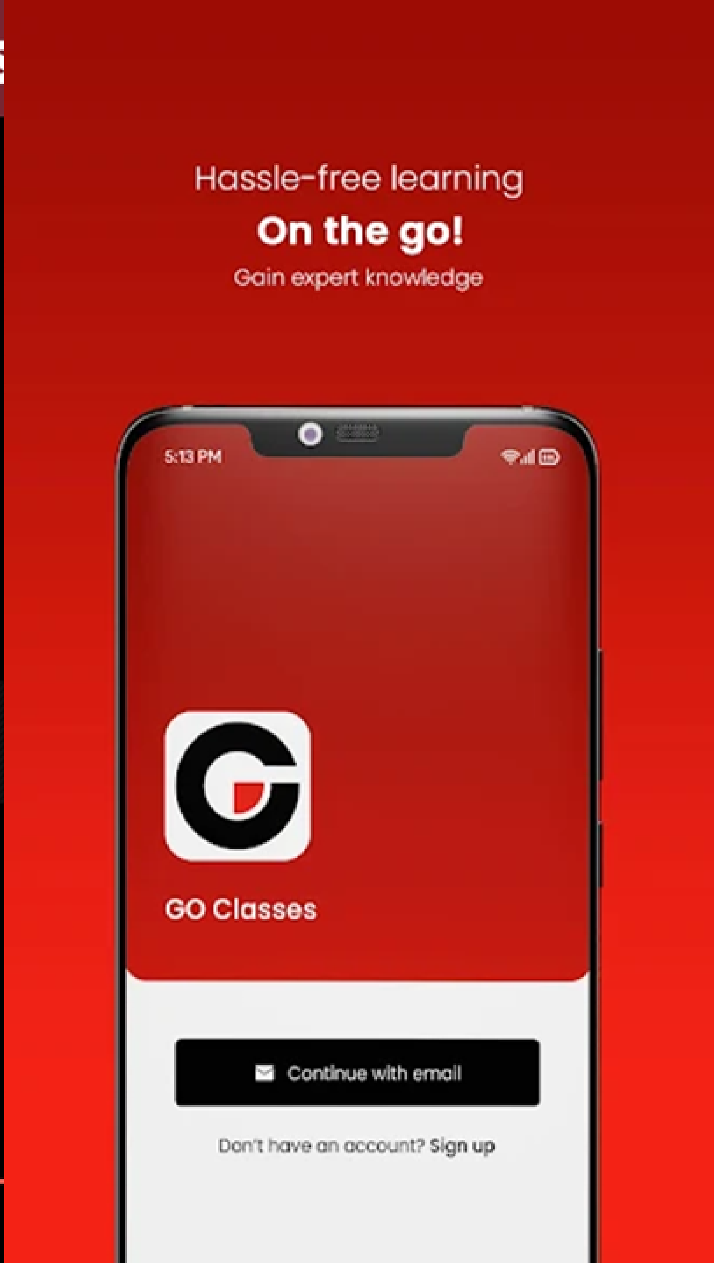

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Next Topic:

Validity, Satisfiability In FOL

Valid $\equiv$ $\boxed{\text{Always True}}$

Satisfiable = Possible to be True

(Sometimes True)

Let M be a First Order Logic Expression.

M is Valid iff M is ALWAYS TRUE (Whatever non-empty domain we take & whatever predicate we take)

M is Satisfiable iff M is possible to be TRUE (for some domain & some predicate)

Is it Valid? Is it satisfiable??

S: $\forall x\, P(x)$

Is it Valid? Is it satisfiable??

S: $\forall x\, P(x)$ → satisfiable

Domain : N

P(x) : $x > -1$ → $\boxed{S = \text{True}}$ ⟹ S : satisfiable

N

1, 2, 3
— — —
— — —

Is it Valid? Is it satisfiable??

S: $\boxed{\forall x\, P(x)}$ → Satisfiable ✓

Domain: Z
P(x) : $x^2 \geq 0$ → $\boxed{S = \text{True}}$

Is it Valid? Is it satisfiable??

S: $\forall x\, P(x)$ → Invalid

Domain: N
P(x) : x is even → $\boxed{S = \text{false}}$

Is it Valid? Is it satisfiable??

S: $\exists x\, P(x)$ → Satisfiable

Domain : N

P(x) : x is even → S = T

Is it Valid? Is it satisfiable??

S: $\exists x\, P(x)$

Domain: N

P(x) : x < -2

S = false

~~Invalid~~

Is it Valid? Is it satisfiable??

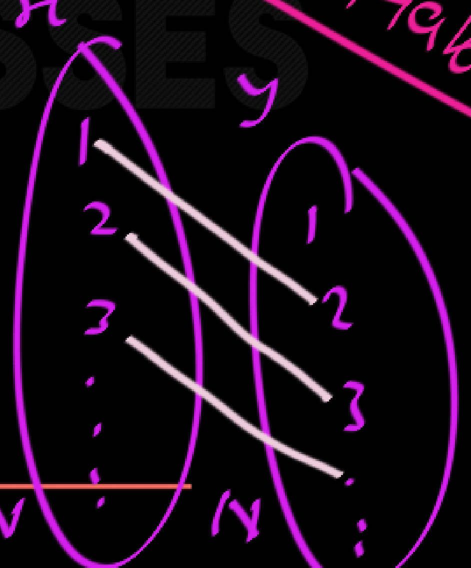
S : $\forall x \exists y P(x, y)$.

Satisfiable ✓

Domain : $\mathbb{N}$

$P(x,y) : x<y$

S = True



$x<y$

Is it Valid? Is it satisfiable??

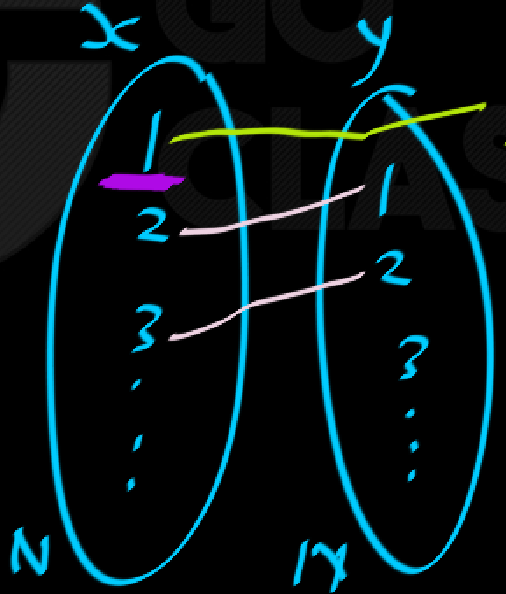
S : $\forall x \exists y P(x, y).$

Invalid

Domain : N

P(x,y) : x > y

S : False



None

x > y

Is it Valid? Is it satisfiable??

$$S: (\underline{\exists x\ Px}) \lor (\underline{\exists y \sim Py})$$

Is it Valid? Is it satisfiable??

$$S: \underbrace{(\exists_x P_x)}_{\alpha} \vee \underbrace{(\exists_y \sim P_y)}_{\beta}$$

$S = \alpha \vee \beta$

satisfiable ✓

Domain: N

P(x): x is prime

α : True

β : True

$S = \overset{T}{\alpha} \vee \overset{T}{\beta} = \text{True}$

Is it Valid? Is it satisfiable??

$S: \underbrace{(\exists_x P_x)}_{\alpha} \vee \underbrace{(\exists_y \sim P_y)}_{\beta}$

$S: \boxed{\alpha \vee \beta}$

$S: \alpha \vee \beta$

$\alpha = F$
$\beta = F$

$\boxed{\alpha : \text{False}}$

$\downarrow$

$\boxed{\text{for every element, } P \text{ false}}$

$\beta = \text{True}$

Domain
a, b, . . .

Is it Valid? Is it satisfiable??

$$S : \boxed{\underbrace{(\exists_x P_x)}_{\alpha} \vee \underbrace{(\exists_y \sim P_y)}_{\beta}}$$

Domain

a, b, c ...

$S = \alpha \vee \beta$

If $S = \text{False}$ $\Rightarrow$ $\alpha = F$, $\beta = F$ $\rightarrow$ Never possible

Never possible

Is it Valid? Is it satisfiable??

S : $\boxed{(\exists_x P_x) \lor (\exists_y \sim P_y)}$

→ Valid, ~~satisfiable~~

Valid $\longrightarrow$ Satisfiable

Valid $\not\longleftarrow$ Satisfiable

Is it Valid? Is it satisfiable??

$$\forall x \exists y (x + y = 0)$$

Is it Valid? Is it satisfiable??

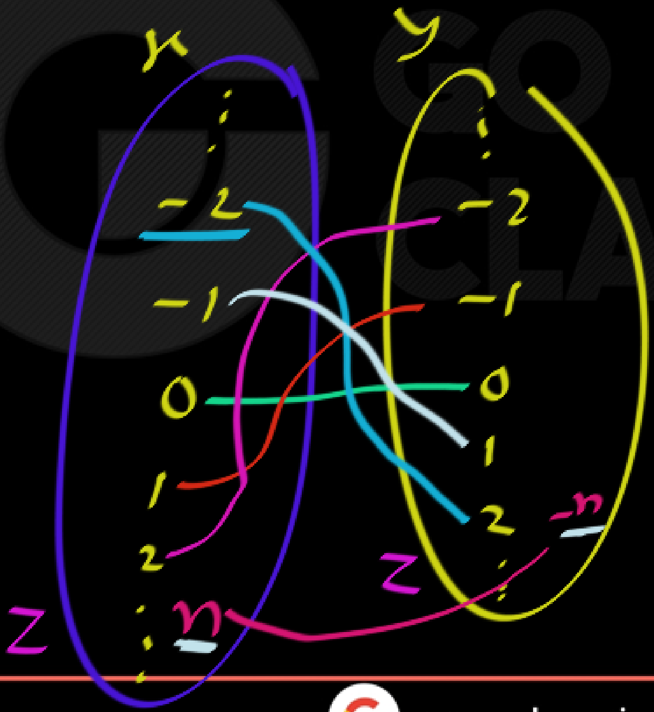
S : $\forall x \exists y (x + y = 0)$

Domain: Z

S = True

Satisfiable

$x + y = 0$

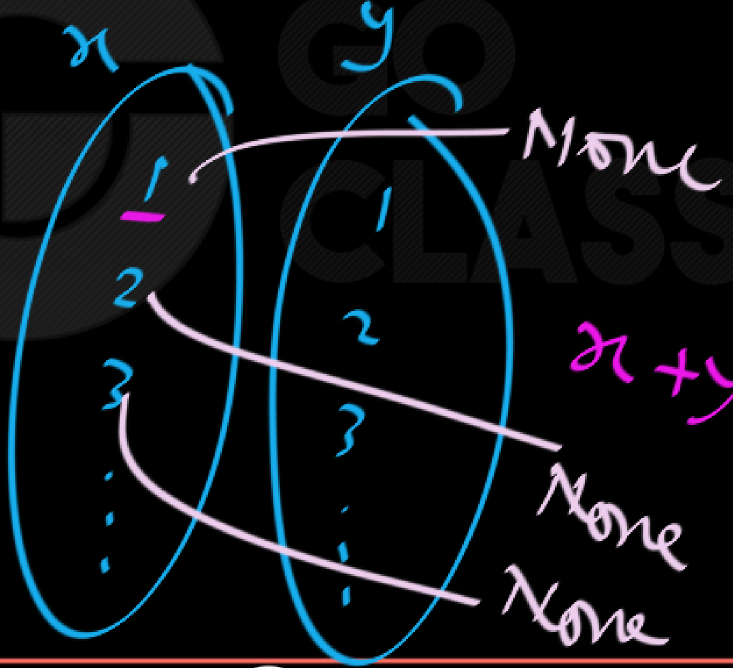


Is it Valid? Is it satisfiable??

S : $\forall x \exists y (x + y = 0)$

$\boxed{\text{Domain: } \mathbb{N}}$

$\boxed{S = \text{False}}$



$x + y = 0$

Invalid

How to check if a FOL Expression M is Valid or Not??

How to check if a FOL Expression M is Valid or Not??

Method 1: Intuition & Logical Thinking

Method 2: A Systematic Procedure

How to check if a FOL Expression M is Valid or Not??

Procedure:

1. Take an abstract domain, like {a,b,c,d,…..}
2. Try to make M false somehow..
   - If you can, M is Invalid.
   - If you can Never, M is Valid.

How to check if a FOL Expression M is Satisfiable or Not??

How to check if a FOL Expression M is Satisfiable or Not??

Method 1: Intuition & Logical Thinking

Method 2: A Systematic Procedure

How to check if a FOL Expression M is Satisfiable or Not??

Procedure:

1. Take an abstract domain, like {a,b,c,d,…..}
2. Try to make M TRUE somehow..
    - If you can, M is Satisfiable.
    - If you can Never, M is Not Satisfiable.

Is it Valid? Is it satisfiable??

$$S: \boxed{(\underbrace{\forall_x P_x}_{\alpha}) \wedge (\underbrace{\exists_y \sim P_y}_{\beta})}$$

$$S: \boxed{\alpha \wedge \beta}$$

Conjunction

To make $S = \text{True}$; $\alpha = T$ & $\beta = T$ Simulataneously

$$S : \underbrace{(\forall x\, Px)}_{\alpha} \land \underbrace{(\exists y \sim Py)}_{\beta}$$

α: For every element in Domain, P is True

β: for at least one element, P is false

Non-Empty Domain

a, b, c

Can't happen together.

S : $\boxed{(\forall x\, Px) \land (\exists y \sim Py)}$

→ invalid

→ Never True

≡ Unsatisfiable

Unsatisfiable → Invalid

NOTE:

1. In a FOL Expression, by default, ALL Variables have Same Domain.

2. Domain in FOL, is ALWAYS Non-Empty.. (unless explicitly gives as empty)

Is it Valid? Is it satisfiable??

✗ ✗

$$\left.\begin{matrix}(\forall_x P_x) \land (\exists_x \sim P_x) \\ \\ (\forall_x P_x) \land (\exists_y \sim P_y)\end{matrix}\right\} \text{Same}$$

Not Satisfiable

NOTE:

1. In a FOL Expression, by default, ALL Variables have Same Domain.

2. Domain in FOL, is ALWAYS Non-Empty..
(unless explicitly gives as empty)

Is it Valid? Is it satisfiable??

$$(\forall_x P_x) \land (\forall_y \sim P_y)$$

$$S : (\underbrace{\forall x\, Px}_{\alpha}) \land (\underbrace{\forall y \sim Py}_{\beta})$$

$S : \alpha \land \beta$

for everyone, P is True

for everyone, P is false

Can't happen together.

Domain

a, b, c

S : $\boxed{(\forall x\, Px) \wedge (\forall y \sim Py)}$

Never True

Unsatisfiable

invalid

Is it Valid? Is it satisfiable??

$$(\exists_x P_x) \wedge (\exists_y \sim P_y)$$

$S : \underbrace{(\exists x\, P x)}_{\alpha} \wedge \underbrace{(\exists y \sim P y)}_{\beta}$

α → for Someone P True

β → for someone P false

Can happen Together

$S : \alpha \wedge \beta$

To make S = True,

$\alpha = T$
& $\beta = T$

Together

Domain

a, b, c, ...

x → Domain

y → Domain

Is it Valid? Is it satisfiable??

$$S : \boxed{\underbrace{(\exists_x P_x)}_{\alpha} \land \underbrace{(\exists_y \sim P_y)}_{\beta}}$$

Domain : N

P(x) : x is prime

$\alpha = T$

witness : 2

$\beta = \text{True}$

witness : 4

$\boxed{S = \text{True}}$

Satisfiable ✓

Is it Valid? Is it satisfiable??

$$S : \underbrace{(\exists_x P_x)}_{\alpha} \wedge \underbrace{(\exists_y \sim P_y)}_{\beta}$$

invalid

Conjunction

~~Valid~~

Domain : N

$P(x)$: $x^2 = -1$ $\rightarrow$ α = false

$\boxed{S = \text{false}}$

Is it Valid? Is it satisfiable??

$$(\forall_x P_x) \vee (\forall_y \sim P_y)$$

$S: \underbrace{(\forall x\, Px)}_{\alpha} \vee \underbrace{(\forall y\, \sim Py)}_{\beta}$

OR

for everyone P is True

for everyone P is false

Domain

a, b, c

$S: \alpha \vee \beta$

S : $\underbrace{(\forall x\, Px)}_{\alpha} \underset{\vee}{\vee} \underbrace{(\forall y \sim Py)}_{\beta}$

Domain: N

$P(x)$: $x \geq 1$

α = True

$\boxed{S = \text{True}}$

S : satisfiable

$S:\ \boxed{\underbrace{(\forall x\, P_x)}_{\alpha} \vee \underbrace{(\forall y \sim P_y)}_{\beta}}$ → Invalid

Domain: N

P(x) : x is prime

→ $\boxed{S = \text{false}}$ → S : Invalid

$\alpha = F$; β = False

$$S : \underbrace{(\forall x \, Px)}_{\alpha} \lor \underbrace{(\forall y \, \sim Py)}_{\beta}$$

Invalid

Satisfiable

a, b, c, -
- -
- -
-

Is it Valid? Is it satisfiable??

S: $[\ \forall x\ P(x)\ ] \rightarrow [\ \exists x\ P(x)\ ]$

$S: \quad [\, \forall x\, P(x) \,] \rightarrow [\, \exists x\, P(x) \,]$

α

β

$S: \boxed{\alpha \rightarrow \beta}$

implication

Always True

Valid

for everyone P is True

for someone P is True

Non-Empty Domain

a, b, c

Q: Assume Domain Empty.

$$S: [\ \underbrace{\forall x\, P(x)}_{\text{True}}\ ] \rightarrow [\ \underbrace{\exists x\, P(x)}_{\text{false}}\ ]$$

??

$\boxed{S: \text{false}}$

S: $\boxed{\forall x\, P(x) \longrightarrow \exists x\, P(x)}$

Valid

Always True

Always Non-Empty Domain

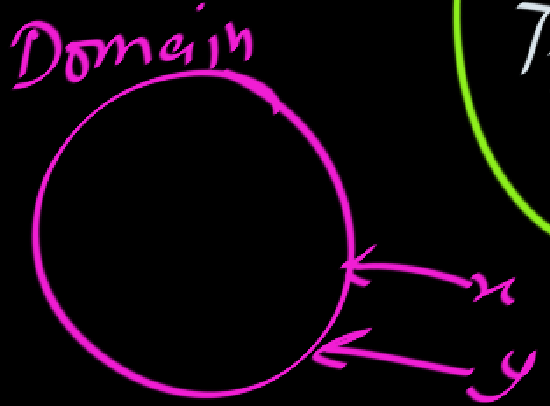
NOTE:

The Domain in FOL is NEVER Empty.

Unless the question explicitly gives an empty domain.

$$\boxed{\forall x\, P(x) \rightarrow \exists y\, P(y)}$$

Valid (Always True)



Because the universe cannot be empty, the formula $\forall x P(x) \Rightarrow \exists y P(y)$ is also valid. If we did allow for the empty universe, it would be merely satisfiable. Excluding the possibility of the empty universe not only makes this intuitively obvious formula true in all interpretations, it avoids the metaphysical question of the need for a logic in a world in which there are no logicians and nothing for them to reason about!

- The sentence $\forall x(P(x) \vee \neg P(x))$ is valid.

- The sentence $\forall x(P(x) \lor \neg P(x))$ is valid.

$\forall_x \left(\underbrace{P(x) \lor \neg P(x)} \right)$

Always True

Always True

Valid

$\boxed{p \lor \bar{p} \equiv \text{True}}$

Domain

a, b, c

- The sentence $\forall x P(x) \vee \forall x \neg P(x)$ is not valid, but is satisfiable.

- The sentence $\exists x(P(x) \land \neg P(x))$ is not satisfiable. Why?

- The sentence $\exists x(P(x) \land \neg P(x))$ is not satisfiable. Why?

$\exists x \left(\underbrace{P(x) \land \neg P(x)}_{\text{Always false}} \right)$

Always false

NOT Satisfiable

invalid

$P \land \neg P \equiv \text{False}$

a, b, c

Next Topic:

Validity of

a FOL Expression Involving Implication