



Sequential Circuits



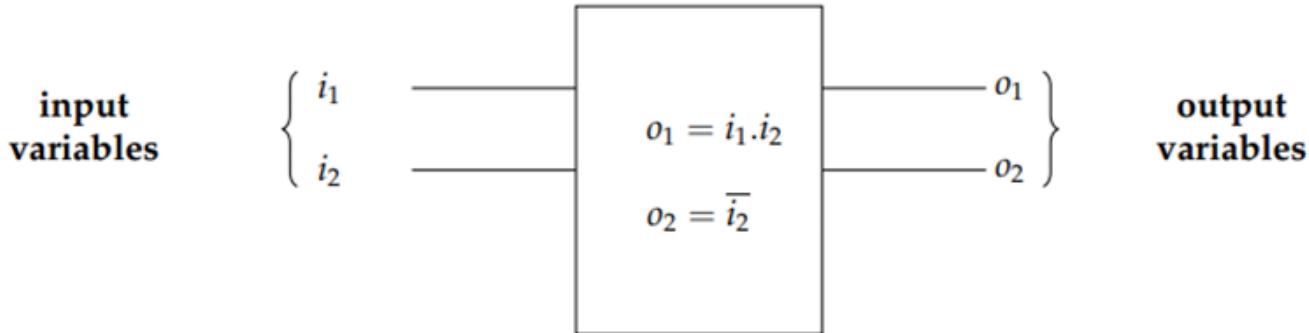
Combinational Circuits Vs Sequential Circuits



Next Topic:

Combinational Circuits Vs Sequential Circuits

Combinational Circuits



Combinational circuits:

- Consists only of logic gates
- The output are determined by the present value of input
- Circuit behavior specified by a set of Boolean functions, Truth-tables, K-maps
- We have learned techniques to analyze and synthesize such circuits
- Examples: Adder, Multiplexers, Encoders, Decoders, etc.



Digital Circuits

Combinational
Circuit

Eg: So far
 $\left\{ \begin{array}{l} \text{logic gates} \\ \text{mux, Decoder,} \\ \text{encoder, Adder} \end{array} \right\}$

Sequential Circuits

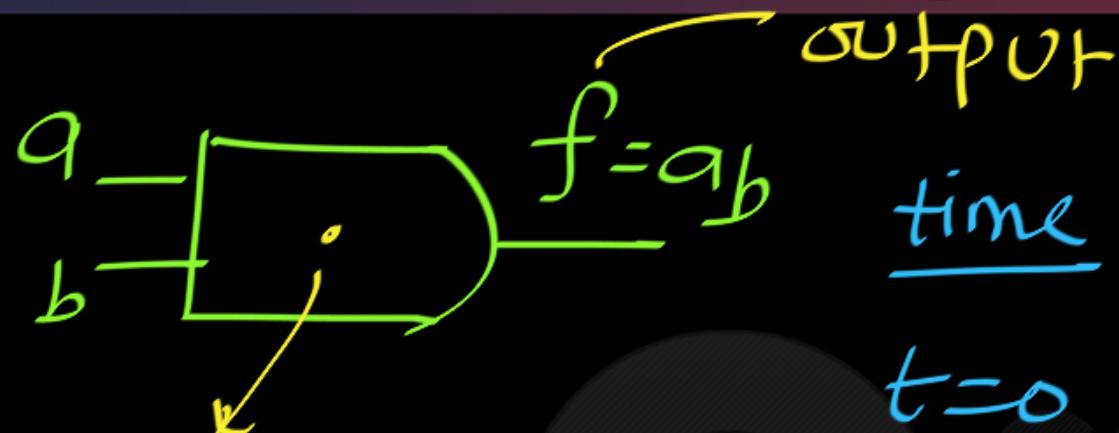
Now onwards:

$\left\{ \begin{array}{l} \text{Counter, Register,} \\ \text{flip flop, latch} \end{array} \right\} ; \left\{ \begin{array}{l} \text{finite state machines} \end{array} \right\}$

Combinational Circuits :

Digital Circuits whose outputs depend only on current inputs .

Current output does not depend on previous outputs / inputs .



Logic gate

Comb. Circuit

WV

Comb. Circuit

Comb. Circuit

Current time

$t=0$

inputs

$a=0$
 $b=0$

output

$f=0$

$t=1$

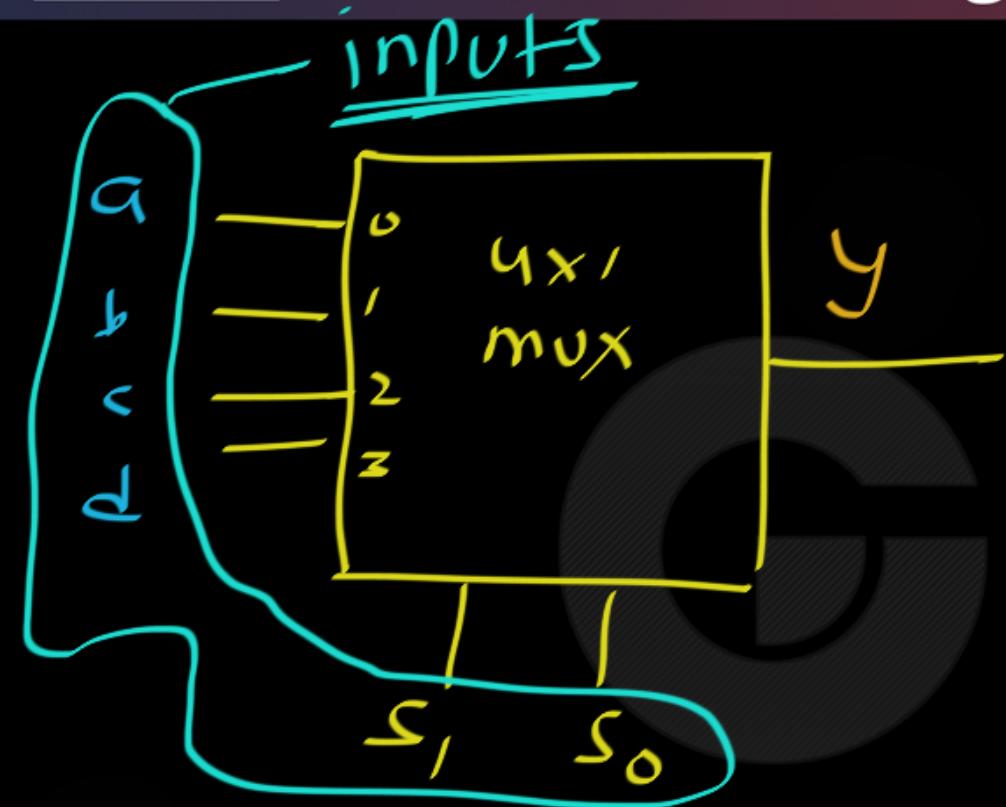
$a=1$
 $b=0$

$f=0$

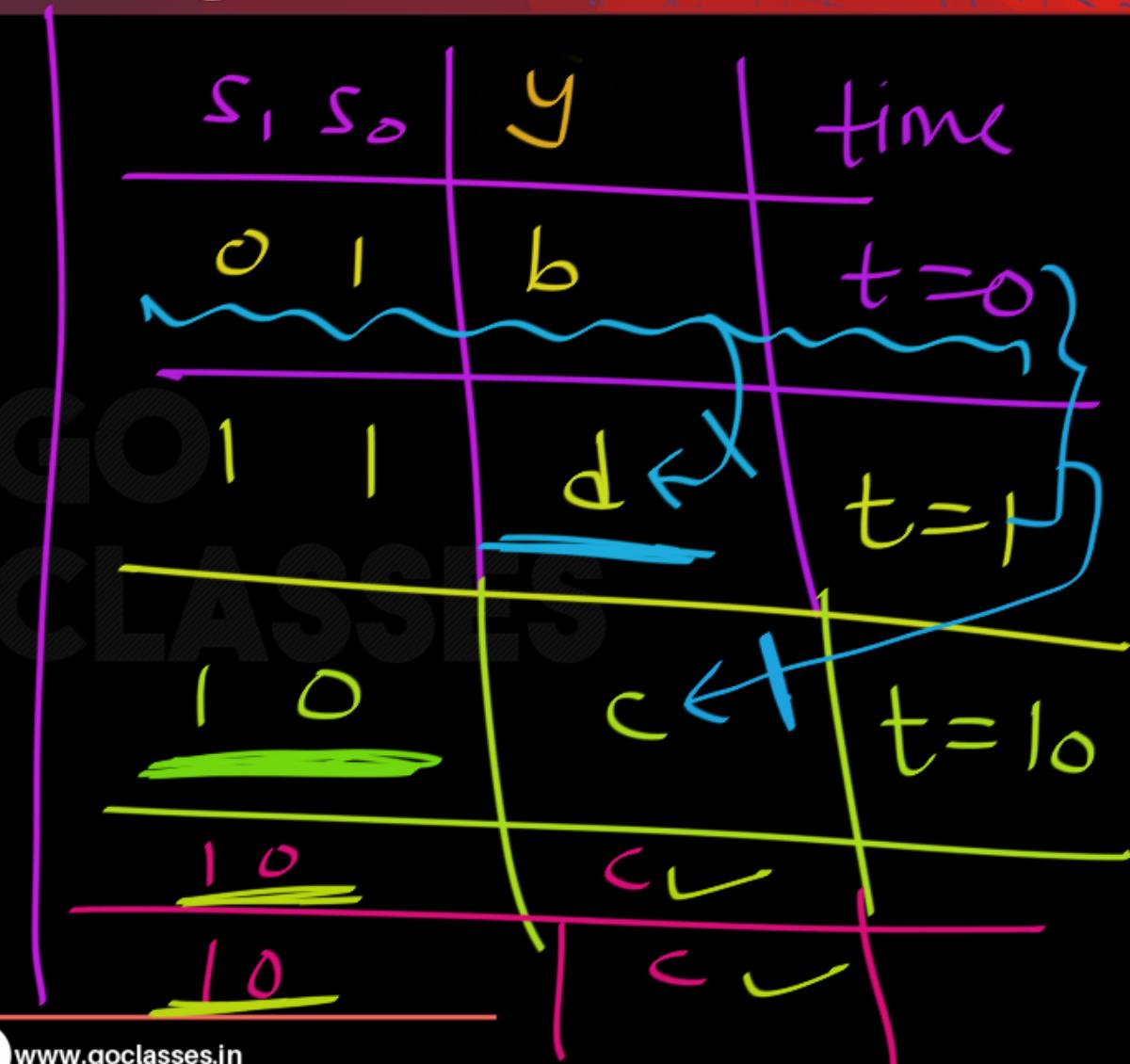
$t=2$

$a=1$
 $b=1$

$f=1$



$$y = f(s_1, s_0, a, b, c, d)$$





Combinational Circuits:

"Living in the Present."

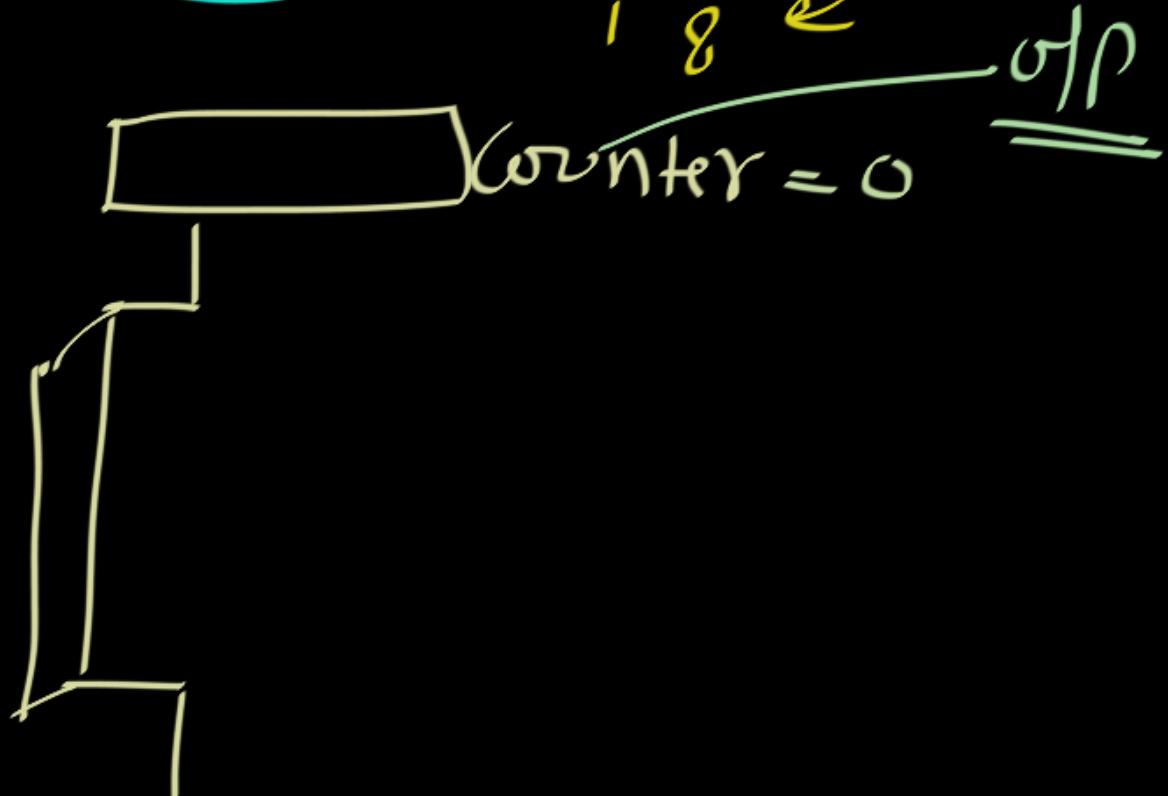


Automatic
Washing machine :-



~~20~~
~~19~~
~~18~~

Example :- Counter
input



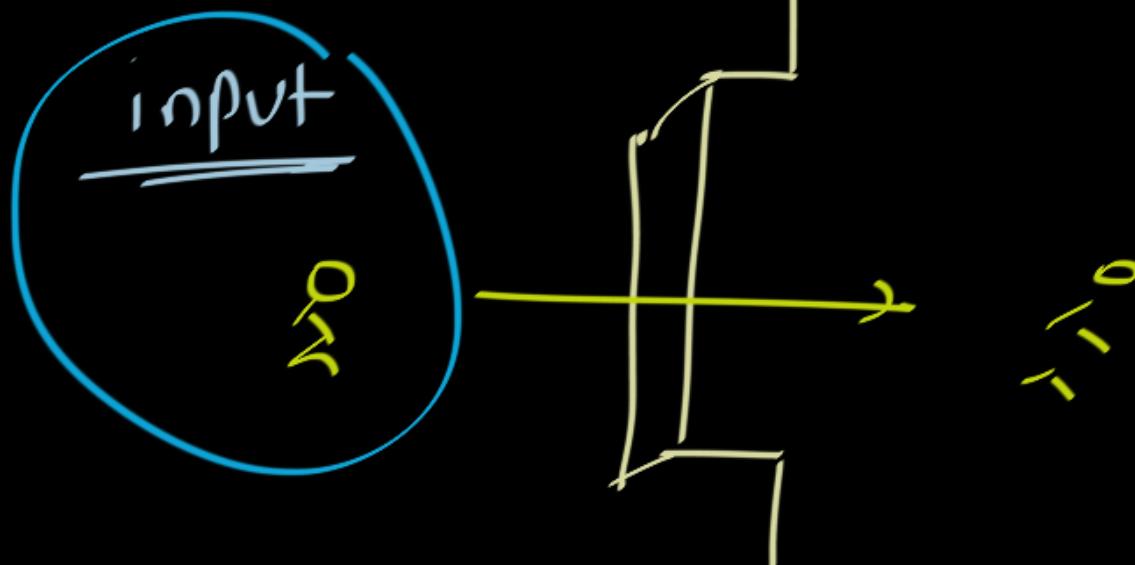
Automatic
Washing machine :-



~~20~~
~~19~~
~~18~~

Example :- Counter

OP
Counter = ~~Q~~ 1



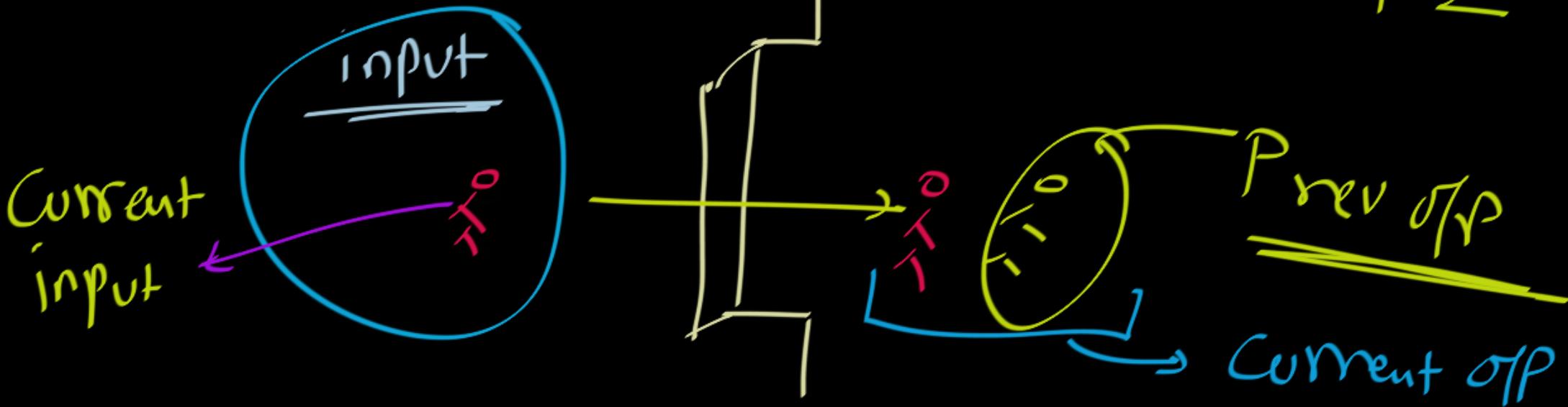
Automatic
Washing machine :-



~~20~~
~~19~~
~~18~~

Example :- Counter

Counter = ~~Q~~ \times 2 \oplus



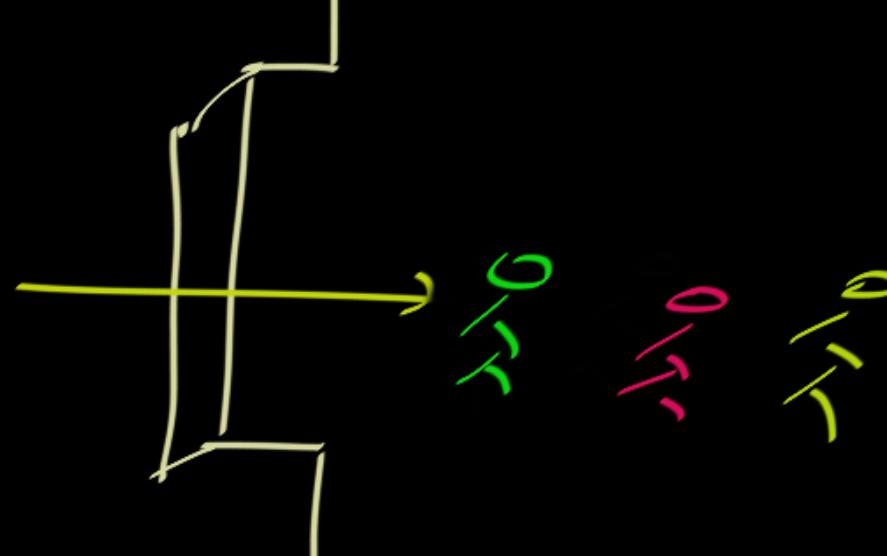
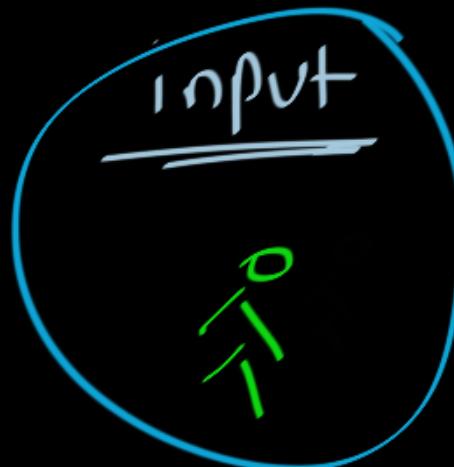
Automatic
Washing machine :-



~~20~~
~~19~~
~~18~~

Example :- Counter

Counter = ~~0~~ \times ~~2~~ $\overline{+}$ ~~3~~



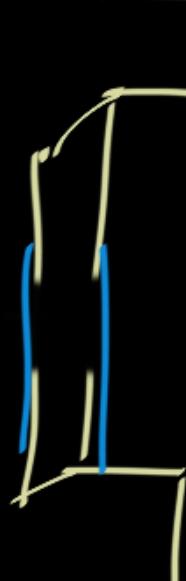
Automatic
Washing machine :-

timer

~~20~~
~~19~~
~~18~~

Example :- Counter

input



Counter

~~0~~
~~1~~
~~2~~
~~3~~

0 0 0

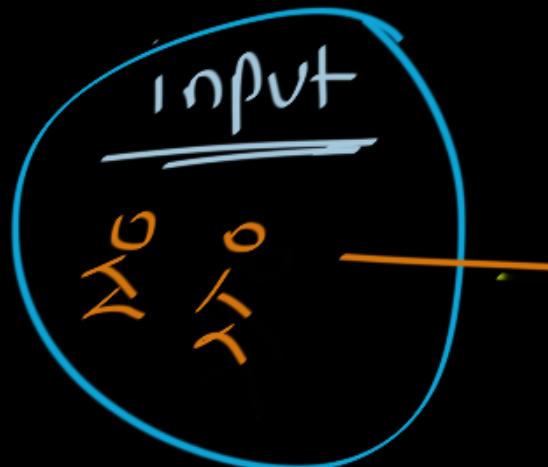
Automatic
Washing machine :-



~~20~~
~~19~~
~~18~~

Example :- Counter

Counter = ~~0~~ \times ~~2~~
~~3~~
~~3~~
~~5~~



Counter ?

Current output = $f\left(\underline{\text{Current input}}, \underline{\text{prev. output}}\right)$

Sequential Circuits

Sequential Logic

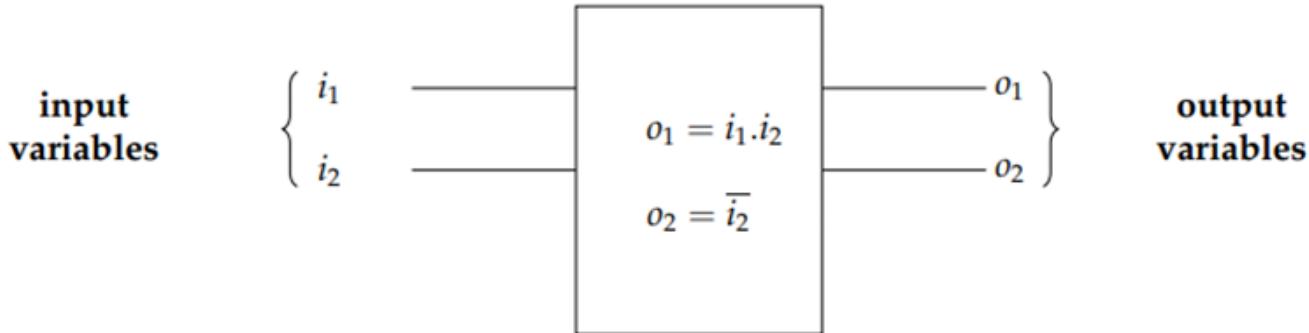
- The logic circuits discussed previously are known as combinational, in that the output depends only on the condition of the latest inputs
- However, we will now introduce a type of logic where the output depends not only on the latest inputs, but also on the condition of earlier inputs. These circuits are known as sequential, and implicitly they contain *memory* elements



Properties of Sequential Circuits

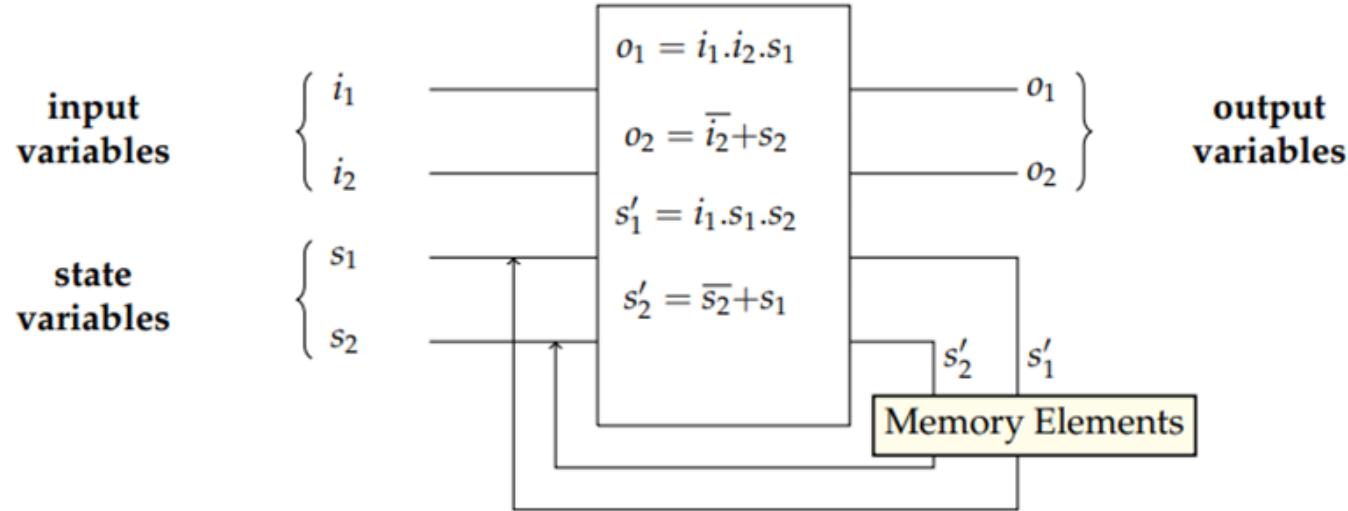
- ◆ So far we have seen Combinational Logic
 - ➡ ■ the output(s) depends only on the current values of the input variables
- ◆ Here we will look at Sequential Logic circuits
 - ➡ ■ the output(s) can depend on present and also past values of the input and the output variables

Combinational Circuits



Combinational circuits:

- Consists only of logic gates
- The output are determined by the present value of input
- Circuit behavior specified by a set of Boolean functions, Truth-tables, K-maps
- We have learned techniques to analyze and synthesize such circuits
- Examples: Adder, Multiplexers, Encoders, Decoders, etc.

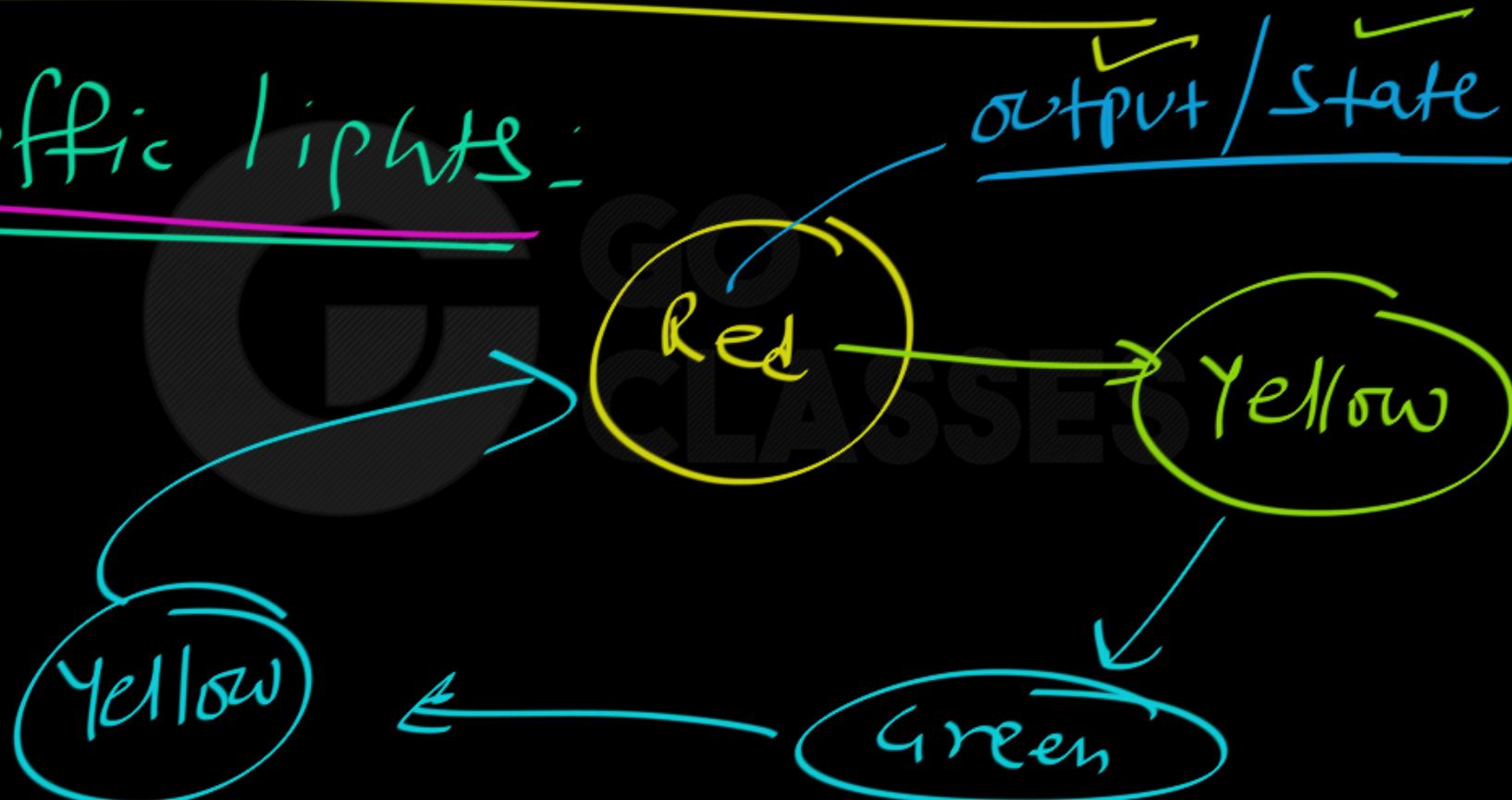


Sequential Circuits:

- Consists of **logic gates** and **storage elements**
- The output are determined by the present value of the **input** and the **state of the storage elements**
- In effect, the output may depend on past values of the input via the state of the storage elements

Real life examples of Seq. Ckts.

① Traffic lights:



Real life examples of Seq Circuits

② Almost Every (useful) Circuit is

Sequential Circuit

IISc

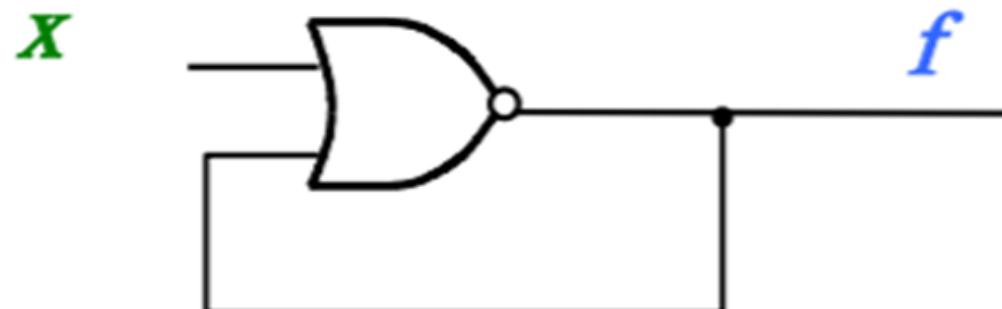
Cricket:

Next output = $f(\text{Current input}, \text{Prev Sop})$



Is it Combinational OR Sequential ?

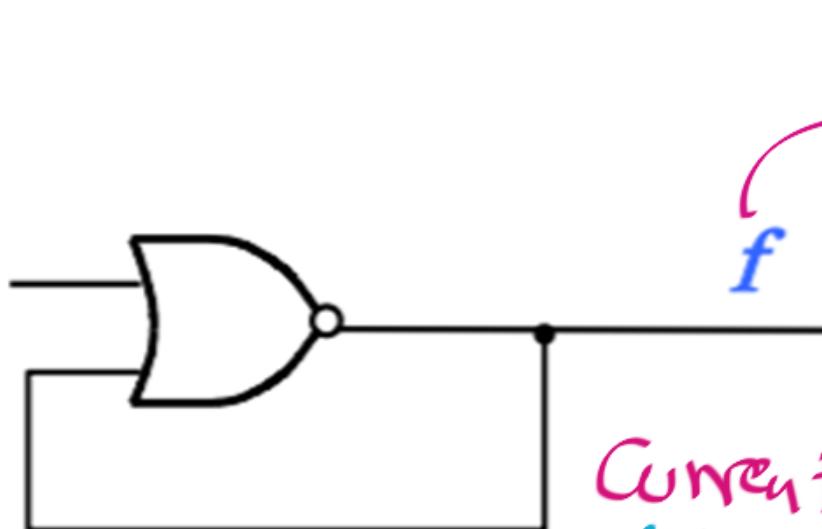
Let's Try to Analyze This Circuit



Seq. Ckt

Let's Try to Analyze This Circuit

input
x

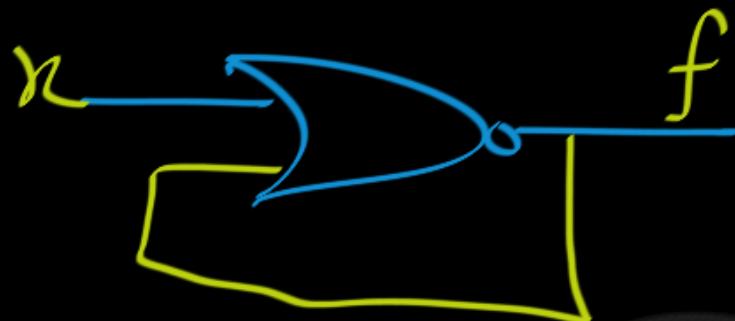


output/state
f

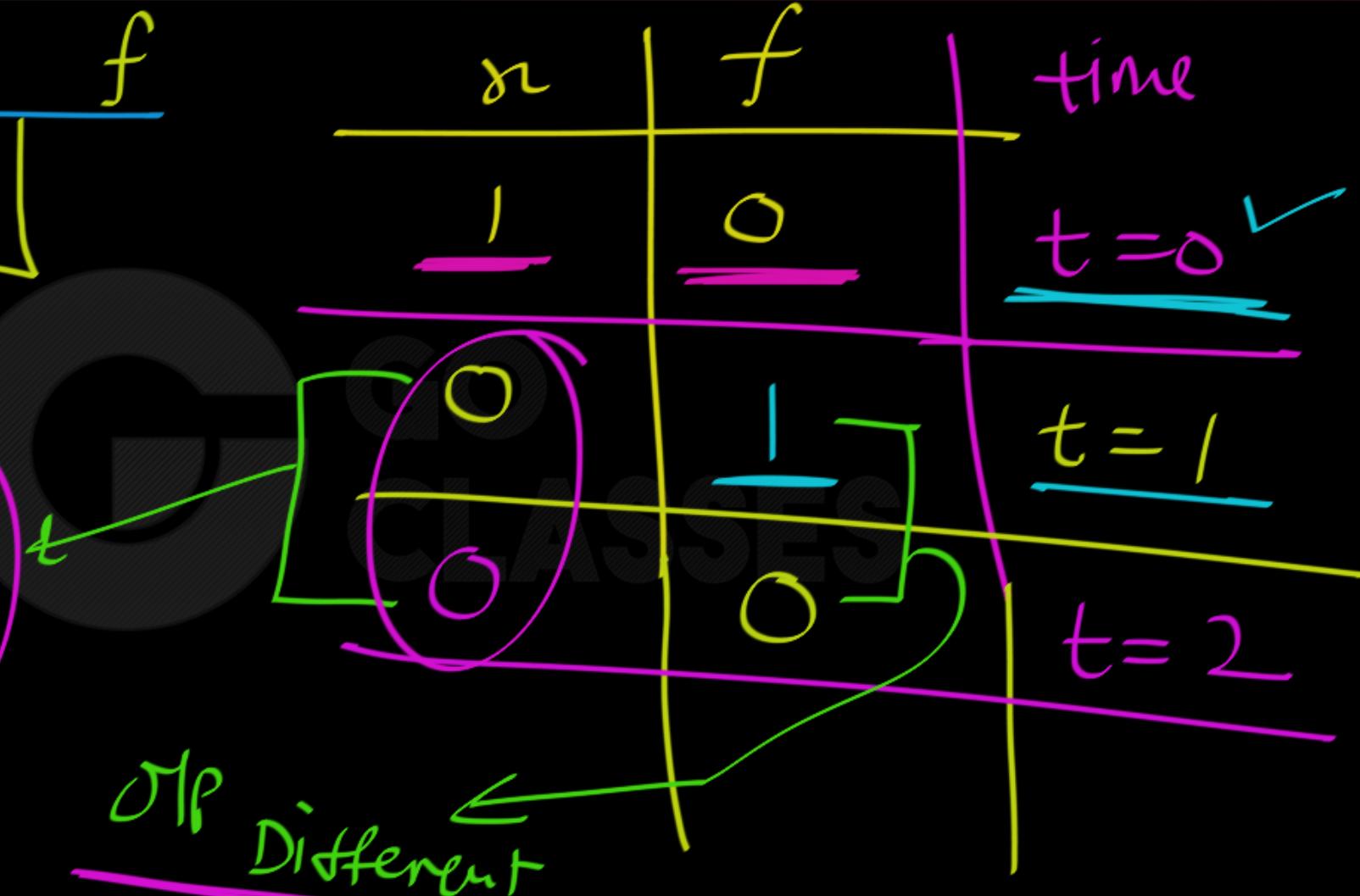
Current
output $f = f(n, f_{\text{prev}})$

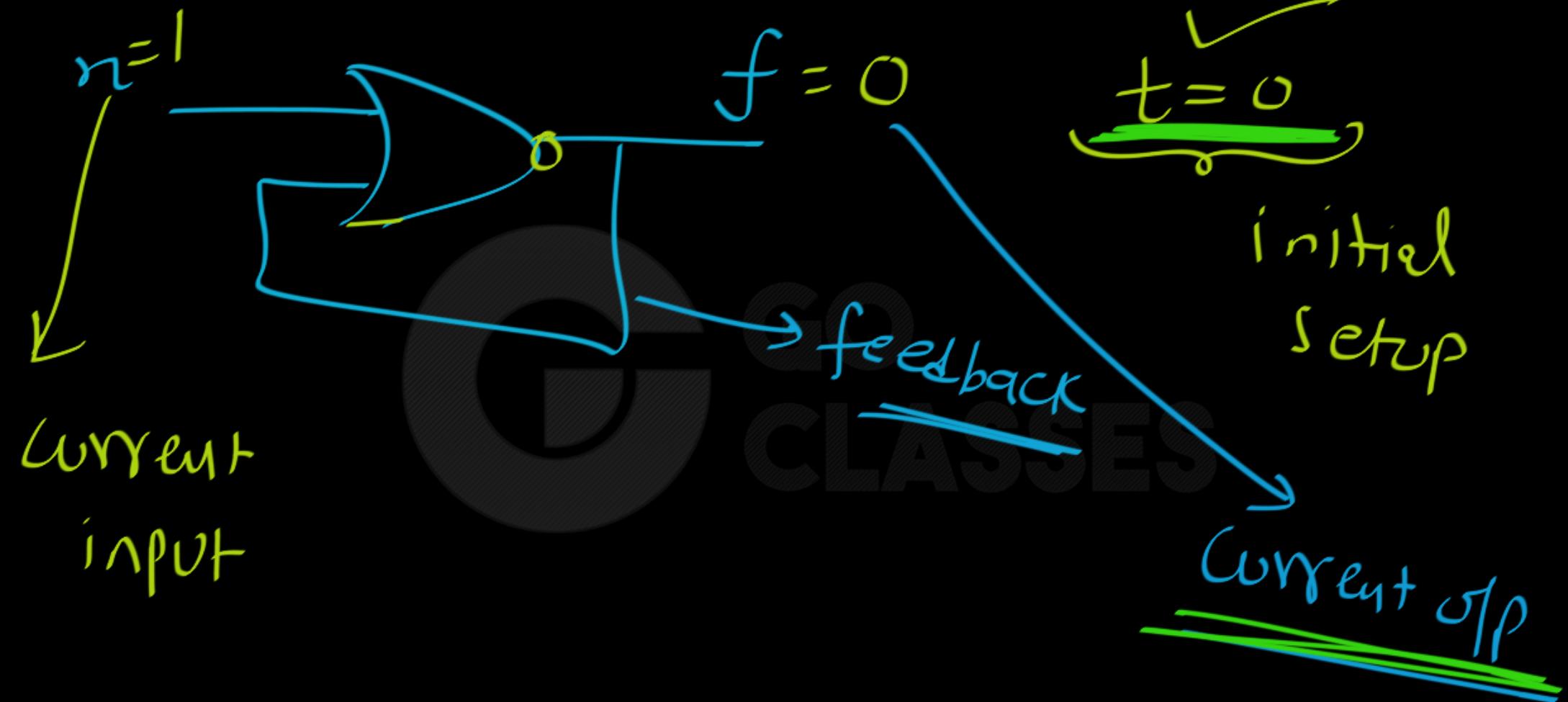


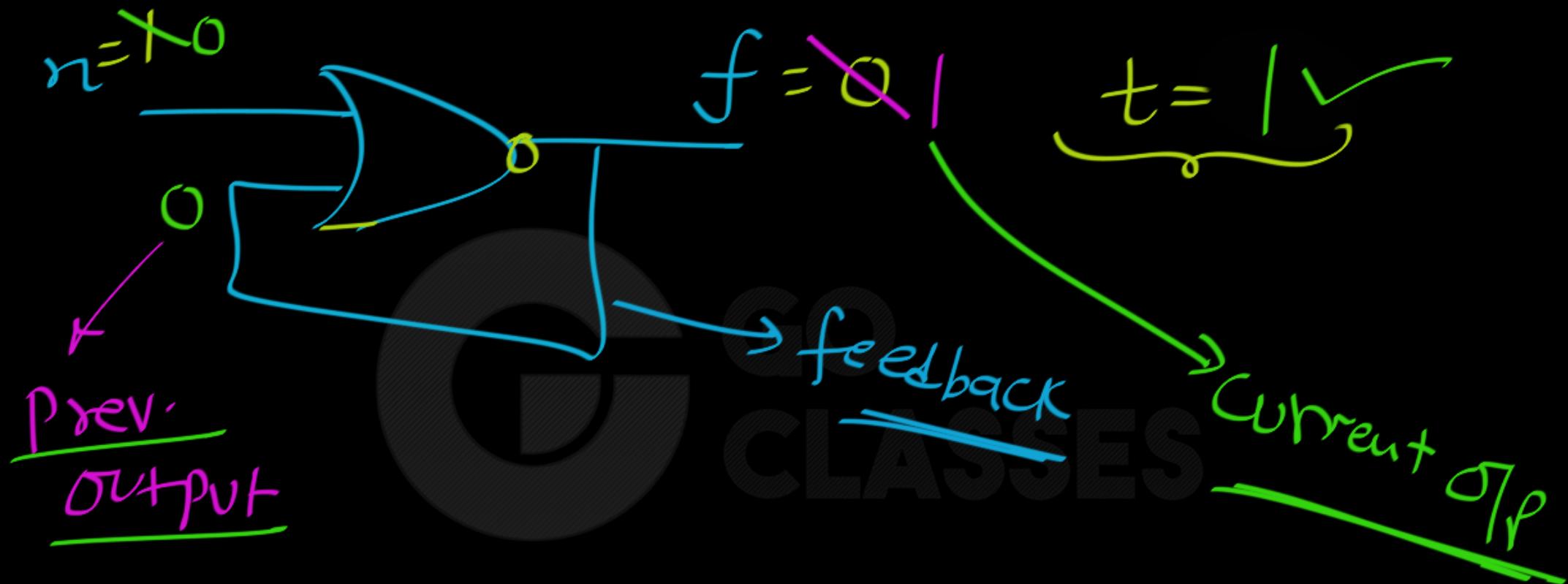
Digital Logic

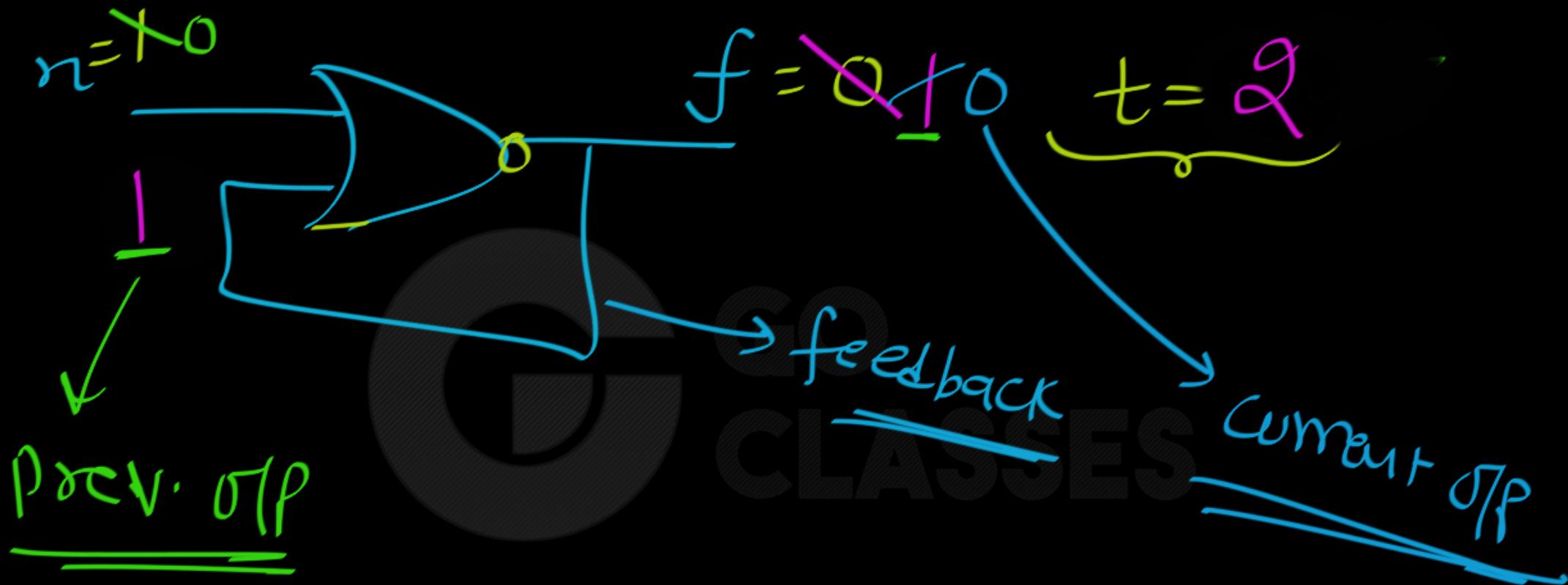


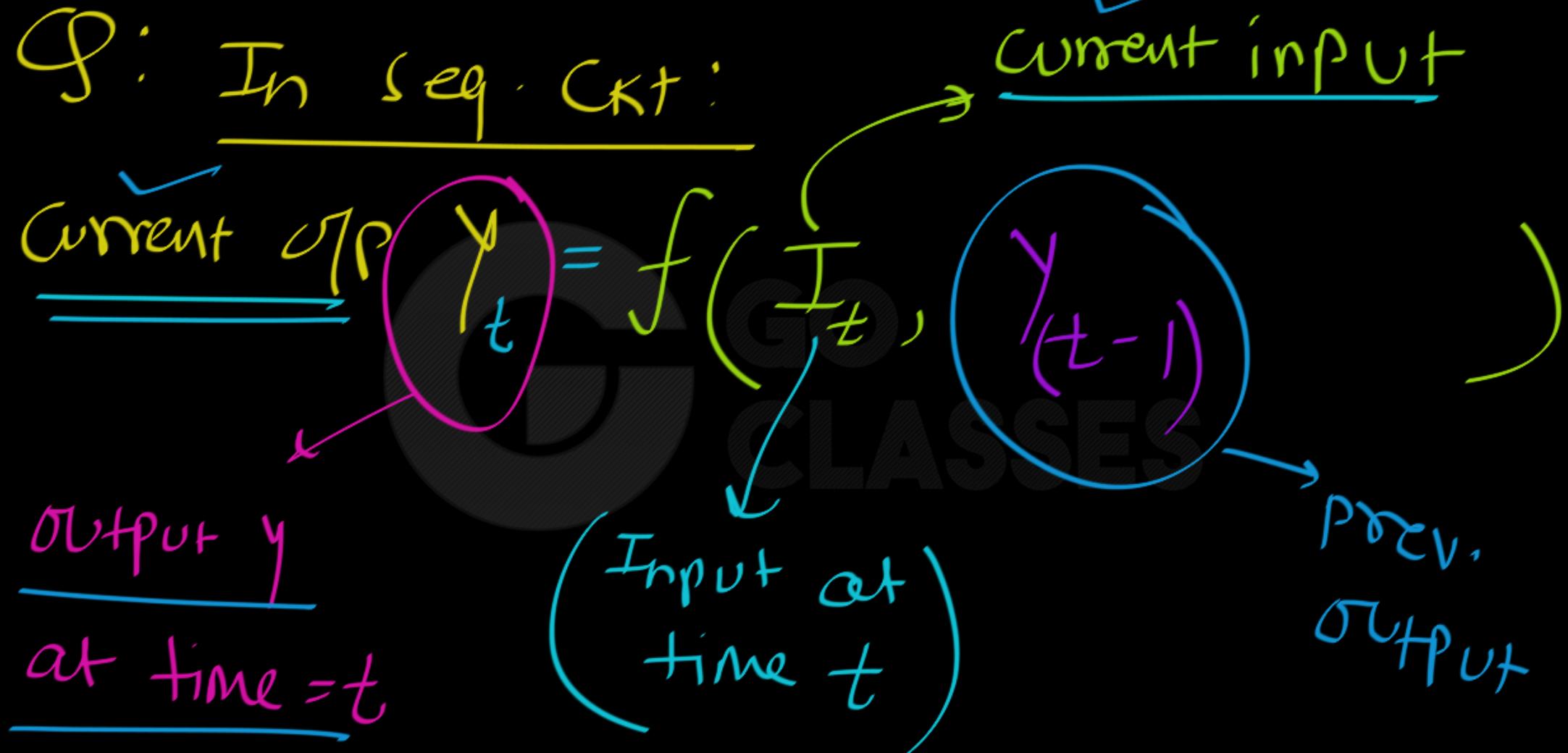
input
same













Q: for seq. CKT:

- a) Current op depends on Prev. output
- b) " " " Prev. all ops.

Q: for seq. CKT:

- ① Current op depends on Prev. output
- ② " " " Prev. all ops.



$y_t = \text{Op } y \text{ at time } t$

$I_t = \text{Input } I \text{ at time } t$

$y_{t-n} = \text{Op } y \text{ at time } t-n$

$$Y_t = f(I_t, Y_{t-1})$$

$$Y_t = f(I_t, f(I_{t-1}, Y_{t-2}))$$

$$Y_t = f(I_t, f(I_{t-1}, f(I_{t-2}, Y_{t-3})))$$



Sequential CKT \rightarrow State = Output

Current off (state) = $f\left(\frac{\text{Current input}}{\text{Past inputs}}, \frac{\text{Past outputs}}{\text{all}}\right)$

Depends on Prev. history



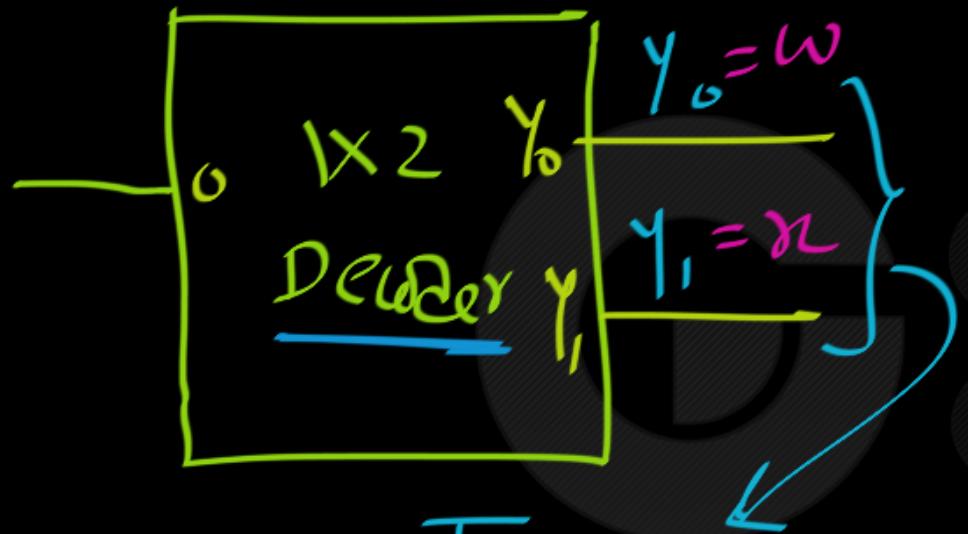
Small small Points which are very important :

In seg Circuit

$$Y_0 = \text{output } Y \text{ at time } t=0$$
$$Y_1 = \text{output } Y \text{ at time } t=1$$

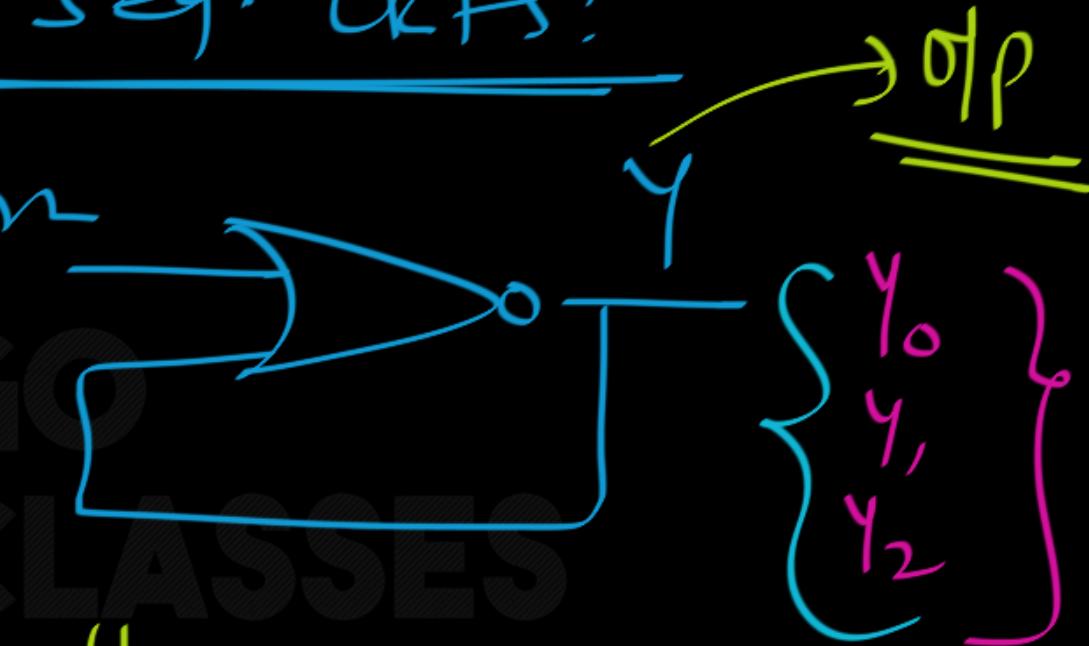


Combinational CKTS



Two
Different OP
lines

Seq. CKTS:



#Output lines = 1

= y



Traffic light:



$y_0 = \text{Red}$

$y_1 = \text{Yellow}$

$y_2 = \text{Green}$

Human Growth: \rightarrow Analogous to

Seq. CKT'

You



One Person

Age = 2

Age = 10

Age = 18

Age = 60

idiot & cute

headach & cute

intelligent & not
cute

Judge
wasted & cute



Different Conventions :

Author 1:

$$\text{Current Output} = Y_t = Y(t)$$

$$\text{Prev Output} = Y_{t-1} = Y(t-1)$$

$$\text{Next Output} = Y_{t+1} = Y(t+1)$$

Different Conventions :

Author 2 :

$$\text{Next op} = Y_n$$

next
op

✓ Current op : Y



Q:

In Sequential Circuits, how to
remember previous output?



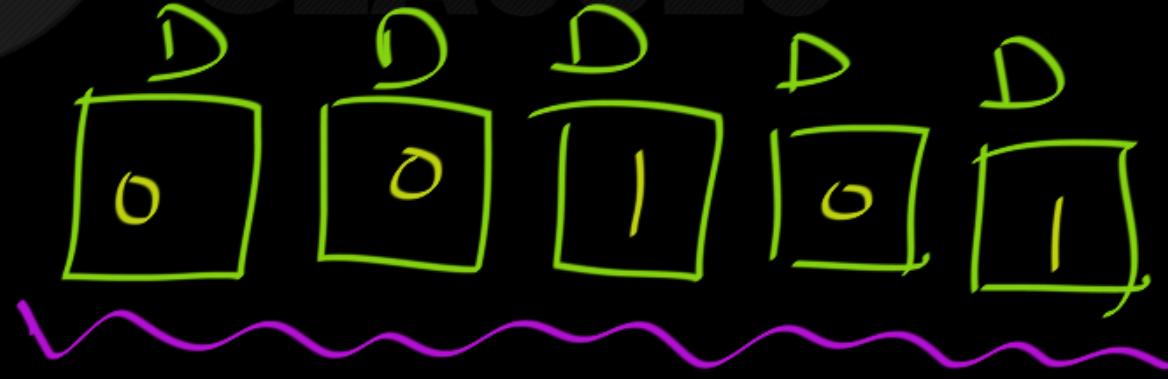
Q:

In Sequential Circuits, how to
remember previous output?

Ans: Some Memory Device/element

What is the most Basic info.
We need to store } \Rightarrow single bit

Device D: \rightarrow Can store one bit (0/1)

To store 00101 \Rightarrow 



Next Topic:

Memory Element/Storage CLASSES



Motivation for Sequential Circuits :

So far, our circuits have been converting inputs to outputs without storing any data.

To do more advanced things (e.g., to build computers), we need components that can store data. Can we make a component that “remembers” using the components that we know already?



One bit storage Device



we need to create
CLASSES



What does “Remembering Information” mean?



What does "Remembering Information" mean?

Store and Retain until it is
changed further.

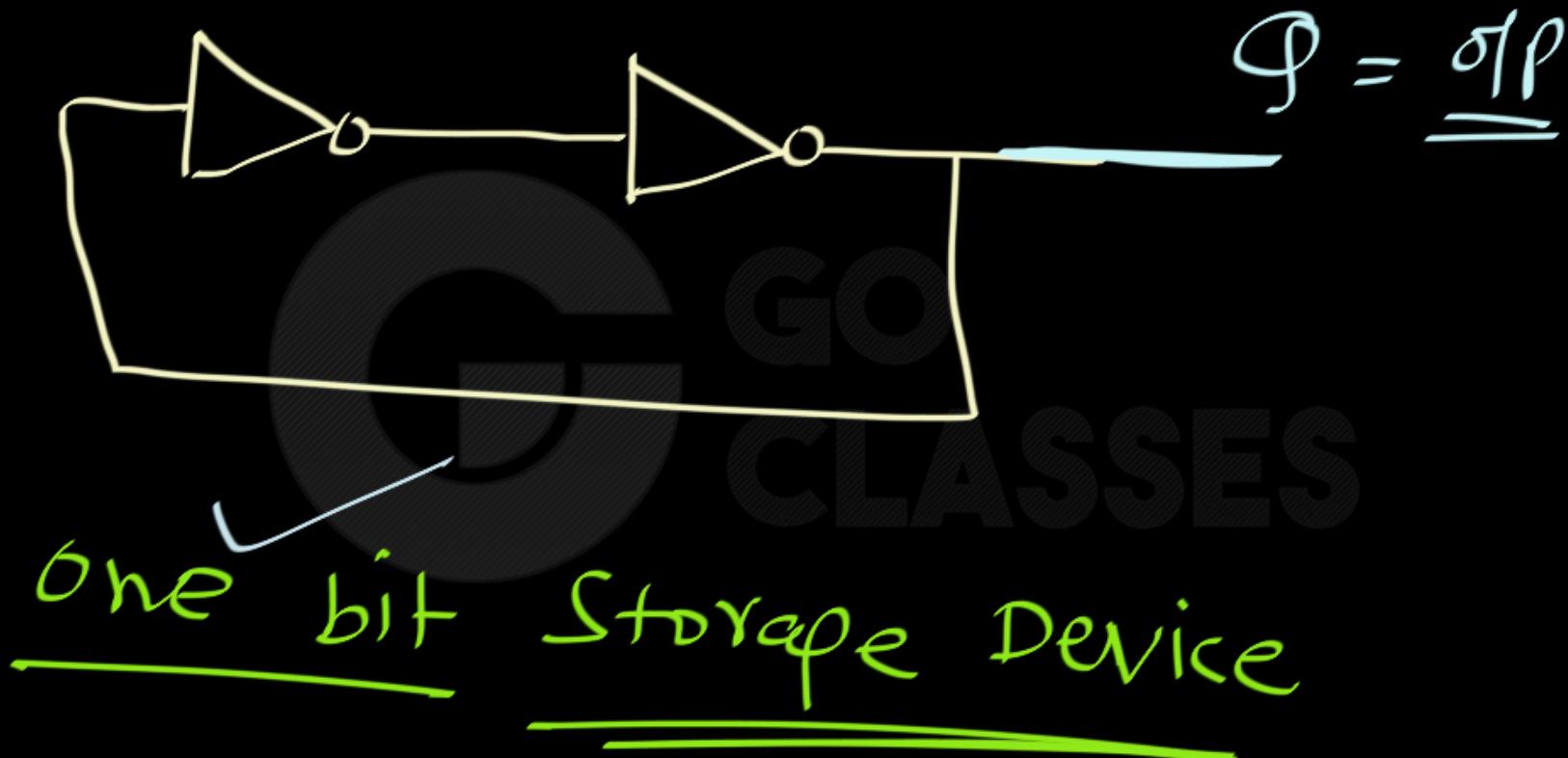
Absence
of Voltage





Next Topic:

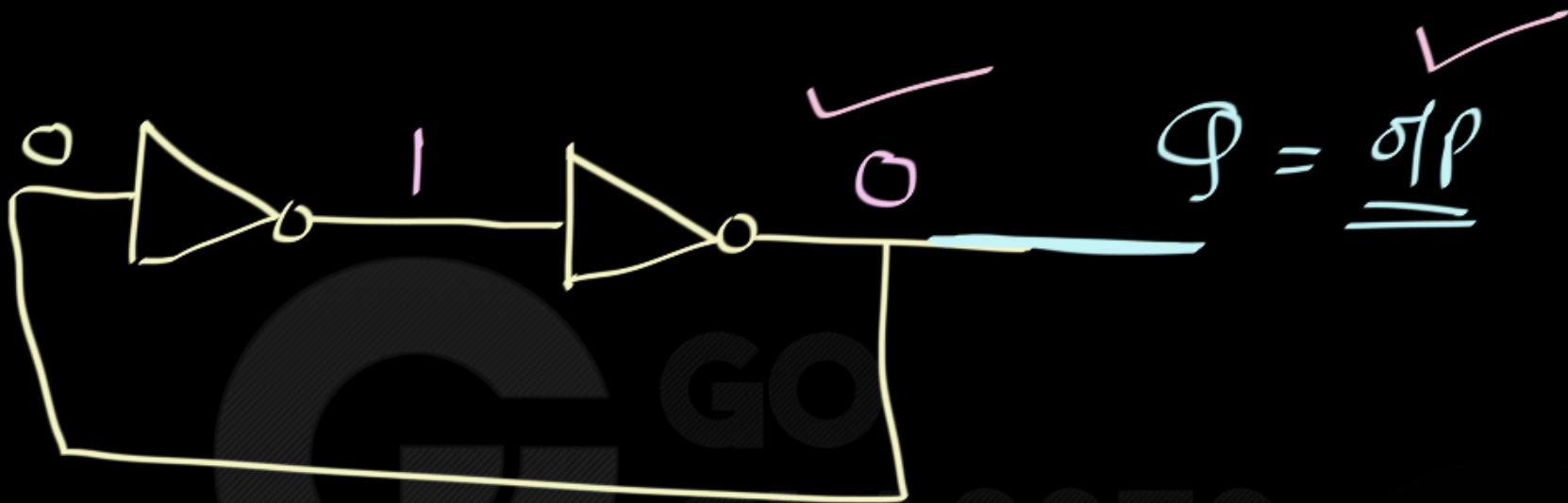
Basic Memory Element/Device For One Bit Storage





GO
CLASSES

Storing 1, Retained



GO
CLASSES

Storing 0, Retained



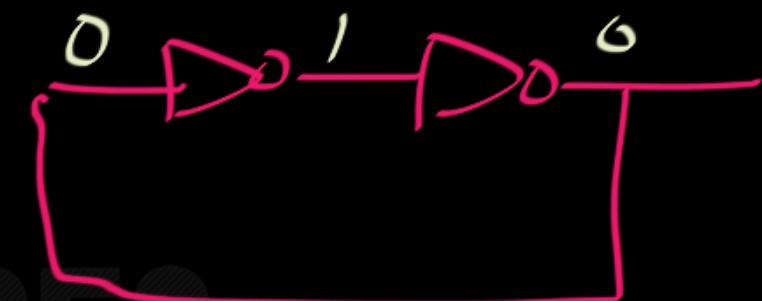
Can we say that this is a 2-bit
Storage Device ?



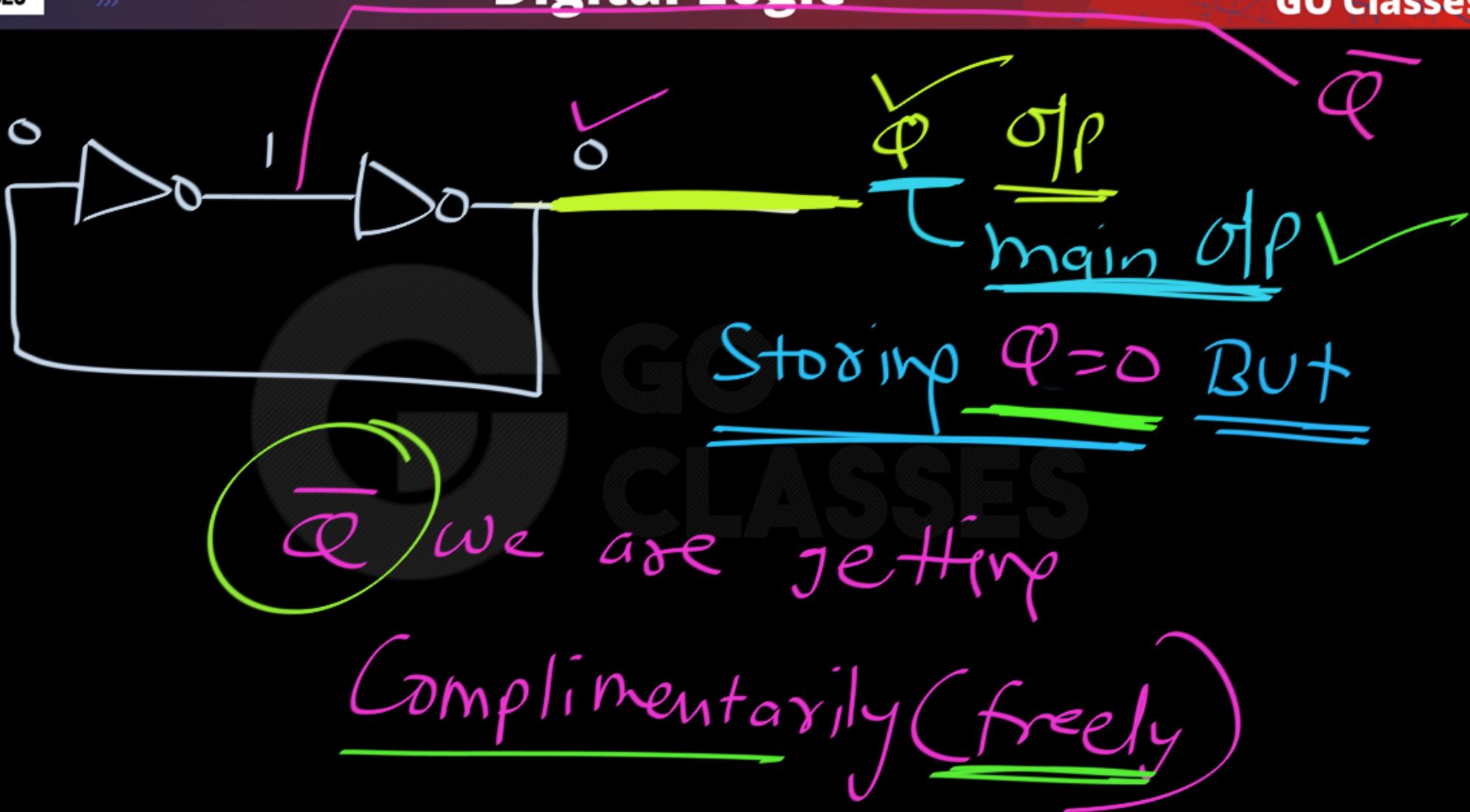
Can we say that this is a 2-bit storage device ?

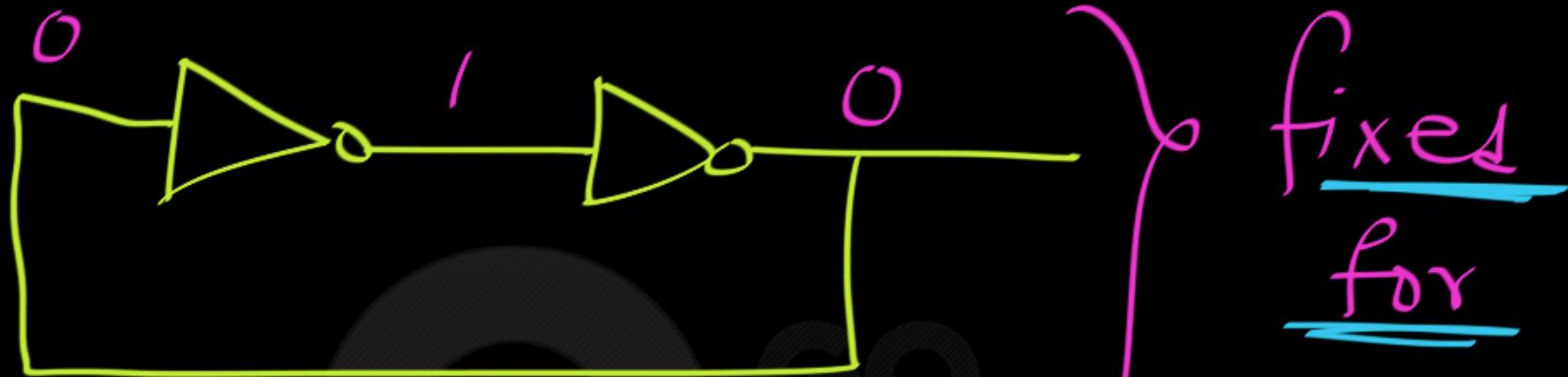


2-bit Storage Device :



0, 1
1, 0



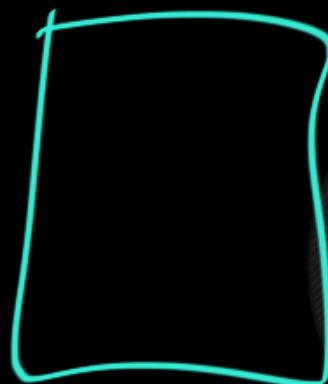


"Cross-Coupled Inverter" time

→ We can store & retain one bit.



Kid Toy

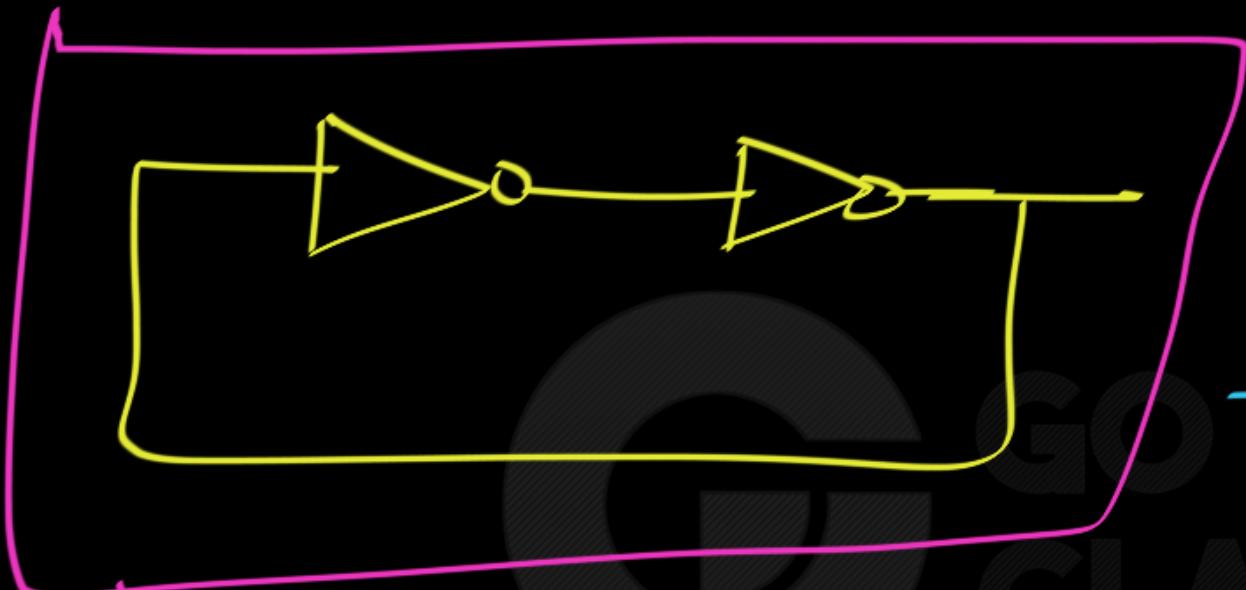


3 scores

Pre-programmed

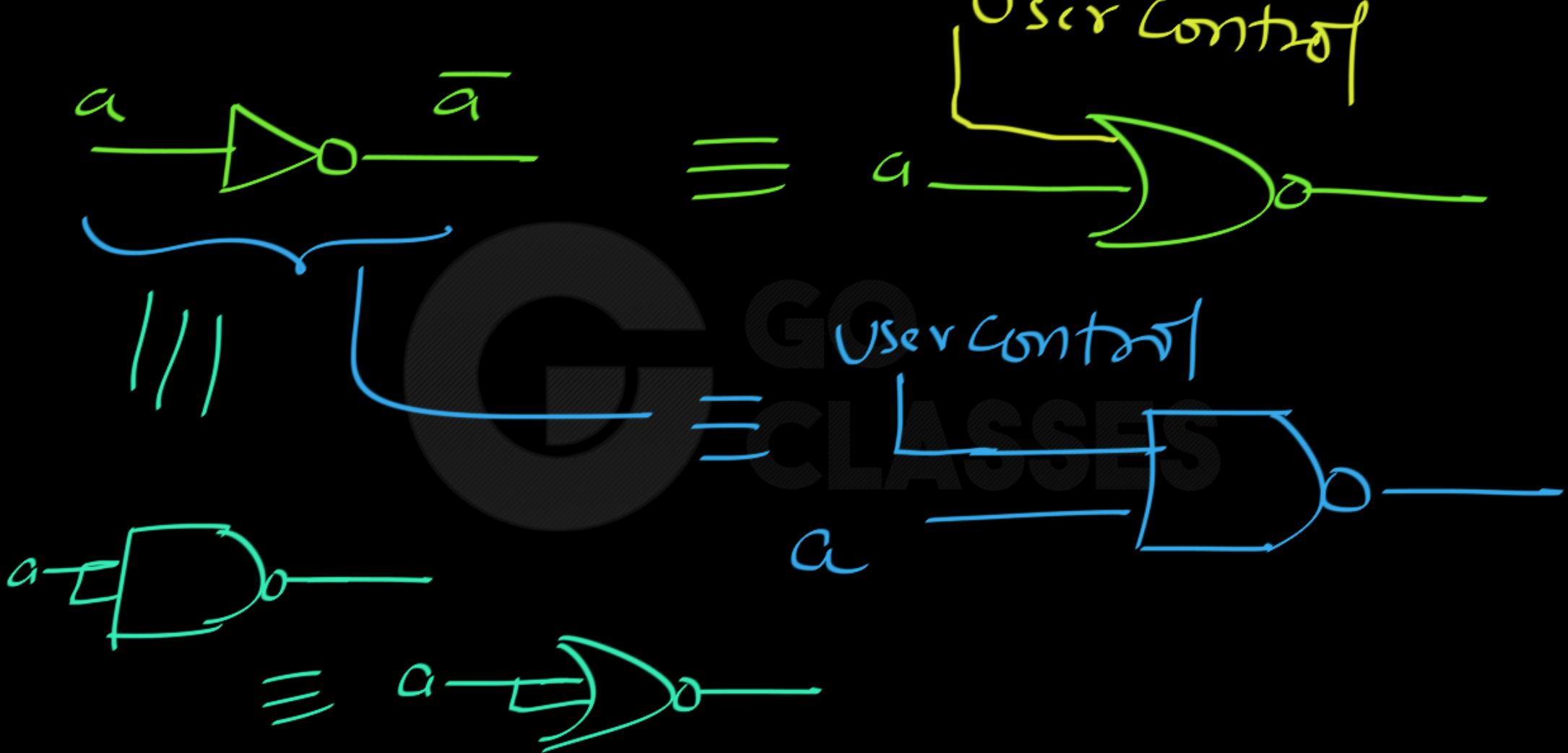
Not in User

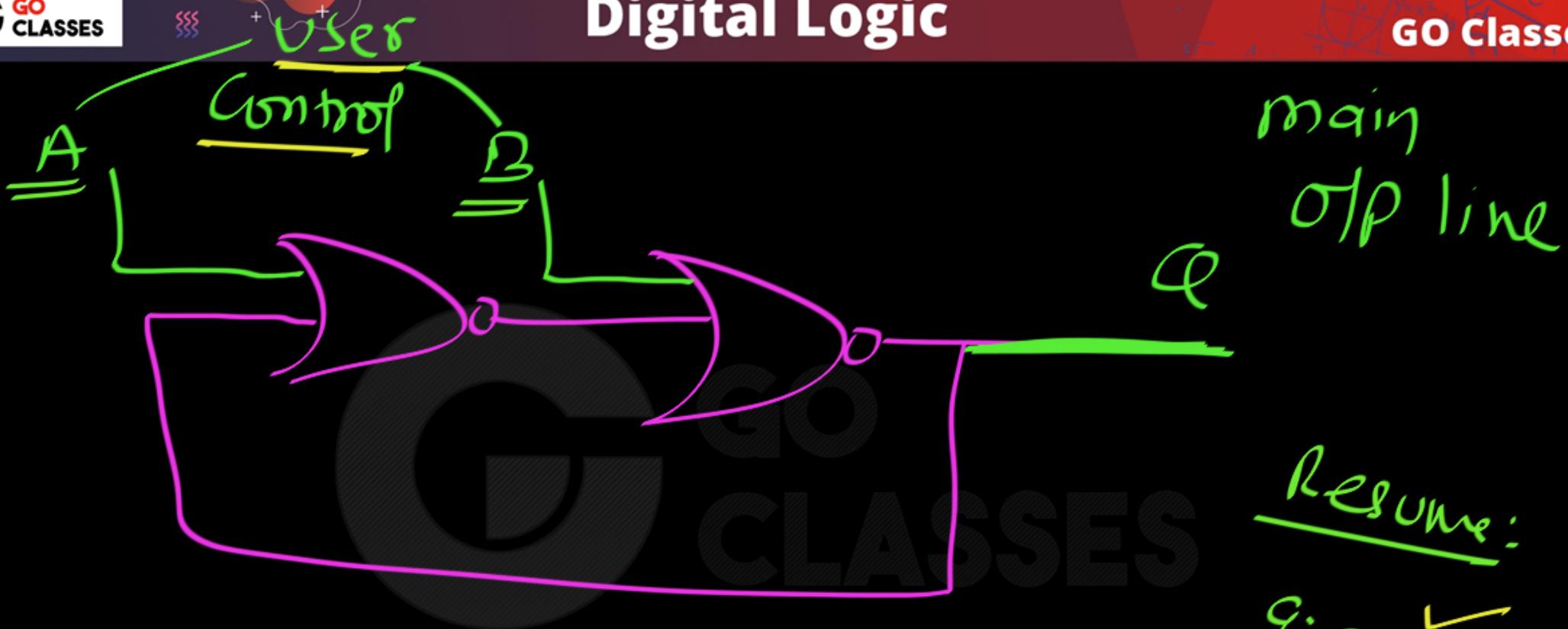
Control



No User
Control.

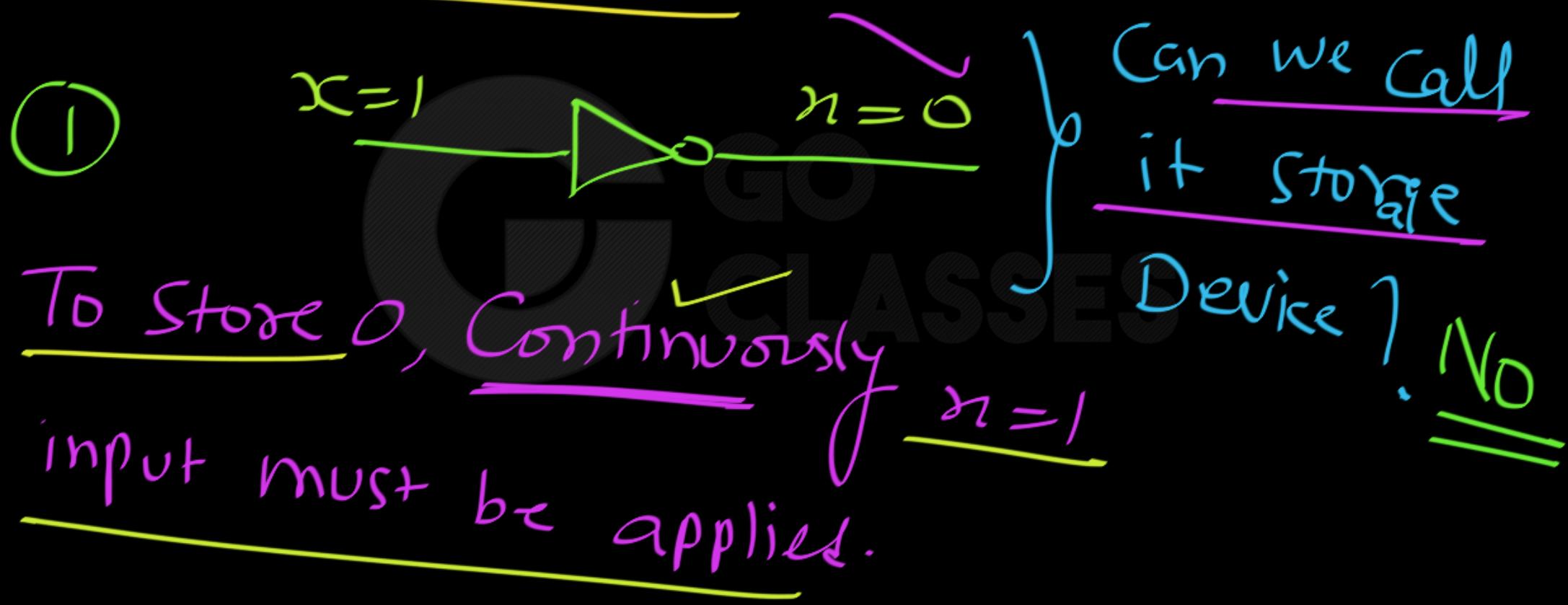
Basic Idea to store one bit.







Store one bit:





(2)

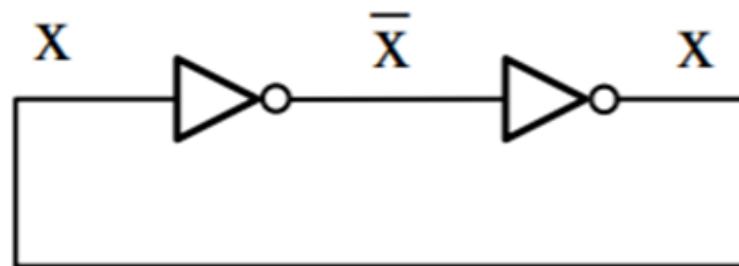


Storing & Retaining

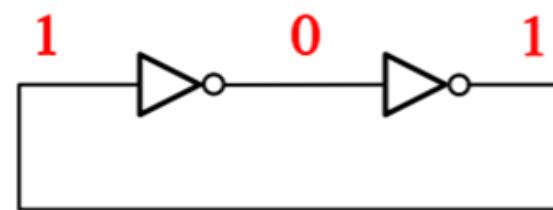
Cross coupled inverter



A simple memory element with NOT Gates

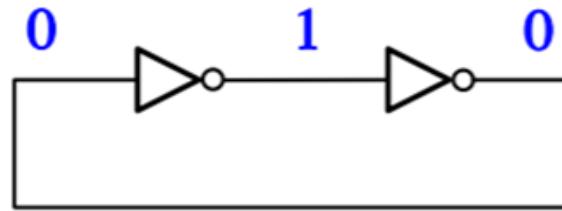


A simple memory element with NOT Gates



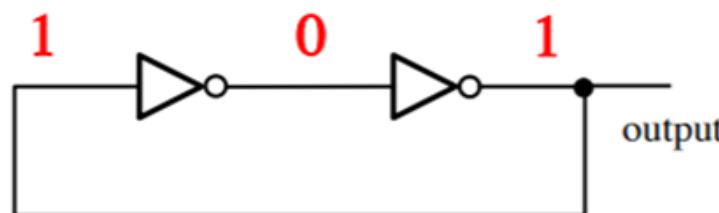
The circuit will stay in this state indefinitely.

A simple memory element with NOT Gates

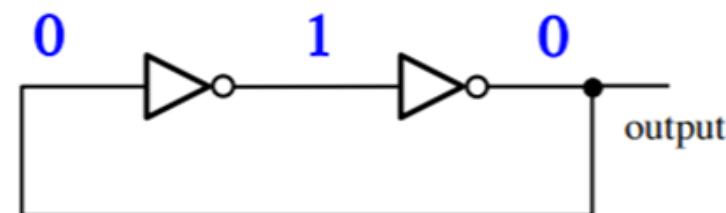


The circuit will stay in this state indefinitely.

This circuit can be in two possible states

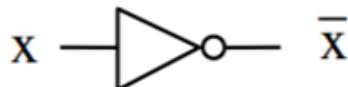


used to store a 1

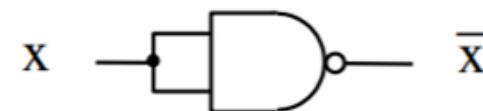


used to store a 0

Building a NOT gate with a NAND gate



x	\bar{x}
0	1
1	0



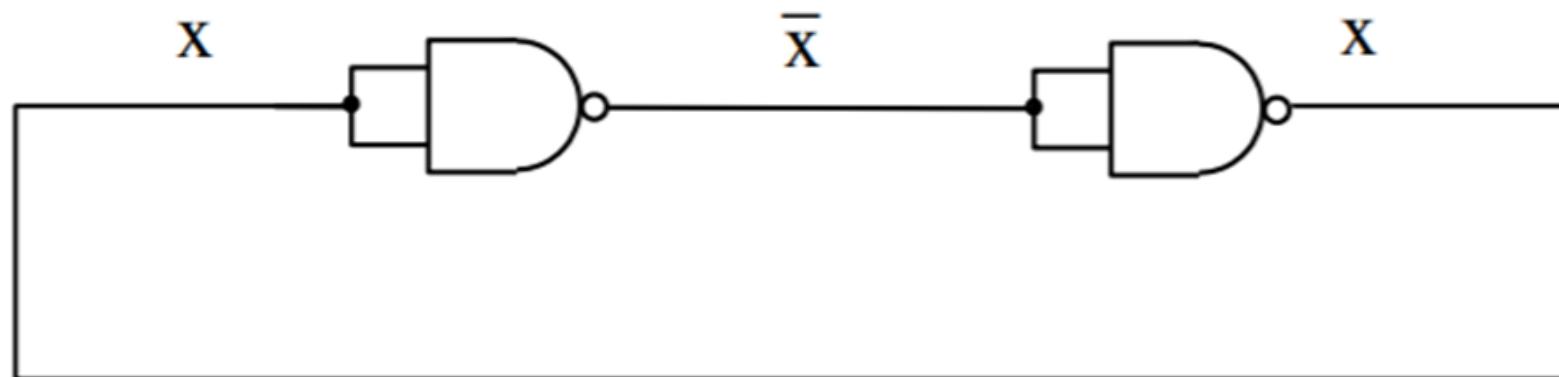
x	x	f
0	0	1
1	1	0

impossible
combinations

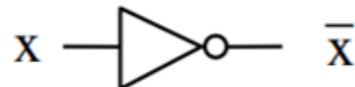
Thus, the two truth tables are equal!



A simple memory element with NAND Gates



Building a NOT gate with a NOR gate



x	\bar{x}
0	1
1	0

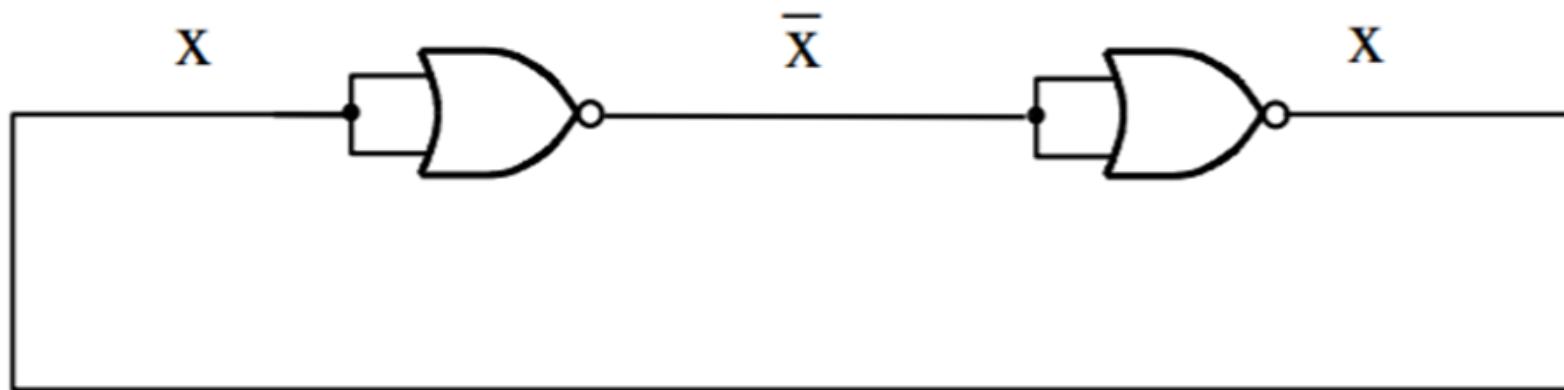


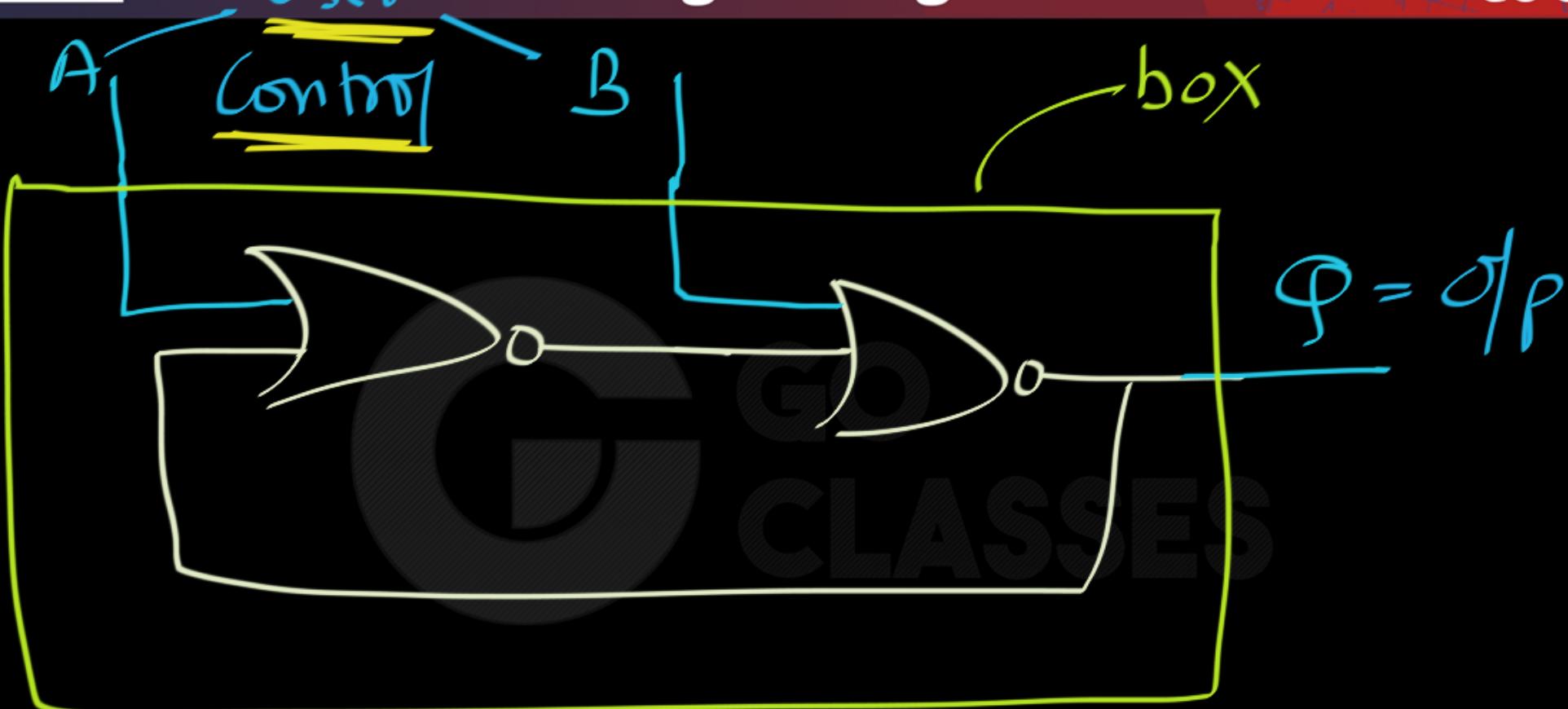
x	x	f
0	0	1
1	1	0

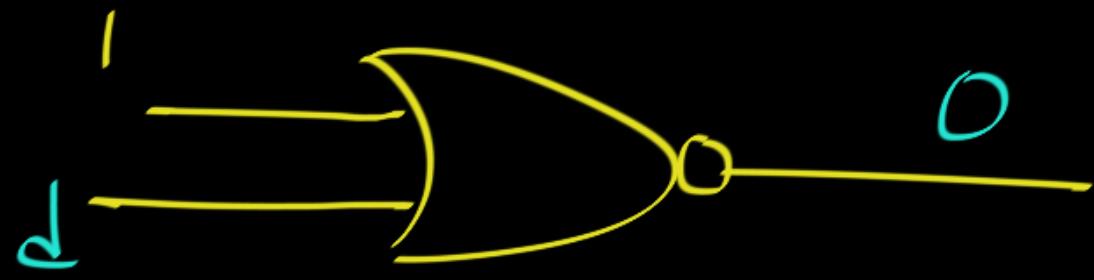
impossible
combinations

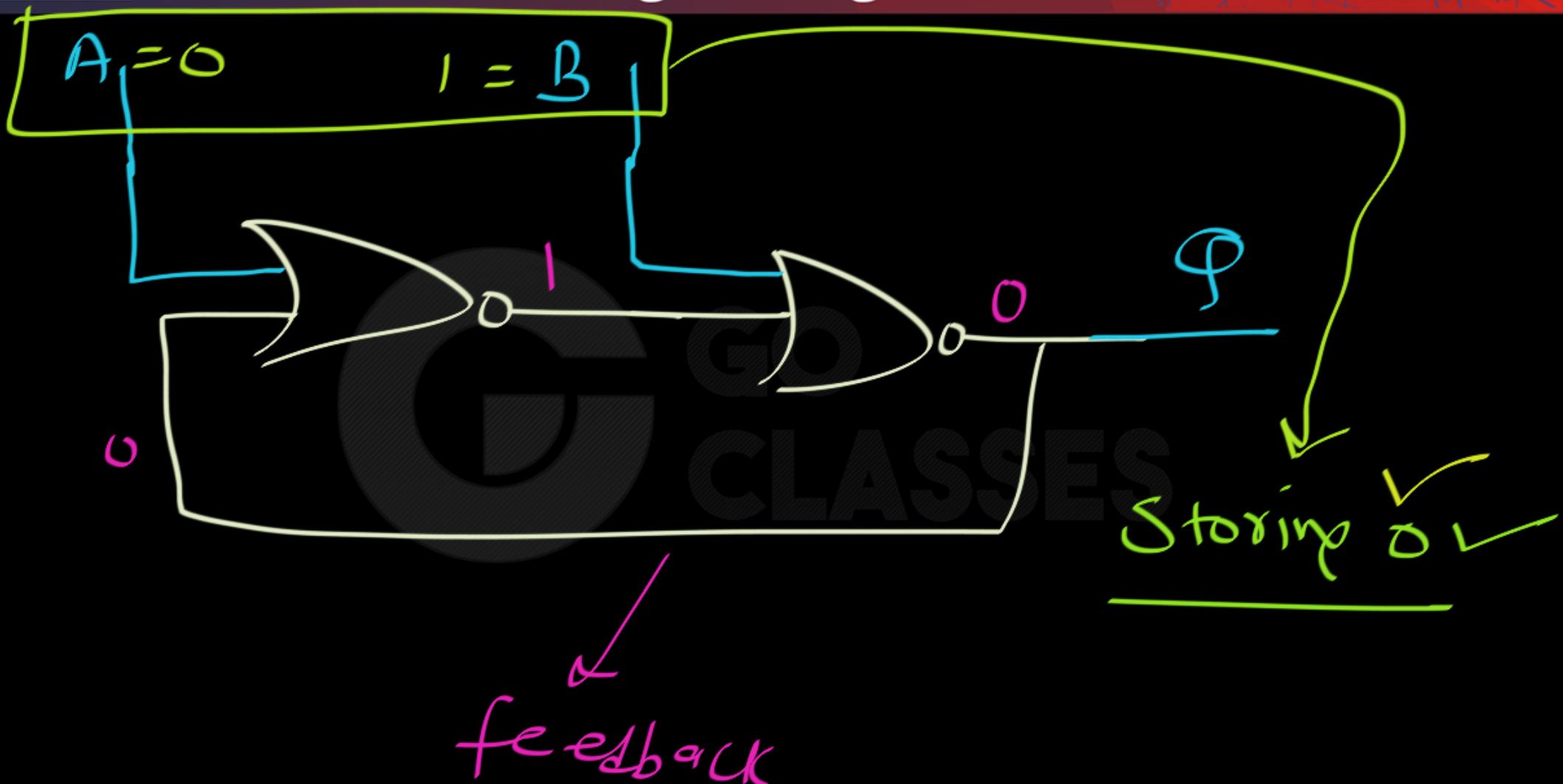
Thus, the two truth tables are equal!

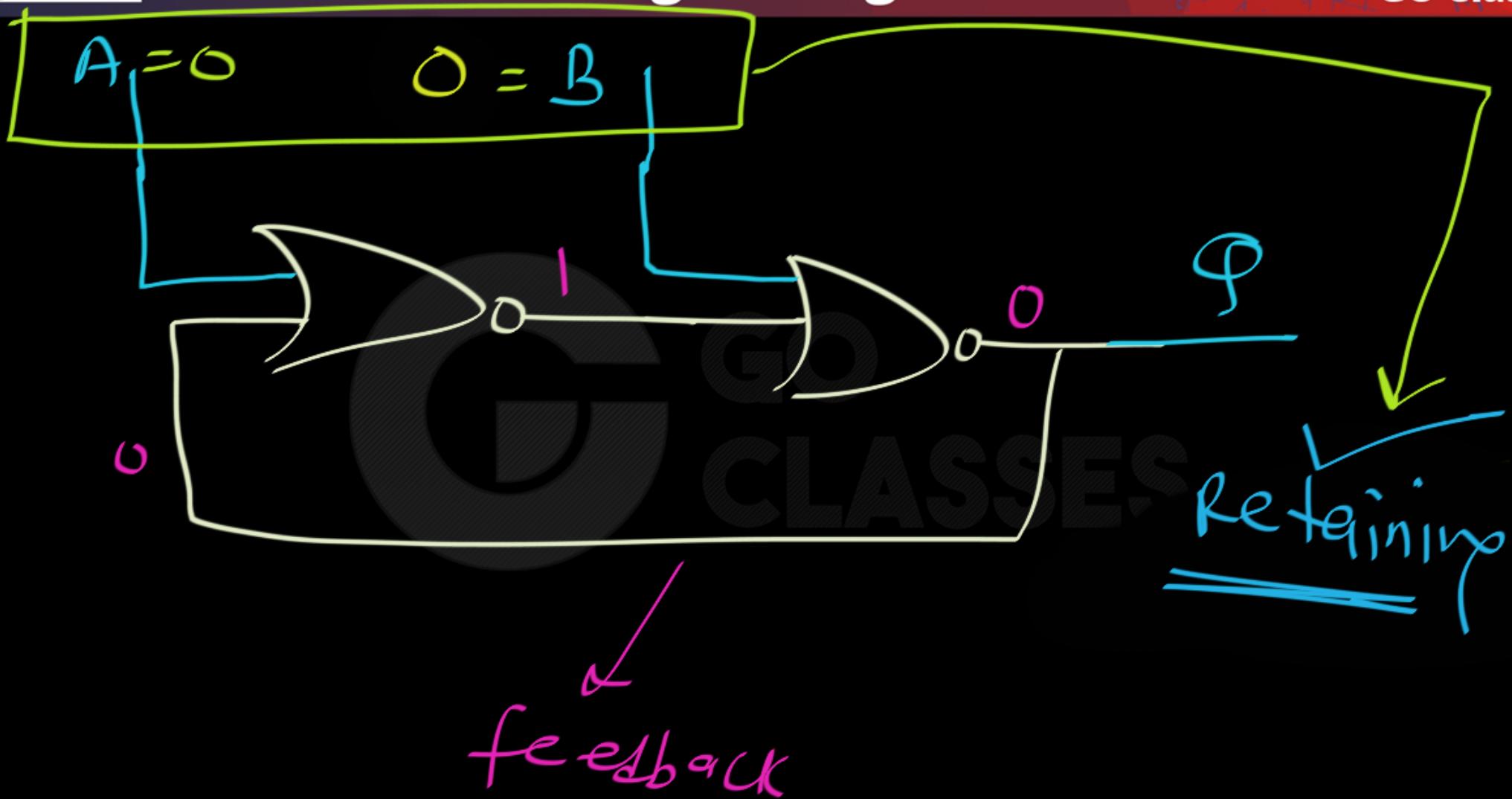
A simple memory element with NOR Gates

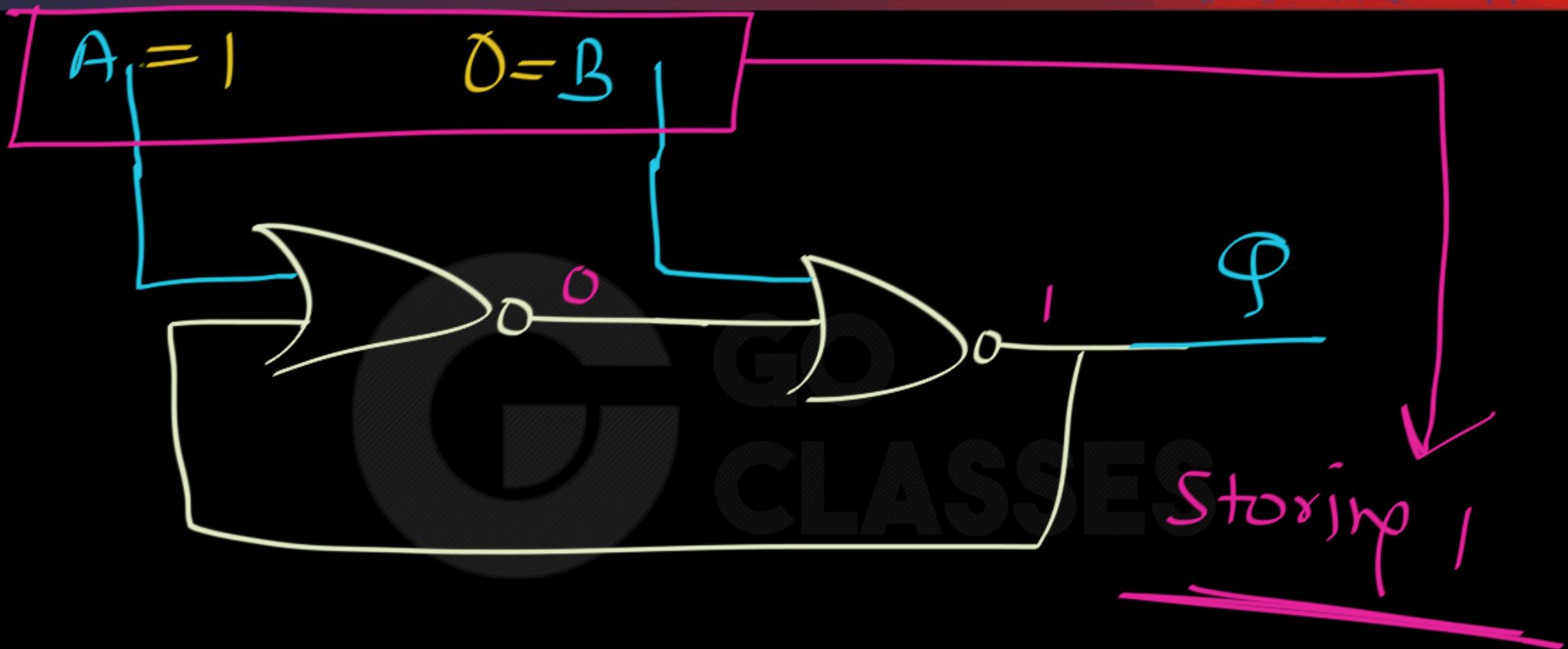


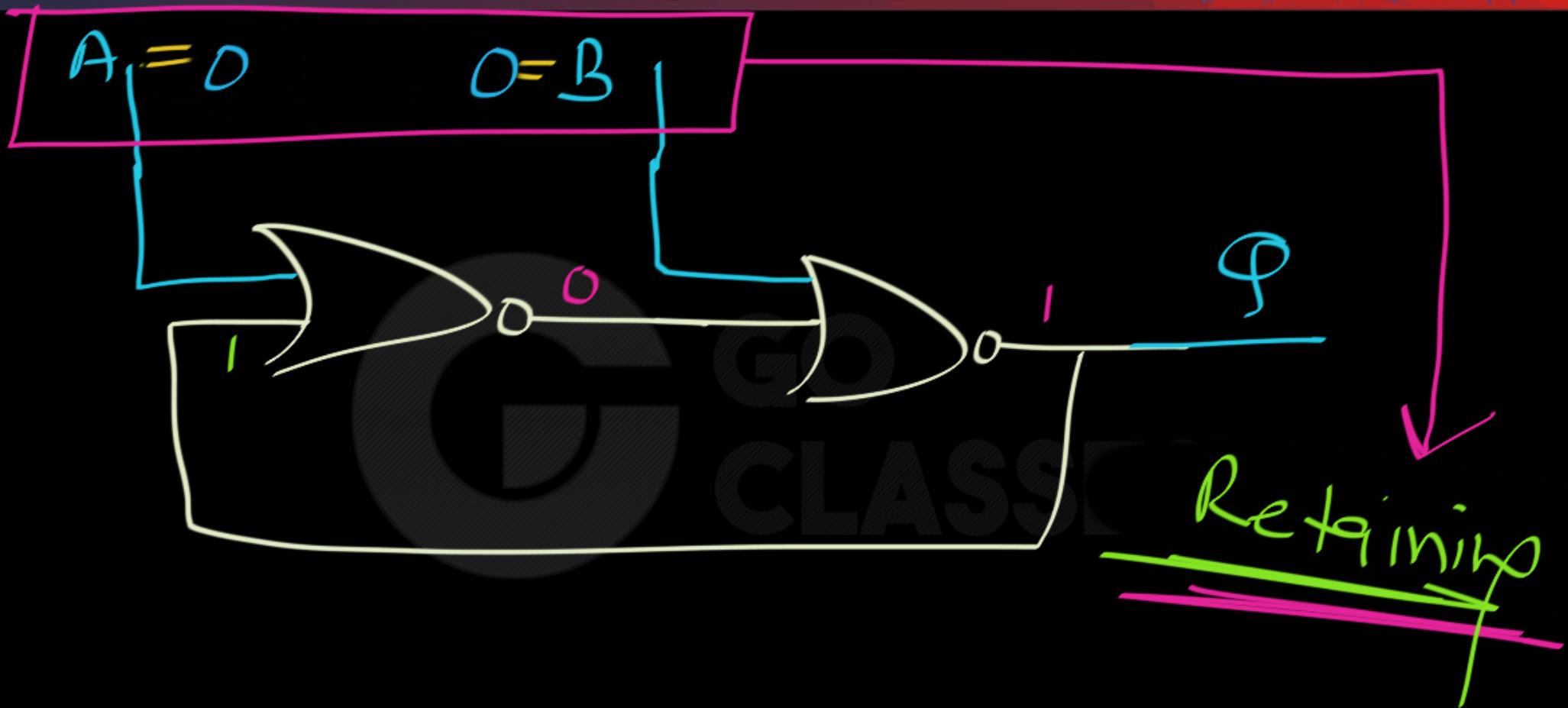


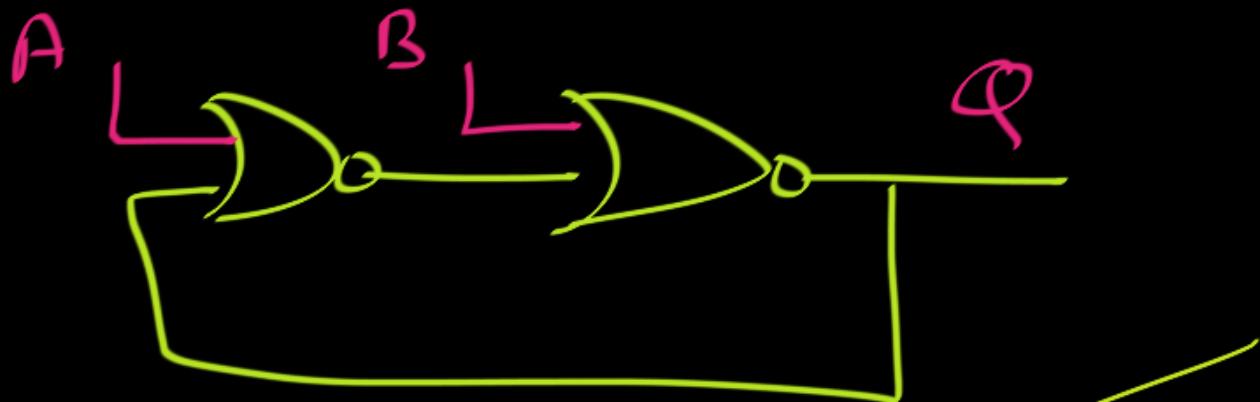












Next off

A	B	Q_n	time	
0	1	0	0	Storing 0
0	0	$Q=0$	1	Retaining
1	0	1 ✓	2	Storing 1
0	0	1	3	retaining

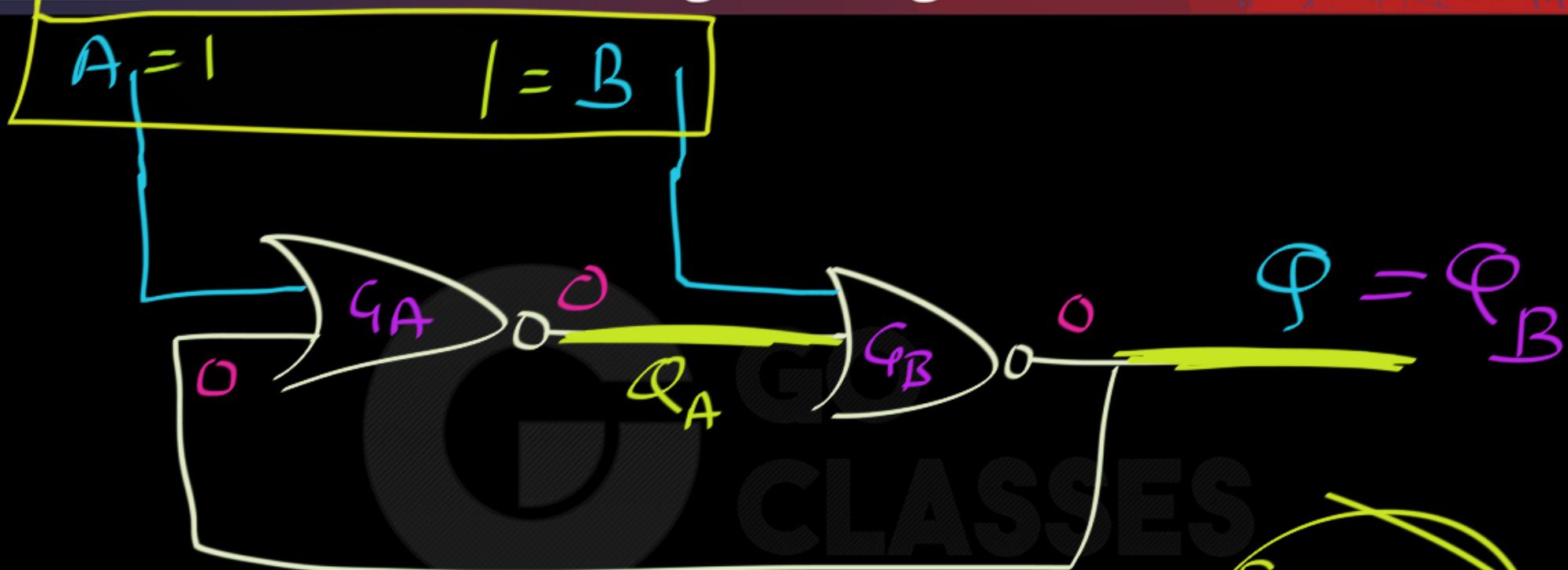
Store 1 : $A = 1, B = 0$

Store 0 : $A = 0, B = 1$

Retain : $A = 0, B = 0$

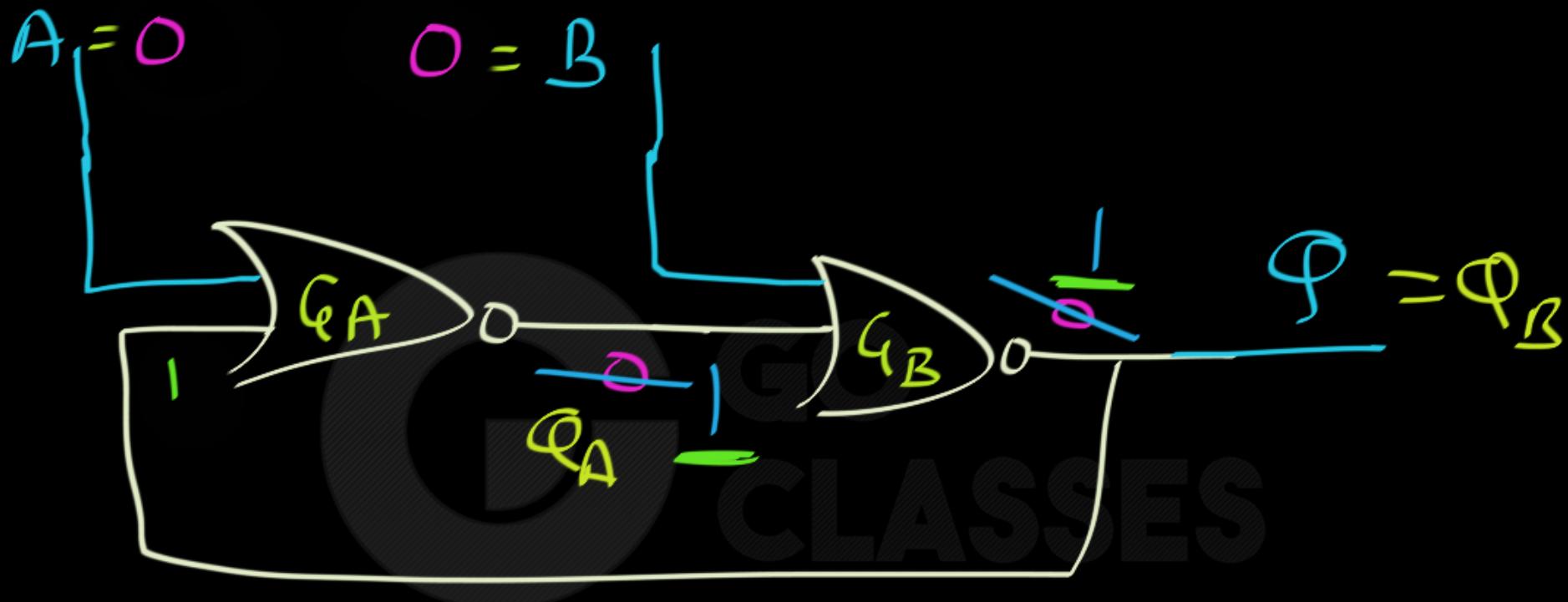
What happens when

$$A = I \quad ?$$
$$B = I \quad ?$$

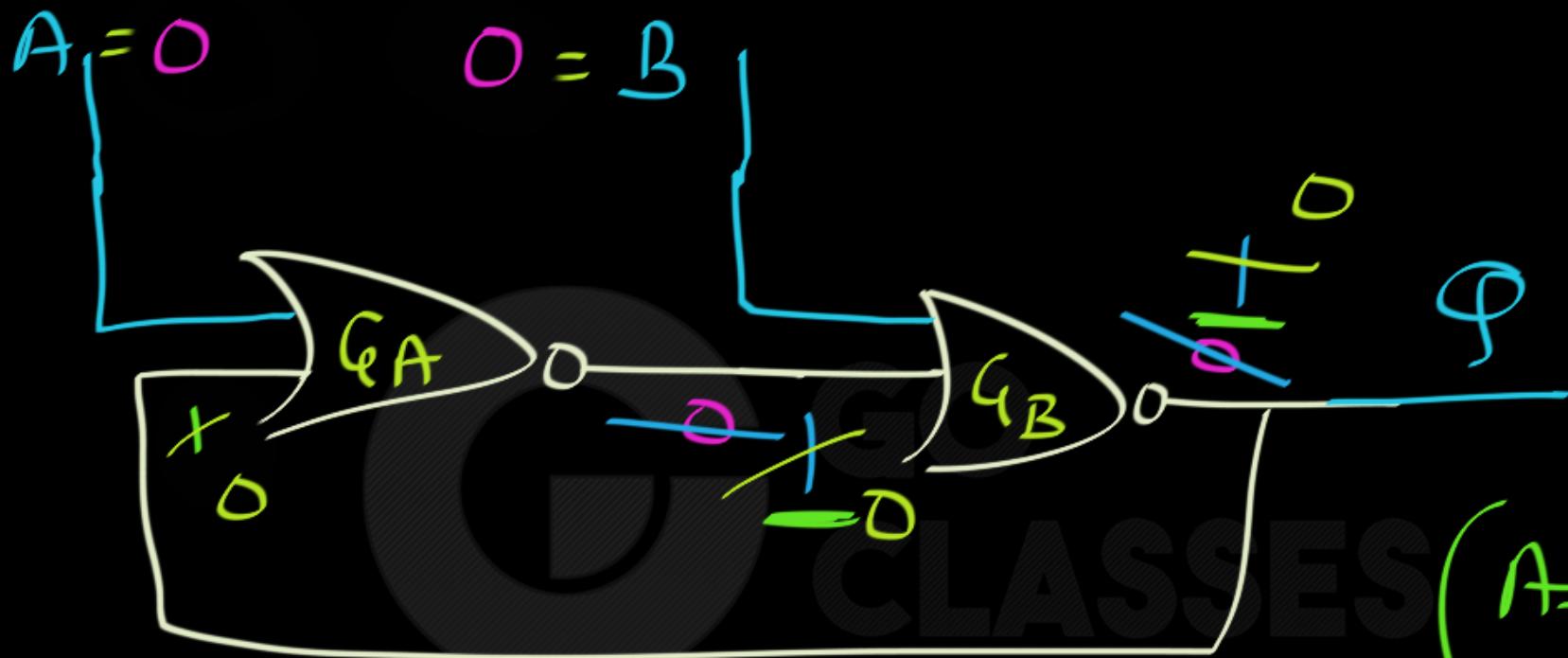


Output $\Phi = 0$

Stable
output



If
 Q_A, Q_B have equal speed :



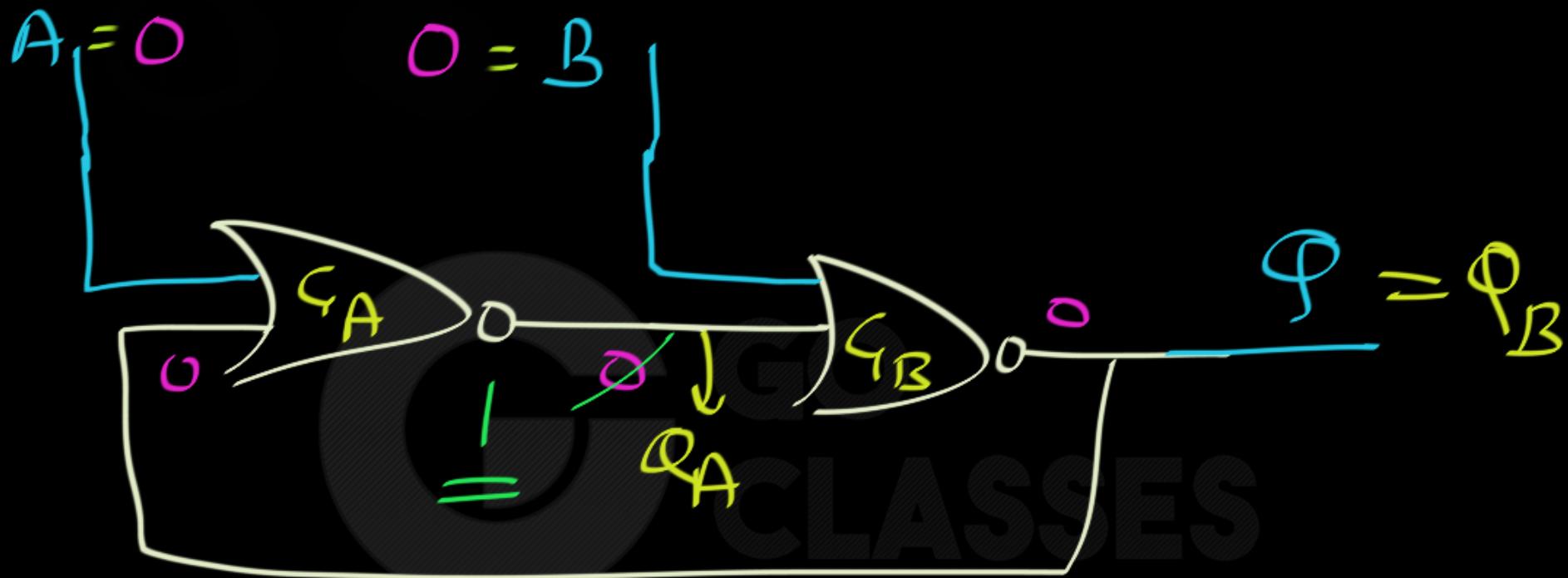
If
 G_A, G_B have equal speed :

(After 1 second)

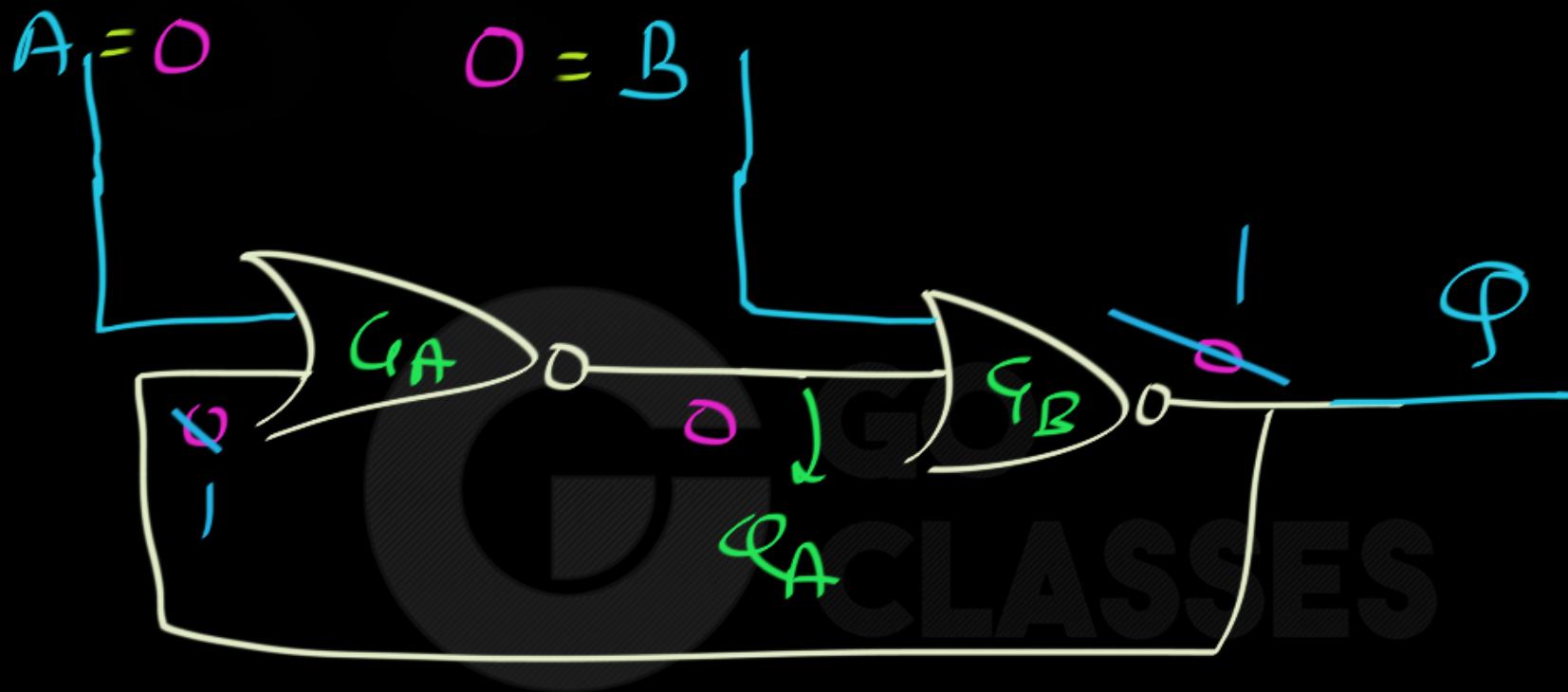
If G_A, G_B have equal speed :

$$\begin{array}{cc} Q_A & Q_B \\ \hline 0 & 0 \\ \hline 1 & 1 \\ \hline 0 & 0 \\ \hline 1 & 1 \end{array}$$

Not
Stable if

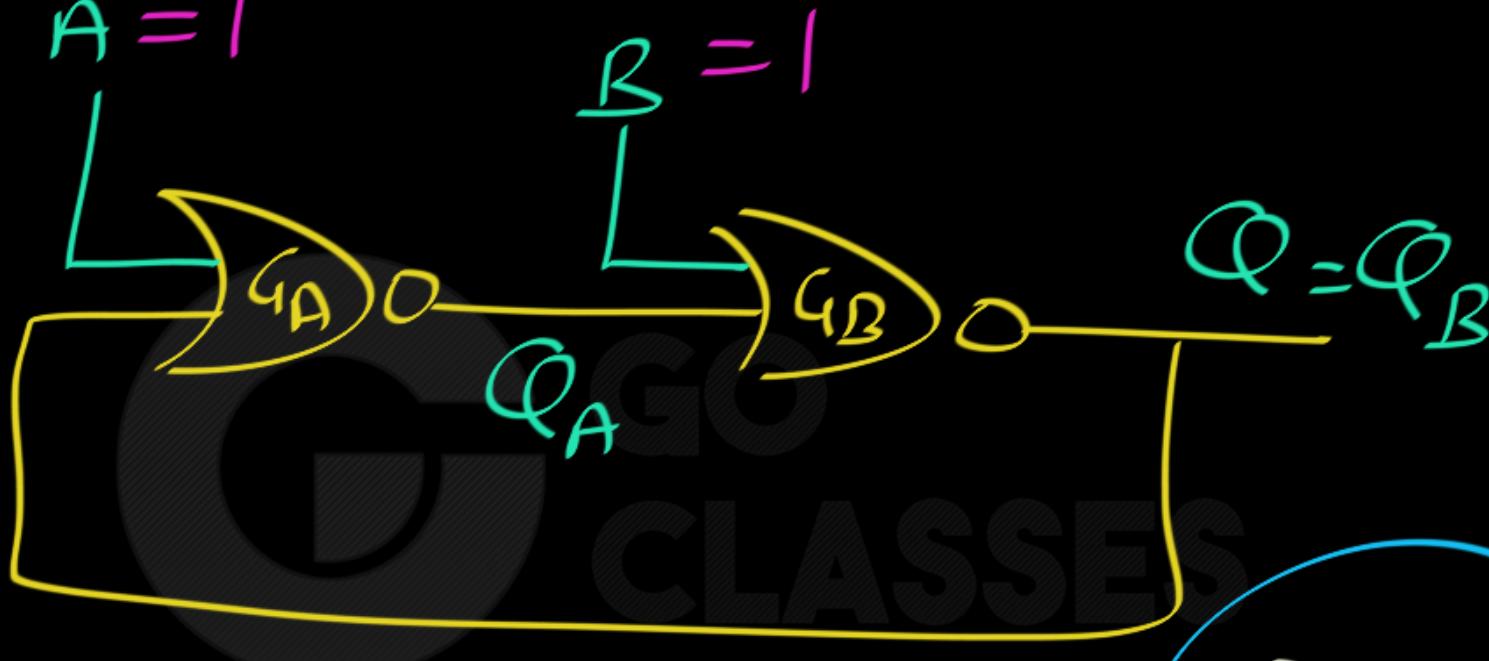


If Q_A speed > Q_B speed

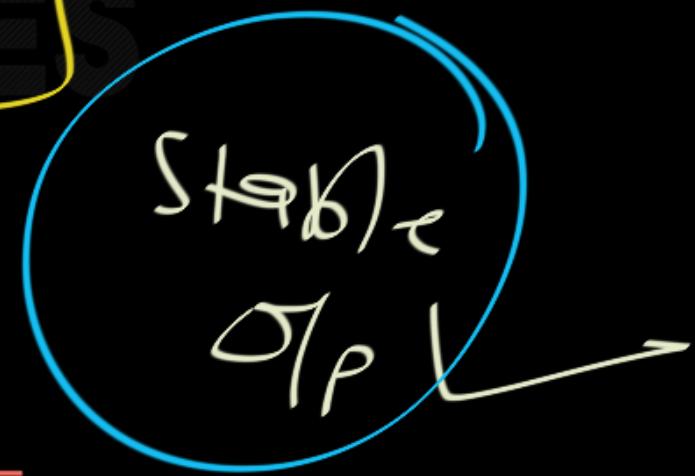


If ζ_B speed > ζ_A speed

Note: $A = 1$



A	B	Q_A	Q_B
1	1	0	0 ✓



But After $A = B = 1$

If You Remove input signal

(If You want to Retain)

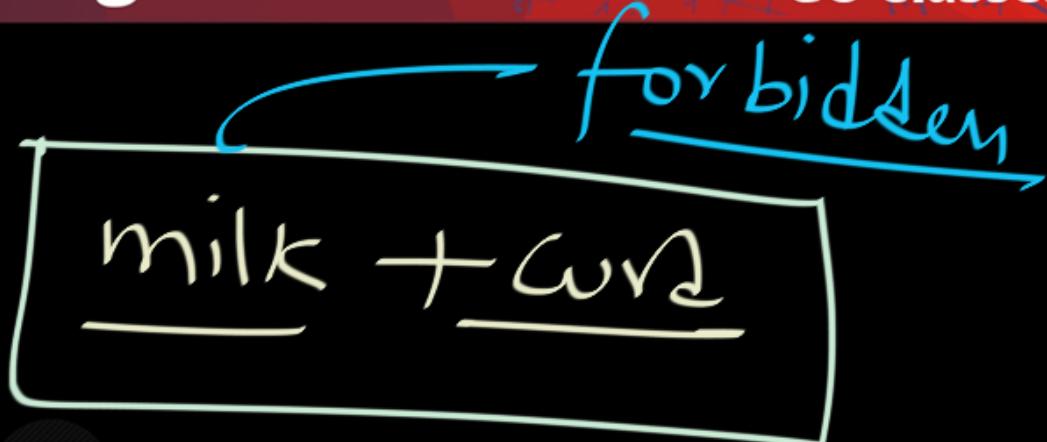
(If You make $A=0, B=0$)

then we Don't know what will happen.



Analogy:

Dinner:



Next Day: Uncertain behaviour
of stomach.

So, $A=1, B=1$ is forbidden.

\checkmark
 $\text{Op} \Rightarrow \text{stable}$

Q_A Q_B
0 0

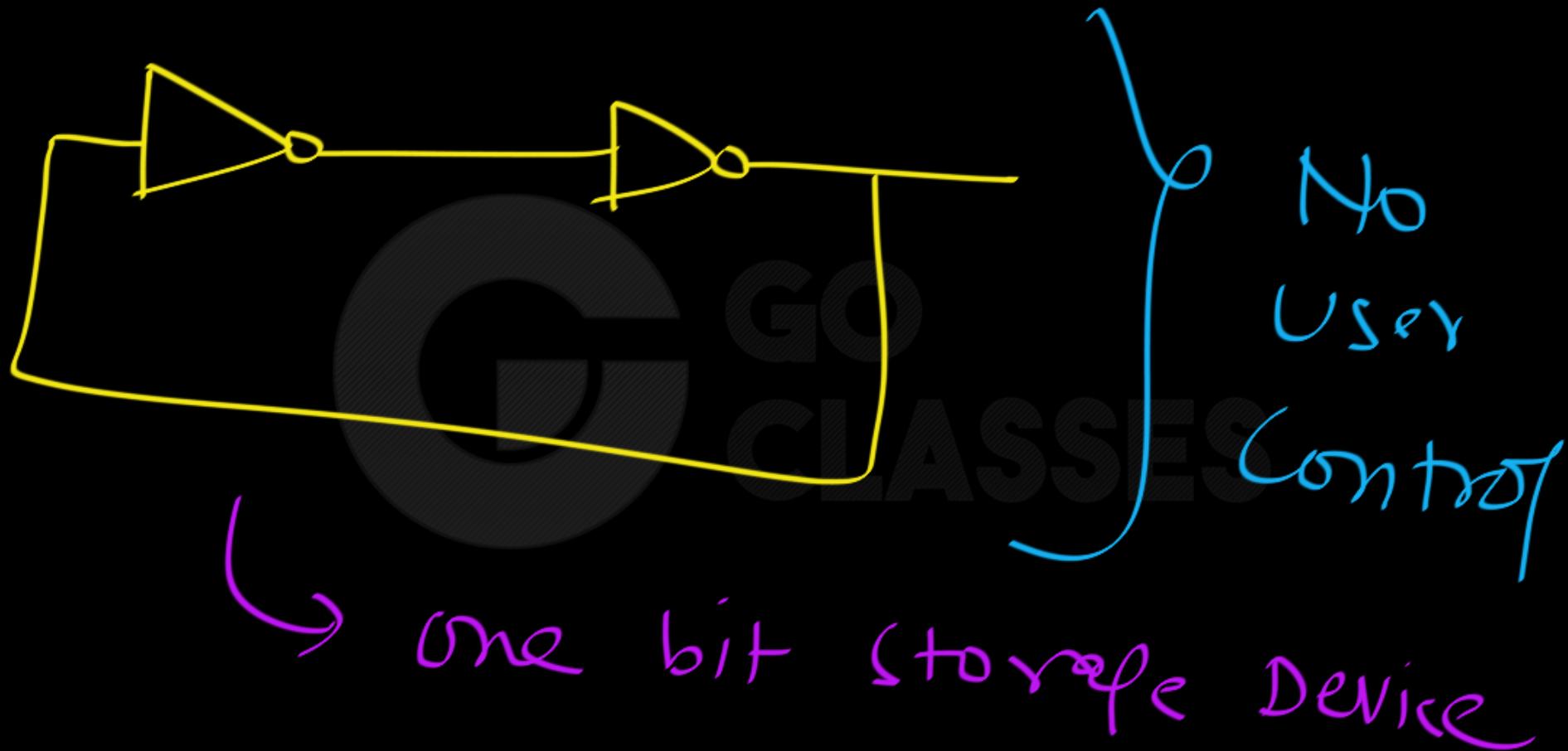
Problem:

$A=1$
 $B=1$

\Rightarrow

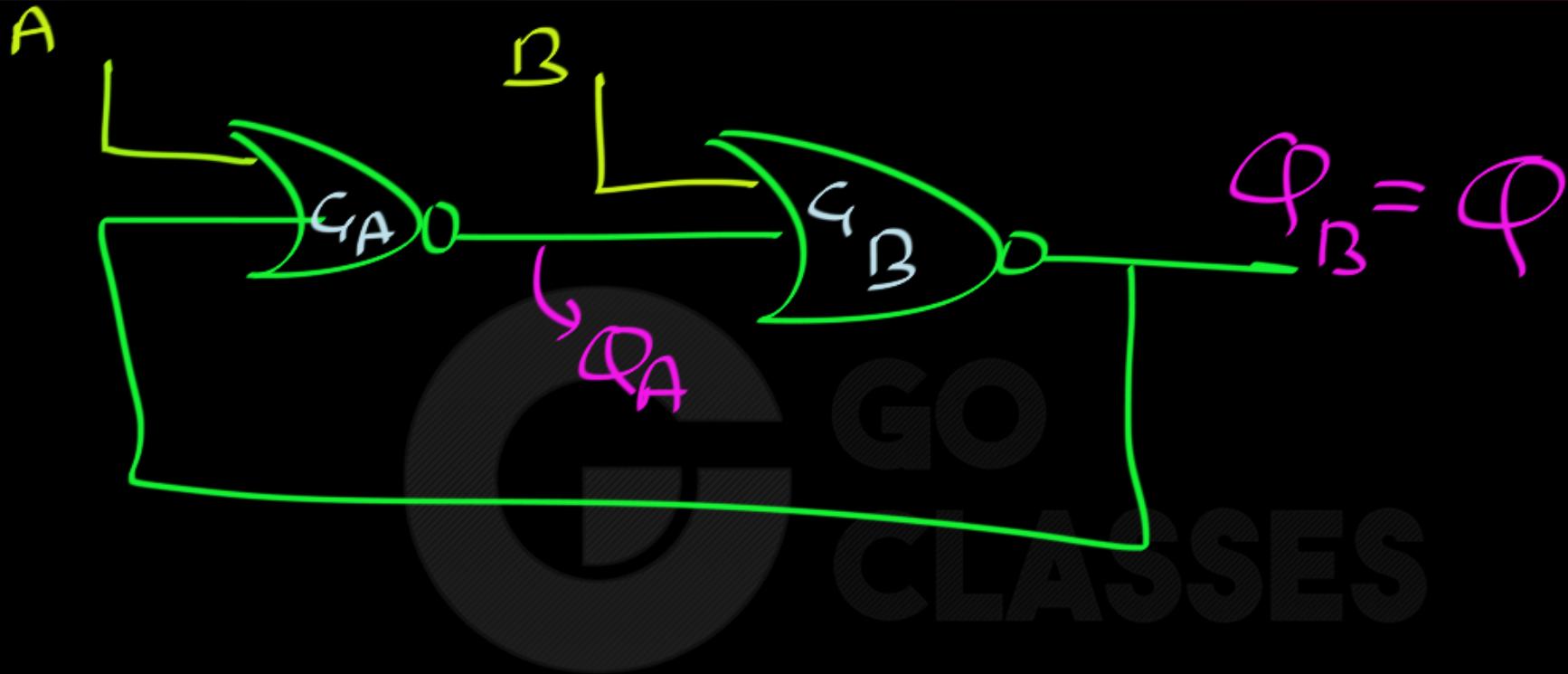
$A=0$
 $B=0$

Unkernf
 $\neg q_{in}$
 δ_{op}





Digital Logic





A	B	$Q_n \rightarrow \underline{\text{next op}}$
0	1	0
1	0	1
0	0	<u>Current O/P</u>
1	1	<u>Q</u> <u>Retain / unchanged</u> <u>O (forbidden)</u>



In Digital Ckts:

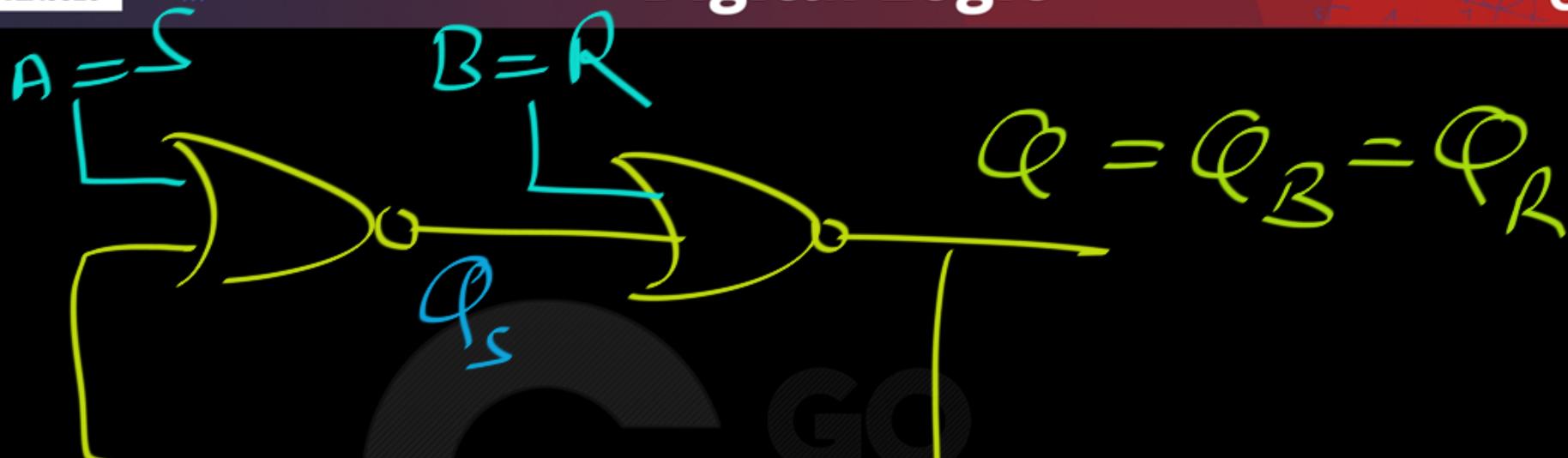
{ Making $S/P = 1$

{ Making $S/P = 0$

Set(ting)

Reset(ting)

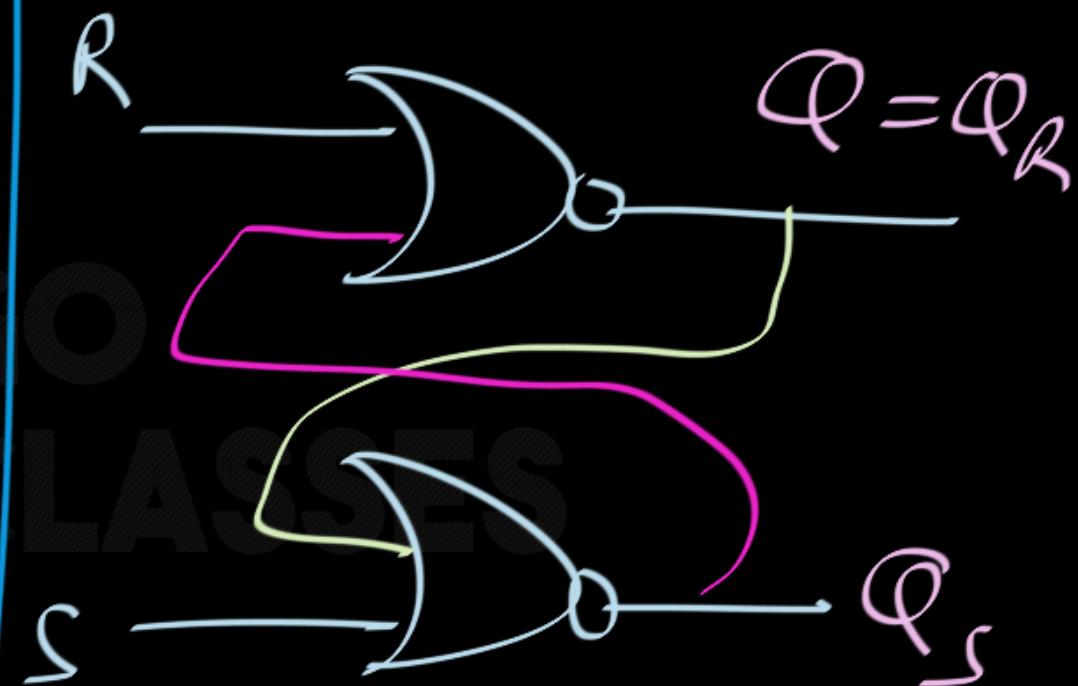
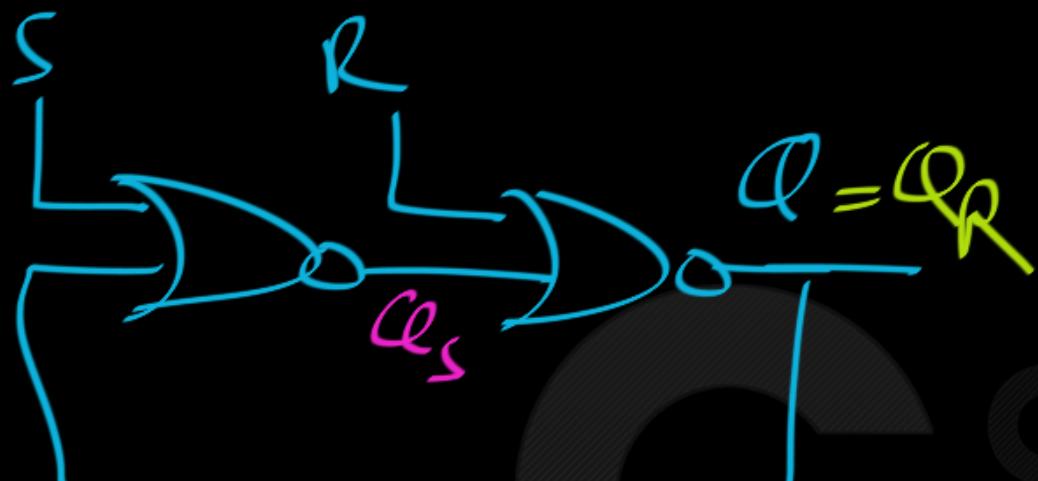
<u>A=S</u>	<u>B=R</u>	$Q_n \rightarrow$ <u>next O/P</u>	SR latch
0	1	(Reset)	
1	0	(Set)	
0	0	<u>Current O/P</u>	
1	1	<u>Retain / unchanged</u>	
		(forbidden)	



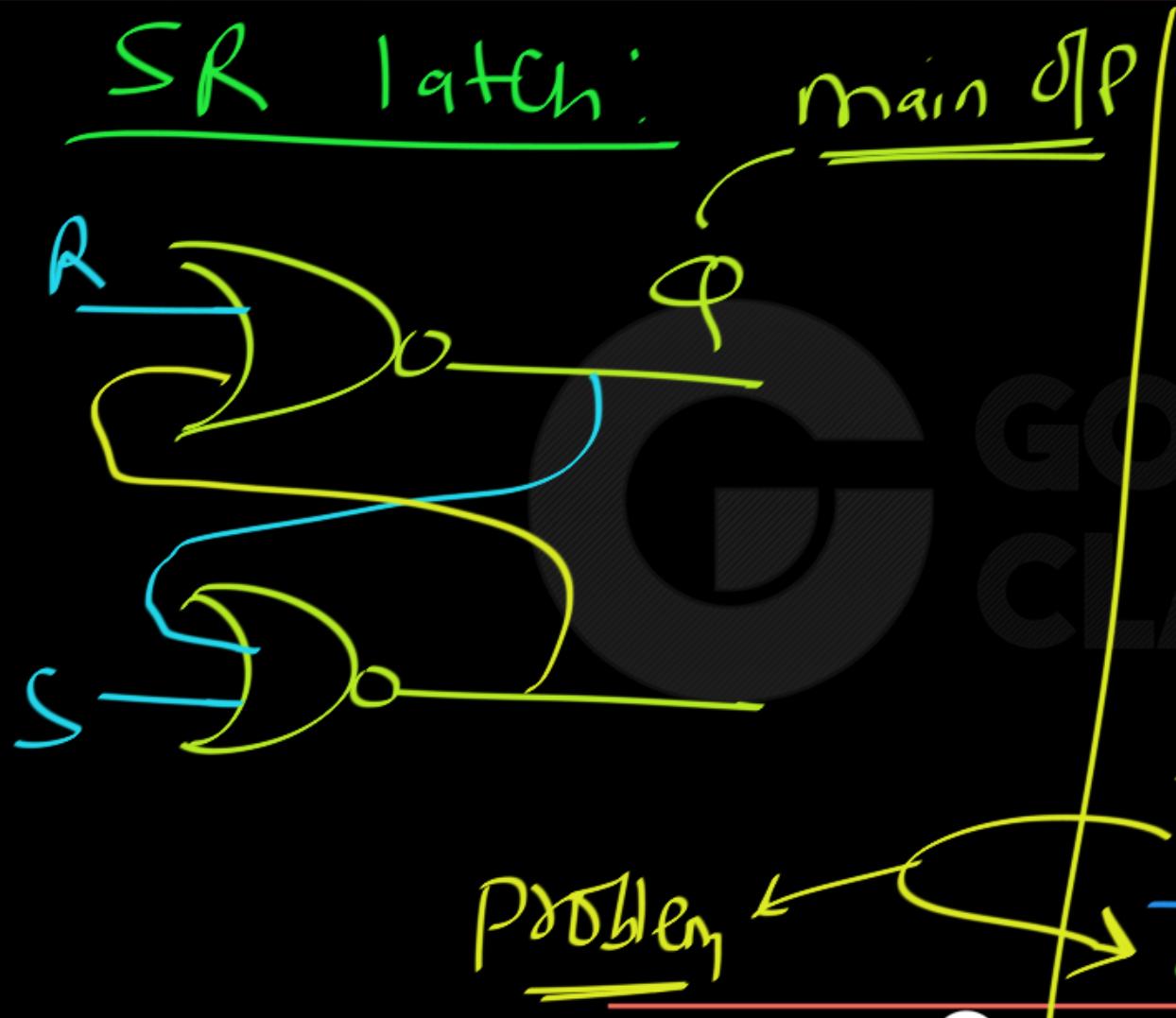
SR latch (Set-Reset latch)



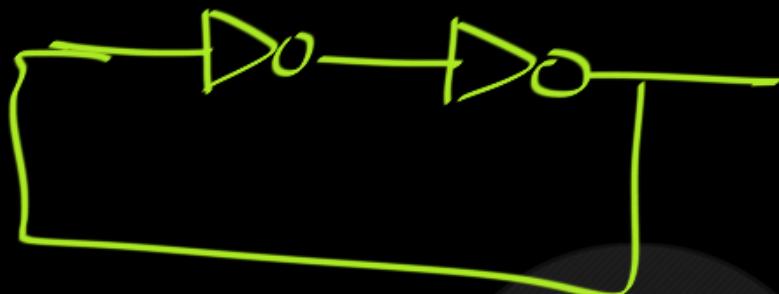
Digital Logic



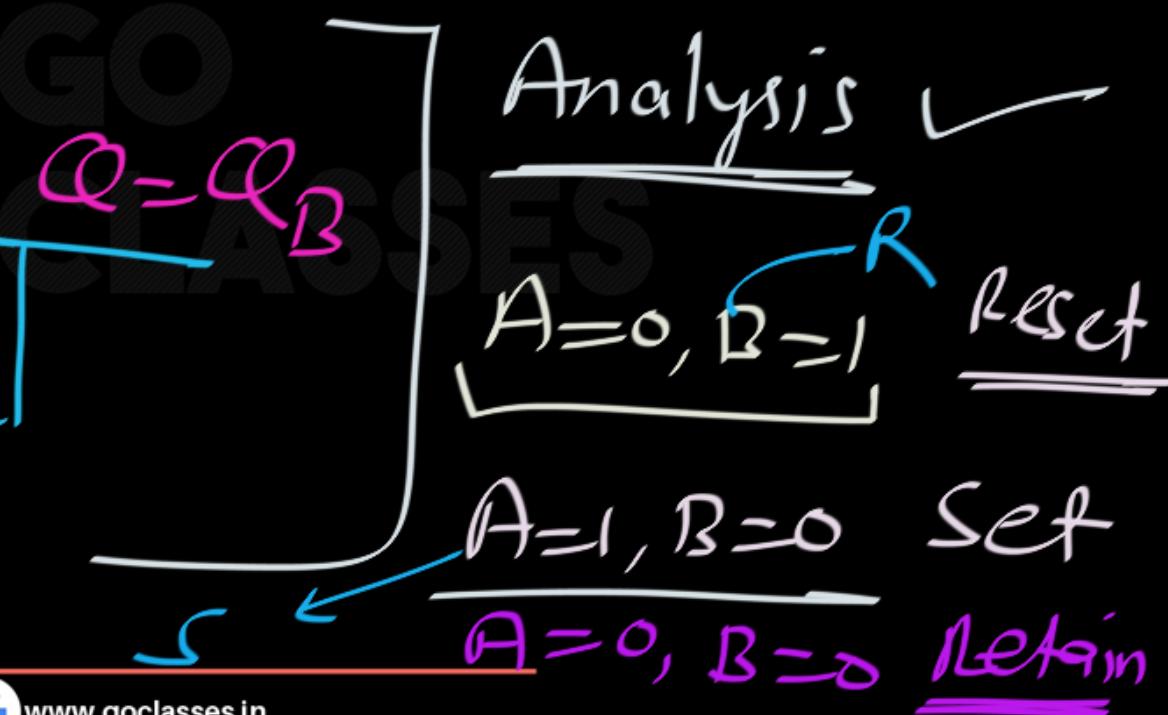
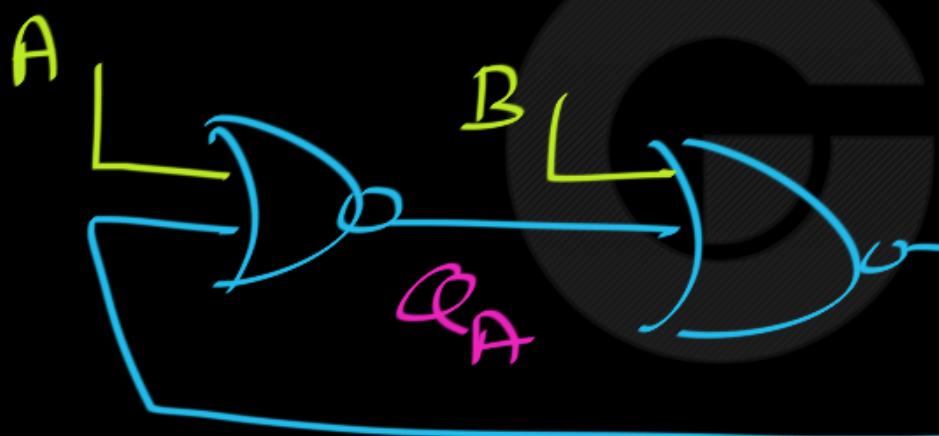
SR lateh



S	R	Q_n	time
1	0	1	0
0	0	$Q = 1$	1
0	1	0	2
0	0	$Q = 0$	3
1	1	0	4
0	0	undefined	5



No User Control



SR latch:

Set ($Q_P = 1$)

Reset ($Q_P = 0$)

S	R	Q_{t+1}	Next Q_P
0	1	0	
1	0	1	
0	0	Q_t	
1	1	0	But is <u>forbidden</u>

Why $S=1, R=1$ is forbidden in

SR latch?

- (A) Uncertain output
- (2) Unstable output
- (3) None of these

Why $S = 1, R = 1$ is forbidden in

SR

latch

Output = 0
Stable

- (A) Uncertain output
- (2) Unstable output
- (3) None of these



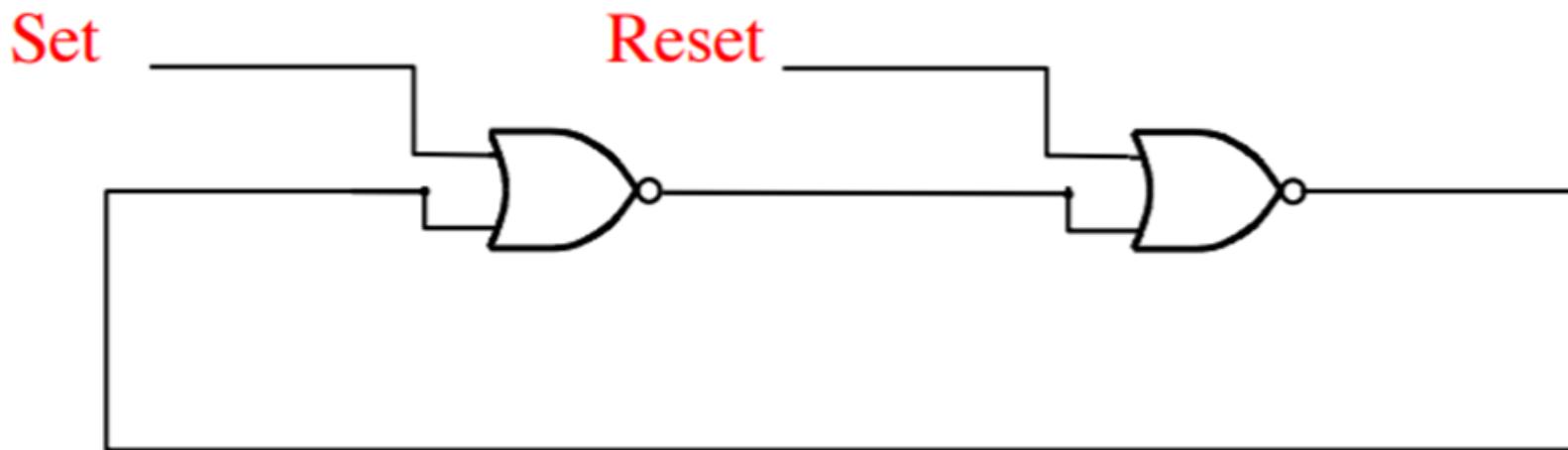
Why $\boxed{S=1, R=1}$ is forbidden in

SR latch?

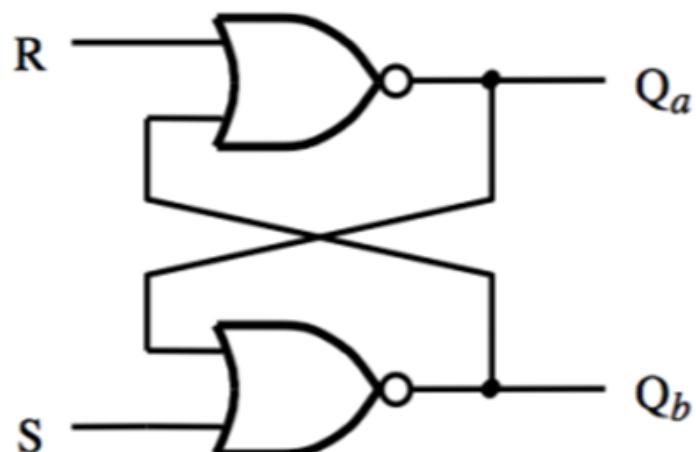
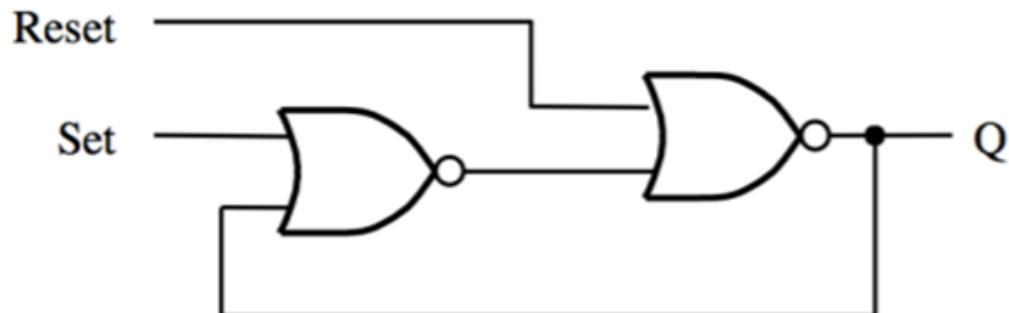
Reason:- After δ $\frac{S=1, R=1}{}$

If we make $\frac{S=0, R=0}{}$ then \checkmark uncertain $\sigma/p \checkmark$

A simple memory element with NOR Gates

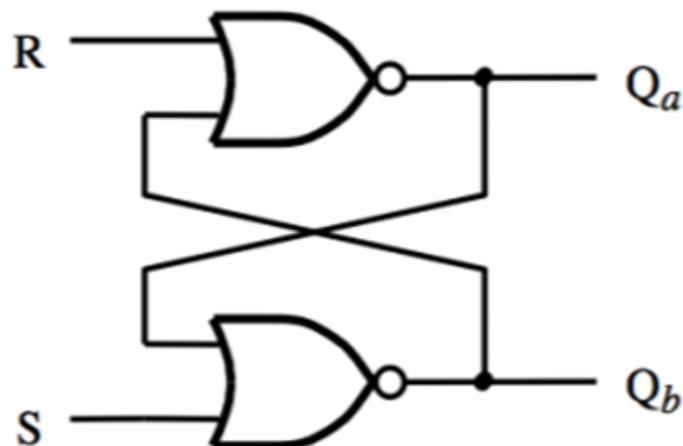
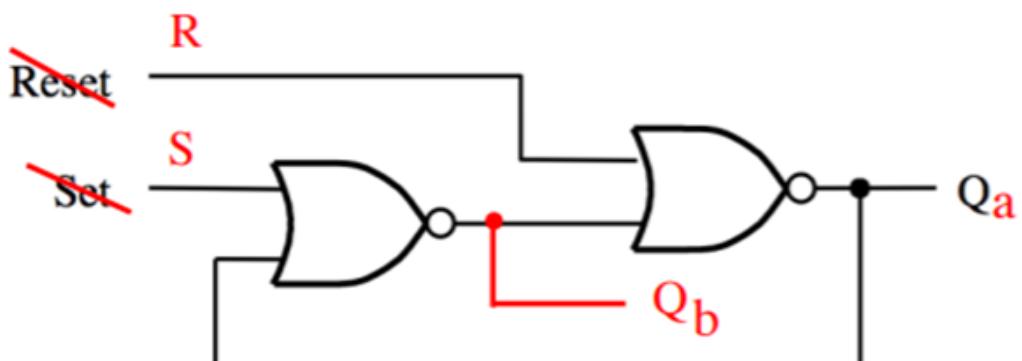


Two Different Ways to Draw the Same Circuit



Two Different Ways to Draw the Same Circuit

SR latch





What is a latch?

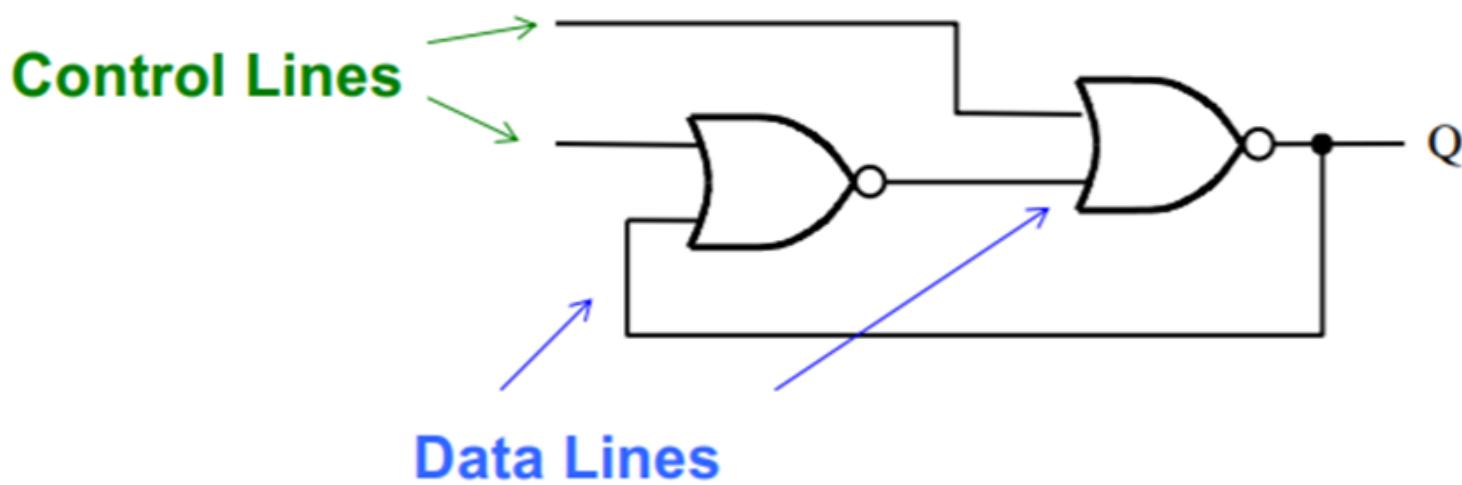




Back to the Basic Latch

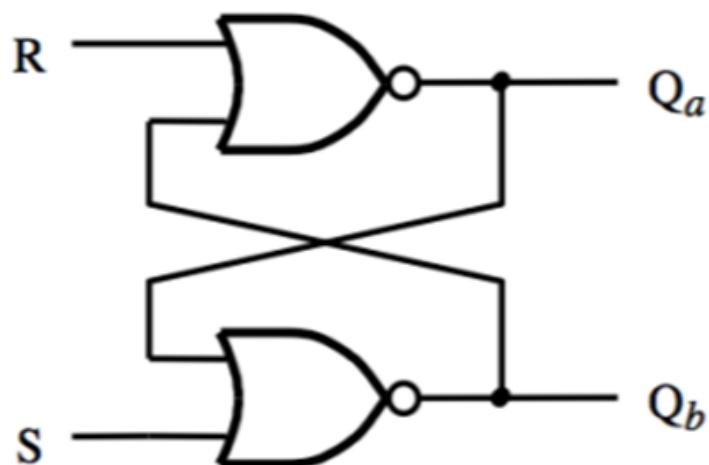
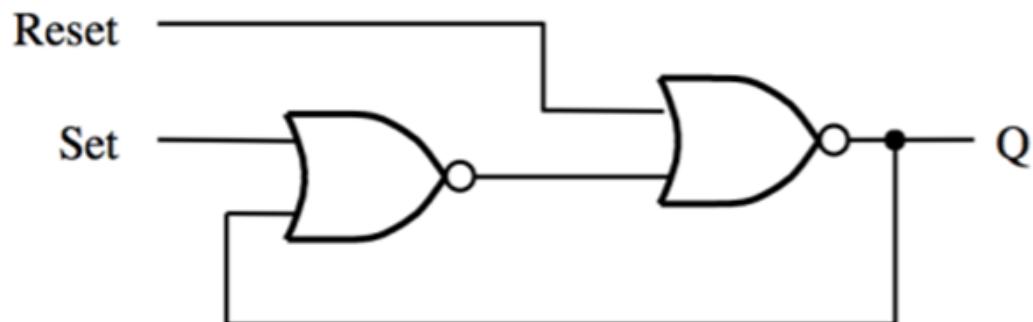


The Basic Latch

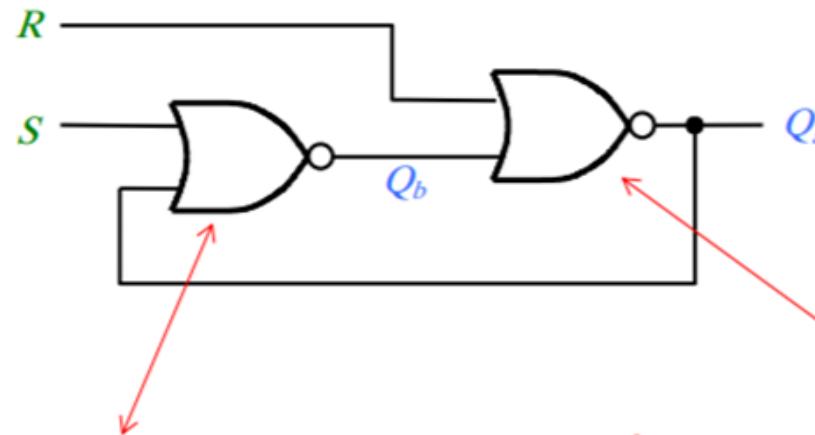




Two Different Ways to Draw the Same Circuit



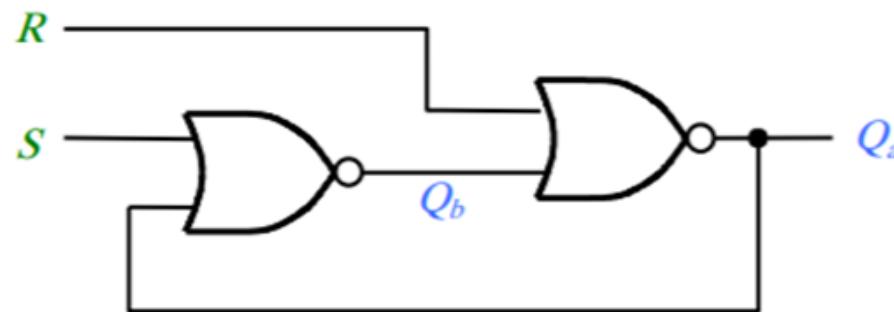
Analyzing The Basic Latch



S	Q_a	$Q_b = \text{NOR}(S, Q_a)$
0	0	1 } \bar{Q}_a
0	1	0 }
1	0	0 } 0
1	1	0 }

R	Q_b	$Q_a = \text{NOR}(R, Q_b)$
0	0	1 } \bar{Q}_b
0	1	0 }
1	0	0 } 0
1	1	0 }

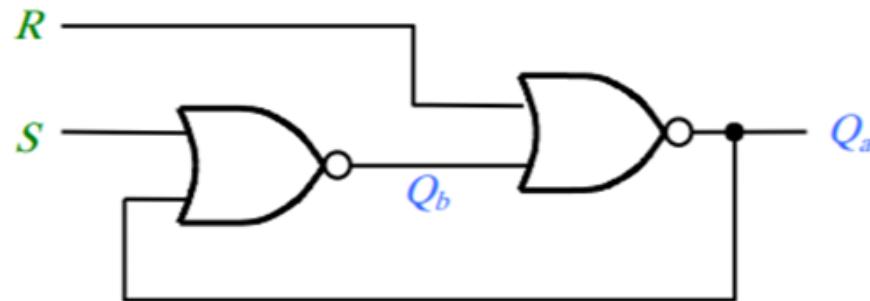
Analyzing The Basic Latch



S	Q_b	\bar{Q}_a
0	0	1
1	1	0

R	Q_a	\bar{Q}_b
0	1	0
1	0	1

Behavior of the Basic Latch



S	R	$Q_a(t+1)$	$Q_b(t+1)$
0	0	$Q_a(t)$	$Q_b(t)$
0	1	0	1
1	0	1	0
1	1	0	0

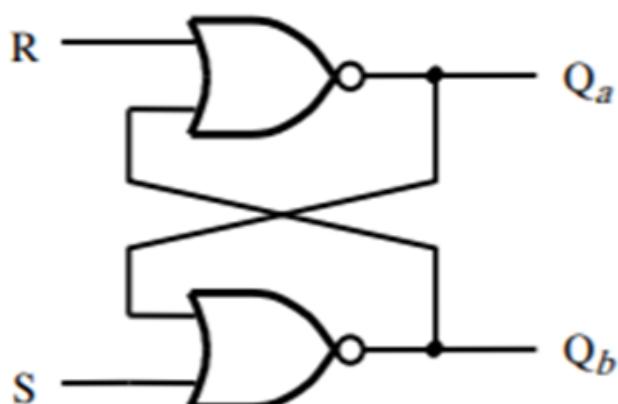
Latch

Reset

Set

Undesirable

Circuit and Characteristic Table



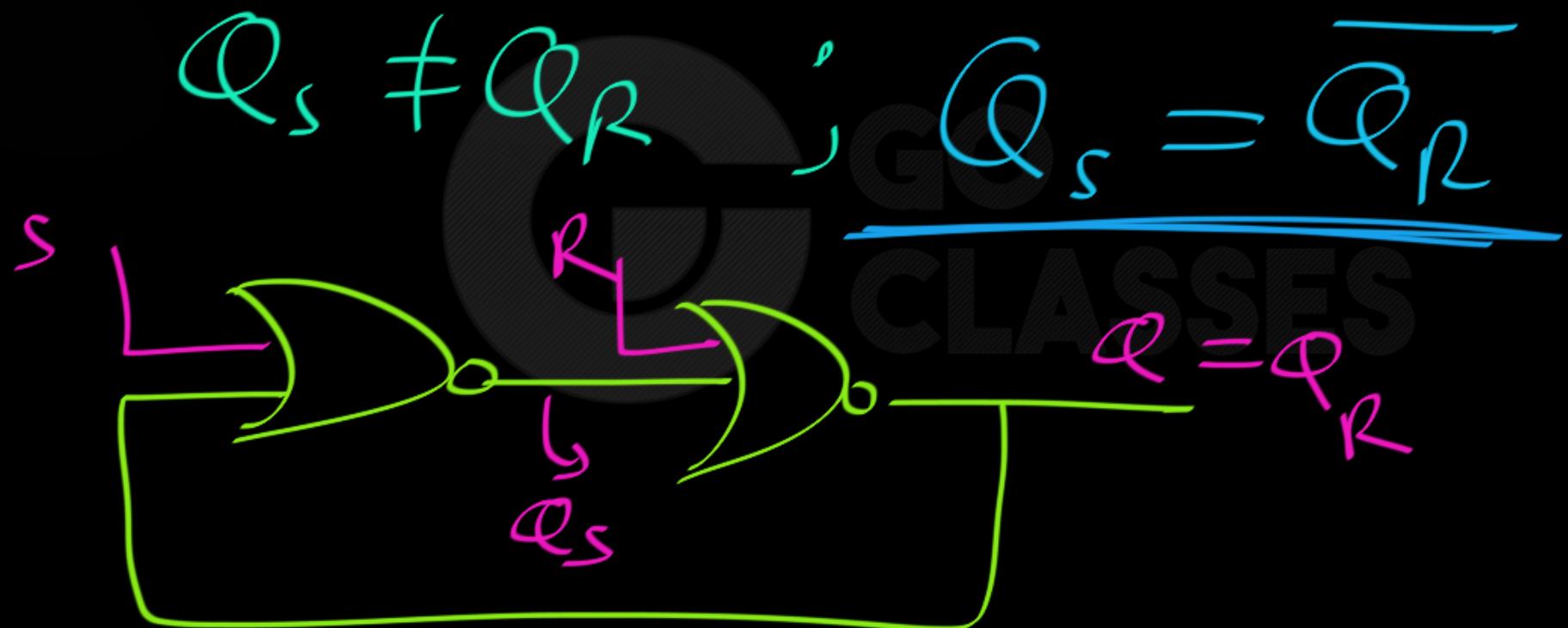
(a) Circuit

S	R	Q_a	Q_b
0	0	0/1	1/0
0	1	0	1
1	0	1	0
1	1	0	0

(b) Characteristic table

Note that Q_a and Q_b are inverses of each other!

Notes: ① for all Allowed Input Combinations



S	R	Q_s	$Q_R = \Phi$	invalid flip comb. (<u>forbidden</u>)
1	1	0	0	
0	1	1	0	
1	0	0	1	
0	0	1	1	$Q_s = \overline{Q_R}$

Note:

If

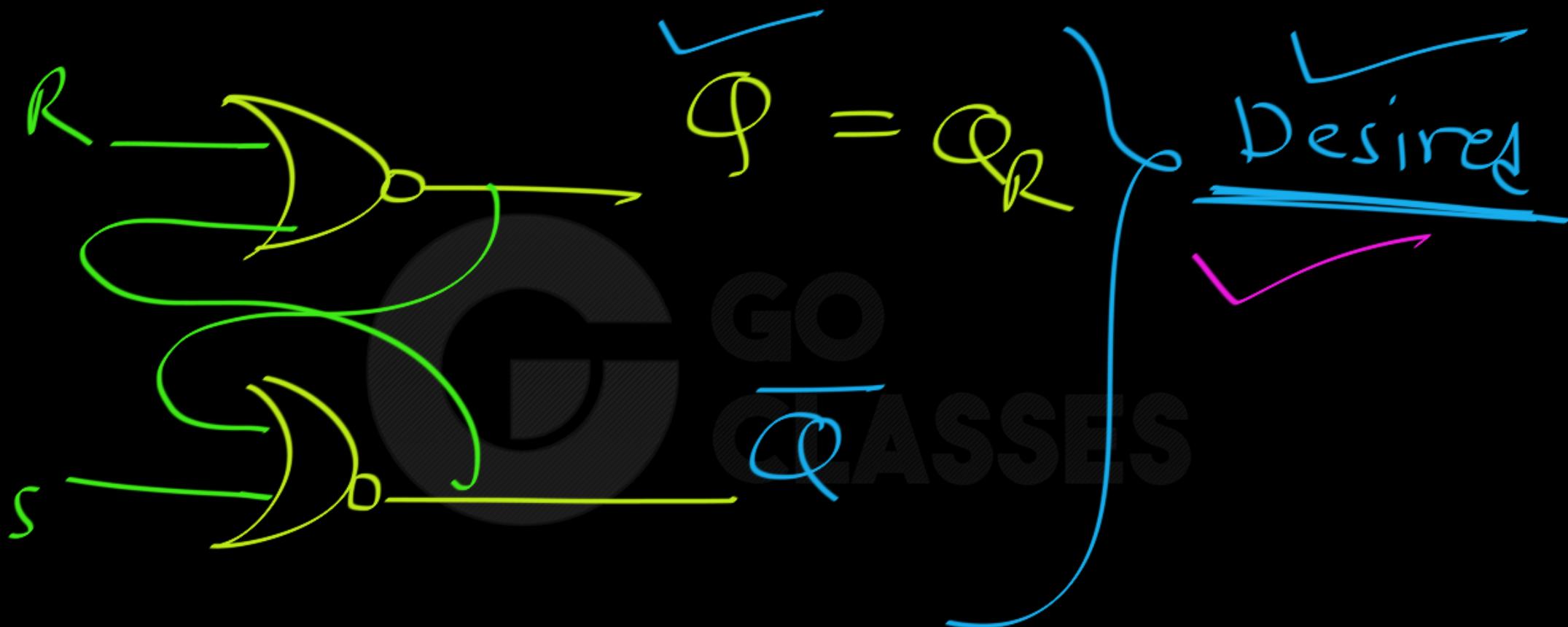
$$Q_s = Q_R$$

If we toy
to retain

✓
Uncertain ✓
behavior ✓

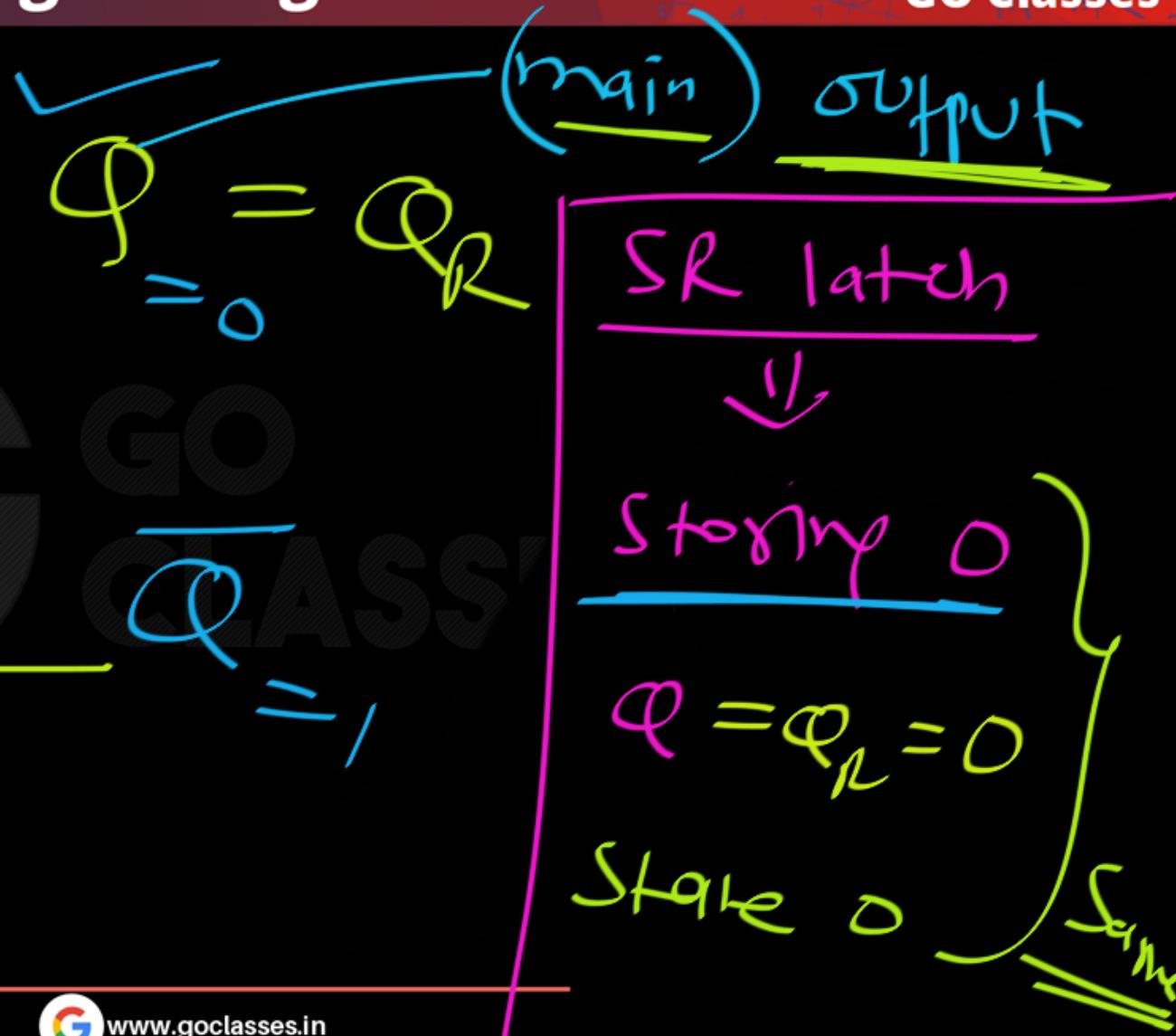
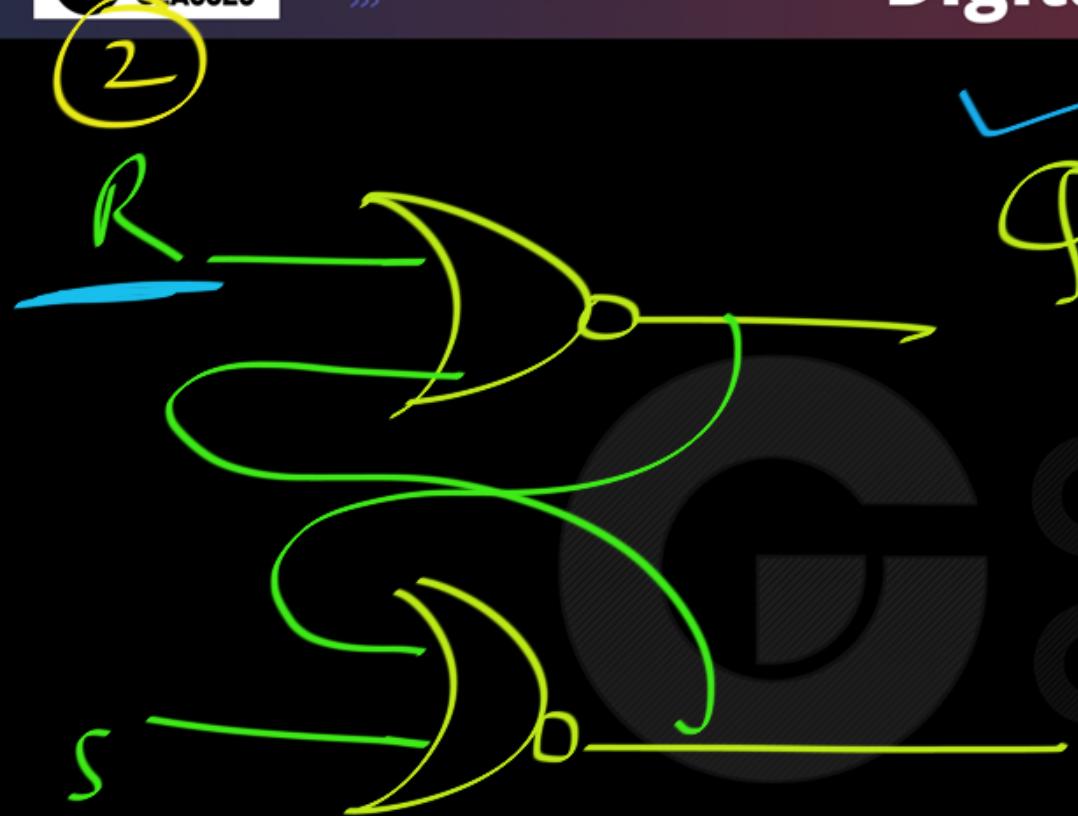


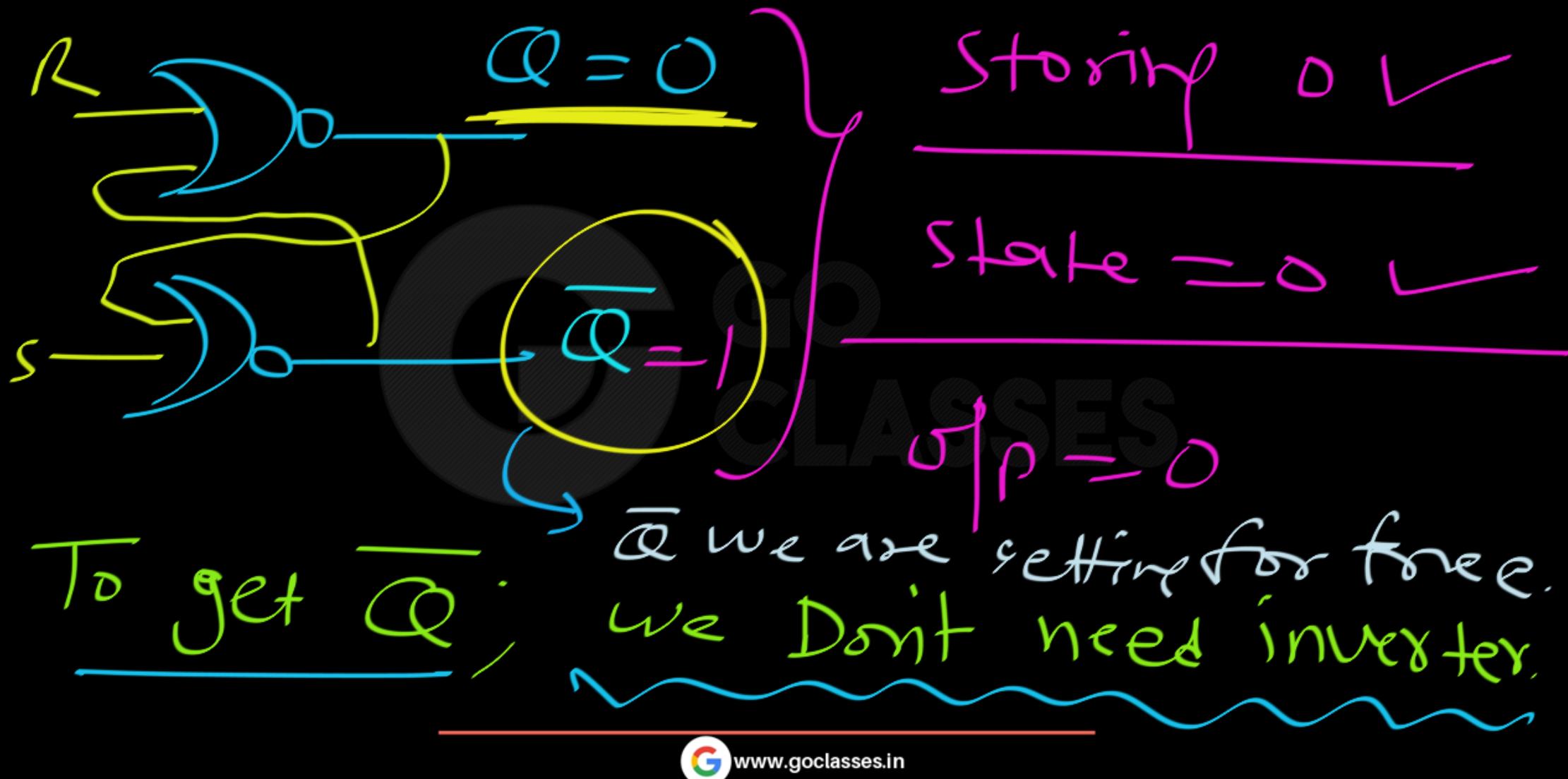
Digital Logic





Digital Logic



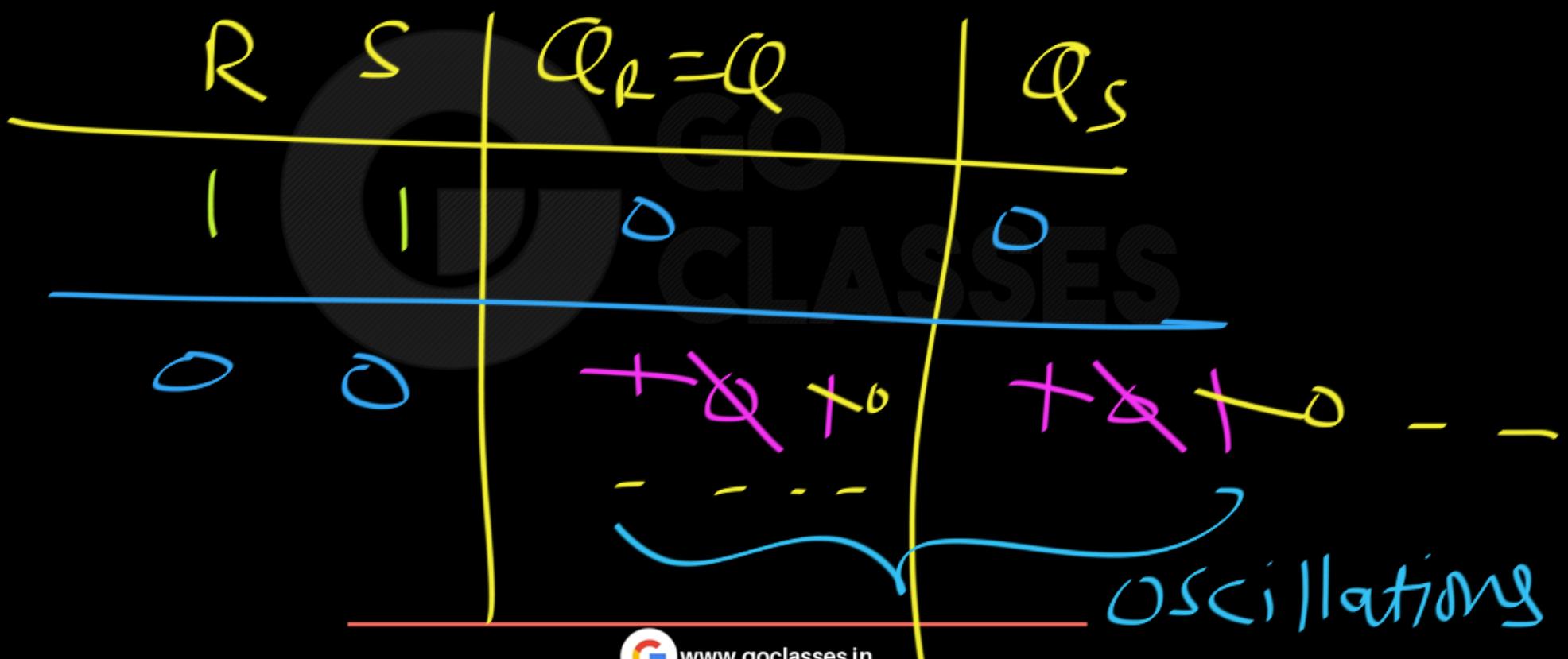


R	S	$Q = QR$	Q_S
0	0	0	0
1	1	0	1

Uncertain/undefined
Oscillating (when both NOR gate have same speed)



If both NOR gate have same speed:



but in Reality ; Both NOR gate
have different speed :

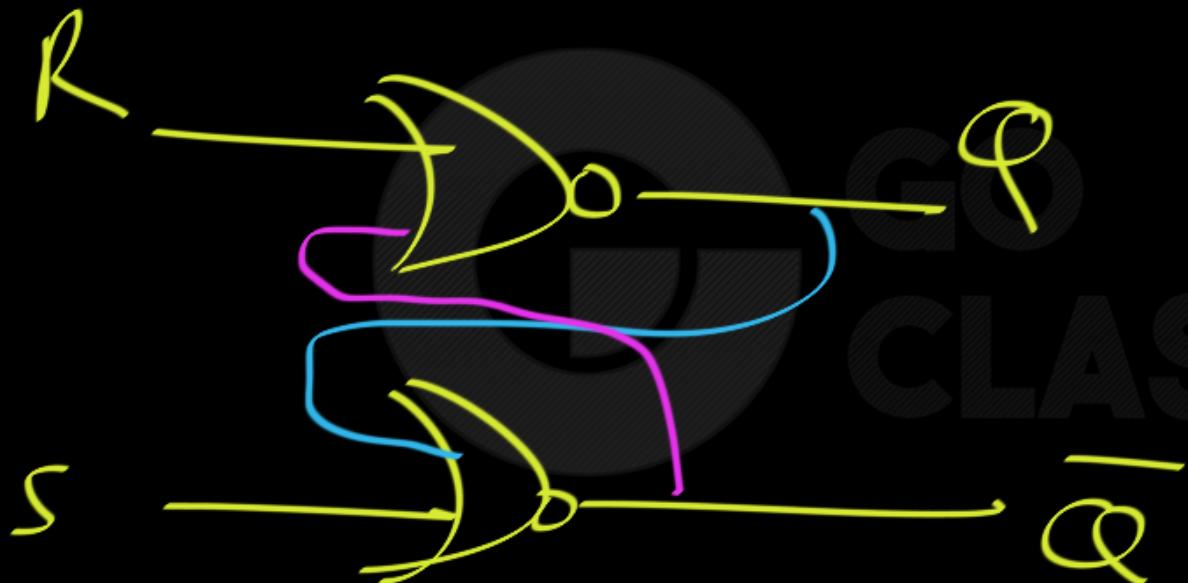
S $Q_R = 0, Q_S = 1$ } Uncertain
OR
 $Q_R = 1, Q_S = 0$

- When S=1 and R=1 both outputs of the latch are equal to 0, i.e., $Q_a=0$ and $Q_b=0$.
- Thus, the two outputs are no longer complements of each other.
- This is undesirable as many of the circuits that we will build later with these latches rely on the assumption that the two outputs are always complements of each other.
- (This is obviously not the case for the basic latch, but we will patch it later to eliminate this problem).

Oscillations and Undesirable States

- An even bigger problem occurs when we transition from $S=R=1$ to $S=R=0$.
- When $S=R=1$ we have $Q_a=Q_b=0$. After the transition to $S=R=0$, however, we get $Q_a=Q_b=1$, which would immediately cause $Q_a=Q_b=0$, and so on.
- If the gate delays and the wire lengths are identical, then this oscillation will continue forever.
- In practice, the oscillation dies down and the output settles into either $Q_a=1$ and $Q_b=0$ or $Q_a=0$ and $Q_b=1$.
- The problem is that we can't predict which one of these two it will settle into.

SR latch: \equiv SR latch using NOR

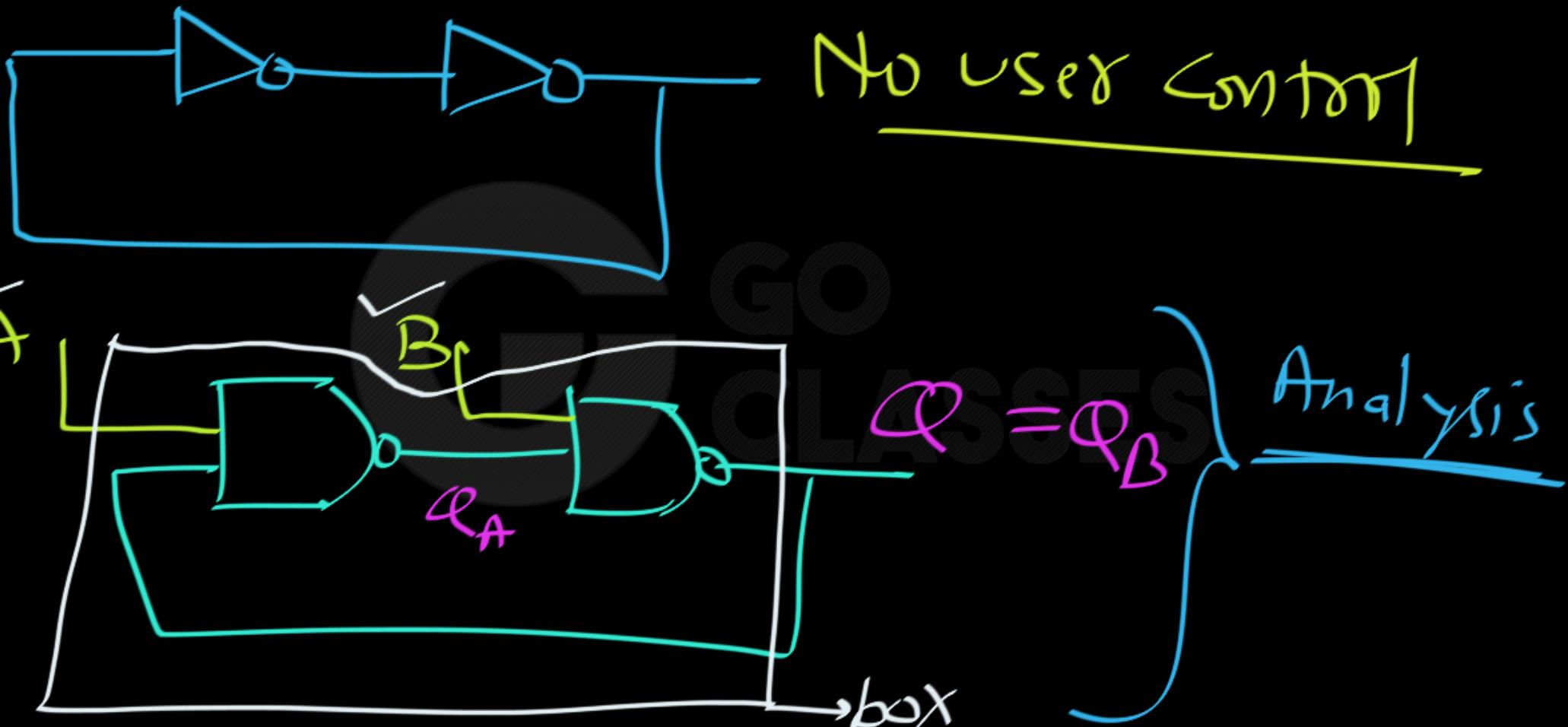


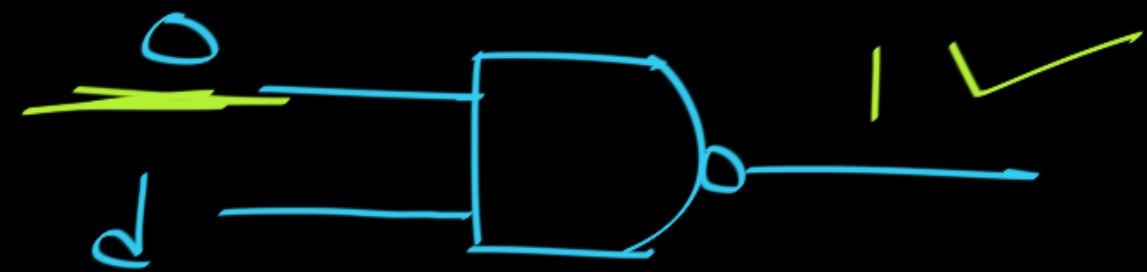


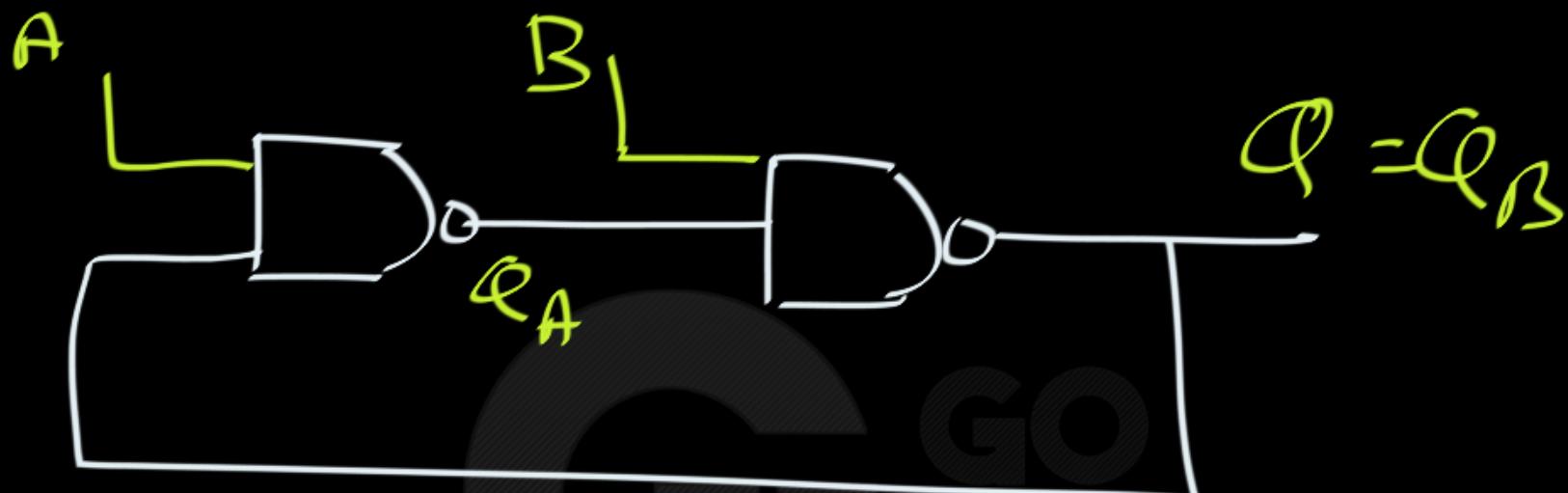
Next Topic:

SR Latch with NAND Gates

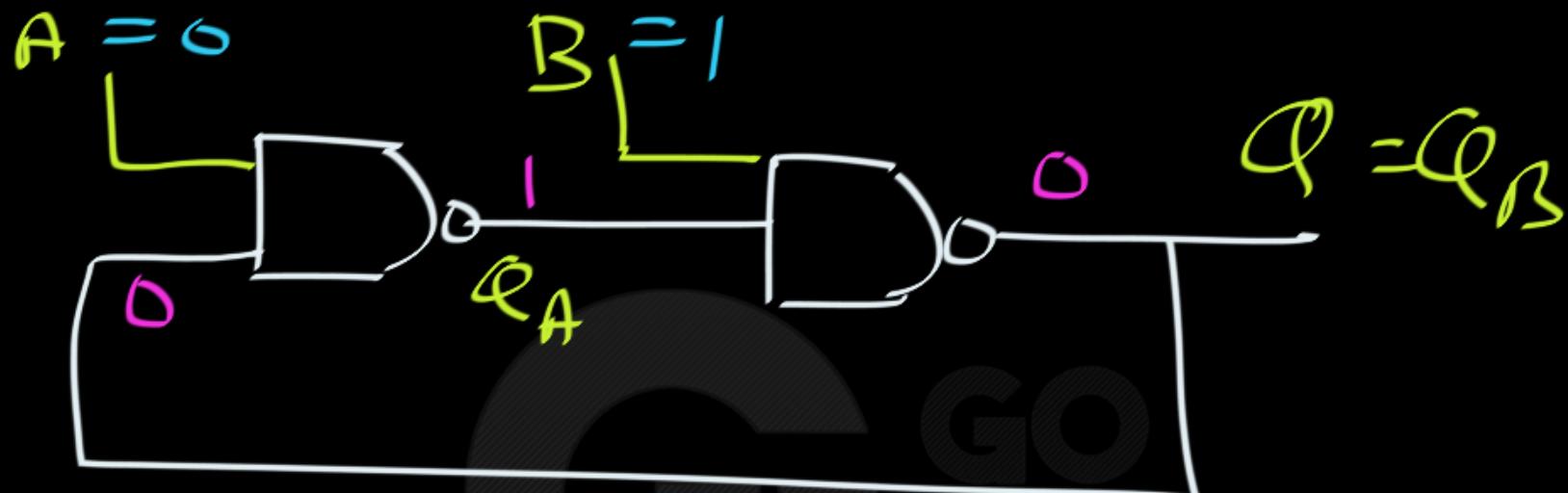






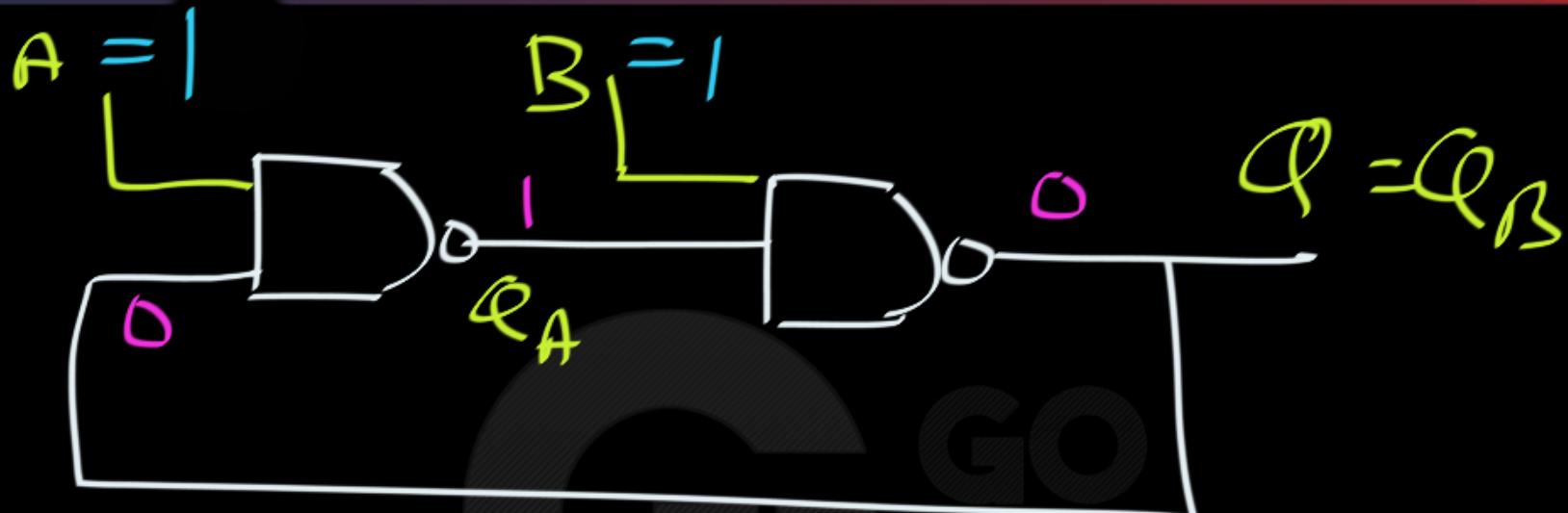


GO
CLASSES



Storing 0 (Reset)

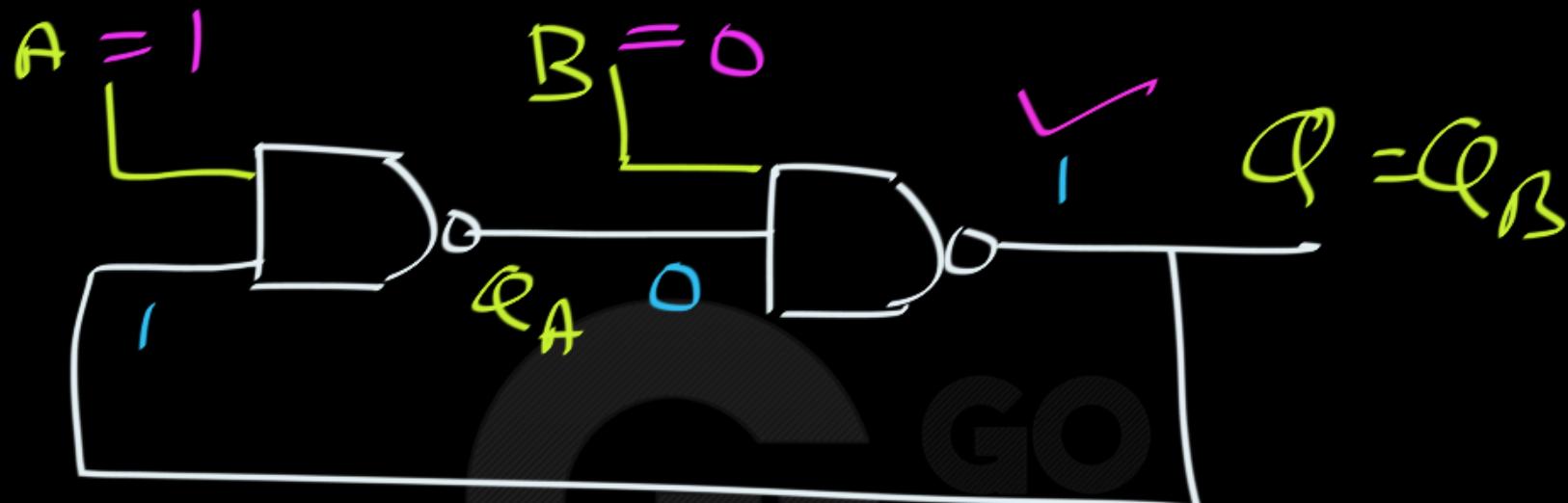
A	B	Q_{t+1}
0	1	0



Retain

$A = 1, B = 1$

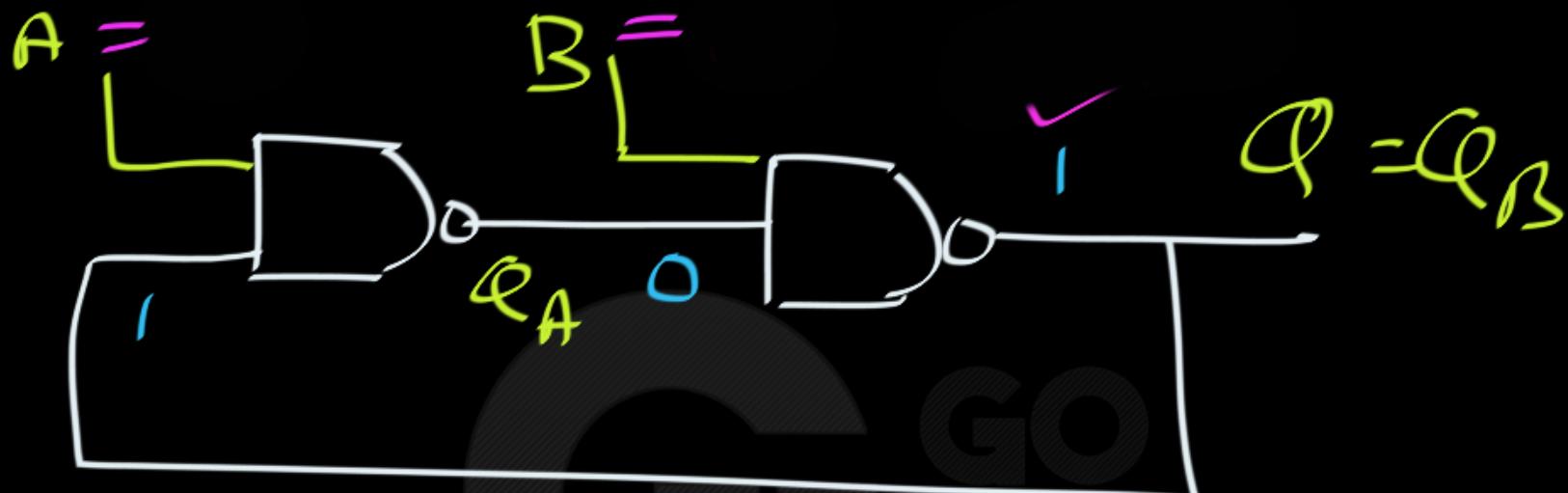
$Q_{t+1} = Q_t \xrightarrow{\text{Retain}} \checkmark$



Storing 1

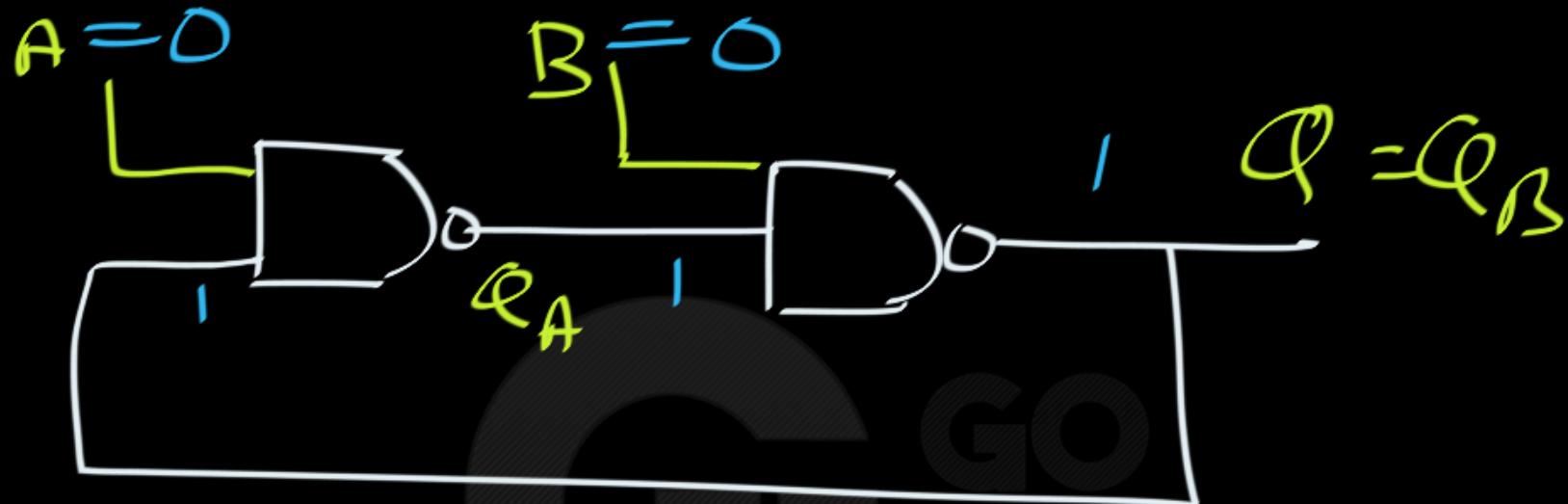
(Set)

A	B	Q_{t+1}
1	0	1
0		1

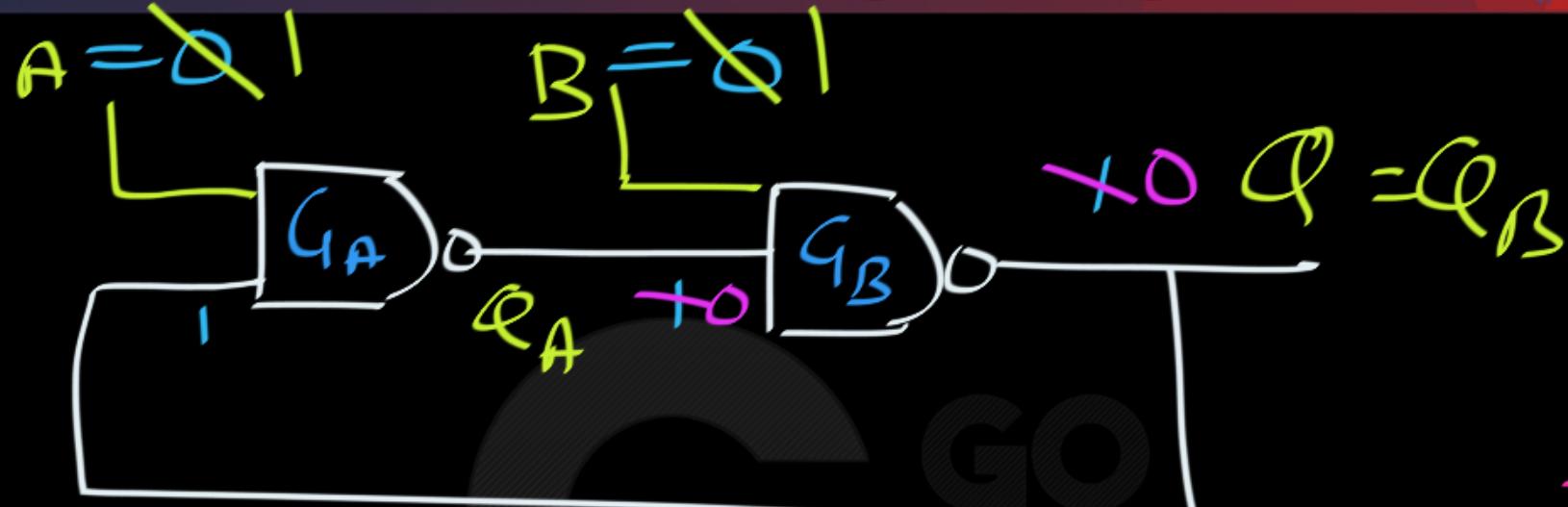


$A = 1, B = 1 \Rightarrow \text{Retain}$
 $Q_{t+1} = Q$

A	B	$Q = Q_B$	Q_A	
0	1	0	1	Reset
1	0	1	0	set
1	1	<u>Retain</u>	<u>Retain</u>	<u>Retain</u>
0	0	?	?	?

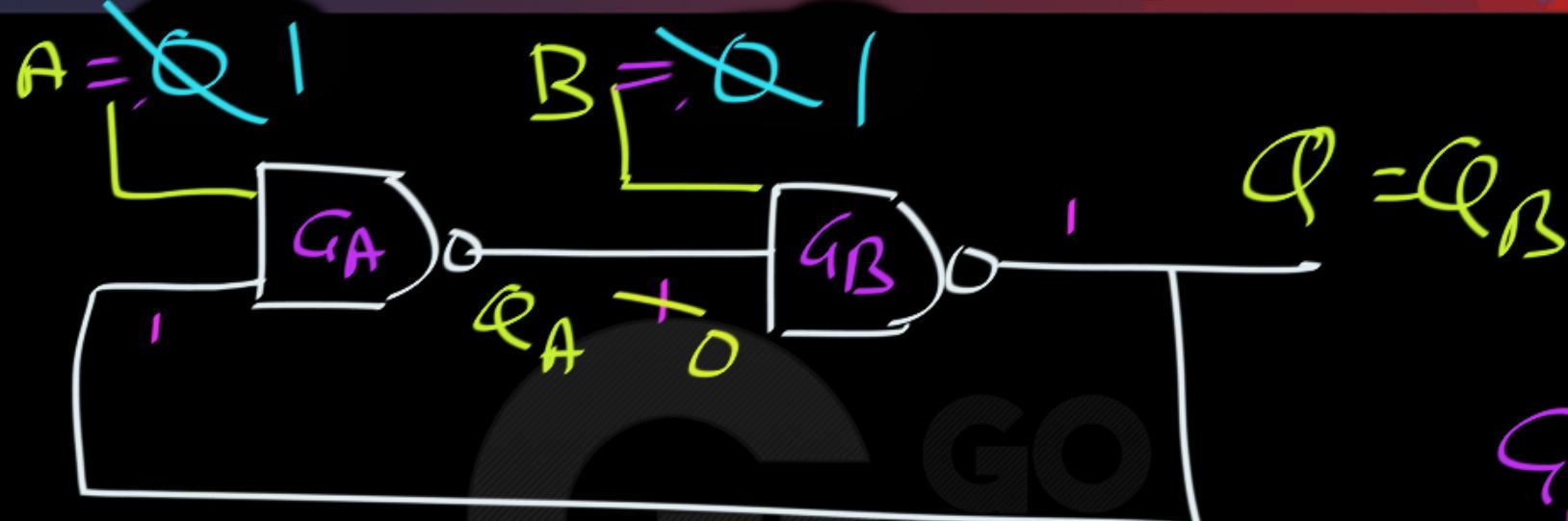


A	B	Q_A	Q_B
0	0	1	1



A	B	Q_A	Q_B
0	0	1	1
1	1	0	0

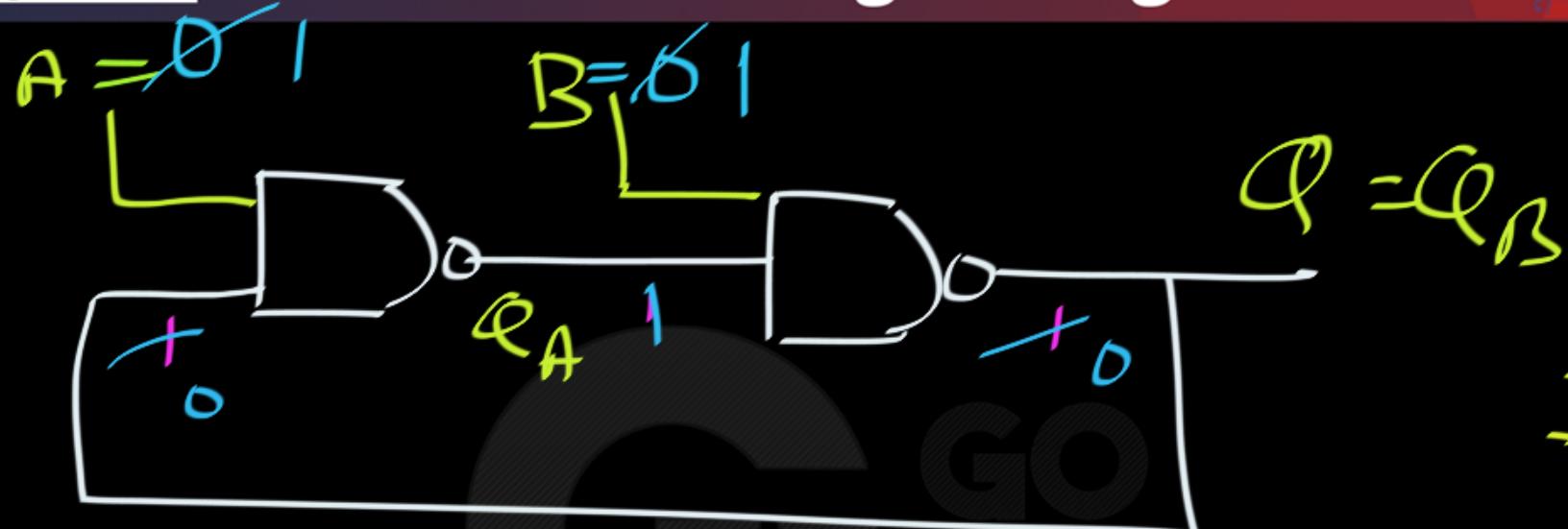
If $Q_A = Q_B$
Speed
↓
oscillation



A timing diagram showing the outputs of Q_A and Q_B over time. The top row shows the initial state (0, 0) followed by a transition to (1, 1). The bottom row shows the final state (0, 1). A green arrow points from the initial state to the final state.

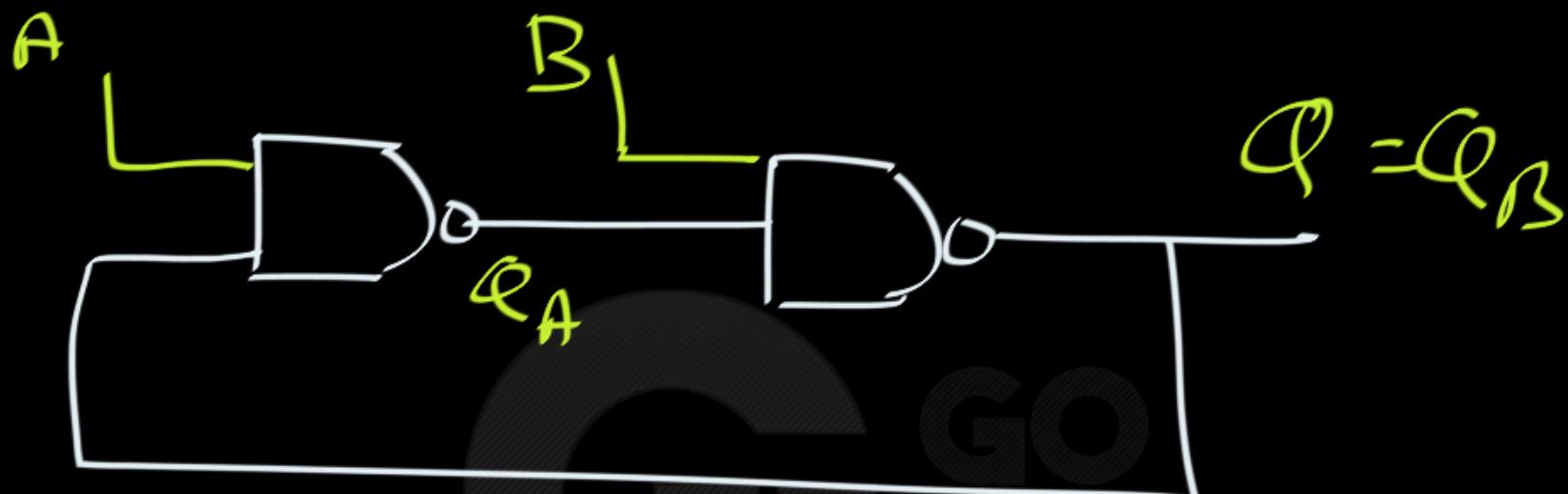
A	B	\overline{Q}_A	\overline{Q}_B
0	0	1	1
1	1	0	1

If
 $\tau_A > \tau_B$
Speed

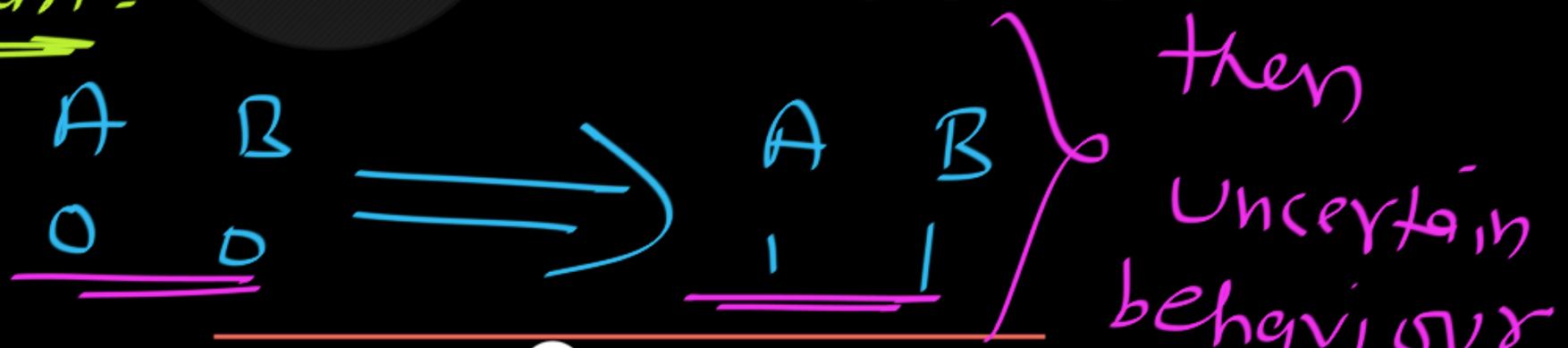


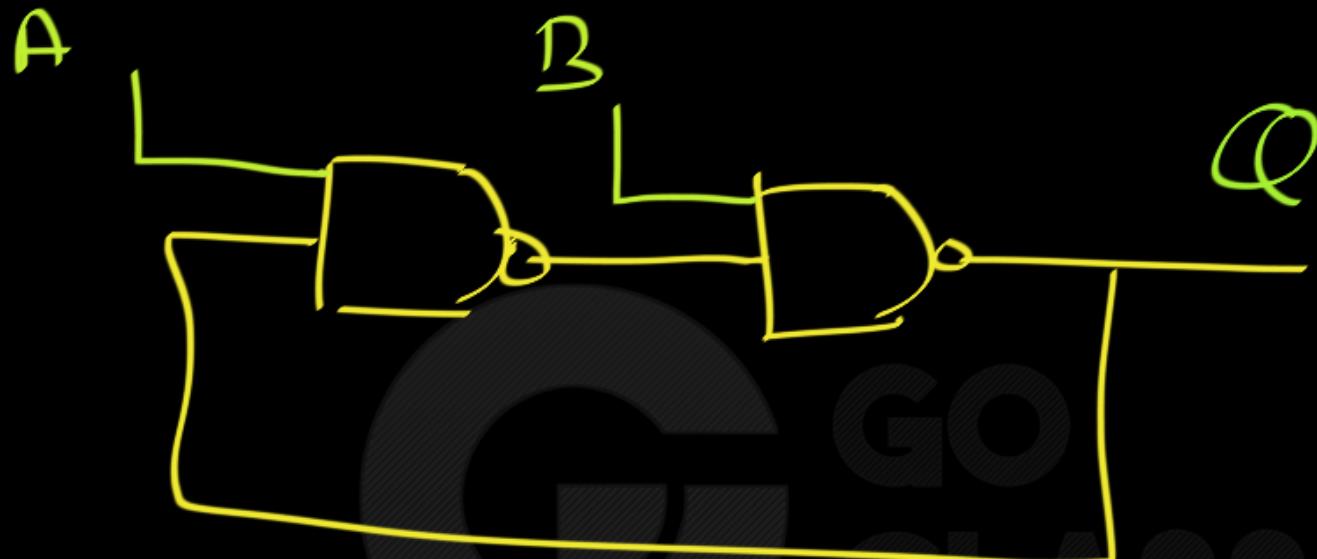
A	B	Q_A	Q_B
0	0	1	1
1	1	1	0

If $Q_B > Q_A$
Speed



Problem:



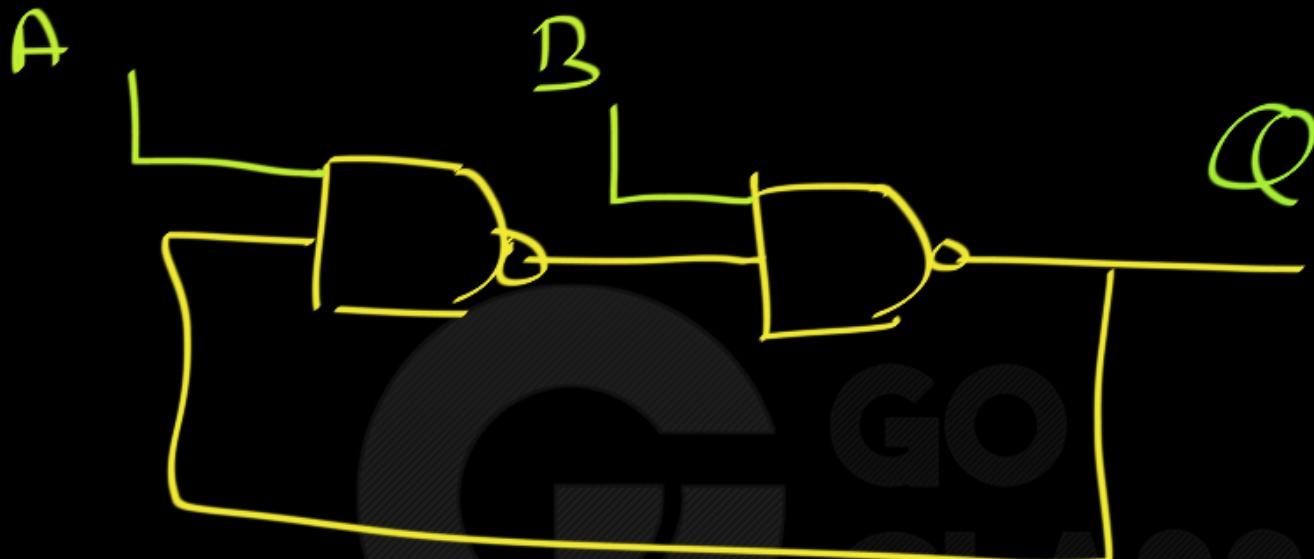


Q: Set it ? ($Q=1$)

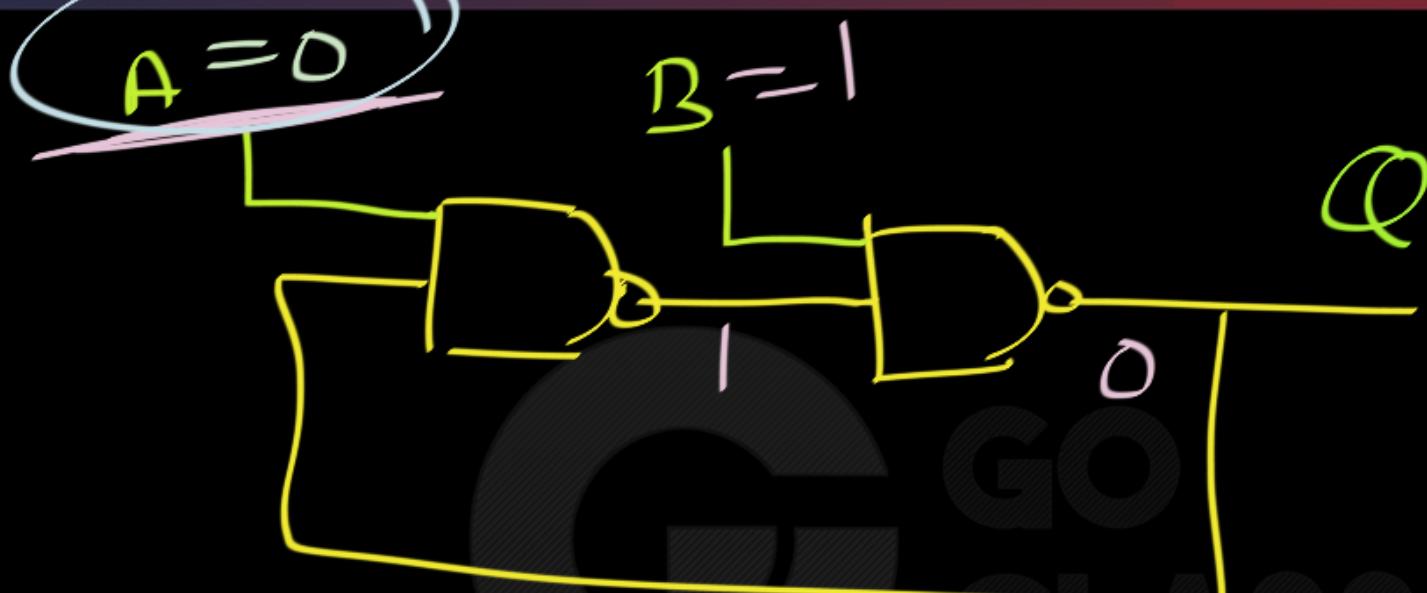


Q : Set it ?

$B \Rightarrow$ working for
Set(tine)
But Active low wise

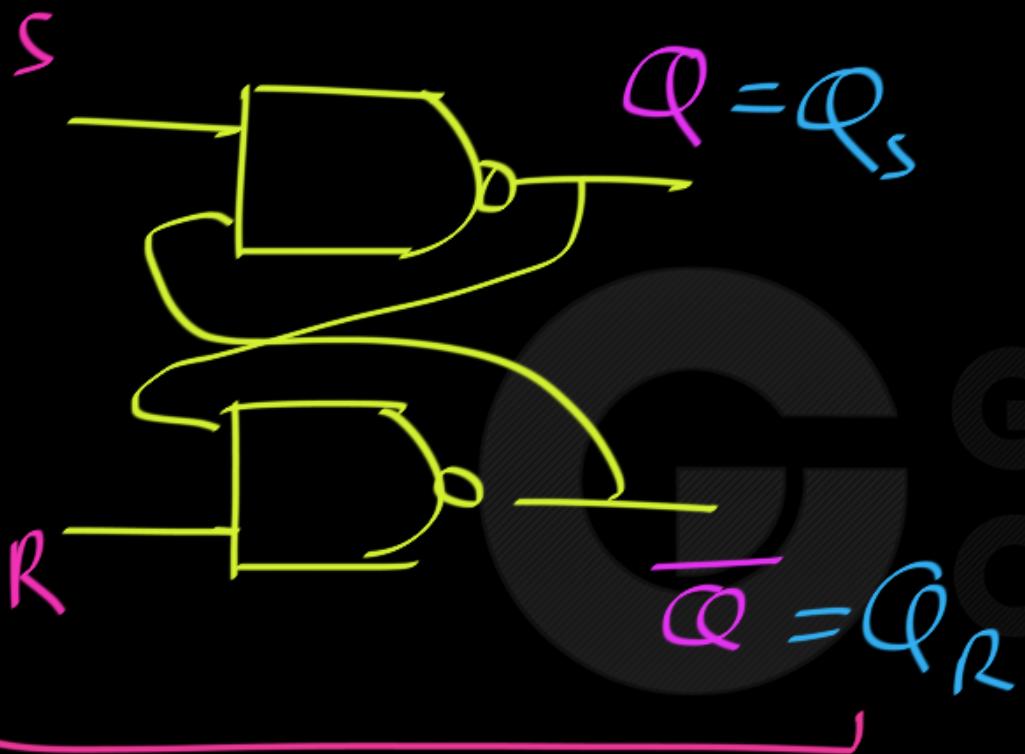


Q : Reset it ? ($Q=0$)



Q : Reset it ? ($Q=0$)

A is working
as Reset
but Active
low wise



SR latch

S	R	Q_{t+1}
0	1	1 (Set)
1	0	0 (Reset)
1	1	Netein = Q_t

$\xrightarrow{\text{forbidden}}$



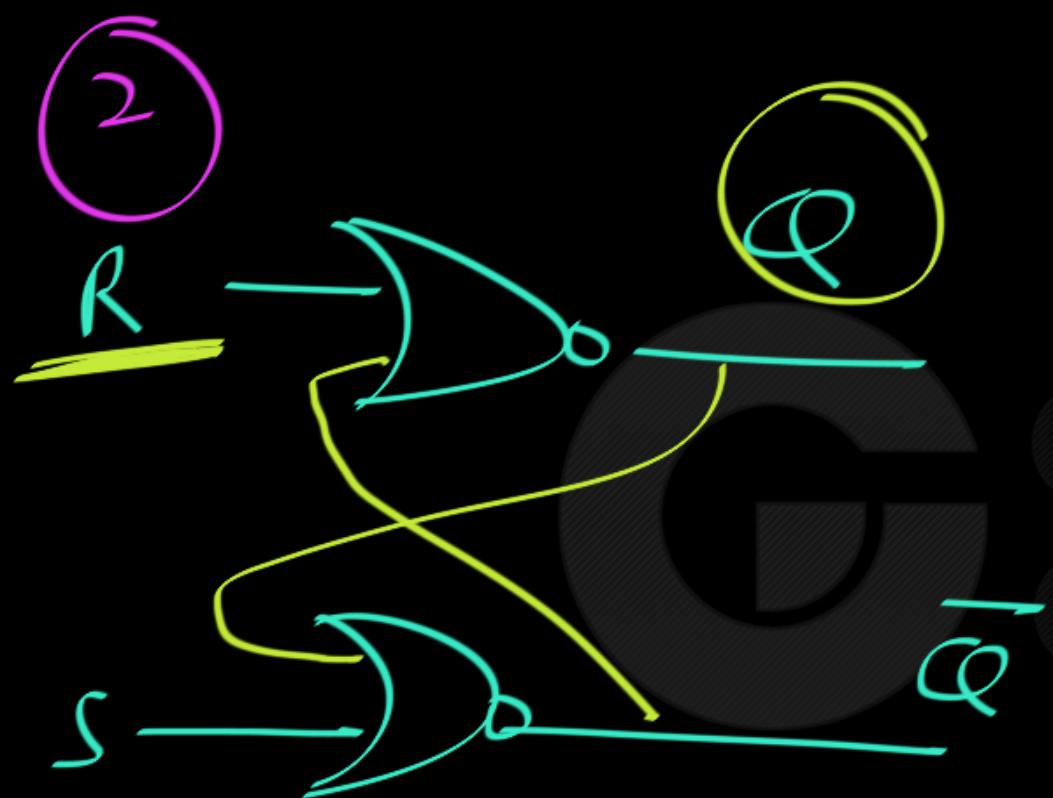
Comparison of SR latch and $\overline{S}\overline{R}$ latch:

① SR latch = SR latch with NOR

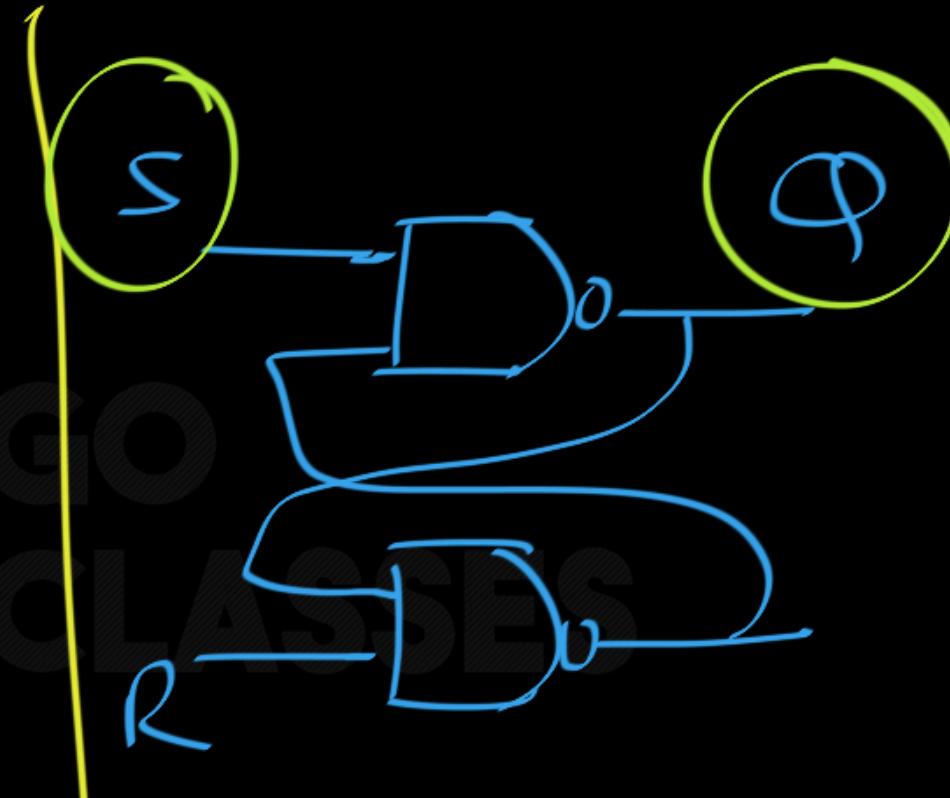
$\overline{S}\overline{R}$ latch = II " NAND



some Authors



SR latch



$\bar{S} \bar{R}$ latch



③ SR latch is Active High.
 \hookrightarrow (SR latch with NOR)

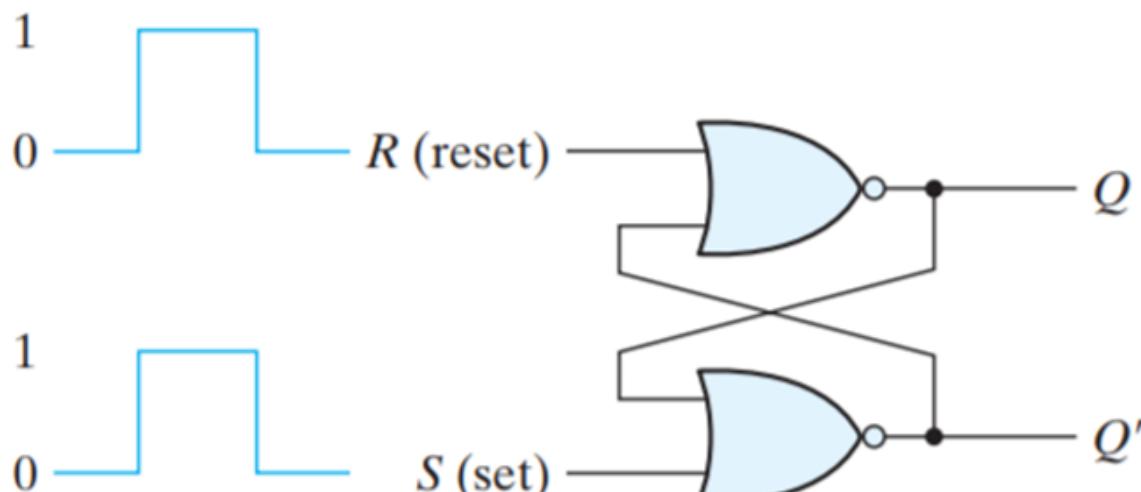
$\overline{S}\overline{R}$ latch is Active low.

<u>S</u>	<u>R</u>	<u>SR latch</u>		<u>SR latch</u>	
		$Q = Q_R$	Q_S	$Q = Q_S$	Q_R
1	0	1 (Set)	0	0 (Reset)	1
0	1	0 (Reset)	1	1 (Set)	0
0	0	Retain	Retain	1	1 (forbidden)
1	1	0 (forbidden)	0	Retain	Retain



SR Latch

The *SR* latch is a circuit with two cross-coupled NOR gates or two cross-coupled NAND gates, and two inputs labeled *S* for set and *R* for reset. The *SR* latch constructed with two cross-coupled NOR gates is shown in Fig. 5.3. The latch has two useful states. When output $Q = 1$ and $Q' = 0$, the latch is said to be in the *set state*. When $Q = 0$ and $Q' = 1$, it is in the *reset state*. Outputs Q and Q' are normally the complement of each other. However, when both inputs are equal to 1 at the same time, a condition in which both outputs are equal to 0 (rather than be mutually complementary) occurs. If both inputs are then switched to 0 simultaneously, the device will enter an unpredictable or undefined state or a metastable state. Consequently, in practical applications, setting both inputs to 1 is forbidden.



(a) Logic diagram

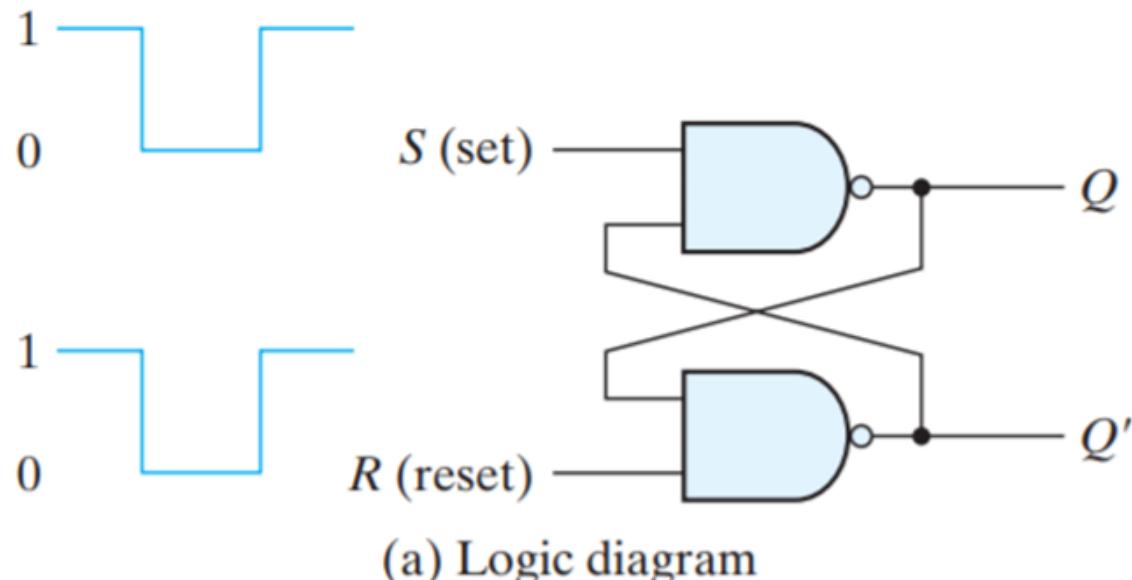
S	R	Q	Q'
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1
1	1	0	0

(after $S = 1, R = 0$)
(after $S = 0, R = 1$)
(forbidden)

(b) Function table

FIGURE 5.3

SR latch with NOR gates



S	R	Q	Q'	
1	0	0	1	
1	1	0	1	(after $S = 1, R = 0$)
0	1	1	0	
1	1	1	0	(after $S = 0, R = 1$)
0	0	1	1	(forbidden)

(b) Function table

FIGURE 5.4
SR latch with NAND gates



In comparing the NAND with the NOR latch, note that the input signals for the NAND require the complement of those values used for the NOR latch. Because the NAND latch requires a 0 signal to change its state, it is sometimes referred to as an $S'R'$ latch. The primes (or, sometimes, bars over the letters) designate the fact that the inputs must be in their complement form to activate the circuit.





Oscillations and Undesirable States

- The basic latch with NAND gates also suffers from oscillation problems, similar to the basic latch implemented with NOR gates.

Note:

Problem?

Not
the
problem

in SR latch :



(forbidden
Combination)

This
Transition is
→ the Problem

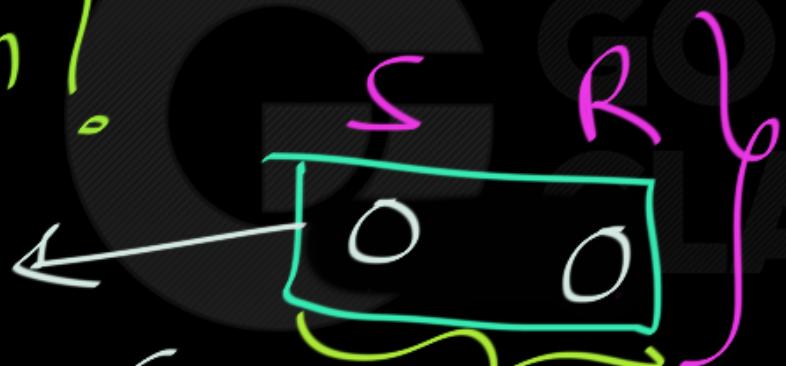
S R
0 0
Retain
Combination

Note:

Problem?

Not
the
problem

in $\overline{S}\overline{R}$ latch :



(forbidden
Combination)

This
Transition is
→ the Problem



Retain
Combination

Note:

in SR latch, $\overline{S}\overline{R}$ latch

Problem?



This Transition
leads
to
Uncertain
behavior.



GATE CSE 2004 | Question: 18, ISRO2007-31

In an *SR* latch made by cross-coupling two NAND gates, if both *S* and *R* inputs are set to 0, then it will result in

- A. $Q = 0, Q' = 1$
- B. $Q = 1, Q' = 0$
- C. $Q = 1, Q' = 1$
- D. Indeterminate states



GATE CSE 2004 | Question: 18, ISRO2007-31

In an SR latch made by cross-coupling two NAND gates, if both S and R inputs are set to 0, then it will result in

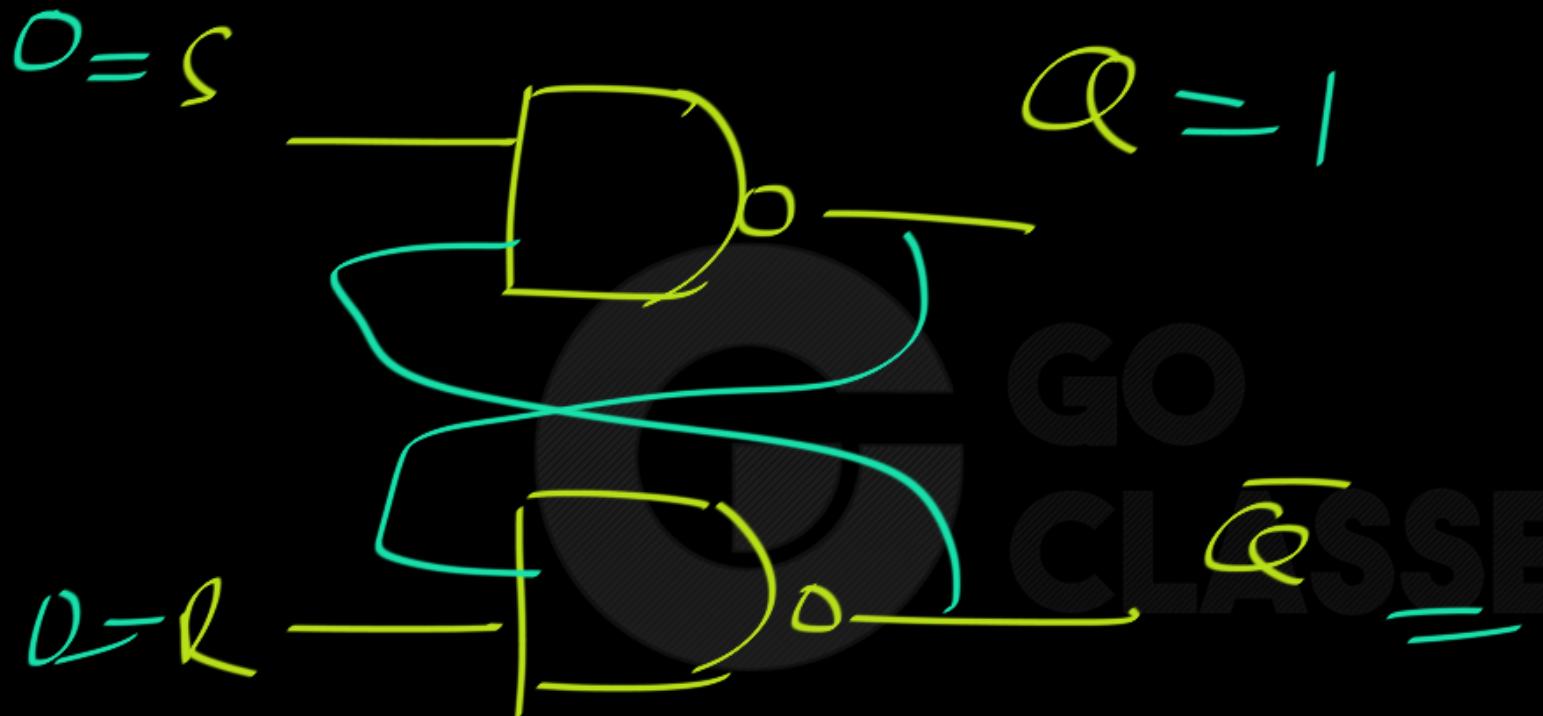
- A. $Q = 0, Q' = 1$
- B. $Q = 1, Q' = 0$
- C. $Q = 1, Q' = 1$ ✓
- D. Indeterminate states

→ SR latch
↓

S	R	Q	\overline{Q}
0	0	1	1

Not the Answer.

majority will Answer this





GATE IT 2007 | Question: 7

Which of the following input sequences for a cross-coupled $R - S$ flip-flop realized with two $NAND$ gates may lead to an oscillation?

- A. 11, 00
- B. 01, 10
- C. 10, 01
- D. 00, 11



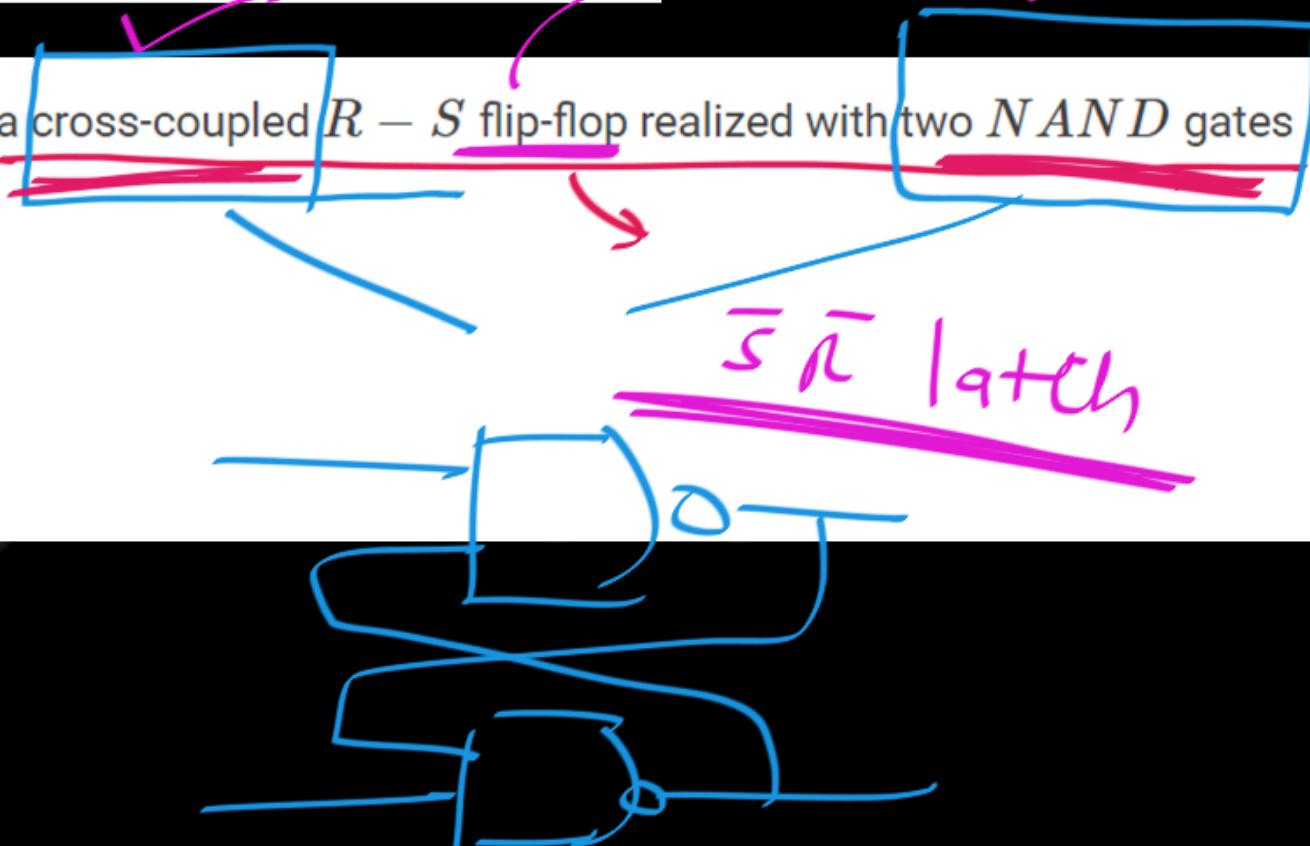
GATE IT 2007 | Question: 7

To Confuse

Which of the following input sequences for a cross-coupled $R - S$ flip-flop realized with two $NAND$ gates may lead to an oscillation?

- A. 11, 00
- B. 01, 10
- C. 10, 01
- D. 00, 11

✓





In an SR latch made by cross-coupling two NAND gates, if both S and R inputs are set to 0, then it will result in

- A. $Q = 0, Q' = 1$
- B. $Q = 1, Q' = 0$
- C. $Q = 1, Q' = 1$ ✓
- D. Indeterminate states

GATE 2004

Which of the following input sequences for a cross-coupled $R - S$ flip-flop realized with two $NAND$ gates may lead to an oscillation?

- A. 11, 00
- B. 01, 10
- C. 10, 01
- D. 00, 11 ✓

GATE 2007