Lecture 9 :

Implicants(I),

Prime Implicants(PI),

Essential Prime Implicants(EPI)


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Next Topic:

Implicants(I),

Prime Implicants(PI),

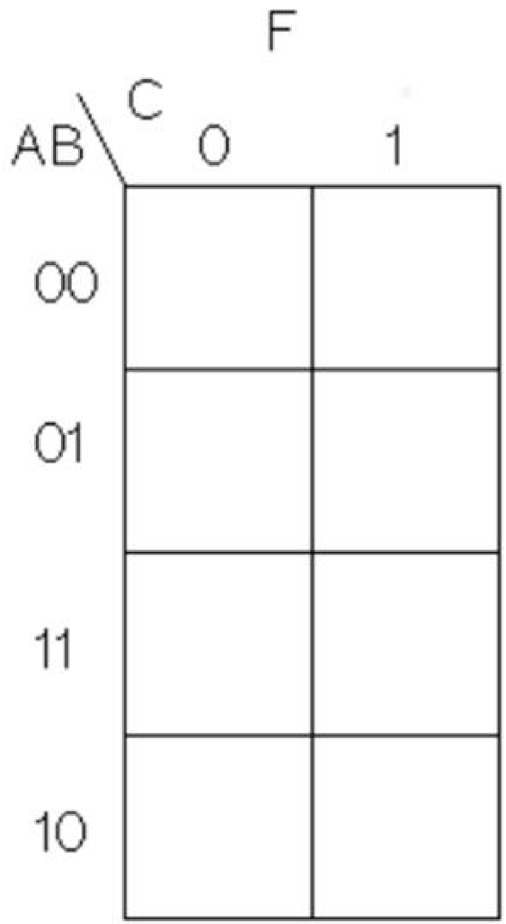
Essential Prime Implicants(EPI)

Karnaugh Maps (K-maps)

- A visual method.
- Good for up to 4 variables. Difficult for 5 or 6 variables. Very difficult for more than 6 variables.
- Start with a truth table for your function.
- A rectangular grid.
- Number of squares is equal to the number of lines in truth table.

K-Maps

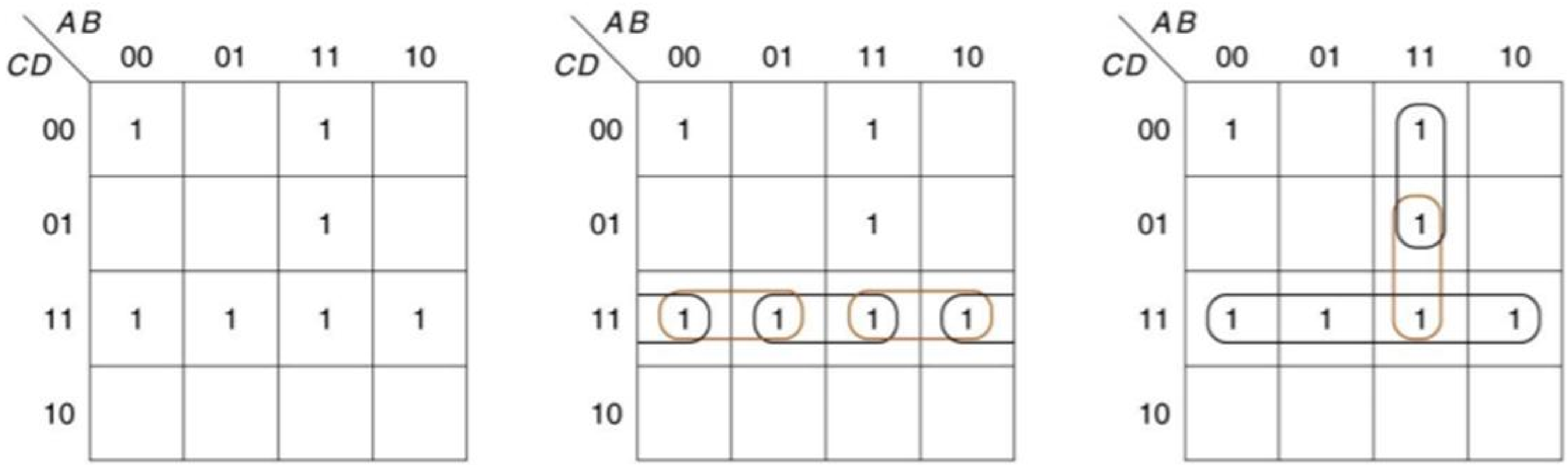
- 3 variable K-map
- Map the equation $\Sigma m(2,3,5,6)$
- Implicant – any rectangular group of 1's where rectangle is a power of 2 on each side.
- Prime Implicant – An implicant that can't "grow larger".



Implicant

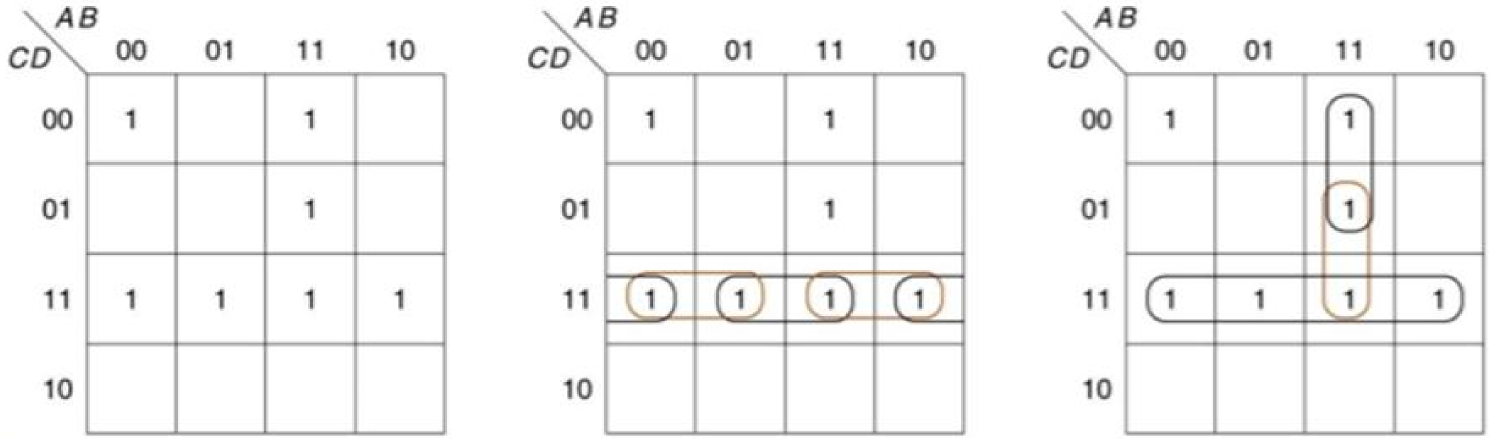
An implicant is a rectangle of 1, 2, 4, 8, . . . (any power of 2) 1's. That rectangle may not include any 0's.

Map 3.12 A function to illustrate definitions.



Implicant -- 2

Map 3.12 A function to illustrate definitions.



The implicants of *F* are

Minterms	*Groups of 2*	*Groups of 4*
$A'B'C'D'$	$A'CD$	CD
$A'B'CD$	BCD	
$A'BCD$	ACD	
$ABC'D'$	$B'CD$	
$ABC'D$	ABC'	
$ABCD$	ABD	
$AB'CD$		

Implicant

An implicant of a function is a product term that can be used in an SOP expression for that function

From the point of view of the map, an implicant is a rectangle of 1,2,4,8...2^n 1's. No 0's may be included

Implicant:

A product term is said to be an implicant of a function if the product term implies the function.

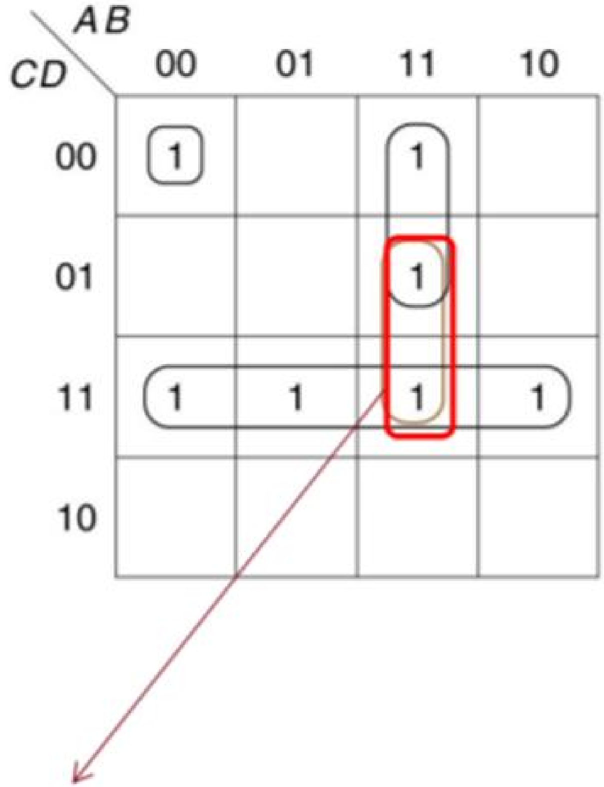
Prime Implicant

A prime implicant is an implicant that (from the point of view of the map) is not fully contained within any one other implicant

Prime Implicant

A *prime implicant* is an implicant that (from the point of view of the map) is not fully contained in any one other implicant.

An *essential prime implicant* is a prime implicant that includes at least one 1 that is not included in any other prime implicant.



prime implicant, but not essential prime implicant

Essential Prime Implicant

An essential prime implicant is a prime implicant that includes at least one 1 that is not in any other prime implicant.

K-Maps

- Essential Prime Implicant – A prime implicant that has one square that is not part of another prime implicant.
- Non-essential Prime Implicant – A prime implicant where every one of its squares is part of another prime implicant.

Example

#Implicants = 5

$\underbrace{3}_{\text{Size 1}} + \underbrace{2}_{\text{Size 2}}$

$x \backslash yz$	00	01	11	10
0	0	0	0	1
1	0	0	1	1

$y\bar{z}$
xy
} Both are prime implicants

Both are Essential.

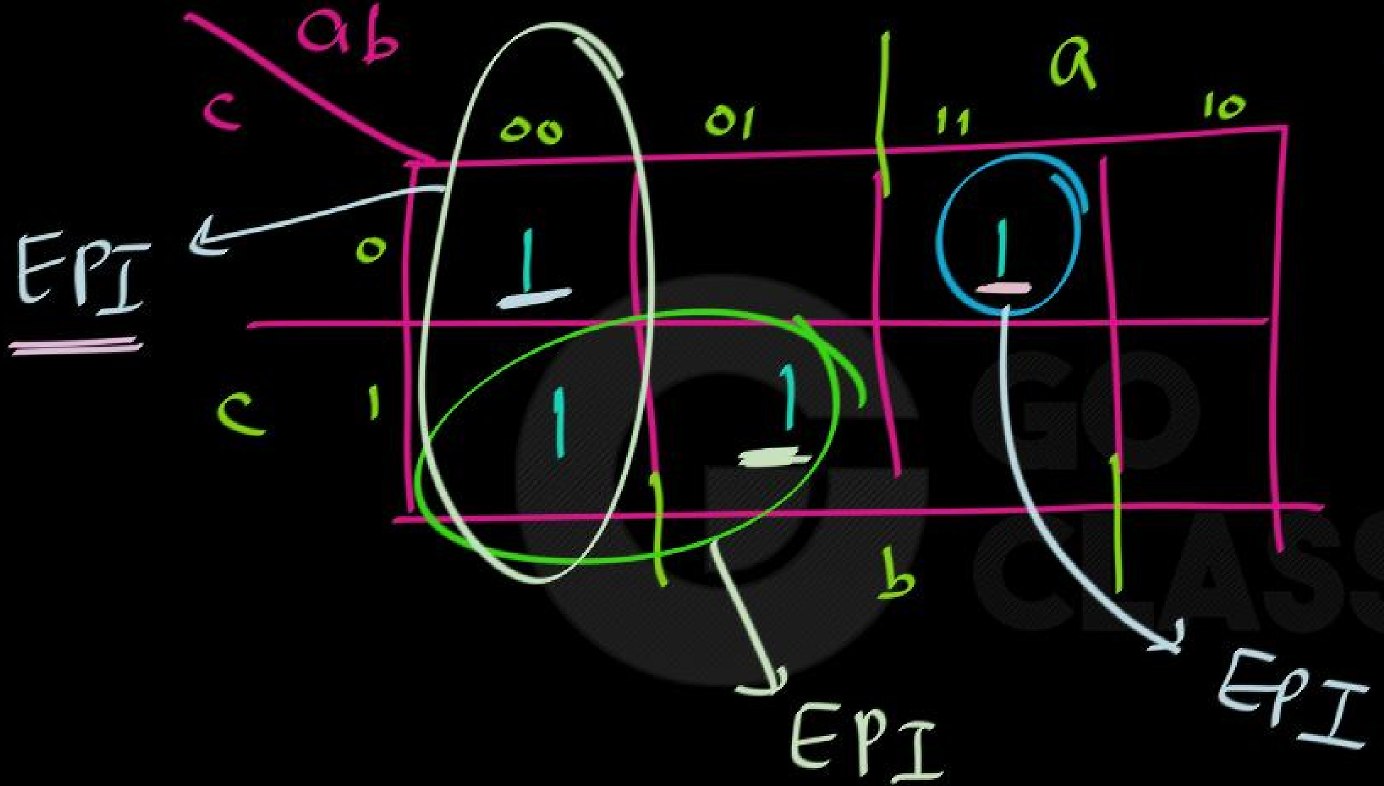
$f = y\bar{z} + xy$ → Unique mSop

How do we know this is minimal?

Essential Prime Implicants

- Some 1-cells appear in only one prime implicant subcube, others appear in more than one.

x \ yz	00	01	11	10
0	1	1	0	0
1	0	1	1	0



#Implicants = 6

$\underbrace{4}_{\text{size1}} + \underbrace{2}_{\text{size2}}$

#PI : 3

#EPI = 3

mSOP: $\bar{a}\bar{b} + c\bar{a} + ab\bar{c}$ → unique mSOP

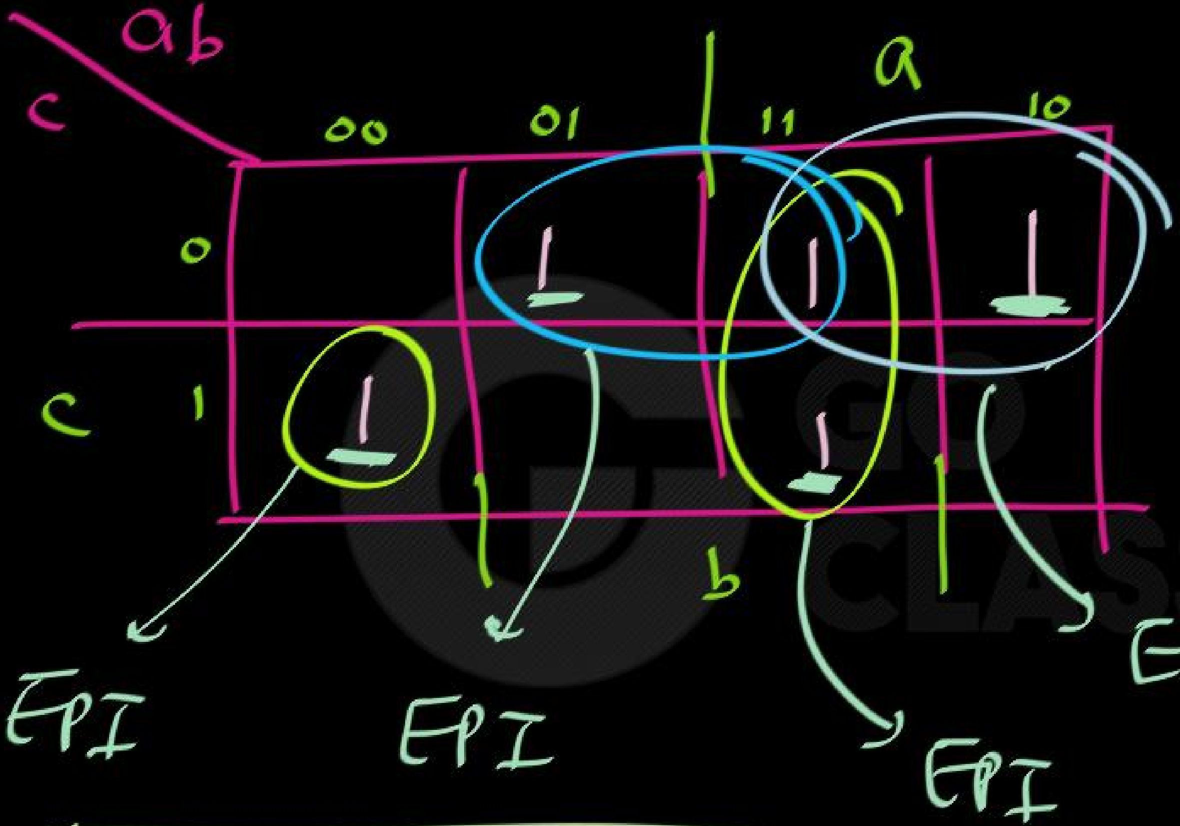
In mSOP;

We have to take "all Essential PIs"

AND if EPIs are Covering all 1's then we get Unique mSOP.

EPI : $\boxed{\text{Essential PI}}$

A PI which Covers at least one "1-cell" uniquely (No other PI Covers it)



$$\#\text{Implicants} = 8$$

$$\underbrace{5}_{\text{size1}} + \underbrace{3}_{\text{size2}}$$

$$\#PI = 4$$

$$\#EPI = 4$$

$\boxed{\text{unique mSOP}}$

$c \backslash ab$	00	01	11	10
0	1	1	1	1
1		1	1	1

a

b

c

#Implicants ?

$\underbrace{7}_{\text{Size 1}} + \underbrace{2+2+1+2}_{\text{Size 2}} \; +1 \; +1$

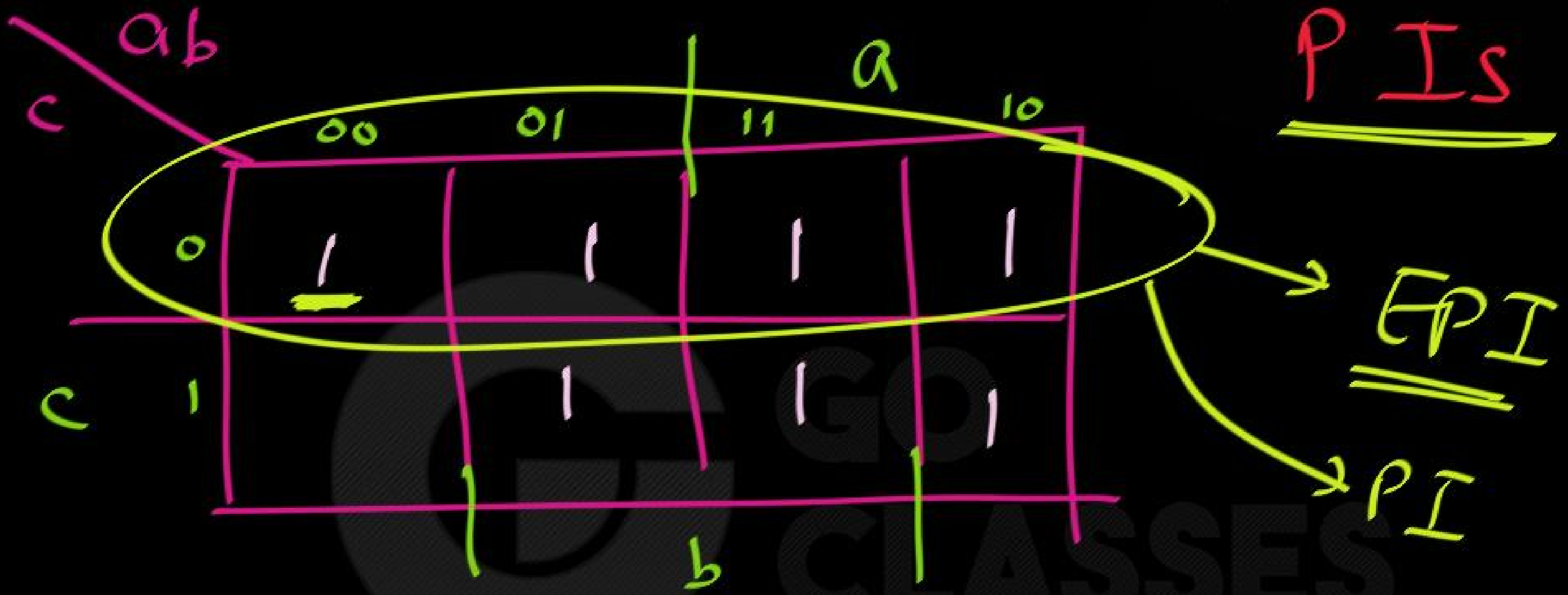
$\underbrace{1+1+1}_{\text{Size 4}}$

$\underbrace{0}_{\text{Size 8}}$

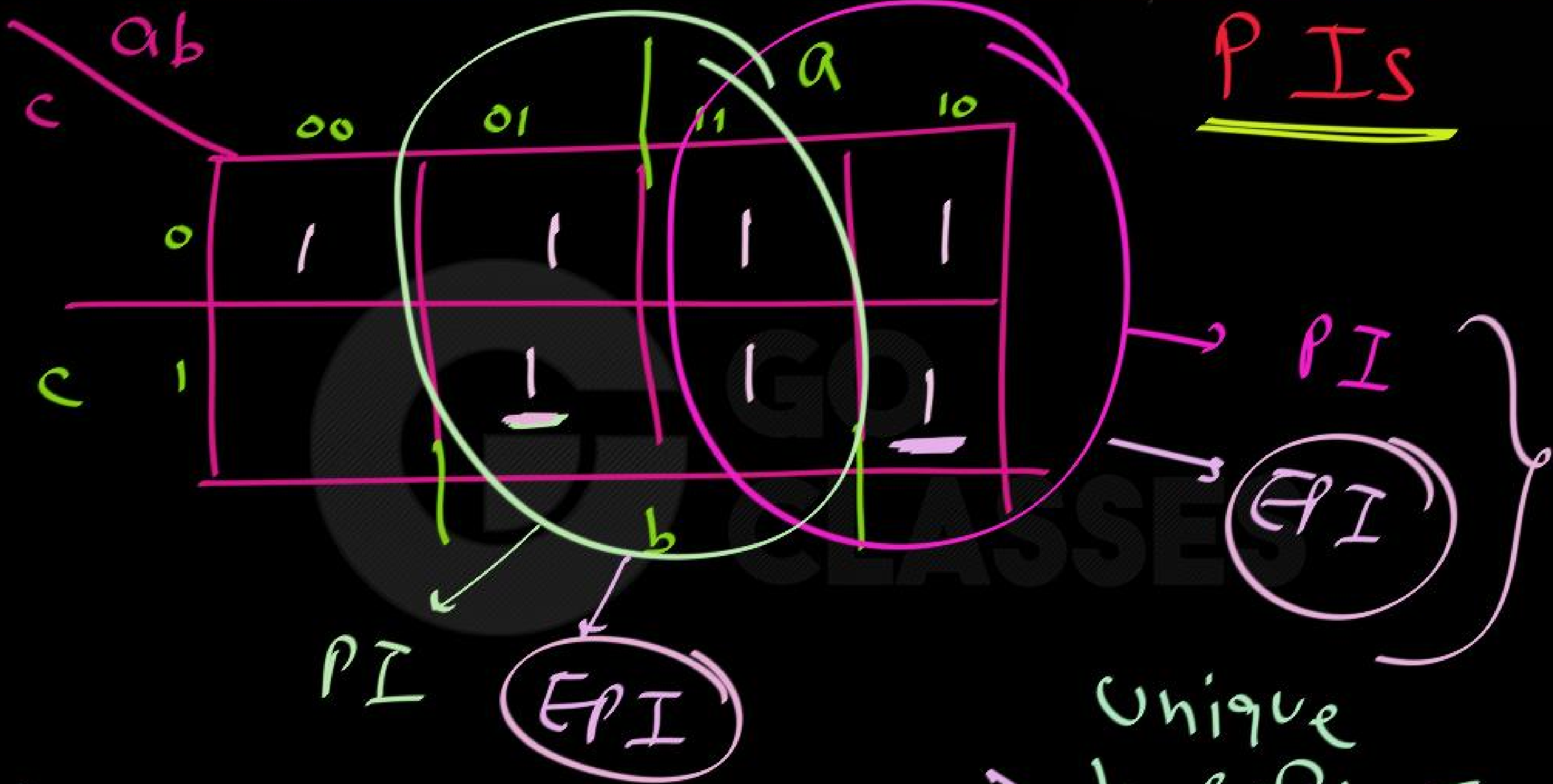
#Implicants = 19

$\bar{a}b$

abc'


ab
c
a
00
01
11
10
0
1
c
b
P Is
EPI
PI

PIs

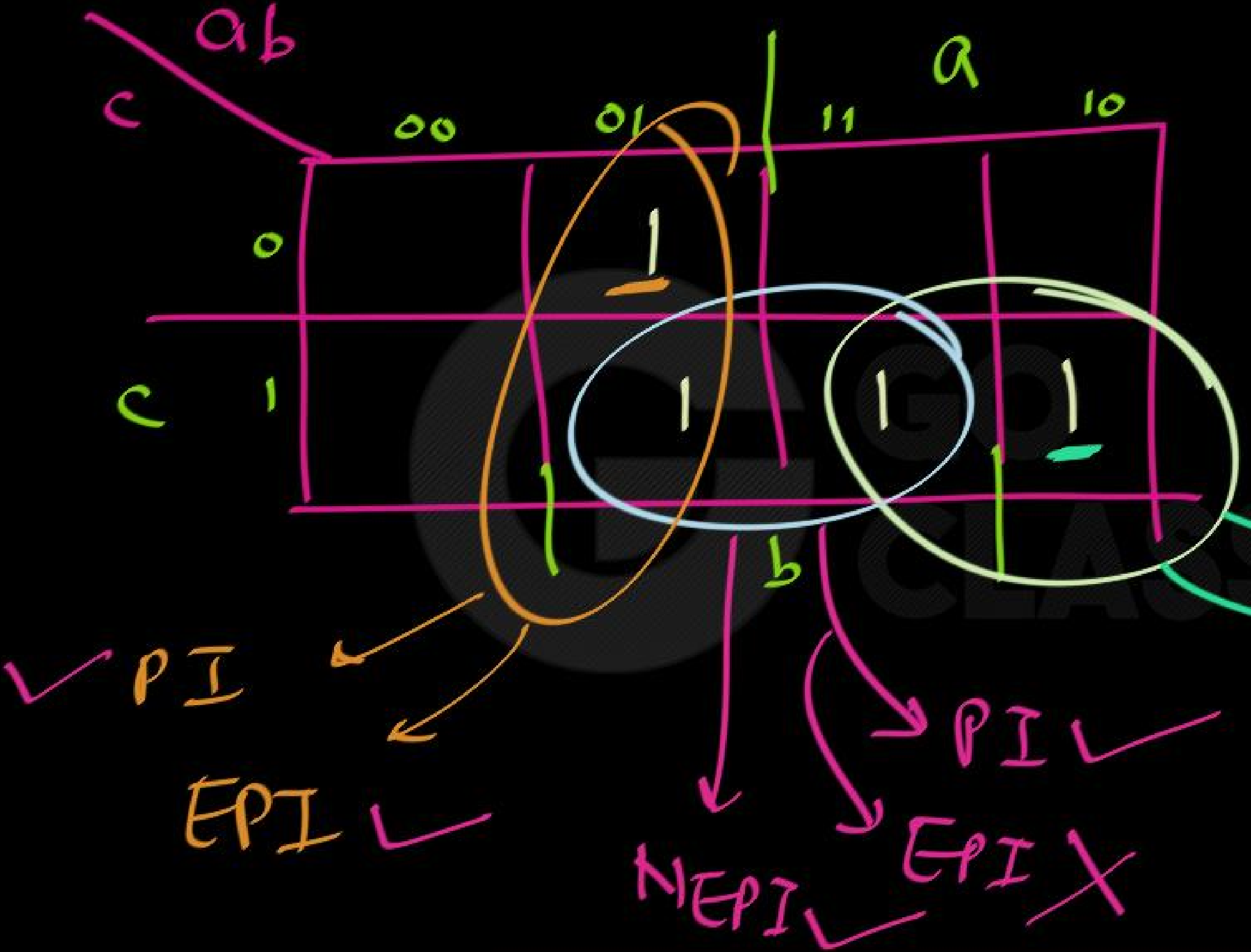


#PI = 3; #EPI = 3 → Unique MSOP

NEPI : Non Essential PI

Not Covering any "1-cell" uniquely.

Those PI which are NOT EPI.

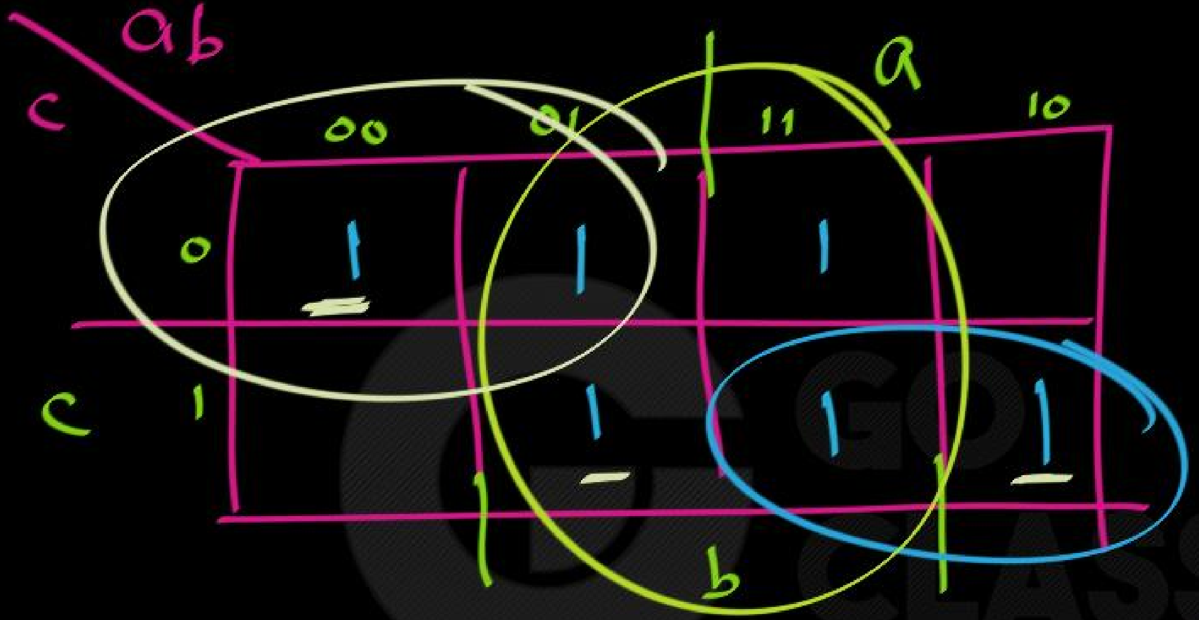


#Implicants = 4 + 3 = 7

#PI = 3 ;

#EPI = 2

#NEPI = 1



PI: $\underbrace{1}_{\text{Size } 4} + 2$

EPI = 3 ✓

Unique msop

Q: If all EPIs Covering Everything then Unique min SOP.

Is the Converse True? No

c \ ab	00	01	11	10
0	1		1	1
c 1	1		1	1

#PI = 2

#EPI = 2

Unique mSOP.

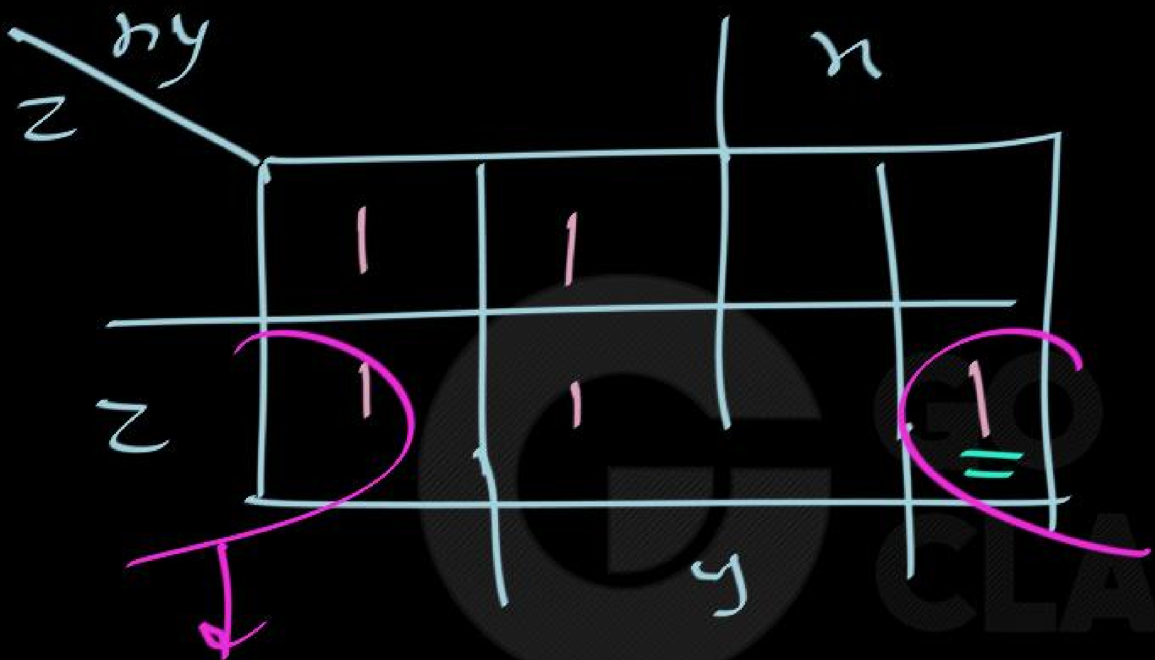
Example

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$\overline{x}\,\overline{y}z$ is also an implicant

$x\overline{y}z$ is an implicant. Is it a prime implicant?

$\overline{y}z$ is an implicant.
Is it a prime implicant?
Yes. $\overline{y}, z$ are not implicants



$\bar{y}z$ = Implicant

EPI

PI

$\bar{x}\bar{y}z$ = Implicant

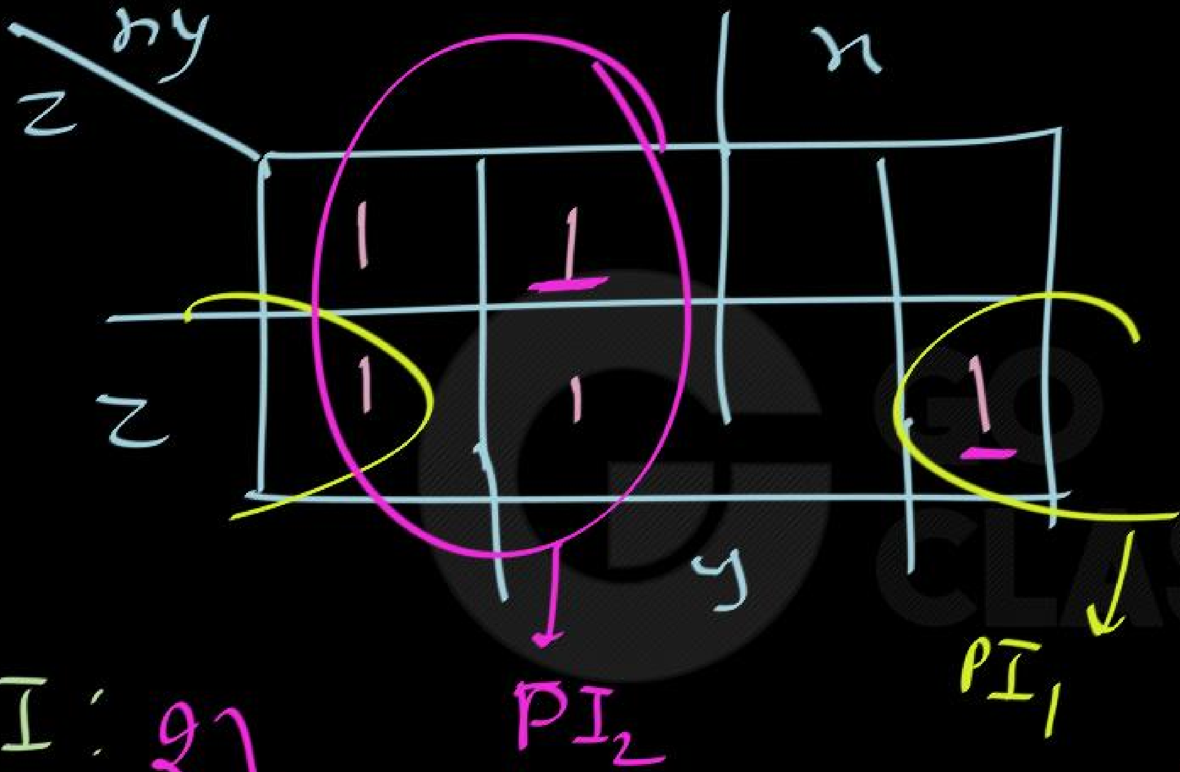
$x\bar{y}z$ = "

$xy\bar{z}$ = NOT Implicant

$\bar{x}$ = Implicant

$\bar{x}y$ = Implicant

$x\bar{z}$ = NOT Implicant



$\bar{x}\bar{y}z$ = NOT PI

$x\bar{y}z$ = " "

$xy\bar{z}$ = " "

$\bar{x}$ = PI

$\bar{x}y$ = NOT PI

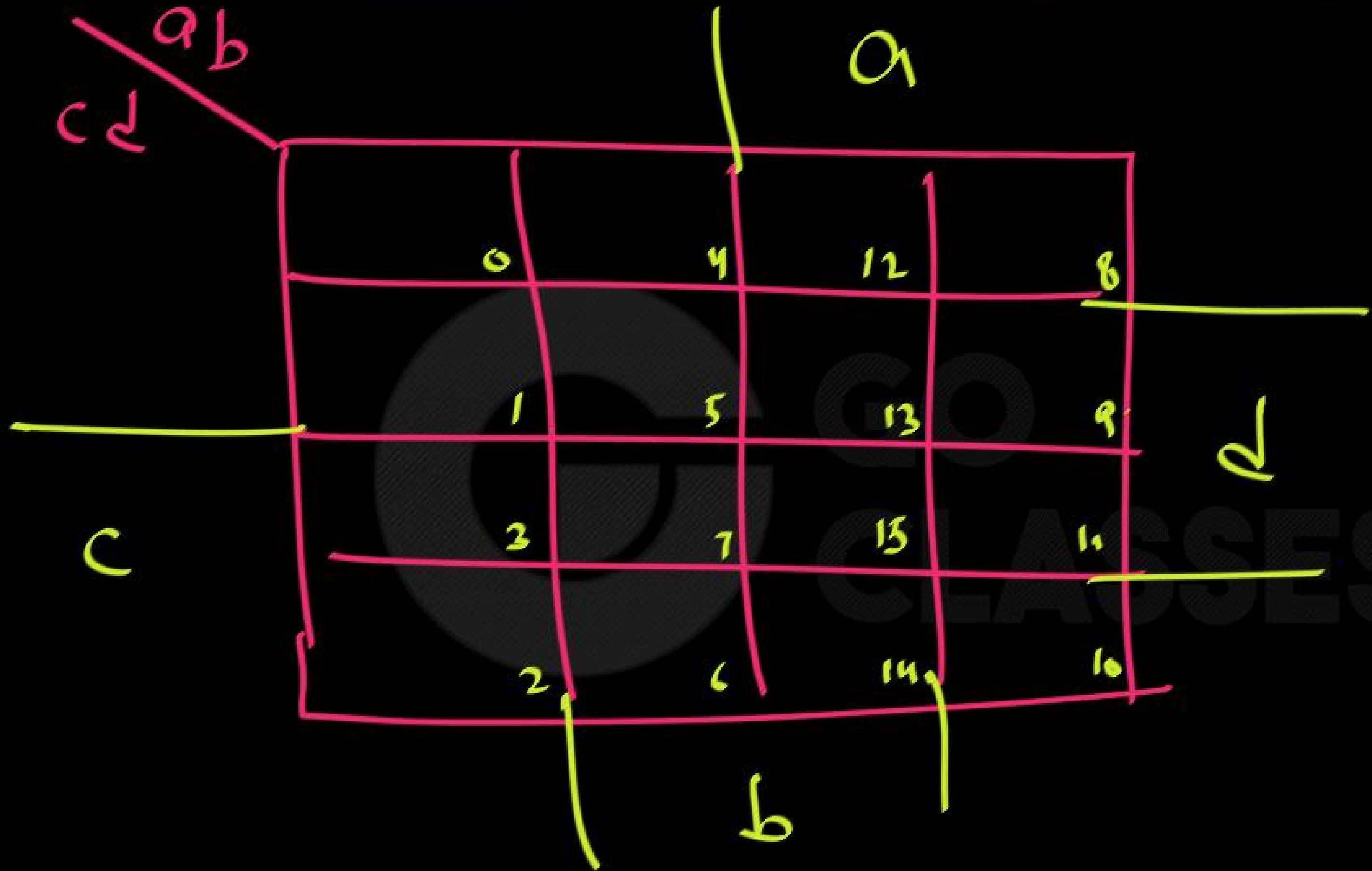
$x\bar{z}$ = NOT PI

PI : 2

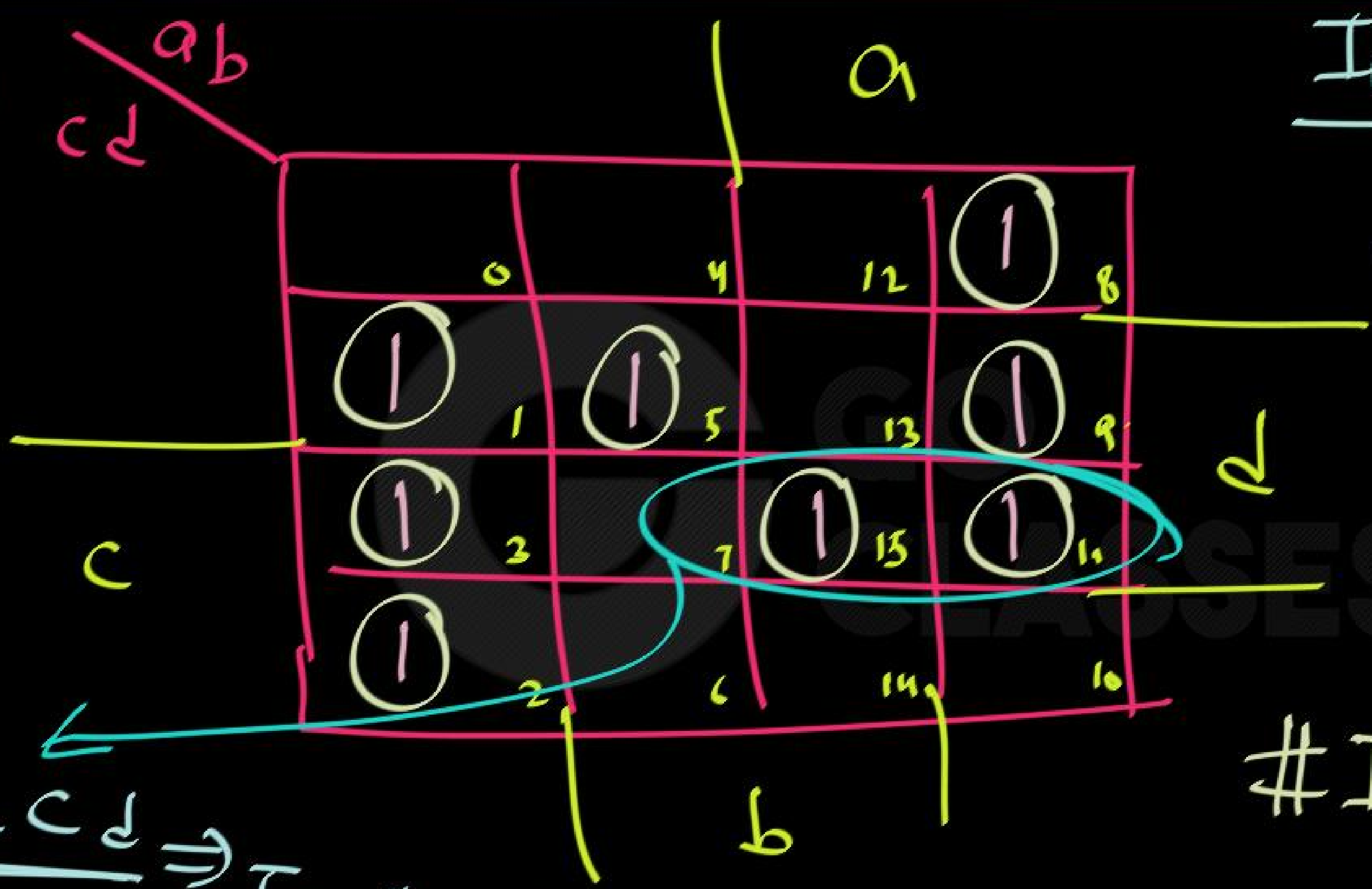
EPI = 2

K-Maps

- Determine essential and non-essential prime implicants: $f(a,b,c,d)$
  - $\Sigma m(1,2,3,5,8,9,11,15)$
  - $\Sigma m(5,7,9,13,14,15)$
  - $\Sigma m(0,1,3,4,6,9,11,14,15)$
  - $\Sigma m(1,5,13,14,15)$
  - $\Sigma m(2,3,5,7,8,10,12,13)$


ab
cd
a
0
4
12
8
1
5
13
9
d
c
3
7
15
11
2
6
14
10
b

1



Implicants =

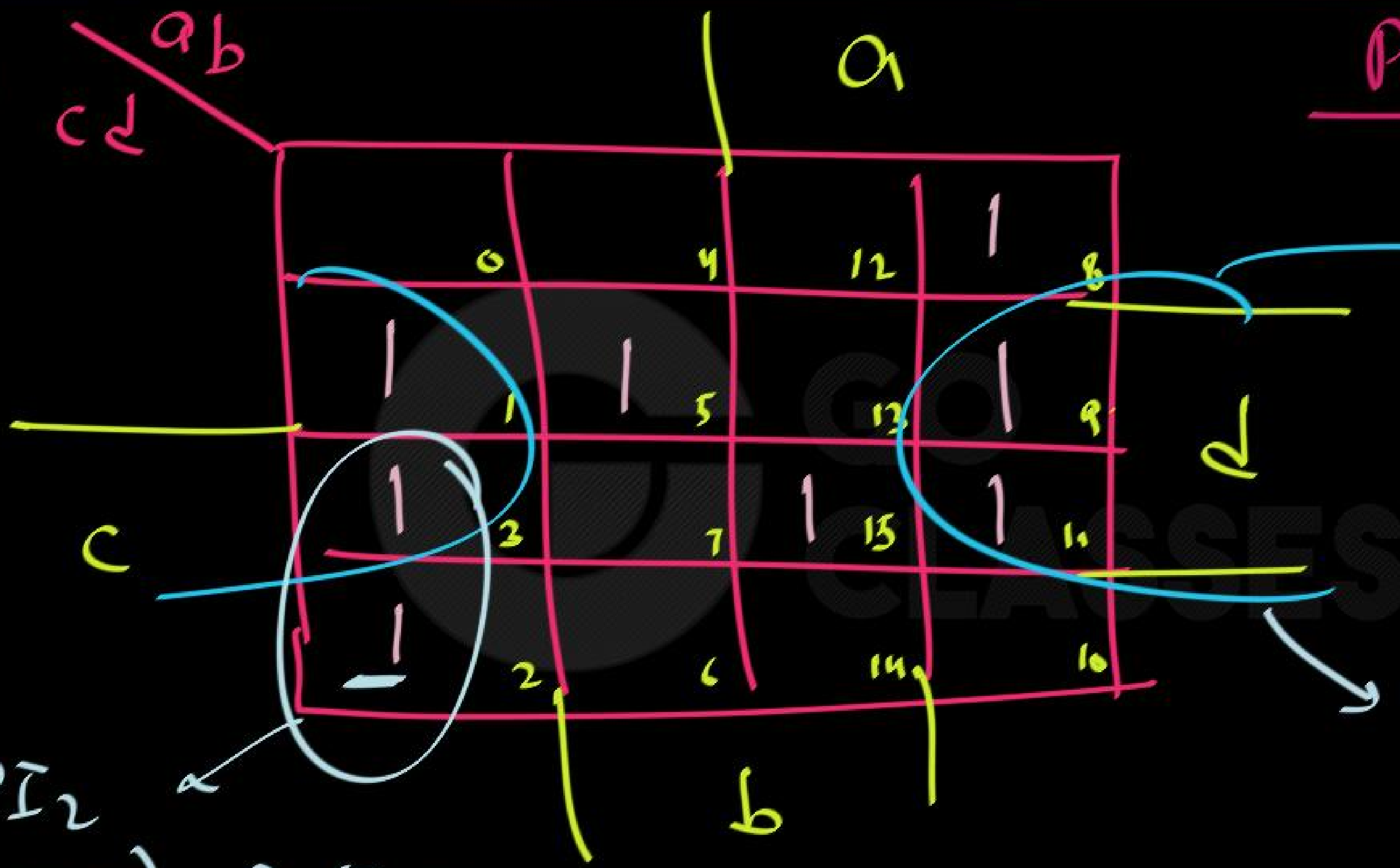
$\underbrace{8}_{\text{size 1}} + \underbrace{1+2+2+2+1}_{\text{size 2}} + \underbrace{1}_{\text{size 4}}$

Implicants = 17

$\underline{acd} \Rightarrow$ Implicant

1

PI :

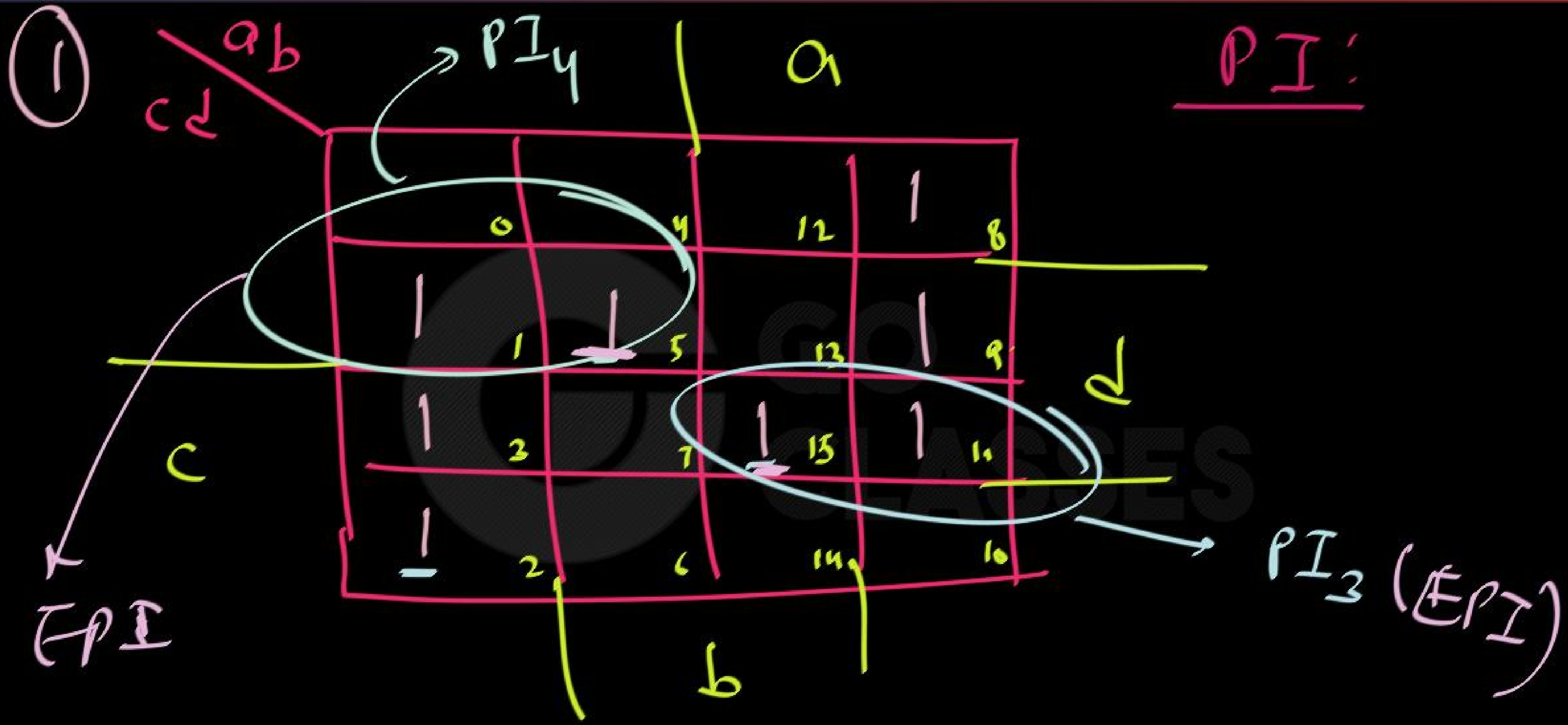


PI_1 $(d\bar{b})$

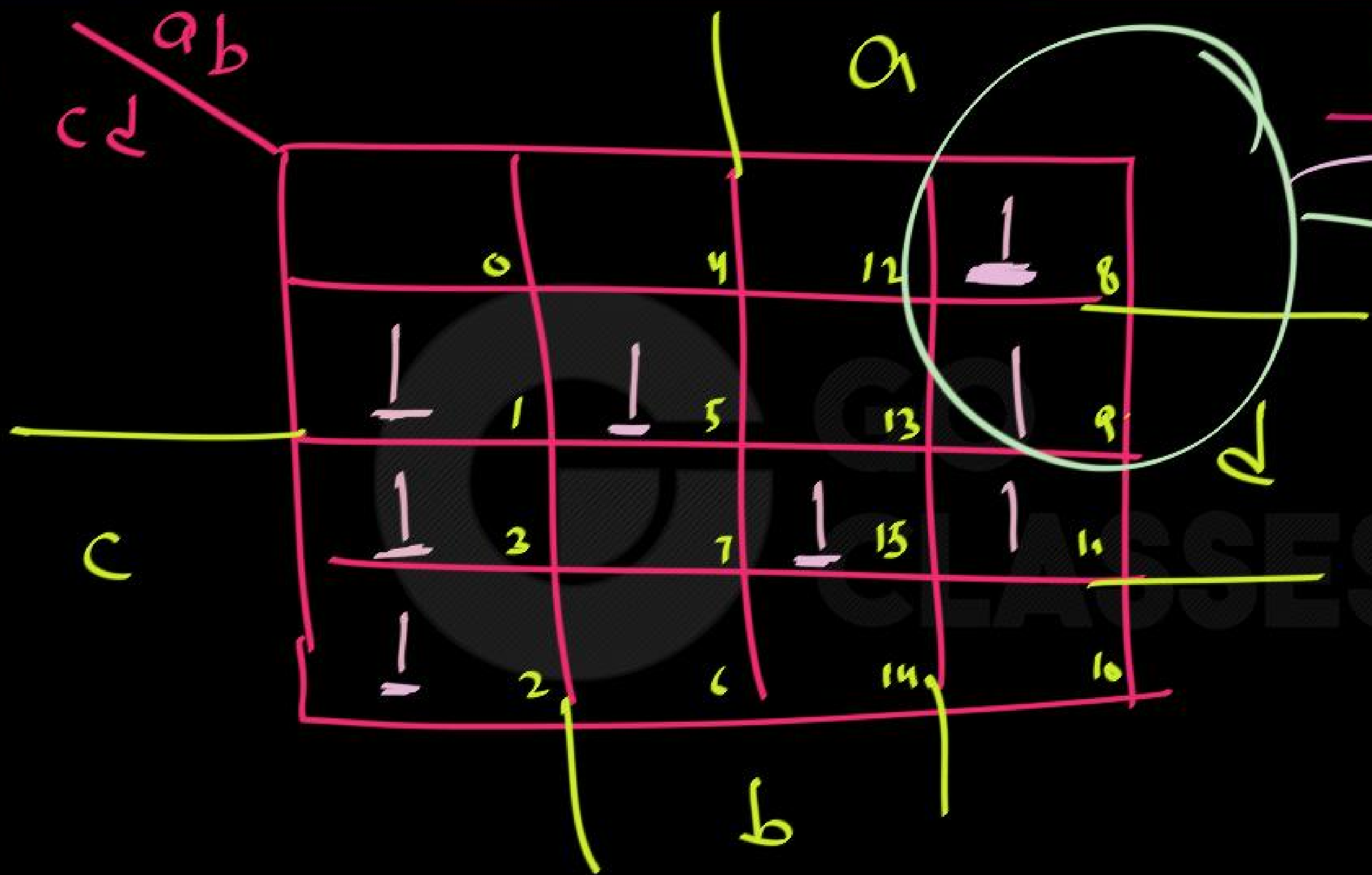
NOT EPI

PI_2

$(c\bar{a}\bar{b}) \Rightarrow EPI$


1
ab
cd
PI_4
a
PI :
0
4
12
8
1
1
1
1
1
5
13
9
d
1
1
1
1
c
3
7
15
11
1
2
6
14
10
PI_3 (EPI)
EPI
b

(1)



PI:

EPI

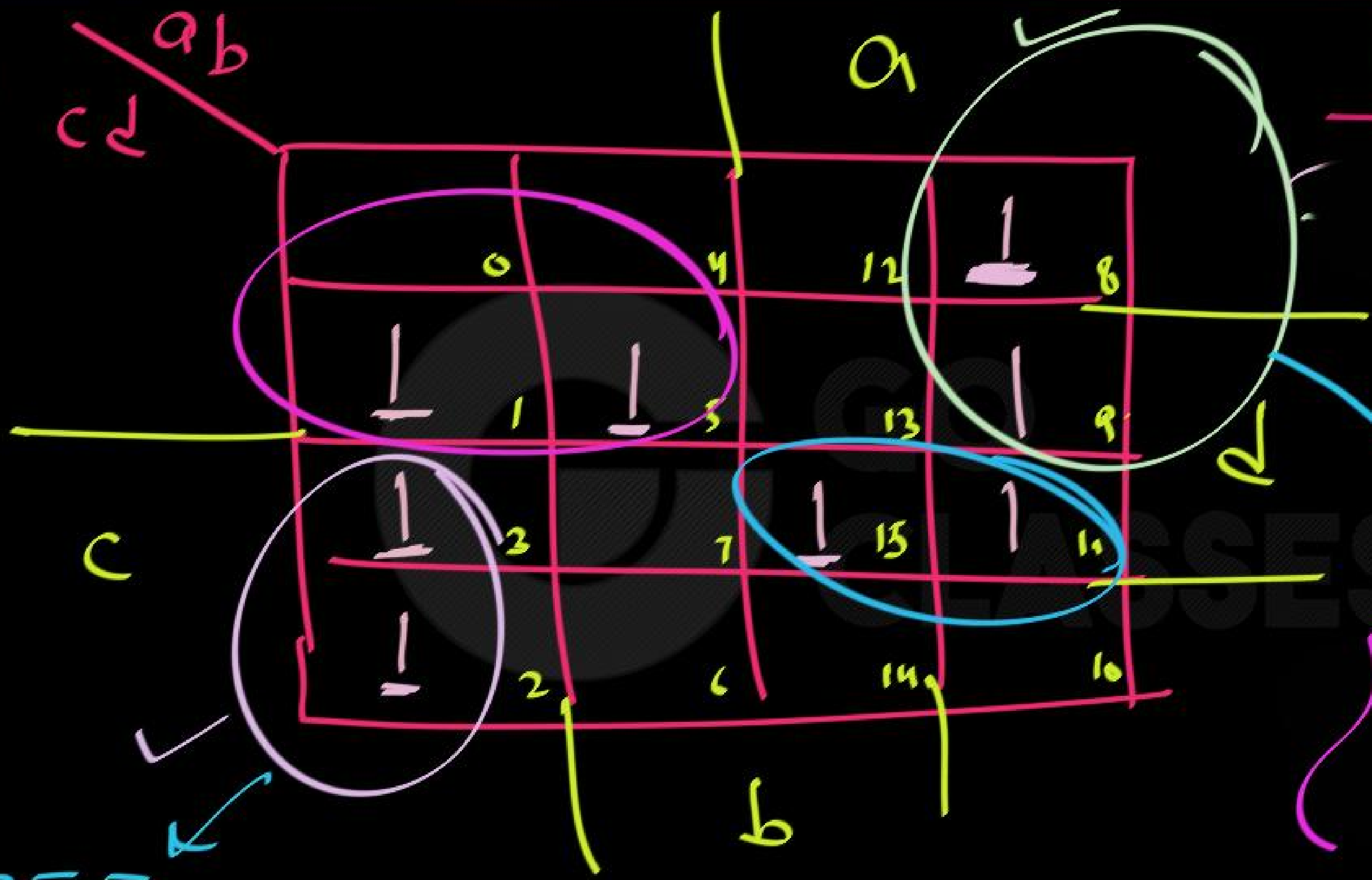
PI

5

#PI = 5

#EPI = 4

1



$c\bar{a}\bar{b}$

PI:

unique

msop:

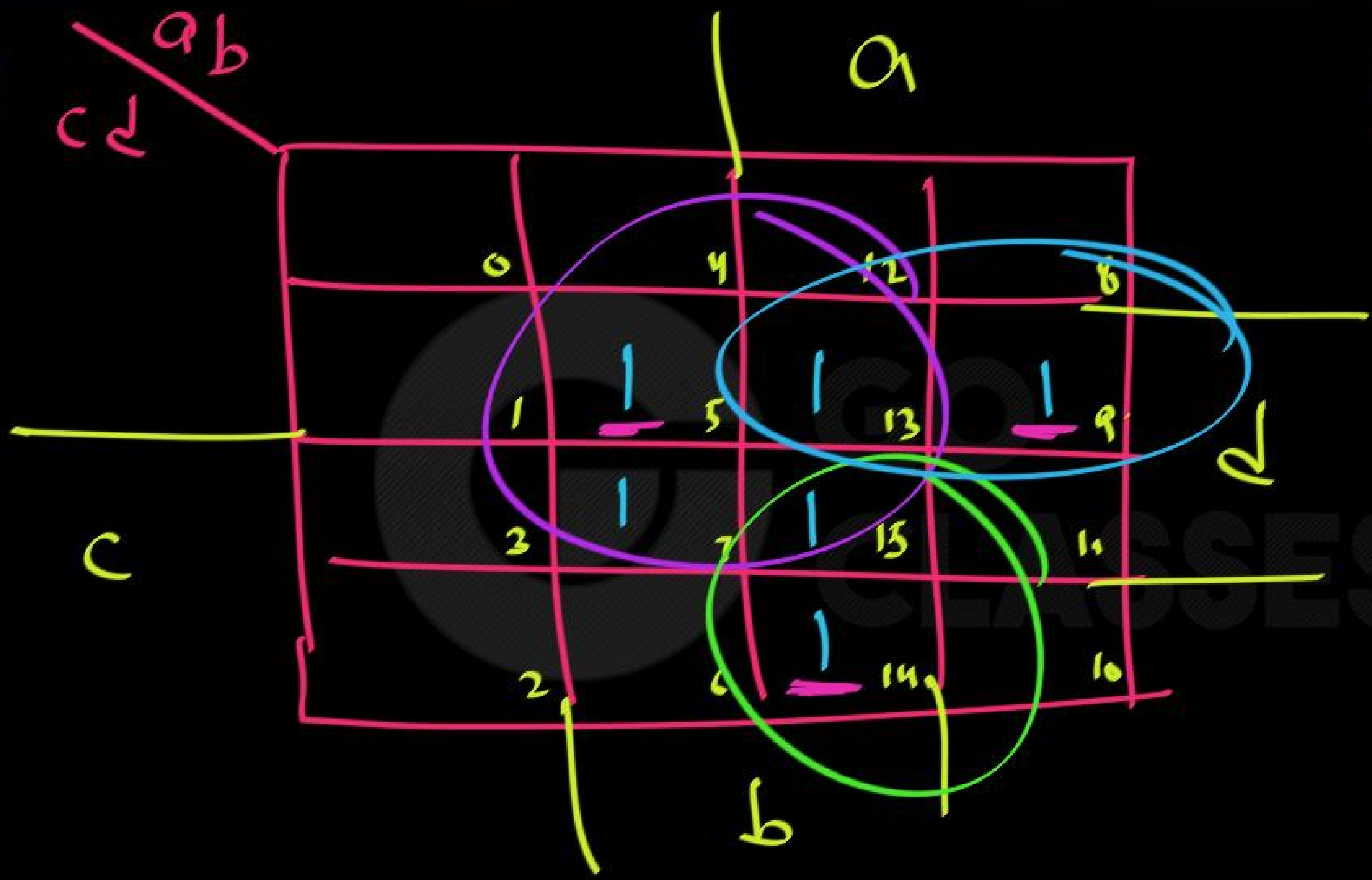
$c\bar{a}\bar{b}$ +

$a\bar{c}\bar{b}$ +

dac +

$\bar{a}d\bar{c}$

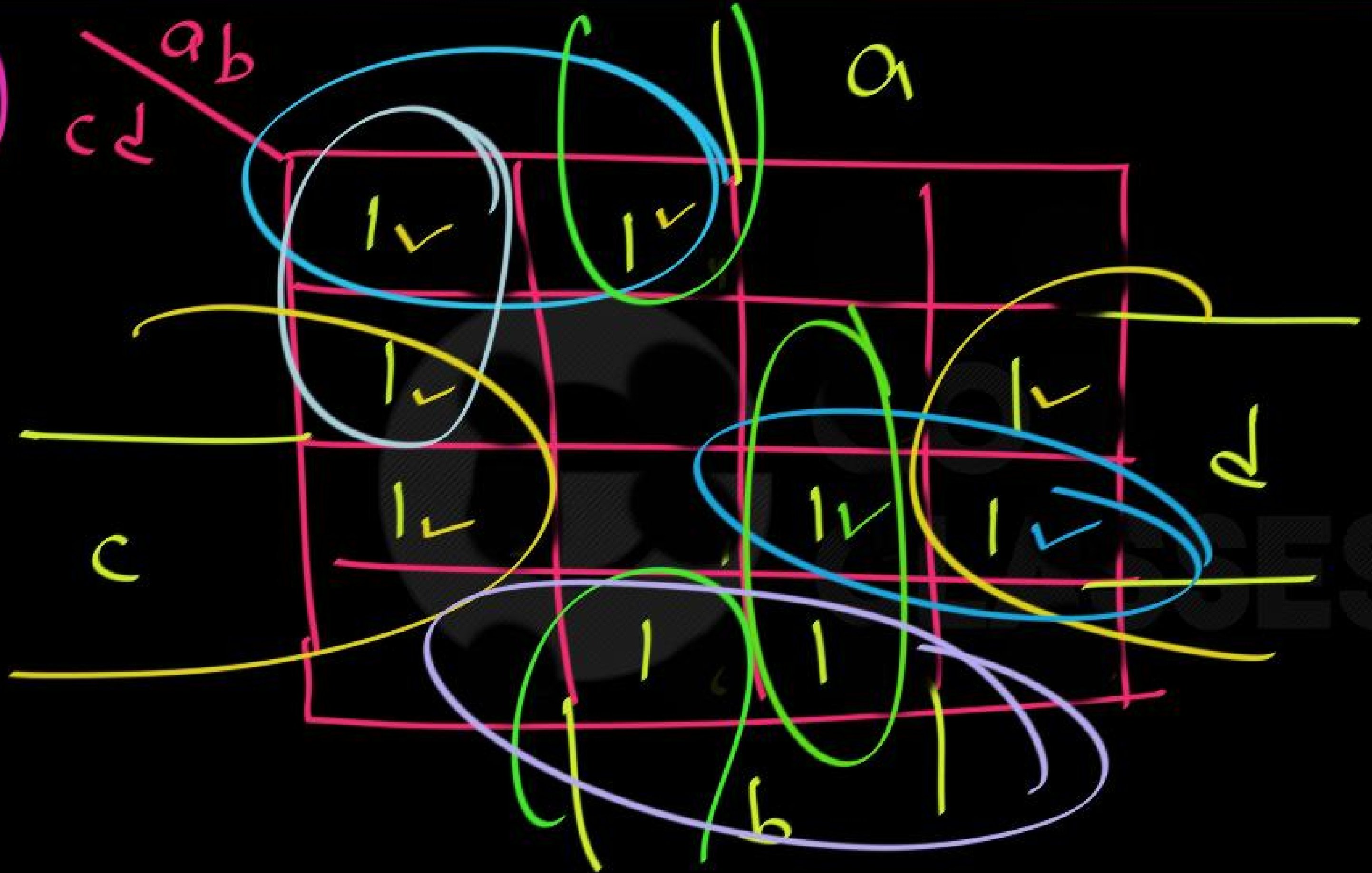
(2)



PI : 3

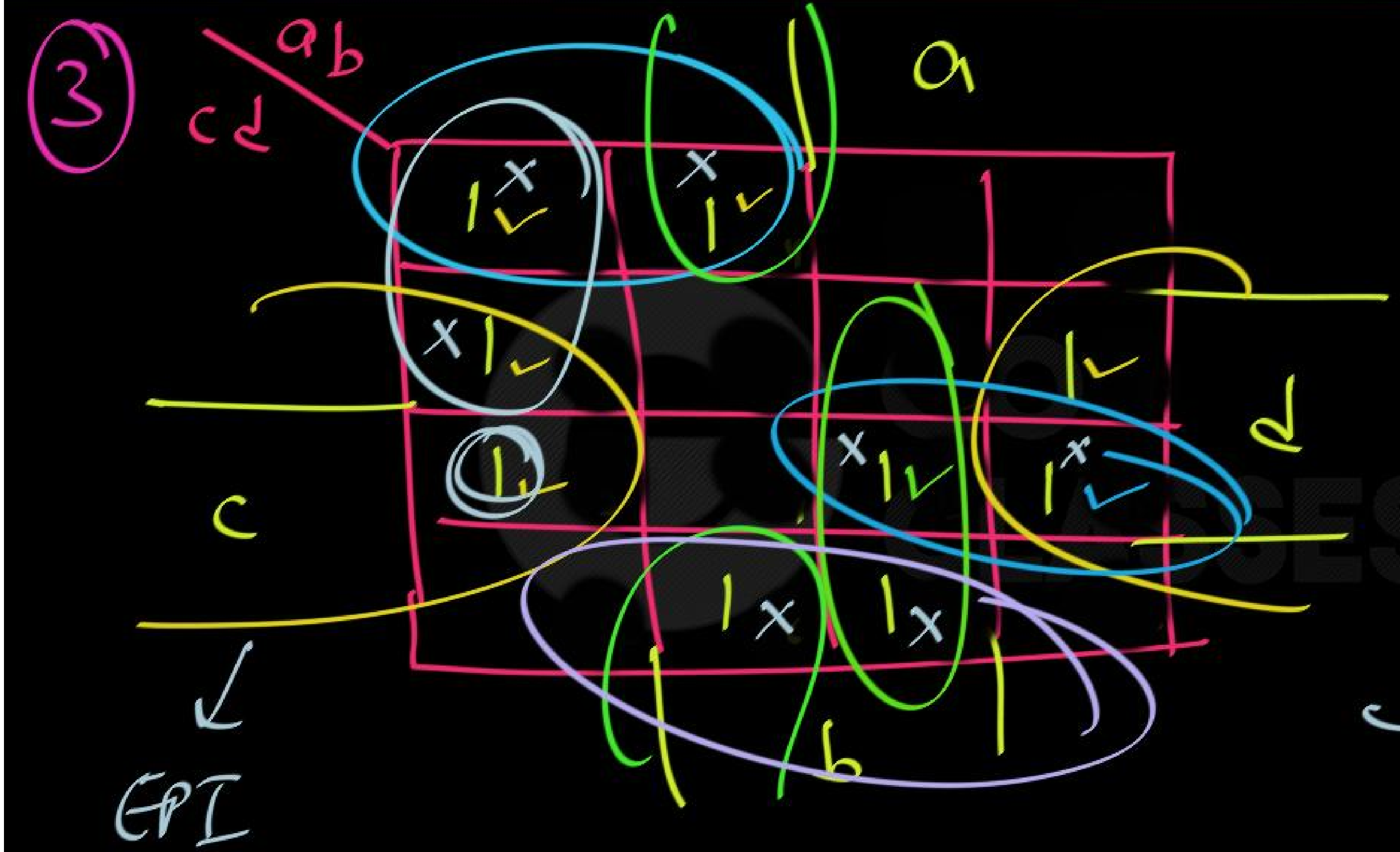
EPI = 3

3



PI

$2 + 1 + 1 + 1 + 1 + 1 + 1 = 7$



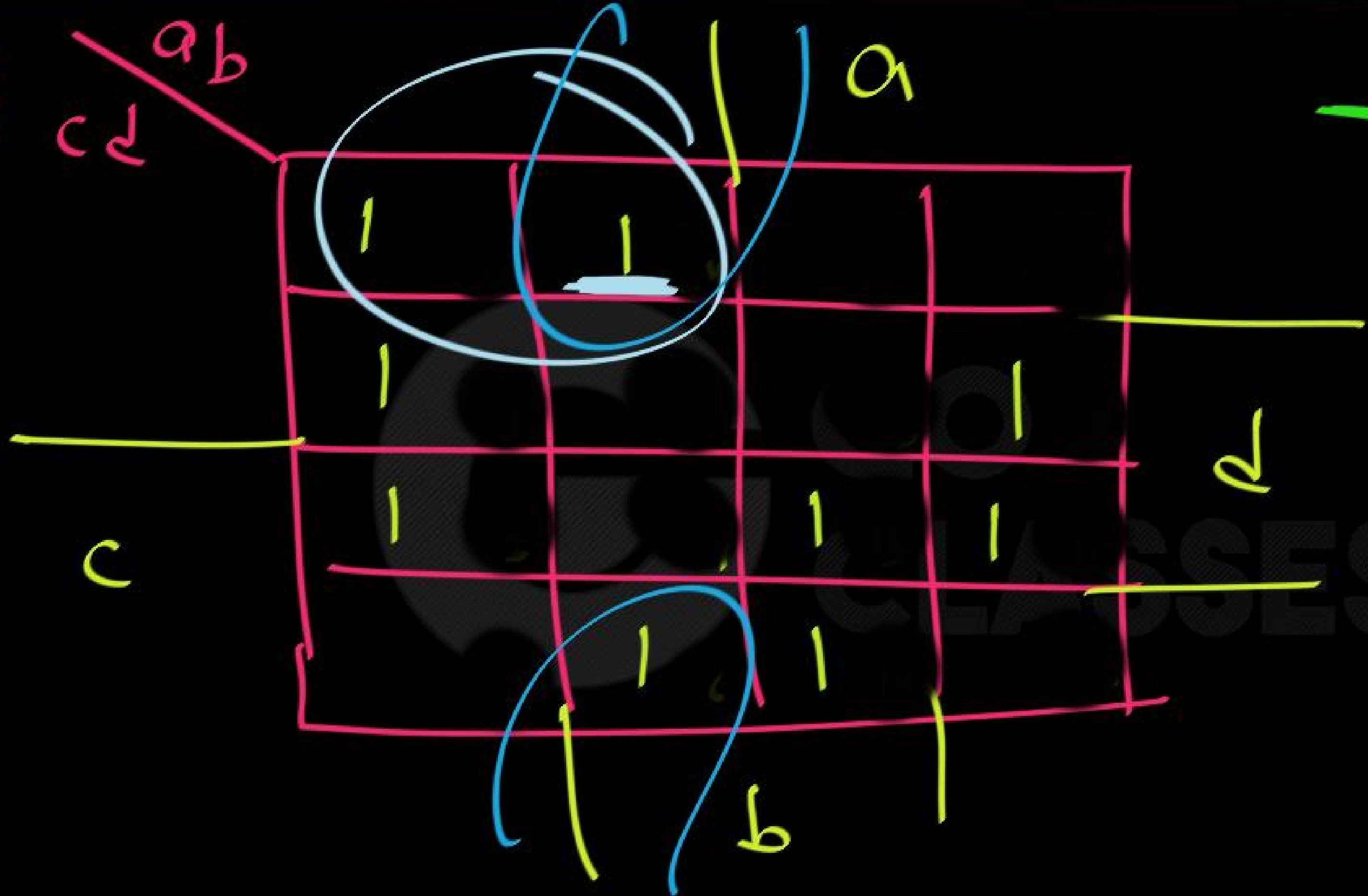
PI

$2 + 1 + 1 + 1 + 1 + 1 + 1$

$= 7$

$EPI = 1$

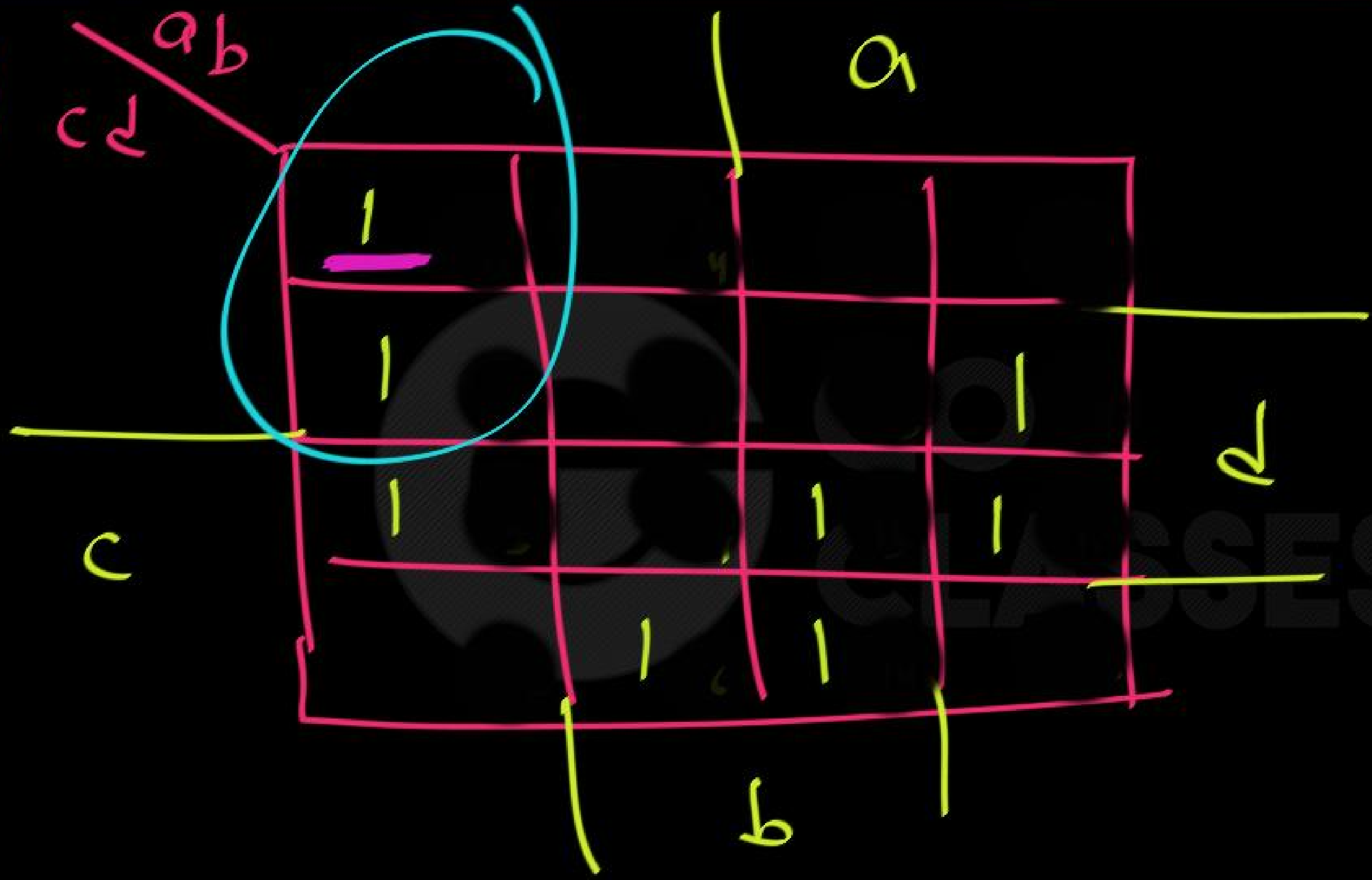
3



PI

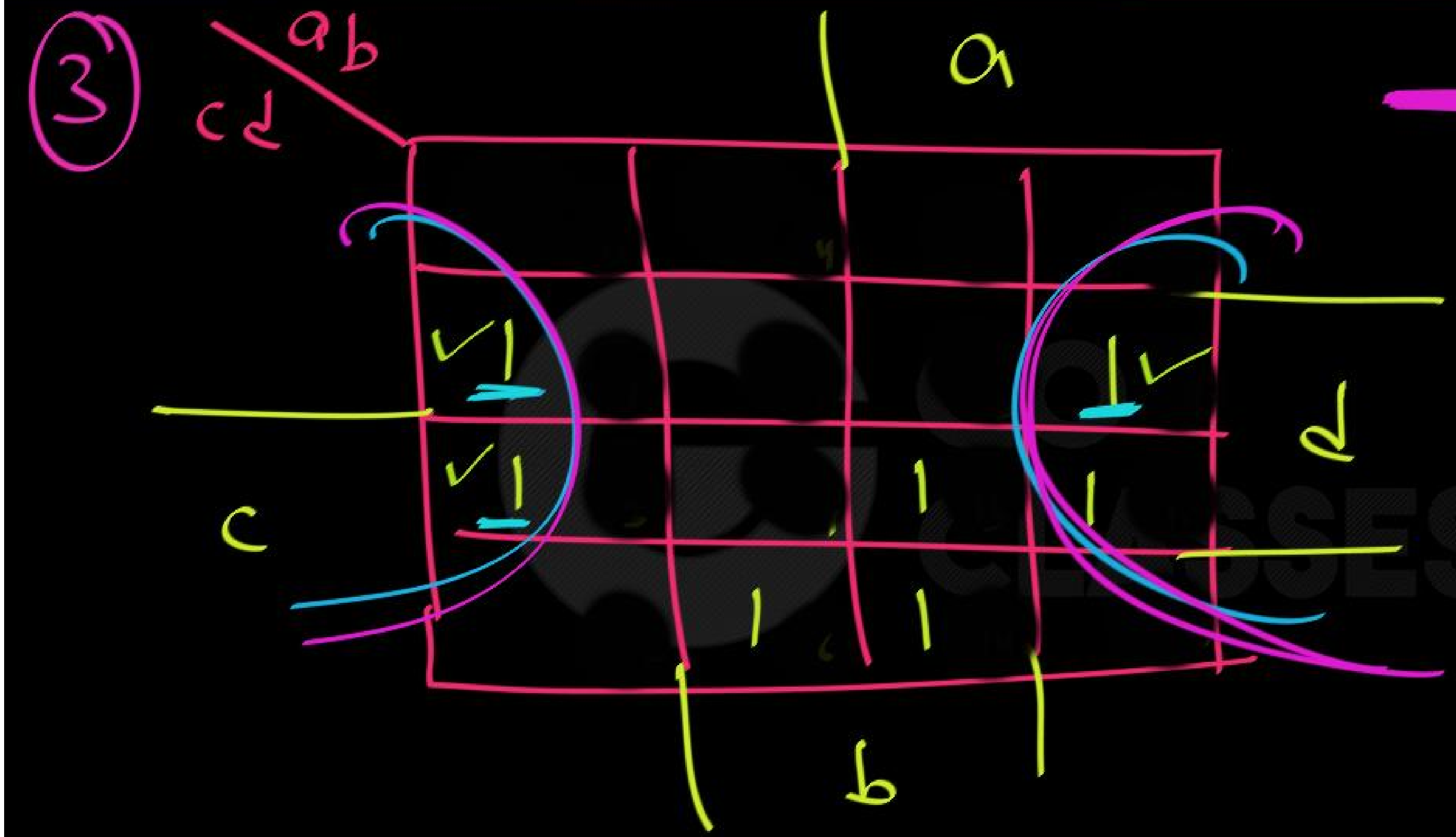
2

3



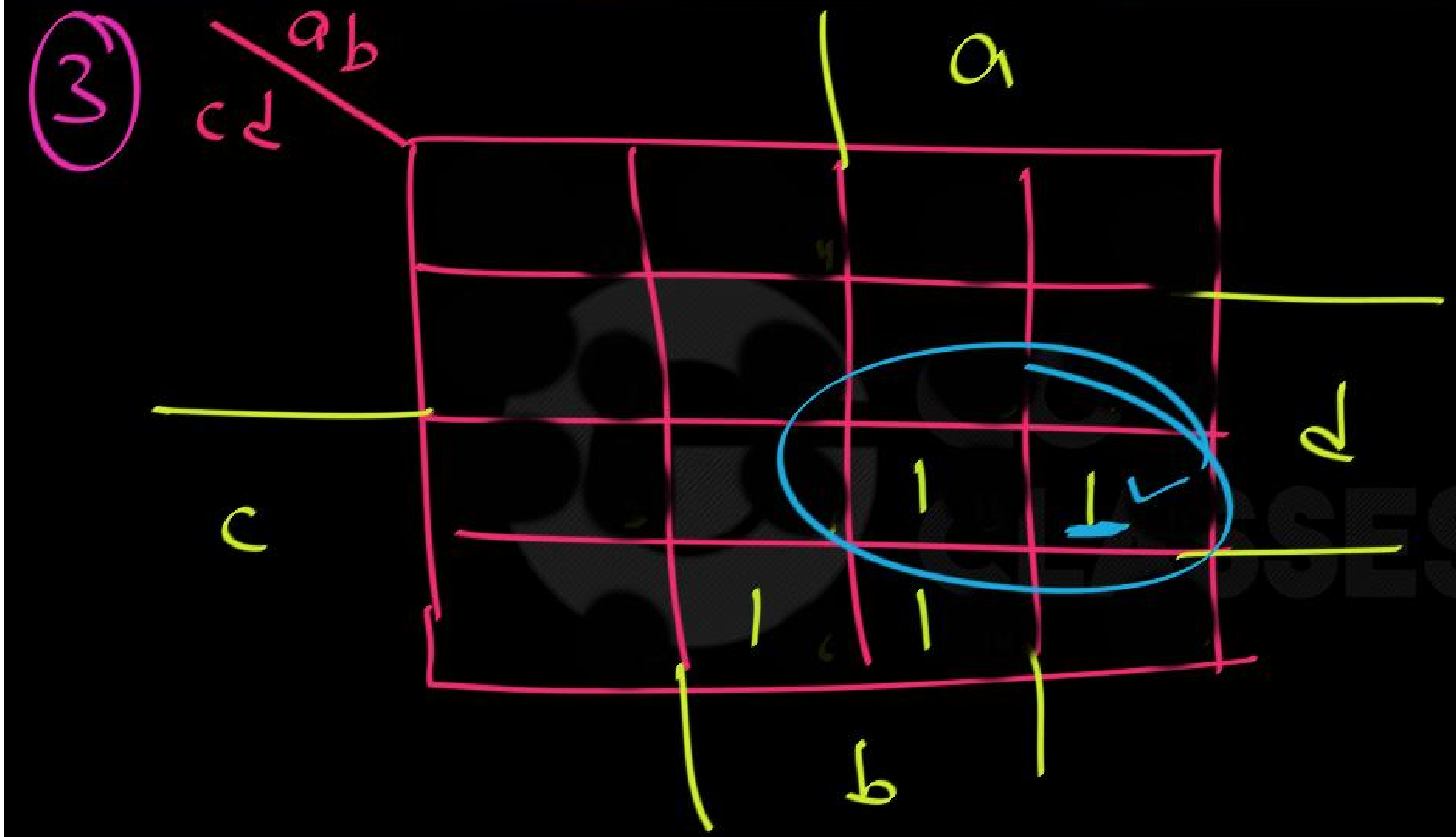
PI

1



PI

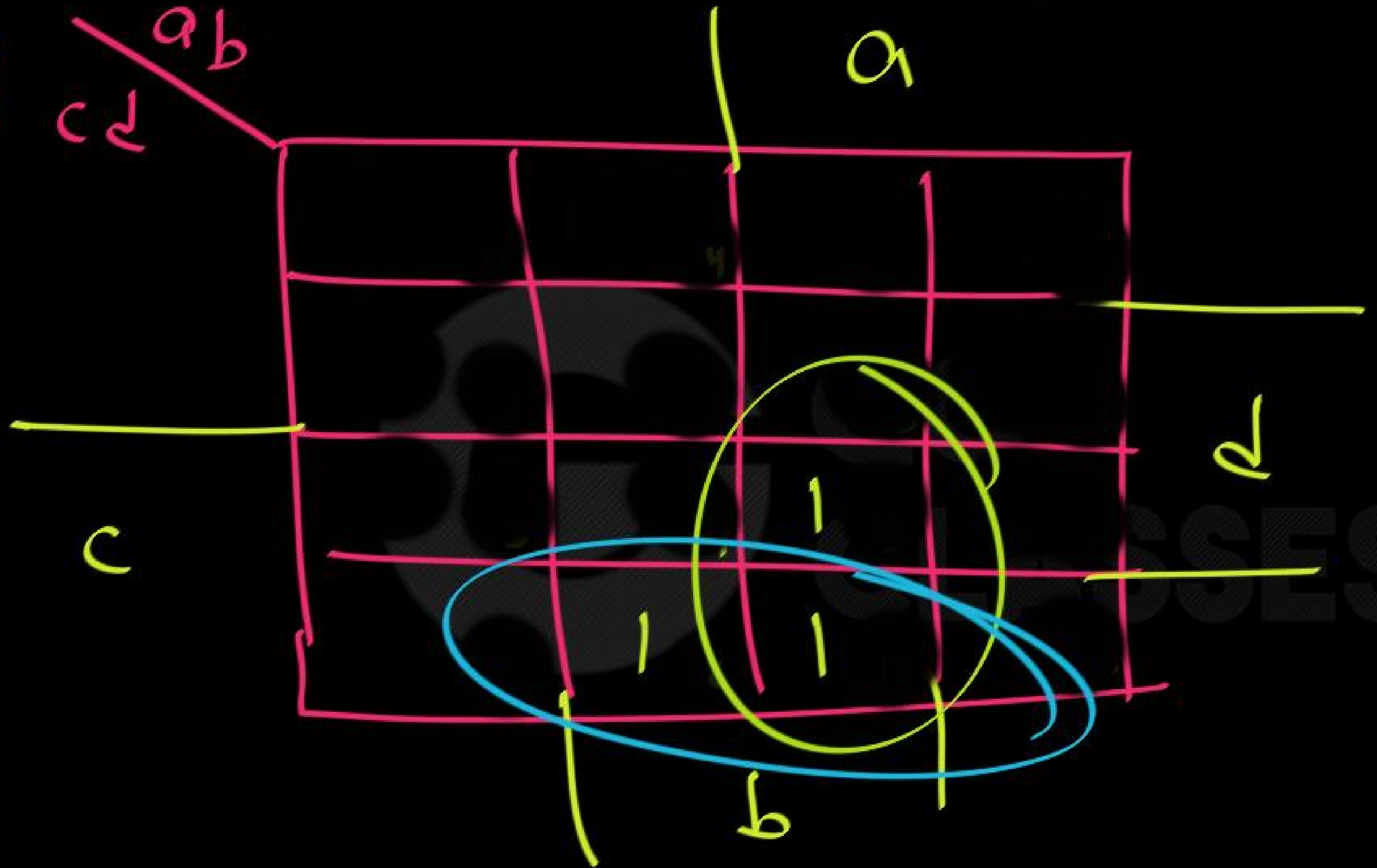
1



PI

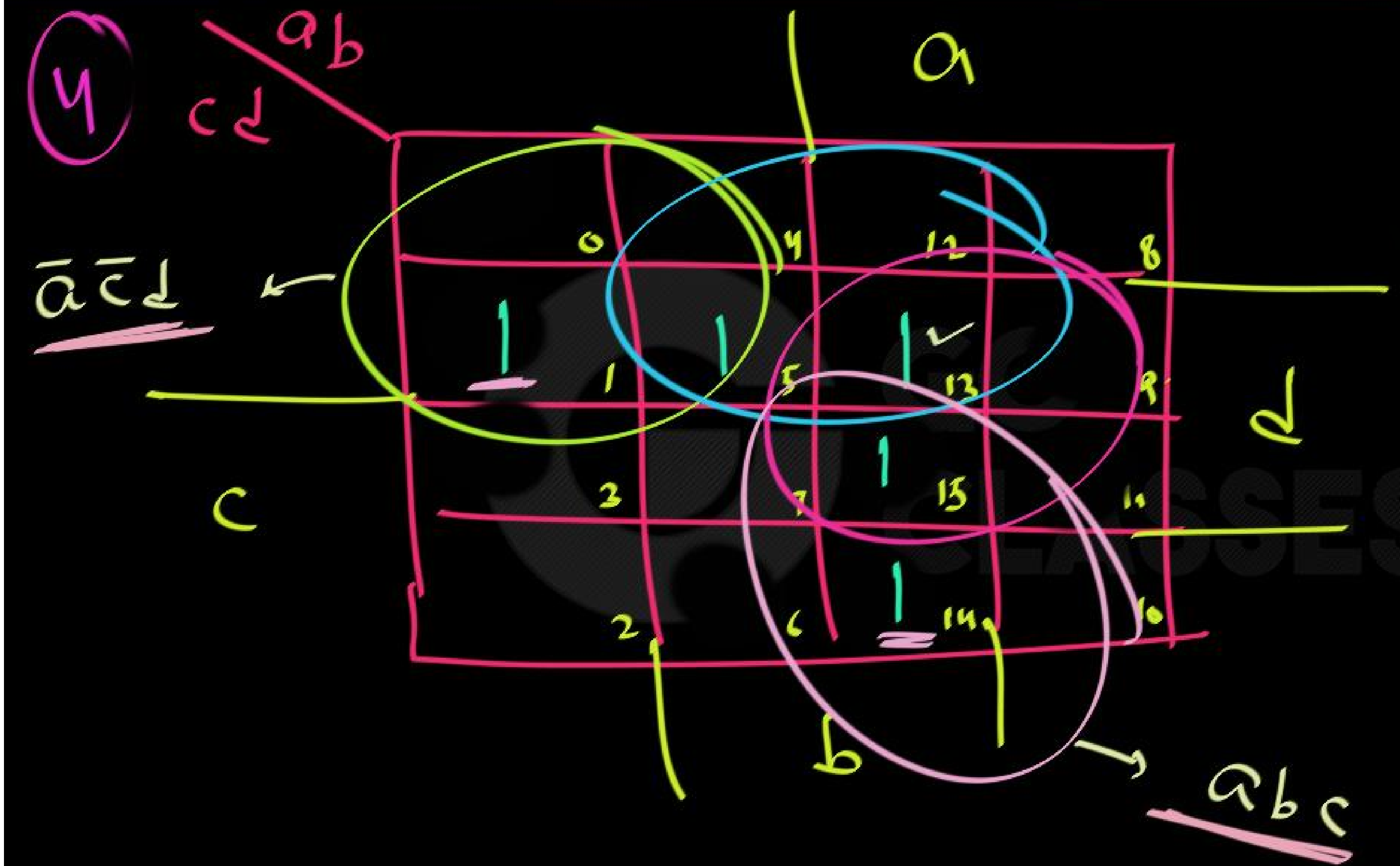
1

3



PI

2



PI : 4

EPI = 2

msop : ⇒ 2 msop

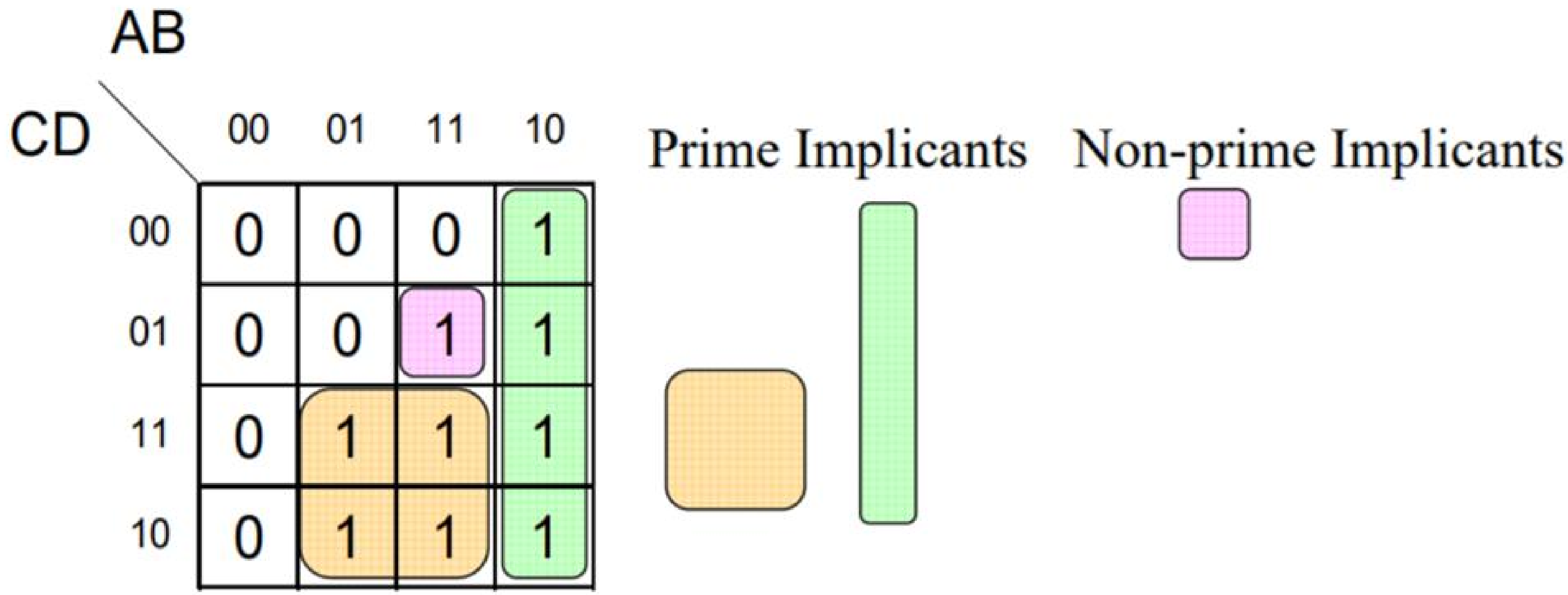
$\bar{a}\bar{c}d + abc$
$+$

K-Maps

HW

- Determine essential and non-essential prime implicants:
  - $\Sigma m(0,1,3,5,6,7,8,9,10,12,14,15)$
  - $\Sigma m(1,2,3,5,6,13,14,15)$

Prime Implicants

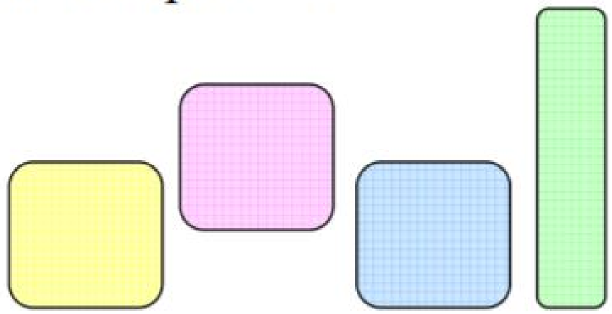


Are there any additional prime implicants in the map that are not shown above?

All The Prime Implicants

CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1

Prime Implicants



When looking for a minimal solution –
only circle prime implicants…
A minimal solution will ***never*** contain
non-prime implicants

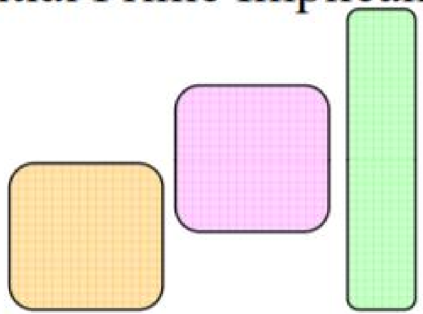
Essential Prime Implicants

CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1

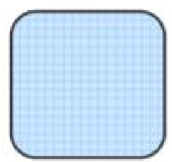
Not all prime implicants are required…

A prime implicant which is the only cover of some 1 is ***essential*** – a minimal solution requires it.

Essential Prime Implicants



Non-essential Prime Implicants



A Minimal Solution Example

CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1

$F = \boxed{AB'} + \boxed{BC} + \boxed{AD}$

Minimum

Not required…

Another Example

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	1	0
10	1	0	0	1

Another Example

PI = 4

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	1	0
10	1	0	0	1

NOT EPI ← PI_1

PI_2 → EPI

#EPI = 3

PI_4 → EPI

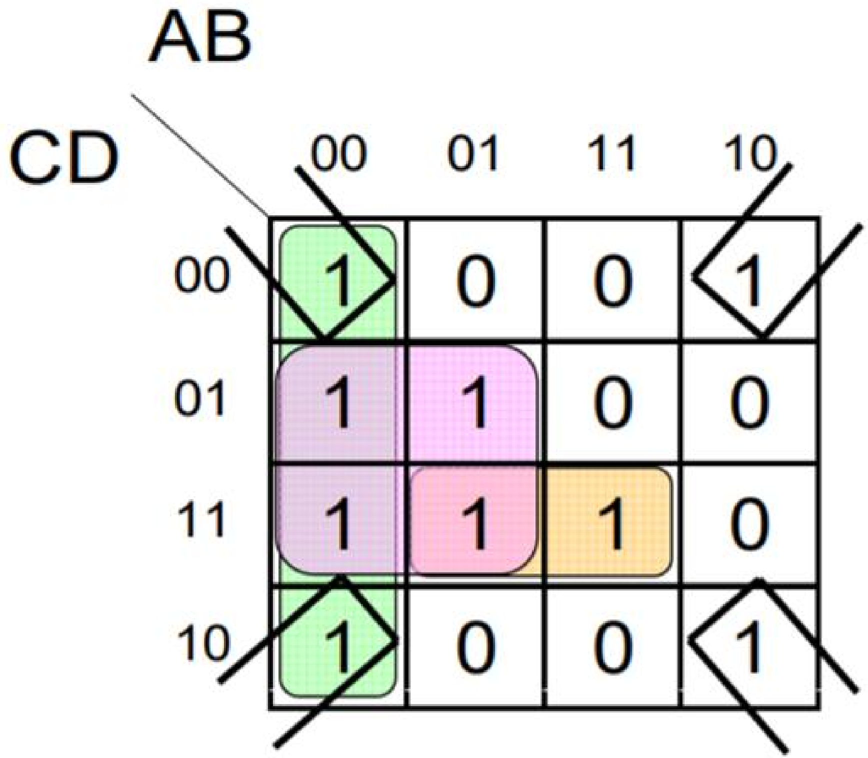
PI_3 → EPI

Note: Always, for once, look at Corner one, fold ones

Another Example

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	1	0
10	1	0	0	1

Another Example



$F = A'D + BCD + B'D'$

Minimum

A'B' is not required…

Every one one of its locations is covered by multiple implicants

After choosing essentials, everything is covered…

Finding the Minimum Sum of Products

1. Find each essential prime implicant and include it in the solution.
2. Determine if any minterms are not yet covered.
3. Find the minimal # of remaining prime implicants which finish the cover.

K-Maps

- An optimized SOP has all of the essential prime implicants. It may have non-essential prime implicants only if the essential prime implicants don't cover all squares.
- There can be more than one simplified SOP due to the selection of non-essential prime implicants.

Yet Another Example
(Use of non-essential primes)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	0	0

Yet Another Example (Use of non-essential primes)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	0	0

fold

$PI_1 \rightarrow EPI = \bar{A}\bar{D}$

$PI_2 \rightarrow EPI = AD$

$PI_3 \rightarrow$ Not EPI

PI = 4

EPI = 2

PI_4 (NOT EPI)

msop:

$$\underbrace{\bar{A}\bar{D} + AD}_{\text{EPIs}} \qquad \underbrace{PI_3 = CD}_{\text{Cover remaining 1-cells with minimum \# PIs.}}$$

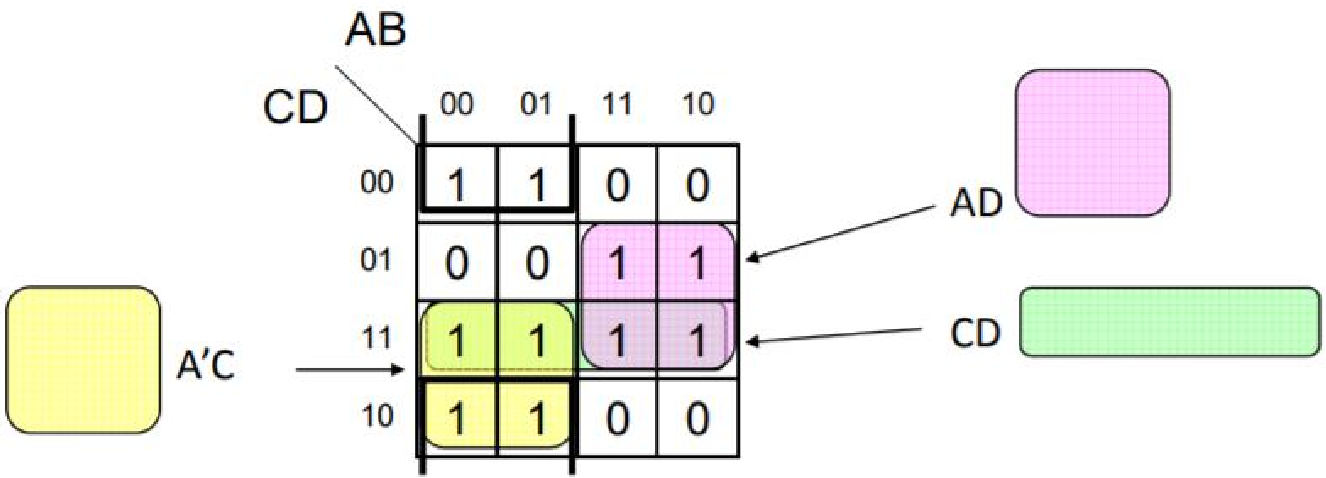
msop:

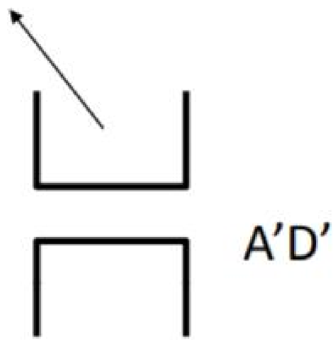
$$\underbrace{\bar{A}\bar{D} + AD}_{\text{EPIs}} \qquad \underbrace{PI_4 = \bar{A}C}_{\text{Cover Remaining 1-cells with minimum \# PIs.}}$$

Yet Another Example
(Use of non-essential primes)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	0	0

Yet Another Example (Use of non-essential primes)





Essentials: $\boxed{A'D'}$ **and AD**

Non-essentials: A'C and CD

Solution: $\boxed{A'D'}$ **+ AD + A'C**

or

$\boxed{A'D'}$ **+ AD + CD**

Three-Variable Maps

- Each cell is adjacent to 3 other cells.
- Imagine the map lying on the surface of a cylinder.

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	0	0

Four-Variable Maps

- Each cell is adjacent to 4 other cells.
- Imagine the map lying on the surface of a torus.

wx \ yz	00	01	11	10
00	1	1	0	1
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

Why do we need these?

The map is supposed to help us find minimum SOP expressions

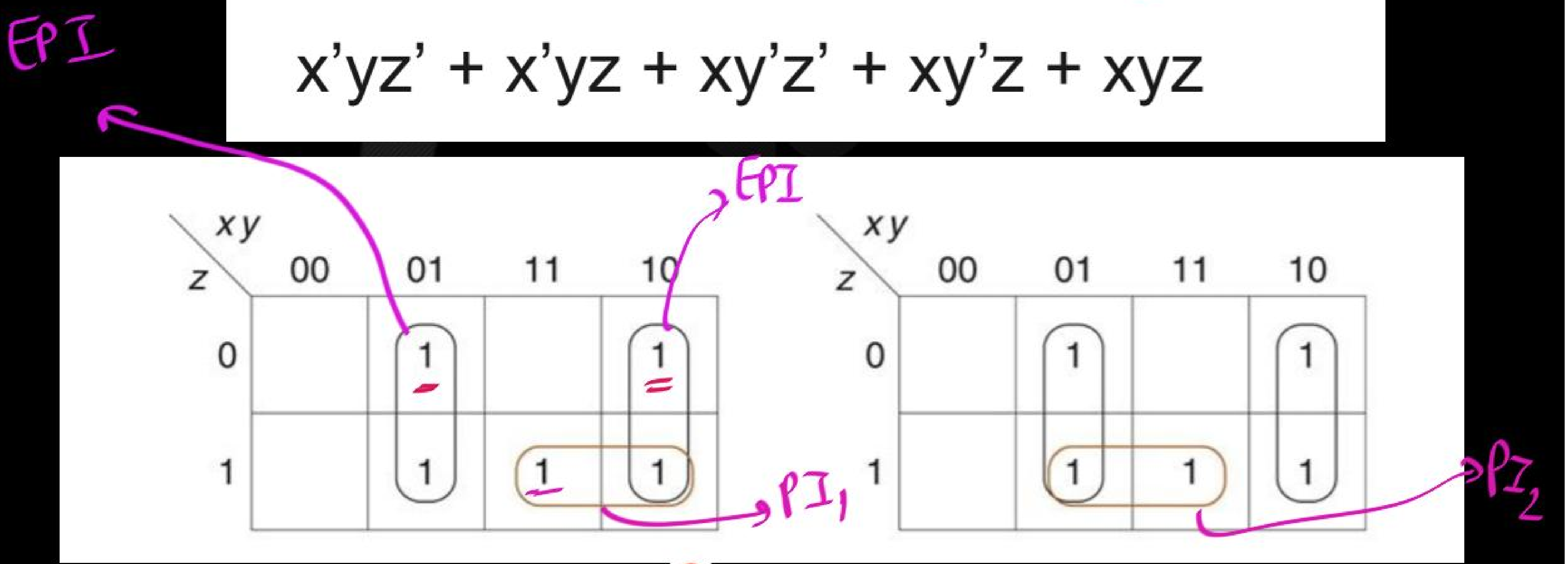
The only product terms we need to care about are prime implicants

Essential prime implicants are the prime implicants that MUST be used in any min SOP expression

Example

PI = 4
EPI = 2

$x'yz' + x'yz + xy'z' + xy'z + xyz$



mSOP

$$\underbrace{x'y + x\bar{y}}_{\text{EPIs}} + \underbrace{PI_1 = zx}_{\text{remaining 1-cells Cover with minimum \#PIs.}}$$

mSOP

$\underbrace{x'y + x\bar{y}}_{\text{EPIs}} + \underbrace{PI_2 = zy}_{\text{remaining 1-cells}}$

Cover with minimum #PIs.

Example

HW

$w'xy'z' + w'xy'z + wxy'z + wx'y'z + w'x'yz + w'xyz + wxyz + wxyz'$

yz \ wx	00	01	11	10
00	1*	1	1*	1*
01		1		
11		1	1	1*
10				

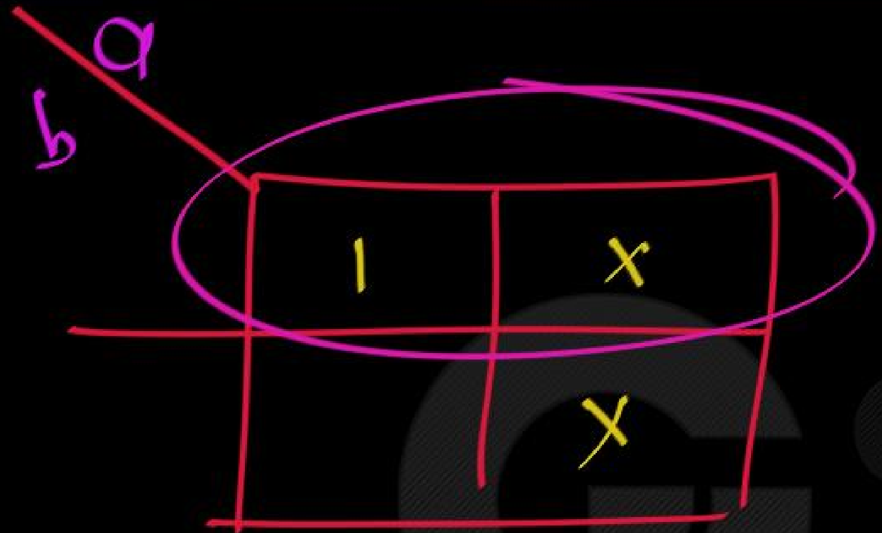
yz \ wx	00	01	11	10
00	1*	1	1*	1*
01		1		
11		1	1	1*
10				

$$f = y'z' + wyz + w'xz$$

Next Topic:

Implicants(I), Prime Implicants(PI), Essential Prime Implicants(EPI)

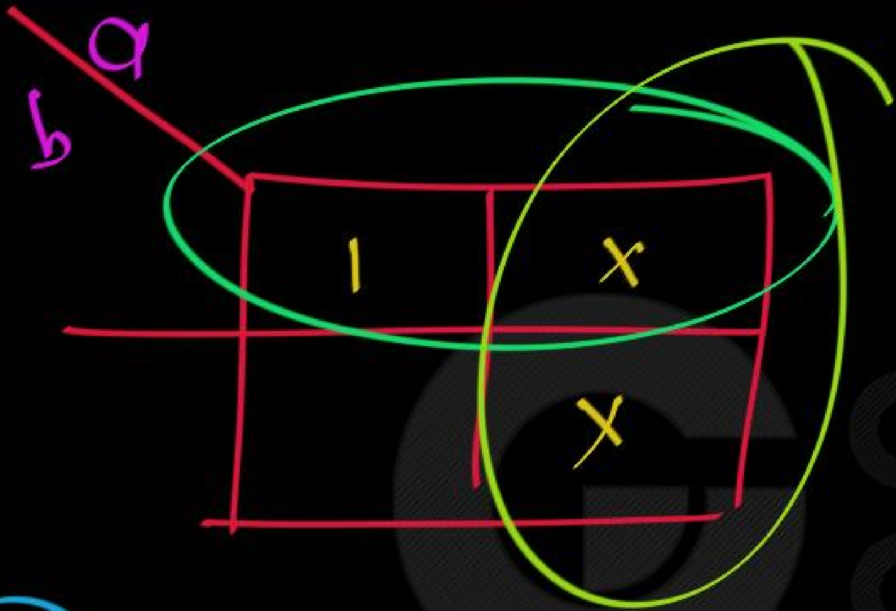
In Case of Don't Cares



Implicants = 5

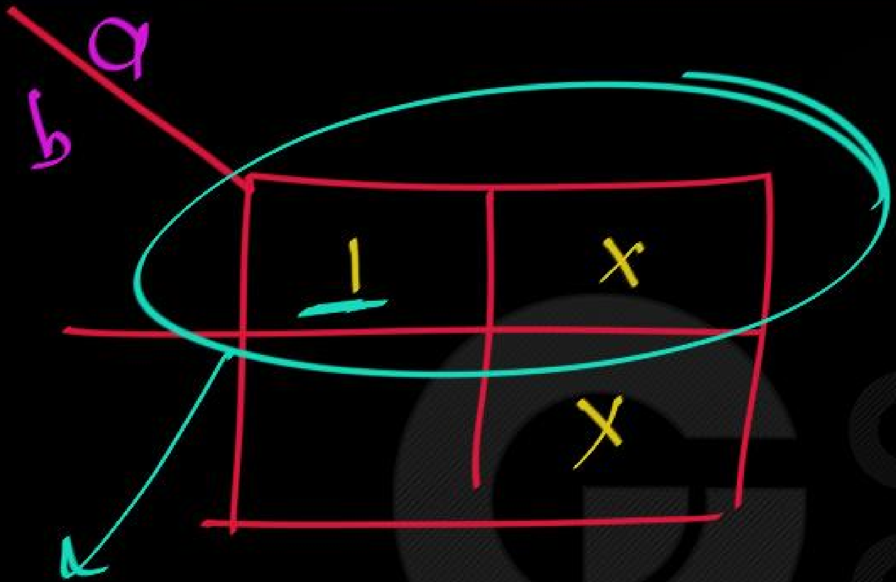
$$\underbrace{3}_{\text{Size 1}} + \underbrace{2}_{\text{Size 2}}$$

"Implicants" : $\longrightarrow$ Consider "X" as 1's



PI : 2

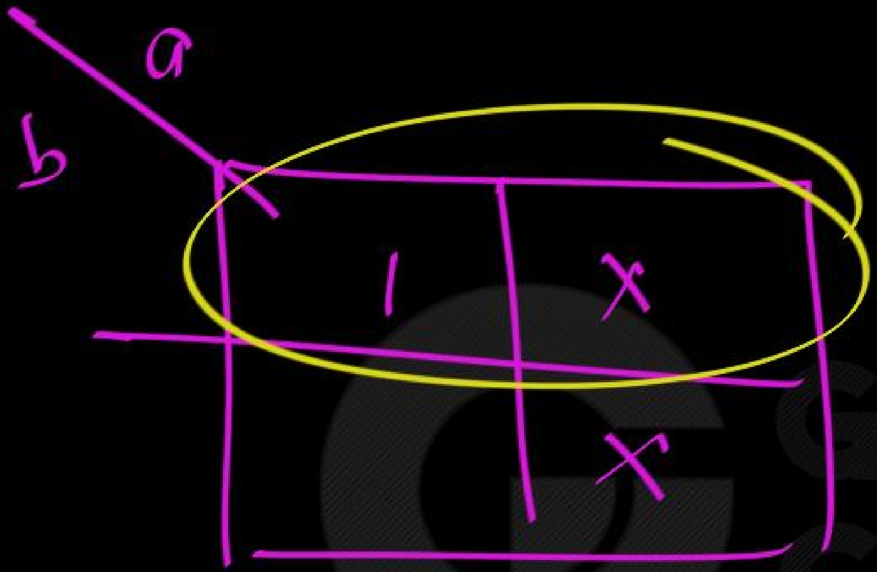
PI $\Rightarrow$ Consider "x" as 1's.



EPI

$\boxed{\#EPI = 1}$

EPI : Essential PI which covers at least one "1-cell" uniquely.



mSOP:

$\bar{b}$

only care about "1-cells"

Don’t cares

Implicants can have don’t cares in them too

An *implicant* is a rectangle of 1, 2, 4, 8, … 1’s or X’s

But not 0's

A *prime implicant* is a rectangle of 1, 2, 4, 8, … 1’s or X’s not included in any one larger rectangle. Thus, from the point of view of finding prime implicants, X’s (don’t cares) are treated as 1’s.

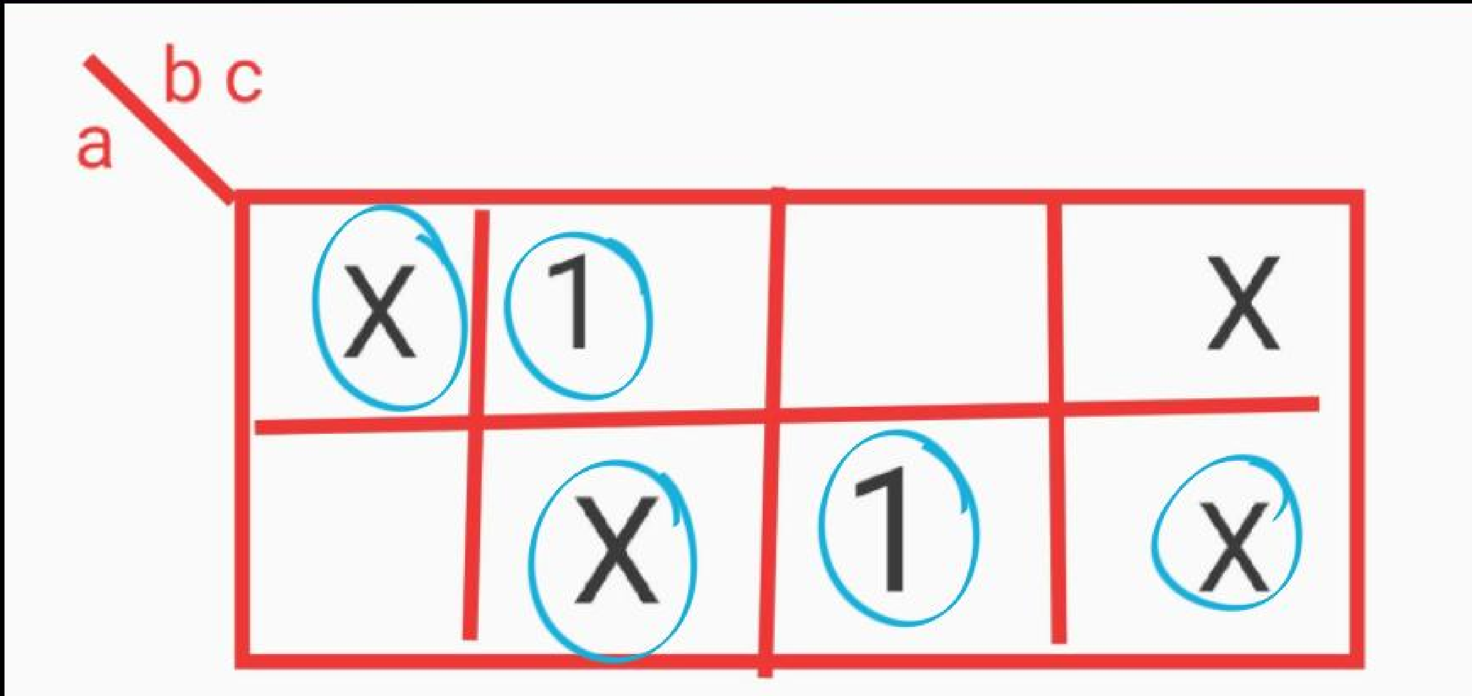
An *essential prime implicant* is a prime implicant that covers at least one 1 not covered by any other prime implicant (as always). Don’t cares (X’s) do not make a prime implicant essential.

Don't-cares, prime implicants, and essential prime implicants

Our terms need to be adapted *very slightly* to account for don't-cares . . .

- *Prime implicant*: Group of 1, 2, 4, 8, etc., 1-cells **and/or X-cells**; group can't be doubled in size without collecting 0-cells.
- *Essential prime implicant*: Same as before.

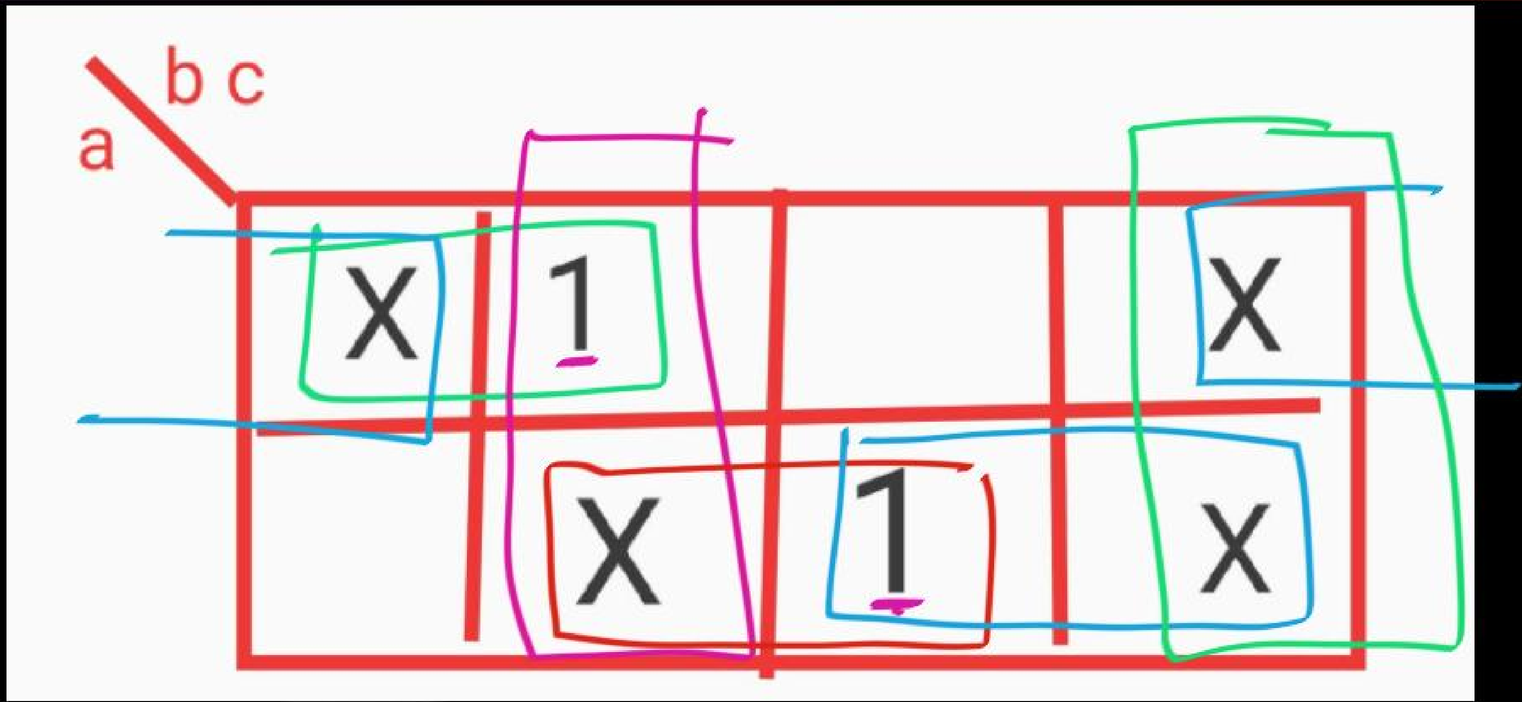
a \ b c				
	X	1		X
		X	1	X



#Implicants: $\underbrace{6}_{\text{size } 1} + \underbrace{2+1+1+1+1}_{\text{size } 2} = 12$

PI

6



No EPI

Example

HW

$$f = SUM[m(1,7,10,11,13)] + SUM[d(5,8,15)]$$

CD \ AB	00	01	11	10
00				X
01	1	X	1	
11		1	X	1
10				1

5 PI, 2 EPI

CD \ AB	00	01	11	10
00				X
01	1*	X	1*	
11		1*	X	1
10				1

minimum

CD \ AB	00	01	11	10
00				X
01	1	X	1	
11		1	X	1
10				1

other p.i.s

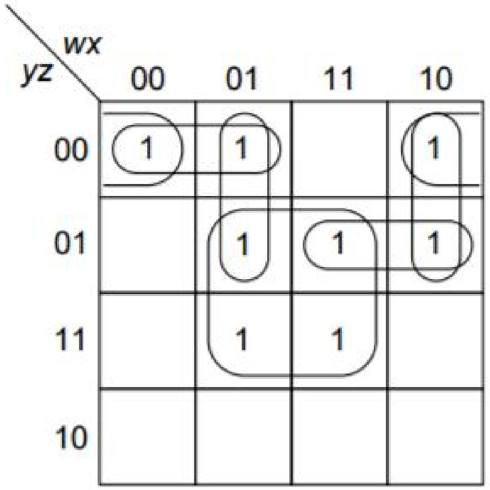
mSOP = $F = \underbrace{BD + A'C'D}_{EPI} + AB'C$

Unique mSOP

Deriving Prime Implicants and Minimal Expressions

Example: for $f(w,x,y,z) = \sum(0,4,5,7,8,9,13,15)$ below, set of prime implicants

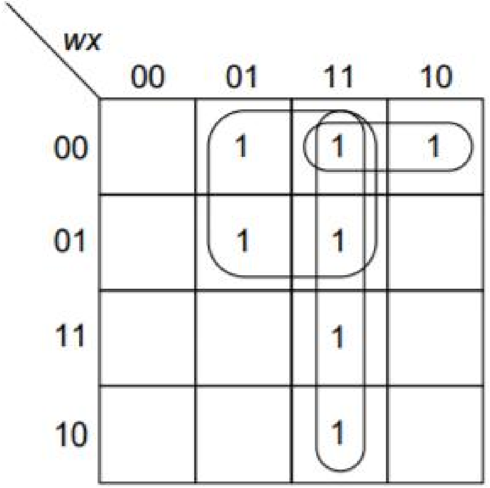
$P = \{xz,\ w'y'z',\ wx'y',\ x'y'z',\ w'xy',\ wy'z\}$



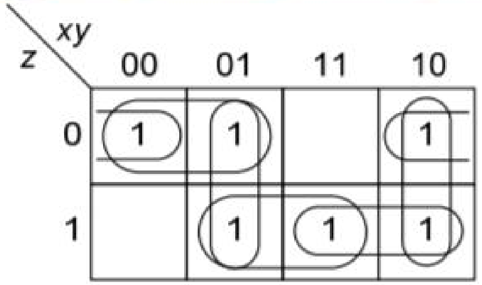
Essential prime implicants: covers at least one minterm not covered by any other prime implicant, e.g., *xz*

- Since all minterms must be covered, all essential prime implicants must be contained in any irredundant expression of the function

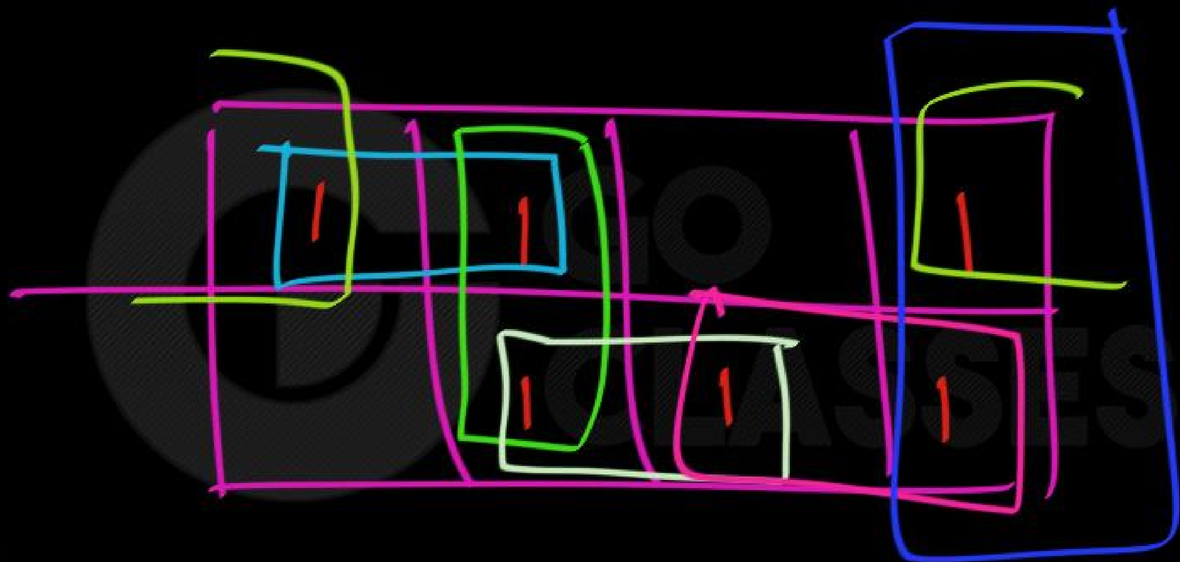
Example: prime implicants of $f(w,x,y,z) = \sum(4,5,8,12,13,14,15)$ are all essential



Example: Cyclic prime implicant chart in which no prime implicant is essential, all prime implicants have the same size, and every 1 cell is covered by exactly two prime implicants

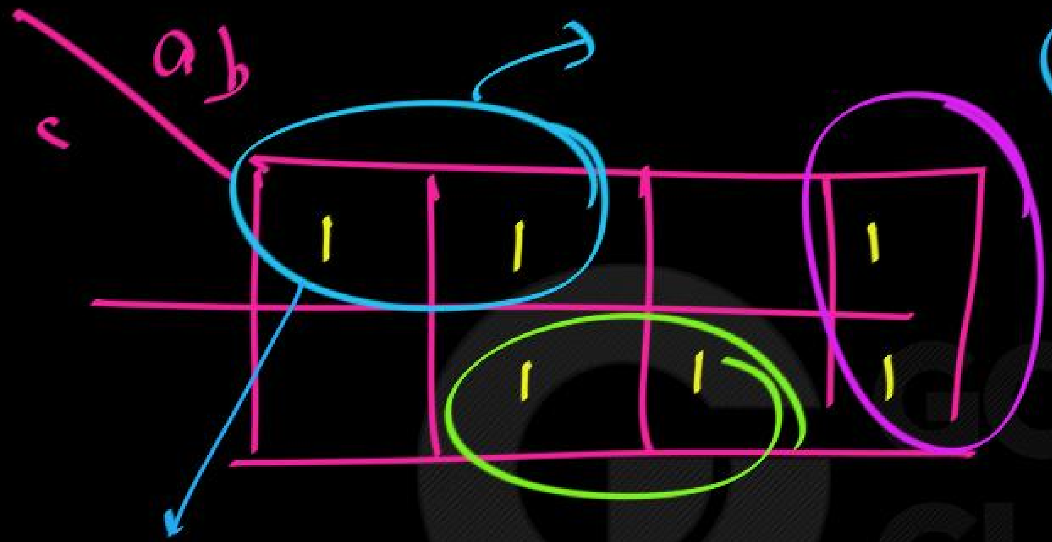


Cyclic k-map: → 2 min SOP → No Common term.



No EPI.

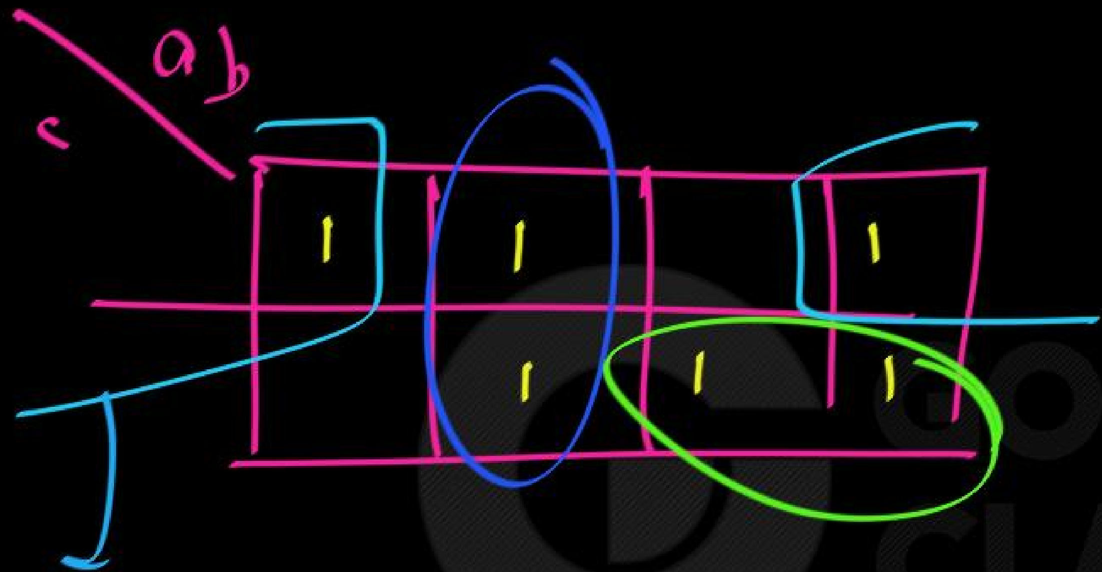
Every PI Same size ; Every 1-cell Covered by Exactly two PI.



Start with this

msop:

$$\underbrace{\bar{a}\bar{c} + cb + a\bar{b}}$$



Start with this

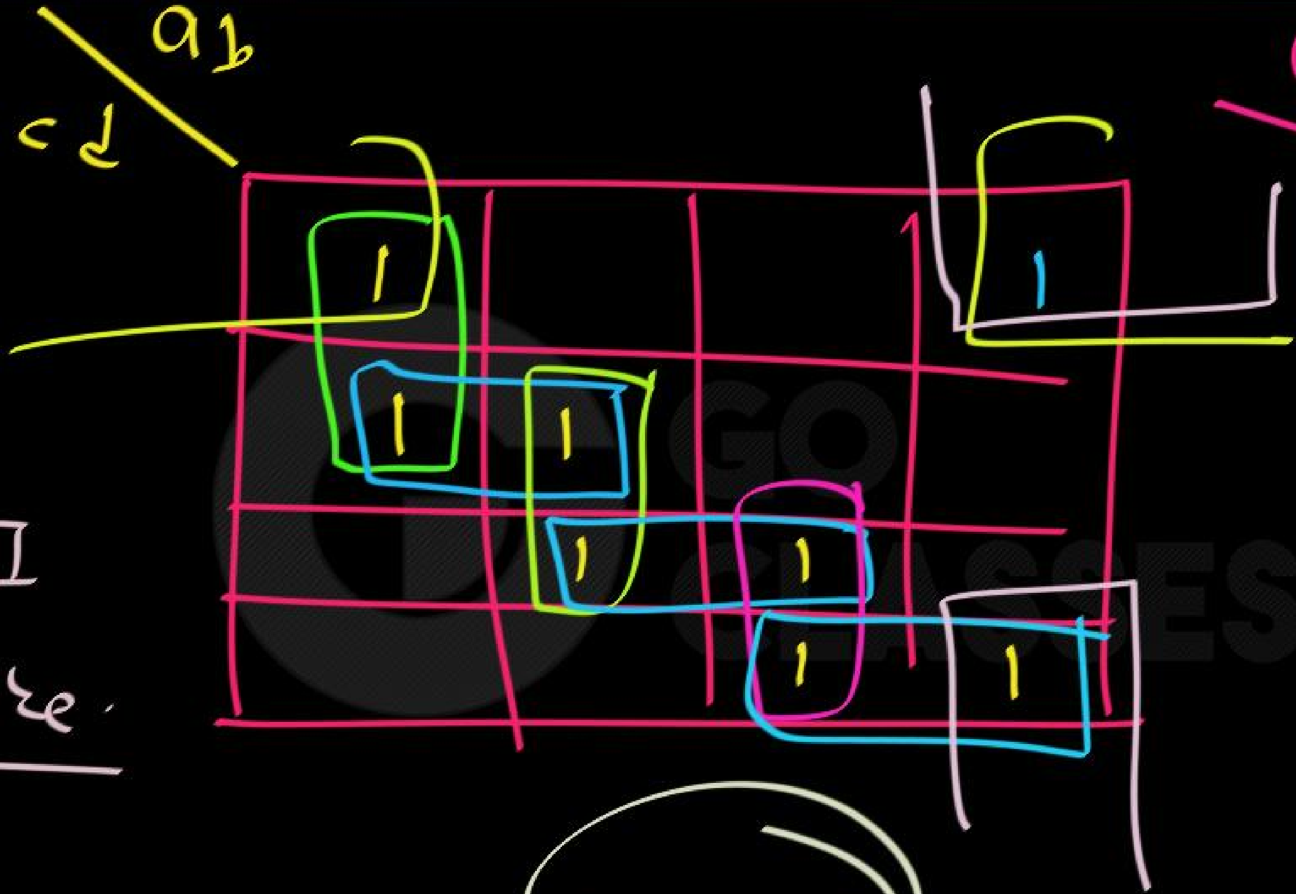
msop:

$\bar{b}\bar{c} + \bar{a}b + ac$

Cyclic k-map:

Cyclic k-map.

cd \ ab				
	1			1
	1	1		
		1	1	
			1	1



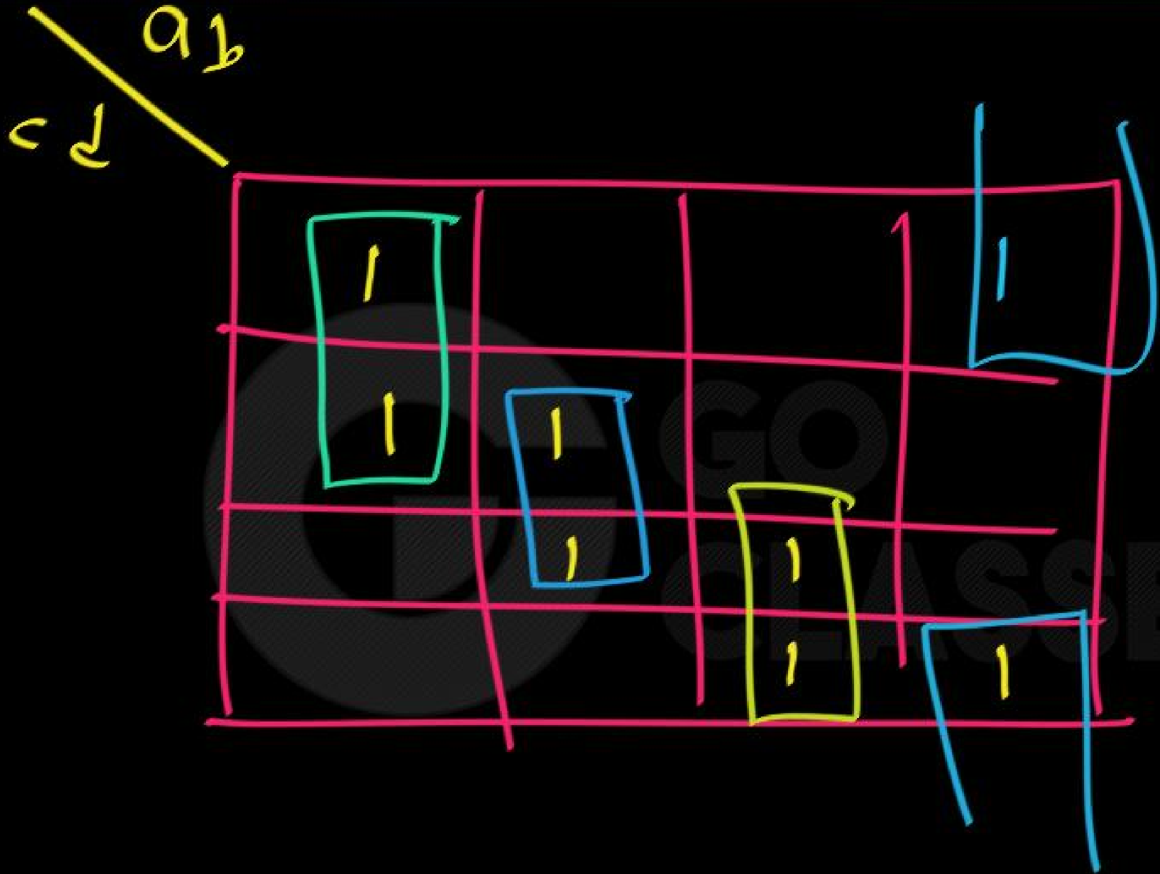
Cyclic k-map.

No EPI

Every PI Same Size.

Every 1-cell Covered by 2 PIs.

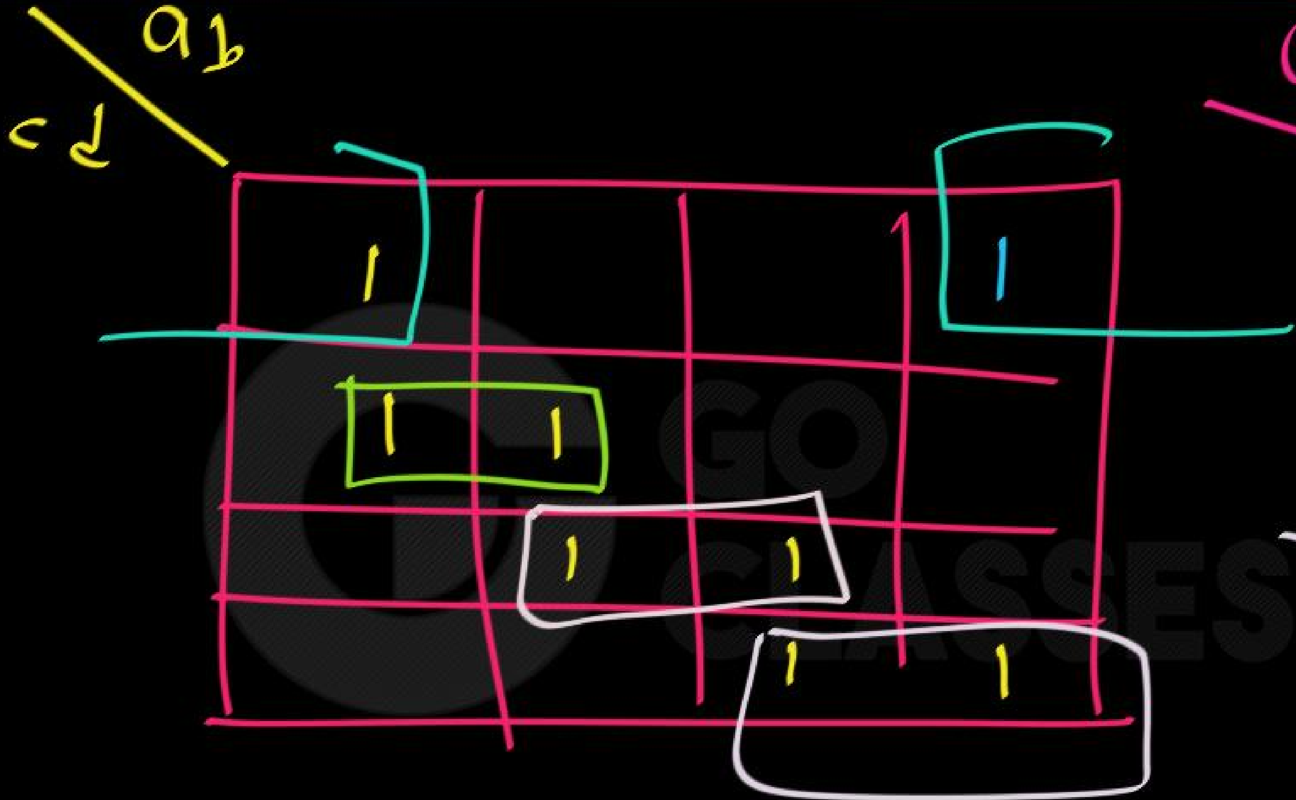
Cyclic k-map: ⇒ 2 msop ⇒ No Common term.



Cyclic k-map:

msop 1

Cyclic k-map.



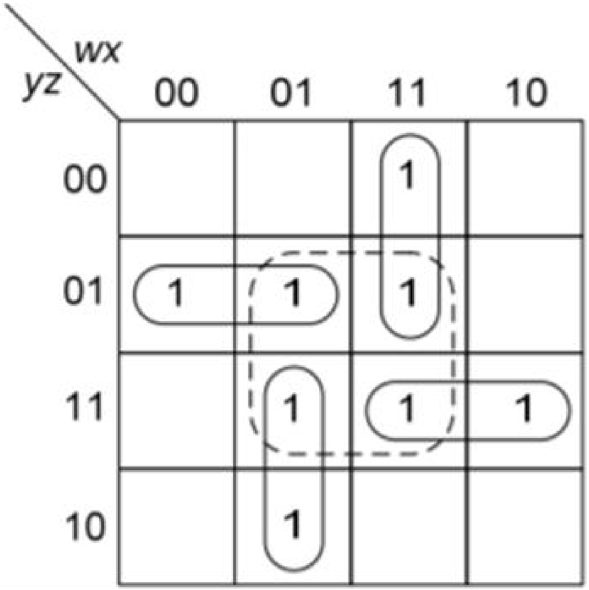
msop 2

Procedure for Deriving Minimal Sum-of-products Expression

Procedure:

1. Obtain all essential prime implicants and include them in the minimal expression
2. Remove all prime implicants which are covered by the sum of some essential prime implicants
3. If the set of prime implicants derived so far covers all the minterms, it yields a unique minimal expression. Otherwise, select additional prime implicants so that the function is covered completely and the total number and size of the added prime implicants are minimal

Example: prime implicant *xz* is covered by the sum of four essential prime implicants, and hence *xz* must not be included in any irredundant expression of the function



Create K-map from Expressions

$$f(x, y, z) = x'yz' + x'yz + xy'z' + xy'z + xyz$$

Create K-map from Expressions

$f(A, B, C, D) = A'B'C'D' + A'BC'D' + AB'C'D' + ABC'D + AB'C'D + A'B'CD + A'BCD + AB'CD + A'B'CD' + ABCD' + AB'CD' = \sum m(0, 2, 3, 4, 7, 8, 9, 10, 11, 13, 14)$

CD \ AB	00	01	11	10
00	A'B'C'D'	A'B C'D'	A B C'D'	A B'C'D'
01	A'B'C'D	A'B C'D	A B C'D	A B'C'D
11	A'B'C D	A'B C D	A B C D	A B'C D
10	A'B'C D'	A'B C D'	A B C D'	A B'C D'

CD \ AB	00	01	11	10
00	1	1		1
01			1	1
11	1	1		1
10	1		1	1

Write out the numerical expression of the following K-map

C \ AB	00	01	11	10
0	0	2 1	6	4 1
1	1	3	7 1	5 1

Get expression from K-Map – Practice 2

Write out the numerical expressions of the following K-maps

$yz \backslash wx$	00	01	11	10
00				1
01	1	1		
11	1	1	1	1
10	1	1		1

f

$yz \backslash wx$	00	01	11	10
00		1	1	1
01				
11			1	
10	1	1	1	

g

$yz \backslash wx$	00	01	11	10
00		1		1
01	1	1		
11	1	1	1	
10		1		

h

$CD \backslash AB$	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

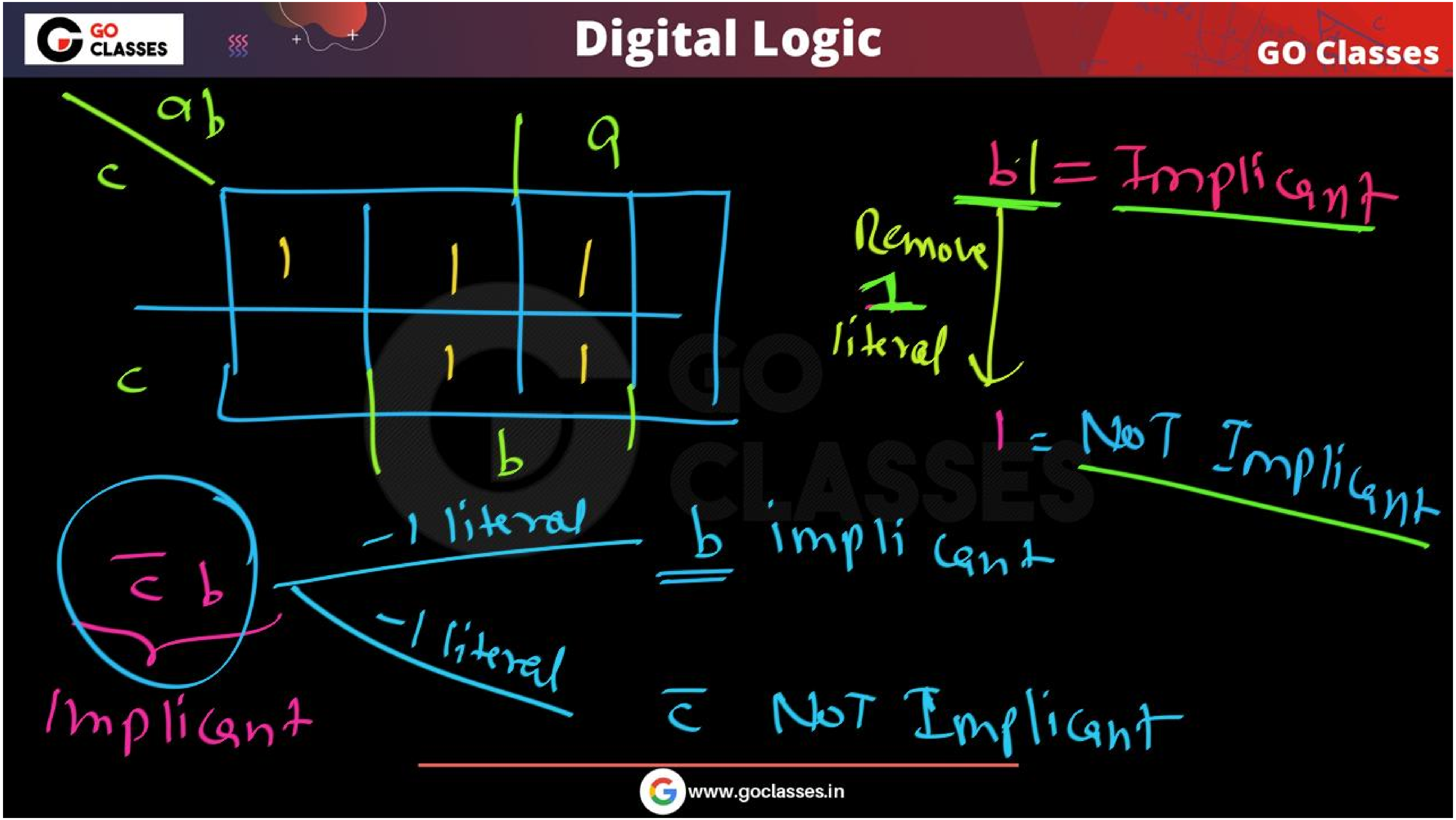
Another Definition of Implicant:

(1) K-map → any cube of 2^m size Covering 1's and/OR X's)

(2) product term which 'implies' function.

Implicant : product term (which implies f)

$\boxed{\text{Implicant} - \text{one literal}}$ ⇒ may or may not be Implicant.



GO CLASSES
Digital Logic
GO Classes
ab
c
a
1
1
1
c
1
1
b
b1 = Implicant
Remove 1 literal
1 = NOT Implicant
$\bar{c}$ b
Implicant
-1 literal
b implicant
-1 literal
$\bar{c}$ NOT Implicant
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$\underbrace{\bar{a}\, b\, \bar{c}}_{\text{Implicant}}$

-1 literal

$b\bar{c}$ = implicant ✓

$\bar{a}\,\bar{c}$ = " ✓

-1 literal

$\bar{a}\, b$ = implicant ✓

$\bar{a}$ → NOT Implicant

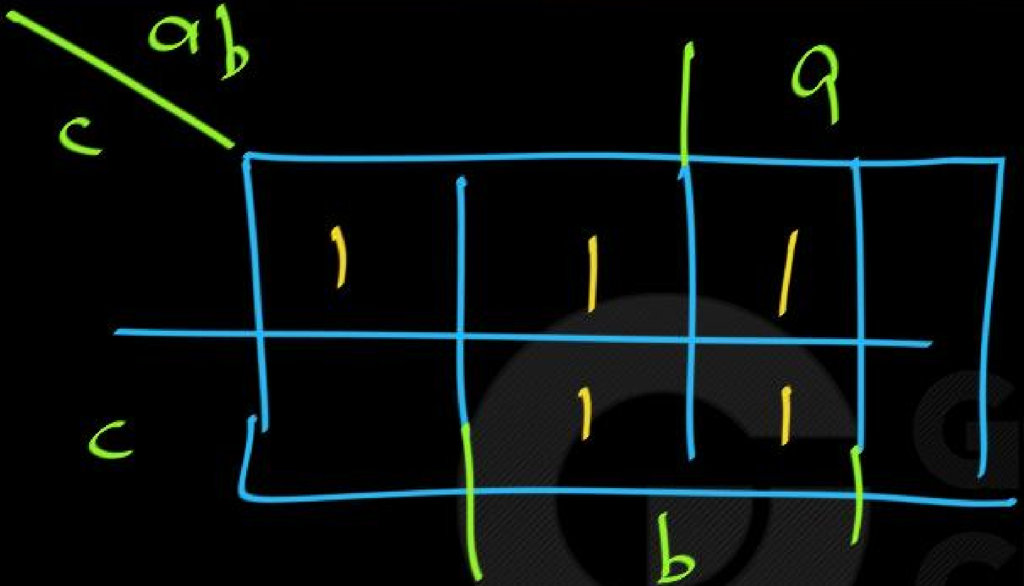
b = implicant ✓

PI : In k-map ; a cube of 1-cells which is as big as possible

PI : product term as small as possible which implies f .

PI – one literal ⇒ Not even be Implicant

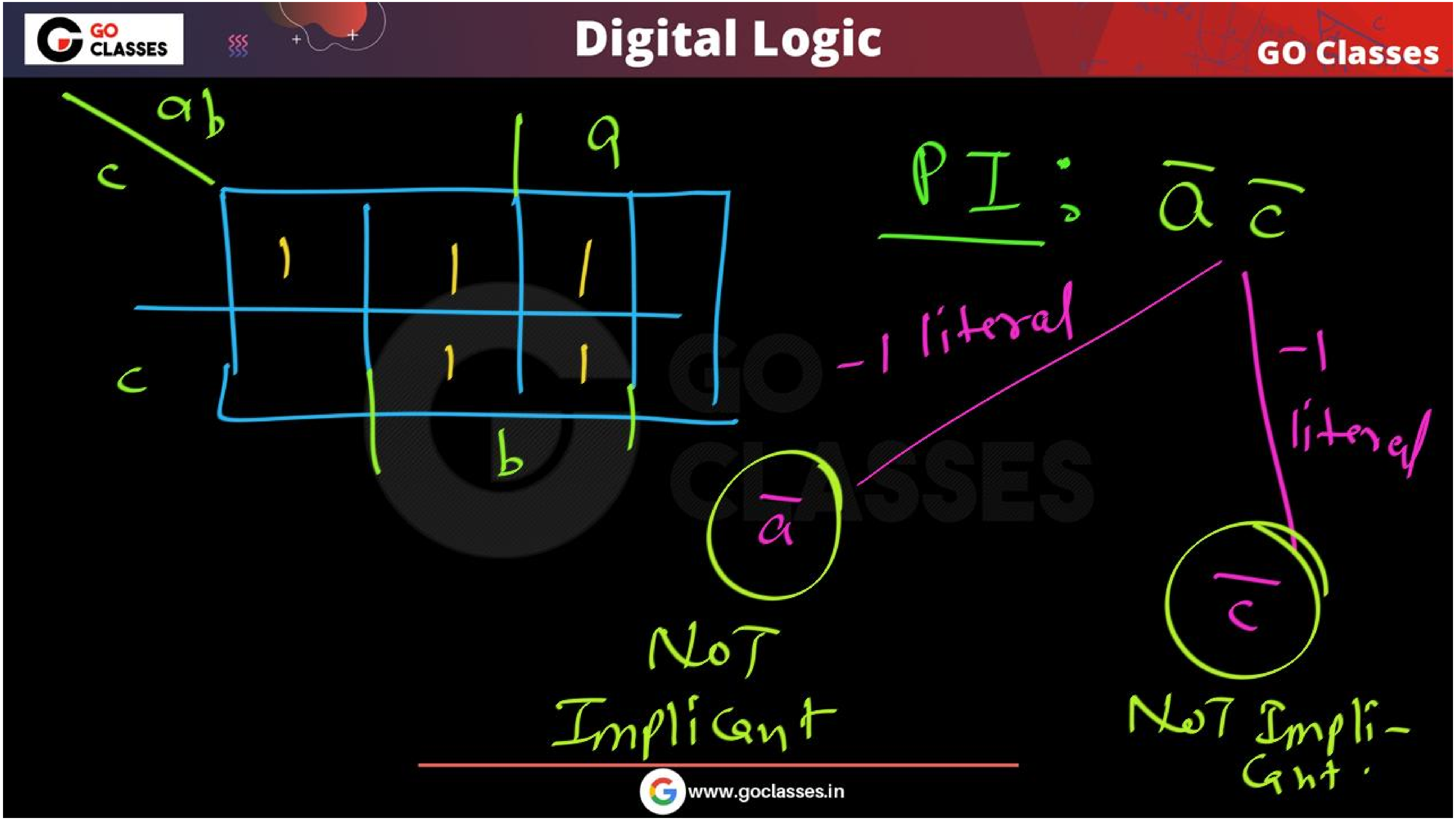
Prod. term
as small as possible



PI : b·1

– 1 literal

1 → NOT Implicant


GO CLASSES
Digital Logic
GO Classes
ab
c
a
c
b
PI : $\bar{a}\bar{c}$
-1 literal
-1 literal
$\bar{a}$
$\bar{c}$
NOT Implicant
NOT Implicant.
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Implicant:

Product term

which implies f.

Prime Implicant :

Product term which Does not have Redundant literal which implies f.

Prime Implicant :

Product term which is as small as possible which implies f.

Another Definition of PI :

Minimal Functions and Their Properties

Implicants: function *f* covers function *g* with the same input variables if *f* has a 1 in every row of the truth table in which *g* has a 1

- If *f* covers *g* and *g* covers *f*, then *f* and *g* are equivalent
- Let *h* be a product of literals. If *f* covers *h*, then *h* is said to imply *f* or *h* is said to be an implicant of *f*, denoted as $h \rightarrow f$

Example: If $f = wx + yz$ and $h = wxy'$, then *f* covers *h* and *h* implies *f*

Prime implicant *p* of function *f*: product term covered by *f* such that the deletion of any literal from *p* results in a new product not covered by *f*

- *p* is a prime implicant if and only if *p* implies *f*, but does not imply any product with fewer literals which in turn also implies *f*

Example: $x'y$ is a prime implicant of $f = x'y + xz + y'z'$ since it is covered by *f* and neither x' nor y alone implies *f*

The Karnaugh Map (K-map)

- K-map is a graphical approach to finding minimum SOP expressions (prime implicants) for function simplification.
- K-map is very useful for small design problems of 3-4 variables (up to 6 variables)

Basic Rules of Karnaugh maps

GO Classes

- Anytime you have N variables, you will have 2^N possible combinations, and 2^N places in a truth table or K-Map.
- In a Karnaugh Map of any size, crossing a vertical or horizontal cell boundary is a change of only one variable -- no matter how many variables there are.
- Each single cell that contains a 1 represents a minterm in the function, and each minterm can be thought of as a "product" term with N variables.
- To combine variables, use groups of **1, 2, 4, 8,** etc. A group of 2 in an N-variable Karnaugh map will give you a "product" term with N-1 variables. A group of 4 will have N-2 variables, etc.
- You will never have a group of 3, a group of 5, etc.

$$F = wxy + yz + x\bar{y}z + w\bar{z}$$

wx\yz	00	01	11	10
00			1	1
01		1	1	
11	1	1	1	1
10			1	1

wx\yz	00	01	11	10
00			1	1
01		1	1	
11	1	1	1	1
10			1	1

When grouping you should only use groups with 2^n cells. Make as large groups as possible. And remember that groups can wrap around from a top edge to a bottom edge and from a left edge to a right edge.

The above map can thus be reduced to 3 groups of 4 cells each, giving:

$$F = w\bar{z} + xz + yz$$

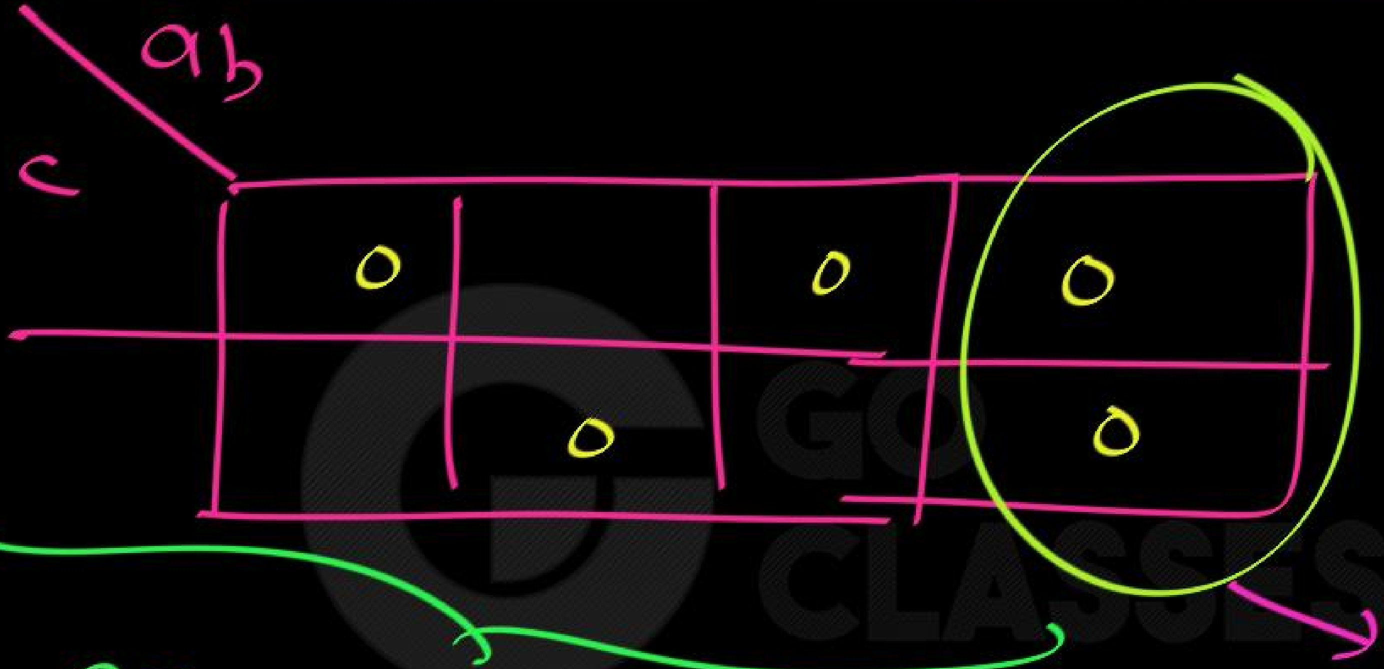
Q:

c \ ab				
	0		0	0
		0		0

#PI ?

#EPI ?

Q:



PI

Not PI.

I, PI, EPI

↳ are for Covering 1-cells.

Q: ab \ c

0	1	0	0
1	0	1	0

#PI = 3

#EPI = 3

NOTE:

The terms Implicant, PI, EPI etc are defined only for SOP expressions (i.e. product terms, covering 1's or X's, BUT not 0's).

Some Authors (But No Standard book)

Define Similar Concepts to Implicant for 0-cell (for sum terms) (for POS)

for 1-cell, x's	for 0-cell, x's
Implicant	Implicate
Prime Implicant	Prime Implicate
EPI	EP "
Every standard book	No standard book.

Note:

In exam, if "Implicate" comes then they have to give Definitions.

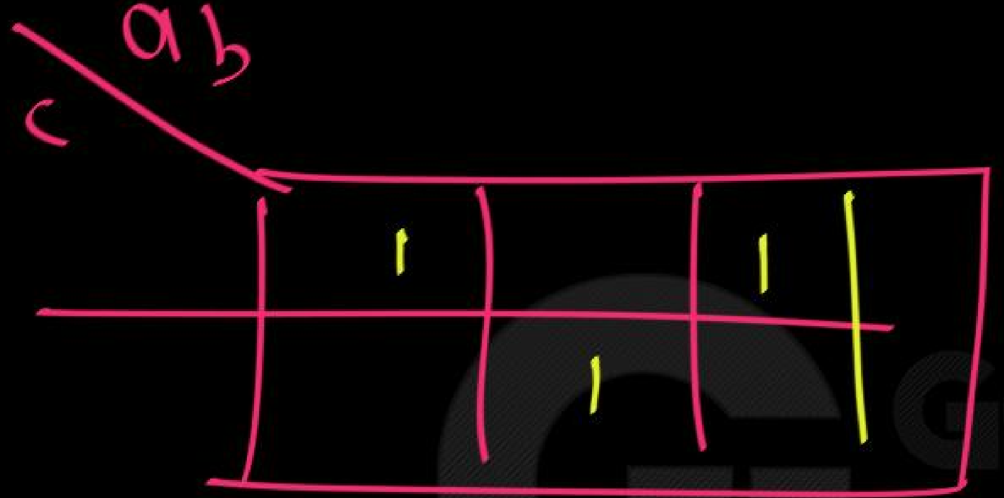
Next Topic:

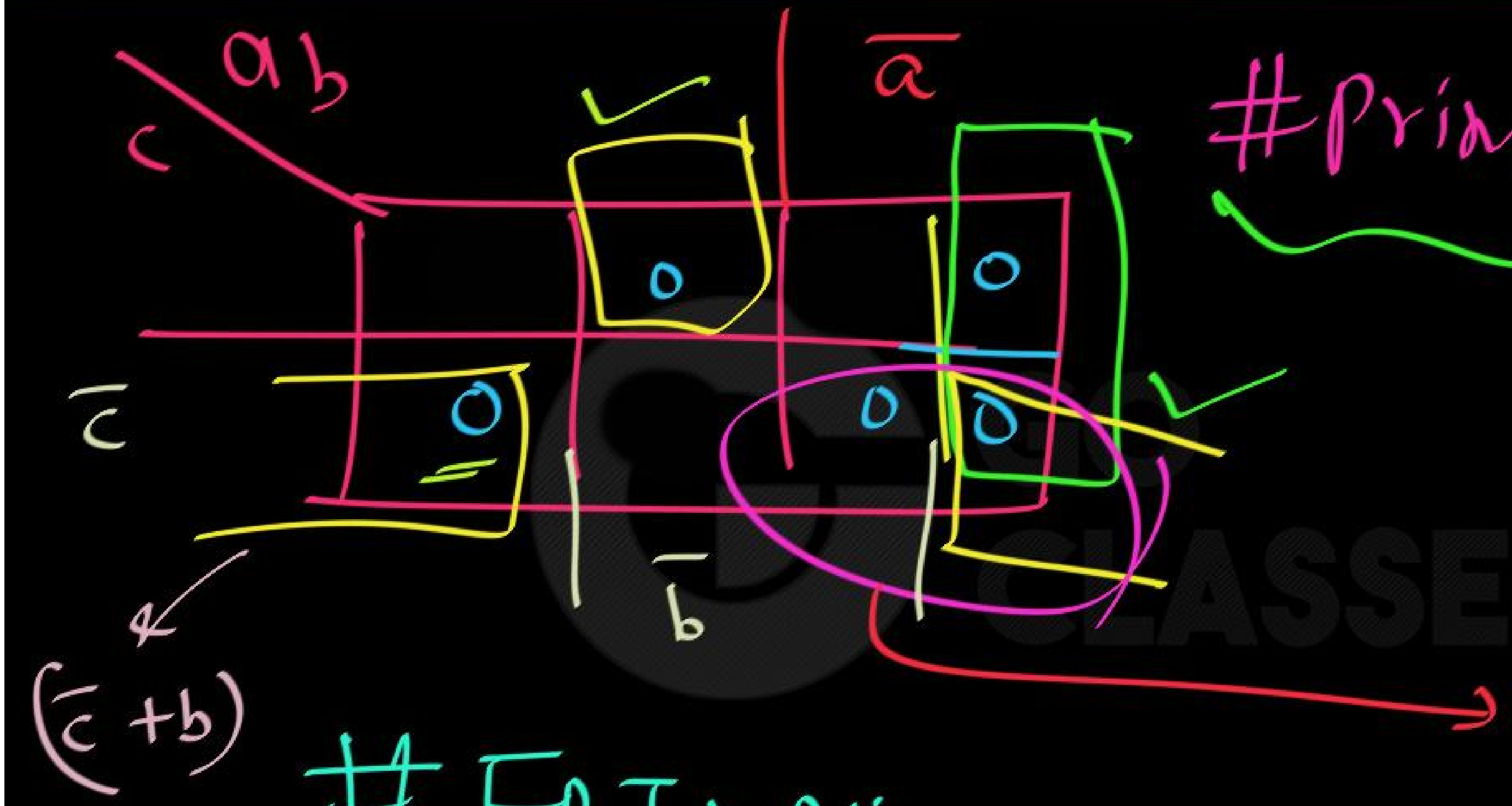
(for 0-cells, x-cells) For POS expression (for sum terms)

Implicats(I), Prime Implicats(PI),

Essential Prime Implicats(EPI)

Prime Implicates?





Prime Implicates?

4 prime Implicate.

EP Implicate = 4

Review Some Definitions

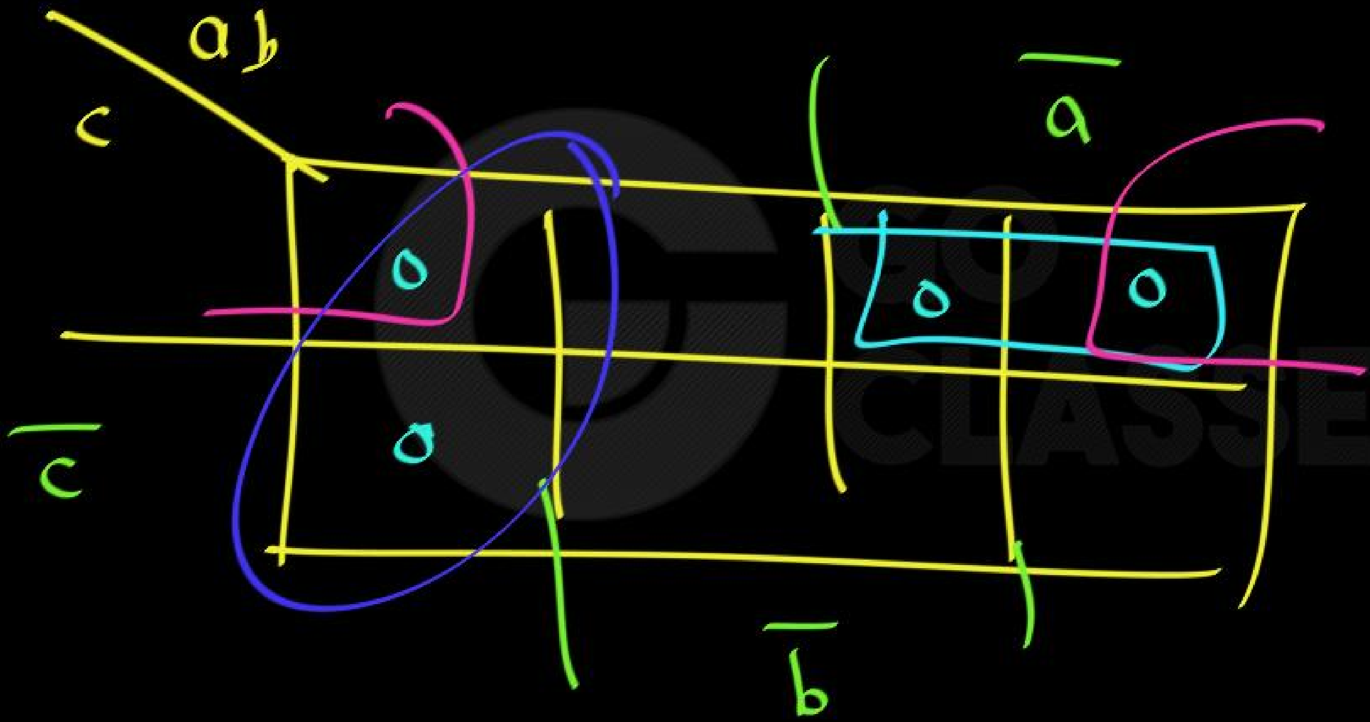
- Complement: variable with a bar over it
  $\bar{A}$, $\bar{B}$, $\bar{C}$
- Literal: variable or its complement
  A, $\bar{A}$, B, $\bar{B}$, C, $\bar{C}$
- Implicant: product of literals which implies f
  $A\bar{B}C$, $A\bar{C}$, BC
- Implicate: sum of literals
  $(\bar{A}+B+C)$, $(\bar{A}+\bar{C})$, $(B+C)$
- Minterm: product that includes all input variables
  $A\bar{B}C$, $AB\bar{C}$, ABC
- Maxterm: sum that includes all input variables
  $(A+B+C)$, $(A+B+C)$, $(A+B+C)$

K-Maps

- To create a simplified POS, select rectangular regions of 0's.
- These are called Implicates.
- A simplified POS is made up of essential prime implicates and maybe non-essential prime implicates.
- Map the equation $\Pi M(0,1,4,6)$

$f : \Pi M(0, 1, 4, 6)$

Prime Implicate :

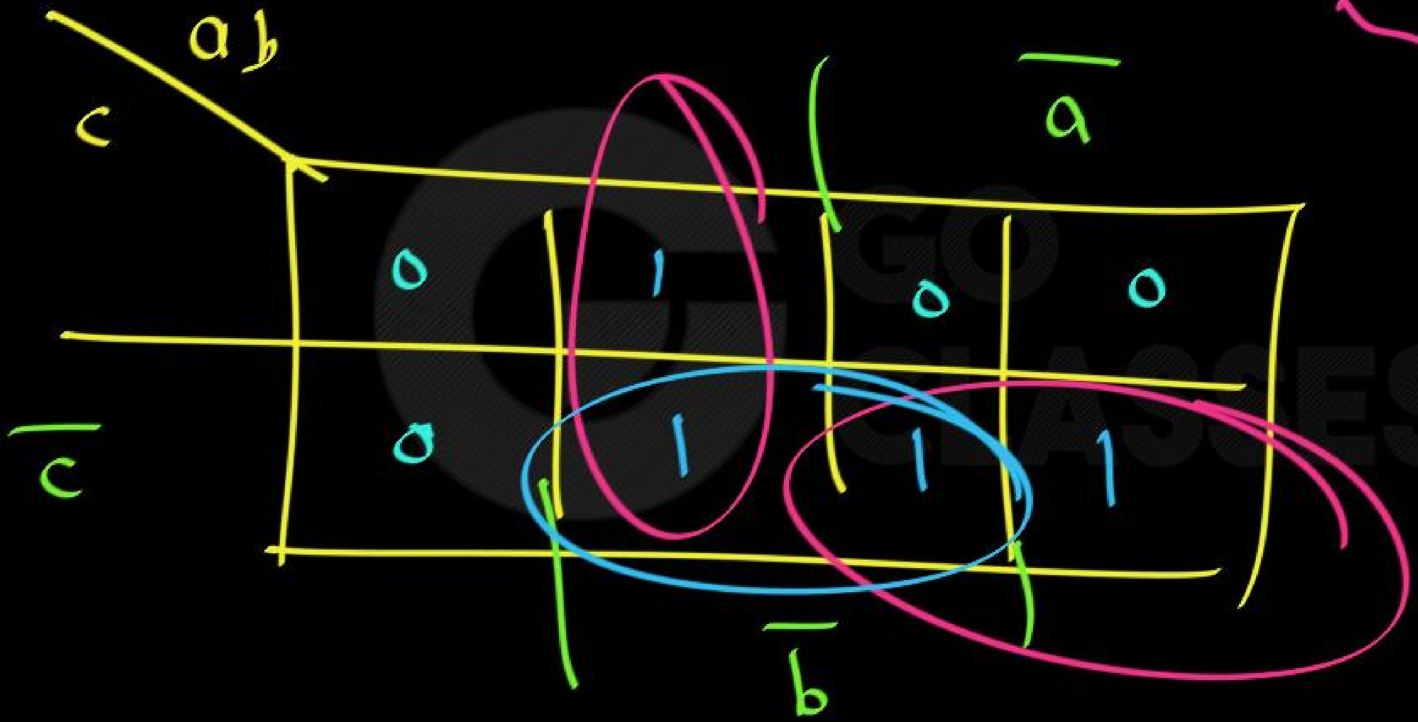


3 prime Implicate

2 EPI

$f: \Pi M(0,1,4,6)$

Prime Implicant



3 PI

2 EPI

K-Maps

- Determine essential and non-essential prime implicates:
  - $\prod M(1,2,3,5,8,9,11,15)$
  - $\prod M(5,7,9,13,14,15)$
  - $\prod M(0,1,3,4,6,9,11,14,15)$
  - $\prod M(1,5,13,14,15)$
  - $\prod M(2,3,5,7,8,10,12,13)$

K-Maps

- Determine essential and non-essential prime implicates:
  - $\prod M(0,1,3,5,6,7,8,9,10,12,14,15)$
  - $\prod M(1,2,3,5,6,13,14,15)$

K-Maps

- Sometimes you don't care if a minTerm or maxTerm is a 0 or 1. We call these “don't cares”.
- Use them to your advantage to create larger prime implicants or implicates.
  - $\Sigma m(3,5,8,12)$, $d(1,2,6,7,10,14)$