Propositional Logic

Next Chapter:

Logical Connectives

(Logical Operators)

- By Deepak Poonia (IISc Bangalore)

Instructor:

Deepak Poonia

IISc Bangalore

GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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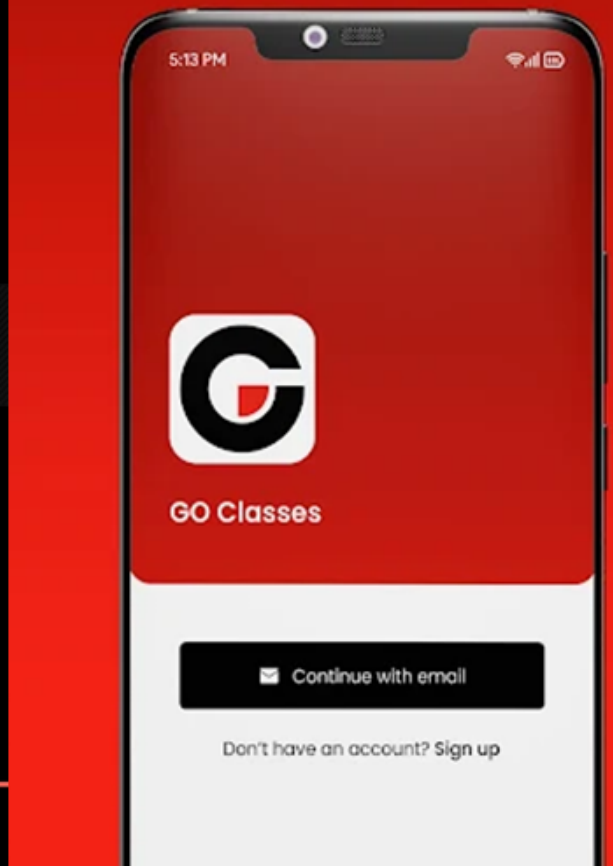

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Propositional Logic

Next Topic:

Logical Connectives

Atomic Propositions == Propositional Variables

p, q, r .....

New propositions, called compound propositions, are formed from existing propositions using logical operators.

A : This book is interesting.

B : I am staying at home.

Atomic statements

This book is interesting, and I am staying at home.

This book is not interesting.

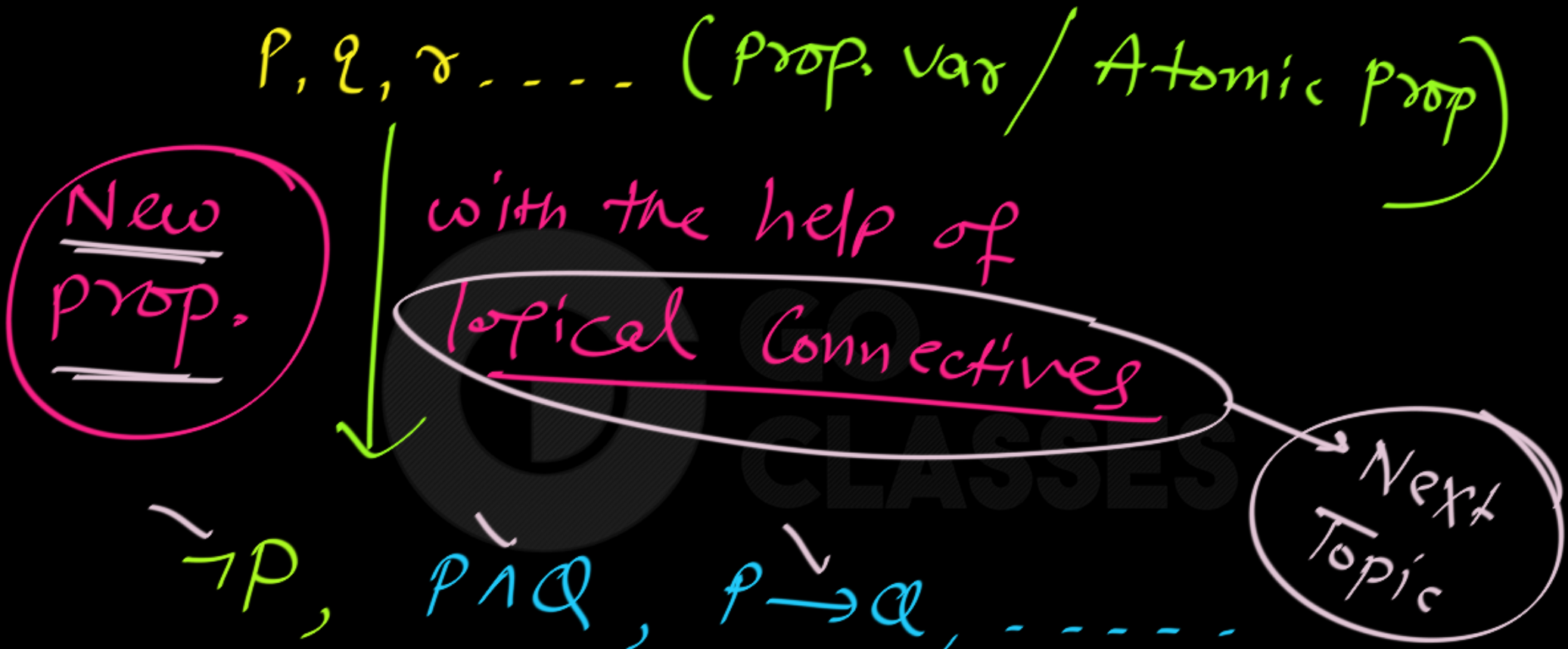
This book is interesting, or I am staying at home.

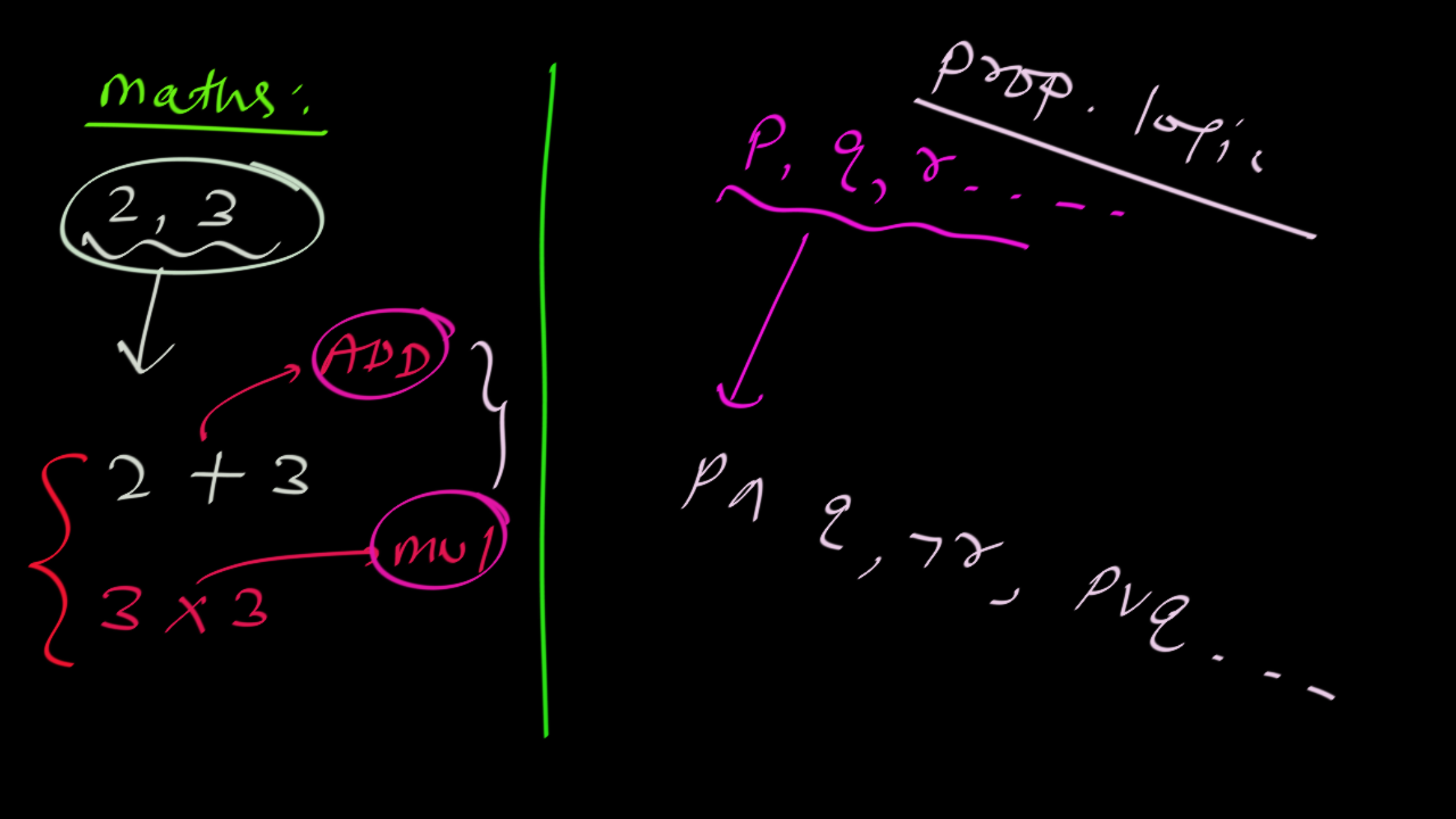
This book is interesting, or I am staying at home but not both.

If this book is interesting, then I am staying at home.

Standard Logical Connectives(Operators):

1. NOT
2. AND
3. OR
4. Exclusive OR
5. Implication
6. Bi-Implication
7. NAND
8. NOR


P, Q, R .... (Prop. Var / Atomic Prop)
New prop.
with the help of
Logical Connectives
Next Topic
¬P, P ∧ Q, P → Q, ...


Maths:
2, 3
ADD
2 + 3
mul
3 x 3
Prop. logic
p, q, r. . . .
p ^ q, ~r, p v q. . .

Next Topic:

Logical Connectives –

1. Negation / NOT

p: This book is interesting.

What is the Negation of this proposition??

p: This book is interesting. → p

This book is NOT interesting → Negation of p → ¬p

It is Not the case that this book is interesting. → ¬p

¬ → Negation Symbol

Proof : p

Negation of p :

$\neg p \equiv \sim p \equiv \overline{p}$

$\equiv p'$

p: This book is interesting.

Negation of p :

This book is Not interesting. $\neg p$

It is Not the case that this book is interesting. $\neg p$

It is Not true that this book is interesting. $\neg p$

It is false that this book is interesting. $\neg p$

This book is interesting, is false. $\neg p$

p: 2 is a prime number.

2 is NOT a prime number. $\neg p$

It is Not the case that 2 is a prime number. $\neg p$

It is Not true that 2 is a prime number. $\neg p$

It is false that 2 is a prime number. $\neg p$

2 is a prime number, is false. $\neg p$

$\neg p$

Negation. Negation Operator, "not", has symbol ¬.

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Various Symbols for Negation:

Proposition : p

Negation of p :

$\neg p \equiv \sim p \equiv \overline{p} \equiv p' \equiv$ Not p

Proposition : p

NOT p.

It is Not the case that p.

It is Not true that p.

It is false that p.

p is false.

$\neg p$

Note :

Prop p

"p is false" $\equiv \neg p$

Find the negation of the proposition

"Michael's PC runs Linux"

and express this in simple English.

Solution: The negation is

"It is not the case that Michael's PC runs Linux."

This negation can be more simply expressed as

"Michael's PC does not run Linux."

Find the negation of the proposition

p "Michael's PC runs Linux"

and express this in simple English.

Solution: The negation is

$\neg p$

"It is not the case that Michael's PC runs Linux."

This negation can be more simply expressed as

"Michael's PC does not run Linux." $\neg p$

Find the negation of the proposition

"Vandana's smartphone has at least 32GB of memory"

and express this in simple English.

Solution: The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

or even more simply as

"Vandana's smartphone has less than 32GB of memory."

Find the negation of the proposition

"Vandana's smartphone has at least 32GB of memory"

and express this in simple English.

Solution: The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

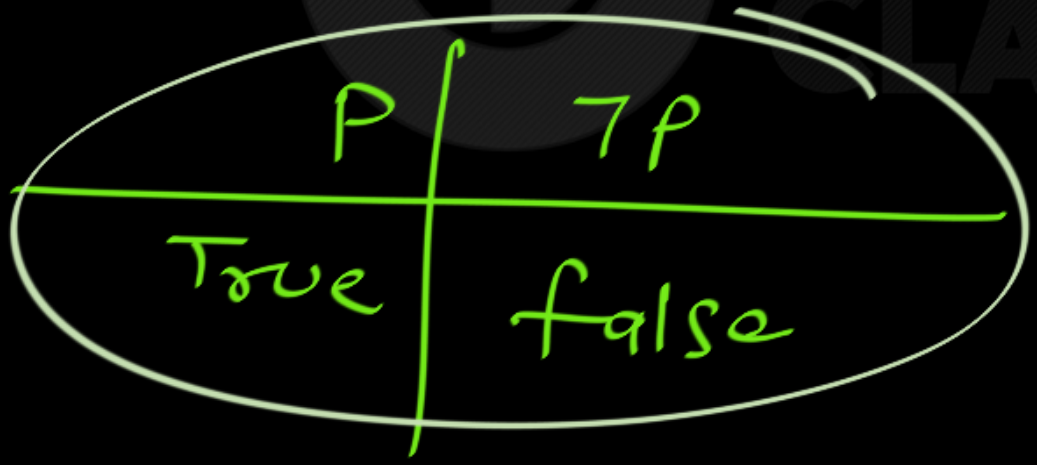
or even more simply as

"Vandana's smartphone has less than 32GB of memory."

Behaviour of ¬ :

p : 2 is a prime number. True

$\neg p$: 2 is NOT a prime number. : False



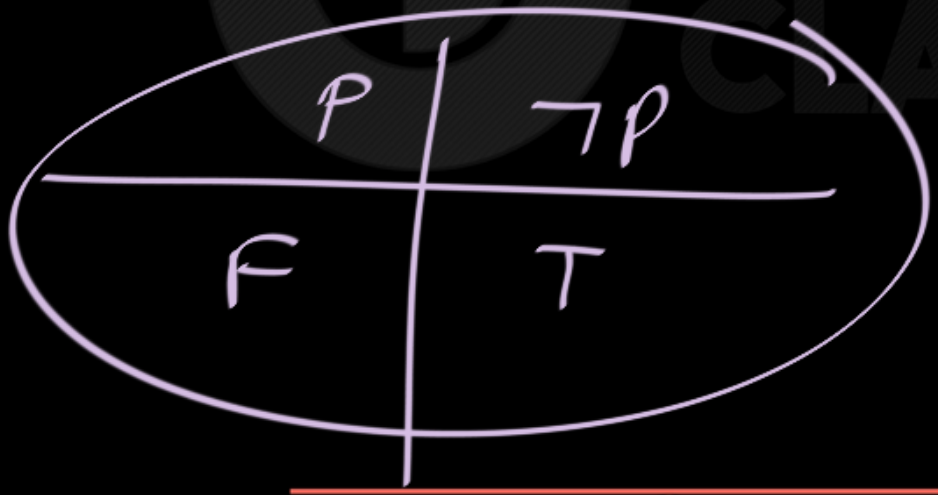
p : New Delhi is the capital of India. True

$\neg p$: New Delhi is NOT the capital of India. false

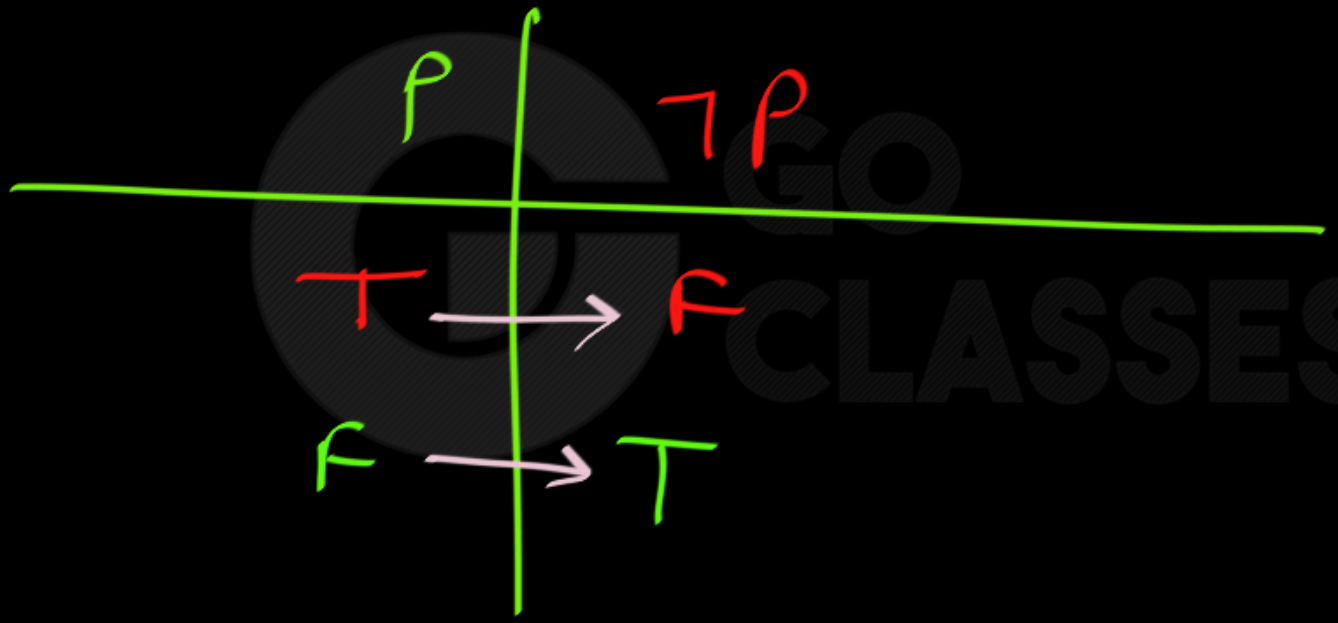
p	$\neg p$
T	F

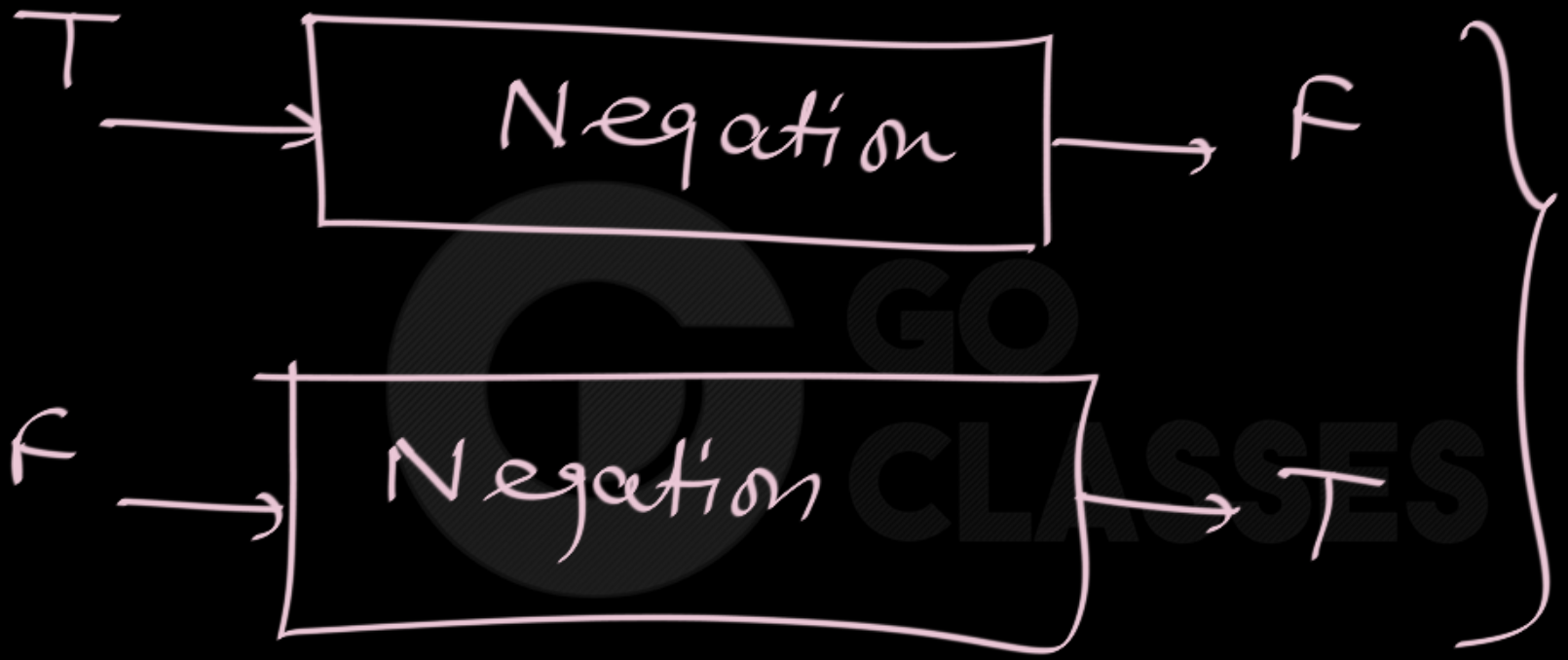
P : Mumbai is the capital of India. False

$\neg P$: Mumbai is NOT the capital of India. True




Prop
P
T
OR
F


P
¬P
T
F
F
T


T
Negation
F
F
Negation
T

2.3 Negation

basic logical operator is negation, a fancy way to say "not."

Definition 5. Let p be a proposition. The *negation* of p, denoted $\neg p$, is a proposition that is true when p is false, and false when p is true.

This operator is fairly straightforward: it simply takes the opposite truth value from p. A truth table for $\neg p$ takes the form:

p	$\neg p$
T	F
F	T

Truth Tables

p	$\neg p$
F	T
T	F

Negation NOT $\neg$

- The **negation** of a proposition p is denoted by $\neg p$ and has this truth table:

p	$\neg p$
T	F
F	T

- **Example**: If p denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."

The negation operator is a unary operator which, when applied to a proposition p, changes the truth value of p.
That is, the negation of a proposition p, denoted by ¬p, is the proposition that is false when p is true and true when p is false.

For example, if p is the statement "I understand this", then its negation would be "I do not understand this" or "It is not the case that I understand this."
Another notation commonly used for the negation of p is ~ p, or p'.

Summary:

Negation / NOT : **Unary operator**

Symbols: $\neg p$, $\sim p$, $\bar{p}$, p'

$\neg p$ → operand

p	¬p
T	F
F	T

Next Topic:

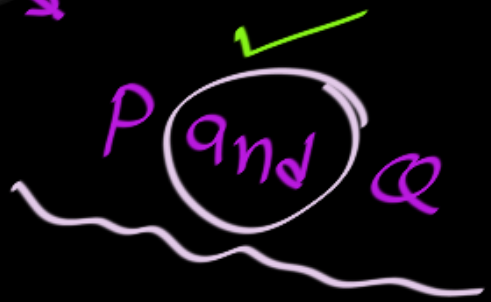
Logical Connectives –

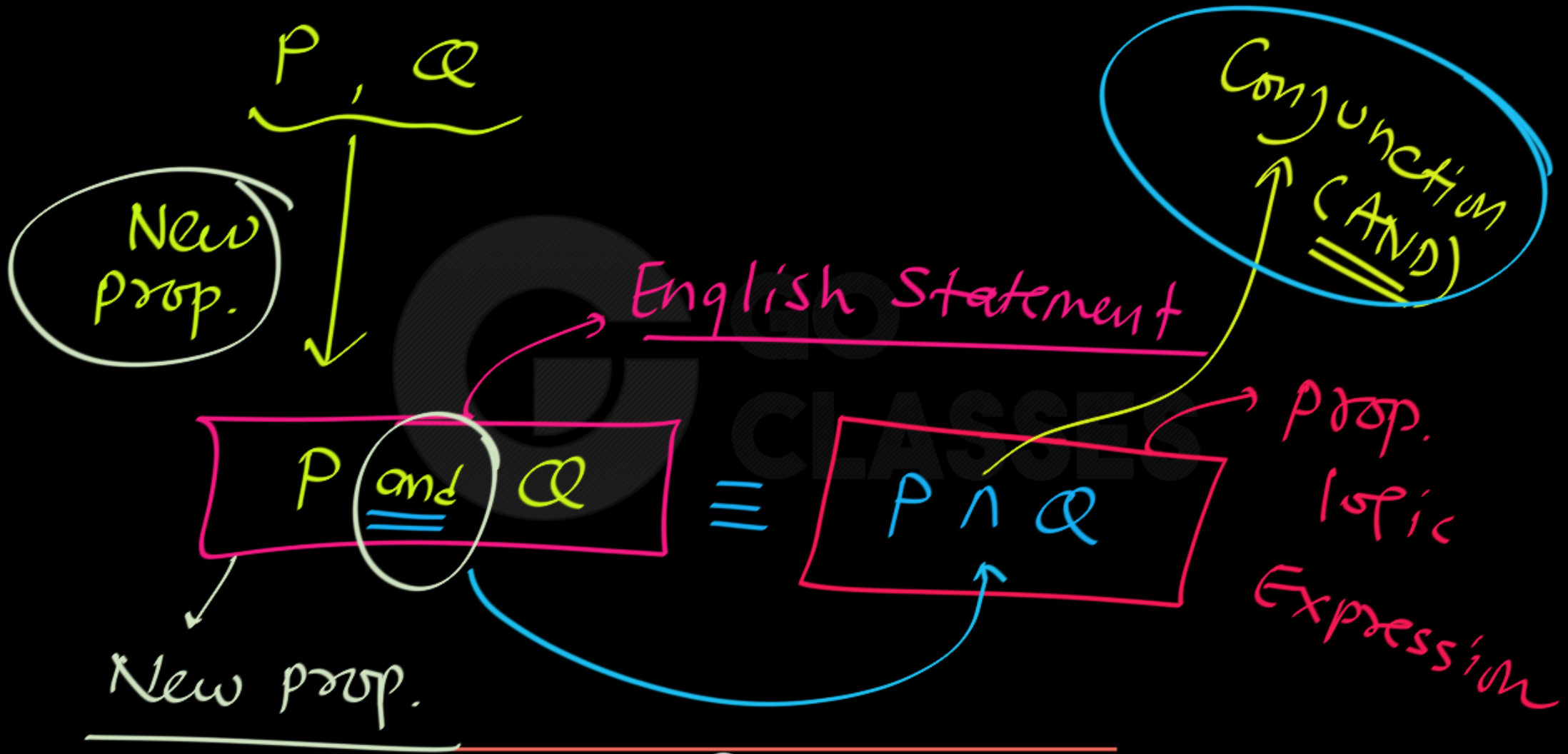
2. Conjunction / AND

P : 9 is divisible by 3.

Q : 5 is an odd number.

9 is divisible by 3 and 5 is an odd number.




P, Q
New prop.
Conjunction (AND)
English Statement
P and Q
≡
P ∧ Q
Prop. logic Expression
New prop.

Conjunction. Conjunction Operator, "and", has symbol $\wedge$.

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

p: This book is interesting. q: I am staying at home.

p ∧ q: This book is interesting, and I am staying at home.

p q

AND (Conjunction)

P : 10 is divisible by 3. → false

Q : 4 is an odd number. → false

P ∧ Q : 10 is divisible by 3 and 4 is an odd number. → false

P = false

Q = false

P	Q	P ∧ Q
F	F	F

P : 9 is divisible by 3. True

Q : 4 is an odd number. false

false

$P \wedge Q$: 9 is divisible by 3 and 4 is an odd number.

AND

P = True

Q = false

P	Q	P∧Q
T	F	F

P : 10 is divisible by 3.

Q : 5 is an odd number.

false

P ∧ Q : 10 is divisible by 3 and 5 is an odd number.

P = false

Q = True

AND

P	Q	P ∧ Q
F	T	F

P : 9 is divisible by 3. → T

Q : 5 is an odd number. → T

True

P ∧ Q : 9 is divisible by 3 and 5 is an odd number.

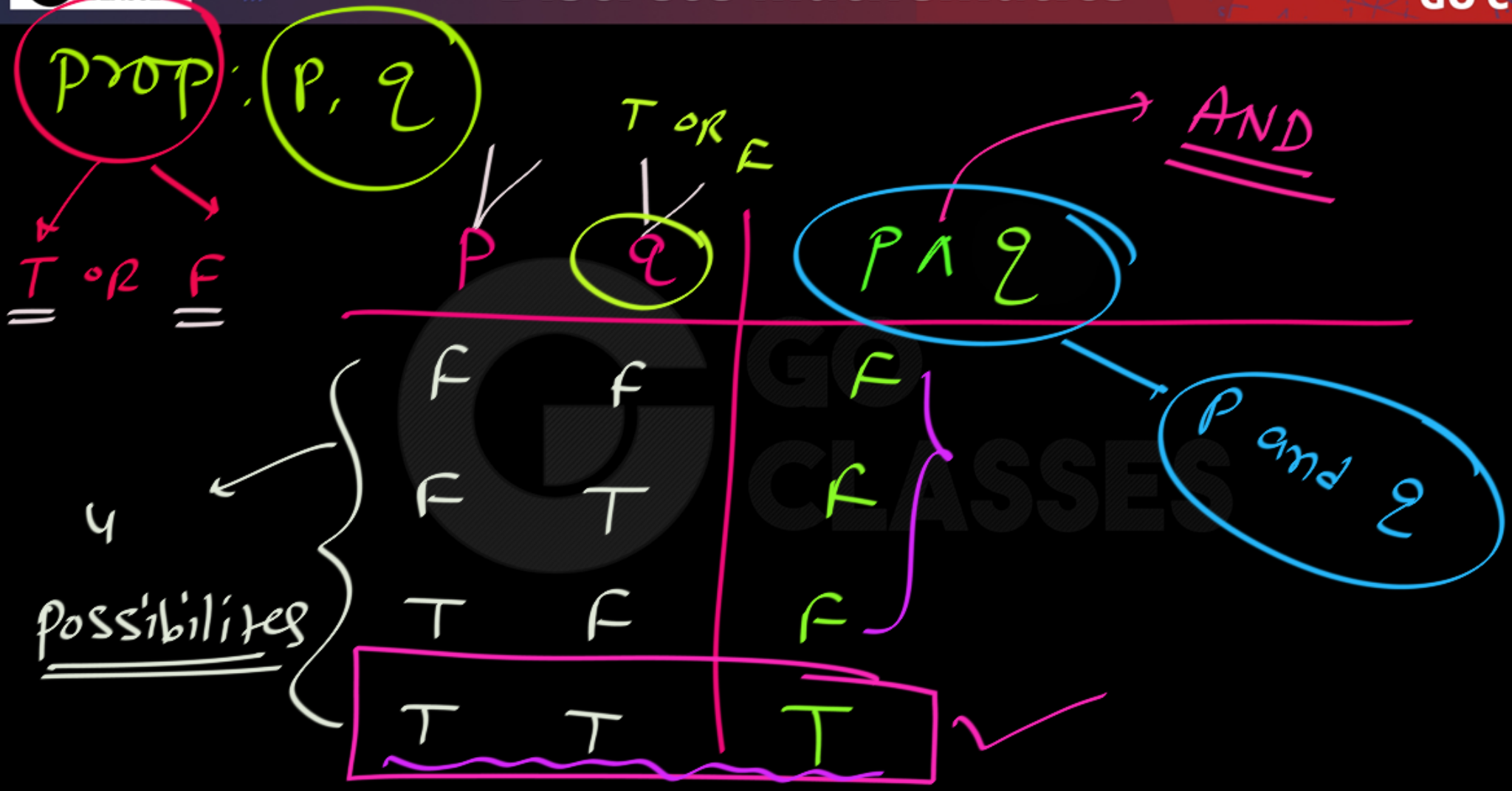
P = True

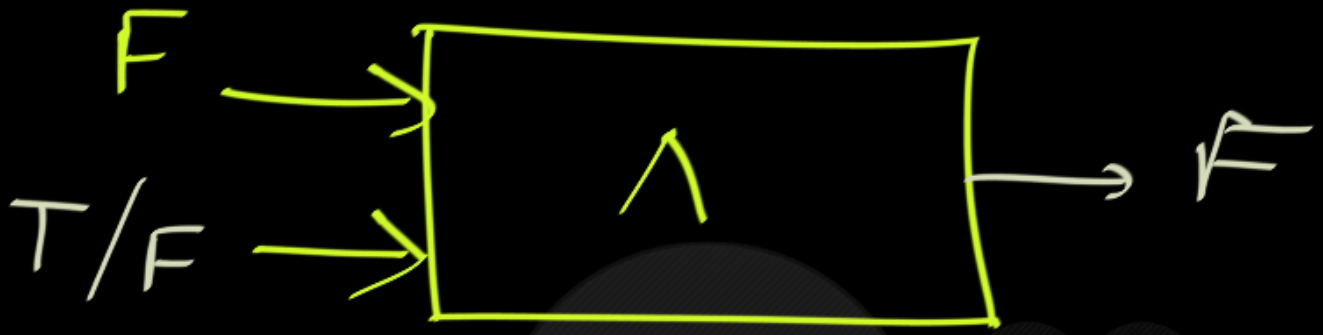
Q = True

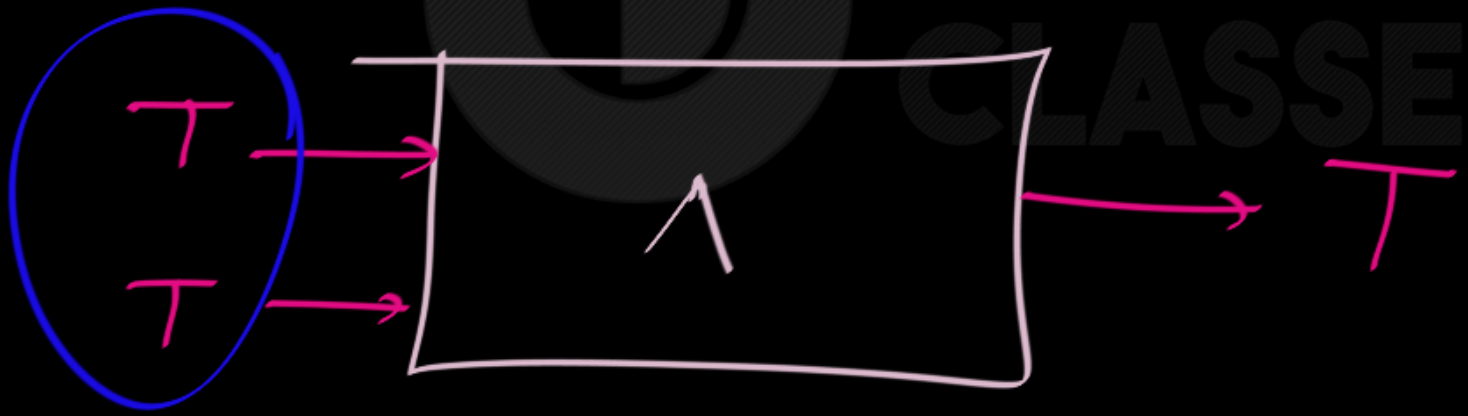
P	Q	P ∧ Q
T	T	T

Truth Table :

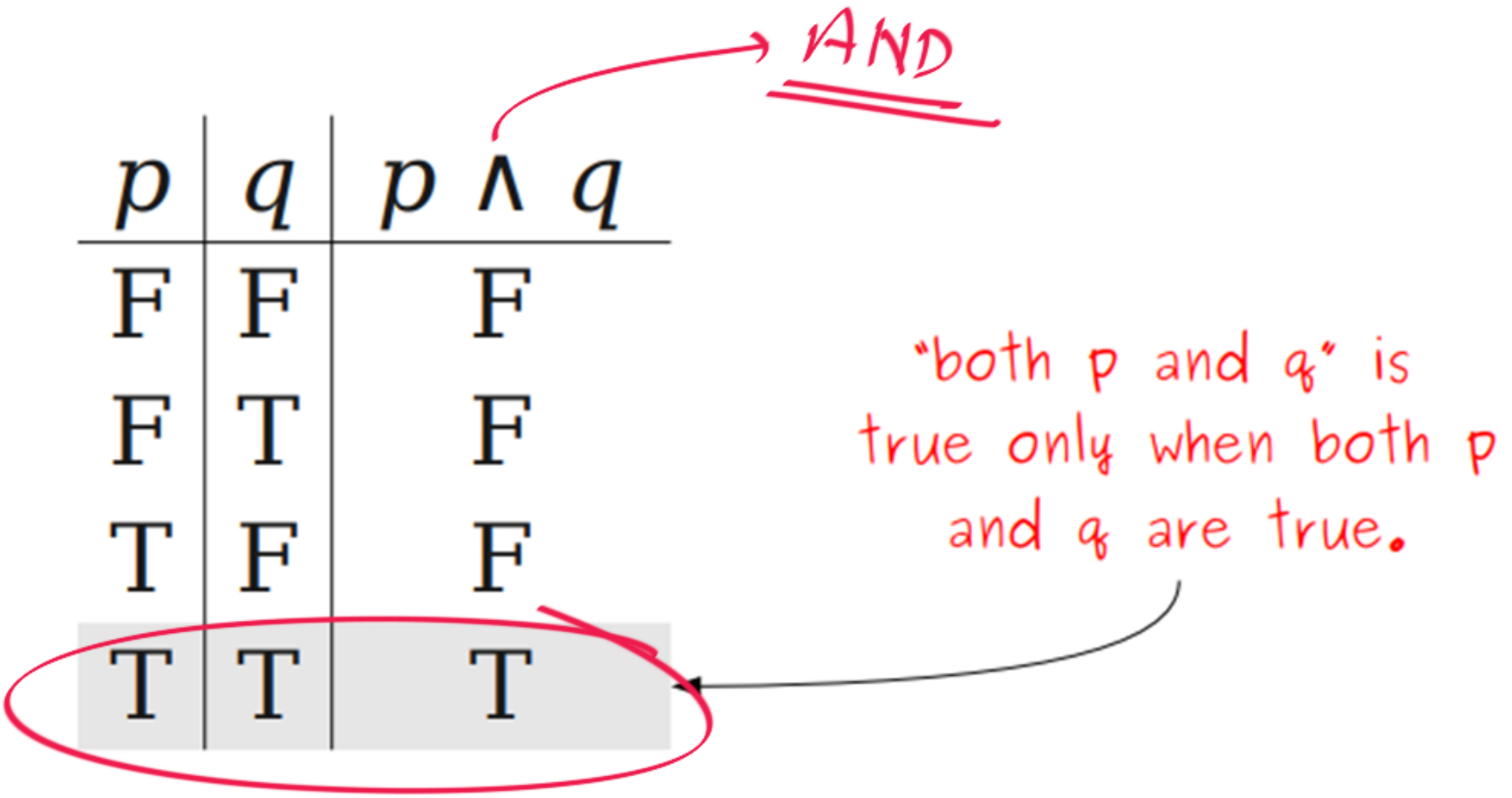
Truth table tells us about the Truth Value of a Compound Proposition for each combination of truth values of atomic propositions.


Prop : P, Q
T OR F
T OR F
P
Q
T OR F
AND
P ∧ Q
P and Q
4
possibilites
f f f
f T f
T f f
T T T


F
T/F
∧
F

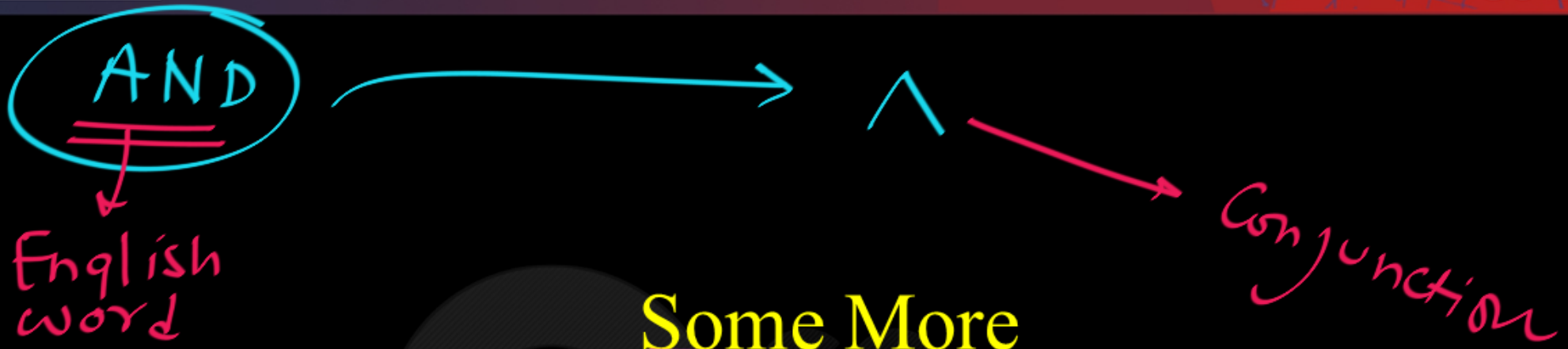

T
T
∧
T

Truth Tables



Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

Solution: The conjunction of these propositions, $p \wedge q$, is the proposition "Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz." This conjunction can be expressed more simply as "Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz." For this conjunction to be true, both conditions given must be true. It is false, when one or both of these conditions are false.



Some More English Words which Translate to Conjunction:

- P : Joe has received below a C in a class.
- Q : Joe has maintained a B average

Joe has maintained a B average even though he did receive a grade below a C in a class.

Joe has maintained a B average and he did receive a grade below a C in a class.

- P : Joe has received below a C in a class.
- Q : Joe has maintained a B average

Joe did receive a grade below a C in a class, however, Joe has maintained a B average.

$P \wedge Q$

Joe did receive a grade below a C in a class, yet, Joe has maintained a B average.

$P \wedge Q$

Joe has maintained a B average though he did receive a grade below a C in a class.

- P : Joe has received below a C in a class.
- Q : Joe has maintained a B average

Joe has maintained a B average, Moreover he received a grade below a C in a class.

Joe received a grade below a C in a class, nevertheless, Joe has maintained a B average.

Joe has maintained a B average BUT he received a grade below a C in a class.

Dictionary

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English

Search for a word

nevertheless

/nɛvəðəˈlɛs/

adverb

in spite of that; notwithstanding; all the same.
"statements which, although literally true, are nevertheless misleading"

Similar: in spite of that/everything | nonetheless | even so | however | but | still

English : AND ≠ BUT ≠ Although ≠ However ≠ Moreover ≠ Yet ≠ Still ≠ Even though ≠ though ≠ Nevertheless ≠ Comma

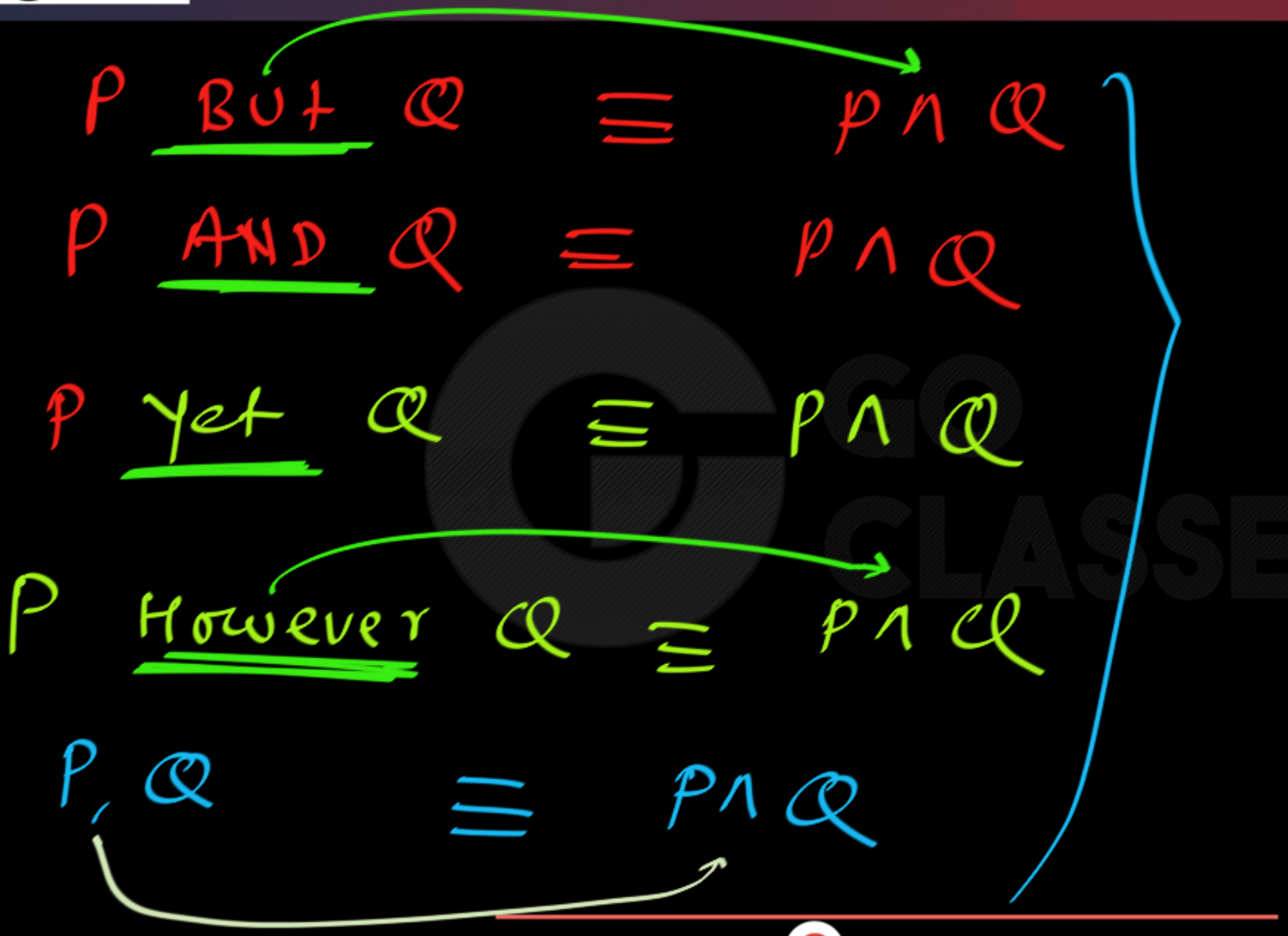
Logic : All Above words Translate to Conjunction.

The conjunction operator is the binary operator which, when applied to two propositions p and q, yields the proposition "p and q", denoted p∧q. The conjunction p∧q of p and q is the proposition that is true when both p and q are true and false otherwise.

NOTE : In Logic, we have -

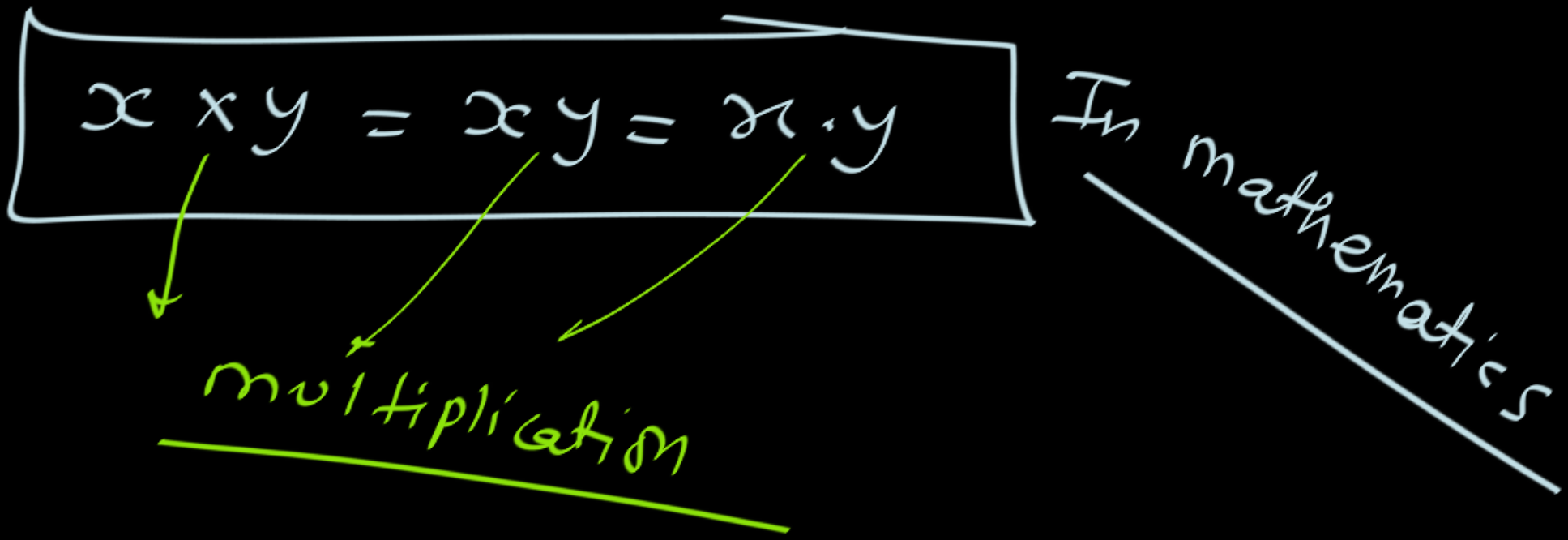
But = AND = Though = Even Though = However = Yet = Nonethless = Neverthless = Moreover

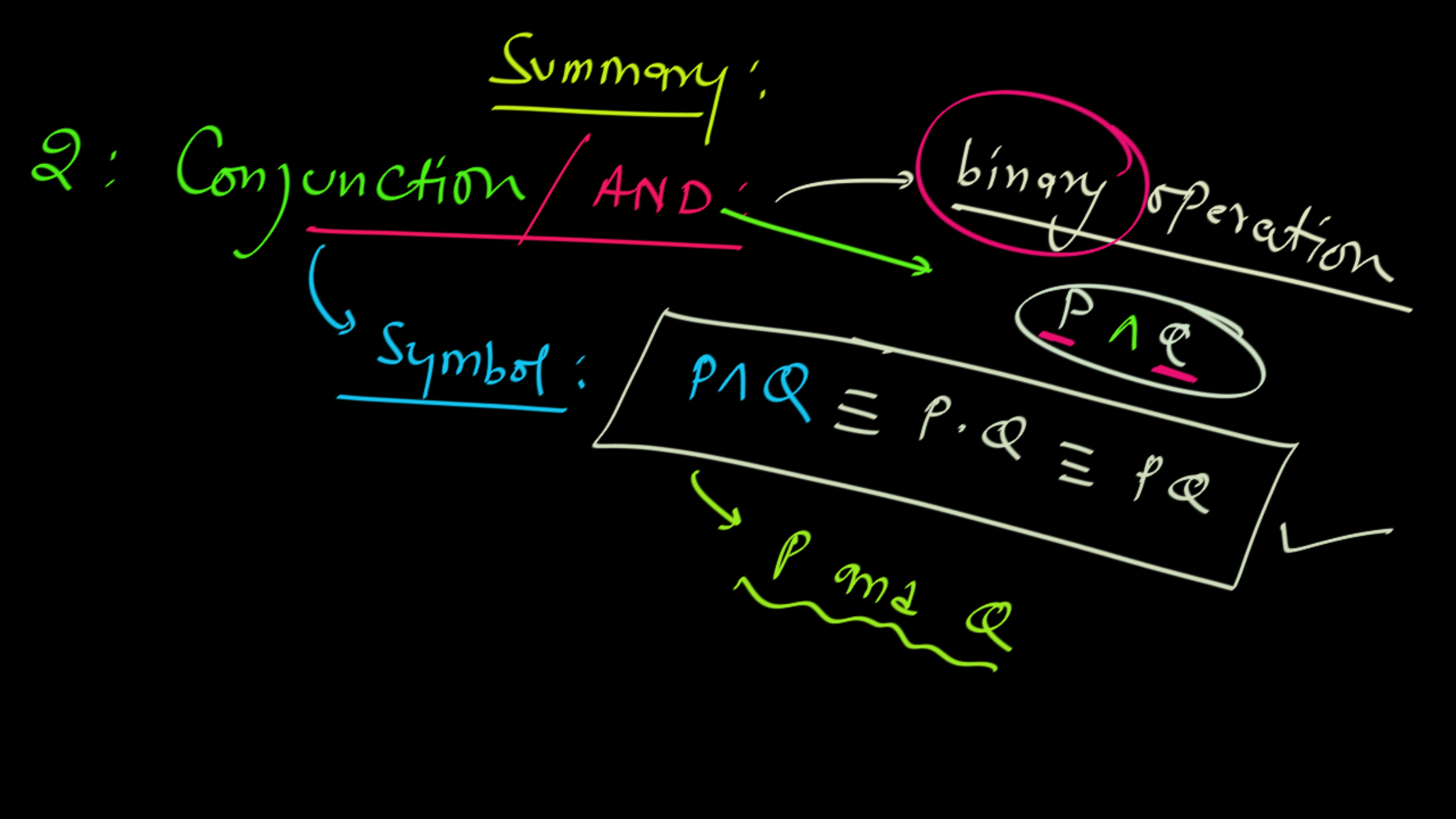
English statements such as "even though", "however", "but", and "yet" all have usages that contrast facts, but they are all translated to "and" in propositional logic.


P But Q ≡ P ∧ Q
P And Q ≡ P ∧ Q
P Yet Q ≡ P ∧ Q
P However Q ≡ P ∧ Q
P, Q ≡ P ∧ Q

2 is even, prime .

, comma ≡ AND


x x y = x y = x · y
In mathematics
multiplication


Summary:
2: Conjunction / AND:
binary operation
Symbol:
P ∧ Q
P ∧ Q ≡ P · Q ≡ PQ
P and Q

Next Topic:

Logical Connectives –

3. Disjunction / OR

P : 9 is divisible by 3.

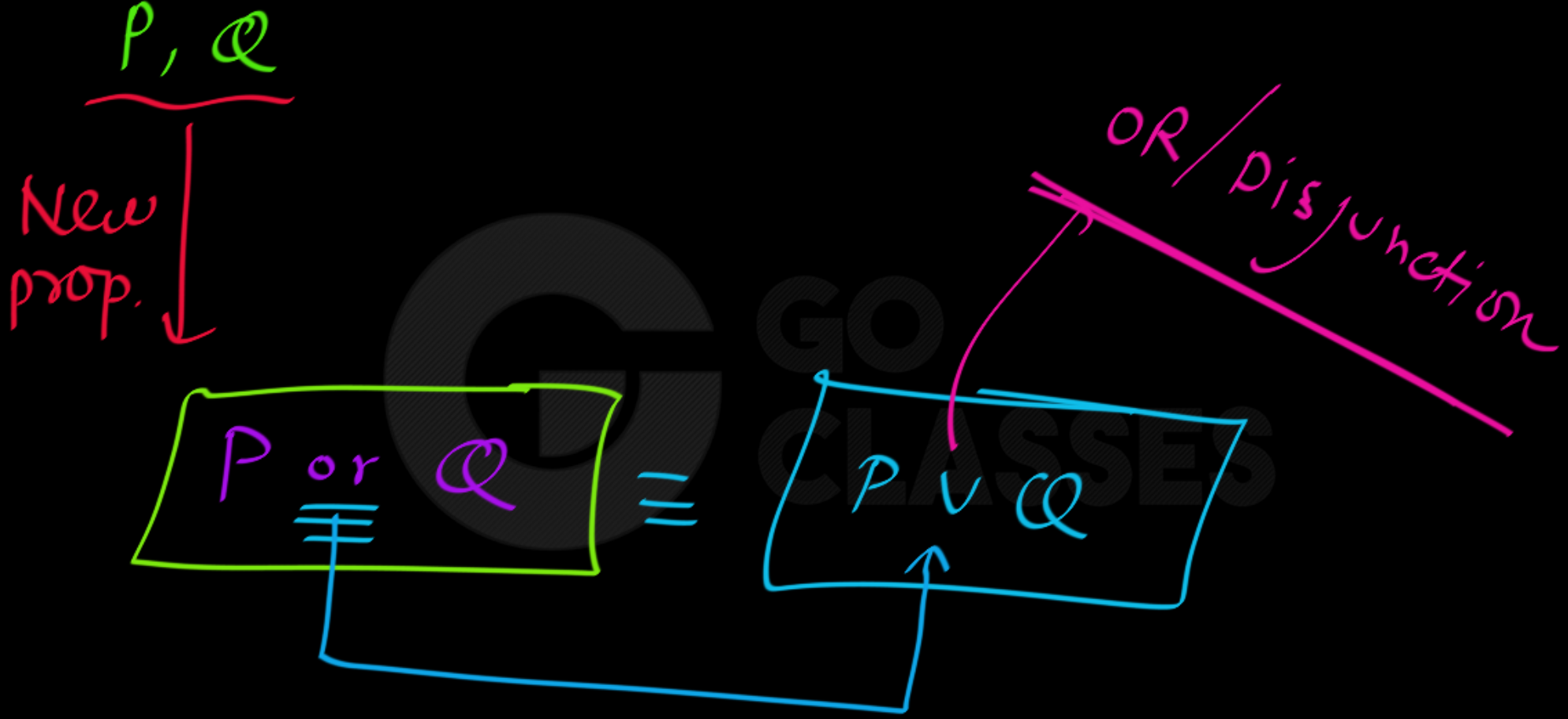
Q : 5 is an odd number.

9 is divisible by 3 OR 5 is an odd number.

P

Q

New Statement


P, Q
New prop.
P or Q
≡
P ∨ Q
OR / Disjunction

Disjunction. Disjunction Operator, inclusive "or", has symbol $\vee$.

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

p: This book is interesting. q: I am staying at home.

p ∨ q: This book is interesting, or I am staying at home.

OR

P : 9 is divisible by 3. → True

Q : 5 is an odd number. → True

P v Q : 9 is divisible by 3 OR 5 is an odd number. → True

P = T

Q = T

P	Q	P v Q
T	T	T

P : 9 is divisible by 3. → True

Q : 4 is an odd number. → false

→ True

P v Q : 9 is divisible by 3 OR 4 is an odd number.

P	Q	P v Q
T	F	T

P : 10 is divisible by 3. — f

Q : 5 is an odd number. → T

True

P v Q : 10 is divisible by 3 OR 5 is an odd number.

false

True

P	Q	P v Q
F	T	T ✓

P : 10 is divisible by 3. → F

Q : 4 is an odd number. → F

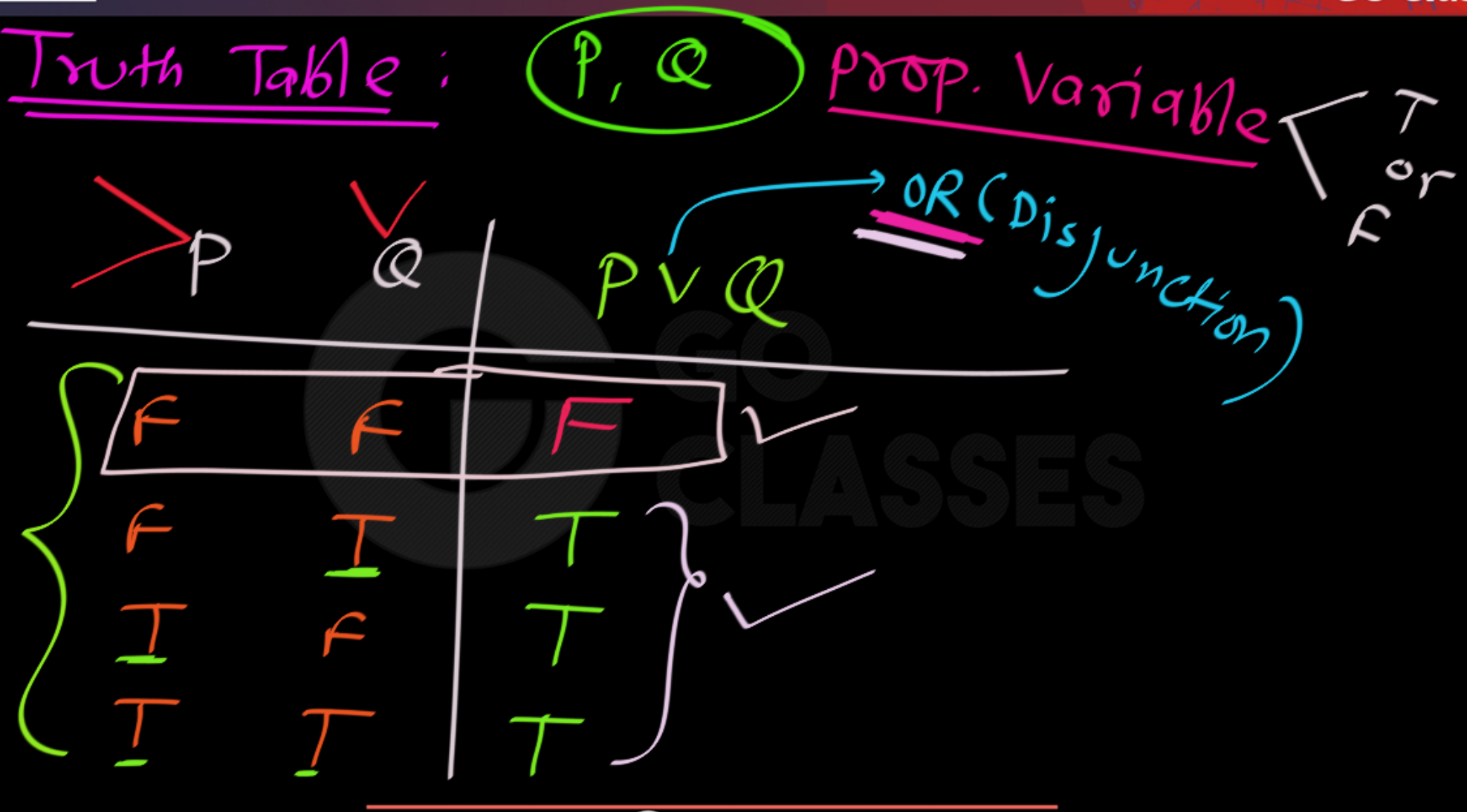
false

P v Q : 10 is divisible by 3 OR 4 is an odd number.

false

false

P	Q	P v Q
F	F	F


Truth Table :
P, Q
Prop. Variable
T or F
P
Q
P ∨ Q
OR (Disjunction)
F F F
F T T
T F T
T T T

Truth Table :

Truth table tells us about the Truth Value of a Compound Proposition for each combination of truth values of atomic propositions.

Truth Tables

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

"p or q" is true even if both p and q are true.
Remember that there are three ways for "p or q" to be true!

Summary :

(3) OR (Disjunction)

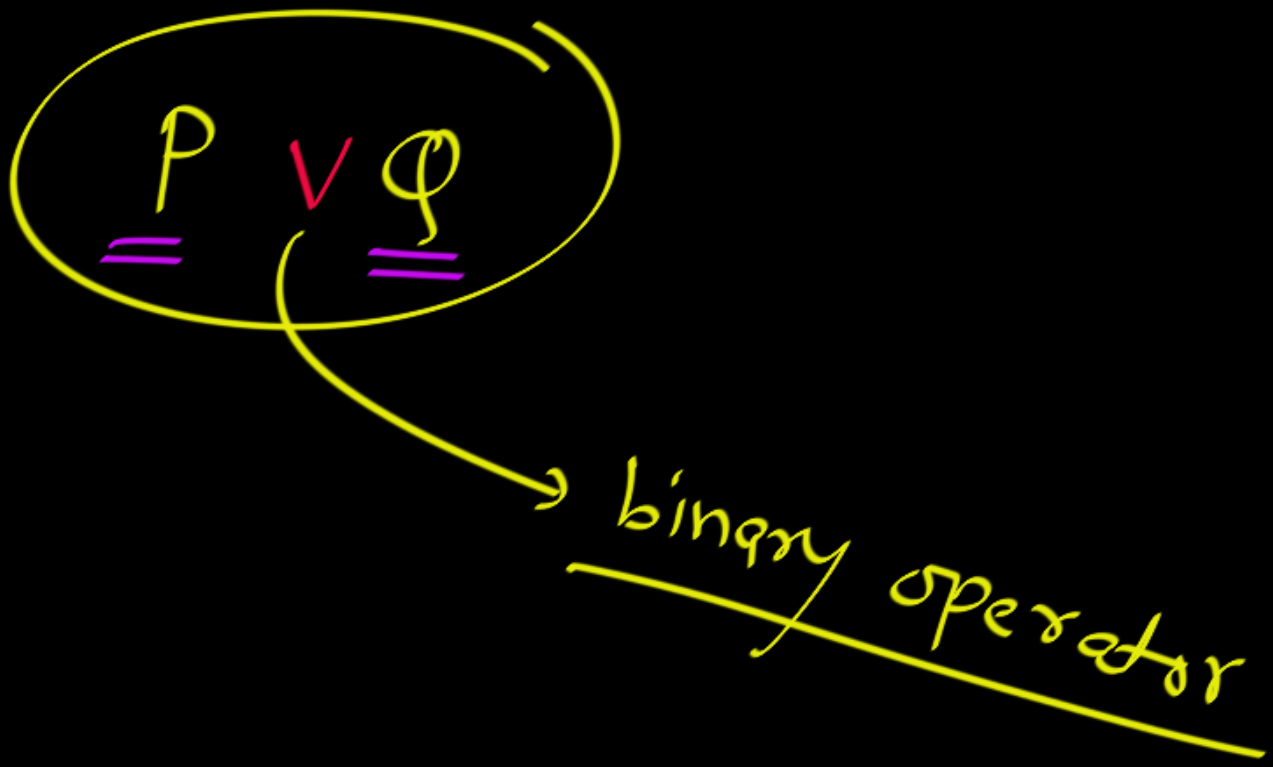
Symbols : $p \vee q$

OR

$p + q \equiv p \vee q$

NOT ADDition

p OR q


P V Q
binary operator

The disjunction operator is the binary operator which, when applied to two propositions p and q, yields the proposition "p or q", denoted p V q. The disjunction p V q of p and q is the proposition that is true when either p is true, q is true, or both are true, and is false otherwise.

NOTE : In Logic –

Or = Inclusive Or

Next Topic:

Logical Connectives –

4. Exclusive OR (xor)