Standard Forms of Boolean Expressions

Minterm, Maxterm, Canonical SOP, Canonical POS

Instructor:

Deepak Poonia
MTech, IISc Bangalore
GATE CSE AIR 53; AIR 67;
AIR 107; AIR 206; AIR 256

Digital Logic Complete Course:

https://www.goclasses.in/courses/Digital-Logic

GO Classes

Test Series

On "GATE Overflow" Website

Join **GO+ GO Classes Combined Test Series** for BEST quality tests, matching GATE CSE Level:

Visit www.gateoverflow.in website to join Test Series.

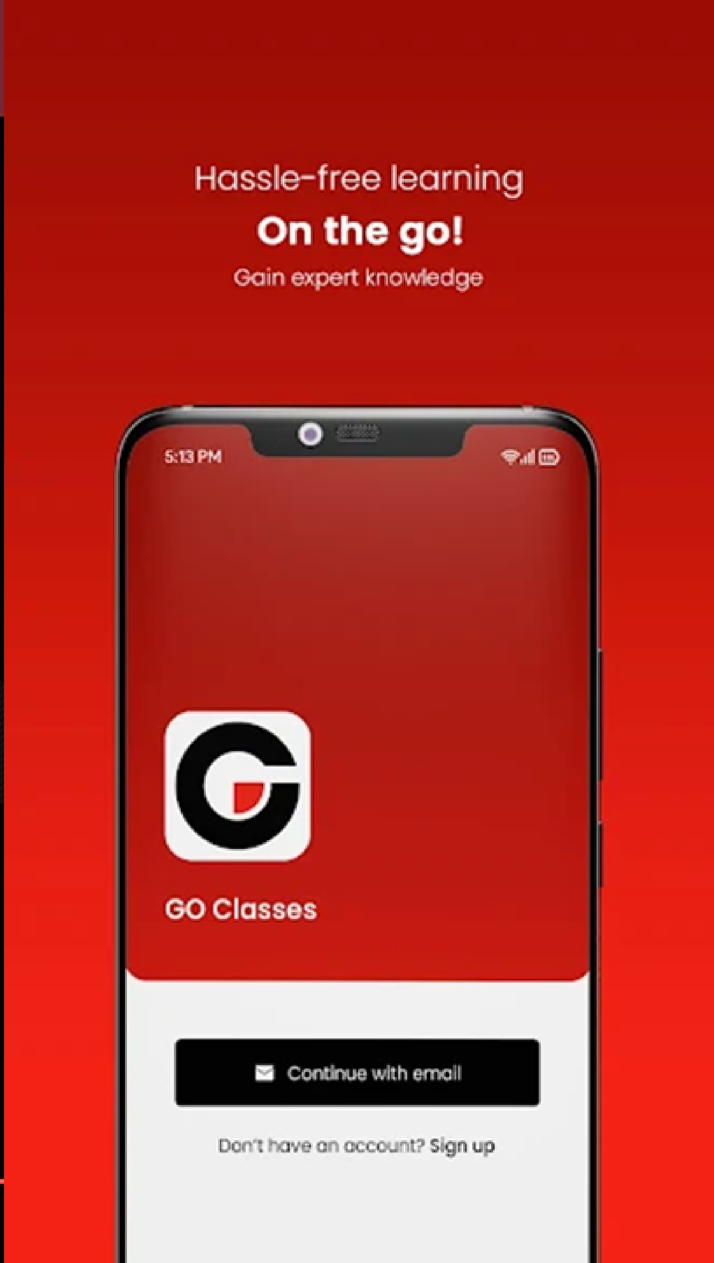
1. **Quality Questions:** No Ambiguity in Questions, All Well-framed questions.
2. Correct, **Detailed Explanation**, Covering Variations of questions.
3. **Video Solutions.**

https://gateoverflow.in/blog/14987/gate-overflow-and-go-classes-test-series-gate-cse-2024

Download the GO Classes Android App:

https://play.google.com/store/apps/details?id=com.goclasses.courses

Search "GO Classes" on Play Store.



NOTE :

Complete Discrete Mathematics & Complete Engineering

Mathematics Courses, by GO Classes, are **FREE** for ALL learners.

Visit here to watch : https://www.goclasses.in/s/store/

SignUp/Login on Goclasses website for free and start learning.

We are on Telegram. **Contact us** for any help.

Link in the Description!!

Join GO Classes **Doubt Discussion** Telegram Group :

Username:

@GATECSE_GOCLASSES

We are on Telegram. **Contact us** for any help.

Join GO Classes Telegram Channel, Username: **@GOCLASSES_CSE**

Join GO Classes **Doubt Discussion** Telegram Group :

Username: **@GATECSE_Goclasses**

(Any doubt related to Goclasses Courses can also be asked here.)

Join GATEOverflow Doubt Discussion Telegram Group :

Username: **@GateOverflow_CSE**

GO CLASSES


GO CLASSES
GO CLASSES
GO CLASSES
GO CLASSES
GO CLASSES
GO CLASSES

Standard Forms of Boolean Expressions

Minterm, Maxterm,

Canonical SOP, Canonical POS

Standard Forms of Boolean Expressions

- Minterm, Maxterm
- Expressing Boolean function using Minterms, Maxterms
- Canonical SOP, Canonical POS

Standard Forms of Boolean Expressions

Minterm, Maxterm

Standard Forms of Boolean Expressions

Minterm, Maxterm

1. Definition

- Minterm
  - ☞An product of literals in which each variable is represented once and only once in either its complemented or uncomplemented form.

- Maxterm
  - ☞An sum of literals in which each variable is represented once and only once in either its complemented or uncomplemented form.

Minterm and Maxterm

- For a switching function, a ***literal*** is defined as a variable in uncomplemented or complemented form.
  - Example: x, x’, y, y’, etc.
- Consider an n-variable switching function f $(x_1, x_2, \ldots, x_n)$.
  - A product term (that is, an AND operation) of all the n literals is called a ***minterm***.
  - A sum term (that is, an OR operation) of all the n literals is called a ***maxterm***.
- Consider a 3-variable function f (A, B, C).
  - Examples of minterm: A’.B’.C’, A.B’.C, A.B.C, etc.
  - Examples of maxterm: (A + B’ + C’), (A’ + B’ + C’), (A + B + C), etc.

Minterms over 2 variables a,b

a product term containing EVERY variable

'Exactly once', in complemented form OR uncomplemented form but not both.

Variables a,b :

ab ✓ ab' ✓ a'b ✓

a'b' ✓

a+b ✗

a ✗

aa'b ✗

Maxterms over 2 variables a,b

① 'Sum' term

② Contains Every Variable Exactly once either in Comp. or uncomp form but not both.

Variables a, b :

$a+b$ ✓ $a'+b$ ✓ ab ✗

$a+b'$ ✓ $a'+b'$ ✓ a ✗ $a+a'+b$ ✗

Standard Forms of Boolean Expressions

Minterm, Maxterm

2. An Important Property of Minterm, Maxterm

Q:

Consider the following minterm over variables a,b,c:

ab'c

For how many input combinations, does it become 1?

$a \cdot \bar{b} \cdot c = 1 \longrightarrow a=1, b=0, c=1$

for all other input Combinations:

$a=1, b=1, c=1 \longrightarrow ab'c = 0$

$a=0, b=0, c=1 \longrightarrow ab'c = 0$

$a=1, b=1, c=0 \longrightarrow ab'c = 0$

_ _ _ _

$\boxed{a \cdot \bar{b} \cdot c} = 1 \longrightarrow a=1, b=0, c=1$

for all other input Combinations, it is false.

Since $ab'c$ is 1 only for one input Combination $a=1$, $b=0$, $c=1$; So, we say $ab'c$ is the minterm associated with $a=1$, $b=0$, $c=1$ input Combination.

Q:

Consider the following minterm over variables a,b,c:

abc

For how many input combinations, does it become 1?

Q:

Consider the following minterm over variables a,b,c:

abc

For how many input combinations, does it become 1? → for only 1 input Combination

a=1, b=1, c=1 ←

minterm: abc

abc is 1 only for $a=1, b=1, c=1$

for all other input combinations; $abc = 0$

So, we say minterm abc is associated with $a=1, b=1, c=1$ input combination.

Q:

Consider the following minterm over variables a,b,c:
a’b’c

For how many input combinations,
does it become 1?

Q:

Consider the following minterm over variables a,b,c:

Associated with

$a'b'c$

For how many input combinations, does it become 1?

for only 1 input Combination

$\boxed{a=0,\ b=0,\ c=1}$

- Properties of minterms and maxterms:
  - A minterm assumes value 1 for exactly one combination of variables.
  - A maxterm assumes the value 0 for exactly one combination of variables.
- Example:

Two Variables: a, b $\longrightarrow$ 4 minterms

	a	b	minterm $a'b'$	minterm $a'b$	minterm ab'	minterm ab
0	0	0	1 ✓	0	0	0
1	0	1	0	1 ✓	0	0
2	1	0	0	0	1 ✓	0
3	1	1	0	0	0	1 ✓

Two Variables: a, b

	a	b	Associated minterm
0	0	0	$a'b'$
1	0	1	$a'b$
2	1	0	ab'
3	1	1	ab

Observation:

for input Combination

$a = 0, b = 1$

Associated minterm

$a'b$

Input Combination:

a	b	c	Associated minterm
0	1	0	$a'bc'$
1	1	0	abc'

Note:

for input combination:

a	b	c	d	e
1	0	0	1	1

minterm

$ab'c'de$

Two Variables: a, b

Another Simple Notation for minterm

	a	b	Associated minterm	
0	0	0	$a'b'$	m_0
1	0	1	$a'b$	m_1
2	1	0	ab'	m_2
3	1	1	ab	m_3

Q:

Consider the following minterm over 5 variables:

$a'bcd'e'$

How to find the Input Combination associated with it?

Q:

Consider the following minterm over 5 variables:

$a'bcd'e'$

How to find the Input Combination associated with it?

$a=0$; $b=1$; $c=1$ $d=0$ $e=0$

Minterms

For a function of n variables, a product term in which each of the n variables appears once is called a *minterm*. The variables may appear in a minterm either in uncomplemented or complemented form. For a given row of the truth table, the minterm is formed by including x_i if $x_i = 1$ and by including $\overline{x}_i$ if $x_i = 0$.

Source:
Fundamentals of Digital Logic with Verilog Design by Stephen Brown and Zvonko Vranesic
Department of Electrical and Computer Engineering, University of Toronto

Minterms

Each row has a unique minterm

X	Y	**Minterm**	$\overline{X}\,\overline{Y}$	$\overline{X}Y$	$X\overline{Y}$	XY
0	0	$\overline{X}\,\overline{Y}$	1	0	0	0
0	1	$\overline{X}Y$	0	1	0	0
1	0	$X\overline{Y}$	0	0	1	0
1	1	XY	0	0	0	1

The minterm is the product term that is 1 for only its row

Q:

Consider the following maxterm over variables a,b,c:

$a+b'+c$

For how many input combinations, does it become 0?

Q:

Consider the following maxterm over variables a,b,c:

$a+b'+c$

For how many input combinations, does it become 0? → for only 1 input Combination

Variables: a, b, c

maxterm: $a + b' + c$

$a + b' + c = 0 \longrightarrow a = 0, b = 1, c = 0$

for all other input Combinations, $a + b' + c$ is 1.

So, Since maxterm $a + b' + c$ is 0 for unique Combination $a = 0, b = 1, c = 0$; So we say maxterm $a + b' + c$ is Associated with this input.

Q:

Consider the following maxterm over variables a,b,c:

$a + b + c$

For how many input combinations,
does it become 0?

Q:

Consider the following maxterm over variables a,b,c:

Associated with

$a + b + c$

For how many input combinations, does it become 0? → for only 1 input combination

$\boxed{a=0, b=0, c=0}$

Q:

Consider the following maxterm over variables a,b,c:

Associated with

$a' + b' + c$

For how many input combinations, does it become 0? for only 1.

$\boxed{a = 1, b = 1, c = 0}$

- Properties of minterms and maxterms:
  - A minterm assumes value 1 for exactly one combination of variables.
  - A maxterm assumes the value 0 for exactly one combination of variables.
- Example:

Two variables a, b : → 4 maxterms

	a	b	maxterm $a+b$	maxterm $a+b'$	maxterm $a'+b$	maxterm $a'+b'$
0	0	0	0 ✓	1	1	1
1	0	1	1	0 ✓	1	1
2	1	0	1	1	0 ✓	1
3	1	1	1	1	1	0 ✓

Two variables a, b: $\rightarrow$ 4 maxterms

	a	b	Associated maxterm
0	0	0	$a+b$
1	0	1	$a+b'$
2	1	0	$a'+b$
3	1	1	$a'+b'$

Observation:

for $a=0, b=1$

maxterm: $a+b'$

for $a=1, b=0$

maxterm: $a'+b$

Observation:

$a_1 \quad a_2 \quad - \quad v \quad - \quad a_n$

Associated maxterm

$- \quad - \quad 1 \quad - \quad - \quad - \longrightarrow \cdots + V' + \cdots$

$- \quad - \quad - \quad 0 \quad - \quad - \quad - \longrightarrow \cdots + V + \cdots$

Two variables a, b : → 4 maxterms

	a	b	Associated maxterm	Another simple Notation for maxterm
0	0	0	$a+b$	M_0
1	0	1	$a+b'$	M_1
2	1	0	$a'+b$	M_2
3	1	1	$a'+b'$	M_3

Q:

Consider the following maxterm over 5 variables:

$a + b' + c + d + e'$

How to find the Input Combination associated with it?

Q:

Consider the following maxterm over 5 variables:

$a + b' + c + d + e'$

How to find the Input Combination associated with it?

$a = 0 \quad b = 1 \quad c = 0 \quad d = 0 \quad e = 1$

$a + b' + c' + d + e'$

Associated

input Combination

Unique Combination for which this maxterm becomes 0.

$\underbrace{a+b'+c'+d+e'}=0 \rightarrow a=0, b=1, c=1, d=0, e=1$

a	b	c	d	e	minterm	maxterm
–	–	–	0	–	$\ldots d' \ldots$	$\ldots + d + \ldots$
–	–	–	1	–	$\ldots d \ldots$	$\ldots + d' + \ldots$

- For Minterms:
  - "1" means the variable is "Not Complemented" and
  - "0" means the variable is "Complemented".
- For Maxterms:
  - "0" means the variable is "Not Complemented" and
  - "1" means the variable is "Complemented".

Maxterms

Each row has a unique maxterm

X	Y	**Maxterm**	$X+Y$	$X+\overline{Y}$	$\overline{X}+Y$	$\overline{X}+\overline{Y}$
0	0	$X+Y$	0	1	1	1
0	1	$X+\overline{Y}$	1	0	1	1
1	0	$\overline{X}+Y$	1	1	0	1
1	1	$\overline{X}+\overline{Y}$	1	1	1	0

The maxterm is the sum term that is 1 for only its row

- Properties of minterms and maxterms:
  - A minterm assumes value 1 for exactly one combination of variables.
  - A maxterm assumes the value 0 for exactly one combination of variables.
- Example:

Standard Forms of Boolean Expressions

Minterm, Maxterm

3. Short Notation

for Minterm, Maxterm

Short Notation for Minterms:

Variables a,b,c

	a	b	c	minterm
3	0	1	1	$a'bc = m_3$
6	1	1	0	$abc' = m_6$
7	1	1	1	$abc = m_7$

m_i

Short Notation for Maxterms:

Variables a,b,c

M_i

	a	b	c	maxterm
3	0	1	1	$a+b'+c' = M_3$
5	1	0	1	$a'+b+c' = M_5$
6	1	1	0	$a'+b'+c = M_6$
7	1	1	1	$a'+b'+c' = M_7$

maxterm → sum term

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

TAKE CARE when using Short Notation (m_i, M_i) for Minterms/Maxterms

TAKE CARE when using Short Notation (m_i, M_i) for Minterms/Maxterms

Depends on Agreed order of Variables.

Short Notation for Minterms:

Variables y, s, b

Which is Ambiguous??

- s'by
- m3

Short Notation for Minterms:

Variables y, s, b

Which is Ambiguous??

- s'by
- m3

"Never" any Ambiguity

Depends on the Agreed order of variables

If Agreed order of variables then $m_3 = ?$

s b y

$f(s, b, y)$

order

Given order

	s	b	y	m_3
3	0	1	1	$s'by$ ✓

If Agreed order of variables } then m_3 = ?

b s y

f(b, s, y)

Order

Given order

	b	s	y	m_3
3	0	1	1	b's y

By Default, Agreed order of Variables is the "Alphabetical order."

In Question:

$f(c, b, a)$ Given then $c, b, a \rightarrow$ order

$f(a, b, c)$ Given then $a, b, c \rightarrow$ order

Standard Order

- Minterms and maxterms are designated with a subscript
- The subscript is a number, corresponding to a binary pattern
- The bits in the pattern represent the complemented or normal state of each variable listed in a standard order.
- All variables will be present in a minterm or maxterm and will be listed in the same order (usually alphabetically)
- Example: For variables a, b, c:
  - Maxterms: $(a + b + \overline{c}), (a + b + c)$
  - Terms: $(b + a + c), a\overline{c}$b, and $(c + b + a)$ are NOT in standard order.
  - Minterms: $a\overline{b}c, abc, a\overline{b}\overline{c}$
  - Terms: $(a + c), \overline{b}c$, and $(\overline{a} + b)$ do not contain all variables

Index Example in Three Variables

- Example: (for three variables)
- Assume the variables are called X, Y, and Z.
- The standard order is X, then Y, then Z.
- The Index 0 (base 10) = 000 (base 2) for three variables). All three variables are complemented for minterm 0 ($\overline{X}, \overline{Y}, \overline{Z}$) and no variables are complemented for Maxterm 0 (X, Y, Z).
  - Minterm 0, called m_0 is $\overline{X}.\overline{Y}.\overline{Z}$
  - Maxterm 0, called M_0 is $(X + Y + Z)$
  - Minterm 6 ?
  - Maxterm 6 ?

Index Example in Three Variables

- Example: (for three variables)
- Assume the variables are called X, Y, and Z.
- The standard order is X, then Y, then Z. — given order
- The Index 0 (base 10) = 000 (base 2) for three variables). All three variables are complemented for minterm 0 $(\bar{X}, \bar{Y}, \bar{Z})$ and no variables are complemented for Maxterm 0 (X, Y, Z).
  - Minterm 0, called m_0 is $\bar{X}.\bar{Y}.\bar{Z}$
  - Maxterm 0, called M_0 is $(X + Y + Z)$
  - Minterm 6 ? → $m_6 = xyz'$
  - Maxterm 6 ? → $M_6 = x' + y' + z$

product term

sum term

$6 = \begin{array}{ccc} x & y & z \\ 1 & 1 & 0 \end{array}$

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\bar{b}\bar{c}\bar{d}$	$a+b+c+d$
1	0001	$\bar{a}\bar{b}\bar{c}d$	?
3	0011	?	$a+b+\bar{c}+\bar{d}$
5	0101	$\bar{a}b\bar{c}d$	$a+\bar{b}+c+\bar{d}$
7	0111	?	$a+\bar{b}+\bar{c}+\bar{d}$
10	1010	$a\bar{b}c\bar{d}$	$\bar{a}+b+\bar{c}+d$
13	1101	$ab\bar{c}d$	?
15	1111	$abcd$	$\bar{a}+\bar{b}+\bar{c}+\bar{d}$

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\bar{b}\bar{c}\bar{d}$	$a+b+c+d$
1	0001	$\bar{a}\bar{b}\bar{c}d$	?
3	0011	?	$a+b+\bar{c}+\bar{d}$
5	0101	$\bar{a}b\bar{c}d$	$a+\bar{b}+c+\bar{d}$
7	0111	?	$a+\bar{b}+\bar{c}+\bar{d}$
10	1010	$a\bar{b}c\bar{d}$	$\bar{a}+b+\bar{c}+d$
13	1101	$ab\bar{c}d$	?
15	1111	$abcd$	$\bar{a}+\bar{b}+\bar{c}+\bar{d}$

$a+b+c+d'$

$a'b'cd$

$a'bcd$

$a'+b'+c+d'$

Boolean Algebra :

Minterm, Maxterm

Relationship

x	y	z	minterm	maxterm
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$

$$\overline{m_0} = M_0$$

$$\overline{x'y'z'} = x+y+z$$

$$\boxed{\overline{m_i} = M_i}$$

$$\boxed{\overline{M_i} = m_i}$$

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterm and Maxterm Relationship

- Review: DeMorgan's Theorem
  $\overline{x.y} = \bar{x} + \bar{y}$ and $\overline{x+y} = \bar{x}.\bar{y}$
- Two-variable example:
  $M_2 = \bar{x} + y$ and $m_2 = x.\bar{y}$
  Thus M_2 is the complement of m_2 and vice-versa.
- Since DeMorgan's Theorem holds for *n* variables, the above holds for terms of *n* variables
- giving:
  $M_i = \overline{m_i}$ and $m_i = \overline{M_i}$
  Thus M_i is the complement of m_i.

Function Tables for Both

- Minterms of 2 variables

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

- Each column in the maxterm function table is the complement of the column in the minterm function table since M_i is the complement of m_i.

Q:

For n variables, how many minterms are there??

A. Exactly 2^n
B. At Most 2^n
C. At Least 2^n
D. Can't Determine.

Q:

For n variables, how many minterms are there??

A. Exactly 2^n
B. At Most 2^n
C. At Least 2^n
D. Can't Determine.

Very Common misconception: for minterm, function value should be 1.

for n variables :

Exactly 2^n minterms

Exactly 2^n maxterms

4 Variables:

minterm:

a b c d

$\underbrace{a\ ,\ \bar{a}}_{2}$ $\underbrace{b\ ,\ \bar{b}}_{2}$ $\underbrace{c\ ,\ \bar{c}}_{2}$ $\underbrace{d\ ,\ d'}_{2}$

2 choices for a

2 choices for b

2 choices for c

2 choices for d

2^4 minterms

4 Variables:

Maxterm:

$$\underset{a}{(\quad)} + \underset{b}{(\quad)} + \underset{c}{(\quad)} + \underset{d}{(\quad)}$$

$\underbrace{a,\ \bar{a}}_{2}$ choices for a

$\underbrace{b,\ \bar{b}}_{2}$ choices for b

$\underbrace{c,\ \bar{c}}_{2}$ choices for c

$\underbrace{d,\ d'}_{2}$ choices for d

$2^4 = 16$ maxterms

Minterms and Maxterms (1/2)

- A minterm of n variables is a product term that contains n literals from all the variables.

  Example: On 2 variables x and y, the minterms are:

  $x' \cdot y'$, $x' \cdot y$, $x \cdot y'$ and $x \cdot y$

- A maxterm of n variables is a sum term that contains n literals from all the variables.

  Example: On 2 variables x and y, the maxterms are:

  $x'+y'$, $x'+y$, $x+y'$ and $x+y$

- In general, with n variables we have 2^n minterms and 2^n maxterms.

Minterms and Maxterms

A binary variable may appear either in its normal form (x) or in its complement form (x'). Now consider two binary variables x and y combined with an AND operation. Since each variable may appear in either form, there are four possible combinations: $x'y'$, $x'y$, xy', and xy. Each of these four AND terms is called a *minterm*, or a *standard product*. In a similar manner, n variables can be combined to form 2^n minterms. The 2^n different minterms may be determined by a method similar to the one shown in Table 2.3 for three variables. The binary numbers from 0 to $2^n - 1$ are listed under the n variables. Each minterm is obtained from an AND term of the n variables, with each variable being primed if the corresponding bit of the binary number is a 0 and unprimed if a 1. A symbol for each minterm is also shown in the table and is of the form m_j, where the subscript j denotes the decimal equivalent of the binary number of the minterm designated.

In a similar fashion, n variables forming an OR term, with each variable being primed or unprimed, provide 2^n possible combinations, called *maxterms*, or *standard sums*. The eight maxterms for three variables, together with their symbolic designations, are listed in Table 2.3. Any 2^n maxterms for n variables may be determined similarly. It is important to note that (1) each maxterm is obtained from an OR term of the n variables, with each variable being unprimed if the corresponding bit is a 0 and primed if a 1, and (2) each maxterm is the complement of its corresponding minterm and vice versa.

3.)Which of the following is a minterm for boolean function f(x, y, z)?

	a.)xy
	b.) xyz
	c.) $x\bar{y}$
d.)	x+y+z

3.)Which of the following is a minterm for boolean function f(x, y, z)?

	a.)xy → Product term, but not minterm (z/$\bar{z}$ missing)
	b.) xyz
	c.) $x\bar{y}$
	d.) x+y+z → a sum term, not product term

3.)Which of the following is a [redacted] for boolean function f(x, y, z)?

Maxterm

	a.)xy
	b.) xyz
	c.) $x\bar{y}$
d.)	x+y+z

Definitions

- Literals x_i or x_i'
- Product Term $x_2x_1'x_0$
- Sum Term $x_2 + x_1' + x_0$
- Minterm of n variables: A product of n literals in which every variable appears exactly once.
- Maxterm of n variables: A sum of n literals in which every variable appears exactly once.

Minterm

– A minterm of n variables = product of n literals in which each variable appears exactly once either in T or F form, but not in both. (Also known as a standard product term)

Standard Forms of Boolean Expressions

Expressing Boolean Functions using Minterms, Maxterms

Standard Forms of Boolean Expressions

1.

Expressing Boolean Functions

using Minterms

Table 2.4
Functions of Three Variables

minterms	*x*	*y*	*z*	**Function** f_1
$x'y'z'$	0	0	0	0
$x'y'z$	0	0	1	1
$x'yz'$	0	1	0	0
$x'yz$	0	1	1	0
$xy'z'$	1	0	0	1
$xy'z$	1	0	1	0
xyz'	1	1	0	0
xyz	1	1	1	1

f_1 is 0 for m_0

f_1 is 0 for minterm m_2

f_1 is 1 for minterm m_4

f_1 is 1 for minterm m_7

Table 2.4
Functions of Three Variables

Maxterms	x	y	z	Function f_1	
$x+y+z$	0	0	0	0	$\rightarrow$ f_1 is 0 for maxterm M_0
$x+y+z'$	0	0	1	1	
$x+y'+z$	0	1	0	0	$\rightarrow$ f_1 is 0 for M_2
$x+y'+z'$	0	1	1	0	
$x'+y+z$	1	0	0	1	$\rightarrow$ f_1 is 1 for maxterm M_4
$x'+y+z'$	1	0	1	0	
$x'+y'+z$	1	1	0	0	
$x'+y'+z'$	1	1	1	1	$\rightarrow$ f_1 is 1 for M_7

Any Boolean Function:

is True for some minterms

is false for other minterms

Any Boolean Function:

is True for some minterms

True-minterms

1-minterms

is false for other minterms

false-minterms

0-minterms

Any Boolean Function:

is True for some max terms

is false for other maxterms

Any Boolean Function:

is True for some max terms

1

True-maxterms (1-max terms)

is false for other maxterms

0

false-maxterms

0 - maxterms

- For a given switching function, and for given values of the input variables,
  - All the minterms that have the value 1 are called ***true minterms***. 1-minterms
  - All the minterms that have the value 0 are called ***false minterms***. 0-minterms
  - All the maxterms that have the value 1 are called ***true maxterms***. 1-maxterms
  - All the maxterms that have the value 0 are called ***false maxterms***. 0-maxterms
- Example:
  - f (x, y, z) = x'.y + x.y.z → True minterms are: x'.y.z', x'.y.z, x.y.z

Source: NPTEL, Prof. I.Sengupta, IITKGP

- For a given switching function, and for given values of the input variables,
  - All the minterms that have the value 1 are called ***true minterms***.
  - All the minterms that have the value 0 are called ***false minterms***.
  - All the maxterms that have the value 1 are called ***true maxterms***.
  - All the maxterms that have the value 0 are called ***false maxterms***.
- Example:
  - f (x, y, z) = x'.y + x.y.z → True minterms are: x'.y.z', x'.y.z, x.y.z

Table 2.4
Functions of Three Variables

min term	max term	x	y	z	Function f_1
m_0	M_0	0	0	0	0
		0	0	1	1
		0	1	0	0
		0	1	1	0
m_4	M_4	1	0	0	1
		1	0	1	0
		1	1	0	0
		1	1	1	1

f_1 is 0 for minterms m_0

f_1 is 0 for maxterm M_0

f_1 is 1 for minterm m_4

f_1 is 1 for maxterm M_4

Table 2.4
Functions of Three Variables

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

for f_1:

1-minterms

(True-minterms)

$m_1 = x'y'z$

$m_4 = xy'z'$

$m_7 = xyz$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

for f_1

0-minterms (false-minterms)

$m_0 = x'y'z'$

$m_2 = x'yz'$

$m_3 = x'yz$

$m_5 = xy'z$

$m_6 = xyz'$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

for f_1

1-maxterms

(True-maxterms)

$M_1 = x+y+z'$

$M_4 = x'+y+z$

$M_7 = x'+y'+z'$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

for f_1
0-maxterms (false-maxterms)

$\left.\begin{array}{l} M_0 \\ M_2 \\ M_3 \\ M_5 \\ M_6 \end{array}\right\}$ 0-max terms.

Express a boolean function

using Minterms :

Eg :

min term	a	b	f(a,b)
a'b'	0	0	0
a'b	0	1	1
ab'	1	0	1
ab	1	1	1

Express f using minterms :

f is 1 iff

a=0, b=1 OR

a=1, b=0 OR

a=1, b=1

Eg:

min term	a	b	$f(a,b)$
$a'b'$	0	0	0
$a'b$	0	1	1 ✓
ab'	1	0	1 ✓
ab	1	1	1 ✓

Express f using minterms:

f is 1 iff

$a'b$

OR

ab'

OR

ab

Eg:

minterm	a	b	f(a,b)
$a'b'$	0	0	0
$a'b$	0	1	1 ✓
ab'	1	0	1 ✓
ab	1	1	1 ✓

1-minterms

Express f using minterms:

$$\boxed{f = a'b + ab' + ab}$$

f can be expressed as a summation of its 1-minterms.

Note:

Any boolean function f can be expressed as a summation (OR) of "those minterms" for which f=1.

Expressing Boolean Functions using Minterms:

A Boolean function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms. For example, the function f_1 in Table 2.4 is determined by expressing the combinations 001, 100, and 111 as $x'y'z$, $xy'z'$, and xyz, respectively. Since each one of these minterms results in $f_1 = 1$, we have

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

Source: Morris Mano, Digital Logic

Sum of Minterms

Previously, we stated that, for n binary variables, one can obtain 2^n distinct minterms and that any Boolean function can be expressed as a sum of minterms. **The minterms whose sum defines the Boolean function are those which give the 1's of the function in a**

truth table. Since the function can be either 1 or 0 for each minterm, and since there are

Source: Morris Mano, Digital Logic

3.5 Canonical Forms

In general, the unique algebraic expression for any Boolean function can be obtained from its truth table by using an OR operator to combined all minterms for which the function is equal to 1.

A **minterm**, denoted as m_i, where $0 \le i < 2^n$, is a product (AND) of the n variables in which each variable is complemented if the value assigned to it is 0, and uncomplemented if it is 1.

1-minterms = minterms for which the function $F = 1$.
0-minterms = minterms for which the function $F = 0$.

Any Boolean function can be expressed as a sum (OR) of its **1-minterms**. A shorthand notation:

F(list of variables) = Σ(list of 1-minterm indices)

$$f = \sum_m (f=1)$$

minterms

OR

$$\boxed{f = \sum_m (f=1)}$$

Table 2.4
Functions of Three Variables

	x	y	z	Function f_1	Function f_2
	0	0	0	0	0
$x'y'z$	0	0	1	1 ✓	0
	0	1	0	0	0
	0	1	1	0	1
$xy'z'$	1	0	0	1 ✓	0
	1	0	1	0	1
	1	1	0	0	1
xyz	1	1	1	1 ✓	1

$\boxed{\sum m(1,4,7)}$

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

Table 2.4
Functions of Three Variables

	x	y	z	Function f_1	Function f_2
	0	0	0	0	0
	0	0	1	1	0
	0	1	0	0	0
$x'yz$	0	1	1	0	1 ✓
	1	0	0	1	0
$xy'z$	1	0	1	0	1 ✓
xyz'	1	1	0	0	1 ✓
xyz	1	1	1	1	1 ✓

Similarly, it may be easily verified that

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7 = \sum_m(3, 5, 6, 7)$$

Misconception;

a	b	f
0	0	0
0	1	1
1	0	1
1	1	0

only these are the minterms

WRONG

Correct Concept:

minterm: a "product term" containing all variables.

	a	b	f
$a'b'$	0	0	0
$a'b$	0	1	1
ab'	1	0	1
ab	1	1	0

These minterms are used to express f.

$$f = a'b + ab' = \sum_m (1,2)$$

Representations of Logic Functions

Generalized Truth table for 3 variables:

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7 maxterm 7

F = Summation of those minterms for which F=1.

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': product of maxterms for which F=0

e.g. if F=1 for rows 0,3,4,6,7, then $F=\underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Representations of Logic Functions

Generalized Truth table for 3 variables:

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7 maxterm 7

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': product of maxterms for which F=0

e.g. if F=1 for rows 0,3,4,6,7, then $F=\underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Conclusion:

To express f:

f = summation (OR) of those minterms for which $f = 1$.

"Minterms of a function:"

from function f point of view:

To express f, "those minterms are used for which $f = 1$".

These minterms are called

"minterms of function f"

Minterms of a function:

- We can implement any function by "**OR**ing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.

web.stanford.edu/class/archive/ee/ee108a/ee108a.1082/reader/ch1to12.pdf

main 89 / 232 153%

If we say that a product is a $\boxed{\textit{minterm of a function}}$ we are saying that it is both a minterm, and an implicant of the function.

function value is 1.

Minterms of a function:

- We can implement any function by "ORing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.

web.stanford.edu/class/archive/ee/ee108a/ee108a.1082/reader/ch1to12.pdf

main 89 / 232 153%

If we say that a product is a *minterm of a function* we are saying that it is both a minterm, and an implicant of the function.

Conclusion: $f(a,b,c)$

If I say m_3, m_6, m_7 are the $\boxed{\text{minterms of } f}$ then it implicitely means f is 1 for m_3, m_6, m_7 & $f=0$ for other minterms.

minterms of f_1:

m_1, m_4, m_7

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

minterms of f_2 =

m_3, m_5, m_6, m_6

Similarly, it may be easily verified that

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

Minterms of a function:

If we say m_1, m_2, m_5 are the minterms of $f(a,b,c)$ then it implicitely means that

for $m_1, m_2, m_5 \longrightarrow f = 1$

for other minterms $\longrightarrow f = 0$

4.) A boolean function with two variables are defined in the table below. The Boolean expression for the function specified in the table is a sum of minterms. The minterms for which rows are included in sum?

x	y	f(x, y)	Row #
0	0	0	1
0	1	1	2
1	0	0	3
1	1	1	4

a.)	1, 2, 4
b.)	2,3
c.)	2, 4
d.)	3, 4

4.) A boolean function with two variables are defined in the table below. The Boolean expression for the function specified in the table is a sum of minterms. The minterms for which rows are included in sum?

Row No.	x	y	f(x, y)
1	0	0	0
2	0	1	1 ✓
3	1	0	0
4	1	1	1 ✓

a.)	1, 2, 4
b.)	2,3
c.)	2, 4 ✓
d.)	3, 4

f can be expressed as Sum of minterms.

which minterms?

Those minterms for which $f = 1$

Row numbers

Row numbers

$$f = \underline{m_1 + m_3} = \sum_m (1, 3)$$

Conclusion:

Expressing Boolean Functions using Minterms:

A Boolean function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms. For example, the function f_1 in Table 2.4 is determined by expressing the combinations 001, 100, and 111 as $x'y'z$, $xy'z'$, and xyz, respectively. Since each one of these minterms results in $f_1 = 1$, we have

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$x'y'z$

$xy'z'$

xyz

$f_1 = \sum_m (1, 4, 7)$

OR

$f_1 = m_1 + m_4 + m_7$

Similarly, it may be easily verified that

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$x'yz$

$xy'z$

xyz'

xyz

$f_2 = \sum m(3,5,6,7)$

$f_2 = m_3 + m_5 + m_6 + m_7$

Similarly, it may be easily verified that

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

Representations of Logic Functions

Generalized Truth table for 3 variables:

$F = \Sigma_m (f=1)$

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7 maxterm 7

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': product of maxterms for which F=0

e.g. if F=1 for rows 0,3,4,6,7, then $F = \underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Minterms of a function:

If a function f is given, then minterms of f

implicitely mean the minterms for which f = 1.

Because

Summation of these minterms describes f.

Minterms of a function:

If a function f is given, then minterms of f are those minterms for which f = 1.

"True Minterms of f" are also called "minterms of f"

A	B	C	S	Co
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$S = \Sigma (1, 2, 4, 7)$

$Co = \Sigma (3, 5, 6, 7)$

$S = \Sigma_m (1, 2, 4, 7)$

Canonical Sum-of-Products Form

- From the truth table, identify all the true minterms.
  - Corresponding to rows for which the output function is 1.
- Take the sum of all the minterms.
- Example for the full adder:

  S = A'.B'.C + A'.B.C' + A.B'.C' + A.B.C

  Co = A.B.C' + A'.B.C + A.B'.C + A.B.C
- We can write down the canonical s-o-p expressions in a compact way by noting down the decimal equivalents of the input combinations:

  $S = \Sigma\ (1, 2, 4, 7)$

  $Co = \Sigma\ (3, 5, 6, 7)$

Express the Boolean function $F = A + B'C$ as a sum of minterms.

Express the Boolean function $F = A + B'C$ as a sum of minterms.

	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$F = A + B'C$

$A = 1 \longrightarrow F = 1$

$B'C = 1 \longrightarrow F = 1$

$\underbrace{B'C}_{} \longrightarrow B = 0, C = 1$

Express the Boolean function $F = A + B'C$ as a sum of minterms.

	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$F = A + B'C$

$F = \Sigma_m(1, 4, 5, 6, 7)$

$= m_1 + m_4 + m_5 + m_6 + m_7$

$= A'B'C + AB'C' + AB'C + m_6 + m_7$

Note:

for any function f :

$\boxed{\text{Sum of minterms}}$ is called

$\boxed{\text{Canonical sum of product}}$ form of f.

for f :

Canonical SOP = Sum of minterms

= minterm expansion = $\sum_m (f=1)$

Express the Boolean function $F = A + B'C$ as a sum of minterms.

$$F = A'B'C + AB'C + AB'C + ABC' + ABC$$
$$= m_1 + m_4 + m_5 + m_6 + m_7$$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

Q: for 2 variables : a, b

How many minterms?

Product term containing all variables.

No function is Given

exactly 4.

$$\left\{\begin{matrix} ab & ab \\ a'b & \\ ab' & \end{matrix}\right\}$$

Q: If function is Given;

	a	b	f
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

$f = 0$ for minterms m_0, m_2

$f = 1$ for minterms m_1, m_3

Q: If function is Given;

	a	b	f
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

f = 0 for minterms m_0, m_2

f = 1 for minterms m_1, m_3

To express f these minterms used.

Q: If function is Given;

	a	b	f
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

To express f, minterm m_1, m_3 are used

m_1, m_3 are called "minterms of f"

a	b	f
0	0	0
0	1	1
1	0	0
1	1	1

m_0, m_1, m_2, m_3 are minterms.

from f point of view

m_1, m_3 are called "minterms of f"

a	b	f
0	0	0
0	1	1
1	0	0
1	1	1

f is 1 iff

a=0, b=1

OR

a=1, b=1

a	b	f
0	0	0
0	1	1
1	0	0
1	1	1

f is 1 iff

$a'b$

OR

ab

a	b	f
0	0	0
0	1	1
1	0	0
1	1	1

$$f = a'b + ab$$

Conclusion:

Any boolean function f can be expressed as summation (OR) of those minterms for which f = 1

Conclusion:

$$f = \sum_m (f = 1)$$

These minterms for which $f = 1$

are called "Minterms of f".

Standard Forms of Boolean Expressions

2.

Expressing Boolean Functions using Maxterms

Table 2.4
Functions of Three Variables

	x	y	z	Function f_1
$x+y+z$	0	0	0	0
$x+y+z'$	0	0	1	1
	0	1	0	0
	0	1	1	0
$x'+y+z$	1	0	0	1
	1	0	1	0
$x'+y'+z$	1	1	0	0
	1	1	1	1

$f_1 = 1$ for which maxterms:

M_1, M_4, M_7

Table 2.4
Functions of Three Variables

	x	y	z	Function f_1
$x+y+z$	0	0	0	0
$x+y+z'$	0	0	1	1
	0	1	0	0
	0	1	1	0
$x'+y+z$	1	0	0	1
	1	0	1	0
$x'+y'+z$	1	1	0	0
	1	1	1	1

$f_1 = 0$ for which maxterms:

M_0, M_2, M_3, M_5, M_6

Any Boolean Function:

is True for some maxterms

is false for other maxterms

- For a given switching function, and for given values of the input variables,
  - All the minterms that have the value 1 are called ***true minterms***.
  - All the minterms that have the value 0 are called ***false minterms***.
  - All the maxterms that have the value 1 are called ***true maxterms***.
  - All the maxterms that have the value 0 are called ***false maxterms***.
- Example:
  - f (x, y, z) = x'.y + x.y.z → True minterms are: x'.y.z', x'.y.z, x.y.z

Source: NPTEL, Prof. I.Sengupta, IITKGP

Table 2.4
Functions of Three Variables

x	*y*	*z*	Function f_1
0	0	0	0 ✓
0	0	1	1
0	1	0	0 ✓
0	1	1	0 ✓
1	0	0	1
1	0	1	0 ✓
1	1	0	0 ✓
1	1	1	1

0-maxterms
(false maxterms)

M_0, M_2, M_3, M_5, M_6

How to Express a function f using maxterms ?

How to Express a function f using maxterms ?

f can be expressed as a Product (AND) of those maxterms for which f = 0.

Expressing Boolean Functions using Maxterms:

$$f = \prod_{M} (f=0)$$

Product (AND)

maxterms

Those maxterms ~~for~~ for which $f=0$.

Maxterms (2)

- **Any Boolean function can be expressed as a product (AND) of its 0-maxterms:**

$$F(\text{list of variables}) = \Pi(\text{list of 0-maxterm indices})$$

Example:

Row Numbers	Variable Values			Function Values	
	x	y	z	F_1	F_1'
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	0

Equation Table

$$\begin{aligned} F_1(x, y, z) &= \Pi(0, 1, 2, 4) \\ &= M_0 M_1 M_2 M_4 \\ &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \end{aligned}$$

$$\begin{aligned} F_1'(x, y, z) &= \Pi(3, 5, 6, 7) \\ &= M_3 M_5 M_6 M_7 \\ &= (x + y' + z')(x' + y + z')(x' + y' + z)(x' + y' + z') \end{aligned}$$

Expressing Boolean Functions using Maxterms:

These examples demonstrate a second property of Boolean algebra: Any Boolean function can be expressed as a product of maxterms (with "product" meaning the ANDing of terms). The procedure for obtaining the product of maxterms directly from the truth table is as follows: Form a maxterm for each combination of the variables that produces a 0 in the function, and then form the AND of all those maxterms. **Boolean functions expressed as a sum of minterms or product of maxterms are said to be in *canonical form*.**

Source: Morris Mano

Expressing Boolean Functions using Maxterms:

Canonical Product-of-Sums Form

- From the truth table, identify all the false maxterms. (0-maxterms)
  - Corresponding to rows for which the output function is 0.
  - For each false maxterm, form a sum term where a variable will appear in uncomplemented (complemented) form if it has value 0 (1) in the row.

- Product of all 0-maxterms.

Source: NPTEL, Prof. I.Sengupta, IITKGP

A **maxterm**, denoted as M_i, where $0 \le i < 2^n$, is a sum (OR) of the n variables (literals) in which each variable is complemented if the value assigned to it is 1, and uncomplemented if it is 0.

1-maxterms = maxterms for which the function $F = 1$.

0-maxterms = maxterms for which the function $F = 0$.

Any Boolean function can be expressed as a product (AND) of its **0-maxterms**. A shorthand notation:

F(list of variables) = Π(list of 0-maxterm indices)

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Express f_1 using maxterms.

$f_1 =$

$M_0 M_2 M_3 M_5 M_6$

$= \prod_m (0, 2, 3, 5, 6)$

$f_1 = \sum_m (1, 4, 7) = \prod_m (0, 2, 3, 5, 6)$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Express f_2 using maxterms.

$f_2 =$

$M_0 M_1 M_2 M_4$

$= \prod_M (0,1,2,4)$

$f_2 = \prod_M (0,1,2,4) = \sum_m (3,5,6,7)$

Note:

we can Express any boolean function f using minterms as well as using maxterms.

Note:

we can Express any boolean function f using minterms : → Just focus on where f = 1.

$$f = \sum_m (f = 1)$$

OR

Note:

we can Express any boolean function f using maxterms → Just focus on where $f = 0$

$$f = \prod_m (f = 0)$$

Note:

we can Express any boolean function f using minterms as well as using maxterms.

$$f = \sum_m (f=1) = \prod_M (f=0)$$

OR

AND

So; Two Representations of Same function f :

$\sum_m (f=1)$ = Sum of minterms = Canonical SOP form

$\prod_M (f=0)$ = Product of Maxterms = Canonical POS form.

So; Two Representations of Same function f :

$\sum_m (f=1)$ = Sum of minterms = Canonical SOP form

product term

$\prod_M (f=0)$ = Product of Maxterms = Canonical POS form.

Sum term

So; Two Representations of Same function f:

Canonical Sum of Product = Sum of minterms
minterm

Canonical Product of Sum = Product of maxterms
maxterm

Representations of Logic Functions

$f = \prod m(f=0)$

Generalized Truth table for 3 variables:

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7 maxterm 7

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': $\boxed{\text{product of maxterms for which F=0}}$

e.g. if F=1 for rows 0,3,4,6,7, then $F=\underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Maxterms of a function:

[redacted]

- We can implement any function by "**OR**ing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.
- We can implement any function by "**AND**ing" the maxterms corresponding to "0" entries in the function table. These are called the maxterms of the function.
- This gives us two canonical forms:
    - **Sum of Minterms (SOM)**
    - **Product of Maxterms (POM)**

for stating any Boolean function.

If I say M_0, M_4, M_6 are $\boxed{\text{maxterms of } f}$ then it implicitly

means: $f = 0$ for M_0, M_4, M_6

$f = 1$ for other maxterms.

maxterms of f_1:

M_0, M_2, M_3, M_5, M_6

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

maxterms of f_2:

M_0, M_1, M_2, M_4

Conclusion:

Expressing Boolean Functions using Maxterms:

$$f = \prod_{M} (f = 0)$$

Representations of Logic Functions

Generalized Truth table for 3 variables:

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7 maxterm 7

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': product of maxterms for which F=0

e.g. if F=1 for rows 0,3,4,6,7, then $F=\underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Maxterms of a function:

If a function f is given, then maxterms of f implicitly mean the maxterms for which f = 0.

Because

Product of these maxterms describes f.

Maxterms of a function:

If a function f is given, then maxterms of f are those maxterms for which f = 0.

"False maxterms of f" are also called "maxterms of f"

Canonical Product-of-Sums Form

- From the truth table, identify all the false maxterms.
  - Corresponding to rows for which the output function is 0
  - For each false maxterm, form a sum term where a variable will appear in uncomplemented (complemented) form if it has value 0 (1) in the row.

– Product of these 0-maxterms.

A	B	C	S	Co
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

S = Π (0, 3, 5, 6)

Co = Π (0, 1, 2, 4)

$S = \prod_m (0, 3, 5, 6)$

$S = \sum_m (1, 2, 4, 7)$

$C_o = \sum_m (3, 5, 6, 7)$

Representations of Logic Functions

Generalized Truth table for 3 variables:

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	X'·Y'·Z'	X+Y+Z
1	0	0	1	F(0,0,1)	X'·Y'·Z	X+Y+Z'
2	0	1	0	F(0,1,0)	X'·Y·Z'	X+Y'+Z
3	1	1	1	F(0,1,1)	X'·Y·Z	X+Y'+Z'
4	1	0	0	F(1,0,0)	X·Y'·Z'	X'+Y+Z
5	1	0	1	F(1,0,1)	X·Y'·Z	X'+Y+Z'
6	1	1	0	F(1,1,0)	X·Y·Z'	X'+Y'+Z
7	1	1	1	F(1,1,1)	X·Y·Z	X'+Y'+Z'

minterm 7   maxterm 7

'Canonical sum' of a logic function: sum of minterms for which F=1.
'Canonical product': product of maxterms for which F=0

e.g. if F=1 for rows 0,3,4,6,7, then $F=\underbrace{\Sigma_{X,Y,Z}(0,3,4,6,7)}_{\text{'minterm list' or 'on-set'}} = \underbrace{\Pi_{X,Y,Z}(1,2,5)}_{\text{'maxterm list' or 'off-set'}}$

Standard Forms of Boolean Expressions

Canonical SOP

Canonical POS

Some Definitions

- The ***canonical sum of products (CSOP)*** form of an expression refers to rewriting the expression as a sum of minterms.
  - *Examples for 3-variables:* $\bar{a}bc + abc$ is a CSOP expression; $\bar{a}b + c$ is not.
- The ***canonical product of sums (CPOS)*** form of an expression refers to rewriting the expression as a product of maxterms.
  - *Examples for 3-variables:* $(\bar{a}+b+c)(a+b+c)$ is a CPOS expression; $(\bar{a}+b)c$ is not.

- There is a close correspondence between the truth table and minterms and maxterms.

Q: for any boolean function f;

Is Csop unique?

Is Cpos unique?

Q: for any boolean function f ;

Is Csop Unique ? → Yes

Is Cpos Unique ? → Yes.

$$\boxed{Csop = \sum_m (f=1)}$$ → Unique expression

$$Cpos = \prod_m (f=0)$$ → Unique expression

Canonical Forms

- Two canonical forms:
  - Sum-of-minterms
  - Product-of-maxterms
- Canonical forms are unique.

Any function f has

(1) Unique CSOP ✓

&

(2) Unique CPOS ✓

Sum-of-minterms and Product-of-maxterms

Two mechanical ways to translate a function's truth table into an expression:

X	Y	**Minterm**	**Maxterm**	F
0	0	$\overline{X}\,\overline{Y}$	$X+Y$	0
0	1	$\overline{X}Y$	$X+\overline{Y}$	1
1	0	$X\overline{Y}$	$\overline{X}+Y$	1
1	1	XY	$\overline{X}+\overline{Y}$	0

The sum of the minterms where the function is 1:

csop form of F

$$F=\overline{X}Y+X\overline{Y} \longrightarrow \text{csop}$$

The product of the maxterms where the function is 0:

cpos form of F

$$F=(X+Y)(\overline{X}+\overline{Y}) \longrightarrow \text{cpos}$$

Minterms and Maxterms: Another Example

The minterm and maxterm representation of functions may look very different:

X	Y	**Minterm**	**Maxterm**	F
0	0	$\overline{X}\,\overline{Y}$	$X+Y$	0
0	1	$\overline{X}\,Y$	$X+\overline{Y}$	1
1	0	$X\,\overline{Y}$	$\overline{X}+Y$	1
1	1	$X\,Y$	$\overline{X}+\overline{Y}$	1

The sum of the minterms where the function is 1:

CSOP

$$F = \overline{X}\,Y + X\,\overline{Y} + X\,Y$$

CSOP (unique)

The product of the maxterms where the function is 0:

CPOS

$$F = X + Y$$

CPOS (unique)

Standard Forms of Boolean Expressions

Canonical SOP Vs SOP

Canonical POS Vs POS

a	b	f
0	0	0
0	1	1
1	0	1
1	1	1

for f :

CSOP : Σ minterms =

$a'b + ab' + ab$ → csop ✓ sop ✓

$f = a + b$ → SOP form

→ ~~csop~~

a + b

SOP Vs CSOP → Sum of minterms

SOP → Sum of product terms

$f = a + b$ ⇒ Not Sum of minterm
But it is SOP.

a minterm? No.

for f :

Is SOP unique ? No

Is CSOP unique ? Yes.

Absorption Law

$x + xy = x$

$f = \underbrace{a+b}_{\substack{\text{SOP} \checkmark \\ \text{CSOP} \times}} = \underbrace{a + ab + b}_{\substack{\text{SOP} \checkmark \\ \text{CSOP} \times}} = \underbrace{a + ab + b + bb}_{\substack{\text{SOP} \checkmark \\ \text{CSOP} \times}}$

for $f = a+b$ → How many sop expressing possible ?

$= a+b+bb$

$= a+b+bb+bbb$

$= \boxed{a+aa+b+bb}$ → sop expression

$=$ –

–

–

–

∞ many

for $f = a + b$

How many CSOP ? $\rightarrow$ Unique

$\rightarrow$ Sum of minterms

$= m_1 + m_2 + m_3$

$= \sum_m (1, 2, 3) = \boxed{a'b + ab' + ab}$

a	b	f
0	0	0
0	1	1
1	0	1
1	1	1

for f :

CPOS : Π maxterms → a sumterm

= $a + b$

= M_0

$a+b$ → POS? Yes. $(a+b)$

$a+b$ → CPOS

$f = a + b$ $\longrightarrow$ NOT unique

POS expressions of f : $(a+b)$ ✓

$(a+b) \cdot (a+b)$ ✓

$(a+b) \cdot (a+a')$ ✓

a	b	f
0	0	0
0	1	0
1	0	1
1	1	1

Same

Expressions for f :

$f = a$ → SOP ✓

→ POS ✓

$= ab' + ab$: CSOP form

$= (a+b).(a+b')$: CPOS form

SOP Also

CSOP → unique.

Example:

$$F_1 = \boxed{xyz + xyz' + xy'z + xy'z'}$$ → CSOP form

$$= xy(z+z') + xy'(z+z')$$ → Neither SOP, NOR POS

$$= xy + xy'$$ → SOP ✓, POS ✗

$$= x(y+y')$$ → POS ✓, SOP ✗

$$= x$$ → SOP ✓, POS ✓

Standard Forms (1)

- Two standard forms
  - Sum-of-products
  - Product-of-sums
- Standard forms are not unique.

- Sum-of-products is an OR expression with product terms that may have less literals than minterms

Example:

$$F = xy + x'yz + xy'z$$

Standard Forms (2)

- Product-of-sums is an AND expression with sum terms that may have less literals than maxterms

Example:

$$F = (x' + y')(x + y' + z')(x' + y + z')$$

Q: Is it Possible for any function f that

$$f = \overline{f} ?$$

Q: Is it Possible for any function f that

$$f = \overline{f} \; ?$$

→ No

Can Never happen for any function.

f	$\overline{f}$
0	1
1	0
0	1
1	0
1	0
0	1

So $f, \overline{f}$ are NEVER Same.

Q: for f;

Is CSOP Complement of CPOS?

φ: for f;

Is Csop Dual of cpos?

Q: for f;

Is CSOP complement of CPOS?

φ : for f ;

Is Csop Dual of Cpos ?

for f ;

CSOP Equivalent of CPOS.

for any f , CSOP, CPOS, POS, SOP,
All are Just Different Representations.

Same function can be written in many Different ways

Equivalent.

Eg: $\boxed{f = a + b}$

Csop of f =

Cpos of f =

Dual of f =

Complement of f =

Eg: $\boxed{f = a+b}$

CSOP of $f = a'b + ab' + ab$ 1

CPOS of $f = a+b$ 2

Dual of $f = ab$ 3

Complement of $f = a'b'$ 4

1 $\equiv$ 2

1 $\not\equiv$ 3

1 $\not\equiv$ 4

3 $\not\equiv$ 4

$f = a + b$

$\text{cpos} = a + b$

$\text{csop} = a'b + ab' + ab$

Simplify csop.

$a'b + \underbrace{ab' + ab}_{a(b'+b)} = a'b + a = a + b$

csop

$$\left.\begin{aligned} x + x'y &= x + y \\ \alpha + \alpha'\beta &= \alpha + \beta \end{aligned}\right\}$$

Note:

CSOP, CPOS, POS, SOP, - - - - etc

are Just Different ways of writing

$\boxed{\text{Same}}$ function.

$f = ab \rightarrow$ CSOP $= ab$ → Equivalent

$\rightarrow$ CPOS $= (a+b)(a'+b)(a+b')$

$\rightarrow$ Dual $= a+b$

$\rightarrow \bar{f} = \bar{a} + \bar{b}$

Is CSOP of f Dual of CPOS of f ? No.

Sum-of-products form

Canonical

Principle DNF = CSOP

- sometimes also called **disjunctive normal form** (DNF) principle
- sometimes also called a ***minterm expansion***

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$

Boolean Algebra $\equiv$ Propositional logic

World of 0,1 — world of T,F

0 ——— F

1 ——— T

Boolean Algebra $\equiv$ Propositional logic

SOP

Disjunction of Conjunction

= Disjunctive Normal form (DNF)

$\boxed{SOP = DNF}$

Boolean Algebra ≡ Propositional logic

POS

Conjuction of Disjuction

= Conjunctive Normal form (CNF)

$\boxed{\text{POS} = \text{CNF}}$

Boolean Alg	Propositional logic
SOP	DNF
POS	CNF
CSOP	CDNF (Principle DNF)
CPOS	CCNF (Principle CNF)

(variables appear once in each minterm)

A	B	C	F	$\overline{F}$	minterm
0	0	0	1	0	m0 $\overline{A}\,\overline{B}\,\overline{C}$
0	0	1	1	0	m1 $\overline{A}\,\overline{B}C$
0	1	0	1	0	m2 $\overline{A}B\overline{C}$
0	1	1	0	1	m3 $\overline{A}BC$
1	0	0	1	0	m4 $A\overline{B}\,\overline{C}$
1	0	1	1	0	m5 $A\overline{B}C$
1	1	0	0	1	m6 $AB\overline{C}$
1	1	1	0	1	m7 ABC

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$
$$= m0 + m1 + m2 + m4 + m5$$
$$= \sum m(1,0,2,4,5)$$

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$
$$= m3 + m6 + m7$$
$$= \sum m(3,6,7)$$

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

$$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')$$
$$= M_0M_2M_4M_5$$

A convenient way to express this function is as follows:

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

Product-of-sums form

Canonical

principle

- sometimes also called **conjunctive normal form** (CNF)
- sometimes also called a **maxterm expansion**

$$F = (A+\overline{B}+\overline{C})\ (\overline{A}+\overline{B}+C)\ (\overline{A}+\overline{B}+\overline{C})$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

Product-of-sums form

Canonical

principle

- sometimes also called **conjunctive normal form** (CNF)
- sometimes also called a **maxterm expansion**

$$F = (A+\overline{B}+\overline{C})\,(\overline{A}+\overline{B}+C)\,(\overline{A}+\overline{B}+\overline{C})$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

$$\overline{F} = (A+B+C)\,(A+B+\overline{C})\,(\overline{A}+B+\overline{C})\,(\overline{A}+B+C)\,(\overline{A}+B+\overline{C})$$

A	B	C	F	$\overline{F}$	maxterm	
0	0	0	1	0	M0	$A+B+C$
0	0	1	1	0	M1	$A+B+\overline{C}$
0	1	0	1	0	M2	$A+\overline{B}+C$
0	1	1	0	1	M3	$A+\overline{B}+\overline{C}$
1	0	0	1	0	M4	$\overline{A}+B+C$
1	0	1	1	0	M5	$\overline{A}+B+\overline{C}$
1	1	0	0	1	M6	$\overline{A}+\overline{B}+C$
1	1	1	0	1	M7	$\overline{A}+\overline{B}+\overline{C}$

$$F = (A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$$
$$= (M3)(M6)(M7)$$
$$= \prod M(3,6,7)$$

$$\overline{F} = (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)(\overline{A}+B+\overline{C})$$
$$= (M0)(M1)(M2)(M4)(M5)$$
$$= \prod M(0,1,2,4,5)$$

Some Observations:

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)		
Product of sums (POS)		

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

Complement of F.

	F	$\overline{F}$
Canonical Sum of products (SOP)	$\Sigma_m(F=1)$	$\Sigma_m(\overline{F}=1)$
Canonical Product of sums (POS)	$\Pi_m(F=0)$	$\Pi_m(\overline{F}=0)$

Standard Form Example

Complement of F.

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Canonical Sum of products (SOP)	$\Sigma_m(F=1)$	$\Sigma_m(F=0)$
Canonical Product of sums (POS)	$\Pi_m(F=0)$	$\Pi_m(F=1)$

Standard Form Example

Complement of F.

	A	B	C	F	$\overline{F}$
0	0	0	0	0	1
1	0	0	1	1	0
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\Sigma(1,3,5,6)$	$\Sigma(0,2,4,7)$
Product of sums (POS)	$\Pi(0,2,4,7)$	$\Pi(1,3,5,6)$

for f :

$$\text{csop} \longrightarrow \text{cpos}$$

$$\Sigma(\underbrace{-----}_{S}) \qquad \Pi\left(\begin{array}{l}\text{Remaining} \\ \text{Indexes} \\ \text{which} \notin S\end{array}\right)$$

Q: $f(a,b,c) = \Sigma(1,5,6)$

$= (POS)?$

Q: $f(a,b,c) = \boxed{\sum(1,5,6)}$ = sum of minterms

= cpos = $\prod(0,2,3,4,7)$ ✓

Tell us that $f=1$ for these combination.

→ So $f=0$ for 0, 2, 3, 4, 7

Q: $f(a,b,c) = \prod(1,2,3,5)$

$= \Sigma(\ !\ )$

Q: $f(a,b,c) = \prod(1, 2, 3, 5)$

$= \sum(0, 4, 6, 7)$ ✓

Tell us that $f = 0$ for these Combinations / Indexes

So $f = 1$ for $0, 4, 6, 7$

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(1,3,5,6)$	$\sum m(0,2,4,7)$
Product of sums (POS)	$\prod M(0,2,4,7)$	$\prod M(1,3,5,6)$

for f :

$$\text{Csop} = \Sigma(\underbrace{\_\ \_\ \_\ \_}_{\in S}) = \text{sum of minterms}$$

$$\text{cpos} = \Pi(\underbrace{\_\ \_\ \_\ \_}_{\notin S}) = \text{Product of maxterms.}$$

for f :

CPOS : $\Pi(\underbrace{- - - -}_{\in S})$: Product of maxterms

CSOP : $\Sigma(\underbrace{- - - -}_{\notin S})$: Sum of minterms.

for f : $f = \Sigma(\underbrace{- - - - -}_{S})$

$\overline{f} = \Sigma(\underbrace{- - - - -}_{?})$

for f : $f = \Sigma(\underbrace{- - - - -}_{S})$

$\bar{f} = \Sigma(\underbrace{- - - - -}_{\notin S})$

$\bar{f} = 1$ for indexes $\notin S$.

S means $f = 1$ for indexes in S.

for f: $f = \Pi(\underbrace{- \; - \; -}_{s})$

$\bar{f} = \Pi(\underbrace{- \; - \; -}_{?})$

for f: $f = \Pi(\underbrace{- - -}_{S})$ → means: $f = 0$ for indexes in S.

$\bar{f} = \Pi(\underbrace{- - -}_{\notin S})$

$\bar{f} = 0$ for indexes $\notin S$

Summary of SOP and POS

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

Canonical Forms

- Two canonical forms:
  - Sum-of-minterms
  - Product-of-maxterms
- Canonical forms are unique.
- Conversion between canonical forms is achieved by:
  - Exchanging Σ and Π
  - Listing all the missing indices

Definition: Any Boolean function that is expressed as a sum of minterms or as a product of maxterms is said to be in its **canonical form**.

To convert from one canonical form to its other **equivalent** form, interchange the symbols Σ and Π, and list the index numbers that were excluded from the original form.

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(1,3,5,6)$	$\sum m(0,2,4,7)$
Product of sums (POS)	$\prod M(0,2,4,7)$	$\prod M(1,3,5,6)$

GO Classes

Converting between canonical forms

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

DeMorgans