Quantitative Aptitude

Floor Function, Ceiling Function

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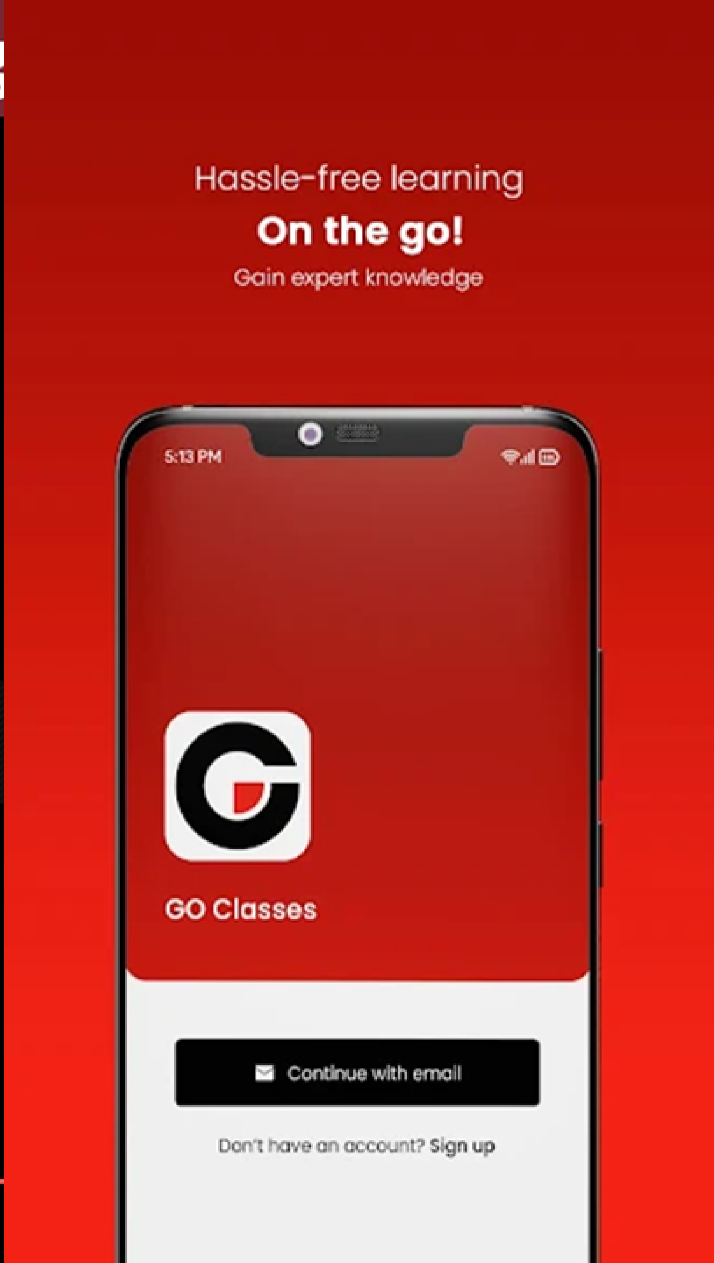

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Quantitative Aptitude

Floor Function, Ceiling Function

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If x is any Real Number ;

$x = 4.5$ $\quad$ $\text{floor}(x) = \lfloor x \rfloor = 4$

$x = 0.5$ $\quad$ $\lfloor x \rfloor = 0$

$x = 4.999$ $\quad$ $\lfloor x \rfloor = 4$

$x = 5.0001$ $\quad$ $\lfloor x \rfloor = 5$

$x = 5$ $\quad$ $\lfloor x \rfloor = 5$

If x is any Real Number ;

floor (x) = floor value of $x = \lfloor x \rfloor$

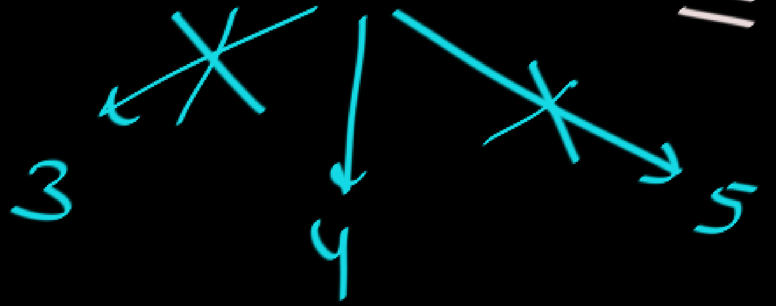
= greatest integer n that is $\leq x$.

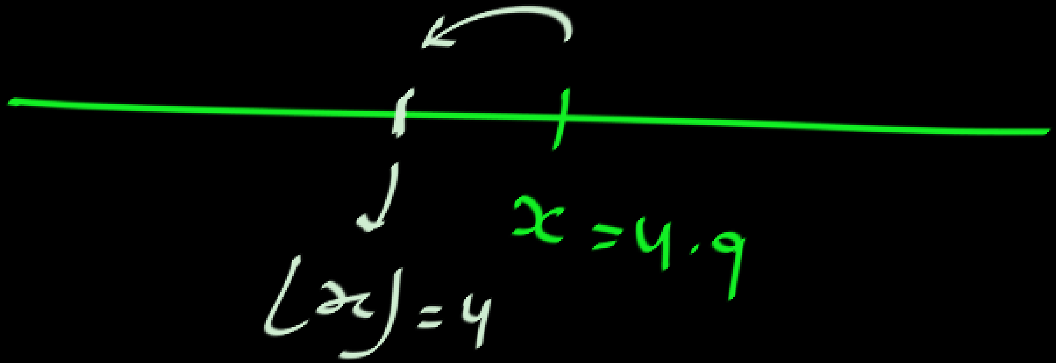
$\lfloor x \rfloor$ = Greatest integer $n \leq x$.

$x = 4.9$

Greatest integer

$n \leq 4.9$





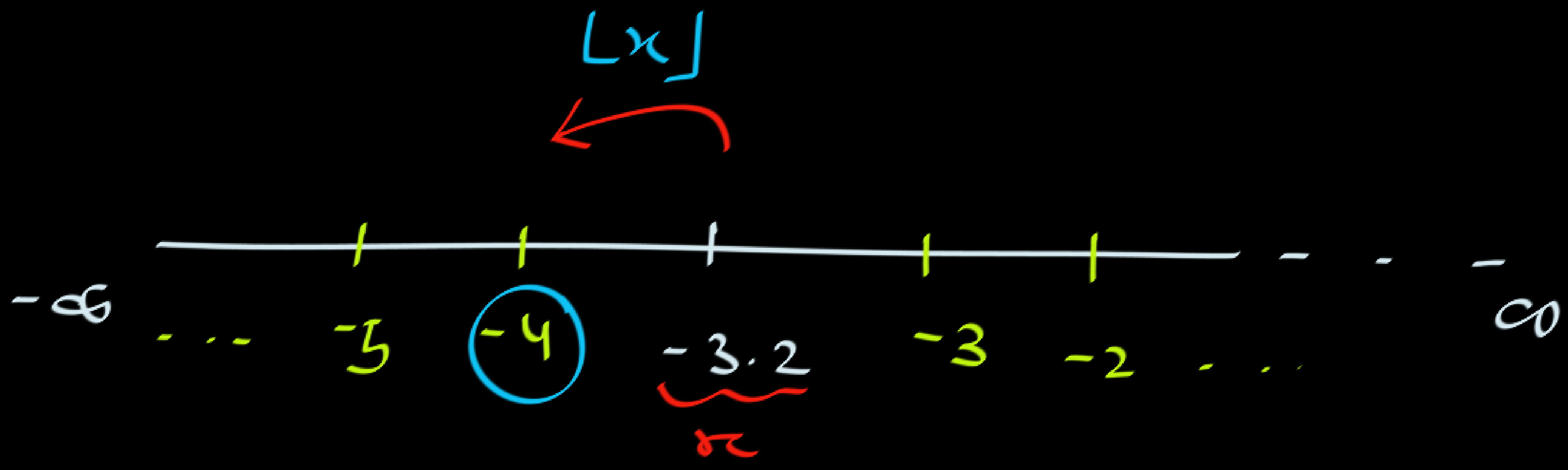
If x is any Real Number ;

$x = 8.12 \longrightarrow \lfloor x \rfloor \longrightarrow 8$

Greatest integer ≤ 8.12

$x = -3.2 \longrightarrow \lfloor x \rfloor \longrightarrow -4$

Greatest integer ≤ -3.2



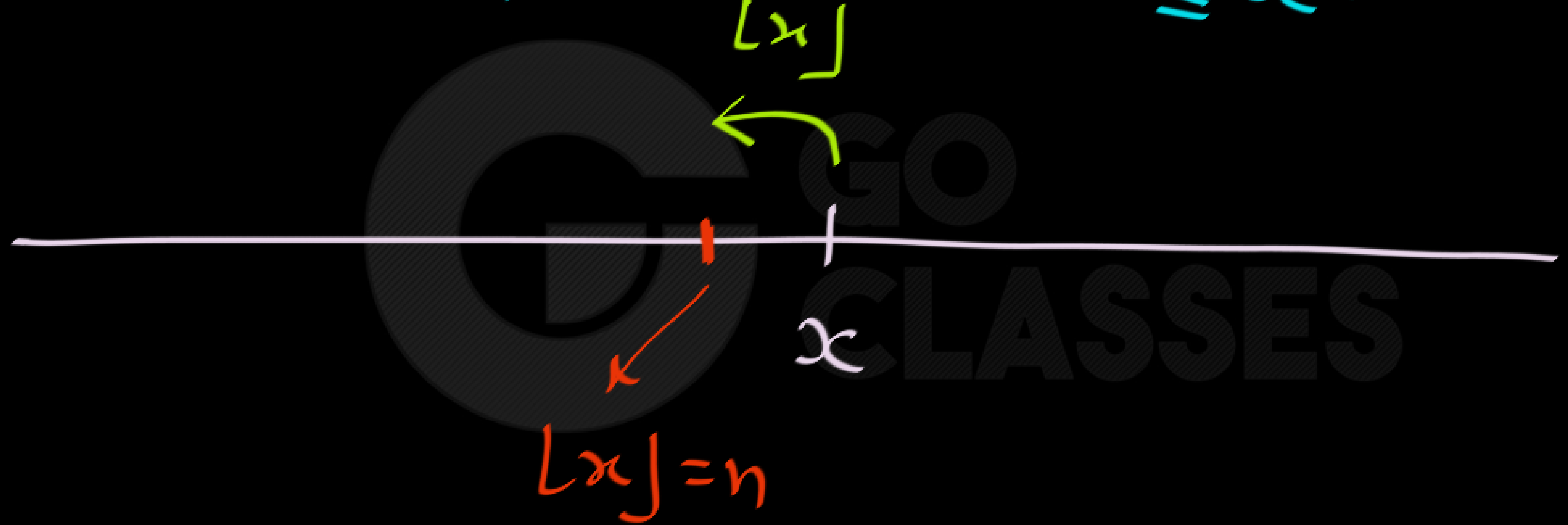
$$-4 < -3.2 < -3$$

If x is any Real Number;

$\lfloor x \rfloor$ = floor value of x = Greatest integer function.

$\lfloor x \rfloor \rightarrow n$ where n is the Greatest integer $\leq x$.

If x is any Real Number ; $\lfloor x \rfloor$ is the nearest integer that is $\leq x$.



To find $\lfloor x \rfloor$, Go to the left of x & find the nearest integer that is $\leq x$.

If x is any Real Number ;

Ceiling(x) = Ceiling value of x = $\lceil x \rceil$

$x = 4.8 \longrightarrow \lceil x \rceil = 5$

$x = 0.1 \longrightarrow \lceil x \rceil = 1$

$x = 1 \longrightarrow \lceil x \rceil = 1$

$x = -1.7 \longrightarrow \lceil x \rceil = -1$

If x is any Real Number ;

Ceiling(x) = Ceiling value of x = $\lceil x \rceil$

$\lceil x \rceil$ is the Least integer n that is $\geq x$.

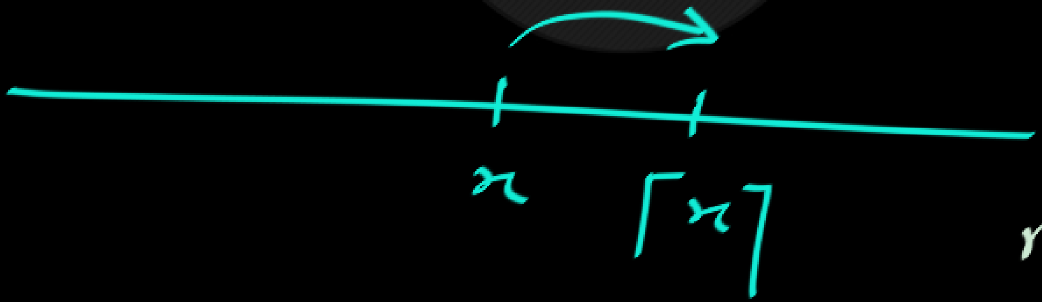
$x = 4.7 \longrightarrow \lceil x \rceil = 5$

Least integer $\geq x$

If x is any Real Number ;

Ceiling(x) = Ceiling value of $x = \lceil x \rceil$

$\lceil x \rceil$ is the nearest integer that is $\geq x$.



To find $\lceil x \rceil$, Go to the Right & find nearest integer that is $\geq x$.

If x is any Real Number ;

Ceiling(x) = Ceiling value of x = $\lceil x \rceil$

$\underbrace{x = 5}$ $\longrightarrow$ $\lceil x \rceil$ = 5 ✓

Nearest integer ≥ 5

If x is any Real Number ;

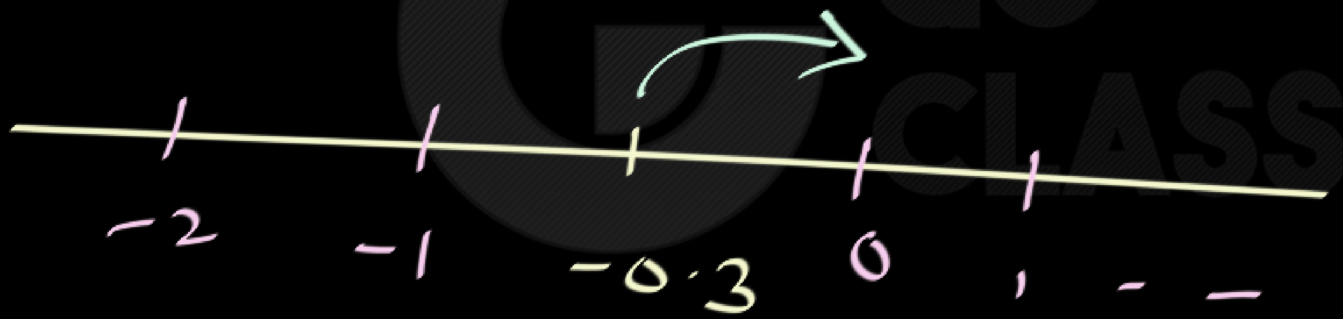
Ceiling(x) = Ceiling value of x = $\lceil x \rceil$

$x = 5.9 \longrightarrow \lceil x \rceil = 6$

Nearest integer ≥ 5.9

If x is any Real Number ;

$x = -0.3 \longrightarrow \lceil x \rceil = 0$



If x is any Real Number ;

$$x = -1.001 \longrightarrow \lceil x \rceil = -1$$

$$x = -1.001 \longrightarrow \lfloor x \rfloor = -2$$

$\lfloor\ \rfloor$ $\lceil\ \rceil$

$$-2 < \underline{-1.001} < -1$$

Some Important Functions

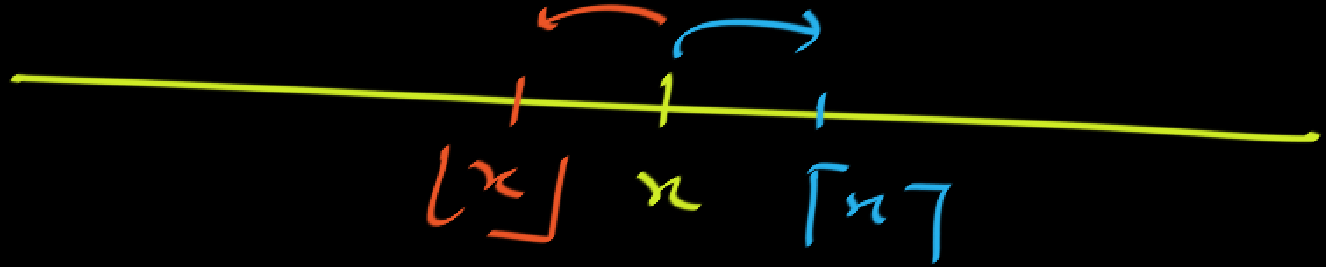
Next, we introduce two important functions in discrete mathematics, namely, the floor and ceiling functions. Let x be a real number. The floor function rounds x down to the closest integer less than or equal to x, and the ceiling function rounds x up to the closest integer greater than or equal to x. These functions are often used when objects are counted. They play an important role in the analysis of the number of steps used by procedures to solve problems of a particular size.

	$\lfloor x \rfloor$	$\lceil x \rceil$
$x = \pi = 3.14\ldots\ldots$	3	4
$x = 4$	4	4
$x = 0$	0	0
$x = -6$	-6	-6
$x = -\pi = -3.14\ldots\ldots$	-4	-3
$x = \frac{1}{2}$	0	1

x : Real Number

$\lfloor x \rfloor = n \longrightarrow$ n is the Greatest int $\leq x$.

$\lceil x \rceil = n \longrightarrow$ n is the Least int $\geq x$.



DEFINITION 12 The *floor function* assigns to the real number x the largest integer that is less than or equal to x. The value of the floor function at x is denoted by $\lfloor x \rfloor$. The *ceiling function* assigns to the real number x the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.

Remark: The floor function is often also called the *greatest integer function*. It is often denoted by $[x]$.

EXAMPLE 26 These are some values of the floor and ceiling functions:

$$\lfloor \tfrac{1}{2} \rfloor = 0,\ \lceil \tfrac{1}{2} \rceil = 1,\ \lfloor -\tfrac{1}{2} \rfloor = -1,\ \lceil -\tfrac{1}{2} \rceil = 0,\ \lfloor 3.1 \rfloor = 3,\ \lceil 3.1 \rceil = 4,\ \lfloor 7 \rfloor = 7,\ \lceil 7 \rceil = 7.$$

Q: for a real number x;

$\lfloor x \rfloor = \lceil x \rceil$ then x?

Q: for a real number x;

$\lfloor x \rfloor = \lceil x \rceil$ then x?

x: integer.

$x = -7 \longrightarrow \lfloor x \rfloor = -7 = \lceil x \rceil$

$x = -7.1 \longrightarrow \lfloor x \rfloor = -8; \lceil x \rceil = -7$

Note:

for any Real number x ;

x is integer $\boxed{\text{if and only if}}$

$$\lfloor x \rfloor = \lceil x \rceil$$

x is integer $\Longleftrightarrow$ $\lceil x \rceil = \lfloor x \rfloor$

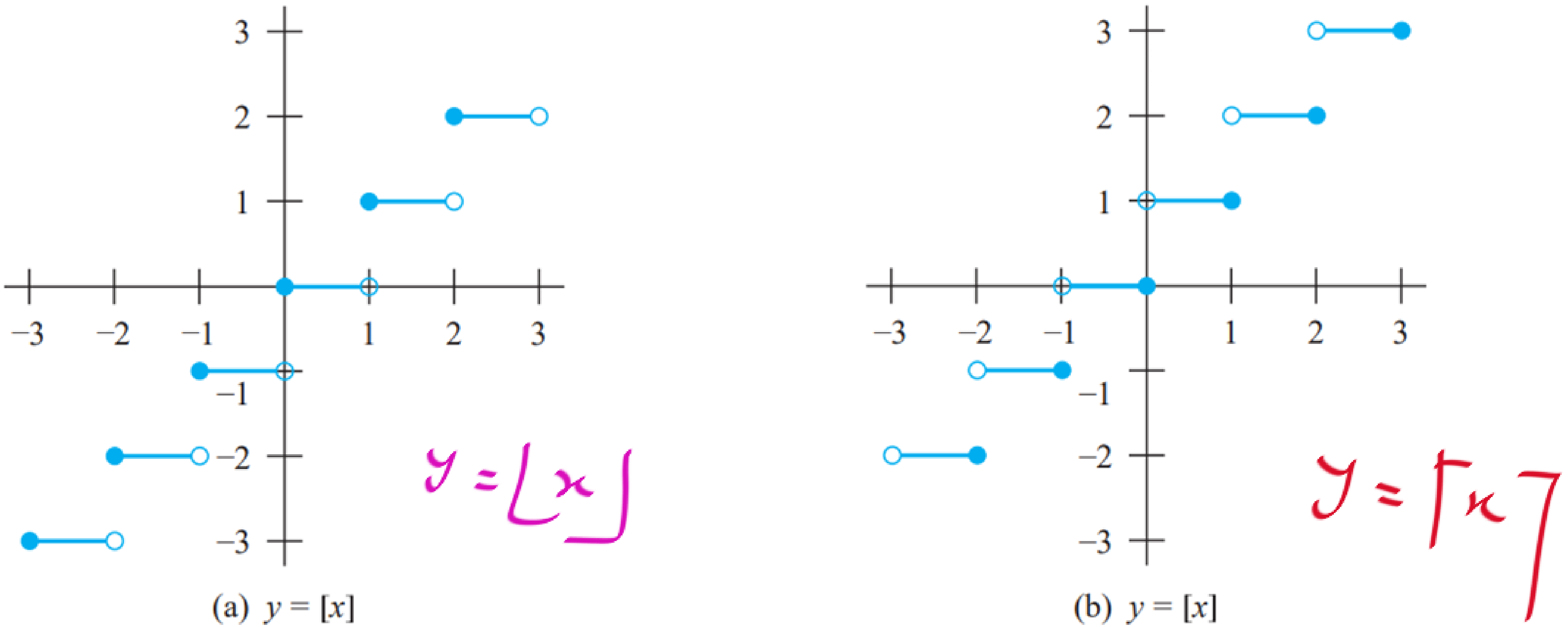
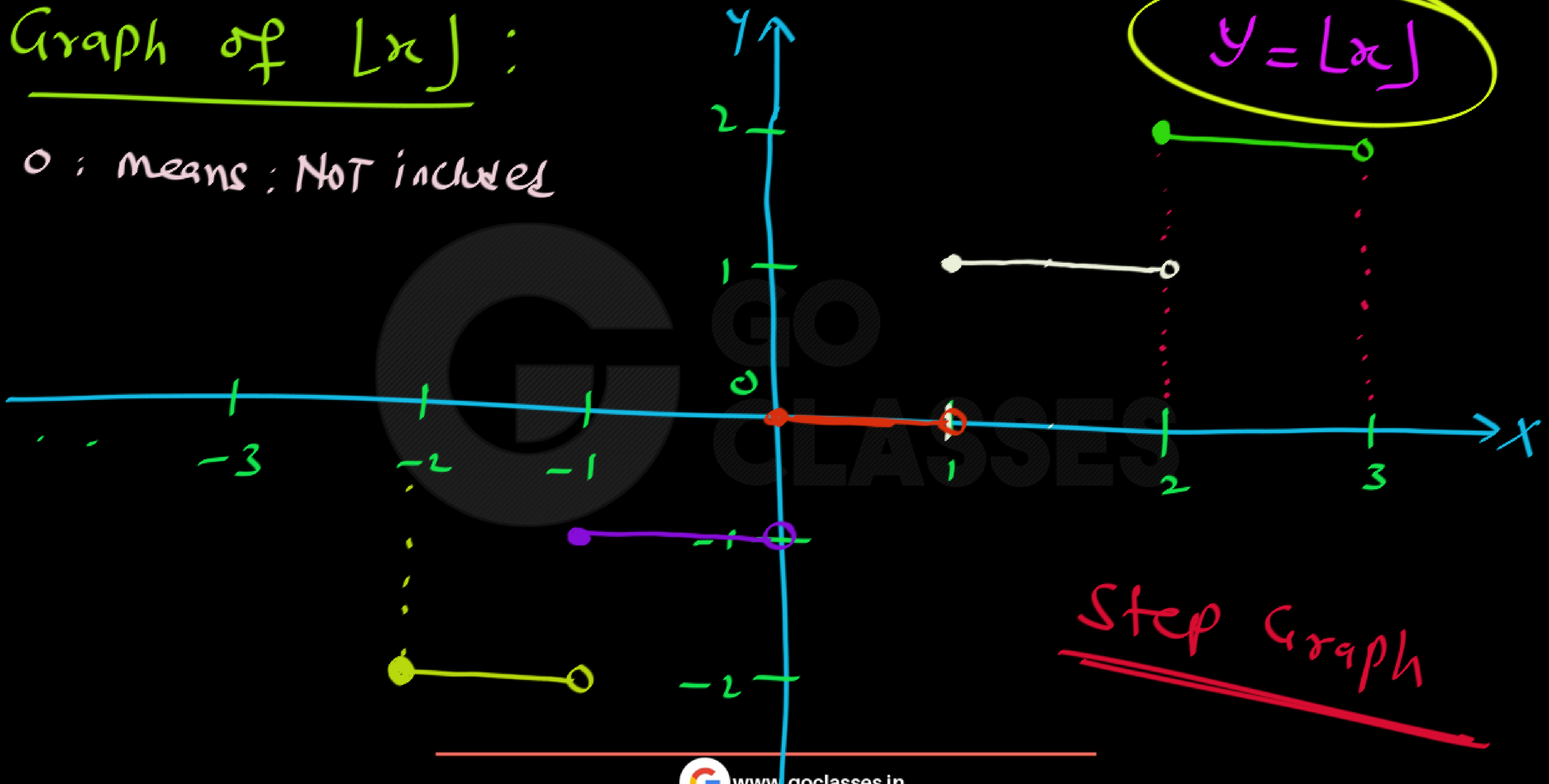


FIGURE 10 **Graphs of the (a) Floor and (b) Ceiling Functions.**

Graph of $\lfloor x \rfloor$:

$y = \lfloor x \rfloor$

o : means : NOT includes



Step Graph

$$\underbrace{1 \le x < 2} \longrightarrow \lfloor x \rfloor = 1$$

$\longrightarrow x = 1.\ldots\ldots$

$$\underbrace{2 \le x < 3} \longrightarrow \lfloor x \rfloor = 2$$

$\longrightarrow x = 2.\ldots\ldots$

$$\underbrace{-4 \le x < -3} \longrightarrow \lfloor x \rfloor = -4$$

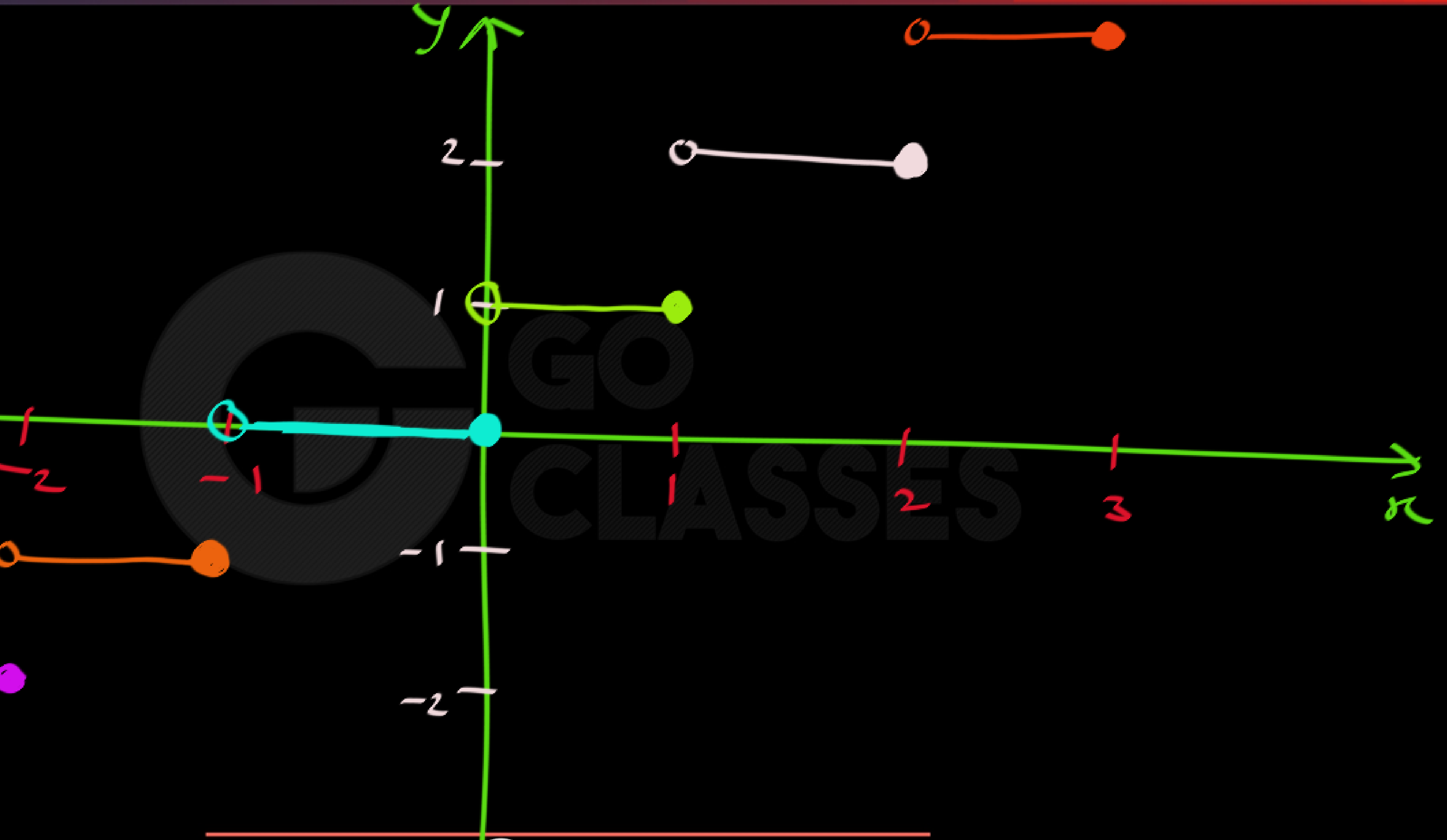
$\longrightarrow x = -4, -3.9, -3.1$

$1 < x \leq 2 \longrightarrow \lceil x \rceil = 2$

$0 < x \leq 1 \longrightarrow \lceil x \rceil = 1$

$-4 < x \leq -3 \longrightarrow \lceil x \rceil = -3$

$y = \lceil x \rceil$



EXAMPLE 27 Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

Send 10 bits : → How many bytes ? = 2

000000 10 11 0111 00

Padding

Data

Byte #1

byte #2

$$\left\lceil \frac{10 \text{ bits}}{\underbrace{8 \text{ bits}}_{1 \text{ byte}}} \right\rceil = 2$$

1 Byte = 8 bits

bit → 0, 1

EXAMPLE 27 Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

Data

Data : 00110100 - - -

1 Byte

1 byte 1 byte

$$\left\lceil \frac{100 \text{ bits}}{8 \text{ bits}} \right\rceil = 13$$

EXAMPLE 27 Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

Solution: To determine the number of bytes needed, we determine the smallest integer that is at least as large as the quotient when 100 is divided by 8, the number of bits in a byte. Consequently, $\lceil 100/8 \rceil = \lceil 12.5 \rceil = 13$ bytes are required. ◀

In any Objective Exam:

$\lceil x \rceil$, $\lfloor x \rfloor$ Questions:

Consider Cases:

$$\left\{\begin{array}{l} x = \text{integer} \\ x = 5.3 \\ x = -5.3 \\ x = 0 \end{array}\right.$$

& Try to eliminate options.

Exercise 3.2.4: Proving facts about the floor and ceiling functions.

Prove or disprove each statement

(a)

If n is an even integer, then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n}{2}$

(b)

If n is an odd integer, then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$

(c)

For any real number x, $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$.

(d)

For any real number x, $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$.

(e)

For any real numbers x and y, $\lceil x \rceil \lceil y \rceil = \lceil xy \rceil$.

Exercise 3.2.4: Proving facts about the floor and ceiling functions.

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Prove or disprove each statement

(a) If n is an even integer, then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n}{2}$ $= \left\lceil \frac{n}{2} \right\rceil$ True

(b) If n is an odd integer, then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$ True

(c) For any real number x, $\lfloor 2x \rfloor = 2\lfloor x \rfloor$. False

(d) For any real number x, $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$. True

(e) For any real numbers x and y, $\lceil x \rceil \lceil y \rceil = \lceil xy \rceil$. False

(a) n : even integer $= 2 * m$ → integer

$n = 0, 2, -2, 4, -4, \_\ \_\ \_\ .$

$\left\lfloor \frac{n}{2} \right\rfloor = \frac{n}{2}$ ✓

$\left\lfloor \frac{n}{2} \right\rfloor = \frac{n}{2} = \left\lceil \frac{n}{2} \right\rceil$ ✓

formal :

$n = 2m$ → integer

$\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{2m}{2} \right\rfloor = \lfloor m \rfloor = m = \frac{n}{2}$

(b) n : odd integer

$$\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$$

$n = 1, -1, 3, -3, \ldots$

$$\left\lfloor \frac{n}{2} \right\rfloor = 0 \qquad \frac{n-1}{2} = 0$$

$$\left\lfloor \frac{n}{2} \right\rfloor = -2 \qquad \frac{n-1}{2} = -2$$

n : odd integer

$n = 3$

$\frac{n-1}{2} = 1$

$\left\lceil \frac{n}{2} \right\rceil = 2$

$\left\lceil \frac{n}{2} \right\rceil = \frac{n-1}{2}$ ✗

$\boxed{\left\lceil \frac{n}{2} \right\rceil = \frac{n+1}{2}}$

(c) for any Real x

$$\lfloor 2x \rfloor = 2\lfloor x \rfloor \quad \text{false}$$

$x = \frac{1}{2}$

Counterexample

$$\lfloor 2x \rfloor = \lfloor 2 * \tfrac{1}{2} \rfloor = 1$$

$$2\lfloor x \rfloor = 2\lfloor 0.5 \rfloor = 2 * 0 = 0$$

(d) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for any Real x

True

integer n

$\boxed{\lfloor n \rfloor = n}$ ✓

for any Real x;

$\lfloor x \rfloor \rightarrow$ integer

$\lceil x \rceil \rightarrow$ integer

(e) for any real x, y

$$\lceil x \rceil \lceil y \rceil = \lceil xy \rceil$$

False.

$x = \frac{1}{2}$, $y = 2$

$\lceil x \rceil \lceil y \rceil = 2$

$\lceil xy \rceil = 1$

$\neq$

Task 1 (1.5 pt)

Reminder.

Floor and Ceiling Functions

Let x be any real number. Then x lies between two integers called the floor and the ceiling of x. Specifically,

$\lfloor x \rfloor$, called the *floor* of x, denotes the greatest integer that does not exceed x.

$\lceil x \rceil$, called the *ceiling* of x, denotes the least integer that is not less than x.

If x is itself an integer, then $\lfloor x \rfloor = \lceil x \rceil$; otherwise $\lfloor x \rfloor + 1 = \lceil x \rceil$. For example,

$$\lfloor 3.14 \rfloor = 3, \quad \left\lfloor \sqrt{5} \right\rfloor = 2, \quad \lfloor -8.5 \rfloor = -9, \quad \lfloor 7 \rfloor = 7, \quad \lfloor -4 \rfloor = -4,$$

$$\lceil 3.14 \rceil = 4, \quad \left\lceil \sqrt{5} \right\rceil = 3, \quad \lceil -8.5 \rceil = -8, \quad \lceil 7 \rceil = 7, \quad \lceil -4 \rceil = -4$$

Properties of Floor, Ceiling Functions:

1. If x is already an integer:

$$x = \lfloor x \rfloor = \lceil x \rceil$$

(2) for any Real value x;

$\lfloor x \rfloor$ = integer

$\lceil x \rceil$ = integer

$\lceil \alpha \rceil$; $\lfloor \alpha \rfloor$ are integers.

$$\lceil \lceil x \rceil \rceil = \lceil x \rceil$$

$$\lceil x \rceil = \lceil \lceil x \rceil \rceil = \lceil \lceil \lceil x \rceil \rceil \rceil = \cdots = \lceil x \rceil$$

$$\left\lceil \left\lfloor \lceil x \rceil \right\rfloor \right\rceil = \lceil x \rceil \quad \checkmark$$

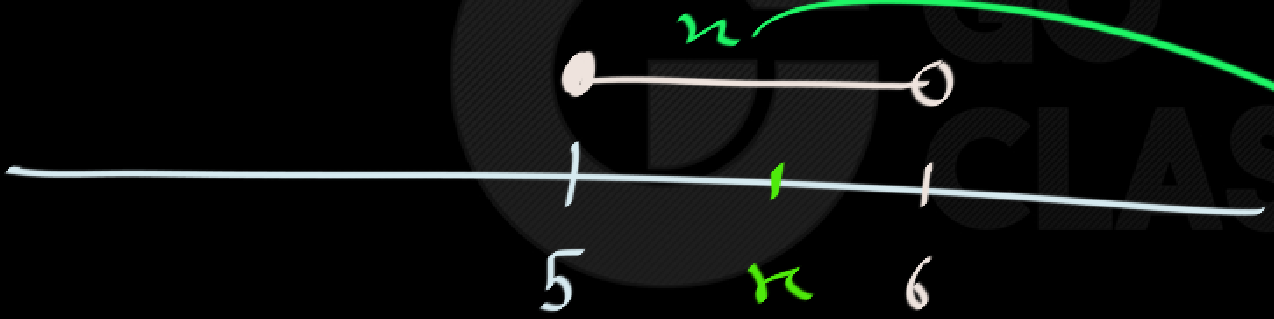
$$\left\lceil \left\lfloor \lfloor x \rfloor \right\rfloor \right\rceil = \lfloor x \rfloor \quad \checkmark$$

TABLE 1 Useful Properties of the Floor and Ceiling Functions.
(n is an integer, x is a real number)

(1a)	$\lfloor x \rfloor = n$ if and only if $n \le x < n+1$
(1b)	$\lceil x \rceil = n$ if and only if $n-1 < x \le n$
(1c)	$\lfloor x \rfloor = n$ if and only if $x-1 < n \le x$
(1d)	$\lceil x \rceil = n$ if and only if $x \le n < x+1$
(2)	$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$
(3a)	$\lfloor -x \rfloor = -\lceil x \rceil$
(3b)	$\lceil -x \rceil = -\lfloor x \rfloor$
(4a)	$\lfloor x+n \rfloor = \lfloor x \rfloor + n$
(4b)	$\lceil x+n \rceil = \lceil x \rceil + n$

1a , 1c ;

$\lfloor x \rfloor = 5 \longleftrightarrow 5 \leq x < 6$



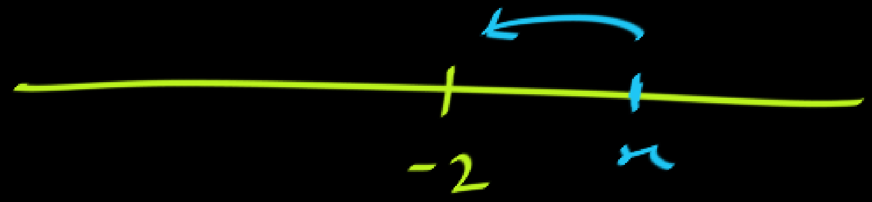
$\lfloor x \rfloor = 5$

$\lfloor x \rfloor$

$$\lfloor x \rfloor = 1 \iff 1 \le x < 2$$

$$\lfloor x \rfloor = 0 \iff 0 \le x < 1$$

$$\lfloor x \rfloor = -2 \iff -2 \le x < -1$$



1a , 1c :

$\lfloor x \rfloor = n \iff n \le x < n+1$

Proof: $\lfloor x \rfloor = n \xrightarrow[\text{Definition}]{\text{from}} n \le x$

Greatest integer that is $\le x$

1a , 1c :

$\lfloor x \rfloor = n \longleftrightarrow n \leq x < n+1$

Proof: $\lfloor x \rfloor = n \xrightarrow[\text{Definition}]{\text{from}} n \leq x$

n : integer

Greatest integer that does not exceed x.

What about any integer > n ?

that will definitely exceed x.

$\lfloor x \rfloor = 5 \longrightarrow$ means 5 is the Largest integer that does not exceed x.

So; any integer > 5 will exceed x

$\rightarrow > x$.

$\lfloor x \rfloor = n \xleftrightarrow[\text{Definition}]{\text{from}} n \le x < n+1$

Largest int that is $\le x$.

integer ;

So; $n+1$ is integer $> n$.

will exceed x

$\lfloor x \rfloor = 5 \longleftrightarrow 5 \leq x < 6$

$\lfloor x \rfloor = -2 \longleftrightarrow -2 \leq x < -1$

$$\boxed{\lfloor x \rfloor = n \longleftrightarrow n \leq x < n+1}$$

$\rightarrow$ Proof by Definition of $\lfloor x \rfloor$.

Note: $\underbrace{\lfloor x \rfloor = n \longleftrightarrow n \le x < n+1}$

Done ✓

find Range of n in terms of x?

Note:

$\lfloor x \rfloor = n \longleftrightarrow n \leq x < n+1$

$x-1 < n$

$n \leq x$

$$\boxed{x-1 < n \leq x}$$

1a, 1c:

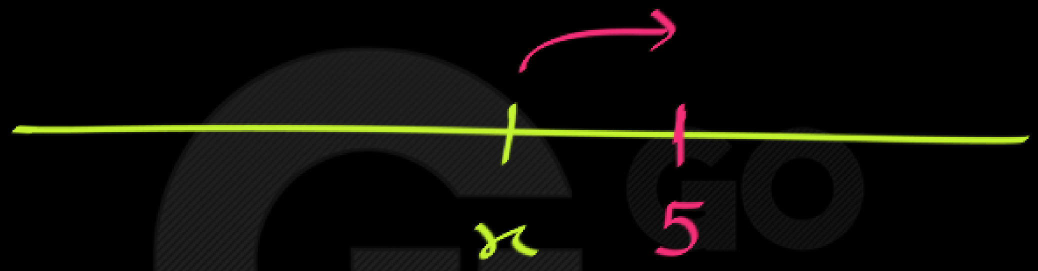
1a: $\lfloor x \rfloor = n$ iff $n \leq x < n+1$

1c $\lfloor x \rfloor = n$ iff $x-1 < n \leq x$

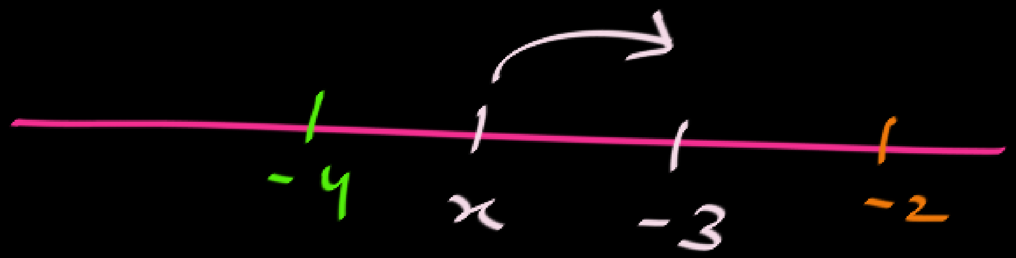
1 b :

$\lceil x \rceil = n$ iff ?

$\lceil x \rceil = 5 \longrightarrow 4 < x \leq 5$

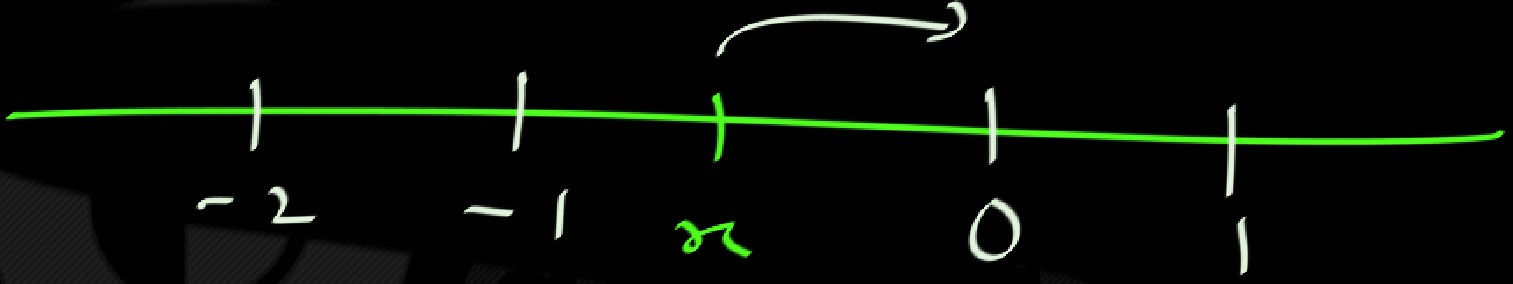


$\lceil x \rceil = -3$ $\quad -4 < x \leq -3$



$\lceil x \rceil = 0 \longrightarrow -1 < x \leq 0$

must be integer



$\lceil x \rceil = 3 \longleftrightarrow 2 < x \leq 3$

$\lceil x \rceil = n \longleftrightarrow n-1 < x \leq n$

integer

$\lceil x \rceil = n$ iff $\underbrace{n-1 < x \leq n}$

Proof by Definition of $\lceil x \rceil$:

$\boxed{\lceil x \rceil = n \longrightarrow \text{Least integer} \geq x.}$

$n \geq x$

Definition of $\lceil x \rceil$

$\lceil x \rceil = n$ Least integer $n \geq x$

n is the Lowest int that is $\geq x$. So any lower integer than n can not be $\geq x$. So; any lower int has to be $< x$.

$x \leq n$

$\lceil x \rceil = n$ → Least integer $n \geq x$

n is the Lowest int that is $\geq x$. So any lower integer than n will be $< x$.

$\lceil x \rceil = n$ → integer

$n-1 < x$

$$\boxed{n-1 < x \leq n}$$

1 b

$\lceil x \rceil = n$ iff $n-1 < x \leq n$

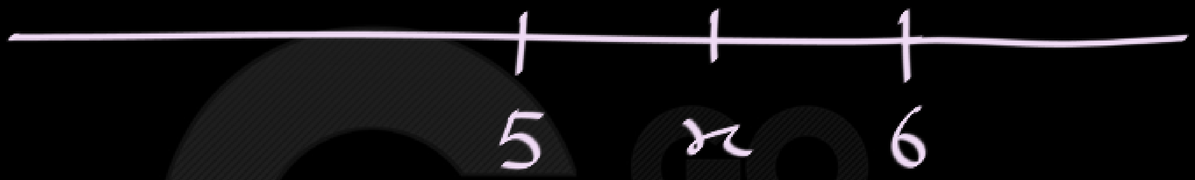
$\lceil x \rceil = n$ iff $x \leq n < x+1$

$n < x+1$

1b $\lceil x \rceil = n$ iff $n-1 < x \leq n$

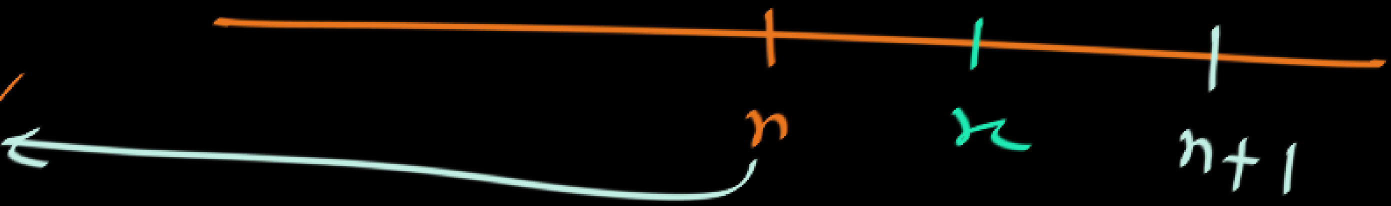
1d $\lceil x \rceil = n$ iff $x \leq n < x+1$

$\lfloor x \rfloor = 5$ $\qquad$ $5 \leq x < 6$



$\lfloor x \rfloor = n$ $\qquad$ $n \leq x < n+1$

integer



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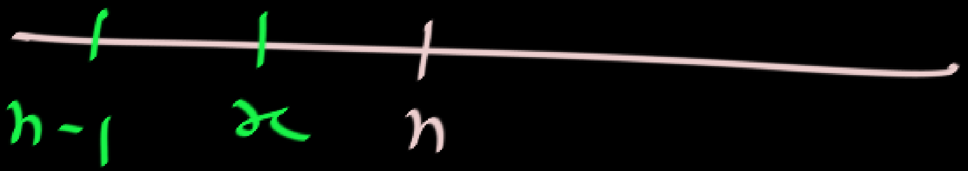
$\lceil x \rceil = 6$ iff $5 < x \leq 6$

$\lceil x \rceil = n$ iff $n-1 < x \leq n$



Just Remember this Photo.

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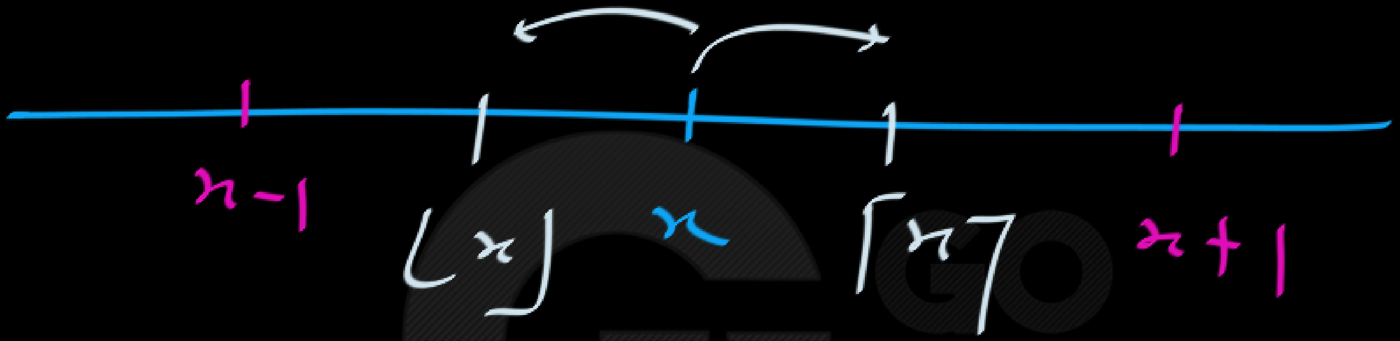
(1a) $\lfloor x \rfloor = n$ if and only if $n \le x < n + 1$	
(1b) $\lceil x \rceil = n$ if and only if $n - 1 < x \le n$	
(1c) $\lfloor x \rfloor = n$ if and only if $x - 1 < n \le x$	
(1d) $\lceil x \rceil = n$ if and only if $x \le n < x + 1$	
(2) $x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$	
(3a) $\lfloor -x \rfloor = -\lceil x \rceil$	
(3b) $\lceil -x \rceil = -\lfloor x \rfloor$	
(4a) $\lfloor x + n \rfloor = \lfloor x \rfloor + n$	
(4b) $\lceil x + n \rceil = \lceil x \rceil + n$	

2

$x-1 \qquad \lfloor x \rfloor \qquad x \qquad \lceil x \rceil \qquad x+1$

$$\boxed{x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1}$$



$$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$$

$x = 5.2$

$\underbrace{4.2}_{x-1}$ $\lfloor x \rfloor = 5$ $x = 5.2$ $\lceil x \rceil = 6$ $\underbrace{6.2}_{x+1}$

$$\boxed{x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1}$$

③ S: $\lfloor -x \rfloor = -\lceil x \rceil$

Cases:

x = integer → 5 → S True ✓

→ 0 → S True ✓

→ −2 → S True ✓

$x = 1.2$ → $\lfloor -1.2 \rfloor = -\lceil 1.2 \rceil$

−2   −2

$x = -1.2$ → S True.

③ $\lfloor -x \rfloor = -\lceil x \rceil$ ✓

$\lceil -x \rceil = -\lfloor x \rfloor$ ✓

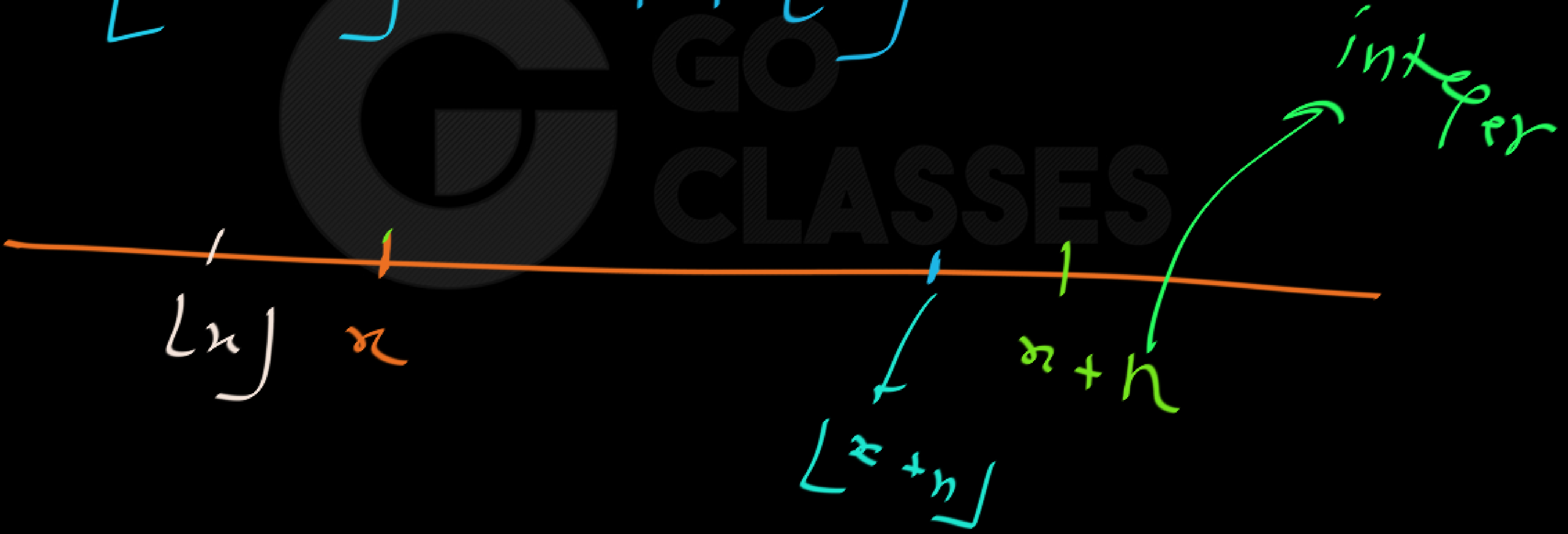
Just Consider $\boxed{\text{all Verity of Cases}}$ to Solve such Questions.

All Varity of Cases:

- integer
  - → 0
  - → +ve
  - → -ve
- non-integer
  - → +ve
  - → -ve

(4) n is integer ; x : Real value.

$$\lfloor n + x \rfloor = n + \lfloor x \rfloor$$ ✓



$x = 2.2 \quad ; \quad n = 7$

$$\lfloor 2.2 + 7 \rfloor = \lfloor 9.2 \rfloor = 9$$

$$= \lfloor 2.2 \rfloor + 7 = 9$$

n : integer ; x : Real

$$\lfloor x + n \rfloor = \lfloor x \rfloor + n$$

$$\lceil x + n \rceil = \lceil x \rceil + n$$

EXAMPLE 30 Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ for all real numbers x and y.

EXAMPLE 30 Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ for all real numbers x and y.

false

$x = \frac{1}{2}$

$y = \frac{1}{2}$

Counterexample

$\lceil x+y \rceil = 1$

$\lceil x \rceil + \lceil y \rceil = 2$

$\neq$

EXAMPLE 30 Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ for all real numbers x and y.

Solution: Although this statement may appear reasonable, it is false. A counterexample is supplied by $x = \frac{1}{2}$ and $y = \frac{1}{2}$. With these values we find that $\lceil x + y \rceil = \lceil \frac{1}{2} + \frac{1}{2} \rceil = \lceil 1 \rceil = 1$, but $\lceil x \rceil + \lceil y \rceil = \lceil \frac{1}{2} \rceil + \lceil \frac{1}{2} \rceil = 1 + 1 = 2$. ◀

Note:

In any objective exam; for Questions about $\lceil\ \rceil$, $\lfloor\ \rfloor$;

Consider all variety of cases & Eliminate options.