Q: In a Group, can we say that
If a*a = a then a = e ? ⇒ Yes

$a * a = a$

$a * a = a * e$

$a = e$

$a * e = a \quad , \forall a$

(Group → left Cancellation)

Q: In a Monoid, can we say that
If a*a = a then a = e ? ⇒ No

$(N, *)$; $a * b = \max(a, b)$

$e = 1$

$5 * 5 = \max(5, 5) = 5$

$5 * 5 = 5$ But $5 \neq e$

Monoid

- Unique identity ✓
- Inverse may not exists for some elements
- left cancellation ✗
- Right " ✗

Group → monoid with inverse

- Unique identity ✓
- Unique inverse ✓
- left Cancellation ✓
- Right " ✓

Group :

$a * c = a * d$

$\bar{a}^{-1} * a * c = a^{*} * a * d$

$e \qquad\qquad e$

$c = d$