Definition :

Definitions are Always "If and only if" statements.

(even if they are written as "If")

Definition of Even number :

a number is even number if it is integer multiple of 2.

Definition of Even number :

a number is even number if it is integer multiple of 2.

even

m2

m2 ⟶ even

m2 $\longrightarrow$ even

even $\longrightarrow$ m2 ?? Yes.

So the Correct statement is :

even if and only if m2

Definition is **Always** a

"If and only if"

Statement.

Implication (one way Statement)

tells you about "Property"

BiImplication tells you the

"Definition".

Definition of a even number :

D1 integer multiple of 2

D2 integer divisible by 2

D3 $\underbrace{n \bmod 2}$ $= 0$

$\underbrace{\text{Prime} > 2}_{A} \longrightarrow \underbrace{\text{odd}}_{B}$ ✓

Is "being odd" Definition of "Prime >2" ?!

No.

But "being odd" is property of "prime >2".

Boy → Human

"being Human" is property of "being boy"

Define a boy :

$\underbrace{\text{boy is a Human.}}_{\text{No Correct Definition}}$

$$\underbrace{P \to Q}$$

P is property of Q ✗

Q " " " P ✓

Every P has property Q.

Properties of Natural numbers :

① integer

② > 0

③ rational

④ real

⑤ odd or even

N → I ✓

N → real ✓

N → rational ✓

Implication tells us Property.

Bi Implication tells us Definition.

Eg: Prime minister of India $\longrightarrow$ Indian

So, PM has property that He/She is Indian,

But Being Indian is Not the Definition of PM of Indian.

Eg: $\underbrace{\text{Prime number greater than 2}}_{P} \longrightarrow \underbrace{\boxed{\text{Odd Number.}}}_{Q}$

Q is property of P.

$\underbrace{P \longleftrightarrow Q}$

P is Definition of Q.

Q " " " P.

P, Q are Definition of each other.

eg: $x = 2 \rightleftarrows x = \text{even prime}$

$$x = 2 \longleftrightarrow x = \text{even prime}$$

So "being 2" is Definition of "even prime"

"being even prime" is " " "being 2"

$\underbrace{P \to Q}$

Q is property of P. (P satisfies property Q)

$P \leftrightarrow Q$

P, Q are Definitions of each other.

Definitions of even prime :

D1 is 2.

D2 Prime < 3

D3 is prime and divisible by 2

So, $\underbrace{P \longrightarrow Q}$

Says that P satisfy property Q.

$P \longleftrightarrow Q$

Says that P, and only P satisfies property Q.

Recap So Far

- A **propositional variable** is a variable that is either true or false.
- The **logical connectives** are
  - Negation: $\neg p$
  - Conjunction: $p \wedge q$
  - Disjunction: $p \vee q$
  - Implication: $p \to q$
  - Biconditional: $p \leftrightarrow q$

$$\left.\begin{array}{l} P \rightarrow Q \\ P \rightarrow R \end{array}\right\} p$$

$\underbrace{\text{Q, R are Properties of P}}$

P satisfies Q, R

If P is True, Q True, R True

Logical Connectives

- **Logical NOT: $\neg p$**
  - Read "**not** p"
  - $\neg p$ is true if and only if p is false.
  - Also called **logical negation**.
- **Logical AND: $p \land q$**
  - Read "p **and** q."
  - $p \land q$ is true if both p and q are true.
  - Also called **logical conjunction**.
- **Logical OR: $p \lor q$**
  - Read "p **or** q."
  - $p \lor q$ is true if at least one of p or q are true (inclusive OR)
  - Also called **logical disjunction**.

Some Popular Boolean Operators

Formal Name	Nickname	Arity	Symbol
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$