

Question1: Solve the following summation

$$\sum_{i=1}^4 8$$



Answer 1

$$\sum_{i=1}^4 8 = 8 + 8 + 8 + 8 = 8(4) = 32$$

Question 2: True/False


$$\sum_{j=i}^n j = \sum_{k=0}^{n-i} i + k$$

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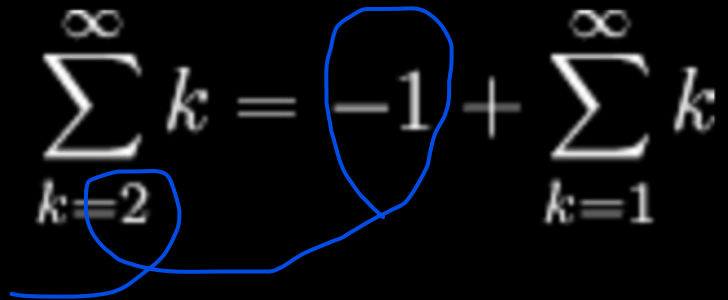
Answer 2: True


$$j=i+k$$

Let $k = j - i$. When $j = i$, $k = 0$ and when $j = n$, $k = n - i$. So we can write


$$\sum_{j=i}^n j = \sum_{k=0}^{n-i} i + k$$

Question 3: True/False

$$\sum_{k=2}^{\infty} k = -1 + \sum_{k=1}^{\infty} k$$


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Answer 3: True

$$\sum_{k=2}^{\infty} k = 2 + 3 + 4 + \dots \infty$$

$$-1 + \sum_{k=1}^{\infty} k = -1 + 1 + 2 + 3 + 4 + \dots \infty = 2 + 3 + 4 + \dots \infty$$

We can easily see that both summations are same

Question 4

Which of the following is same as given summation ?

$$\sum_{i=1}^5 i^2$$

A.

$$\sum_{i=0}^4 (i+1)^2$$

B.

$$\sum_{i=2}^6 (i-1)^2$$

C.

Both

D.

None

Answer 4

All of the below 3 represent same summation.

One way to check is to unroll them. Or you can use method given in class

$$\sum_{i=1}^5 i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\sum_{i=0}^4 (i+1)^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\sum_{i=2}^6 (i-1)^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

Question 5

Use your intuition to write the long sum in sigma notation

$$1 + 2 + 3 + 4 + 5$$



Answer

$$1 + 2 + 3 + 4 + 5 = \sum_{k=1}^5 k .$$

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Question 6

Use your intuition to write the long sum in sigma notation

$$1 + 4 + 9 + 16 + 25 + 36$$

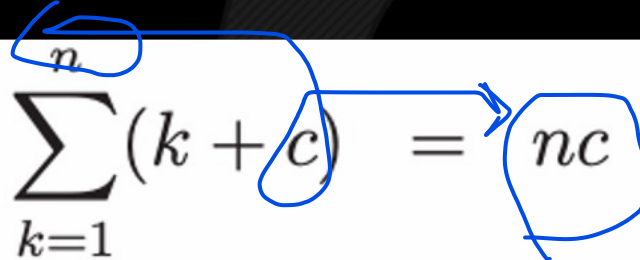
Handwritten blue annotations below the sum: 3^2 under 9, 1^2 under 1, 2^2 under 4, 4^2 under 16, 5^2 under 25, and 6^2 under 36.

Answer

$$1 + 4 + 9 + 16 + 25 + 36 = \sum_{k=1}^6 k^2 .$$

Question 7

True/False


$$\sum_{k=1}^n (k + c) = nc + \sum_{k=1}^n k$$

Answer: True

$$\sum_{k=1}^n (k + c) = \sum_{k=1}^n k + \sum_{k=1}^n c = \sum_{k=1}^n k + cn$$

Bonus Question

Use your intuition to write the long sum in sigma notation

$$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots + \frac{1}{100}$$

Answer

In this example, the first term -1 can also be written as a fraction $-\frac{1}{1}$. We also notice that the signs of the terms alternate, with a minus sign for the odd-numbered terms and a plus sign for the even-numbered terms. So we can take care of the sign by using $(-1)^k$, which is -1 when k is odd, and $+1$ when k is even. We can therefore write the sum as

$$(-1)^1 \frac{1}{1} + (-1)^2 \frac{1}{2} + (-1)^3 \frac{1}{3} + (-1)^4 \frac{1}{4} + \dots + (-1)^{100} \frac{1}{100}.$$

We can now see that k -th term is $(-1)^k 1/k$, and that there are 100 terms, so we would write the sum in sigma notation as

$$\sum_{k=1}^{100} (-1)^k \frac{1}{k}.$$