Partial Order Relations

Recap

GLB/Meet, LUB/Join

Website : https://www.goclasses.in/

Finding LUB,GLB of One element:

$$\left.\begin{array}{l} a \vee a = a \\ a \wedge a = a \end{array}\right\}$$

$$a \vee a = LUB\{a\} = LUB\{a, a\} = a \cup a = a$$

$$a \wedge a = GLB\{a\} = GLB\{a, a\} = a \cap a = a$$

Finding LUB,GLB of Two elements:

When element are Comparable:

If $a R b$ then

$a \vee b = b$

$a \wedge b = a$

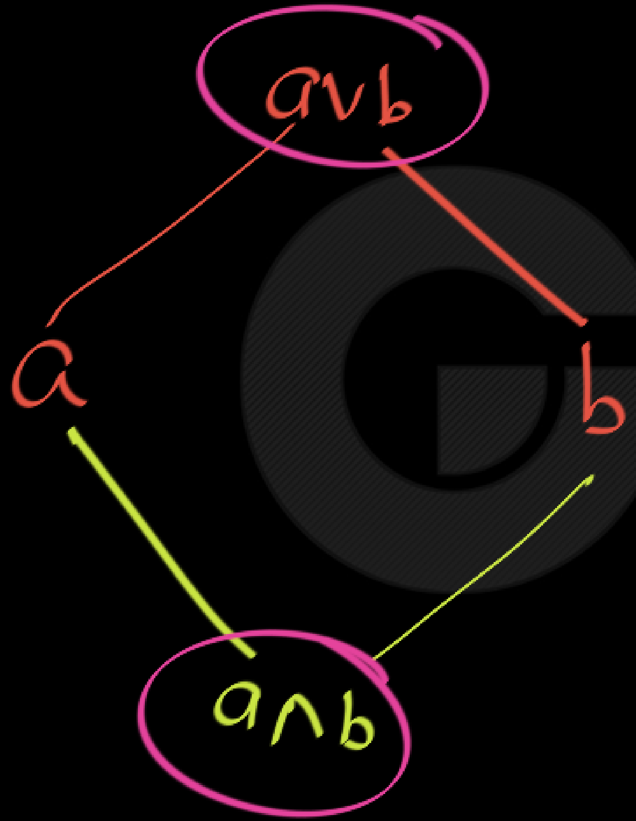
Finding LUB,GLB of Two elements:

When element are Incomparable:

For LUB, unique first joining point in upper direction.

For GLB, unique first meeting point in down direction.

$a \not R b, \quad b \not R a$



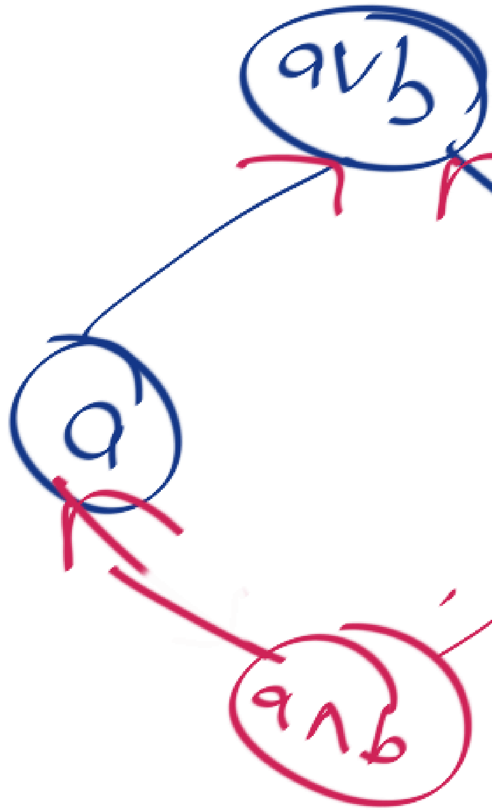
$a \; R \; (a \vee b)$

$b \; R \; (a \vee b)$

$(a \wedge b) \; R \; a$

$(a \wedge b) \; R \; b$

$(a \wedge b) \; R \; (a \vee b)$



LUB

upper

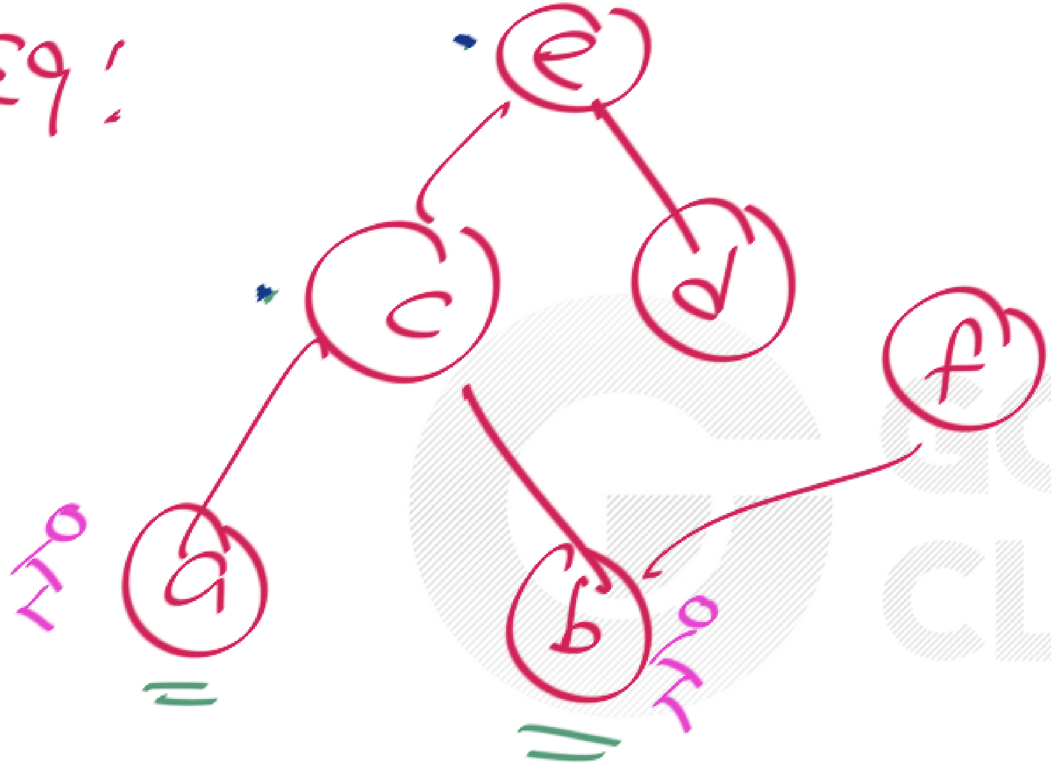
least

GLB

lower

Greatest

Eg:



$d \vee f = DNE$

$a \vee a = a$

$b \wedge b = b$

$a \vee b = c$

$a \wedge b = DNE$

$c \vee d = e$

$c \wedge d = DNE$

bRe

$b \vee e = e$

$b \wedge e = b$

$UB\{a,b\} = \{c, e\}$

Finding LUB,GLB of more than Two elements:

Method 1:

For LUB, find UB then in that UB, find Least element.

For GLB, find LB then in that LB, find Greatest element.

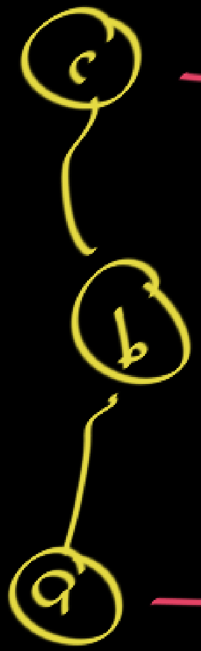
Finding LUB,GLB of more than Two elements:

Method 2:

Same as for “two elements”. For ex:

GLB(S) = Unique first meeting point for ALL the elements S.

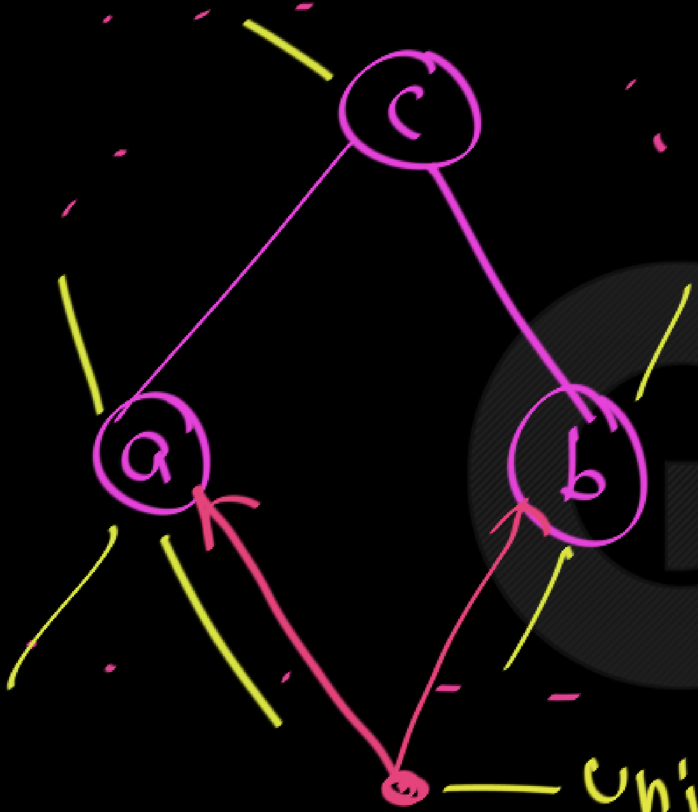
LUB $\{a, b, c\} = ?$



$a \vee b \vee c = \text{LUB}\{a, b, c\}$

$a \wedge b \wedge c = \text{GLB}\{a, b, c\}$

Case 1 $\quad aRbRc$



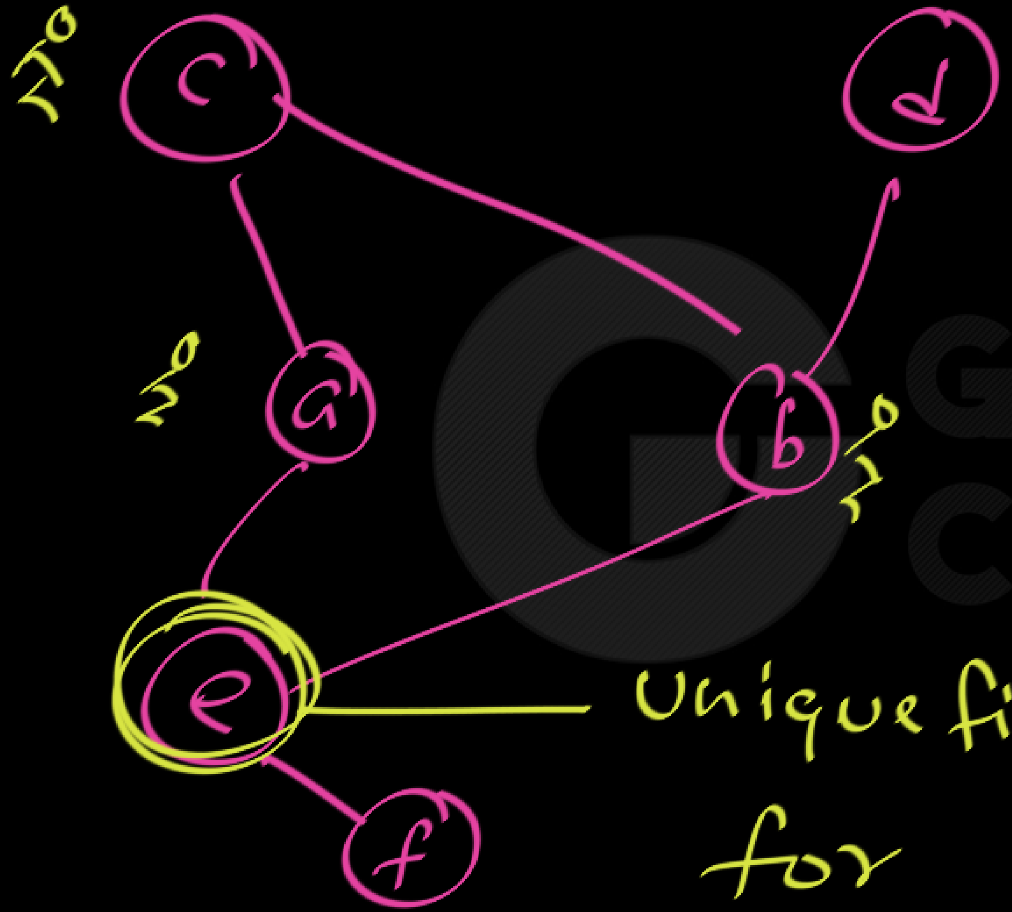
$a \vee b \vee c = LUB\{a, b, c\}$

$= c$

$GLB\{a, b, c\} = a \wedge b$

$= GLB\{a, b\}$

Unique first meeting point for a, b, c

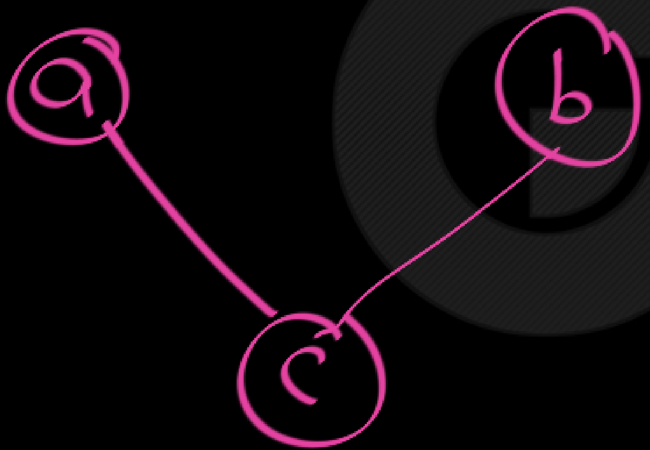


$a \wedge b = e$

$a \wedge b \wedge c = GLB\{a, b, c\}$

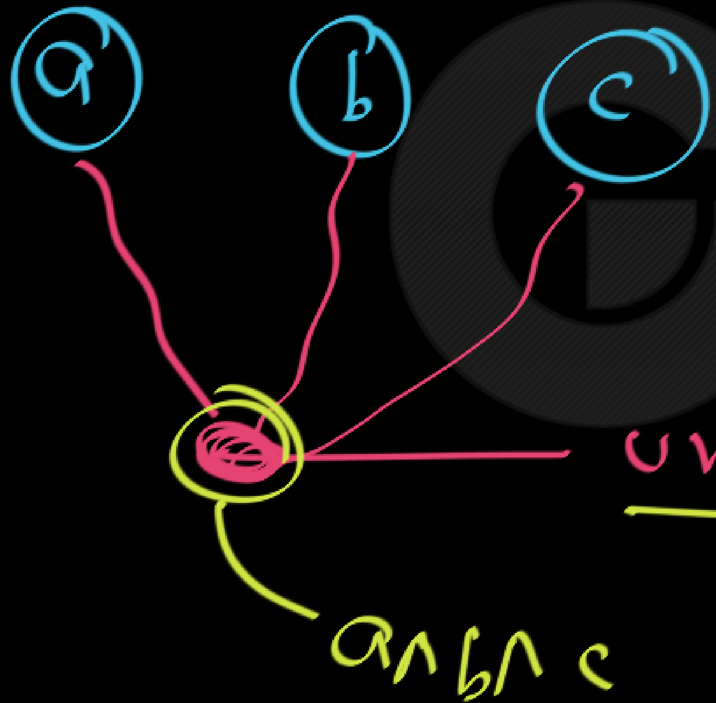
$= e$

Unique first meeting point for all a, b, c



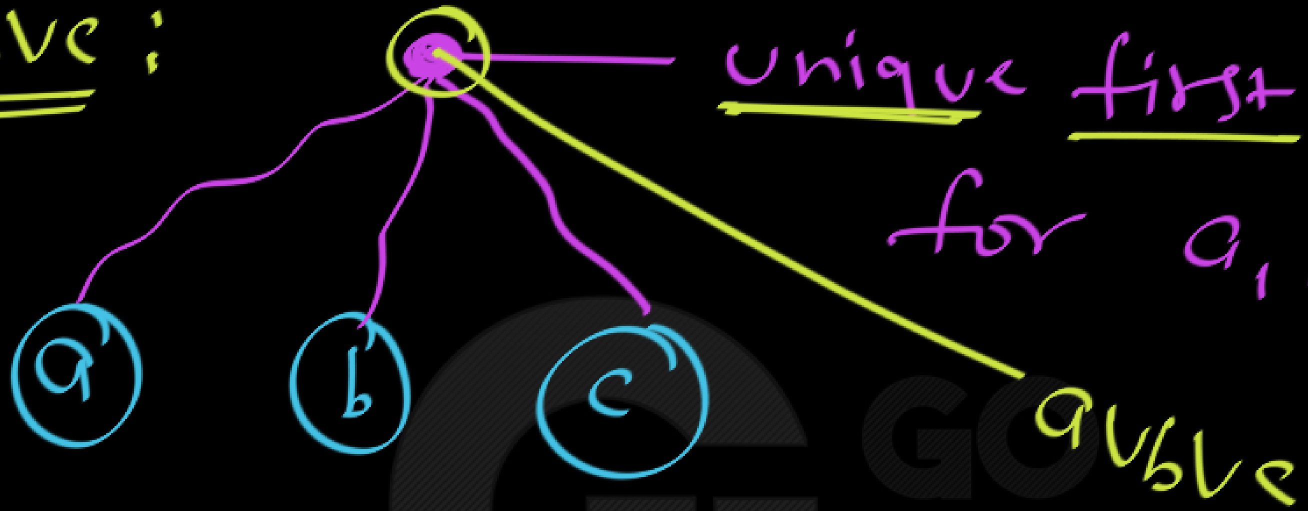
$$LUB\{a,b,c\} = a \vee b \vee c$$
$$= a \vee b$$
$$= LUB\{a,b\}$$
$$GLB\{a,b,c\} = a \wedge b \wedge c$$
$$= c$$

$a \wedge b \wedge c$:

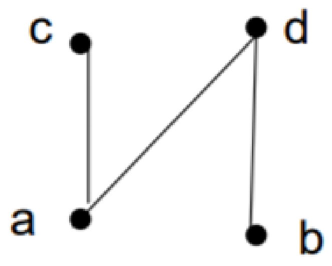


unique first meeting point for a, b, c

avbvc :



unique first Joining point for a, b, c in up Direction

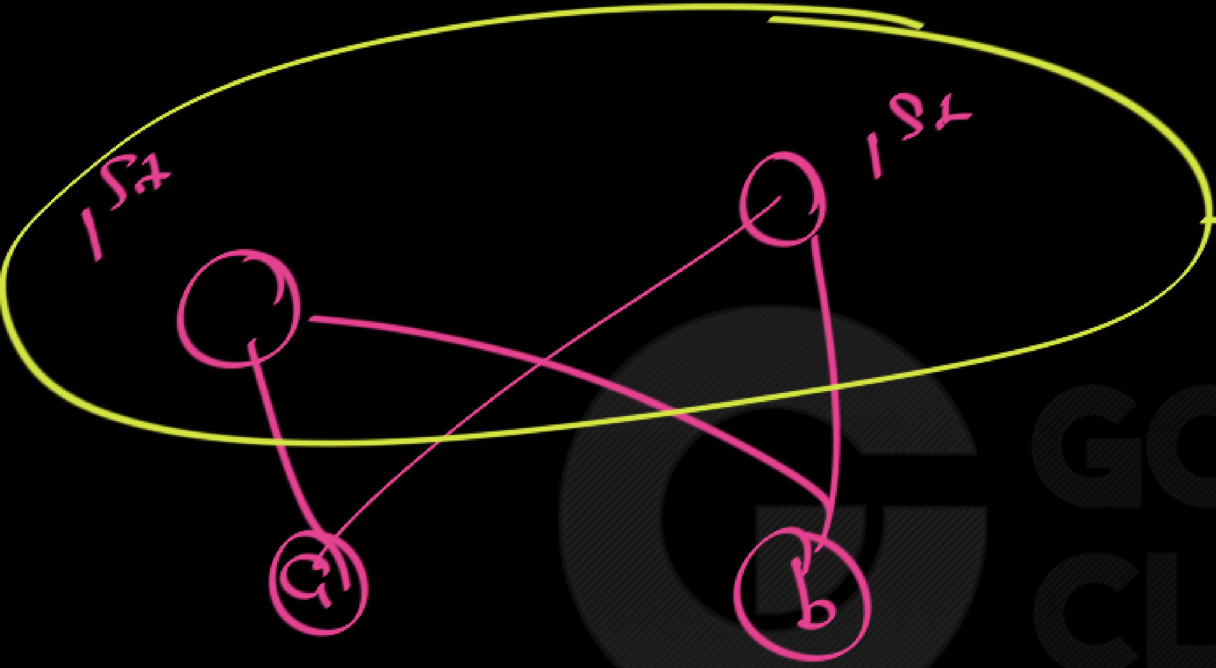


$c \vee c = c$
$b \wedge b = b$
$x \vee x = x$
$x \wedge x = x$

$a \vee c = c$
$a \wedge c = a$
a, c Comparable
$a R c$

$a \vee b = d$
$a \wedge b = DNE$

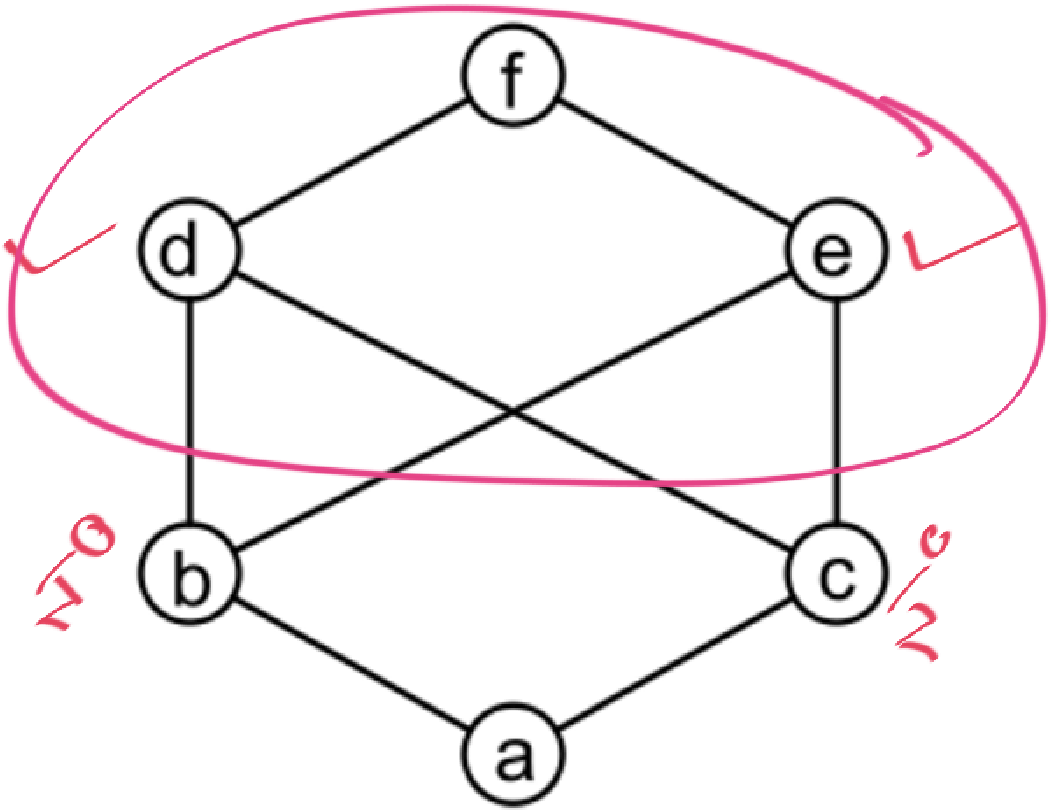
$c \vee b = DNE$
$c \wedge b = DNE$



not unique first Joining point, so

$a \vee b = \text{DNE}$

$a \vee b = \text{Join}\{a, b\}$



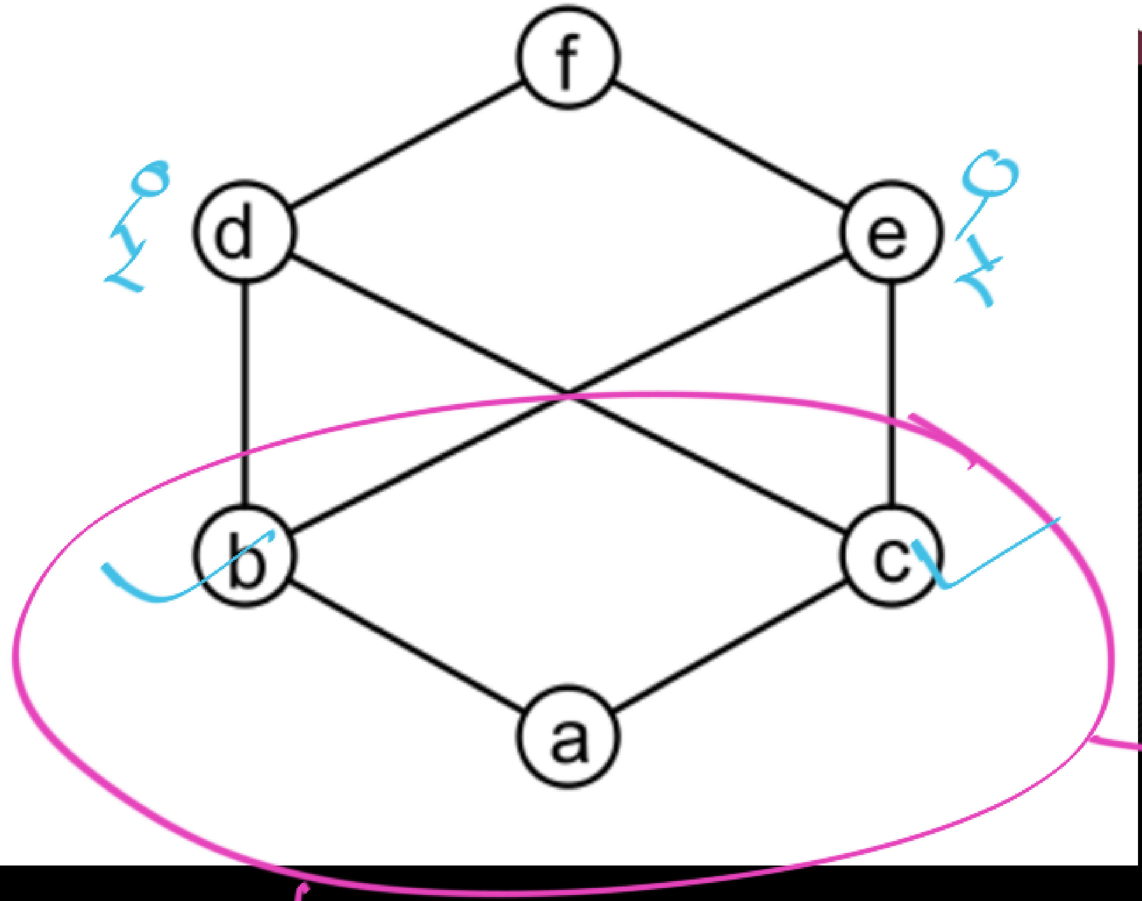
$b \vee c = DNE$

$UB\{b, c\} = \boxed{\{d, e, f\}}$

least element = DNE

$b \wedge c = a$; $b \wedge a = a$; $d \wedge a = a$

$d \vee a = d$



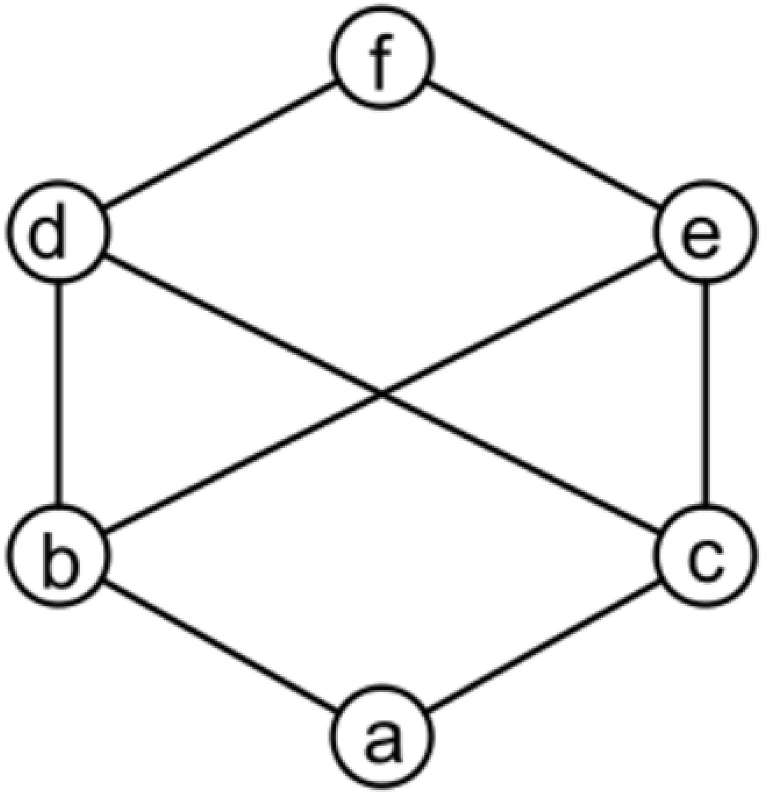
$d \wedge e =$ DNE

GLB$\{d, e\}$ = DNE

$d \wedge e =$ meet$\{d, e\}$

In this, No Greatest element

LB$\{d, e\}$



$d \vee c = d$

$d \wedge c = c$

$f \vee b = f$

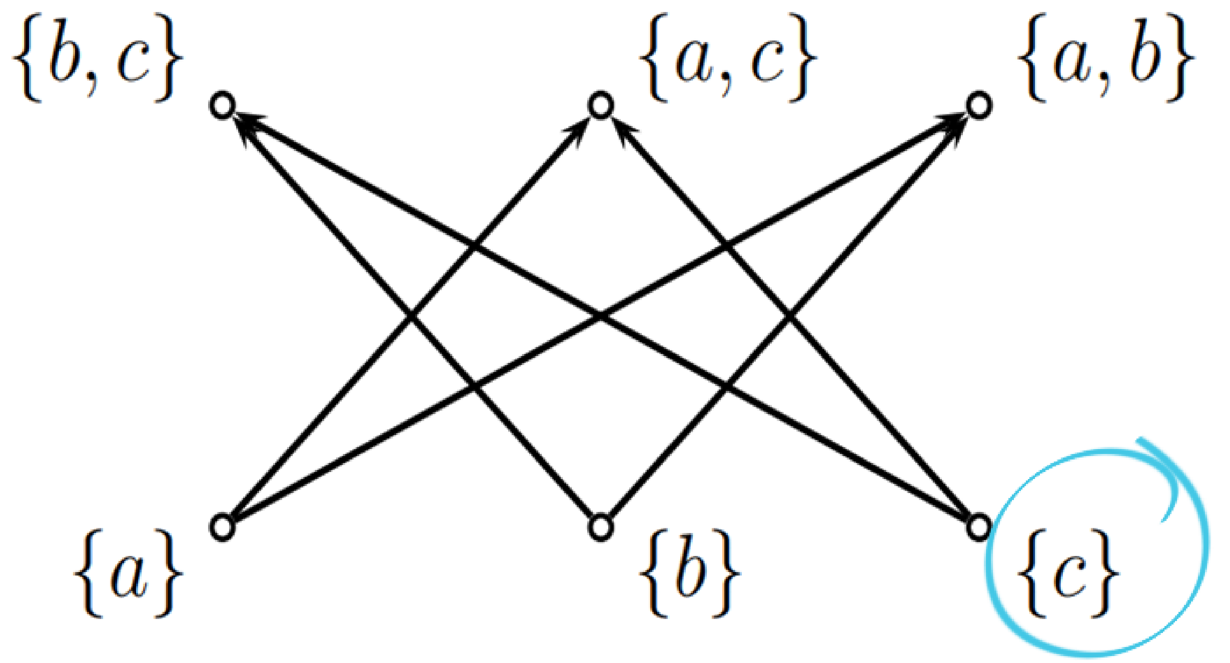
$f \wedge b = b$

$b \vee e = e$

$b \wedge e = b$

LUB{ {c} } = {c}

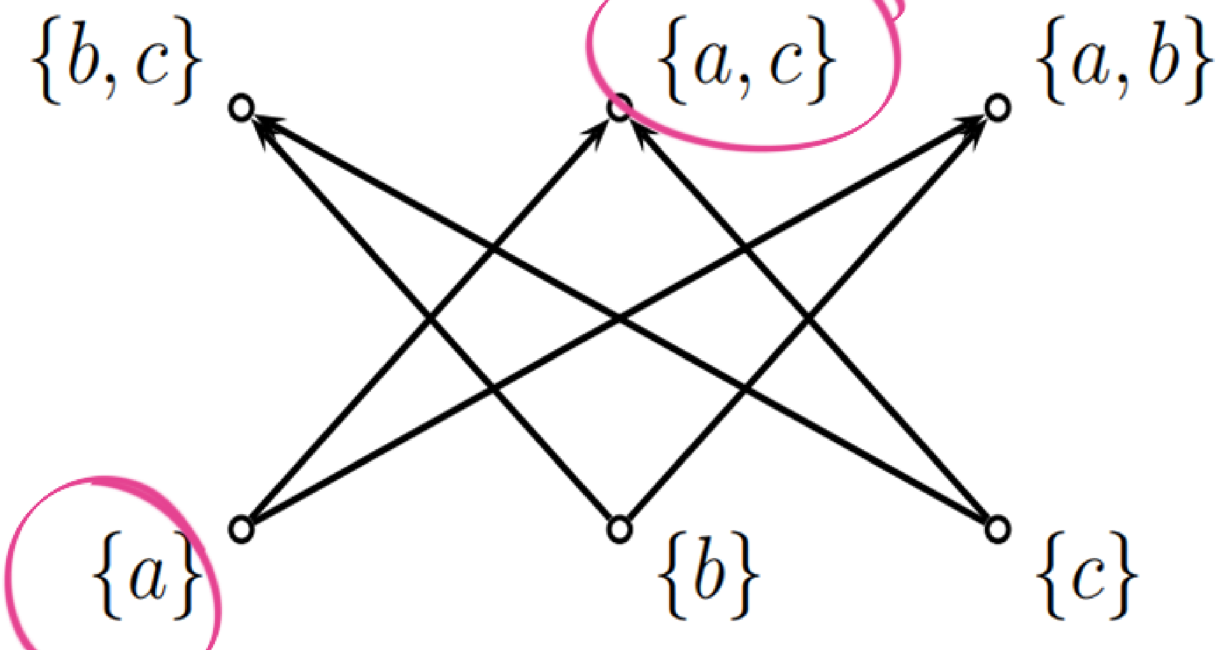
GLB{ {c} } = {c}



LUB{ {a,c},{a} } = {a, c}

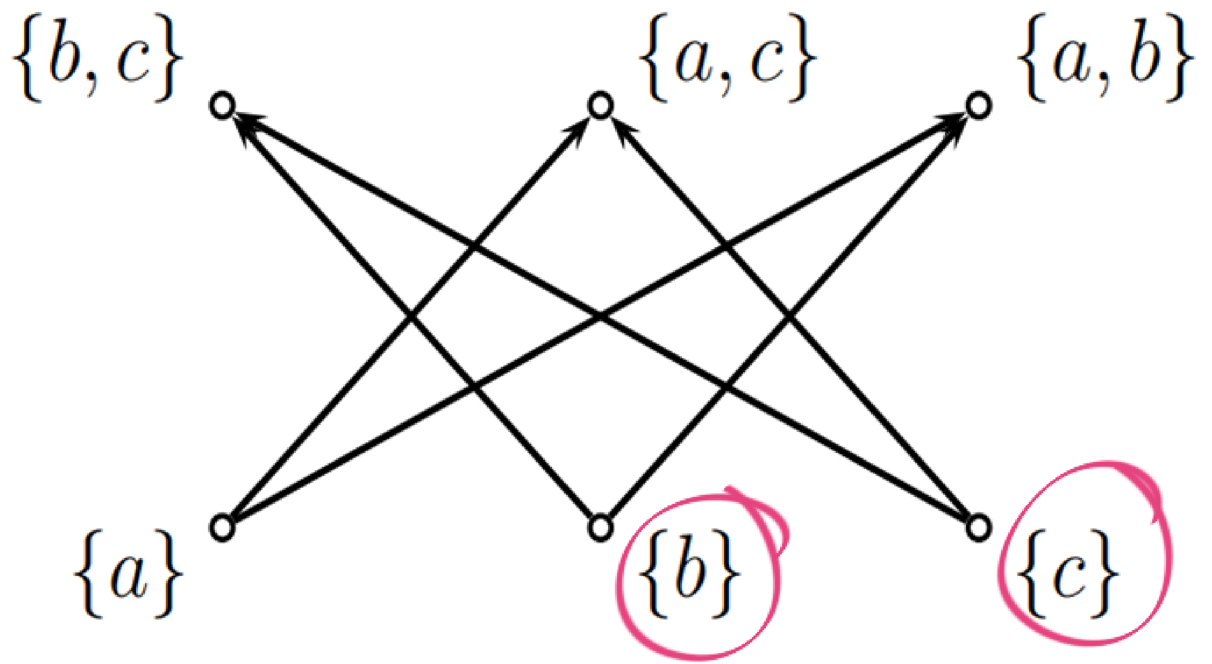
GLB{ {a,c},{a} } = {a}

{a} R {a,c}



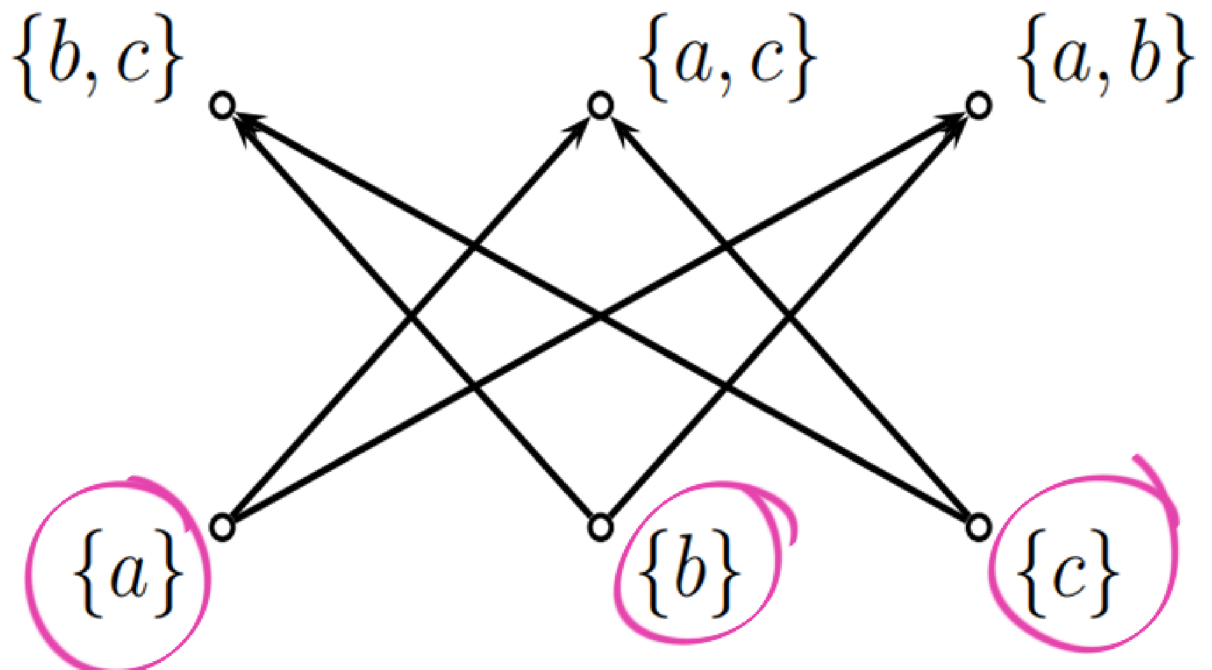
LUB{ {b},{c} } = {b, c}

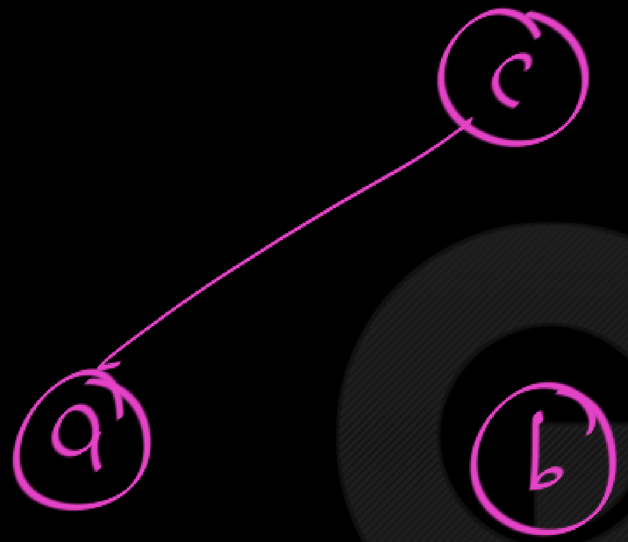
GLB{ {b},{c} } = DNE



LUB{ {a},{b},{c} } = DNE

GLB{ {a},{b},{c} } = DNE





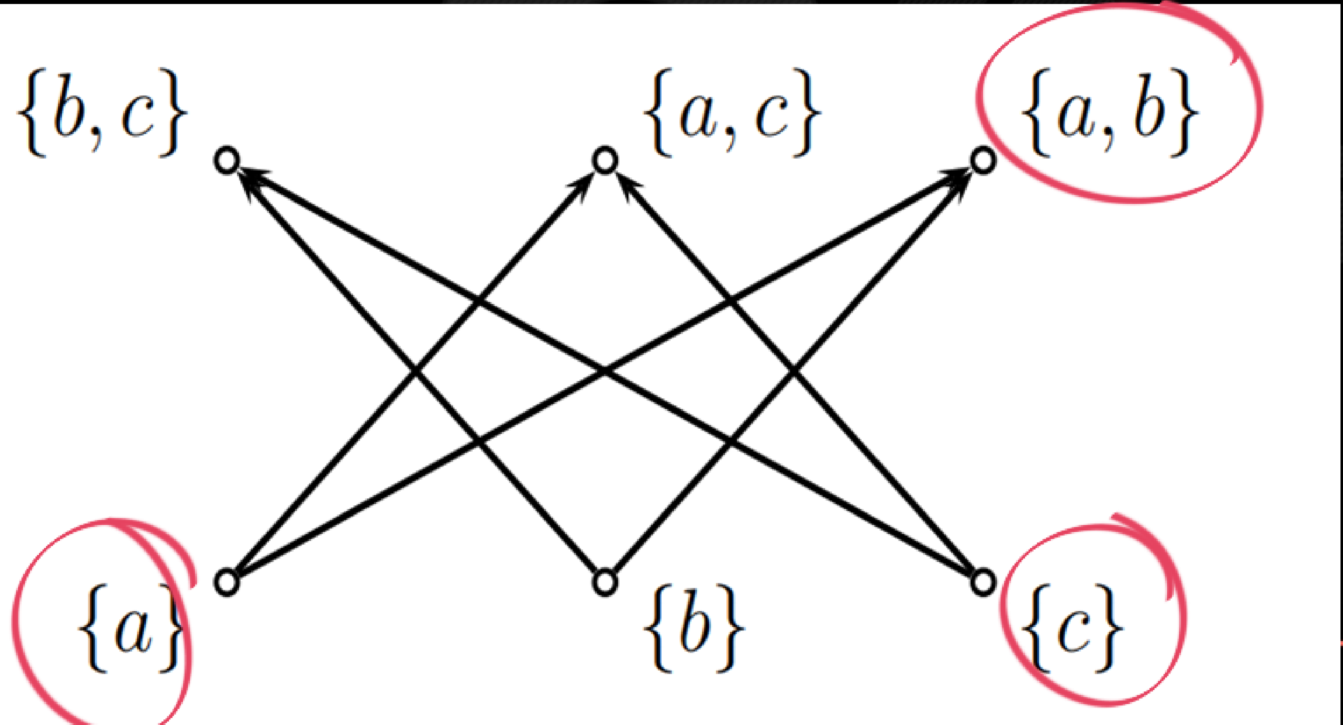
$LUB\{a, b, c\} =$

$LUB\{c, b\}$

$GLB\{a, b, c\} = a \cap b$

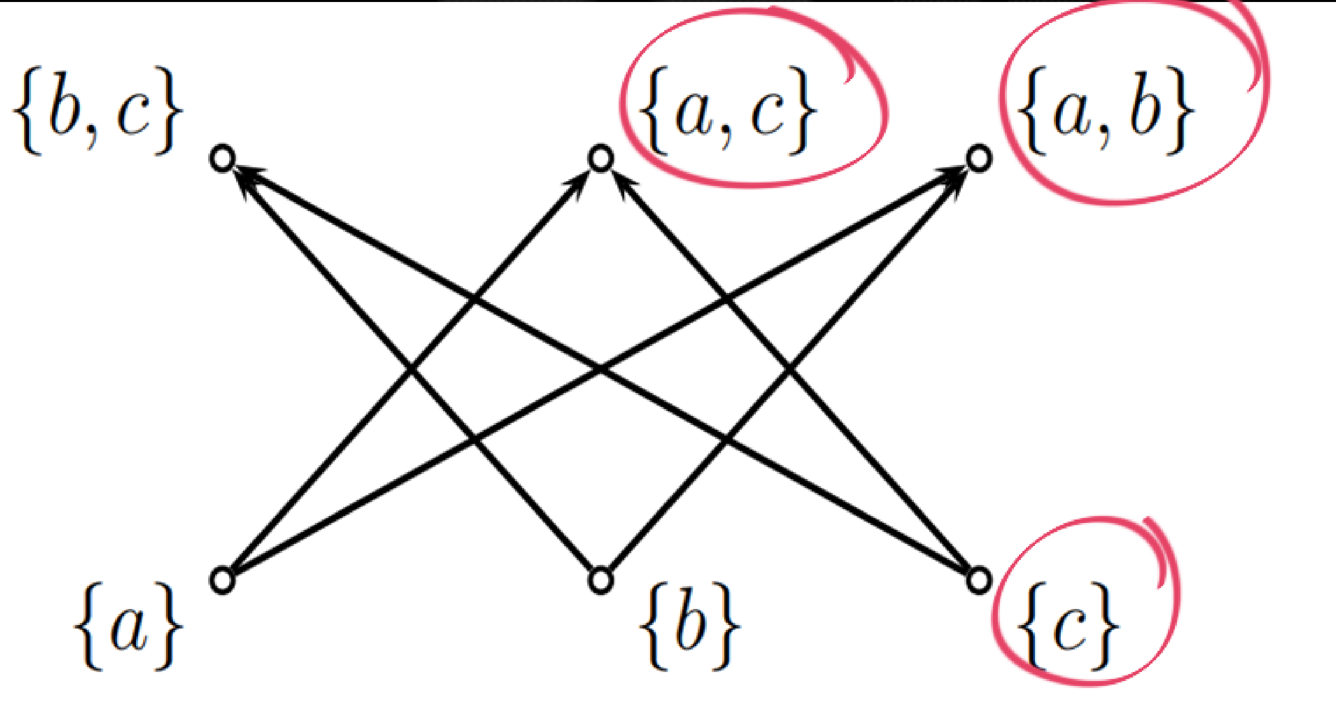
LUB{ {a},{a,b},{c} } = LUB{{a,b}, {c}} = DNE

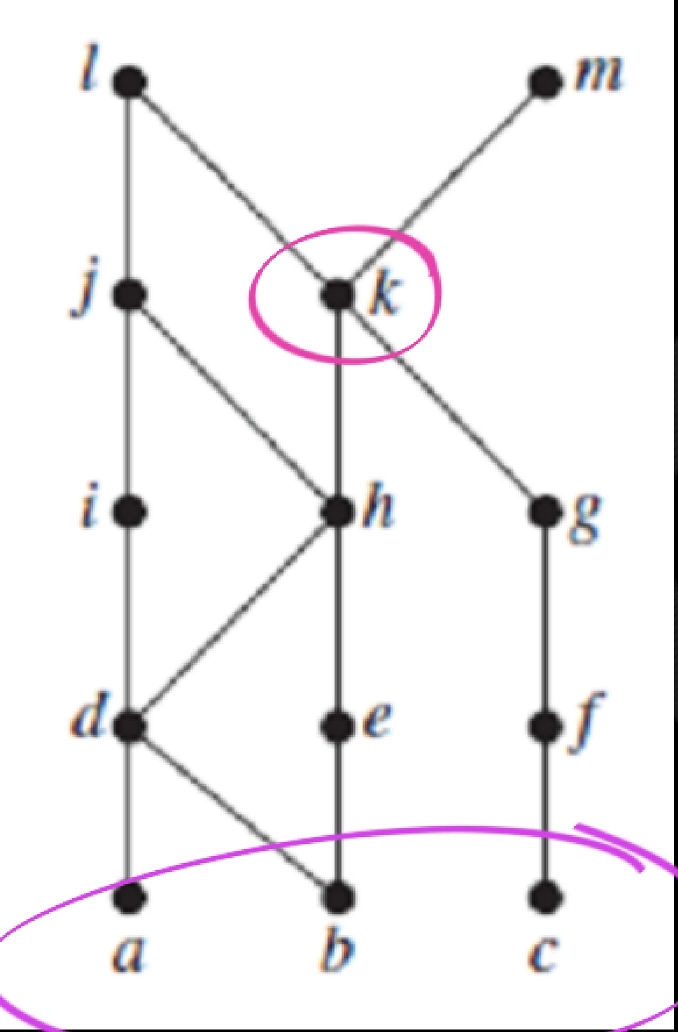
GLB{ {a},{a,b},{c} } = DNE



LUB{ {a,c},{a,b},{c} } = DNE

GLB{ {a,c},{a,b},{c} } = DNE





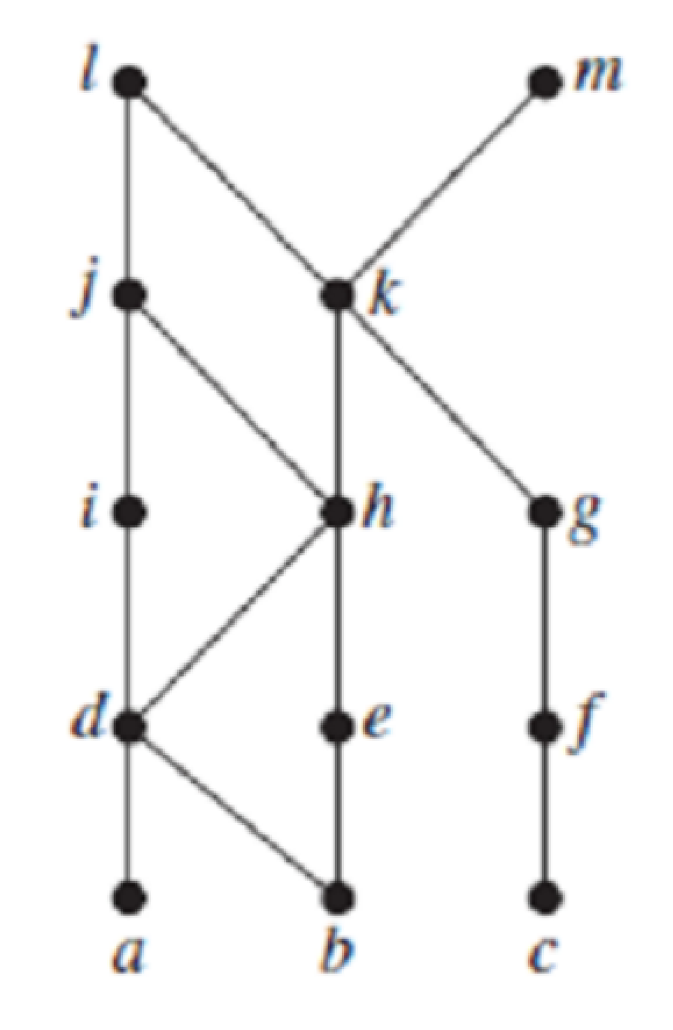
$k \wedge k = k$
$k \vee k = k$

$d \vee k = k$
$d \wedge k = d$

$d \; R \; k$

$a \vee b \vee c = \text{LUB}\{a, b, c\}$

$= k$



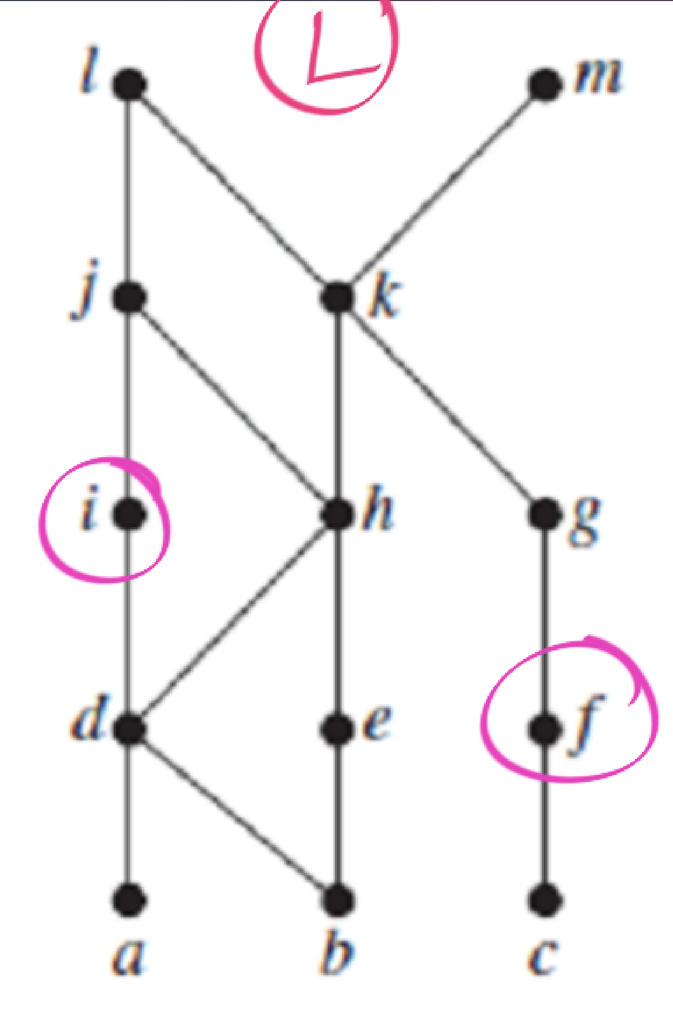
$a \wedge b \wedge c = GLB\{a, b, c\}$
$= DNE$

$a \vee i \vee l = l$

$a \wedge i \wedge l = a$

$b \vee h \vee k = k$

$b \wedge h \wedge k = b$



$a \vee d \vee i \vee f = i \vee f$
$= l$

$a \wedge d \wedge i \wedge f = a \wedge f =$
DNE

$\bigvee L = \text{DNE}$

$\bigwedge L = \text{DNE}$

Q: Poset P ;

$\bigvee P$ = LUB of all elements

= Greatest element

Q: $\bigwedge P$ = Least element

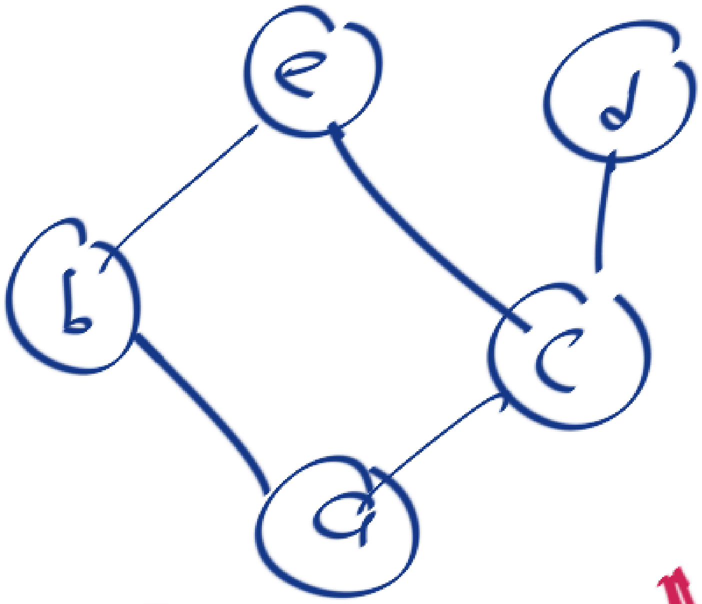
Partial Order Relations

Next Topic:

Hasse Diagram of Total Order Relations

Total Order/TOSET/Chain/Linear Order

Website : https://www.goclasses.in/



which is True? cRd

1. aRa
2. ~~bRa~~
3. aRd
4. aRc
5. ~~eRd~~
6. $b \not R d$
7. ~~$b \not R e$~~
8. ~~eRb~~
9. ~~$b \not R c$~~

HD

$d \not R c$

$(\{1,2,3,4,5\}, >=)$

5 R 4 $5 \geq 4$

4 R 3 $4 \geq 3$

2 R 5 ✗

5 R 2 ✓

Total order

1
|
2
|
3
|
4
|
5

Chain / Straight line / linear order

$\forall$ a, b ; a, b are Comparable

$(\{1,2,3,4,5\}, <=)$ — Total order

2 R 5 ✓

3, 1 are Comparable

1 R 3

5
|
4
|
3
|
2
|
1

Chain / Straight line / Linear order

$(Z, <=)$ — TO

$\vdots$
2
1
0
−1
−2
$\vdots$

100, −2000 are comparable

−2000 R 100

maximal = DNE
minimal = DNE

$\boxed{\text{minimal} = -\infty}$

nonsense

$(N, <=)$ − TOR

$\vdots$
4
3
2
1

minimal = 1

least = 1

maximal = DNE

$(R, <=)$ — TOR
$\underline{\underline{TT}}$

$a \in R$
$b \in R$
then
$a <= b$ OR
$b \leq a$

$\vdots$
1
0
-1
$\vdots$

π, e
$e R \pi$
$e \leq \pi$

$(Z^-, <=)$ — TOR

maximal = -1

-1
-2
-3
$\vdots$

Greatest = -1

least = DNE

$([0,1], <=)$ — TOR

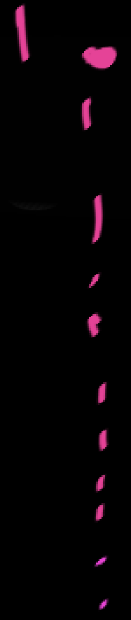
maximal — 1 — Greatest

minimal — 0 — least

$((0,1], <=)$

Greatest = 1

Least = DNE



$([0,1), <=)$ — TORL

Greatest = DNE
Least = 0

$((0,1), <=)$

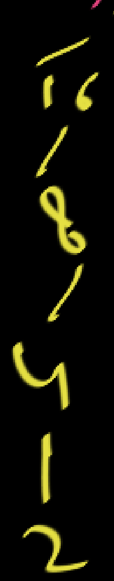
Greatest = DNE
least = "

0.6
0.5

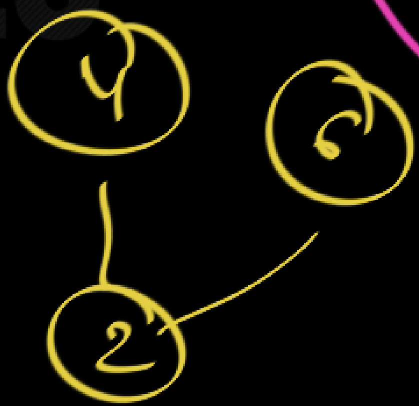
$(A, |)$

Give set A such that $(A, |)$ is TOR.

$A = \{2, 4, 8, 16, 32\}$ ✓



$\{2, 4, 6\}$ ✗



$4 \not{R} 6$

$6 \not{R} 4$

$(\mathbb{N}, |)$ — not TOR

2, 3 are not Comparable.

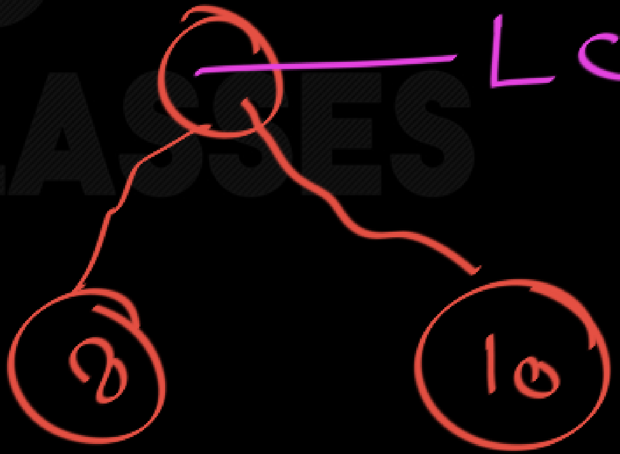
$5 \vee 105 = 105$

$2 \vee 3 = 6$

$10 \vee 8 = 40$

$10 \wedge 8 = 2$

$5 \wedge 105 = 5$



Lcm{8,10}

LCM

→ least Common Multiple

GCD

→ Greatest Common Divisor

(A, Subset) Give A, $|A| = 4$, such that

$(A, \subseteq)$ is TOR.

~~$A = \{a, b, c, d\}$~~

$A = \{ \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\} \}$

$(A, \subseteq)$ is TOR

$$\begin{array}{c}\{a,b,c\}\\ | \\ \{a,b,c\}\\ | \\ \{a,b\}\\ | \\ \{a\}\end{array}$$

$$(\text{set of sets}, \subseteq)$$

$$\left(\left\{\underset{\text{set}}{\downarrow}, \underset{\text{set}}{\downarrow}, \underset{\text{set}}{\downarrow}, \ldots\right\}, \subseteq\right)$$

$(P(A), \subseteq)$ is TOR iff $|A| \leq 1$

$P(A)$ — Base set

$A = \phi$

HD:

ϕ

TOR ✓

$A = \{a\}$ — not Base set

HD:

$\{a\}$

|

ϕ

TOR ✓

$(P(A), \subseteq)$

not TOR.

$A = \{a, b\}$

$|A| \geq 2$

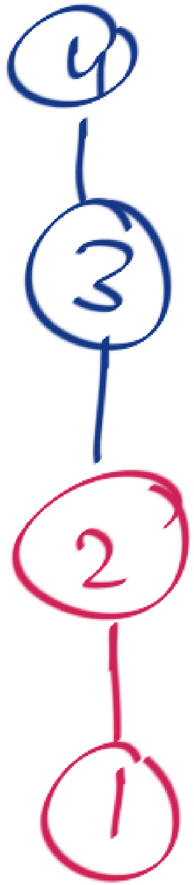
$A = \{a, b, c\}$

$\{a\}, \{b\}$ are not Comparable.

$(P\{a,b,c,d\}, \subseteq)$ is NOT TOR

$\{a\}, \{b\}$ are not Comparable.

$(\{1,2,3,4\}, \leq)$



Total order or Chain (Straight line)

$1R2$

$1R3$

$1R4$

$2R3$

$(\{1,2,3,4\}, \leq)$

Poset and Every two elements are Comparable = Toset

Total order $\equiv$ Chain $\equiv$ linear order $\equiv$ Toset

$\Downarrow$

Poset
+
Every two
elements
Comparable

$\Downarrow$

Because
HD of Toset
is straight line
(ALL elements
in single line)

TOR → HD is Chain.

d
c
b
a

all pairs are Comparable

TOR $\equiv$ Chain $\equiv$ Linear Order

$\equiv$ Straight Line

Partial Order Relations

Next Topic:

Lattices

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