Propositional Logic

Homework 4

Tautology Equivalence

- By Deepak Poonia (IISc Bangalore)

Instructor:

Deepak Poonia

IISc Bangalore

GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

Discrete Mathematics Complete Course:

https://www.goclasses.in/courses/IDiscrete-Mathematics-Course

Join GO Classes **GATE CSE Complete Course** now:

https://www.goclasses.in/s/pages/gatecompletecourse

1. Quality Learning: No Rote-Learning. Understand Everything, from basics, In-depth, with variations.

2. Daily Homeworks, Quality Practice Sets, Weekly Quizzes.

3. Summary Lectures for Quick Revision.

4. Detailed Video Solutions of Previous ALL GATE Questions.

5. Doubt Resolution, Revision, Practice, a lot more.

Download the GO Classes Android App:

https://play.google.com/store/apps/details?id=com.goclasses.courses

Search "GO Classes" on Play Store.

Hassle-free learning
On the go!
Gain expert knowledge

NOTE :

Complete Discrete Mathematics & **Complete Engineering**

Mathematics Courses, by GO Classes, are **FREE** for ALL learners.

Visit here to watch : https://www.goclasses.in/s/store/

SignUp/Login on Goclasses website for free and start learning.

We are on Telegram. Contact us for any help.

Link in the Description!!

Join GO Classes **Doubt Discussion** Telegram Group :

Username:

@GATECSE_GOCLASSES

We are on Telegram. Contact us for any help.

Join GO Classes Telegram Channel, Username: @GOCLASSES_CSE

Join GO Classes Doubt Discussion Telegram Group :

Username: @GATECSE_Goclasses

(Any doubt related to Goclasses Courses can also be asked here.)

Join GATEOverflow Doubt Discussion Telegram Group :

Username: @GateOverflow_CSE

Propositional Logic

Homework 4

Tautology Equivalence

- By Deepak Poonia (IISc Bangalore)

Detailed Solutions of this Homework

Will be discussed in the **"Live Doubts Session-5"** of

Goclasses Discrete Mathematics Course-2024"

Discrete Mathematics Complete Course:

https://www.goclasses.in/courses/IDiscrete-

Mathematics-Course

1. [*30pts*] Which of the following compound propositions are a tautology? You may use a truth table, but are not required to. To prove a proposition is NOT a tautology you need just give one setting for p and q for which the compound proposition is false.

   (a) $\neg\neg p \leftrightarrow p$

   (b) $(p \lor q) \land (\neg p \land \neg q)$

   (c) $p \to q \leftrightarrow \neg p \lor q$

   (d) $(p \lor q) \land \neg p$

   (e) $((p \to q) \land (q \to r)) \to (p \to r)$

   (f) $\neg(p \lor q) \land (\neg p \land \neg q)$

2   Logical equivalence

a) Use a truth table to demonstrate that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.   (1 mark)

b) Use a truth table to demonstrate that $p \wedge q \equiv \neg(\neg p \vee \neg q)$.   (1 mark)

c) Use substitutions and the transitive property of logical equivalence to show that

$$p \rightarrow q \equiv \neg(p \wedge \neg q).$$

Provide a justification for each step (i.e. substitution or transitivity, and what logical equivalences you are using.)   (1 mark)

d) Use substitutions and the transitive property of logical equivalence to show that

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p).$$

Provide a justification for each step (i.e. substitution or transitivity, and what logical equivalences you are using.)   (1 mark)

3. **Proof using propositional logic properties**

Using propositional logic properties and other logical equivalences (not truth tables), prove the following statements:

1. $(p \vee q) \vee (p \vee \neg q)$ is a tautology
2. $((p \rightarrow r) \wedge (q \rightarrow r) \wedge (p \vee q)) \rightarrow r$ is a tautology
3. $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction
4. $\neg(q \rightarrow p) \wedge (p \wedge q \wedge s \rightarrow r) \wedge p$ is a contradiction
5. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

2. (4 points) Use propositional logic to prove the validity of the *disjunctive syllogism* rule, which is written as (note: don't use disjunctive syllogism in its own proof!):

$$(P \vee Q) \wedge P' \to Q$$

3. (4 points each) Prove the validity of each of the following arguments using propositional logic:

   (a) $(A' \to B') \wedge B \wedge (A \to C) \to C$

   (b) $(A \vee B) \wedge (A \to C) \wedge (B \to C) \to C$

   (c) $(Y \to Z') \wedge (X' \to Y) \wedge (Y \to (X \to W)) \wedge (Y \to Z) \to (Y \to W)$

3. Proof using properties

(a) Use propositional logic properties to prove that the following propositions are tautologies (without using truth tables):

1. $(p \lor q) \lor (p \lor \neg q)$
2. $(\neg(\neg x)) \leftrightarrow x$
3. $((p \to q) \land (q \to r)) \to (p \to r)$

(b) Use propositional logic properties to prove that the following propositions are contradictions (without using truth tables):

1. $(p \lor q) \land (\neg p \land \neg q)$
2. $\neg((p \to False) \to \neg p)$
3. $\neg(q \to p) \land (p \land q \land s \to r) \land p$