Set Theory

Next Topic:

Subsets and Power Sets

Website : https://www.goclasses.in/

$S = \{a, b, c\}$ — Collection

$A = \{a, b\}$ SubCollection

$B = \{a\}$ SubCollection

A is subset of S.

B " " " S.

$\left.\begin{array}{l} A \subseteq S \\ B \subseteq S \end{array}\right\}$ ✓

$S = \{a, b, c\}$

Subsets of S :

$\{\ \}$, $\{b, c\}$, $\{a, c\}$, $\{a, b\}$,

$\{c\}$, $\{a\}$, $\{b\}$, $\{a, b, c\}$

$\{a, b, c, 1\}$ ✗ $\{a, b, 1\}$ ✗

Subsets

- A set S is called a ***subset*** of a set T (denoted $\boldsymbol{S \subseteq T}$) if all elements of S are also elements of T.
- Examples:
  - $\{ 1, 2, 3 \} \subseteq \{ 1, 2, 3, 4 \}$
  - $\{ b, c \} \subseteq \{ a, b, c, d \}$
  - $\{ \mathrm{H, He, Li} \} \subseteq \{ \mathrm{H, He, Li} \}$
  - $\mathbb{N} \subseteq \mathbb{Z}$ (*every natural number is an integer*)
  - $\mathbb{Z} \subseteq \mathbb{R}$ (*every integer is a real number*)

Set P, set Q

$P \subseteq Q$ means

Every element of P is in Q.

$$\forall_x \left(x \in P \longrightarrow x \in Q \right)$$

$P = \{1, 2\} \subseteq \{1, 2, 3\} = Q$

$P \subseteq Q$ ✓

$Q \subseteq P$ ✗

$P = \{1, 2\}$ $\quad$ $Q = \{1, 3\}$

$P \subseteq Q$ ✗ $\quad$ $Q \subseteq P$ ✗

$2 \in P$ But $2 \notin Q$ $\quad$ $3 \in Q$ but $3 \notin P$.

$P = \{1, 2, 3\} \subseteq \{1, 2, 3\} = Q$

$\forall x$

$x \in P \longrightarrow x \in Q$

$P \subseteq Q$

$\{1, 2, 3\} \subseteq \{1, 2, 3\}$

① $S \subseteq S$ ✓

$$S = \{a_1, a_2, --, a_n\}$$

$$S = \{a_1, a_2, --, a_n\}$$

$S \subseteq S$ ✓

② $\phi \subseteq S$ ✓ (Just Remove all elements of S)

$\phi \subseteq S$ ✓

$\phi \subseteq \phi$ ✓

$S \subseteq S$ ✓

$\phi \rightarrow$ Empty set

$S = \{1, 2, 3\}$ — elements: 1 ✓, 2 ✓, 3 ✓, $\{1,2\}$ ✗

$P = \{\{1, 2\}\}$ — elements: $\{1, 2\}$

$P \subseteq S$? ✗

$\{\{\}\} \subseteq S$ ✗

$\{\{\}\}$ — elements: $\{\}$

Note: $\phi = \{\}$ is subset of Every set.

$S = \{a, b\}$

$\phi \subseteq S$ ✓

$\phi \neq \{\phi\}$

$\{\phi\} \subseteq S$ ✗

$\{\} \subseteq S$ ✓

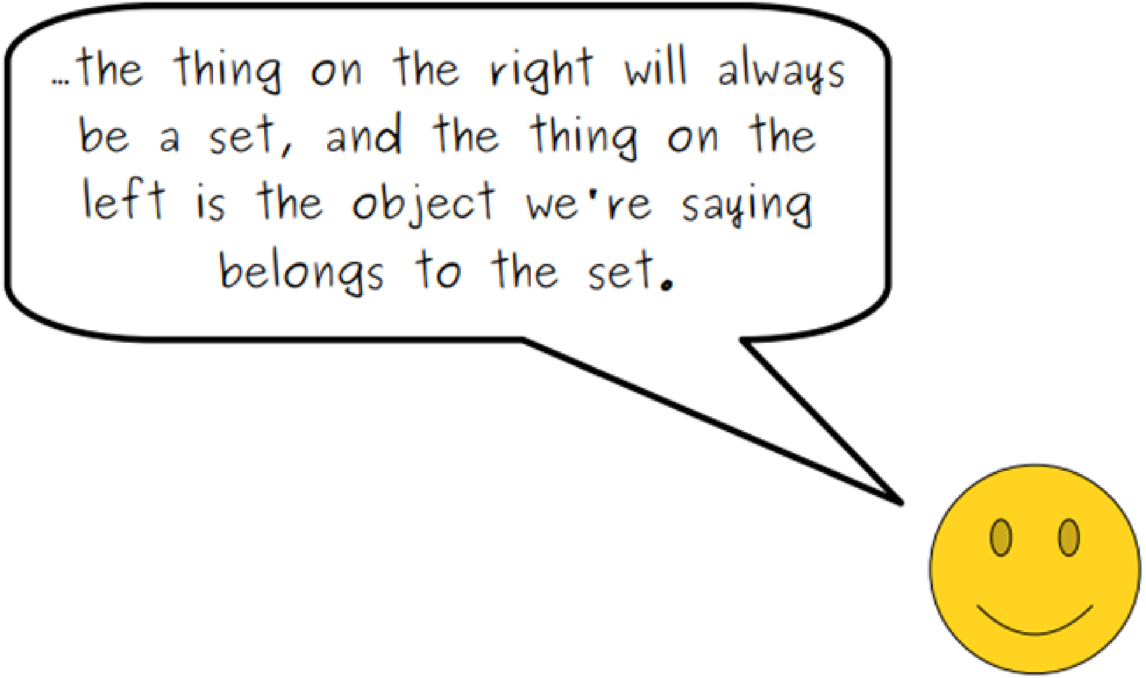
Very Imp: $S \subseteq Q$ means

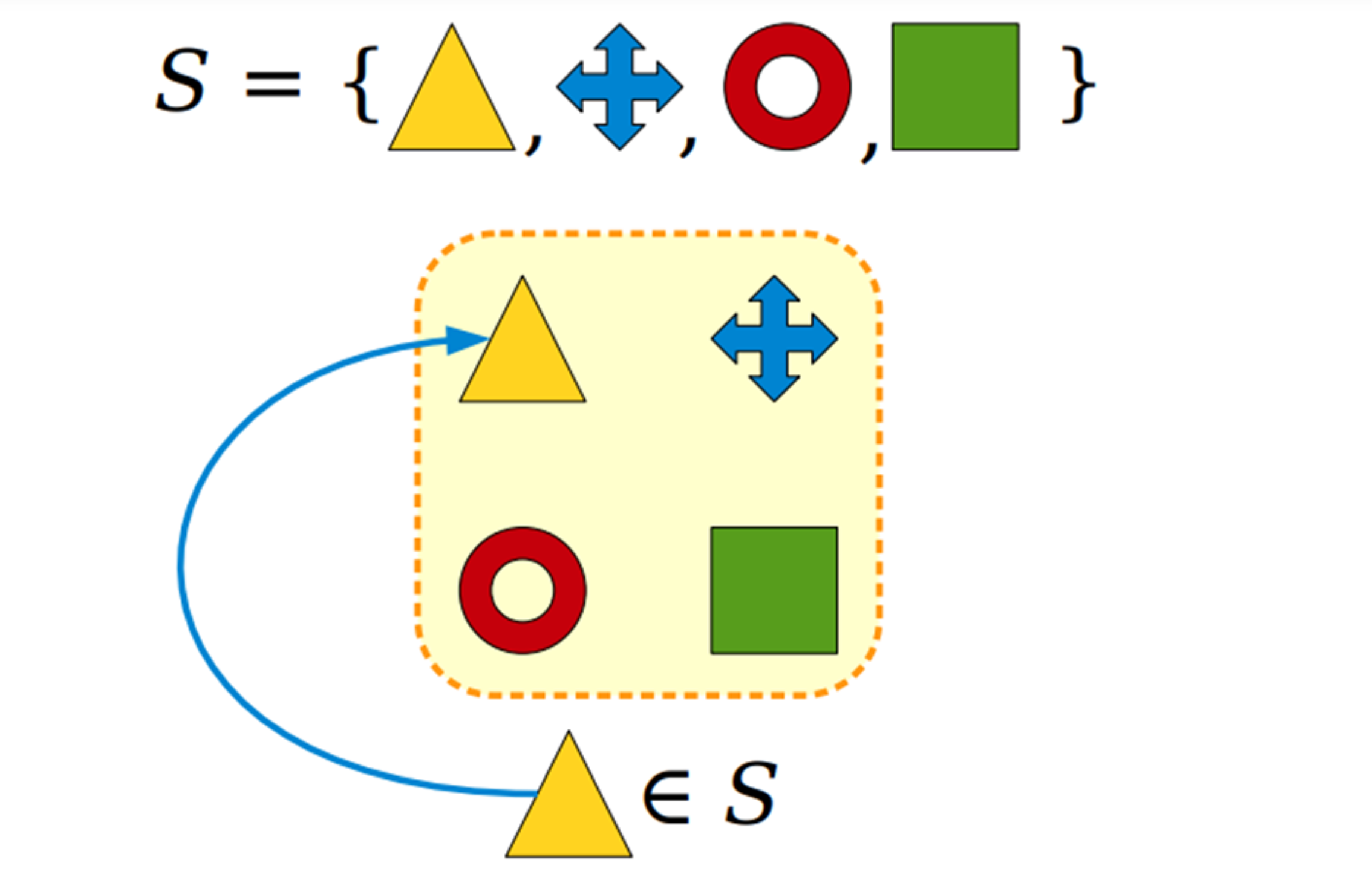
Every element of S

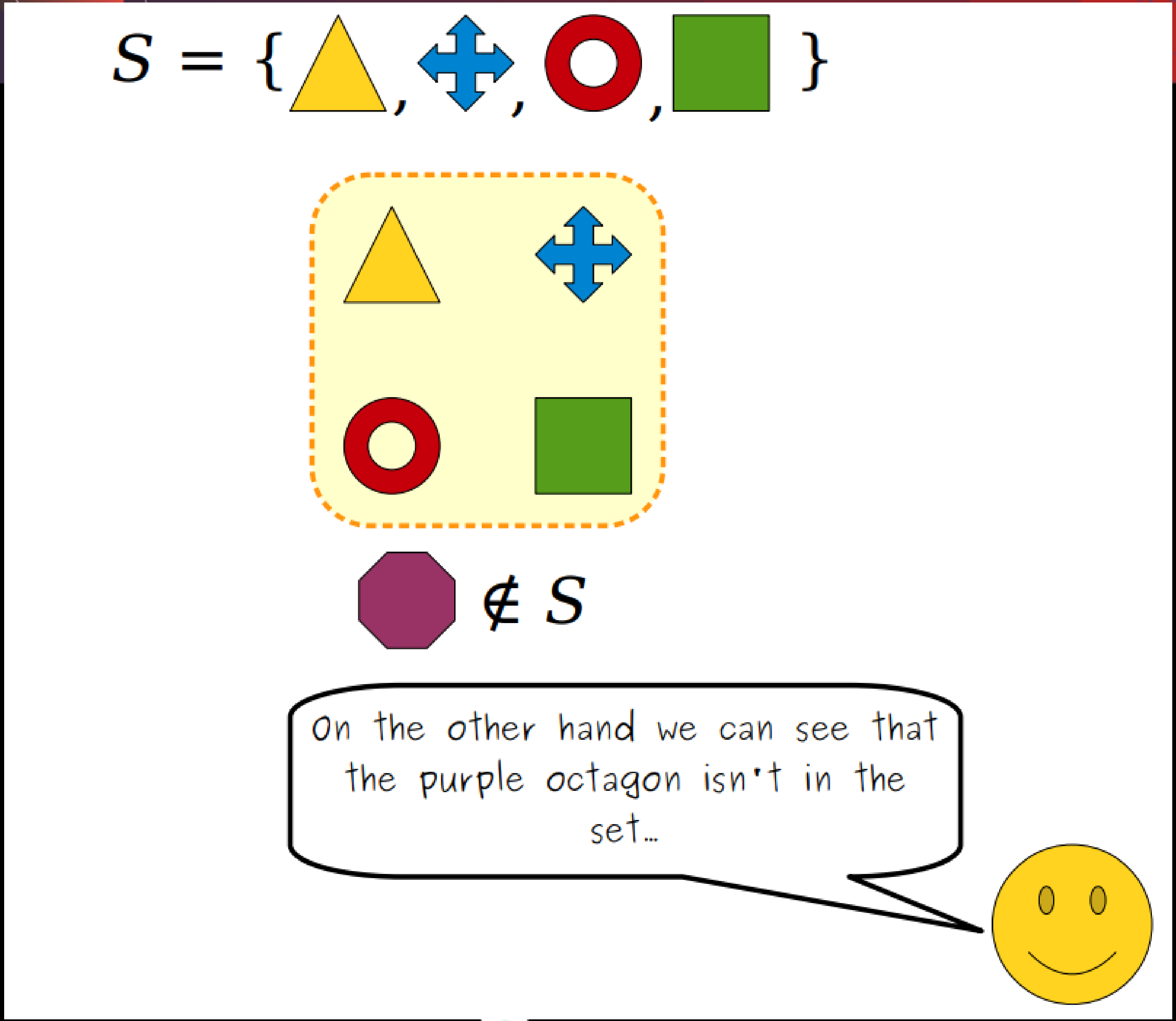
is element of Q.

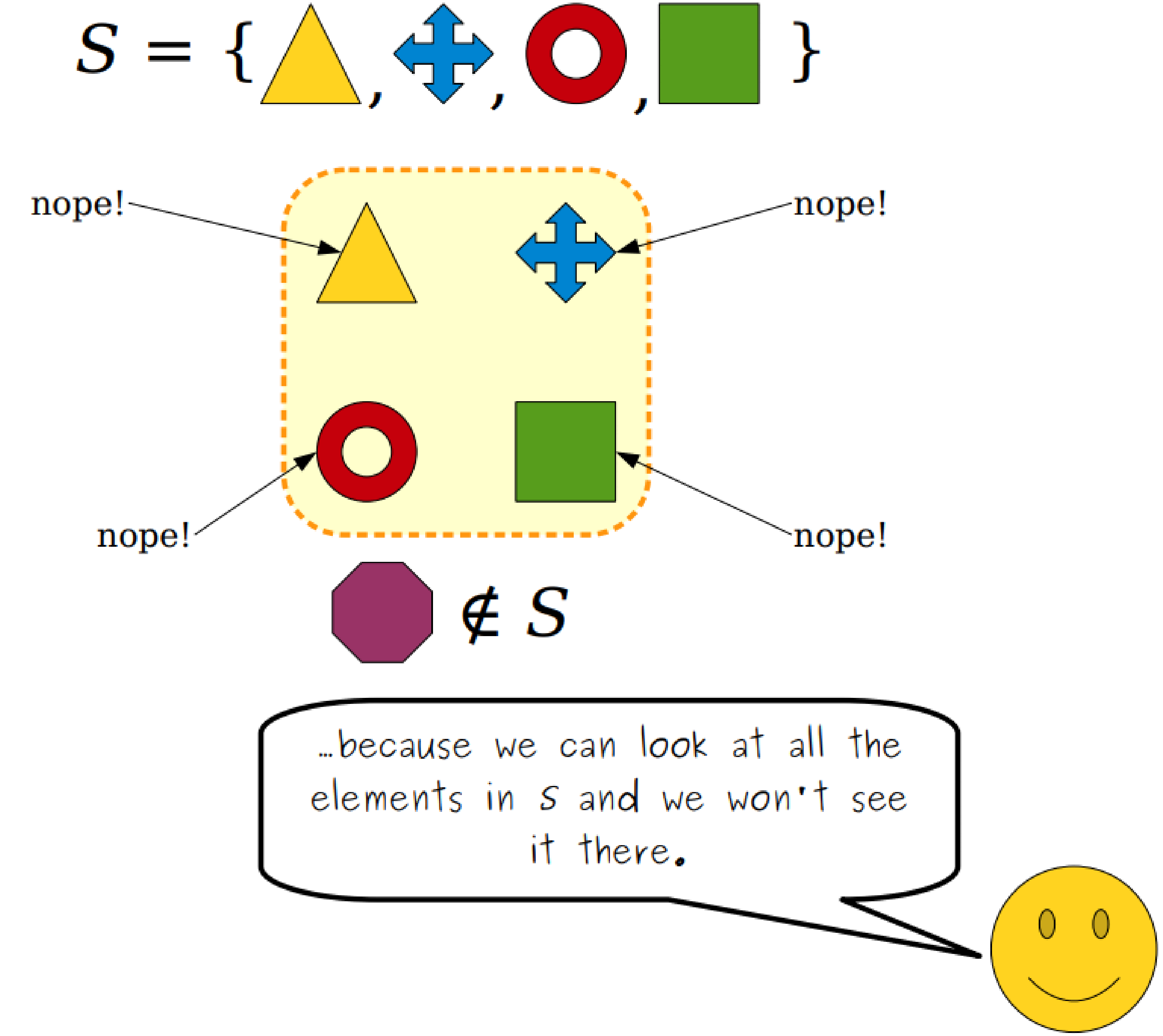
$$x \in S$$

This object is in this set

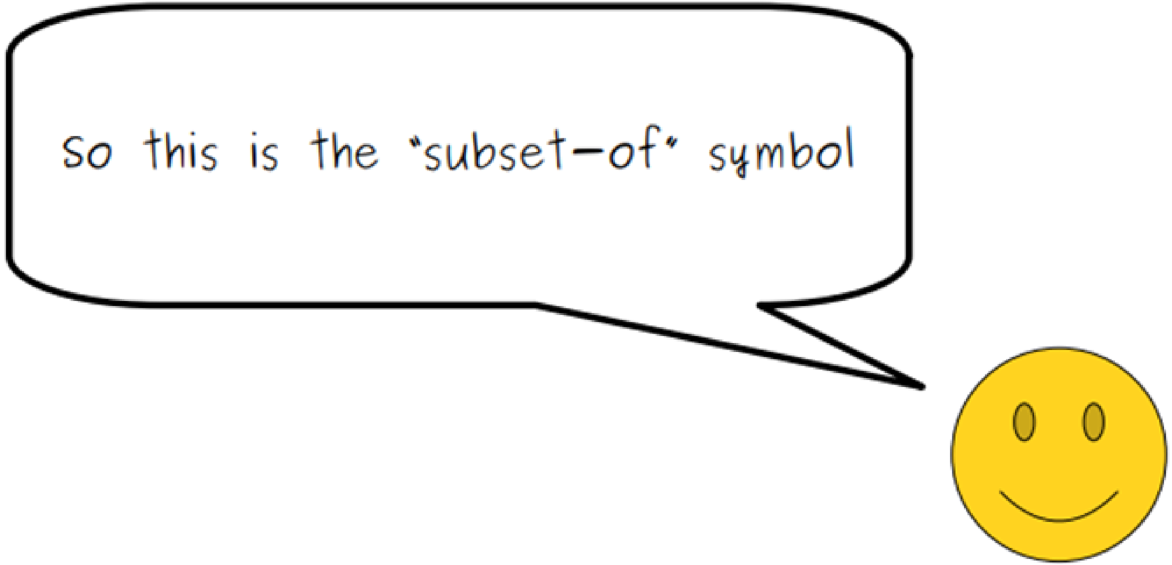



S = { , , , }
∈ S


S = { , , , }
∉ S
On the other hand we can see that the purple octagon isn't in the set...

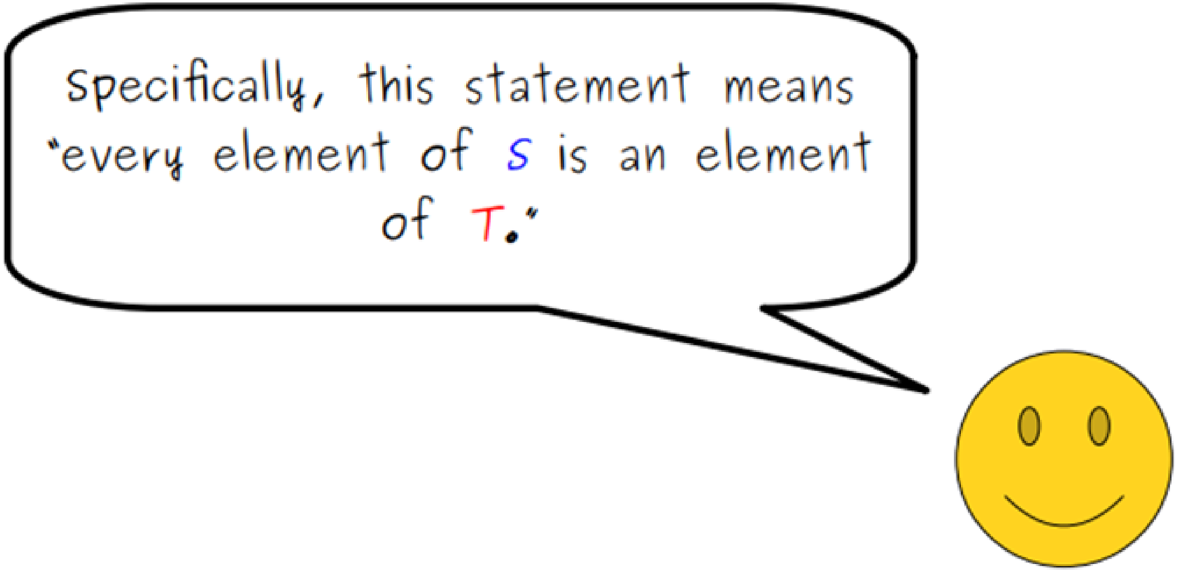

S = { , , , }
nope!
nope!
nope!
nope!
∉ S
...because we can look at all the elements in S and we won't see it there.

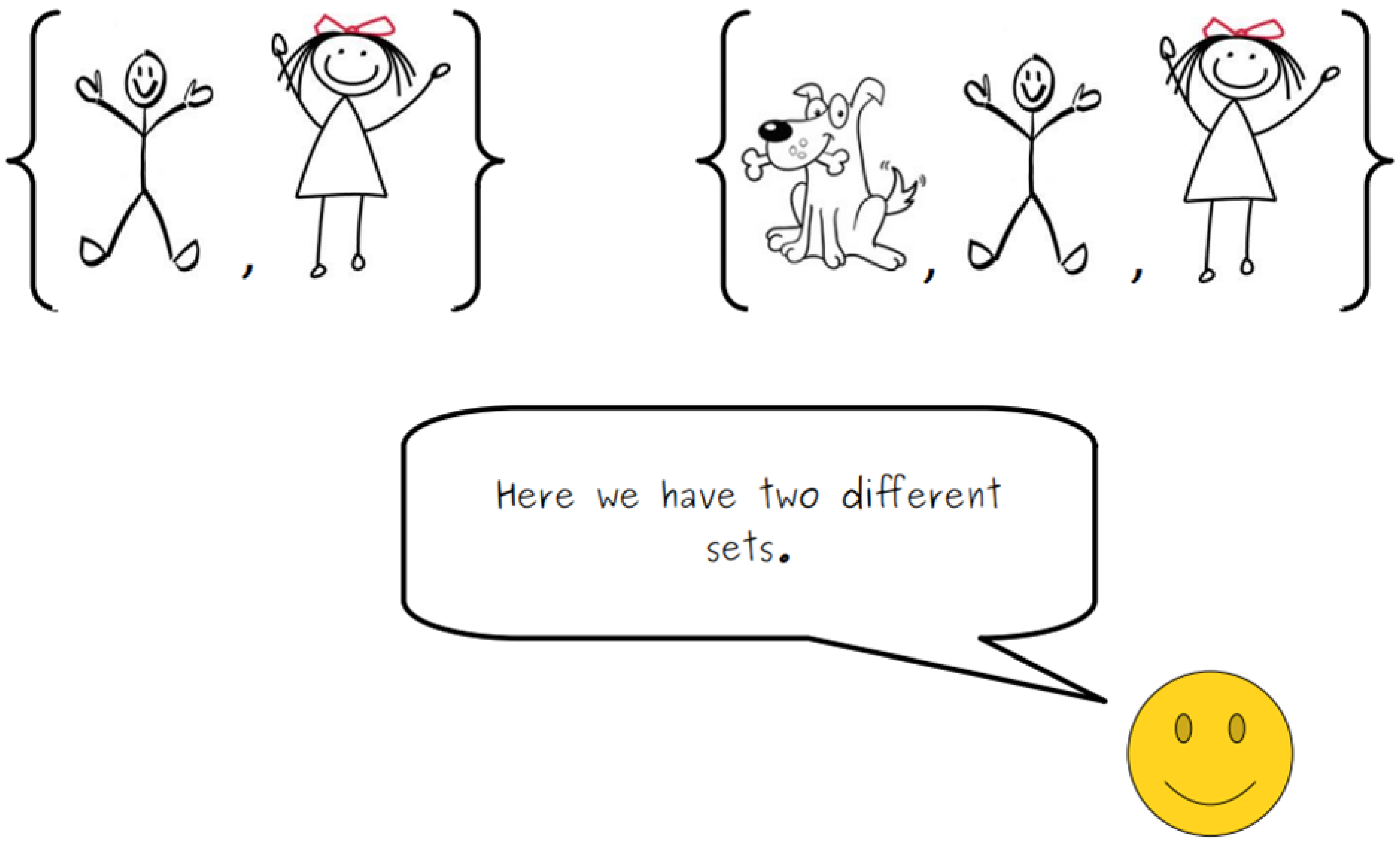
$\subseteq$

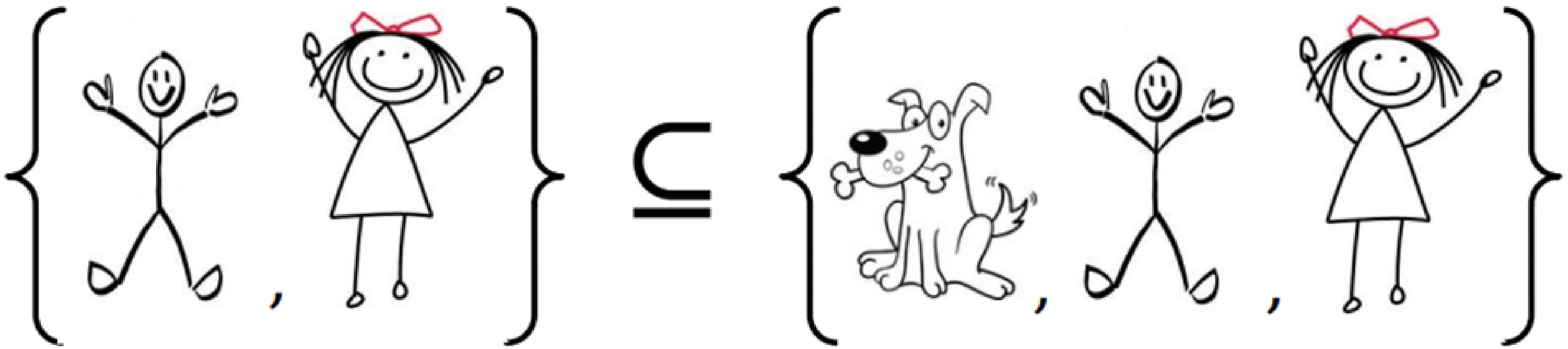


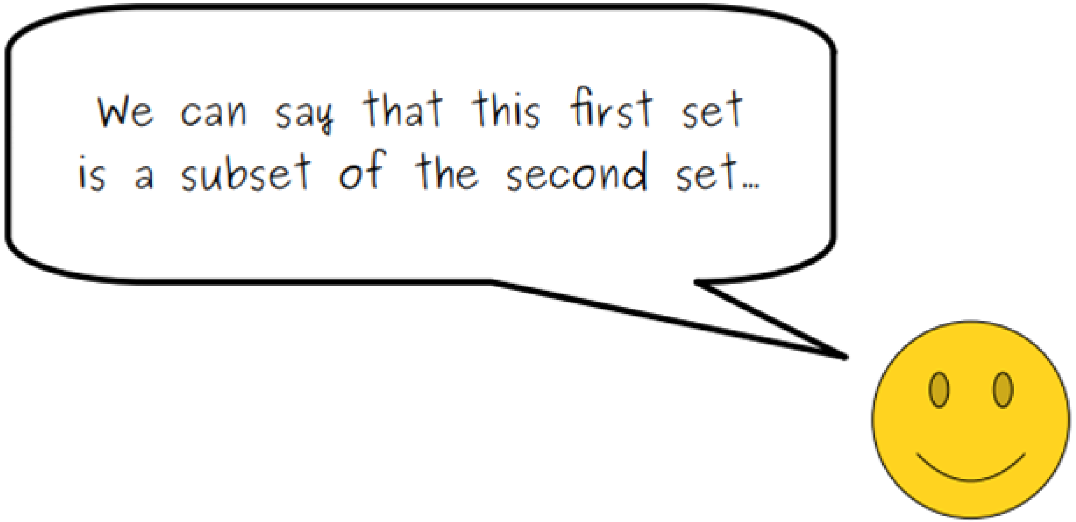
$$S \subseteq T$$

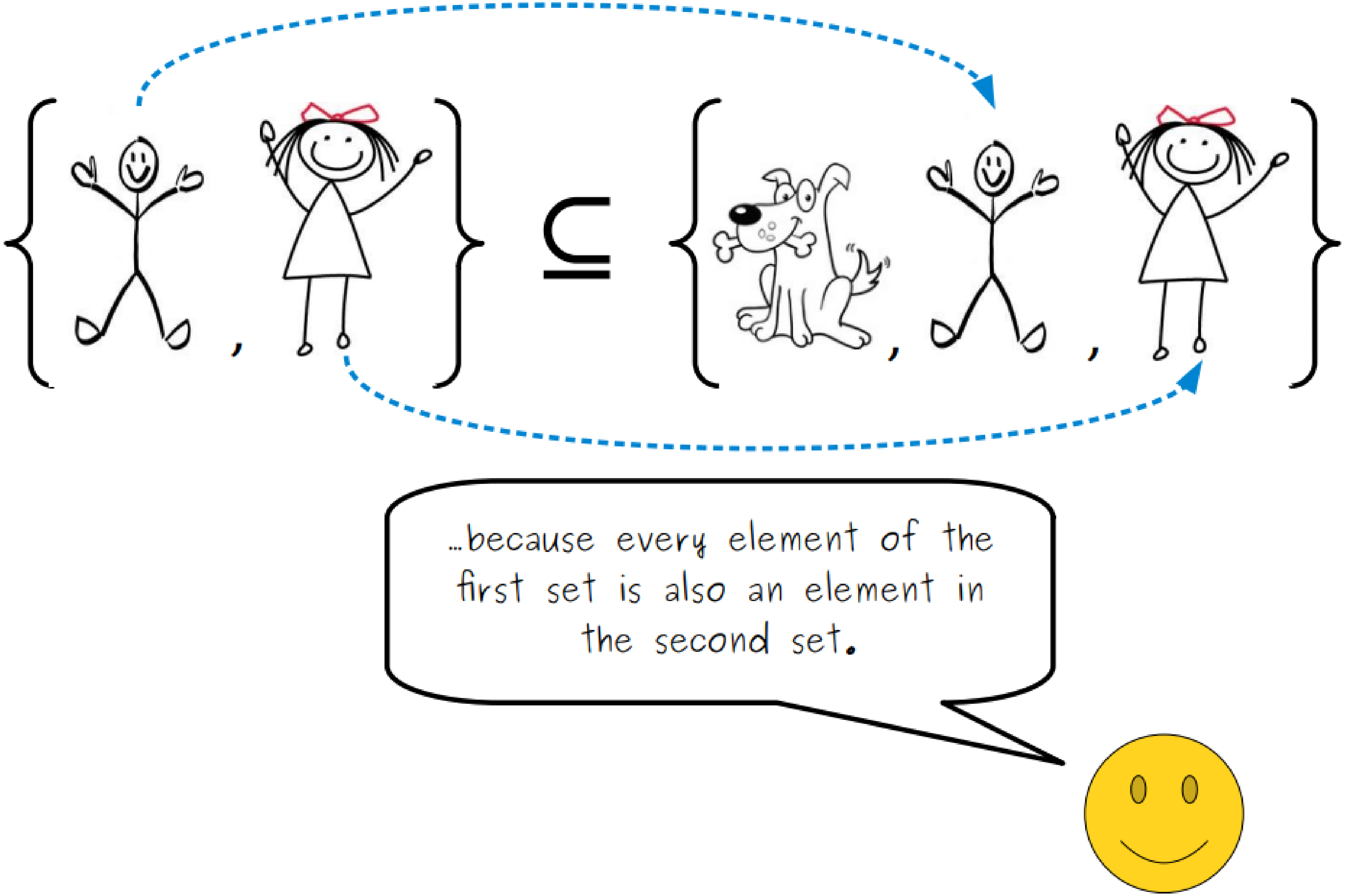
Every object in this set is in this set

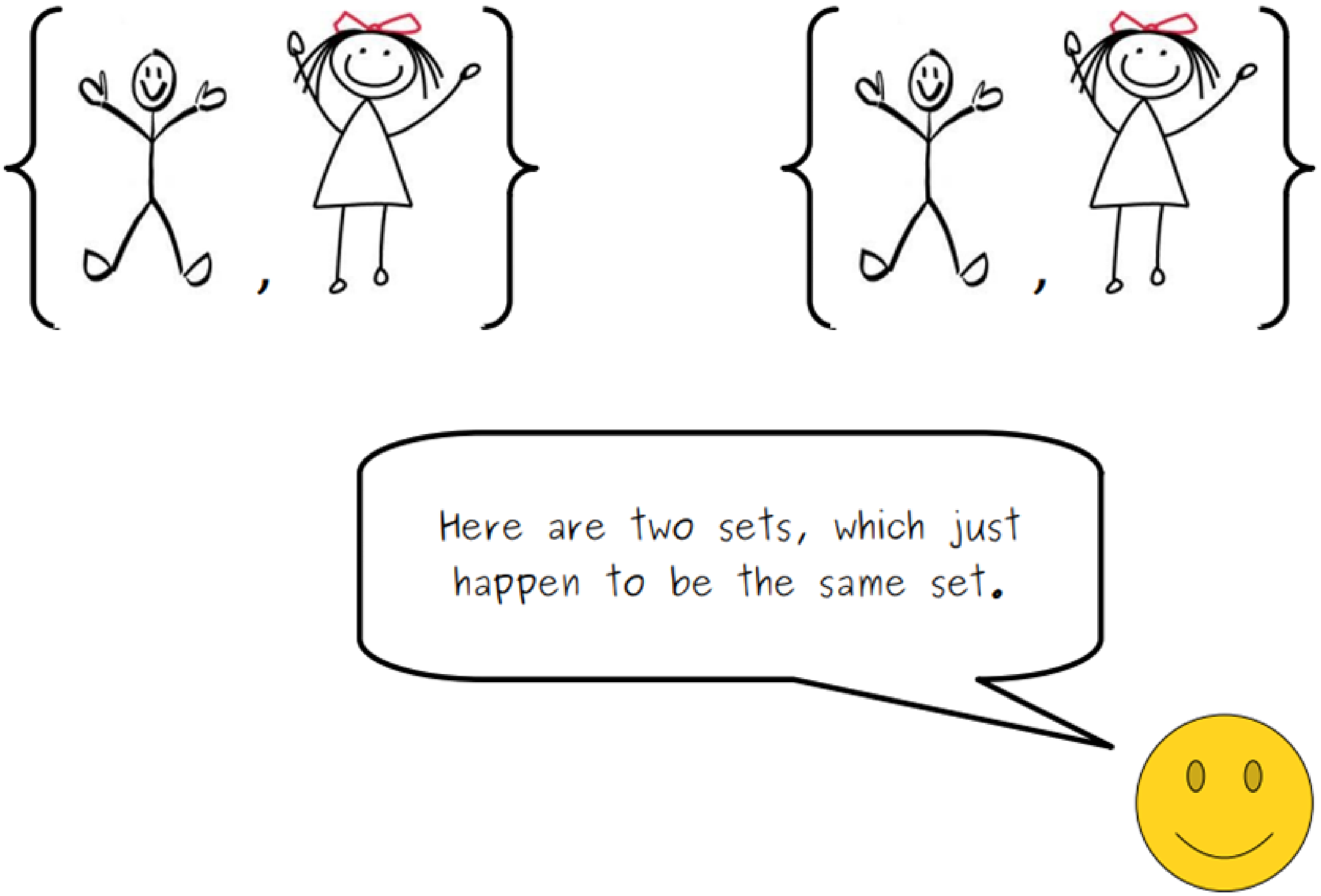


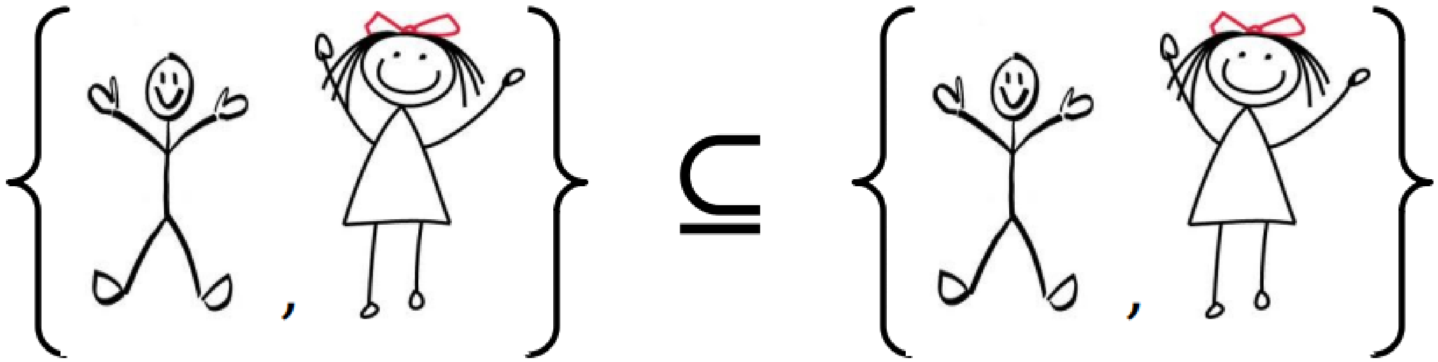

Here we have two different sets.

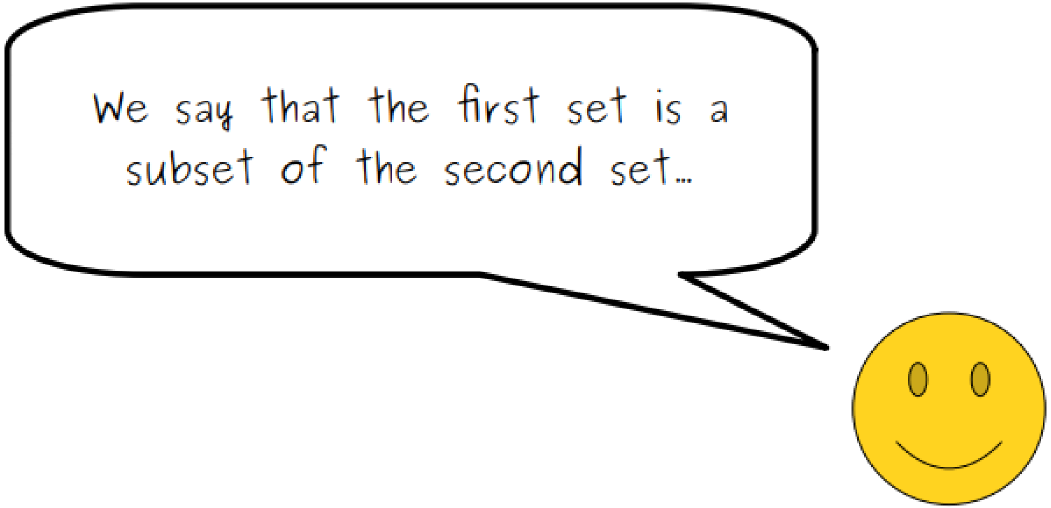


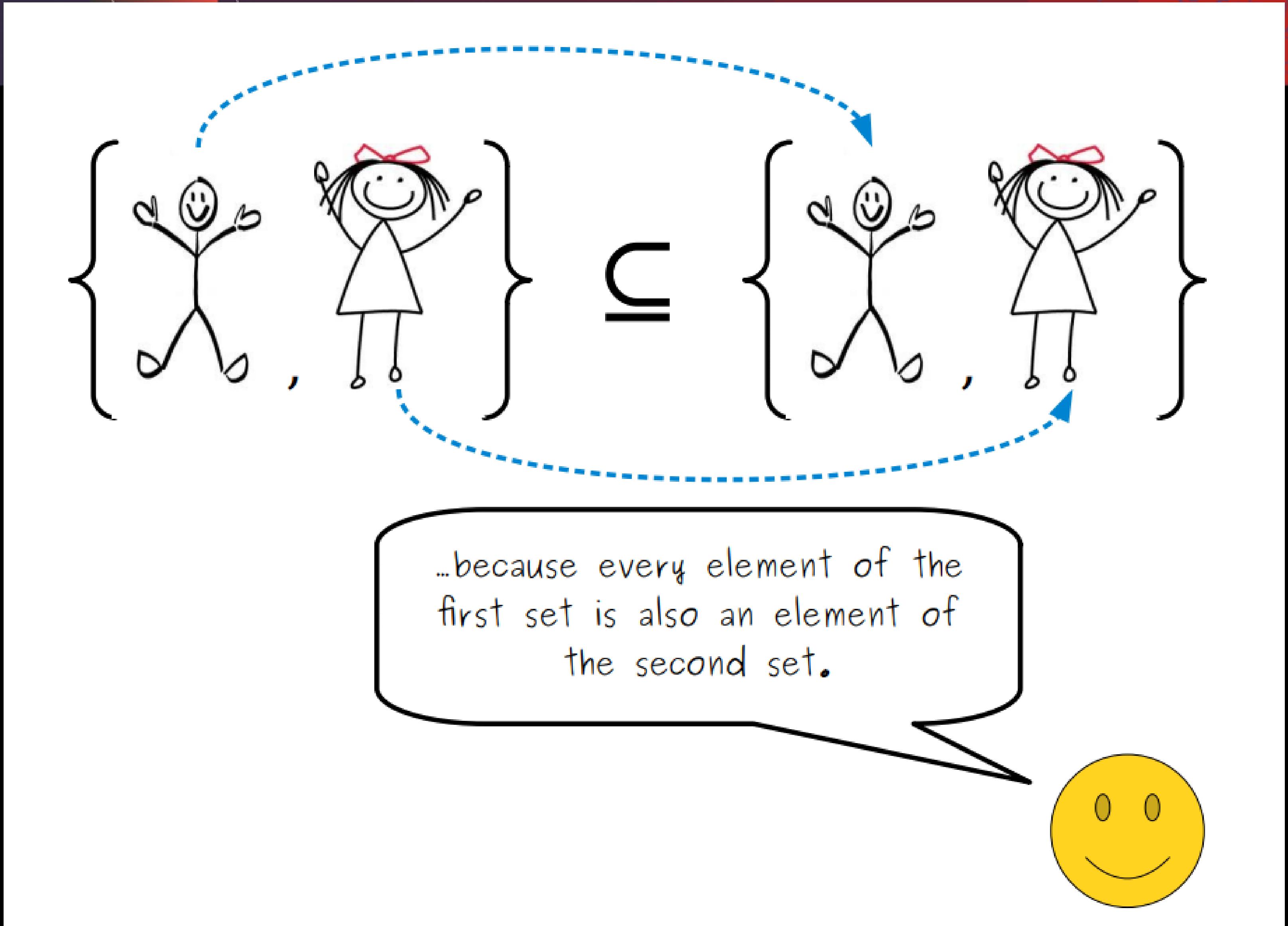

We can say that this first set is a subset of the second set...


$\subseteq$
...because every element of the first set is also an element in the second set.


Here are two sets, which just happen to be the same set.



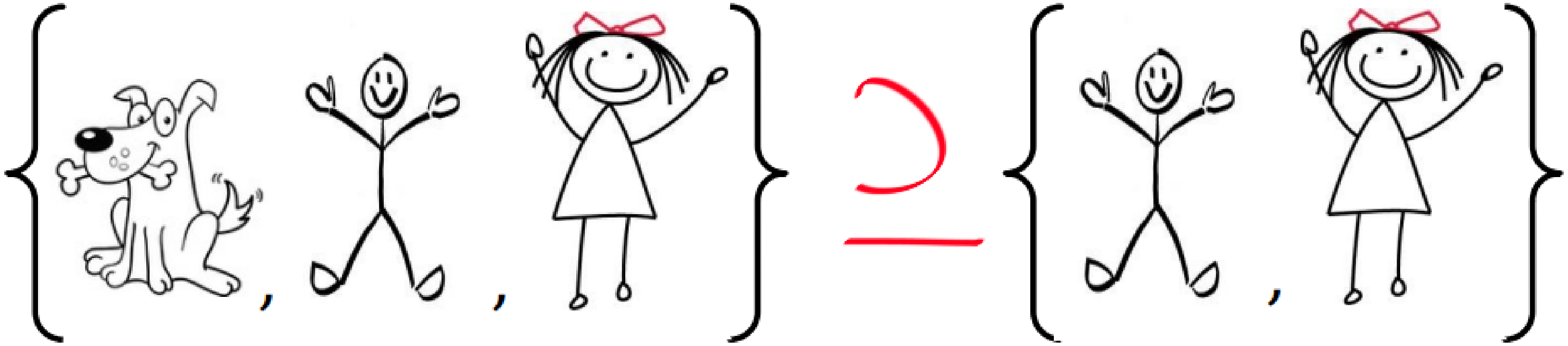

We say that the first set is a subset of the second set...


⊆
...because every element of the first set is also an element of the second set.

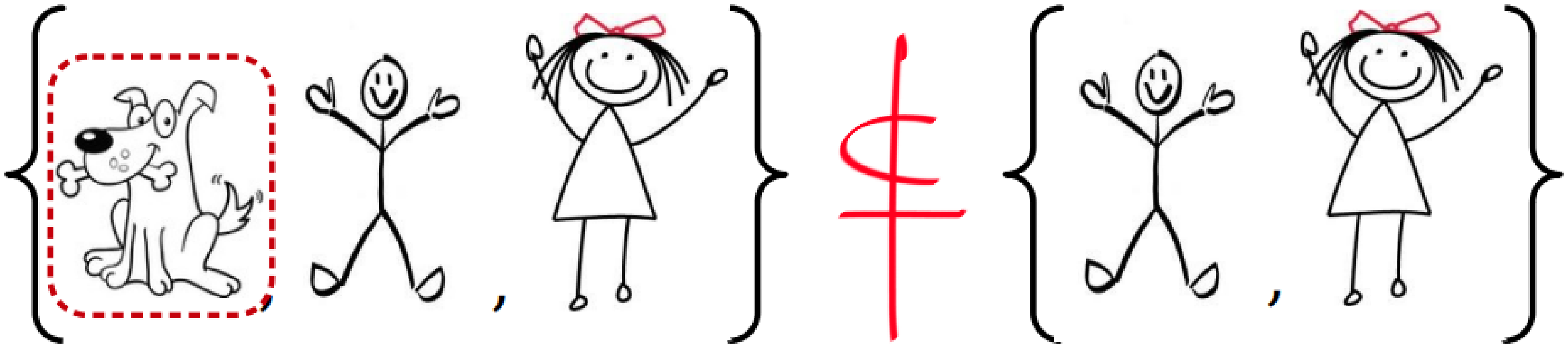
Note: for Every set S,

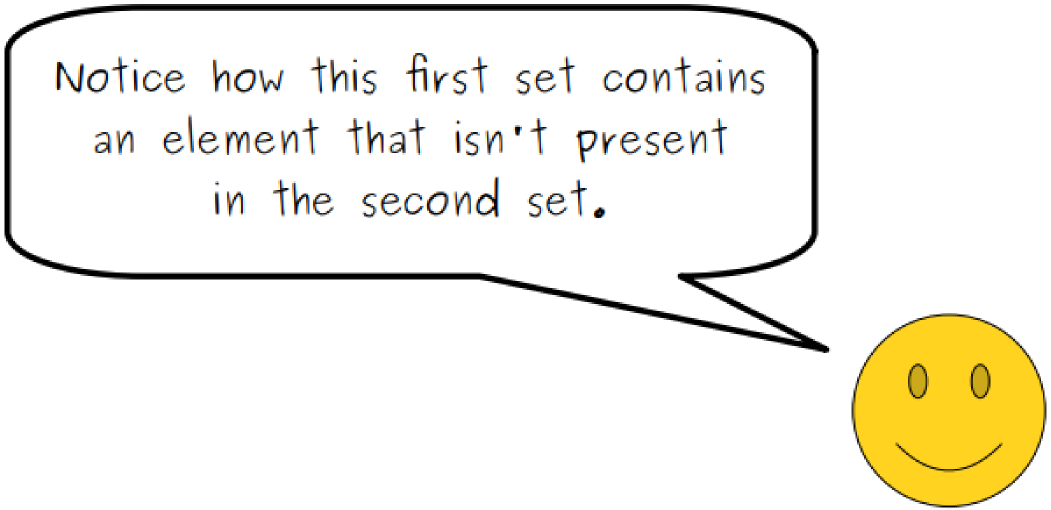
$S \subseteq S$ ✓

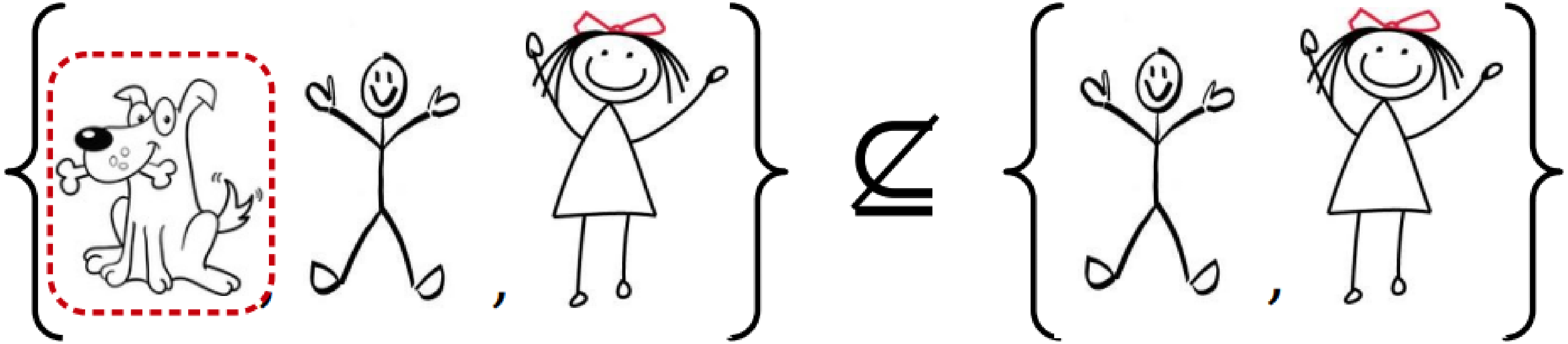
$\phi \subseteq S$ ✓



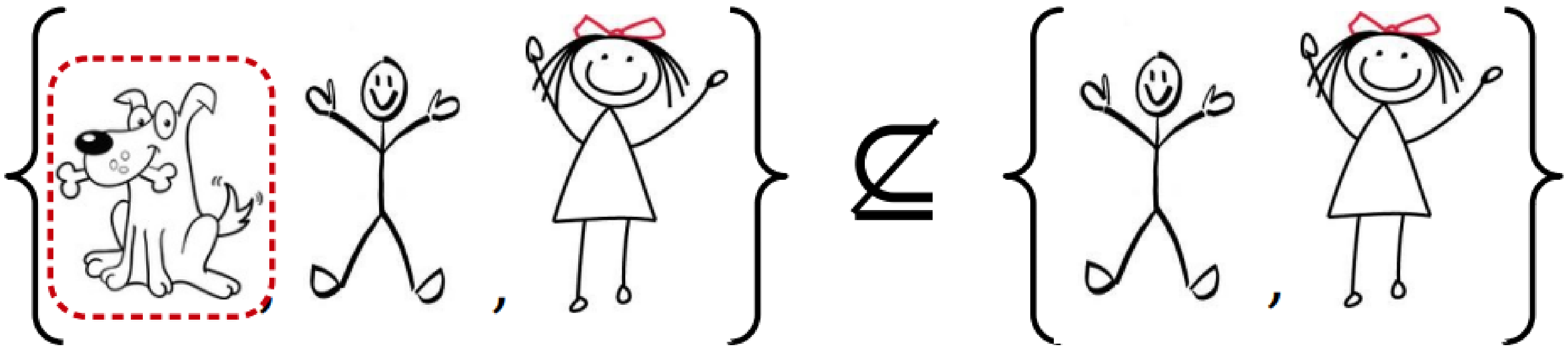
Here are two sets that we talked about earlier, but presented in the reverse order.

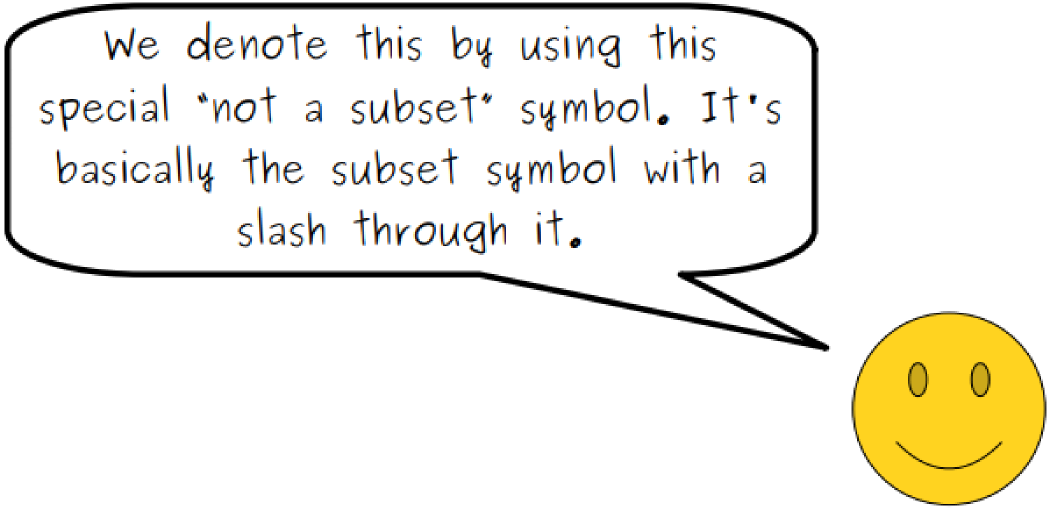



Notice how this first set contains an element that isn't present in the second set.



As a result, this first set is not a subset of the second set.



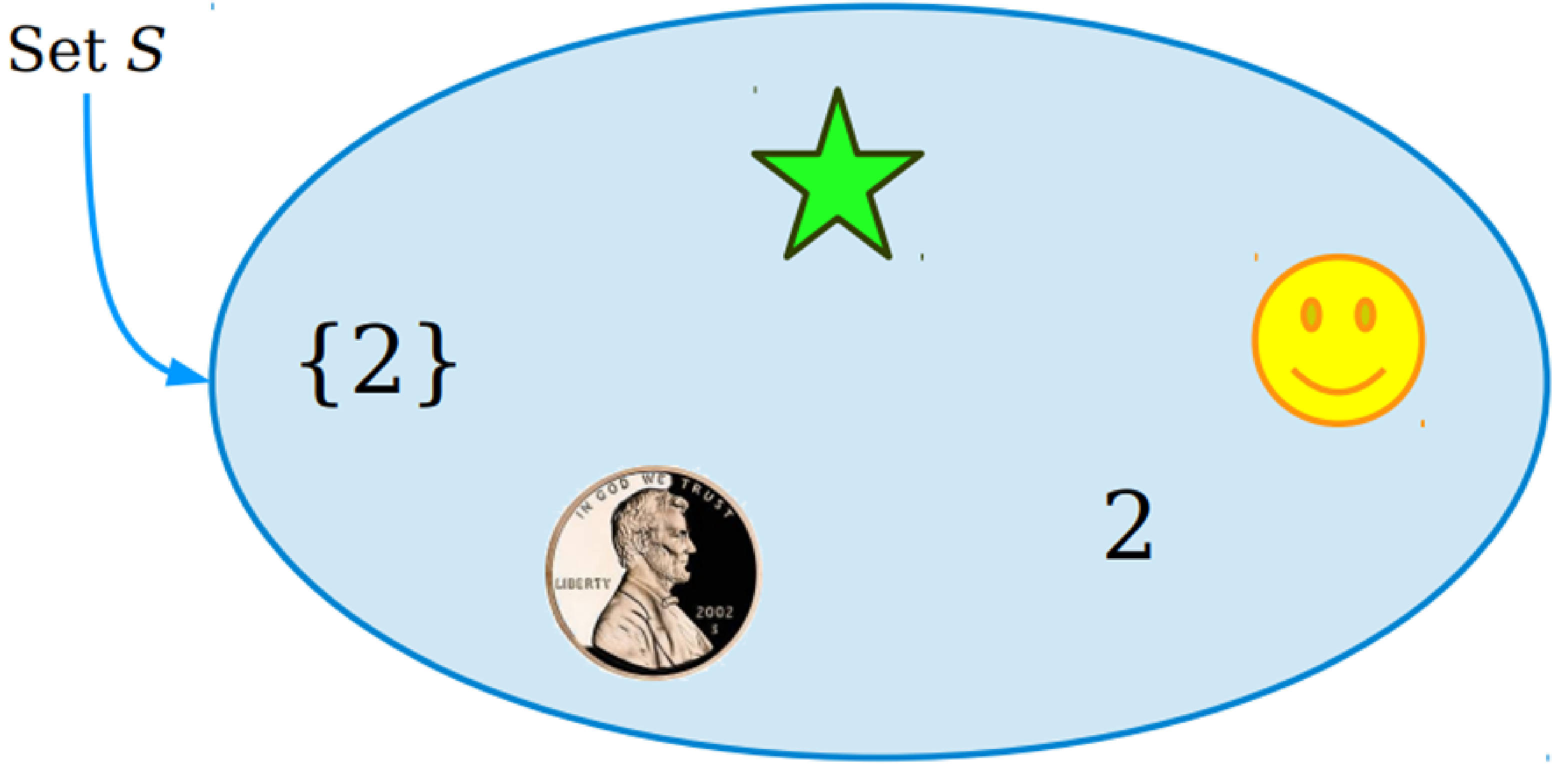

We denote this by using this special "not a subset" symbol. It's basically the subset symbol with a slash through it.

Note: while checking subset : Always Apply Definition.

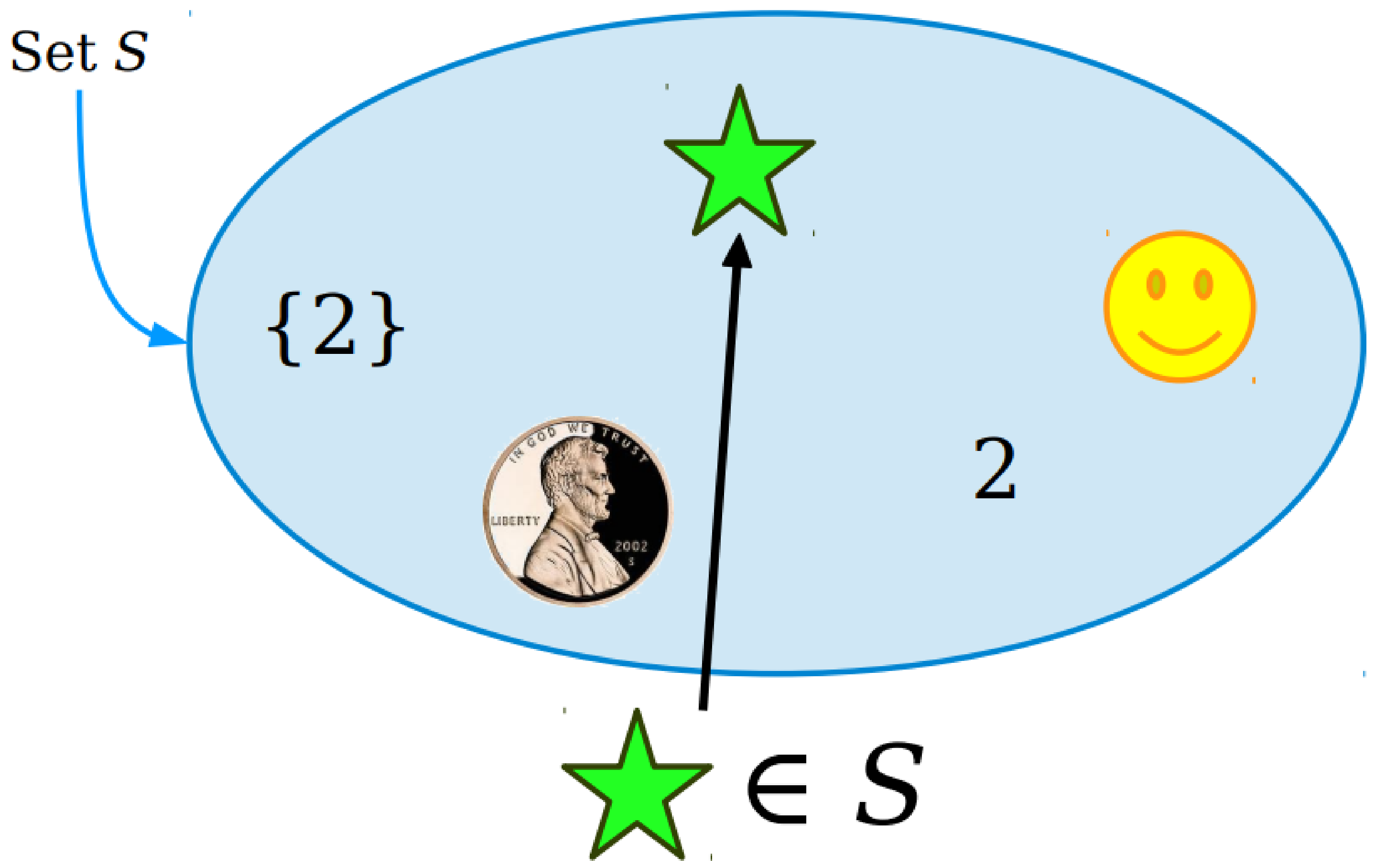
$$S \subseteq \phi$$

$$\forall x, \quad x \in S \longrightarrow x \in \phi$$

Subsets and Elements



Subsets and Elements



$S = \{1, 2, \{2\}, 3\}$ — elements:

1 ✓
2 ✓
$\{2\}$ ✓
3 ✓
~~$\{3\}$~~
~~ϕ~~

$1 \in S$ ✓

$2 \in S$ ✓

$\{2\} \in S$ ✓

$\{3\} \in S$ ✗

$\phi \in S$ ✗

$1 \subseteq S$ ✗

$\{1\} \subseteq S$ ✓

$\{1, 2\} \subseteq S$ ✓

$\{2\} \subseteq S$ ✓

$\phi \subseteq S$ ✓

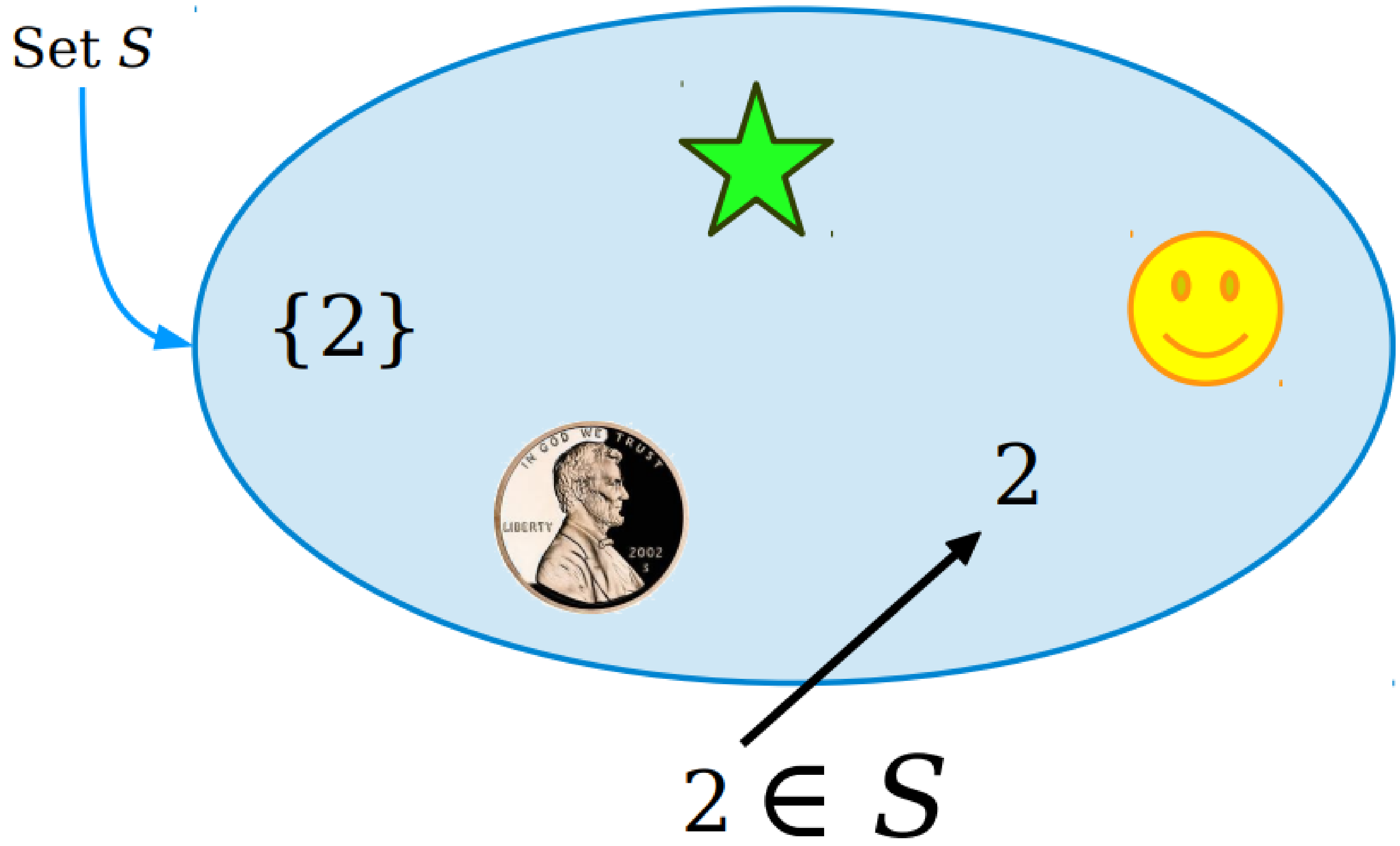
$\{\{2\}\} \subseteq S$ ✓

$\{\phi\} \subseteq S$ ✗

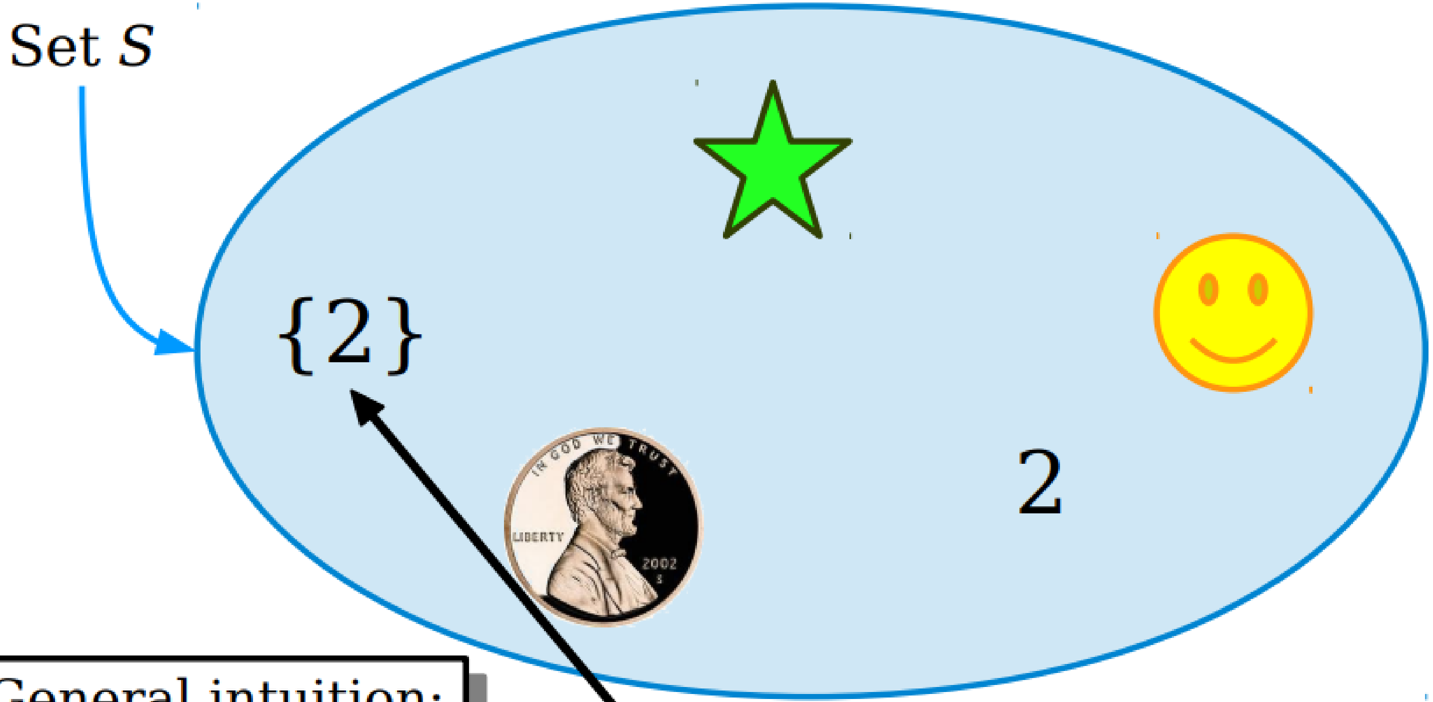
$3 \subseteq S$ ✗

not even a set

Subsets and Elements



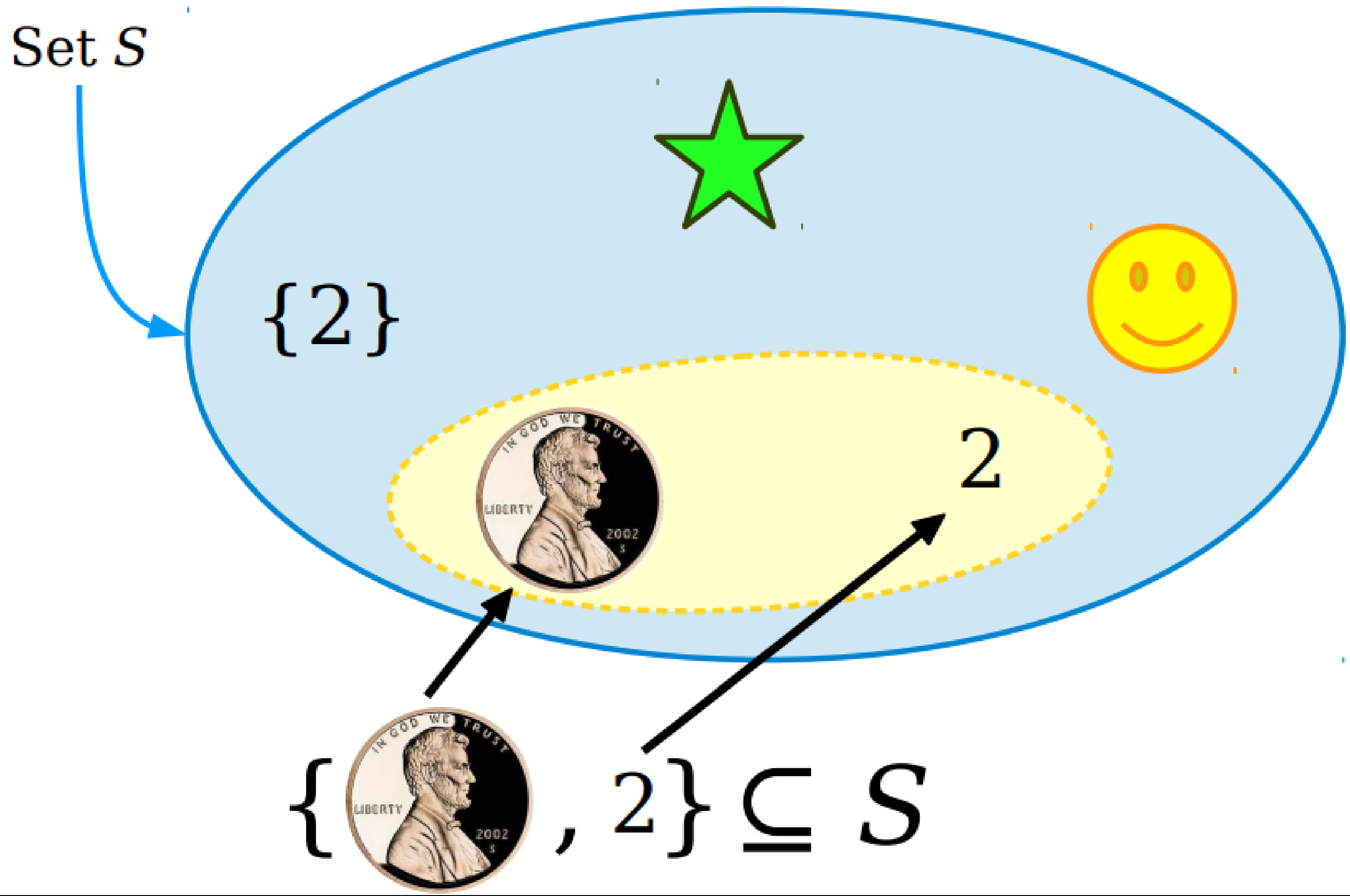
Subsets and Elements



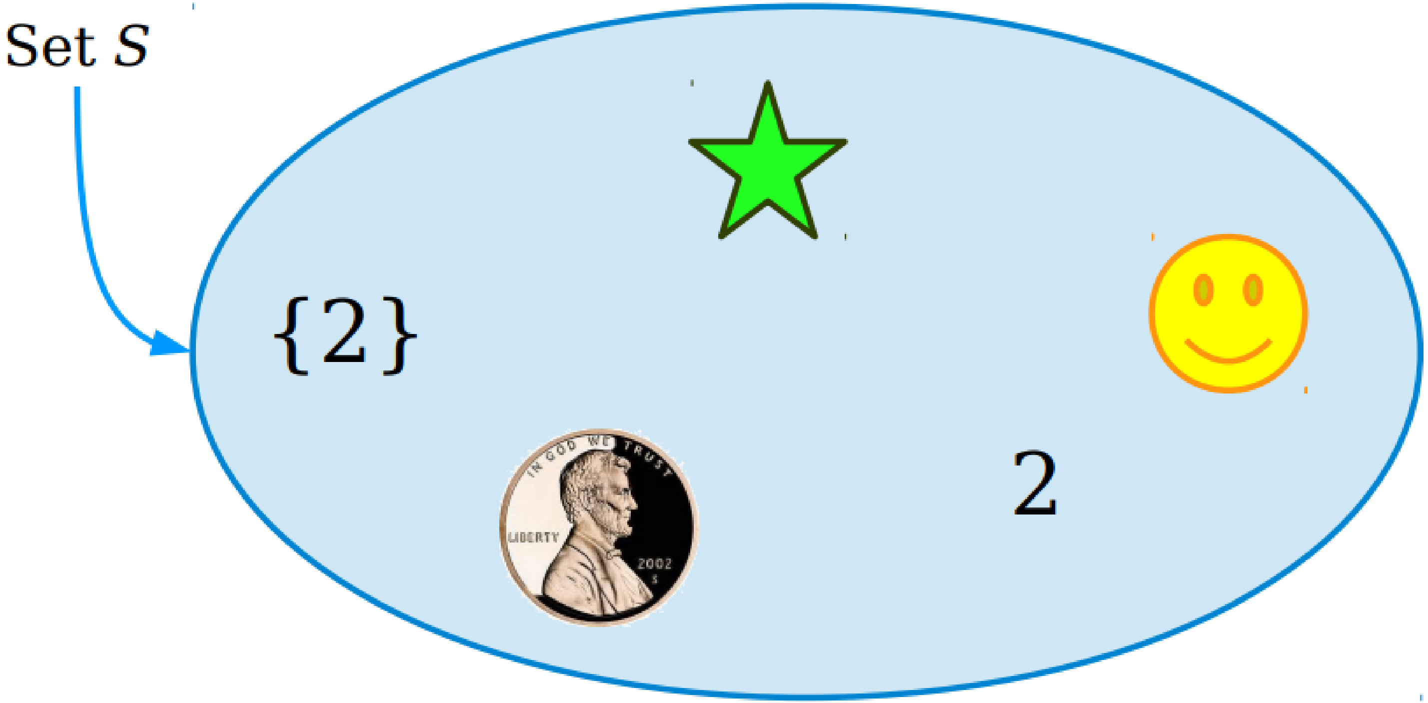
General intuition: $x \in S$ means you can ***point at x inside of S***.

$\{2\} \in S$

Subsets and Elements

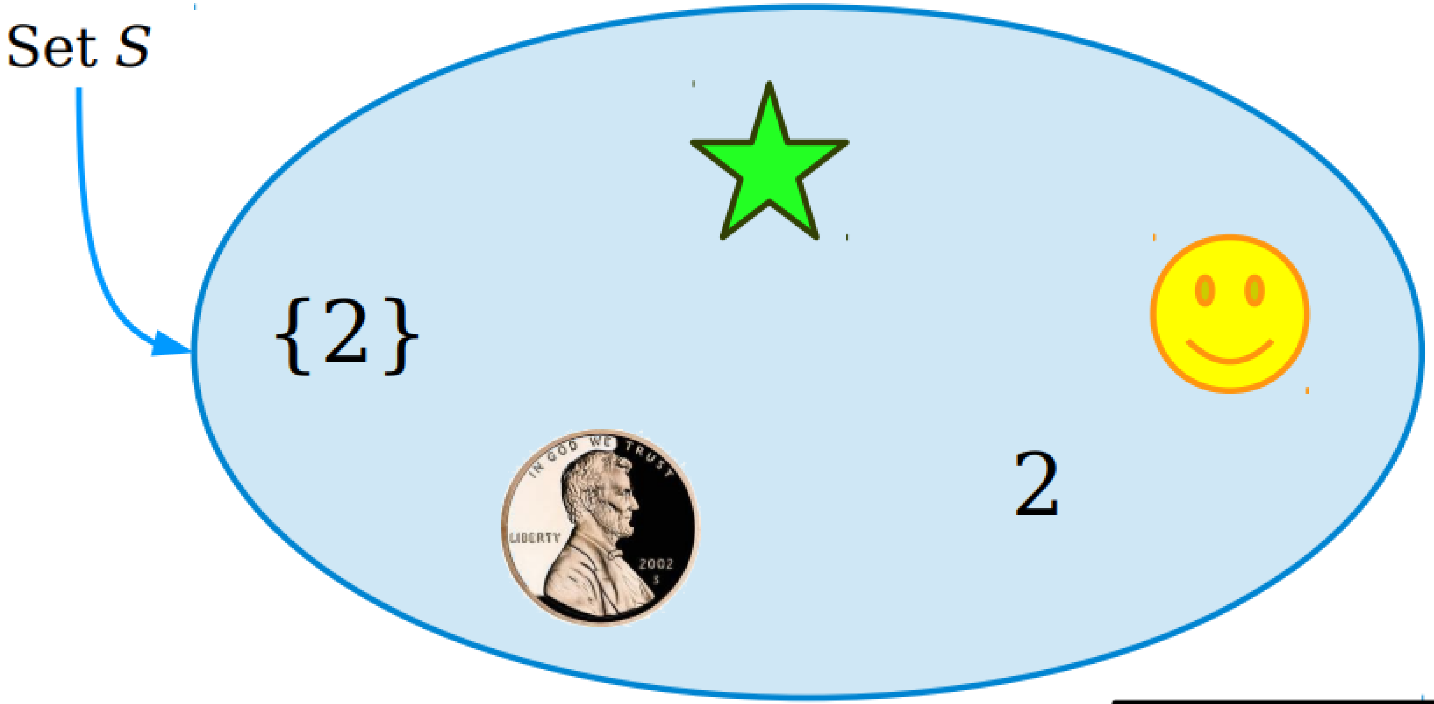


Subsets and Elements



$\{$  $, 2\} \notin S$

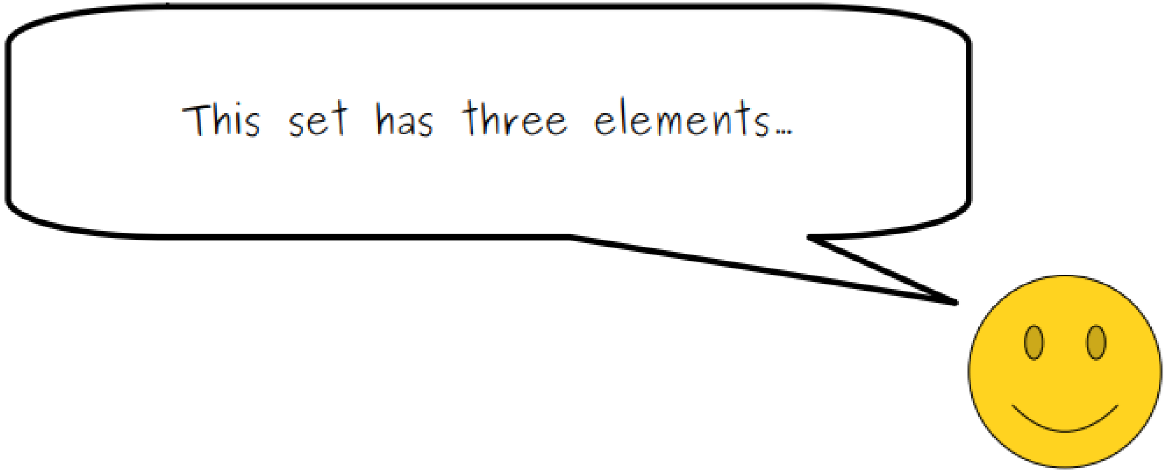
Subsets and Elements



$$2 \nsubseteq S$$

(Since 2 isn't a set.)

$S = \{ 1, \{2, 3\}, 4 \}$



1. $\{1\} \subseteq S$ ✓
2. $\{1\} \in S$ ✗
3. $\{2\} \subseteq S$ ✗
4. $\{2,3\} \subseteq S$ ✗
5. $\{\{2,3\}\} \subseteq S$ ✓

Subsets and Elements

- We say that $\boldsymbol{S \in T}$ if, among the elements of T, one of them is *exactly* the object S.
- We say that $\boldsymbol{S \subseteq T}$ if S is a set and every element of S is also an element of T. (S has to be a set for the statement $S \subseteq T$ to be true.)
  - Although these concepts are similar, ***they are not the same!*** Not all elements of a set are subsets of that set and vice-versa.

What About the Empty Set?

- A set S is a **subset** of a set T (denoted $S \subseteq T$) if all elements of S are also elements of T.
- Is $\emptyset \subseteq S$ for any set S?
- **Yes**: The above statement is always true.
- **Vacuous truth**: A statement that is true because it does not apply to anything.
  - "All unicorns are blue."
  - "All unicorns are pink."

Proper Subsets

- By definition, any set is a subset of itself. *(Why?)*
- A **proper subset** of a set S is a set T such that
  - $T \subseteq S$
  - $T \neq S$
- There are multiple notations for this; they all mean the same thing:
  - $T \subsetneq S$
  - $T \subset S$

$S = \{1, 2\}$

Proper subsets of S

ϕ, $\{1\}$, $\{2\}$

Definition 2.12 *For two sets S and T we say that S* **is a subset of** *T if each element of S is also an element of T. In formal notation $S \subseteq T$ if for all $x \in S$ we have $x \in T$.*

If $S \subseteq T$ then we also say T contains S which can be written $T \supseteq S$. If $S \subseteq T$ and $S \neq T$ then we write $S \subset T$ and we say S is a *proper* subset of T.

Example 2.9 Subsets

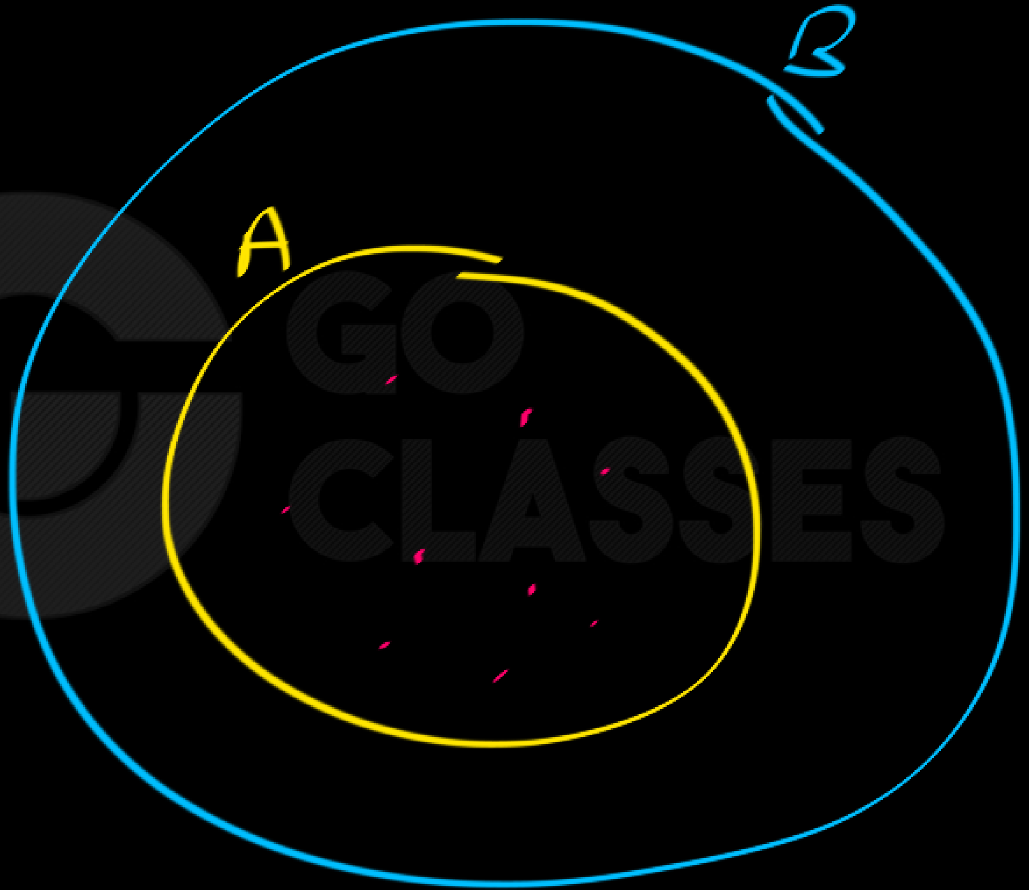
If $A = \{a, b, c\}$ *then* A *has eight different subsets:*

$$\emptyset \qquad \{a\} \qquad \{b\} \qquad \{c\}$$

$$\{a, b\} \quad \{a, c\} \quad \{b, c\} \quad \{a, b, c\}$$

Notice that $A \subseteq A$ *and in fact each set is a subset of itself. The empty set* $\emptyset$ *is a subset of every set.*

$A \subseteq B$



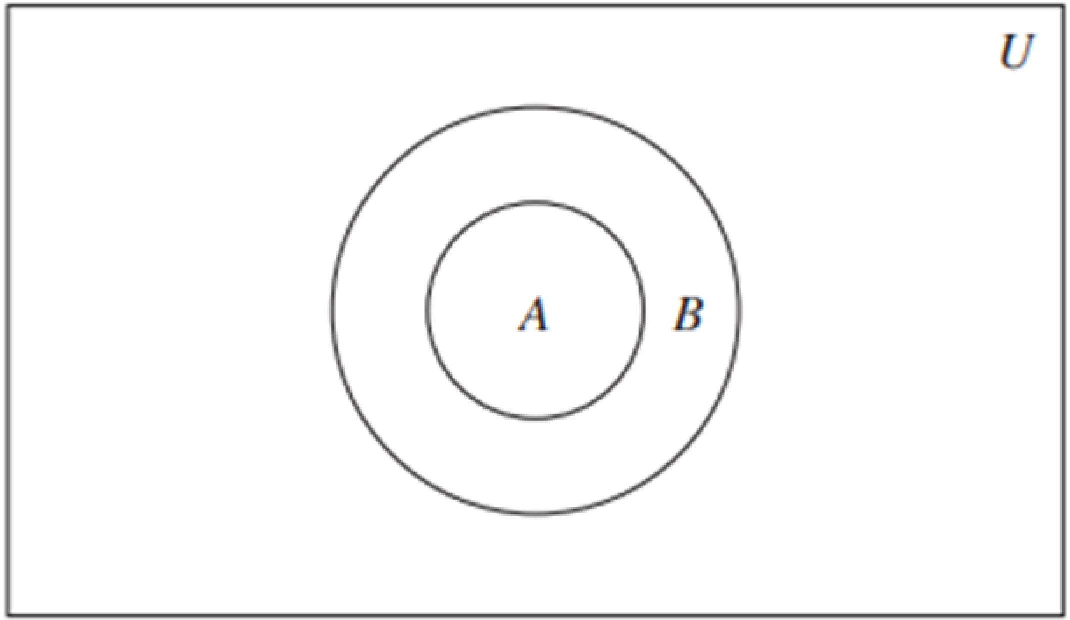


FIGURE 2 Venn Diagram Showing that A Is a Subset of B.