



Digital Logic :

Lecture 4 :

Boolean Algebra, Logic gates

Algebraic Simplification



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Digital Logic :

Recap :

Dual of an expression,

Boolean Algebra Laws



Laws of Boolean Algebra

Duality: A dual of a Boolean expression is derived by interchanging OR and AND operations, and 0s and 1s (literals are left unchanged).

$$\{F(x_1, x_2, \dots, x_n, 0, 1, +, \bullet)\}^D = \{F(x_1, x_2, \dots, x_n, 1, 0, \bullet, +)\}$$

Any law that is true for an expression is also true for its dual.

Operations with 0 and 1:

$$1. x + 0 = x \quad x * 1 = x$$

$$2. x + 1 = 1 \quad x * 0 = 0$$

Idempotent Law:

$$3. x + x = x \quad x * x = x$$

Involution Law:

$$4. (x')' = x$$

Laws of Complementarity:

$$5. x + x' = 1 \quad x * x' = 0$$

Commutative Law:

$$6. x + y = y + x \quad x * y = y * x$$





Laws of Boolean Algebra (cont.)

Associative Laws:

$$(x + y) + z = x + (y + z) \quad x \cdot y \cdot z = x \cdot (y \cdot z)$$

Distributive Laws:

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z) \quad x + (y \cdot z) = (x + y)(x + z)$$

"Simplification" Theorems:

$$x \cdot y + x \cdot y' = x$$

$$x + x \cdot y = x$$

$$(x + y) \cdot (x + y') = x$$

$$x \cdot (x + y) = x$$

DeMorgan's Law:

$$(x + y + z + \dots)' = x'y'z'$$

$$(x \cdot y \cdot z \cdot \dots)' = x' + y' + z'$$



Single Variable Theorems

- $x \cdot 0 = 0$
- $x + 1 = 1$
- $x \cdot 1 = x$
- $x + 0 = x$
- $x \cdot x = x$
- $x + x = x$
- $x + !x = 1$
- $x \cdot !x = 0$
- $x \cdot x \cdot x \cdot \dots \cdot x = x$
- $!!x = x$



Two Variable Theorems

- $x \cdot y = y \cdot x$
- $x + y = y + x$
- Both are commutative

Three Variable Theorems

■ Associative Laws

$$\square \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$\square \quad x + (y + z) = (x + y) + z$$

■ Distributive Law

$$\square \quad x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

$$\blacksquare \quad x + (y \cdot z) = (x + y) \cdot (x + z)$$





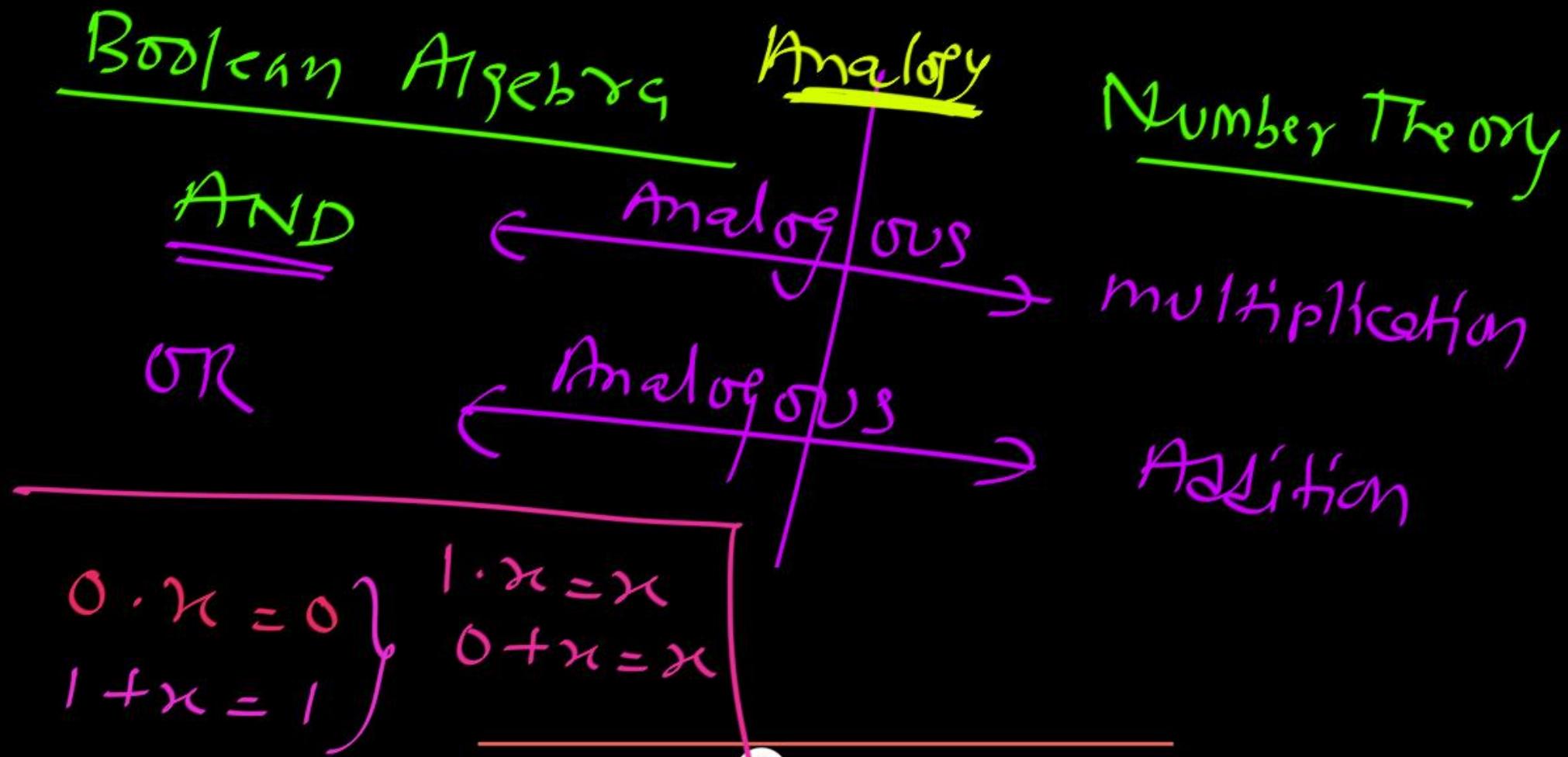
Theorem for Multiplying and Factoring:

$$(x + y)(x' + z) = xz + x'y$$

Consensus Theorem:

$$xy + yz + x'z = (x + y)(y + z)(x' + z)$$

$$xy + x'z = (x + y)(x' + z)$$



Boolean AlgebraNumber Theory

$$\left. \begin{array}{l} 0 \cdot 0 = 0 ; 1 \cdot 1 = 1 \\ 1 \cdot 0 = 0 ; 0 \cdot 1 = 0 \end{array} \right\} \xrightarrow{\text{AND}} \begin{array}{l} 0 + 0 = 0 \\ 0 + 1 = 1 \\ 1 + 0 = 1 \\ 1 + 1 = 1 \end{array}$$

$$\xleftarrow{\text{Analogy}} \left. \begin{array}{l} 0 \times 0 = 0 \\ 0 \times 1 = 0 \\ 1 \times 1 = 1 \end{array} \right\} \xrightarrow{\text{Multiplication}}$$

$$\left. \begin{array}{l} 0+0=0 \\ 0+1=1=1+0 \\ 1+1=2 \end{array} \right\} \xleftarrow{\text{Addition}}$$

Q: Number of Boolean functions for
 a, b, c, d , such that $f = \bar{f}$?

Q: Number of Boolean functions for

a, b, c, d , such that $f = \bar{f}$?

Ans:

0

f is NEVER equal to its
Complement.

Boolean Algebra :

Next Topic :

Simplification of Boolean Functions

Algebraic Simplification

= Simplification Using Boolean Algebra



①

$$\underbrace{\alpha b + \alpha c}_{\alpha(b+c)} = \alpha(b+c)$$

Distributive
property

ef: $\underbrace{\alpha + \alpha c}_{\frac{1}{1}(\alpha + \alpha c)} = \alpha \left(\frac{1+c}{1} \right) = \alpha \cdot 1 = \alpha$

②

$$a(b+c) = ab + ac$$

$$(a+b)(c+d) = ac + ad + bc + bd$$



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Simplify

$$A \overline{B} D + A \overline{B} \overline{D} =$$



$$A \overline{B} D + A \overline{B} \overline{D} =$$

$$A \overline{B} (D + \overline{D})$$

$D + \overline{D}$ = 1

$$= A \overline{B}$$



$$A \overline{B} D + A \overline{B} \overline{D} =$$

$$A \overline{B} (D + \overline{D})$$

$$A \overline{B} (1)$$

$$A \overline{B}$$

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$$(A + B)(A + C)$$



$$(A + B)(A + C)$$

$$\cancel{AA} + A\cancel{C} + BA + BC$$

$$\cancel{A} + \cancel{AC} + \cancel{AB} + BC$$

$$\underline{\underline{A + BC}} \checkmark = (A + B)(A + C)$$

$$\begin{aligned} AA &= A \\ A + A\cancel{C} &= A \end{aligned}$$

$$\begin{aligned} A + A\cancel{C} &= A(1 + \cancel{C}) \\ &= A \end{aligned}$$

$$\begin{aligned}(A + B)(A + C) &= AA + AC + AB + BC \\&= A + AC + AB + BC \\&= A(1 + C + B) + BC \\&= A \cdot 1 + BC \\&= A + BC\end{aligned}$$



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$$(\bar{A} + B)(A + B) =$$



$$(\underline{\bar{A}} + B)(\underline{A} + B) =$$

$\circ \quad (\underline{\bar{A}}\underline{A}) + \bar{A}B + BA + (\underline{B}\underline{B}) = B$

$$\boxed{B + A\bar{B}} + B\bar{A}$$

$$= \boxed{B + B\bar{A}} = B \checkmark$$

$$\begin{aligned} 0 + \alpha &= \alpha \\ B + (\underline{B}\underline{\alpha}) &= B \\ &= \underline{\underline{B}} \end{aligned}$$



$$(\bar{A} + B)(A + B) = B (\bar{A} + A)$$

$$B (1)$$

$$B$$

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Absorption Law:

$$\cancel{\alpha + \beta} + \cancel{\alpha \gamma} + \cancel{\alpha \zeta}$$

$$= \alpha \checkmark$$

$$\text{EP: } \alpha \bar{b} c + \alpha \bar{b} c d + \alpha c d \bar{e} \bar{b}$$

$$= \alpha \bar{b} c \checkmark$$



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$$A\bar{C} + AB\bar{C} =$$



$$A\bar{C} + AB\bar{C} =$$

$$A\bar{C}(1+B)$$

$$= A\bar{C}$$

$$\overline{A\bar{C}} + \overline{A\bar{C}B} = \overline{\overline{A\bar{C}}} = \alpha$$

$$= A\bar{C} \checkmark$$



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$$\begin{aligned} A\bar{C} + AB\bar{C} &= A\bar{C} (1 + B) \\ &= A\bar{C} (1) \\ &= A\bar{C} \end{aligned}$$

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DeMorgan Law:

Exp E

find \overline{E} .

$$\overline{(a \bar{b})} = \bar{a} + b \quad \text{L}$$



1. Simplify the following expressions using Boolean Algebra.

- a. $A + \overline{A}$
- b. $A(\overline{A} + B)$
- c. $(\overline{A} + \overline{B})(\overline{A} + B)$
- d. $\overline{A} + \overline{AB}$
- e. $\overline{A(ABCD)}$

1. Simplify the following expressions using Boolean Algebra.

a. $A + \bar{A} = 1$

b. $A(\bar{A} + B) = 0 + AB = AB$

c. $(\bar{A} + \bar{B})(\bar{A} + B) = \underline{\bar{A}} + \underline{\bar{A}}B + \underline{\bar{A}}\bar{B} + 0 = \bar{A}$

d. $\bar{A} + \bar{A}B = \bar{A}$

e. $\bar{A}(ABCD) = \bar{A}(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

$$= (\cancel{\bar{A}}) + \cancel{\bar{A}}\bar{B} + \cancel{\bar{A}}\bar{C} + \cancel{\bar{A}}\bar{D} = \bar{A}(1 + \dots)$$

$$= \cancel{\bar{A}} \checkmark = \bar{A}$$

③
$$\begin{aligned} & (\bar{A} + \bar{B})(\bar{A} + B) \\ &= (\overline{AB})(\overline{A}\bar{B}) \end{aligned}$$

Absorption Law: $\alpha + \alpha\beta + \alpha\gamma + \alpha\zeta = \alpha$



- a. $F = (XY + Z)(Y + XZ)$
- b. $F = (AB + C)(B + \overline{C}D)$
- c. $F = A(\overline{B}CD + \overline{B}\overline{D} + D(\overline{A} + AB))$

Hw



1. Simplify the Boolean expression using Boolean algebra.

a. $(A + B) + \overline{B}$.

b. $AA + BC + B\overline{C}$.

c. $\overline{A} + C + AB$.

d. $\overline{A}(B + AC)$.

1. Simplify the Boolean expression using Boolean algebra.

a. $(A + B) + \bar{B} = A + (B + \bar{B}) = 1 \checkmark$

b. $\cancel{AA} + \underline{BC + B\bar{C}} = A + B$
 $\qquad\qquad\qquad \swarrow B$

c. $\bar{A} + C + \cancel{AB} = \bar{A} + B + C$

d. $\bar{A}(B + AC) = \bar{A}B + \cancel{\bar{A}AC} = \bar{A}B$

$$\begin{aligned} & A + \bar{A}B \\ &= A + B \end{aligned}$$



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Simplify $A(\bar{A} + B) + A\bar{B}$. Show each stage of your working.



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Simplify $\underline{\underline{A(\bar{A} + B)}} + A\bar{B}$. Show each stage of your working.

$$\begin{aligned} O &= \cancel{A\bar{A}} + AB + A\bar{B} \\ &= \cancel{AB} + A\bar{B} = A(\cancel{B + \bar{B}}) \\ &= A \checkmark \end{aligned}$$



Using Boolean algebra, simplify the following expressions:

(d) $(A + \bar{A})(AB + A\bar{B}\bar{C})$

Using Boolean algebra, simplify each expression:

(b) $\overline{A}\overline{B}C + \overline{(A + B + \bar{C})} + \overline{A}\overline{B}\overline{C}D$

(d) $ABCD + AB(\overline{C}\overline{D}) + (\overline{A}\overline{B})CD$

Using Boolean algebra, simplify the following expressions:

$$(d) \frac{(A + \bar{A})(AB + A\bar{B}\bar{C})}{I} = A\bar{B}$$

Absorption

Using Boolean algebra, simplify each expression:

$$(b) \overline{ABC} + \overline{(A + B + \bar{C})} + \overline{AB}\overline{CD}$$

$$(d) ABCD + AB(\overline{CD}) + (\overline{AB})CD$$

b) $\overline{A} \overline{B} C + \overbrace{(\overline{A} + \overline{B} + \overline{C})}^{\circ} + \overline{A} \overline{B} \overline{C} D$

 $= \overline{\overline{A} \overline{B} C} + \overline{\overline{A} \overline{B} C} + \overline{A} \overline{B} \overline{C} D$
 $= \overline{\overline{A} \overline{B} C} + \overline{\overline{A} \overline{B} \overline{C}} D$
 $= \overline{\overline{A} \overline{B}} (C + \cancel{\overline{C} D}) = \underline{\underline{\overline{A} \overline{B}}} (C + D)$

$$\alpha + \alpha \beta = \alpha \quad \checkmark$$

$$\underbrace{\bar{A} \bar{B} C}_{\alpha} + \underbrace{\bar{A} \bar{B} \bar{C} D}_{\alpha?} \quad \left. \begin{array}{l} \text{Absorption} \\ \text{Law} \end{array} \right\} X$$



Problem #01] Using Boolean algebra techniques, simplify as much as possible.

(a) $A(A + B)$

(b) $A(\bar{A} + AB)$

(c) $BC + \bar{B}C$

(d) $A(A + \bar{A}B)$

(e) $\bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}$



Problem #01] Using Boolean algebra techniques, simplify as much as possible.

(a) $A(A + B)$

(b) $A(\bar{A} + AB)$

(c) $BC + \bar{B}C$

(d) ~~$A(A + \bar{A}B)$~~

(e) $\underline{\bar{A}\bar{B}C} + \underline{\bar{A}B\bar{C}} + \underline{\bar{A}\bar{B}\bar{C}}$

(a) A

(b) AB

(c)

$$A \bar{B}(c) + \bar{A}(c) = c(\bar{A} + A\bar{B})$$

$$= c\bar{A} + c\bar{B} \checkmark$$

$$\underline{\underline{\alpha}} + \underline{\underline{\alpha}}' \beta = \underline{\underline{\alpha + \beta}}$$

$$\alpha + \alpha \beta = \alpha$$

$$\boxed{\alpha + \bar{\alpha} = 1}$$

$$\boxed{\alpha \bar{\alpha} = 0}$$



Using Boolean algebra techniques, simplify the following expressions as much as possible:

$$(a) A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + A\bar{B}CDE$$

$$(b) (B + \bar{B})(BC + BCD)$$

$$(c) BCDE + BC(\overline{DE}) + (\overline{BC})DE$$

$$(d) \bar{B}\bar{C}D + (\overline{B+C+D}) + \bar{B}\bar{C}\bar{D}E$$



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Using Boolean algebra techniques, simplify the following expressions as much as possible:

$$(a) \overline{A}\bar{B} + A\bar{B}\bar{C} + \overline{A}\bar{B}\bar{C}\bar{D} + \overline{A}\bar{B}CDE$$

$$(b) \overline{(B+\bar{B})}(\underline{BC} + \underline{BC}\bar{D})$$

$$(c) \overline{BCDE} + BC(\overline{DE}) + (\overline{BC})DE = \cancel{\overline{BC}} + \cancel{DE}$$

$$(d) \bar{B}\bar{C}\bar{D} + \cancel{(\overline{B+C+D})} + \bar{B}\bar{C}\bar{D}E$$

(a) $A\bar{B} + A\bar{B}\bar{C} = A(\bar{B} + \cancel{\bar{B}\bar{C}}) = A(\bar{B} + \bar{C})$

(b) $B\bar{C}$

(c) $\cancel{B}\bar{C} \left(\underbrace{DE + \overline{DE}}_1 \right) + \cancel{B}\bar{C} DE = \cancel{B}\bar{C} + \cancel{\cancel{B}\bar{C} DE}$

$$\overline{BC} + \overline{\overline{BC}}DE = \cancel{C} + \cancel{\overline{C}}DE$$

~~\cancel{C}~~ ~~$\cancel{\overline{C}}$~~

$$= \underline{\underline{C}} + \underline{\underline{DE}} \quad \checkmark$$

\textcircled{D} $\overline{B}\overline{C}D + \overline{B}\overline{C}\overline{D} + \overline{B}\overline{C}\overline{D}\overline{E}$

$$= \underline{\underline{B}}\underline{\underline{C}}D + \underline{\underline{B}}\underline{\underline{C}}\overline{D} = \overline{B}\overline{C}(D + \overline{D}) = \overline{B}\overline{C}$$



YuvRq)

Question 3 (21 marks)

Using Boolean algebra, simplify the following expressions. Please state the Boolean identity or the Boolean theorem applied there, next to each simplification step.

- i. $Z = \underline{\underline{A + \bar{A}}}. \underline{\underline{A \cdot B + A \cdot B \cdot \bar{C}}} = \underline{\underline{AB}} \quad \checkmark$
- ii. $F = (A + \bar{B} \cdot C) \cdot (A + B \cdot \bar{C})$
- iii. $X = \bar{A} \cdot B \cdot \bar{C} + \bar{A} \cdot C + \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{C}$
- iv. $Z = (A \cdot B \cdot C) \cdot [A \cdot B + \bar{C} \cdot (B \cdot C + A \cdot C)]$
- v. $F = (A + \bar{B}) \cdot (A + C)$
- vi. $Y = \overline{\overline{A + B \cdot \bar{C}}} + C \cdot D + B \cdot C$
- vii. $Z = \overline{X \cdot \bar{Y} \cdot (\bar{\bar{W}} + \bar{\bar{Y}})}$

(a) $x = \underline{ABC + \bar{A}C} = C(\bar{A} + A\cancel{B}) = C(\bar{A} + B)$

Mw

(b) $y = (Q + R)(\bar{Q} + \bar{R})$

(c) $w = ABC + A\bar{B}C + \bar{A}$

Gavimq

(d) $q = \overline{RST}(\overline{R} + S + T)$

(e) $x = \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C$

(f) $z = (B + \bar{C})(\bar{B} + C) + \bar{\bar{A}} + B + \bar{C}$

(g) $y = (\bar{C} + D) + \bar{A}CD + A\bar{B}\bar{C} + A\bar{B}CD + ACD$

(h) $x = AB(\bar{\bar{C}}D) + \bar{A}BD + \bar{B}\bar{C}\bar{D}$



Using Boolean algebra, simplify the following expressions:

- (a) $(\bar{A} + B)(A + C)$ (b) $A\bar{B} + A\bar{B}C + A\bar{B}CD + A\bar{B}CDE$
(c) $BC + \overline{BCD} + B$ (d) $(B + \bar{B})(BC + BCD)$
(e) $BC + (\bar{B} + \bar{C})D + BC$

Pawan
HCO
Arpan

Using Boolean algebra, simplify the following expressions:

- (a) $CE + C(E + F) + \bar{E}(E + G)$ (b) $\bar{B}\bar{C}D + (\bar{B} + \bar{C} + \bar{D}) + \bar{B}\bar{C}\bar{D}E$
(c) $(C + CD)(C + \bar{C}D)(C + E)$ (d) $BCDE + BC(\bar{D}\bar{E}) + (\bar{B}\bar{C})DE$
(e) $BCD[BC + \bar{D}(CD + BD)]$



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$$F = \overline{X}YZ + \overline{XY}\overline{Z} + XZ$$



$$F = \overline{XYZ} + \overline{XY\bar{Z}} + XZ$$

$$\overline{x}\overline{y}(z+\overline{z}) + xz$$

$$\overline{x}\overline{y} + xz \checkmark$$

$$F = \overline{XYZ} + \overline{XY}\overline{Z} + XZ$$

$$\overline{XY}(Z + \overline{Z}) + XZ \quad (\text{by reverse distribution})$$

$$\overline{XY} + XZ \quad (\text{by complementarity})$$

$$\overline{XY} + XZ \quad (\text{by identity})$$



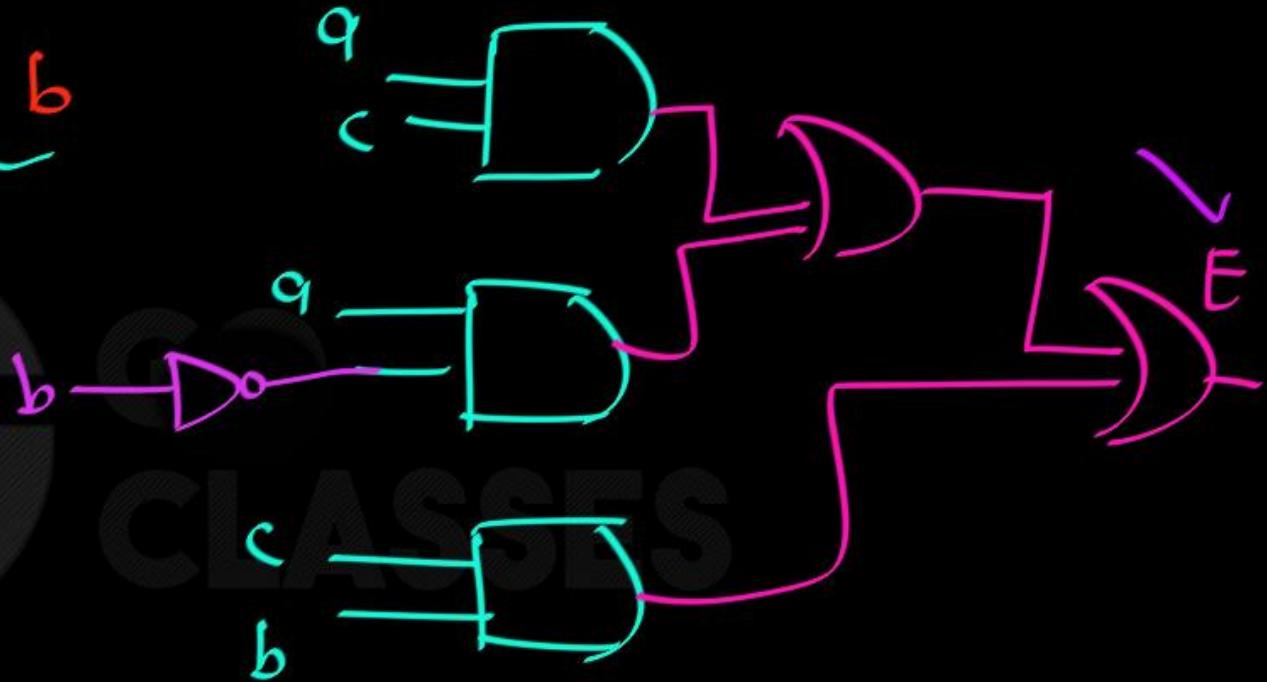
Boolean Algebra :

Next Topic :

Why Simplify a Boolean Function?

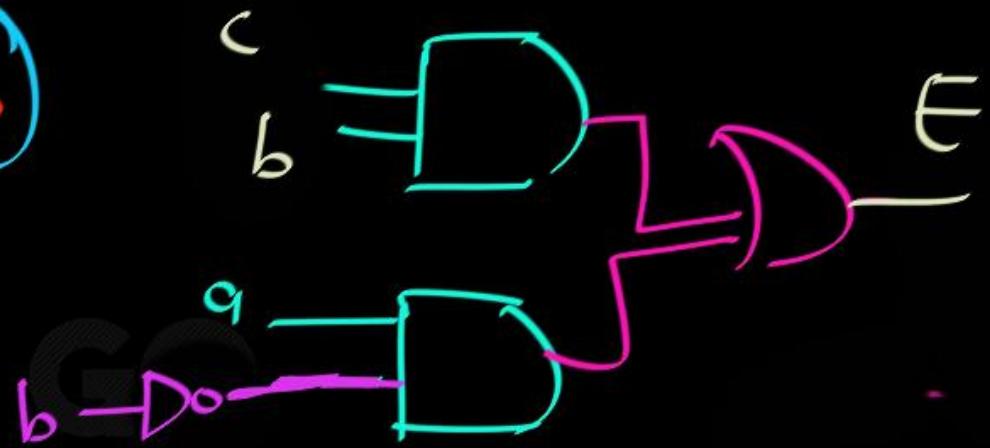


$$E = \overbrace{ac + a\bar{b} + cb}^{\text{wavy line}}$$



$$E = \underline{\underline{ac}} + \underline{a}\underline{\bar{b}} + \underline{\underline{c}}\underline{\bar{b}}$$

$$\underline{\underline{E = a\bar{b} + c\bar{b}}}$$





Digital Logic

In each case, one expression can be replaced by a simpler one. Since each expression corresponds to a circuit of logic gates, simplifying an expression leads to simplifying the corresponding logic circuit.

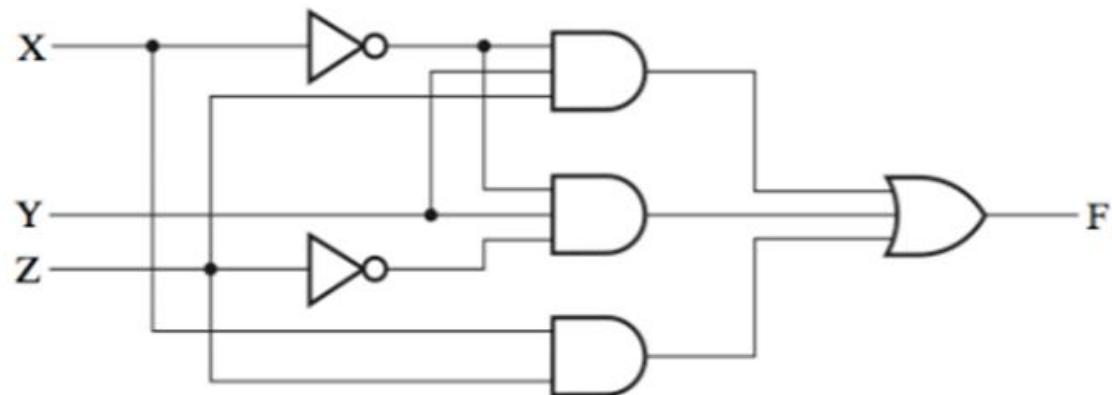


$$F = \overline{XYZ} + \overline{XY}\overline{Z} + XZ$$

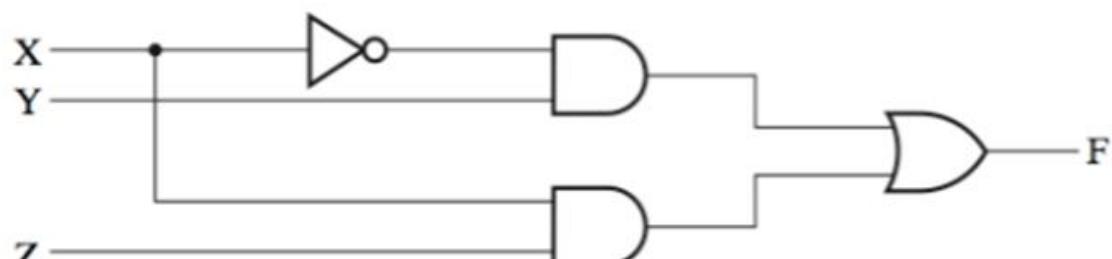
$$\overline{XY}(Z + \overline{Z}) + XZ \quad (\text{by reverse distribution})$$

$$\overline{XY} + XZ \quad (\text{by complementarity})$$

$$\overline{XY} + XZ \quad (\text{by identity})$$



$$(a) F = \overline{XYZ} + \overline{XY\bar{Z}} + XZ$$



$$(b) F = \overline{XY} + XZ$$

Literal: $x, \bar{x}, y, \bar{y}, z, \bar{z}$

Variable:

x, y, z

$x \Rightarrow$ Variable ✓

$\bar{x} \Rightarrow$ Not Variable

Literal: a variable in its original or

complementary form is called literal.

Literal Count ^(LC) tells you how big is your expression. e.g.: $f = \underbrace{a+a+a+b}_{\text{LC=4}}$

Each appearance of a variable or its complement in an expression will be referred to as a *literal*. Thus, the following expression, which has three variables, has 10 literals:

$$ab'c + a'b + a'bc' + b'c' \rightarrow \# \text{Variables} = 3$$

$$f = a + b \Rightarrow LC = 2 \quad | \quad f = \underbrace{a \downarrow b' \downarrow c + a' \downarrow b + a' \downarrow b' \downarrow c + b' \downarrow c}_{LC = 10}$$

$$E: \underbrace{a + \bar{a} + a + \bar{a}}_{\text{Literal count of } E} \rightarrow \text{over only one variable}$$

Literal count of $E = 4$

$$E_2: 1$$

$$LC(E_2) = 0$$

$$E_1 : a + b + \cancel{b} \equiv E_2 : a + \cancel{b}$$

\downarrow

$L_C = 3$ $L_C = 2$

Simplification / minimization of an Expression:
(minimize) Reducing the Literal Count.

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C$$

How big
is the Expression

← →
Literal Count = 8
#Variables = 3

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C$$

$$\underbrace{B + \overline{B}}_{=} = 1$$

Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C$$

$$B(\overline{A} + A) + \overline{B}(\overline{C} + C) \quad (\text{by distribution})$$

$$B + \overline{B} \quad (\text{by complementarity})$$

$$1 \quad (\text{by complementarity})$$



Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\bar{X} + Y = \bar{X}\bar{Y} + \bar{X}Y + XY$$

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Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\overline{X} + Y = \overline{XY} + \overline{XY} + XY$$

~~\overline{XY}~~ + ~~\overline{XY}~~

$$= \overline{x} + y$$

Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\bar{X} + Y = \bar{X}\bar{Y} + \bar{X}Y + XY$$

$$\bar{X}\bar{Y} + \bar{X}Y + \bar{X}Y + XY \quad (\text{by idempotence})$$

$$\bar{X}(\bar{Y} + Y) + Y(\bar{X} + X) \quad (\text{by distribution})$$

$$\bar{X}_1 + Y_1 \quad (\text{by null})$$

$$\bar{X} + Y \quad (\text{by identity})$$



Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$



Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$\overline{X}Y(Z + \overline{Z}) + XZ \quad (\text{by reverse distribution})$$

$$\overline{X}Y + XZ \quad (\text{by complementarity})$$

$$\overline{X}Y + XZ \quad (\text{by identity})$$



Example



- Using the Theorems and Laws of Boolean algebra,
Prove the following.

$$(A+B) \cdot (A+A'B') \cdot C + (A' \cdot (B+C'))' + A' \cdot B + A \cdot B \cdot C = A+B+C$$

using

by Case method ✓

Boolean Algebra



Case 1 : $A = 0 \checkmark$

$$\begin{aligned} LHS &= B + (B+C) \\ &= B + \cancel{B}C = \underline{\underline{B+C}} \end{aligned}$$

$$\begin{aligned} \underline{\underline{RHS}} &= B+C \checkmark \\ &\quad \text{Same} \end{aligned}$$

Case 2 : $A = 1$

$$\begin{aligned} LHS &= C + 1 + - + - \\ &= 1 \\ RHS &= 1 \quad \text{Same} \end{aligned}$$



Algebraic Simplification

HW

- Using Boolean algebra techniques, simplify this expression: $AB + A(B + C) + B(B + C)$

Algebraic Simplification

- Using Boolean algebra techniques, simplify this expression: $AB + A(B + C) + B(B + C)$

- Solution

$$= AB + AB + AC + BB + BC \quad (\text{Distributive law})$$

$$= AB + AB + AC + B + BC \quad (B \cdot B = B)$$

$$= AB + AC + B + BC \quad (AB + AB = AB)$$

$$= AB + AC + B \quad (B + BC = B)$$

$$= B + AC \quad (AB + B = B)$$



Algebraic Simplification

- Minimize the following Boolean expression using Algebraic Simplification

$$F(A,B,C) = A'B + BC' + BC + AB'C'$$

Algebraic Simplification

- Minimize the following Boolean expression using Algebraic Simplification

$$F(A,B,C) = A'B + BC' + BC + AB'C'$$

- Solution

$$= A'B + (BC' + BC') + BC + AB'C' \quad [\text{indeponent law}]$$

$$= A'B + (BC' + BC) + (BC' + AB'C')$$

$$= A'B + B(C' + C) + C'(B + AB')$$

$$= A'B + B \cdot 1 + C' (B + A)$$

$$= B(A' + 1) + C'(B + A)$$

$$= B + C'(B + A) \quad [A' + 1 = 1]$$

$$= B + BC' + AC'$$

$$= B(1 + C') + AC'$$

$$= B + AC' \quad [1 + C' = 1]$$



Algebraic Simplification

- Simplify: $C + (BC)'$

Algebraic Simplification

□ Simplify: $C + (BC)'$

$$= C + (BC)'$$

Original Expression

$$= C + (B' + C')$$

DeMorgan's Law.

$$= (C + C') + B'$$

Commutative, Associative Laws.

$$= 1 + B'$$

Complement Law.

$$= 1$$

Identity Law.



NOTE:

Cancellation Laws do NOT hold in Boolean Algebra.

Ex: If $a_b = a_c$ then $b = c$? No.

~~$\frac{0 \cdot 1}{0} = \frac{0 \cdot 0}{0}$~~ $\Rightarrow 1 \neq 0$

If $a+b = a+c$ then $b = c$? No.

~~$1+0 = 1+1$~~ $\Rightarrow 0 \neq 1$

If $a \oplus b = a \oplus c \Rightarrow b = c ?$

If $a \odot b = \overline{a} \odot c \Rightarrow b = c ?$

$$\text{If } \underline{a} \oplus b = \underline{a} \oplus c \Rightarrow b = c ? \checkmark$$

$$\underline{a=0} \rightarrow b = c$$

$$\underline{a=1} \quad \bar{b} = \bar{c} \Rightarrow b = c$$

$$\text{If } \underline{\underline{a} \odot b} = \underline{\underline{\bar{a} \odot c}} \Rightarrow \boxed{b = \bar{c}} ?$$

$$\underline{\underline{a=0}} \Rightarrow \bar{b} = c \checkmark$$

$$\underline{\underline{a=1}} \Rightarrow b = \bar{c} \Rightarrow \bar{b} = c$$

If $a \odot b = a \odot c \Rightarrow b = c ?$

If $a + b = a + c \text{ and } \underline{a=0} \Rightarrow b = c ?$

$$\text{If } \underbrace{a \odot b}_{a=0 \Rightarrow \bar{b}=\bar{c}} = a \odot c \Rightarrow b=c ? \checkmark$$

$a=1 \Rightarrow b=c$

$$\text{If } \underbrace{a+b}_{\text{and } \underline{a=0}} = a+c \Rightarrow b=c ? \checkmark$$

As we have previously observed, some of the theorems of Boolean algebra are not true for ordinary algebra. Similarly, some of the theorems of ordinary algebra are *not* true for Boolean algebra. Consider, for example, the cancellation law for ordinary algebra:

$$\text{If } x + y = x + z, \quad \text{then} \quad y = z \quad (3-31)$$

The cancellation law is *not* true for Boolean algebra. We will demonstrate this by constructing a counterexample in which $x + y = x + z$ but $y \neq z$. Let $x = 1, y = 0, z = 1$. Then,

$$1 + 0 = 1 + 1 \text{ but } 0 \neq 1$$

In ordinary algebra, the cancellation law for multiplication is

$$\text{If } xy = xz, \quad \text{then} \quad y = z \quad (3-32)$$

This law is valid provided $x \neq 0$.



In Boolean algebra, the cancellation law for multiplication is also *not* valid when $x = 0$. (Let $x = 0, y = 0, z = 1$; then $0 \cdot 0 = 0 \cdot 1$, but $0 \neq 1$). Because $x = 0$ about half of the time in switching algebra, the cancellation law for multiplication cannot be used.

Even though Statements (3-31) and (3-32) are generally false for Boolean algebra, the converses

$$\text{If } y = z, \quad \text{then} \quad x + y = x + z \quad (3-33)$$

$$\text{If } y = z, \quad \text{then} \quad xy = xz \quad (3-34)$$

$$X \oplus 0 = ? = X$$

0 → 0
1 → 1

Associativity of \oplus

Prove using by-case method

True

$$\varphi: a(b \oplus c) = ab \oplus ac$$

Case 1:

$$a = 0$$

$$LHS = 0$$

$$RHS = 0 \oplus 0 = 0$$

Same

Case 2: $a = 1$

$$LHS = b \oplus c$$

$$RHS = b \oplus c$$

Same

$$a \oplus b = a' \oplus b'$$

$$\neq \overbrace{\overline{a \oplus b}}^{\text{III}}$$

Prove:

$$a=0$$

$$\text{LHS} = b ; \text{RHS} = b$$

$$a=1$$

$$\text{LHS} = \overline{b} ; \text{RHS} = \overline{b}$$



The following theorems apply to exclusive OR:

$$X \oplus 0 = X$$

$$X \oplus 1 = X'$$

$$X \oplus X = 0$$

$$X \oplus X' = 1$$

$$X \oplus Y = Y \oplus X \text{ (commutative law)}$$

$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z \text{ (associative law)}$$

$$X(Y \oplus Z) = XY \oplus XZ \text{ (distributive law)}$$

$$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$$

$$\overline{(a \oplus b)} = a \odot b = a' \oplus b = a \oplus b'$$

$$= a' \odot b'$$

Note:

$$\overline{a \oplus b} = \overline{a'} \oplus \overline{b'}$$

$a \neq b \rightarrow \text{Diff}$

$a = b \rightarrow \text{Same}$

$$\overline{a \oplus b} = \overline{a'} \oplus \overline{b'}$$

$a \neq b \rightarrow \text{Same}$

$a = b \rightarrow \text{Different}$

NOTE:

$$a \oplus b = a' \oplus b'$$

$$a \odot b = a' \odot b'$$

$$a' \oplus b = a \oplus b' = a \odot b = \overline{(a + b)}$$

$$a' \odot b = a \odot b' = a \oplus b = \overline{a \odot b}$$



Any of these theorems can be proved by using a truth table or by replacing $X \oplus Y$ with one of the equivalent expressions from Equation (3-7). Proof of the distributive law follows:

$$\begin{aligned}XY \oplus XZ &= XY(XZ)' + (XY)'XZ = XY(X' + Z') + (X' + Y')XZ \\&= XYZ' + XY'Z \\&= X(YZ' + Y'Z) = X(Y \oplus Z)\end{aligned}$$





Equivalence is the complement of exclusive OR:

$$\begin{aligned}(X \oplus Y)' &= (X'Y + XY')' = (X + Y')(X' + Y) \\ &= XY + X'Y' = (X \equiv Y)\end{aligned}\tag{3-18}$$

Just as for exclusive OR, equivalence is commutative and associative.



$$a \oplus b = \overline{a \odot b} = ab + \bar{a}\bar{b} =$$

$$\overline{a \odot b} = ab + \bar{a}\bar{b}$$

$$(\bar{a} + \bar{b})(a + b)$$

$$a \oplus b = \bar{a}b + a\bar{b} = (a+b)(\bar{a}+\bar{b})$$

$$a \odot b = ab + \bar{a}\bar{b} = (a+b)(\bar{a}+b)$$

$$a \odot b = \overline{a \oplus b} = (\bar{a}b + a\bar{b})$$

$$= (a + \bar{b})(\bar{a} + b)$$

$$a \oplus b = \bar{a}b + a\bar{b}$$



13a. $(X + Y) \cdot (\bar{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\bar{X} + Z)$

Consensus Law

13b. $XY + \bar{X}Z + YZ = XY + \bar{X}Z$

Consensus Law

14a. $X \oplus Y = (X + \bar{Y}) \cdot (\bar{X} + Y)$

14b. $X \oplus Y = \bar{X}Y + X\bar{Y}$

XOR Gate

15a. $X \odot Y = (X + Y) \cdot (\bar{X} \cdot \bar{Y})$

15b. $X \odot Y = \bar{X}\bar{Y} + XY$

XNOR Gate

15c. $X \odot Y = (X + Y) \cdot (\bar{X} + \bar{Y})$

XNOR Gate

DeMorgan's Theorem

$$\overline{(X_1 + X_2 + \dots + X_n)} = \overline{X_1} \cdot \overline{X_2} \cdot \dots \cdot \overline{X_n}$$

$$\overline{(X_1 \cdot X_2 \cdot \dots \cdot X_n)} = \overline{X_1} + \overline{X_2} + \dots + \overline{X_n}$$

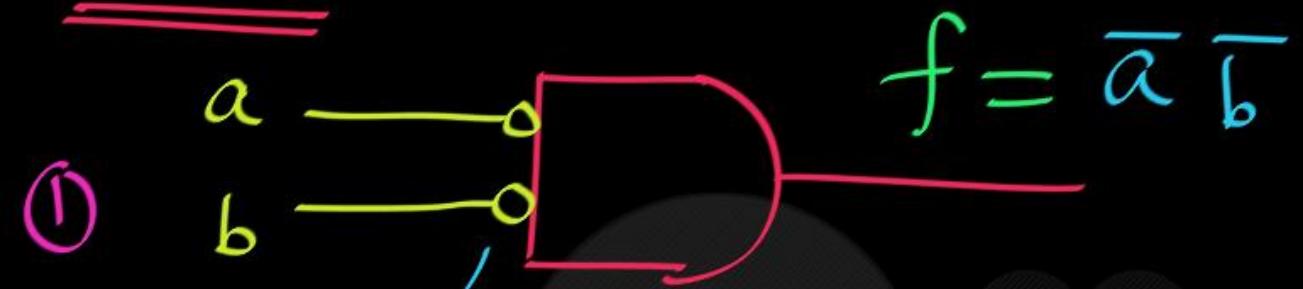
Remember:

$$\overline{X \cdot Y} \neq \overline{X} \cdot \overline{Y}$$

$$\overline{X + Y} \neq \overline{X} + \overline{Y}$$



Note:

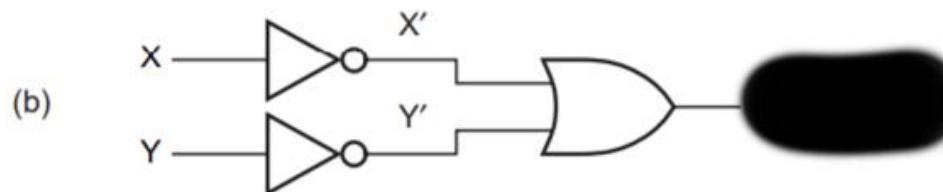
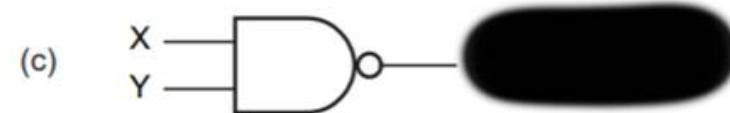
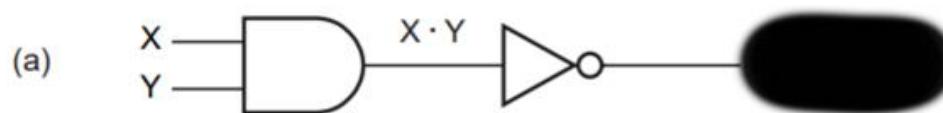


bubble \equiv NOT



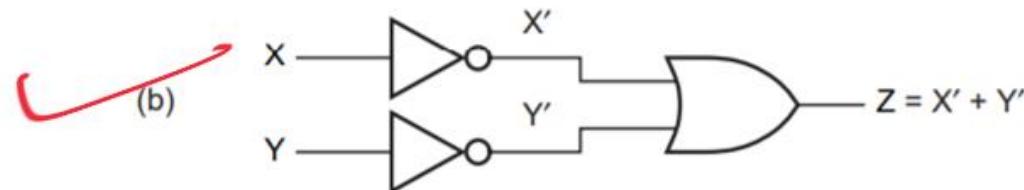
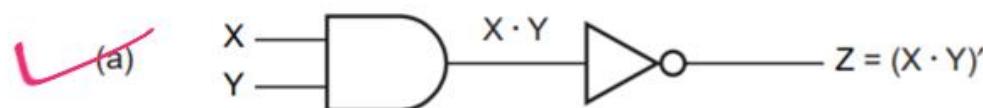
DeMorgan's theorems

$$(X \cdot Y)' = X' + Y'$$

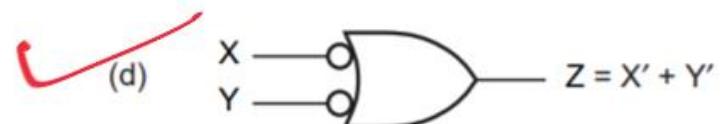


$$\overline{(xy)} = \bar{x} + \bar{y}$$

$$\underline{(X \cdot Y)'} = X' + Y':$$

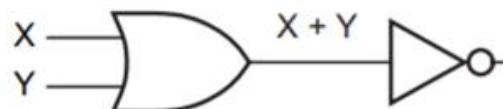


NAND = NOT of AND

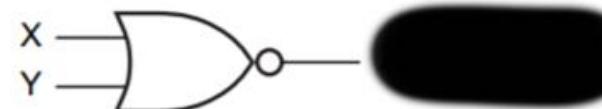


$$(X + Y)' = X' \cdot Y'$$

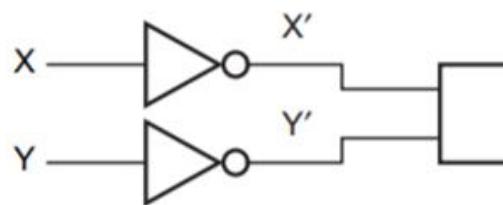
(a)



(c)



(b)

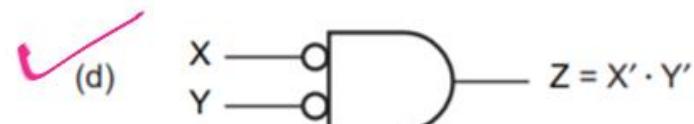
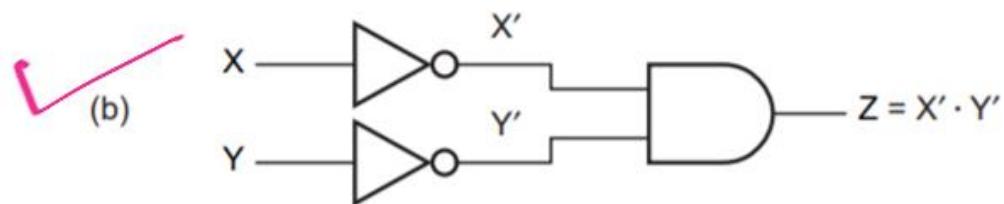
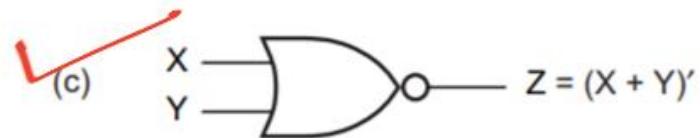
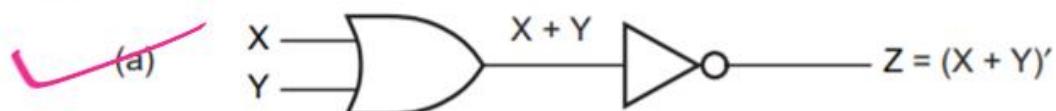


(d)



$$\text{NOR} = \text{NOT of OR} = \overline{x+y} = \bar{x} \bar{y}$$

$$\underline{\underline{(X+Y)'}} = X' \cdot Y':$$



NAND

Standard

$$X = \overline{A \cdot B}$$



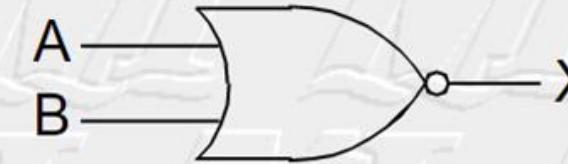
AND

$$X = A \cdot B$$



NOR

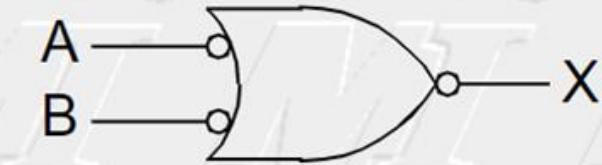
$$X = \overline{A + B}$$

**DeMorgan's**

$$X = \overline{\overline{A} + \overline{B}}$$



$$X = \overline{\overline{A} \cdot \overline{B}}$$



$$X = \overline{\overline{A} \cdot \overline{B}}$$



OR

$$X = A + B$$



$$X = \overline{\overline{A}} \cdot \overline{\overline{B}}$$



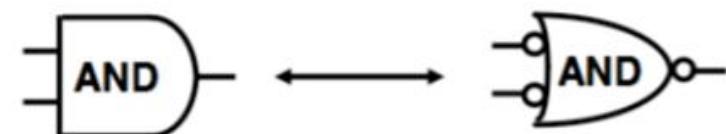
$$A + B$$

$$\overline{\overline{A}} \cdot \overline{\overline{B}}$$

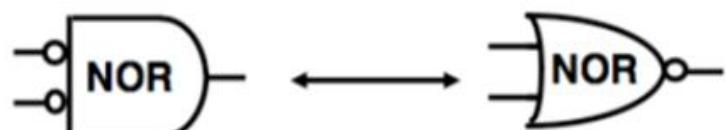
$$\overline{X} + \overline{Y} = \overline{XY}$$



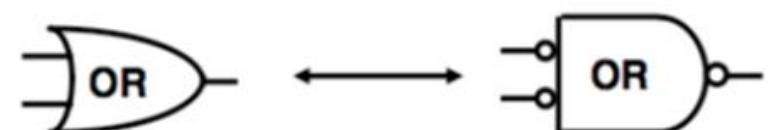
$$XY = \overline{\overline{X} + \overline{Y}}$$



$$\overline{XY} = \overline{X} + \overline{Y}$$



$$X + Y = \overline{\overline{XY}}$$



$$\underline{a = a + a = a + 0 = a \oplus 0 = a \odot 1}$$

$$\underline{a' = a' + \bar{a} = \bar{a} \bar{a} = \bar{a} + 0 = a \oplus 1 = a \odot 0}$$

$$\underline{a \oplus b = a' \oplus b' = \overline{a \odot b} = a'b + a\bar{b}}$$

$$\underline{0 = a \cdot 0 = a\bar{a} = a \oplus a = a \odot \bar{a}}$$

<u>$f(a, b)$</u>		f
a	b	
0	0	0/1
0	1	0/1
1	0	0/1
1	1	0/1

for every Row (input combination) we have 2 choices for $f(a, b)$



Q:

Number of Different Boolean Functions on 1 Boolean variables?



Q:

$f(a)$ ✓
 Number of Different Boolean Functions on 1 Boolean variables?

a	f_0 ✓	f_1	f_2	f_3
{ 0	0	0	1	1
1 } + 0	1	1	0	1

Ans: 4

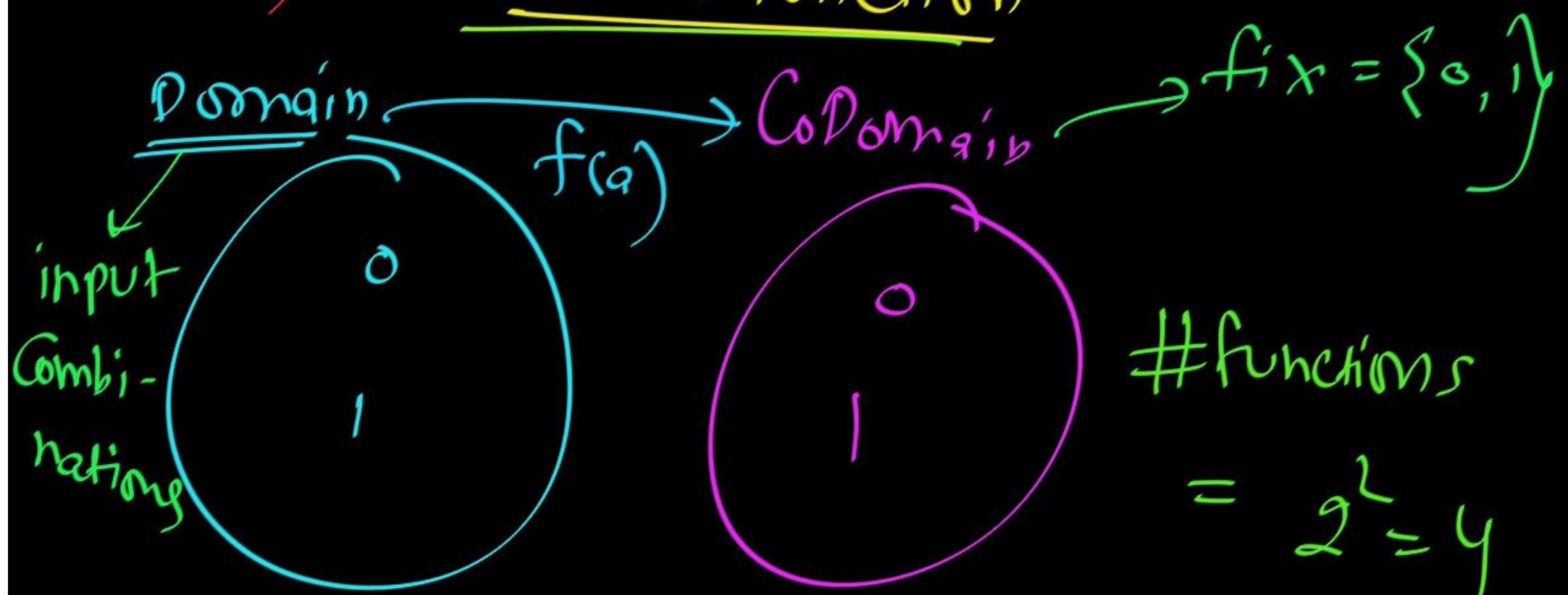
$$\begin{aligned}
 f_0 &= 0 \\
 f_1 &= a \\
 f_2 &= \bar{a} \\
 f_3 &= 1
 \end{aligned}
 \quad \left. \begin{array}{l} \text{Diff} \\ \parallel \end{array} \right\}$$

$$a \oplus (a + q) = a \oplus a = 0 = f_0$$

$$a \odot (a \oplus \bar{q}) = a \odot 1 = a = f_1$$

$$\underline{\underline{(a \oplus 1)}} \odot \underline{\underline{(\bar{q} + 1)}} = \bar{q} = f_2$$
$$q + \bar{q} = 1 = f_3 \quad \boxed{\underline{\underline{q\bar{q}} = 0 = f_0}}$$

$f(a) = \underline{\text{Boolean function}}$





Q:

Number of Different Boolean Functions on 0 Boolean variables?



Q:

Number of Different Boolean Functions on 0 Boolean variables?

Q. \rightarrow Constant Boolean functions

$$\begin{aligned}f_0 &= 0 \\f_1 &= 1\end{aligned}$$



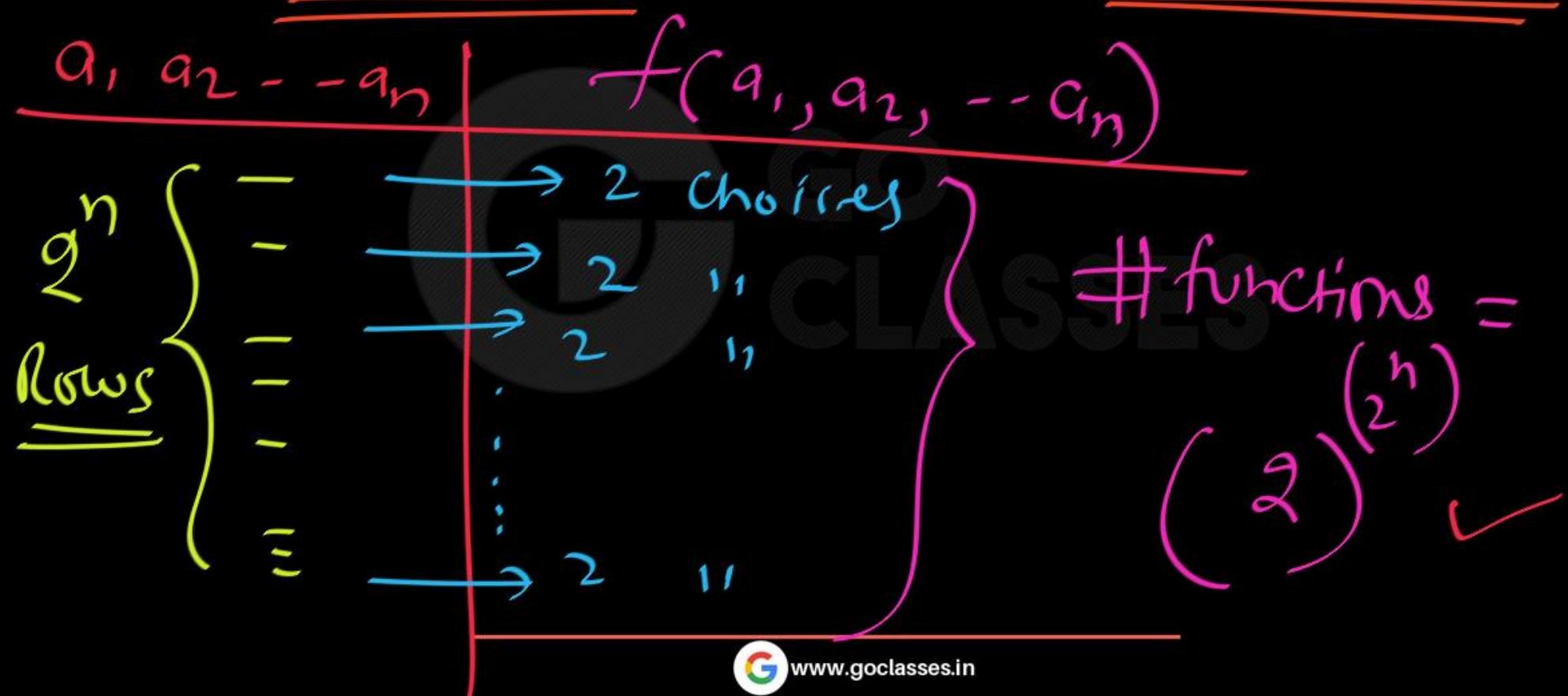
Q:

Number of Different Boolean Functions on n Boolean variables? ✓



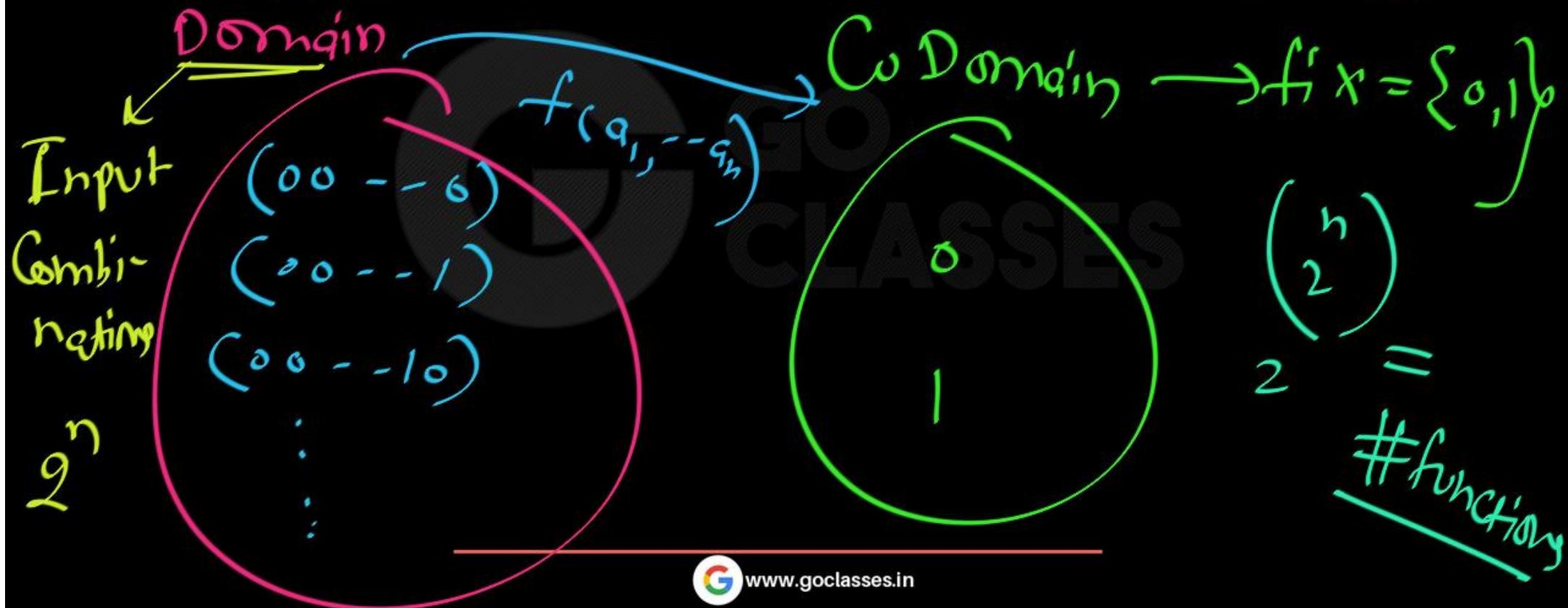
Q:

Number of Different Boolean Functions on n Boolean variables?



Q:

Number of Different Boolean Functions on n Boolean variables?



On n boolean variables ;

$$\# \text{Boolean functions} = 2^{(2^n)}$$



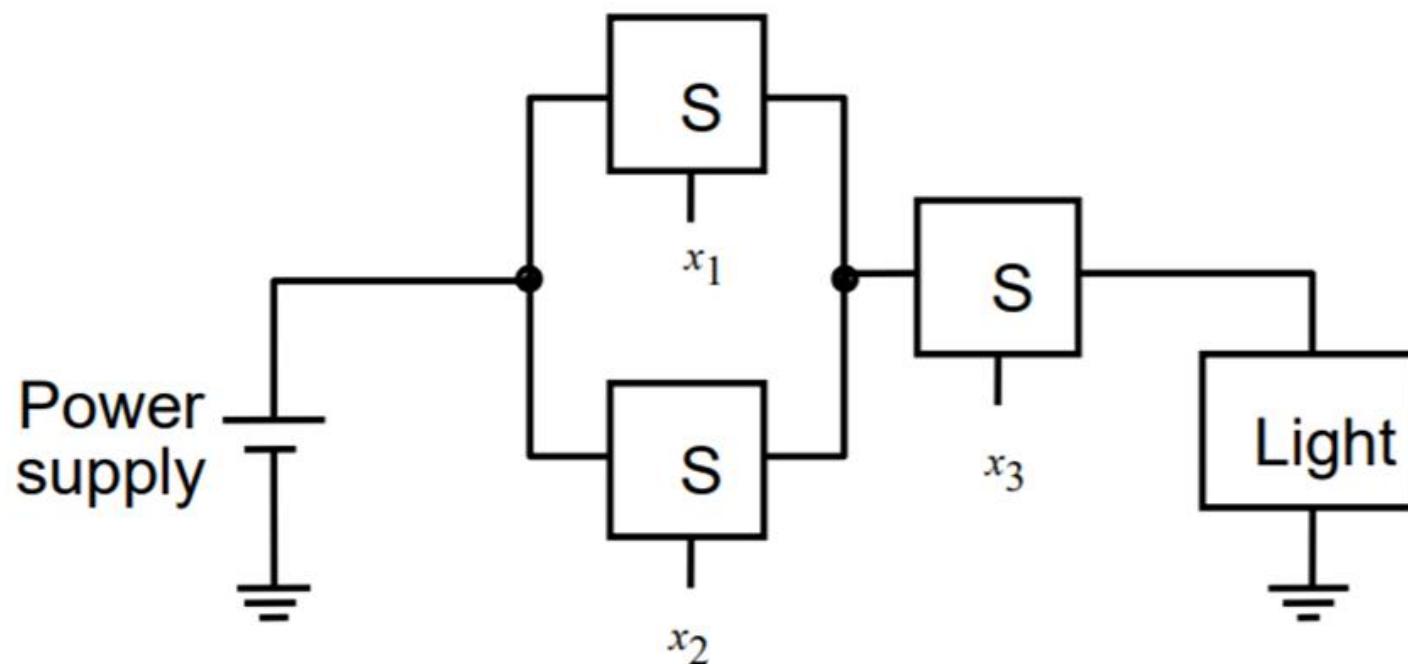
Boolean Algebra :

Next Topic :

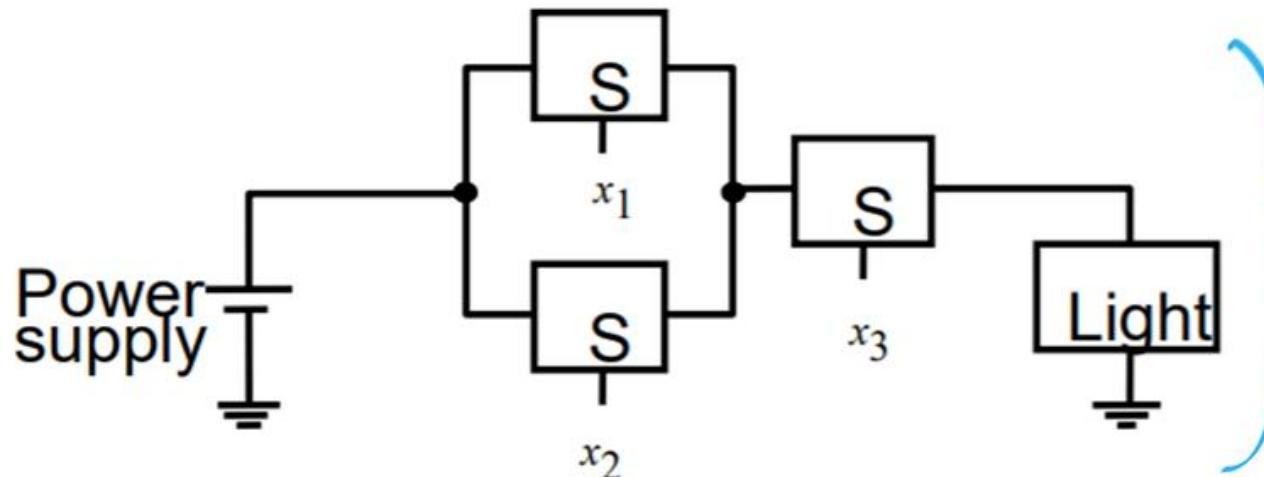
Switches, Boolean Expressions &

Logic gate circuit implementation

A Series Parallel Circuit



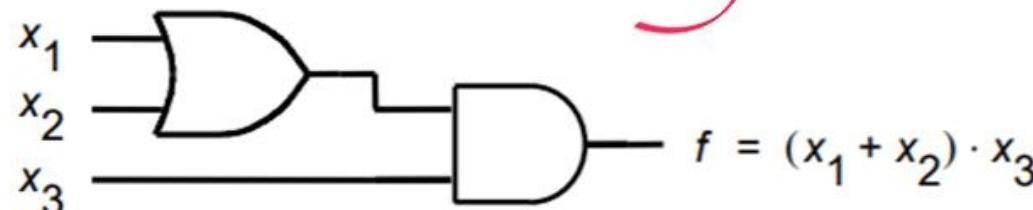
Boolean Functions



Physical circuit

$$\begin{aligned}f &= (x_1 \text{ OR } x_2) \text{ AND } (x_3) \\&= (x_1 + x_2) \cdot x_3\end{aligned}$$

logical behaviour

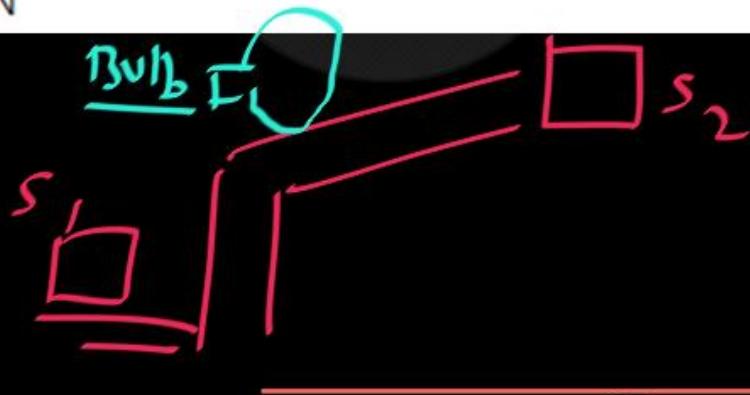


logic gate implementation



Design Problem 1

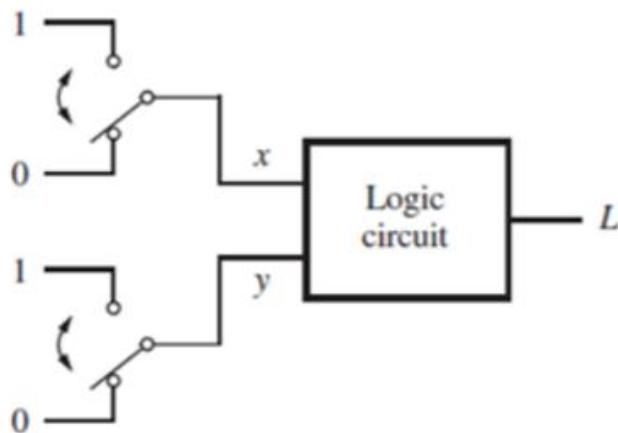
- Design a circuit for staircase light control using a two-way switch
- If both switches are OFF or ON, light is off
- If one of them is ON and another is OFF, the light is ON





Design Problem 1

- Design a circuit for staircase light control using a two-way switch
- If both switches are OFF or ON, light is off
- If one of them is ON and another is OFF, the light is ON



(a) Two switches that control a light

x	y	L
0	0	0
0	1	1
1	0	1
1	1	0

(b) Truth table



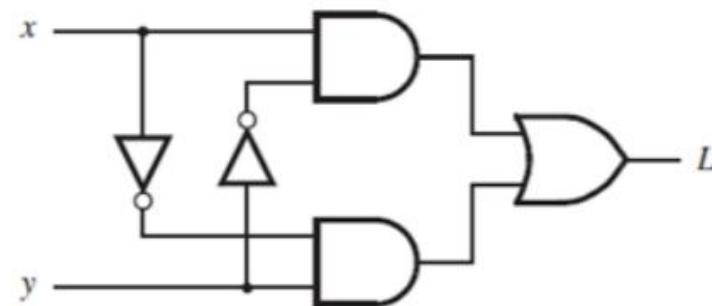


Boolean Expression

- Lamp is ON when
 - Either x is ON and y is OFF
 - Or x is OFF and y is ON

Boolean Expression

- Lamp is ON when
 - Either x is ON and y is OFF
 - Or x is OFF and y is ON
- $L = x \cdot y' + x' \cdot y$



(c) Logic network



(d) XOR gate symbol

Design Problem 2

- Design a circuit which realizes the following truth table

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

$f = ?$



Design Problem 2

- Design a circuit which realizes the following truth table

$$\begin{array}{l} n=0 \Rightarrow \bar{n} \\ n=1 \Rightarrow n \end{array}$$

Next
lecture

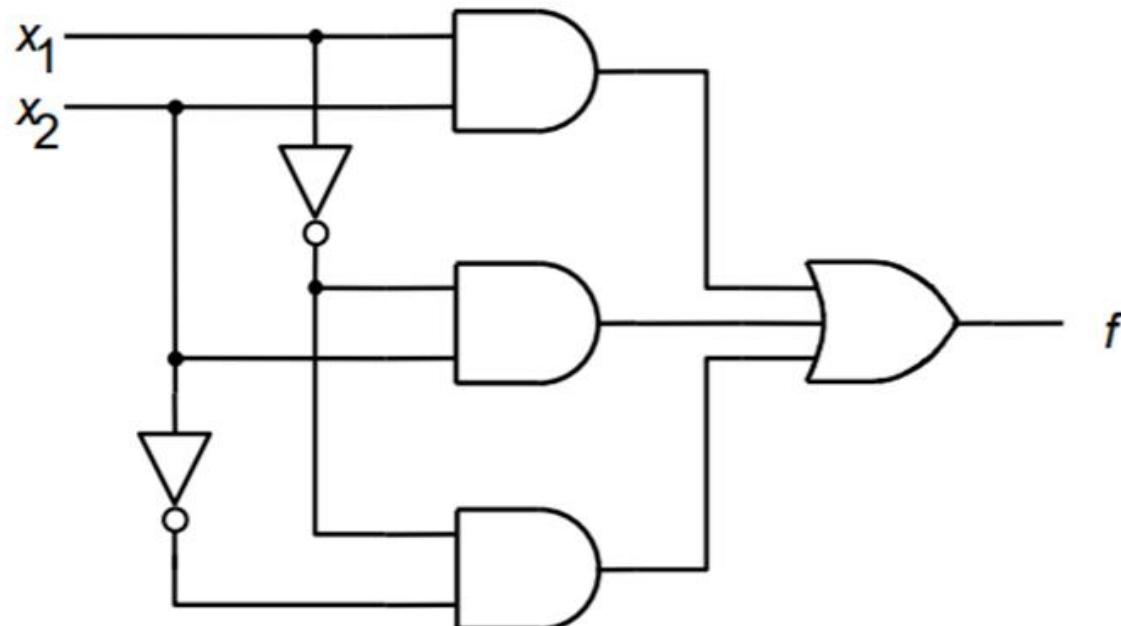
x_1	x_2	$f(x_1, x_2)$
0	0	1 ✓
0	1	1 ✓
1	0	0
1	1	1 ✓

$$f = ? = \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + x_1 \bar{x}_2$$



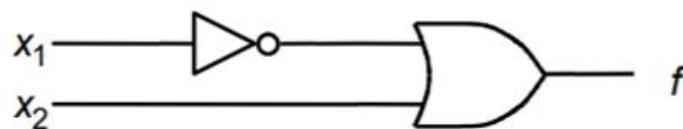
Design Problem 2 (contd.)

$$\blacksquare f = x_1' \cdot x_2' + x_1' \cdot x_2 + x_1 \cdot x_2$$



Power of Simplification

- $$\begin{aligned} f &= x_1' \cdot x_2' + x_1' \cdot x_2 + x_1 \cdot x_2 \\ &= x_1' \cdot x_2' + x_1' \cdot x_2 + x_1' \cdot x_2 + x_1 \cdot x_2 \\ &= x_1' \cdot (x_2' + x_2) + (x_1' + x_1) \cdot x_2 \\ &= x_1' \cdot 1 + 1 \cdot x_2 \\ &= x_1' + x_2 \end{aligned}$$



Minimal-cost realization

