



Digital Logic :

Next Topic :

Decimal & Binary Number Systems

Website : <https://www.goclasses.in/>



Number System:

A System of Counting

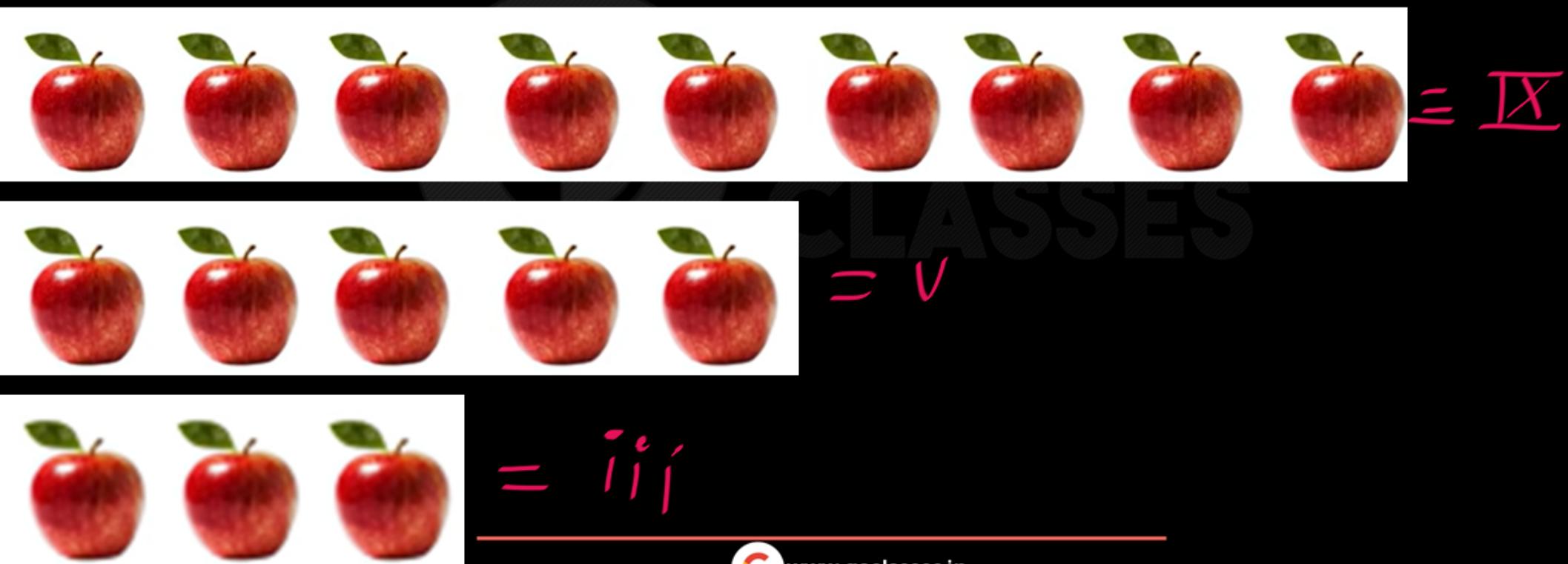
Number systems are simply ways to count things.

Or ways to represent numbers.



Number System:

o. Roman System





Number System:

1. Unary System

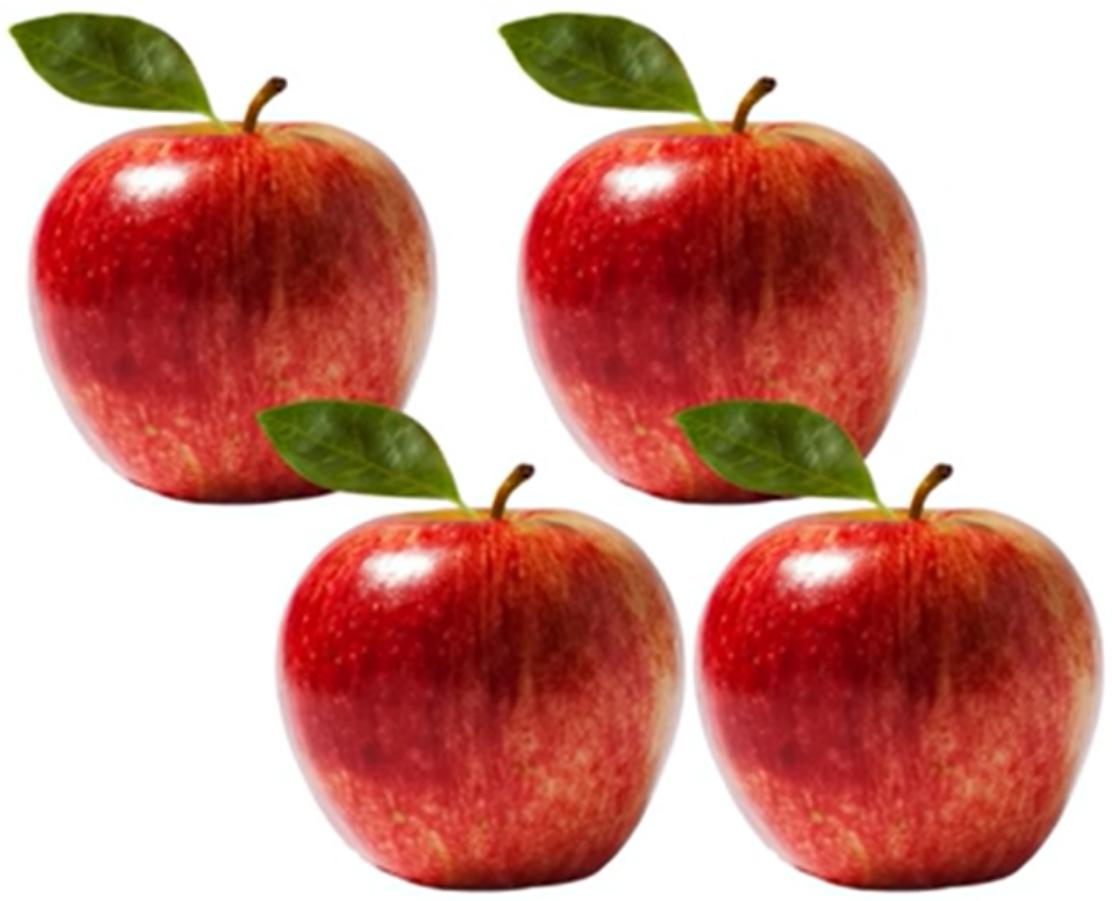
How small babies count?

3 = "|||"

One Symbol = {1}



Digital Logic





Digital Logic

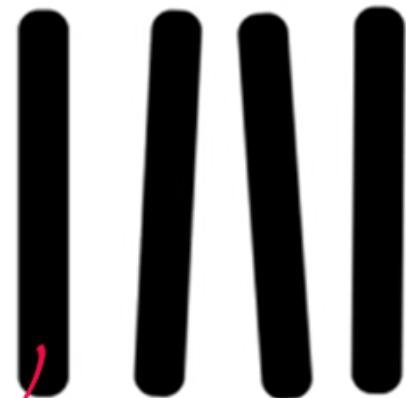
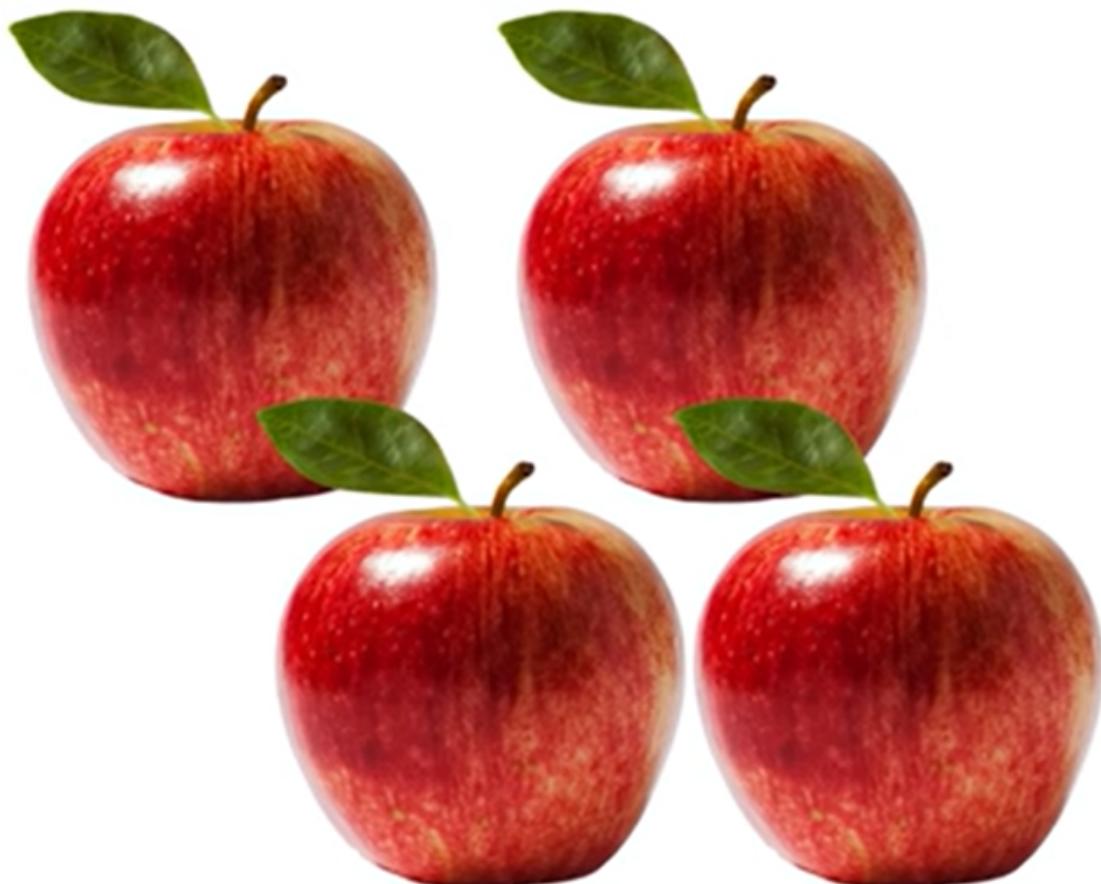


= //

ASSES



Digital Logic



Symbol



Number System:

2. Decimal System

How a normal human counts?

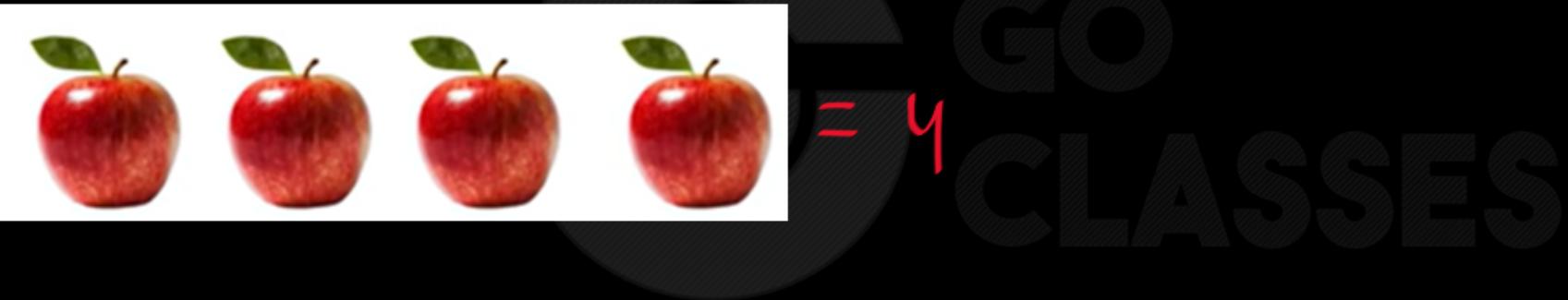
10 symbols (0,1,2,3,...,9)

- Decimal system uses 10 symbols (**digits**)
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- The **decimal system** is the number system that we use everyday



Digital Logic



9 =



8 =



7 =



6 =



5 =



4 =





Decimal system:

Symbols: 0, 1, 2, ..., 9 ✓
↳ Digits





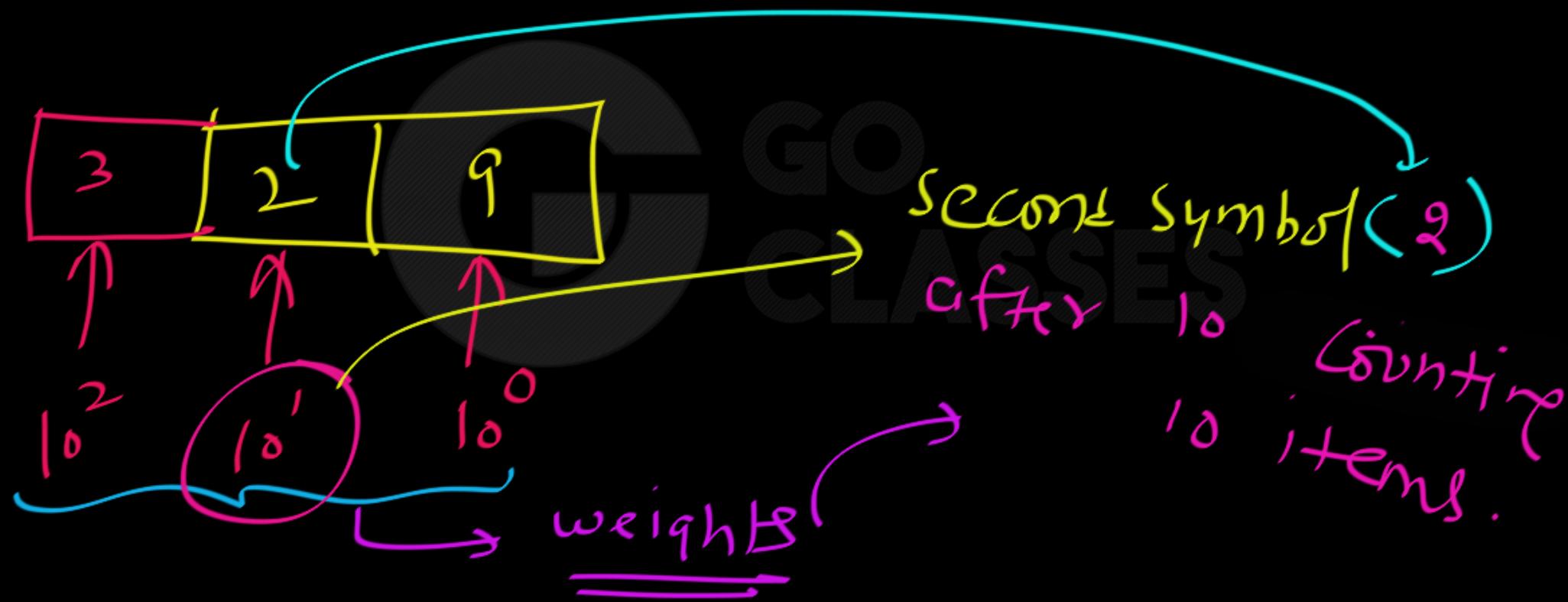
The base of the number system is defined as the total number of symbols(digits) available in the number system.

Decimal system \Rightarrow Base=10

The base value of a number system is the number of different values the set has before repeating itself. For example, decimal has a base of ten values, 0 to 9.

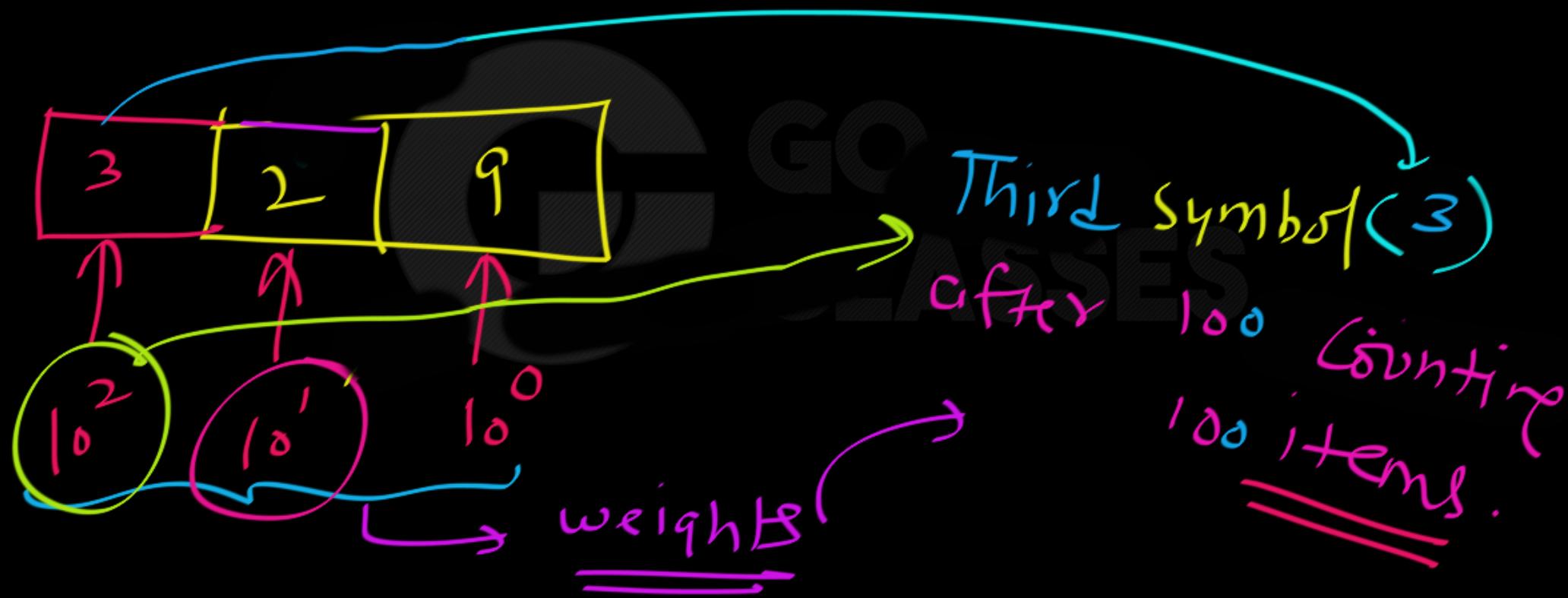


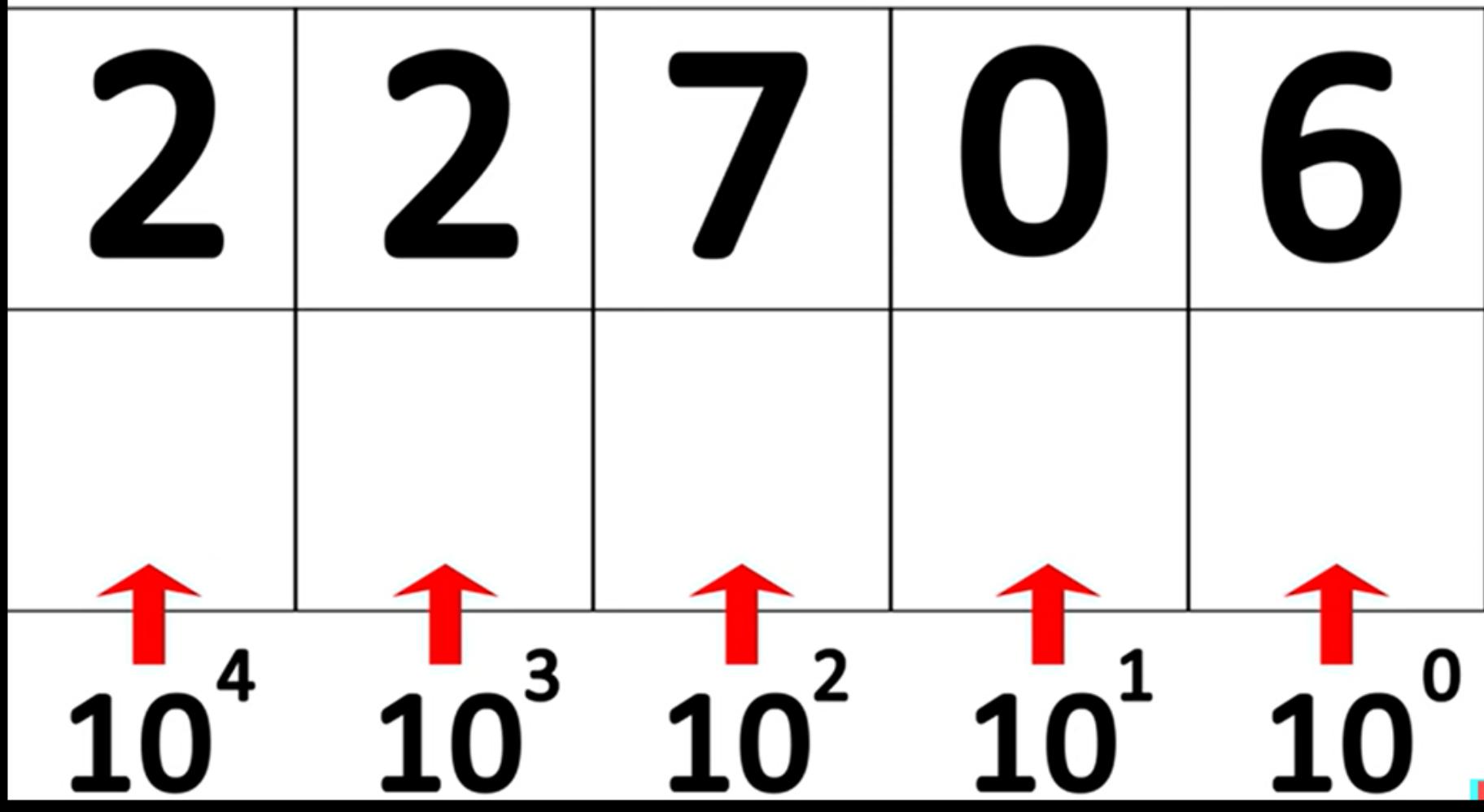
$$\underbrace{3 \ 2 \ 9}_{\text{in base 10}} = 3 \times 10^2 + 2 \times 10 + 9 \times 10^0$$





$$\underbrace{3 \ 2 \ 9}_{\text{in base 10}} = 3 \times 10^2 + 2 \times 10 + 9 \times 10^0$$







$$(22706)_{10} = 2 \times 10^4 + 2 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 6 \times 10^0$$

10 → tell us the number system

$$= \underline{\underline{22706}}$$

$$\left(\begin{array}{c} 1000 \\ \hline 10 \end{array}\right) = \frac{1 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 0 \times 10^0}{\hline}$$
$$= \underline{\underline{1000}}$$



(a b c d e)₁₀ → Decimal system

$$= e \times 10^0 + d \times 10^1 + c \times 10^2 + b \times 10^3 + a \times 10^4$$

2 (5) 6

→ second symbol we are using
after counting 10 items.

→ 5 tells us How many 10's we have counted? $\Rightarrow 5 \checkmark$

$$\begin{array}{r} \textcircled{22} \\ + \quad 9 \\ \hline 10 \\ \hline 21 \end{array}$$

A hand-drawn addition problem. It starts with a circled '22' at the top left. A horizontal line connects it to a vertical '+' sign. Below the '+' is a '9'. To the right of the '9' is a '10' above a vertical line. A curved arrow points from the '22' down to the '10'. Another curved arrow points from the '9' down to the '10'. To the right of the '10' is an equals sign '='. Below the '10' is a '19' above another vertical line. A curly brace is positioned under the '9' and '19', spanning both vertical lines. Below this brace, the word 'tens' is written in red, with a red underline underneath it.

$$1 \text{ } 2 \text{ } 5 \text{ } 5 =$$

How many loops we have
Counted } $= 2$

How many loops we have
Counted ? $= 1$

$$1 \ 2 \ 5 \ 5 = \frac{1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 5 \times 10^0}{10^3 \ 10^2 \ 10^1 \ 10^0}$$



Number System:

3. Binary System: The Computer Number System

How a computers counts?

2 symbols (0,1) (bits)



Binary

0
1

No symbol

1 0
1 1

Decimal

0

1

2

2

3

Binary

1 0 0

1 0 1

1 1 0

1 1 1

1 0 0 0

Decimal

4

5

6

7

8

Base Ten:
Greater than 9=New Digit

Binary:
Greater than 1=New Digit

**Binary
Number**

0

**Amount
of Things**

Binary Number

1

Amount of Things



Binary Number

10

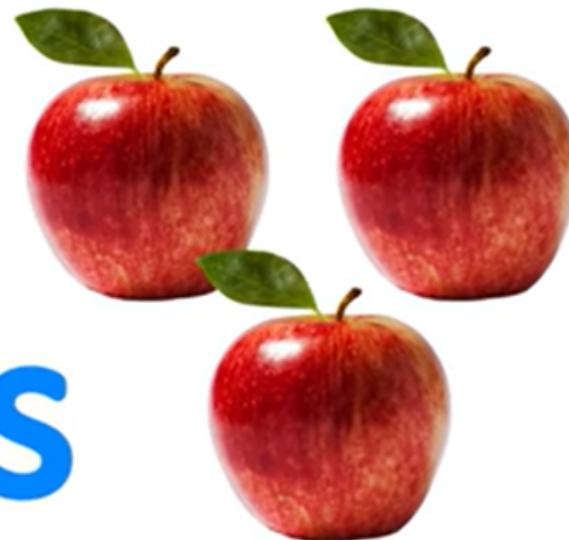
Amount of Things



Binary Number

11

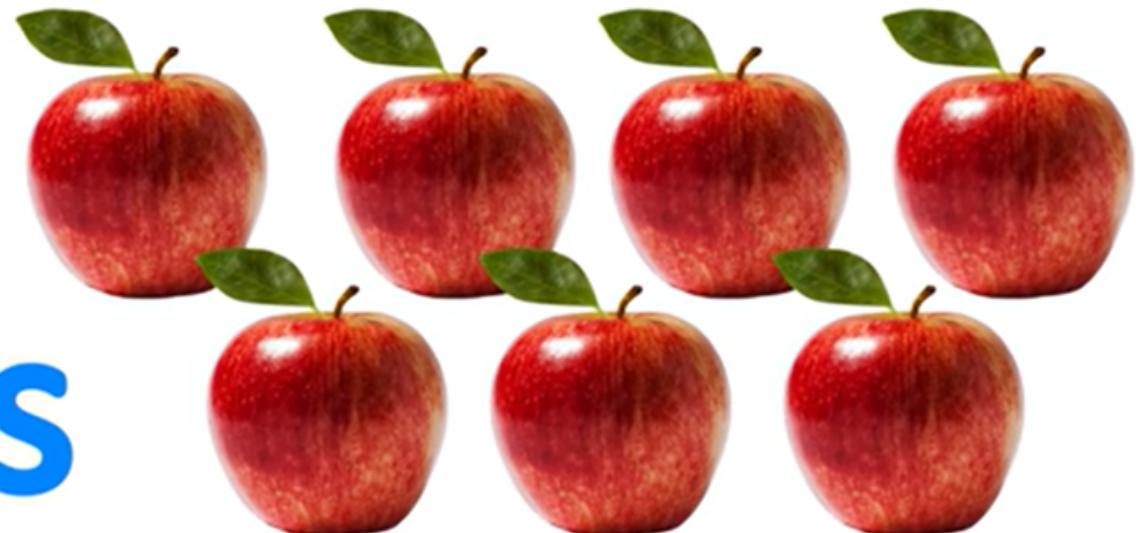
Amount
of Things



**Binary
Number**

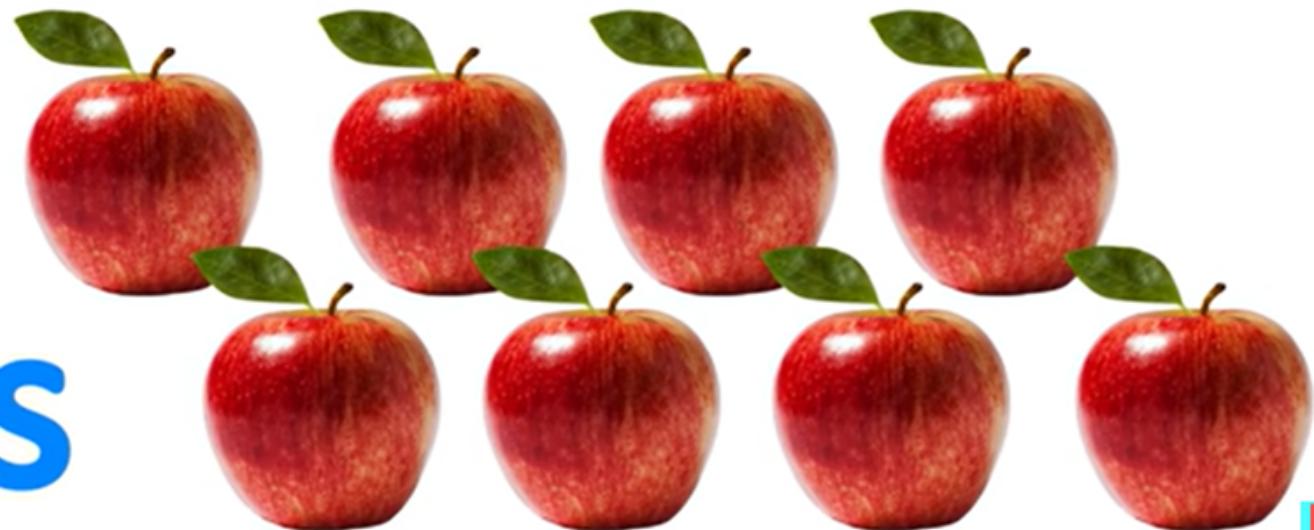
111

**Amount
of Things**



**Binary
Number 1000**

**Amount
of Things**





Binary System:

find out the weights

0 0 0 1

Second symbol used after
Counting 2 items.

4th symbol used after Counting 8 items.



Digital Logic

$$\left(\overline{a_2} \overline{a_1} \overline{a_0} \right)_2$$

↓ ↓ ↓
 $2^2 2^1 1$

A hand-drawn binary number $(\overline{a_2} \overline{a_1} \overline{a_0})_2$ is shown. Below it, a vertical stack of three arrows points downwards, labeled 2^2 , 2^1 , and 1 from bottom to top, indicating the value of each bit position.

1 0 1 1
↓ ↓ ↓ ↓
 $2^3 2^2 2^1 2^0$

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0

1

1

0

0

1

 2^5  2^4  2^3  2^2  2^1  2^0

$$(1010)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 8 + 2 + 1 = 11$$

tells us

Binary System

$$(11)_{10}$$

$$(11)_2 = 1 \times 2^1 + 1 \times 2^0 = (3)_{10}$$

Number Systems and Conversion

When we write decimal (base 10) numbers, we use a positional notation; each digit is multiplied by an appropriate power of 10 depending on its position in the number.
For example,

$$(22003)_{10} =$$

Diagram illustrating the conversion of the decimal number 22003 to its base 10 representation. The digits 2, 2, 0, 0, and 3 are shown above a horizontal line. Below the line, arrows point down to powers of 10: 10^4 , 10^3 , 10^2 , 10^1 , and 10^0 . The powers of 10 are written vertically below the line.

$$\underline{2 \times 10^4 + 2 \times 10^3 + 3 \times 10^0}$$



Number Systems and Conversion

Similarly, for binary (base 2) numbers, each binary digit is multiplied by the appropriate power of 2:

$$\begin{array}{r} (1 \ 0 \ 1)_2 \\ \times 2^2 \quad 2^1 \quad 2^0 \\ \hline = (5)_{10} \end{array}$$

$$= \underbrace{1 \times 2^2 + 1 \times 2^0}_{= 4 + 1} = 5 \checkmark$$



Digital Logic :

Next Sub-Topic :

Binary to Decimal Conversion

(Same as you read a decimal number - Easy Peasy)



$$(1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1)_2 = (?)_{10} ?$$

A green arrow points from the bottom left towards the binary number, and another green arrow points from the bottom right towards the decimal representation.

$$\begin{aligned} & 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 \\ &= 64 + 16 + 8 + 2 + 1 = 91 \text{ Applies} \end{aligned}$$

Binary numbers to Decimal Number

$$(N)_2 = (\underline{\underline{1}}\underline{\underline{1100110}})_2$$

2^6

decimal value is given by,

$$\begin{aligned}(N)_2 &= \underline{1 \times 2^7} + \underline{1 \times 2^6} + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 \\&\quad + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0\end{aligned}$$

$$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 = \underline{\underline{(230)}_{10}}$$

$$\underline{\underline{14}}_{10} = (?)_2 \checkmark$$

$$\begin{array}{r} 2 | 14 \\ \underline{-2} \quad 7 \\ \hline 2 | 7 \\ \underline{-2} \quad 3 \\ \hline 2 | 3 \\ \underline{-2} \quad 1 \\ \hline 0 \end{array}$$

remainder

A blue arrow points from the remainder '0' to the right side of the equation.

$$\underline{\underline{14}}_{10} = (?)_2$$

$\underline{\underline{14}}_{10} = (?)_2$

14 Apples



Digital Logic :

Next Sub-Topic :

Decimal to Binary Conversion

(Do you understand or have you by-hearted?)



Decimal —————→ Binary

Method : Successive Division by 2.

Write Remainders in Reverse Order.

$$(27)_{10} \rightarrow (?)_2 = (\underline{\underline{11011}})_2$$

remainder

2	27
2	13
2	6
2	3
2	1
	0

binary of 27.

$$(32)_{10} \rightarrow (?)_2 = (\underline{\underline{1000\ 00}})_2$$

Nemqin de 88

	32
2	16
2	8
2	4
2	2
2	1
2	0

binary of 32.

$$(a \underline{b} c \underline{d}) = (N)_{10}$$

$a, b, c, d \in \{0, 1\}$

$$N = [a \times 2^3 + b \times 2^2 + c \times 2^1 + d]$$

How to find "d"?

$$d = \underline{N \bmod 2}$$

$$N = a \times 2^3 + b \times 2^2 + c \times 2^1 + d$$

$$\left(\frac{N}{2} \right) \rightarrow \begin{array}{l} \text{Quotient } q_1 = \underline{\underline{ax^2 + bx^1 + c}} \\ \text{Remainder } r_1 = \underline{\underline{d}} \end{array}$$



$$(a \ b \ c \ d)_2 = (N)_{10}$$

$$N = \boxed{a \times 2^3 + b \times 2^2 + c \times 2^1 + d}$$

find c_1

$$c = Q_1 \text{ mod } 2$$

$$\underline{\underline{Q_1}} = \text{Quotient}\left(\frac{N}{2}\right)$$

$$Q_1 = \boxed{ax^2 + bx^1 + c}$$

$$\frac{Q_1}{2} \rightarrow \text{Quotient } Q_2 = ax^2 + b$$
$$\downarrow \text{Remainder } R_2 = c$$

$$Q_2 = a \times 2 + b$$

find b:

$$\underline{b = Q_2 - m \underline{Q_2}}$$



This process is continued until we finally obtain a_n . Note that the remainder obtained at each division step is one of the desired digits and the least significant digit is obtained first.



$$\underline{\underline{(27)_{10}}} = ?_2 = \underline{\underline{(q_4 q_3 q_2 q_1 q_0)}}$$

$$\begin{array}{r}
 2 \Big| 27 = N \\
 \hline
 2 \Big| 13 = Q_1 \\
 \hline
 2 \Big| 6 = Q_2 \\
 \hline
 2 \Big| 3 = Q_3 \\
 \hline
 2 \Big| 1 \quad q_4 \\
 \hline
 0 \quad q_5
 \end{array}$$

remainder

$$R_1 = \underline{\underline{1}} = q_0$$

$$R_2 = \underline{\underline{1}} = q_1$$

$$R_3 = \underline{\underline{0}} = q_2$$

$$R_4 = \underline{\underline{1}} = q_3$$

$$R_5 = \underline{\underline{1}} = q_4$$

$$(11011)_2$$

$$\begin{array}{c}
 q_0 \\
 \uparrow \\
 q_1 \\
 \uparrow \\
 q_2 \\
 \uparrow \\
 q_3 \\
 \uparrow \\
 q_4
 \end{array}$$

$$\begin{array}{r}
 & N = 44 \\
 \hline
 2 & | 44 \\
 \hline
 2 & | 22 Q_1 \\
 \hline
 2 & | 11 Q_2 \\
 \hline
 2 & | 5 \quad Q_3 \\
 \hline
 2 & | 2 \quad Q_4 \\
 \hline
 2 & | 1 \quad Q_5 \\
 \hline
 0 & | 0 \quad Q_6
 \end{array}$$

Remainder

$$R_1 = 0 = a_0$$

$$R_2 = 0 = a_1$$

$$R_3 = 1 = a_2$$

$$R_4 = 1 = a_3$$

$$R_5 = 0 = a_4$$

$$R_6 = 1 = a_5$$

$$\begin{aligned}
 (N = 44) &= (Q_5 \underline{Q_4} \underline{Q_3} \underline{Q_2} \underline{Q_1} \underline{Q_0})_{10} \\
 &= (101100)_2
 \end{aligned}$$



Convert 53_{10} to binary.

$$(a_5, a_4, a_3, a_2, a_1, a_0)$$

2 | 53
 2 | 26
 2 | 13
 2 | 6
 2 | 3
 2 | 1
 0

2

↓ ↓ ↓ ↓ ↓

1 1 0 1 0 1

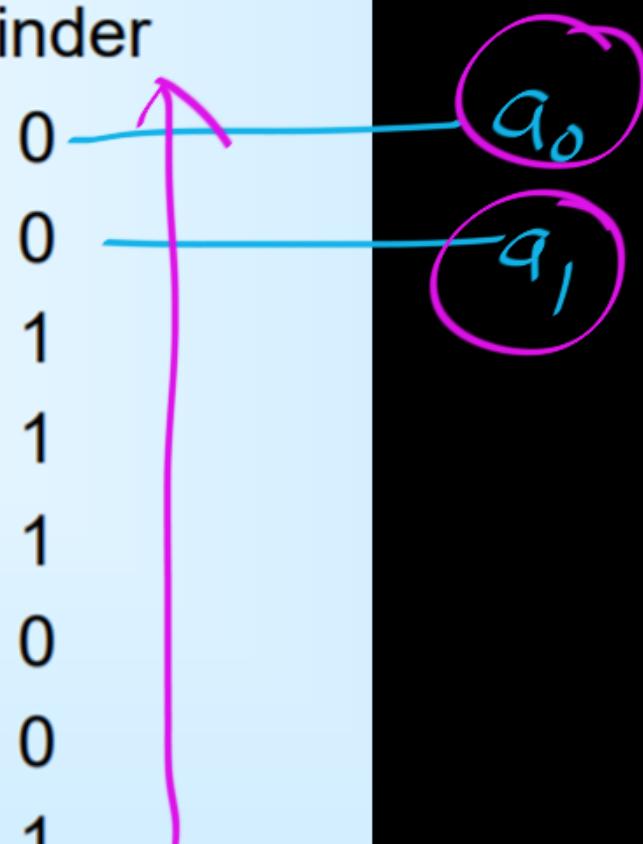
N

$Q_1 \quad Q_2$

$$\begin{array}{r}
 2 \overline{)53} \\
 2 \overline{)26} \qquad \text{rem.} = \underline{1} = a_0 \checkmark \\
 2 \overline{)13} \qquad \text{rem.} = \underline{0} = a_1 \checkmark \\
 2 \overline{)6} \qquad \text{rem.} = \underline{1} = a_2 \qquad 53_{10} = 110101_2 \\
 2 \overline{)3} \qquad \text{rem.} = \underline{0} = a_3 \\
 2 \overline{)1} \qquad \text{rem.} = \underline{1} = a_4 \\
 0 \qquad \text{rem.} = \underline{1} = a_5
 \end{array}$$

	Quotient	Remainder
$N=156 \div 2$	78	
$78 \div 2$	39	
$39 \div 2$	19	1
$19 \div 2$	9	1
$9 \div 2$	4	1
$4 \div 2$	2	0
$2 \div 2$	1	0
$1 \div 2$	0	1

$(156)_{10} = (\underline{\underline{10011100}})_2$ q_0



Binary:

$$(1101)_2$$

2^3 ←

The binary number $(1101)_2$ is shown. A green arrow points from the leftmost digit '1' to the label 2^3 , indicating its weight.

→ 4th symbol we use after counting & items.

Weight

$$= 2^1$$

= Why = We use second symbol after counting 2 items.



$$\underbrace{\square \square \square \square}_{6} = (5)_{10} = (101)_2$$

Apples

$$\begin{aligned} &= (V)_{\text{Roman}} = (1111)_{\text{Binary}} \\ &= (\text{FIVE})_{\text{English}} \end{aligned}$$

Summary - 2^2

$$(\overline{a} \overline{b} \overline{c} \overline{d} \overline{e})_2 = (\quad)_{10}$$

Diagram illustrating the conversion of a binary number to its decimal equivalent:

- The binary number is $(\overline{a} \overline{b} \overline{c} \overline{d} \overline{e})_2$.
- The powers of 2 are indicated below the digits: $2^4, 2^3, 2^2, 2^1, 2^0$.
- The value of each digit is multiplied by its corresponding power of 2:
 - $\overline{a} \times 2^4$
 - $\overline{b} \times 2^3$
 - $\overline{c} \times 2^2$
 - $\overline{d} \times 2^1$
 - $\overline{e} \times 2^0$
- The result is the decimal value $(\quad)_{10}$.

$$\begin{aligned} &= \overline{a} \times 2^4 + \overline{b} \times 2^3 + \overline{c} \times 2^2 \\ &\quad + \overline{d} \times 2^1 + \overline{e} \times 2^0 \end{aligned}$$



Digital Logic :

Next Sub-Topic :

Decimal <---> Binary Conversion

(Do Quickly; Don't apply the previous methods)



① Remember Binary of 0—15.

Decimal

0
1
2
3
4
5

Binary

0
1
10
11
100
101

Decimal

6
7
8
9
10

Binary

110
111
1000
1001
1010



① Remember Binary of 0—15.

Decimal

11

12

13

14

15

Binary

1011

1100

1101

1110

1111

Decimal

6

7

8

9

10

Binary

110

111

1000

1001

1010



② Power of 2

Decimal

$$2^0 = 1$$

$$\frac{1}{2} = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

Binary

1

10

100

1000

$$(2^4)_{10} = (10000)_2$$

$$(2^n)_{10} = (1 \underbrace{00 \dots 0}_{n \text{ zeros}})_2$$

$$(2^{10})_{10} = (\underline{1024})_{10} = (\underline{\underline{100000\ 00000}})_2$$

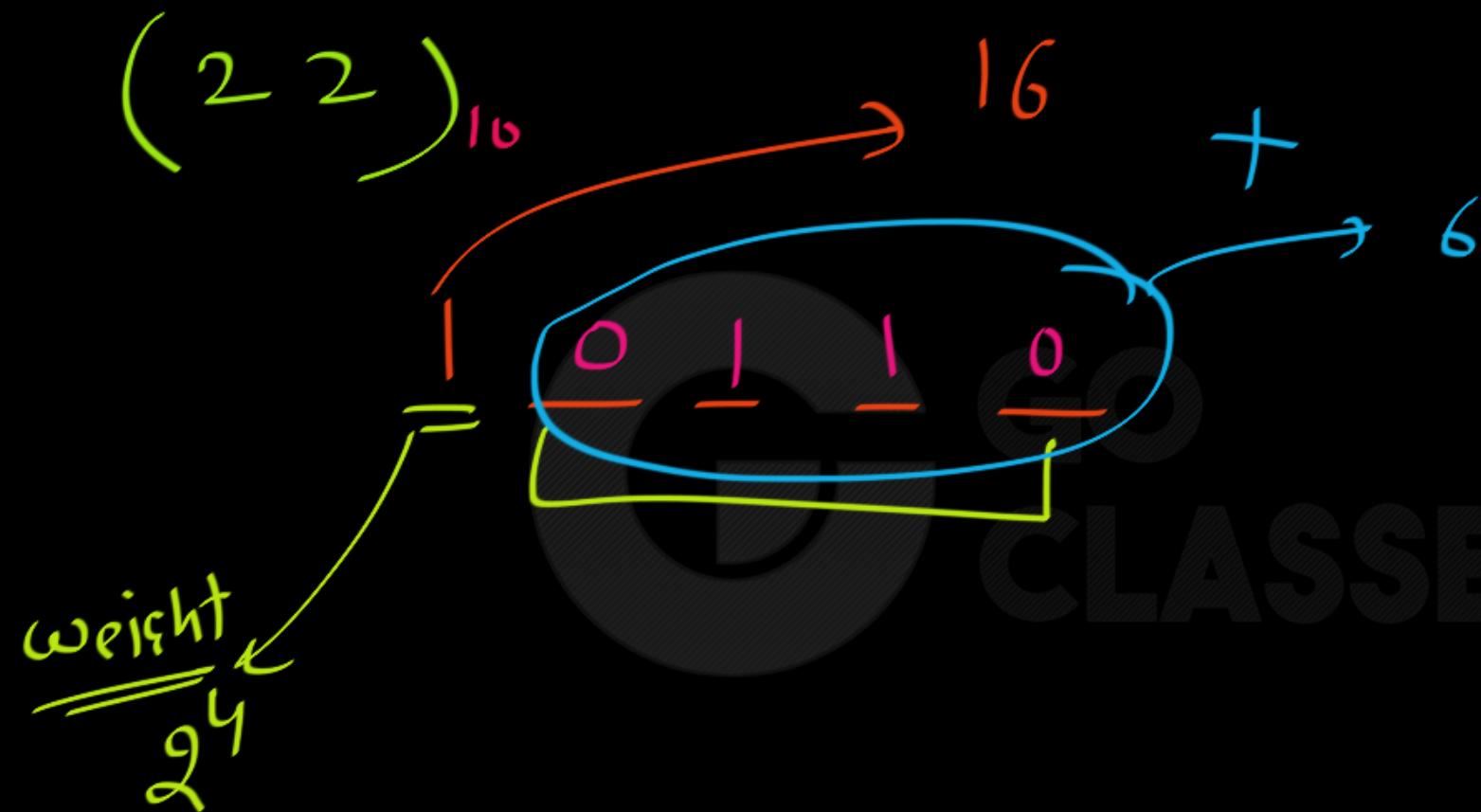


③ $(\underline{\underline{17}})_{10} = (?)_2$

$2^4 + 1$

$\Rightarrow 16 + 1$

$= \underline{\underline{0 \ 0 \ 0 \ 1}}$



$(\underline{\underline{80}})_{10} = 64 + \underline{\underline{16}} = 80$

$\underline{\underline{2^7}} = 128$

Space



$$(11)_{10} = 8 + 3$$

weight
 2^3

$$\begin{array}{r} (\overline{1 \ 6 \ 5})_{10} = \\ \text{---} \\ \begin{array}{c} 1 \ 0 \\ | \\ 0 \ 0 \ 1 \ 0 \ 1 \\ | \quad | \\ 5 \ 5 \text{ space} \end{array} \\ \begin{array}{c} 128 + \\ 32 \\ \hline 160 \end{array} \end{array}$$

$2^7 = 128$

$= (10100101)_2$



$$(5_{10})_{2} = 1 \begin{array}{r} 0 \\ - 0 \\ \hline 0 \end{array} + 1 \begin{array}{r} 1 \\ - 0 \\ \hline 1 \end{array}$$

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$$\underline{\underline{5/2 = 2}}$$

9 SPACES



$$y_0 = \overline{1} \underline{0} \underline{1} \underline{0} \underline{0} \underline{0}$$

Diagram showing the binary representation of 40: 101000. A red arrow labeled '32' points from the second digit to the bottom, and another red arrow labeled '8' points from the fourth digit to the bottom.

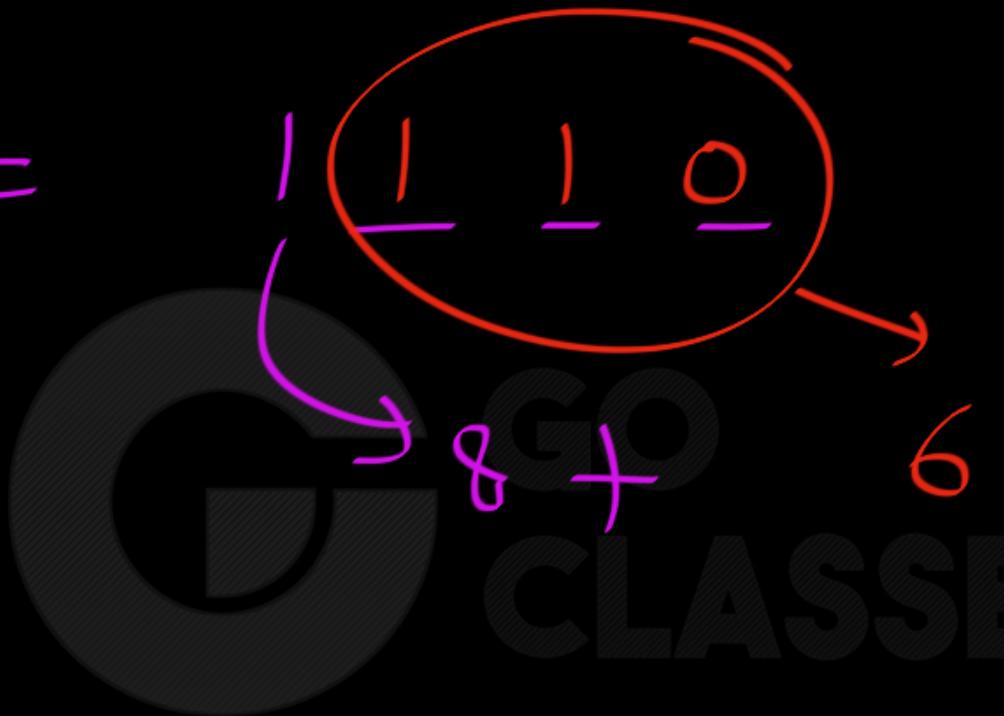
$$y_1 = \overline{1} \underline{0} \underline{1} \underline{0} \underline{0} \underline{1}$$

Diagram showing the binary representation of 41: 101001. A red arrow labeled '32' points from the second digit to the bottom, and another red arrow labeled '1' points from the fifth digit to the bottom.



Digital Logic

(14)
10





Bit Permutations

<u>1 bit</u>	<u>2 bits</u>	<u>3 bits</u>	<u>4 bits</u>	
0	00	000	0000	1000
1	01	001	0001	1001
	10	010	0010	1010
	11	011	0011	1011
		100	0100	1100
		101	0101	1101
		110	0110	1110
		111	0111	1111

Each additional bit doubles the number of possible permutations



0

|

$$8 = \underline{\underline{1000}} \quad \text{milestone} \checkmark$$

$$9 = \underline{1001}$$

$$\begin{array}{r} \underline{\underline{10}} = \underline{\underline{10}} \underline{\underline{10}} = \text{milestone} \checkmark \\ 11 = \underline{10} \underline{11} \end{array}$$



12 = 1100 — milestone

13 = 1101

14 = 1110

15 = 1111 — milestone

16 = 10000 — 11

17 = 10001 ↗



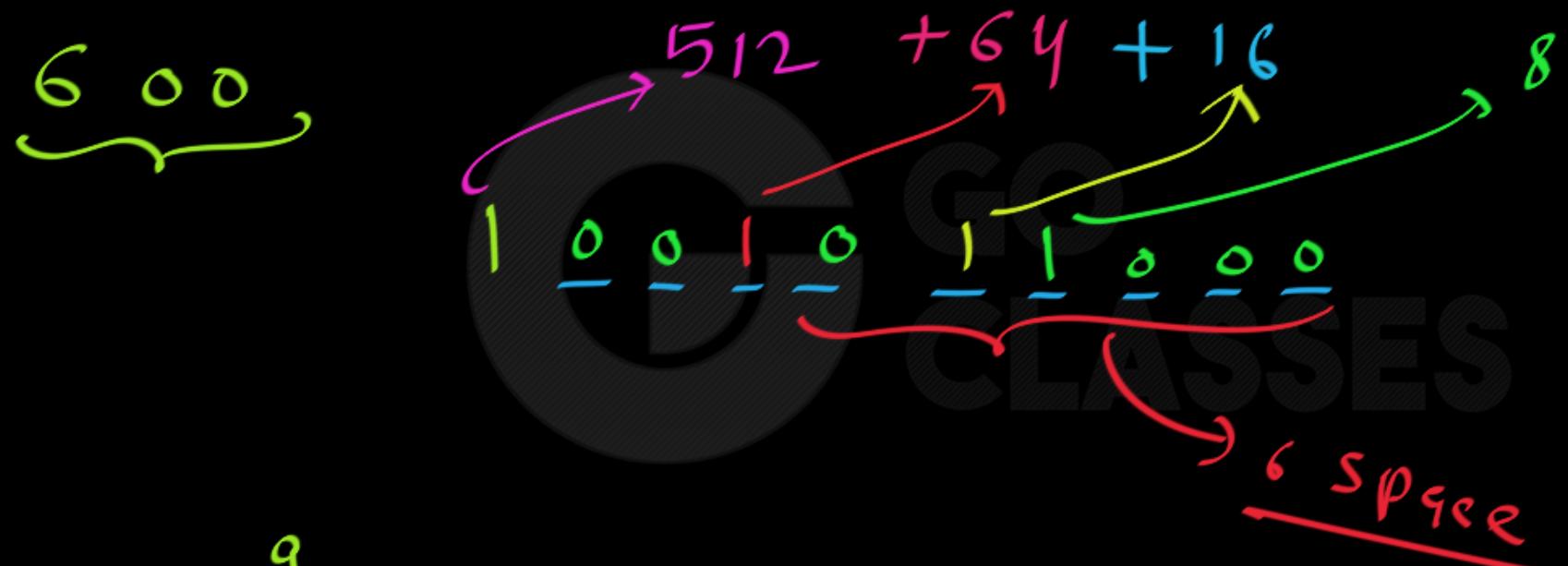
Digital Logic :

Recap :

Decimal & Binary Number Systems

Website : <https://www.goclasses.in/>

$$(600)_{10} = (?)_2 = (1001011000)_2$$



$$\underbrace{512 = 2^9}_{\text{;}} ; 2^6 = 64$$

$$(194)_{10} = (?)_2 = (11000010)_2$$

128 + 64

1 0 0 0 0 0 1 0
7 Spice

$$128 = 2^7$$

$$(128)_{10} = (1000\ 0000)_2$$

$$(250)_{10} = (1111\ 1010)_2$$

↓
lower
closest

$$128 = 2^7$$

$$256 = 2^8$$

$$\begin{array}{ccccccccc} & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ \downarrow & & & & & & & & \\ 128 & + & 64 & + & 32 & + & 16 & & \\ & & & & & & & & \end{array}$$

Not the lower closest

$$\begin{array}{r} \cancel{2^3} = 8 = 1 \ 000 \\ \cancel{2^4} = 16 = 1 \ 0000 \end{array} \quad \left| \begin{array}{l} 2^3 - 1 = 7 = 111 \\ 2^4 - 1 = 15 = 1111 \end{array} \right.$$

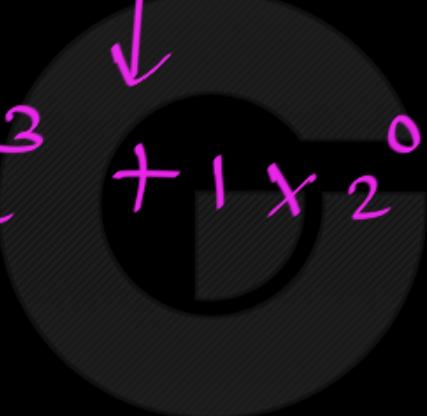
$$2^n = 1 \underbrace{000 \dots 0}_{n \text{ zeros}} - 0$$

$$2^n - 1 = \underbrace{111 \dots 1}_{n \text{ ones}}$$



$$(11011\overline{001})_2 = (?)_{10}$$

1 $x_2^6 + 1x_2^4 + 1x_2^3 + 1x_2^0$
1 x_2^7



$$\begin{aligned} &= 128 + 64 + 16 + 8 + 1 \\ &= \text{GO CLASSES} \end{aligned}$$



$$(1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0)_2 = (?)_{10}$$

↓ ↓ ↓
 2^8 $\underline{64}$ $\underline{\underline{32}}$ $\underline{2}$

GO
CLASSES

$$256 + 64 + 32 + 2$$