

# Modeling and simulation of a simple building energy system

V.S.K.V. Harish

Student –ASHRAE, CIBSE, Research Scholar  
Alternate Hydro Energy Centre  
Indian Institute of Technology Roorkee,  
Uttarakhand -247667, India.  
harishvskv.iitr@gmail.com, rkhrvdah@iitr.ac.in

Arun Kumar

Professor and MNRE Chair Professor (Renewable Energy)  
Alternate Hydro Energy Centre  
Indian Institute of Technology Roorkee  
Uttarakhand-247667, India.

**Abstract**—A mathematical model for building energy systems (BES) is developed which maps the energy transfer processes occurring within the building space. Construction elements making up the building space and the heating and cooling plant responsible for thermal comfort of the occupants are also modeled. This involved quantification of linkages between temperature and humidity conditions and level occupancy (number of occupants, occupancy schedule) within building space. Thermal energy transfer processes of conductive, convective, and radiative heat balance for each surface of the construction elements and a convective heat balance for the building space are modeled. Building space zone is modelled for both sensible and latent thermal energy transfer. State space approach is used to model the building construction elements such as walls, with the parameters estimated using a nonlinear time invariant optimization algorithm with constraints. HVAC system is modelled with a control valve, heat emitter, occupancy driven ventilation controlled through a PID controller. A complete building energy system (BES) modeling procedure based on first principles of building physics is presented. BES model is simulated using MATLAB/Simulink and the results depict the temperature variations within the building space at less computational times.

**Keywords**—Building energy system, Building space, MATLAB, non-linear optimization, lumped capacitance model.

## I. INTRODUCTION

Energy requirements for heating and cooling of residential, commercial and industrial spaces constitute a major fraction of end-use energy consumption and thus, Heating, Ventilation and Air-Conditioning (HVAC) systems are the major consumers of electricity. Conservation of energy is required in alleviating the inefficient use of electricity by keeping a proper trade off with the users comfort levels. Different control techniques together with intelligent building technology (Building Automation Systems) are used to incorporate the popular demand side management (DSM) techniques thereby, improving the energy efficiency of buildings [1-2]. Most significant part in design of control engineering is modelling. In almost all control projects, it is crucial to have good precise models of the systems in order to design and tune the controllers and to simulate their performance.

Modeling of energy transfer processes within building spaces finds its significance in simulations of a building / plant system for load prediction or cost saving estimates. Building energy models reported in literature involve modeling of a single slab (or a wall) [3], HVAC system modeling, modeling of occupants and other causal heating factors and detailed modeling of a complete building. A three stage process for model formulation was illustrated in [4]. Building energy models developed using first principles of building physics have been used for design, energy forecasting and prediction, planning the building energy consumption to deal with capacity needs and to ensure energy efficiency and demand response [5-10]. Experimental analysis and collection of necessary data is majorly performed for building information modeling and for fault detection and diagnosis activities (FDD) [11]. A simple or multivariable mathematical/analytical analysis is performed for calibration of building energy models [12-14].

## II. BUILDING ENERGY SYSTEMS

Building energy systems can be defined as those which are responsible for consumption of energy in buildings [15], [16]. These can be any physical equipment or machinery or can be a process or a combination of them.

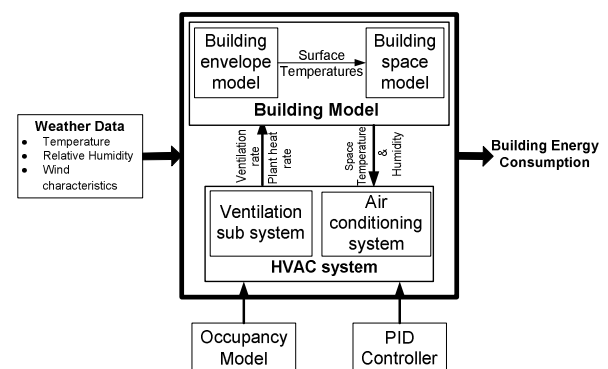


Fig.1. Overall structure of Building Energy System (BES) model

Fig. 1 shows a simplified representation of the developed building energy model.

Sensible heat transfer takes place within the conditioned room space through conduction, convection, and/or radiation whereas latent heat transfer occurs due to transfer of moisture (emission of water vapour by in room equipment and occupants) in and out of the room space [17]. The building elements of a perimeter room, responsible for heat and mass transfer processes are given in the table 1.

TABLE I. BUILDING ELEMENTS AND MODES OF HEAT AND MASS TRANSFER PROCESSES

Heat and mass transfer processes	Building elements
Conduction and/or radiation heat transfer	External wall, roof, ceiling & floor slabs and internal partition wall, doors, skylights.
Conduction heat transfer and solar radiation transmission.	Window glazing
Conduction and/or radiation heat transfer and moisture dissipation	Occupants, lights, and other equipment
Convection heat and mass transfer	Infiltration from outside and adjoin rooms/lobby

### III. BUILDING ENERGY MODEL

Physical processes occurring within a building space that encounter heat and mass transfer processes are modeled using mathematical equations. The complete building model is comprised of the following sub systems. A mathematical energy model of a building space is developed taking into account following assumptions:

- A. Construction elements of the building like wall, roof, etc are lumped into a single thermal capacitance.
- B. Temperature of the layers of the walls and roof are the same and are equal to the indoor air temperature.
- C. Heat loss occurs only due to ventilation (including heat loss due to small air gaps).
- D. Air inside the room is well mixed.
- E. Heat flow within the building materials is isotropic.
- F. Thermo-physical properties of building materials do not change with change in temperature.
- G. Heat flow across the thickness of each wall or any other slab making up the building space is considered.

A simple mathematical model of a conditioned building space is developed using heat balance equation derived from the first law of thermodynamics. The model developed is a single zone model. As per the first law of thermodynamics:

$$\Delta U = \delta Q_{\text{heat}} + \delta W_{\text{sys}} \quad (1)$$

where,  $\Delta U$  is the change in internal (already available) energy of the system under study (Joules),  $Q_{\text{heat}}$  is the amount of heat absorbed or dissipated (Joules), and  $W_{\text{sys}}$  is the work done by or to the systems (Joules).

Now, using above equation for a building zone under study

$$\frac{dU(t)}{dt} = C_{th} \frac{dT_i(t)}{dt} \quad (2)$$

For the present study of modelling of a conditioned space we have,

$$C_{th} \frac{dT_i(t)}{dt} = Q_{\text{vent}} + \phi(t) \quad (3)$$

where

$$\phi(t) = -\frac{1}{R_{th}}(T_i(t) - T_o(t)) \quad (4)$$

Thus, the energy balance equation finally takes the form of:

$$C_{th} \frac{dT_i(t)}{dt} = Q_{\text{vent}} - \frac{1}{R_{th}}(T_i(t) - T_o(t)) \quad (5)$$

where,  $C_{th}$  is the total/global thermal capacitance of the conditioned space (J/°C),  $R_{th}$  is the total / global thermal resistance (°C/W),  $T_i(t)$  is the room air temperature (°C),  $T_o(t)$  is the outside air temperature (°C),  $Q_{\text{vent}}$  is the ventilated heat flow through the conditioned space in the zone under study,  $\phi(t)$  is the flow of heat due to temperature variations between outside & indoor environment (W).

The values  $C_{th}$  &  $R_{th}$  are material dependent. These depend on the thermo physical characteristics of the building construction elements such as walls, roof, etc.

i.e.,

$$R_{th} = r_{si} + r_{so} + \sum_{i=1}^n \frac{x_i}{k_i} \quad (6)$$

$$C_{th} = \sum_{i=1}^n x_i \rho_i C_{p_i} \quad (7)$$

where  $r_{si}$  &  $r_{so}$  are internal and external surface resistances of the building construction elements, respectively (°C/W),  $x$  is the layer thickness (m),  $k$  is the thermal conductivity of the layer (W/m-K),  $\rho$  is the density of the layer (kg/m<sup>3</sup>),  $C_p$  is the thermal capacity of the layer (J/kg-K),  $n$  is the number of layers in the construction element.

Using the available weather data of outdoor air dry bulb temperature, relative humidity, global horizontal solar

radiation and wind characteristics, equations (1) – (7) are mathematically modeled in MATLAB/Simulink.

Energy balance equation for building space zone model is as follows:

$$\rho_a C_{pa} V_{bs} \frac{dT_{bs}}{dt} = Q_{casual} + Q_{hvac} - \frac{10 \times A_{ow} \times (T_{cs} - T_{ows})}{R_{ow}} - \frac{10 \times A_{adfw} \times (T_{cs} - T_{adfw})}{R_{adfw}} - \frac{10 \times A_{adfw1} \times (T_{cs} - T_{adfw1})}{R_{adfw1}} - \frac{10 \times A_{roof} \times (T_{cs} - T_{roof})}{R_{roof}} - \frac{10 \times A_{floor} \times (T_{cs} - T_{floor})}{R_{floor}} - A_{win} \times U_{win} \times (T_{cs} - T_{out}) - 0.33 \times N \times V_{cs} \times (T_{cs} - T_{out}) \quad (8)$$

where,

$\rho_a$ ,  $C_{pa}$ : density (kg/m<sup>3</sup>) & specific heat capacity (J/kg-K) of air in the building space, respectively

$Q_{casual}$ : casual heat (due to occupants) gain (W)

$Q_{hvac}$ : heat output from HVAC system. +ve if heating and –ve if cooling (W)

$A$ : Cross-sectional area of respective constructional elements (sq. mt.)

$T_{cs}$ : Temperature of the building space to be conditioned by HVAC system (deg C)

$U_{win}$ : Window U-value (W/m<sup>2</sup>-K)

$N$ : No. of air changes per hour from the HVAC system (per hour).

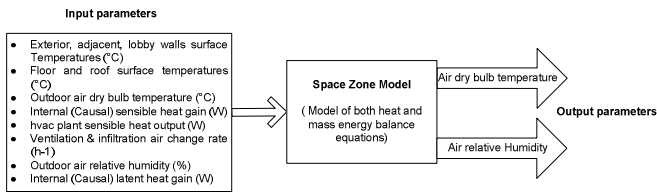


Fig. 2. Building space zone modeling structure

#### IV. BUILDING ENVELOPE MODEL

American Society of Heating, Refrigerating and Air-conditioning engineers (ASHRAE) defines a building envelope as an enclosure providing the necessary physical separation between the indoor and outdoor environments [17]. Building model is developed to map the dynamics of both heat and mass transfer processes, which act as load on to a HVAC system, are modelled. Energy models for the construction elements making up the conditioned space, indoor environment including causal and non-causal heat gains are

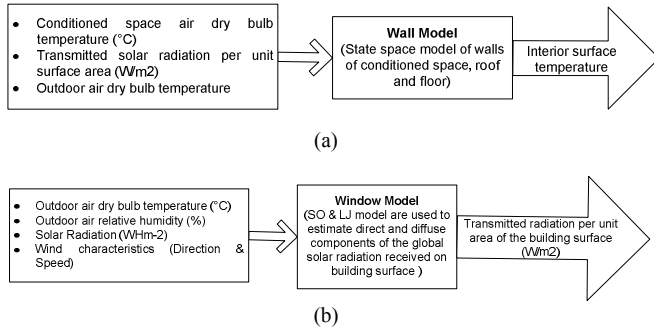


Fig. 3. Building envelope modeling structure (a) wall model, (b) window model

developed.

Construction elements are made up of multilayer materials, thermo-physical properties of whose are given in appendix 1. The input and output variables along with parameters of the wall model is shown in fig. 3 (a)

For a multilayer wall or any other construction element, its overall thermal resistance and capacitance needs to be allocated to one or more number of resistors and capacitors, which indicates the order of the R-C network model. Gouda et al [19] suggested that a 2nd order model with three resistances and two capacitances (3R2C), as shown in fig. 4(a), is accurate enough to model a wall.

ODEs for the 3R2C model can be given as:

$$y_1 C_{th} \dot{T}_1 = \frac{T_{out} - T_1}{x_1 R_{th}} - \frac{T_1 - T_2}{x_2 R_{th}} \quad (8)$$

$$y_2 C_{th} \dot{T}_2 = \frac{T_1 - T_2}{x_2 R_{th}} - \frac{T_2 - T_{BS}}{x_3 R_{th}} + Q_{hvac} \quad (9)$$

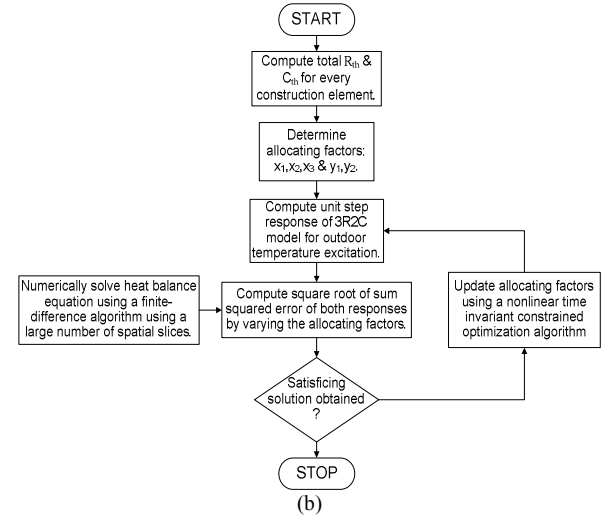
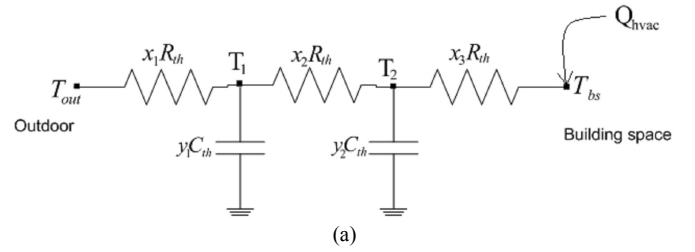


Fig. 4. (a) Lumped capacitance (3R2C) modeling structure for a multilayered wall; (b) Flowchart for the optimization algorithm to identify the parameters of the 3R2C model

The state equations for the thermal energy balance as per equations (8) & (9):

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \end{pmatrix} = \begin{pmatrix} \frac{-1}{y_1 R_{th} C_{th}} \left( \frac{1}{x_1} + \frac{1}{x_2} \right) & \frac{1}{x_2 y_1 R_{th} C_{th}} \\ \frac{1}{x_2 y_2 R_{th} C_{th}} & \frac{-1}{y_2 R_{th} C_{th}} \left( \frac{1}{x_2} + \frac{1}{x_3} \right) \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{x_1 y_1 R_{th} C_{th}} & 0 & 0 \\ 0 & \frac{1}{x_3 y_2 R_{th} C_{th}} & \frac{1}{y_2 C_{th}} \end{pmatrix} \begin{pmatrix} T_{out} \\ Q_{hvac} \\ T_{bs} \end{pmatrix} \quad (10)$$

Output state variable for a particular multilayer wall will be its internal surface temperature i.e.,  $T_s$  (external being the surface under the influence of outside environment). Input variables to the model are the: outdoor air temperature ( $T_{out}$ ), transmitted solar radiation per unit surface area ( $H_T$ ) and the building space conditioned temperature ( $T_{CS}$ ). For linear modelling applications with constant parameters of the governing equations state space model of the wall can be conveniently expressed in matrix form as:

$$\dot{T} = AT + BU \quad (11)$$

$$T_{bs} = CT + DU \quad (12)$$

where,

$\dot{T} \in$  two-dimensional vector matrix of first order derivatives of nodal temperatures.

$T \in$  two-dimensional vector matrix of interlayer surface temperatures.

$T_{bs} \in$  one-dimensional output matrix of temperature of the inner surface of the building envelope.

$A \in$  Co-efficient matrix

$B \in$  Vector matrix of input coefficients

$C \in$  Vector matrix of output coefficients

$D \in$  Coupling matrix of input and output coefficients

Now, accounting for HVAC plant heat rate,  $Q_{HVAC}$  the A, B, C, D state matrices can be deduced as:

$$A = \begin{pmatrix} \frac{-1}{y_1 R_{th} C_{th}} \left( \frac{1}{x_1} + \frac{1}{x_2} \right) & \frac{1}{x_2 y_1 R_{th} C_{th}} \\ \frac{1}{x_2 y_2 R_{th} C_{th}} & \frac{-1}{y_2 R_{th} C_{th}} \left( \frac{1}{x_2} + \frac{1}{x_3} \right) \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{x_1 y_1 R_{th} C_{th}} & 0 & 0 \\ 0 & \frac{1}{x_3 y_2 R_{th} C_{th}} & \frac{1}{y_2 C_{th}} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

An optimization fitting algorithm is developed for converting the complex thermal network of a multi-zone building into a compact reduced-order state-space representation that will more readily enable implementation and assessment of advanced control concepts. Parameters of the lumped 3R2C model are identified using a nonlinear time invariant constrained optimization algorithm [20] as shown in fig. 4(b). The optimization run compares the step response of a standard model to the step response of the target 3R2C model. The main objective of the algorithm is to find optimum values of the allocating factors.

The objective function for the optimization problem is given as:

Minimize:

$$J(x_1, x_2, x_3, y_1, y_2) = \sum_1^N (T_{FDM} - T_{3R2C})^2 \quad (13)$$

$$\text{subject to: } \begin{array}{ll} x_1 + x_2 + x_3 = 1 & ; \quad x_1, x_2, x_3 \geq 0 \\ y_1 + y_2 = 1 & ; \quad y_1, y_2 \geq 0 \end{array}$$

$$\text{Upper boundary conditions: } \begin{array}{l} C_1, C_2 = C_{th}/100 \\ R_1, R_2, R_3 = R_{th}/100. \end{array}$$

$$\text{Lower boundary conditions: } \begin{array}{l} C_1, C_2 = 0.99C_{th} \\ R_1, R_2, R_3 = 0.99R_{th}. \end{array}$$

where,

$T_{FDM}$  = Temperature step response of the finite difference model ( $^{\circ}C$ )

$T_{3R2C}$  = Temperature step response of the 3R2C model ( $^{\circ}C$ )

$x_1, x_2, x_3; y_1, y_2$  = Allocating factors for the Rs and Cs of 3R2C model

$T_1, T_2$  = Nodal temperatures ( $^{\circ}C$ )

$Q_{HVAC}$  = HVAC plant heat rate (W)

## V. SIMULATION RESULTS

Developed BES model is simulated in MATLAB/Simulink with the architecture as shown in fig. 1. The input and the influencing control parameters with their numerical values

along with the weather data of the perimeter building space has been taken from [17].

Inputs to the BES model such as outdoor air temperature, solar radiation are given in fig. 5(a) and (b), respectively.

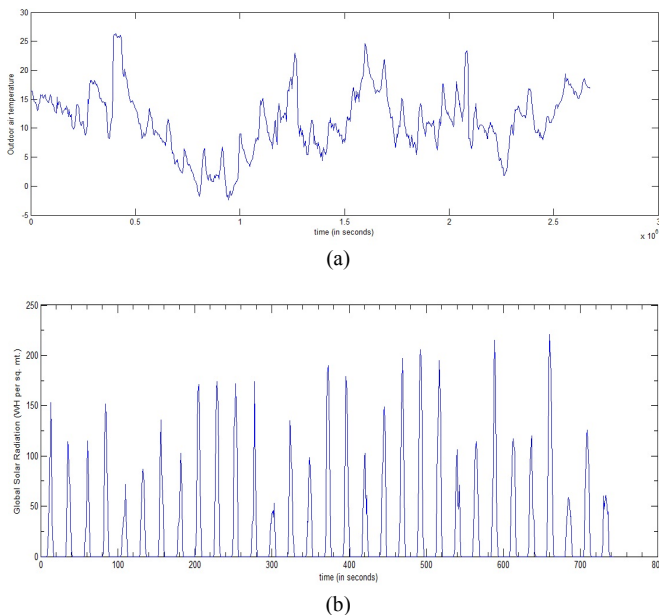


Fig. 5. (a) Outdoor weather air temperature (in deg celsius); (b) Global horizontal solar radiation (Wh /m<sup>2</sup>)

Variations in temperature of the conditioned space are shown in fig. 6.

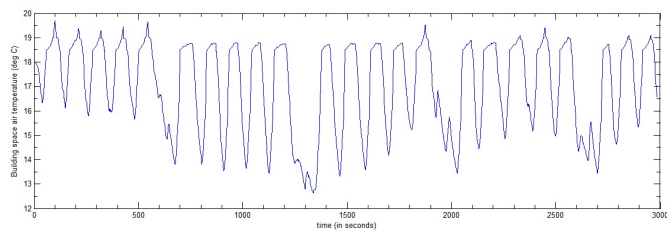


Fig. 6. Building space conditioned air temperature (in deg Celsius)

## VI. CONCLUSION

An energy transfer model for a building space conditioned with a HVAC system is developed. The dynamic processes which act as load to a HVAC system are mapped into the model. This involved quantification of linkages between temperature and humidity conditions and level occupancy (number of occupants, occupancy schedule) within building space. Energy transfer processes of conductive, convective, and radiative heat balance for each surface of the construction elements and a convective heat balance for the building space are modeled. Building space zone is modelled for both sensible and latent thermal energy transfer. State space approach is used to model the building construction elements such as walls, etc making up the conditioned room. Parameters of the elements are estimated using a non-linear optimization problem with constraints. HVAC system is modelled with a control valve, heat emitter, occupancy driven ventilation controlled through a PID controller. Parameters affecting the

energy consumption of a building such as the thermal properties of materials of construction, orientation, planning, and design specifications; climatic parameters of temperature, radiation, and air movement and occupancy factors such as the functional use of a building space are also modelled which would establish the required comfort conditions and resultant energy loads. The model simulations show that MATLAB/Simulink can model the building energy systems and thus, can provide a good platform to develop the necessary control techniques to develop control strategies to save energy and maintain comfort and productivity for the energy systems of buildings.

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