A SIMPLE SIMULATION MODEL OF BUILDING TRANSIENT HEAT CONDUCTION FOR BUILDING ENVIRONMENT EVALUATION

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ABSTRACT
Based on circuit principle, this paper presents a simple simulation model (3R2C model) of transient heat transfer through building envelope, which is applied to simulate and analyze the process of multilayer envelope transient heat transfer. A transfer matrix of a wall is deduced by Laplace transform on the basis of 3R2C model. The theoretical frequency characteristics of the wall are calculated. Via proper identification algorithm, the polynomial coefficients for the transfer matrix are estimated from the theoretical frequency characteristics. Then, the resistances and capacitances of the model are determined by an optimization algorithm, which makes polynomial s-transfer function based on the model equivalent to theoretical transcendental s-transfer function of the wall completely. Examples demonstrate that the 3R2C model is accurate, simple and efficient in calculating building transient heat transfer.

INDEX TERMS
3R2C model; thermal response factors; transient heat transfer

INTRODUCTION
Air conditioning systems have been widely used in residential, commercial and office buildings. The application of air-conditioning system might improve the thermal comfort of occupied space, and also results in some problems such as more energy consumption and bad indoor air quality. Thus, more and more attention is focused on energy consumption, thermal comfort and indoor air quality of air-conditioning buildings.

It is convenient and low-cost to test, commission and evaluate HVAC system control strategies by the dynamic simulation of HVAC system. Dynamic models, which are convenient to use and represent well the dynamic characteristics in all the aspects of concerns (Shengwei Wang and Youming Chen 2002). Heat conduction model of building constructions is the most important part of indoor environment evaluation, building energy analysis and HVAC system simulation programs. This paper presents a simple and efficient model (3R2C) to stimulate and analyze the process of envelope dynamic heat conduction. Response factors method and conduction transfer function (CTF) method are the approaches commonly used to calculate space heating/cooling load. However, the conventional method for calculating the response factors and the conduction transfer function coefficients (CTFs) may lead to incorrect and inefficient computation, because there are risks missing a few roots, especially in the case where two adjacent roots are close together when searching for a large number of characteristic roots of *B(s)* by numerical iteration. A frequency-domain regression (FDR) method (Youming Chen 2002) is used to estimate the 3R2C model of building multiplayer constructions in this paper.

ESTABLISHMENT OF 3R2C DYNAMIC SIMULATION MODEL
The 3R2C model considers a building as a thermal network which is analogous to an electric circuit network , as shown in Fig. 1. Then, we get a linear thermal network and can derive equations from the energy balance of each node. The thermal resistance of a multiplayer construction can be given by Eq. (1).

$$R = \frac{1}{U} = R_{in} + \sum_{i=1}^{N}\frac{d_i}{l_i} + R_{out} = \frac{1}{a_{in}} + \sum_{i=1}^{N}\frac{d_i}{l_i} + \frac{1}{a_{out}} \quad m^2 \cdot {}^\circ C/W \tag{1}$$

Where N is the layer number of the solid wall, a_{in} and a_{out} are the inside and outside surface thermal

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diffusivity of the wall, respectively. d_i and l_i are the thickness and thermal conductivity for each layer, and U is the U factor or thermal transmittance of the wall. Based on the knowledge of electrology, we can get Eq.(2).

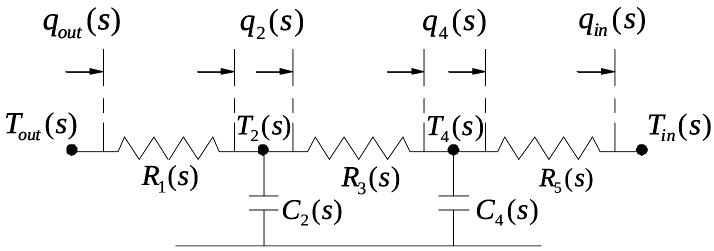

Figure 1. *3R2C simulation model for multiplayer building constructions*

$$q = C\frac{dT}{dt} \tag{2}$$

Where C is the thermal capacitance of the multiplayer construction. As the surface air films (including the air films inside constructions) can be considered as pure thermal resistance, the thermal capacitance is zero. Thus, $C = \sum_{i=1}^{N} C_{pi} \cdot r_i \cdot d_i$, where C_{pi}, r_i and d_i are the material specific heat, density and thickness of the wall respectively. Based on Kirchhoff's current law, from Eq.(1) and Eq(2), we can derive the following equations.

$$C_2\frac{dT_2}{dt} = \frac{T_{out} - T_2}{R_1} - \frac{T_2 - T_4}{R_3} \tag{3}$$

$$C_4\frac{dT_4}{dt} = \frac{T_2 - T_4}{R_3} - \frac{T_4 - T_{in}}{R_5} \tag{4}$$

$$q_{out} = \frac{T_{out} - T_2}{R_1} \tag{5}$$

$$q_{in} = \frac{T_4 - T_{in}}{R_5} \tag{6}$$

Applying Laplace transform on Eq.(3)-(6), we can obtain the following equations.

$$C_2(sT_2(s) - t_2(0)) = \frac{T_{out}(s) - T_2(s)}{R_1} - \frac{T_2(s) - T_4(s)}{R_3} \tag{7}$$

$$C_4(sT_4(s) - t_4(0)) = \frac{T_2(s) - T_4(s)}{R_3} - \frac{T_4(s) - T_{in}(s)}{R_5} \tag{8}$$

$$Q_{out}(s) = \frac{T_{out}(s) - T_2(s)}{R_1} \tag{9}$$

$$Q_{in}(s) = \frac{T_4(s) - T_{in}(s)}{R_5} \tag{10}$$

Based on Eq.(9) and (10), we can get Eq.(11) and (12).

$$T_2(s) = -R_1 Q_{out}(s) + T_{out}(s) \tag{11}$$

$$T_4(s) = R_5 Q_{in}(s) + T_{in}(s) \tag{12}$$

Substituting Eq. (11) and (12) into Eq. (7) and (8), the transmission equation (13) in terms of Laplace variable *s* relates the temperatures and heat flows at both sides of the wall.

$$\begin{bmatrix} T_{in}(s) \\ Q_{in}(s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} T_{out}(s) \\ Q_{out}(s) \end{bmatrix} \tag{13}$$

Where the elements of transmission matrix can be given as follows.

$$\begin{aligned}
&A(s) = C_2C_4R_3R_5s^2 + (C_2 + C_4)R_5s + 1 \\
&B(s) = C_2C_4R_1R_3R_5s^2 + (C_2 + C_4)R_1R_5s + (C_2R_1 + C_4R_5)R_3s + R_1 + R_3 + R_5 \\
&C(s) = -(C_2C_4R_3s^2 + (C_2 + C_4)s) \\
&D(s) = -(C_2C_4R_1R_3s^2 + (C_2 + C_4)R_1s + C_4R_3s + 1)
\end{aligned}$$

Therefore, the transfer functions of external, cross and internal heat conduction of the multiplayer constructions, $G_X(s), G_Y(s)$ and $G_Z(s)$ can be expressed as Eq. (14).

$$\left.\begin{aligned}
G_X(s) &= \frac{A(s)}{B(s)} \\
G_Y(s) &= \frac{1}{B(s)} \\
G_Z(s) &= \frac{D(s)}{B(s)}
\end{aligned}\right\} \tag{14}$$

DETERMINATION OF POLYNOMIAL S-TRANSFER FUNCTIONS

Applying the FDR method to the 3R2C model, we can obtain the estimated polynomial *s*-transfer coefficients of the 3R2C dynamic simulation model, $\hat{b}_0$, $\hat{a}_1$ and $\hat{a}_2$. Substituting $G(jw)$ with $G_X(jw)$, $G_Y(jw)$ and $G_Z(jw)$, we can find the polynomial s-transfer functions of external, cross and internal heat conduction, respectively. From the values of coefficients $\hat{b}_0, \hat{a}_1$ and $\hat{a}_2$, we can calculate the values of design variables R_3、R_5、 C_2 and C_4 based on equations(15)-(19).

$$R = R_1 + R_3 + R_5 \tag{15}$$

$$C = C_2 + C_4 \tag{16}$$

$$\hat{b}_0 = \frac{1}{R_1 + R_3 + R_5} \tag{17}$$

$$\hat{a}_1 = \frac{(C_2 + C_4)R_1R_5 + (C_2R_1 + C_4R_5)R_3}{R_1 + R_3 + R_5} \tag{18}$$

$$\hat{a}_2 = \frac{C_2C_4R_1R_3R_5}{R_1 + R_3 + R_5} \tag{19}$$

Thus, an optimization object function is set up, as shown in Eq.(20).

$$F(R_1, R_3, R_5, C_2, C_4) = (\hat{a}_1 - \frac{(C_2 + C_4)R_1R_5 + (C_2R_1 + C_4R_5)R_3}{R_1 + R_3 + R_5})^2 + (\hat{a}_2 - \frac{C_2C_4R_1R_3R_5}{R_1 + R_3 + R_5})^2 \tag{20}$$

DETERMINATION OF 3R2C MODEL PARAMETER

Based on the calculated values of R_1, R_3, R_5, C_2 and C_4, substituting jw ($j = \sqrt{-1}$) for *s* into the polynomial *s*-transfer function, we can obtain the complex function $G(jw)$, which is the frequency characteristic of cross heat conduction. In this paper, ASHRAE wall group 6 (1997) is taken as example, whose parameters are shown as Table 1. A comparison is made numerically between theoretical frequency characteristics of the wall (ASHRAE wall group 6) and those of 3R2C dynamic simulation model, 3R2C model (a) put forward by Seem and Klein(1989) and 3R2C model (b) put forward by Braun and Chaturvedi. The comparison results are shown in Figure.2 and 3.From Figure.2 and 3, it is shown that except for the phase lag at some points in the high frequency range, the frequency responses of the 3R2C model obtained by the FDR method are almost completely in agreement with the theoretical frequency responses. Therefore, from the point of view of frequency-response characteristics, the 3R2C model is completely equivalent to the other models.

VERIFICATION OF 3R2C MODEL

In order to prove the efficiency and validity of the 3R2C dynamic simulation model, it is used to calculated the response factors of a three layered concrete construction, consisting of two surface layers of concrete and a middle layer of insulation material. In SI units, the thickness and thermal properties of each layer are listed in Table 2.

Table 1. *The parameters of ASHRAE wall group 6*

	L /mm	l /Wm^{-1}K^{-1}	r /kgm^{-3}	C_p /Jkg^{-1}K^{-1}	R /m^{2}KW^{-1}
Outside surface film					0.0586
Stucco	25.39	0.6924	1858	8368	0.0367
High density concrete	101.59	1.7310	2243	8368	0.0587
Insulation	25.30	0.0433	32	8368	0.5846
Plaster	19.05	0.7270	1602	8368	0.0262
Inside surface film					0.1206

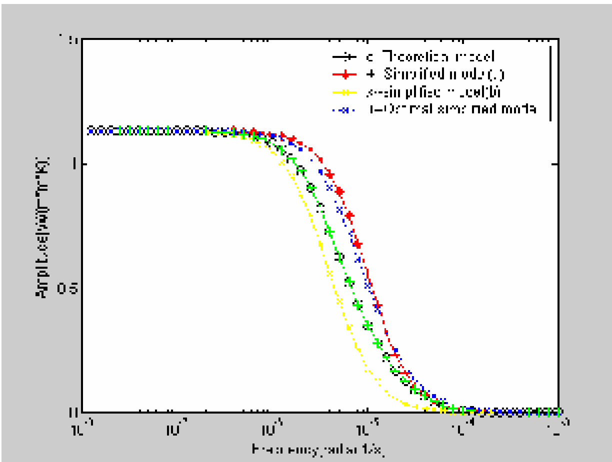

Figure2. *Amplitude of frequency characteristic*

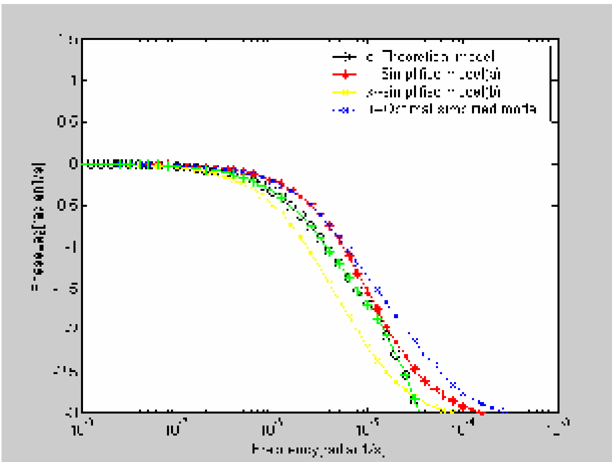

Figure 3. *Phase lag of frequency characteristic*

***Table 2.** Details of a three-layered wall*

	L /m	l /Wm^{-1}K^{-1}	r /kgm^{-3}	C_p /Jkg^{-1}K^{-1}	R /m^{2}KW^{-1}
Outside surface film					0.0500
Concrete	0.089	1.7300	2235	1106	0.0514
Insulation	0.127	0.0744	24	992	1.7070
Concrete	0.089	1.7300	2235	1106	0.0514
Insulation surface film					0.1600

***Table 3.** The response factors* $Y(k)$ $wm^{-2}k^{-1}$ *(the first 68 items)*

k	*Y(k)*	*k*	*Y(k)*	*k*	*Y(k)*
0	0.00150574721130	24	0.00856360418642	48	0.00201258342979
1	0.00796678273273	25	0.00806450246608	49	0.00189462544192
2	0.01343263263537	26	0.00759385610415	50	0.00178358078420
3	0.01689898438841	27	0.00715022298319	51	0.00167904433825
4	0.01895827258206	28	0.00673218163814	52	0.00158063470824
5	0.02003370219895	29	0.00633834812316	53	0.00148799283663
6	0.02042723631175	30	0.00596738677351	54	0.00140078069931
7	0.02035399661528	31	0.00561801666836	55	0.00131868007598
8	0.01996692264412	32	0.00528901508478	56	0.00124139139174
9	0.01937444702757	33	0.00497921886360	57	0.00116863262584
10	0.01865316371108	34	0.00468752434322	58	0.00110013828404
11	0.01785690653203	35	0.00441288632793	59	0.00103565843076
12	0.01702325436325	36	0.00415431642183	60	0.00097495777782
13	0.01617819140704	37	0.00391088096191	61	0.00091781482636
14	0.01533944500022	38	0.00368169871525	62	0.00086402105894
15	0.01451887543562	39	0.00346593845486	63	0.00081338017891
16	0.01372418629836	40	0.00326281649401	64	0.00076570739433
17	0.01296014781128	41	0.00307159423348	65	0.00072082874373
18	0.01222947119249	42	0.00289157575803	66	0.00067858046146
19	0.01153343295901	43	0.00272210550617	67	0.00063880838012
20	0.01087232010024	44	0.00256256602791	68	0.00060136736810
$\sum Y(j)$	0.48702082351403				
Actual U	0.49507905756198				
Error of U	1.63%				

From the above calculations, the summation of thermal response factors is very close to the actual U of the wall. The almost negligible difference between two U values demonstrated the validity of the 3R2C dynamic simulation model.

CONCLUSIONS

By using the 3R2C dynamic simulation model, there is no need to numerically search for a large number of roots of the hyperbolic characteristic equations, and no need to evaluate the same number of residues. So it avoids miscalculation due to missing some roots, and its computation is simple, efficient and straightforward. This model is easy and convenient to use and implement in building environment evaluation.

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