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Mathematical Models for Measuring Environmental Tobacco Smoke Concentrations in A Single Room

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ABSTRACT

This study developed analytical models for a single zone using a macroscopic approach to predict indoor air pollutant concentration in a room. The law of conservation of mass was applied to the box model representing a room apartment with real time measured parameter to develop four models with multi-smoker effect (MSE) and removal mechanisms (RM) other than the normal ventilation effect commonly considered in most box model version. The environmental tobacco smoke (ETS) models, were derived by solving the differential equations analytically to obtain the popular box model and including RMs like dry deposition, chemical removal and filtration. The developed models are a combination of the multiple-smoker version, the sink effect version with dry deposition and chemical removal effect. Data developed using these models show that the inclusion of more than one RM produced a more precise and realistic result close to what obtains in reality than other models with lesser or no RM. Most box models only consider ventilation and filtration which is not enough to determine what real occur in nature. This study seeks to establish the importance of RMs in an indoor air quality (IAQ) model by considering some of the physico-chemical characteristics and behaviour of the contaminants as it affect contaminant concentration in a room. The particle deposition count was also established to determine the number of particulate matter (PM) removed from the micro-environment or deposited on measured surface area over a given time after the smoking activity. MATLAB codes were developed to evaluate the models and the generated graphs and data predicted how indoor ETS concentration can be managed for human health and safety. The model compared favorably with findings of other researchers working on ETS modeling but further validation can be done using real time concentration monitors.

Keywords: *Environmental Tobacco Smoke, Indoor Air Quality, Contaminant, Particulate Matter, Sink, Dry Deposition, Average Smoking Count And Chemical Removal.*

1. INTRODUCTION

Tobacco smoking is a growing public health problem in the developing world, among which Nigeria is one. It is an age long social habit that has claimed so many live worldwide. It is a universal problem which though may have peculiar geographical approaches in terms of solutions, but yet remain one with universal determination in tackling [1]. In Nigeria, 4.5 million adults are tobacco addict according to the National Bureau of Statistics. About 93 million sticks of cigarette are produced yearly in the country and every one of those cigarettes is consumed in Nigeria. World Health Organisation (WHO) estimates that about 1.3 billion people in the world are currently smoking and most of them are in developing countries. Tobacco kills close to five million people yearly worldwide with over 70 percent occurring in developing countries including Nigeria. It is the cause of death of 17.7 per cent of all deaths in developed countries. By 2020, the World Health Organisation (WHO) expects the worldwide death toll from smoking to reach 10 million. [1]

Tobacco is an agricultural product from the fresh leaves of the plant in the genus Nicotiana, which is commercially available in dried, cured, and natural forms. The combustion of this material is a flameless reaction which gives rise to the pollutants in the environmental tobacco smoke (ETS). Besides cigarettes, cigars, stem pipe and hookah also produces ETS [2]. Tobacco is responsible for over 25 diseases in man, including hypertension, heart attack, cancer and other conditions such as asthma and emphysema. It is also responsible for some pregnancy-related problems and other conditions such as tuberculosis, blindness, deafness, nutritional and psychological disorders. Tobacco kills 50 per cent of lifetime smokers and half of these deaths occur among people in their middle age (35-69years) [1]. The government has banned tobacco smoking in public places but the habit still persist in homes, offices and our surroundings and still poses great harm to both the active and passive smokers. The effect of smoking in homes to the smokers, their family members and neighbours remains an issue of great concern.

The modeling of indoor contaminants is greatly influenced by the physico-chemical characteristics and behaviour of the contaminants. These characteristics are infiltration/penetration, deposition, re-suspension, particle formation, coagulation and phase change. Some of these behaviours are analyzed in some IAQ modeling like infiltration and deposition. In this paper we seek to analyze as many of these characteristic behaviours as possible to take us closer to what actual transpire within the room environment. The inclusion of these behaviours will produces a more precise and accurate result of the contaminant concentration within the modeled space. Analyzing all these behaviours in a single IAQ mathematical or computational model will be excellent and produce very accurate result.

For decades effort has been made by several researchers to develop mathematical model that can predict, measure and validate the concentration of environmental tobacco smoke (ETS) in homes and other indoor environment. In most research the mass balance models are developed based on the conservation of mass. It is believed that the constituent mass of the ETS is conserved in the indoor environment by the in/out movement of matter through the windows and doors of the environment. . Various versions of the box model has been used by researchers namely the time-averaged version, the rectangular source emission time function version, the impulse time function version, the source proximate effect version, mixing factor version, the multiple-smoker version using minute by minute smoking count and the sink (removal) effect version.

Wayne [4] reviews the development of the mass balance model and its application to predicting indoor pollutant concentrations from cigarette smoke and derives the time-averaged version of the model from the basic laws of conservation of mass. In 2003, Wayne et al [5], developed an analytical solutions to multi compartment indoor air quality models for predicting indoor air pollutant concentrations in the home, Their model uses Laplace transform methods to solve the mass balance equations for two interconnected compartments, obtaining analytical solutions that can be applied without a computer. In 2010, Radha et al [8] presented in their research paper the use of mass balance based IAQ and ventilation modeling technique to predict the indoor respirable suspended particulate matter (RSPM) concentrations in naturally ventilated classrooms of urban school buildings. They considered different size of particulates, mainly PM_{10}, $PM_{2.5}$ and $PM_{1.0}$ and their infiltration characteristics and noted that the final indoor concentration of particulates are influenced by the temporal and spatial variations of the indoor sources and sinks, physico-chemical transformations, occupants' indoor activities and background concentration of particulates.

This article employs the mass balance equation to develop a simple mathematical model for predicting indoor tobacco smoke contaminants in a local Nigerian-single room apartment. The single box model is a macroscopic model which assumes that the air in the room is sufficiently well mixed and the contaminants concentration constant at every point in the room. The American Society for Testing Materials (ASTM) judged a room to be well mixed if the concentrations at multiple points were within ±10% overall average concentration (ASTM E 741) [4]. Four analytical models were developed with multi-smoker effect (MSE) and removal mechanisms (RM). The environmental tobacco smoke (ETS) models, were derived by solving the differential equations analytically to obtain the popular box model and including RMs like dry deposition (on room walls, floor and ceiling), filtration and chemical removal. The models are (a) MSE with ventilation effect (b) MSE with ventilation and sink effect (c) MSE with ventilation, deposition and chemical removal effect (d) MSE with ventilation, sink, dry deposition and chemical removal effect. The indoor contaminants from tobacco smoke are carbon monoxide (CO), respirable suspended particles (RSP or $PM_{3.5}$), and particulate polycyclic aromatic hydrocarbons (PPAH). Validation of the model can be carried out using real-time monitors for the continuous measurements of CO, RSP, and PPAH in the room.

2. MODEL ROOM DESCRIPTION

The room has floor area of 10.8m^2 and room volume of 32.4m^3. The room has one window and one door. Details shown in Figure 1 below. The room is nominally occupied from 6 p.m. to 9 a.m. Monday through Saturday and nearly 24hours on Sundays.

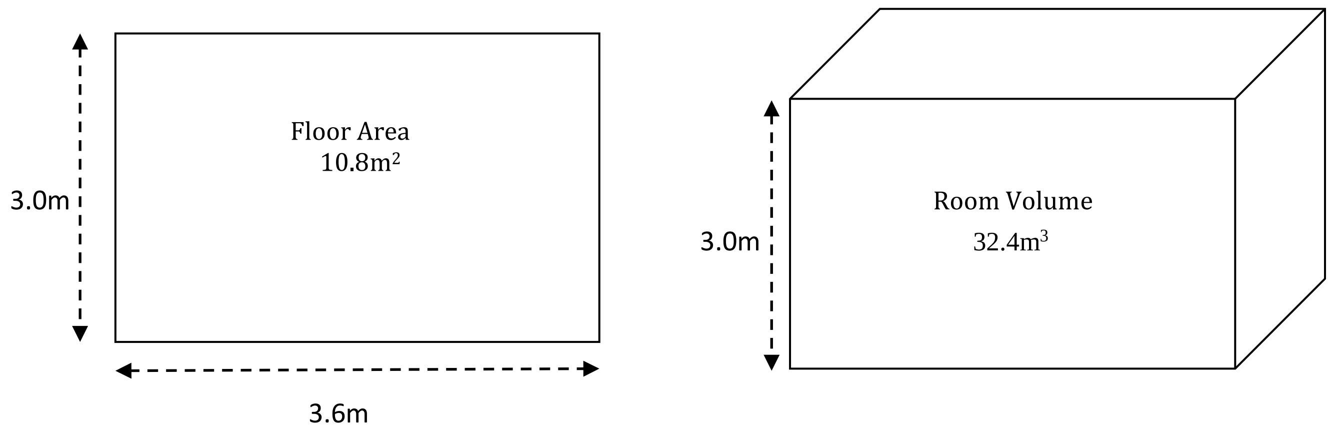

Figure 1: Room Dimension

3. METHODOLOGY

Mathematical formulation from Philip (7) is used to develop the model.

a. Macroscopic approach: well-mixed IAQ box model (single zone with no RM)

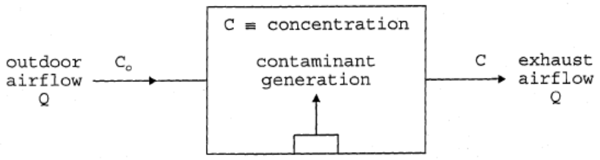

Figure 2: Modeled room represented with a box without RMs.

For small time interval (Δt),

$$\Delta M = G\Delta t + QC_0\Delta t - QC\Delta t \qquad (1)$$

where

ΔM = change in the amount of contaminant mass inside the box during time Δt, in μg

G = generation rate of contaminant in μg/min

Q = effective outdoor and exhaust airflow rate in m^3/min

C_0 = outdoor contaminant concentration in μg/m^3

C = contaminant concentration inside the box and exiting the box in μg/m^3

Simplifying equation (1) gives

$$v\frac{\Delta C}{\Delta t} = G + QC_0 - QC \qquad (2)$$

If Δt is very small, ΔC will also be very small and a differential equation can be written

$$v\frac{dC}{dt} = G + QC_0 - QC \qquad (3)$$

Assuming Q and G are constant, this equation can be solved by integration

$$\int_{C_i}^{C} \frac{dC}{G+QC_0 - QC} = \int_0^t \frac{dt}{v} \qquad (4)$$

where C_i is the initial contaminant concentration inside the box

Integrating gives

$$-\frac{1}{Q}\ln\left(G + QC_0 - QC\right)\Big\{\begin{matrix}C\\C_i\end{matrix} = \frac{t}{v}\Big\{\begin{matrix}t\\0\end{matrix} \qquad (5)$$

or

$$\ln\left(\frac{G + QC_0 - QC}{G + QC_0 - QC_i}\right) = -\frac{Q}{v}t \qquad (6)$$

Solving the equation for C gives.

$$C = \left(C_0 + \frac{G}{Q}\right) + \left(C_i - \frac{G}{Q} - C_0\right)e^{-\lambda t} \qquad (7)$$

This is the mathematical model with ventilation effect only.

where

$\lambda = \frac{Q}{V}$ is the outdoor air exchange rate accounting only for ventilation.

First term on the right side of equation is
1. Independent of time
2. Independent of volume
3. Independent of initial concentration

Second term on the right side of equation
1. Gets smaller as time advances
2. Equals zero at steady state ($t = \infty$)

Significance of λ (outdoor airflow to volume ratio)
1. Defines the *outdoor exchange rate*
2. Specified in air changes per hour (ach)
3. Reciprocal is the *mean residence time* of the air (and contaminant) in the box.

3.2. Macroscopic approach: well-mixed IAQ box model (single zone with sink)

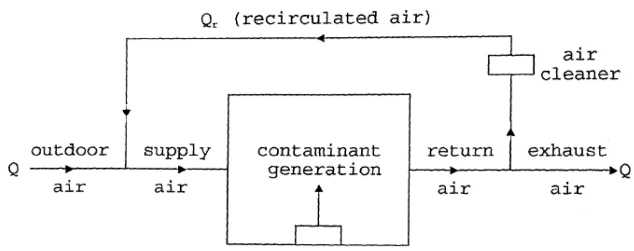

Figure 3: Modeled room represented with a box with sink.

The indoor sink is air cleaners/filters incorporated through an HVAC or filtration system. To include sink equation (1) becomes

$$\Delta M = G\Delta t + QC_0\Delta t - QC\Delta t - Q_r\eta C\Delta t \qquad (8)$$

where

Qr = recirculation airflow passing through an air cleaner in m^3/min

η = air cleaner efficiency (fraction of contaminant removed by air cleaner)

Following the same procedure of section 3.1 as above we have

$$v\frac{\Delta C}{\Delta t} = G + QC_0 - QC - Q_r\eta C$$

Simplifying further gives

$$v\frac{dC}{dt} = G + QC_0 - QC - Q_r\eta C$$

Solving the equation for C gives.

$$C = \frac{(G + QC_0) - (G + QC_0 - QC_i - Q_r\eta C_i)e^{-\lambda_1 t}}{(Q + Q_r\eta)} \quad (9)$$

This is the mathematical model with ventilation and sink effect.

Where

$\lambda_1 = \lambda + \lambda_r$ is the particle decay rate accounting for ventilation and filtration.

$\lambda = \frac{Q}{V}$ is the outdoor air exchange rate

$\lambda_r = \frac{Q_r\eta}{V}$ is the sink air exchange rate

3.3. Macroscopic approach: well-mixed IAQ box model (single zone with dry deposition and chemical removal)

To include sink and dry deposition effects equation (1) becomes

$$\Delta M = G\Delta t + QC_0\Delta t - QC\Delta t - \omega A\Delta t - \frac{C}{T_c}v\Delta t \quad (10)$$

$$\Delta M = G\Delta t + QC_0\Delta t - QC\Delta t - v_d CA\Delta t - \frac{C}{T_c}v\Delta t \quad (11)$$

where

$\omega = v_d$ C = deposition rate in kg/m^2 min

v_d = deposition velocity in m/min

A = deposition surface area in m^2

T_c = time constant in min

Following the same procedure of section 3.1 as above we have

$$v\frac{\Delta C}{\Delta t} = G + QC_0 - QC - v_d CA - \frac{C}{T_c}v$$

Simplifying further gives

$$v\frac{dC}{dt} = G + QC_0 - QC - v_d CA - \frac{C}{T_c}v$$

Solving the equation for C gives.

$$C = \frac{(G + QC_0) - \left(G + QC_0 - QC_i - v_d AC_i - \frac{C_i}{T_c}v\right)e^{-\lambda_2 t}}{\left(Q + v_d A + {}^{v}/_{T_c}\right)} \quad (12)$$

This is the mathematical model with ventilation, dry deposition and chemical removal effect.

where

$\lambda_2 = \lambda + \lambda_d + \lambda_c$ is the decay rate accounting for ventilation, deposition and chemical removal

$\lambda_d = \frac{v_d A}{V}$ is the particle deposition count

$\lambda_c = \frac{1}{T_c}$ is the chemical removal rate

With λ_d it is easy to determine the number of particulate matter deposited on either of the six room surfaces in a given time.

3.4. Macroscopic approach: well-mixed IAQ box model (single zone with sink, dry deposition and chemical removal)

To include sink, dry deposition and chemical removal effects equation (1) becomes

$$\Delta M = G\Delta t + QC_0\Delta t - QC\Delta t - Q_r\eta C\Delta t - v_d CA\Delta t - \frac{C}{T_c}v\Delta t \quad (13)$$

Following the same procedure of section 3.1 as above we have

$$v\frac{\Delta C}{\Delta t} = G + QC_0 - QC - Q_r\eta C - v_d CA - \frac{C}{T_c}v$$

Simplifying further gives

$$v\frac{dC}{dt} = G + QC_0 - QC - Q_r\eta C - v_d CA - \frac{C}{T_c}v$$

Solving the equation for C gives.

$$C = \frac{(G + QC_0) - \left(G + QC_0 - QC_i - Q_r\eta C_i - v_d AC_i - \frac{C_i}{T_c}v\right)e^{-\lambda_3 t}}{\left(Q + Q_r\eta + v_d A + {}^{v}/_{T_c}\right)} \quad (14)$$

where

$\lambda_3 = \lambda + \lambda_r + \lambda_d + \lambda_c$ is the decay rate accounting for ventilation, filtration, deposition and chemical removal

This is the mathematical model with ventilation, sink, dry deposition and chemical removal effect.

Equation (7), (9), (12) and (14) can be re-written by introducing the Average Smoking Count (ASC), n_{ave} according to Wayne (1999)

$$C = \left(C_0 + \frac{n_{ave}G_{cig}}{Q}\right) + \left(C_i - \frac{n_{ave}G_{cig}}{Q} - C_0\right)e^{-\lambda t} \quad (15)$$

$$C = \frac{(n_{ave}G_{cig} + QC_o) - (n_{ave}G_{cig} + QC_0 - QC_i - Q_r\eta C_i)e^{-\lambda_1 t}}{(Q + Q_r\eta)} \quad (16)$$

$$C = \frac{(n_{ave}G_{cig} + QC_o) - (n_{ave}G_{cig} + QC_0 - QC_i - Q_r\eta C_i - v_d AC_i)e^{-\lambda_2 t}}{\left(Q + v_d A + {}^{v}/_{T_c}\right)} \quad 17)$$

$$C = \frac{(n_{ave}G_{cig} + QC_o) - \left(n_{ave}G_{cig} + QC_0 - QC_i - Q_r\eta C_i - v_d AC_i - \frac{C_i}{T_c}v\right)e^{-\lambda_3 t}}{\left(Q + Q_r\eta + v_d A + {}^{v}/_{T_c}\right)} \quad (18)$$

where

$n_{ave} = \frac{G}{G_{cig}}$, therefore $G = n_{ave}G_{cig}$

G_{cig} is the average source strength per cigarette

At $C_o = C_i = 0$, i.e. no initial and outdoor concentration in the micro-environment the four model equations (15) to (18) reduces to the equations below

$$C = \left(\frac{n_{ave}G_{cig}}{Q}\right) - \left(\frac{n_{ave}G_{cig}}{Q}\right)e^{-\lambda t} \quad (19)$$

$$C = \frac{(n_{ave}G_{cig}) - (n_{ave}G_{cig})e^{-\lambda_1 t}}{(Q + Q_r\eta)} \quad (20)$$

$$C = \frac{(n_{ave}G_{cig}) - (n_{ave}G_{cig})e^{-\lambda_2 t}}{\left(Q + v_d A + {}^{v}/_{T_c}\right)} \quad (21)$$

$$C = \frac{(n_{ave}G_{cig}) - (n_{ave}G_{cig})e^{-\lambda_3 t}}{\left(Q + Q_r\eta + v_d A + {}^{v}/_{T_c}\right)} \quad (22)$$

These are the required four IAQ mathematical models that determine the steady state concentration (maximum concentration) at which the room occupants is exposed.

At $C_o = G_{cig} = 0$, i.e. cessation of smoking activity and no outdoor concentration in the micro-environment, the four model equations (15) to (18) reduces to the equations below

$$C = (C_i)e^{-\lambda t} \quad (23)$$

$$C = \frac{(QC_i + Q_r\eta C_i)e^{-\lambda_1 t}}{(Q + Q_r\eta)} \quad (24)$$

$$C = \frac{(QC_i + Q_r\eta C_i + v_d AC_i)e^{-\lambda_2 t}}{\left(Q + v_d A + {}^{v}/_{T_c}\right)} \quad (25)$$

$$C = \frac{\left(QC_i + Q_r\eta C_i + v_d AC_i + \frac{C_i}{T_c}v\right)e^{-\lambda_3 t}}{\left(Q + Q_r\eta + v_d A + {}^{v}/_{T_c}\right)} \quad (26)$$

These are the required four IAQ mathematical models that determine concentration decay after reaching steady state. C_i is the steady state value obtained from equation (19) to (22) above.

Table 1

Simulation Data

Parameter	Unit	Typical value	Min.	Max.	Source
Effective air flowrate (Q)	m^3/min		0.5	80	Wayne R. Ott, 1999
Recirculation air flowrate (Q_r)	m^3/min	5			Estimated
Air cleaner efficiency (η)		(50%) 0.5			Estimated
Room volume (v)	m^3	32.4			Measured
Total room area (A)	m^2	61.2			Measured
The average source strength per cigarette (for RSP) (G_{cig})	mg/min	1.43			Wayne R. Ott, 1999
The average source strength per cigarette (for CO) (G_{cig})	mg/min	11.9			Wayne R. Ott, 1999
Average smoking count (n_{ave})			0.33	5	Wayne R. Ott, 1999
Deposition velocity (v_d)	m/min	0.6			Steven R. H et al, 1982
Time constant (T_c)	min	167	60	2880	Steven R. H et al, 1982

4. RESULTS AND DISCUSSIONS

Table 2

Simulation for PM 2.5 with Model with Ventilation

Concentration, C against Average Smoking Count, n_{ave} and Effective Air Flowrate Q

n_{ave}	Effective Air Flowrate Q, m^3/min																	
	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00
0.33	795.40	460.23	235.81	157.30	117.97	94.38	78.65	67.41	58.99	52.43	47.19	23.60	15.73	11.80	9.44	7.87	6.74	5.90
0.50	1205.15	697.32	357.28	238.33	178.75	143.00	119.17	102.14	89.37	79.44	71.50	35.75	23.83	17.88	14.30	11.92	10.21	8.94
0.67	1614.90	934.41	478.76	319.36	239.52	191.62	159.68	136.87	119.76	106.46	95.81	47.91	31.94	23.95	19.16	15.97	13.69	11.98
1.00	2410.30	1394.65	714.56	476.66	357.50	286.00	238.33	204.29	178.75	158.89	143.00	71.50	47.67	35.75	28.60	23.83	20.43	17.88
1.33	3205.70	1854.88	950.37	633.96	475.47	380.38	316.98	271.70	237.74	211.32	190.19	95.10	63.40	47.55	38.04	31.70	27.17	23.77
1.50	3615.45	2091.97	1071.84	714.99	536.25	429.00	357.50	306.43	268.12	238.33	214.50	107.25	71.50	53.63	42.90	35.75	30.64	26.81
1.67	4025.20	2329.06	1193.32	796.02	597.02	477.62	398.02	341.16	298.51	265.34	238.81	119.41	79.60	59.70	47.76	39.80	34.12	29.85
2.00	4820.60	2789.29	1429.13	953.32	715.00	572.00	476.67	408.57	357.50	317.78	286.00	143.00	95.33	71.50	57.20	47.67	40.86	35.75
2.33	5616.00	3249.52	1664.93	1110.62	832.97	666.38	555.32	475.99	416.49	370.21	333.19	166.60	111.06	83.30	66.64	55.53	47.60	41.65
2.50	6025.75	3486.61	1786.41	1191.65	893.75	715.00	595.83	510.71	446.87	397.22	357.50	178.75	119.17	89.38	71.50	59.58	51.07	44.69
2.67	6435.51	3723.70	1907.88	1272.68	954.52	763.62	636.35	545.44	477.26	424.23	381.81	190.91	127.27	95.45	76.36	63.64	54.54	47.73
3.00	7230.91	4183.94	2143.69	1429.98	1072.50	858.00	715.00	612.86	536.25	476.67	429.00	214.50	143.00	107.25	85.80	71.50	61.29	53.63
3.33	8026.30	4644.17	2379.49	1587.28	1190.47	952.38	793.65	680.27	595.24	529.10	476.19	238.10	158.73	119.05	95.24	79.37	68.03	59.52
3.50	8436.06	4881.26	2500.97	1668.31	1251.25	1001.00	834.17	715.00	625.62	556.11	500.50	250.25	166.83	125.13	100.10	83.42	71.50	62.56
3.67	8845.81	5118.35	2622.45	1749.34	1312.02	1049.62	874.68	749.73	656.01	583.12	524.81	262.41	174.94	131.20	104.96	87.47	74.97	65.60
4.00	9641.21	5578.58	2858.25	1906.64	1430.00	1144.00	953.33	817.14	715.00	635.56	572.00	286.00	190.67	143.00	114.40	95.33	81.71	71.50
4.33	10436.61	6038.81	3094.06	2063.94	1547.97	1238.38	1031.98	884.56	773.99	687.99	619.19	309.60	206.40	154.80	123.84	103.20	88.46	77.40
4.50	10846.36	6275.90	3215.53	2144.97	1608.75	1287.00	1072.50	919.29	804.37	715.00	643.50	321.75	214.50	160.88	128.70	107.25	91.93	80.44
4.67	11256.11	6512.99	3337.01	2226.00	1669.52	1335.62	1113.02	954.01	834.76	742.01	667.81	333.91	222.60	166.95	133.56	111.30	95.40	83.48
5.00	12051.51	6973.23	3572.81	2383.30	1787.50	1430.00	1191.67	1021.43	893.75	794.44	715.00	357.50	238.33	178.75	143.00	119.17	102.14	89.38

Based on a source strength of 1.43 mg per min per cigarette from Wayne. (4).

$C_0 = 0\ \mu g/m^3$

$C_i = 0\ \mu g/m^3$

t = 120min

calculated for v = 32.4m^3

MODEL - MSE with ventilation only

$$C = \left(\frac{n_{ave} G_{cig}}{Q}\right) - \left(\frac{n_{ave} G_{cig}}{Q}\right) e^{-\lambda t}$$

Table 3

Simulation for PM 2.5 with Model with Ventilation and Sink

Concentration, C against Average Smoking Count, n_{ave} and Effective Air Flowrate Q

n_{ave}	Effective Air Flowrate Q, m^3/min																	
	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00
0.33	157.30	134.83	104.87	85.80	72.60	62.92	55.52	49.67	44.94	41.03	37.75	20.97	14.52	11.10	8.99	7.55	6.51	5.72
0.50	238.33	204.29	158.89	130.00	110.00	95.33	84.12	75.26	68.10	62.17	57.20	31.78	22.00	16.82	13.62	11.44	9.86	8.67
0.67	319.36	273.74	212.91	174.20	147.40	127.75	112.72	100.85	91.25	83.31	76.65	42.58	29.48	22.54	18.25	15.33	13.22	11.61
1.00	476.66	408.57	317.78	260.00	220.00	190.67	168.24	150.53	136.19	124.35	114.40	63.56	44.00	33.65	27.24	22.88	19.72	17.33
1.33	633.96	543.40	422.64	345.80	292.60	253.59	223.75	200.20	181.13	165.38	152.15	84.53	58.52	44.75	36.23	30.43	26.23	23.05
1.50	714.99	612.86	476.67	390.00	330.00	286.00	252.35	225.79	204.29	186.52	171.60	95.33	66.00	50.47	40.86	34.32	29.59	26.00
1.67	796.02	682.31	530.69	434.20	367.40	318.41	280.95	251.38	227.44	207.66	191.05	106.14	73.48	56.19	45.49	38.21	32.94	28.95
2.00	953.32	817.14	635.56	520.00	440.00	381.33	336.47	301.05	272.38	248.70	228.80	127.11	88.00	67.29	54.48	45.76	39.45	34.67
2.33	1110.62	951.97	740.42	605.80	512.60	444.25	391.99	350.73	317.32	289.73	266.55	148.08	102.52	78.40	63.46	53.31	45.96	40.39
2.50	1191.65	1021.43	794.44	650.00	550.00	476.67	420.59	376.32	340.48	310.87	286.00	158.89	110.00	84.12	68.10	57.20	49.31	43.33
2.67	1272.68	1090.88	848.47	694.20	587.40	509.08	449.19	401.91	363.63	332.01	305.45	169.69	117.48	89.84	72.73	61.09	52.66	46.28
3.00	1429.98	1225.71	953.33	780.00	660.00	572.00	504.71	451.58	408.57	373.04	343.20	190.67	132.00	100.94	81.71	68.64	59.17	52.00
3.33	1587.28	1360.54	1058.20	865.80	732.60	634.92	560.22	501.25	453.51	414.08	380.95	211.64	146.52	112.04	90.70	76.19	65.68	57.72
3.50	1668.31	1430.00	1112.22	910.00	770.00	667.33	588.82	526.84	476.67	435.22	400.40	222.44	154.00	117.76	95.33	80.08	69.03	60.67
3.67	1749.34	1499.45	1166.24	954.20	807.40	699.75	617.42	552.43	499.82	456.36	419.85	233.25	161.48	123.48	99.96	83.97	72.39	63.61
4.00	1906.64	1634.28	1271.11	1040.00	880.00	762.67	672.94	602.11	544.76	497.39	457.60	254.22	176.00	134.59	108.95	91.52	78.90	69.33
4.33	2063.94	1769.11	1375.98	1125.80	952.60	825.59	728.46	651.78	589.70	538.43	495.35	275.20	190.52	145.69	117.94	99.07	85.41	75.05
4.50	2144.97	1838.57	1430.00	1170.00	990.00	858.00	757.06	677.37	612.86	559.57	514.80	286.00	198.00	151.41	122.57	102.96	88.76	78.00
4.67	2226.00	1908.02	1484.02	1214.20	1027.40	890.41	785.66	702.96	636.01	580.70	534.25	296.80	205.48	157.13	127.20	106.85	92.11	80.95
5.00	2383.30	2042.85	1588.89	1300.00	1100.00	953.33	841.18	752.63	680.95	621.74	572.00	317.78	220.00	168.24	136.19	114.40	98.62	86.67

Based on a source strength of 1.43 mg per min per cigarette from Wayne. (4).

$C_0 = 0\ \mu g/m^3$

$C_i = 0\ \mu g/m^3$

t = 120min

calculated for v = 32.4m^3

MODEL - MSE with ventilation and sink effect

$$C = \frac{(n_{ave} G_{cig}) - (n_{ave} G_{cig}) e^{-\lambda_1 t}}{(Q + Q_r \eta)}$$

Table 4

Simulation for PM 2.5 with Model with MSE, Ventilation, deposition and chemical removal

Concentration, C against Average Smoking Count, n_{ave} and Effective Air Flowrate Q

n_{ave}	Effective Air Flowrate Q, m^3/min																	
	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00
0.33	12.61	12.45	12.13	11.82	11.54	11.26	11.00	10.75	10.51	10.28	10.06	8.29	7.05	6.14	5.43	4.87	4.41	4.04
0.50	19.11	18.86	18.38	17.92	17.48	17.06	16.66	16.28	15.92	15.57	15.24	12.56	10.69	9.30	8.23	7.38	6.69	6.12
0.67	25.61	25.27	24.62	24.01	23.42	22.86	22.33	21.82	21.33	20.87	20.42	16.84	14.32	12.46	11.02	9.89	8.96	8.20
1.00	38.23	37.72	36.75	35.83	34.95	34.12	33.33	32.57	31.84	31.15	30.48	25.13	21.37	18.59	16.45	14.76	13.38	12.23
1.33	50.84	50.17	48.88	47.65	46.49	45.38	44.32	43.31	42.35	41.43	40.54	33.42	28.42	24.73	21.88	19.63	17.79	16.27
1.50	57.34	56.58	55.13	53.75	52.43	51.18	49.99	48.85	47.76	46.72	45.73	37.69	32.06	27.89	24.68	22.13	20.06	18.35
1.67	63.84	62.99	61.37	59.84	58.37	56.98	55.65	54.39	53.18	52.02	50.91	41.96	35.69	31.05	27.48	24.64	22.34	20.43
2.00	76.45	75.44	73.50	71.66	69.91	68.24	66.65	65.13	63.68	62.30	60.97	50.25	42.74	37.19	32.91	29.51	26.75	24.46
2.33	89.06	87.89	85.63	83.49	81.44	79.50	77.65	75.88	74.19	72.57	71.03	58.55	49.80	43.32	38.34	34.38	31.17	28.50
2.50	95.56	94.30	91.88	89.58	87.39	85.30	83.31	81.42	79.60	77.87	76.21	62.82	53.43	46.48	41.13	36.89	33.44	30.58
2.67	102.06	100.71	98.13	95.67	93.33	91.10	88.98	86.95	85.02	83.16	81.39	67.09	57.06	49.64	43.93	39.40	35.71	32.66
3.00	114.68	113.16	110.25	107.49	104.86	102.36	99.98	97.70	95.52	93.44	91.45	75.38	64.12	55.78	49.36	44.27	40.13	36.69
3.33	127.29	125.61	122.38	119.32	116.40	113.62	110.97	108.45	106.03	103.72	101.51	83.67	71.17	61.92	54.79	49.14	44.54	40.73
3.50	133.79	132.02	128.63	125.41	122.34	119.42	116.64	113.98	111.45	109.02	106.69	87.95	74.80	65.08	57.59	51.65	46.82	42.81
3.67	140.29	138.44	134.88	131.50	128.28	125.22	122.30	119.52	116.86	114.31	111.88	92.22	78.44	68.24	60.39	54.15	49.09	44.89
4.00	152.90	150.88	147.01	143.32	139.82	136.48	133.30	130.27	127.37	124.59	121.94	100.51	85.49	74.37	65.82	59.02	53.50	48.93
4.33	165.51	163.33	159.13	155.15	151.35	147.74	144.30	141.01	137.87	134.87	132.00	108.80	92.54	80.51	71.24	63.89	57.92	52.96
4.50	172.01	169.74	165.38	161.24	157.30	153.54	149.97	146.55	143.29	140.17	137.18	113.07	96.17	83.67	74.04	66.40	60.19	55.04
4.67	178.51	176.16	171.63	167.33	163.24	159.34	155.63	152.09	148.70	145.46	142.36	117.34	99.81	86.83	76.84	68.91	62.46	57.12
5.00	191.13	188.60	183.76	179.15	174.77	170.60	166.63	162.83	159.21	155.74	152.42	125.64	106.86	92.97	82.27	73.78	66.88	61.16

Based on a source strength of 1.43 mg per min per cigarette from Wayne. (4).

$C_0 = 0\ \mu g/m^3$

$C_i = 0\ \mu g/m^3$

t = 120min

calculated for v = 32.4m^3

MODEL - MSE with ventilation, deposition and chemical removal effect

$$C = \frac{(n_{ave}\,G_{cig}) - (n_{ave}\,G_{cig})e^{-\lambda_2 t}}{\left(Q + v_d A + {}^{v}/_{T_c}\right)}$$

Table 5

Simulation for PM 2.5 with Model with MSE, Ventilation, sink, deposition and chemical removal

Concentration, C against Average Smoking Count, n_{ave} and Effective Air Flowrate Q

n_{ave}	Effective Air Flowrate Q, m^3/min																	
	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00
0.33	11.82	11.68	11.40	11.13	10.87	10.63	10.39	10.17	9.95	9.75	9.55	7.94	6.80	5.94	5.28	4.75	4.31	3.95
0.50	17.92	17.69	17.27	16.86	16.47	16.10	15.75	15.41	15.08	14.77	14.47	12.04	10.30	9.00	8.00	7.19	6.54	5.99
0.67	24.01	23.71	23.14	22.59	22.07	21.57	21.10	20.64	20.21	19.79	19.39	16.13	13.80	12.07	10.72	9.64	8.76	8.02
1.00	35.83	35.39	34.53	33.72	32.94	32.20	31.49	30.81	30.16	29.54	28.94	24.07	20.60	18.01	15.99	14.38	13.07	11.98
1.33	47.65	47.07	45.93	44.85	43.81	42.83	41.88	40.98	40.12	39.29	38.49	32.01	27.40	23.95	21.27	19.13	17.38	15.93
1.50	53.75	53.08	51.80	50.58	49.41	48.30	47.24	46.22	45.24	44.31	43.41	36.11	30.90	27.01	23.99	21.58	19.61	17.96
1.67	59.84	59.10	57.67	56.31	55.01	53.77	52.59	51.46	50.37	49.33	48.33	40.20	34.41	30.07	26.71	24.02	21.83	20.00
2.00	71.66	70.77	69.07	67.44	65.88	64.40	62.98	61.62	60.32	59.08	57.88	48.14	41.20	36.02	31.99	28.77	26.14	23.95
2.33	83.49	82.45	80.46	78.56	76.75	75.03	73.37	71.79	70.28	68.83	67.43	56.08	48.00	41.96	37.27	33.52	30.45	27.90
2.50	89.58	88.47	86.33	84.30	82.35	80.50	78.73	77.03	75.41	73.85	72.35	60.18	51.51	45.02	39.98	35.96	32.68	29.94
2.67	95.67	94.48	92.20	90.03	87.95	85.97	84.08	82.27	80.53	78.87	77.27	64.27	55.01	48.08	42.70	38.41	34.90	31.97
3.00	107.49	106.16	103.60	101.16	98.83	96.60	94.47	92.44	90.49	88.62	86.82	72.21	61.81	54.02	47.98	43.15	39.21	35.93
3.33	119.32	117.84	114.99	112.28	109.70	107.23	104.86	102.61	100.44	98.37	96.38	80.15	68.61	59.97	53.26	47.90	43.52	39.88
3.50	125.41	123.86	120.86	118.01	115.30	112.70	110.22	107.84	105.57	103.39	101.30	84.25	72.11	63.03	55.98	50.35	45.75	41.91
3.67	131.50	129.87	126.74	123.75	120.90	118.17	115.57	113.08	110.70	108.41	106.22	88.34	75.61	66.09	58.70	52.79	47.97	43.95
4.00	143.32	141.55	138.13	134.87	131.77	128.80	125.96	123.25	120.65	118.16	115.77	96.28	82.41	72.03	63.97	57.54	52.28	47.90
4.33	155.15	153.23	149.53	146.00	142.64	139.43	136.36	133.42	130.60	127.91	125.32	104.22	89.21	77.97	69.25	62.29	56.59	51.85
4.50	161.24	159.24	155.40	151.73	148.24	144.90	141.71	138.66	135.73	132.93	130.24	108.32	92.71	81.04	71.97	64.73	58.82	53.89
4.67	167.33	165.26	161.27	157.47	153.84	150.37	147.06	143.89	140.86	137.95	135.16	112.41	96.21	84.10	74.69	67.18	61.04	55.93
5.00	179.15	176.94	172.66	168.59	164.71	161.00	157.45	154.06	150.81	147.70	144.71	120.35	103.01	90.04	79.97	71.92	65.35	59.88

Based on a source strength of 1.43 mg per min per cigarette from Wayne. (4).

$C_0 = 0\ \mu g/m^3$

$C_i = 0\ \mu g/m^3$

t = 120min

calculated for v = 32.4m^3

MODEL - MSE with ventilation, sink, deposition and chemical removal effec

$$C = \frac{(n_{ave}\,G_{cig}) - (n_{ave}\,G_{cig})e^{-\lambda_3 t}}{\left(Q + Q_r\eta + v_d A + {}^{v}/_{T_c}\right)}$$

Table 6

Simulation for PM 2.5 with Model with Ventilation

Concentration, C against Average Smoking Count, n_{ave} and Time, t

n_{ave}	Time (min)											
	5.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	100.00	120.00
0.33	50.75	74.21	90.07	93.46	94.18	94.34	94.37	94.38	94.38	94.38	94.38	94.38
0.50	76.89	112.44	136.47	141.60	142.70	142.94	142.99	143.00	143.00	143.00	143.00	143.00
0.67	103.03	150.66	182.87	189.75	191.22	191.53	191.60	191.62	191.62	191.62	191.62	191.62
1.00	153.78	224.87	272.93	283.21	285.40	285.87	285.97	285.99	286.00	286.00	286.00	286.00
1.33	204.52	299.08	363.00	376.67	379.59	380.21	380.34	380.37	380.38	380.38	380.38	380.38
1.50	230.67	337.31	409.40	424.81	428.10	428.81	428.96	428.99	429.00	429.00	429.00	429.00
1.67	256.81	375.53	455.80	472.96	476.62	477.41	477.57	477.61	477.62	477.62	477.62	477.62
2.00	307.55	449.74	545.87	566.41	570.81	571.74	571.95	571.99	572.00	572.00	572.00	572.00
2.33	358.30	523.95	635.94	659.87	664.99	666.08	666.32	666.37	666.38	666.38	666.38	666.38
2.50	384.44	562.18	682.34	708.02	713.51	714.68	714.93	714.99	715.00	715.00	715.00	715.00

Based on a source strength of 1.43 mg per min per cigarette from Wayne. (4).

$C_0 = 0\ \mu g/m^3$

$C_i = 0\ \mu g/m^3$

calculated for v = 32.4m^3

MODEL - MSE with ventilation only

$$C = \left(\frac{n_{ave}G_{cig}}{Q}\right) - \left(\frac{n_{ave}G_{cig}}{Q}\right)e^{-\lambda t}$$

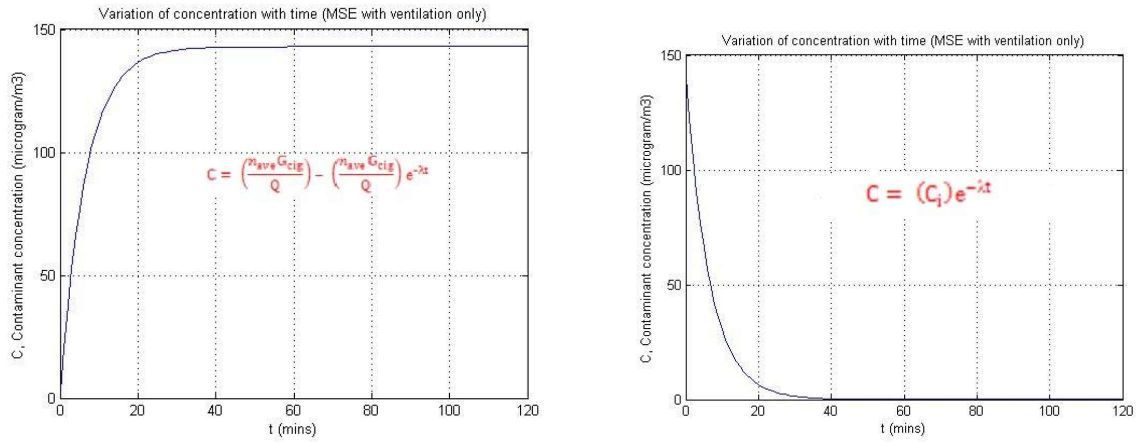

Figure 4 and 5

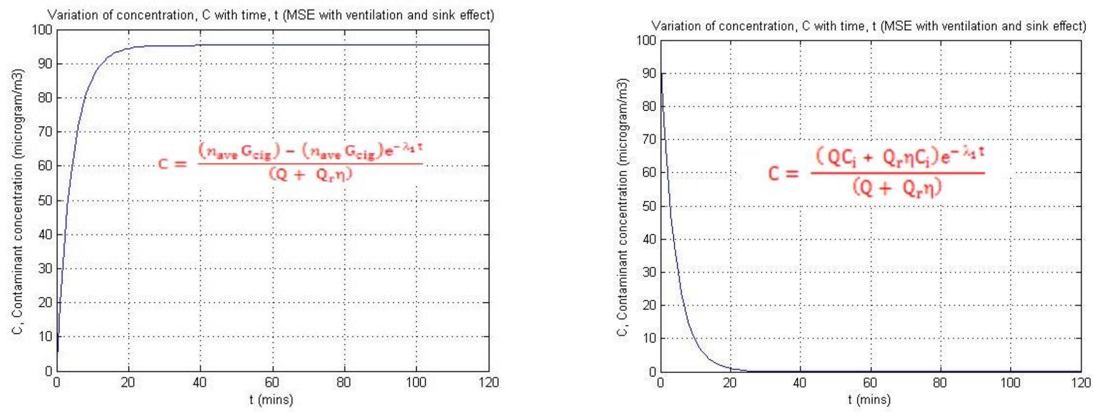

Figure 6 and 7

RSP is consdered in the application of the ETS models. According to the U.S. Environmental Protection Agency's (U.S. EPA), the National Ambient Air Quality Standard (NAAQS) for fine particles ($PM_{2.5}$), which is very close in size to RSP ($PM_{3.5}$), is 65µg/m^3 for 24 hr. Because the mass concentration of $PM_{3.5}$ includes the mass concentration of $PM_{2.5}$, maintaining $PM_{3.5}$ below a certain concentration insures that $PM_{2.5}$ also will be below that concentration. [4].

The purpose of this paper is to develop simple mathematical models that can be used to determine contaminant concentration levels in a micro-environment. This determination will assist in establishing risk levels of occupants in the environment. In achieving this four models were developed to simulate smoking activities in the modeled space. The four models were applied over a period of 120 min (2hrs) using data from Wayne [4]. The models differ

from one another in the number of RMs input into the model. This enables us to show the influence of this RMs in an IAQ model. In most IAQ models one or two of these RMs are considered while in reality all these of these RMs contribute simultaneously in reduction of indoor contaminants. Fromme [9] describes all the possible physico-chemical behaviours of contaminants in a given space. If these characteristics take place within the space over a given period, then a true representation of indoor concentration levels cannot be properly modeled without considering all these behaviours. Radha et al [8] also noted the influence of these physico-chemical transformations on the final concentration of PM in the micro-enviromeent.

In the four developed models, equation (15) and (16) represented what is mostly considered by other researchers while equations (17) and (18) took them some steps further for more realistic results. The four models are further divided into two parts depending on the applied conditions. The first part, comprising of equations (19) to (22) is considered for no initial and outdoor concentration. These models give the final or maximum concentration attained when the smoking activity pattern is maintained within a given period. Each ASC used for the simulation describes the smoking pattern within the period. The final concentration is also known as the STEADY STATE VALUE (SSV). This is the maximum possible indoor occupant exposure level within a given period and specific values of input RMs. The second part, comprising of equations (23) to (26) is considered for no outdoor concentration and cessation of smoking activities. These determine the decay pattern of contaminant after reaching steady state when the source is removed.

The result of simulations carried out using the models is presented in Table 2 to 6 and Figure 4 to 7. Table 2 compare perfectly with the analysis by Wayne [4]. The data in Table 2 to 5 are all SSVs showing exposure level at different flowrates. . These tables show progressive improvement in concentration levels due to the presence of increasing number RMs. The yellow portion of the tables shows safe levels according to NAAQS guideline. The unshaded portion shows concentration at levels very dangerous to human health. Table 4 and 5 shows more exact values of concentration levels. For environment with natural or mechanical ventilation, Table 4 shows that safe levels can be reached at low flowrate. And for environment including filtration systems Table 5 indicates that better IAQ will be achieved faster at very low flowrate. This discovery provides a very good energy saving option for environment using mechanical ventilation or a combination of mechanical ventilation and filtration system. In contrast, Table 2 and 3 does not show good IAQ levels and energy saving options.

Table 6 shows variation of concentration with ASC and time. The SSV is reached after 60mins (1hr) of commencement of smoking activity. The concentration remains constant (at $t = \infty$) as long as the smoking pattern and flowrate is maintained. The blue portion of the table show the SSVs which correspond to the blue portion in Table 3. Figure 4 to 7 gives the concentration build and decay pattern. Figure 4 and 5 correspond to the red portion of Table 2 while Figure 6 and 7 correspond to the red portion of Table 3.

5. CONCLUSION

In course of this paper we have been able to show that a simple, easy to use IAQ model can be developed using the law of conservation of mass. These macroscopic models can be used to determine build and decay pattern of concentration in a micro-environment. More importantly, we have established that an IAQ model cannot produce a near perfect result without taken into consideration the physico-chemical transformations which take place within the micro-environment. Our findings reveals that the more of these transformations considered the better the result of the simulation. The improved results also showed that considerable energy savings can be achieved by operating energy consuming IAQ improving devices at low values and also achieving good IAQ levels for human safety and health.

Our models also produced the deposition count (as against the deposition rate) which gives the number of PMs deposited on room walls, ceiling and floor per time. The deposition rate according Steven R. H et al is the mass per unit area per unit time. This is to establish the difference from the deposition rate in Wayne [4]. However, to be within standard one needs to combine effective air flow rate and ASC within the yellow-shaded portion of the Tables. Otherwise, take a conservative approach within the unshaded portion with little or no margin of safety. For homes without HVAC or filtration systems and relying only on natural ventilation, Table 4 gives a more realistic result useful for predicting IAQ levels of safety for room occupants. On the other hand, home with these systems, as shown in Table 5, stand a better chance of achieve much better IAQ levels with possible energy savings.

The models though used for a single residential home, is very much applicable and useful for restaurants, night clubs, airport smoking lounge and other smoking facilities. This research, however, did not consider all the possible transformations in the micro-environment. There is need as a way of further improvement to include effect of infiltration, exfiltration and re-suspension in a single IAQ model to produce a perfect result.

REFERENCES

[1] Fakoya O. (2008) Nigeria: The Health, Economic and Social Menace of Smoking - Time for Concerted Actions.

[2] Desalu O, Olokoba A, Danburam A, Salawu F and Issa B. (2007) Epidemiology Of Tobacco Smoking Among Adults Population In North-East Nigeria. The Internet Journal of Epidemiology. ISPUB.com / IJE/6/1/9680. Volume 6 Number 1.

[3] Salawu F, Danburam A, Issa B and Agbo J. (2009) Cigarette smoking habits among adolescents in northeast, Nigeria. The Internet Journal of Epidemiology. ISPUB.com / IJE/8/1/4789. Volume 8 Number 1.

[4] Ott W. R. (1999) Mathematical Models for Predicting Indoor Air Quality from Smoking Activity. Environ Health Perspective Volume 107 Supplementary 2: 375-381.

[5] Ott W. R., Klepeis N. E. and Switzer P. (2003) Analytical Solutions to Compartmental Indoor Air Quality Models with Application to Environmental Tobacco Smoke Concentrations Measured in a House. Journal of the Air & Waste Management Association, 53:8, 918-936, DOI: 10.1080/10473289.2003.10466248.

[6] WHO guidelines for indoor air quality: selected pollutants. (2010). The WHO European Centre for Environment and Health, Bonn Office, ISBN 978 92 890 0213 4.

[7] Demokritou P. Introduction to Indoor Air Quality Modeling," Indoor Environmental Quality & Health. Harvard School of Public Health.

[8] Goyal R. and Khare M. (2010) Indoor Air Quality and Ventilation Modeling. Indo-French Indoor Air Quality Seminar, Nantes.

[9] Fromme H. (2012) Particles in the Indoor Environment. Air Quality – Monitoring and Modeling, InTech ISBN 978-953-51-0161-1.

[10] Hanna S. R. Briggs G. A. Hsker Jr R. P. (1982) Handbook on Atmospheric Diffusion" Office of Health and Environmental Research, Ofilce of Energy Research, Technical information Center, U.S. Department of Energy, DOE/TIC-11223 (D E82002045), ISBN 0-87079-127-3.

[11] Furtaw Jr. E. J. Pandian M. D. Nelson D. R. and Behar J. V. (1996) Modeling Indoor Air Concentrations Near Emission Sources in Imperfectly Mixed Rooms. Journal of the Air & Waste Management Association, 46:9, 861-868, DOI: 10.1080/10473289.1996.10467522.

[12] Johnson M. Edwards R. Morawska L. Smith K. R. and Nicas M. WHO IAQ guidelines: household fuel combustion – Review 3: Model linking emissions and air quality.

[13] Geber A. Hofen-Hohloch A. V. Schulze J. and Groneberg D. A. (2015) Tobacco smoke particles and indoor air quality (ToPIQ-II) – a modified study protocol and first results. Journal of Occupational Medicine and Toxicology, 10:5DOI 10.1186/s12995-015-0047-8.