# Mathematical Models for Predicting Indoor Air Quality from Smoking Activity

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Much progress has been made over four decades in developing, testing, and evaluating the performance of mathematical models for predicting pollutant concentrations from smoking in indoor settings. Although largely overlooked by the regulatory community, these models provide regulators and risk assessors with practical tools for quantitatively estimating the exposure level that people receive indoors for a given level of smoking activity. This article reviews the development of the mass balance model and its application to predicting indoor pollutant concentrations from cigarette smoke and derives the time-averaged version of the model from the basic laws of conservation of mass. A simple table is provided of computed respirable particulate concentrations for any indoor location for which the active smoking count, volume, and concentration decay rate (deposition rate combined with air exchange rate) are known. Using the indoor ventilatory air exchange rate causes slightly higher indoor concentrations and therefore errs on the side of protecting health, since it excludes particle deposition effects, whereas using the observed particle decay rate gives a more accurate prediction of indoor concentrations. This table permits easy comparisons of indoor concentrations with air quality guidelines and indoor standards for different combinations of active smoking counts and air exchange rates. The published literature on mathematical models of environmental tobacco smoke also is reviewed and indicates that these models generally give good agreement between predicted concentrations and actual indoor measurements. — *Environ Health Perspect* 107(Suppl 2): 375–381 (1999). *http://ehpnet1.niehs.nih.gov/docs/1999/Suppl-2/375-381ott/abstract.html*

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Considerable progress has been made over four decades in developing, testing, and validating mathematical models to predict the pollutant concentrations present in indoor settings due to smoking activity. Many of these models are summarized in a review by Repace (*1*) and a more recent technical paper (*2*) showing that all the models have a similar mathematical structure. All these mathematical models use the mass balance equation, which is based theoretically on the physical law of conservation of mass, to calculate the concentrations in indoor settings from a knowledge of the source strength (quantity emitted by the tobacco source per unit time), the volume of the indoor setting (for example, cubic meters), the effective air exchange rate [quantity of replacement air infiltrating per unit time expressed as air changes per hour (ach)], and the nonventilatory pollutant loss rate (quantity of pollutant lost indoors to particle deposition or chemical reactions). The mass balance model, or "mass balance law," is based on the law of conservation of mass: in classical physics, mass can be neither created nor destroyed. This law accounts for all the mass emitted, present, or lost, and it allows prediction of indoor pollutant concentrations based on the net flow of mass.

This article reviews the history of the development of the mass balance model and its application to predicting indoor concentrations from cigarette smoking. We derive the time-averaged form of the model theoretically from the law of conservation of mass and illustrate how to predict indoor respirable suspended particle (RSP) concentrations from cigarettes using a convenient table. Other versions of the model, such as the recursive, instantaneous, and minute-by-minute forms, are solutions to the same basic differential equation and are discussed elsewhere in the literature (*3–5*).

Experimental results show that these models can predict indoor pollutant concentrations from smoking activity in indoor settings with high accuracy (*4–6*). These models offer an easy-to-use, practical tool for predicting indoor air quality levels with acceptable accuracy for estimating the health risks of smoking indoors. The models are especially useful for calculating the conditions required to achieve and maintain acceptable indoor air quality standards. These models allow one to predict indoor pollutant concentrations from a knowledge of the smoking activity, which can be counted easily by an observer, and from other variables that are easily measured or estimated in real settings such as offices, taverns, smoking lounges, stores, etc.

## Historical Background

Various investigators have used the mass balance equation to predict pollutant concentrations from tobacco smoke sources in indoor locations for over four decades. In 1960 Brief (*7*) proposed a simple graph to determine transient concentrations for pollutants in indoor settings that is based on an exponential decay as a function of time. Soon afterwards, Turk (*8*) proposed a general equation for calculating the concentrations in a chamber that includes both exterior and interior sources, as well as the removal effect of pollutants by air treatment systems. In 1972, Bridge and Corn (*9*) reported that a solution to the equations proposed by Turk (*8*) adequately predicts the level of tobacco smoke in occupied spaces. In 1974, Jones and Fagan (*10*) used Turk's equation to calculate carbon monoxide (CO) concentrations from cigarette smoke plotted versus time in an office building and a single-family dwelling. In 1980 Ishizu (*11*) examined experimentally the inclusion of a "mixing factor" into these

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Abbreviations used: ach, air changes per hour; ASC, active smoking count; CO, carbon monoxide; U.S. EPA, U.S. Environmental Protection Agency; ETS, environmental tobacco smoke; $g$, pollutant emission rate; $g_{ave}$, $\bar{g}$, average pollutant emission rate; $g_{cig}$, pollutant emission rate for a cigarette; $g(T)$, average source emission rate over time $T$; NAAQS, national ambient air quality standard; $n(t)$ number of active smokers (ASC) in a room at time $t$; $n_{ave}$, average active smoking count; $PM_{2.5}$, concentration of particles of size 2.5 μ and smaller; $PM_{3.5}$, concentration of particles of size 3.5 μ and smaller; $PM_{10}$, concentration of particles of size 10 microns and smaller; RSP, respirable suspended particles; $Q_{in}(T)$, amount of pollutant entering the mixing volume over time $T$; $Q_{out}(T)$, amount of pollutant exiting the mixing volume over time $T$; $SF_6$, sulfur hexafluoride; $t$, time; $T$, special time period of interest; $\tau$, air residence time; $w$, air flow rate; $z(t)$, $z$, pollutant concentration at time $t$, $z(T)$, average concentration over time $T$; $\Delta z$, change in concentration; $\phi$, air exchange rate; $\phi_P$, particle decay rate.

models; Repace and Lowrey (*12*) also developed a modification of the Turk equation incorporating a mixing factor. In 1992 Ott et al. (*4*) developed an instantaneous form of the model for use with a single cigarette smoked according to any time sequence. In 1996, Klepeis et al. (*5*) developed a multiple-smoker version of the instantaneous model and validated it using minute-by-minute smoking counts and real-time measurements of particle concentrations in public smoking lounges. Ott et al. (*13*) showed theoretically that a mathematical trend correction term should be incorporated into the time-averaged version of the model to make it exact, and they tested the model's performance on 76 visits to a 521-$m^3$ sports tavern over a 3-year period. Two literature reviews describe these indoor air quality models for cigarette smoke in greater detail (*1,2*).

The concentrations of pollutants from environmental tobacco smoke (ETS) in a large mixing volume such as a room have been observed to follow a specific equation (*14*) with an exponential form once a cigarette is ignited; similarly, the concentrations have been observed to decay exponentially once the cigarette ends (*1–2,4–16*). Many studies have included measurements of CO and RSP (or $PM_{3.5}$, particulate matter 3.5 μ in diameter or less) as well as other constituents of tobacco smoke (*15–19*). The exponential functions observed in indoor settings (*14*) are valid solutions to the mass balance equation for the case of a source that emits at a fixed rate when it is on—and at zero rate when it is off—with a fixed effective air exchange rate. This source can be viewed as a rectangular input time series (concentration as a function of time) to the mass balance model. The mass balance equation can be derived theoretically in just a few steps from the principles of conservation of mass and entering and leaving the mixing volume and the assumption that the concentration at any instant of time is uniform throughout the mixing volume.

## Smoking Activity Patterns

Each smoker ordinarily engages in a sequential smoking activity pattern over time: one cigarette is smoked after another, with a recovery period between each cigarette. Thus, a person in a space (an office, an automobile, a smoking lounge, a restaurant) with a smoker is exposed to a time series of concentrations resulting from a succession of cigarettes that reflects the smoking activity patterns of the smoker.

If there are many smokers, indoor air quality modeling requires data on their smoking activities. If more than five smokers are present, then many observers may be needed to record the exact beginning and end of each person's cigarette. Collecting this type of activity data in real settings is difficult because the multiple observers may interfere with the activities of the smokers. A simpler approach than using multiple observers is to count the number of cigarettes being actively smoked in the room at fixed time intervals. Using this approach, an investigator typically walks around the room at fixed time periods (for example, once every 5 min) and counts the total number of persons and the total number of cigarettes that are actively burning. The average of these counts—the active smoking count (ASC) represented mathematically as $n(t)$—provides a convenient measure of the amount of smoking activity in any indoor microenvironment (*5,13*). If one person were present who smoked a cigarette for 10 min every 20 min, we would conclude that a cigarette is being smoked, on the average, one-third of the time, and the average ASC would be $n_{ave} = 1/3$ cigarette. If three such persons were present in the room, then the ASC would be $n_{ave} = 1.0$ cigarettes, the same result as for a single cigarette smoked continuously over the entire sampling period.

Ott et al. (*4*) apply the basic mass balance model to the case of a sequence of cigarettes smoked one after another; they evaluate the validity of the model in a chamber with a smoking machine and an automobile with a real smoker. They also theoretically derive general expressions for the minimum, maximum, and mean pollutant concentration in a well-mixed microenvironment for any cigarette smoking activity pattern. In these field experiments in an automobile and a chamber, the predicted pollutant concentration as a function of time (the calculated time series of concentrations) shows excellent agreement with concentrations measured using instruments such as high resolution, real-time electrochemical CO monitors (*20*) capable of monitoring continuously or in real time.

The equations in Ott et al. (*4*) are consistent with previous ETS indoor air quality models derived by Repace (*21*) and Repace and Lowrey (*12,15*). Repace (*1*) described a person with uniform smoking activity (fixed cigarette duration and same time between cigarettes) as a habitual smoker. Ott et al. (*4*) derive general expressions for the habitual smoker and for multiple habitual smokers. Repace (*1*) considers the special case in which the habitual smoker smokes two cigarettes per hour, which is based on a national average smoking rate. When the parameter values used in Repace's habitual smoker model are substituted into the time series model of Ott et al. (*4*), the two models agree. Thus, recently published research establishes the universality of models for predicting concentrations indoors, such as the models of Repace and Lowrey (*12,15*), although a mixing factor appears in some of the earlier models that is not used in the later ones. In summary, all the models in the literature are essentially the same because they are derived from the same basic equation describing conservation of mass, the mass balance model.

The exponential solutions to the mass balance equation (*14*) provide a theoretical basis for calculating all the parameters of the model—air exchange rate, source strength, and sink removal terms—in a single experiment (*4*). The air exchange rate can be determined from the exponential decay of concentrations in the microenvironment. The source strength can be determined from the equilibrium concentration with known smoking activity. The sink removal term for pollutants that adhere to surfaces, such as particles, can be determined by subtracting the particle decay rate from the decay rate for a pollutant that has no surface sinks, such as CO or sulfur hexafluoride ($SF_6$).

As discussed by Klepeis (*6*), several investigators have studied an important assumption in the mass balance model—that the pollutant is well mixed using experimental measurements at multiple points in chambers (*4,22,23*), at a tavern (*13,24*), and at airport smoking lounges (*5*). Mage and Ott (*24*) describe each indoor air pollution episode as separable into an alpha period, a beta period, and a gamma period. The alpha period describes the time during which the cigarette source is actively burning; the room is not yet well mixed because high concentrations occur very close to this emission point source, as we might expect. When the cigarette is extinguished, a transition time period can be identified—the beta period—in which the room changes from poorly mixed to a well-mixed state in which the ratio of the standard deviation of the concentrations throughout the room to the mean is less than 0.10 (that is, the spatial coefficient of variation is less than 10%). After the beta

period, the room enters the gamma or well-mixed period, which lasts until the room concentration is 1% of its maximum value attained at the beginning of the beta period. During the gamma period, the source no longer is emitting, and concentrations decay exponentially at all locations in the mixing volume. For a cigarette, the alpha period is equal to the duration of active smoking, or 7–10 min, which typically is much less than the gamma period.

Use of the ratio of the standard deviation-to-mean of 0.10 as a criterion for a well-mixed room is based on research by Baughman et al. (*22*), who released $SF_6$ tracer gas in a room with a low air exchange rate and measured concentrations at 41 different points. Prior to development of the Baughman-Gadgil-Nazaroff criterion for a well-mixed microenvironment, the American Society for Testing Materials (ASTM) judged a room to be well mixed if the concentrations at multiple points were within ±10% overall average concentration (ASTM E 741) (*24*). These two criteria for uniform mixing are very similar, although they are theoretically different from a mathematical standpoint.

Using measurements of CO and RSP in a 521-$m^3$ tavern after four cigars were smoked at a central table, Mage and Ott (*24*) show experimentally that the average concentrations measured at three separated points in the tavern were very similar when averaged over the entire episode. A person sitting at either of two corner booths in the tavern 12 m (about 36 ft) apart would receive an exposure within ± 10% of the exposure at the central table where the cigars were actually smoked. The uniformity of concentration occurs because pollutants rapidly mix in indoor microenvironments because of natural convection and the effect of human movement on air mixing within the setting. Like other real settings I have studied with smokers present, this tavern satisfactorily met the theoretical criteria for uniform mixing that allowed the mass balance equation to be applied.

Klepeis et al. (*5*) report similar uniformity of concentrations in two airport public smoking lounges. They visited the two lounges 5 times each and counted smokers every minute while recording CO and RSP concentrations at 1- and 2-min intervals, respectively. They used three piezoelectric microbalances to measure fine particle concentrations at three widely spaced points in these lounges and report that the average difference in fine particle concentrations across the room was 12%. Their airport lounge studies permit estimates of the average source strength per cigarette under realistic smoking conditions, which were 11.9 mg/min for CO and 1.43 mg/min for RSP (*5*). Again, these lounges, like other locations where smokers typically are present, were sufficiently well mixed to allow the mass balance model to be applied with high accuracy.

Mage and Ott (*24*) note that the cigarette is especially well suited for making accurate predictions using the mass balance equation in most typical settings because of its short alpha period compared with its typical gamma period. Typically, each cigarette is smoked for only 7 to 10 min, whereas the average residence time [time required for the concentration to reach 1/e, or 37% times its maximum value; (*14*)] of the air in most indoor microenvironments is much greater than 10 min. The ventilatory residence time is the reciprocal of the air exchange rate. An indoor location with an air exchange rate of two air changes per hour (ach) has a residence time of 30 min, and the gamma period for this situation is $4.6 \times 30$ min = 138 min, since 138 min is required to reach 1% of the initial maximum concentration. The gamma period multiplier results from the natural logarithm of 1%, or ln (0.01) = 4.6. If we ignore the beta period, which appears to be quite small in real settings where experiments have been conducted, then the proportion of the total episode in which the air is poorly mixed will be (10 min)/(10 min + 138 min) = 0.068, or poorly mixed for only 6.8% of the total time. If multiple cigarettes are smoked, then each cigarette, by the law of superposition, will contribute concentrations that are poorly mixed for only a relatively short time (for example, 10 min vs 138 min). The short alpha period relative to the gamma period for each cigarette helps explain theoretically why Klepeis et al. (*5*) found such excellent agreement between predicted and observed concentrations in airport smoking lounges, even when multiple smokers were present. Each cigarette has its own brief alpha period relative to the longer gamma period. On the basis of the law of superposition (the concentrations caused by each source of the same pollutant in an indoor setting are linearly additive), each cigarette's poorly mixed time period is relatively small. Thus, for the cigarette, the requirement that the room be well mixed is almost always met in typical locations where smokers are found, and even the minute-by-minute concentrations predicted by the mass balance equations usually follow the minute-by-minute observed concentrations quite well (*5*).

## Derivation of Mass Balance Equation

Several books discuss the derivation of the mass balance equation and its application to predicting indoor air pollutant concentrations (*25–27*), and a variety of scientific papers discuss its use in determining source emissions and predicting indoor air quality levels (*28–31*). Many investigators also have applied the mass balance equation to predicting ETS from cigarette smoking in indoor settings (*1,2,4–13, 15,16*), and, where measurements are available, their results usually are in good agreement with observed and predicted indoor concentrations from smoking.

Although derivation and application of this model are described in the scientific literature, the model can be derived theoretically in only a few mathematical steps. Because these steps give insight into the basis for the model and are easy to follow, it is instructive to present them here. Consider the well-mixed volume such as a room or a chamber in Figure 1 with an internal concentration $z = z(t)$. Suppose that $Q_{in}(T)$ denotes the total amount of

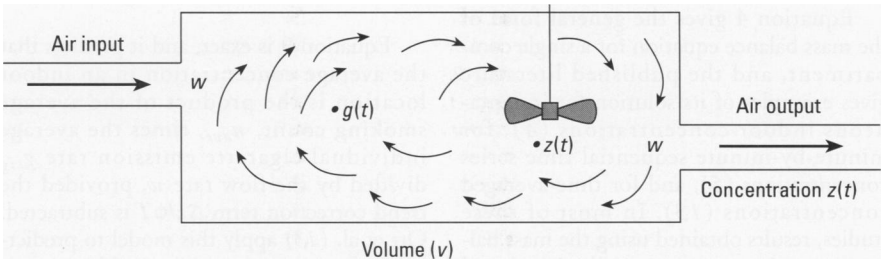


**Figure 1.** Chamber containing cigarette source $g(t)$ with input and output air flow rate $w$. This drawing represents a typical room (an office, lounge, automobile, tavern) with air that is well-mixed because of normal convection and human activities.

pollutant that has entered the mixing volume, $v$, from the initial time $t = 0$ to some time, $t = T$, and $Q_{out}(T)$ denotes the total amount of pollutant that has been removed from time $t = 0$ to time $t = T$:

$$Q(T)_{in} - Q(T)_{out} = vz(T) \quad [1]$$

This equation states that the difference between the total amount of pollutant that has entered the room and the amount that has departed at time $T$ is the amount remaining in the room, $vz(T)$, the product of the volume, $v$, and the concentration $z(T)$, which is everywhere the same throughout the room based on the assumption of uniform mixing. If a cigarette emitting at a rate (mass/time) of $g$ at time $t$ is the only source within the room and if the exit air flow rate (volume/time) is $w$, then the mass exit rate (mass/time) is $wz(T)$. Simplifying the notation by letting $z = z(t)$, Equation 1 can be then be written as:

$$\int_0^T g dt - \int_0^T wz dt = vz \quad [2]$$

Differentiating Equation 2, we obtain:

$$g - wz = v\frac{dz}{dt}, \quad [3]$$

where

$v$ = volume,
$w$ = air flow rate (volume/time),
$z$ = concentration (mass/volume), and
$g$ = cigarette emission rate (mass/time).

Rearranging the terms and substituting $\phi = w/v$:

$$\frac{1}{\phi}\frac{dz}{dt} + z = \frac{g}{w} \quad [4]$$

where

$\frac{w}{v} = 1/\tau = \phi$
$\tau$ = *air residence* time, and
$\phi$ = *air exchange* rate.

Equation 4 gives the general form of the mass balance equation for a single compartment, and the published literature gives examples of its solution for instantaneous indoor concentrations (*4*), for minute-by-minute sequential time series concentrations (*5*), and for time-averaged concentrations (*13*). In most of these studies, results obtained using the mass balance equation compare well with actual measurements. Because health concerns about air quality usually involve air quality standards, which specify concentrations over particular averaging times, the following discussion deals with the exact time-weighted average solution of the model, which is useful for many applications.

Consider an important refinement to Equation 1 that includes an initial condition $z(0)$ time $t = 0$:

$$Q(T)_{in} - Q(T)_{out} = vz(T) - vz(0) \quad [5]$$

Suppose we now divide all terms in this equation by $T$:

$$\frac{1}{T}\int_0^T g dt - \frac{1}{T}\int_0^T wz dt = \frac{v[z(T) - z(0)]}{T} \quad [6]$$

Inspection of Equation 6 shows that the first term is the time-weighted average of the source strength, whereas the second term is the time-weighted average of the product of the flow rate $w$ and the average concentration inside the mixing volume. The right side of the equation, on the other hand, is the change in concentration over the averaging time $T$, or $\Delta z = [z(T) - z(0)]$. In the following equations, the bars above the terms denote the average value of the terms.

$$\overline{g(T)} - \overline{wz(T)} = \frac{v\Delta z}{T} \quad [7]$$

These equations are discussed elsewhere in the literature in greater detail (*2,4,13*). Solving Equation 7 for the average concentration within the mixing volume, we obtain the following important result:

$$\overline{z(T)} = \frac{\bar{g}}{w} - \frac{v\Delta z}{wT} \quad [8]$$

If the average source strength per cigarette is $g_{cig}$ and the average smoking count is $n_{ave}$, then $g = n_{ave}g_{cig}$, and substituting $\phi = w/v$, then Equation 8 can be written as follows:

$$\overline{z(T)} = \frac{n_{ave}g_{cig}}{w} - \frac{\Delta z}{\phi T} \quad [9]$$

Equation 9 is exact, and it predicts that the average concentration in an indoor location is the product of the average smoking count, $n_{ave}$, times the average individual cigarette emission rate $g_{cig}$ divided by the flow rate $w$, provided the trend correction term $\Delta z/\phi T$ is subtracted. Ott et al. (*13*) apply this model to predicting indoor concentrations inside a sports tavern where measurements of particle concentrations were available on 76 dates over a 3-year period.

If the initial and final concentrations in the room are the same, then $\Delta z = 0$ and the trend correction term disappears. Similarly, if $T$ is very large compared with $1/\phi$, then the trend correction term can be ignored because it approaches zero. Since $\phi = 1/\tau$, this trend correction term is negligible if $T$ is very large relative to $\tau$, or $T >> \tau$. For this case, this result can be stated in words as the mass balance law: The average concentration in a well-mixed indoor setting is computed as the source strength divided by the product of the volume of the setting and the air exchange rate of the setting.

## Application of ETS Model

We can illustrate application of the model by using emission rates of various pollutants from cigarettes, which are found in the literature (*4,5,32–37*). To illustrate this model by applying it to respirable suspended particles (RSP or $PM_{3.5}$), we choose the value for $g_{cig} = 1.43$ mg/min from Klepeis et al. (*5*) based on experimental data with real smokers on 10 visits to smoking lounges at two international airports. To make the model's application for predicting RSP concentrations as convenient as possible, this article includes a table (Table 1) giving the average RSP concentration in $\mu g/m^3$ as a function of different active smoking counts (ASCs) ranging from $n_{ave} = 0.33$ cigarettes to $n_{ave} = 10.0$ cigarettes. The values of $w$ are listed at the top of Table 1 ranging from 0.5 to 80 $m^3$/min. For particles, we compute the effective air flow rate $w = \phi_P v$ instead of $w = \phi v$, where $\phi_P$ is the particle decay rate [1/Time; (*4*)] and $v$ is the volume (Length$^3$). Thus, $\phi_P$ is the effective air exchange rate that takes into account both particle deposition and the ventilatory air exchange rate. Thus, $\phi_P > \phi$, and Klepeis et al. (*5*) report $\phi_P = 1.2\ \phi$ for smoking lounges.

The average indoor RSP concentration based on Equation 9 for different ASC levels and effective air flow rates is summarized in Table 1, which facilitates computation in practical applications. Two areas of this table are shaded, one darker than the other. Maintaining indoor RSP concentrations below 150 $\mu g/m^3$ requires one to avoid combinations of effective air flow rates and ASCs in the darkest shaded region. Similarly, maintaining indoor RSP concentrations below 65 $\mu g/m^3$ requires one to avoid combinations of effective air flow rates and ASCs in the lightly shaded region. The U.S. Environmental Protection Agency's (U.S. EPA) new National Ambient Air Quality Standard (NAAQS) for fine

**Table 1.** Respirable particle concentration (μg/m$^3$) in a room as a function of air flow rate $w$ (m$^3$/min) and the active smoking count $n_{ave}$ (average number of cigarettes smoked).[a]

| $n_{ave}$ | Effective air flow rate $w=\phi v$, m$^3$/min | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| 0.33 | 953 | 477 | 238 | 159 | 119 | 95.3 | 79.4 | 68.1 | 59.6 | 5..0 | 47.7 | 23.8 | 15.9 | 11.9 | 9.5 | 7.9 | 6.8 | 6.0 |
| 0.50 | 1430 | 715 | 358 | 238 | 179 | 143 | 119 | 102 | 89.4 | 79.4 | 71.5 | 35.8 | 23.8 | 17.9 | 14.3 | 11.9 | 10.2 | 8.9 |
| 0.67 | 1907 | 953 | 477 | 318 | 238 | 191 | 159 | 136 | 119 | 106 | 95.3 | 47.7 | 31.8 | 23.8 | 19.1 | 15.9 | 13.6 | 11.9 |
| 1.00 | 2860 | 430 | 715 | 447 | 358 | 286 | 238 | 204 | 179 | 159 | 143 | 71.5 | 47.7 | 35.8 | 28.6 | 23.8 | 20.4 | 17.9 |
| 1.33 | 3813 | 1907 | 953 | 636 | 477 | 381 | 318 | 272 | 238 | 212 | 191 | 95.3 | 63.6 | 47.7 | 38.1 | 31.8 | 27.7 | 23.8 |
| 1.50 | 4290 | 2145 | 1073 | 715 | 536 | 429 | 358 | 306 | 268 | 238 | 214 | 107 | 71.5 | 53.6 | 42.9 | 35.8 | 30.6 | 26.8 |
| 1.67 | 4767 | 2383 | 1192 | 794 | 596 | 477 | 397 | 341 | 298 | 265 | 238 | 119 | 79.4 | 59.6 | 47.7 | 39.7 | 34.0 | 29.8 |
| 2.00 | 5720 | 2860 | 1430 | 953 | 715 | 572 | 477 | 409 | 358 | 318 | 286 | 143 | 95.3 | 71.5 | 57.2 | 47.7 | 40.9 | 35.8 |
| 2.33 | 6673 | 3337 | 1668 | 1112 | 834 | 667 | 556 | 477 | 417 | 371 | 334 | 167 | 111 | 83.4 | 66.7 | 55.6 | 47.7 | 41.7 |
| 2.50 | 7150 | 3575 | 1788 | 1192 | 894 | 715 | 596 | 511 | 447 | 397 | 358 | 179 | 119 | 89.4 | 71.5 | 59.6 | 51.1 | 44.7 |
| 2.67 | 7627 | 3813 | 1907 | 1271 | 953 | 763 | 636 | 545 | 477 | 424 | 381 | 191 | 127 | 95.3 | 76.3 | 63.6 | 54.5 | 47.7 |
| 3.00 | 8580 | 4290 | 2145 | 1430 | 1073 | 858 | 715 | 613 | 536 | 477 | 429 | 215 | 143 | 107 | 85.8 | 71.5 | 61.3 | 53.6 |
| 3.33 | 9533 | 4767 | 2383 | 1589 | 1192 | 953 | 794 | 681 | 596 | 530 | 477 | 238 | 159 | 119 | 95.3 | 79.4 | 68.1 | 59.6 |
| 3.50 | 10010 | 5005 | 2503 | 1668 | 1251 | 1001 | 834 | 715 | 626 | 556 | 501 | 250 | 167 | 125 | 100 | 83.4 | 71.5 | 62.6 |
| 3.67 | 10487 | 5243 | 2622 | 1748 | 1311 | 1049 | 874 | 749 | 655 | 583 | 524 | 262 | 175 | 131 | 105 | 87.4 | 74.9 | 65.5 |
| 4.00 | 11440 | 5720 | 2860 | 1907 | 1430 | 1144 | 953 | 817 | 715 | 636 | 572 | 286 | 191 | 143 | 114 | 95.3 | 81.7 | 71.5 |
| 4.33 | 12393 | 6197 | 3098 | 2065 | 1549 | 1239 | 1033 | 885 | 775 | 689 | 620 | 310 | 207 | 155 | 124 | 103 | 88.5 | 77.5 |
| 4.50 | 12870 | 6435 | 3218 | 2145 | 1609 | 1287 | 1073 | 919 | 804 | 715 | 644 | 322 | 215 | 161 | 129 | 108 | 91.9 | 80.4 |
| 4.67 | 13348 | 6673 | 3337 | 2224 | 1668 | 1335 | 1112 | 953 | 834 | 742 | 667 | 334 | 222 | 167 | 134 | 111 | 95.3 | 83.4 |
| 6.00 | 14300 | 7150 | 3575 | 2383 | 1788 | 1430 | 1192 | 1021 | 894 | 794 | 715 | 358 | 238 | 179 | 143 | 119 | 102 | 89.4 |
| 5.33 | 15253 | 7627 | 3813 | 2542 | 1907 | 1525 | 1271 | 1090 | 953 | 847 | 763 | 381 | 254 | 191 | 153 | 127 | 109 | 95.3 |
| 5.50 | 15370 | 7865 | 3933 | 2622 | 1966 | 1573 | 1311 | 1124 | 983 | 874 | 787 | 393 | 262 | 197 | 157 | 131 | 112 | 98.3 |
| 5.67 | 16206 | 8103 | 4052 | 2701 | 2026 | 1621 | 1351 | 1158 | 1013 | 900 | 810 | 405 | 270 | 203 | 162 | 135 | 116 | 101 |
| 6.00 | 17160 | 8580 | 4290 | 2860 | 2145 | 1716 | 1430 | 1226 | 1073 | 953 | 858 | 429 | 286 | 215 | 172 | 143 | 123 | 107 |
| 6.33 | 18113 | 9057 | 4528 | 3019 | 2264 | 1811 | 1509 | 1294 | 1132 | 1006 | 906 | 453 | 302 | 226 | 181 | 151 | 129 | 113 |
| 6.50 | 18590 | 9295 | 4648 | 3098 | 2324 | 1859 | 1549 | 1328 | 1162 | 1033 | 930 | 465 | 310 | 232 | 186 | 155 | 133 | 116 |
| 6.67 | 19066 | 9533 | 4767 | 3178 | 2383 | 1907 | 1589 | 1362 | 1192 | 1059 | 953 | 477 | 318 | 238 | 191 | 160 | 136 | 119 |
| 7.00 | 20020 | 10010 | 5005 | 3337 | 2503 | 2002 | 1668 | 1430 | 1251 | 1112 | 1001 | 500 | 334 | 250 | 200 | 167 | 143 | 125 |
| 7.33 | 20973 | 10487 | 5243 | 3496 | 2622 | 2097 | 1748 | 1498 | 1311 | 1165 | 1049 | 524 | 350 | 262 | 210 | 175 | 150 | 131 |
| 7.50 | 21450 | 10725 | 5363 | 3575 | 2681 | 2145 | 1788 | 1532 | 1341 | 1192 | 1073 | 536 | 358 | 268 | 215 | 179 | 153 | 134 |
| 7.67 | 21927 | 10963 | 5482 | 3654 | 2741 | 2193 | 1828 | 1566 | 1370 | 1218 | 1096 | 548 | 365 | 274 | 219 | 183 | 157 | 137 |
| 8.00 | 22880 | 11440 | 5720 | 3813 | 2860 | 2288 | 1907 | 1634 | 1430 | 1271 | 1144 | 572 | 381 | 286 | 229 | 191 | 163 | 143 |
| 8.33 | 23833 | 11917 | 5958 | 3972 | 2979 | 2383 | 1986 | 1702 | 1490 | 1324 | 1192 | 596 | 397 | 298 | 238 | 199 | 170 | 149 |
| 8.50 | 24310 | 12155 | 6078 | 4052 | 3039 | 2461 | 2026 | 1736 | 1519 | 1351 | 1216 | 608 | 405 | 304 | 243 | 203 | 174 | 152 |
| 8.67 | 24787 | 12393 | 6197 | 4131 | 3098 | 2479 | 2066 | 1770 | 1549 | 1377 | 1239 | 620 | 143 | 310 | 284 | 207 | 177 | 155 |
| 9.00 | 25740 | 12870 | 6435 | 4290 | 3218 | 2574 | 2145 | 1839 | 1609 | 1430 | 1287 | 644 | 429 | 322 | 257 | 215 | 184 | 161 |
| 9.33 | 26693 | 13347 | 6674 | 4449 | 3337 | 2669 | 2224 | 1907 | 1668 | 1483 | 1335 | 667 | 445 | 334 | 267 | 222 | 191 | 167 |
| 9.50 | 27170 | 13585 | 6793 | 4528 | 3396 | 2717 | 2264 | 1941 | 1698 | 1509 | 1359 | 679 | 453 | 340 | 272 | 226 | 194 | 170 |
| 9.67 | 27647 | 13812 | 6912 | 4608 | 3456 | 2765 | 2304 | 1975 | 1728 | 1536 | 1382 | 691 | 461 | 346 | 276 | 360 | 197 | 173 |
| 10.00 | 28600 | 14300 | 7150 | 4767 | 3575 | 2860 | 2383 | 2043 | 1788 | 1584 | 1460 | 715 | 477 | 358 | 286 | 238 | 204 | 17 |

[a]Based on a source strength of 1.43 mg per min per cigarette from Klepeis et al. (*5*).

particles ($PM_{2.5}$), which is very close in size to RSP ($PM_{3.5}$), is 65 μg/m$^3$ for 24 hr. Because the mass concentration of $PM_{3.5}$ includes the mass concentration of $PM_{2.5}$, maintaining $PM_{3.5}$ below a certain concentration insures that $PM_{2.5}$ also will be below that concentration. Maintaining RSP concentrations indoors below this outdoor NAAQS requires a room to have values of $w$ that lie in the unshaded area of the table. For an ASC value of 0.33 cigarettes, the effective fresh air flow rate must be $w = 8$ m$^3$/min or greater. For an ASC value of 3.33 cigarettes, for example, the effective fresh air flow rate must be $w = 80$ m$^3$/min or greater. The required effective air exchange rate is obtained for a given volume $v$ by computing $\phi_p = w/v$ using the volume $v$ of the indoor location that is being studied. These predictions can be used by ventilation engineers to design smoking lounges and other smoking facilities.

The NAAQS level has an averaging period of 24 hr, and a person typically may be in a smoking environment for only 8 hr. If we assume that the person's exposure to RSP from smoking activity is zero for the remaining 16 hr, then a level of 65 μg/m$^3$ for 8 hr would not violate the NAAQS. Due to the lower levels encountered outside the smoking environment, this person's 24-hr average exposure actually would be $1/3 \times 65$ μg/m$^3$ = 21.7 μg/m$^3$. Thus, Table 1 shading patterns include a fairly large margin of safety—a 3-fold factor that is especially protective of public health. If one chooses a less conservative approach—no margin of safety—the NAAQS values should be multiplied by 3 to give $3 \times 65$ μg/m$^3$ = 195 μg/m$^3$. By finding the RSP concentrations in Table 1 below 195 μg/m$^3$, one then can find the combinations of fresh air flow rates and smoking activities that maintain air quality below this less stringent version of the federal air quality standard for fine particles. As a compromise between these two versions of the NAAQS—65 μg/m$^3$ for 24 hr and 195 μg/m$^3$ for 8 hr—one might choose 150 μg/m$^3$ for 8 hr as a reasonable indoor air quality guideline or standard. The federal outdoor 24-hr NAAQS for particles less than 10 μ ($PM_{10}$) in diameter also is 150 μg/m$^3$.

Using Table 1, RSP concentrations at or below 150 μg/m$^3$ can be maintained at an ASC value of 2.0 cigarettes and an effective air flow rate of $w = 20$ m$^3$/min or also at an ASC value of 4.0 cigarettes and a flow rate of $w = 40$ m$^3$/min. For a room of 200 m$^3$, these values correspond to $\phi_P$ = (20 m$^3$/min) × (60 min/hr)/(200 m$^3$) = 6 ach and $\phi_P$ = (40 m$^3$/min) × (60 min/hr)/(200 m$^3$) = 12 ach. [Note that $\phi_P$ is expressed in the same units as the ventilatory air exchange rate $\phi$, ach or hr$^{-1}$; (*4*)]. For the desired particle concentration to be maintained at or below the federal NAAQS of 65 μg/m$^3$, Table 1 gives $w = 50$ m$^3$/min for 2.0 cigarettes and $w > 80$ m$^3$/min for 4.0 cigarettes, which, for a 200-m$^3$ room, correspond to $\phi_P$ = (50 m$^3$/min) × (60 min/hr)/(200 m$^3$) = 15 ach and $\phi_P > 24$ ach, respectively.

Two other indoor air quality examples help to illustrate the application of this table. Assume that the ASC is 2.0 cigarettes and that the location to be modeled has a volume of 500 m$^3$ and a particle decay rate of $\phi_P = 6$ hr$^{-1}$. We calculate the resulting air flow rate as $w$ = (6 hr$^{-1}$) × (500 m$^3$)/(60 min/hr) = 50 m$^3$/min. Referring to the column marked $w = 50$ m$^3$/min and the row marked 2.0 cigarettes in Table 1, we see that the average RSP concentration from this level of smoking activity is 57.2 μg/m$^3$.

A second example illustrates how the table can be used to predict concentrations within a residence. Many homes have air exchange rates as low as 1 or 2 ach and volumes of about 200 m$^3$. Suppose that the particle deposition rate for such a home is $\phi_P = 1.2$ hr$^{-1}$. If a person smokes an average of two 10-min cigarettes per hour in this home during waking hours, the active smoking count during this period would be $n_{ave}$ = (2 cigarettes/hr) × (10 min/60 min) = 0.33. Referring to Table 1, we find the top row marked $n_{ave} = 0.33$, and the corresponding column marked $w$ = (1.2 hr$^{-1}$) × (200 m$^3$)/(60 min/hr) = 4 m$^3$/min and we find the predicted concentration is 119 μg/m$^3$.

Since $w$ for particles includes both the effect of the ventilatory air exchange rate and the particle deposition rate, using only the fresh air ventilatory air exchange rate to compute $w$ will give higher RSP concentrations than the true values and will cause the table to be conservative (i.e., protective of health). Using the correct value of the particle decay parameter $\phi_P$ will give the exact RSP concentration, based on the source strengths reported by Klepeis et al. (*5*) for real smokers in realistic settings. Again, these predictions can be used by ventilation specialists to design smoking lounges.

## Conclusion

Our findings support the following conclusions: *a*) mathematical models for predicting ETS in indoor settings have a long history of development, with derivations based on the underlying physical theory of the process involved—conservation of mass—and with experimental results confirming the validity of the predictions of the models; *b*) the models have similar structures because all are based on the same physical law of conservation of mass; *c*) the concentrations predicted by the models agree well with measured values in real settings, both on a minute-by-minute (sequential time series) basis and for longer term averages; and *d*) the models are especially useful for determining the ventilation parameters required to meet indoor air quality standards for given smoking activity levels.

To illustrate application of the model, a table is included to calculate the indoor RSP concentration in any microenvironment for which the volume and decay rate are known. These models offer a practical, easy-to-apply methodology with acceptable accuracy for estimating the concentrations in indoor settings caused by indoor smoking activity. If one seeks to achieve adequate indoor air quality by maintaining pollutant concentrations below certain air quality standards for a given mixing volume and smoking activity, then such a model allows one to determine the minimum effective fresh air exchange rate that will be required of the ventilation system. One way to achieve desirable indoor air quality is by adopting indoor air quality standards—concentration levels indoors that should not be exceeded by indoor smoking areas—and to establish parameters that will meet these standards. In the absence of specific indoor air quality standards, it is reasonable to apply the NAAQS adopted by the U.S. EPA as guidelines for indoor settings. These mathematical ETS models allow one to predict concentrations that can be compared directly with these standards. These indoor modeling approaches are general in that they apply to any indoor location in which the ACS, volume, and air exchange rate are known or can be estimated. They also are useful for the engineering design of specialized smoking facilities and indoor lounges. The models have great potential for assisting building designers and public health specialists in achieving and maintaining adequate levels of indoor air quality in a scientifically valid manner. Although follow-up measurements could be undertaken to assess indoor air quality levels in such settings, the models have performed so well in our experiments with real smokers in real settings that such measurements

actually may not be necessary, and the models can be used to predict the unknown concentrations. These indoor air quality models also serve as the building blocks for developing larger total human exposure models designed to predict the frequency distribution of exposures across large populations and for making health risk assessments.

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14. After the cigarette is ignited, the concentration in the room follows the equation
$$z(t) = (g/w)\{1 - \exp(-\phi t)\}$$
where $g$ is the pollutant emission rate, $\phi$ is the effective air exchange rate, and $w = \phi v$ is the effective fresh air flow rate where $v$ is the volume of the room. At the instant that the cigarette burns out, $t = t_s$, the maximum concentration will be given by
$$z_{max} = (g/w)\{1 - \exp(-\phi t_s)\}$$
Thereafter, the concentration will decay following the relationship
$$z(t) = z_{max}\{\exp(-\phi[t - t_s])\}$$
Here, "exp" indicates the base of the natural logarithm, $e = 2.71828\ldots$, raised to the power given in parentheses.
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