Optimal HVAC Scheduling Using Phase-Change Material as a Demand Response Resource

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***Abstract*—In this paper, the effectiveness of phase-change material (PCM) for demand response in a residential building is evaluated. PCM serves as a thermal energy storage which stores latent heat (during preheating) in the building's envelope and releases it for use in response to varying price signals. With the PCM, the heating, ventilation and air conditioning (HVAC) system is optimally scheduled through a *home energy management system* (HEMS) leading to a reduced winter heating cost and a more comfortable indoor temperature profile. The outdoor temperature on one of the coldest days in Sydney (26th June, 2016, based on average daily temperature) is used as a case study. The effects of PCM thickness, heating tariff scheme and the preferences of the home owner on performance of the PCM are investigated. The analysis of the results shows that with cost minimization as priority, considerable cost savings can be achieved with PCM and with thermal comfort as priority, a better indoor temperature profile is also achieved with PCM.**

NOMENCLATURE

DR	Demand response
PCM	Phase-change material
TOU	Time of use tariff
HEMS	Home energy management system
HVAC	Heating, ventilation and air conditioning
$\dot{Q}_\mathrm{h}$	Heat flow from heater into the room (W)
$\dot{Q}_\mathrm{l}$	Heat loss from the room to outdoor (W)
$\dot{Q}_\mathrm{wi}$	Heat exchange between wall and indoor (W)
$\dot{Q}_\mathrm{wo}$	Heat exchange between wall and outdoor (W)
$\dot{Q}_\mathrm{ipcm}$	Heat exchange between PCM and indoor (W)
$\dot{Q}_\mathrm{wpcm}$	Heat exchange between wall and PCM (W)
η_h	Efficiency of the heater
S_h	Operational status of the heater $\in$ [0,1]
P_h	Power input to the heater (W)
T_w	Temperature of the wall (K)
T_in	Temperature of the indoor air (K)
T_out	Outdoor temperature (K)
T_pcm	Temperature of the phase-change material (K)
T_m	PCM Peak melting temperature (K)
T_s	Desired room set temperature (K)
R_T	Total resistance of the wall (K/W)
C_T	Total thermal capacity of the wall (J/K)
A	Total wall area (m^2)
r_si	Inside surface wall resistance ($\mathrm{m}^2\,\mathrm{K/W}$)
r_so	Outside surface wall resistance ($\mathrm{m}^2\,\mathrm{K/W}$)
d_l	Thickness of the lth wall section (m)
ρ_l	Density of the lth wall section ($\mathrm{kg/m}^3$)
λ_l	Thermal conductivity of the lth wall section ($\mathrm{W/m\,K}$)
r_l	Thermal resistance of the lth wall section ($\mathrm{m}^2\,\mathrm{K/W}$)
c_l	Specific heat capacity of the lth wall section ($\mathrm{J/kg\,K}$)
C_w	Heat capacity of wall (J/K)
C_pcm	Heat capacity of PCM (J/K)
C_in	Heat capacity of Indoor air (J/K)
R_io	Resistance between indoor and outdoor (K/W)
R_wo	Resistance between wall and outdoor (K/W)
R_wi	Resistance between indoor and wall (K/W)
R_wpcm	Resistance between wall and PCM (K/W)
R_ipcm	Resistance between indoor and PCM (K/W)
a_n, b_n, c_n	Constants in the PCM specific heat capacity function where n ranges from 1 to 8
R_k	Resistance of the kth resistor (K/W)
f_k	Resistance factors (sum up to 1), $k = 1, 2, 3$
n	number of layers that make up wall
R_k^*, C_k	resistances and thermal capacitances of individual wall layers
$\mathrm{C_g}$	Heating cost ($\$/\mathrm{kWh}$)
x	Varying heater power (continuous)
i	Ranges from 1 to N (where N is the number of time steps)
Δt	Each time step = 24/1440 (for N = 1440)
u, w	Weighting factor for the two parts of the objective function

I. INTRODUCTION

Heating, ventilation and air conditioning loads in buildings constitute a large proportion of the energy demand of households and, therefore, are a good candidate for demand response [1]. With home energy management systems, a HVAC system can be automatically scheduled to respond to varying price signals, with the aim of reducing heating cost. Phase-change material is used to enhance the thermal inertia of buildings, by storing latent heat in the building's envelope. Latent heat storage gives higher storage density when compared to sensible heat storage, with a smaller temperature swing between heat storage and release [2]. It essentially enables a reduction in the cost of heating/cooling as the heater/air conditioner may not be used all the time or kept on at full power, while keeping the room temperature close to the desired one. The use of PCM

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for energy savings and thermal comfort has been studied in the recent literature. In [3], [4], [5], [6], the effectiveness of PCM for improving building thermal comfort and for energy savings has been demonstrated considering different scenarios. In [7], the use of PCM for optimal operation of HVAC system was studied but not in the context of DR. There is very little discussion of the use of building thermal inertia for DR. In [8], [9], the use of a building's thermal inertia for DR was investigated, but not with PCM. In light of this shortcoming, this research evaluates PCM performance for DR with a detailed thermal modelling of the PCM and the building. The rest of this paper is organized as follows: Section 2 details the model of PCM's thermal behaviour in buildings. Section 3 describes the optimization model and Section 4 shows the results. In Section 5, the results are discussed and in Section 6 the conclusions are drawn.

II. Thermal Modelling

A. Building thermal model

The thermal model approach utilised in this work is the lumped capacitance method (also known as the RC model, analogous to an electrical resistive-capacitive network), described in [10]. In this method, the building is seen to comprise of several layers of capacitances and resistances lumped together to single equivalent capacitance and resistance values. This method gives an acceptable degree of accuracy for short time-horizon simulations of building thermal systems. It has been applied in [9], [11] and [12] for the thermal modelling of a building. The building studied here is the same as the one described in the MATLAB Simulink example, "Thermal Model of a House" [13]. This work utilised similar building physical parameters (length, width and height of building). However, we have improved the model to depict the energy flow associated with the building wall, represented as an additional differential equation.

The equations describing the thermal dynamics of the improved building model are given in (1) to (6) below:

$$\dot{Q}_{\mathrm{h}} = \eta_{\mathrm{h}} S_{\mathrm{h}} P_{\mathrm{h}} \tag{1}$$

$$\dot{Q}_{\mathrm{l}} = \frac{T_{\mathrm{in}} - T_{\mathrm{out}}}{R_{\mathrm{io}}} \tag{2}$$

$$\dot{Q}_{\mathrm{wo}} = \frac{T_{\mathrm{w}} - T_{\mathrm{out}}}{R_{\mathrm{wo}}} \tag{3}$$

$$\dot{Q}_{\mathrm{wi}} = \frac{T_{\mathrm{in}} - T_{\mathrm{w}}}{R_{\mathrm{wi}}} \tag{4}$$

$$\dot{T}_{\mathrm{w}} = \frac{1}{C_{\mathrm{w}}}\Big(\dot{Q}_{\mathrm{wi}} - \dot{Q}_{\mathrm{wo}}\Big) \tag{5}$$

$$\dot{T}_{\mathrm{in}} = \frac{1}{C_{\mathrm{in}}}\Big(\dot{Q}_{\mathrm{h}} - \dot{Q}_{\mathrm{l}} - \dot{Q}_{\mathrm{wi}}\Big) \tag{6}$$

When phase-change materials are added to the building's (wall) envelope, Equations (3) to (6) are modified accordingly and another differential equation, (10), is added for the PCM:

TABLE I
Wall Parameters [14] [15]

Element	d_l (m)	λ_l (W/m K)	ρ_l (kg/m^3)	c_l (J/kg K)	r_l (m^2 K/W)
Outer Surface	–	–	–	–	0.0400
External brickwork	0.122	0.84	1700	800	0.1452
Cavity Insulation	0.050	0.03	30	1764	1.6667
Blockwork	0.112	0.51	1400	1000	0.2196
Plaster	0.013	0.16	600	1000	0.0720
Inner Surface	–	–	–	–	0.1300

$$\dot{Q}_{\mathrm{wpcm}} = \frac{T_{\mathrm{pcm}} - T_{\mathrm{w}}}{R_{\mathrm{wpcm}}} \tag{7}$$

$$\dot{Q}_{\mathrm{ipcm}} = \frac{T_{\mathrm{in}} - T_{\mathrm{pcm}}}{R_{\mathrm{ipcm}}} \tag{8}$$

$$\dot{T}_{\mathrm{w}} = \frac{1}{C_{\mathrm{w}}}\Big(\dot{Q}_{\mathrm{wpcm}} - \dot{Q}_{\mathrm{wo}}\Big) \tag{9}$$

$$\dot{T}_{\mathrm{pcm}} = \frac{1}{C_{\mathrm{pcm}}}\Big(\dot{Q}_{\mathrm{ipcm}} - \dot{Q}_{\mathrm{wpcm}}\Big) \tag{10}$$

$$\dot{T}_{\mathrm{in}} = \frac{1}{C_{\mathrm{in}}}\Big(\dot{Q}_{\mathrm{h}} - \dot{Q}_{\mathrm{l}} - \dot{Q}_{\mathrm{ipcm}}\Big) \tag{11}$$

B. Building wall parameters

The building wall is made up of 4 main sections: The external brick, a cavity insulation, a block work and plaster. These sections are of different thickness, with different technical properties as given in Table I ([14], [15]). In order to calculate the total thermal resistance and capacitance of the block as a whole, we employ the formulas given in (12) and (13) [14].

$$R_{\mathrm{T}} = \Big(r_{\mathrm{si}} + r_{\mathrm{so}} + \sum_{l=1}^{n} \frac{d_l}{\lambda_l}\Big)/A \tag{12}$$

$$C_{\mathrm{T}} = C_{\mathrm{w}} = A \sum_{l=1}^{n} (d_l \rho_l c_l) \tag{13}$$

C. Modeling the phase-change material (PCM)

The specific heat capacity of the phase-change material is dependent on its temperature in a nonlinear manner. The honeycomb PCM model, used as a case study in [16] is adopted for this work and its specific heat capacity is modeled as a function of temperature as given in (14) below.

$$c_{\mathrm{pcm}} = \begin{cases} 1200 + 18800 e^{-\left(\frac{T_{\mathrm{m}} - T_{\mathrm{pcm}}}{1.5}\right)} & \text{if } T_{\mathrm{pcm}} < T_{\mathrm{m}} \\ 1300 + 18700 e^{-4\left(T_{\mathrm{m}} - T_{\mathrm{pcm}}\right)^2} & \text{if } T_{\mathrm{pcm}} \geq T_{\mathrm{m}} \end{cases} \tag{14}$$

The values of c_{pcm} (in J/kg °C) were obtained for corresponding values of T_{pcm} in the range 18 °C to 30 °C (with $T_{\mathrm{m}} = 27.6$ °C). Curve fitting techniques in MATLAB were applied to get a general formula for all PCM temperatures. The sum of sines function in (15) gave the curve of best fit.

$$c_{\mathrm{pcm}}(i) = a_1 \sin(b_1 T_{\mathrm{pcm}}(i) + c_1) + a_2 \sin(b_2 T_{pcm}(i) + c_2) + \\ .... + a_8 \sin(b_8 T_{\mathrm{pcm}}(i) + c_8) \tag{15}$$

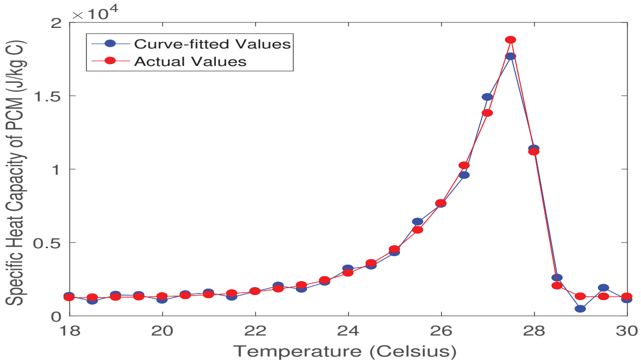

Fig. 1. Actual and curve-fitted values of PCM specific heat capacity

Figure 1 shows a comparison of the actual and curve-fitted values of the PCM specific heat capacity. It shows that the curve-fitted results derived from the sum of sines function gives a good approximation of the actual values.

D. RC model of the wall without PCM

The first-order lumped capacitance method was used to model the building without PCM. This model consists of two lumped resistances and one lumped capacitance (known as 2R1C model) as shown in Figure 2. The two resistances R_{out} and R_{in} add to make up the equivalent wall resistance R_T but are shared in a proportion determined by the 'accessibility factor, α'. This factor, defined in [17] is given in (16) to (19).

$$R_{\mathrm{in}} = \alpha R_{\mathrm{T}} = R_{\mathrm{wi}} \tag{16}$$

$$R_{\mathrm{out}} = (1-\alpha) R_{\mathrm{T}} = R_{\mathrm{wo}} \tag{17}$$

$$\alpha = 1 - \frac{1}{R_{\mathrm{T}} C_{\mathrm{w}}} \sum_{k=1}^{n} R_k^* C_k \tag{18}$$

$$R_k^* = \sum_{j=1}^{n-1} R_j + R_k/2 \tag{19}$$

Intuitively, α should be ≥ 0.5 but < 1. Authors in [18] suggested an α value of 0.5 assuming equal distribution of the resistances if the wall has low thickness. This is a reasonable assumption and is therefore adopted in this work. A more accurate α value can be obtained by applying (16) and (17) if the thermal resistance and capacitance values of all individual layers are available.

E. RC model of the wall with PCM

The second-order lumped capacitance method was used to model the building with PCM. This model consists of three lumped thermal resistances and two lumped thermal capacitances (known as 3R2C model) as shown in Figure 3. Table II shows the wall properties with PCM. Authors in [14] applied (20) to divide the total resistances into three parts according to the 3R2C lumped parameter approach. These factors (which sum up to 1) were results of a second-order element parameter optimization for high thermal capacity rooms.

$$R_k = f_k R_{\mathrm{T}} \tag{20}$$

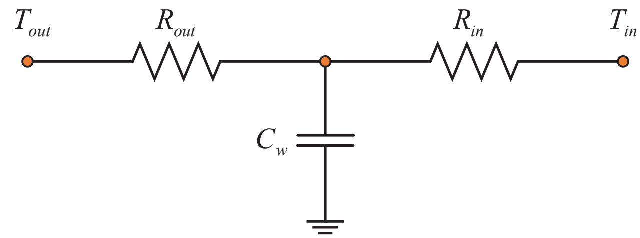

Fig. 2. First-order RC model

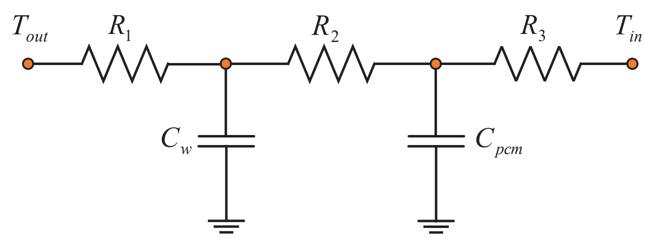

Fig. 3. Second-order RC model

TABLE II
WALL PARAMETERS WITH PCM [14] [15]

Element	d_l (m)	λ_l (W/m K)	ρ_l (kg/m^3)	c_l (J/kg K)	r_l (m^2 K/W)
Outer Surface	–	–	–	–	0.0400
External brickwork	0.122	0.84	1700	800	0.1452
Cavity Insulation	0.050	0.03	30	1764	1.6667
Blockwork	0.112	0.51	1400	1000	0.2196
Plaster	0.013	0.16	600	1000	0.0720
PCM	0.050	2.80	545	Varies	0.0179
Gypsum Plasterboard	0.025	0.16	800	840	0.1588
Inner Surface	–	–	–	–	0.1300

In this work, a simpler approach is adopted. The values of R_k will be calculated from the thermal resistances of the first-order RC model (using R_{in} and R_{out}) and resistance values from Table II. Equation (21) shows the R_k values.

$$\begin{aligned} R_1 &= R_{\mathrm{out}} = R_{\mathrm{wo}} \\ R_2 &= R_{\mathrm{in}} + \frac{R_{\mathrm{pcm}}}{2} = R_{\mathrm{wpcm}} \\ R_3 &= \frac{R_{\mathrm{pcm}}}{2} + R_{\mathrm{gypsum}} = R_{\mathrm{ipcm}} \end{aligned} \tag{21}$$

This approach is based on the assumption that the thermal capacity of both the wall (without PCM) and the PCM is in the mid-point of the respective elements.

F. Assumptions

In this work, the following assumptions are made:

- The weather (winter) profile for Sydney on 26th June 2016 is used as outdoor temperature data.
- The utilization of PCM (embedded in the building wall) is considered with the HVAC operating as a heater during winter only. It is assumed that heater operates continuously with varying power levels.

- Indoor temperature comfort bounds (shown as the dark dotted lines in Figure 6) are in accordance with ASHRAE [19] specifications, and taken as 19 to 23 °C.
- Heat input due to solar radiation, infiltration and low mass envelope components in the building are neglected.

III. Optimization Model

The optimization problem comprises of two parts. First, the minimization of the heating cost and second, the minimization of the temperature difference between the desired and the actual indoor temperature. There are two weighting factors (u and w) to reflect the importance of each part of the objective to the home owner. The optimization problem (objective function and constraints) is written as in (22) to (26) below:

$$\underset{\substack{S_{\mathrm{h}}^i, T_{\mathrm{in}}^i, x^i,\\ T_{\mathrm{pcm}}^i, c_{\mathrm{pcm}}^i}}{\text{minimize}} \quad u \sum_{i=1}^{N} C_{\mathrm{g}}^i x^i \Delta t + w \sum_{i=1}^{N} (T_{\mathrm{in}}^i - T_{\mathrm{s}})^2 \tag{22}$$

$$\text{subject to eqs. (1) and (9) to (11)} \quad \text{and} \tag{23}$$

$$0 \leq x^i \leq P_{\mathrm{h}}^i S_{\mathrm{h}}^i \tag{24}$$

$$T_{\mathrm{in}}^{\min} \leq T_{\mathrm{in}}^i \leq T_{\mathrm{in}}^{\max} \tag{25}$$

$$S_{\mathrm{h}}^i \in [0, 1], \qquad i \in N \tag{26}$$

Equations (5), (6) and (9) to (11) are represented as temperature-time difference equations (as linear approximations of the actual differential equations), so Δt is made sufficiently small in the AMPL [20] optimization model. The weighting factors are set according to the preference of the home owner. With $u = 1$ and $w = 0$ (scenario 1), heating cost minimization is a priority and with $u = 0$ and $w = 1$ (scenario 2), the thermal comfort of the home is a priority. Simulations are performed for these two scenarios and the indoor temperature and heating cost values are recorded for a day. Simulations were also done for both a constant and a time varying heating cost. This is to establish the benefits of ToU for DR. It is expected that more savings will exist, using PCM with time-varying tariff since there is an incentive for reducing heating at periods when the heating cost is high.

IV. Results and Discussions

For the two scenarios depicting the home owner's preference, and for each heating tariff schemes (ToU and flat rate tariff),[1] the heating cost and indoor temperature values were compared for the building types. Because both parts of the objective function are in different units, the heating cost for scenario 2 (unit of w converted to thermal discomfort cost) is calculated manually by multiplying the varying power input and the cost at each time step, and summing them up to get the total cost. Table III compares the results of the two heating tariff schemes while Figure 4 shows the percentage reduction in cost for the two PCM types (as compared to the case with no PCM). The heating schedules and indoor temperature profiles are shown in Figures 5 and 6 respectively.

[1]The ToU tariff used in this work is higher and varies from 0.11 to 0.48 \$/kWh, while the flat tariff is set at 0.11 \$/kWh

TABLE III
Comparison of Results of Different Heating Tariff Schemes

Tariff	Scenario	Heating Cost (\$) No PCM	PCM 0.02m	PCM 0.05m
ToU	1. u = 1, w = 0	39.74	35.25	34.16
	2. u = 0, w = 1	51.31	47.98	49.20
Flat	1. u = 1, w = 0	20.05	18.30	18.38
	2. u = 0, w = 1	24.63	23.48	24.05

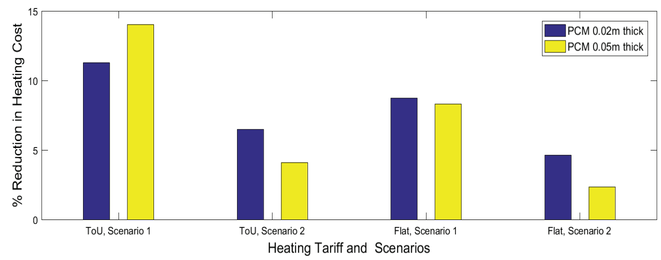

Fig. 4. Percentage Reduction in Heating Cost with PCM

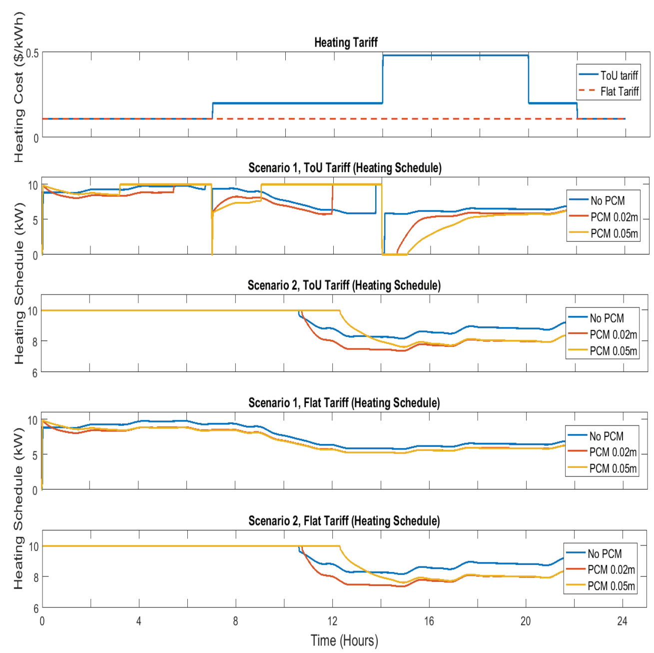

Fig. 5. Heating Tariff and Heating Schedules

From the optimization results, it can be deduced that the use of PCM, results in lower heating cost. This reduction is more for the time-varying (ToU) heating tariff, since there is an incentive for reduced heating at periods of high heating cost (see Figure 4). This is a form of demand response, encouraging home owners to install PCM in their buildings. If the priority of the home owner is cost reduction (scenario 1), the optimizer sets the indoor temperature at the minimum acceptable level (see Figure 6), which results in lower heating costs. On the other hand, if thermal comfort is priority, the optimizer sets the

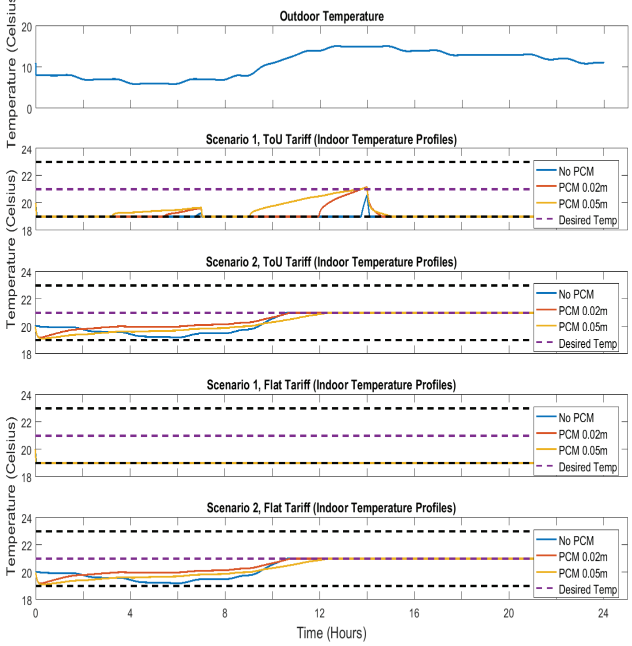

Fig. 6. Outdoor and Indoor Temperature Profiles

indoor temperature exactly equal to the desired level, T_s; this results in higher heating cost for both heating tariff schemes.

Comparing PCM with different thickness in Table III and Figure 4, we can deduce the following: With cost minimization as priority, greater savings is achieved with the PCM of higher thickness, for ToU tariff scheme. This is so because the thicker PCM saves more heat for use during periods of higher heating cost, and so reduces heating while maintaining temperature at $T_{\rm in}^{\rm min}$. In other cases, the 0.02m thick PCM results in lower heating cost. When thermal comfort is priority (scenario 2), there is no incentive for using a thicker PCM. Similar to the results obtained in [16], using a thicker PCM does not necessarily imply improved thermal comfort level. To better evaluate the effect of PCM thickness, simulations should be performed for a longer time horizon (with varying outdoor temperature). However, PCM improved the thermal comfort for all scenarios as compared to the case without PCM.

V. Conclusions

In all, it has been shown that PCM will serve as a good DR resource for ToU heating tariff. A cost-benefit analysis should be done by the end-user to weigh the cost of installing the PCM against the potential long-term savings and thermal comfort it provides. From the results, we can conclude:

- The building with PCM gives a better indoor temperature profile (for both heating tariff schemes) when thermal comfort is priority. With cost minimization as priority and with ToU tariff, the PCM reduces cost in so far as the room temperature is within the stipulated bounds.
- With time-varying heating tariff, considerable cost savings can be achieved with the use of PCM of both thickness sizes, while maintaining the room temperature within the stipulated bounds. Whereas, with flat heating tariff, cost savings are not very significant.

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