Question-8.2.1

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: Find the area of the circle $4x^2 + 4y^2 = 9$, which is interior to the parabola $x^2 = 4y$

Solution:

Trapezoidal rule: It is a numerical method for approximating the definite integral of a function. The classic definition is as follows:
Given a function $f(x)$ that is continuous in the interval $[a, b]$, the trapezoidal rule approximates the integral by dividing the interval $[a, b]$ into n sub intervals of equal width h=$\frac{b-a}{n}$. The x-values at the sub interval boundaries are:

$$x_0 = a, x_1 = a + h, x_2 = a + 2h, \cdots, x_n = b. \tag{0.1}$$

The trapezoidal rule then computes the integral as

$$\int_a^b f(x)\ dx \approx \frac{h}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right) \tag{0.2}$$

First we need to calculate the point of intersections of the two given curves

$$y = \frac{x^2}{4} \tag{0.3}$$

$$x^2 + y^2 = \frac{9}{4} \tag{0.4}$$

$$x^2 + \frac{x^4}{16} = \frac{9}{4} \tag{0.5}$$

$$16x^2 + x^4 = 36 \tag{0.6}$$

on solving the equation we get x values as $\sqrt{2}$ and $-\sqrt{2}$
so the points of intersection of both curves are $\left(\pm\sqrt{2}, \frac{1}{2}\right)$
The interval $\left[-\sqrt{2}, \sqrt{2}\right]$ is divided into 1000 sub intervals each of width h=$\frac{2\sqrt{2}}{1000}$ then the points in the interval can be written as

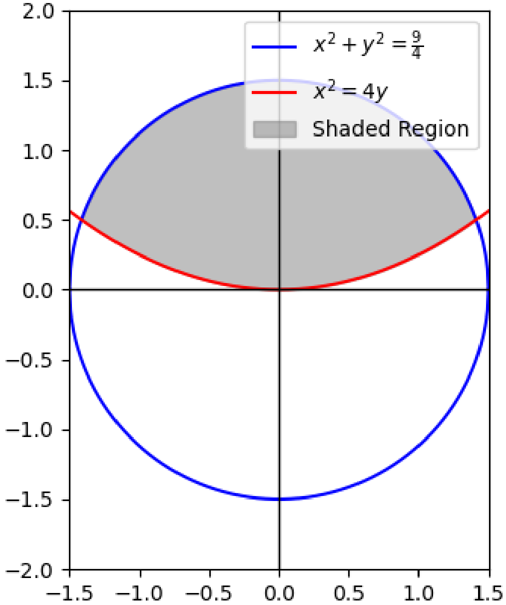
$$x_i = -\sqrt{2} + i \cdot h \tag{0.7}$$

$$\text{Area} \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \tag{0.8}$$

$$f(x_i) = \sqrt{\frac{9}{4} - x_i^2} - \frac{x_i^2}{4} \tag{0.9}$$

Using a c code to find the area using trapezoidal rule i got the area as

$$A \approx 3.005356 \tag{0.10}$$



If the area is calculated directly using integration method the area would be around 3.063 which is so close to the one we found using trapezoidal rule