Question-10.4.2.1.2

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: Find the roots of the equation $2x^2 + x - 6 = 0$

Solution:

We can solve the above equation using fixed point iterations. First we separate x, from the above equation and make an update equation of the below sort.

$$x = g(x) \implies x_{n+1} = g(x_n) \tag{0.1}$$

Applying the above update equation on our equation, we get

$$x_{n+1} = 6 - 2x_n^2 \tag{0.2}$$

$$\tag{0.3}$$

Now we take an initial value x_0 and iterate the above update equation. But we realize that the updated values always approach infinity for any initial value.

Thus we will alternatively use Newton's Method for solving equations.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.4}$$

Where we define $f(x)$ as,

$$f(x) = 2x^2 + x - 6 \tag{0.5}$$

$$f'(x) = 4x + 1 \tag{0.6}$$

Thus, the new update equation is,

$$x_{n+1} = x_n - \frac{2x_n^2 + x_n - 6}{4x_n + 1} \tag{0.7}$$

Taking an initial guess of $x_0 = -4$ we can see that x_n converges at the 3rd iteration with x as

$$x = -2 \tag{0.8}$$

Taking an initial guess of $x_0 = 1$ we can see that x_n converges at the 3rd iteration with x as

$$x = \frac{3}{2} \tag{0.9}$$

CODING LOGIC FOR FINDING EIGENVALUES :-

The quadratic equation

$$2x^2 + x - 6 = 0 \tag{0.10}$$

is rewritten in matrix form:

$$\text{Matrix} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \tag{0.11}$$

$$a = 2, \quad b = 1, \quad c = -6. \tag{0.12}$$

Substituting the values of a, b and c, the matrix becomes:
Let

$$\text{A} = \begin{pmatrix} 0 & 3 \\ 1 & -\frac{1}{2} \end{pmatrix} \tag{0.13}$$

QR-Algorithm

The QR method is an iterative algorithm used to compute the eigenvalues of a square matrix A. The algorithm works as follows:

1) Initialization
   Let $A_0 = A$, where A is the given matrix.
2) QR Decomposition
   For each iteration $k = 0, 1, 2, \ldots$:
   a) Compute the QR decomposition of A_k, such that:

   $$A_k = Q_k R_k \tag{2.1}$$

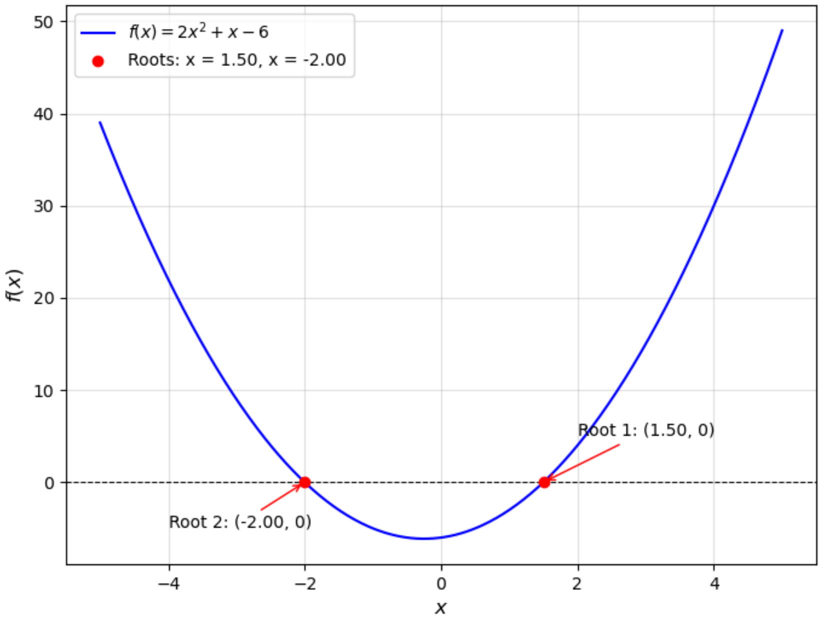
   where:
   i) Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).
   ii) R_k is an upper triangular matrix.

   The decomposition ensures $A_k = Q_k R_k$.
   b) Form the next matrix A_{k+1} as:

   $$A_{k+1} = R_k Q_k \tag{2.2}$$

3) Convergence
   Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.

4) The eigenvalues of matrix will be the roots of the equation.



$f(x) = 2x^2 + x - 6$
Roots: x = 1.50, x = -2.00
Root 1: (1.50, 0)
Root 2: (-2.00, 0)
50
40
30
20
10
0
−4
−2
0
2
4
$f(x)$
x