

Question-10.3.2.1.1

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Solution:

Let the number of boys be x and the number of girls be y . From the question, we can frame the following equations.

$$x + y = 10 \quad (0.1)$$

$$y = x + 4 \quad (0.2)$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} \quad (0.3)$$

$$(0.4)$$

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \quad (0.5)$$

The upper triangular matrix U is found by row reducing A ,

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \quad (0.6)$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \quad (0.7)$$

l_{21} is the multiplier used to zero a_{21} , so $l_{21} = -1$.

Now

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \quad (0.8)$$

Now we can get the solution to our problem by the two step process,

$$L\mathbf{y} = \mathbf{b} \quad (0.9)$$

$$U\mathbf{x} = \mathbf{y} \quad (0.10)$$

Using forward substitution to solve the first equation,

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} \quad (0.11)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix} \quad (0.12)$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix} \quad (0.13)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \quad (0.14)$$

Therefore the number of girls are 7 and boys are 3

