

Question-9.4.9

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: $\frac{dy}{dx} = \sin^{-1} x$

Solution:

Integrate on both sides

$$\int dy = \int \sin^{-1} x \, dx \quad (0.1)$$

Using integration by parts

$$y = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx \quad (0.2)$$

$$t = \sqrt{1-x^2} \quad (0.3)$$

$$dt = \frac{-2x}{\sqrt{1-x^2}} \, dx \quad (0.4)$$

$$y = x \sin^{-1} x + \int \frac{dt}{2\sqrt{t}} \quad (0.5)$$

$$y = x \sin^{-1} x + \sqrt{t} + c \quad (0.6)$$

substituting value of t gives

$$y = x \sin^{-1} x + \sqrt{1-x^2} + c \quad (0.7)$$

Let the initial conditions be $X_0 = 0, Y_0 = 1$

$$1 = 0 + 1 + c \quad (0.8)$$

$$c = 0 \quad (0.9)$$

Final equation of the curve

$$Y = x \sin^{-1} x + \sqrt{1-x^2} \quad (0.10)$$

Now let us this computationally from the definition of $\frac{dy}{dx}$

$$Y_{n+1} = Y_n + \frac{dy}{dx} \cdot h \quad (0.11)$$

From the differential equation

$$\frac{dy}{dx} = \frac{y_n - x_n}{y_n + x_n} \cdot h \quad (0.12)$$

$$y_{n+1} = y_n + \left(\frac{y_n - x_n}{y_n + x_n} \right) \cdot h \quad (0.13)$$

BY taking $x_1=0$ and $y_1=1$ and $h=0.01$ going till $x=0.5$ by iterating through the loop and finding y_2, y_3, y_4, \dots and plotting the graph. we can verify the function we got by solving the differential equation mathematically

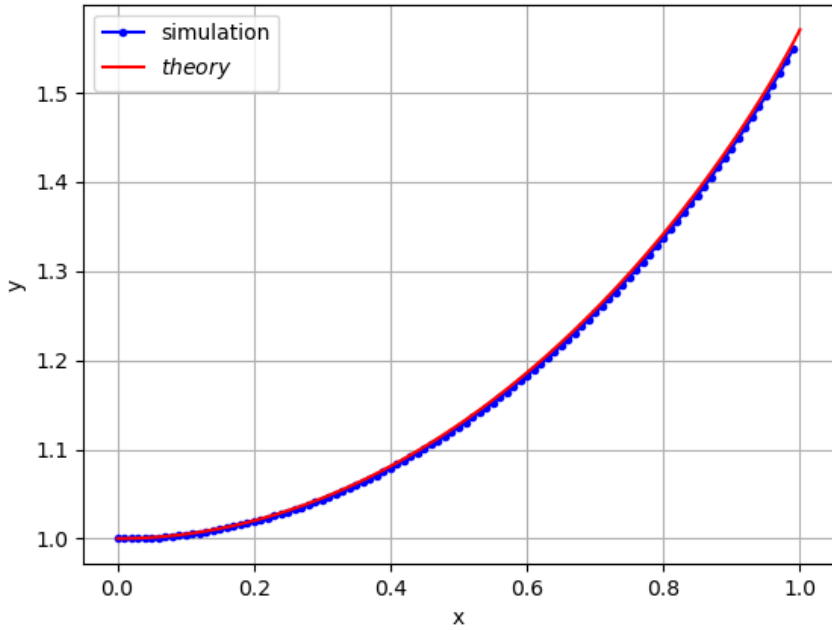


Fig. 0.1