Question-10.4.2.1.2

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: Find the roots of the equation $2x^2 + x - 6 = 0$

Solution:

We can solve the above equation using fixed point iterations. First we separate x, from the above equation and make an update equation of the below sort.

$$x = g(x) \implies x_{n+1} = g(x_n) \tag{0.1}$$

Applying the above update equation on our equation, we get

$$x_{n+1} = 6 - 2x_n^2 \tag{0.2}$$

$$\tag{0.3}$$

Now we take an initial value x_0 and iterate the above update equation. But we realize that the updated values always approach infinity for any initial value.

Thus we will alternatively use Newton's Method for solving equations.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.4}$$

Where we define $f(x)$ as,

$$f(x) = 2x^2 + x - 6 \tag{0.5}$$

$$f'(x) = 4x + 1 \tag{0.6}$$

Thus, the new update equation is,

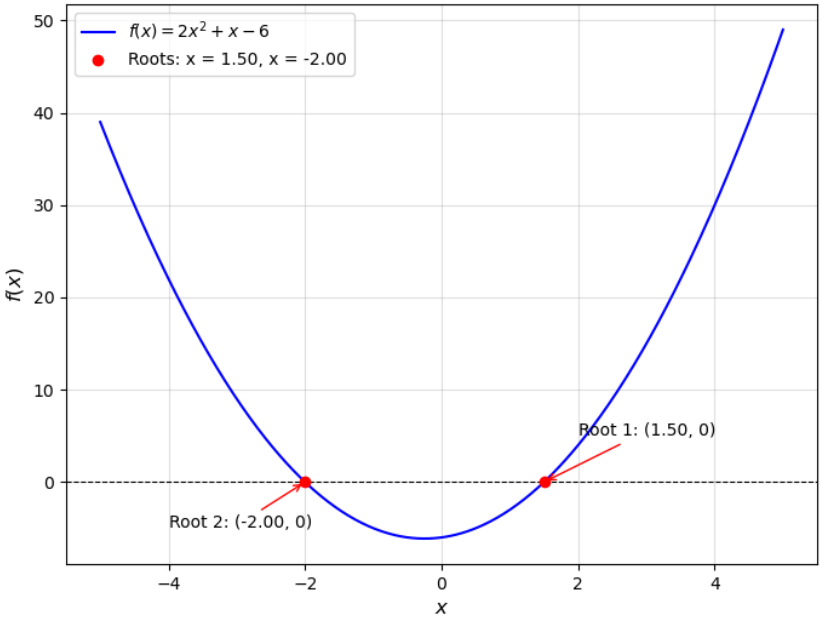
$$x_{n+1} = x_n - \frac{2x_n^2 + x_n - 6}{4x_n + 1} \tag{0.7}$$

Taking an initial guess of $x_0 = -4$ we can see that x_n converges at the 3rd iteration with x as

$$x = -2 \tag{0.8}$$

Taking an initial guess of $x_0 = 1$ we can see that x_n converges at the 3rd iteration with x as

$$x = \frac{3}{2} \tag{0.9}$$


$f(x) = 2x^2 + x - 6$
Roots: x = 1.50, x = -2.00
Root 1: (1.50, 0)
Root 2: (-2.00, 0)
$f(x)$
x
50
40
30
20
10
0
−4
−2
0
2
4