Question - 9-9.4-4

EE24BTECH11038 - Malakala Bala Subrahmanya Aravind

Question:
Find the area of the region bounded by $y^2 = 2x$ and $x^2 + y^2 = 4x$.
Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of circle

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ u_1 = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, \ f_1 = 0 \tag{0.1}$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ u_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \ f_2 = -4 \tag{0.2}$$

The intersection of two conics with parameters $V_i, u_i, f_i, \ i = 1, 2$ is defined as

$$x^T (V_1 + \mu V_2) x + 2 (u_1 + \mu u_2)^T x + (f_1 + \mu f_2) = 0 \tag{0.3}$$

Solving this the points of intersection are

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \ \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{0.4}$$

Area between the curves is,

$$2 \int_0^2 \left(2 + \sqrt{4 - y^2} - \frac{y^2}{2} \right) dy \tag{0.5}$$

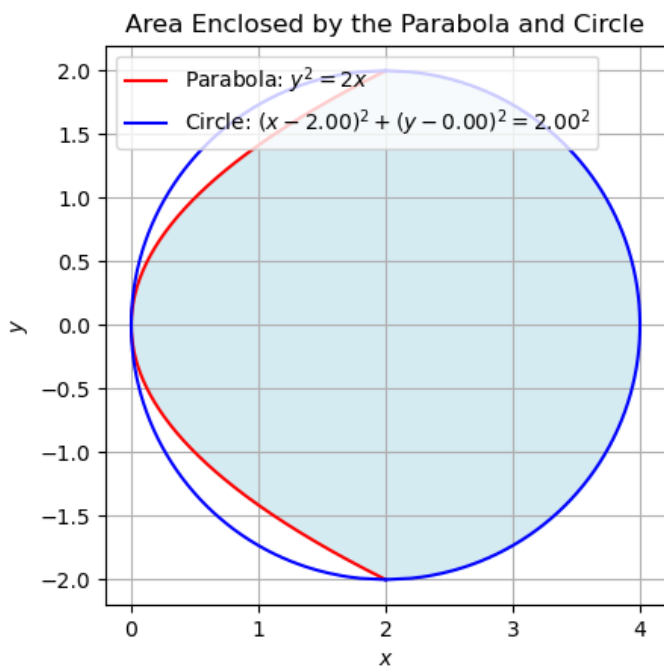By solving the integration, we get area is equal to 11.62 sq.units

Fig. 0.1