

# 2021-June

## Session-26-06-2022-shift-1-1-15

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

- 1) Let  $f(x) = \frac{x-1}{x+1}$ ,  $x \in \mathbf{R} - \{0, -1, 1\}$ . If  $f^{n+1}(x) = f(f^n(x))$  for all  $n \in \mathbf{N}$ , then  $f^6(6) + f^7(7)$  is equal to :
  - $\frac{7}{6}$
  - $-\frac{3}{2}$
  - $\frac{7}{12}$
  - $-\frac{11}{12}$
- 2) Let  $A = \left\{ z \in \mathbf{C} : \left| \frac{z+1}{z-1} < 1 \right| \right\}$  and  $B = \left\{ z \in \mathbf{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3} \right\}$  then  $A \cap B$  is
  - a portion of circle centred at  $\left(0, -\frac{1}{\sqrt{3}}\right)$  that lies in the second and third quadrants only
  - a portion of circle centred at  $\left(0, -\frac{1}{\sqrt{3}}\right)$  that lies in the second quadrant only
  - an empty set
  - a portion of circle of radius  $\frac{2}{\sqrt{3}}$  that lies in the third quadrant only
- 3) Let  $A$  be a  $3 \times 3$  invertible matrix. If  $|adj(24A)| = |adj(3adj(2A))|$  then  $|A|^2$  is equal to
  - $6^6$
  - $2^{12}$
  - $2^6$
  - 1
- 4) The order pair  $(a, b)$ , for which the system of linear equations

$$3x - 2y + z = b$$

$$5x - 8y + 9z = 3$$

$$2x + y + az = -1$$

has no solution is :

- $\left(3, \frac{1}{3}\right)$
- $\left(-3, \frac{1}{3}\right)$
- $\left(-3, -\frac{1}{3}\right)$
- $\left(3, -\frac{1}{3}\right)$

5) The remainder when  $(2021)^{2023}$  is divided by 7 is :

- a) 1
- b) 5
- c) 5
- d) 6

6)  $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$  is equal to :

- a)  $\sqrt{2}$
- b)  $-\sqrt{2}$
- c)  $\frac{1}{\sqrt{2}}$
- d)  $-\frac{1}{\sqrt{2}}$

7) Let  $f, g: \mathbf{R} \rightarrow \mathbf{R}$  be two real valued functions defined as

$$f(x) = \begin{cases} -|x+3|, & x < 0 \\ e^x, & x \geq 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 + k_1 x, & x < 0 \\ 4x + k_2, & x \geq 0 \end{cases}$$

where  $k_1, k_2$  are real constants. If  $(gof)$  is differentiable at  $x=0$ , then  $(gof)(-4) + (gof)4$  is equal to :

- a)  $4(e^4 + 1)$
- b)  $2(2e^4 + 1)$
- c)  $4e^4$
- d)  $2(2e^4 - 1)$

8) The sum of absolute minimum and absolute maximum values of the function  $f(x) = |3x - x^2 + 2| - x$  in the interval  $[-1, 2]$

- a)  $\frac{\sqrt{17}+3}{2}$
- b)  $\frac{\sqrt{17}+5}{2}$
- c) 5
- d)  $\frac{9-\sqrt{17}}{2}$

9) Let  $S$  be set of all natural numbers, for which the line  $\frac{x}{a} + \frac{y}{b} = 2$  is a tangent to the curve  $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$  at the point  $(a, b), ab \neq 0$ . Then:

- a)  $S = \emptyset$
- b)  $n(S) = 1$
- c)  $S = \{2k : k \in \mathbf{N}\}$
- d)  $S = \mathbf{N}$

- 10) The area bounded by the curve  $y=|x^2 - 9|$  and the line  $y=3$  is :
- $4(2\sqrt{3} + \sqrt{6} - 4)$
  - $4(4\sqrt{3} + \sqrt{6} - 4)$
  - $8(4\sqrt{3} + 3\sqrt{6} - 9)$
  - $4(4\sqrt{3} + \sqrt{6} - 9)$
- 11) Let  $\mathbf{R}$  be point  $(3, 7)$  and let  $\mathbf{P}$  and  $\mathbf{Q}$  be two points on the line  $x+y=5$  such that  $PQR$  is an equilateral triangle. Then the area of  $\triangle PQR$
- $\frac{25}{4\sqrt{3}}$
  - $\frac{25\sqrt{3}}{2}$
  - $\frac{25}{\sqrt{3}}$
  - $\frac{25}{2\sqrt{3}}$
- 12) Let  $C$  be a circle passing through the points  $\mathbf{A}(2, -1)$  and  $\mathbf{B}(3, 4)$ . The line segment  $AB$  is not a diameter of  $C$ . If  $r$  is radius of  $C$  and its centre lies on the circle  $(x-5)^2 + (y-1)^2 = \frac{13}{2}$ , then  $r^2$  is equal to:
- 32
  - $\frac{65}{2}$
  - $\frac{61}{2}$
  - 30
- 13) Let the normal at the point  $\mathbf{P}$  on the parabola  $Y^2 = 6X$  pass through the point  $(5, -8)$ . If the tangent at  $\mathbf{P}$  to the parabola intersects at its directrix at the point  $\mathbf{Q}$  the the ordinate of point  $\mathbf{Q}$
- 3
  - $-\frac{9}{4}$
  - $-\frac{5}{2}$
  - 2
- 14) if the two lines  $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$  and  $l_2 : \frac{x-1}{2} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$  are perpendicular then the angle between the lines  $l_2$  and  $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$  is :
- $\cos^{-1} \frac{29}{4}$
  - $\sec^{-1} \frac{29}{4}$
  - $\cos^{-1} \frac{2}{29}$
  - $\cos^{-1} \frac{2}{\sqrt{29}}$
- 15) Let the plane  $2x+3y+z+20=0$  be rotated through a right angle about its line of intersection with the plane  $x-3y+5z=8$ . If the mirror image of the point  $(2, -\frac{1}{2}, 2)$  in the rotated plane is  $\mathbf{B}(a, b, c)$  then :
- $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$
  - $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$
  - $\frac{a}{8} = \frac{b}{5} = \frac{c}{4}$
  - $\frac{a}{8} = \frac{b}{5} = \frac{c}{2}$