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EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

53) Let **X** be a random variable with uniform distribution on the interval $(-1, 1)$ and **X** = $(\mathbf{X} + 1)^2$. Then the probability density function f(y) of **Y**, over the interval $(0, 4)$, is
   a) $\frac{3\sqrt{y}}{16}$
   b) $\frac{1}{4\sqrt{y}}$
   c) $\frac{1}{6\sqrt{y}}$
   d) $\frac{1}{\sqrt{y}}$

54) Let **S** be the solid whose base is the region in the XY-plane bounded by the curves y = x^2 and y = 8-x^2, and whose cross-sections perpendicular to the x-axis are squares. Then the volume of **S** is

55) Consider the trinomial distribution with the probability mass function $P(X = x, Y = y) = \frac{7!}{x!y!(7-x-y)!}(0.6)^x(0.2)^y(0.2)^{7-x-y}, x \geq 0, y \geq 0, \text{ and} x + y \leq 7$ Then $E(Y|X = 3)$ is equal to

56) Let $Y_i = \alpha + \beta x_i + \in_i$, where i = 1, 2, 3, 4, x_i's are fixed covariates and $\in$'s are independent and identically distributed standard normal random variables. Here, α and β are unknown parameters. Let ϕ be the cumulative distribution function of the standard normal distribution and $\phi(1.96) = 0.975$. Given the following observations,

Y_i	3	-2.5	5	-5
X_i	1	-2	3	-2

TABLE 56: variables used

the length (rounded off to two decimal places of the shortest) 95% confidence interval for β based on its least squares estimator is equal to

57) Consider a discrete time Markov chain on the state space $\{1, 2, 3\}$ with one-step transition probability matrix $\begin{pmatrix} 0 & 0.2 & 0.8 \\ 0.5 & 0 & 0.5 \\ 0.6 & 0.4 & 0 \end{pmatrix}$ Then the period of the Markov chain is

58) Suppose customers arrive at an ATM facility according to a Poisson process with rate 5 customers per hour. The probability (rounded off to two decimal places) that no customer arrives at the ATM facility from 1:00 pm to 1:18 pm is

59) Let **X** be a random variable with characteristic function $\phi_x(.)$ such that $\phi_x(2\pi) = 1$. Let **Z** denote the set of integers. Then $P(X \in \mathbf{Z})$ is equal to

60) Let X_1 be a random sample of size 1 from uniform distribution over $\left(\theta, \theta^2\right)$, where $\theta > 1$. To test $\mathbf{H}_0: \theta = 2$ against $\mathbf{H}_1: \theta = 3$, reject $\mathbf{H}_0$ if and only if $X_1 > 3.5$. Let α and β be the size and the power, respectively, of this test. Then $\alpha + \beta$ is equal to

61) Let $Y_i = \beta_0 + \beta_1 x_i + \in_i$, i = $1, \cdots, n$ where x_i's are fixed covariates, and $\in$'s are uncorrelated random variables with mean zero and constant variance. Suppose that $\hat{\beta}_0$ and $\hat{\beta}_1$ are the least squares estimators of the unknown parameters β_0 and β_1, respectively. If $\sum_{i=1}^{n} x_i = 0$, then the correlation between $\hat{\beta}_0$ and $\hat{\beta}_1$ is equal to

62) Let $f: \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = \left(3x^2 + 4\right)\cos x$ then $\lim_{h\to 0} \frac{f(h)+f(-h)-8}{h^2}$ is equal to

63) The maximum value of $(x-1)^2 + (y-2)^2$ subject to the constraint $x^2 + y^2 \leq 45$ is equal to

64) Let $x_1, \cdots x_{10}$ be independent and identically distributed normal random variables with mean 0 and variance 2. Then $E\left(\frac{x_1^2}{x_1^2 + \cdots x_{10}^2}\right)$ is equal to

65) Let **I** be the 4×4 identity matrix and $v = (1, 2, 3, 4)^t$, where t denotes the transpose. Then the determinant of $\mathbf{I} + vv^t$ is equal to