Question-9-9.2-19

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

Solution:

Description	Equation
circle	$x^2 + y^2 = a^2$
Line	$x = \frac{a}{\sqrt{2}}$
Point of intersections	$\mathbf{A, B}$

TABLE 0: variables used

The given circle can be expressed as a conic with parameters

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = 0, f = -a^2 \tag{0.1}$$

Line parameters are

$$\mathbf{h} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix}, \mathbf{m} = \mathbf{e}_3 \tag{0.2}$$

$$\mathbf{A} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{a}{\sqrt{2}} & \frac{a}{\sqrt{2}} \end{pmatrix} \tag{0.3}$$

From the (0.1) the area of the portion is given by

$$ar(APQ) = 2ar(APR) \tag{0.4}$$

$$= 2 \int_0^{\frac{a}{\sqrt{2}}} \left(\sqrt{a^2 - x^2} \right) dx \tag{0.5}$$

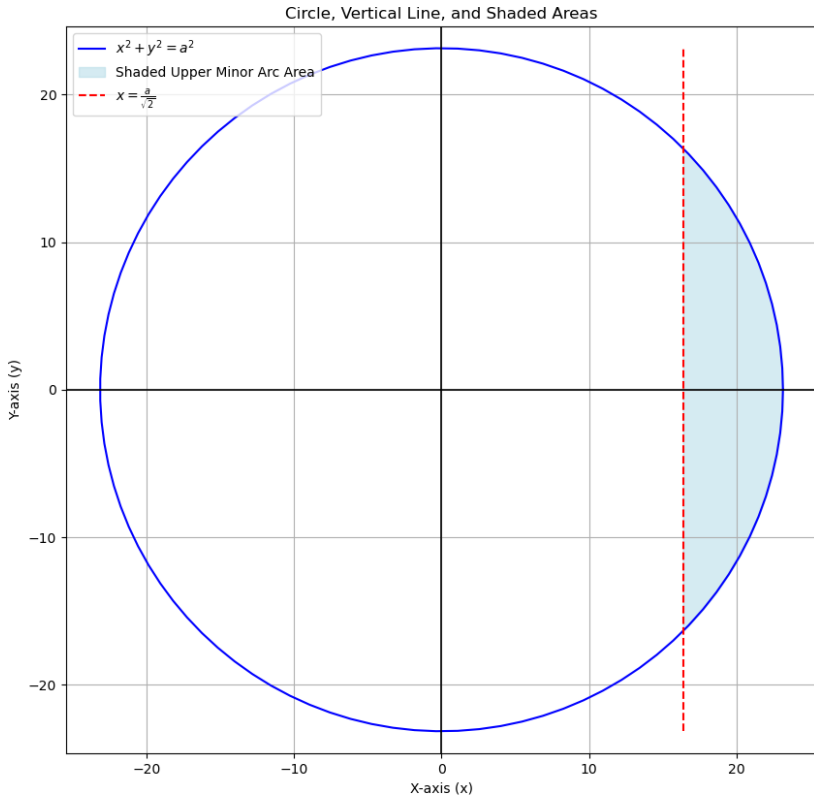$$= \frac{a^2}{2} \left(1 + \frac{\pi}{2} \right) \tag{0.6}$$

Fig. 0.1