2018-MA-53 to 65

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

53) Let f(x) = $Z^3e^{Z^2}$ for Z $\in$ **C** and let γ be the circle z = $e^{i\theta}$, where θ varies from 0 to 4π. Then

$$\frac{1}{2\pi i}\oint_{\gamma}\frac{f'(z)}{f(z)}\,dz = \cdots$$

54) Let S be the surface of the solid

$$\mathbf{V} = \{(x,y,z) : 0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3\}$$

Let $\hat{n}$ denote the unit outward normal to S and let

$$\mathbf{F}(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k},\ \ (x,y,z) \in \mathbf{V}$$

Then the surface integral $\iint_S \mathbf{F}.\hat{n}\,dS$ equals to

55) Let A be a 3×3 matrix with real entries. If three solutions of the linear system of differential equations $\hat{x}(t) = Ax(t)$ are given by

$$\begin{pmatrix} e^t - e^{2t} \\ -e^t + e^{2t} \\ e^t + e^{2t} \end{pmatrix}, \begin{pmatrix} -e^{-t} - e^{2t} \\ -e^{-t} + e^{2t} \\ e^{-t} + e^{2t} \end{pmatrix}, \begin{pmatrix} e^{-t} + 2e^t \\ e^{-t} - 2e^t \\ -e^{-t} + 2e^t \end{pmatrix}$$

then the sum of the diagonal entries of A is equal to

56) If $y_1(x) = e^{-x^2}$ is a solution of the differential equation

$$xy'' + \alpha y' + \beta x^3 y = 0$$

for some real α, β, then $\alpha\beta$

57) Let $L^2([0,1])$ be the Hilbert space of all real valued square integrable functions on $(0,1)$ with the usual inner product. Let ϕ be the linear functional on $L_2(\{0,1\})$ defined by

$$\phi(f) = \int_{\frac{1}{4}}^{\frac{3}{4}} 3\sqrt{2}f\ d\mu$$

where μ denotes the Lebesgue measure on $[0,1]$. Then $||\phi||$ =

58) Let U be an orthonormal set in a Hilbert space H and let x $\in$ H be such that $||x||$ = 2. Consider the set

$$E = \left\{u \in \mathbf{U} : |\langle x,u\rangle| \ge \frac{1}{4}\right\}$$

Then the maximum possible number of elements in E is

59) If p(x) = 2 - $(x+1)$ + x$(x+1)$ - βx$(x+1)(x-\alpha)$ interpolates the points (x,y) in the table then α+ β

x	-1	0	1	2
y	2	1	2	7

TABLE 59: variables used

60) If $\sin(\pi x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$ for $0 < x < 1$, then $(a_0 + a_1)\pi =$
61) For n=1,2,$\cdots$, let $f_n(x) = \frac{2nx^{n-1}}{1+x}$, $x \in [0, 1]$. Then $\lim_{n\to\infty} \int_0^1 f_n(x)$
62) Let X_1, X_2, X_3, X_4 be independent exponential random variables with mean 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, respectively. Then Y = min(X_1 ,X_2 ,X_3, X_4) has exponential distribution with mean equal to
63) Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbf{P}(X = Y)$ correct up to three decimal places is
64) Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $\mathbf{P}(X = k)$ = p$(1 - p)^{k-1}$, k = 1, 2,$\cdots$ Then the value of P($X_1 = 2|X_1 + X_2 = 4$) correct up to three decimal places is
65) A certain commodity is produced by the manufacturing plants P_1 and P_2 whose capacities are 6 and 5 units, respectively. The commodity is shipped to markets $M_1, M_2, M_3$and M_4 whose requirements are 1, 2, 3 and 5 units, respectively. The transportation cost per unit from plant P_i to market M_j is as follows: Then the optimal cost of transportation

	M_1	M_2	M_3	M_4	
P_1	1	3	5	8	6
P_2	2	5	6	7	5
	1	2	3	5	

TABLE 65: variables used