

Question-9-7.2-19

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Question:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

Solution:

| Conic | Equation |
|--------|--------------------------|
| circle | $x^2 + y^2 = a^2$ |
| Line | $x = \frac{a}{\sqrt{2}}$ |

TABLE 0: variables used

The given circle can be expressed as a conic with parameters

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = 0, f = -a^2 \quad (0.1)$$

Line parameters are

$$\mathbf{h} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix}, \mathbf{m} = \mathbf{e}_3 \quad (0.2)$$

Let the point of intersections with the line to the conic be \mathbf{A} and \mathbf{B}

$$\mathbf{A} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{a}{\sqrt{2}} & \frac{a}{\sqrt{2}} \end{pmatrix} \quad (0.3)$$

From the Fig.0.1 the area of the portion is given by

$$ar(APQ) = 2ar(APR) \quad (0.4)$$

$$= 2 \int_0^{\frac{a}{\sqrt{2}}} (\sqrt{a^2 - x^2}) dx \quad (0.5)$$

$$= \frac{a^2}{2} \left(1 + \frac{\pi}{2} \right) \quad (0.6)$$

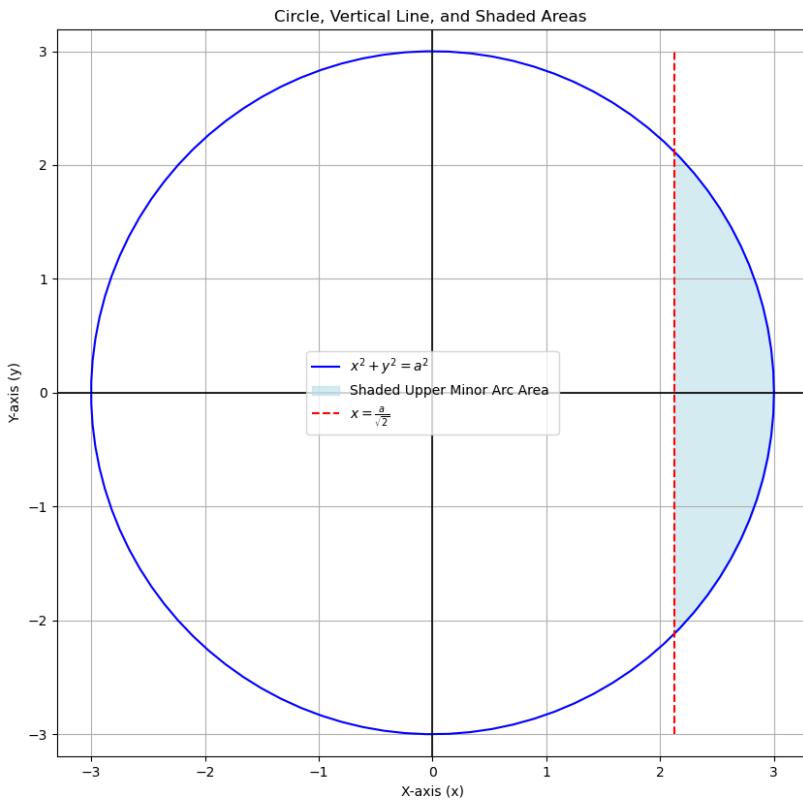


Fig. 0.1