

Question - 9-9.4-4

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Question:

Find the area of the region bounded by $y^2 = 2x$ and $x^2 + y^2 = 4x$.

Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of circle
P_1, P_2	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad u_1 = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, \quad f_1 = 0 \quad (0.1)$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad f_2 = -4 \quad (0.2)$$

The intersection of two conics with parameters $V_i, u_i, f_i, i = 1, 2$ is defined as

$$x^T (V_1 + \mu V_2) x + 2(u_1 + \mu u_2)^T x + (f_1 + \mu f_2) = 0 \quad (0.3)$$

Solving this the points of intersection are

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (0.4)$$

Area between the curves is,

$$2 \int_0^2 \left(2 + \sqrt{4 - y^2} - \frac{y^2}{2} \right) dy \quad (0.5)$$

By solving the integration, we get area is equal to 11.62 sq.units

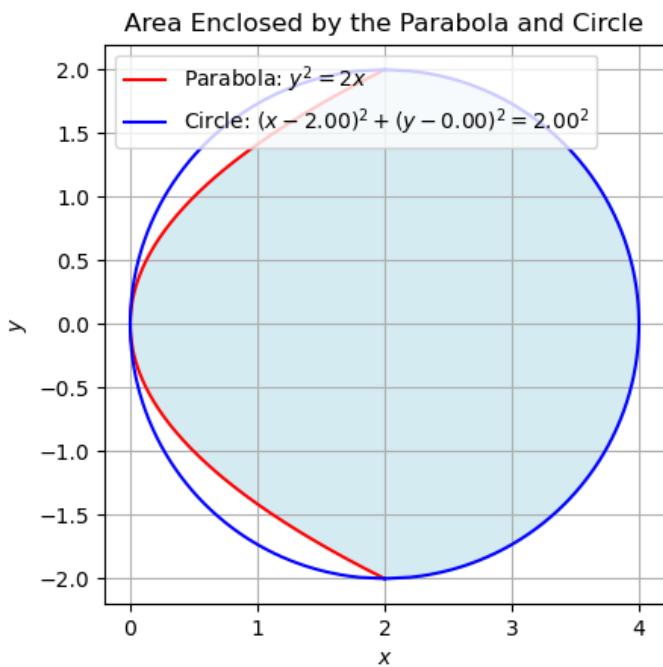


Fig. 0.1