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# Optimal scheduling in a yogurt production line based on mixed integer linear programming

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**Abstract**

A Mixed Integer Linear Programming (MILP) model is proposed in this paper that targets the optimal production scheduling in a single yogurt production line. The model takes into account all the standard constraints encountered in production scheduling (material balances, inventory limitations, machinery capacity, labor shifts and manpower restrictions). Furthermore, it considers special features that characterize yogurt production, which are limitations in production sequencing mainly due to different fat contents and flavors of various products and sequence-dependent setup times and costs. The objective function that is minimized considers all major sources of variable cost that depend on the production schedule, i.e. changeover cost, inventory cost and labor cost. The model is applied to a yogurt production line of a major dairy industry and the results are presented and discussed.


*Keywords:* Production scheduling; Yogurt production; Mixed integer linear programming; Sequence-dependent setups

## 1. Introduction

The increasing variety of products offered by the food industry has helped the industry to respond to market trends, but at the same time has resulted in a more complex production process, calling for greater flexibility and more efficient coordination of existing resources. A critical function for the efficient management of production in a food manufacturing company is production scheduling. Scheduling refers to the strategies of allocating equipment, utility and manpower resources over time to execute processing tasks required to manufacture one or several products (Pinto & Grossmann, 1998).

Several restrictions encountered in everyday production complicate the scheduling problem. The available machine time and man hours constitute a significant restriction to the problem. Another issue in the scheduling process is accomplishment of the production targets. There are several possible modes of operation and the choice among them depends on the goal sought, which could be the optimization of production earliness or tardiness or the optimization of the incurring production profit or cost. The chosen mode will affect demand fulfillment and, if unsuccessful, it could result to shortage and lost sales or stock surplus and increased inventory. There could also be limitations in the production sequence. These either arise from technical issues, or they are rules set empirically by the production engineers in order to simplify the scheduling process. Technical issues can arise due to configuration modifications in the machinery during transitions or due to changes in the packaging materials. Both require a changeover time, during which the production is seized. In some cases the equipment has to be cleaned during transitions, which additionally creates a changeover cost due to losses of product quantities and consumption of utilities.

Especially in industrial yogurt production, there is a wide variety of products that differ in features like fat content, the whey used to produce the mixture, the flavor, the size of the container or the language on the label (for goods to be exported). As pointed out by Nakhla (1995)

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**Nomenclature**

*Indices*
$i$ days
$j, k, l$ products

*Parameters*
$N$ scheduling horizon (days)
$P$ number of products
$demand(i,j)$ demand for product $j$ on day $i$ (1000 cups)
$csetup(j,k)$ changeover cost from product $j$ to product $k$ (€)
$tsetup(j,k)$ changeover time from product $j$ to product $k$ (h)
$cstorage$ storage cost (€/1000 cups/day)
$cost8$, $cost16$, $cost24$ labor costs for the three shifts (€/h)
$u(j)$ machine speed for product $j$ (1000 cups/h)
$openinv(i,j)$, $tarinv(i,j)$ opening and target inventory level of product $j$ at the end of day $i$ (1000 cups)
$M(j)$, $\mu(j)$ maximum and minimum production lots (1000 cups)

*Decision variables*
$prod(i,j)$ produced quantity of product $j$ on day $i$ (1000 cups)
$inv(i,j)$ inventory level of product $j$ at the end of day $i$ (1000 cups)
$Time(i)$ total utilization of machine, including changeover times on day $i$ (h)
$BIN(i,j)$ production of product $j$ on day $i$ (1/0)
$BINSETUP(i,j,k)$ changeover from product $j$ to product $k$ on day $i$ (1/0)

regarding scheduling operations at a yogurt production line, an empirical law is that the succession of products must follow an increasing fat level. For example, if skimmed milk is used to produce unflavored yogurt, it should be produced before full-fat milk in order to reduce setup time and the wasted quantity. Similarly, plain yogurt is processed before flavored. The opposite would require significant cleaning time and costs in order to make sure that no flavor or coloring would be transferred to the plain yogurt that follows.

Much of the scheduling research is directed towards problems with sequence-independent transitions. A scheduling methodology that incorporates sequence-dependent changeovers would be closer to reality and increase its effectiveness to a great extent. This can be justified as follows: If a sequence of operations requires excessive changeover time, it should not be preferred, despite the fact that its setup cost may be small. Therefore, when deciding among possible operations and their sequences, it is not sufficient to consider only one aspect, that is to say, time or cost, as this could lead to solutions with high costs or inappropriate timing and infeasible schedules respectively. Especially in the food industry, product-dependent transition costs and times, where cost is not proportional to time, are quite often. For instance, two products may be of the same flavor and differ only in the packaging material. In that case, setup time would normally be small and there would be no changeover cost, as no product quantity is lost. Alternatively, if flavors are different between two products, it is usually necessary to clean up the equipment in order to avoid mixing flavors and to maintain high hygienic standards. This results in consumption of utilities and material losses, but also idling of the machine and reduction of the available machine time. Setup cost in this situation would be high; setup time would vary depending on the cleaning procedure. A comprehensive review of scheduling problems that consider sequence-dependent transitions between products can be found in Reklaitis (2000) and Allahverdi et al. (1999).

Another important issue that must be considered in yogurt production scheduling is the rather short life-cycle of the products that must be consumed in a matter of weeks or even days. Furthermore, customers prefer that the yogurt they buy is as fresh as possible. This pushes towards a just-in-time mode of operation, which makes the supply chain more susceptible to fluctuations in demand.

All the above factors need to be taken into consideration in the design and implementation of the scheduling process. Only a few papers have been published concerning production planning or scheduling models in the dairy industry and they are focused on milk processing (Schuermann & Kannan, 1977; Sullivan & Secrest, 1985). To our knowledge, there is no paper that deals with the development and formulation of production planning or scheduling models specifically for yogurt production lines. The objective of this paper is to propose a concise formulation of the scheduling problem oriented at the food industry and in particular to yogurt production. The model takes into account the aforementioned issues, such as sequencing limitations and sequence-dependent changeover times and costs and aims to the minimization of an objective function that considers all the major sources of cost: changeover cost, labor cost and inventory cost. Despite the fact that production scheduling usually has to manage a great number of machines which are often interconnected in a complicated way, the single machine problem can be regarded as a building block in the development and understanding of more complicated systems and is addressed in this work. A final important issue in modeling a problem is the representation of the time domain. Continuous-time representation was employed in this work, taking into account the suitability of the scheduling problem to this type of time representation. Compared to discrete-time models, continuous-time models are more compact, use less binary variables and have a lower computational cost.

The rest of the paper is structured as follows: In the next section the motivating example that led to the development of the model is briefly described. The characteristics of the problem and the detailed model formulation are presented in Section 3. In Section 4, the computational results of the model in a particular case study are presented. The paper ends with the concluding remarks.

## 2. Motivating example

This study was stimulated by the effort to address the scheduling needs in the yogurt production lines of a major dairy company located in Athens, Greece. The particular problem that was addressed in this work was to optimally schedule the yogurt production operations on a single machine over a six-day horizon, with a maximum of three 8-h shifts daily. Daily production time cannot exceed 23 h, since all machinery must be cleaned at the end of the day and this takes 1-h time. For the same reason a production lot cannot involve more than one days. There are 18 different products with due dates within the week. The requested quantities represent local demands, but also orders from overseas. Production must always abide by the requested quantities and due dates, that is, the quantities must be met and no tardiness is allowed. Earliness is possible, but is burdened with an inventory holding cost, so it is an optimization decision whether it is beneficial to produce earlier or just-in-time. The system should also take the available inventory at the beginning of the scheduling horizon into consideration. The sought-for application should serve as a tool for the production scheduling department to be used in the beginning of the week or at any other time during the week. Its output should be the complete production schedule for the scheduling horizon.

The yogurt production problem is a typical example of the special sequencing restrictions found in the food industry. The machine under investigation manufactures numerous products that differ in package size or language, flavor, fat content, protein content, or presence of ingredients that increase the functional value. For that reason, it is not possible to group the products into families, as there is little uniformity among the products in transition costs and times. Another factor to be considered is the production speed of the machine for each product. A smaller speed is used for certain products with special packaging material in order to avoid machine breakdowns or to reduce losses due to damaged packages. Therefore, product-dependent production speeds should be accommodated in the model.

The finished products of the dairy industry are not merely stored in a warehouse; they are kept refrigerated. That results to a significant inventory holding cost, which must be taken into account in the optimization procedure. Furthermore, as demand is relatively steady, production is often intense and takes place in three shifts, which correspond to different labor costs. However, if the machine is not in operation, the personnel can be assigned to another production line. Therefore, labor cost depends on machine utilization time. Other costs that do not vary greatly with time or according to the specific product manufactured, like raw materials and utilities, were ignored in this study.

## 3. Problem definition – model formulation

The problem that is examined in this paper has the following structure: Given (i) the daily demand of each product, (ii) the starting inventory, (iii) setup costs and times for the transitions between products, (iv) the production speed of each product, (v) the inventory holding cost, (vi) the labor cost for the three working shifts, and (vii) the sequencing limitations, the objective of the model is to decide and calculate (i) the products to be manufactured in each day and their respective quantities, (ii) the machine time (starting time and ending time) utilized by each product and (iii) the inventory quantities of each product at the end of each day. At the same time, the constraints of the problem should be met. There are restrictions in: (i) production demands, (ii) due dates of orders, (iii) sequencing of operations, (iv) available machine time and man hours.

More precisely, the basic characteristics of the proposed scheduling tool are the following:

i. *Model formulation and time representation:* The formulation that is presented in this paper uses binary variables to indicate whether setup between two products takes place or not. The total scheduling horizon is separated into discrete-time periods, whose length is equal to one day. Within each time period, continuous-time formulation was preferred in order to reduce the size of the model and the required solution time and increase the accuracy of the model regarding time representation.
ii. *Food industry-specific:* The methodology that is presented in this work is oriented towards the food industry, as it takes into consideration a set of production restrictions that are frequent in food and especially yogurt production practice. For instance, the job sequence is determined by fat content, flavor and added ingredients. All products are placed in a succession with a ranking from the first to the last product to be produced. This succession must always be followed, although one or more products can be omitted and not be produced. Moreover, transitions between products are sequence dependent, meaning that both changeover times and costs are considered as sequence dependent. One more industry-specific requirement is the fact that at the end of the day all the equipment must be cleaned. So, production is seized at the end of each day. This is the reason for which each day is considered as a discrete-time period. For the same reason, each production lot must start and finish in the same day, although production

in the next day may start with the last product that is manufactured in the current day.

iii. *Demand satisfaction:* Demand is product-specific and is considered daily, that is, there are product orders and due dates within the scheduling horizon, often multiple within the week. Early production is possible but tardiness is not allowed.

iv. *Decision variables:* A number of model characteristics are not fixed or pre-determined in order to allow the solution algorithm to search for a combination of their values that would optimize the objective function. Such decision variables are the size and number of lots of each product, the processing time of each lot and the day and time of production. In that way, the optimization algorithm can decide for instance whether to assign one or more orders per lot or to group demands in order to reduce changeover costs (provided that all due dates are met).

v. *Objective function:* The objective function not only takes makespan into consideration, but also includes setup costs, standard and overtime labor costs and inventory costs. Thus, the procedure is directed towards solutions that realistically reduce the total production cost. Furthermore, daily utilization of manpower and inventory levels are separately calculated and can provide valuable information for plant operation.

The model is formulated as a Mixed Integer Linear Programming (MILP) problem, which is explained in details in the sequel. In order to define completely the MILP model we list separately the parameters, the decision variables, the constraints that must be satisfied and the objective function that describes the sources of production cost.

### 3.1. Parameters

- scheduling horizon,
- number and sequencing of products,
- demand of each product for each day,
- setup time and cost for each possible transition,
- storage cost of a unit of product for a day,
- labor cost for each shift,
- machine speed for each product,
- opening inventories and target inventories at the end of the scheduling horizon.

### 3.2. Decision variables

The optimal values of the decision variables are provided by the solution of the optimization problem and can be grouped into continuous variables and binary variables. For each day in the scheduling horizon, the optimal values of the following variables are obtained:

*Continuous variables*

- The produced quantity of each product.
- The inventory level of each product at the end of the day.
- The total utilization of the machine including the setup times.

*Binary variables*

- Binary variables (one for each product) indicating whether the respective product is to be produced in the particular day.
- Binary variables (one for each possible transition) indicating whether the respective changeover will take place or not.

### 3.3. Objective function – minimization of variable cost

$$\begin{aligned}
&\sum_i \sum_j \sum_l csetup(j,l) \cdot BINSETUP(i,j,l) \\
&\quad + \sum_i \sum_j inv(i,j) \cdot cstorage + \sum_i cost8 \cdot Time(i) \\
&\quad + \sum_i (cost16 - cost8) \cdot (Time(i) - 8 + a(i)) \\
&\quad + \sum_i (cost24 - cost16) \cdot (Time(i) - 16 + b(i)) \qquad (1)
\end{aligned}$$

The objective function represents the production cost, which is comprised of the setup costs, the inventory holding costs and the labor costs for all days of the scheduling horizon. Raw material and utility costs do not depend on any particular schedule and are not included in the objective function.

### 3.4. Constraints

The constraints that must be satisfied are expressed by the following set of equations. The names of the parameters and variables are explained in the nomenclature.

- Relationship between continuous variables and binary variables

$$prod(i,j) \leqslant M(j) \cdot BIN(i,j) \quad \forall i,j \qquad (2)$$

$$prod(i,j) \geqslant \mu(j) \cdot BIN(i,j) \quad \forall i,j \qquad (3)$$

where $M(j)$ and $\mu(j)$ indicate the maximum and the smallest lot sizes allowed. The above inequalities express the restriction that production of product $j$ in day $i$ is allowed ($prod(i,j) > 0$) if and only if the binary variable $BIN(i,j)$ takes the value of 1. Similarly, product $j$ is not manufactured in day $i$ ($prod(i,j) = 0$), if and only if the binary variable $BIN(i,j)$ takes the value of 0.

- Total material balance for each product throughout the scheduling horizon

$$openinv(j) + \sum_{i=1}^{N} prod(i,j) = \sum_{i=1}^{N} demand(i,j) + inv(N,j) \quad \forall j \qquad (4)$$

The summation of produced quantities of product $j$ throughout the production horizon plus the initial inventory must equal the sum of demand of all days plus the inventory of product $j$ at the end of the last day.

- Earliness is possible, but no tardiness is allowed

$$inv(1,j) = openinv(j) + prod(1,j) - demand(1,j) \quad \forall j$$
$$inv(1,j) \geqslant 0 \quad \forall j \tag{5}$$

$$inv(i,j) = inv(i-1,j) + prod(i,j) - demand(i,j) \quad i > 1, \forall j$$
$$inv(i,j) \geqslant 0 \quad i > 1, \forall j \tag{6}$$

Constraints (5) and (6) calculate the daily inventory levels for each product $j$, while at the same time require that daily demands are satisfied and no tardiness is allowed. At the end of the first day, the inventory must equal the initial inventory plus any produced quantity, reduced by the demand of that day (Eq. 5). On any other day, the inventory must be equal to the inventory level of the previous day plus any produced quantity, reduced by that day's demand.

- Satisfaction of the target inventory level at the end of the scheduling horizon

$$inv(N,j) = tarinv(j) \quad \forall j \tag{7}$$

The inventory levels at the end of the scheduling horizon must meet the targets. We should mention however, that the user can modify this constraint by substituting the equality sign by an inequality sign in case he just wants

Table 1
Investigation of correctness of set (11)–(14)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *BIN*(*i*,*j*) | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| $\sum_{k=j+1}^{l-1} BIN(i,k)$ | 0 | ⩾1 | 0 | ⩾1 | 0 | ⩾1 | 0 | ⩾1 |
| *BIN*(*i*,*l*) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| *BINSETUP*(*i*,*j*,*l*) | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

Table 2
Production sequence and machine speed

| Priority | Product | Machine speed (in 1000 cups/h) |
|---|---|---|
| 1 | P1 | 12 |
| 2 | P2 | 12 |
| 3 | P3 | 12 |
| 4 | P4 | 12 |
| 5 | P5 | 12 |
| 6 | P6 | 12 |
| 7 | P7 | 12 |
| 8 | P8 | 12 |
| 9 | P9 | 12 |
| 10 | P10 | 12 |
| 11 | P11 | 12 |
| 12 | P12 | 12 |
| 13 | P13 | 12 |
| 14 | P14 | 12 |
| 15 | P15 | 12 |
| 16 | P16 | 10 |
| 17 | P17 | 10 |
| 18 | P18 | 10 |

Table 3
Changeover costs (in €)

| From | To P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | P15 | P16 | P17 | P18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P1 | | 132.230 | 119.69 | 117.3 | 101.08 | 126.43 | 127.15 | 101.37 | 93.266 | 95.149 | 94.792 | 93.235 | 135.37 | 128.23 | 137.58 | 128.64 | 94.697 | 105.31 |
| P2 | | | 97.835 | 132.4 | 126.25 | 137.75 | 122.54 | 106.39 | 108.76 | 97.916 | 121.85 | 117.24 | 123.51 | 108.86 | 103.01 | 104.86 | 122.5 | 108.53 |
| P3 | | | | 130.1 | 93.412 | 122.82 | 136.99 | 134.97 | 108.68 | 110.68 | 112.15 | 131.82 | 138.09 | 135.02 | 115.73 | 98.893 | 137.61 | 125.34 |
| P4 | | | | | 138.21 | 127.12 | 131.64 | 105.69 | 114.2 | 118.02 | 93.319 | 97.266 | 98.149 | 99.172 | 121.82 | 124.54 | 112.89 | 98.418 |
| P5 | | | | | | 107.25 | 113.5 | 102.58 | 138.47 | 103.43 | 108.71 | 98.576 | 127.43 | 108.42 | 110.05 | 103.2 | 116.84 | 130.69 |
| P6 | | | | | | | 121.49 | 111.65 | 107.1 | 129.21 | 102.46 | 93.402 | 108.7 | 135.87 | 114.33 | 112.89 | 93.324 | 113.31 |
| P7 | | | | | | | | 132.12 | 102.63 | 109.39 | 136.24 | 131.2 | 112.71 | 115.8 | 127.52 | 132.18 | 114.69 | 126.11 |
| P8 | | | | | | | | | 119.24 | 91.549 | 121.47 | 96.699 | 91.928 | 94.515 | 96.31 | 134.07 | 110.88 | 139.74 |
| P9 | | | | | | | | | | 119.28 | 133.92 | 134.24 | 118.12 | 126.77 | 92.153 | 125 | 104.61 | 108.13 |
| P10 | | | | | | | | | | | 122.08 | 115.74 | 108.62 | 90.236 | 108.55 | 127.78 | 104.48 | 126.54 |
| P11 | | | | | | | | | | | | 138.18 | 129.64 | 120.16 | 124.67 | 138.73 | 127.69 | 122.48 |
| P12 | | | | | | | | | | | | | 129.76 | 137.84 | 136.79 | 110.11 | 94.84 | 124.07 |
| P13 | | | | | | | | | | | | | | 109.87 | 113.88 | 96.564 | 93.846 | 90.381 |
| P14 | | | | | | | | | | | | | | | 96.455 | 126.24 | 126.05 | 122.71 |
| P15 | | | | | | | | | | | | | | | | 134.98 | 128.25 | 137.26 |
| P16 | | | | | | | | | | | | | | | | | 122.9 | 120.66 |
| P17 | | | | | | | | | | | | | | | | | | 129.15 |
| P18 | | | | | | | | | | | | | | | | | | |

to pose an upper or lower limit in the inventory level at the end of the scheduling horizon.

- Time calculations

$$Time(i) = \sum_{j=1}^{P} \frac{prod(i,j)}{u(j)} + \sum_{j=1}^{P} \sum_{l=1}^{P} tsetup(j,l) \cdot binsetup(i,j,l) \quad \forall i \tag{8}$$

$$Time(i) \leqslant 23 \quad \forall i$$

For each day $i$, the first term in Eq. (8) summates the production times for all different products $j$. It should be noted that the term is linear since the speed $u(j)$ is a constant parameter and not a variable. The total machine time for each day $Time(i)$ includes the aforementioned production time and the time used for setups, which is reflected by the second term in Eq. (8). Eq. (8) implicitly implies that each product is produced at most once per day. Otherwise the job sequence is violated. Moreover, one production lot cannot involve more than one day, since all equipment must be cleaned up at the end of the day. Given that three 8-h shifts are available and considering the 1-h cleaning time at the end of each day, the total machine utilization time for each day cannot exceed 23 h, as the second part of the above equation indicates.

$$Time(i) - 8 + a(i) \geqslant 0 \quad \forall i \tag{9}$$

$$Time(i) - 16 + b(i) \geqslant 0 \quad \forall i \tag{10}$$

The above two inequalities introduce the intermediate variables $a(i)$ and $b(i)$ which are utilized in the objective function in order to calculate the labor cost.

- Setup constraints

We need the binary variable $BINSETUP(i,j,l)$ to take the value of 1 if and only if there is a changeover from product $j$ to product $l$ on day $i$. This is achieved by the following set of inequalities:

$$BINSETUP(i,j,l) \leqslant 1 + (1 - BIN(i,j)) + (1 - BIN(i,l)) - \lambda \sum_{k=j+1}^{l-1} BIN(i,k) \quad \forall i,j,\ \forall l > j \tag{11}$$

$$BINSETUP(i,j,l) \geqslant BIN(i,j) + BIN(i,l) - 1 - \sum_{k=j+1}^{l-1} BIN(i,k) \quad \forall i,j,\ \forall l > j \tag{12}$$

$$BINSETUP(i,j,l) \leqslant BIN(i,j) \quad \forall i,j,\ \forall l > j \tag{13}$$

$$BINSETUP(i,j,l) \leqslant BIN(i,l) \quad \forall i,j,\ \forall l > j \tag{14}$$

where $\lambda$ is a sufficiently small number. In order to confirm that the set of inequalities (11)–(14) always produces the correct value of the binary variable, we generated Table 1 that examines all the different cases, which may be encountered. It is shown that for each case there is only one possible value of BINSETUP ($i$, $j$, $l$), which is becoming equal to 1, only if $BIN(i,j)$ is 1 and $BIN(i,l)$ is 1 and additionally all binary variables $BIN(i,k)$, $k = j+1, \ldots, l-1$ are equal to 0.

The next Eq. (15) indicates that no transitions that violate the given sequence of production are allowed:

$$BINSETUP(i,j,l) = 0 \quad \forall i,j,\ \forall l \leqslant j \tag{15}$$

The final constraint (16) ensures that the number of products to be produced on each day $i$ minus the number of setups must be less or equal to 1. In fact (16) is an equality in all cases except from the situation when the machine remains idle for an entire day. In this extreme case both the number of products and the number of setups are zero. Eq. (16) does

Table 4
Change over times (in h)

| From | To | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | P15 | P16 | P17 | P18 |
| P1 | | 0.185 | 0.239 | 0.192 | 0.221 | 0.182 | 0.177 | 0.211 | 0.244 | 0.196 | 0.220 | 0.195 | 0.185 | 0.205 | 0.221 | 0.195 | 0.217 | 0.174 |
| P2 | | | 0.220 | 0.199 | 0.224 | 0.201 | 0.236 | 0.218 | 0.215 | 0.210 | 0.233 | 0.188 | 0.243 | 0.226 | 0.190 | 0.228 | 0.240 | 0.218 |
| P3 | | | | 0.171 | 0.232 | 0.195 | 0.185 | 0.247 | 0.222 | 0.215 | 0.206 | 0.203 | 0.244 | 0.219 | 0.182 | 0.246 | 0.230 | 0.201 |
| P4 | | | | | 0.200 | 0.183 | 0.206 | 0.236 | 0.232 | 0.231 | 0.212 | 0.194 | 0.171 | 0.194 | 0.222 | 0.180 | 0.200 | 0.187 |
| P5 | | | | | | 0.242 | 0.171 | 0.195 | 0.178 | 0.232 | 0.184 | 0.224 | 0.231 | 0.238 | 0.246 | 0.175 | 0.228 | 0.185 |
| P6 | | | | | | | 0.195 | 0.217 | 0.170 | 0.209 | 0.180 | 0.245 | 0.246 | 0.179 | 0.235 | 0.180 | 0.183 | 0.176 |
| P7 | | | | | | | | 0.180 | 0.213 | 0.234 | 0.188 | 0.197 | 0.235 | 0.193 | 0.244 | 0.183 | 0.247 | 0.171 |
| P8 | | | | | | | | | 0.171 | 0.208 | 0.178 | 0.215 | 0.244 | 0.178 | 0.195 | 0.243 | 0.186 | 0.233 |
| P9 | | | | | | | | | | 0.186 | 0.181 | 0.180 | 0.186 | 0.202 | 0.192 | 0.181 | 0.232 | 0.171 |
| P10 | | | | | | | | | | | 0.207 | 0.184 | 0.224 | 0.197 | 0.213 | 0.219 | 0.219 | 0.240 |
| P11 | | | | | | | | | | | | 0.192 | 0.244 | 0.246 | 0.183 | 0.192 | 0.183 | 0.198 |
| P12 | | | | | | | | | | | | | 0.198 | 0.237 | 0.187 | 0.188 | 0.172 | 0.228 |
| P13 | | | | | | | | | | | | | | 0.191 | 0.187 | 0.227 | 0.193 | 0.247 |
| P14 | | | | | | | | | | | | | | | 0.222 | 0.214 | 0.248 | 0.182 |
| P15 | | | | | | | | | | | | | | | | 0.245 | 0.246 | 0.183 |
| P16 | | | | | | | | | | | | | | | | | 0.188 | 0.195 |
| P17 | | | | | | | | | | | | | | | | | | 0.172 |
| P18 | | | | | | | | | | | | | | | | | | |

not actually introduce an additional piece of information to the model, but was found to accelerate the convergence of the optimization algorithm considerably:

$$\sum_{j=1}^{P} BIN(i,j) - \sum_{j=1}^{P} \sum_{l=1}^{P} BINSETUP(i,j,l) \leqslant 1 \quad \forall i \tag{16}$$

Table 5
Labor and inventory costs

| | | |
|---|---|---|
| *cost*8 | 50 | €/h |
| *cost*16 | 70 | €/h |
| *cost*24 | 120 | €/h |
| *cstorage* | 10 | (€/1000 cups)/day |

## 4. Case study

The case study presented here concerns a specific yogurt production line, where 18 products are produced. The products are indicated in Table 2, along with the production speeds. All the products are plain or fruit added cow milk yogurts, which also differ in fat content and in the packaging material. The problem for a six-day scheduling horizon, consists of 9.265 constraints and 1.368 variables, of which 1.026 are binary variables. The proposed tool was utilized to calculate the optimal production schedule for a week, for which the complete list of data are shown in Tables 3–6. Both opening and target inventories are set to zero for all 18 different products in this case study, which in fact means that the actual inventory levels at the beginning of the time window are assumed as minimum

Table 6
Production demand during the scheduling horizon (in 1000 cups)

| Product | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Total demand |
|---|---|---|---|---|---|---|---|
| P1 | – | 10 | – | – | 12 | – | 22 |
| P2 | – | – | 20 | – | – | 12 | 32 |
| P3 | 50 | 50 | 10 | 20 | 5 | 13 | 148 |
| P4 | 10 | 20 | 50 | 12 | 2 | 20 | 114 |
| P5 | – | 12 | – | – | 20 | – | 32 |
| P6 | 5 | 20 | 40 | 10 | 5 | 20 | 100 |
| P7 | 60 | – | – | – | 50 | – | 110 |
| P8 | – | – | – | 15 | – | – | 15 |
| P9 | 2 | 4 | 60 | – | 5 | 30 | 101 |
| P10 | 12 | – | – | 20 | 23 | 2 | 57 |
| P11 | – | 12 | 4 | 54 | 30 | – | 100 |
| P12 | – | – | 20 | – | – | – | 20 |
| P13 | – | 50 | – | 20 | 13 | 3 | 86 |
| P14 | 5 | 5 | 5 | 5 | 10 | 12 | 42 |
| P15 | 20 | – | – | – | 2 | – | 22 |
| P16 | – | – | 60 | 40 | 10 | – | 110 |
| P17 | – | 30 | – | – | – | 30 | 60 |
| P18 | 2 | 5 | – | 54 | 23 | 2 | 86 |

Table 7
The calculated production schedule (in 1000 cups)

| Product | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| P1 | – | 10 | – | – | 12 | – |
| P2 | – | 20 | – | – | – | 12 |
| P3 | 50 | 60 | – | 25 | – | 13 |
| P4 | 10 | 20 | 64 | – | – | 20 |
| P5 | – | 12 | – | – | 20 | – |
| P6 | 5 | 20 | 40 | 15 | – | 20 |
| P7 | 60 | – | – | – | 50 | – |
| P8 | – | – | – | 15 | – | – |
| P9 | 6 | – | 65 | – | – | 30 |
| P10 | 12 | – | – | 20 | 25 | – |
| P11 | – | 16 | – | 54 | 30 | – |
| P12 | – | – | 20 | – | – | – |
| P13 | – | 50 | – | 20 | 16 | – |
| P14 | 20 | – | – | – | 10 | 12 |
| P15 | 22 | – | – | – | – | – |
| P16 | – | – | 63 | 37 | 10 | – |
| P17 | – | 30 | – | – | – | 30 |
| P18 | 7 | – | – | 54 | 25 | – |
| Total produced quantity | 192 | 238 | 252 | 240 | 198 | 137 |

Table 8
Daily production time including setup times (in h)

| Product | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| P1 | – | 0.83 | – | – | 1.00 | – |
| P2 | – | 1.67 | – | – | – | 1.00 |
| P3 | 4.17 | 5.00 | – | 2.08 | – | 1.08 |
| P4 | 0.83 | 1.67 | 5.33 | – | – | 1.67 |
| P5 | – | 1.00 | – | – | 1.67 | – |
| P6 | 0.42 | 1.67 | 3.33 | 1.25 | – | 1.67 |
| P7 | 5.00 | – | – | – | 4.17 | – |
| P8 | – | – | – | 1.25 | – | – |
| P9 | 0.50 | – | 5.42 | – | – | 2.50 |
| P10 | 1.00 | – | – | 1.67 | 2.08 | – |
| P11 | – | 1.33 | – | 4.50 | 2.50 | – |
| P12 | – | – | 1.67 | – | – | – |
| P13 | – | 4.17 | – | 1.67 | 1.33 | – |
| P14 | 1.67 | – | – | – | 0.83 | 1.00 |
| P15 | 1.83 | – | – | – | – | – |
| P16 | – | – | 6.31 | 3.69 | 1.00 | – |
| P17 | – | 3.00 | – | – | – | 3.00 |
| P18 | 0.70 | – | – | 5.40 | 2.50 | – |
| Total production time (including setup times) | 17.67 | 21.97 | 22.78 | 23.00 | 18.76 | 13.11 |

safety bounds that must be satisfied at the end of each day. Moreover, the inventory levels at the end of the scheduling horizon should match the stored quantities at the beginning of the time window. The maximum and minimum production lots were set to 276,000 (the number of cups produced in 23 h with the maximum production speed) and 1000 cups respectively for all products. The MILP optimization problem that was formulated was solved using the CPLEX algorithm on a Pentium 4 2.8 GHz computer and the global optimum solution was obtained in less than 15 s. The optimal production schedule, daily machine utilization and daily inventory levels are depicted respectively in Tables 7–9.

The production schedule contains the decisions concerning the products to be manufactured every day and the respective quantities. As can be observed, production is accommodated towards the minimization of cost. For instance, as far as product 3 (P3) is concerned, there is a demand every day throughout this 6-day scheduling horizon. However, according to the optimal schedule, P3 is produced only in four days during the week. The excess quantities are used to cover the rest of the demand. More specifically Wednesday's demand for P3 is satisfied by Tuesday's production. Similarly, in order to meet the demand on Friday an additional 5.000 cups P3 are produced on Thursday, which remain in storage for one day.

Table 8 gives the daily machine utilization time allocated to each product. Additionally, the total time that the machine is utilized every day (including the changeover times) is shown in the last row of the same table. It can be observed that in all days except Saturday, operation of the machine exceeds 16 h, so that two full shifts and a part of the third shift are needed. These results are also given in a more graphical representation in Fig. 1, which

Table 9
Inventory levels beyond the safety limits at the end of each day (in 1000 cups)

| Product | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| P1 | – | – | – | – | – | – |
| P2 | – | 20 | – | – | – | – |
| P3 | – | 10 | – | 5 | – | – |
| P4 | – | – | 14 | 2 | – | – |
| P5 | – | – | – | – | – | – |
| P6 | – | – | – | 5 | – | – |
| P7 | – | – | – | – | – | – |
| P8 | – | – | – | – | – | – |
| P9 | 4 | – | 5 | 5 | – | – |
| P10 | – | – | – | – | 2 | – |
| P11 | – | 4 | – | – | – | – |
| P12 | – | – | – | – | – | – |
| P13 | – | – | – | – | 3 | – |
| P14 | 15 | 10 | 5 | – | – | – |
| P15 | 2 | 2 | 2 | 2 | – | – |
| P16 | – | – | 3 | – | – | – |
| P17 | – | – | – | – | – | – |
| P18 | 5 | – | – | – | 2 | – |

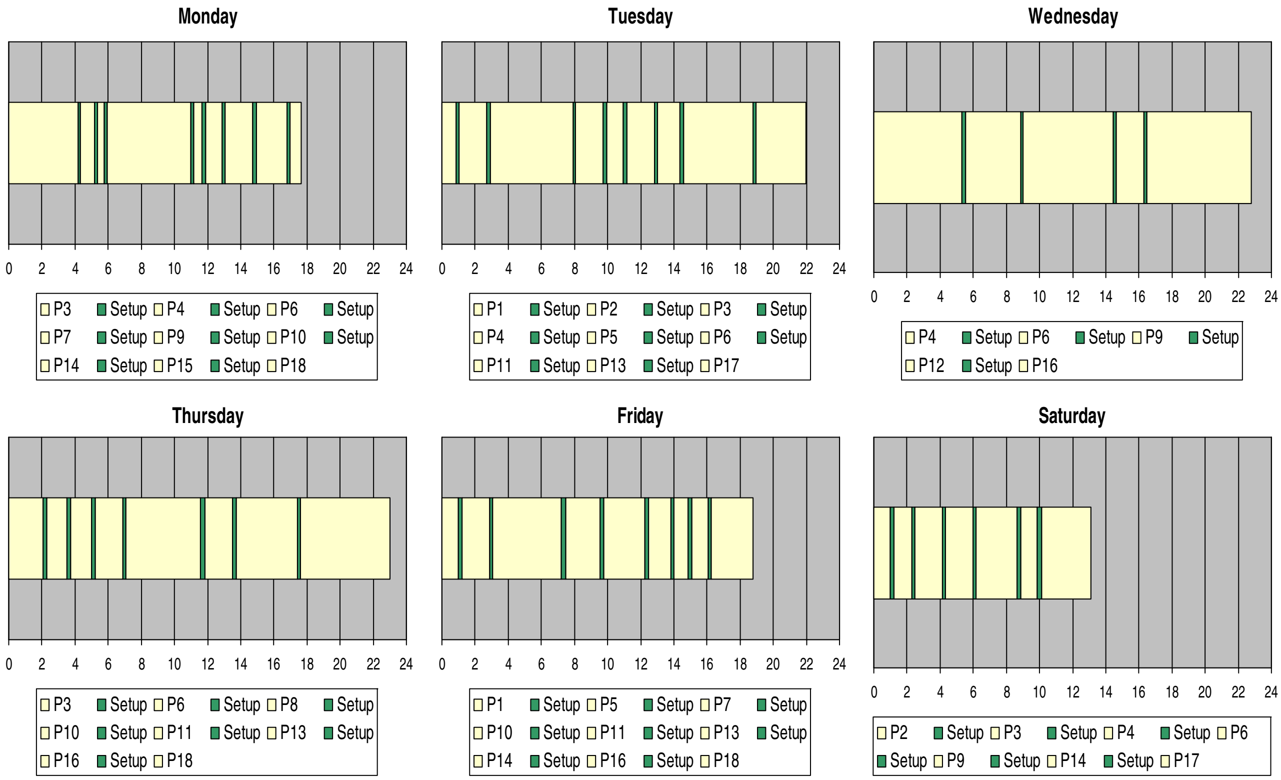


Fig. 1. Gantt charts for the six-day scheduling horizon.

shows detailed Gantt charts for the six days of the scheduling horizon. In these charts both production times for the different products and changeover times between products can be observed. Finally, Table 9 provides full knowledge of the inventory profile throughout the scheduling horizon. This information can contribute to a more efficient and realistic planning of the plant warehouse.

## 5. Conclusions

The problem of production scheduling for a yogurt production line of a dairy industry was studied in this work. The specific restrictions of the yogurt production process were taken into consideration in the formulation of a production scheduling optimization problem. The problem contains only linear equalities and inequalities, so that the global optimum solution is reached in a very short time. Another important aspect is that the optimization criterion is not restricted to time consideration (i.e. production time or earliness/tardiness), but includes production sequence-dependent costs, labor costs and inventory holding costs, in order to represent more realistically the production cost while achieving production goals.

The model produces the complete production schedule for a selected future horizon, including the sequence of products that should be produced every day and the respective quantities and the inventory levels at the end of each day. Future extensions of this research will be directed towards the simultaneous scheduling of parallel machines that is a set of machines, where due to technical restrictions there are dependencies on the operations across the machines. Ultimately, a production planning system that would include all production lines in a plant, including raw material and inventory capacity considerations, would offer a complete scheduling tool for the dairy industry.

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