Extending the Mehra-Prescott Model: A Rare Disasters Approach

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Abstract

The equity premium puzzle, first identified by Mehra and Prescott (1985), remains a central challenge in asset pricing. A promising explanation involves rare economic disasters. We extend the Mehra-Prescott model by introducing a disaster state with state-dependent disaster probabilities, disaster persistence, and an updated empirical model calibration incorporating the Great Recession and COVID-19 pandemic. Our model improves the fit of average risk-free rates and risk premia to historical U.S. data, showing that rare disasters lower average risk-free rates and raise average risk premia. We examine how disaster probability, magnitude, and persistence affect these outcomes, finding that more frequent or severe disasters better explain the equity premium puzzle, while persistence has ambiguous effects. Additionally, our model generates state-dependent yield curves, with higher shorter-maturity interest rates for low-growth and disaster states, underscoring broader implications of rare disasters for interest rate dynamics. Finally, we introduce a generalized method of moments (GMM) model calibration that captures asymmetric disaster probabilities and disaster persistence but struggles to resolve the equity premium puzzle.

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1 Introduction

The equity premium puzzle, first identified by Mehra and Prescott (1985), remains a fundamental challenge in asset pricing. Standard consumption-based models struggle to explain the historically high risk premium and low risk-free rate observed in the U.S. asset market, using reasonable levels of risk aversion. One promising approach to resolving this puzzle introduces rare economic disasters in asset pricing models. Rietz (1988) first proposed that even a small probability of catastrophic economic downturns could significantly increase the risk premium, without requiring unrealistically high levels of risk aversion. Barro (2006) expanded on this by using historical data to estimate the probabilities and severities of such rare disasters.

Building on this foundation, in this paper, we extend the Mehra-Prescott model by introducing a disaster state, alongside state-dependent disaster probabilities, disaster persistence, and an updated empirical model calibration reflecting more recent economic disasters, namely the Great Recession and COVID-19 pandemic. We then evaluate how well our model explains key aspects of the U.S. asset market, including the unconditional risk-free rate, unconditional risk premium, and yield curve. Additionally, we analyze the effects of changes in disaster probability, disaster magnitude, and disaster persistence on risk-free rates and risk premia. Finally, we introduce a generalized method of moments (GMM) approach to calibrating model parameters and compare its results to those of our empirical approach.

This paper contributes to the growing literature on the role of economic disasters in understanding asset puzzles. Gabaix (2012) directly built upon Rietz (1988) and Barro (2006) by introducing an asset pricing model with time-varying disaster severity and Epstein-Zin preferences, addressing asset puzzles ranging from the equity premium puzzle to high put prices being followed by high stock returns. Gourio (2012) integrated rare disasters into a real business cycle model, showing not only significant implications for asset prices but also broader macroeconomic implications for employment and output. Nakamura et al. (2013) further refined the rare disaster framework by allowing disasters to unfold over several years and to be systematically followed by recoveries, with cross-country correlations in disaster timing. These advancements underscore the robustness of disaster-based explanations, motivating our own rare disaster framework and calibration methods.

2 Theoretical Framework

2.1 Model

We model an economy where consumption growth follows a stochastic process governed by a three-state Markov chain x_t, with the transition matrix:

$$\mathbf{P} = \begin{bmatrix} \phi & 1-\phi-d_1 & d_1 \\ 1-\phi-d_2 & \phi & d_2 \\ \frac{1}{2}-\frac{1}{2}p & \frac{1}{2}-\frac{1}{2}p & p \end{bmatrix}.$$

A special case of this model, corresponding to Rietz (1988), occurs when $d_1 = d_2$ and $p = 0$. Consumption growth follows a stationary process:

$$\frac{C_{t+1}}{C_t} = \exp(g_C(x_t, x_{t+1})),$$

represented by the matrix Γ_C with elements:

$$[\Gamma_C]_{ij} = \exp(g_C(x_t = e_i, x_{t+1} = e_j)).$$

Preferences are modeled using a constant relative risk aversion (CRRA) utility function with time preference β and relative risk aversion γ:

$$u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}, \qquad 1 \neq \gamma > 0$$
$$u(c) = \log(c), \qquad \gamma = 1.$$

This yields the stochastic discount factor (SDF):

$$\frac{S_{t+1}}{S_t} = \beta \frac{u'(C_{t+1})}{u'(C_t)} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \beta \exp(-\gamma g_C(x_t, x_{t+1})),$$

represented by the matrix Γ_S with elements:

$$[\Gamma_S]_{ij} = \beta \exp(-\gamma g_C(x_t = e_i, x_{t+1} = e_j)) = \beta [\Gamma_C]_{ij}^{-\gamma},$$

where the $-\gamma$ power applies element-wise. We specifically define Γ_C as:

$$\Gamma_C = \begin{bmatrix} 1+\mu+\delta & 1+\mu-\delta & \psi(1+\mu) \\ 1+\mu+\delta & 1+\mu-\delta & \psi(1+\mu) \\ 1+\mu+\delta & 1+\mu-\delta & \psi(1+\mu) \end{bmatrix},$$

where $\psi(1+\mu) < 1+\mu-\delta < 1+\mu+\delta$. That is, e_1 is the high-growth state, e_2 the low-growth state, and e_3 the disaster state.

2.2 Risk-Free Rate and Risk Premium

The risk-free rate is:

$$R_{t+1}^{f} = \left(E_t\left[\frac{S_{t+1}}{S_t}\right]\right)^{-1},$$

represented by the matrix $\mathbf{R}^f$ with elements:

$$[\mathbf{R}^f]_{ij} = ([\mathbf{P} * \Gamma_S]_i \mathbf{1})^{-1}.$$

By averaging the risk-free rate across the stationary distribution π, we obtain the average risk-free rate:

$$E[R_{t+1}^{f}] = \sum_{i=1}^{3} \pi_i ([\mathbf{P} * \Gamma_S]_i \mathbf{1})^{-1}.$$

The equity return[1] is:

$$R_{t+1}^{c} = \frac{Q_{t+1} + C_{t+1}}{Q_t} = \frac{q(x_{t+1}) + 1}{q(x_t)} \frac{C_{t+1}}{C_t},$$

represented by the matrix $\mathbf{R}^c$ with elements:

$$[\mathbf{R}^c]_{ij} = \frac{\mathbf{q}_j + 1}{\mathbf{q}_i} [\Gamma_C]_{ij}.$$

Thus, the average risk premium is:

$$E[R_{t+1}^{c} - R_{t+1}^{f}] = \sum_{i=1}^{3} \pi_i \sum_{j=1}^{3} ([\mathbf{R}^c]_{ij} - [\mathbf{R}^f]_{ij}) [\mathbf{P}]_{ij}.$$

2.3 Term Structure of Interest Rates

The time-t price of a zero-coupon bond with maturity T is recursively defined as:

$$Q_t^{[T]} = E_t\left[\frac{S_{t+1}}{S_t} Q_{t+1}^{[T-1]}\right], \text{ where } Q_t^{[0]} = 1.$$

From these prices, we obtain the yields:

$$y_t^{[T]} = -\frac{1}{T} \log Q_t^{[T]}.$$

This results in a state-dependent term structure. That is, we get distinct yield curves for $x_t = e_1$, $x_t = e_2$, and $x_t = e_3$.

[1]We acknowledge that aggregate consumption is less volatile than aggregate dividends, which likely results in our model underestimating the risk premium. However, to maintain comparability with Mehra and Prescott (1985), Rietz (1988), Barro (2006), etc., we continue to use aggregate consumption. Future work should examine the use of aggregate dividends in our model.

3 Methodology and Data

In this paper, we fix $\mu = 0.018$, $\delta = 0.036$, and $\phi = 0.43$. This is the calibration Mehra and Prescott (1985) found to mimic the dynamics of U.S. aggregate consumption. We experiment with the calibration of disaster-specific parameters d_1, d_2, p, and ψ. Also following Mehra and Prescott (1985), we compute the average risk-free rate and average risk premium across different preference parameters $\beta \in (0, 1)$ and $\gamma \in (0, 10)$, focusing on modeling the U.S. asset market.

3.1 Empirical Calibration

Barro (2006) defines a disaster as a "15 percent or greater decline in real per capita GDP," accompanied by a drop in real consumer expenditure. Using Maddison (2003)'s historical dataset, Barro identified one U.S. disaster in the twentieth century: a 31% fall in real per capita GDP from 1929 to 1933, corresponding to the Great Depression. This implies an annual disaster probability of 0.010 for the U.S. (one disaster over 100 years).

To update this estimate, we analyzed U.S. real per capita GDP data from 2001 to 2024. While no new events met Barro's 15% threshold, lowering the threshold to 5% revealed two additional disasters: a 5.1% fall from 2007:Q4 to 2009:Q2, corresponding to the Great Recession, and a 9.3% fall from 2019:Q4 to 2020:Q2, corresponding to the COVID-19 pandemic. Thus, we calibrate $d_1, d_2 = 0.024$ (three disasters over 124 years). Notably, lowering the threshold does not change the number of disasters identified before 2001.

For disaster-state consumption contraction parameter ψ, we use an average of the observed contractions: 31% (Great Depression), 5% (Great Recession), and 9% (COVID-19 pandemic), calibrating $\psi = 0.85$. As an empirical value for p is less well-documented, we initially calibrate $p = 0$.

3.2 GMM Calibration

In addition to the empirical calibration described above, we estimate d_1, d_2, p, and ψ using generalized method of moments (GMM), first applied in the asset pricing context by Hansen and Singleton (1982). GMM estimates parameters θ that best fit moment conditions $f(X_{t+1}; \theta)$, where X_{t+1} is the available data. If θ_0 is the "true" θ, then:

$$\theta_0 = \text{argmin}_\theta E[f(X_{t+1}; \theta)]' W E[f(X_{t+1}; \theta)],$$

where W is an arbitrary positive definite weighting matrix.

In our case, $\theta = (d_1, d_2, p, \psi)$ and $X_{t+1} = (C_{t+1}/C_t, R^f_{t+1}, R^c_{t+1})$. C is "Real personal consumption expenditures per capita" (A794RX0Q048SBEA) from FRED. R^f_{t+1} and R^c_{t+1} come from asset returns data on Kenneth French's website, originally used in Fama and French (1993). Specifically, we take the following monthly returns:

- RF: The nominal risk-free rate, equivalent to the U.S. one-month Treasury yield (this corresponds to R^f).
- Mkt-RF: The excess return on a broadly constructed U.S. stock portfolio (Mkt-RF + RF corresponds to R^c).

Then, we compute quarterly returns:

$$R^N_{t+1} = \prod_{j=1}^{3} \left(1 + \frac{\tilde{r}_{t+1,j}}{100}\right),$$

where $\tilde{r}_{t+1,j}$ are the monthly returns in quarter $t+1$. Finally, we deflate the returns:

$$R_{t+1} = \frac{R^N_{t+1}}{1 + \pi_{t+1}},$$

where π is "Personal Consumption Expenditures: Chain-type Price Index" (PCECTPI) from FRED. A limitation is that this series starts in 1947:Q1, thereby excluding the Great Depression.

We impose the following moment conditions, equating "theoretical" (strictly model-dependent) moment $h(\theta)$ and "empirical" (strictly data-dependent) moment $k(X_{t+1})$.

$$E_t\left[\frac{S_{t+1}}{S_t} R^f_{t+1}\right] = 1 \implies E_t\left[\frac{S_{t+1}}{S_t}\right] = \frac{1}{R^f_{t+1}} \implies \left(E_t\left[\frac{S_{t+1}}{S_t}\right]\right)^{-1} = R^f_{t+1}$$

$$\implies (1)\ \underbrace{E\left[\left(E_t\left[\frac{S_{t+1}}{S_t}\right]\right)^{-1}\right]}_{h(\theta)} = \underbrace{\frac{1}{T}\sum_{t=0}^{T-1} R^f_{t+1}}_{k(X_{t+1})}$$

$$(2)\ \underbrace{Var\left[\left(E_t\left[\frac{S_{t+1}}{S_t}\right]^{-1}\right)\right]}_{h(\theta)} = \underbrace{Var[R^f_{t+1}]}_{k(X_{t+1})}$$

$$R^c_{t+1} = \frac{Q_{t+1} + C_{t+1}}{Q_t} = \frac{q_{t+1} + 1}{q_t}\frac{C_{t+1}}{C_t}$$

$$\implies (3)\ \underbrace{E[R^c_{t+1}]}_{h(\theta)} = \underbrace{E\left[\frac{q_{t+1} + 1}{q_t}\frac{C_{t+1}}{C_t}\right]}_{k(X_{t+1})}$$

$$(4)\quad \underbrace{E[(R^e_{t+1} - E[R^e_{t+1}])^2]}_{h(\theta)} = \underbrace{\frac{1}{T}\sum_{t=0}^{T-1}(R^e_{t+1} - E[R^e_{t+1}])^2}_{k(X_{t+1})}$$

We refer to these conditions as f_1, f_2, f_3, and f_4. f_1 tries to ensure our model fits the average risk-free rate, f_2 the variance of the risk-free rate, f_3 the average equity return, and f_4 the variance of the risk premium. Note that our GMM calibration allows for $d_1 \neq d_2$ and $p \neq 0$, unlike our empirical calibration.

4 Results

4.1 Empirically-Calibrated Model

In this section, model parameters are set to $\phi = 0.43, \mu = 0.018$, $\delta = 0.036$, $d_1 = 0.024$, $d_2 = 0.024$, $p = 0$, and $\psi = 0.85$, unless otherwise specified. The admissible regions in the plots that follow represent the average risk-free rate and average risk premium computed over $\beta \in (0, 1)$ and $\gamma \in (0, 10)$.

We find that adding a disaster state to the Mehra-Prescott model meaningfully changes asset market dynamics. Specifically, incorporating rare disasters decreases the average risk-free rate and increases the average risk premium, relative to the original model, for most values of β and γ.[2] Figure 1 illustrates this change, showing that our empirically-calibrated model raises the admissible region, falling a bit short of the historical average risk premium of 5% to 7%.

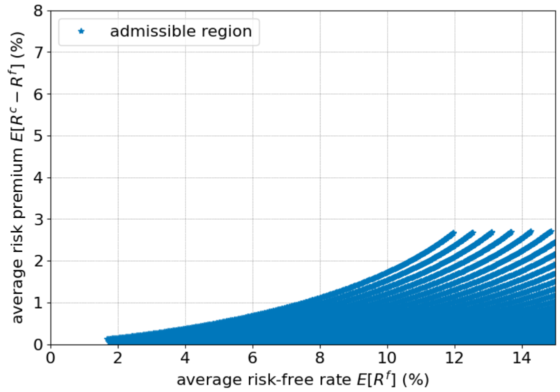
We further explore the effects of changing d_1, d_2, ψ, and p in our model. First, we reduce d_1 and d_2 from 0.024 to 0.010, consistent with Barro (2006). This changes the stationary distribution π from $(0.4883, 0.4883, 0.0234)'$ to $(0.4950, 0.4950, 0.0099)'$, and yields a higher average risk-free rate and a lower average risk premium.[3] This aligns with intuition: less frequent disasters reduce the risk perceived by investors, thus lowering their required compensation for risk. Figure 2 illustrates this change, resulting in a lower admissible region.

Similarly, if disasters are more severe (ψ decreases), the average risk-free rate should decrease and the average risk premium should increase, which is exactly what we observe when we reduce

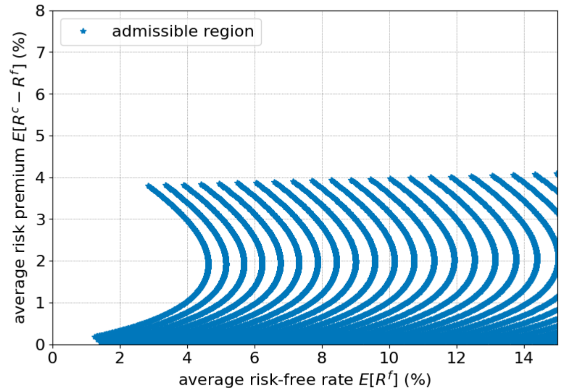
[2]A few exceptions occur when $\gamma = 0$, where investors have no risk aversion and would likely not react to the possibility of disasters.

[3]Again, a few exceptions occur when $\gamma = 0$.

Figure 1: Mehra-Prescott Model Without and With a Disaster State

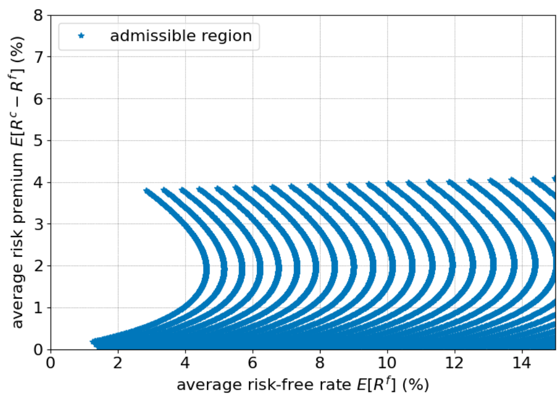


(a) Without Disaster State

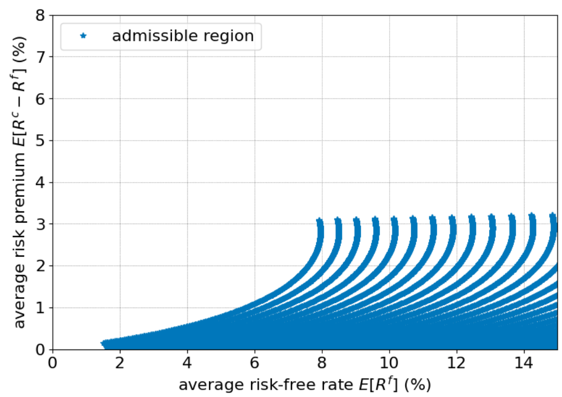


(b) With Disaster State

Figure 2: Effect of a Lower Disaster Probability



(a) $d_1, d_2 = 0.024$

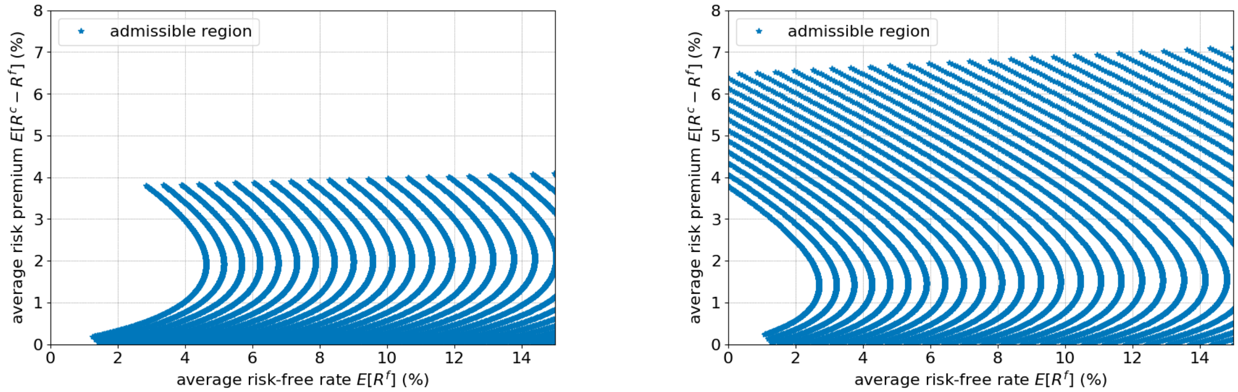


(b) $d_1, d_2 = 0.010$

ψ from 0.85 to 0.80. Notably, this change raises the admissible region to the historical average risk premium of 5% to 7% (see Figure 3 below), suggesting that $\psi = 0.80$ may better capture U.S. asset market dynamics.

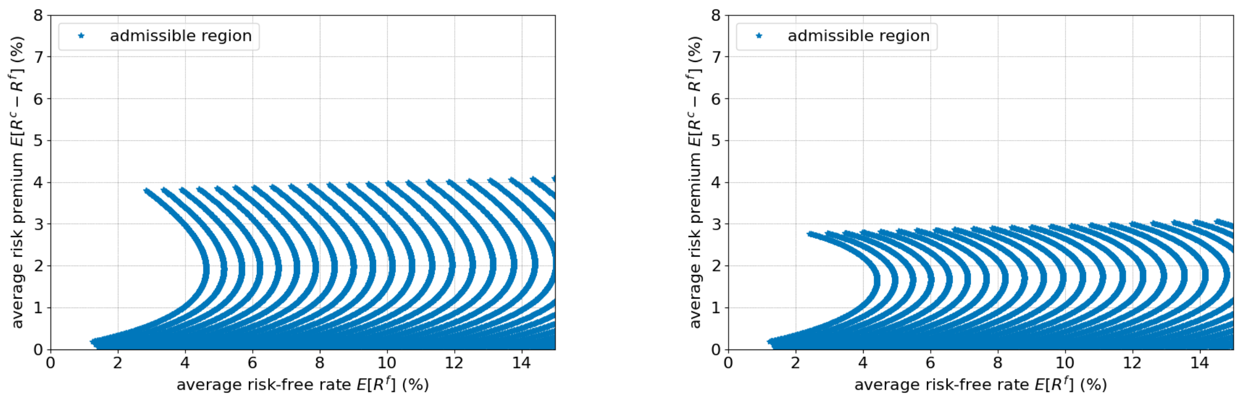
Finally, we increase p from 0 to 0.05. This results in the stationary distribution π changing slightly from $(0.4883, 0.4883, 0.0234)'$ to $(0.4877, 0.4877, 0.0246)'$. Interestingly, the average risk premium increases about 35% of the time and decreases in the remaining 65%, with no clear pattern in β and γ. Since there are more instances where the risk premium decreases, the admissible region falls (see Figure 4). We are unsure why this is the case. We expected the average risk premium to generally increase, as the probability of staying in the disaster state has increased. Future work should examine the cause(s) of this phenomenon.

Figure 3: Effect of a More Severe Disaster



(a) $\psi = 0.85$ (b) $\psi = 0.80$

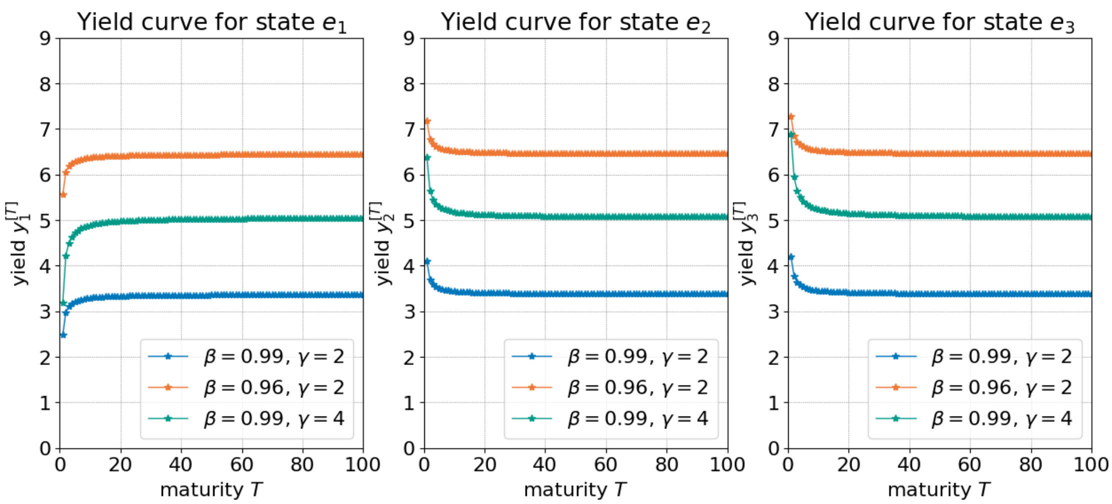
Figure 4: Effect of a Higher Disaster Persistence Probability



(a) $p = 0$ (b) $p = 0.05$

Turning to the term structure of interest rates, Figure 5 presents state-dependent yield curves. The left panel corresponds to the high-growth state (e_1), the middle panel to the low-growth state (e_2), and the right panel to the disaster state (e_3). As expected, shorter-maturity interest rates are lower for the high-growth state and higher for the low-growth state. This aligns with the elasticity of substitution effect, where higher expected growth leads to higher borrowing demand, which in turn pushes interest rates up to clear the risk-free asset market. The disaster state resembles the low-growth state but has slightly higher shorter-maturity interest rates. This is likely due to the larger gap between the high-growth and disaster states compared to the high-growth and low-growth states.

Figure 5: State-Dependent Yield Curves



4.2 GMM-Calibrated Model

Complementing our empirical calibration, we calibrate d_1, d_2, p, and ψ using GMM. Given four moment conditions, we estimate these parameters across $\beta \in (0, 1)$ and $\gamma \in (0, 10)$ and compare the results to those of our empirical calibration.

Table 1 presents reasonable calibrations found by GMM. Notably, the GMM calibration seems to capture asymmetry in disaster probabilities d_1 and d_2, with $d_2 > d_1$ suggesting that a disaster is more likely to follow a low-growth state than a high-growth state. The presence of positive estimates for p also supports the idea of disaster persistence, which we were unable to capture in our empirical calibration.

Despite these promising findings, our GMM calibration does not directly solve the equity premium puzzle. As seen in Table 2, the GMM calibration fails to produce average risk premia close to 5% to 7%. The primary source of error seems to be moment condition f_3, which tries to fit the average equity return. Furthermore, the GMM calibration sometimes results in negative average risk-free rates, indicating model misspecification. We experimented with weighting matrices other than the identity matrix to downweight f_3, but this did little to change the resulting risk-free rates and risk premia. Further research is needed to determine whether GMM is a suitable calibration method for our rare disaster model.

Table 1: GMM Results for Fixed (β, γ)

(β, γ)	ψ	d_1	d_2	p
(0.6, 0)	0.7646	0.0250	0.0254	0.0002
(0.6, 9)	0.6964	0.0517	0.0806	0.0316
(0.7, 0)	0.7011	0.0247	0.0271	0.0002
(0.8, 8)	0.6554	0.0125	0.0175	0.0350
(0.9, 10)	0.7918	0.0246	0.0564	0.0000
(1.0, 1)	0.8394	0.0241	0.0251	0.0000
(1.0, 5)	0.7100	0.0197	0.0262	0.0002
(1.0, 9)	0.8264	0.0131	0.0486	0.0001
(1.0, 10)	0.8532	0.0093	0.0609	0.0075

Table 2: Empirical vs. GMM Average Risk-Free Rate and Risk Premium

	Average Risk-Free Rate		Average Risk Premium	
(β, γ)	Empirical	GMM	Empirical	GMM
(0.6, 0)	0.6667	0.6667	0.0000	0.0000
(0.6, 9)	0.7286	-0.2466	0.0464	0.2581
(0.7, 0)	0.4286	0.4286	0.000	0.000
(0.8, 8)	0.3036	-0.0238	0.0313	0.0355
(0.9, 10)	0.1428	-0.0675	0.0408	0.0802
(1.0, 1)	0.0126	0.0121	NA	0.0019
(1.0, 5)	0.0438	-0.0244	0.0134	0.0365
(1.0, 9)	0.0371	-0.0183	0.0321	0.0309
(1.0, 10)	0.0285	-0.0101	0.0380	0.0227

5 Conclusion

In this paper, we extended the Mehra-Prescott model by incorporating a disaster state with state-dependent disaster probabilities, disaster persistence, and an updated empirical calibration that includes the Great Recession and COVID-19 pandemic. Our results demonstrated that introducing a disaster state significantly affects asset market dynamics, decreasing the average risk-free rate and increasing the average risk premium. This suggests that rare disasters remain a compelling explanation for the equity premium puzzle.

Our empirical calibration showed that increasing disaster probability (higher d_1 and d_2) or disaster severity (lower ψ) generally improves our model's ability to produce realistic average risk-free rates and average risk premia. However, the effect of increasing disaster persistence (higher p) on the average risk premium was ambiguous, requiring further research. Additionally, our GMM calibration supported the idea that disasters are more likely to follow low-growth states than high-growth states and that disasters can persist. Nevertheless, the GMM calibration failed to produce realistic average risk premia and sometimes produced negative average risk-free rates, indicating potential model misspecification.

Beyond risk-free rates and risk premia, our model captures state-dependent yield curves, where shorter-maturity interest rates are higher for low-growth and disaster states. This highlights the role of rare disasters in also shaping interest rate dynamics.

Future research should explore several directions. First, incorporating aggregate dividends instead of consumption growth may improve our model's ability to match observed risk premia. Second, refining the GMM calibration to address model misspecification and conduct inference is a crucial next step. Third, introducing other risk preferences, such as habit formation or Epstein-Zin preferences, could provide deeper insights into the relationship between disasters and asset market dynamics. Fourth, allowing for not only state-dependent disaster probabilities but also state-dependent disaster severities, similar to Gabaix (2012), may improve our model's performance. Finally, while this paper focuses on U.S. data, extending our model to international settings, like Nakamura et al. (2013), could offer a broader perspective on rare disaster dynamics in global markets.

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