Topics: Normal distribution, Functions of Random Variables

1. **The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**

   a) 0.3875
   b) 0.2676
   c) 0.5
   d) 0.6987

Ans. Let us denote the time required by 'x'. We have μ=45, σ=8 after the 10 min wait the service manager has 50 minutes to complete the work within an hour.
So, probability that the service manager cannot meet his commitment is P(x>50)

P(x>50) = 1- P(x<=50).

Z score associated with x=50,

$$\text{Z-Score} = \frac{x-\mu}{\sigma}$$

$$= \frac{50-45}{8}$$

$$= \underline{\underline{0.625}}$$

Corresponding probability, p = 0.7324 (from Z-Table)

P(x>50) = 1-0.7324 $\underline{\underline{= 0.2676}}$

probability that the service manager cannot meet his commitment = 0.2676 [Option (B)]

2. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.**
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. For the given normal distribution, μ= 38 and σ= 6. Let us denote the number of employees by 'x'.

A. Probability that an employee is older than 44 = P(x>44)

P (x > 44) = 1 - P(x ≤ 44).

At x=44,

Z-Score = $\frac{x-\mu}{\sigma}$

= $\frac{44-38}{6}$

= 1

Corresponding probability, p = 0.8413 (from Z-Table)

P(x>44) = 1-0.8413 = 0.1587

At x=38,

Z-Score = $\frac{x-\mu}{\sigma}$ = 0

Corresponding probability, p = 0.5(from Z-Table)

P(38<x<44) = 0.8413-0.5 = 0.3413

P(38<x<44) > P(x > 44). Therefore, the statement A is FALSE.

B. Probability that an employee is older than 30 = P(x<30)

At x=44,

Z-Score = $\frac{x-\mu}{\sigma}$

= $\frac{30-38}{6}$

= -1.33

Corresponding probability, p = 0.0918 (from Z-Table)

9.18% of total 400 employees is 36.72 ≅ 36.

Therefore statement B is TRUE.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans.

As we know that if $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ are two independent random variables then

$X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ and

$X_1 - X_2 \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

Generally,

$a_1X_1 + a_2X_2 \sim N(a_1\mu_1 + a_2\mu_2, a_1\sigma_1^2 + a_2\sigma_2^2)$

Therefore, in the question

$2X_1 \sim N(2u, 4\sigma^2)$ and

$X_1 + X_2 \sim N(0, 2\sigma^2)$

$2X_1 - (X_1 + X_2) = N(4\mu, 6\sigma^2)$

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- **a) 90.5, 105.9**
- **b) 80.2, 119.8**
- **c) 22, 78**
- **d) 48.5, 151.5**
- **e) 90.1, 109.9**

Ans. Since we need to find out the values of a and b, which are symmetric about the mean, Also, the probability of random variable taking a value between them is 0.99
In case of symmetric probability = 0.99, to get symmetry about mean

$$\frac{(1-0.99)}{2} = 0.005$$

Z-score corresponding to the probability is -2.57.

Now finding the a, b values

$(20 \times (-2.57)) \pm 100$ gives the values (48.6, 151.4)

So, Option D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

- **A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
- **B. Specify the 5th percentile of profit (in Rupees) for the company**
- **C. Which of the two divisions has a larger probability of making a loss in a given year?**

Ans. **A)**

In [2]:
```
# The total profit can be estimated by adding the two given profits
mean = (5+7)*45
std = np.sqrt(3**2+4**2)*45
```

In [3]:
```
total_profit = ss.norm.interval(0.95,mean,std)
print("The total profit is in the range",total_profit,"Million Rupees")
```

```
The total profit is in the range (99.00810347848784, 980.9918965215122) Million Rupees
```

B)

In [4]:
```
# Finding the 5th Percentile
print("The 5th Percentile of Profit is",ss.scoreatpercentile(total_profit,5),"Million Rupees")
```

```
The 5th Percentile of Profit is 143.10729313063905 Million Rupees
```

C)

In [5]:
```
#The probability of making a loss for both divisions
print('The probability of Division 1 making a loss is',ss.norm.cdf(0,5,3)," and")
print('The probability of Division 2 making a loss is',ss.norm.cdf(0,7,4))
```

```
The probability of Division 1 making a loss is 0.0477903522728147  and
The probability of Division 2 making a loss is 0.040059156863817086
```

- Since the probability that Profit < 0 is higher for Division 1, it has the larger chances of making a loss.