Derivation of my Velocity Model

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The net force when taking drag into account,

$$F_{net} = W - D,$$

where W is weight and D is drag, can be manipulated as follows to show that drag depends on the square of speed:

$$m\frac{dv}{dt} = W - D$$

$$\frac{dv}{dt} = \frac{W - D}{m}$$

$$\frac{dv}{dt} = g - \frac{C_d \rho A}{2m}(v^2),$$

where g = 9.80 $\frac{m}{s^2}$, C_d = coefficient of drag, A= front area, m = mass, and v= velocity.

This yields an ODE that can be solved as follows

$$dv = (g - \frac{C_d \rho A}{2m}(v^2))dt$$

$$\int_{v_i}^{v_f} dv = \int_0^t (g - \frac{C_d \rho A}{2m}(v^2))dt.$$

Given that v and g are vectors whose only nonzero components are their j components in this experiment, the following substitution may be performed:

$v^2 = v_i^2 + 2gr$, where r = y-position, so

$$\int_{v_i}^{v_f} dv = \int_0^t \left(g - \frac{C_d \rho A}{2m}\left(v^2\right)\right)dt \rightarrow \int_{v_i}^{v_f} dv = \int_0^t \left(g - \frac{C_d \rho A}{2m}\left(v_i^{\,2} + 2gr\right)\right)dt$$

$$v_f = gt - \frac{C_d A}{2m}\left(v_i^{\,2} t + 2grt\right) + gt + v_i .$$