Derivation of my Velocity Model

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The net force when taking drag into account,

$$F_{net} = W - D,$$

where W is weight and D is drag, can be manipulated as follows to show that drag depends on the square of speed:

$$m\frac{dv}{dt} = W - D$$

$$\frac{dv}{dt} = \frac{W - D}{m}$$

$$\frac{dv}{dt} = g - \frac{C_d \rho A}{2m}(v^2),$$

where g = 9.80 $\frac{m}{s^2}$, C_d = coefficient of drag, A= front area, m = mass, ρ = density (which we are taking to be 1atm), and v= velocity.

This yields an ODE that can be solved as follows

$$dv = (g - \frac{C_d A}{2m}(v^2))dt$$

$$\int_{v_i}^{v_f} dv = \int_0^t (g - \frac{C_d A}{2m}(v^2))dt.$$

Given that v and g are vectors whose only nonzero components are their j components in this experiment, the following substitution may be performed:

$v^2 = v_i^2 + 2gr$, where r = y-position, so

$$\int_{v_i}^{v_f} dv = \int_0^t \left(g - \frac{C_d A}{2m}\left(v^2\right)\right)dt \rightarrow \int_{v_i}^{v_f} dv = \int_0^t \left(g - \frac{C_d A}{2m}\left(v_i^2 + 2gr\right)\right)dt$$

I) $v_f = v_i + gt - \frac{C_d A}{2m} t\left(v_i^2 + 2gr\right)$.

II) $v_f = -\frac{C_d A t}{2m}$

It is easy to see from **I** that this equation is a modified form of $v_f = v_i + at$, wherein a quantity is being subtracted at every value of time t, which is consistent with our notion of how drag affects velocity. It is also easy to see from **II** that, if v_i is taken as the variable, the velocity will follow a downward-facing parabola, which is consistent with our notion of projectile motion.

In my computer model, each iteration of final velocity will be substituted into the position update formula $r_f = r_i + vt$ to iterate position at each time step.