Complex Potential Function for a Point Vortex

Given a coordinate $z = x + iy$ in the complex plane, a point vortex is described by

$$w = \frac{-i\Gamma}{2\pi}\ln(z)$$

Where Γ is the circulation.

The derivative gives the velocity components in the x direction (u) and y direction (v).

$$\frac{dw}{dz} = \frac{-i\Gamma}{2\pi}\cdot\frac{1}{z} = u - iv$$

Interaction Between 2 or more Vortices

Given N vortices, each of their motion is described by

$$\frac{dx_j}{dt} = -\frac{1}{2\pi}\sum_{\substack{i=1\\ i\neq j}}^{N}\frac{\Gamma_i(\mathrm{y}_j - \mathrm{y}_i)}{r_{ij}^2}$$

$$\frac{dy_j}{dt} = \frac{1}{2\pi}\sum_{\substack{i=1\\ i\neq j}}^{N}\frac{\Gamma_i(\mathrm{x}_j - \mathrm{x}_i)}{r_{ij}^2}$$

Where $(\mathrm{x}_j, \mathrm{y}_j)$ is the position of the jth vortex and r_{ij} is the distance between the jth and ith vortices.