ERASMUS MUNDUS MASTER WAVES

Computational Modelling

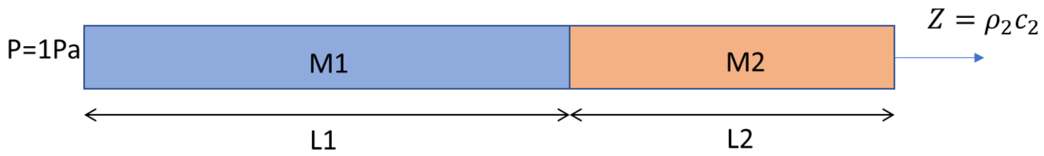
2024/2025

PRACTICAL WORK 1

NOTE: A1 and A2 are the last 2 digits of your student identification number (from UC)

(for example: SID 1234567 -> A1=7 and A2=6)

Consider a 1D problem governed by the Helmholtz equation $\frac{\partial^2 p}{\partial x^2} + \left(\frac{\omega}{c}\right)^2 p = 0$, in which p is the acoustic pressure, c the sound propagation velocity and ω the angular frequency. In this problem, consider the following scenario, in which the propagation medium is composed of two different fluids, with the following arrangement and boundary conditions:



Both fluids have the same density, and the sound propagation velocity in each fluid is given by:

FLUID M1: c=1500.0+10.0*A1-A2 (m/s); $\rho = 1000 \mathrm{kg/m^3}$

FLUID M2: c=(1800.0+20.0*A2*A1)x(1+0.02*i*(2+$\sqrt{A1}$)) (m/s) ; $\rho = 1000 \mathrm{kg/m^3}$

The dimensions L1 and L2 are given by:

L1=20+A1/0.5 (m)

L2=15+A2/0.5 (m)

The excitation frequency is: 500.0+A2 x 20.0+A1 (Hz)

For this case:

a) For a generic discretization (with spacing Δx), write the Finite Difference Method (FDM) equations at an internal node of M1, at an internal node of M2, and at the extreme nodes where the boundary conditions are prescribed.

b) Consider now a finite element discretization, with 50 linear elements in fluid M1 and 50 in fluid M2. For the node connecting M1 and M2, write the acoustic stiffness and mass matrices of the two elements connected to the node, and establish the corresponding equation of the Finite Element Method (FEM) at that node.

c) Starting from the Matlab codes used in the classes for the **FDM**, adapting it, and also making use of the analytical solution given in the appendix:

   1. Calculate the response using 200 points distributed throughout the domain using the finite difference method, and compare the response with the given analytical solution.

2. For discretizations starting at 100 points and then progressively increasing this number (use at least 10 different discretizations), calculate the average error in the numerical estimation of the pressure (P) throughout the domain, and represent the result as a convergence curve (in a Log-Log scale), representing the number of points in the horizontal axis and the absolute error in the vertical axis.

d) Starting from the Matlab codes used in the classes for the **FEM**, adapting them, and also making use of the analytical solution given in the appendix:
   1. Discretize the domain using a maximum element size of 0.5m, and compute the solution using the FEM. Compare the response with the given analytical solution.
   2. For successively finer discretizations, starting with an element size of approximately 1.0 m and then reducing the element size (use at least 10 different discretizations), calculate the average error in the numerical estimation of the pressure (P) throughout the domain, and represent the result as a convergence curve (in a Log-Log scale), representing the total number of nodes in the horizontal axis and the absolute error in the vertical axis.

e) Compare the convergence curves calculated for the FDM and FEM, superimposing both curves in the same plot. Comment the obtained results.

f) Using the FEM with adequate discretizations, compute the response of the system for frequencies between 1 Hz and the excitation frequency given above, considering two different cases for the boundary conditions of the system:
   1. The Robin boundary condition given above at the rightmost node ($Z = \rho_2 c_2$);
   2. A Neumann boundary condition at the rightmost node, with $v_n = 0$.

   For each case, present the results graphically:

   1. at the interface point between M1 and M2 in terms of the real, imaginary and absolute values of the acoustic pressure for each frequency.
   2. in a colormap plot, for position x frequency, in terms of a sound pressure level with a reference 1 Pa (20*log10(P/1.0)).

   Analyze and comment the results.

NOTE: for convergence analysis consider that the average absolute error is given by:

$$\bar{\varepsilon} = \frac{\sum \left| p_{analytical(x_i)} - p_{numeric(x_i)} \right|}{N}$$

APPENDIX

```
function P=solution_1D(f,c1,c2,L1,L2,x)
      % f-frequency
% c1,c2 – propagation velocities
      % L1,L2 – length of each médium
      % x – position of receivers
w=2*pi*f;
k1=w/c1;
k2=w/c2;
A=complex(zeros(4));
A(1,:)=[1 exp(-1i*k1*L1) 0 0];
A(2,:)=[exp(-1i*k1*L1)  exp(0)  -exp(0)  -exp(-1i*k2*(L2))];
A(3,:)=[(-1i*k1)*exp(-1i*k1*L1)  -(-1i*k1)*exp(0)  -(-
1i*k2)*exp(0)  (-1i*k2)*exp(-1i*k2*(L2))];
A(4,:)=[0 0 (-1/1i/w)*(-1i*k2)*exp(-1i*k2*L2)*c2-exp(-1i*k2*L2)
-(-1/1i/w)*(-1i*k2)*c2-1];
B=[1;0;0;0];
X=A\B;
P=zeros(numel(x),1);
for ii=1:numel(x)
    if(x(ii)<=L1 & x(ii)>=0)
        P(ii)=X(1)*exp(-1i*k1*x(ii))+X(2)*exp(-1i*k1*abs(x(ii)-
L1));
    elseif(x(ii)<=L1+L2)
        P(ii)=X(3)*exp(-1i*k2*abs(x(ii)-L1))+X(4)*exp(-
1i*k2*abs(x(ii)-L1-L2));
    else
        P(ii)=NaN;
    end
end
return
end
```