VOLATILITY ARBITRAGE STRATEGY

A TOPICS COURSE REPORT FOR
FRE-GY 6901 – Volatility Models

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1. Executive Summary:

The course, **"Volatility Models",** touches upon pricing of different volatility models and calibrating them to market data. This includes different types of volatility like local, stochastic and rough volatility and models such as Black-Scholes, Heston and Bergomi.

In this report, we use the VIX index which is the 30 day implied volatility of the US Market. We explore mispricings between the implied and realized volatility in the US Market using a **Volatility Arbitrage Strategy** on the SPY option chain. We enter our long and short positions based on the mispricings.

We simulate the future option prices for three markets scenarios using the Black-Scholes model to determine the fair option price. We calculate the PnL for each delta-neutral position and conduct further analysis. Finally, we create plots to better understand the data and pick better option contracts.

2. Introduction:

Volatility arbitrage stands as a cornerstone in the realm of financial strategies, offering a unique avenue for capitalizing on the inherent fluctuations within markets. Volatility, often considered the essence of uncertainty in financial instruments' prices, manifests itself as the variance or standard deviation of these prices over a given period. In this dynamic environment, volatility arbitrage strategies seek to exploit disparities between implied and realized volatility, harnessing the potential mispricing of financial derivatives such as options. Through sophisticated models and careful execution, practitioners of volatility arbitrage aim to generate consistent returns regardless of market direction, relying instead on the amplitude of price swings for profitability.

Understanding the pivotal role of volatility in financial markets is paramount to grasping the essence of volatility arbitrage. Volatility serves as a key parameter in option pricing models, influencing the premiums investors are willing to pay for the right to buy or sell an underlying asset at a predetermined price. Implied volatility, derived from option prices, reflects market participants' expectations of future price movements. Realized volatility, on the other hand, represents the actual volatility observed over a specific time frame. Discrepancies between implied and realized volatility levels present opportunities for volatility arbitrageurs to capitalize on mispricings, by either buying or selling options to profit from subsequent adjustments in their prices.

The pursuit of alpha through volatility arbitrage has garnered significant attention from both institutional and individual investors, drawn by the promise of uncorrelated returns and risk mitigation benefits. The strategy's ability to thrive in diverse market conditions, including periods of market turbulence or stability, underscores its appeal as a valuable portfolio diversification tool. However, navigating the complexities of volatility arbitrage demands a comprehensive understanding of market dynamics, sophisticated quantitative modeling techniques, and adept risk management practices. In this report, we delve into the methodology, results, analysis, and implications of a volatility arbitrage strategy, shedding light on its efficacy in capturing opportunities within the ever-evolving landscape of financial markets.

3. Methodology:

The volatility arbitrage strategy employed in this study is primarily focused on exploiting discrepancies between implied and realized volatility levels in the options market. The strategy involves constructing a portfolio of options positions designed to profit from changes in volatility levels, irrespective of the direction of the underlying asset's price movement. The key principle underlying the strategy is mean reversion in volatility, where periods of high volatility are expected to revert to lower levels and vice versa. This mean reversion behavior is utilized to identify mispricings in options, allowing for the implementation of trades that capture potential profit opportunities as volatility adjusts.

The project utilizes historical price data for the VIX index and the option chain of SPY ETF sourced from Yahoo Finance. The data includes daily price data for the VIX index from 2023-05-01 to 2024-05-01. The data also includes the entire option chain for SPY for available expiration dates.

Realized volatility, also known as historical volatility, is a measure of the actual price fluctuations observed in the past for a financial asset over a specific period. It quantifies the degree of variability or dispersion in the price movements of the asset over time. In this case we will define the realized volatility to be the mean of the VIX index for the past one year. This comes out to be 14.35%.

The **"expected move"** of a stock refers to the anticipated range within which market participants believe the stock price is likely to fluctuate over a specified period. This concept is particularly relevant in options trading, where it helps traders assess the potential magnitude of price swings and make informed decisions regarding options strategies. The expected move can be calculated using the ATM Call Price and ATM Put Price for the respective expiration dates.

$$\boldsymbol{Expected\ Move = 0.85 \times (ATM\ Call\ Price + ATM\ Put\ Price).}$$

0.85 is a skewness adjustment factor which is applied to account for asymmetry.

The signals are generated based on the mispricing between the implied volatility of the options and the realized volatility. The option chain provides us with the implied volatility of all the open contracts for all the expiry dates. If the $\boldsymbol{Implied\ Volatility > Realized\ Volatility}$, then we sell the option. (–1) If the $\boldsymbol{Implied\ Volatility \leq\ Realized\ Volatility}$, then we buy the option. (+1)

A delta-neutral portfolio is one where the total delta exposure from options positions is offset by an equal and opposite delta exposure from positions in the underlying asset. By delta hedging, traders aim to maintain a delta-neutral position, effectively eliminating the impact of small price movements in the underlying asset on the overall portfolio value. In the volatility arbitrage strategy, we will take a delta neutral position, as we are not concerned of the direction but by the magnitude of the move. 1 options contract of SPY is equivalent to a position in 100 shares of SPY. This gives us the initial investment needed for each option contract.

Option Type	Signal	Delta-Neutral Portfolio
Call	1	Long Call, Short SPY
Call	-1	Short Call, Long SPY
Put	1	Long Put, Long SPY
Put	-1	Short Put, Short SPY

We define three market scenarios; Bullish, Bearish and Choppy:

- **i.** Bullish is the current price + the expected move.
- **ii.** Bearish is the current price – the expected move.
- **iii.** Choppy is no change in the price.

The Black-Scholes formula calculates the fair value of an option based on the current market conditions, time to expiration, strike price, risk-free interest rate, and volatility. The formula indicates that the price of an option is influenced by the probability of the underlying asset's price reaching certain levels (captured by the terms $N(d_1)$ and $N(d_2)$).

For a European call option: $\boldsymbol{C = S_0 N(d_1) - Ke^{-rt} N(d_2)}$.

For a European put option: $\boldsymbol{P = Ke^{-rt} N(-d_2) - S_0 N(-d_1)}$.

Where $N(x)$ is the cumulative distribution function of the standard normal distribution, and (d_1) and (d_2) are calculated as follows:

$$\boldsymbol{d_1 = \frac{ln\frac{S_0}{K} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}}$$

$$\boldsymbol{d_2 = d_1 - \sigma\sqrt{T}}$$

Using the Black-Scholes Model, we calculate theoretical prices for the call and put options for all three scenarios. We then calculate the final portfolio value for all three market scenarios using the Black-Scholes price as the final option value and determine the PnL.

The options contracts have been filtered out to remove outliers and illiquid contracts

- **i.** Strike Price in the range of 440 and 560.
- **ii.** Volume greater than 2500
- **iii.** Last price greater than 0.25.

4. Results and Analysis:

Since there is no historical data for the option chains, we cannot conduct backtesting for this strategy. But based on the results generated for the entire option chain, we can make certain predictions to determine which type of contract is the best bet for an investment.

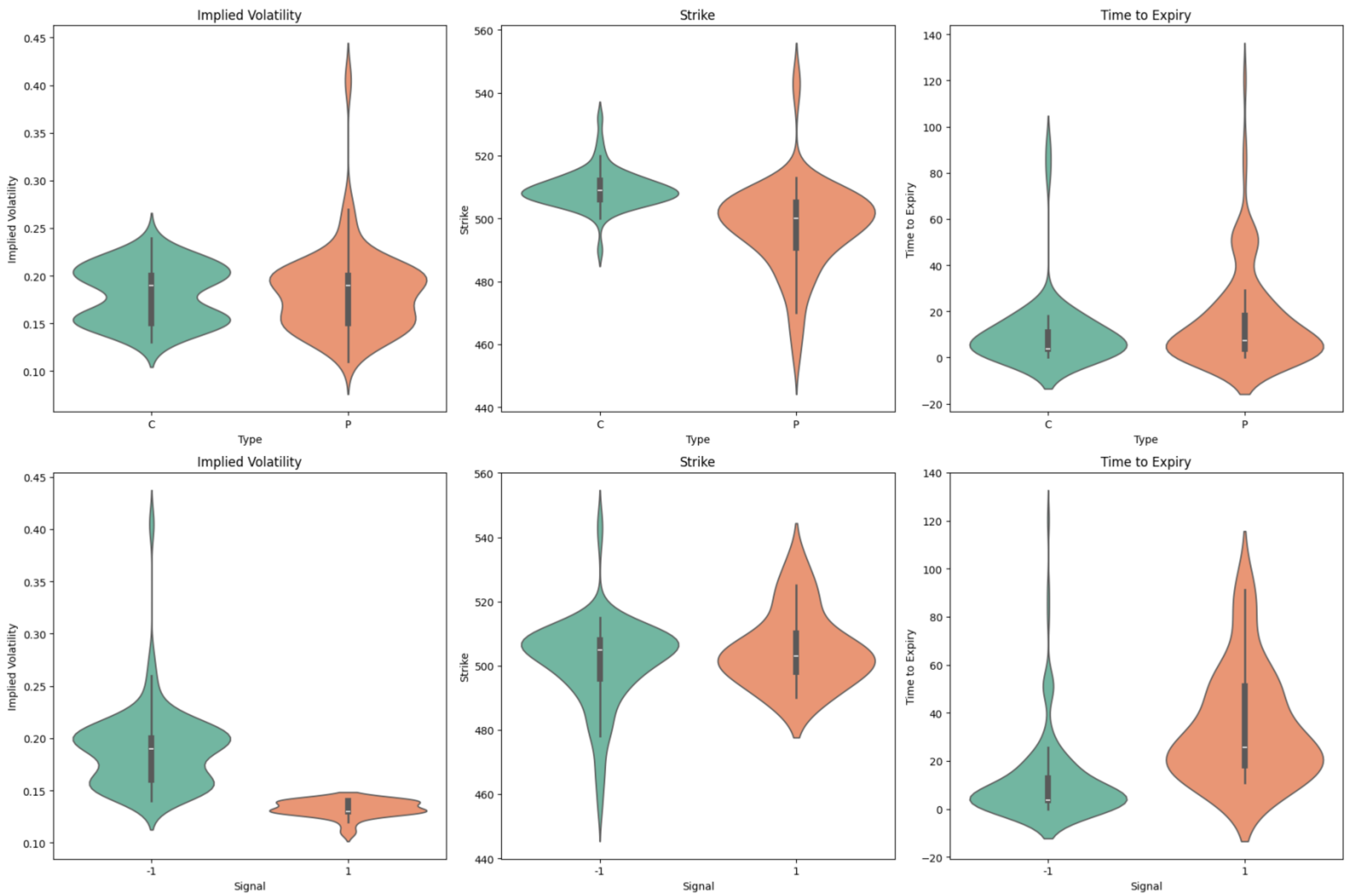
Performance metrics are calculated and analyzed to provide insights into the risk-return profile of the strategy.

Key performance metrics include:

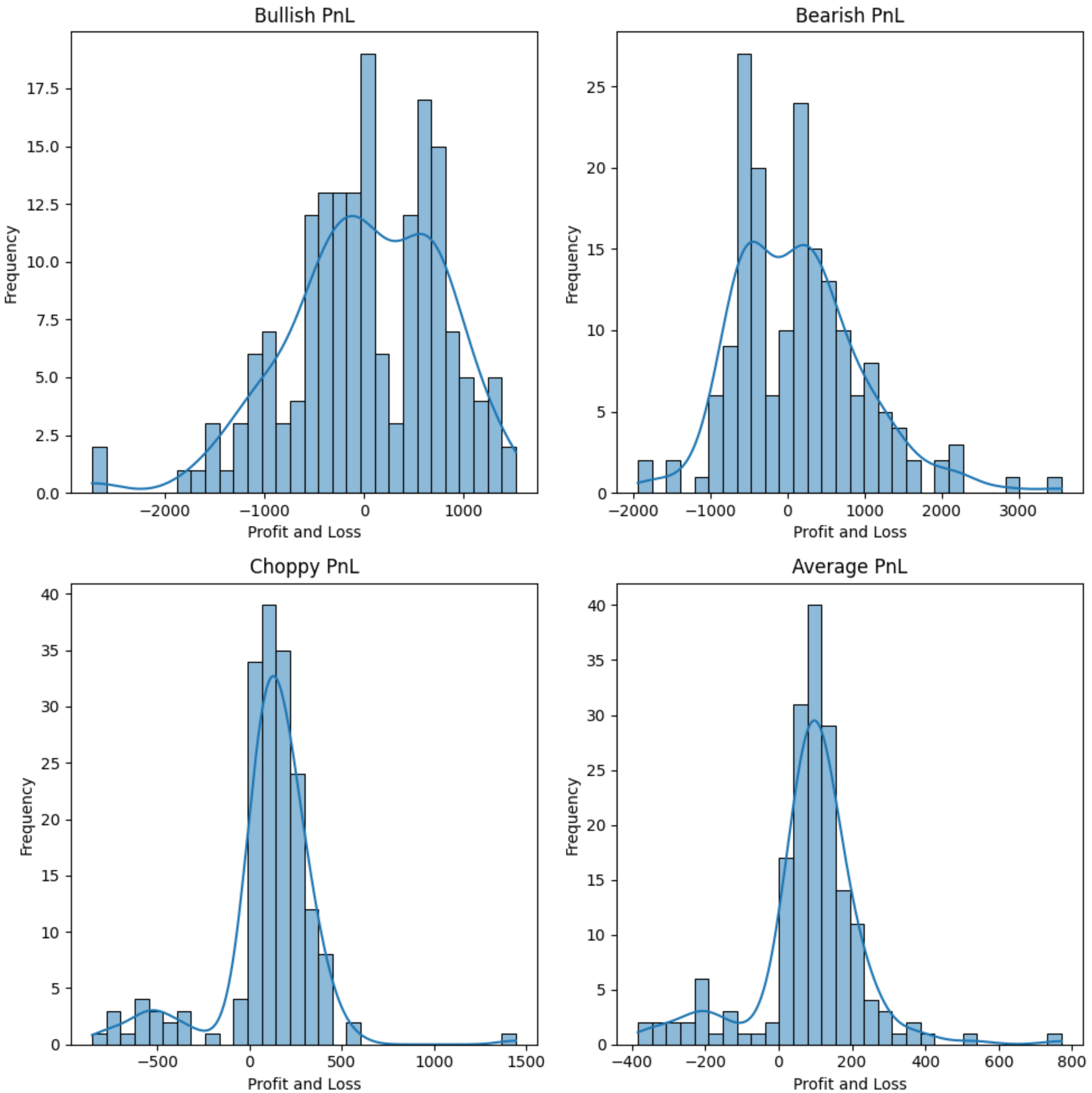
Performance Metrics	Bullish	Bearish	Choppy	Average
Total Profit	2134.0	25436.0	17797.0	15122.35
Average Profit	12.06	143.71	100.55	85.44
Max Profit	1522.0	3555.0	1445.0	772.0
Max Loss	-2728.0	-1936.0	-852.0	-383.67
Win Rate	53.67	58.19	87.57	87.57
Long Call Win Rate	0.0	2.26	0.0	0.0
Short Call Win Rate	32.2	0.56	32.2	32.2
Long Put Win Rate	9.6	0.0	0.0	0.0
Short Put Win Rate	11.86	55.37	55.37	55.37

There are 177 trades and our average total profit is $15122.35 on an investment of $50000. The market is currently in a downtrend and that explains why the bearish results are higher compared to the bullish and choppy ones. Shorts are winning over longs which explains that the volatility is high. In general, puts are known to have a higher volatility compared to calls, this is seen in the win rates as puts has double that of calls. The average win rate of the strategy is 87.57% with each scenario having a win rate over 50%. The max loss is $383.67, which is less than 1%, considering all the investments are about $50000.

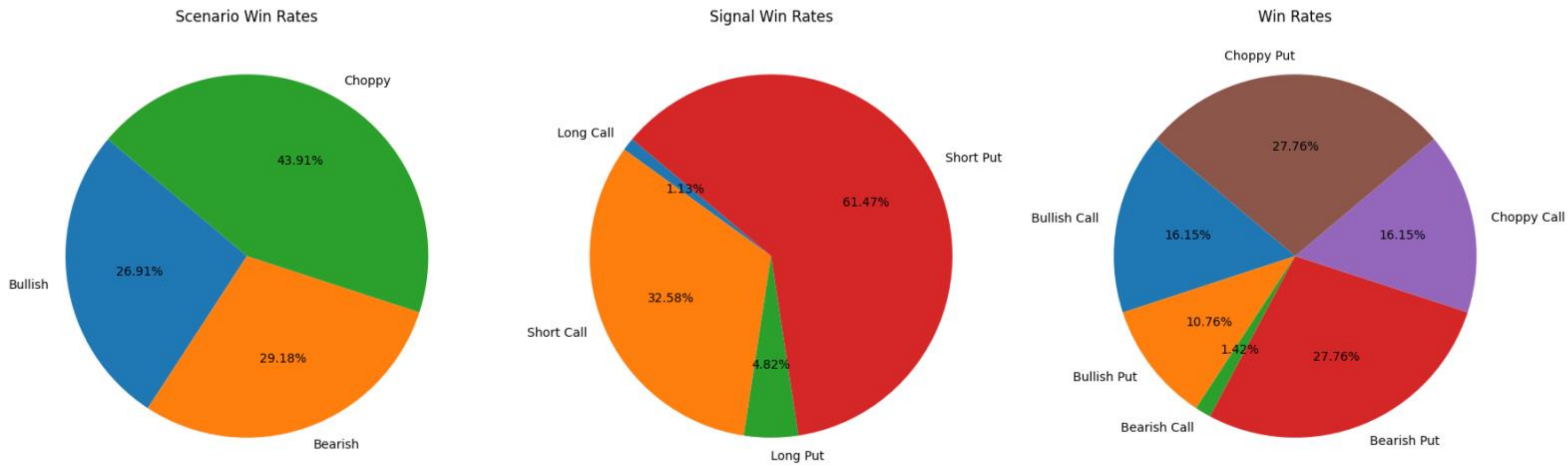
The violin plot is a type of visualization that combines aspects of a box plot and a kernel density plot. The width of the violin plot tell us about the density of the data at that level. Wider the distribution, higher is the density and vice versa. The shape of the violin plot reflects the spread or variability of the data. A wider violin indicates greater dispersion, while a narrower violin suggests less variability. As seen in the first plot, puts have a higher much higher implied volatility compared to calls. Puts also have a much larger set of strikes and a longer time to expiry as seen in plots 2 and 3. As seen in plot 4, shorts have a higher volatility than longs. Shorts also have a much larger set of strikes and a longer time to expiry as seen in plots 5 and 6.



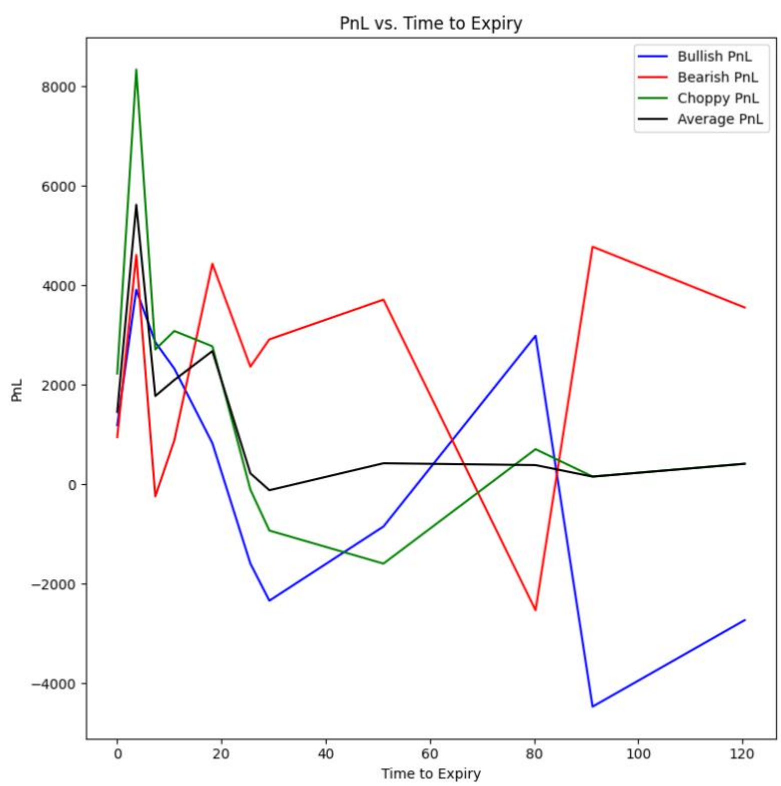
A histogram is a graphical representation of the distribution of numerical data. It consists of a series of adjacent bars, where the height of each bar corresponds to the frequency of data points falling within a specified interval, often referred to as a "bin." The symmetry of the histogram can indicate whether the data is skewed to the left (negative skewness) or right (positive skewness). We can see that the Bullish PnL graph is left–tailed while the Bearish PnL is right–tailed. The Choppy PnL and Average PnL is slightly skewed to the left.



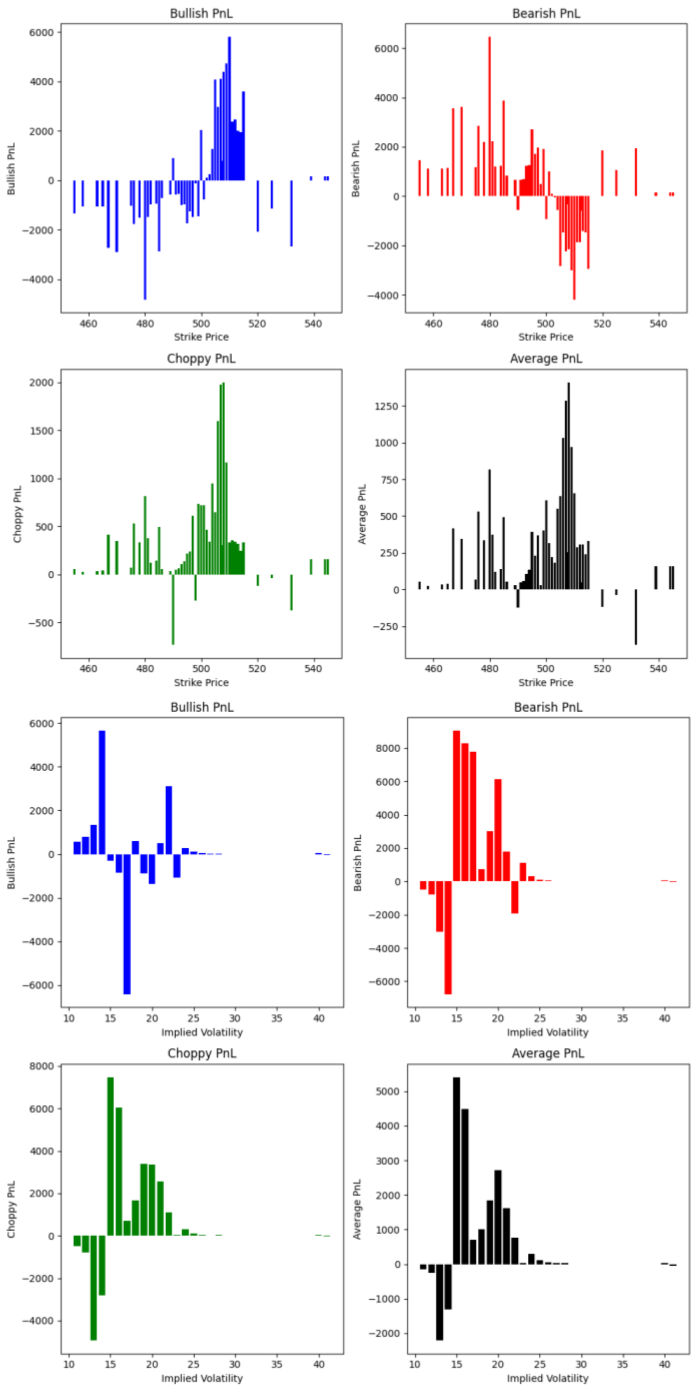
The pie plot tells us about the percentage contribution of each contributor. In this case, we can see that Puts have a 66.29% weightage while Calls has 33.71%. Short positions contribute a staggering 94.05% while long positions are only 5.95%. The Bullish Scenario contributes 26.91%, Bearish contributes 29.18% and Choppy contributes 43.91%. The Bearish and Choppy Put has the highest weightage (27.76%) while the Bearish Call only has 1.42%.



We can see from this graph that all cases are profitable for a nearer to date expiry with the Choppy case being the highest. The bearish scenario is highly profitable for long term expiries as well but this is negated by the bullish scenario. Thus, we can say that the ideal time to expiry is between 0-20 days.



As seen in the first plot, the Bullish case makes a profit for strikes greater than 500, Bearish makes a profit for strikes less than 500 and the Choppy and Average case rarely make a loss. Deep OTM options are not as profitable as the ones closer to the strike price. We also make huge profits for Implied volatility greater than 15 in every case but the Bullish one as volatility falls in a bullish scenario. Thus, we can say that the ideal strike price is $\pm$ 5% from the current price and the ideal volatility is between 15 and 20.



5. Conclusion:

Volatility arbitrage is a challenging yet potentially rewarding trading strategy that offers opportunities for profit in financial markets. By capitalizing on discrepancies between implied and realized volatility, traders can exploit mispricings and generate returns while managing risks effectively. However, success in volatility arbitrage requires a deep understanding of options pricing, quantitative modeling, risk management principles, and market dynamics.

While volatility arbitrage is not without risks, it can serve as a valuable addition to a diversified trading or investment portfolio. By incorporating volatility arbitrage strategies alongside other trading approaches, investors can potentially enhance risk-adjusted returns and achieve greater consistency in performance.

In conclusion, volatility arbitrage represents a sophisticated and nuanced approach to trading that requires skill, experience, and discipline. While it may not be suitable for all investors, those willing to commit the necessary time and resources to understand and implement volatility arbitrage strategies may find it to be a valuable tool for navigating today's complex and dynamic financial markets.

6. References:

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https://www.ssga.com/us/en/intermediary/etfs/funds/spdr-sp-500-etf-trust-spy

7. Appendix:

Volatility Smile (Of all the option contracts of SPY)