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DeepLearning.AI

Derivatives and Optimization

Machine learning motivation

Machine Learning Motivation

Machine Learning Motivation



Machine Learning Motivation



Number of Bedrooms

Machine Learning Motivation



1



2

Number of Bedrooms

Machine Learning Motivation



1

\$150,000



2

\$250,000

Number of Bedrooms

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has



1

\$150,000



2

\$250,000

Number of Bedrooms

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms
1
2
3
5
6
7
8
9
10

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms	Price of House
1	\$150,000
2	\$250,000
3	\$350,000
5	\$600,000
6	\$650,000
7	\$750,000
8	\$800,000
9	??
10	\$1,050,000

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms	Price of House
1	\$150,000
2	\$250,000
3	\$350,000
5	\$600,000
6	\$650,000
7	\$750,000
8	\$800,000
9	??
10	\$1,050,000

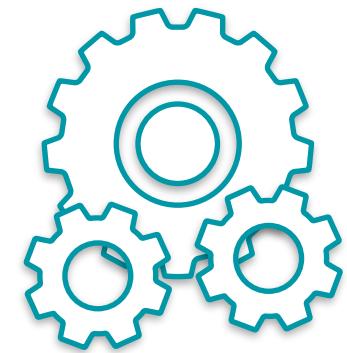
9 ???

Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

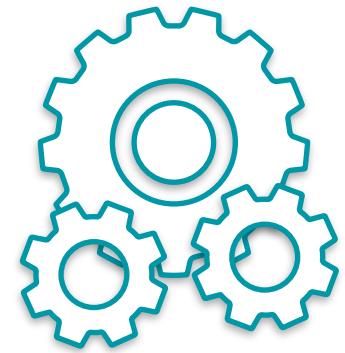
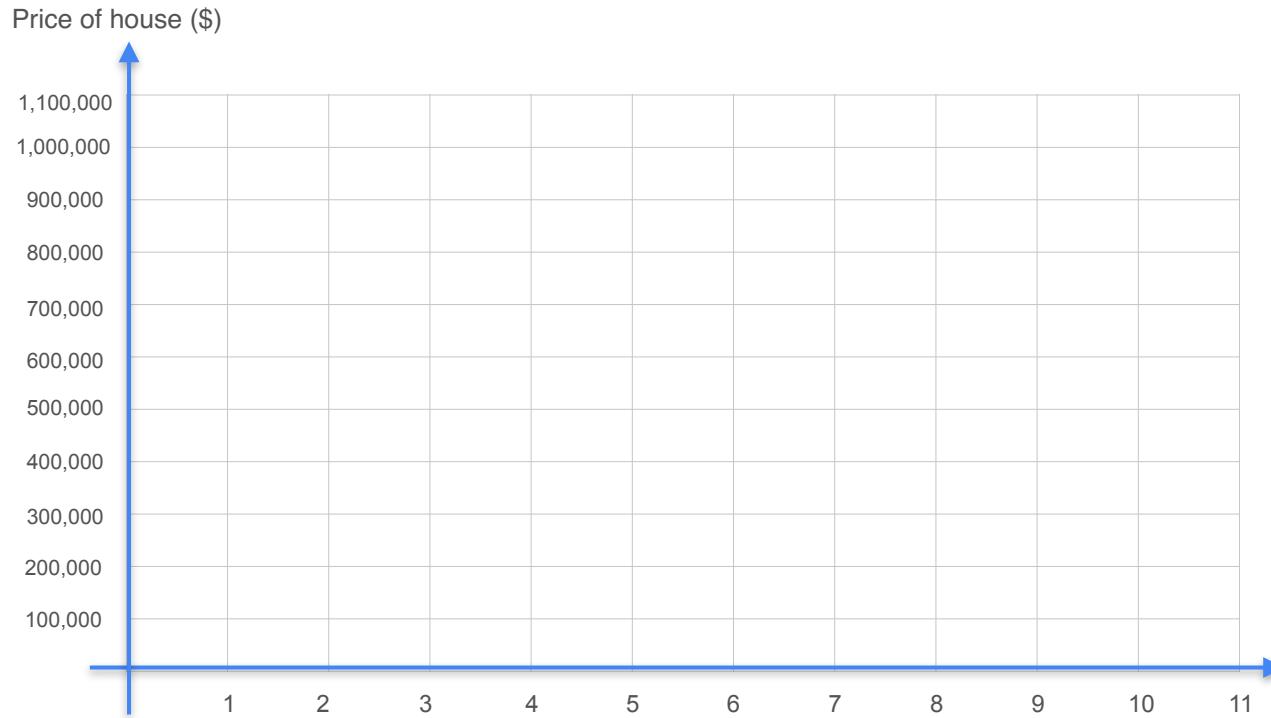
9 ???

Number of Bedrooms	Price of House
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8	\$800,000
9	??
10	\$1,050,000



Machine Learning
Model

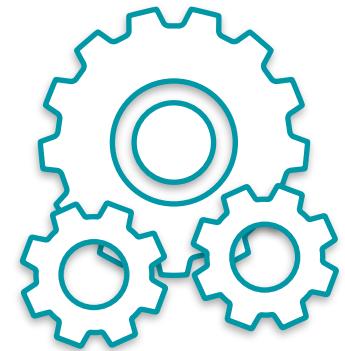
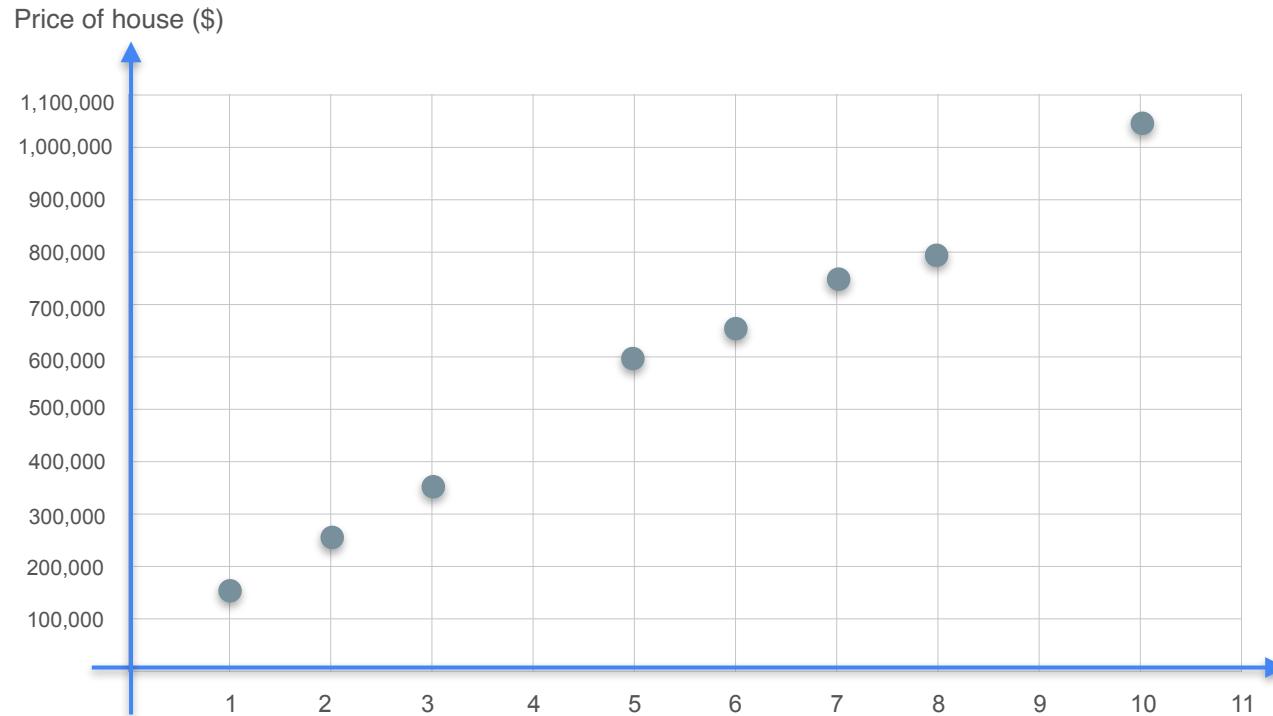
Machine Learning Motivation



Machine Learning
Model

*Number of
Bedrooms*

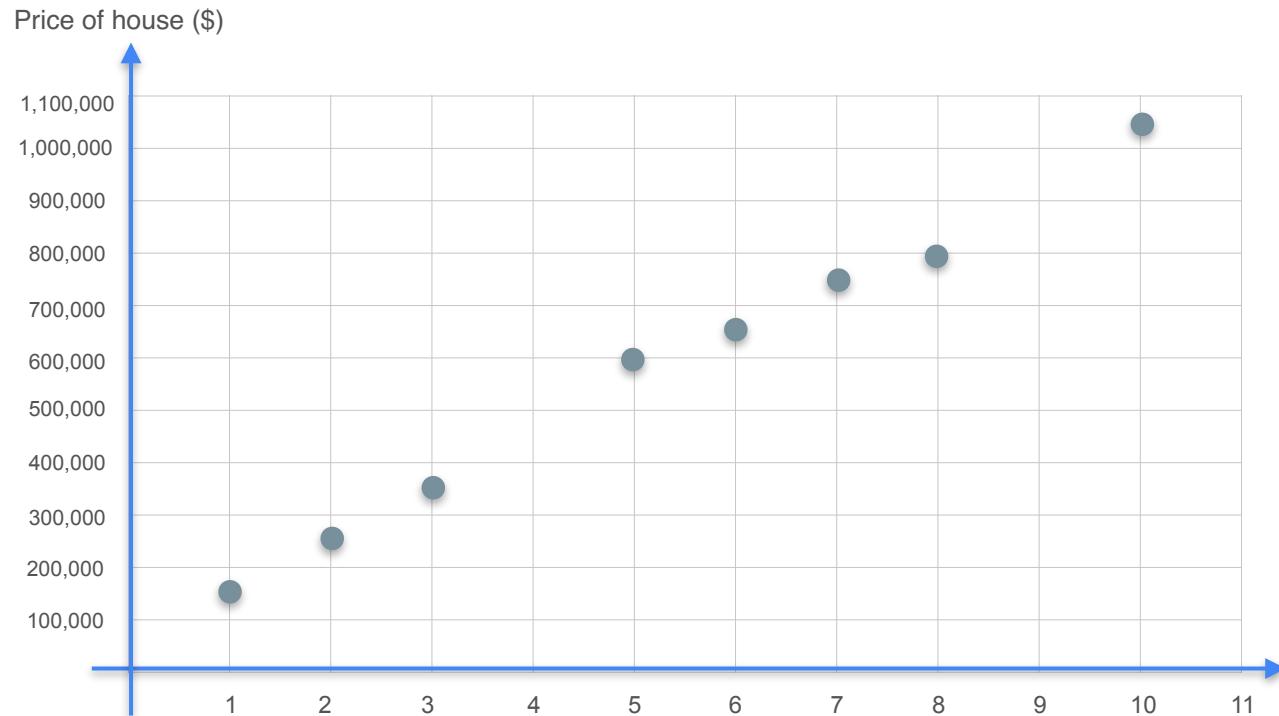
Machine Learning Motivation



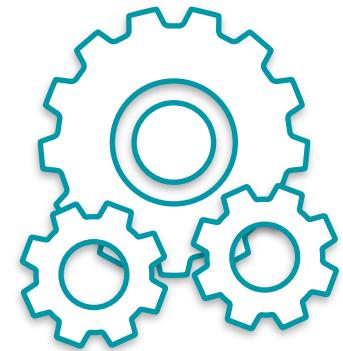
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



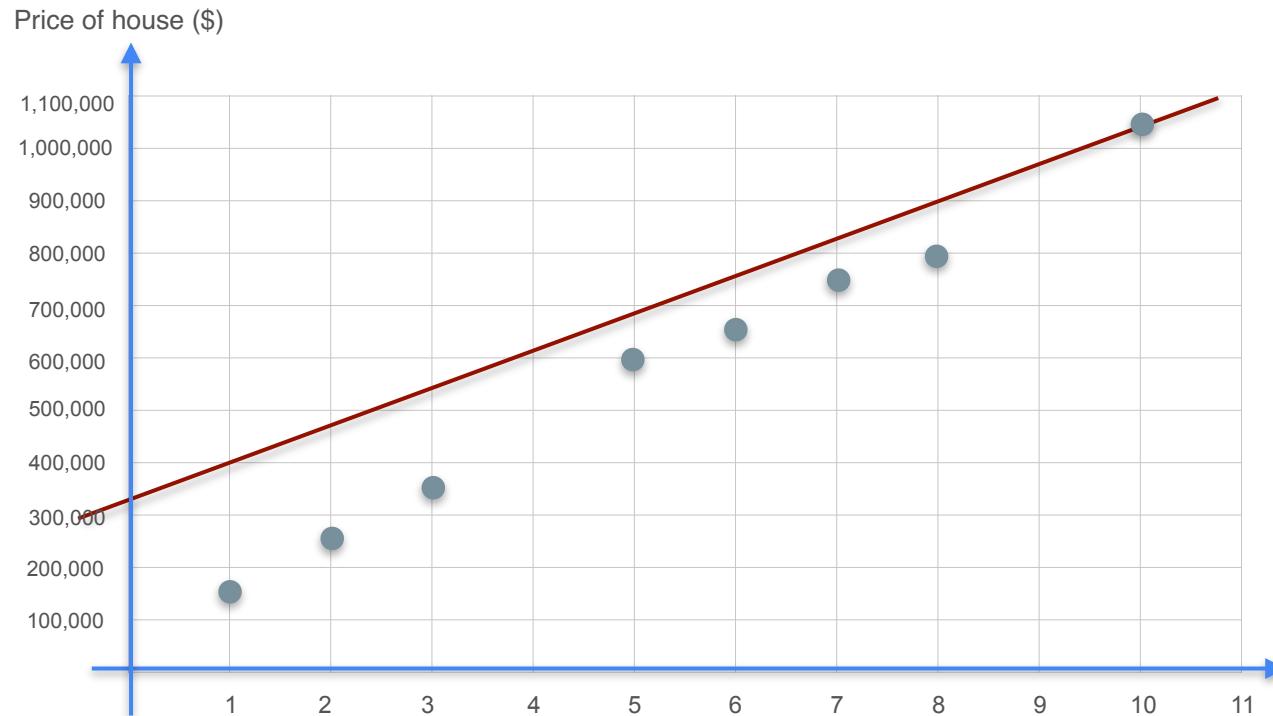
Model Training



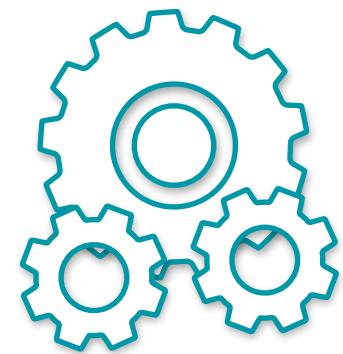
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



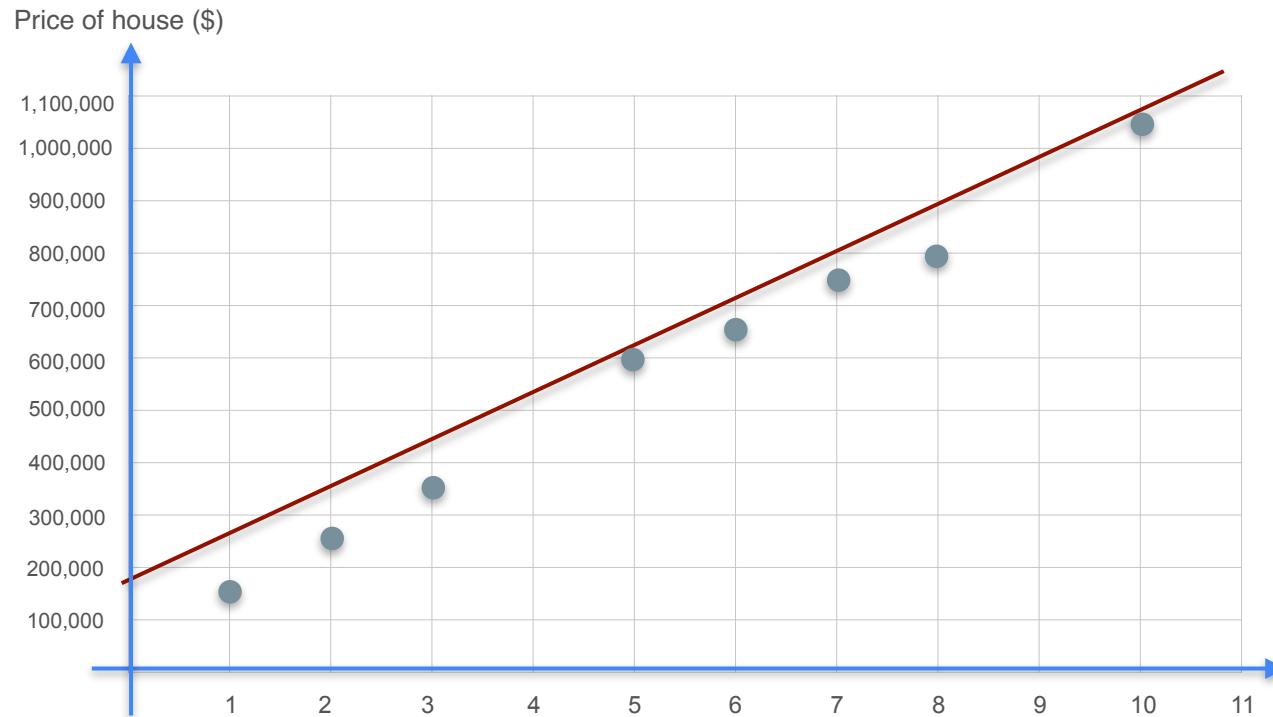
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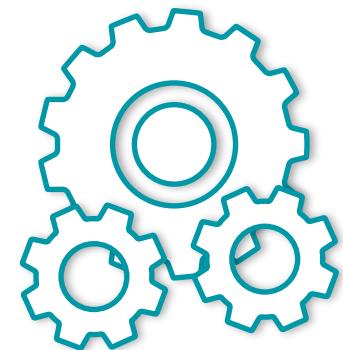
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



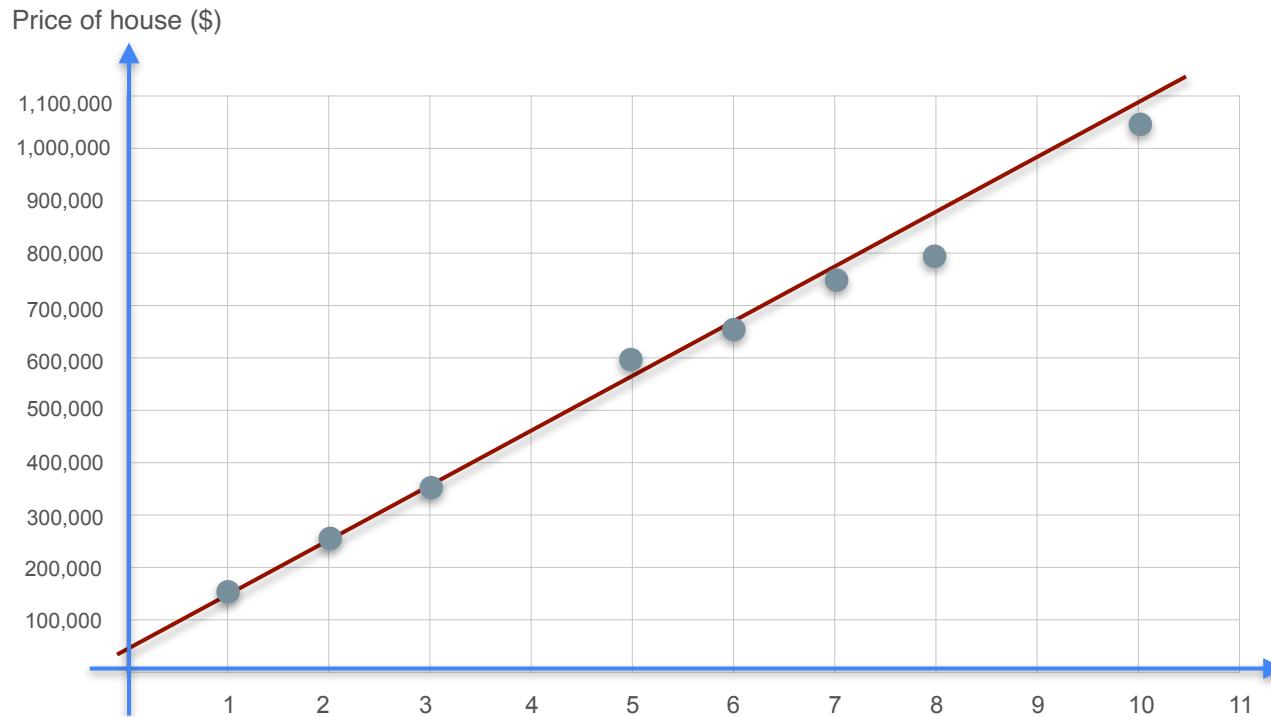
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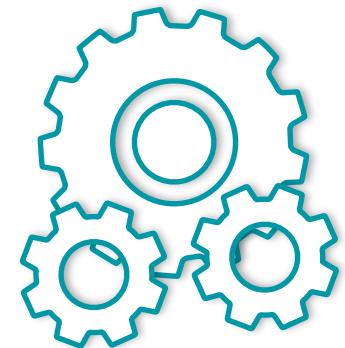
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



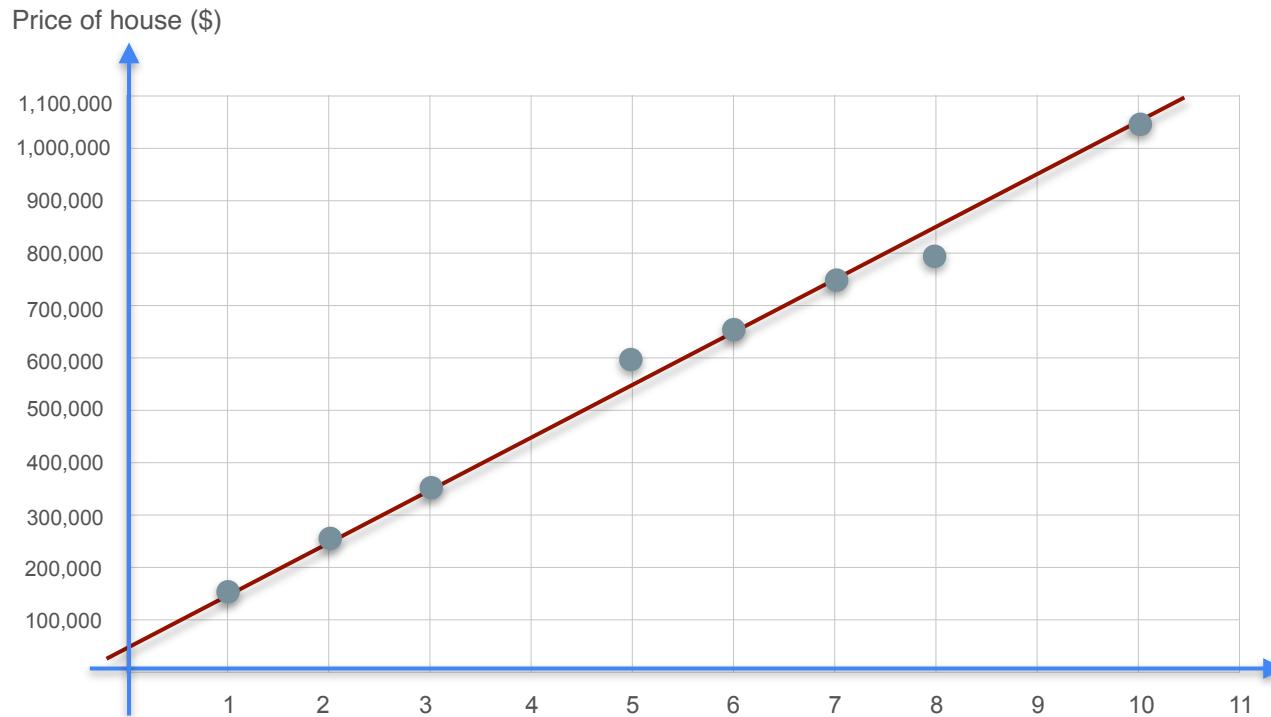
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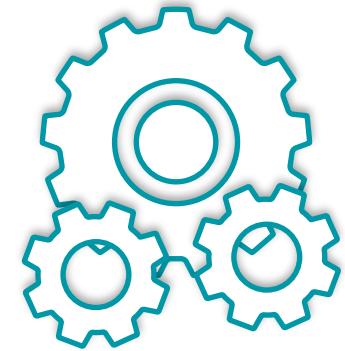
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



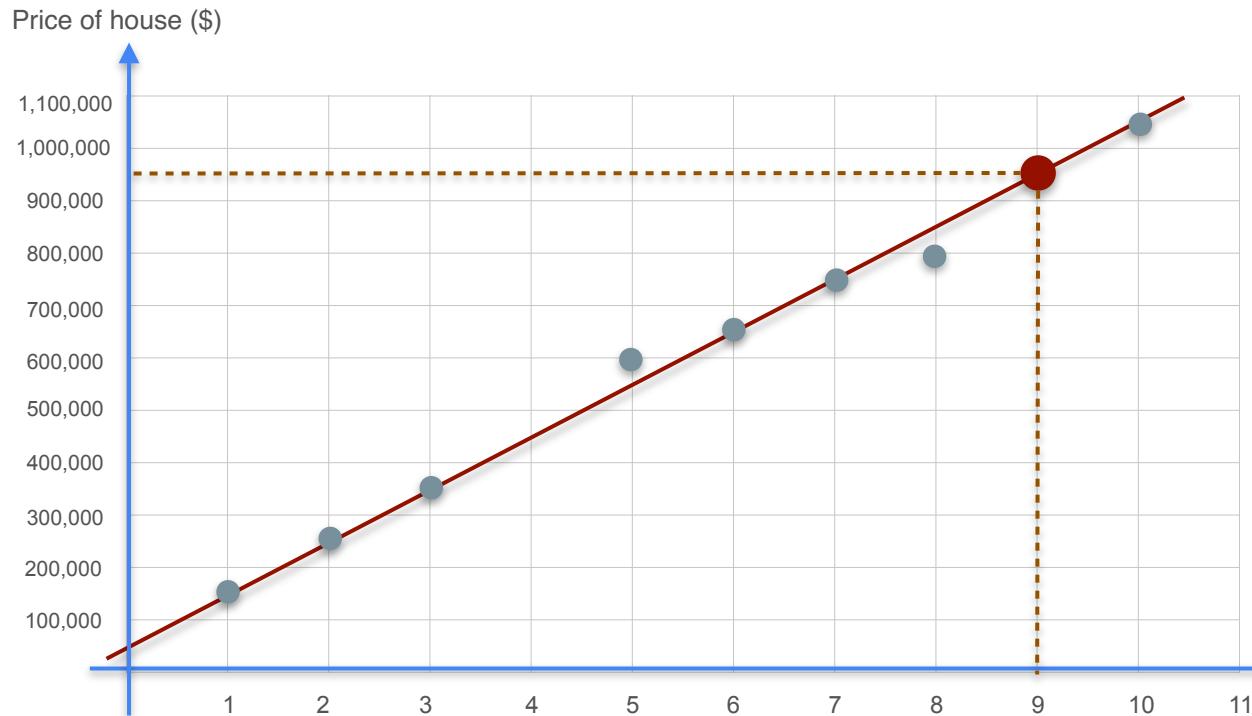
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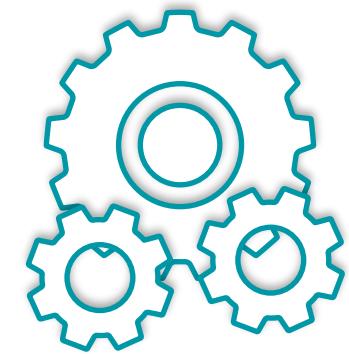
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



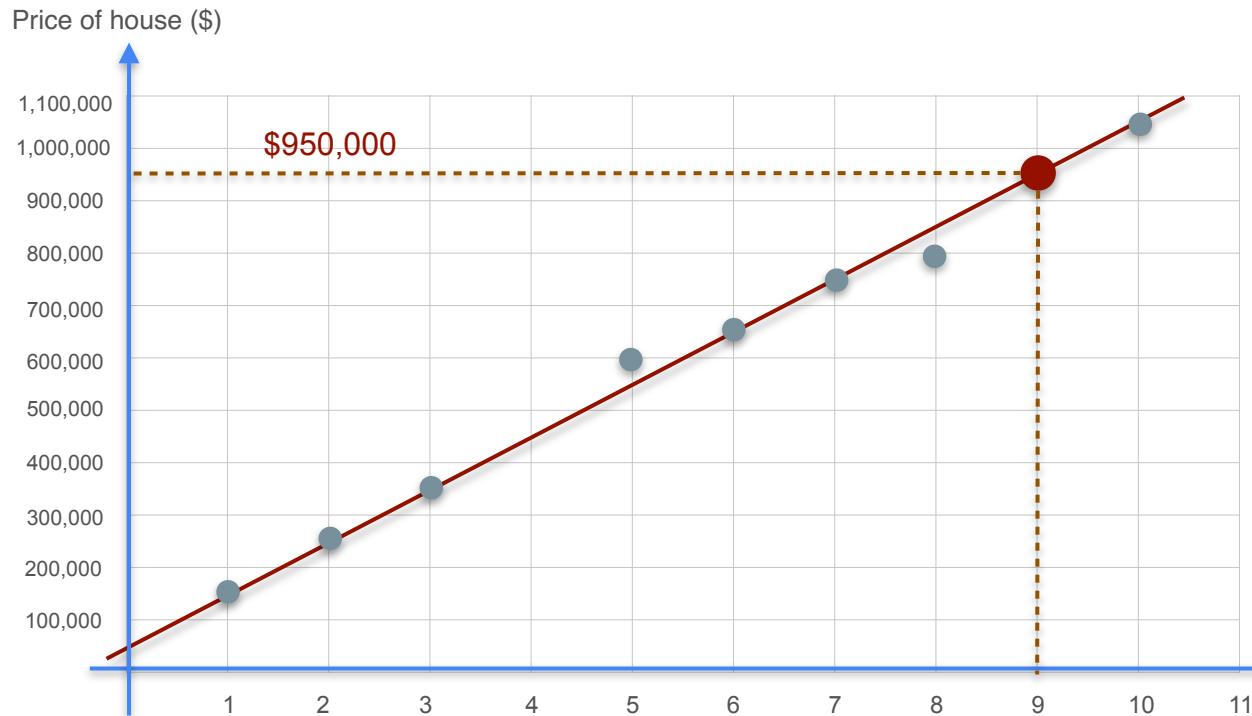
Model Training



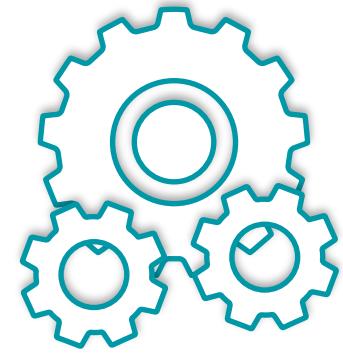
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



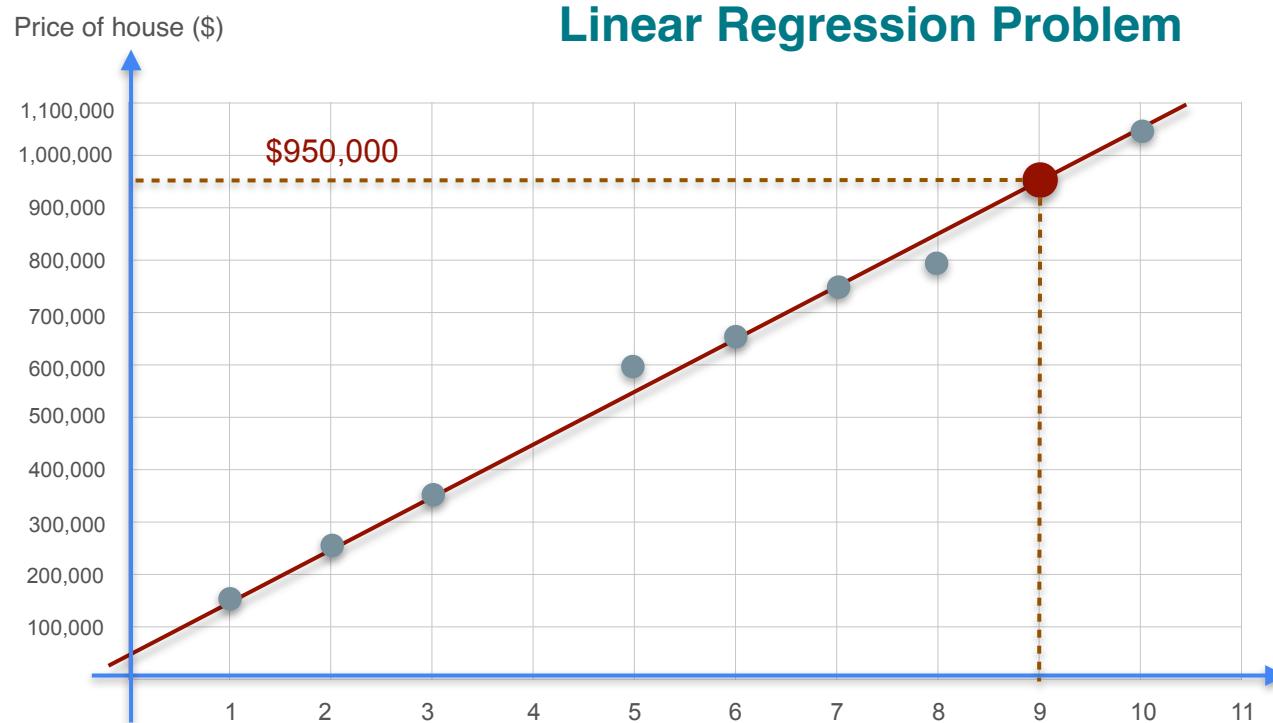
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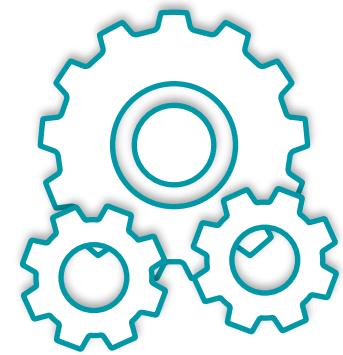
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation



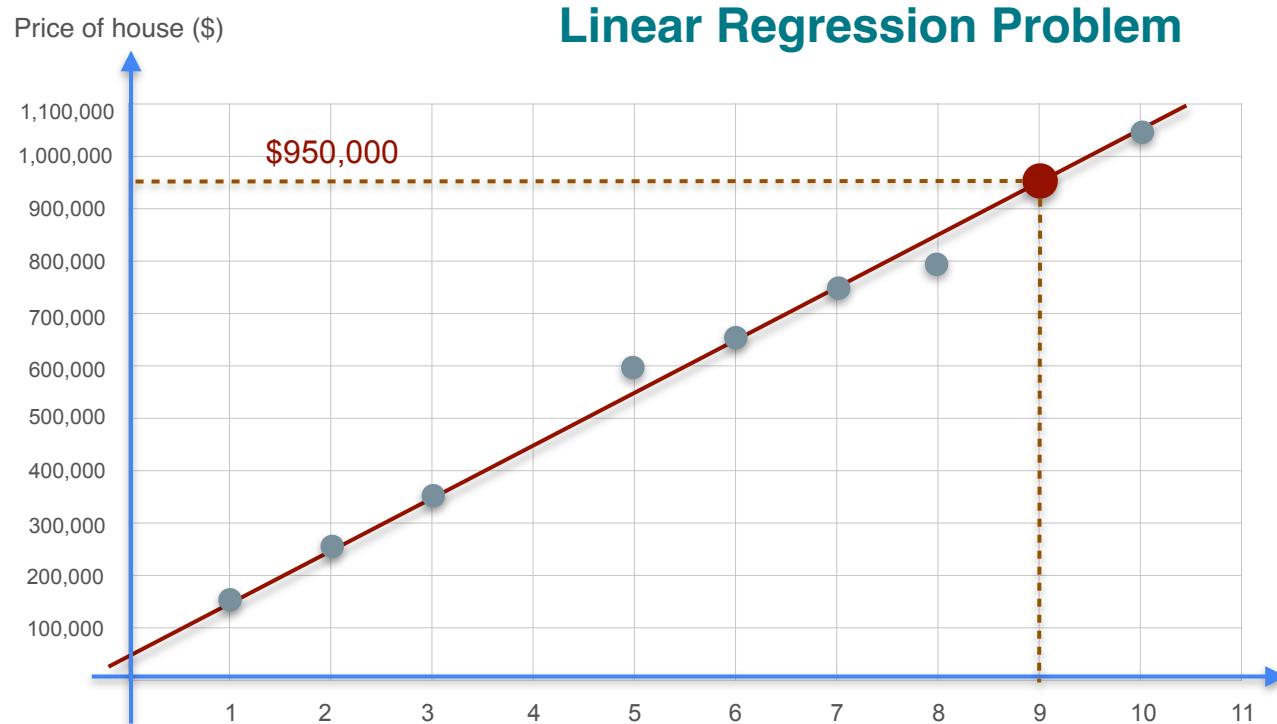
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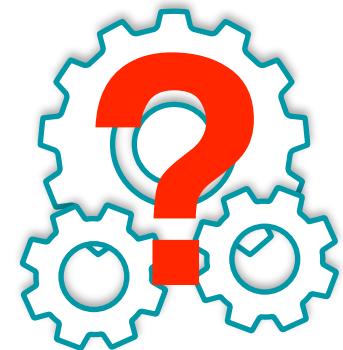
Machine Learning
Model

*Number of
Bedrooms*

Machine Learning Motivation

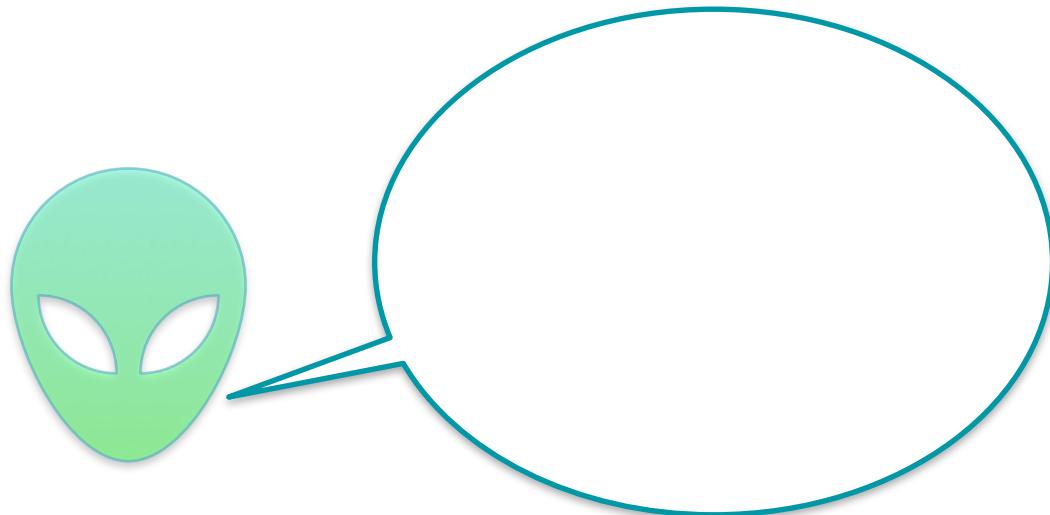


Model Training

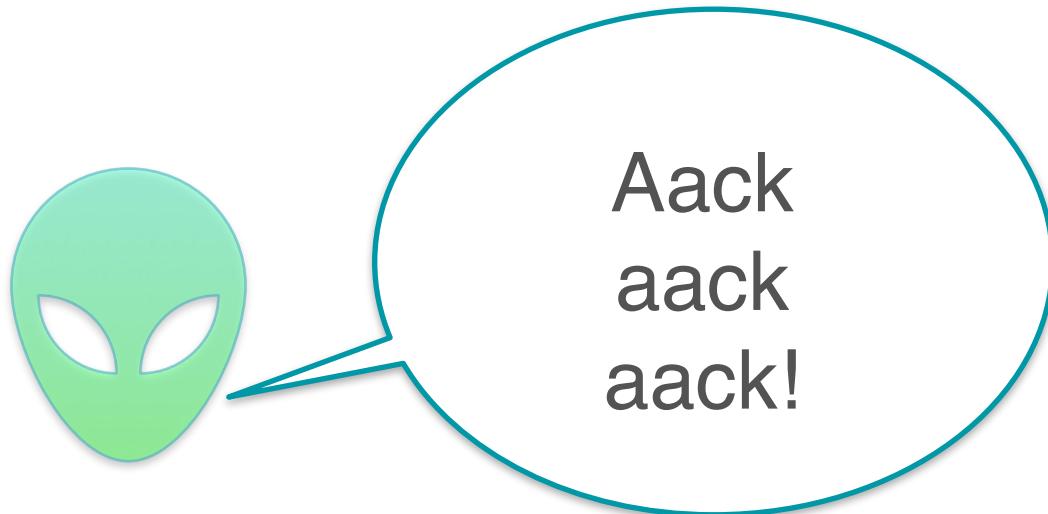


Machine Learning
Model

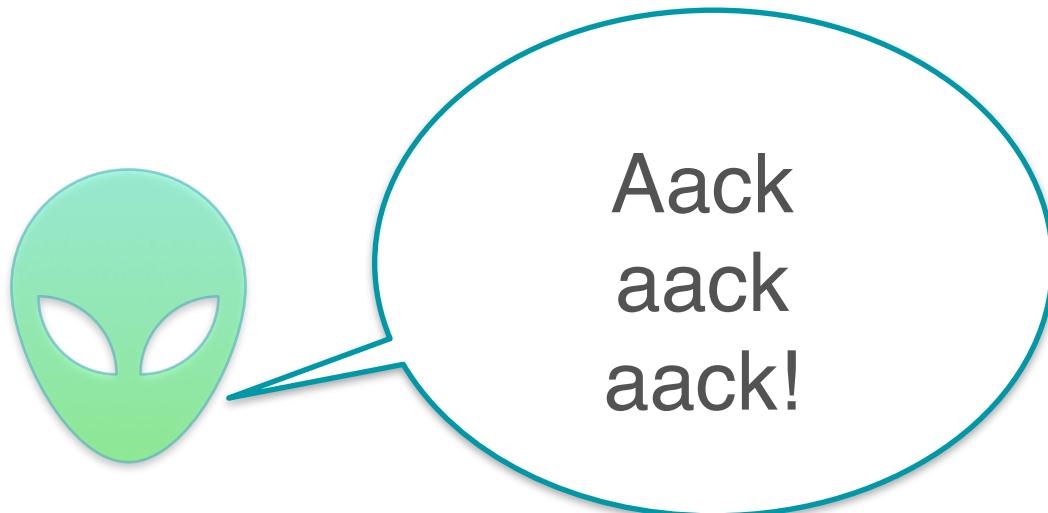
Machine Learning Motivation



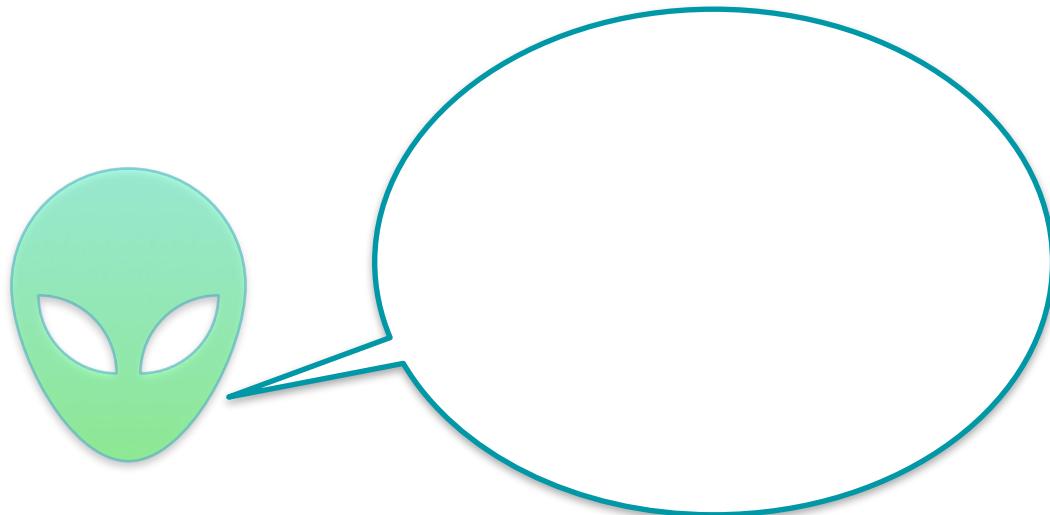
Machine Learning Motivation



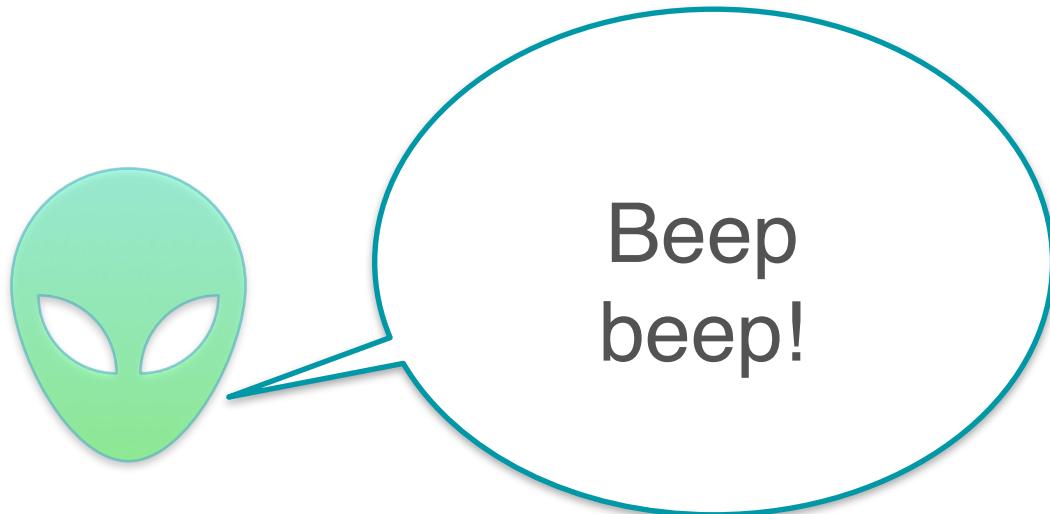
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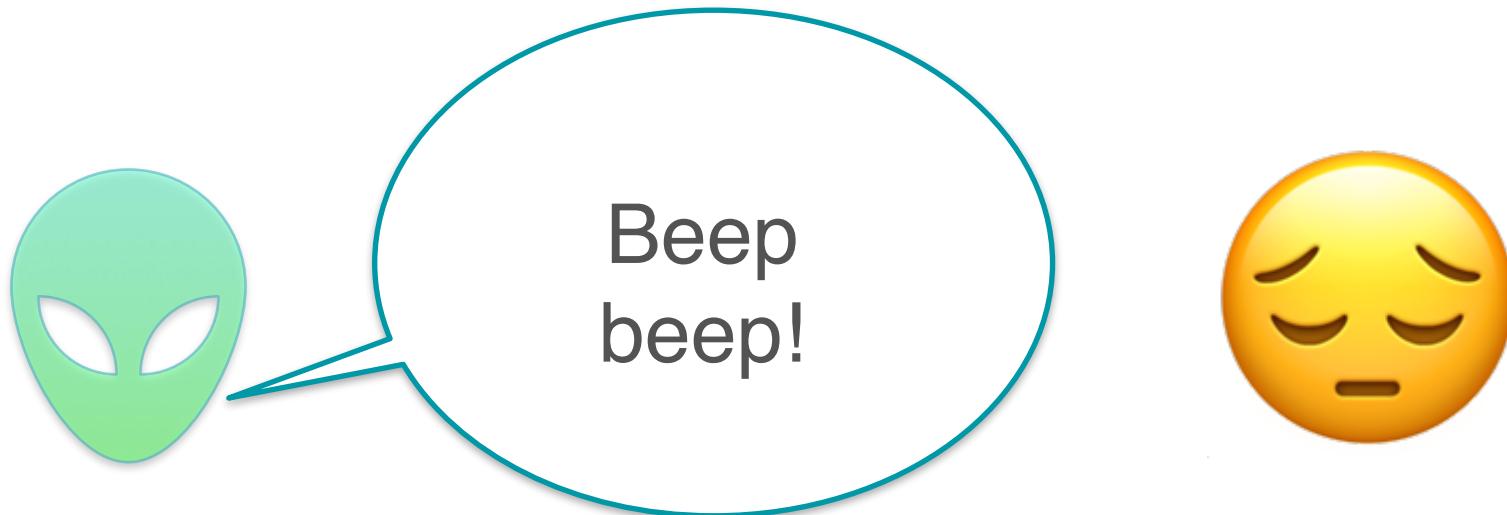
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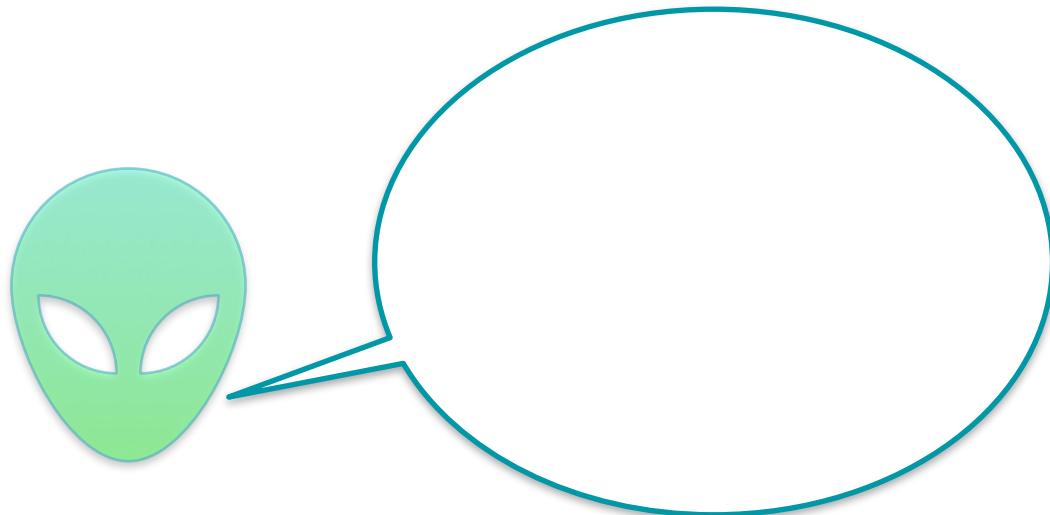
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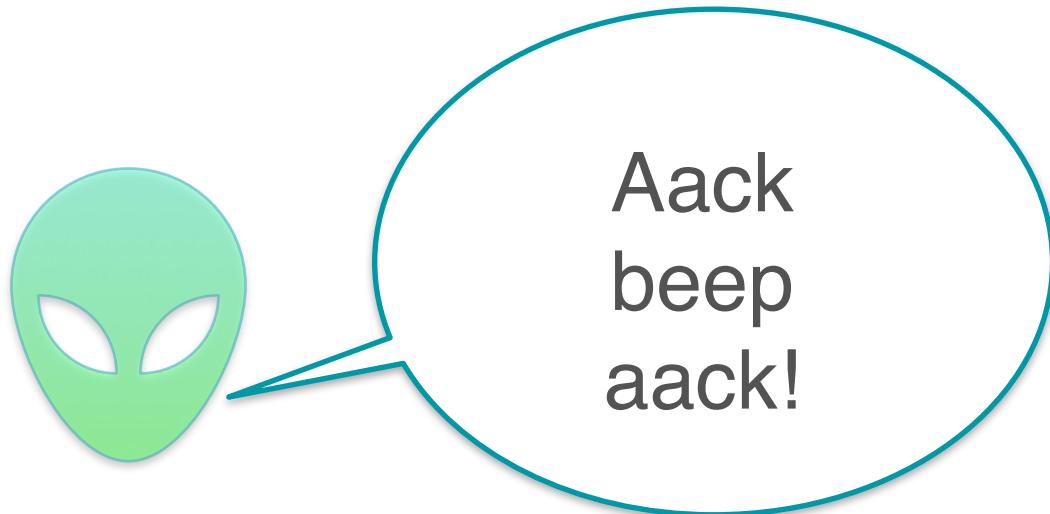
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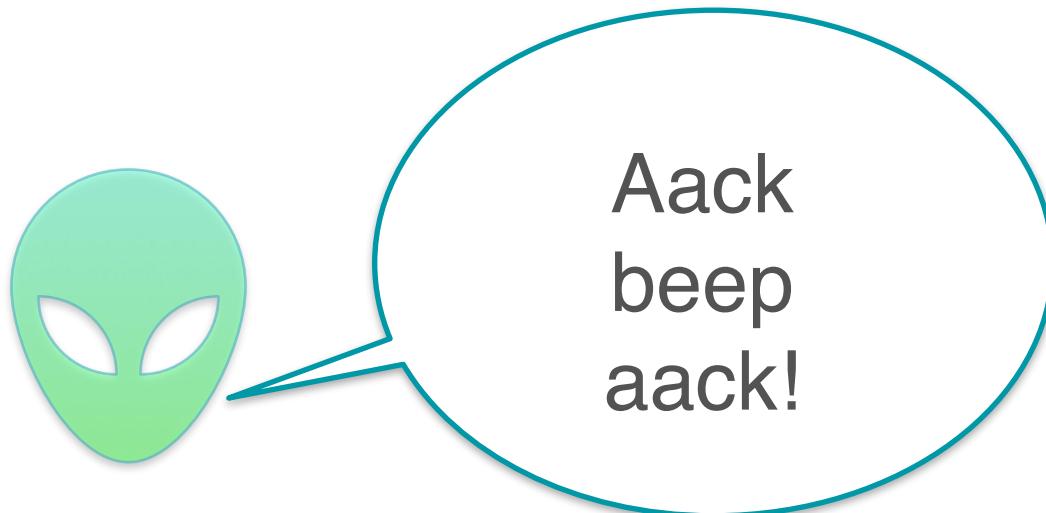
Machine Learning Motivation



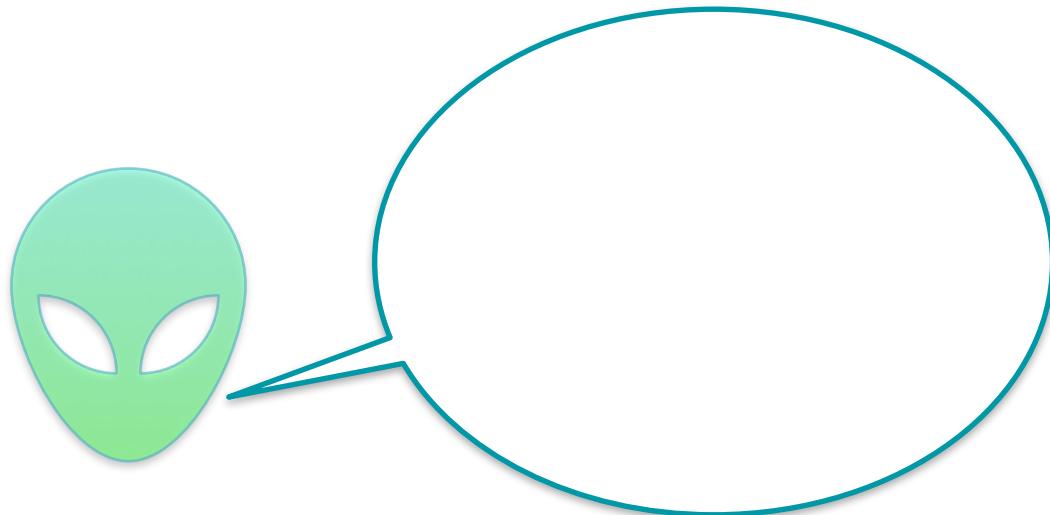
Machine Learning Motivation



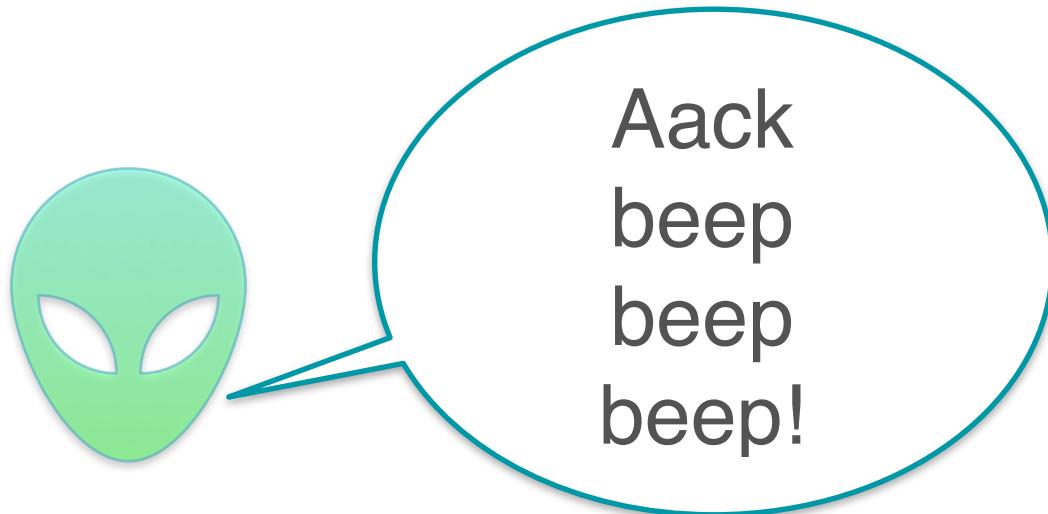
Machine Learning Motivation



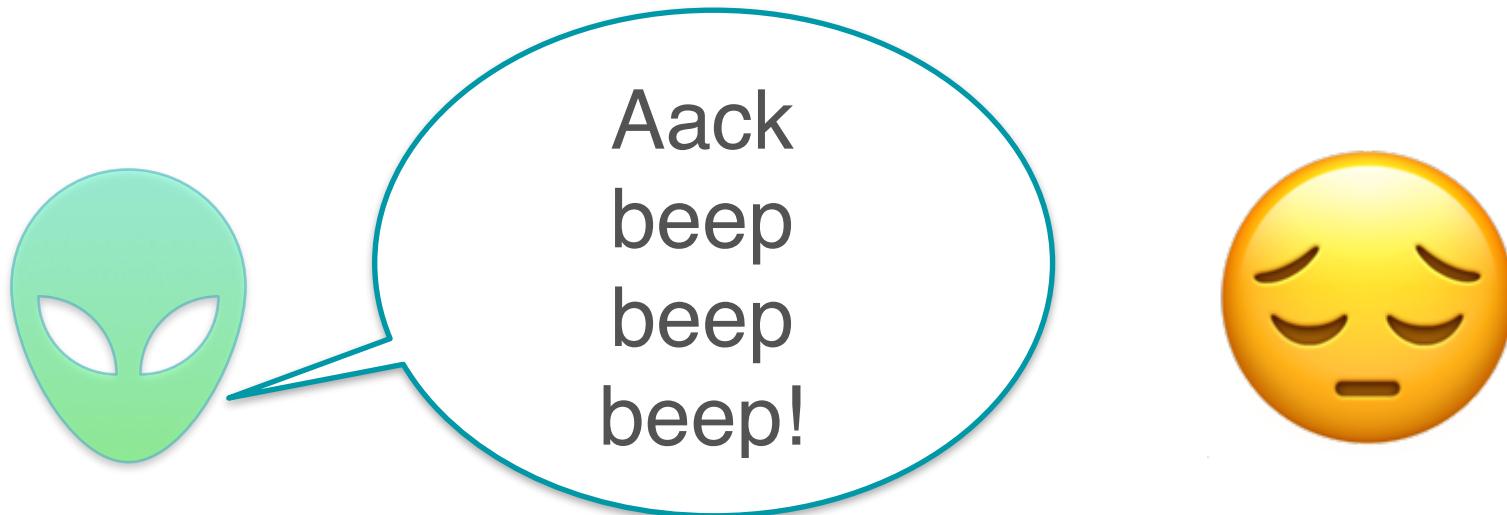
Machine Learning Motivation



Machine Learning Motivation



Machine Learning Motivation



Machine Learning Motivation



Machine Learning Motivation



Sentence

Aack aack aack!

Beep beep!

Aack beep aack!

*Aack beep beep
beep!*

Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>
<i>Aack aack aack!</i>	3	0

Beep beep! 0 2

<i>Aack beep aack!</i>	2	1
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*Aack beep beep
beep!* 1 3

Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

Beep beep! 0 2 😞

<i>Aack beep aack!</i>	2	1	😊
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*Aack beep beep
beep!* 1 3 😞

Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

A classification problem

Beep beep! 0 2 😞

<i>Aack beep aack!</i>	2	1	😊
------------------------	---	---	---

*Aack beep beep
beep!* 1 3 😞

Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

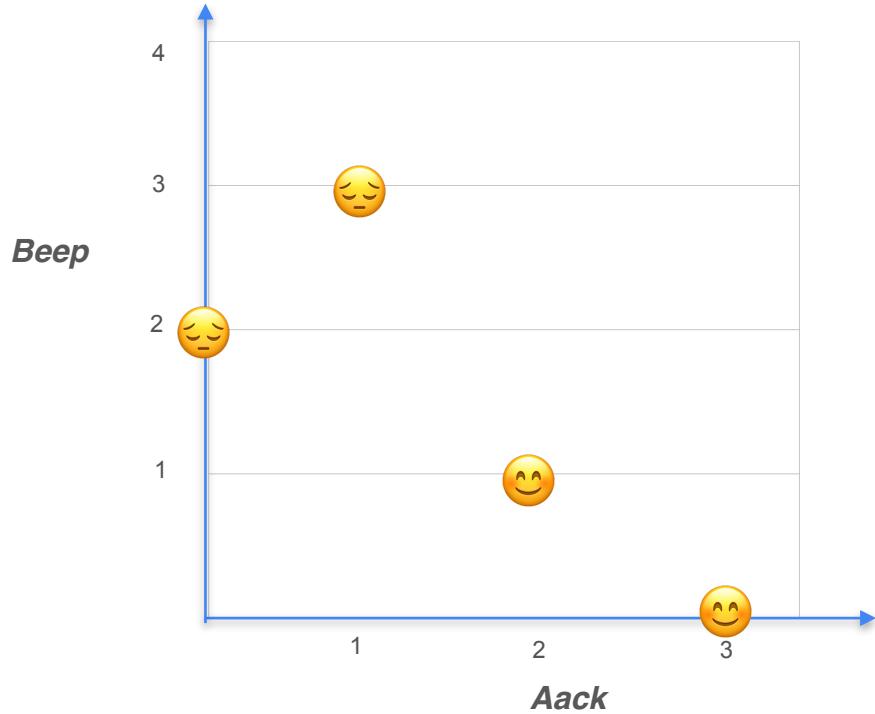
A classification problem

<i>Beep beep!</i>	0	2	😴
<i>Aack beep aack!</i>	2	1	😊

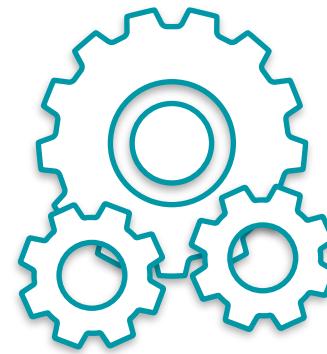
Sentiment analysis

<i>Aack beep beep beep!</i>	1	3	😴
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Machine Learning Motivation

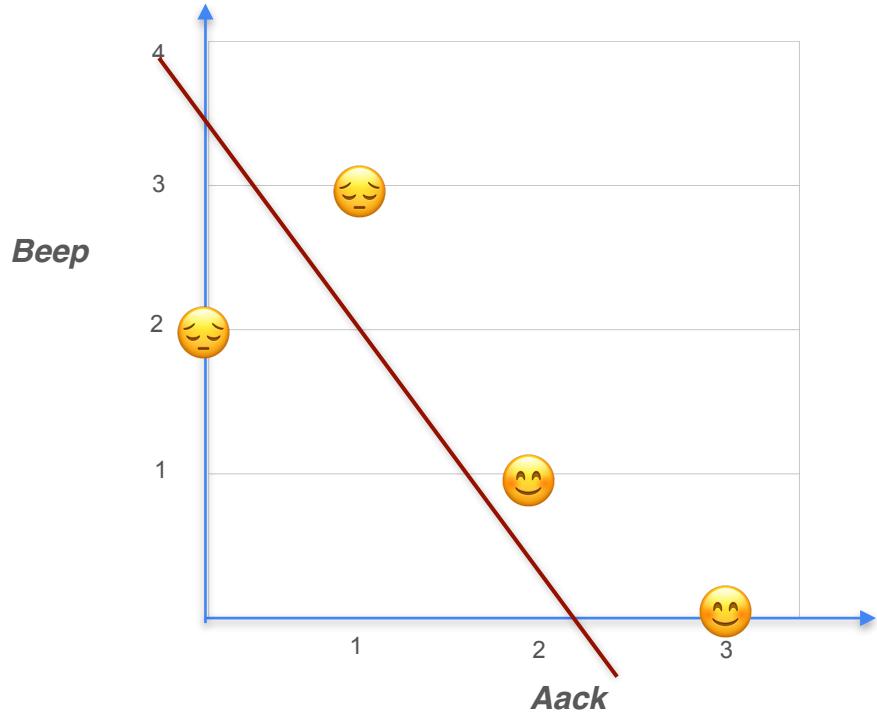


Model Training

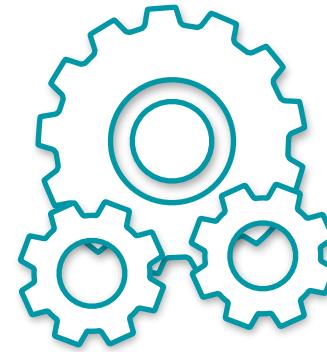


Machine Learning
Model

Machine Learning Motivation

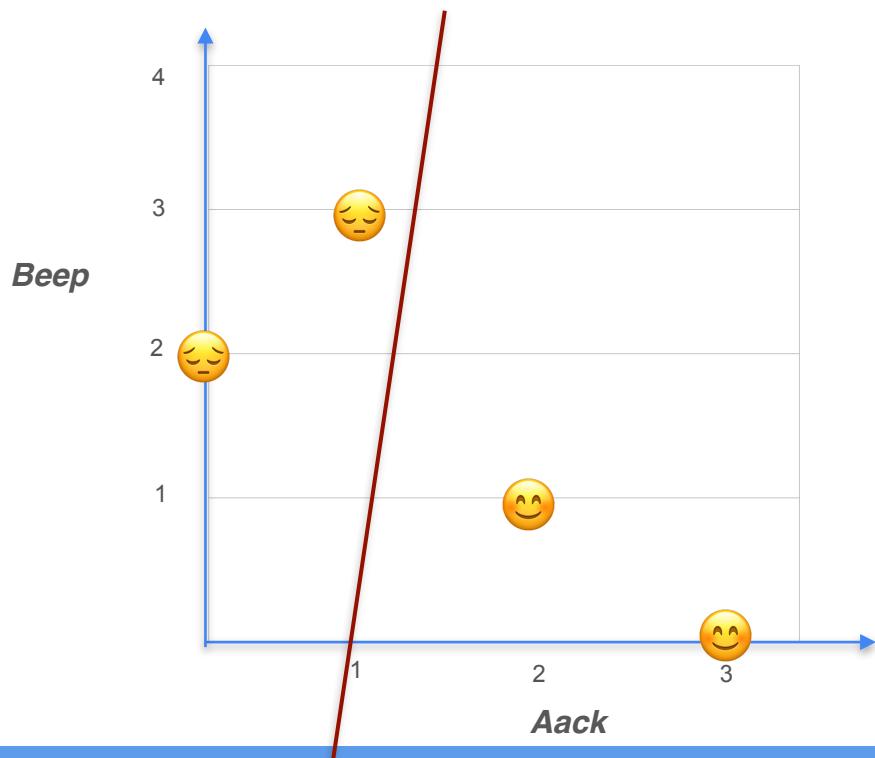


Model Training

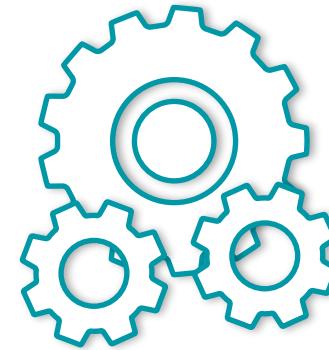


Machine Learning
Model

Machine Learning Motivation

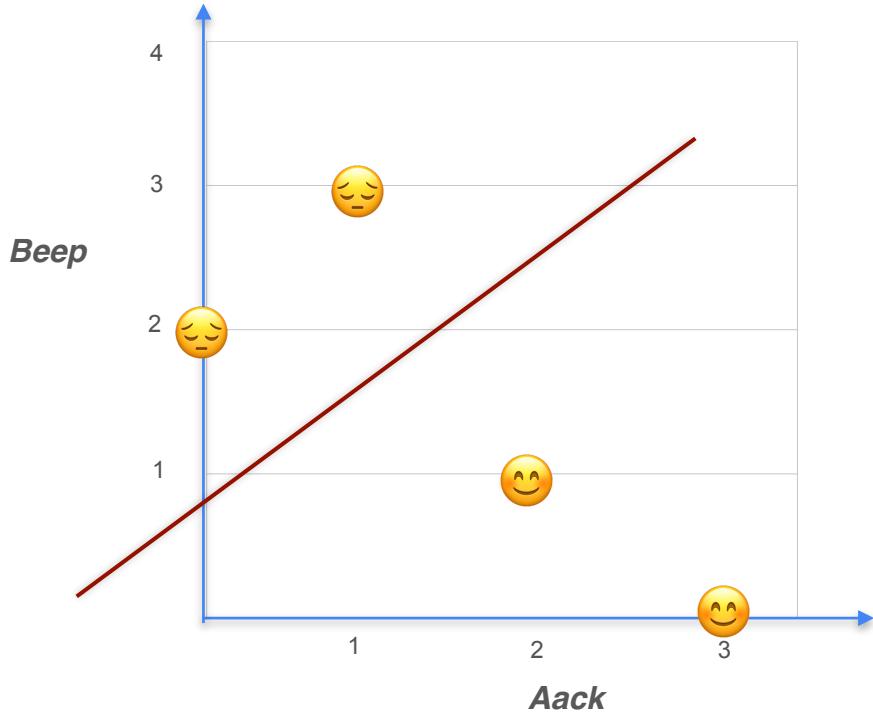


Model Training

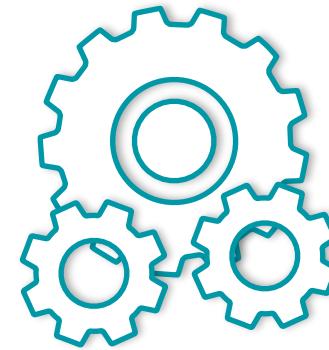


Machine Learning
Model

Machine Learning Motivation

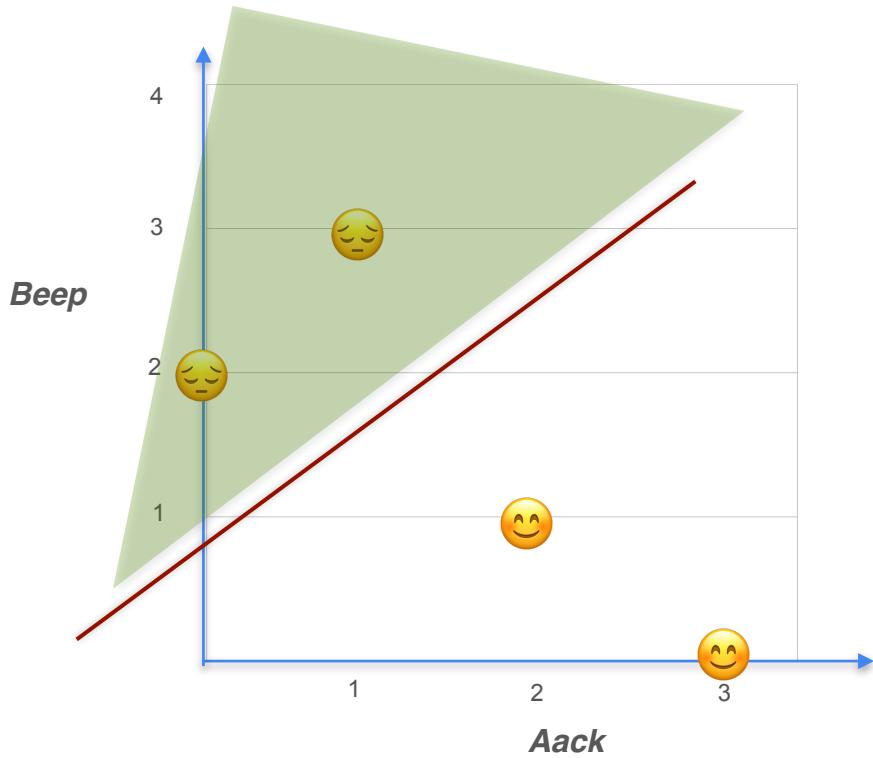


Model Training

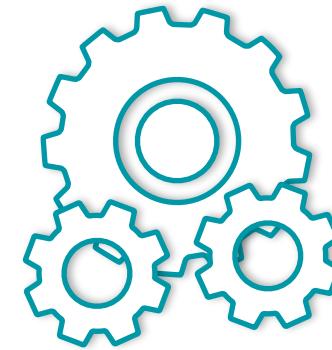


Machine Learning
Model

Machine Learning Motivation

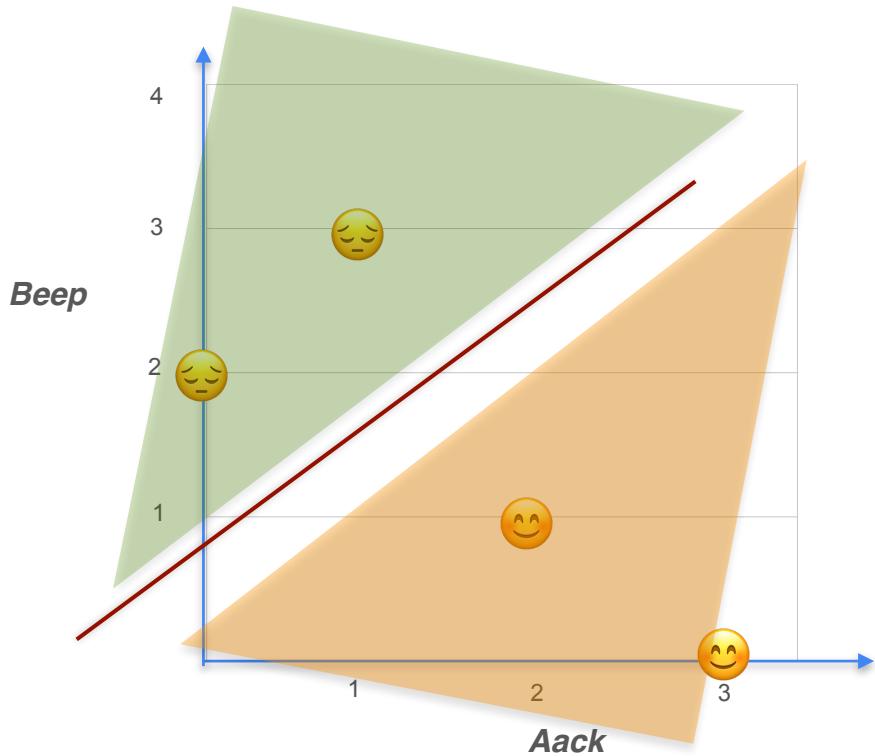


Model Training

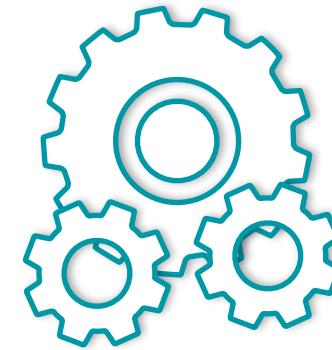


Machine Learning Model

Machine Learning Motivation

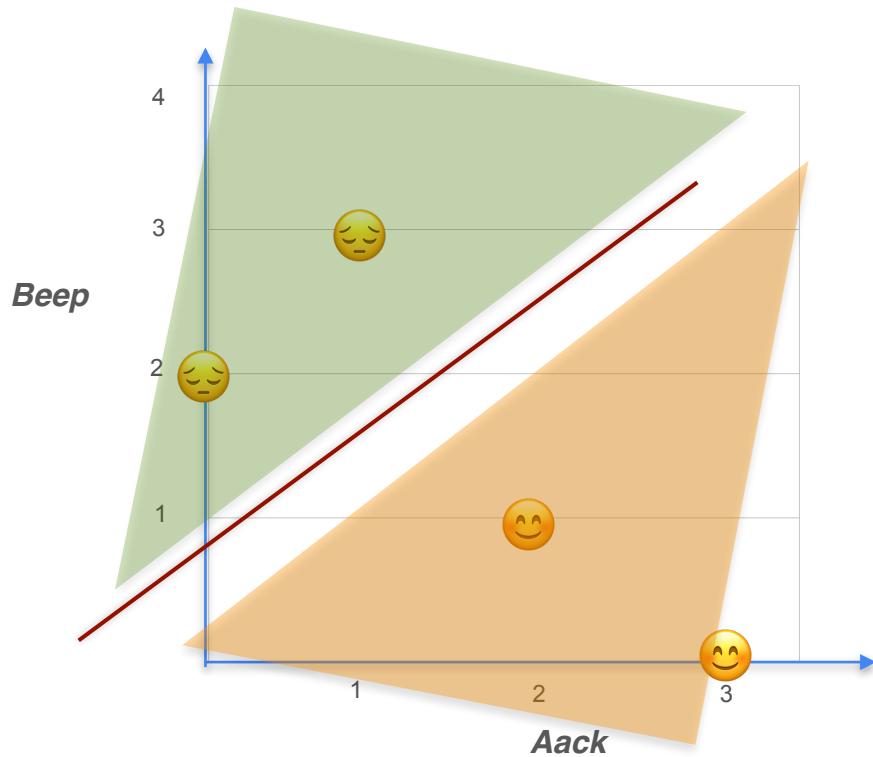


Model Training

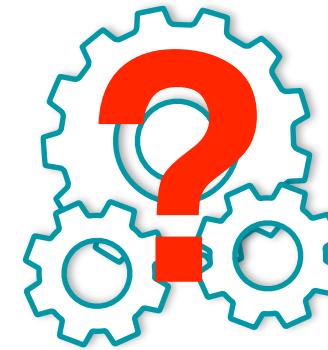


Machine Learning
Model

Machine Learning Motivation

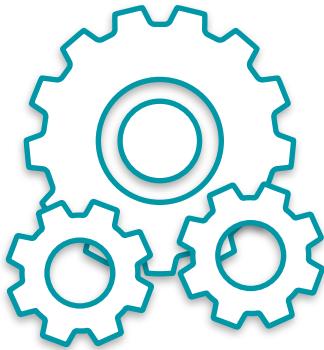


Model Training



Machine Learning
Model

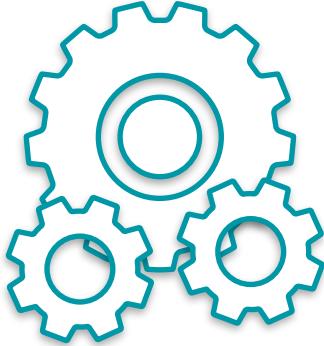
Machine Learning Motivation



Machine Learning
Model

Machine Learning Motivation

Maths concepts used in training a model



Machine Learning
Model

Machine Learning Motivation

Maths concepts used in training a model

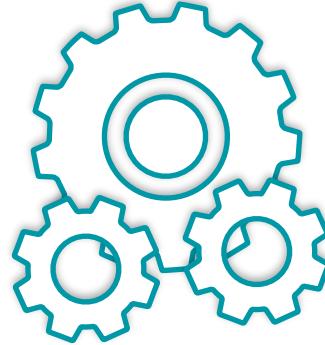
Gradients

Derivatives

Optimization

Loss and Cost functions

Gradient Descent



Machine Learning
Model

Machine Learning Motivation

Maths concepts used in training a model

Gradients

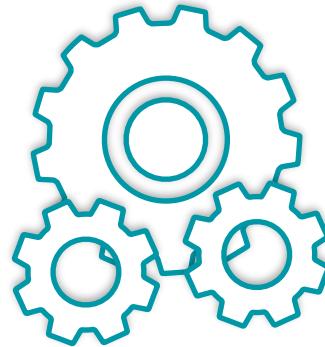
Derivatives

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Gradient Descent

Linear Regression



Machine Learning
Model

Machine Learning Motivation

Maths concepts used in training a model

Gradients

Derivatives

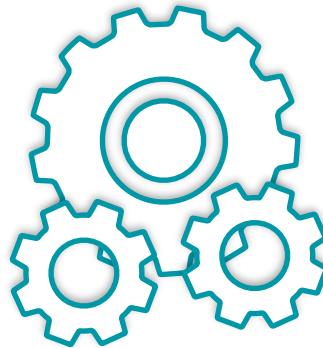
Optimization

Loss and Cost functions

Gradient Descent

Linear Regression

Classification



Machine Learning
Model

Machine Learning Motivation

Maths concepts used in training a model

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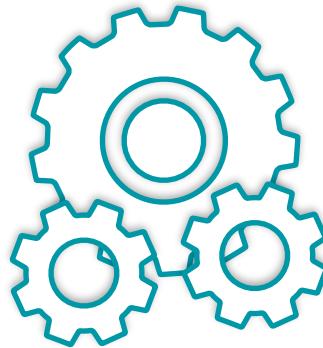
Loss and Cost functions

Gradient Descent

Linear Regression

Classification

Neural Networks



Machine Learning
Model



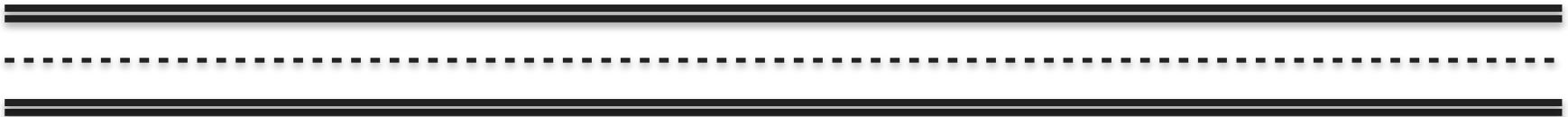
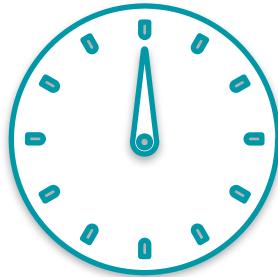
DeepLearning.AI

Derivatives and Optimization

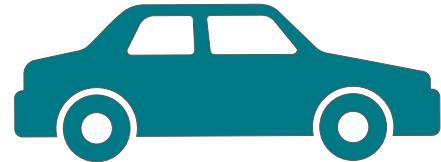
Introduction to derivatives

Introduction to Derivatives

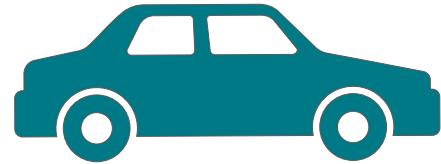
Introduction to Derivatives



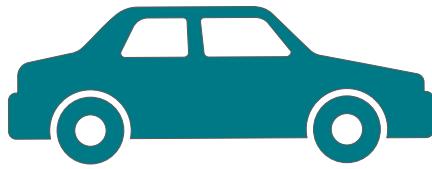
Introduction to Derivatives



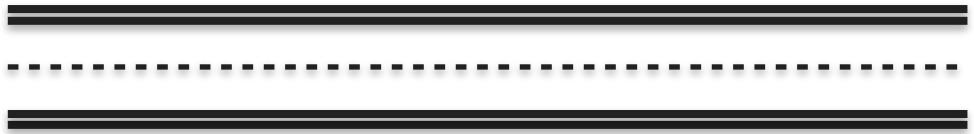
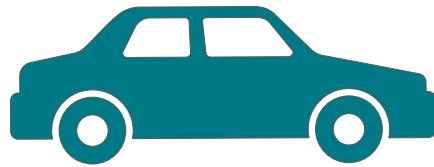
Introduction to Derivatives



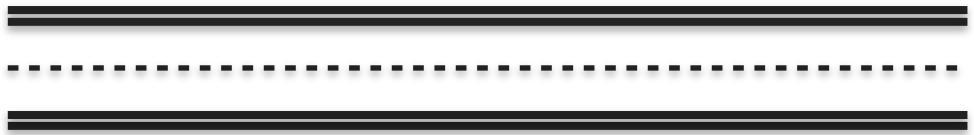
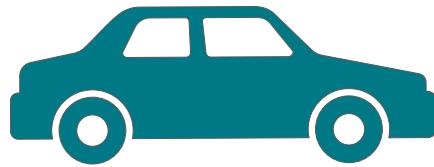
Introduction to Derivatives



Introduction to Derivatives

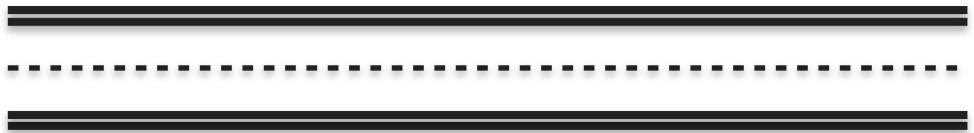
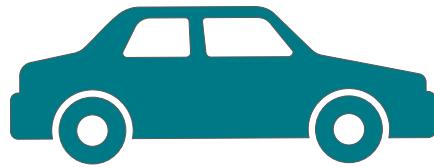
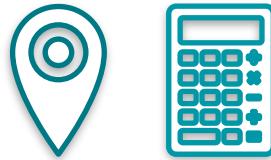


Introduction to Derivatives



Introduction to Derivatives

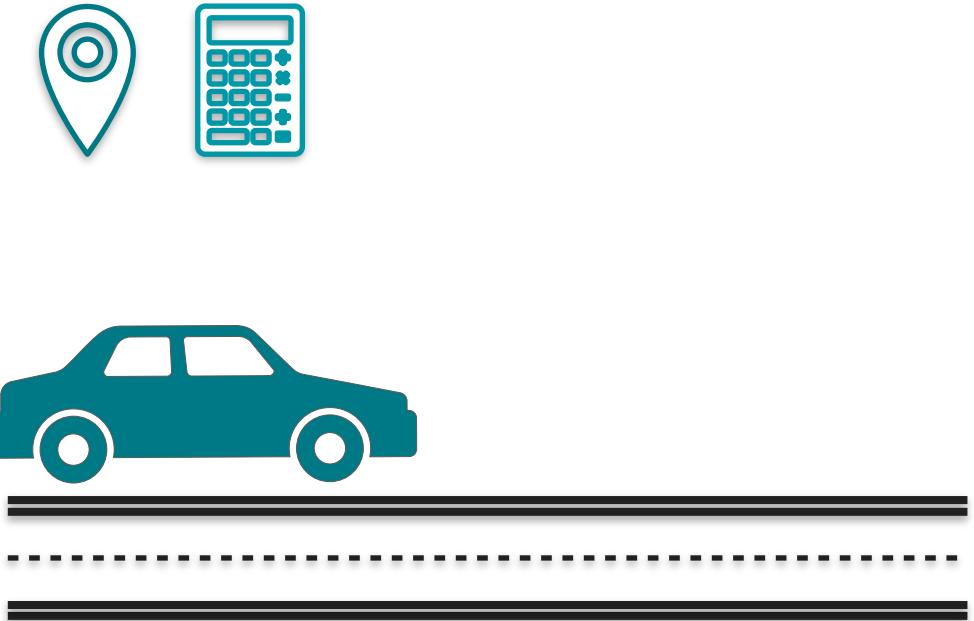
Every 5 seconds



Introduction to Derivatives

Every 5 seconds

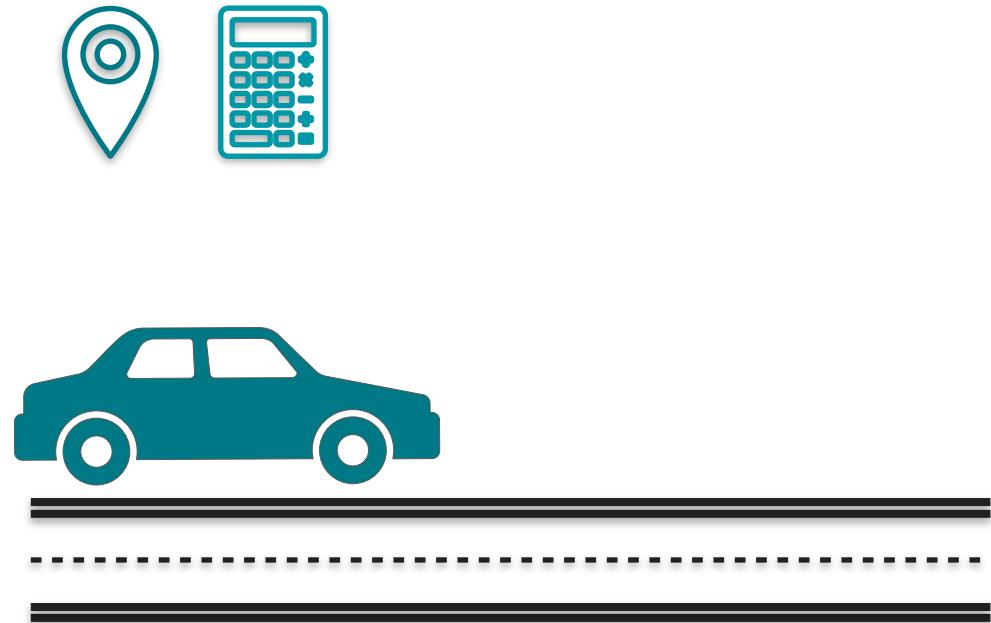
t (seconds)
0
5
10
15
20
25
30
35
40
45
50
55
60



Introduction to Derivatives

Every 5 seconds

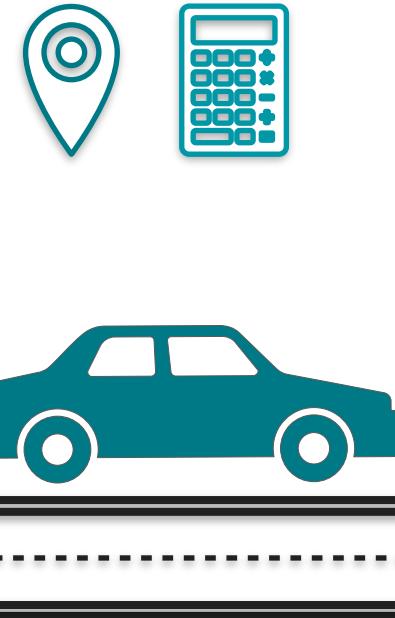
t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



Introduction to Derivatives

Every 5 seconds

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



Is the car moving at a constant speed?

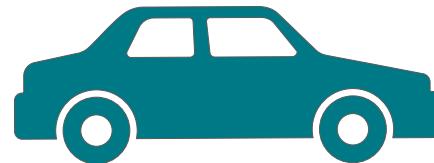
Introduction to Derivatives

Every 5 seconds

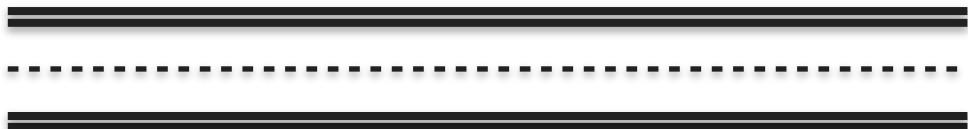
t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



Is the car moving at a constant speed?



Hint: Look at the distance values in the table



Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds: $202 - 122 = 80$ meters

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds: $202 - 122 = 80$ meters

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds: $202 - 122 = 80$ meters

15 - 20 seconds: $265 - 202 = 63$ meters

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds: $202 - 122 = 80$ meters

15 - 20 seconds: $265 - 202 = 63$ meters

Not traveling at constant speed

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly **$t = 12.5 \text{ seconds}$** ?

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly **$t = 12.5 \text{ seconds}$** ?

Hint: velocity = distance traveled / time taken

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly **$t = 12.5 \text{ seconds}$** ?

Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly $t = 12.5 \text{ seconds}$?

Quiz : Slope of a Line 1

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Quiz : Slope of a Line 1

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

What was the average velocity of the car on the interval from 10 to 15 seconds?

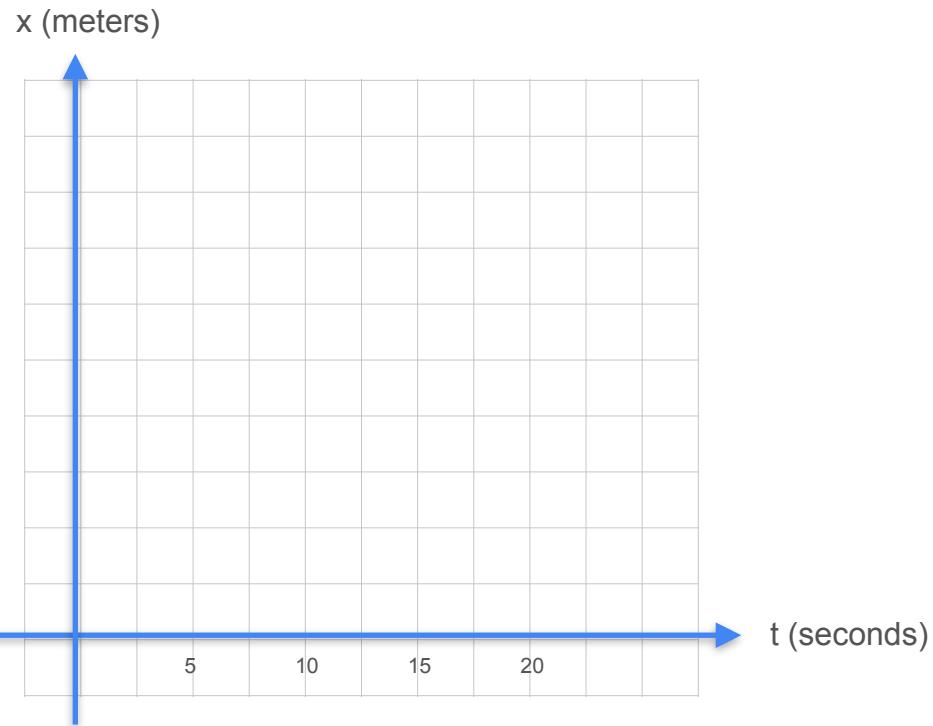
Quiz : Slope of a Line 1 - Solution

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

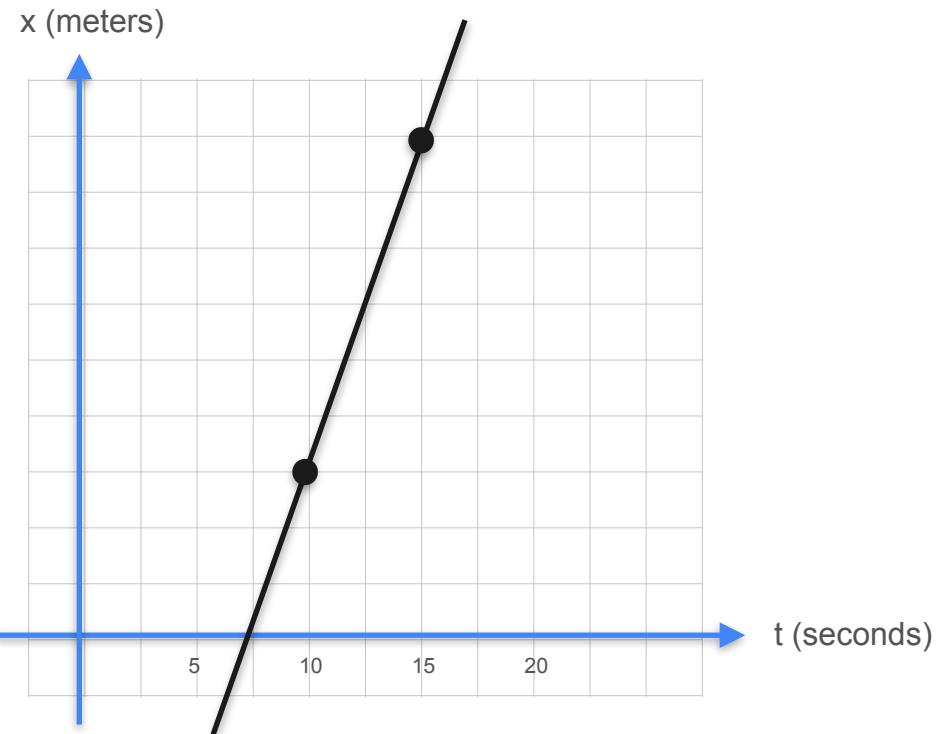
What was the average velocity of the car on the interval from 10 to 15 seconds?

Calculating the Slope

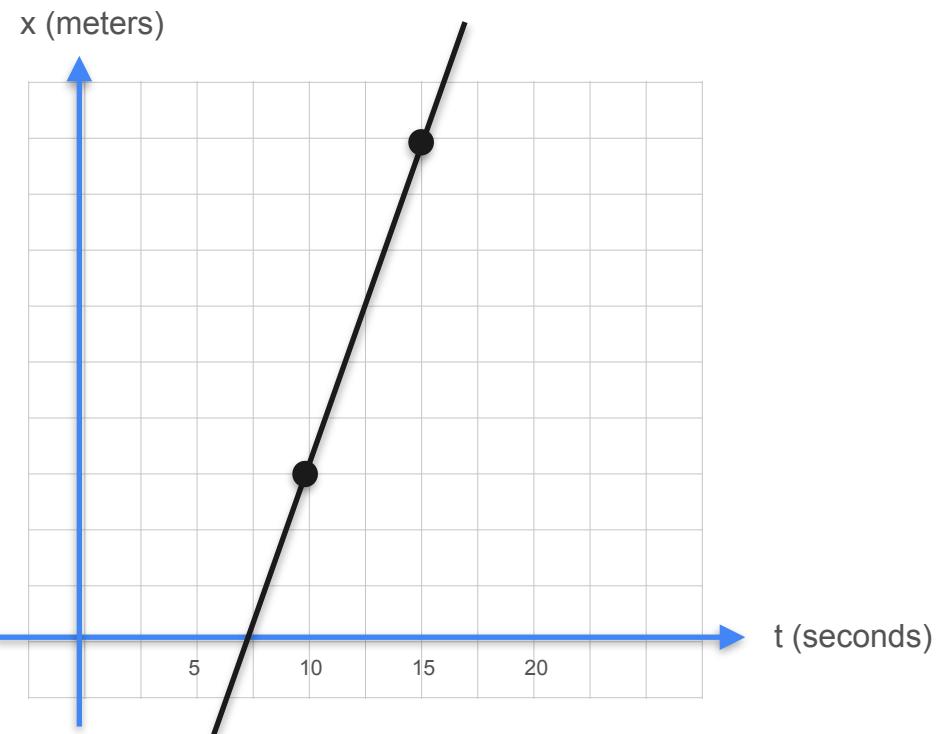
Calculating the Slope



Calculating the Slope

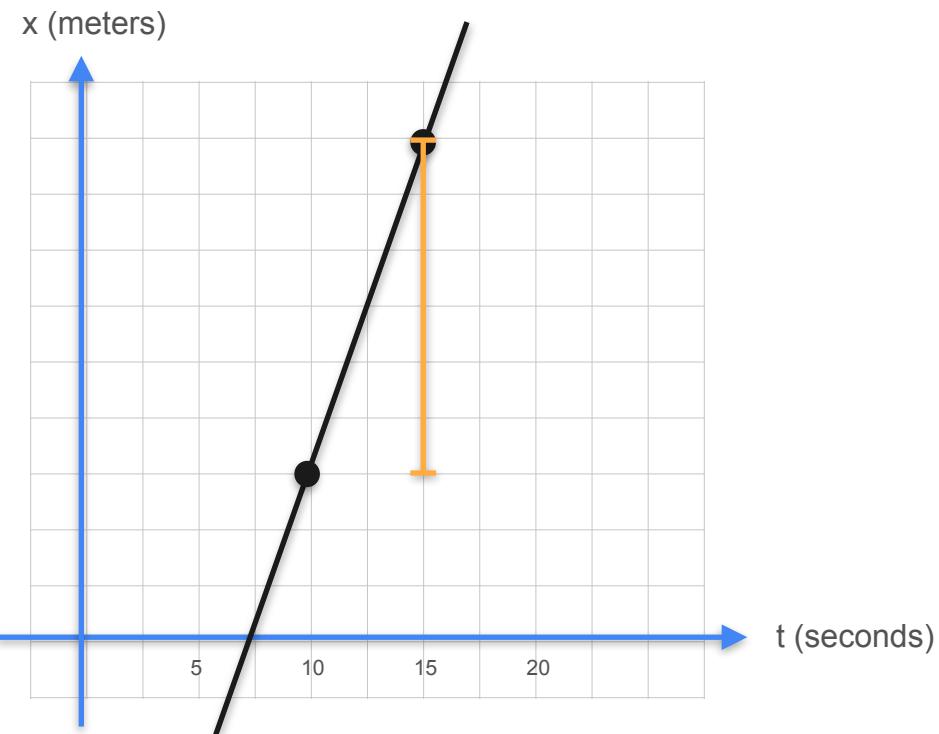


Calculating the Slope



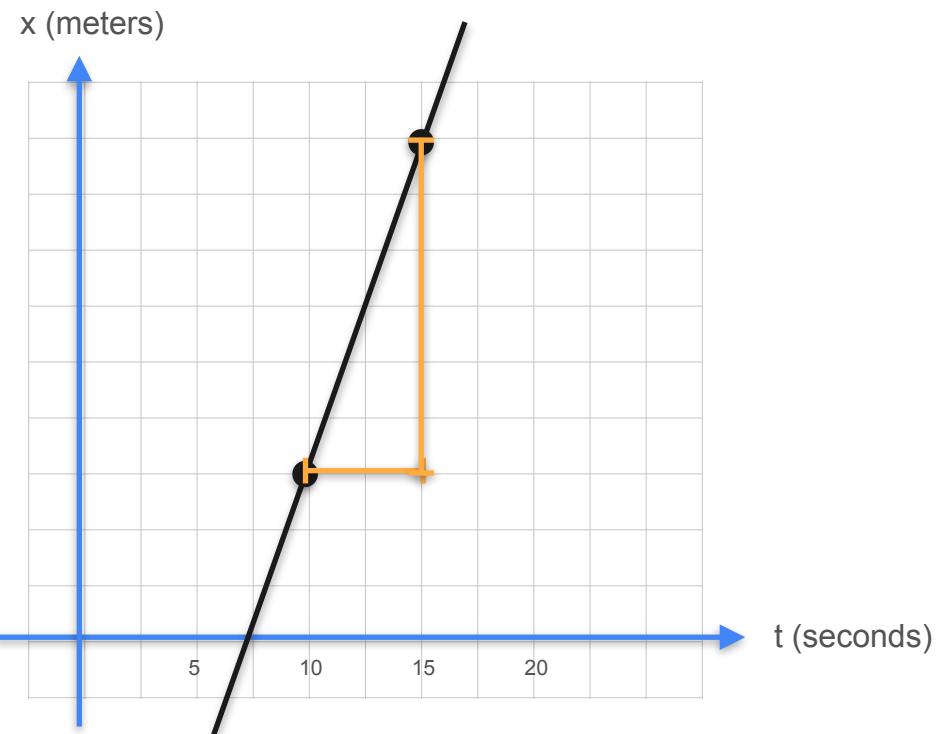
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Calculating the Slope



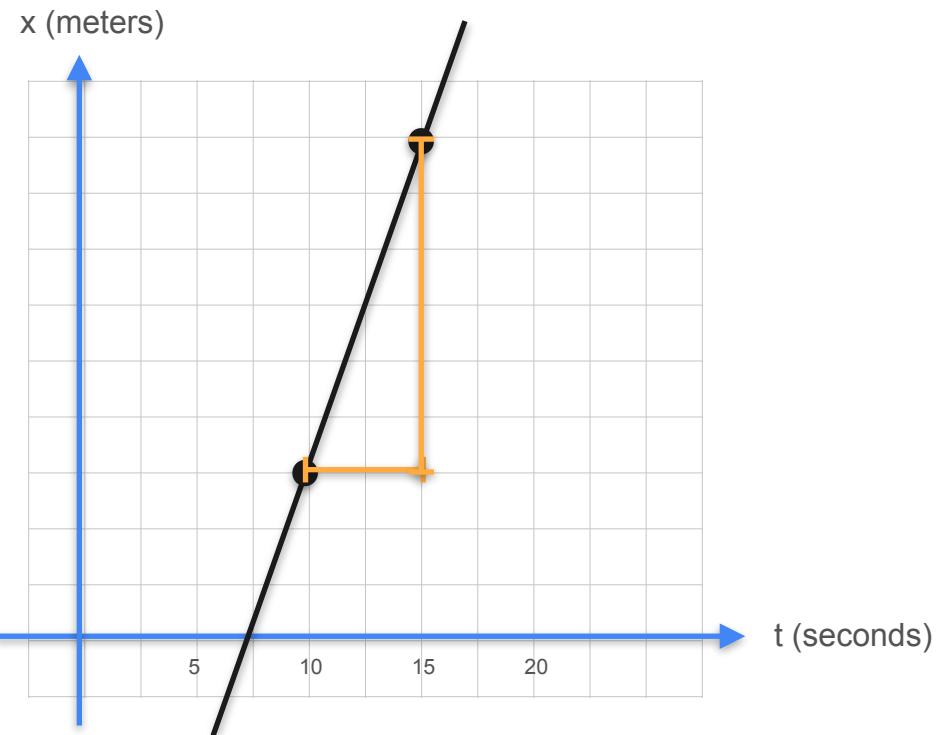
$$slope = \frac{rise}{run}$$

Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$


Calculating the Slope

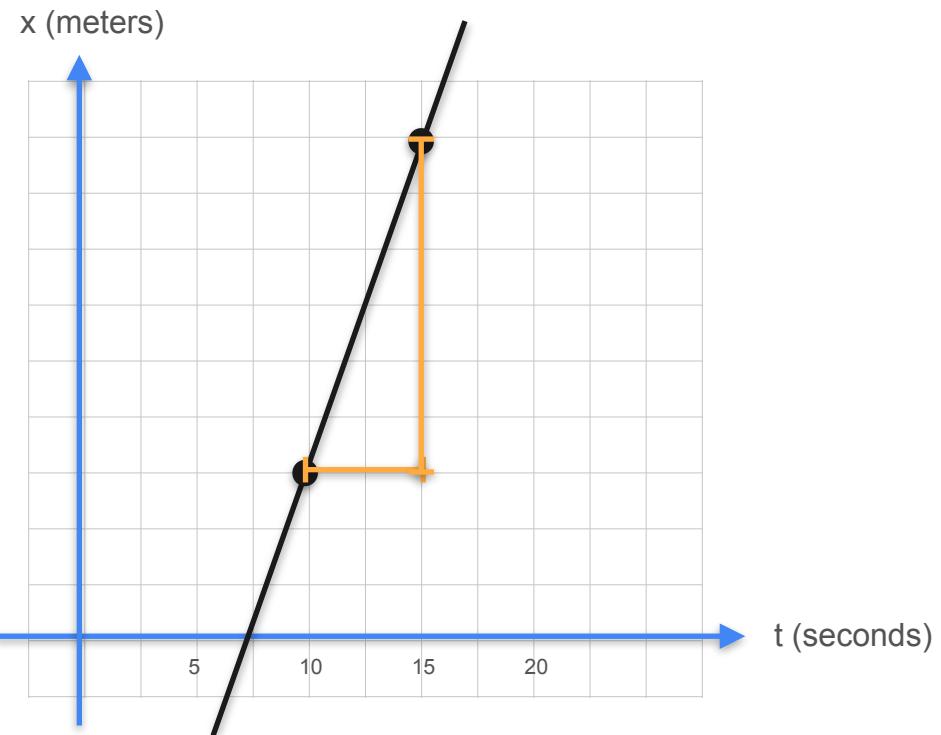


$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

A diagram showing a right triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run". Blue arrows point from the labels to the respective sides.

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

Calculating the Slope



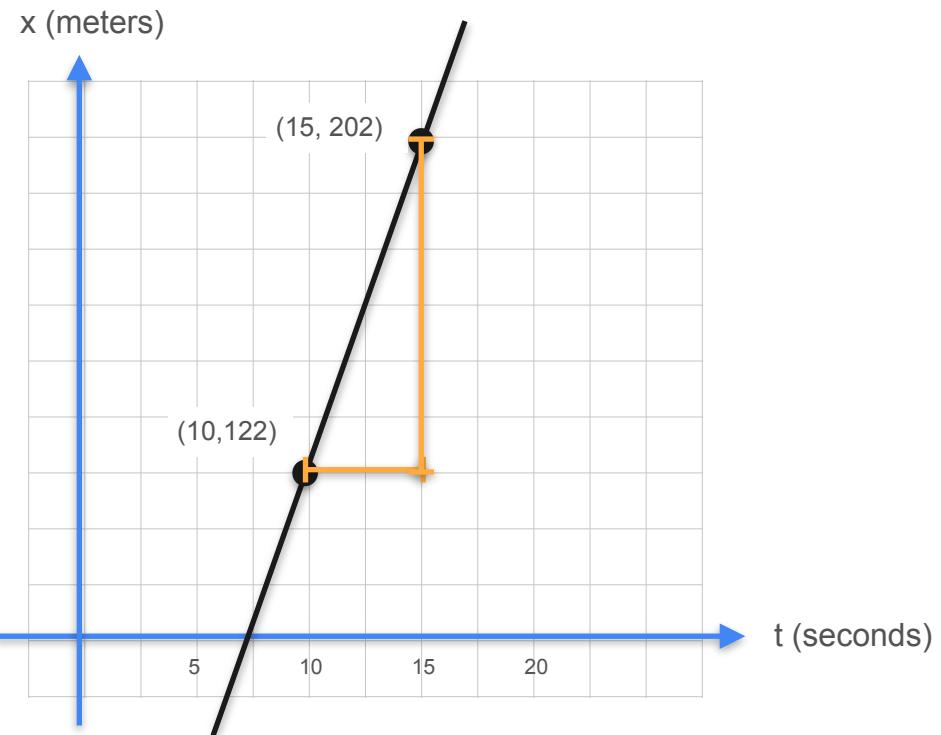
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

A diagram showing a right-angled triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run".

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

Calculating the Slope

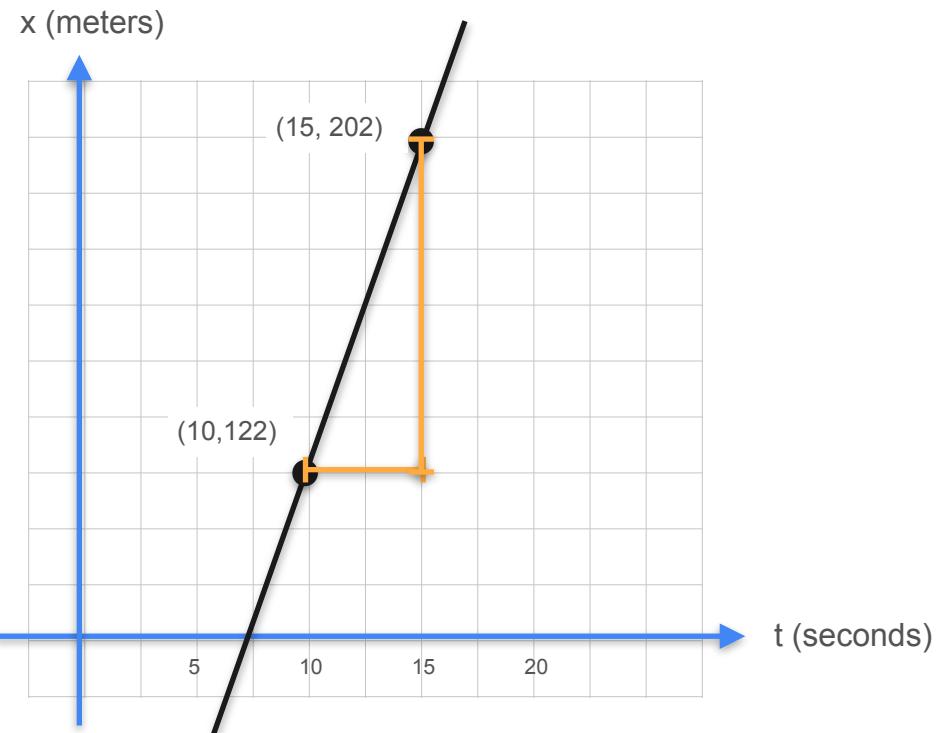


$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

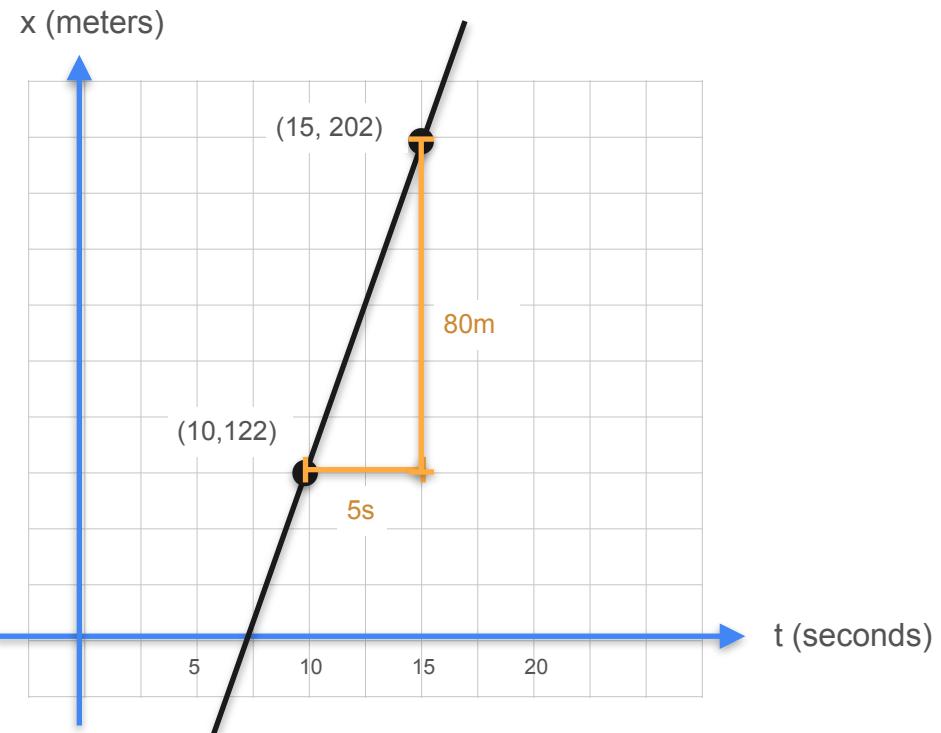
A diagram showing a right triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run".

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

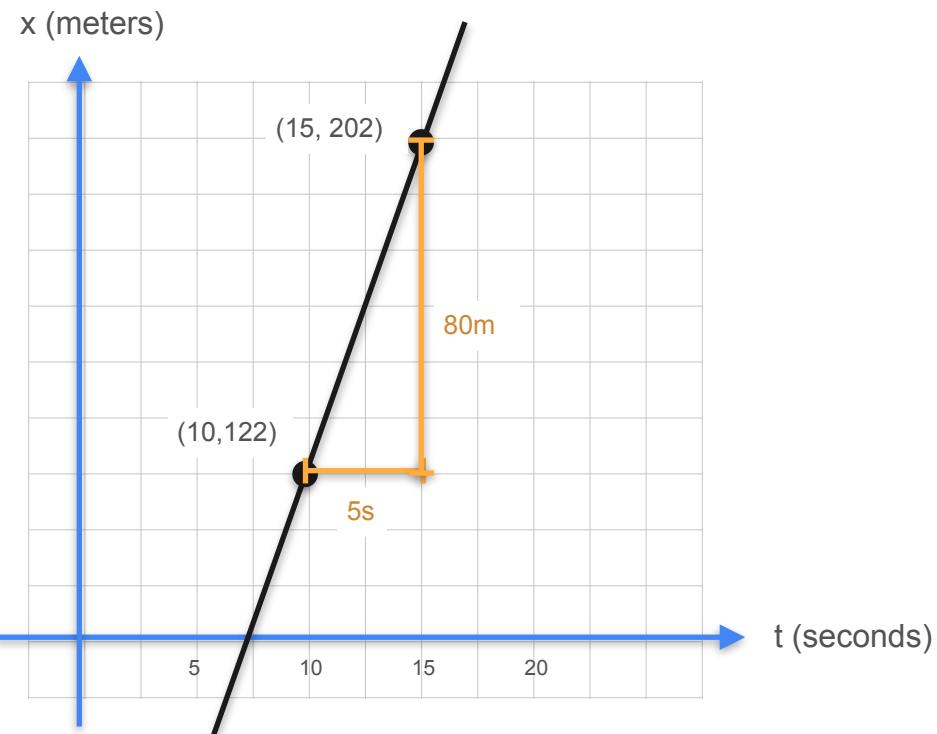
A diagram illustrating the slope formula. It shows a right triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run". Blue arrows point from the labels to the respective sides of the triangle.

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

A small diagram of a right triangle is shown, with a blue arrow pointing up the vertical leg labeled "rise" and a blue arrow pointing right along the horizontal leg labeled "run".

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

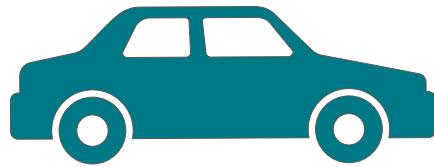
$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

$$\text{slope} = \frac{80m}{5s} = 16m/s$$

Introduction to Derivatives

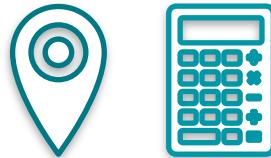


Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

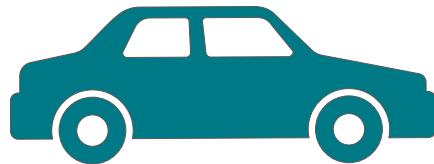


Introduction to Derivatives

Every second



Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?



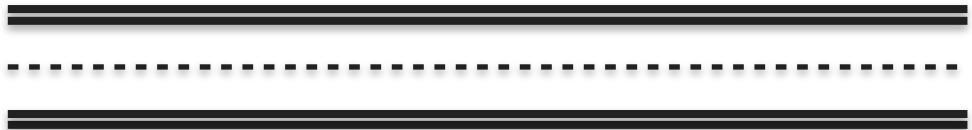
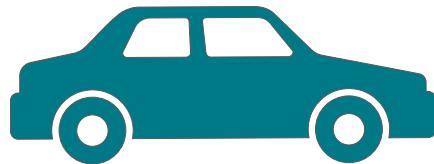
Introduction to Derivatives

Every second

t (s)
10
11
12
13
14
15
16
17
18
19
20



Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?



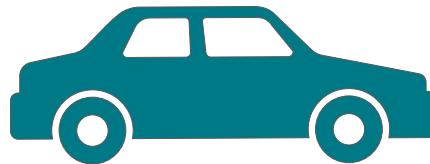
Introduction to Derivatives

Every second

t (s)	x(m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265



Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?



Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

$$\text{slope} = \frac{170m - 155m}{1s}$$

Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time $t = 12.5$ seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

$$\text{slope} = \frac{170m - 155m}{1s}$$

$$\text{slope} = 15m/s$$



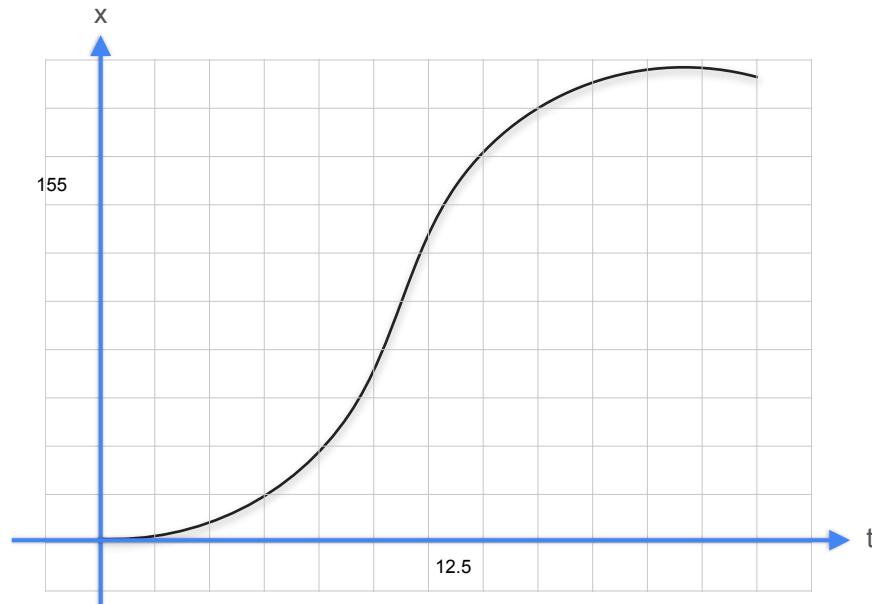
DeepLearning.AI

Derivatives and Optimization

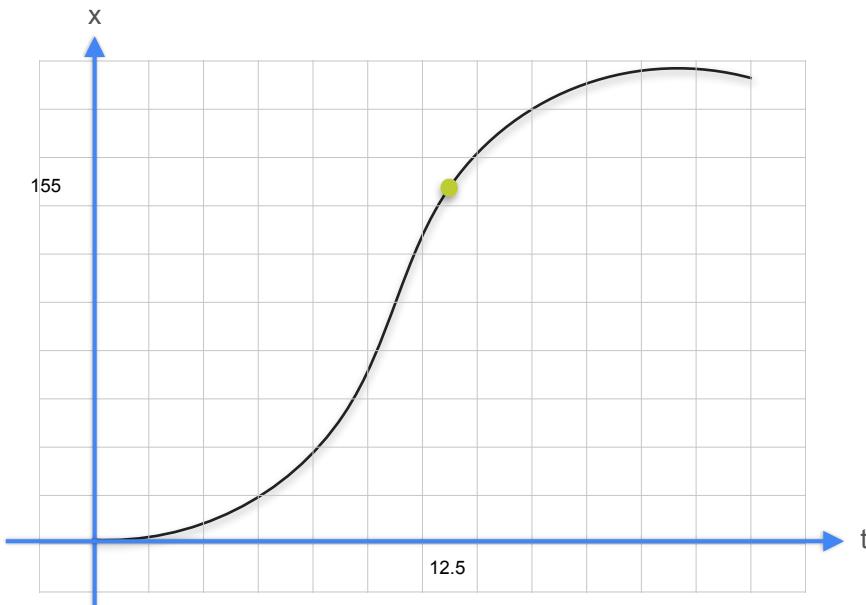
Derivatives and tangents

The derivative

The derivative

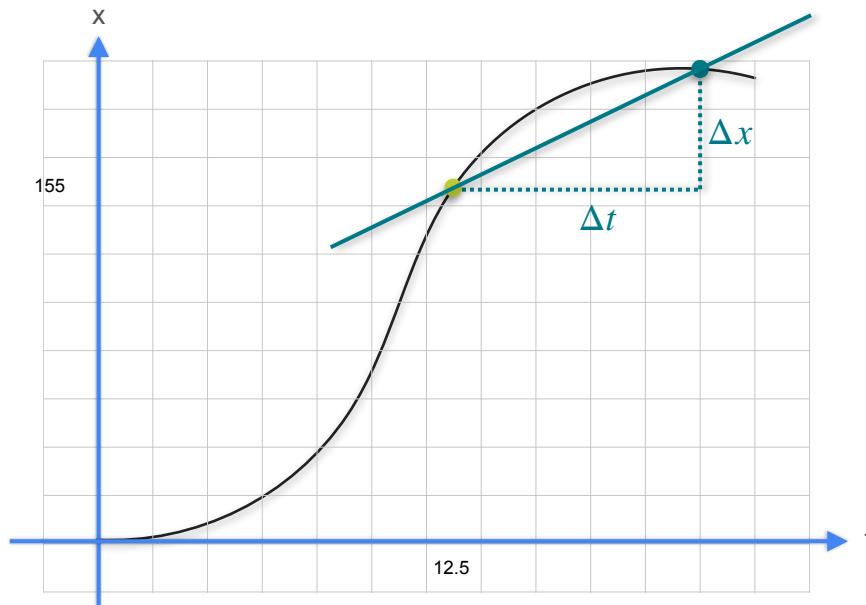


The derivative



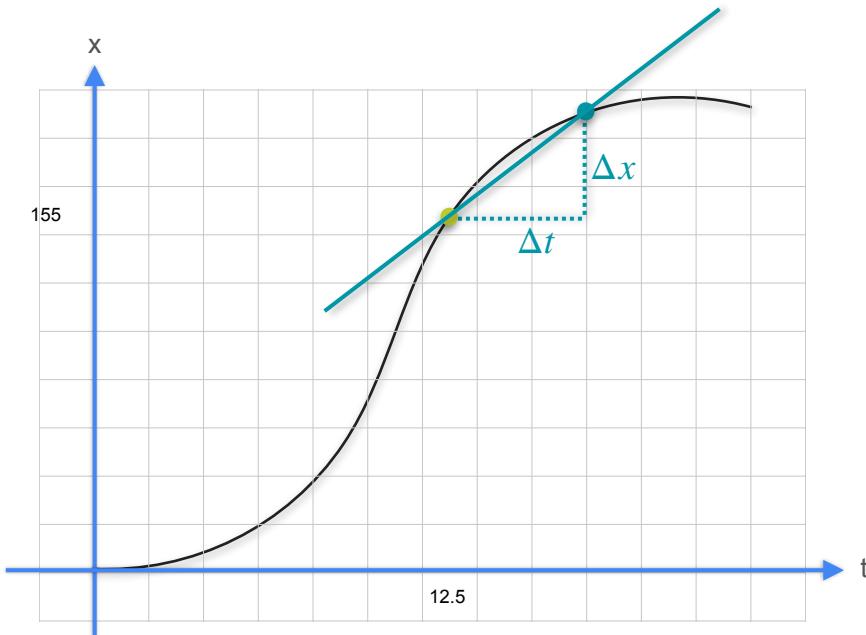
The derivative

$$\frac{\Delta x}{\Delta t}$$



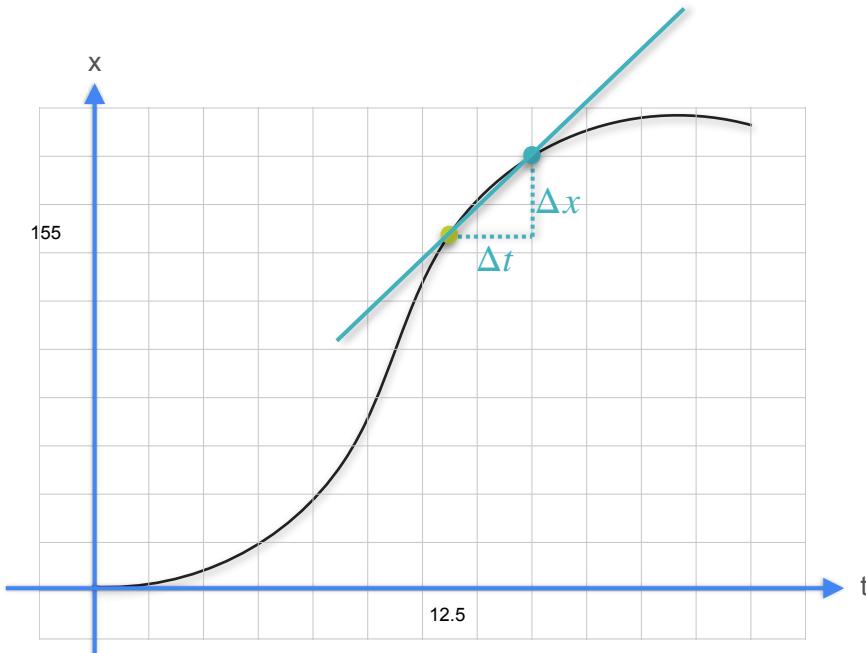
The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



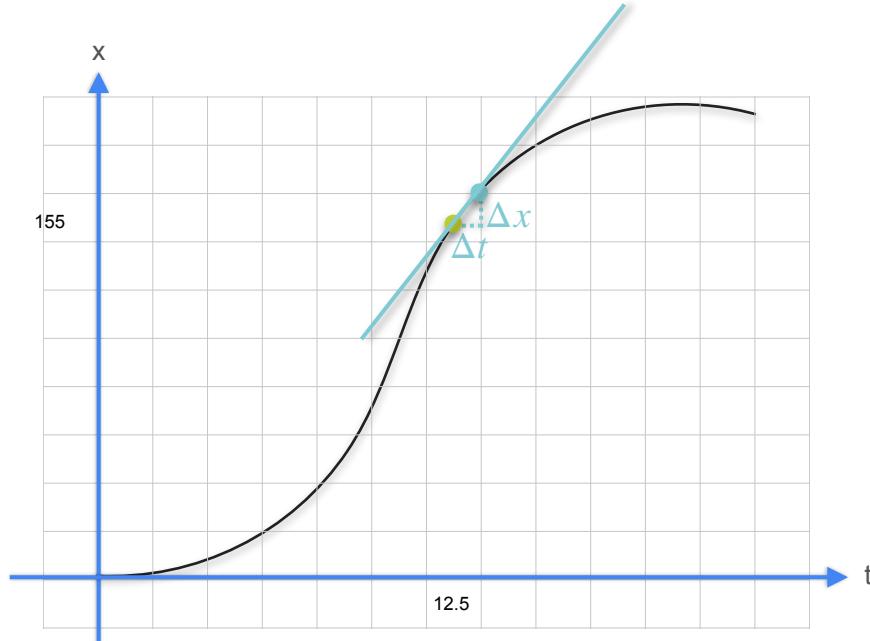
The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



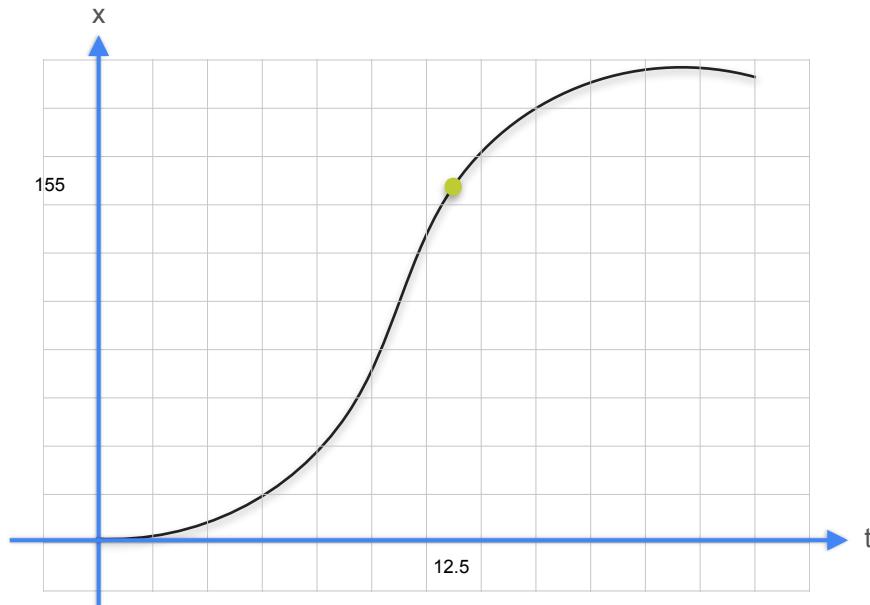
The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



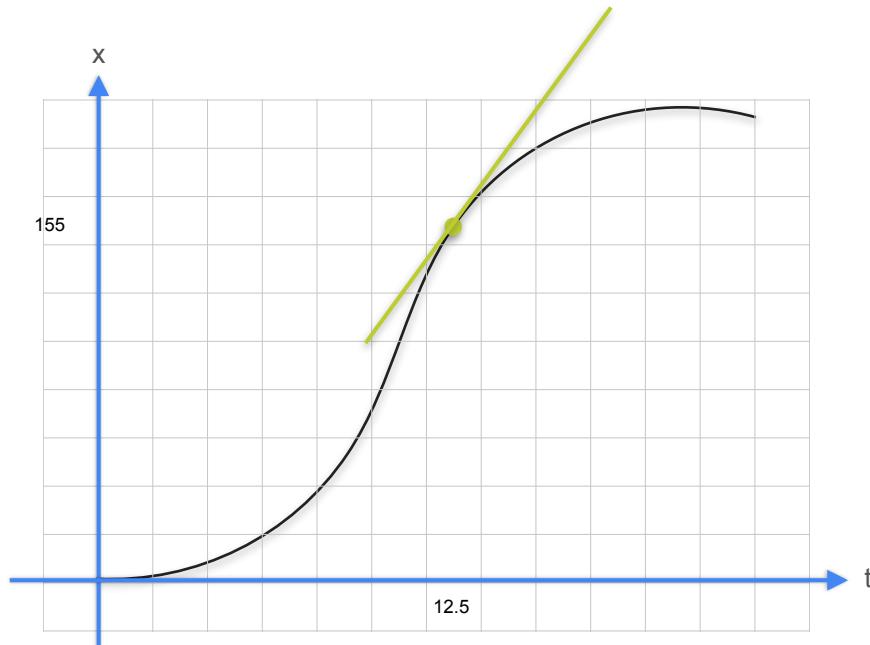
The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



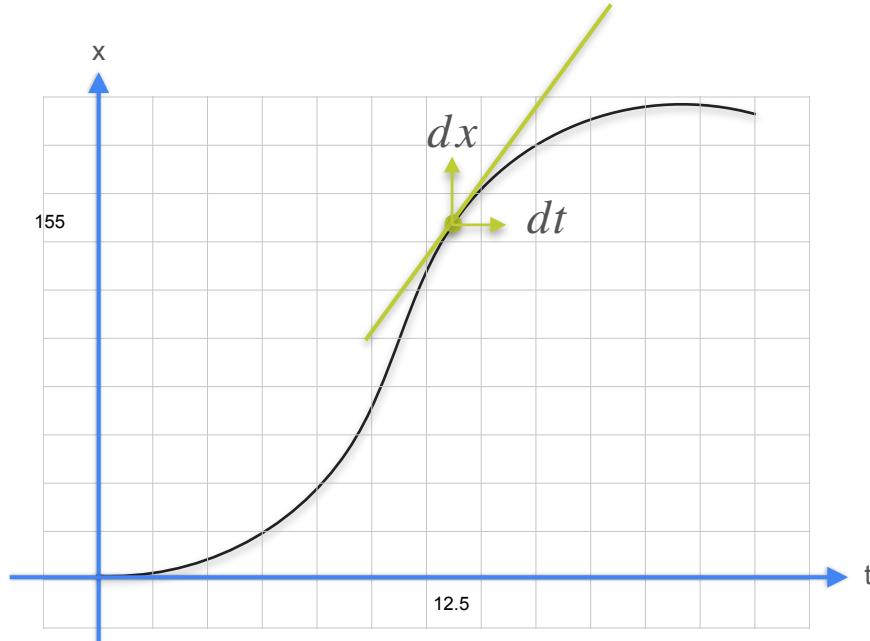
The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



The derivative

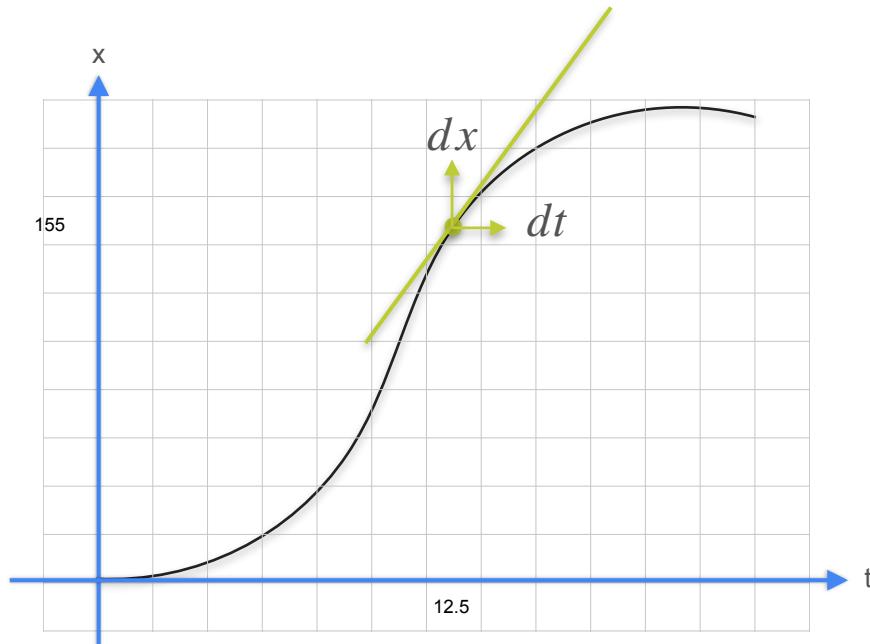
$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$

Derivative





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Derivatives and Optimization

Slopes, maxima, and minima

Zero Slope

t (s)	x (m)
19	265
20	265

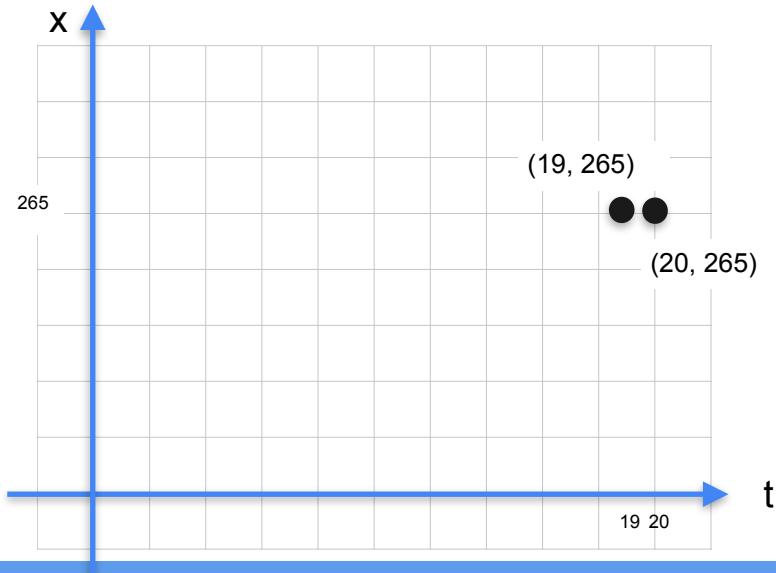
Zero Slope

t (s)	x (m)
19	265
20	265



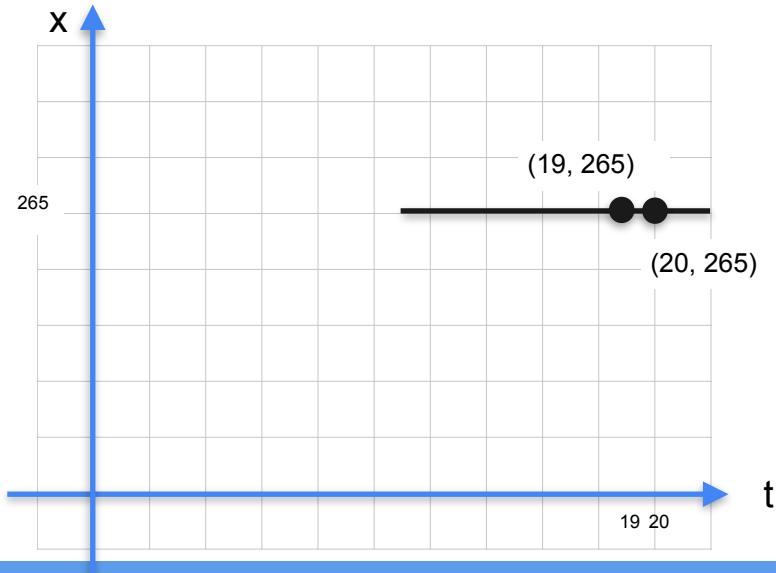
Zero Slope

t (s)	x (m)
19	265
20	265



Zero Slope

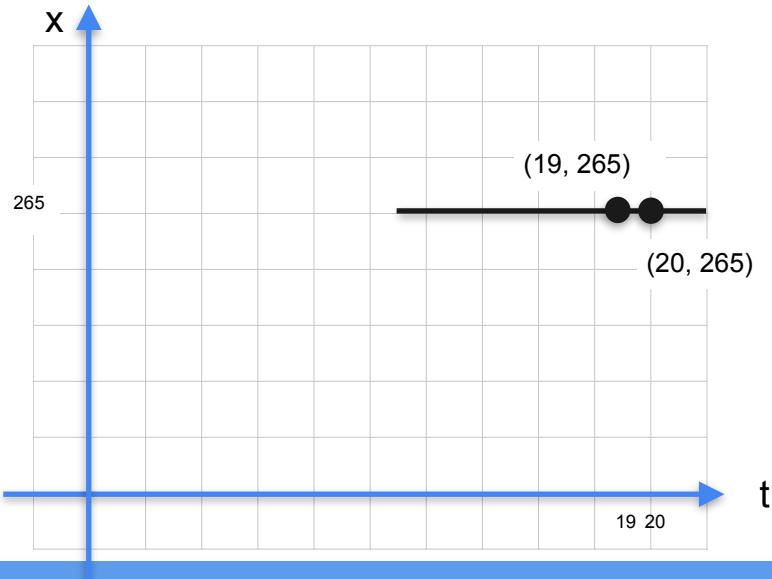
t (s)	x (m)
19	265
20	265



Zero Slope

t (s)	x (m)
19	265
20	265

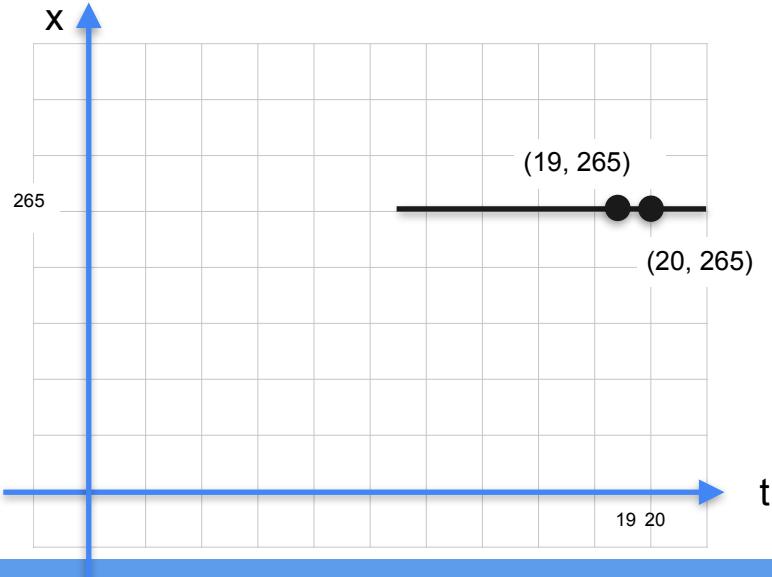
$$slope = \frac{\Delta x}{\Delta t}$$



Zero Slope

t (s)	x (m)
19	265
20	265

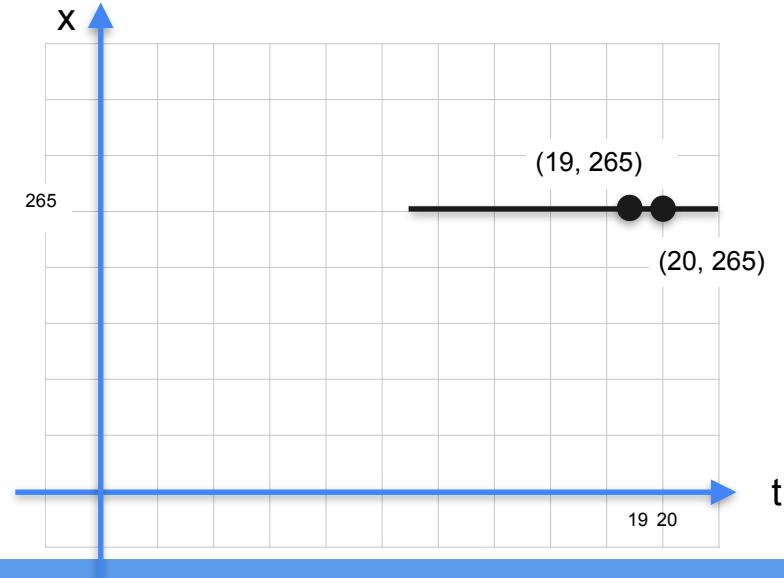
$$slope = \frac{\Delta x}{\Delta t}$$



$$slope = \frac{x(20) - x(19)}{20 - 19}$$

Zero Slope

t (s)	x (m)
19	265
20	265



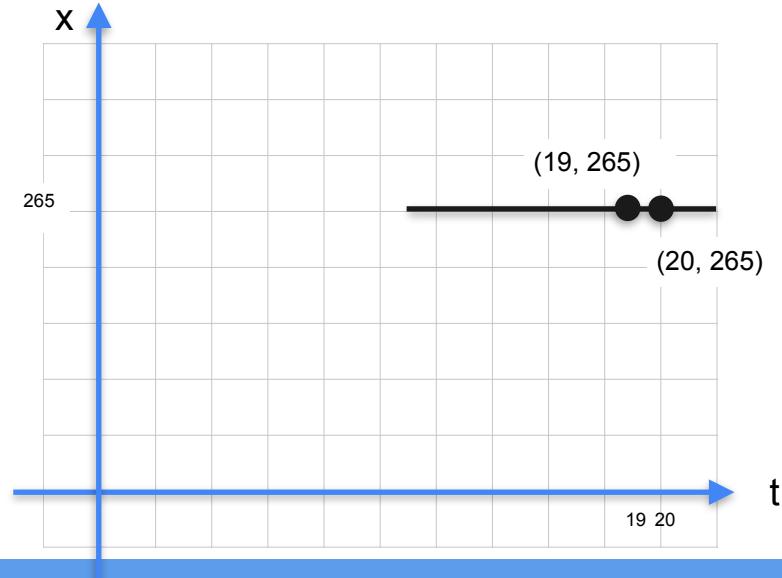
$$slope = \frac{\Delta x}{\Delta t}$$

$$slope = \frac{x(20) - x(19)}{20 - 19}$$

$$slope = \frac{265m - 265m}{1s}$$

Zero Slope

t (s)	x (m)
19	265
20	265



$$slope = \frac{\Delta x}{\Delta t}$$

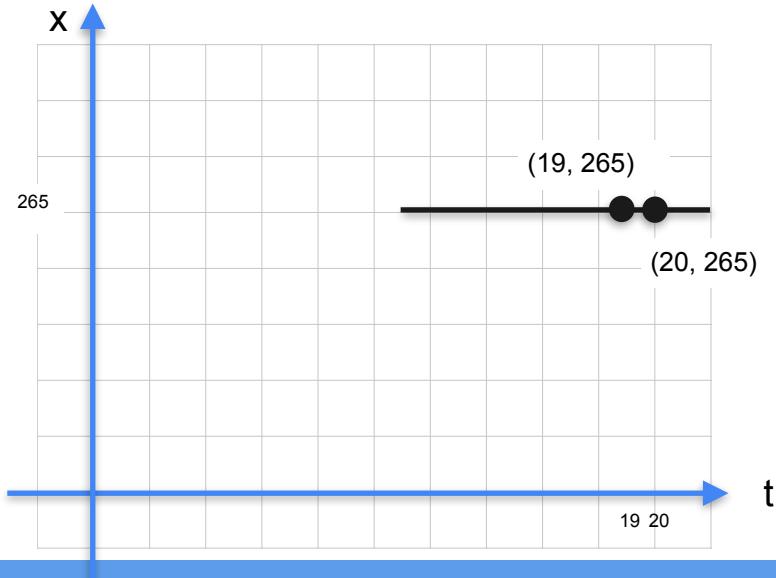
$$slope = \frac{x(20) - x(19)}{20 - 19}$$

$$slope = \frac{265m - 265m}{1s}$$

$$slope = \frac{0}{1}$$

Zero Slope

t (s)	x (m)
19	265
20	265



$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(20) - x(19)}{20 - 19}$$

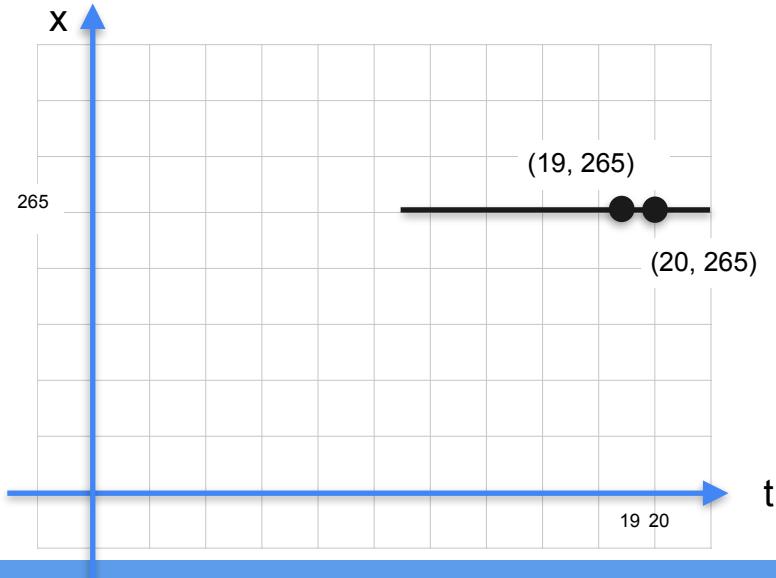
$$\text{slope} = \frac{265m - 265m}{1s}$$

$$\text{slope} = \frac{0}{1}$$

No rise in distance

Zero Slope

t (s)	x (m)
19	265
20	265



$$\text{slope} = \frac{\Delta x}{\Delta t}$$

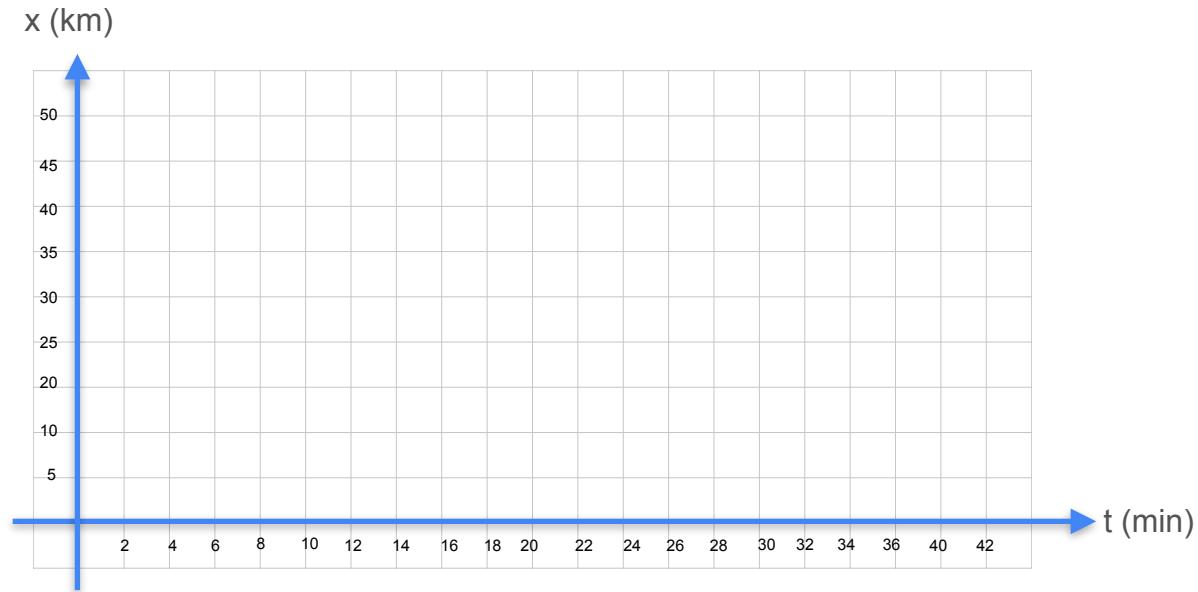
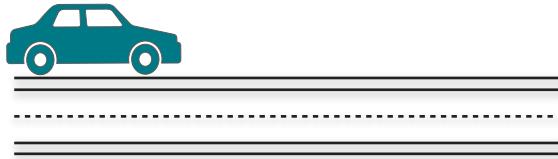
$$\text{slope} = \frac{x(20) - x(19)}{20 - 19}$$

$$\text{slope} = \frac{265m - 265m}{1s}$$

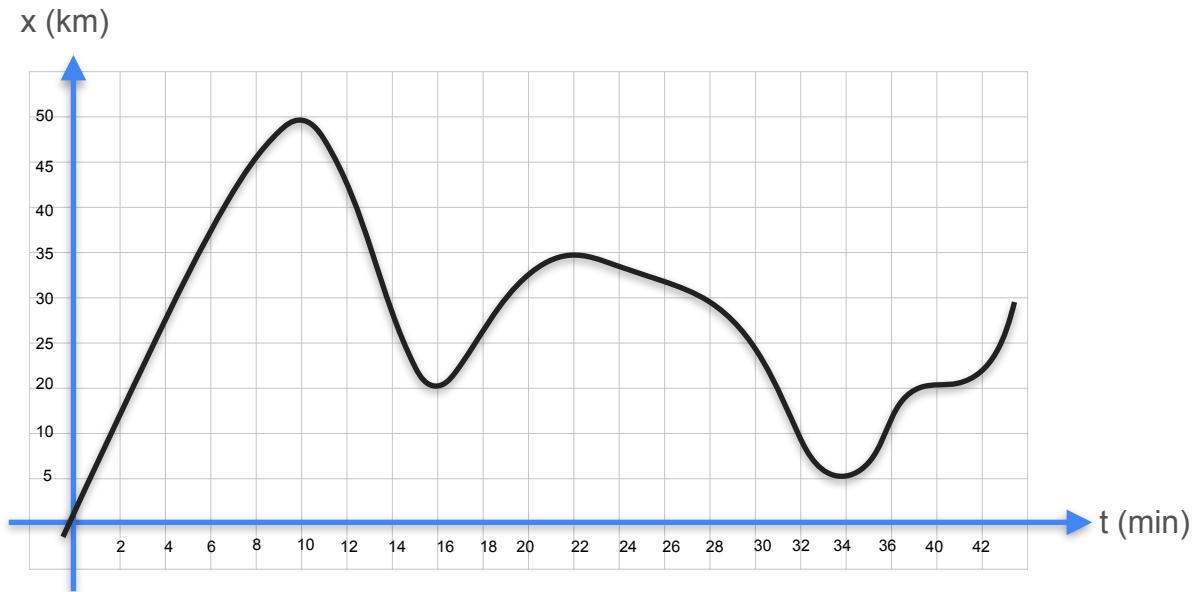
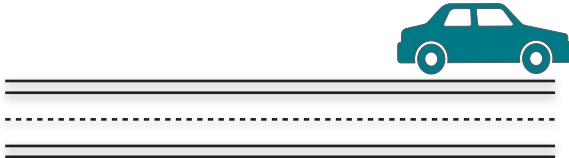
$$\text{slope} = \frac{0}{1} \quad \text{No rise in distance}$$

$$\text{slope} = 0 \text{ m/s}$$

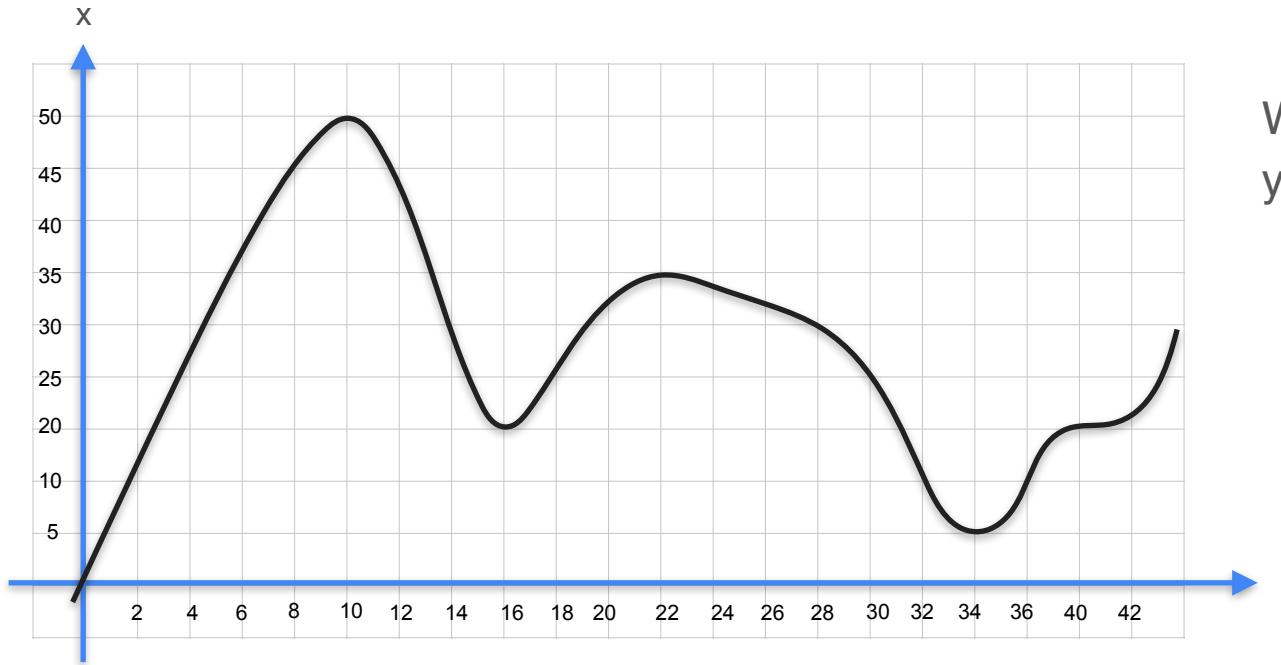
Minima and Maxima



Minima and Maxima

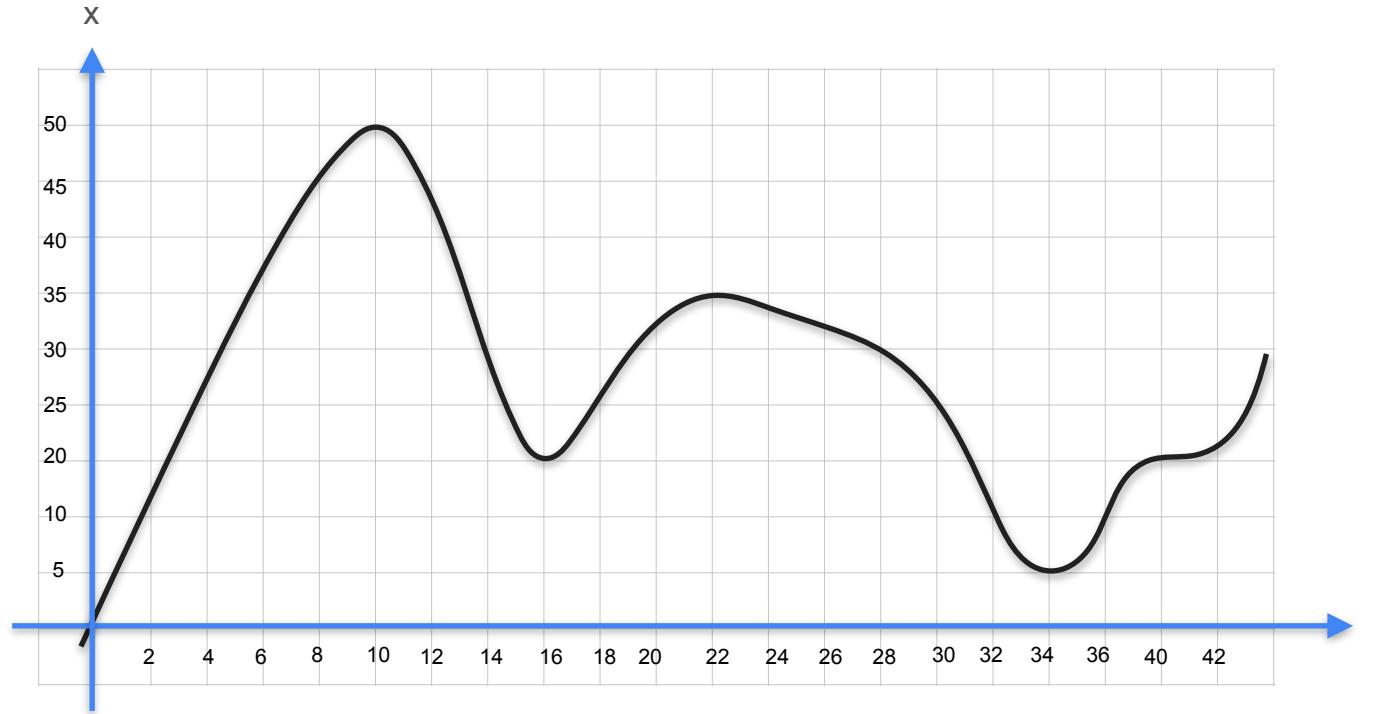


Quiz : Minima and Maxima 1

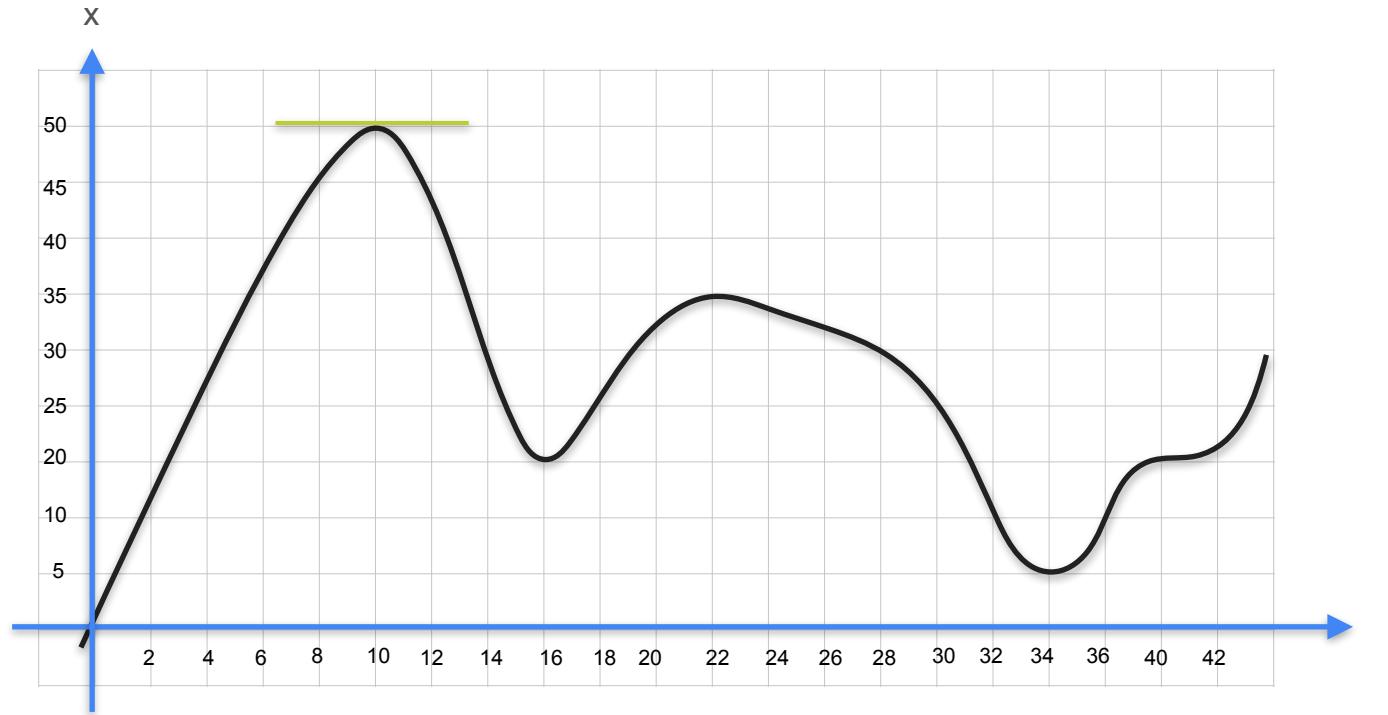


Where was the velocity of your car zero?

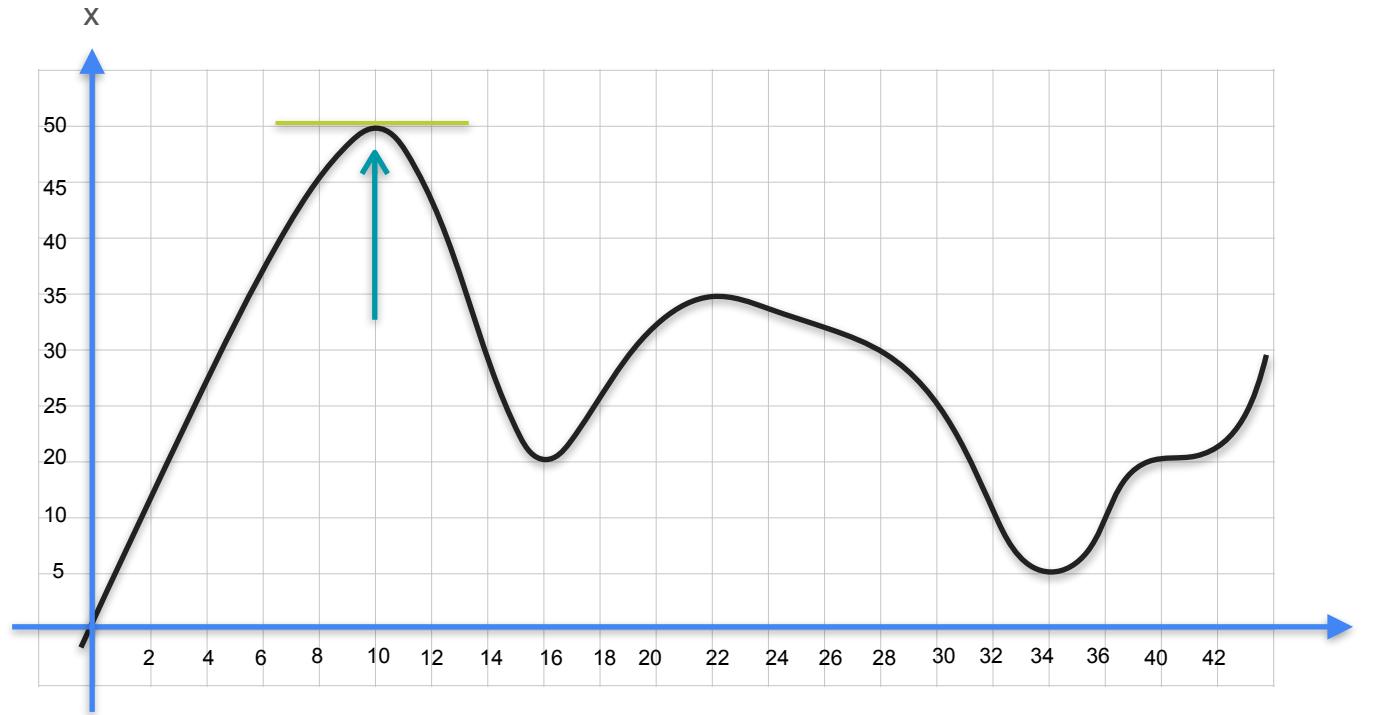
Quiz : Minima and Maxima 1 - Solution



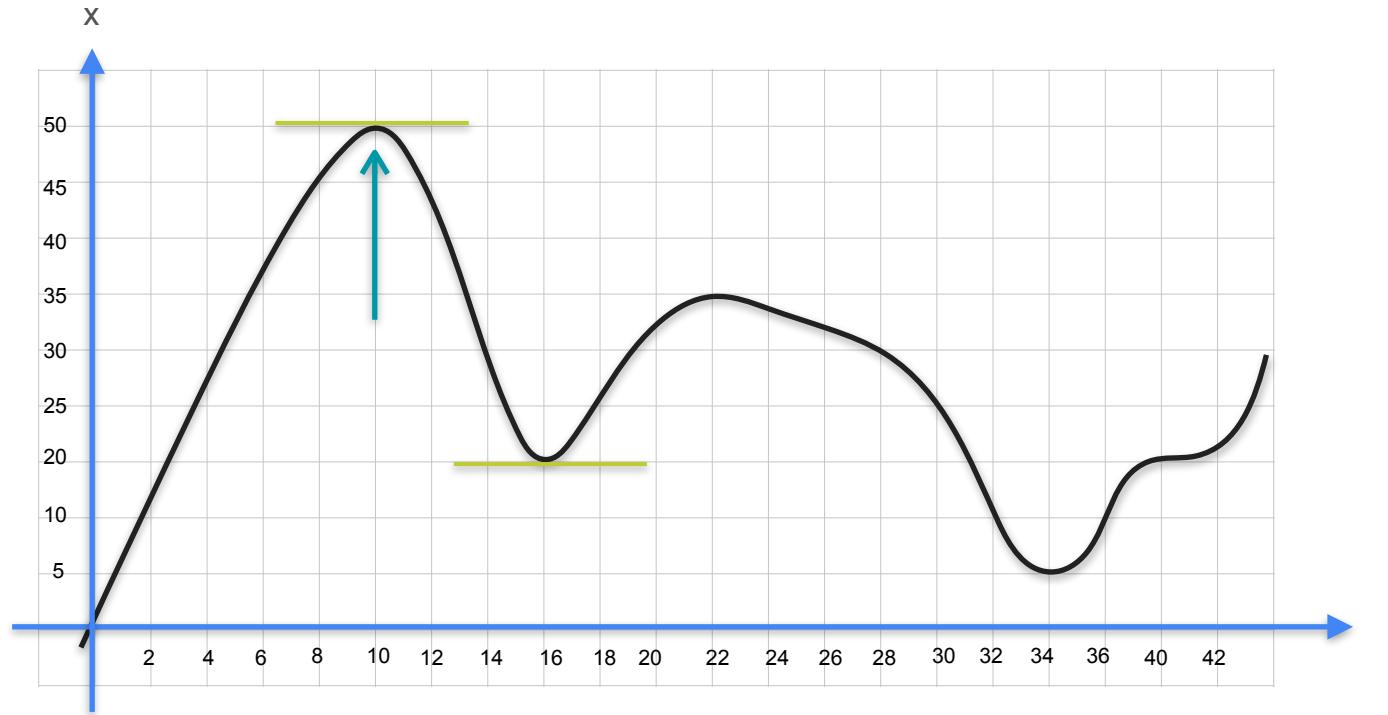
Quiz : Minima and Maxima 1 - Solution



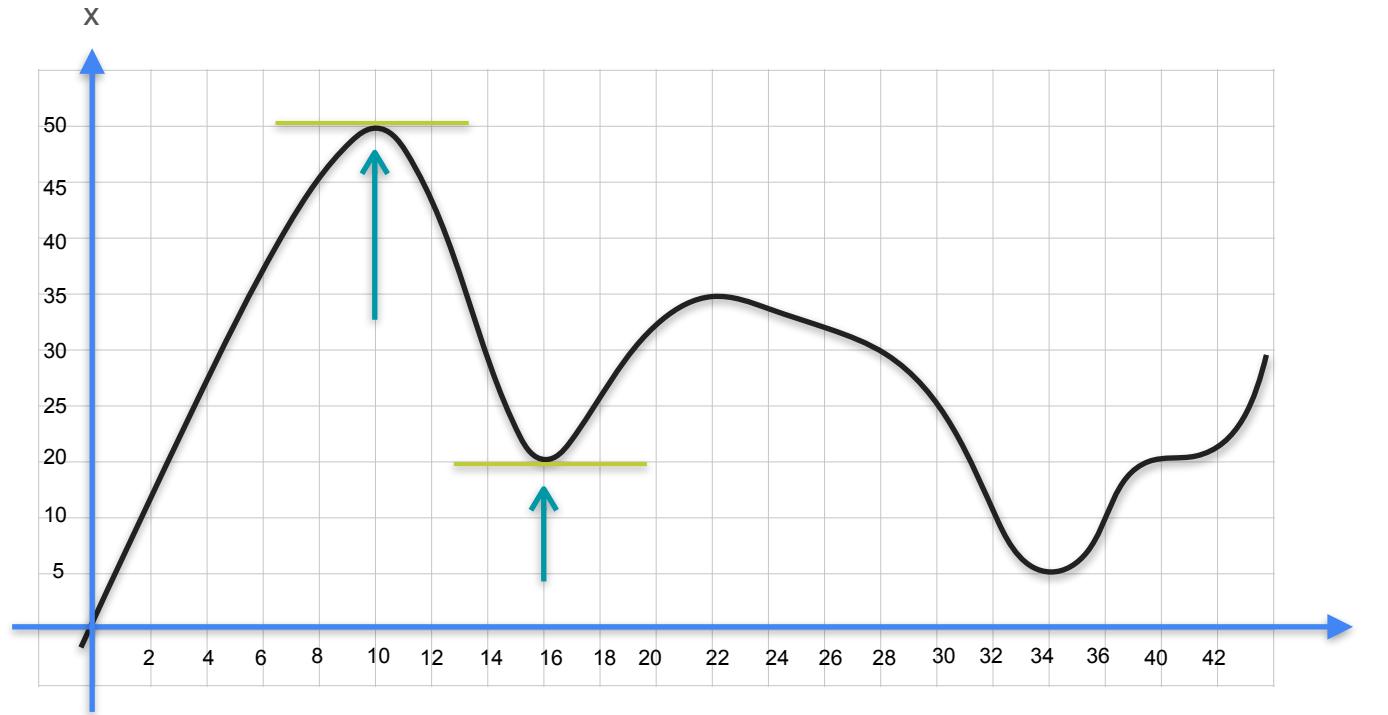
Quiz : Minima and Maxima 1 - Solution



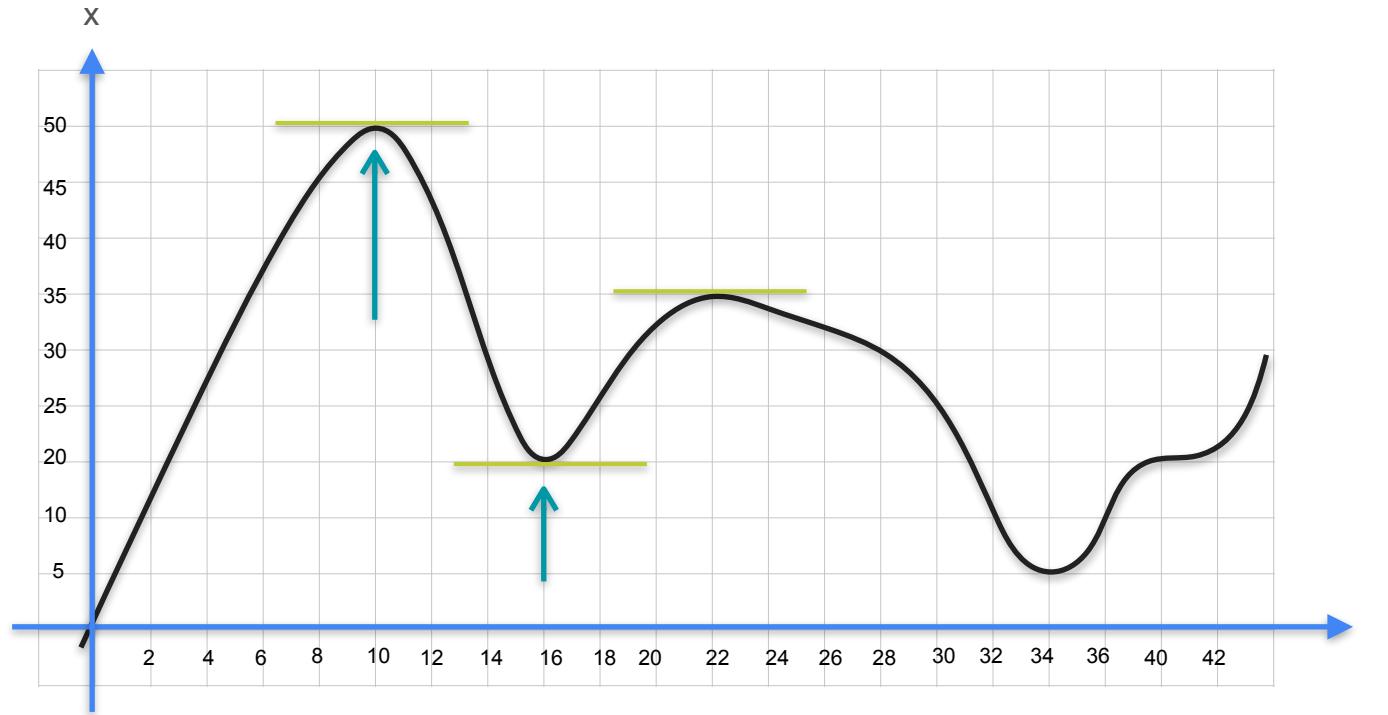
Quiz : Minima and Maxima 1 - Solution



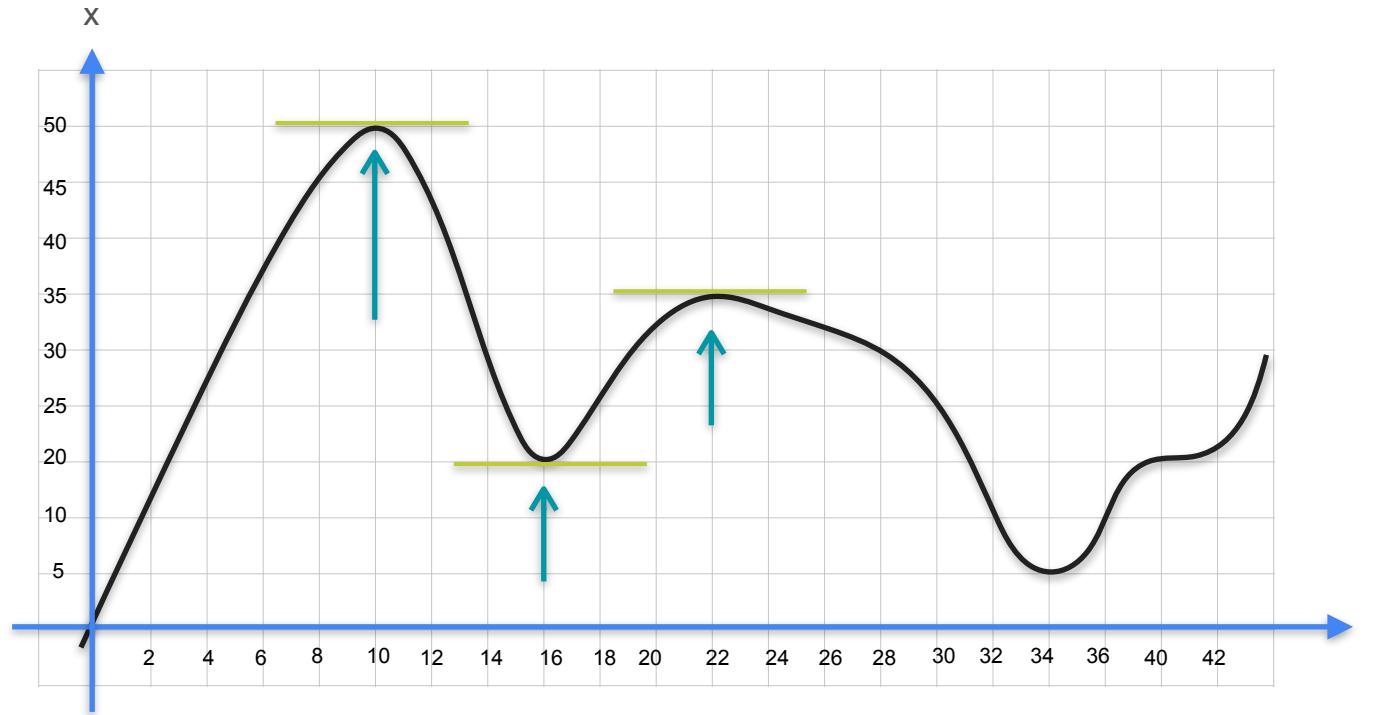
Quiz : Minima and Maxima 1 - Solution



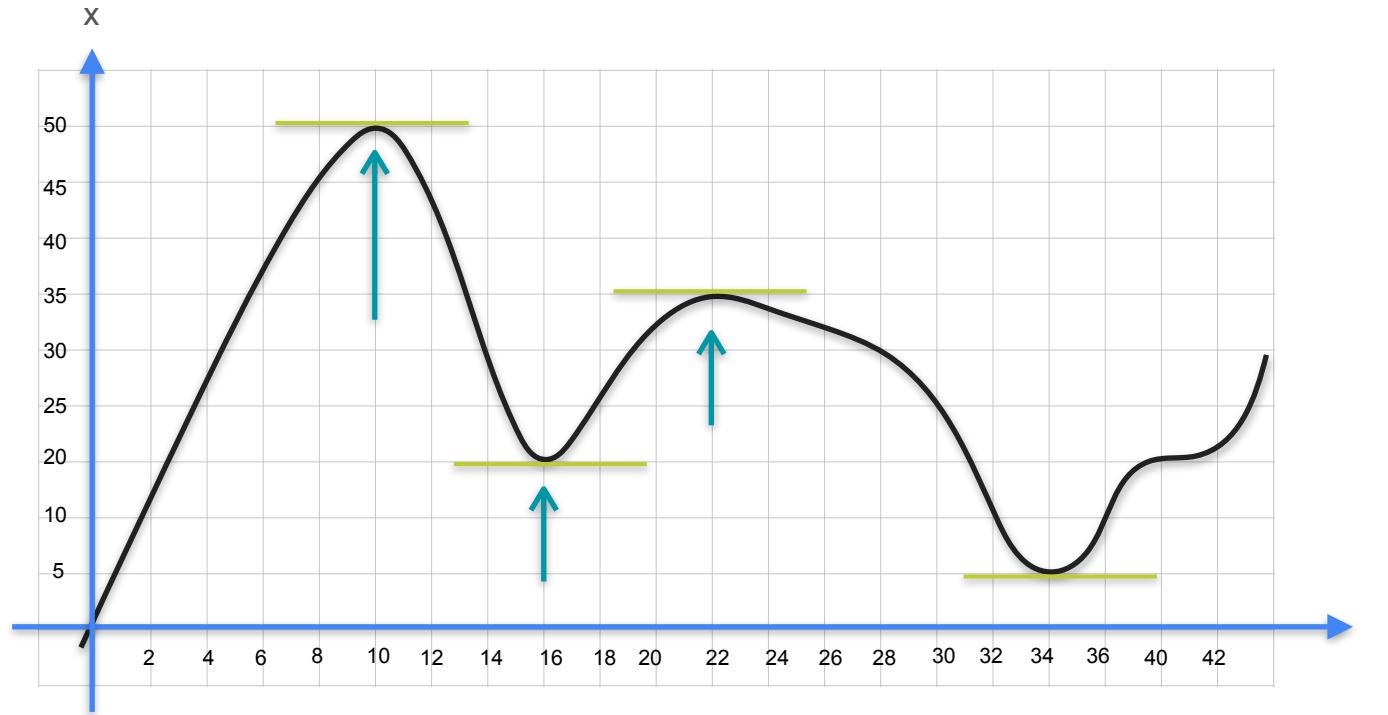
Quiz : Minima and Maxima 1 - Solution



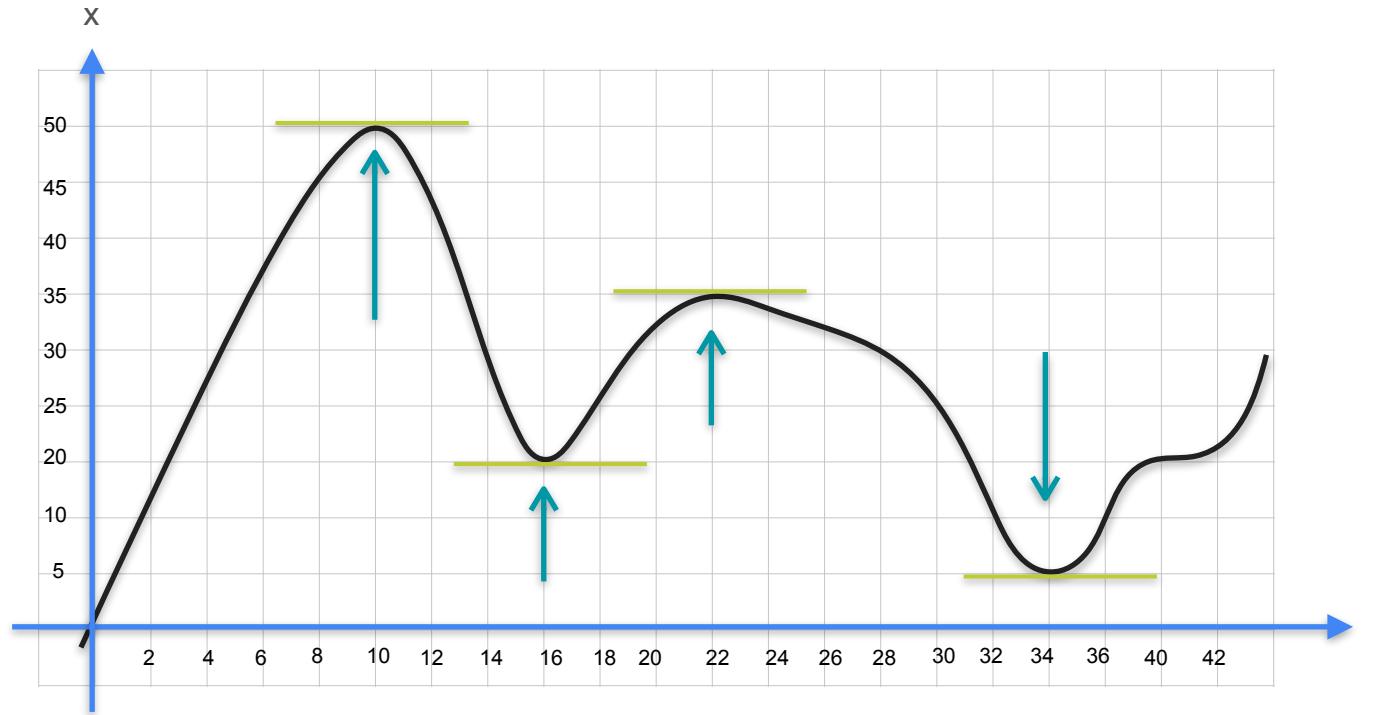
Quiz : Minima and Maxima 1 - Solution



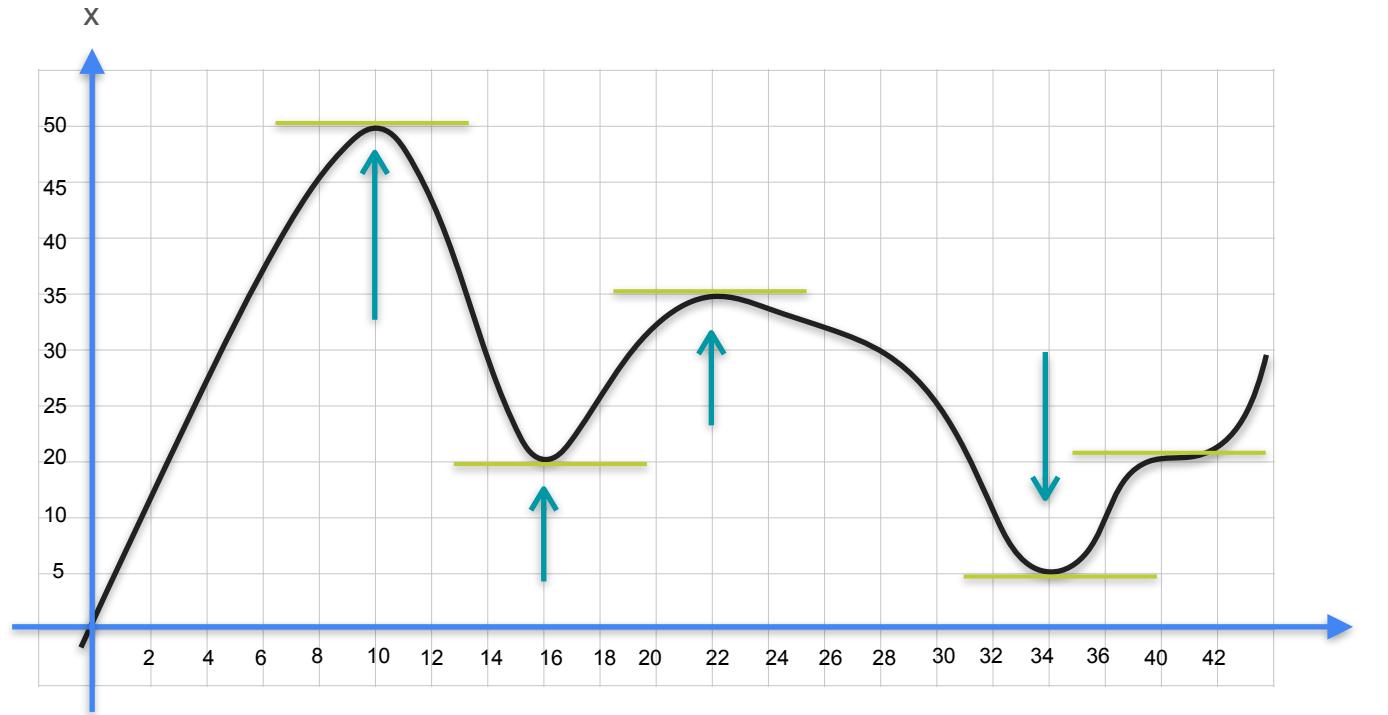
Quiz : Minima and Maxima 1 - Solution



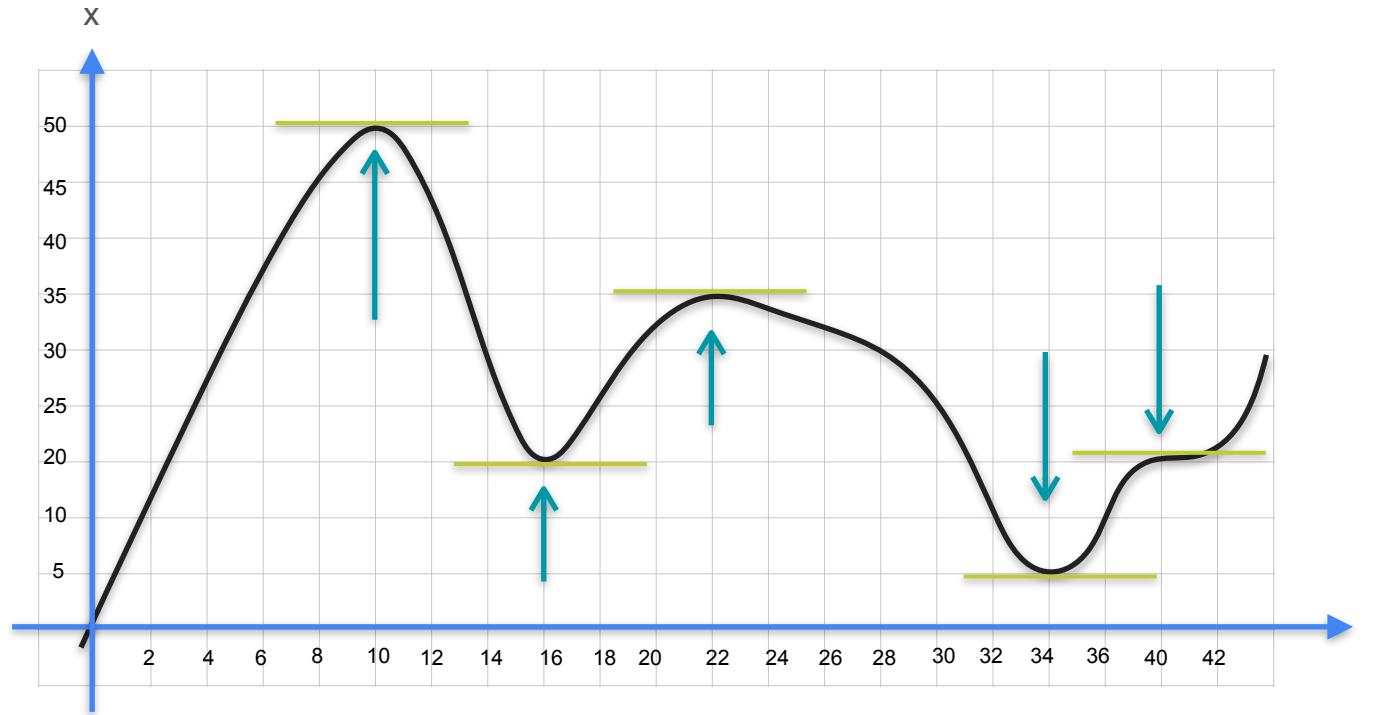
Quiz : Minima and Maxima 1 - Solution



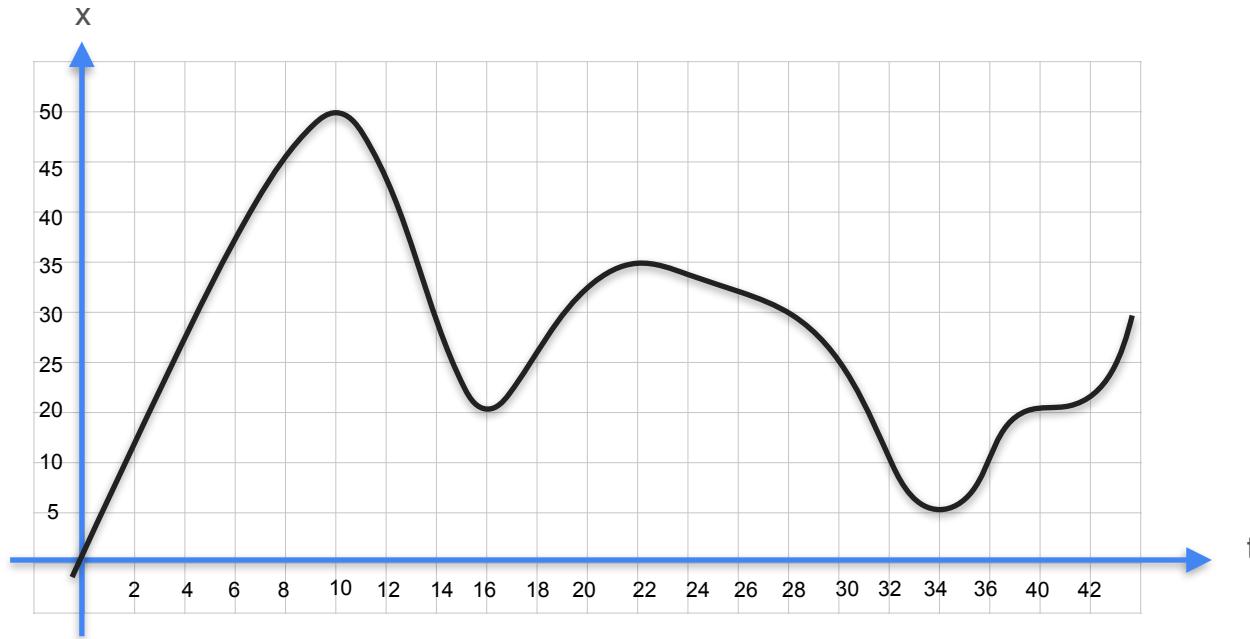
Quiz : Minima and Maxima 1 - Solution



Quiz : Minima and Maxima 1 - Solution

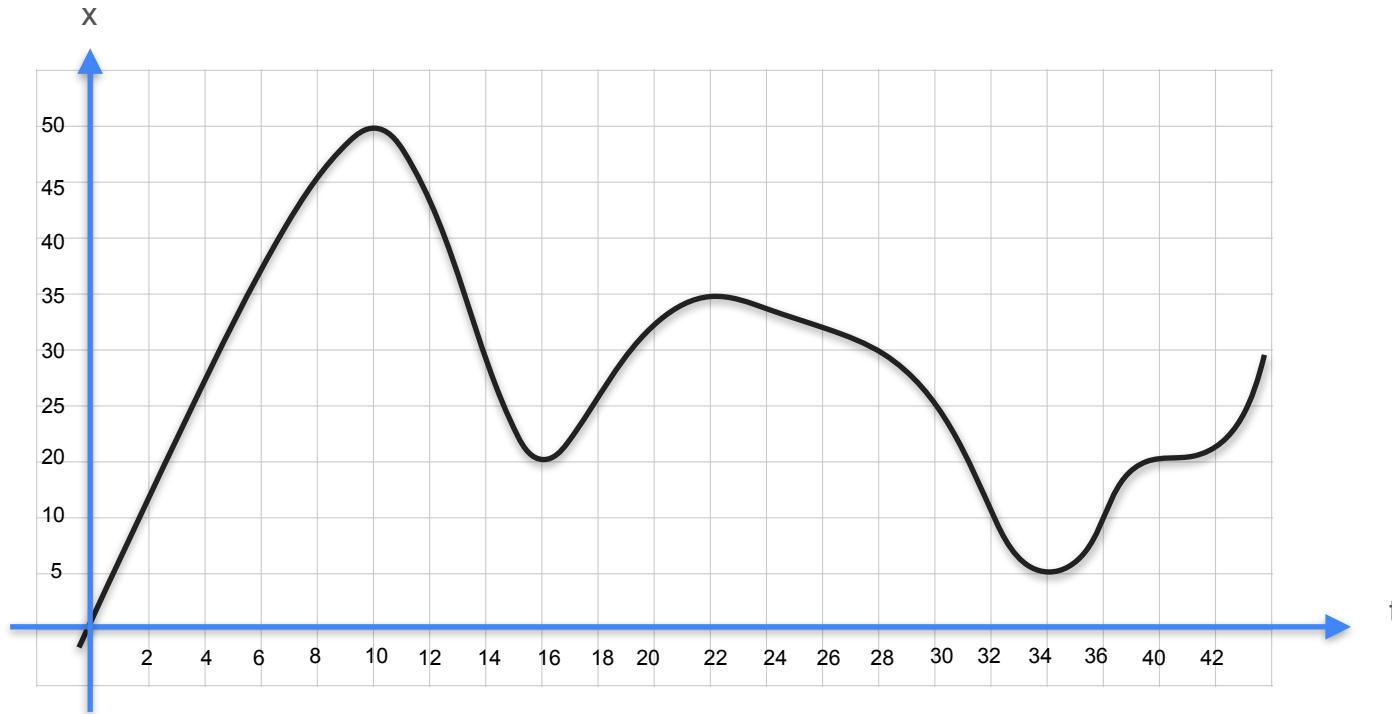


Quiz : Concept of Derivatives 3

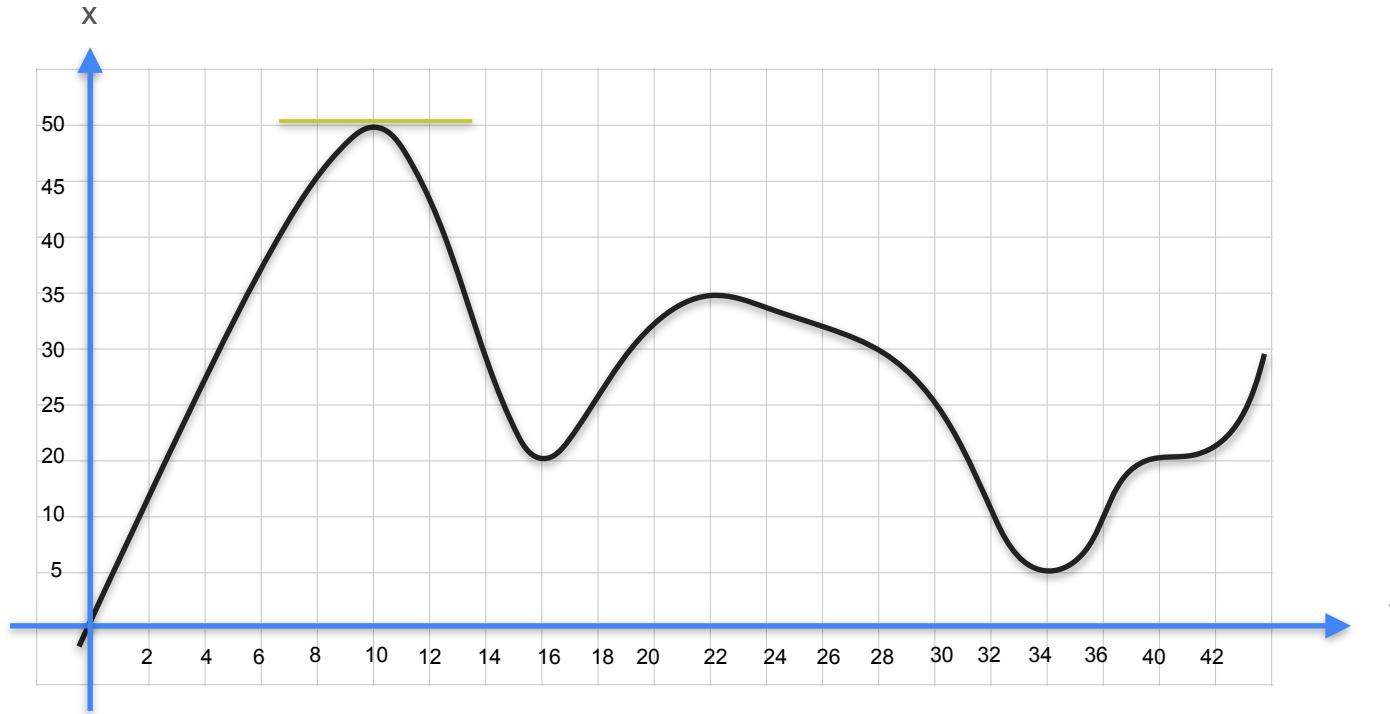


At what time was the car farthest from its starting point?

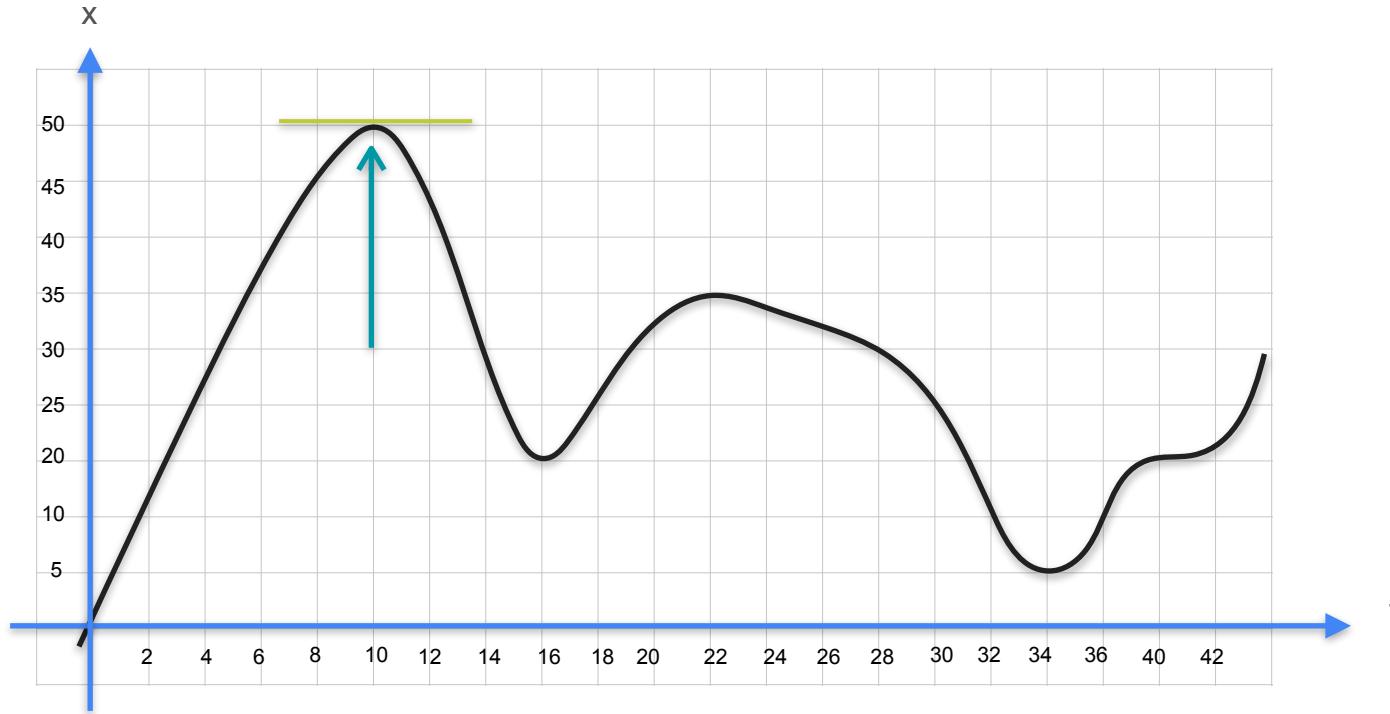
Quiz : Maxima & Minima - Solution



Quiz : Maxima & Minima - Solution



Quiz : Maxima & Minima - Solution



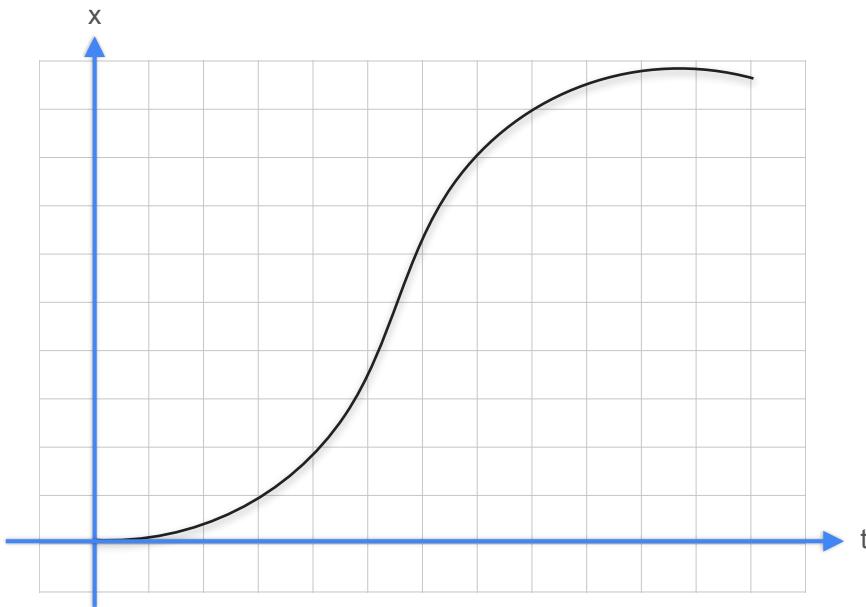


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Derivatives and Optimization

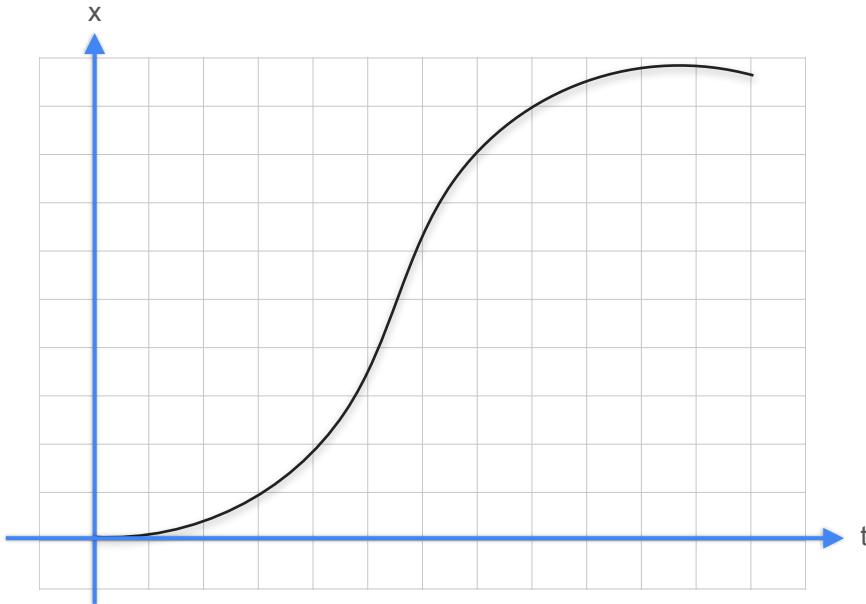
Derivative notation

Derivatives



Derivatives

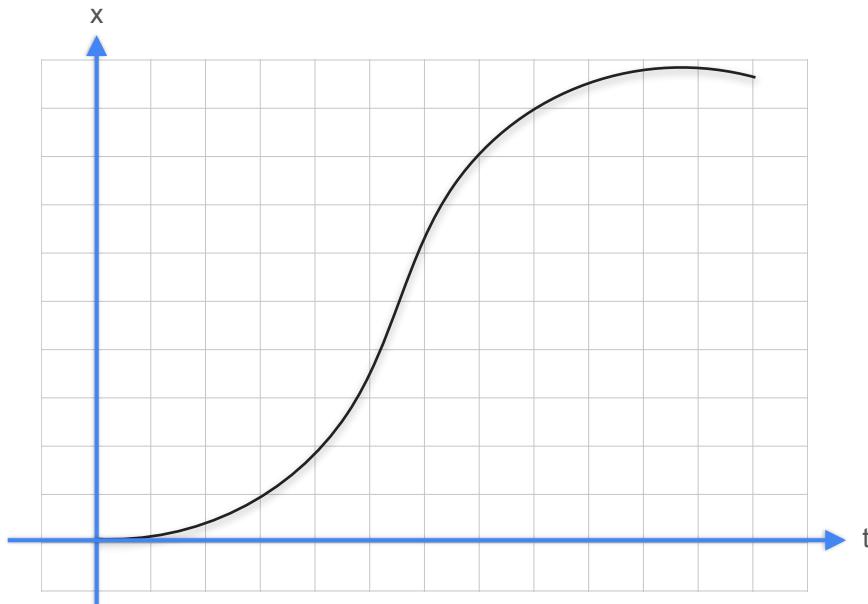
$$\text{slope} = \frac{\text{change in distance}}{\text{change in time}}$$



Derivatives

$$\text{slope} = \frac{\text{change in distance}}{\text{change in time}}$$

$$\text{slope} = \frac{\Delta x}{\Delta t}$$



Derivatives

$$\text{slope} = \frac{\text{change in distance}}{\text{change in time}}$$

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope at a point} = \frac{dx}{dt}$$

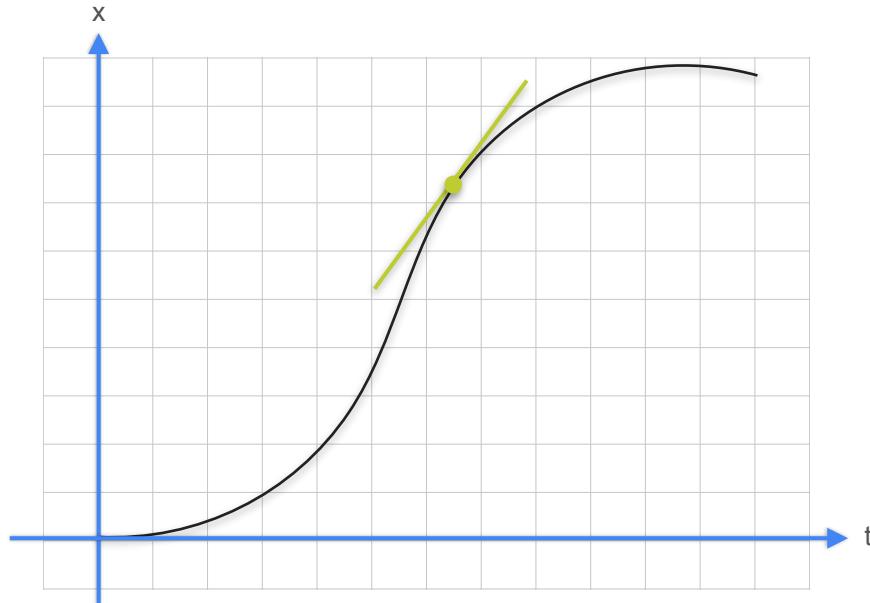


Derivatives

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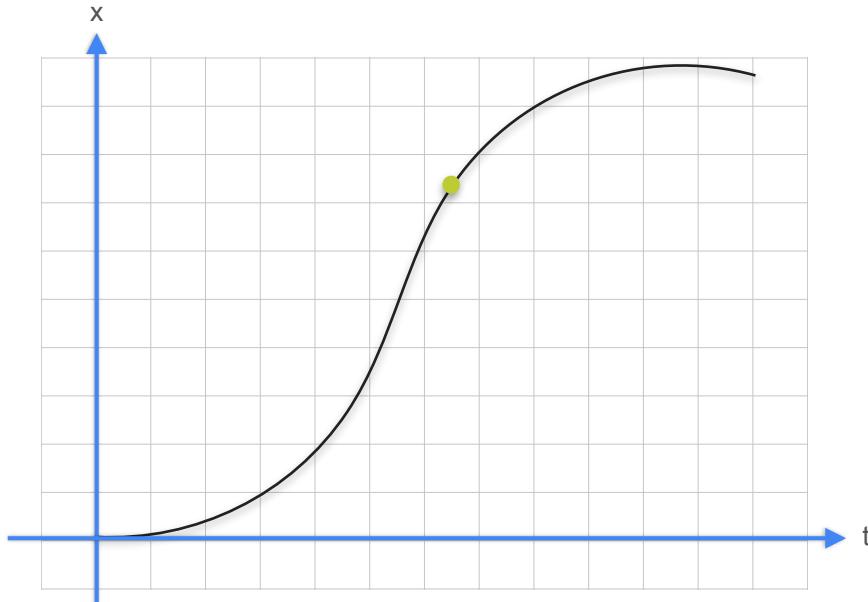


Derivatives

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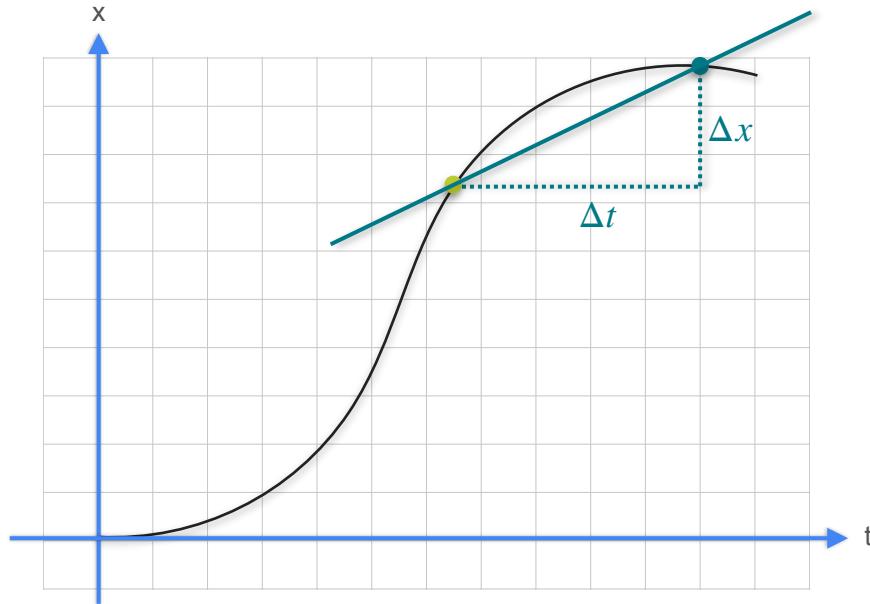


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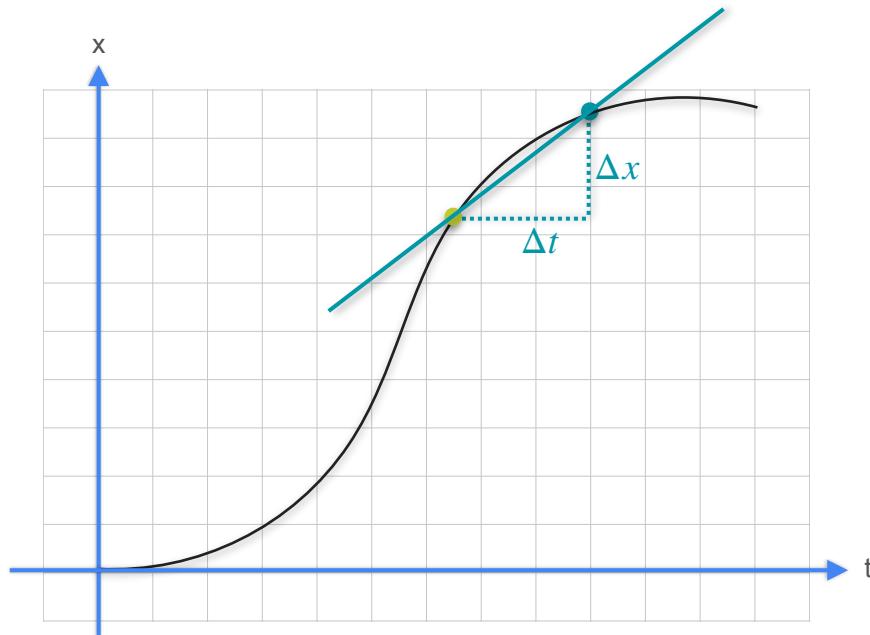


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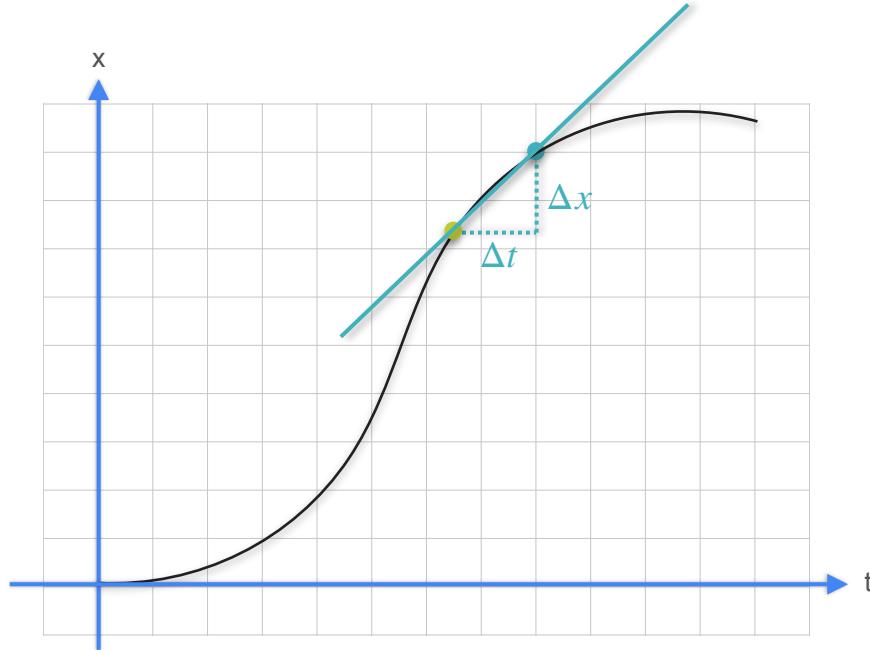


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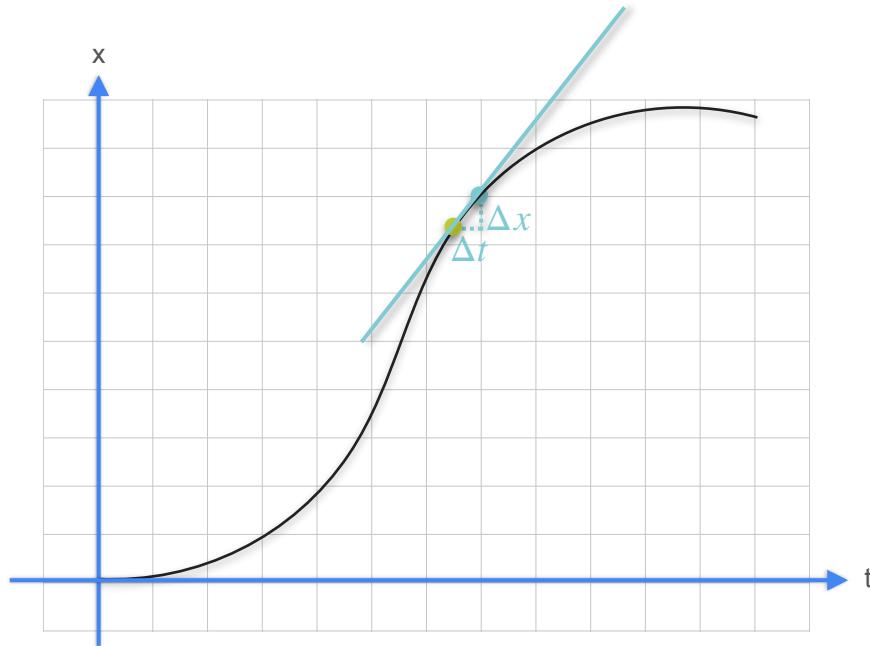


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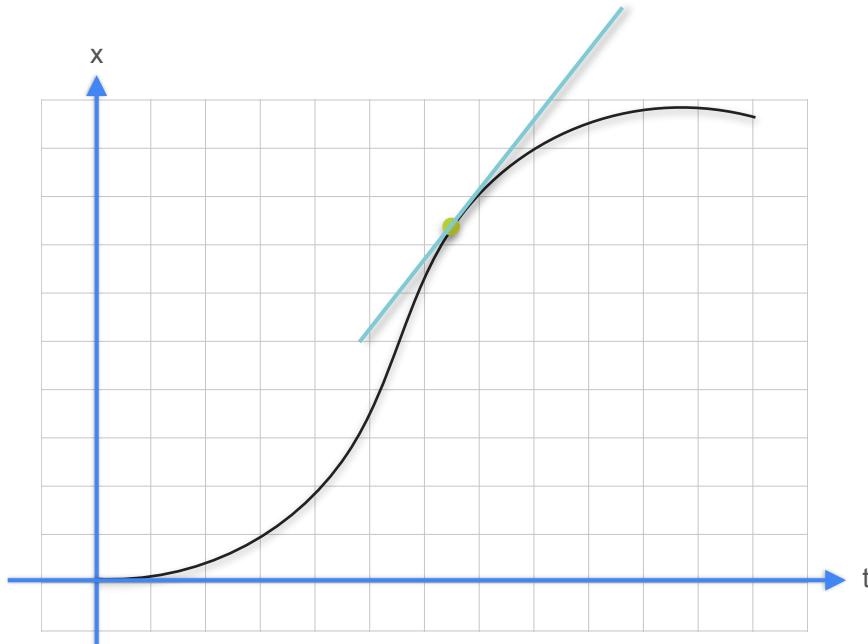


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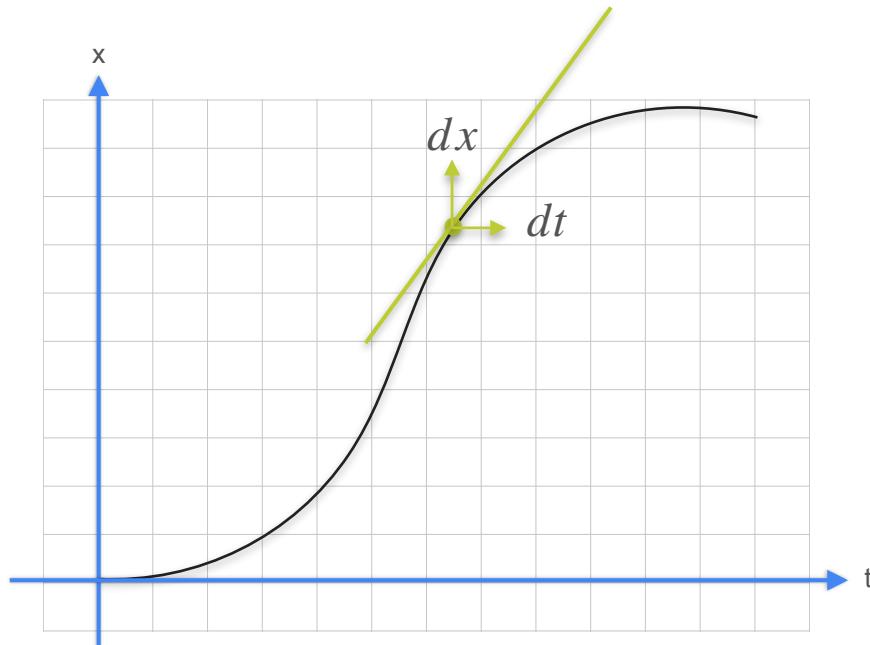


Derivatives

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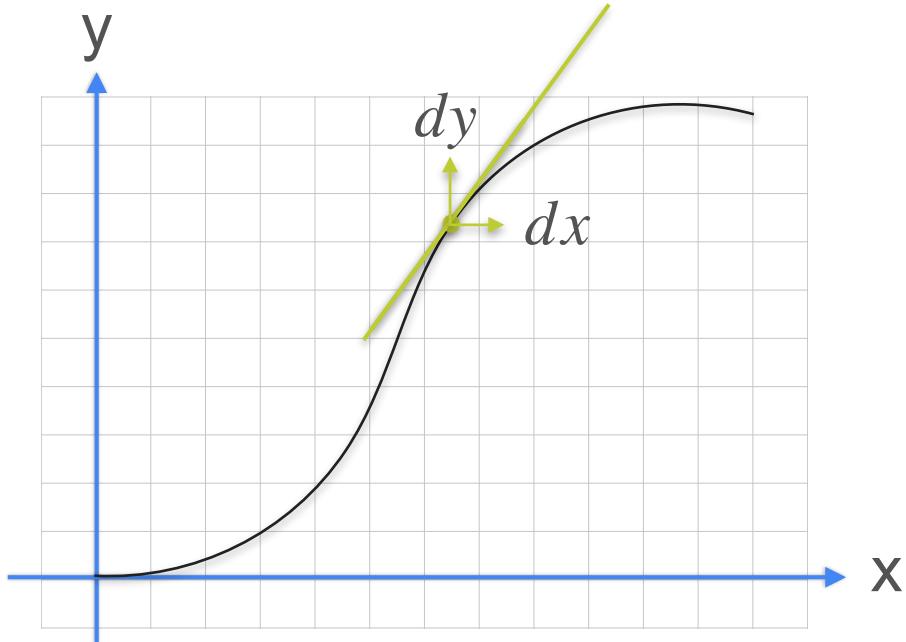


Derivatives

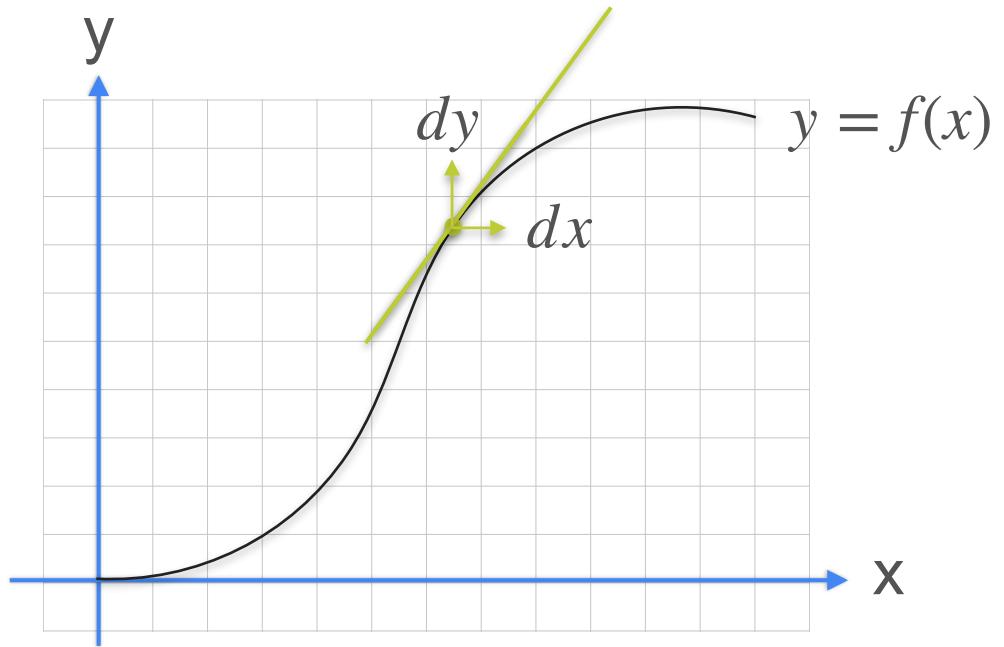
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\text{slope at a point} = \frac{dy}{dx}$$

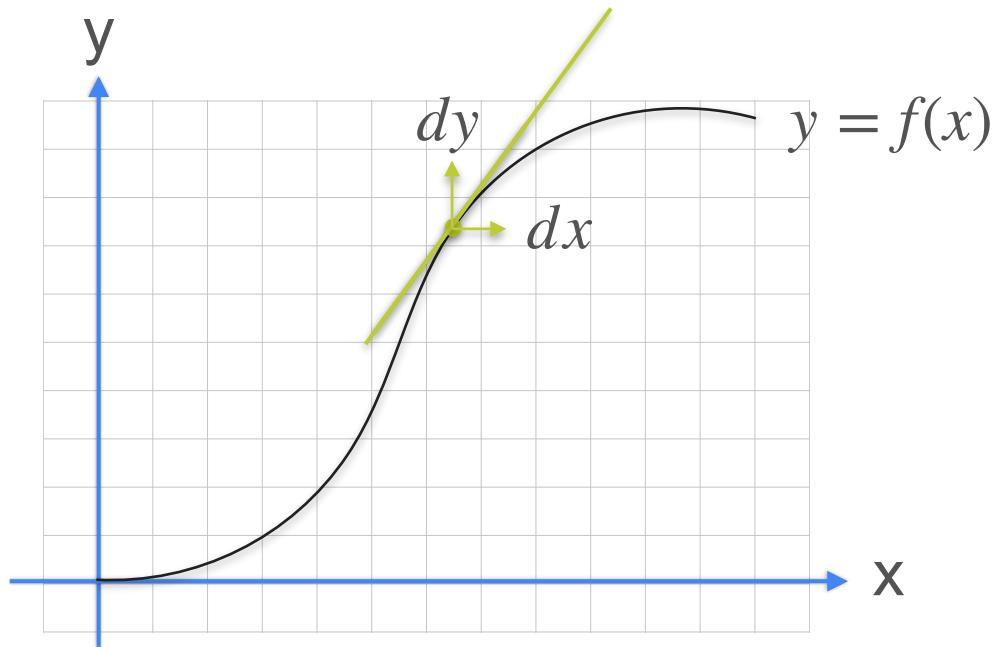


Derivatives



Derivatives

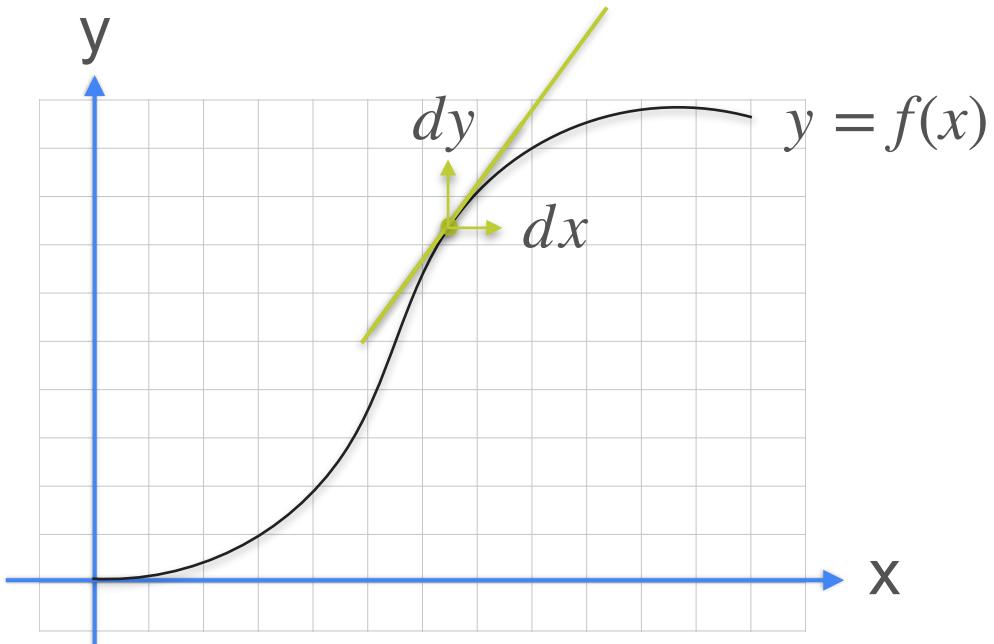
$$y = f(x)$$



Derivatives

$$y = f(x)$$

Derivative of f is expressed as:

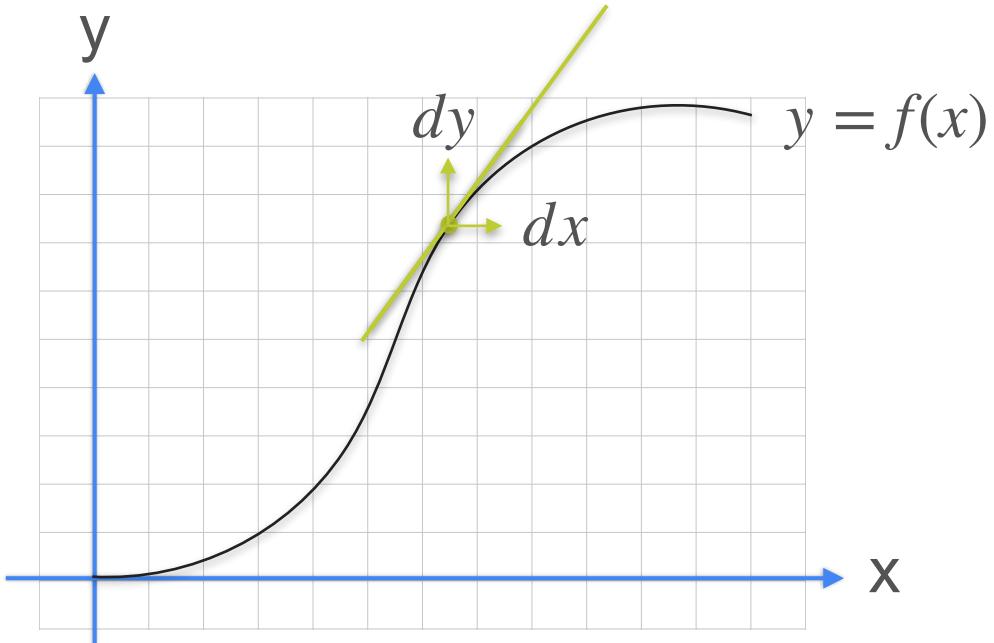


Derivatives

$$y = f(x)$$

Derivative of f is expressed as:

$$\frac{d}{dx}f(x)$$

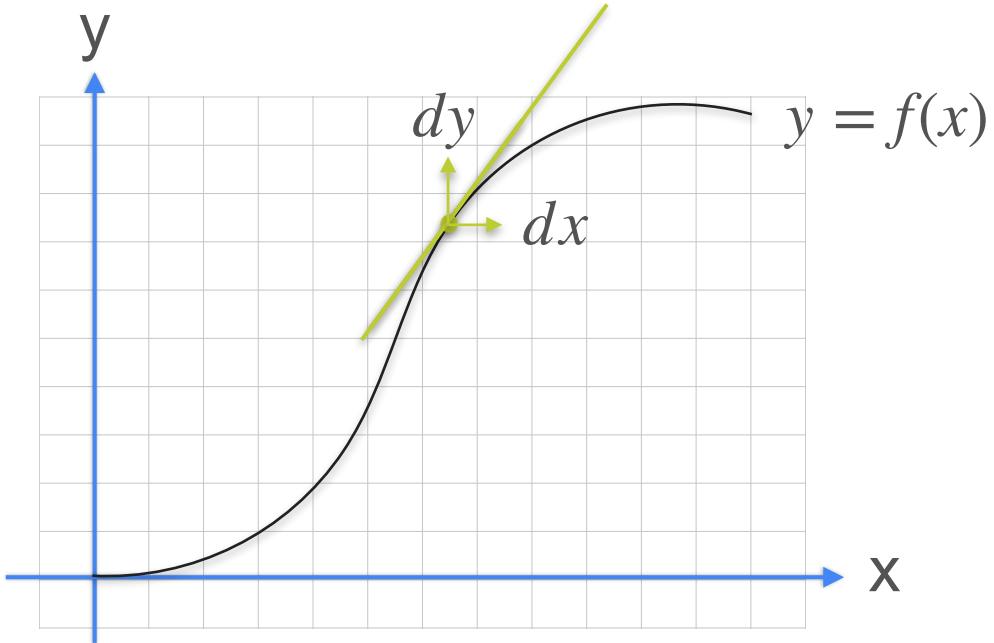


Derivatives

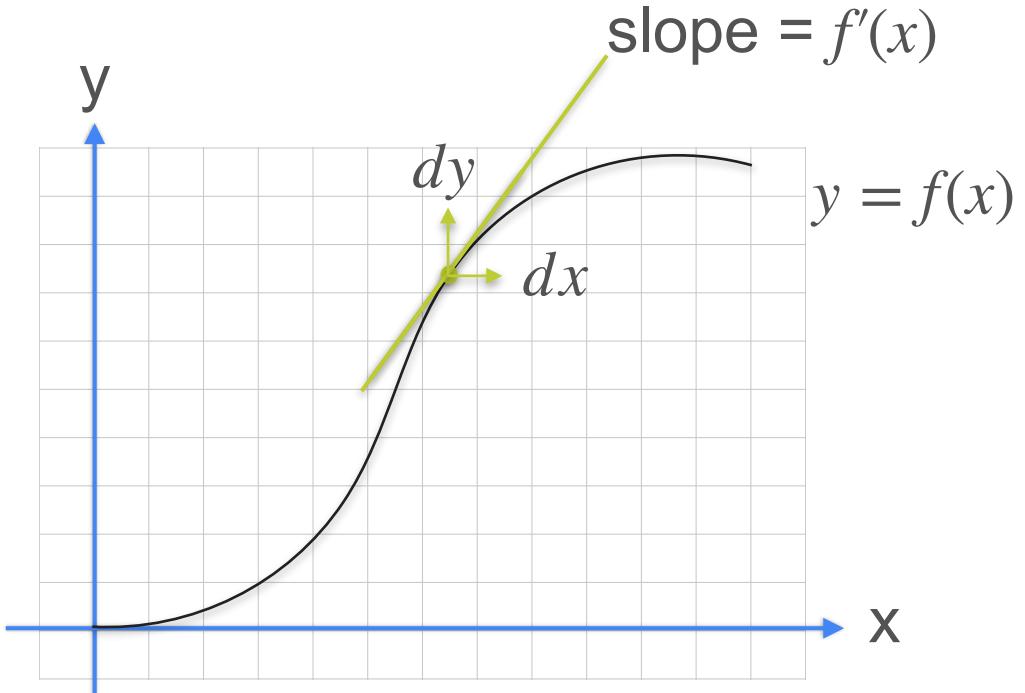
$$y = f(x)$$

Derivative of f is expressed as:

$$\frac{d}{dx}f(x) = \frac{dy}{dx}$$

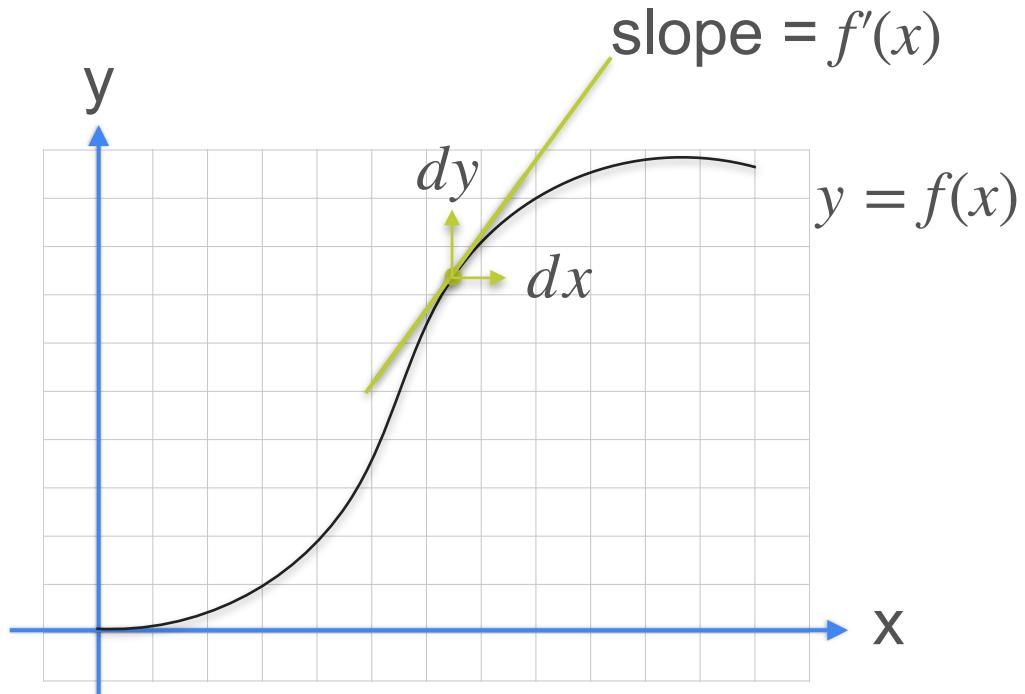


Derivatives: Lagrange's and Leibniz's Notation



Derivatives: Lagrange's and Leibniz's Notation

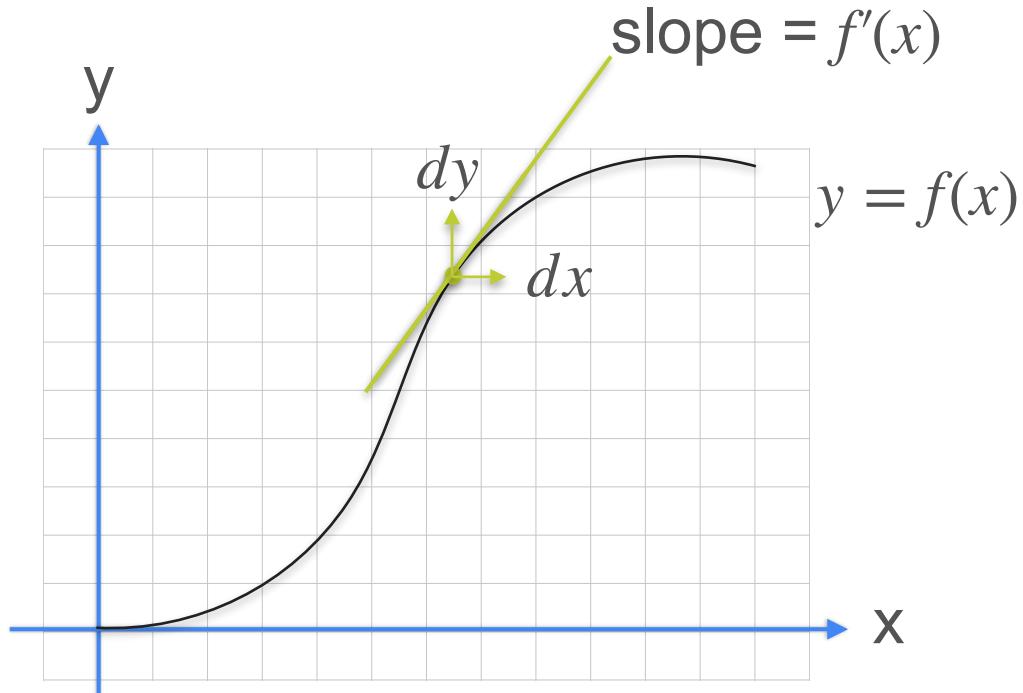
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Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

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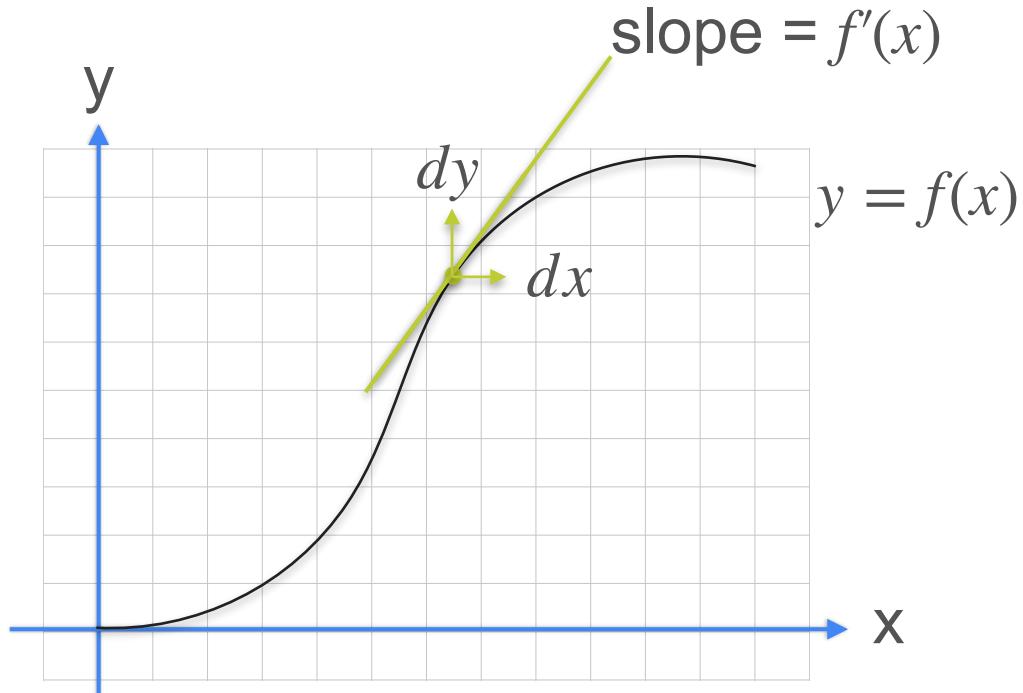


Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of f is expressed as:

$$f'(x)$$

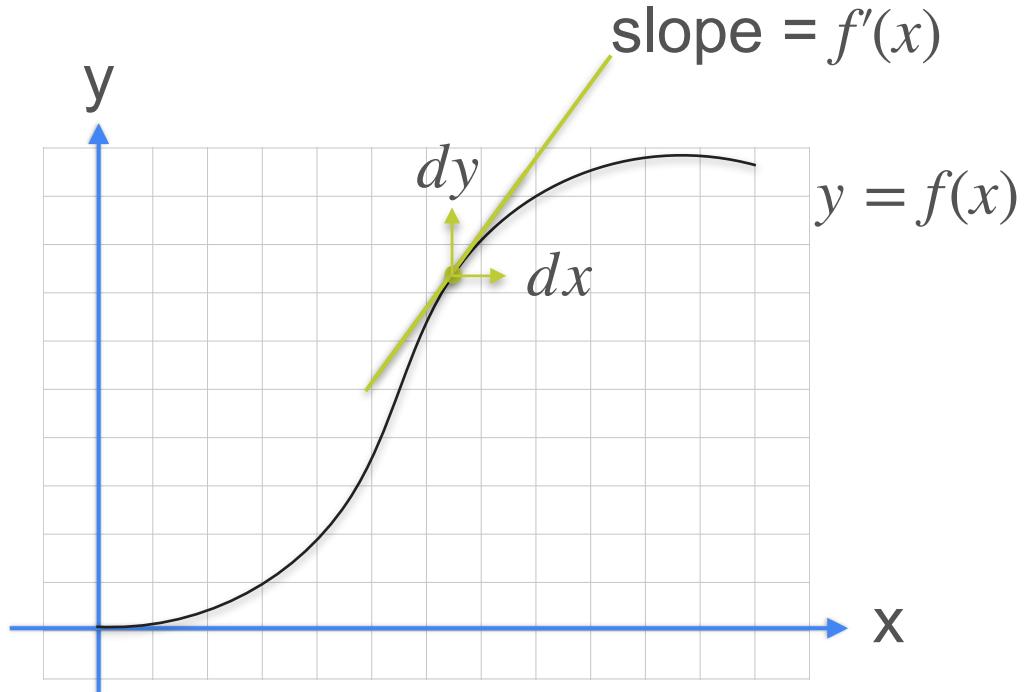


Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of f is expressed as:

$f'(x)$ **Lagrange's notation**



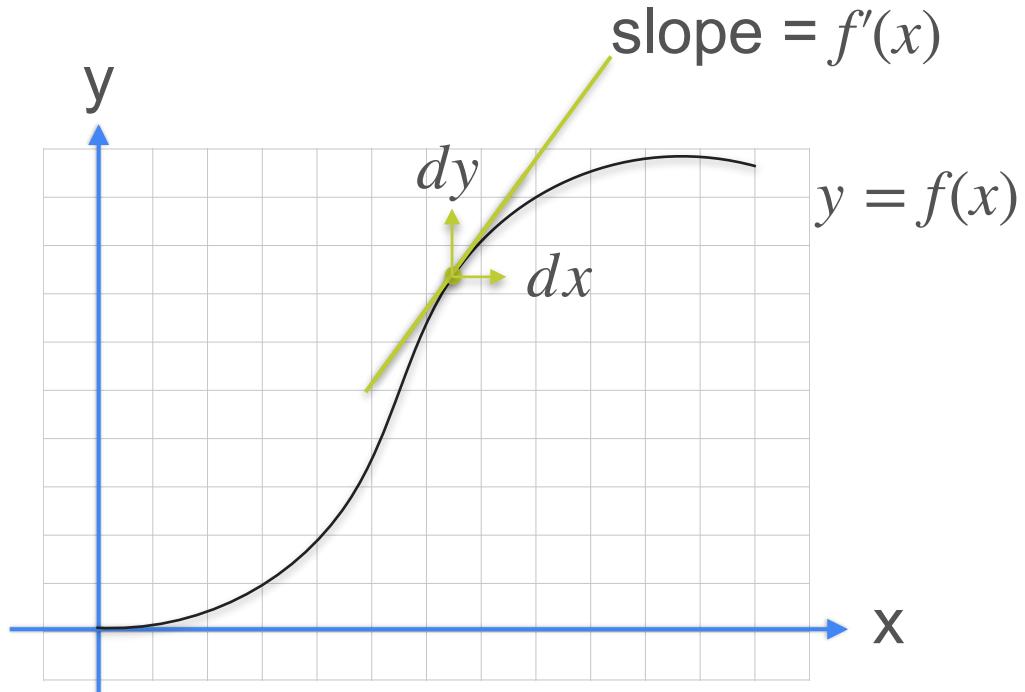
Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of f is expressed as:

$f'(x)$ **Lagrange's notation**

$$\frac{dy}{dx}$$



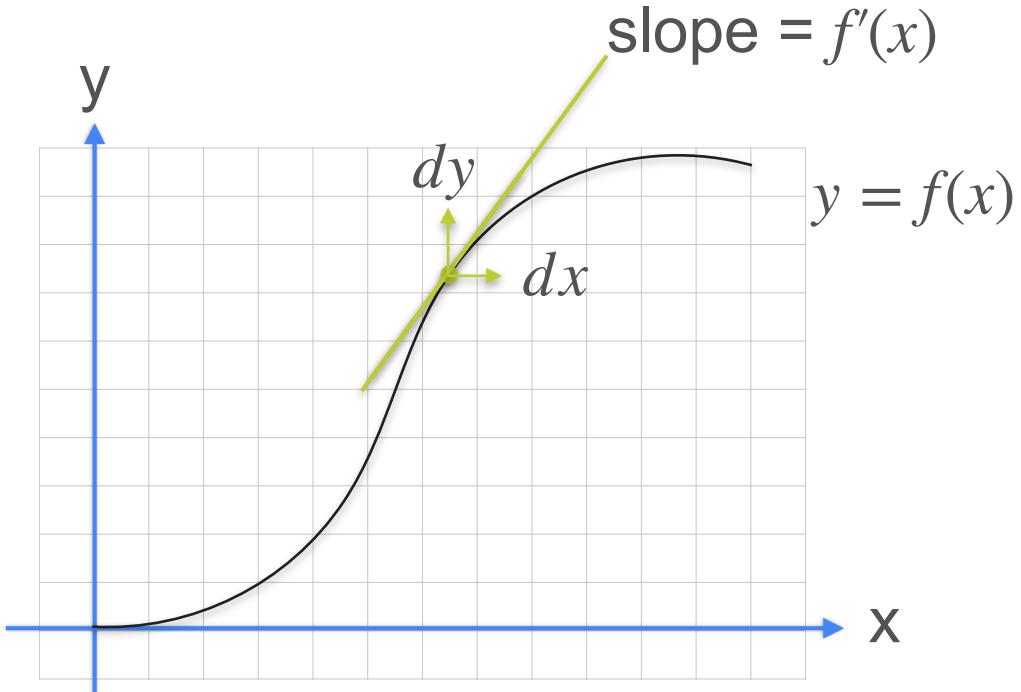
Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of f is expressed as:

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$$\frac{dy}{dx} = \frac{d}{dx}f(x)$$



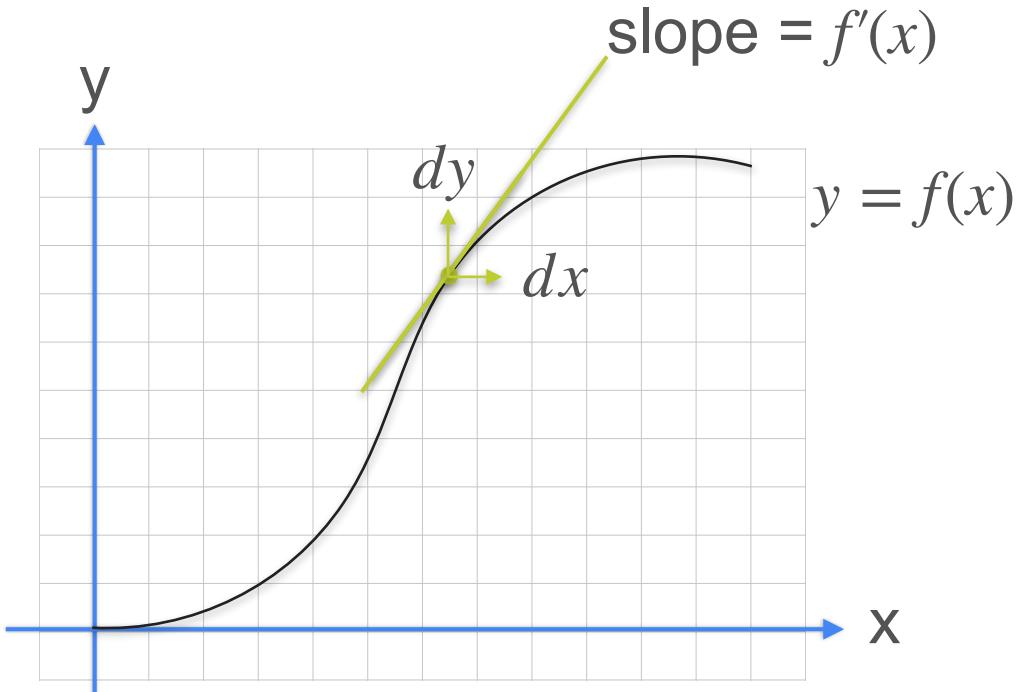
Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of f is expressed as:

$f'(x)$ **Lagrange's notation**

$\frac{dy}{dx} = \frac{d}{dx}f(x)$ **Leibniz's notation**





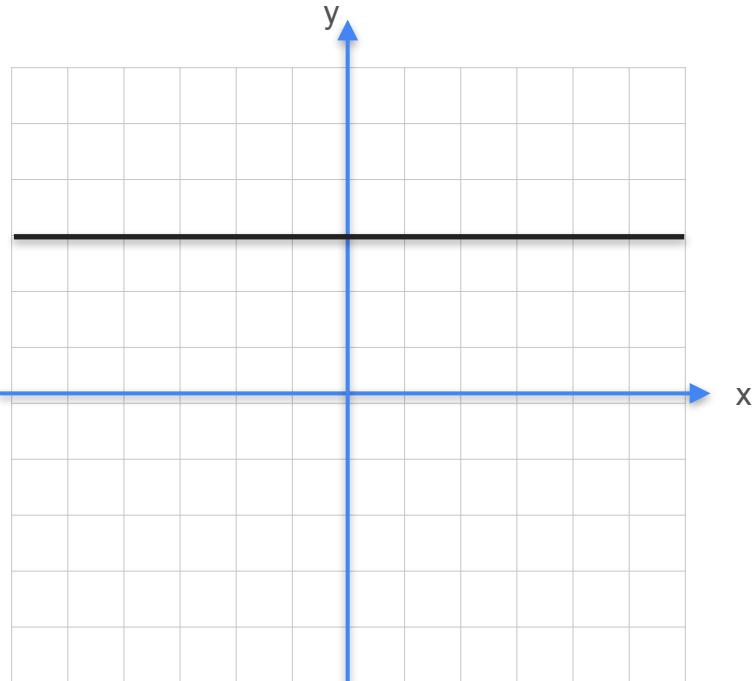
DeepLearning.AI

Derivatives and Optimization

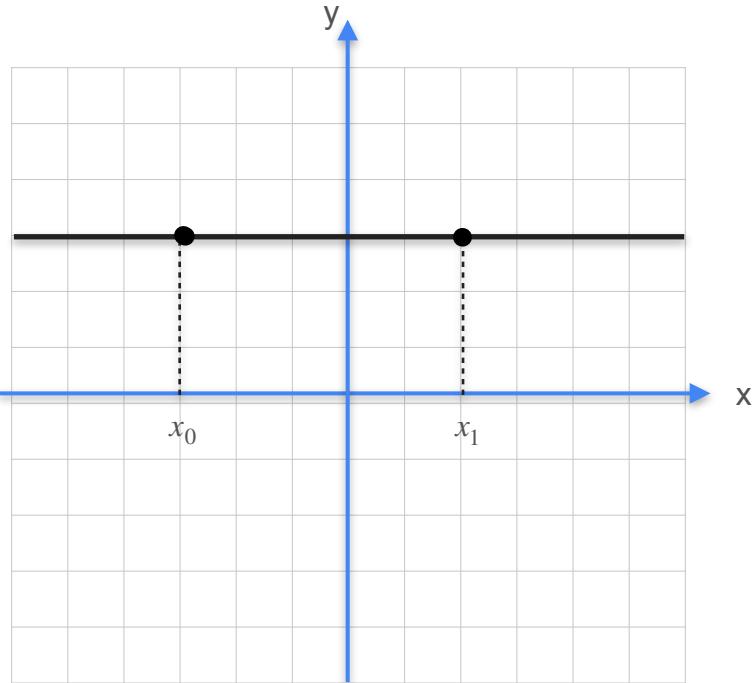
**Some common derivatives:
Lines**

Derivative of a Constant

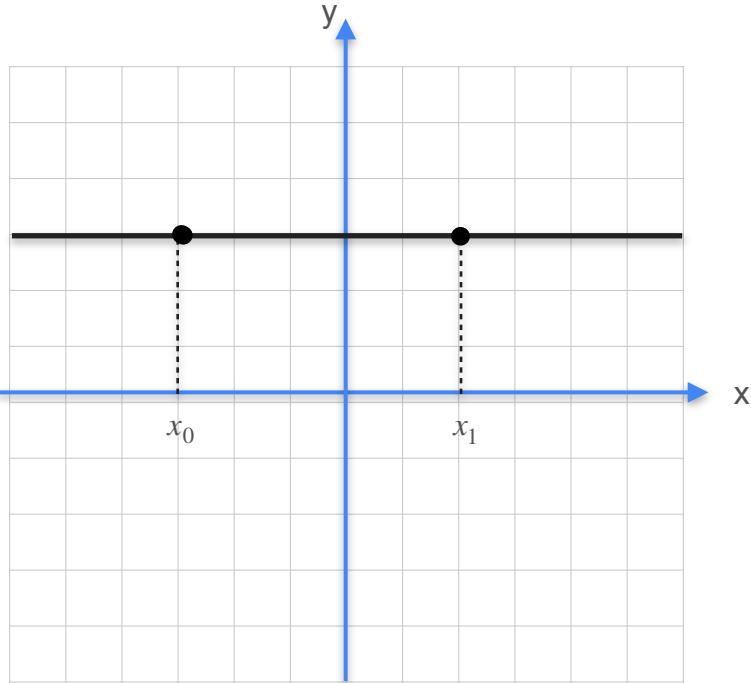
Derivative of a Constant



Derivative of a Constant

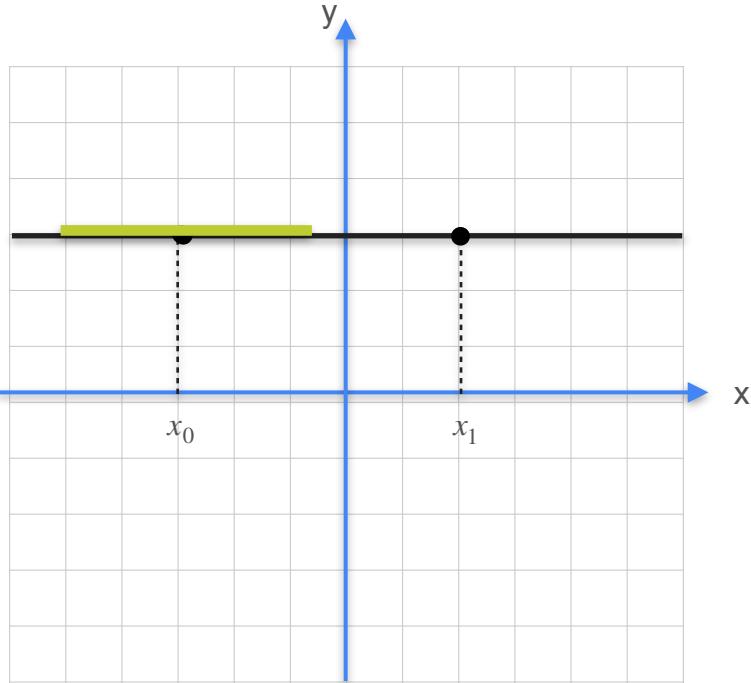


Derivative of a Constant



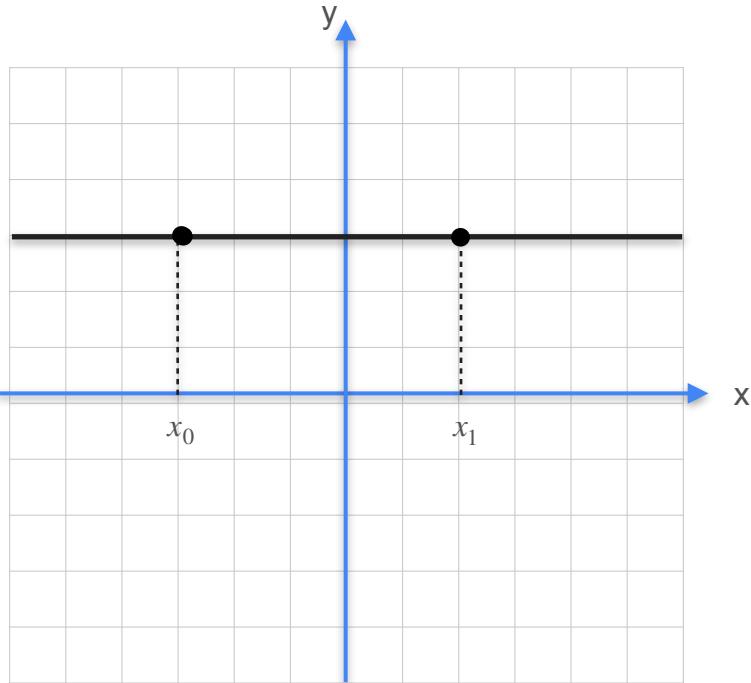
$$y = f(x) = c$$

Derivative of a Constant



$$y = f(x) = c$$

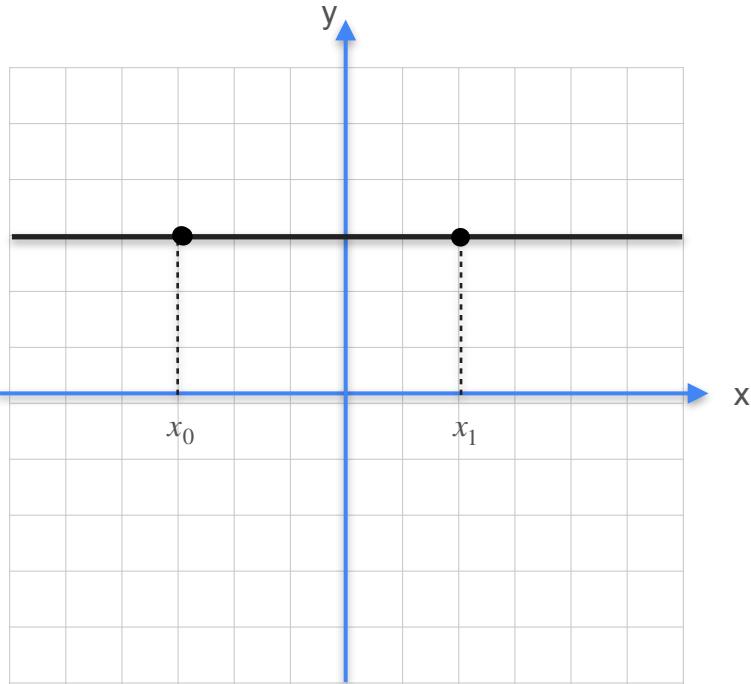
Derivative of a Constant



$$y = f(x) = c$$

Slope?

Derivative of a Constant

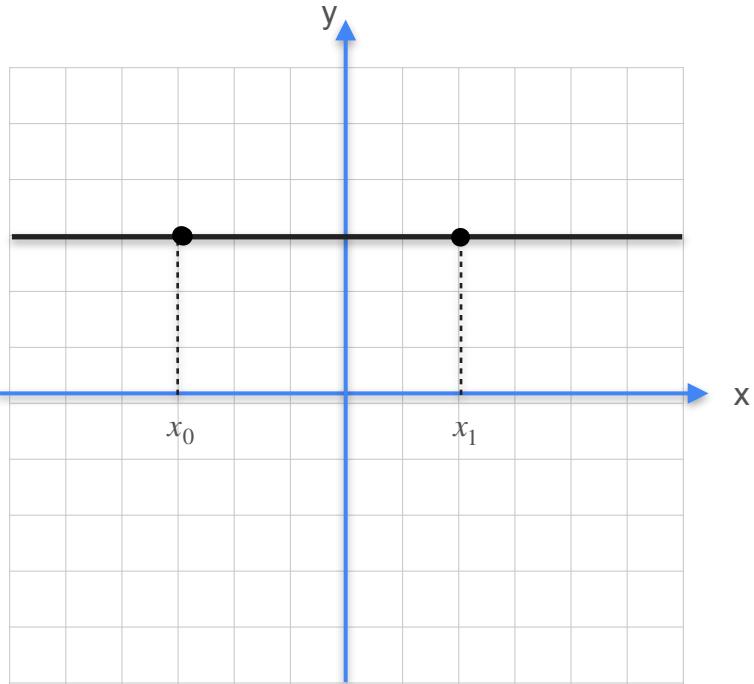


$$y = f(x) = c$$

Slope?

$$\frac{\Delta y}{\Delta x}$$

Derivative of a Constant

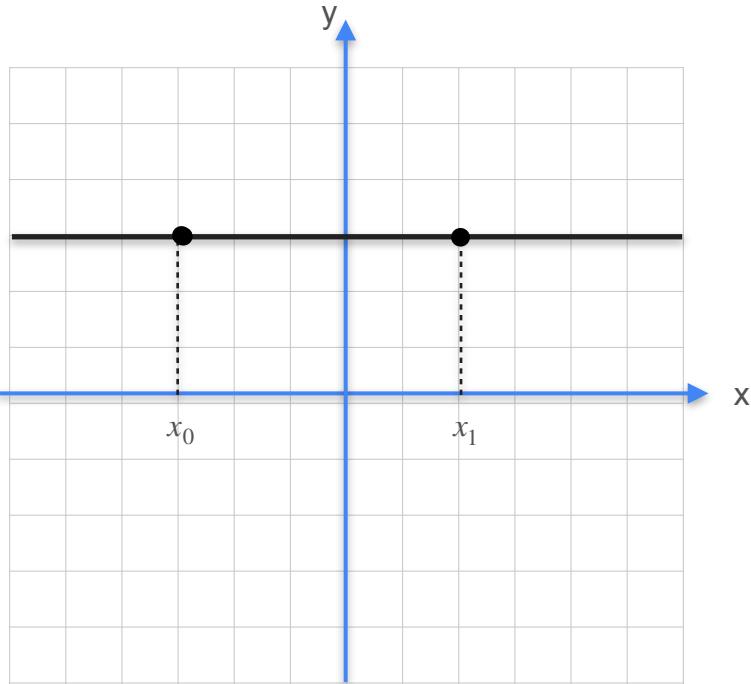


$$y = f(x) = c$$

Slope?

$$\frac{\Delta y}{\Delta x} = \frac{c - c}{x_1 - x_0} = 0$$

Derivative of a Constant



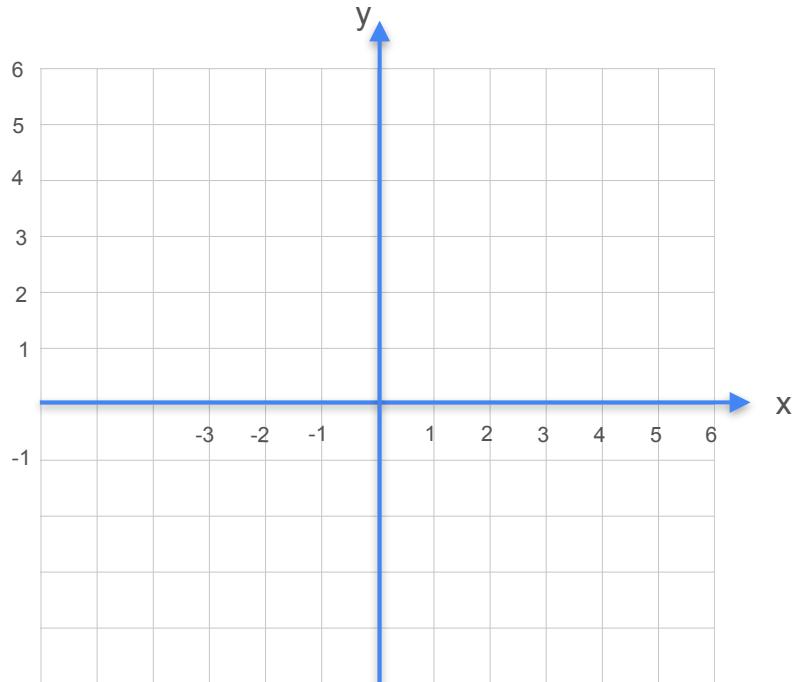
$$y = f(x) = c$$

Slope?

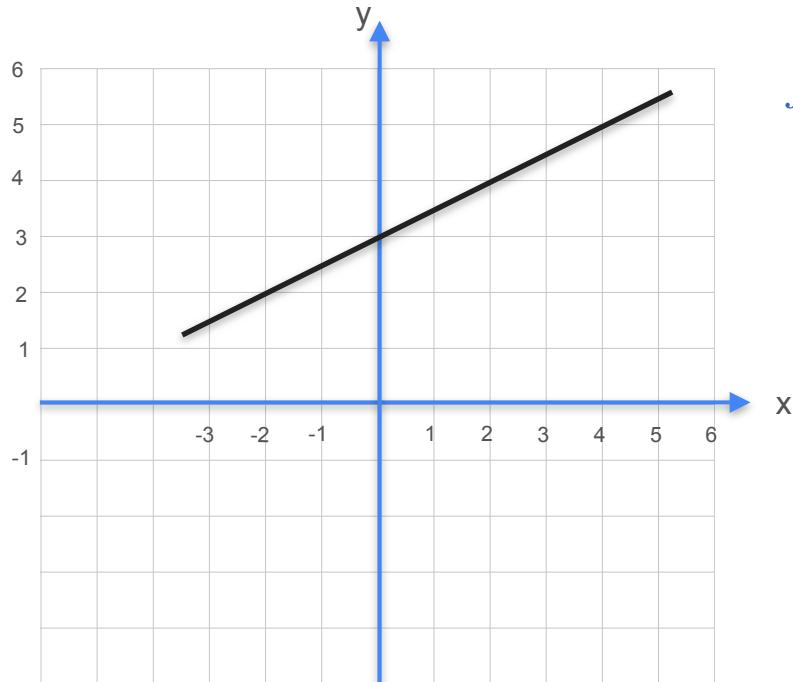
$$\frac{\Delta y}{\Delta x} = \frac{c - c}{x_1 - x_0} = 0 \rightarrow f'(x) = 0$$

Derivative of a Line

Derivative of a Line

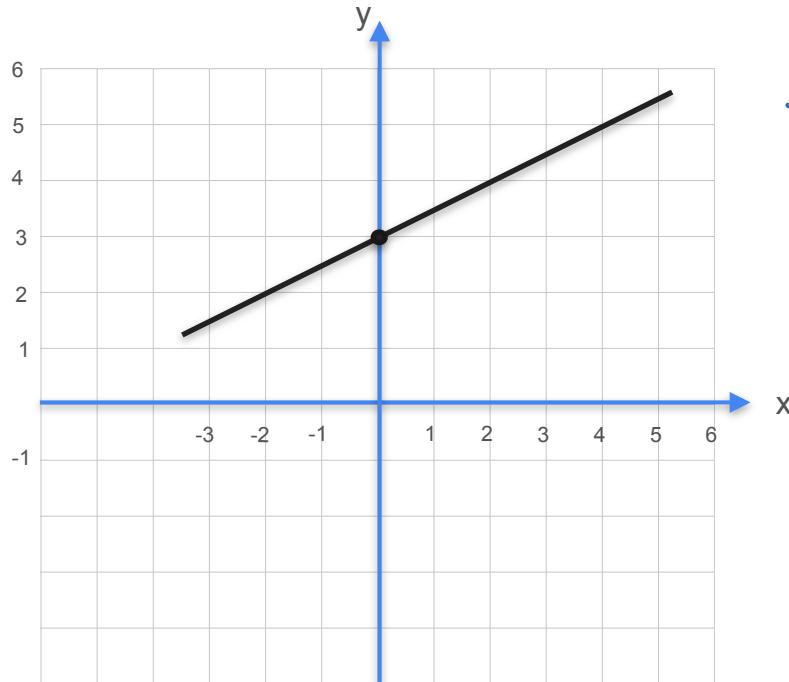


Derivative of a Line



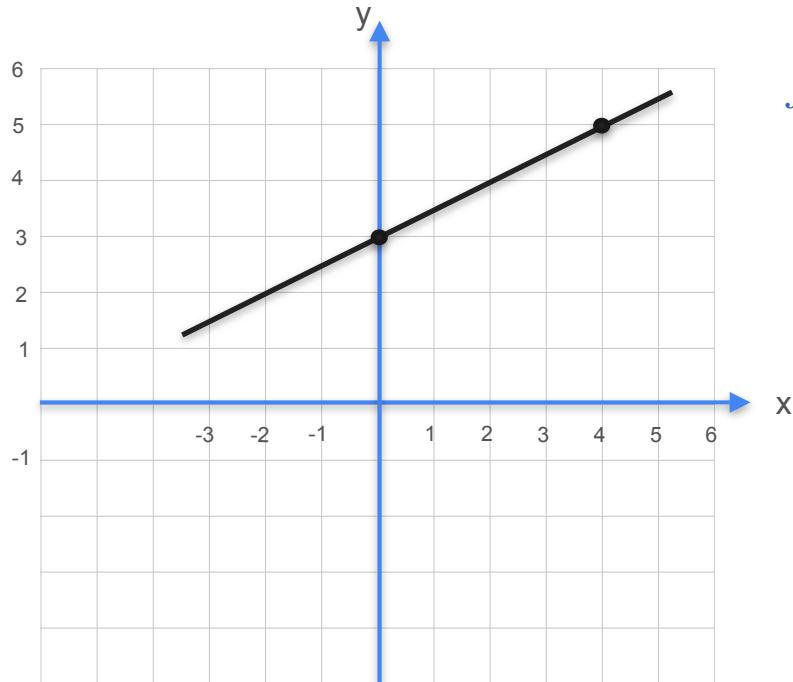
$$f(x) = ax + b$$

Derivative of a Line



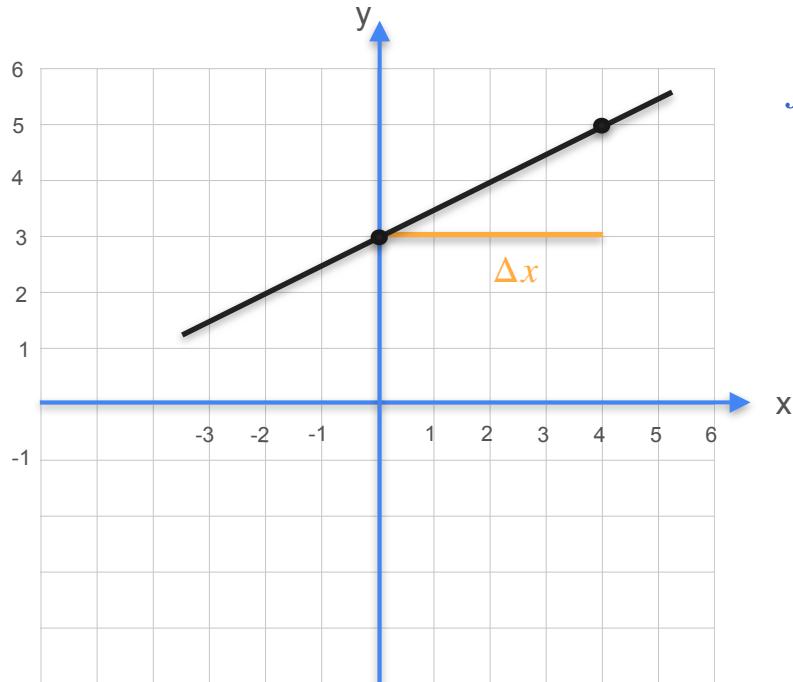
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Derivative of a Line



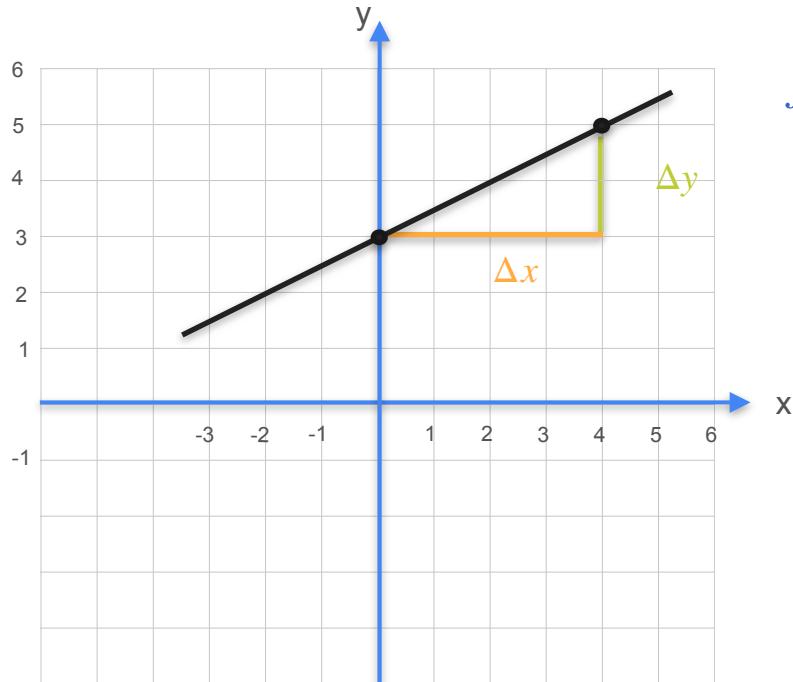
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Derivative of a Line



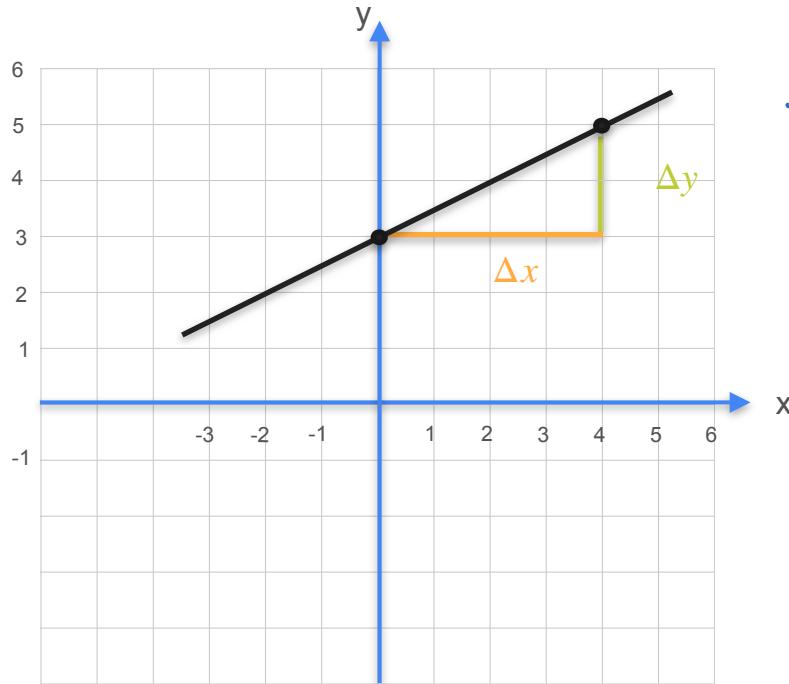
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Derivative of a Line



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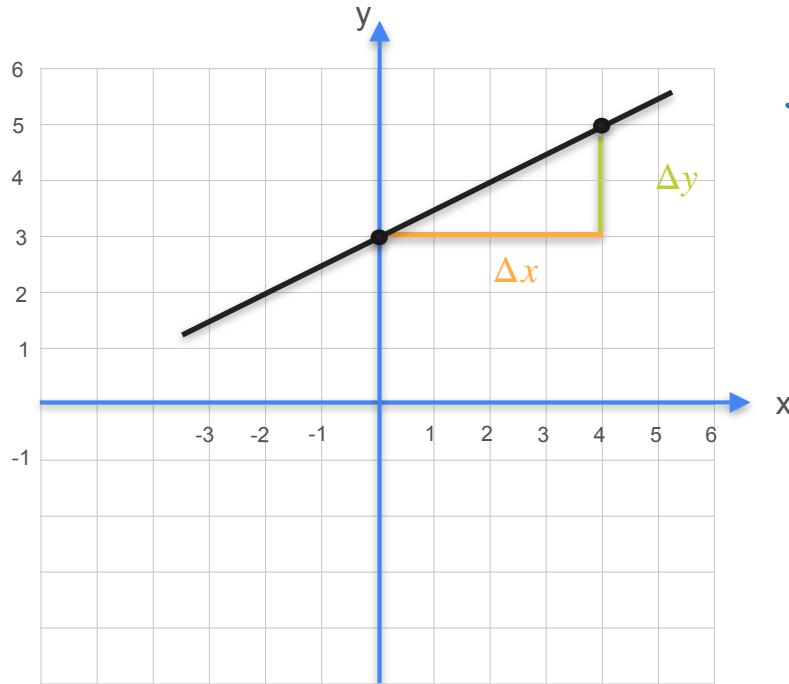
Derivative of a Line



$$f(x) = ax + b$$

$$\frac{\Delta y}{\Delta x}$$

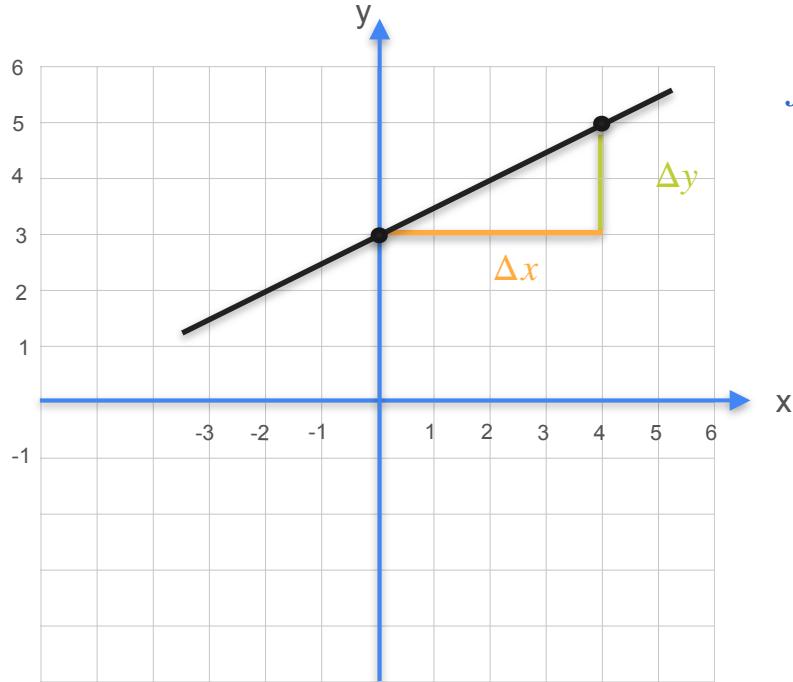
Derivative of a Line



$$f(x) = ax + b$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

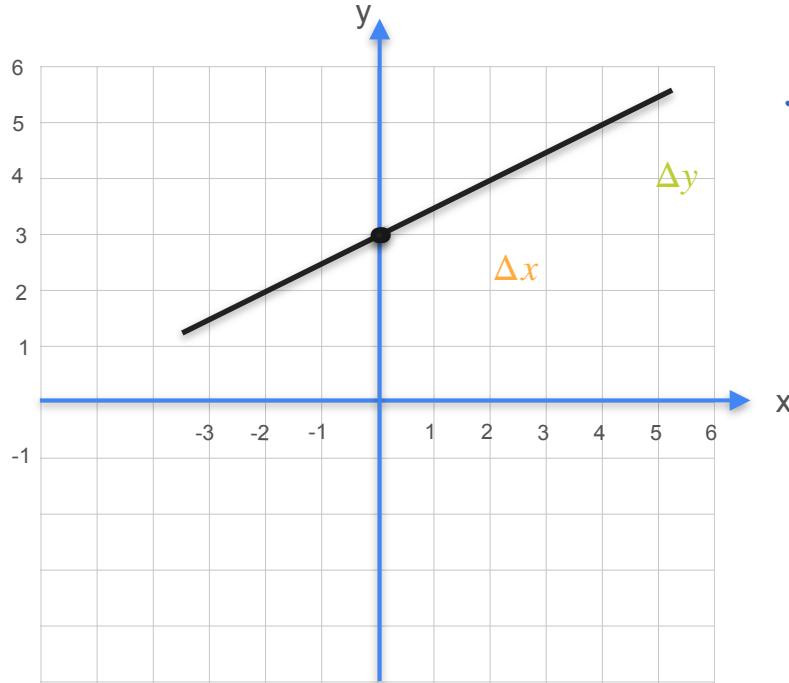
Derivative of a Line



$$f(x) = ax + b$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

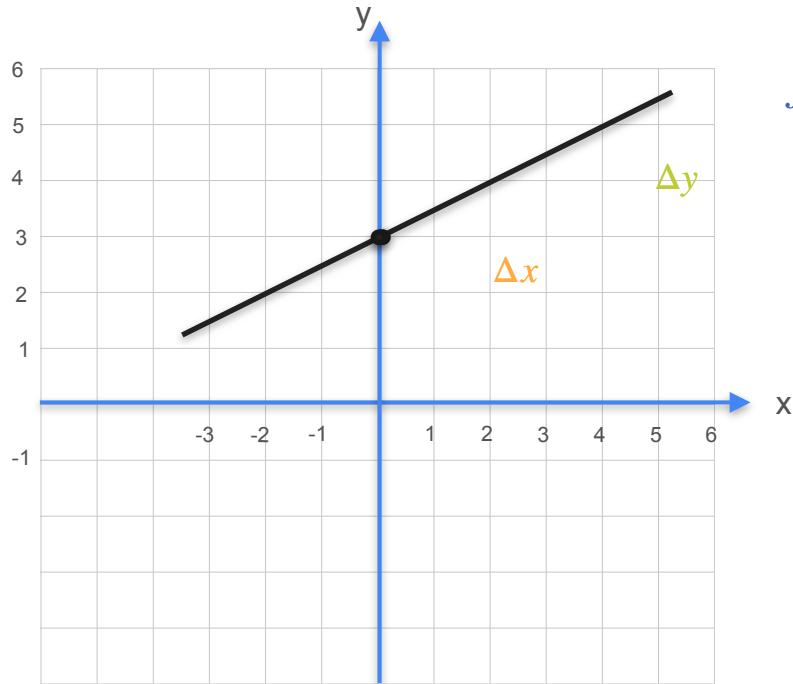
Derivative of a Line



$$f(x) = ax + b$$

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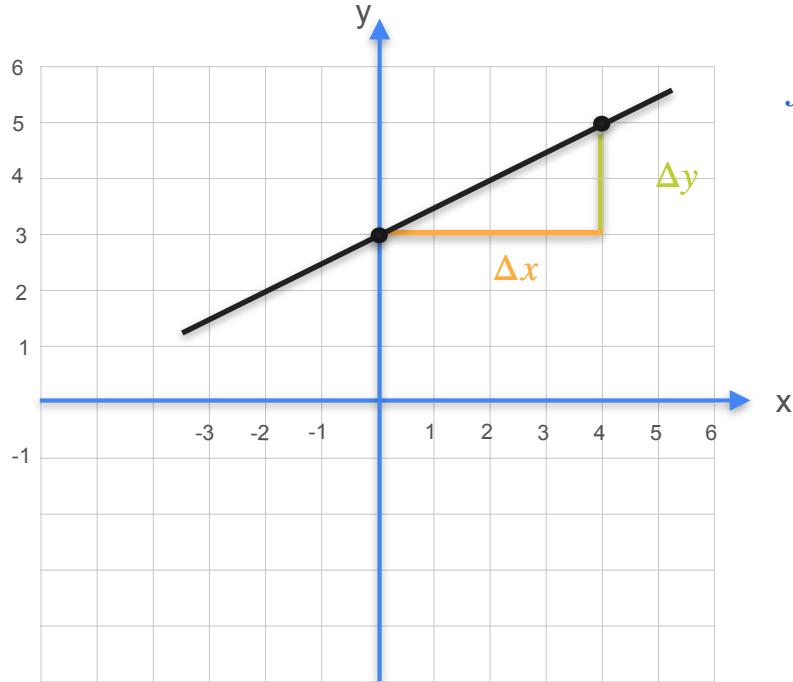
Derivative of a Line



$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

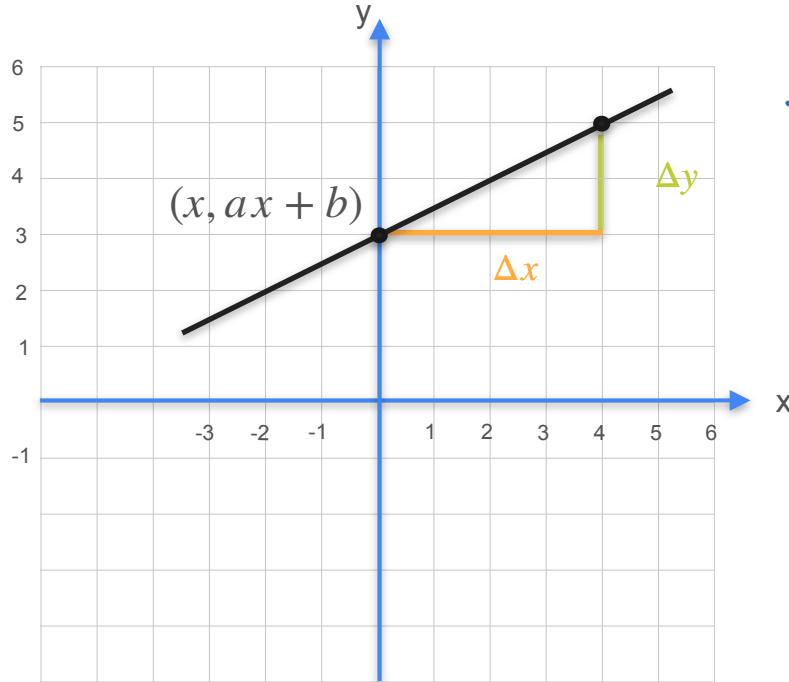
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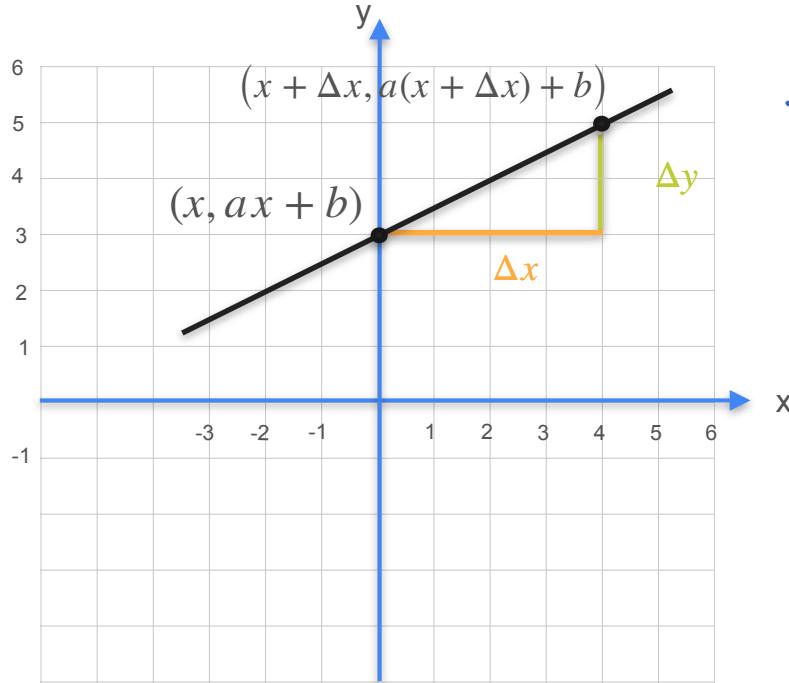
Derivative of a Line



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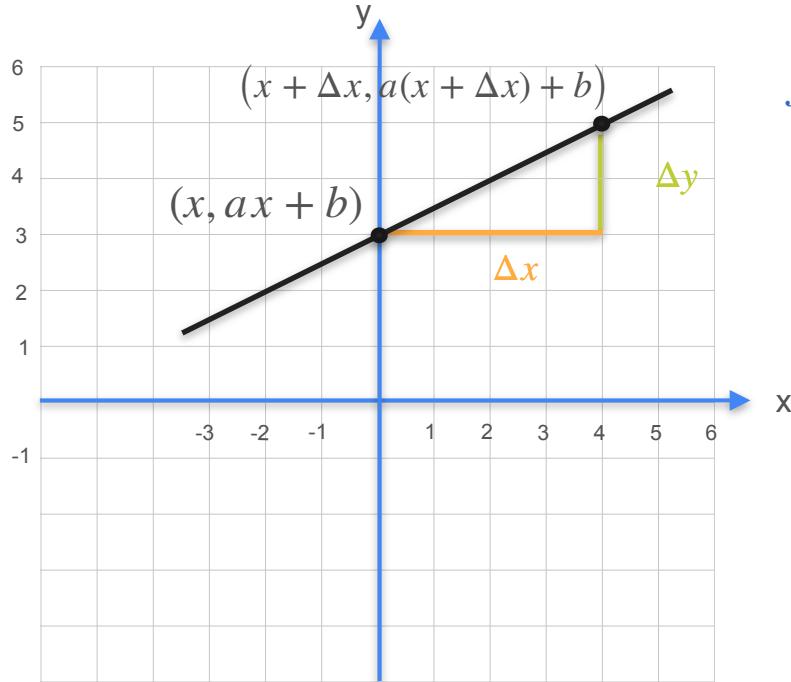
Derivative of a Line



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Derivative of a Line

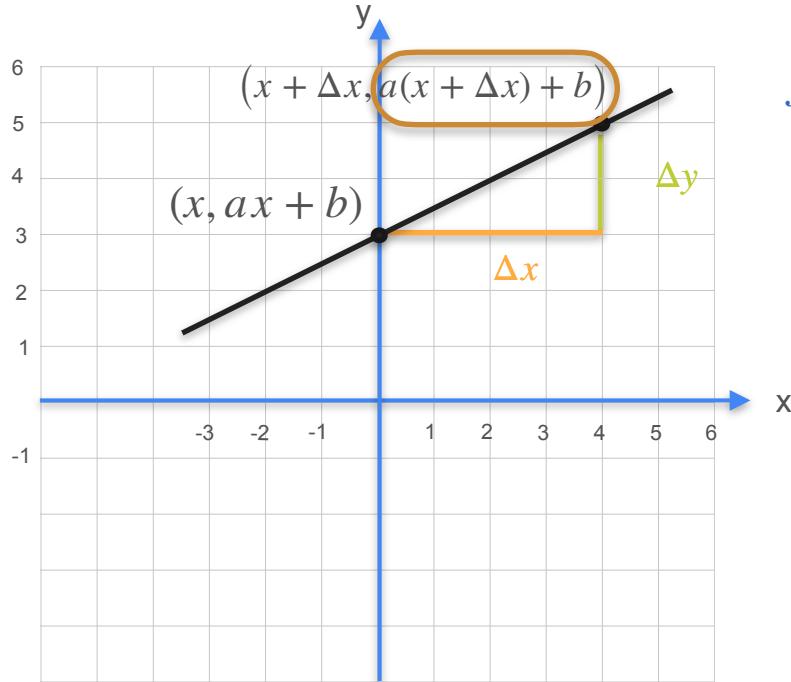


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x}$$

Derivative of a Line

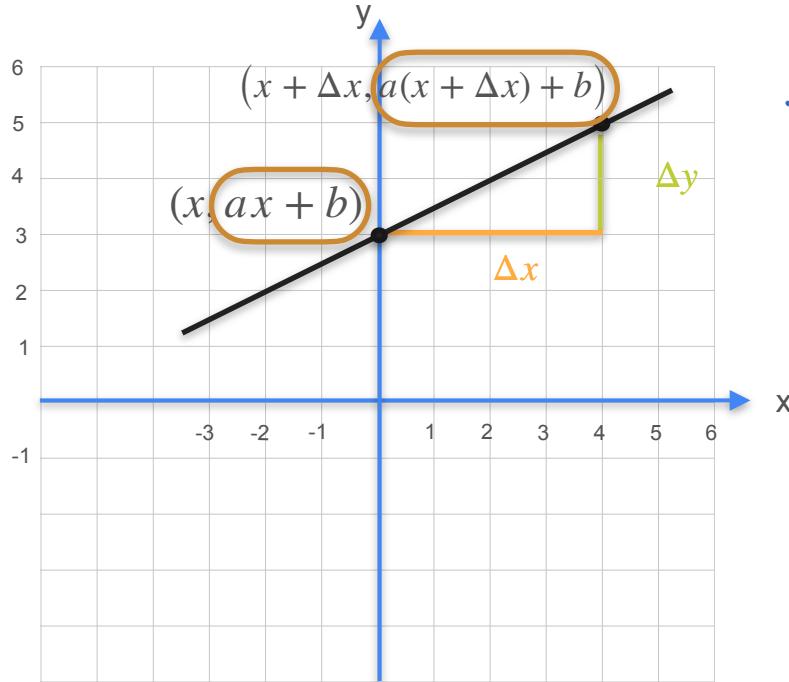


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

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$$\frac{\Delta y}{\Delta x}$$

Derivative of a Line

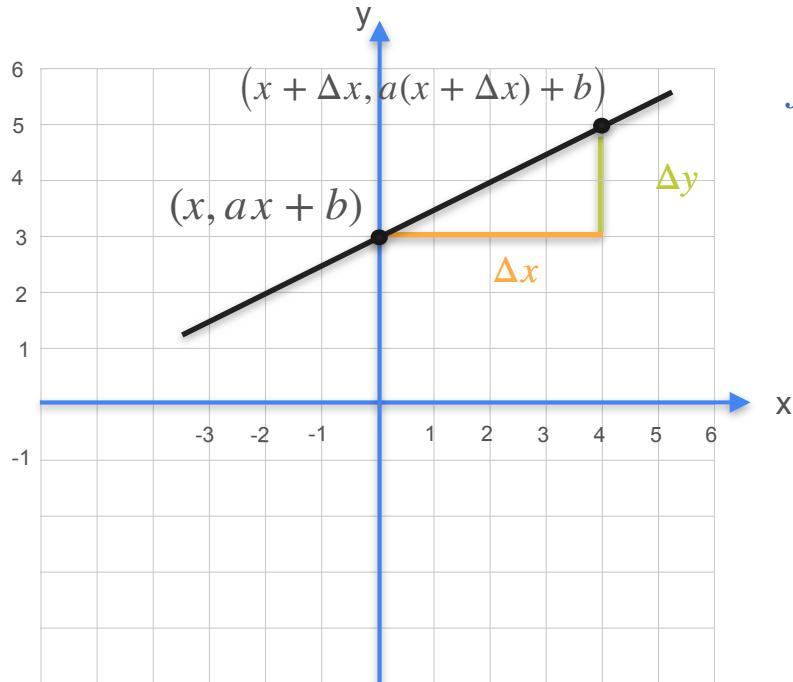


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Derivative of a Line

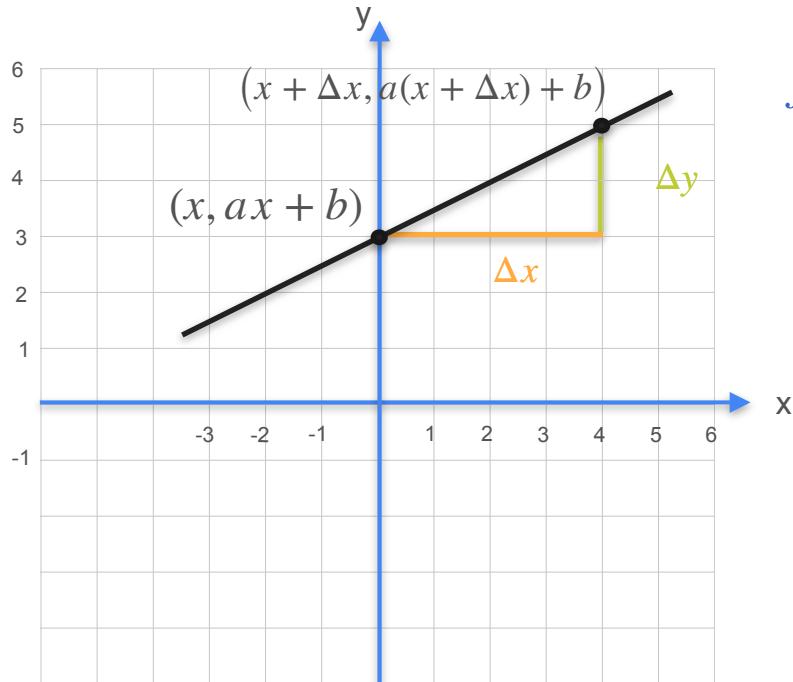


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

Derivative of a Line

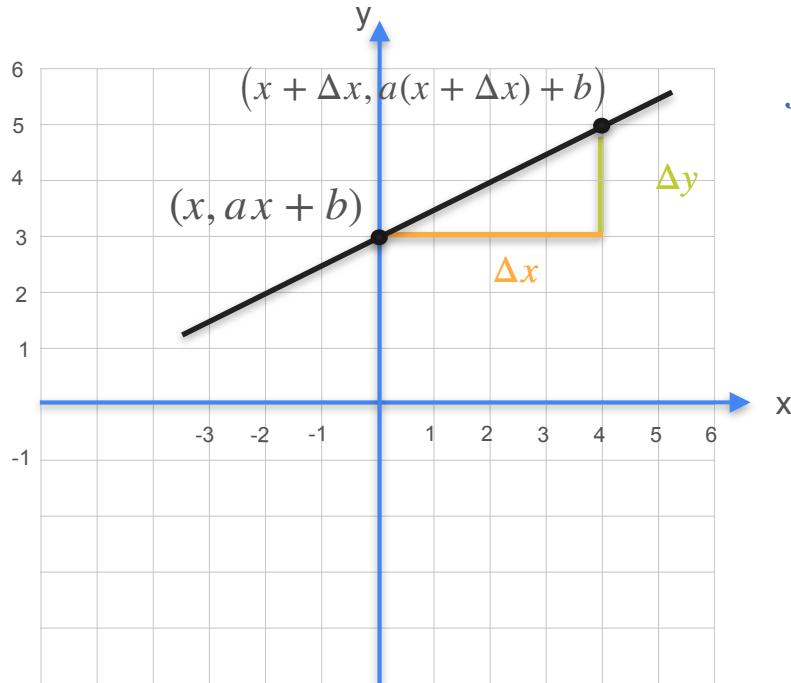


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

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Derivative of a Line

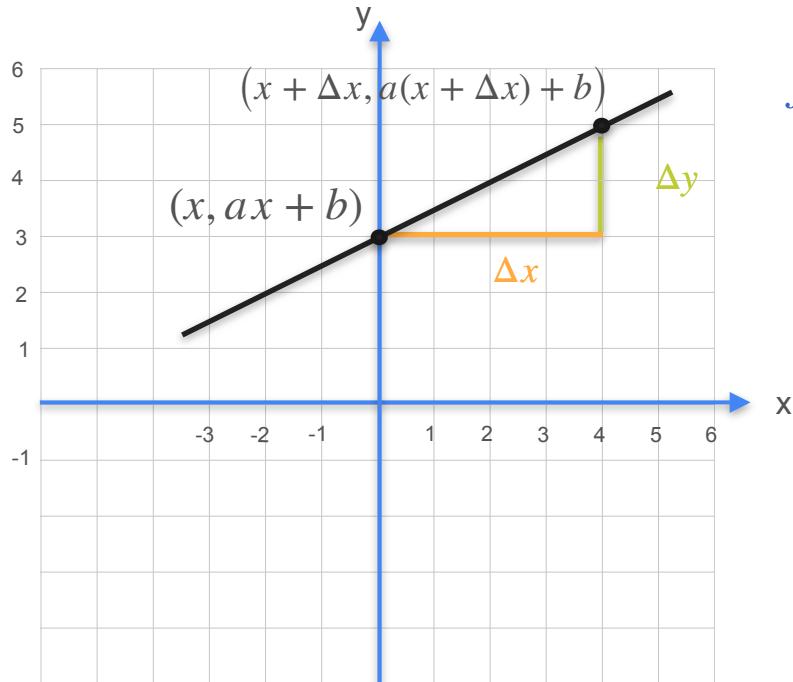


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

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Derivative of a Line



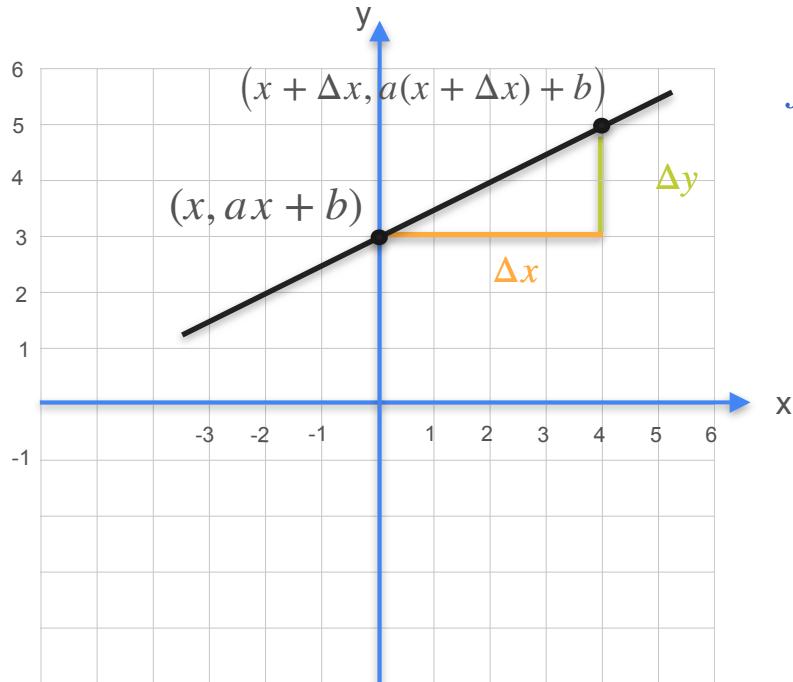
$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

$$= a \frac{\Delta x}{\Delta x}$$

Derivative of a Line

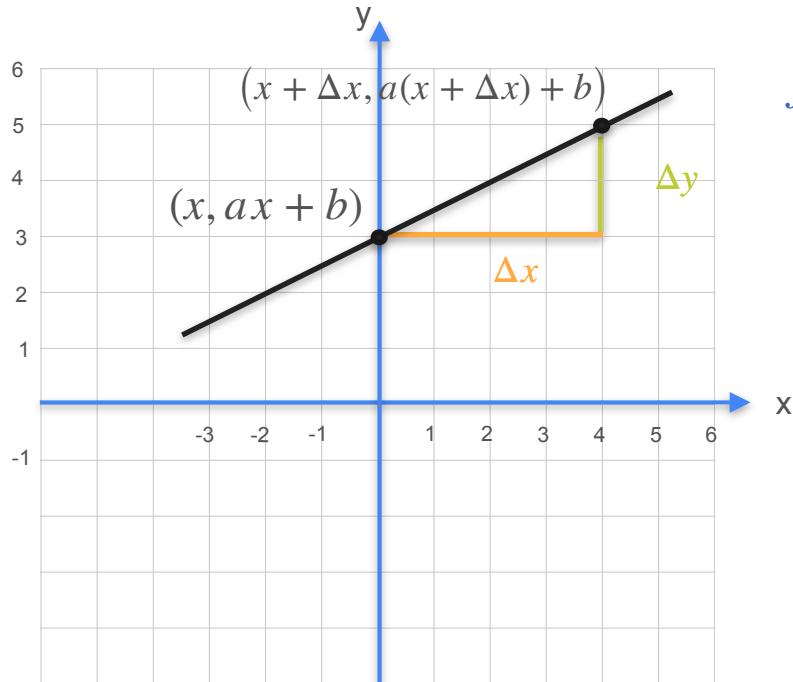


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\Delta x}{\Delta x}\end{aligned}$$

Derivative of a Line

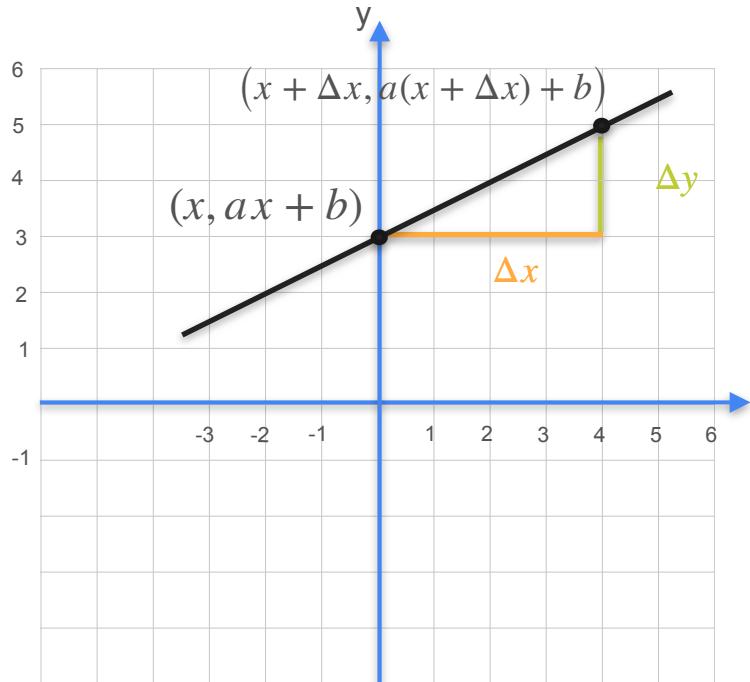


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\cancel{\Delta x}}{\cancel{\Delta x}} = a\end{aligned}$$

Derivative of a Line



$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned} x \frac{df}{dx} &= \frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\Delta x}{\Delta x} = \boxed{a} \end{aligned}$$



DeepLearning.AI

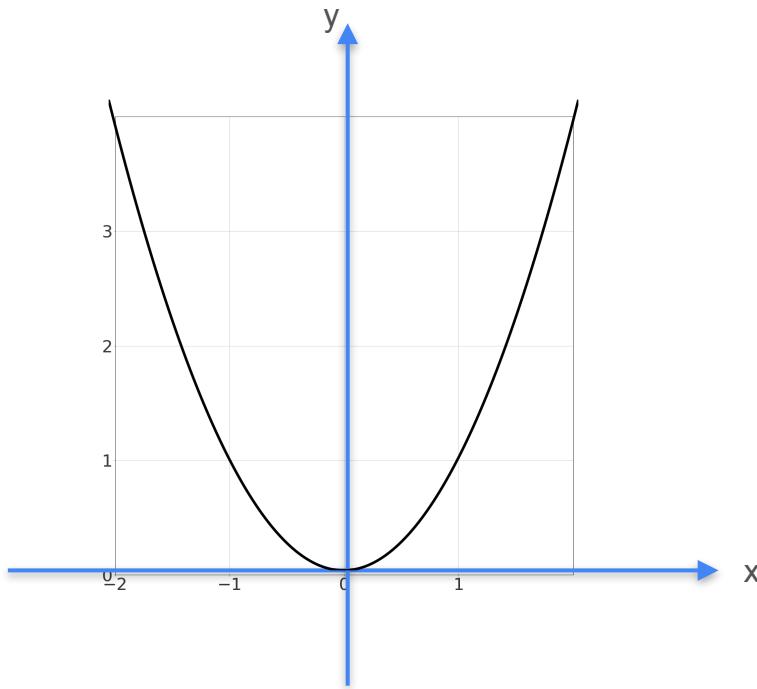
Derivatives and Optimization

**Some common derivatives:
Quadratics**

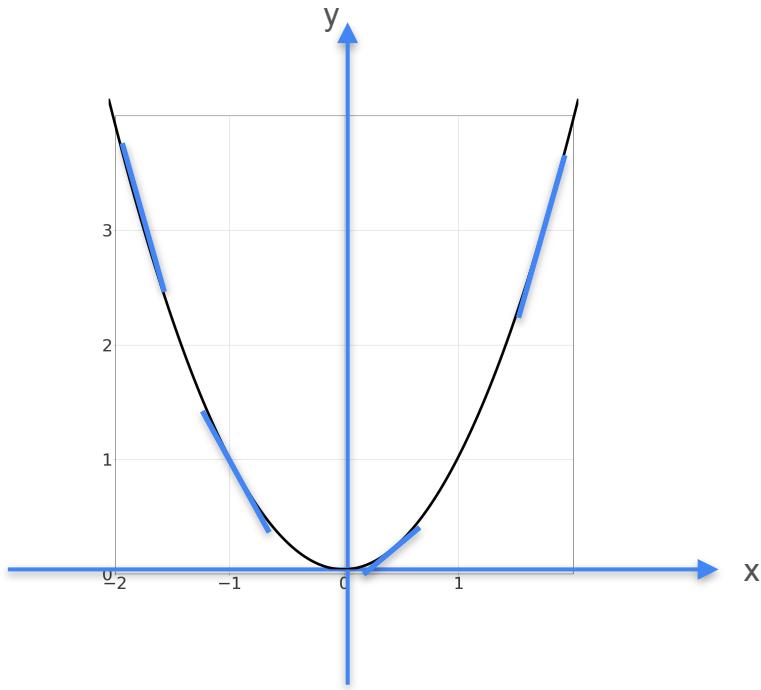
Derivative of Quadratic Functions

Derivative of Quadratic Functions

Quadratics: $y = f(x) = x^2$



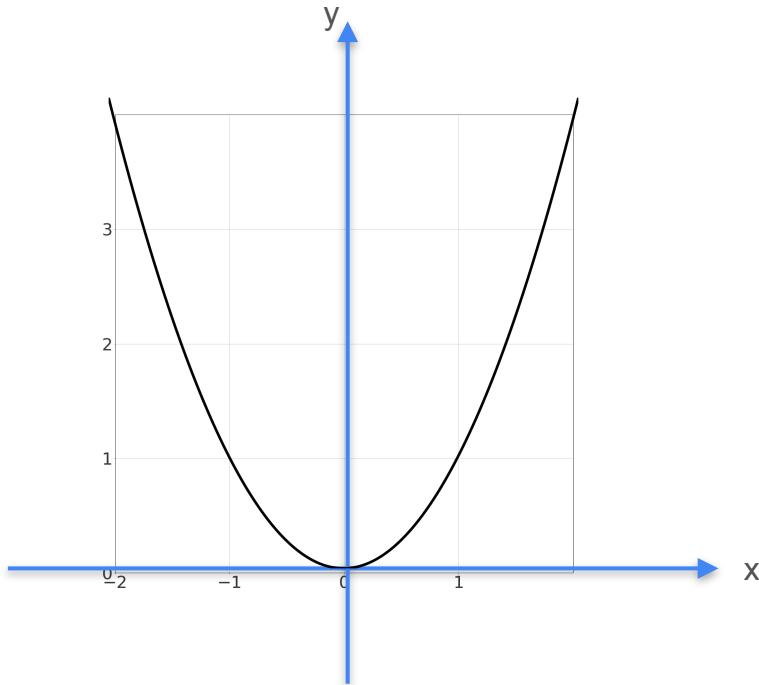
Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x}$

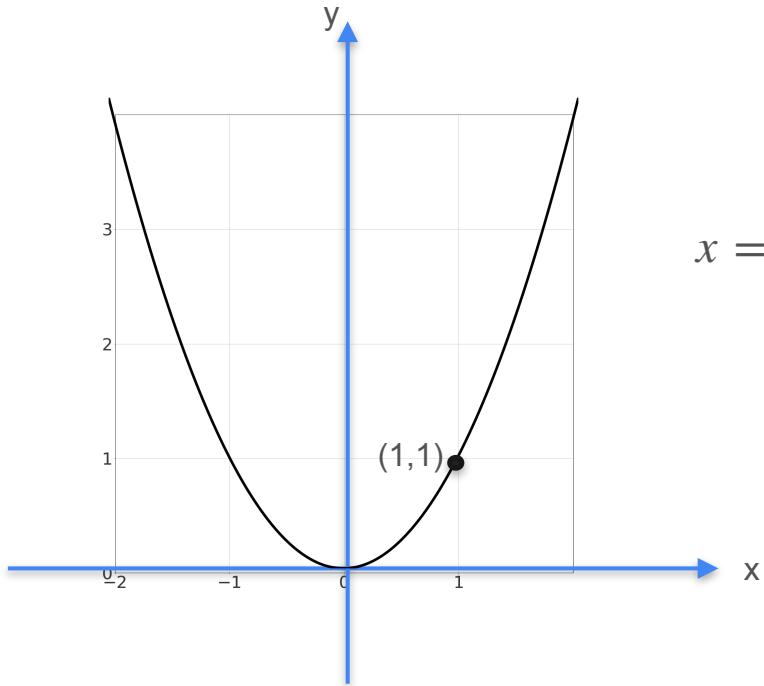
Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Derivative of Quadratic Functions

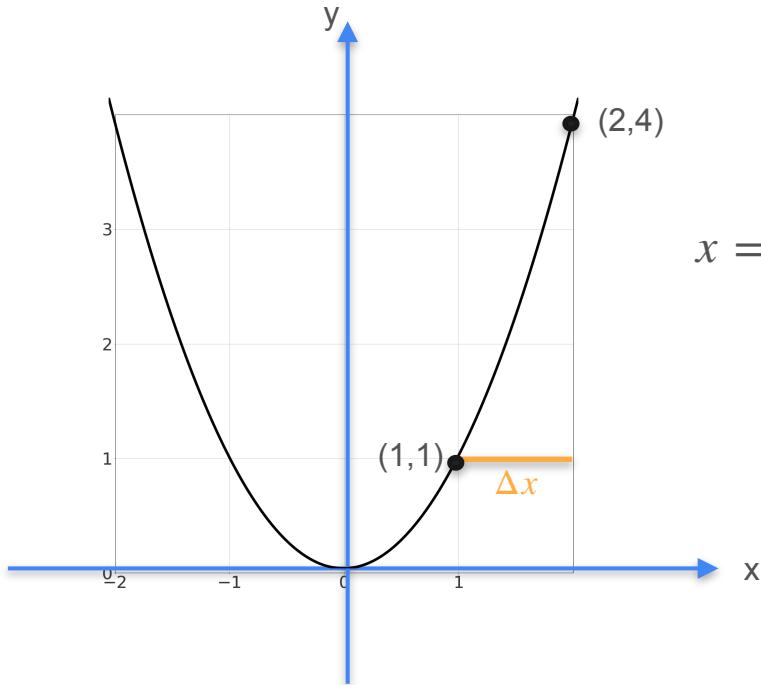


Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$$x = 1$$

Derivative of Quadratic Functions



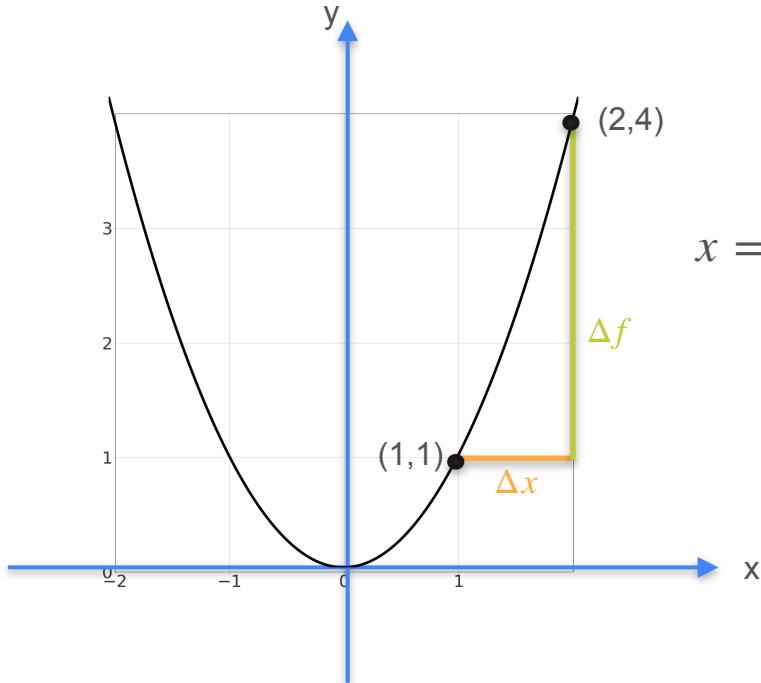
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$$x = 1$$

Δx	1.0
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Derivative of Quadratic Functions



$$x = 1$$

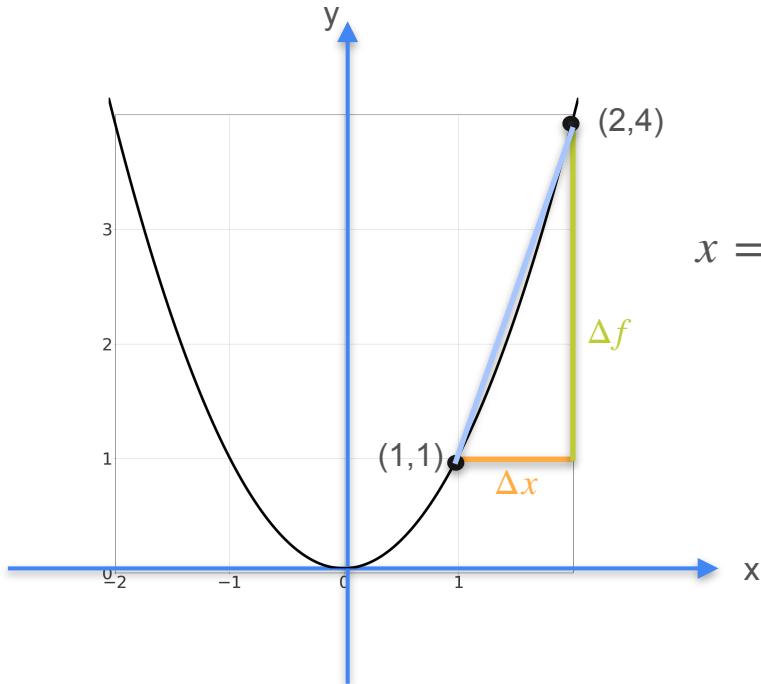
Δx	1.0
Δf	3

$$(1 + 1)^2 - 1^2 = 4 - 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

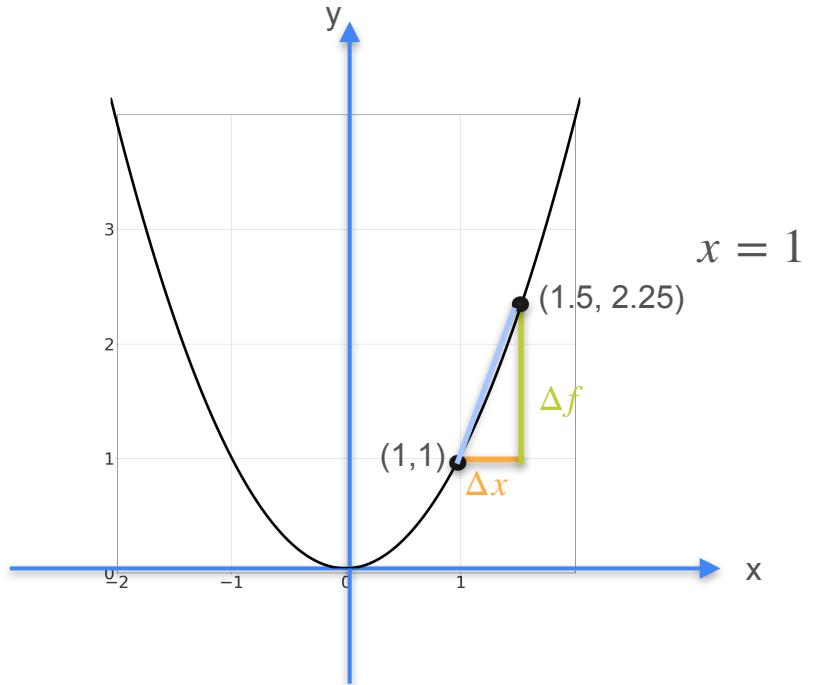
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0
Δf	3
Slope	3

$$(1 + 1)^2 - 1^2 = 4 - 1$$

$$\frac{3}{1}$$

Derivative of Quadratic Functions

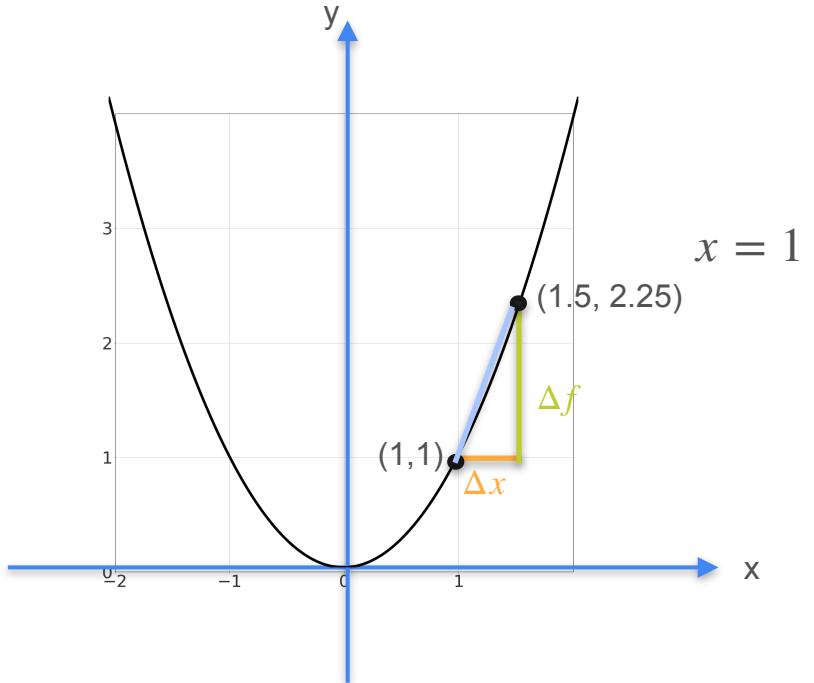


Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2
Δf	3	
Slope	3	

Derivative of Quadratic Functions



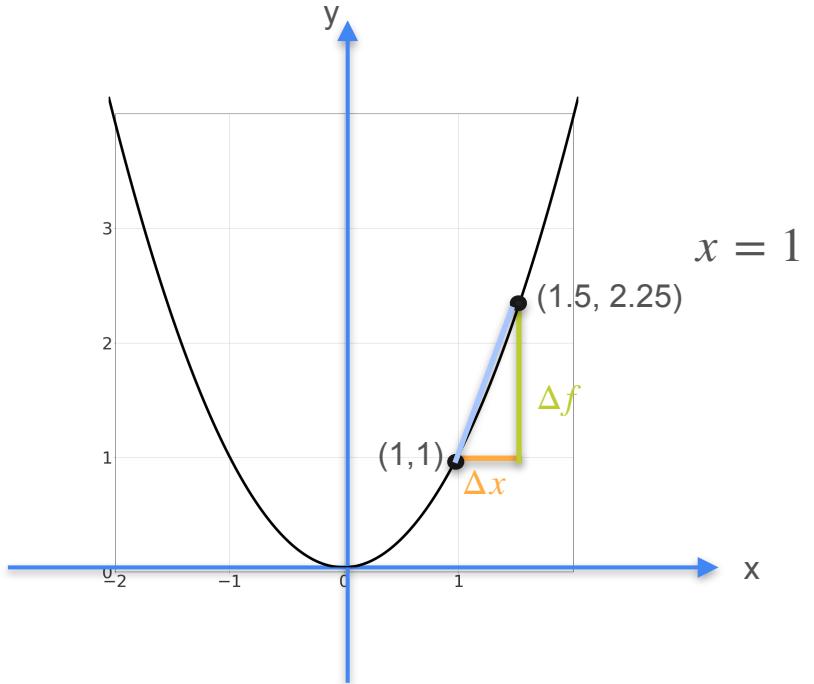
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	$1/2$
Δf	3	1.25
Slope	3	

$$(1 + 0.5)^2 - 1^2 = 2.25 - 1$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

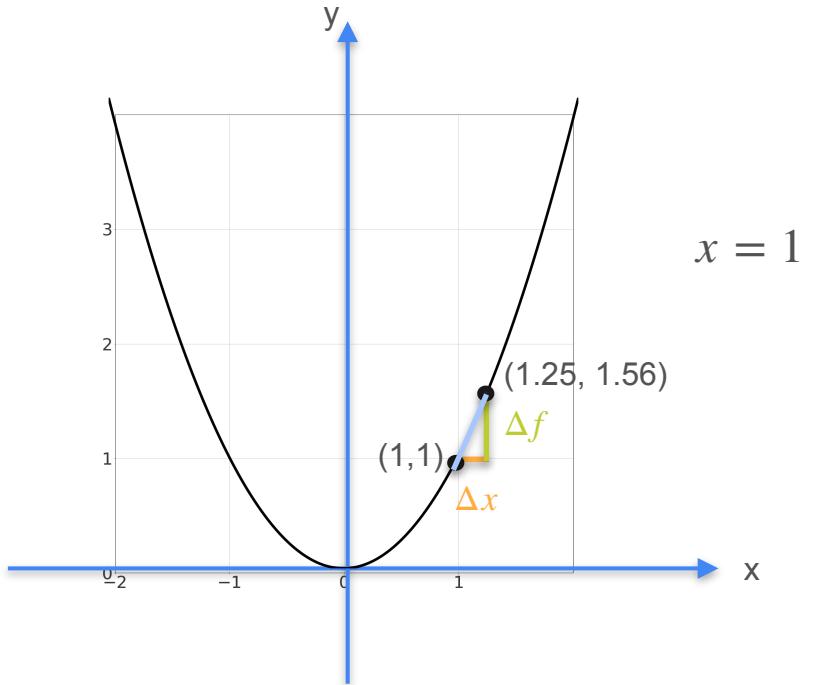
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	$1/2$
Δf	3	1.25
Slope	3	2.5

$$(1 + 0.5)^2 - 1^2 = 2.25 - 1$$

$$\frac{1.25}{0.5}$$

Derivative of Quadratic Functions



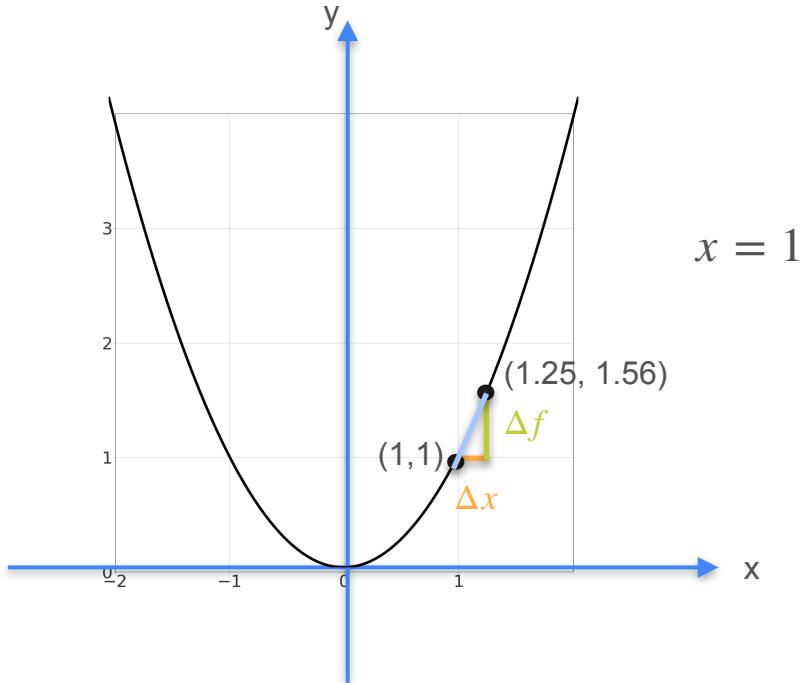
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	$1/2$	$1/4$
Δf	3	1.25	
Slope	3	2.5	

Derivative of Quadratic Functions



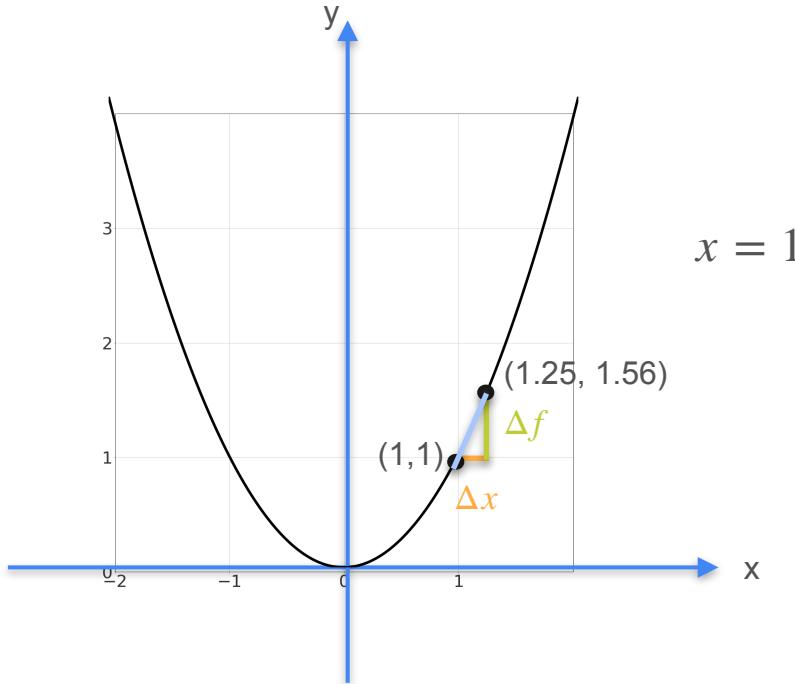
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	3	1.25	0.562
Slope	3	2.5	

$$(1 + 0.25)^2 - 1^2 = 1.56 - 1$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

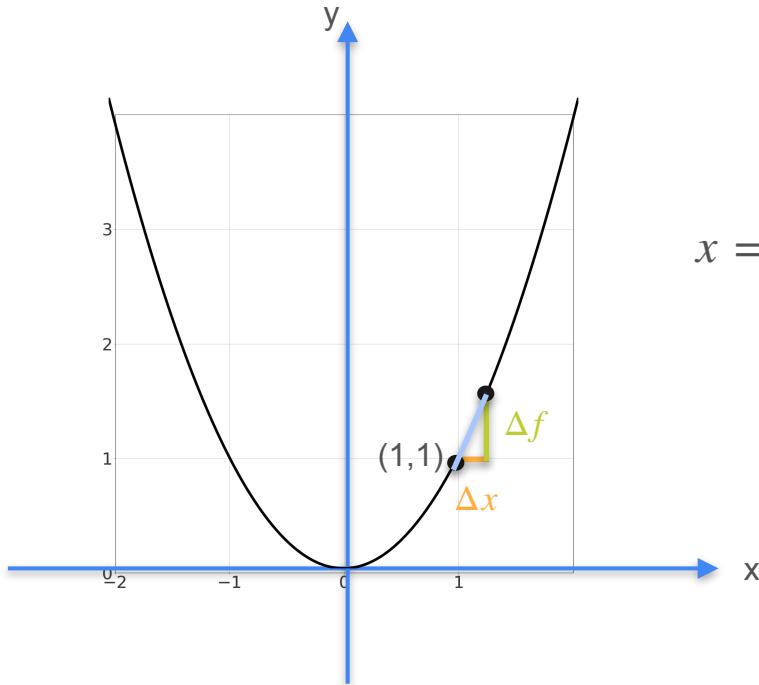
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	3	1.25	0.562
Slope	3	2.5	2.25

$$(1 + 0.25)^2 - 1^2 = 1.56 - 1$$

$$\frac{0.56}{0.25}$$

Derivative of Quadratic Functions



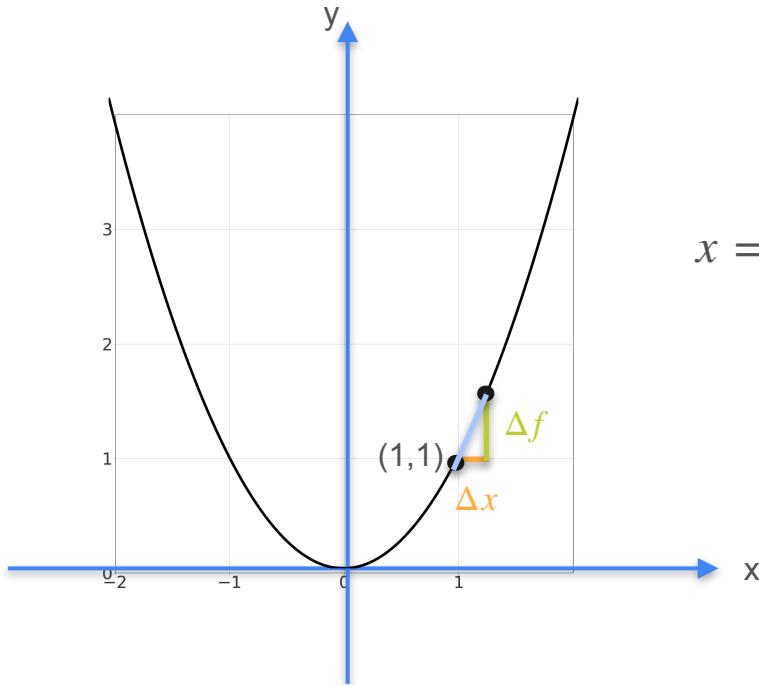
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	3	1.25	0.562
Slope	3	2.5	2.25

Derivative of Quadratic Functions



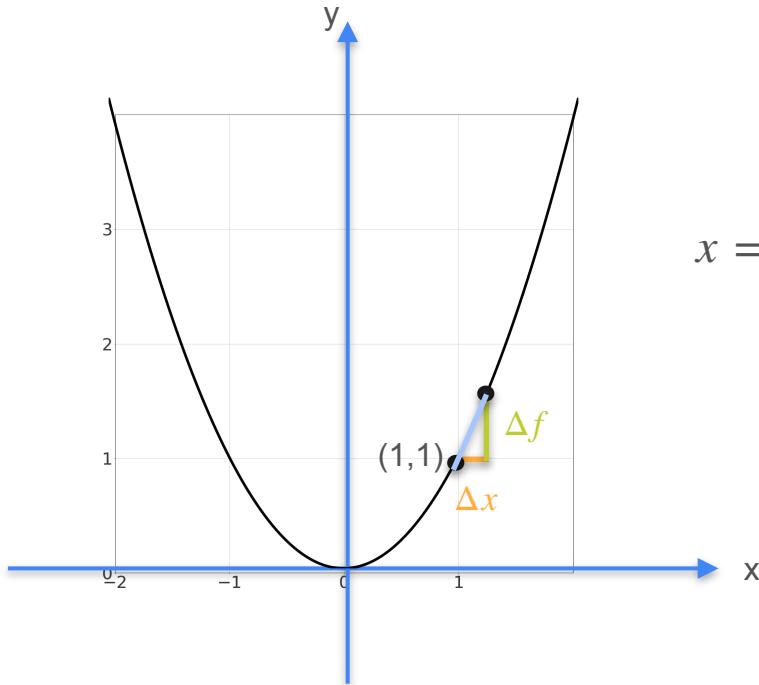
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	3	1.25	0.562	0.265	
Slope	3	2.5	2.25	2.125	

Derivative of Quadratic Functions



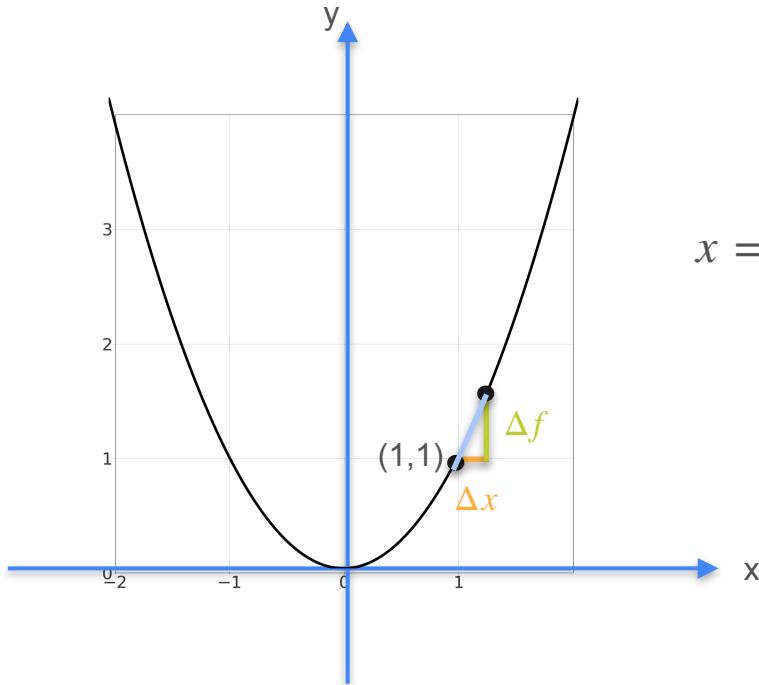
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	3	1.25	0.562	0.265	0.128
Slope	3	2.5	2.25	2.125	

Derivative of Quadratic Functions



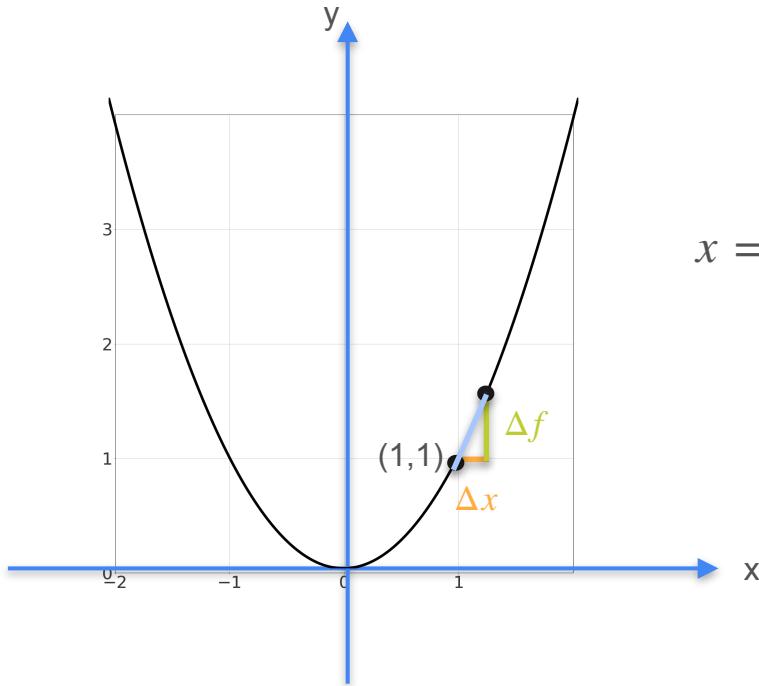
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	3	1.25	0.562	0.265	0.128
Slope	3	2.5	2.25	2.125	2.065

Derivative of Quadratic Functions



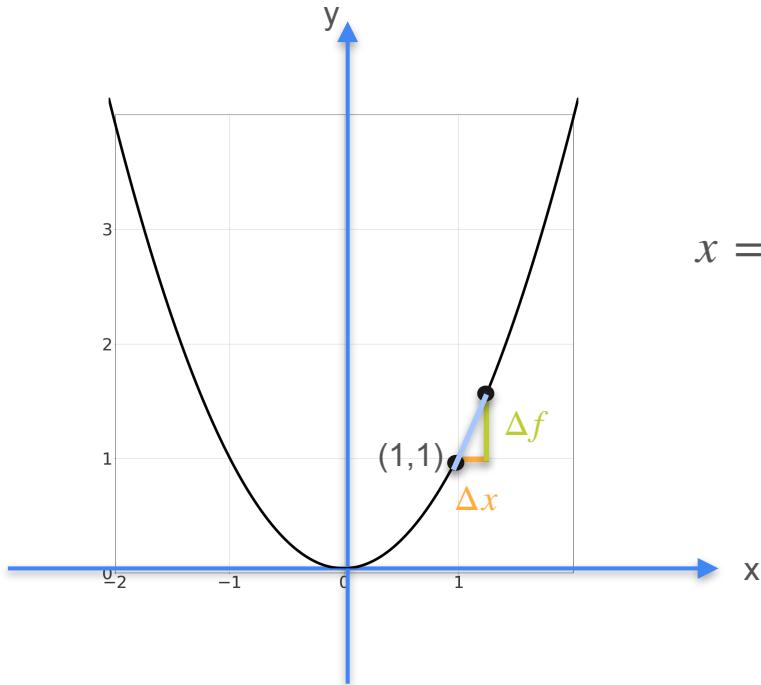
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3	1.25	0.562	0.265	0.128	
Slope	3	2.5	2.25	2.125	2.065	

Derivative of Quadratic Functions



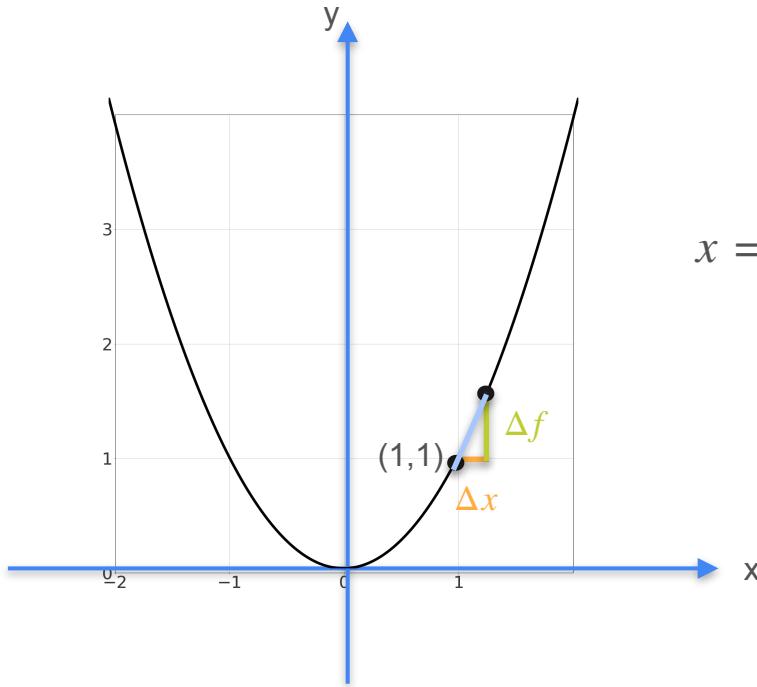
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	

Derivative of Quadratic Functions



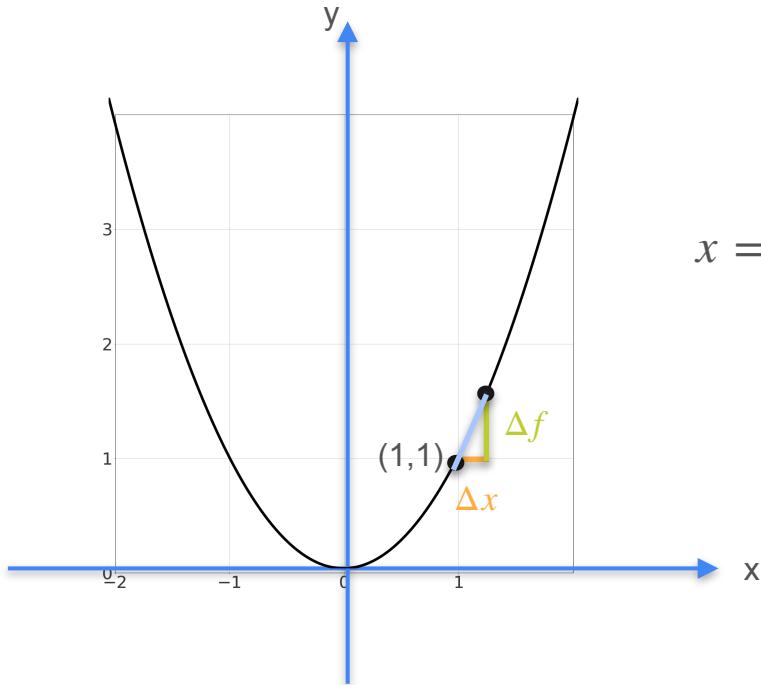
$$x = 1$$

Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

Derivative of Quadratic Functions



$$x = 1$$

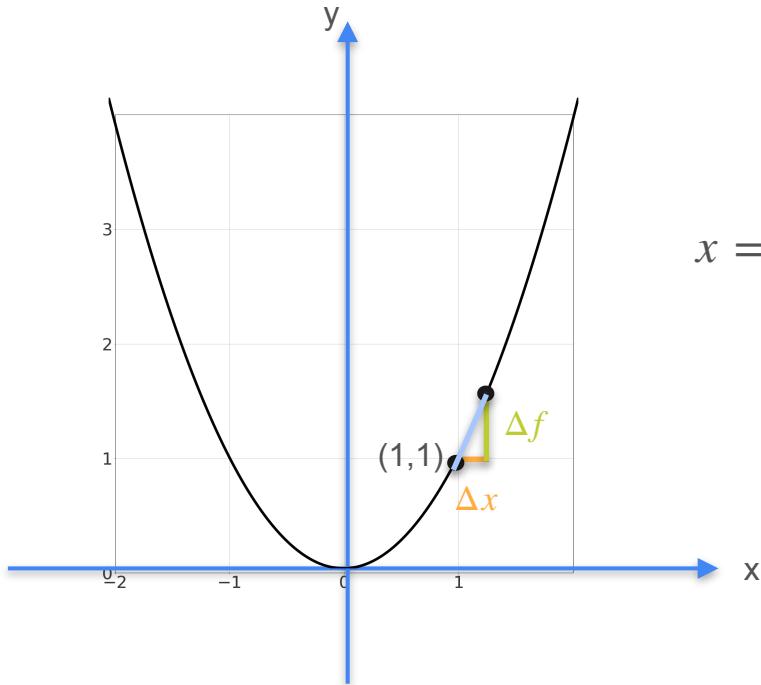
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

$$f'(1) = \frac{d}{dx} f(1) = 2$$

Derivative of Quadratic Functions



$$x = 1$$

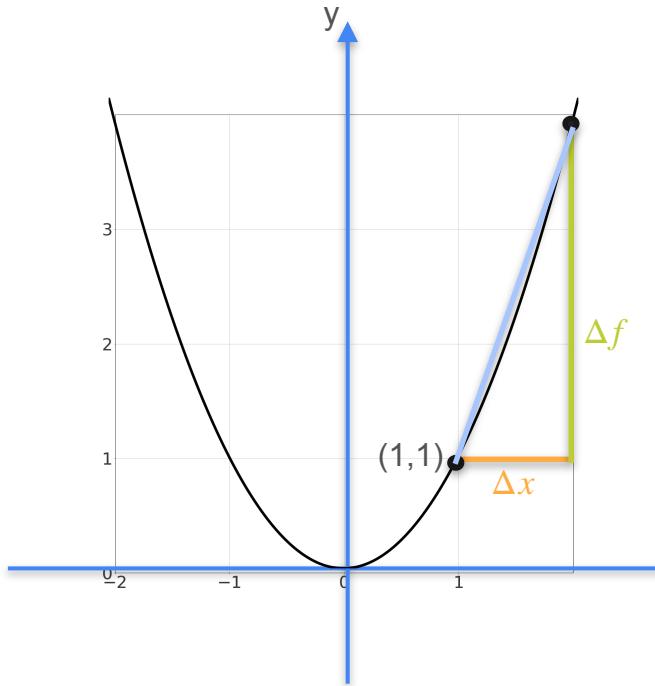
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

$$f'(1) = \frac{d}{dx} f(1) = 2 = 2 \times 1$$

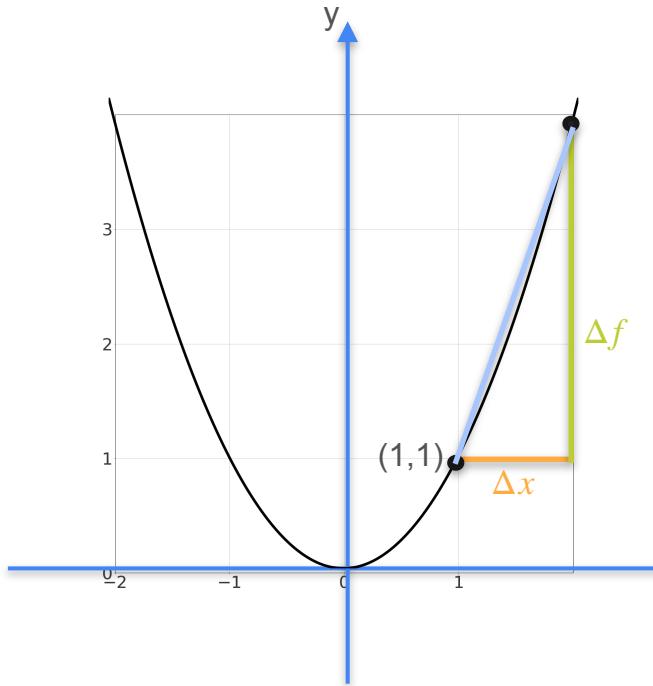
Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

Derivative of Quadratic Functions

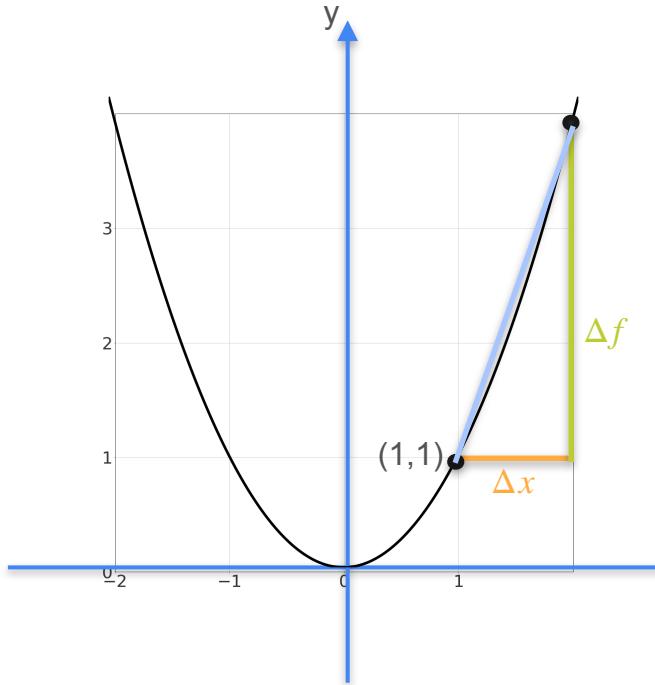


Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x}$$

Derivative of Quadratic Functions

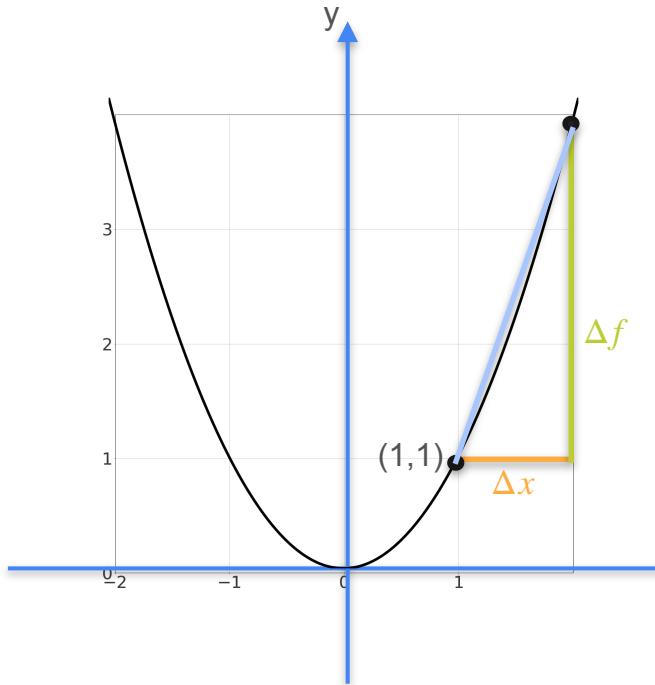


Quadratics: $y = f(x) = x^2$

Slope:
$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

Derivative of Quadratic Functions



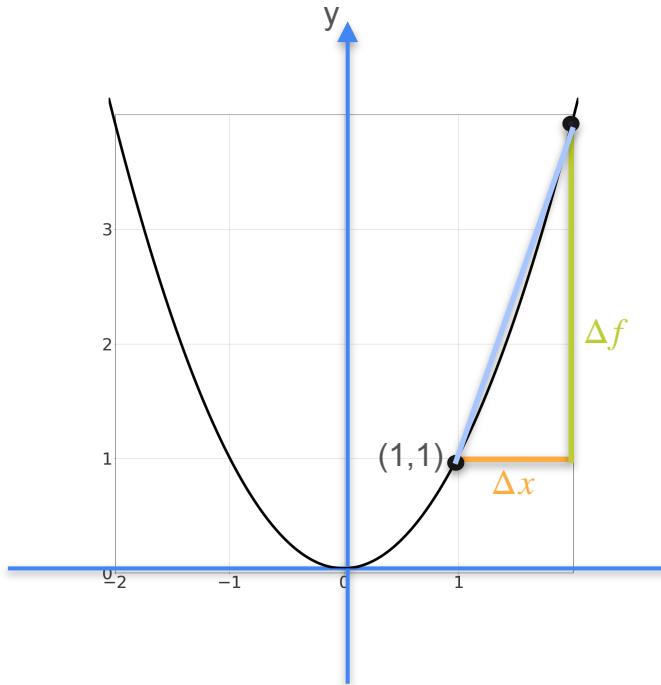
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

Derivative of Quadratic Functions



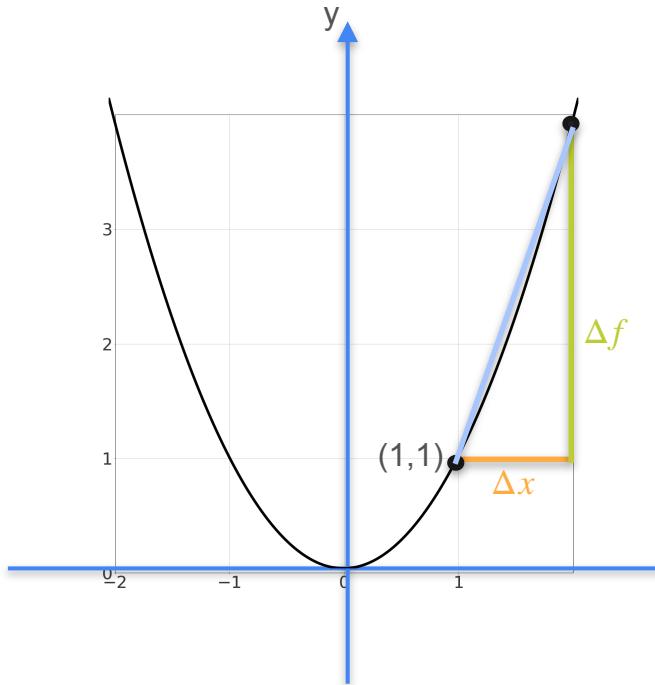
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Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

Derivative of Quadratic Functions



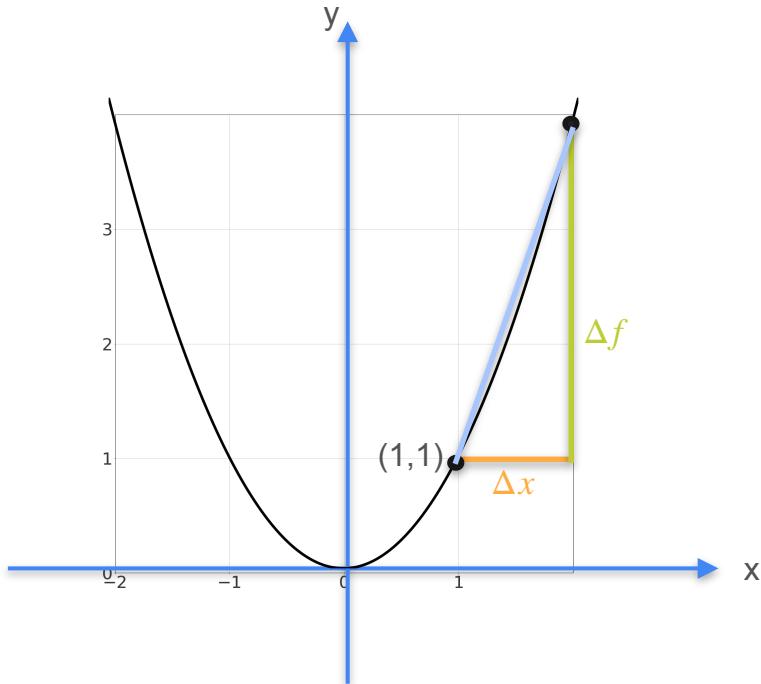
Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

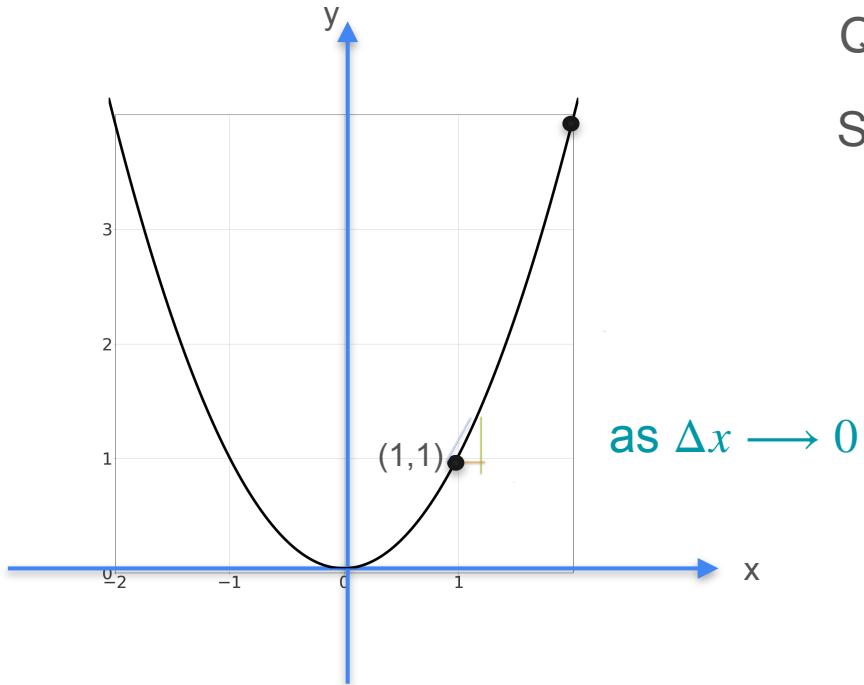
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \Delta x$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

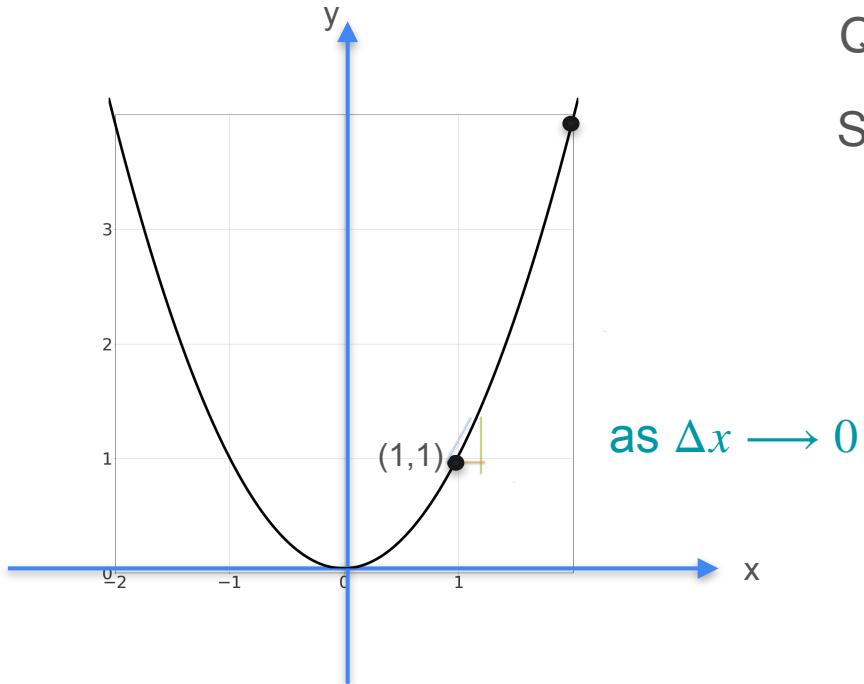
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \Delta x$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

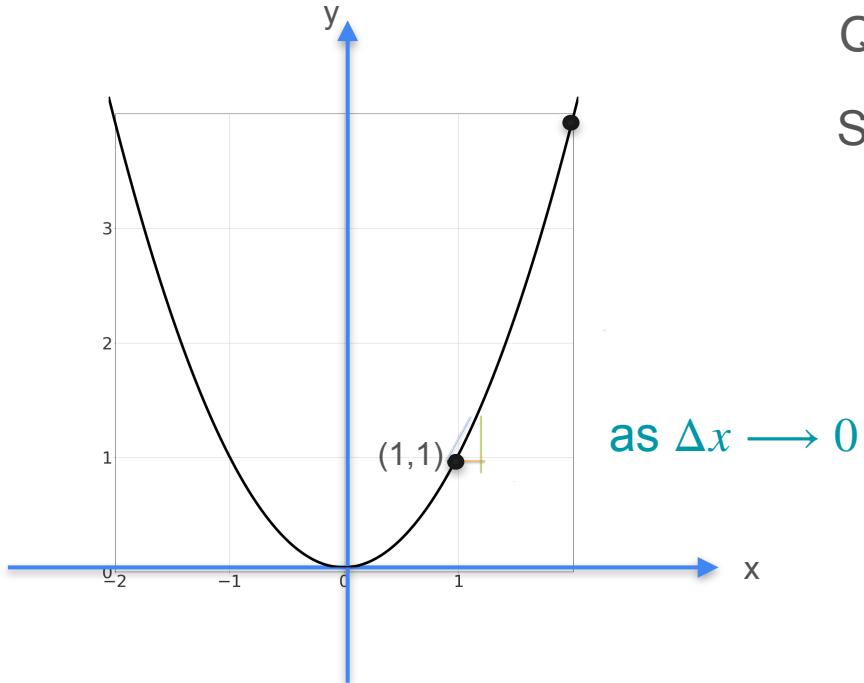
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \boxed{\Delta x} \xrightarrow[0]{}$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

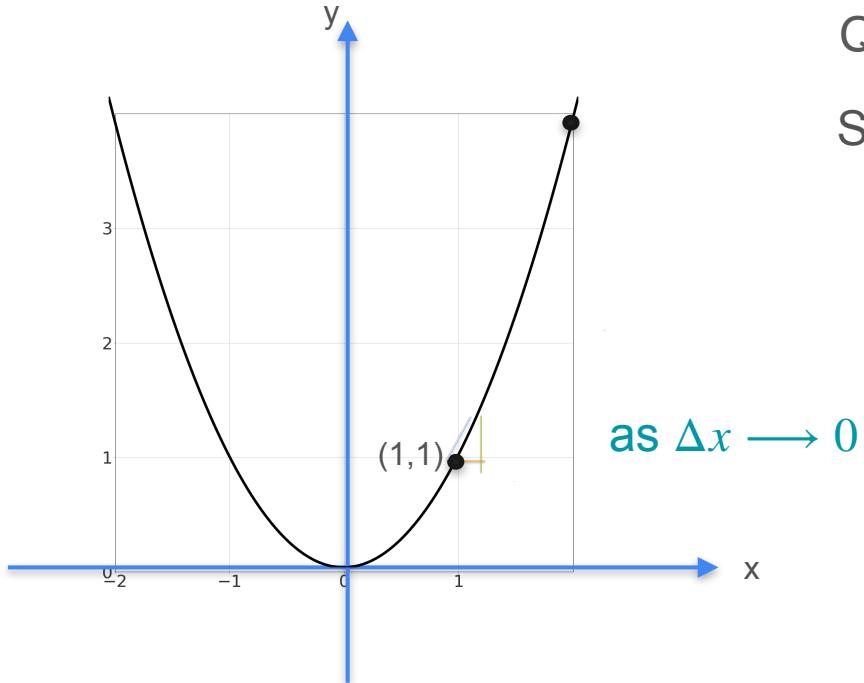
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \boxed{\Delta x} \xrightarrow[0]{}$$

Derivative of Quadratic Functions

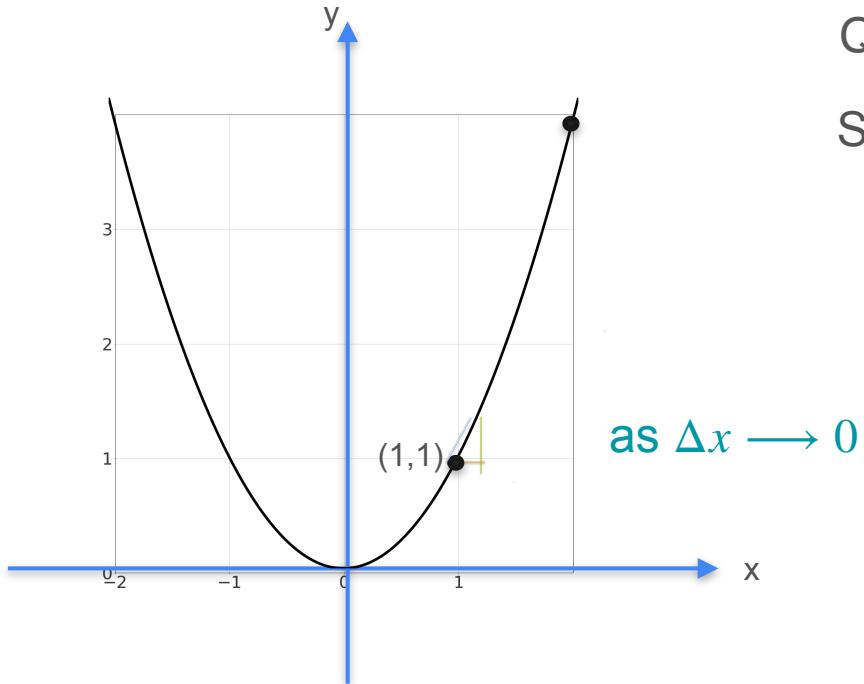


Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= 2x + \Delta x\end{aligned}$$

Derivative of Quadratic Functions



Quadratics: $y = f(x) = x^2$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= 2x + \Delta x\end{aligned}$$

$$f(x) = x^2 \longrightarrow f'(x) = 2x$$



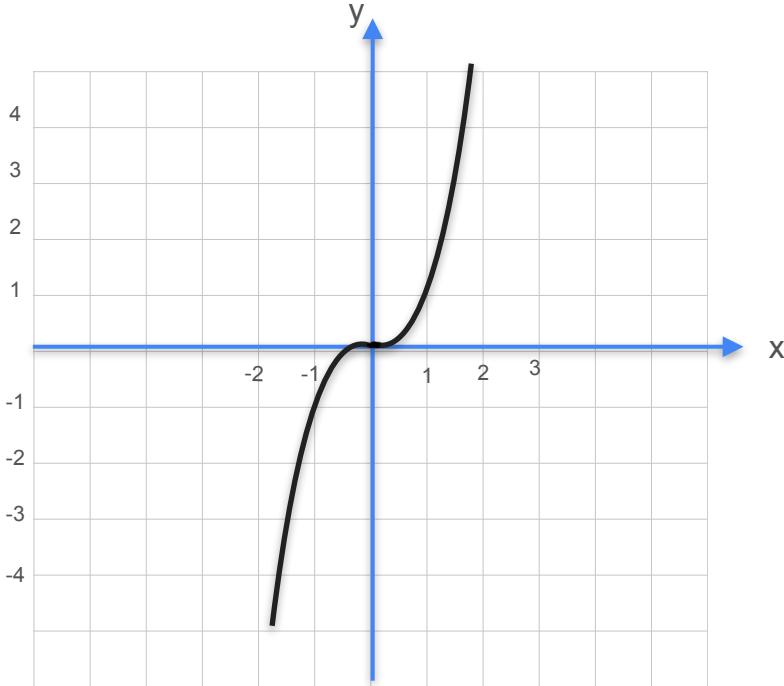
DeepLearning.AI

Derivatives and Optimization

**Some common derivatives:
Higher degree polynomials**

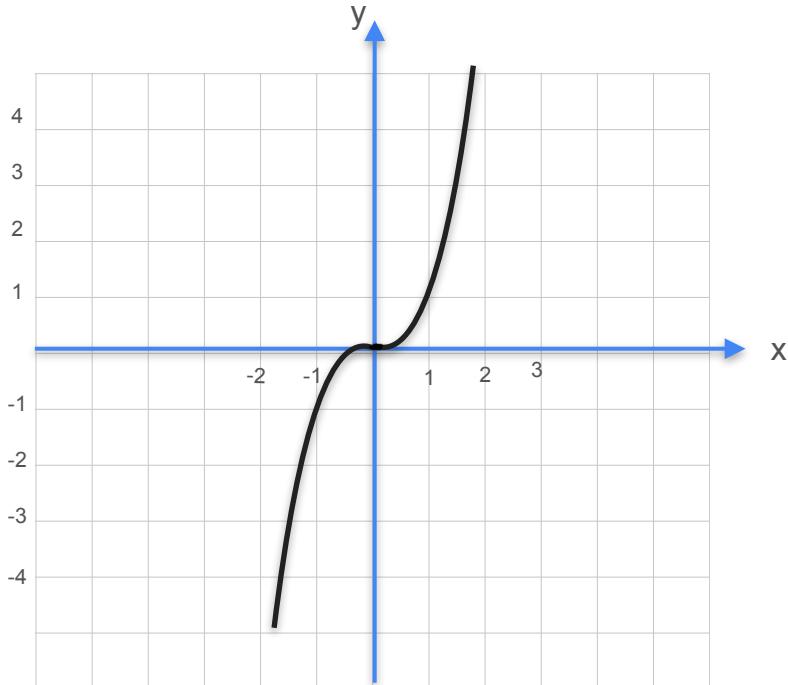
Derivative of Cubic Functions

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

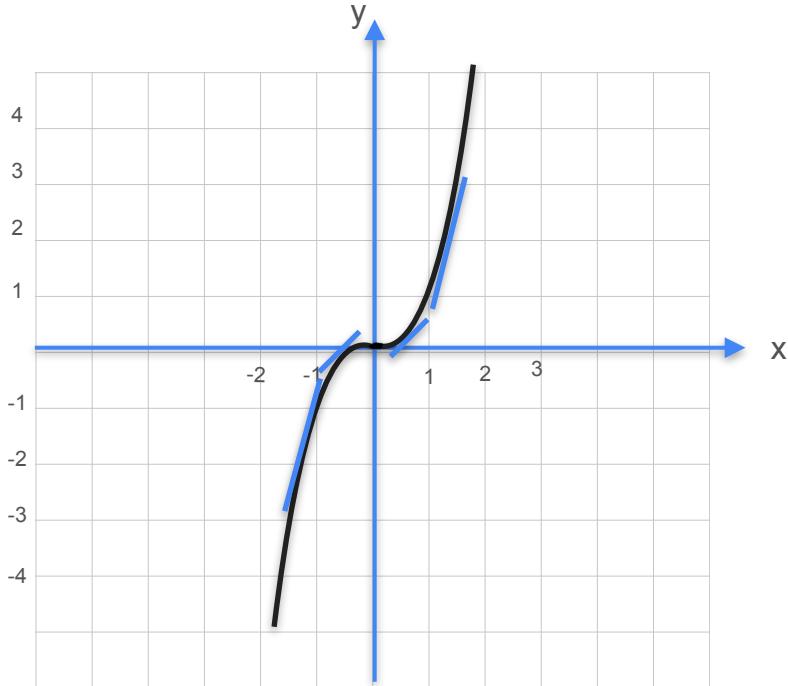
Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x}$

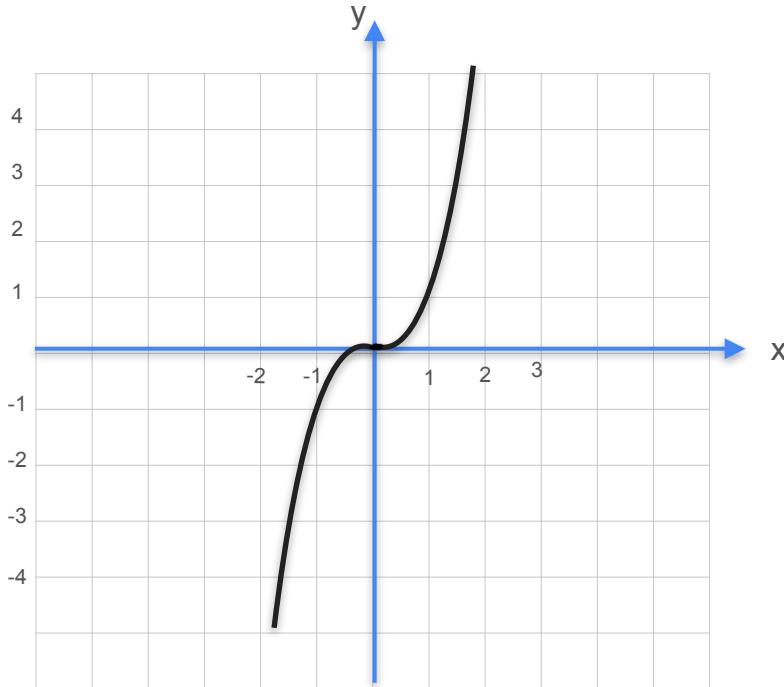
Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x}$

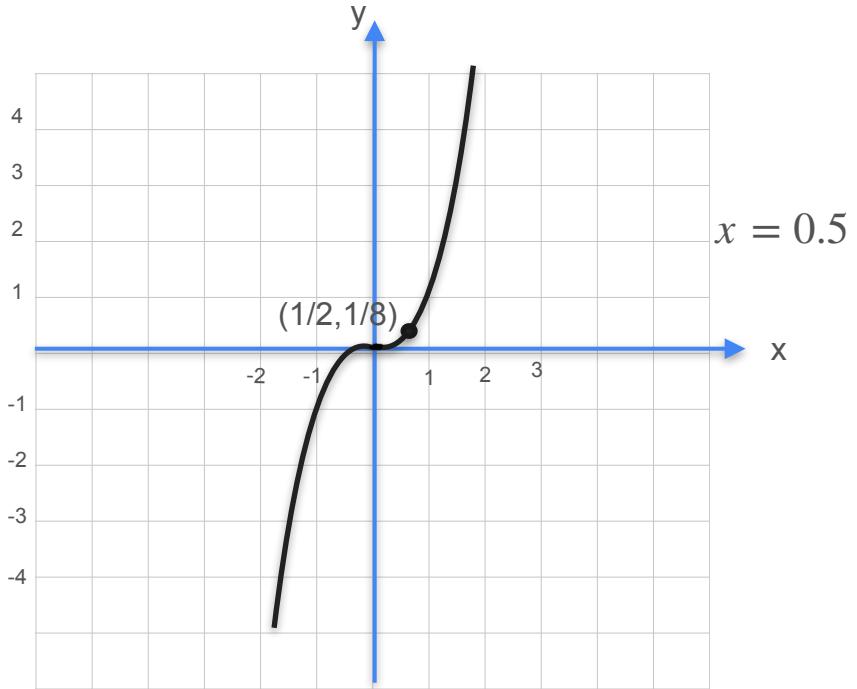
Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

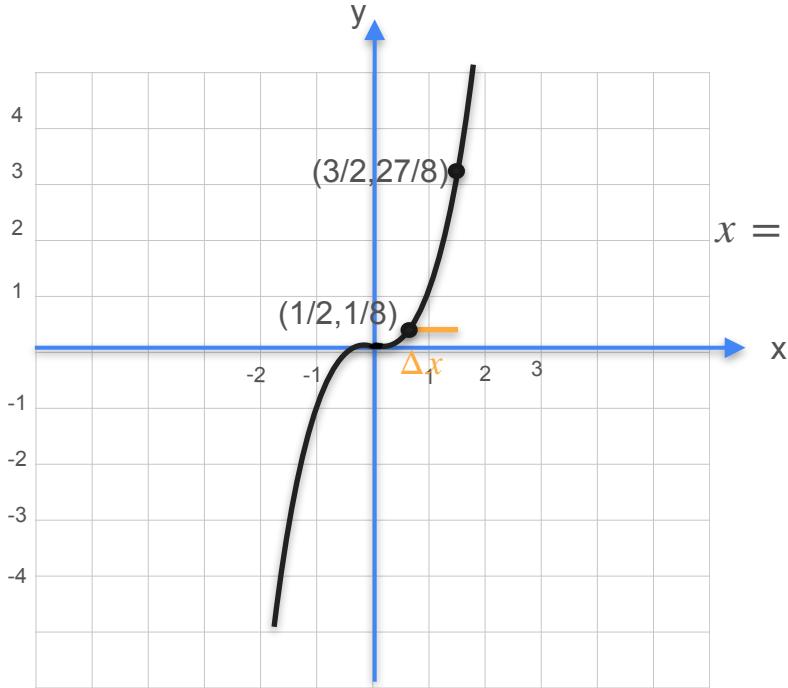
Derivative of Cubic Functions



$$\text{Cubic: } y = f(x) = x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

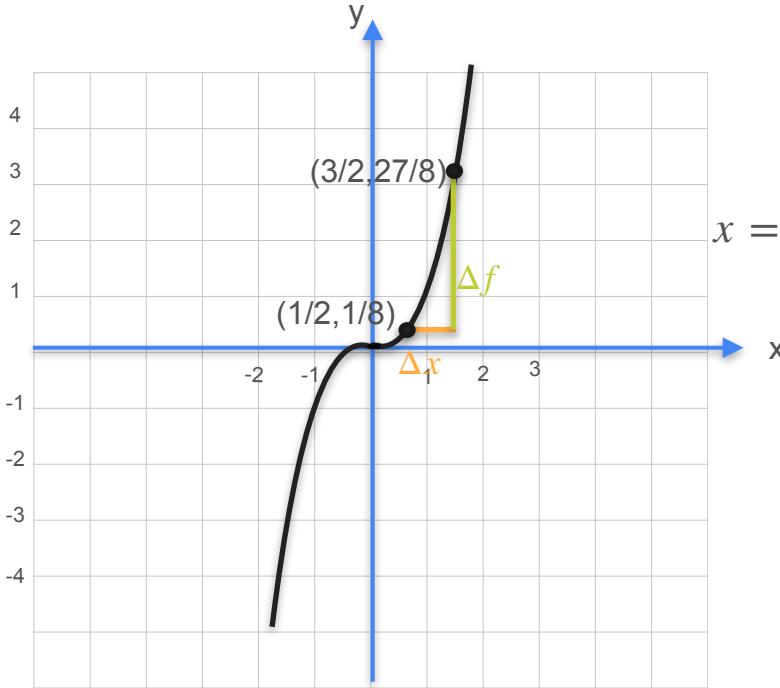
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

Δx

1.0

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

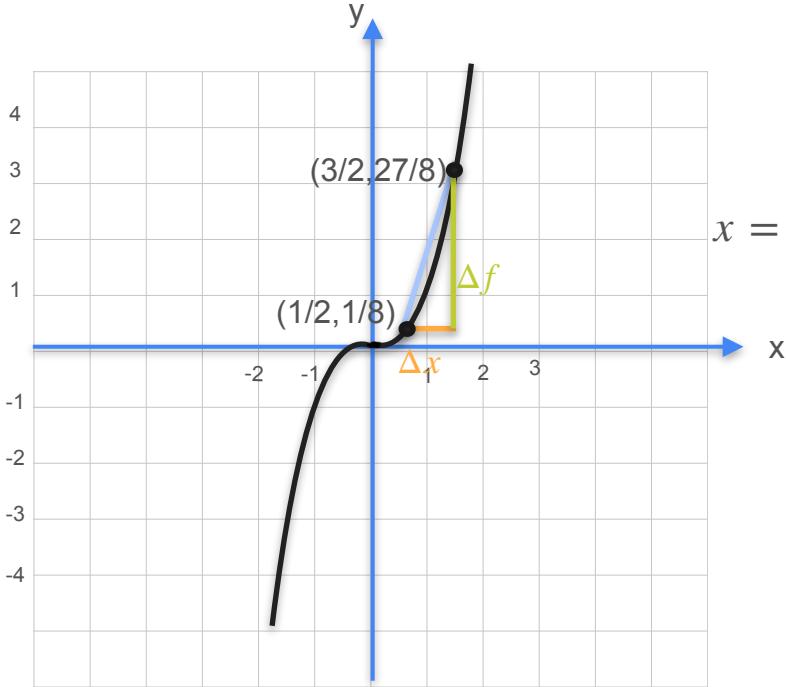
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$$x = 0.5$$

Δx	1.0
Δf	3.25

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

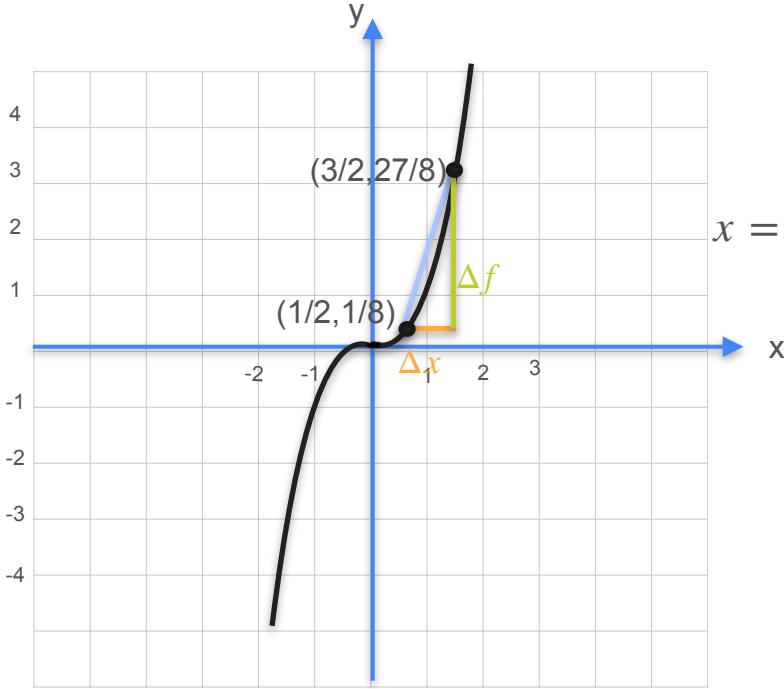
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$$x = 0.5$$

Δx	1.0
Δf	3.25
Slope	

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

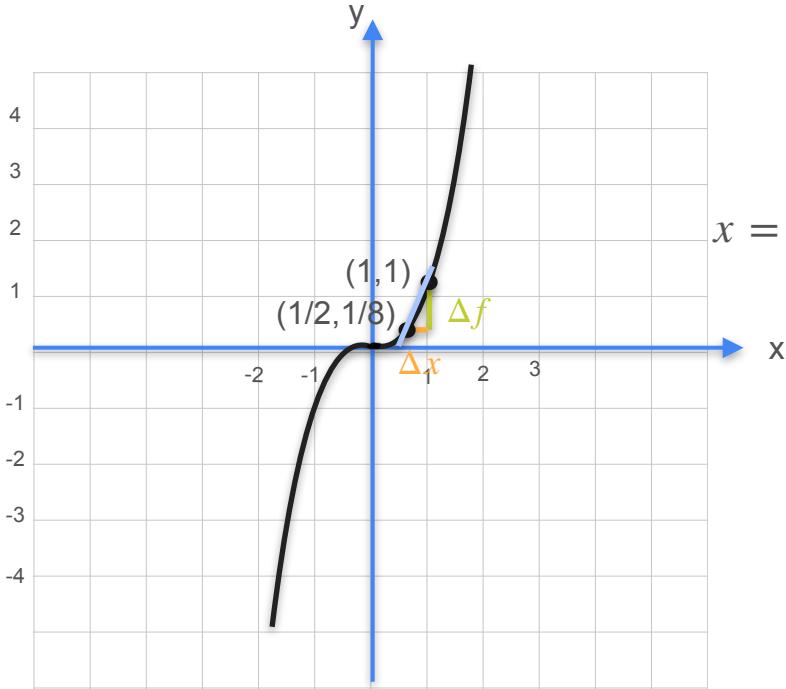
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$$x = 0.5$$

Δx	1.0
Δf	3.25
Slope	3.25

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$
$$\frac{3.25}{1}$$

Derivative of Cubic Functions

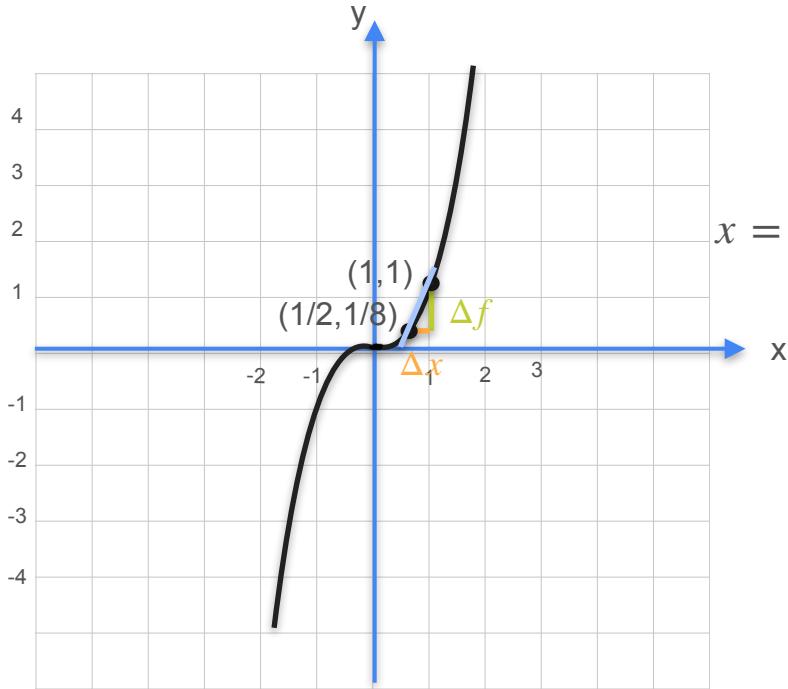


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0
Δf	3.25
Slope	3.25

Derivative of Cubic Functions



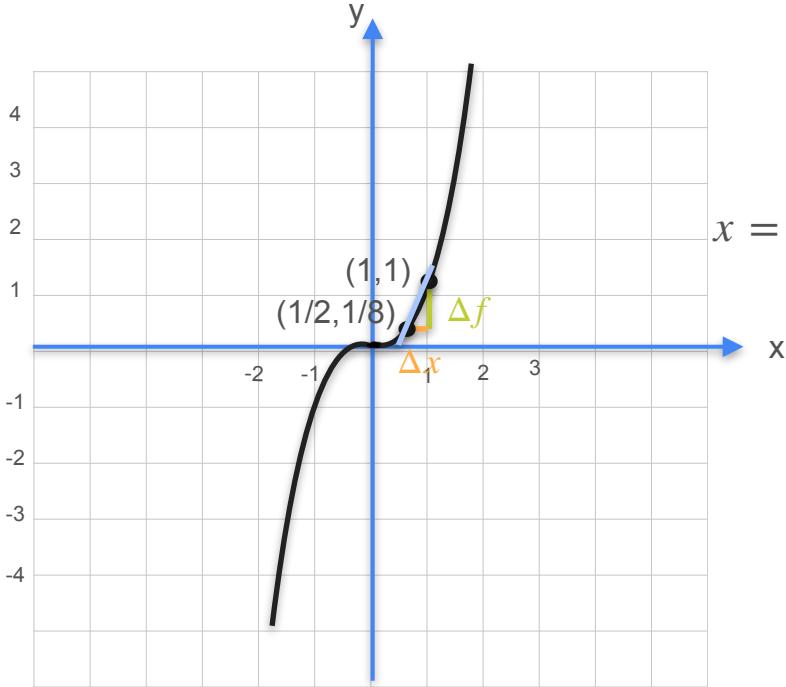
$$x = 0.5$$

Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$
Δf	3.25	
Slope	3.25	

Derivative of Cubic Functions



$$x = 0.5$$

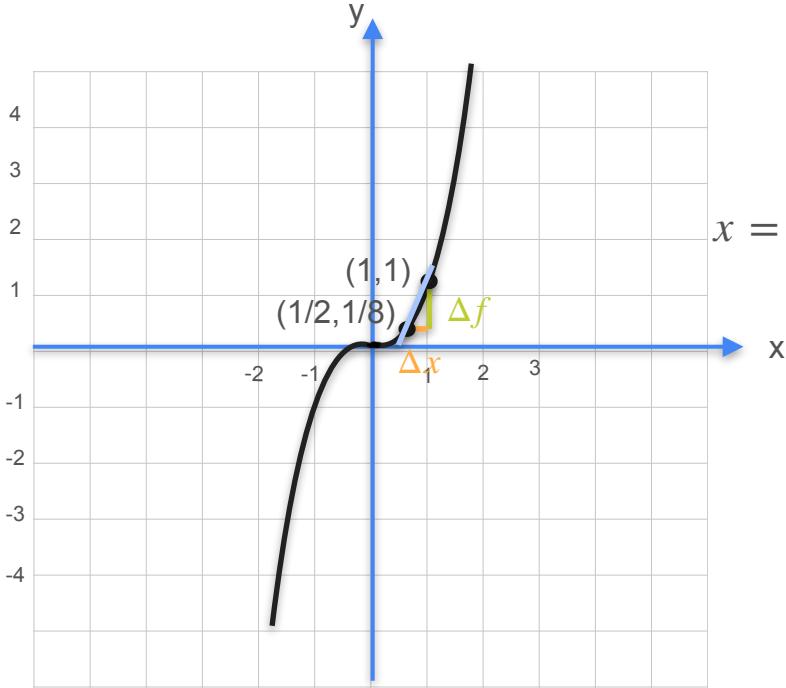
Δx	1.0	1/2
Δf	3.25	0.86
Slope	3.25	

$$\left(\frac{1}{2} + \frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8}$$

Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

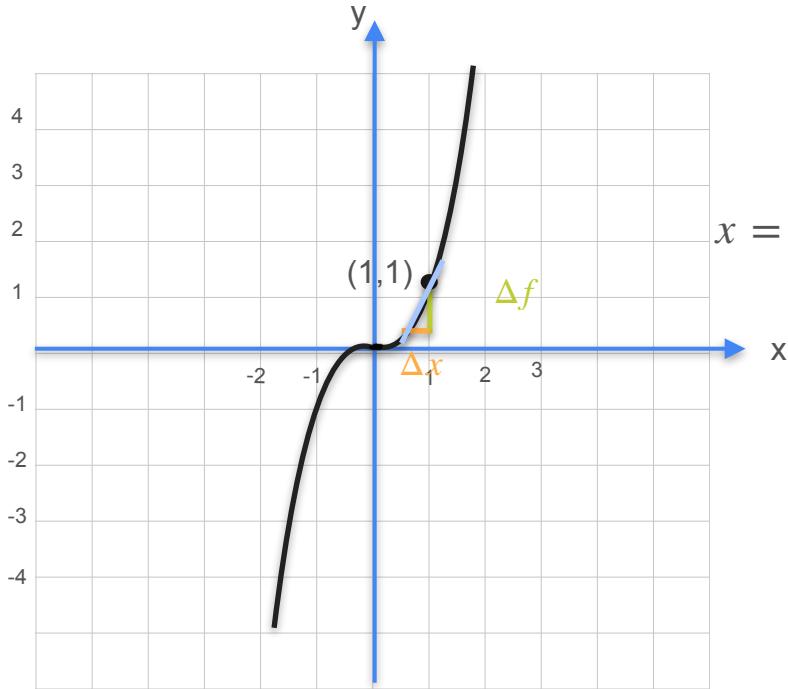
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2
Δf	3.25	0.86
Slope	3.25	1.75

$$\left(\frac{1}{2} + \frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8}$$

$$\frac{0.86}{0.5}$$

Derivative of Cubic Functions

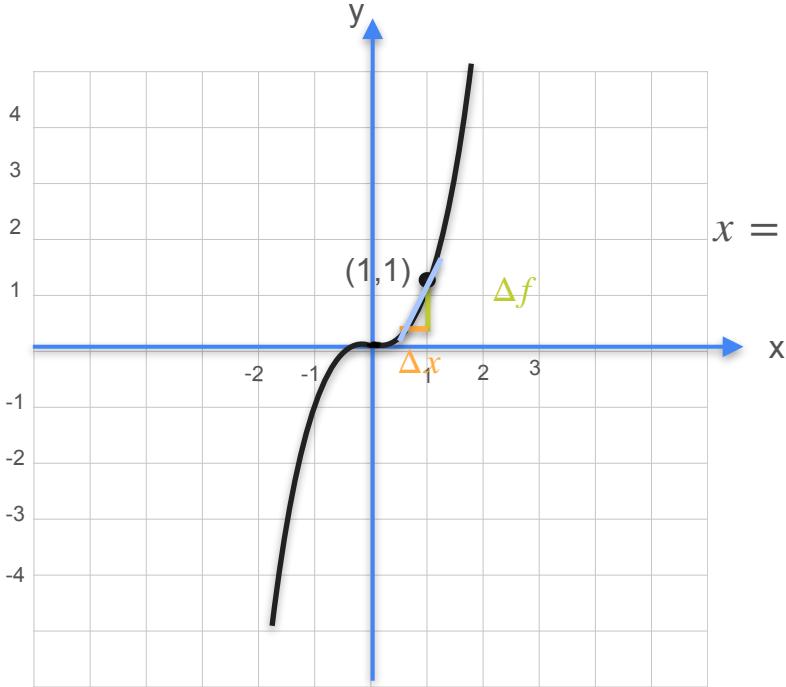


Cubic: $y = f(x) = x^3$

Slope:
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Δx	1.0	1/2
Δf	3.25	0.86
Slope	3.25	1.75

Derivative of Cubic Functions

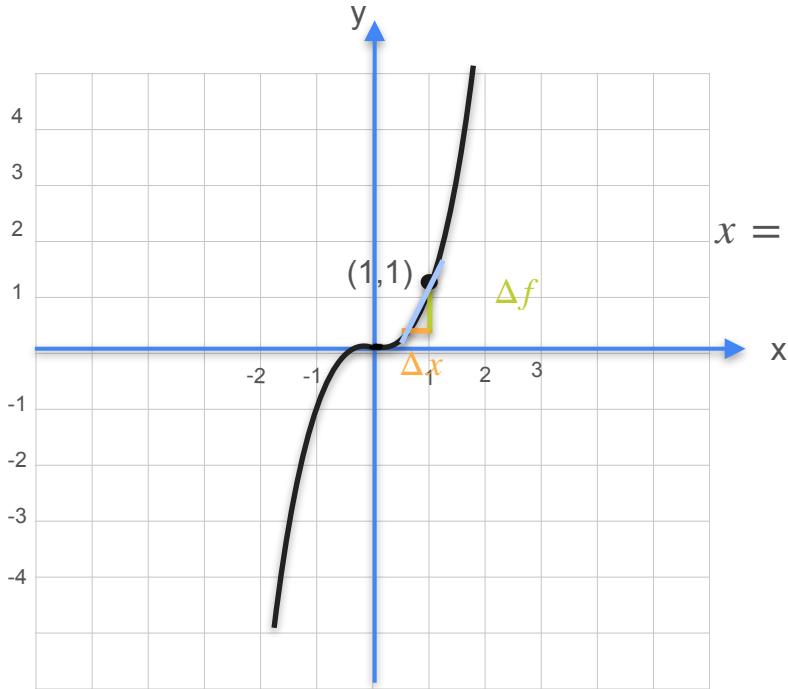


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$
Δf	3.25	0.86	
Slope	3.25	1.75	

Derivative of Cubic Functions

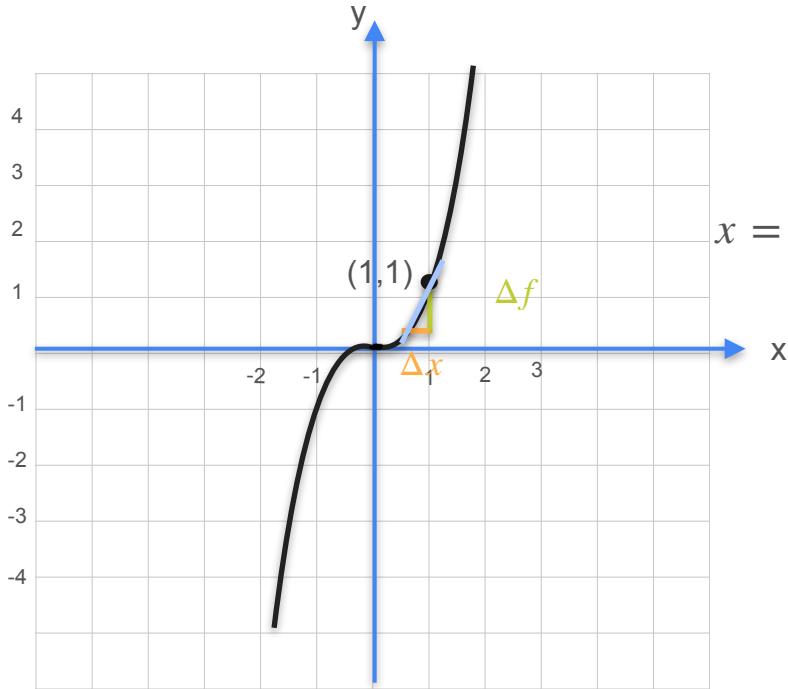


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$
Δf	3.25	0.86	0.30
Slope	3.25	1.75	

Derivative of Cubic Functions

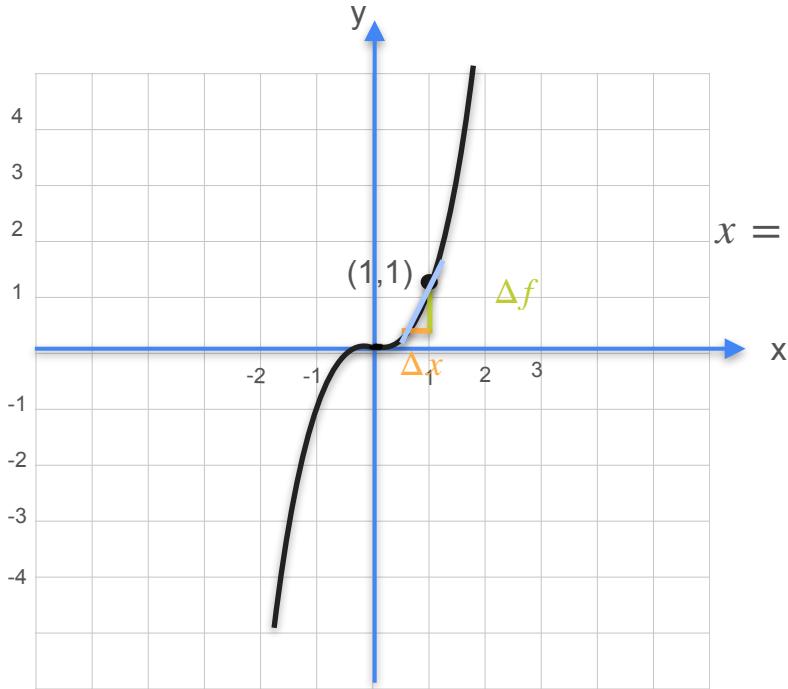


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$
Δf	3.25	0.86	0.30
Slope	3.25	1.75	1.188

Derivative of Cubic Functions



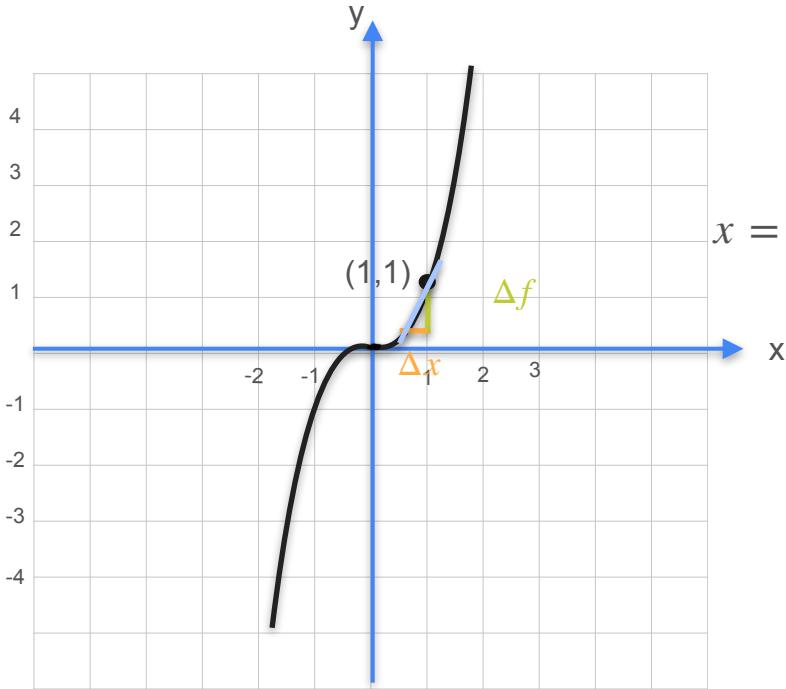
$$x = 0.5$$

Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$	$1/8$
Δf	3.25	0.86	0.30	
Slope	3.25	1.75	1.188	

Derivative of Cubic Functions

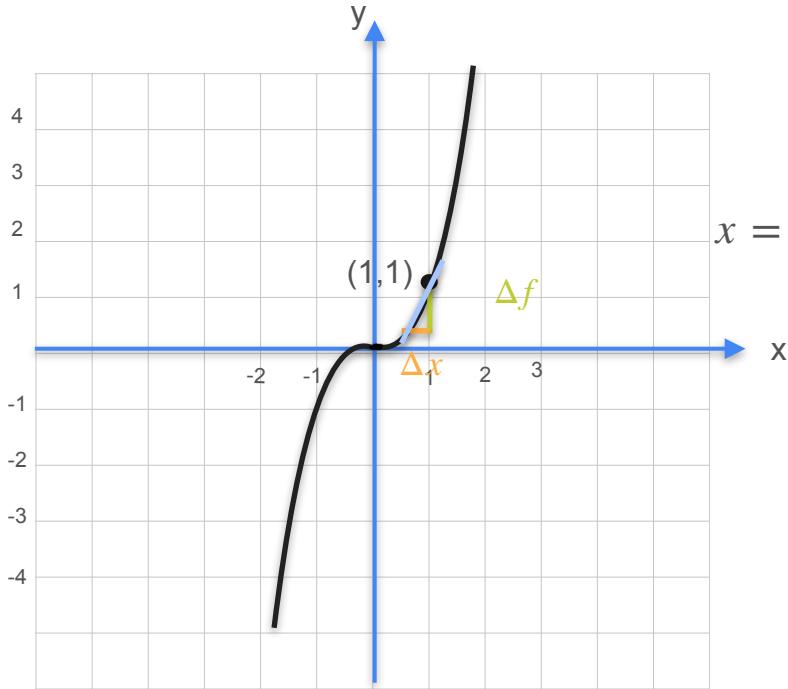


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$	$1/8$
Δf	3.25	0.86	0.30	0.12
Slope	3.25	1.75	1.188	

Derivative of Cubic Functions

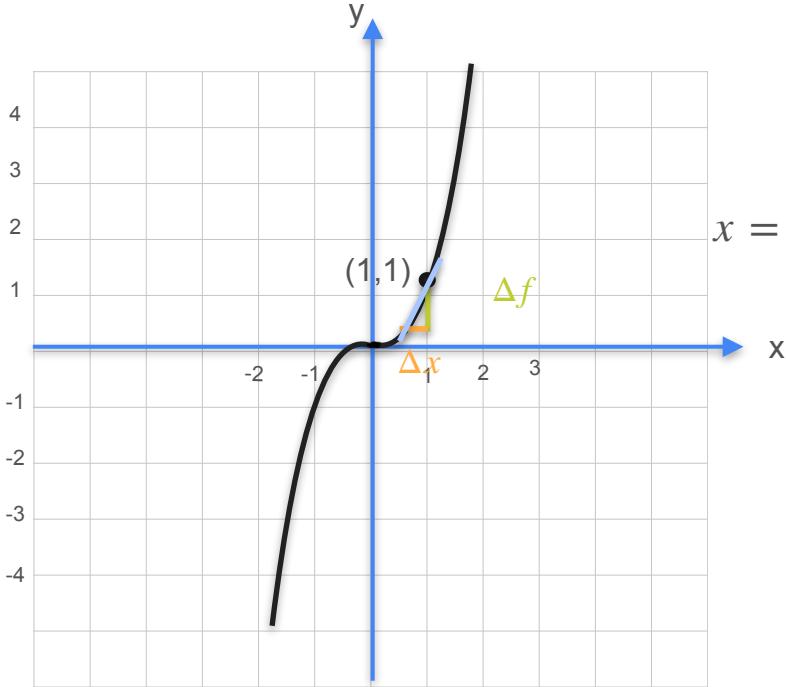


$$\text{Cubic: } y = f(x) = x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Δx	1.0	$1/2$	$1/4$	$1/8$
Δf	3.25	0.86	0.30	0.12
Slope	3.25	1.75	1.188	0.95

Derivative of Cubic Functions

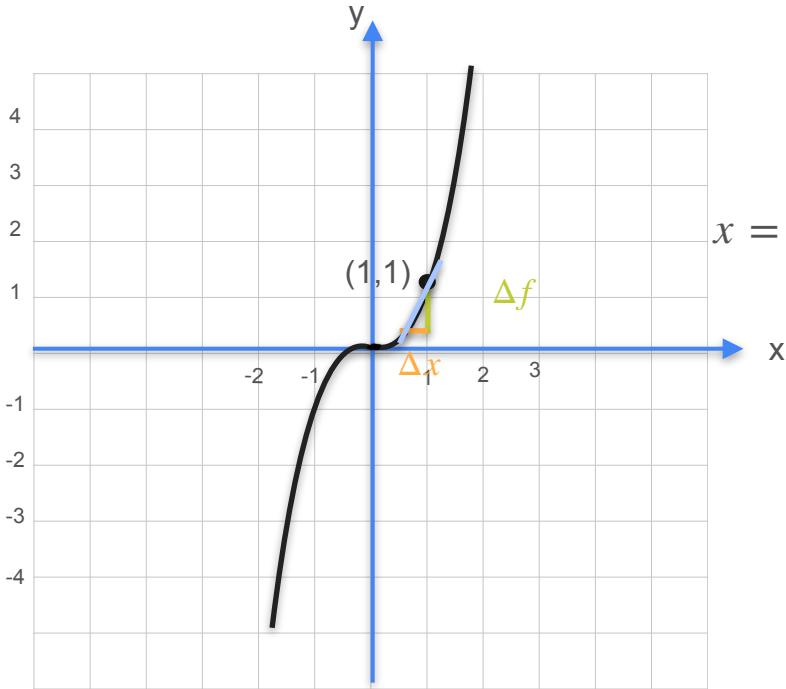


$$\text{Cubic: } y = f(x) = x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$
Δf	3.25	0.86	0.30	0.12	
Slope	3.25	1.75	1.188	0.95	

Derivative of Cubic Functions

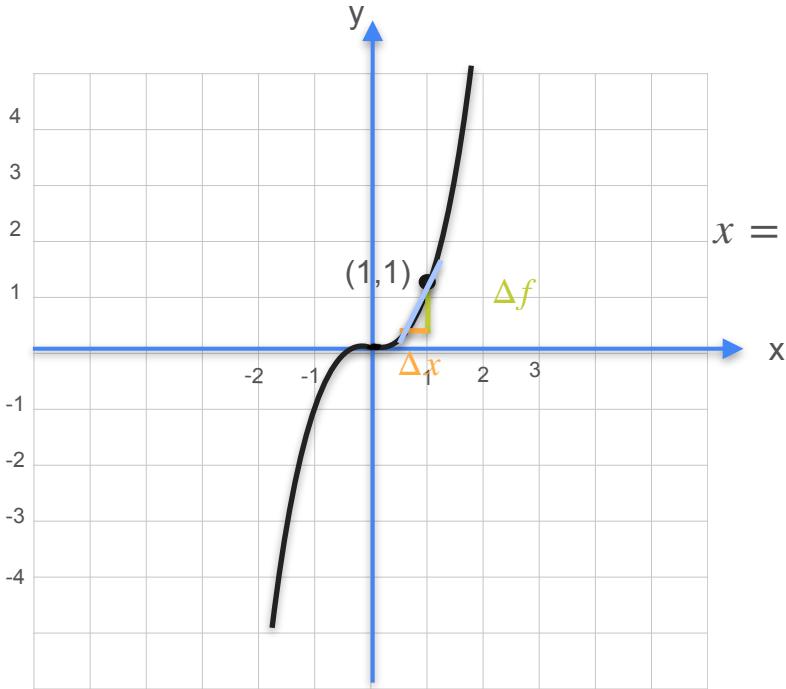


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$
Δf	3.25	0.86	0.30	0.12	0.05
Slope	3.25	1.75	1.188	0.95	

Derivative of Cubic Functions

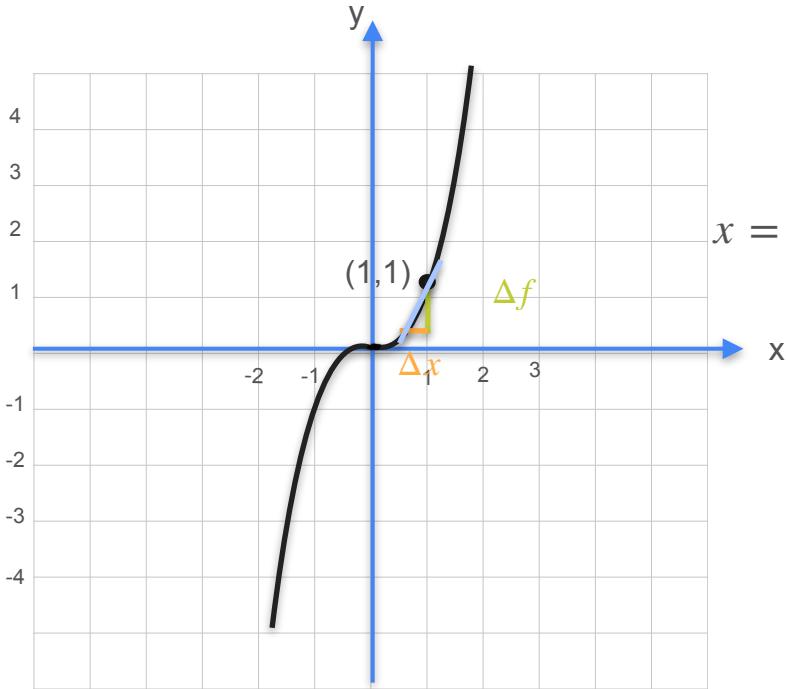


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$
Δf	3.25	0.86	0.30	0.12	0.05
Slope	3.25	1.75	1.188	0.95	0.85

Derivative of Cubic Functions

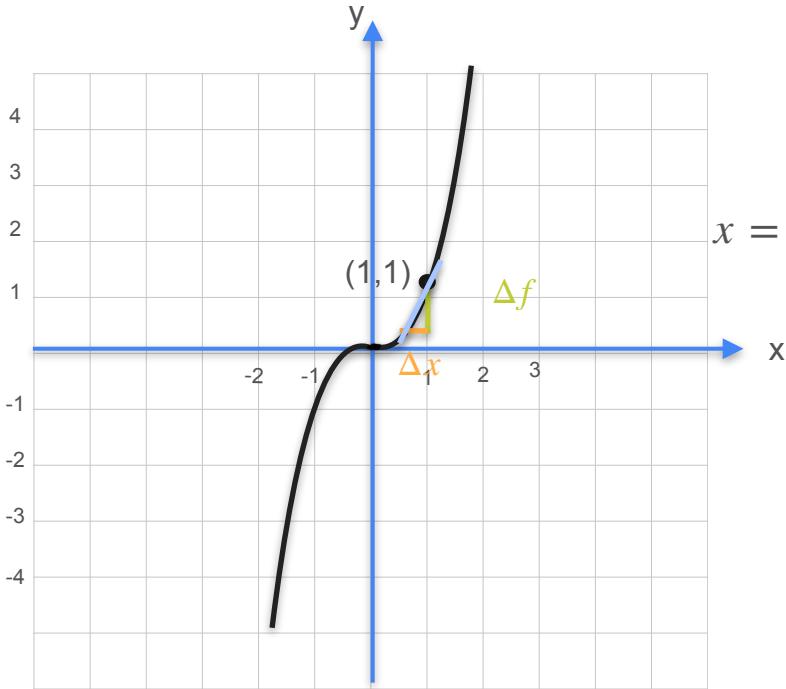


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
Δf	3.25	0.86	0.30	0.12	0.05	
Slope	3.25	1.75	1.188	0.95	0.85	

Derivative of Cubic Functions

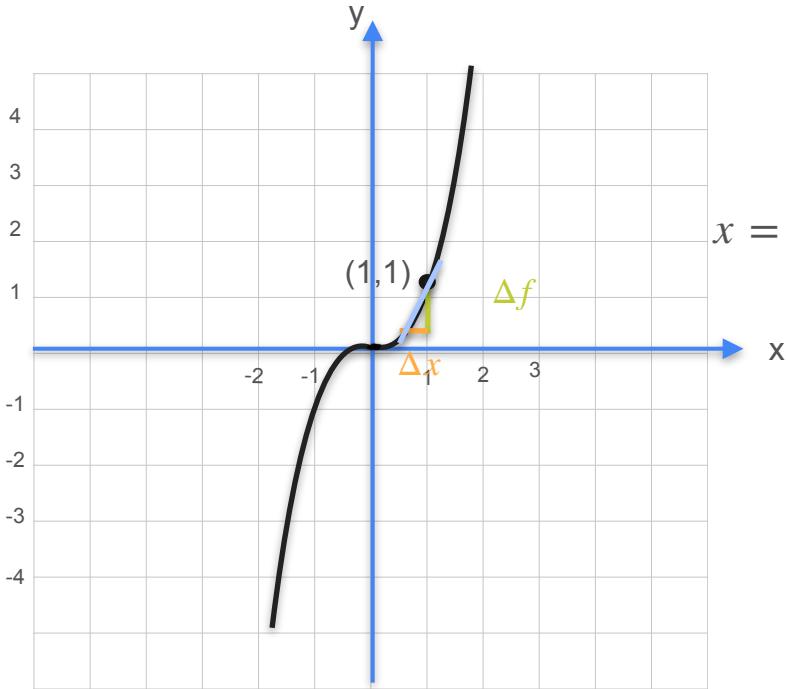


Cubic: $y = f(x) = x^3$

Slope:
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	

Derivative of Cubic Functions

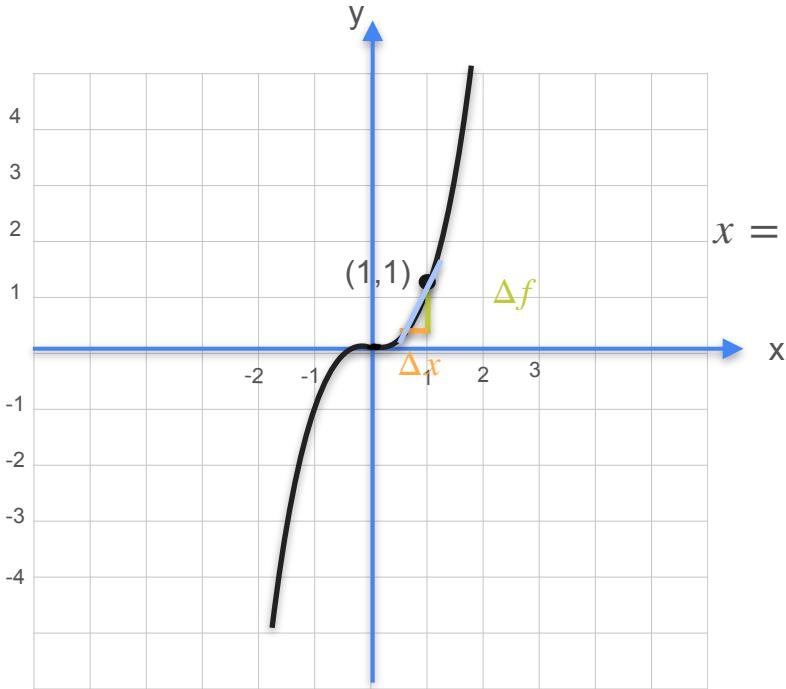


Cubic: $y = f(x) = x^3$

Slope:
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

Δx	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

Derivative of Cubic Functions



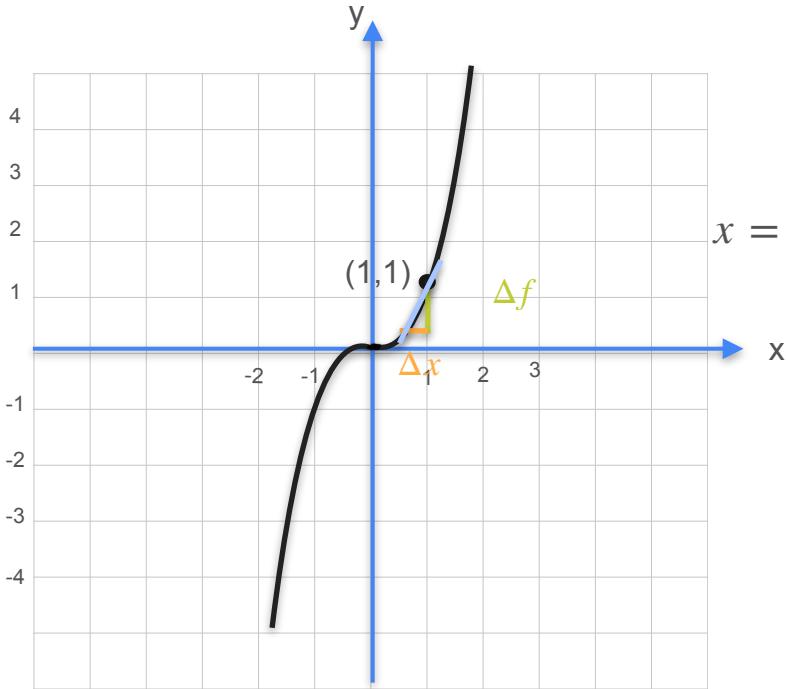
Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

$$f'(0.5) = \frac{d}{dx} f(0.5) = 0.75 = 3 \times 0.5^2$$

Derivative of Cubic Functions



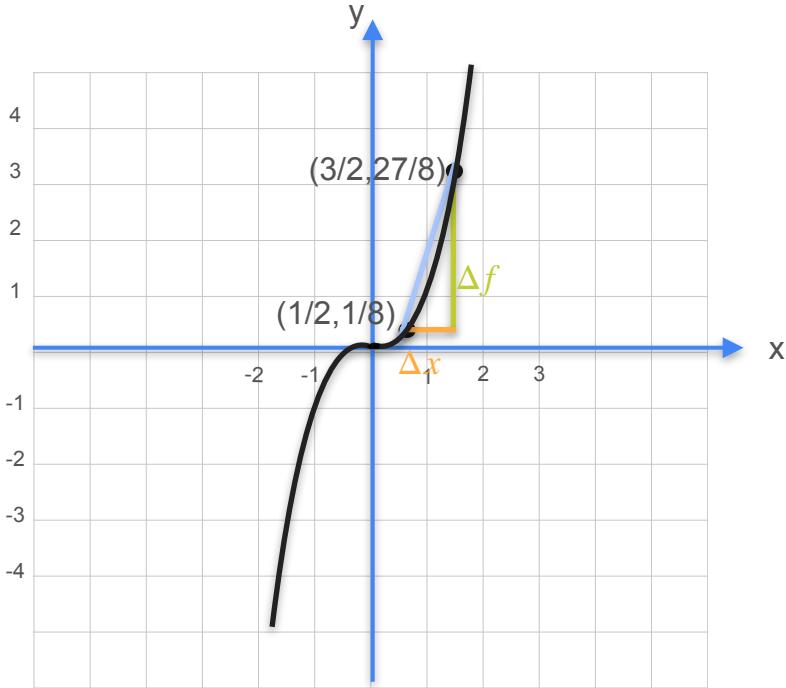
Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

$$f'(0.5) = \frac{d}{dx} f(0.5) = 0.75 = 3 \times 0.5^2 = 3 \times 0.5^2$$

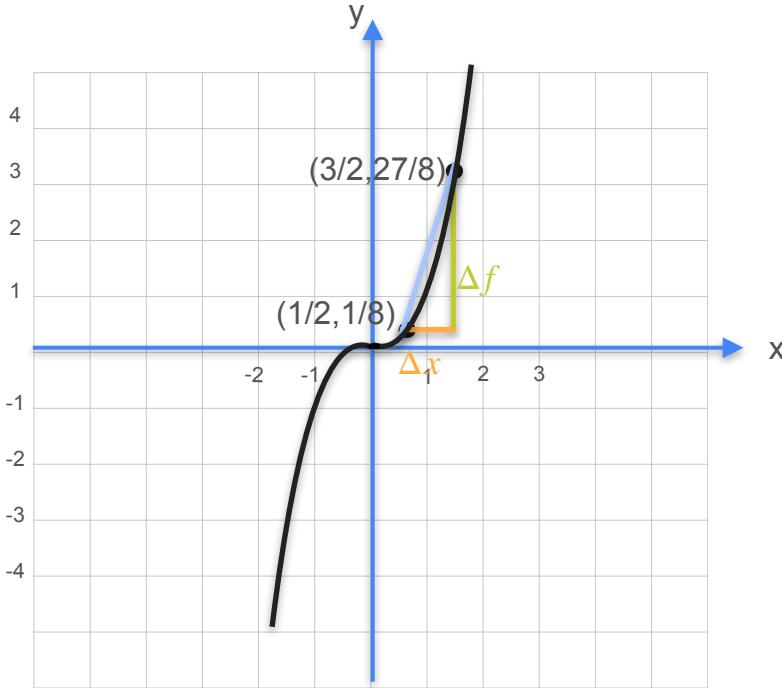
Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

Slope:
$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Derivative of Cubic Functions

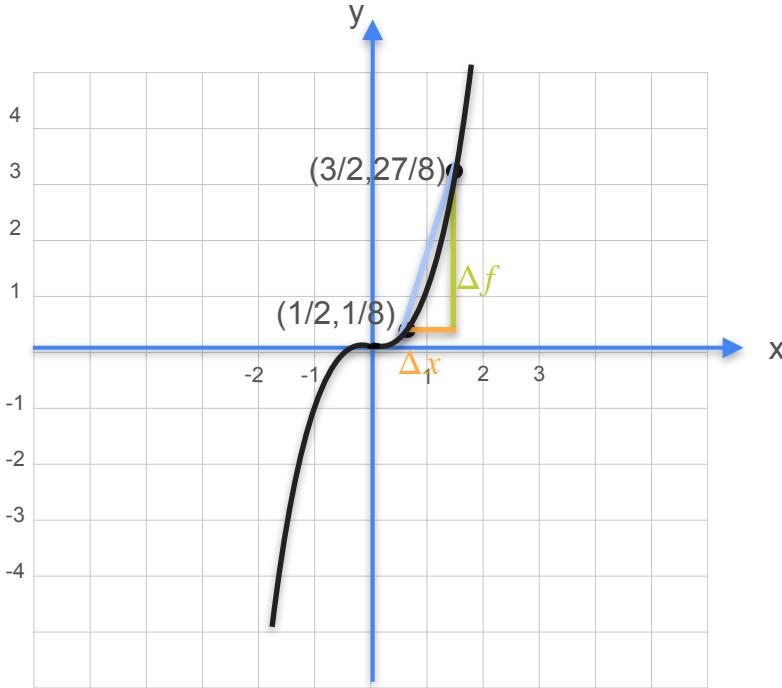


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x}$$

Derivative of Cubic Functions

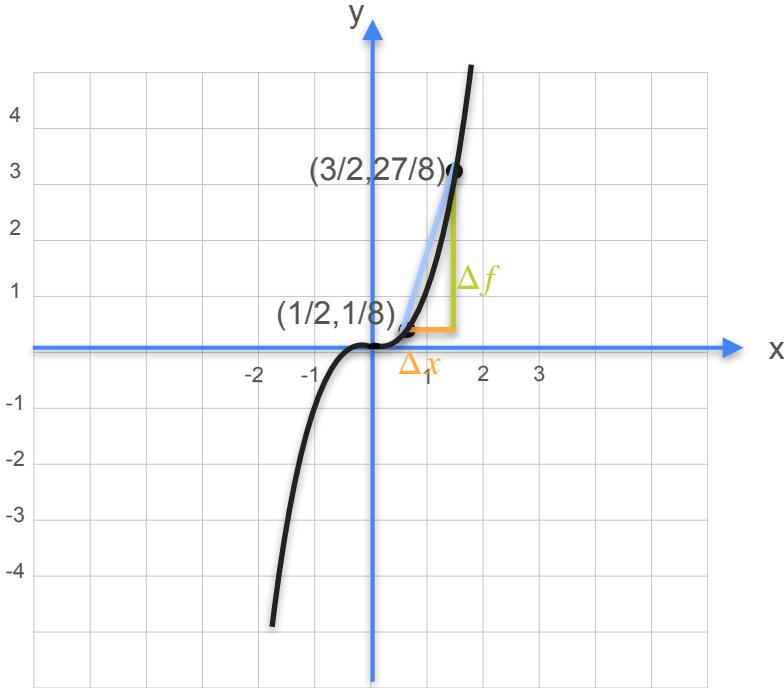


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

Derivative of Cubic Functions

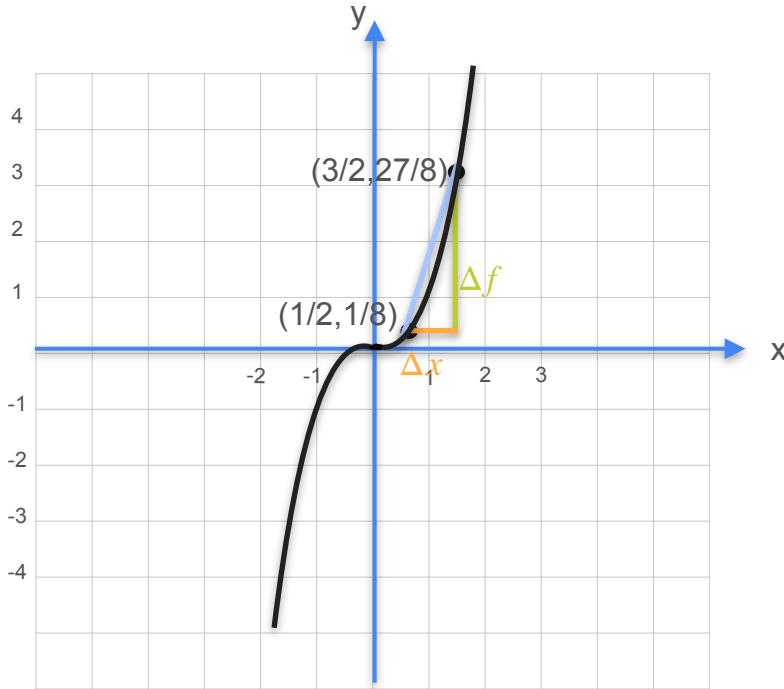


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

Derivative of Cubic Functions



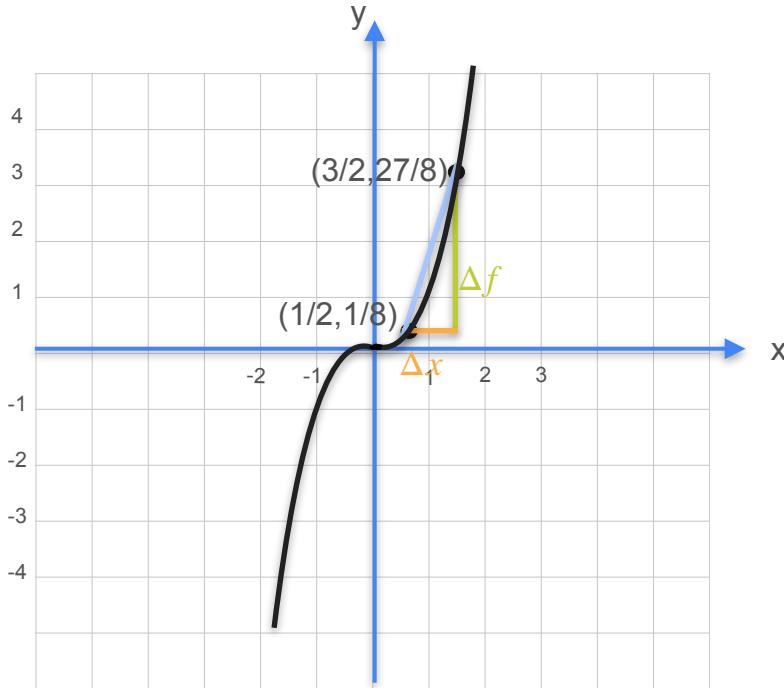
Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

Derivative of Cubic Functions

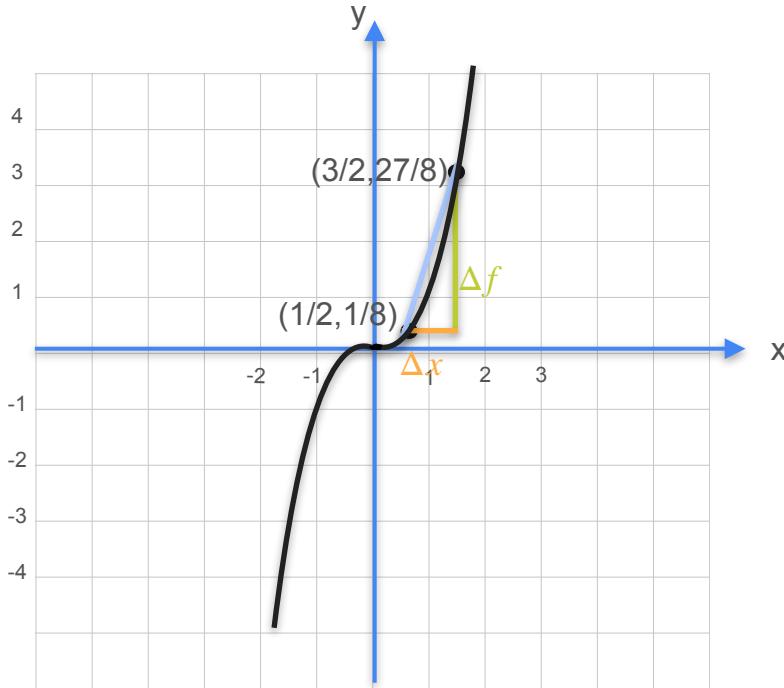


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

Derivative of Cubic Functions

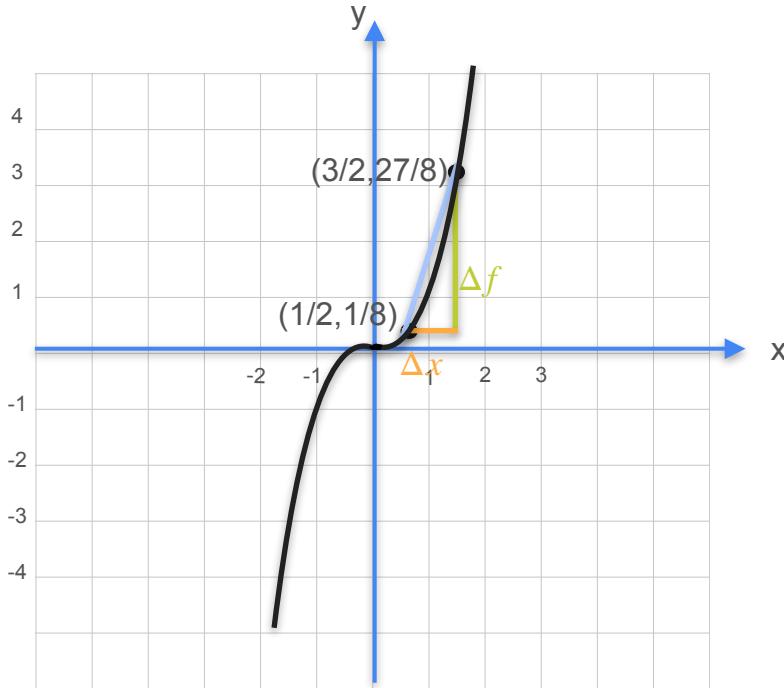


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

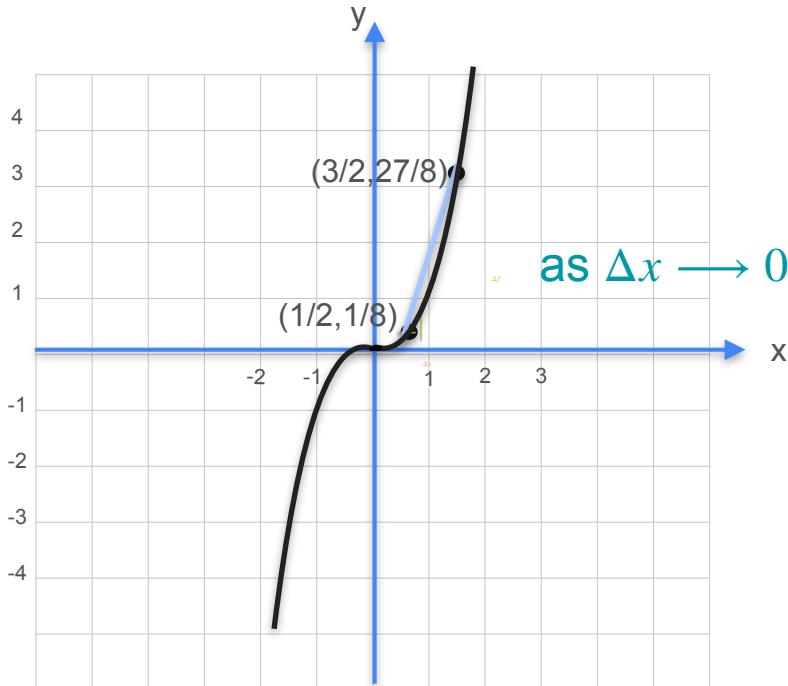
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

$$= 3x\Delta x + 3x^2 + \Delta x^2$$

Derivative of Cubic Functions

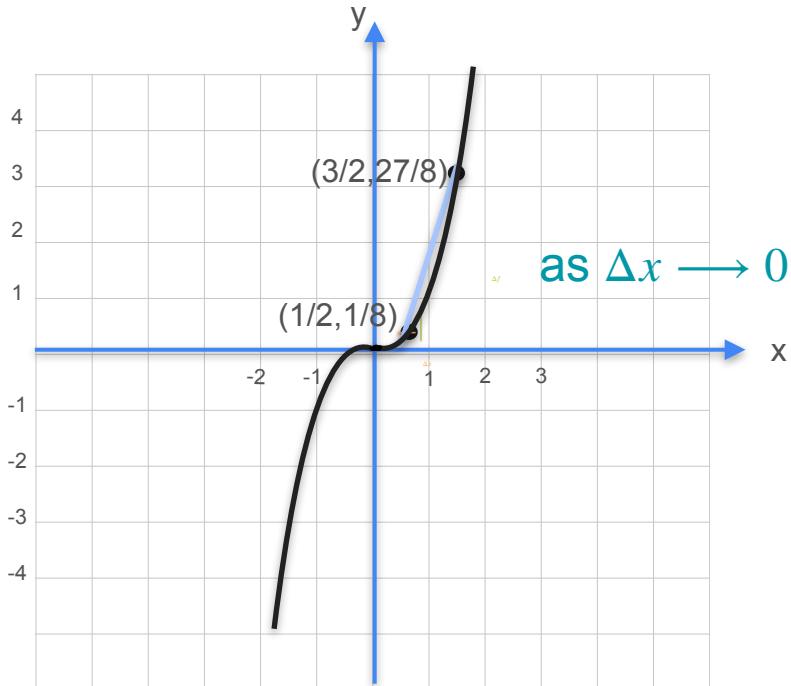


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{\Delta f}{\Delta x} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

Derivative of Cubic Functions

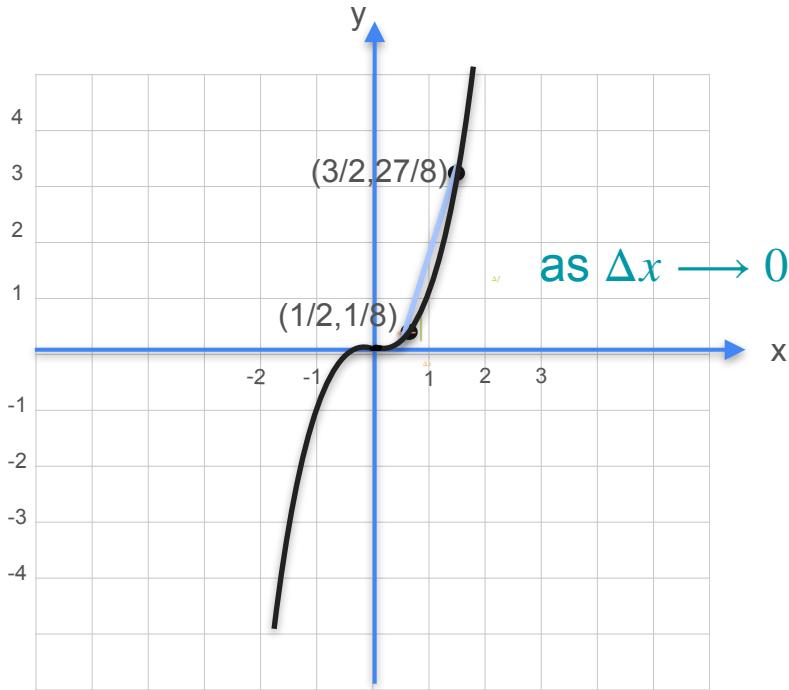


Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

Derivative of Cubic Functions



Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

$$f(x) = x^3 \rightarrow f'(x) = 3x^2$$



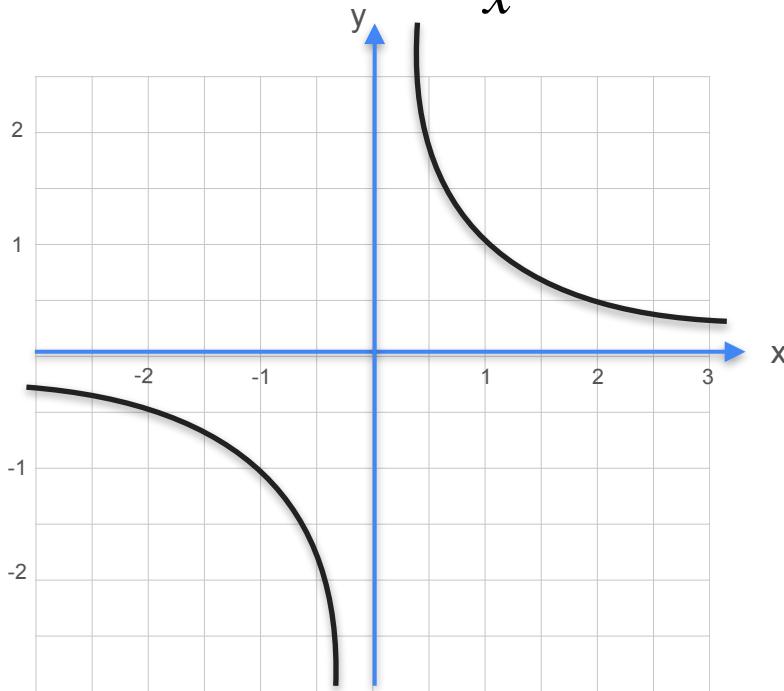
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Derivatives and Optimization

**Some common derivatives:
Other power functions**

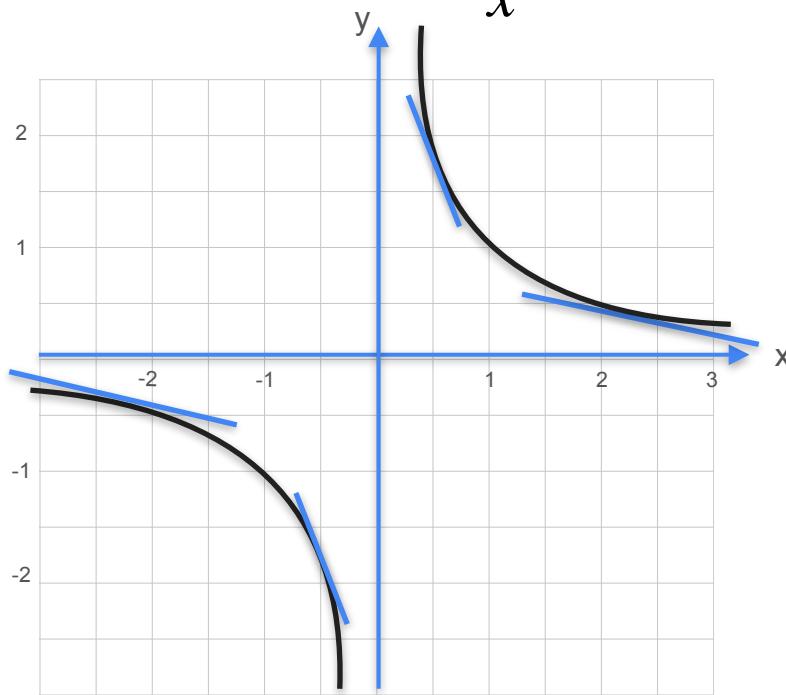
Derivative of $\frac{1}{x}$

Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

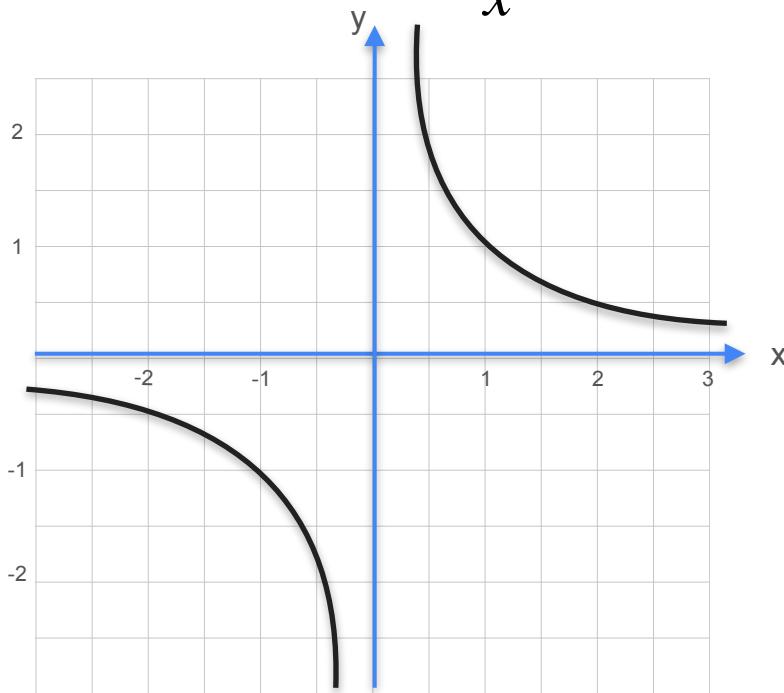
Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope: $\frac{\Delta f}{\Delta x}$

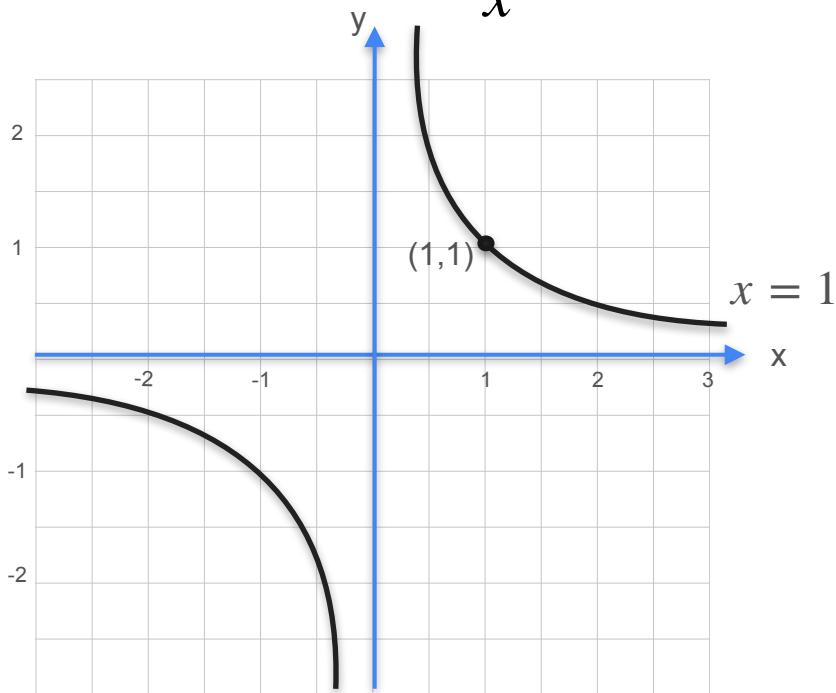
Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Derivative of $\frac{1}{x}$

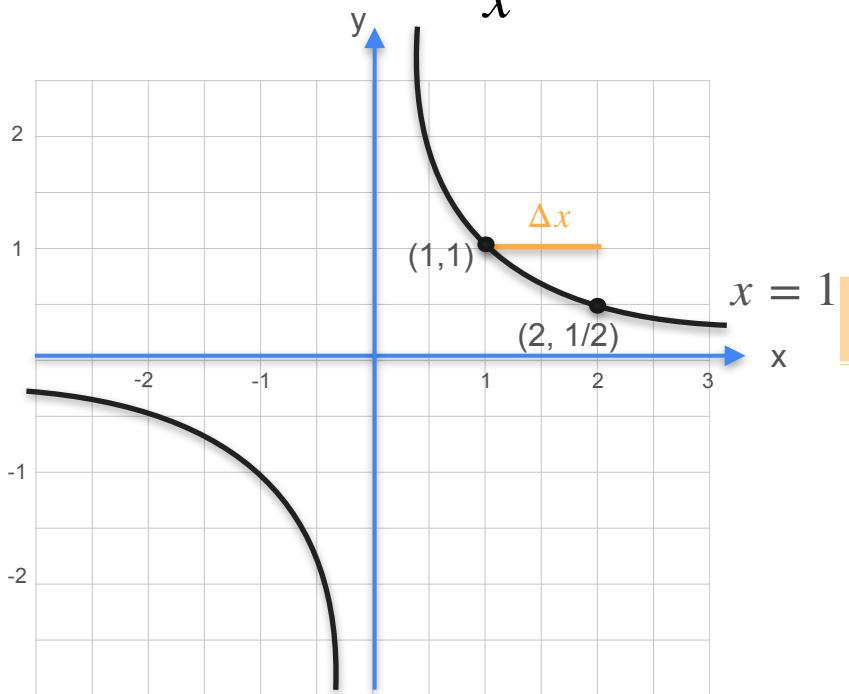


Slope:

$$y = f(x) = x^{-1} = \frac{1}{x}$$
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$$

.

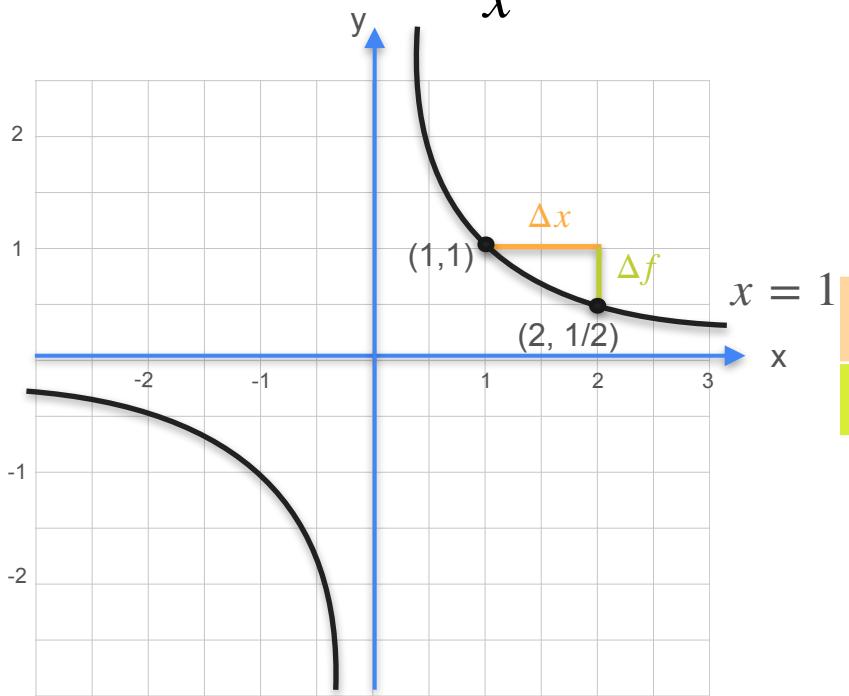
Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$
$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$$

Δx 1.0

Derivative of $\frac{1}{x}$

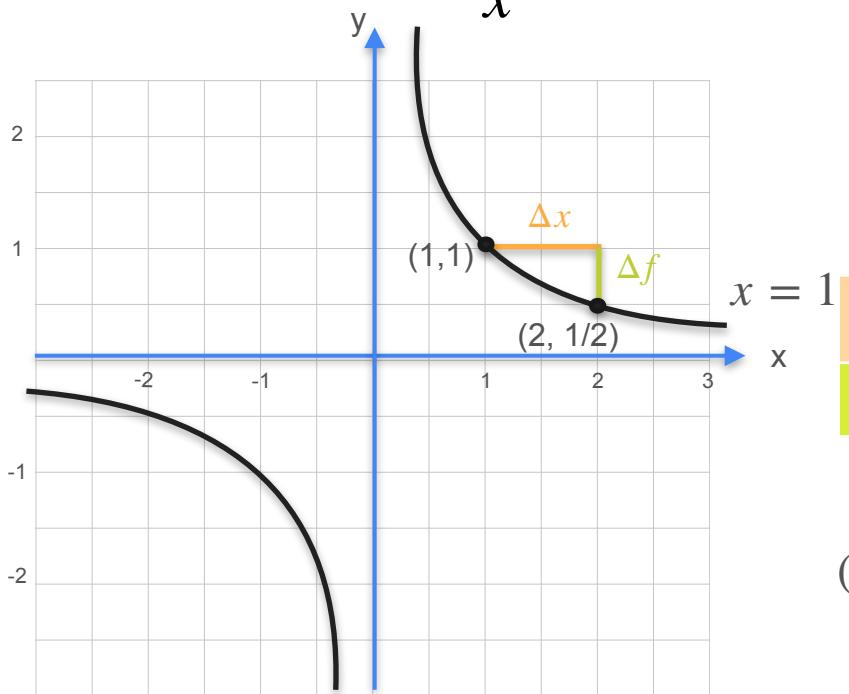


Slope:

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$$

Δx	1.0
Δf	

Derivative of $\frac{1}{x}$



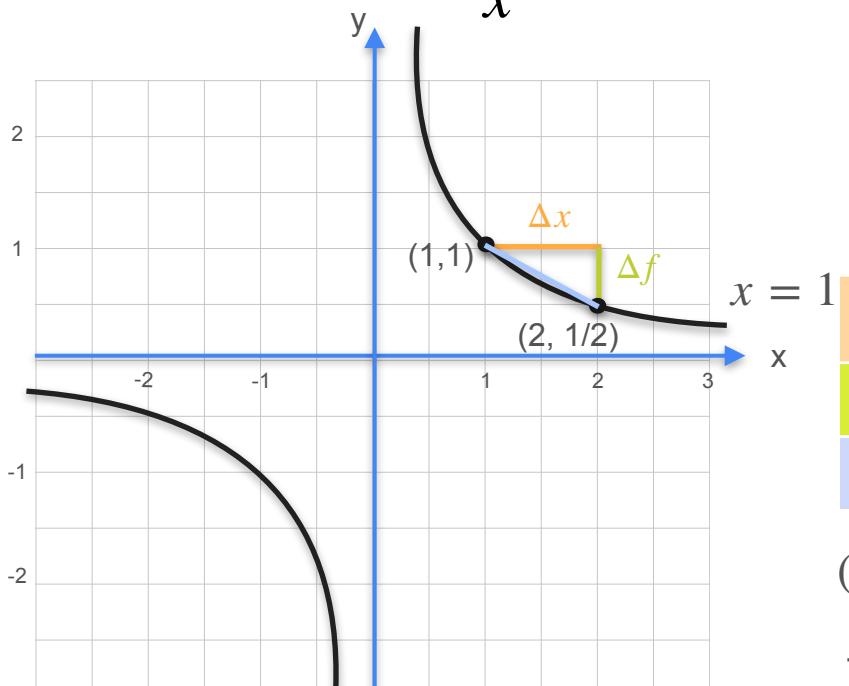
$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0
Δf	-0.5

$$(1 + 1)^{-1} - 1^{-1} = \frac{1}{2} - 1$$

Derivative of $\frac{1}{x}$

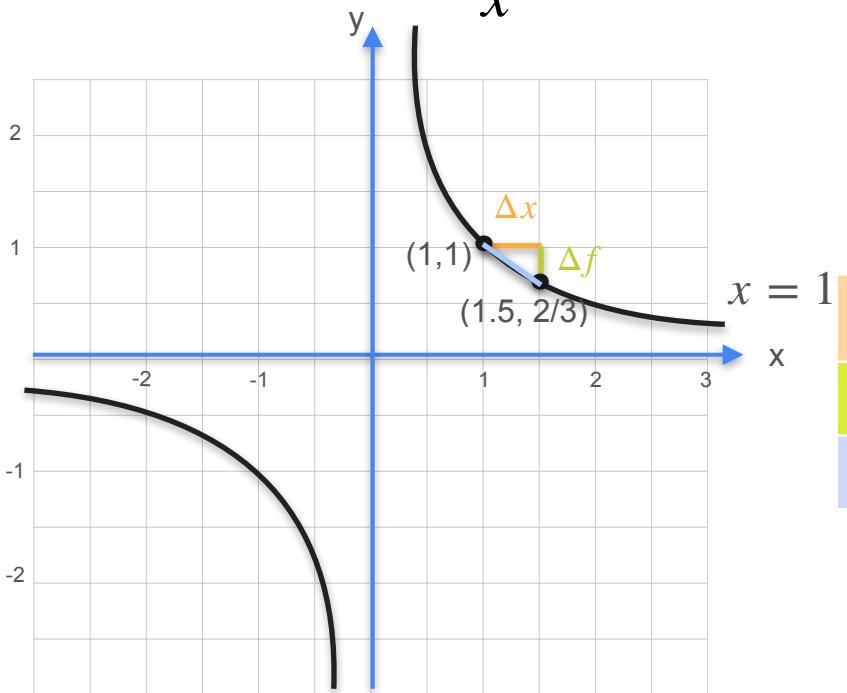


Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0
Δf	-0.5
Slope	-0.5

$$\begin{aligned}(1 + 1)^{-1} - 1^{-1} &= \frac{1}{2} - 1 \\ -0.5 \\ \hline 1\end{aligned}$$

Derivative of $\frac{1}{x}$

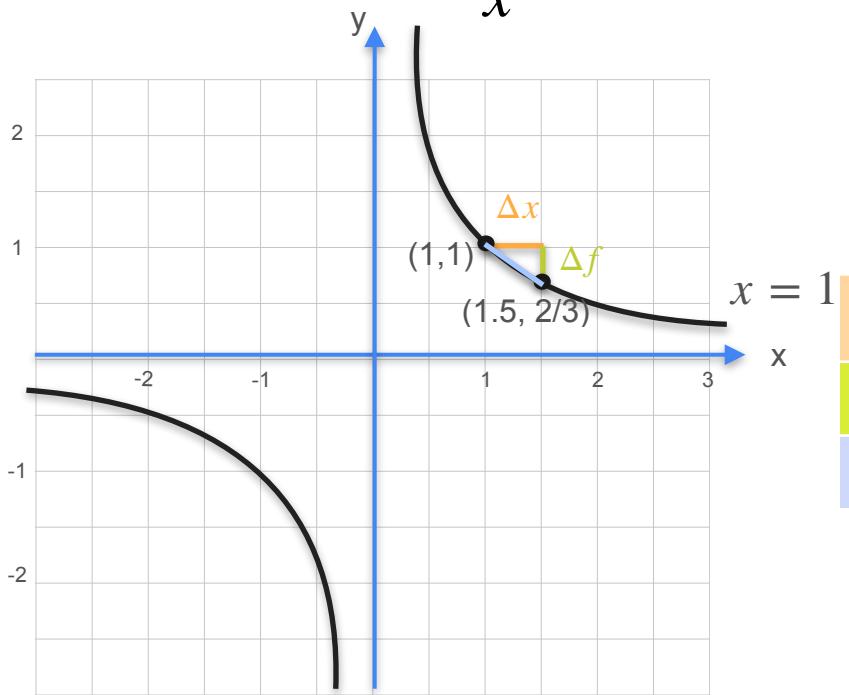


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0
Δf	-0.5
Slope	-0.5

Derivative of $\frac{1}{x}$

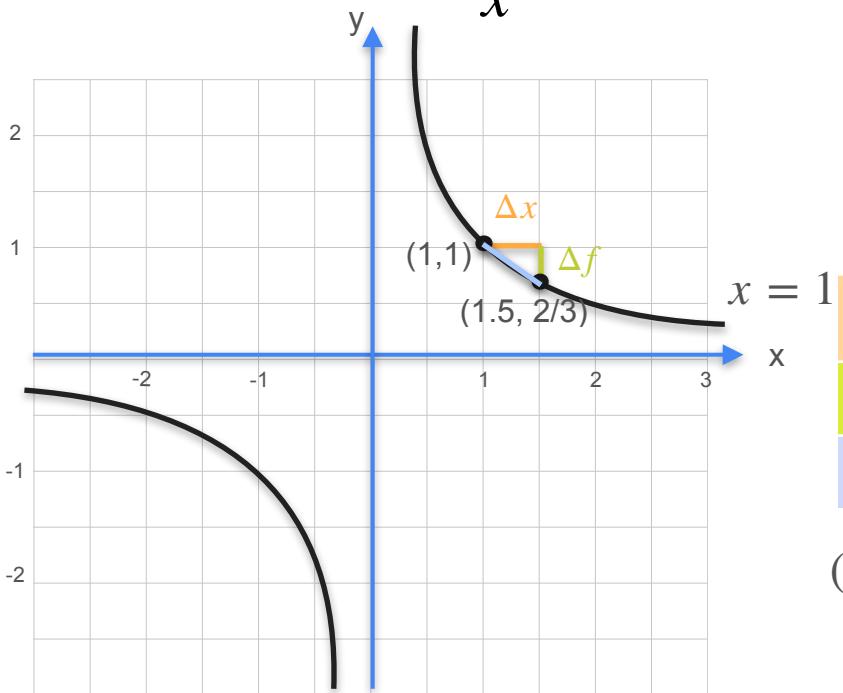


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	$1/2$
Δf	-0.5	
Slope	-0.5	

Derivative of $\frac{1}{x}$



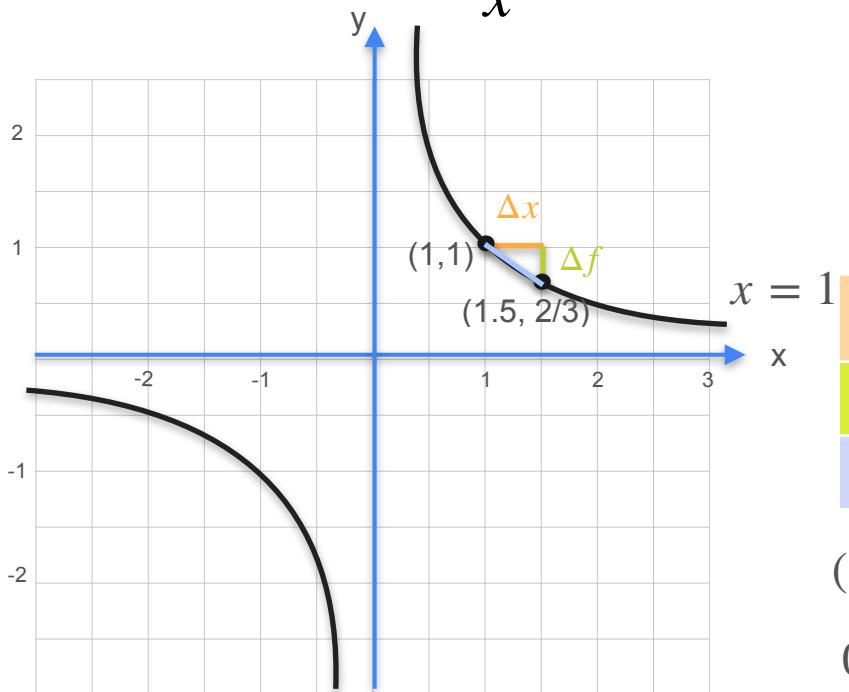
Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	1/2
Δf	-0.5	-0.33
Slope	-0.5	

$$(1 + 0.5)^{-1} - 1^{-1} = \frac{2}{3} - 1$$

Derivative of $\frac{1}{x}$



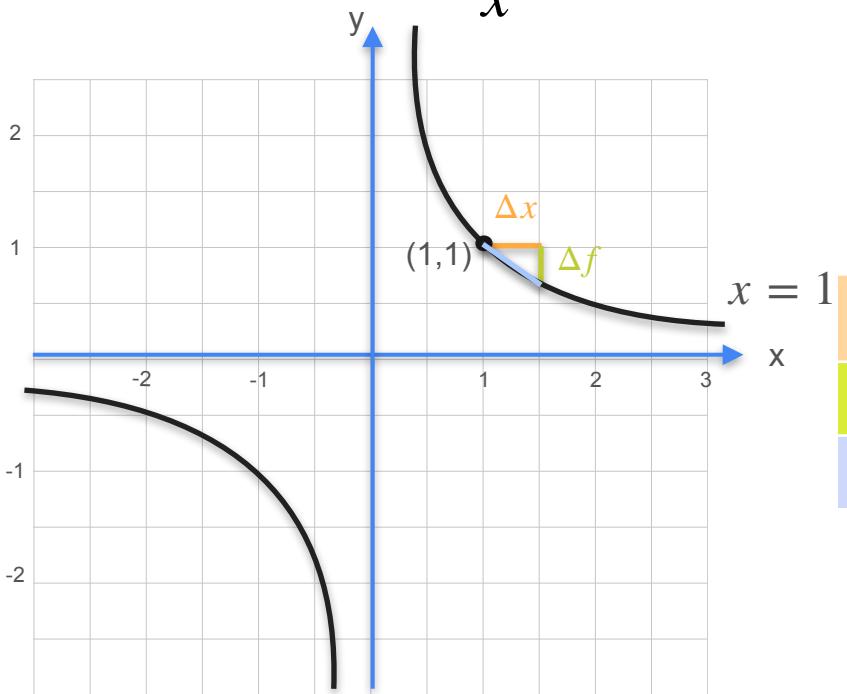
Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	$1/2$
Δf	-0.5	-0.33
Slope	-0.5	-0.67

$$(1 + 0.5)^{-1} - 1^{-1} = \frac{2}{3} - 1$$
$$\frac{0.33}{0.5}$$

Derivative of $\frac{1}{x}$

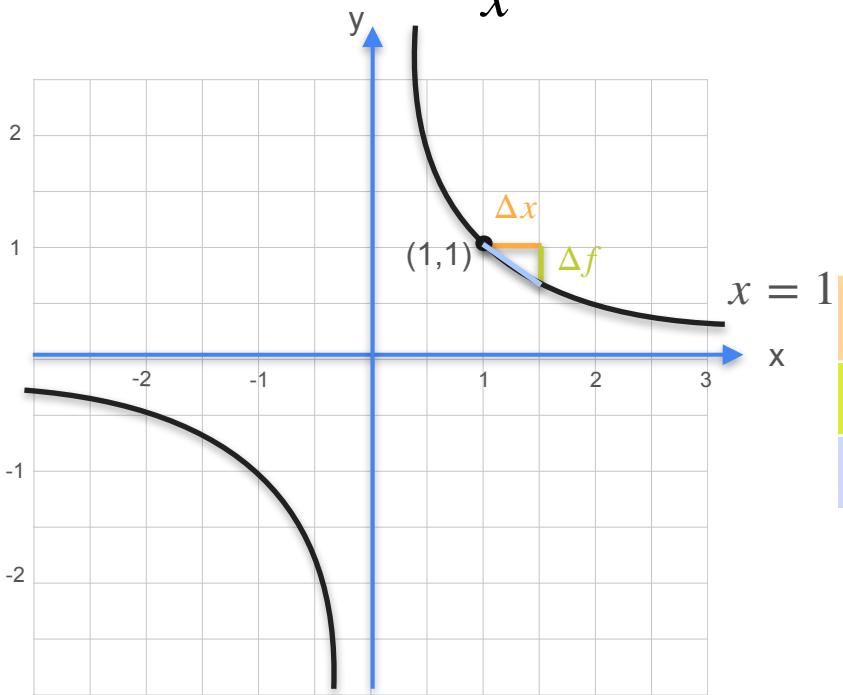


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	$1/2$
Δf	-0.5	-0.33
Slope	-0.5	-0.67

Derivative of $\frac{1}{x}$

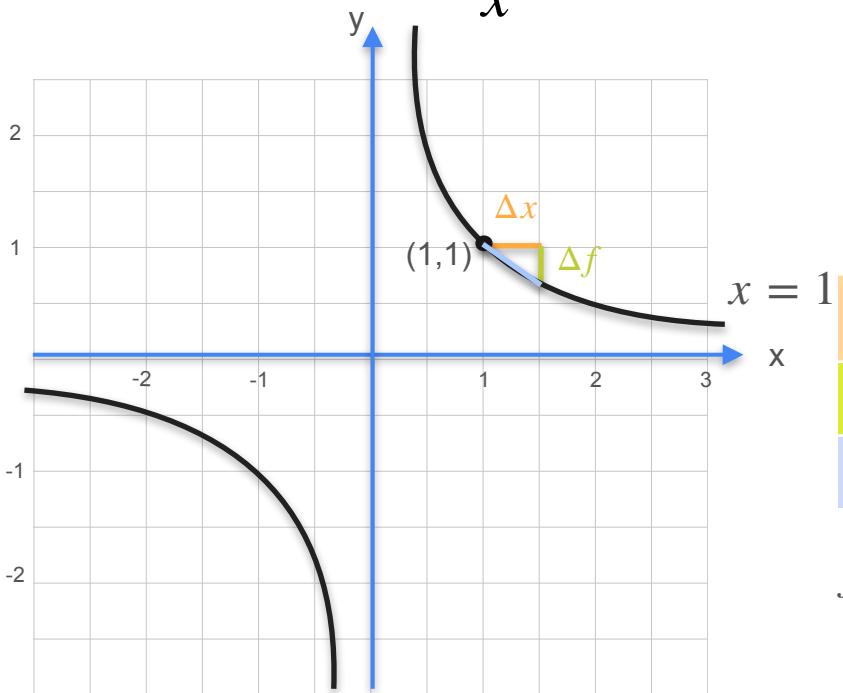


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	-0.5	-0.33	-0.2	-0.11	-0.06	-0.001
Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

Derivative of $\frac{1}{x}$



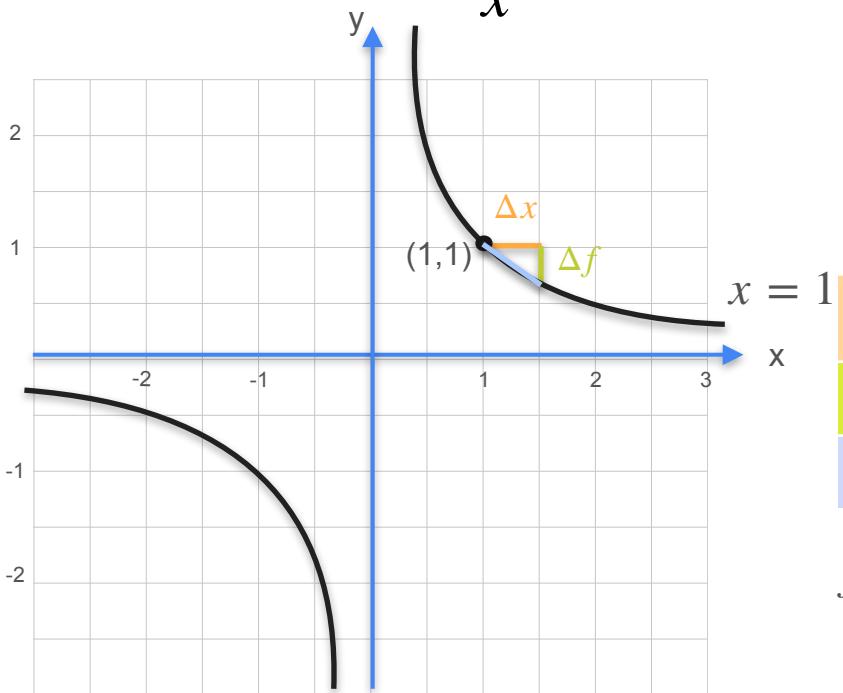
Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	-0.5	-0.33	-0.2	-0.11	-0.06	-0.001
Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

$$f'(1) = \frac{d}{dx} f(1) = -1$$

Derivative of $\frac{1}{x}$



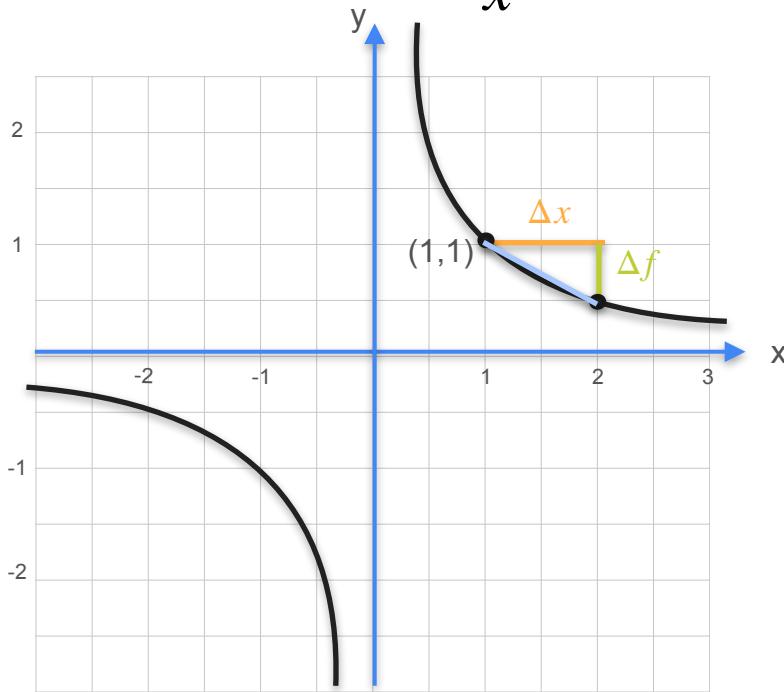
Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	-0.5	-0.33	-0.2	-0.11	-0.06	-0.001
Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

$$f'(1) = \frac{d}{dx} f(1) = -1 = -1 \times 1^2$$

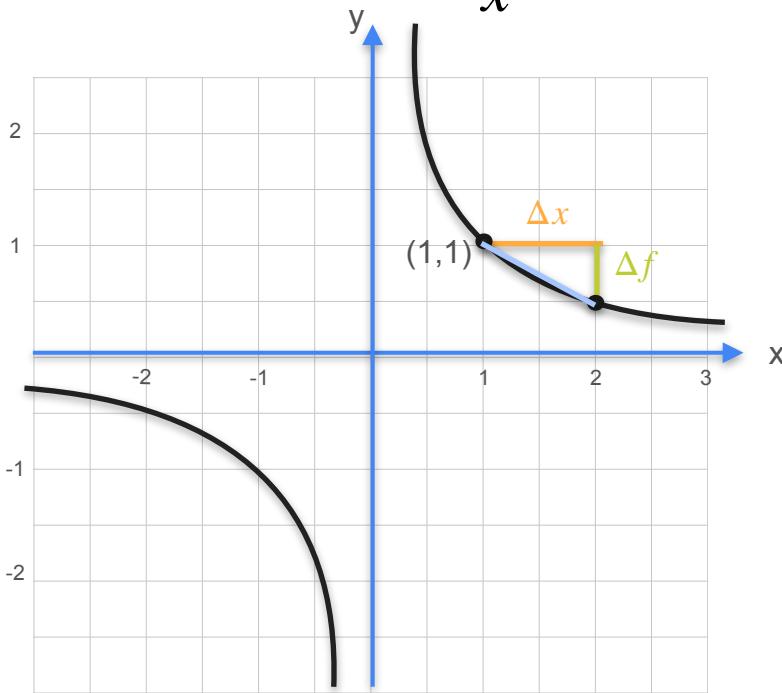
Derivative of $\frac{1}{x}$



Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

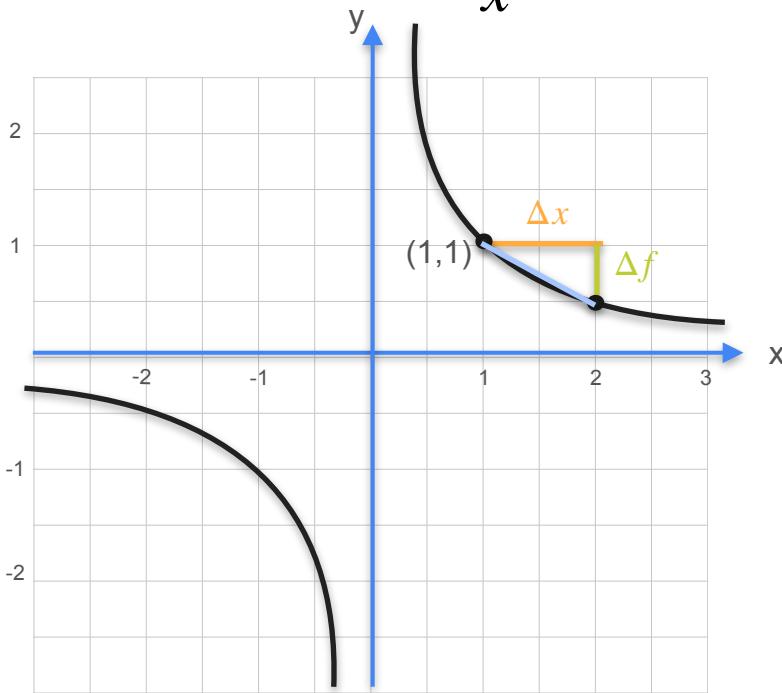
Derivative of $\frac{1}{x}$



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Derivative of $\frac{1}{x}$

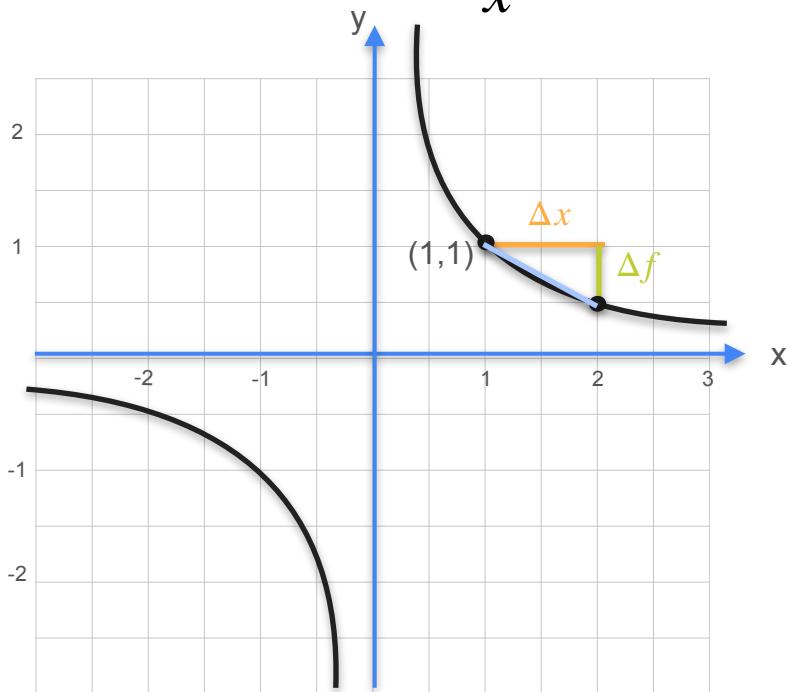


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

Derivative of $\frac{1}{x}$



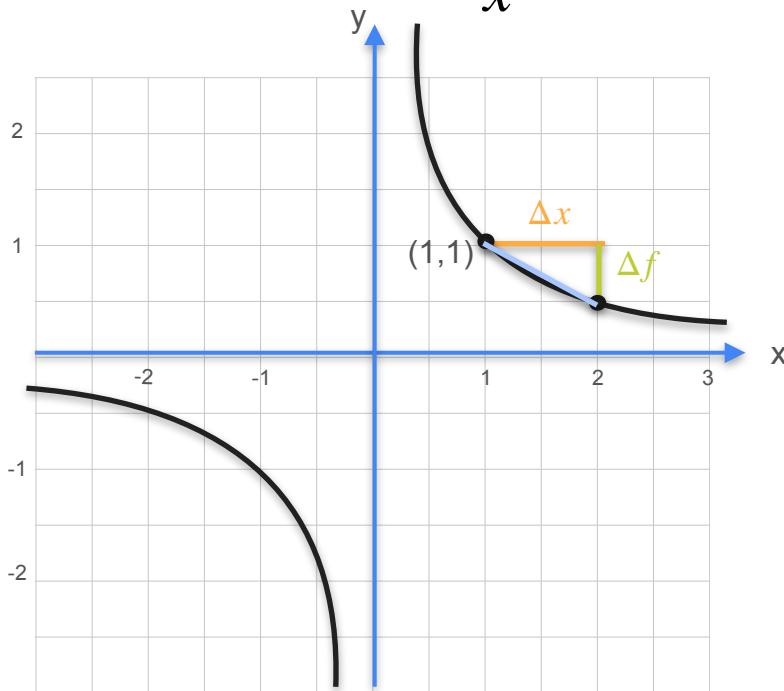
Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x}$$

Derivative of $\frac{1}{x}$



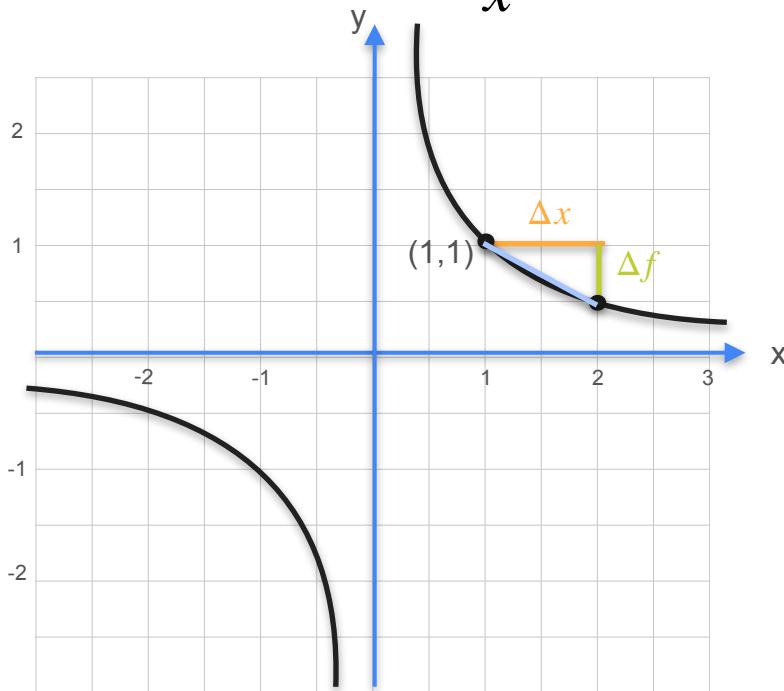
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$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x}$$

Derivative of $\frac{1}{x}$



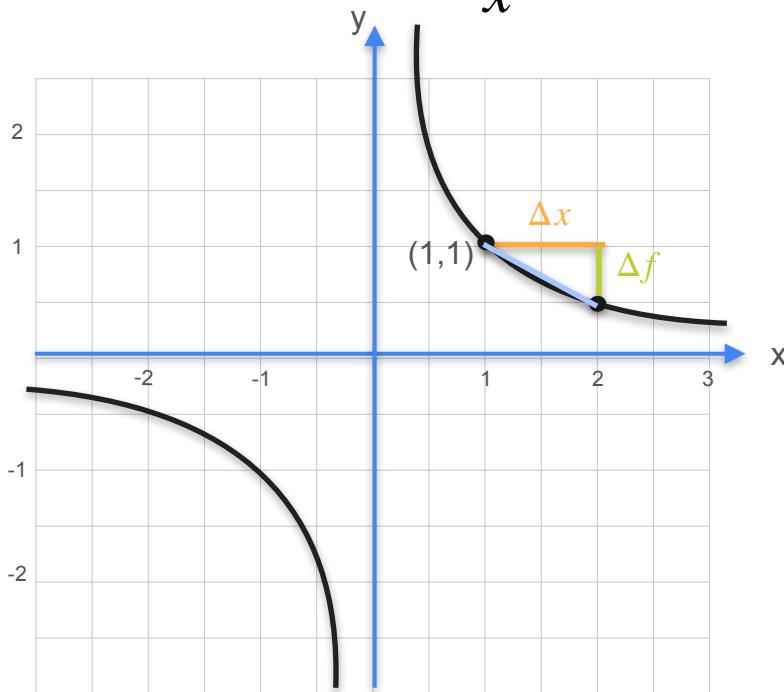
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Derivative of $\frac{1}{x}$



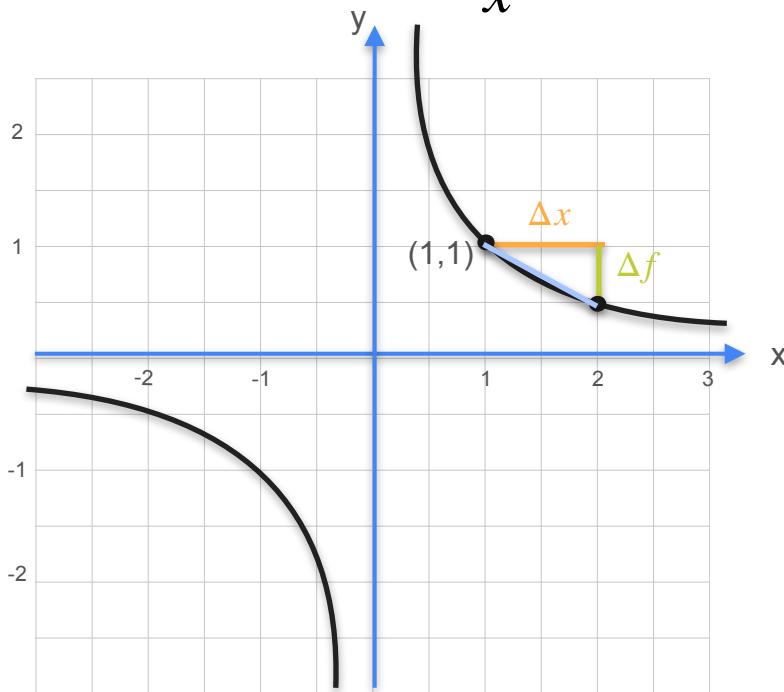
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Derivative of $\frac{1}{x}$



Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

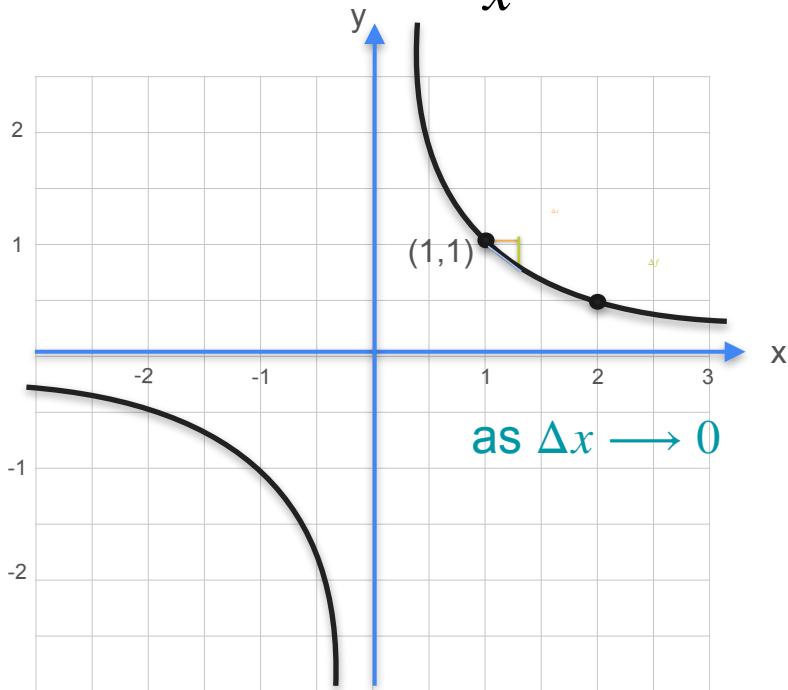
Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x}$$

$$= -\frac{1}{x^2 + x\Delta x}$$

Derivative of $\frac{1}{x}$



Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

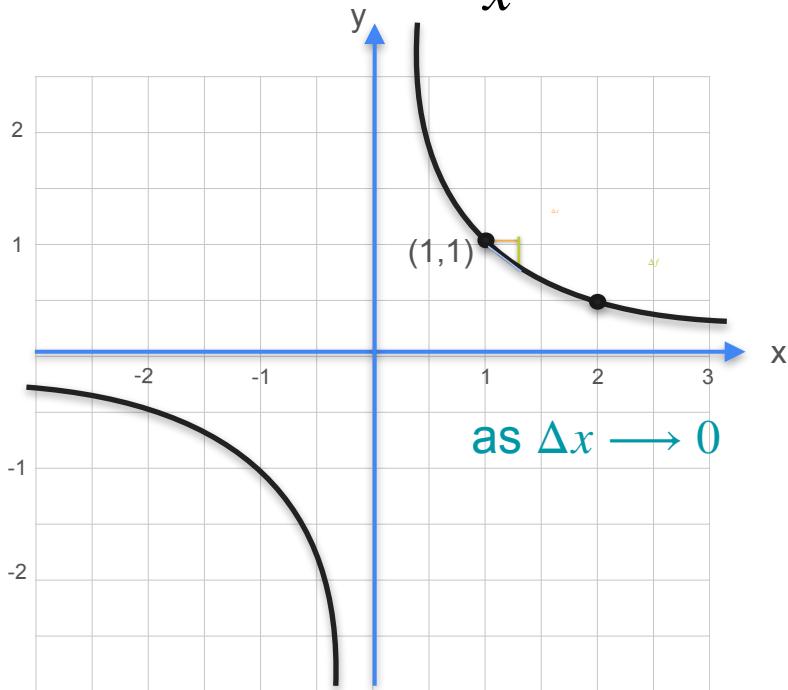
Slope:
$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

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$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x}$$

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Derivative of $\frac{1}{x}$

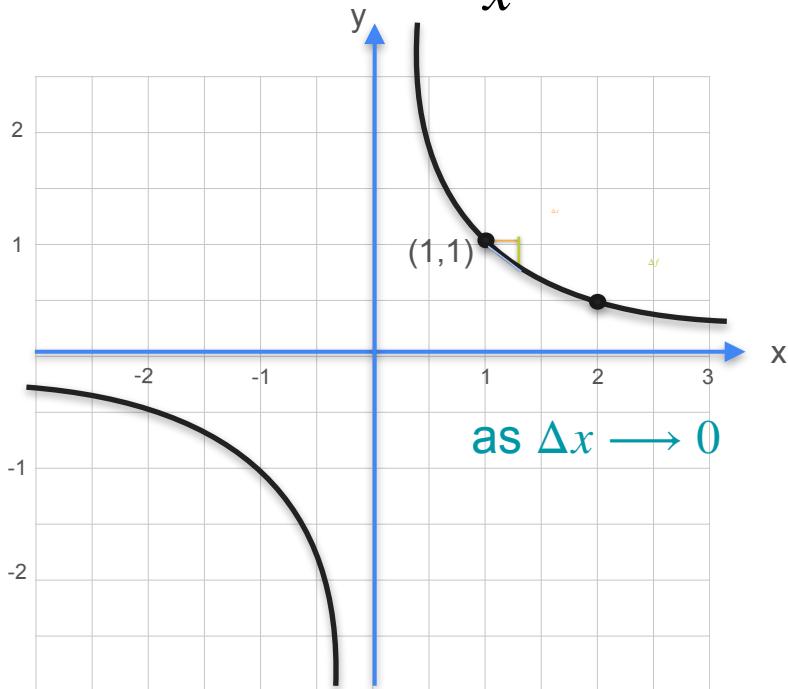


Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x} \\ &= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x} \\ &= -\frac{1}{x^2 + x\Delta x} \quad 0\end{aligned}$$

Derivative of $\frac{1}{x}$



Inverse: $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

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$$f(x) = x^{-1} \rightarrow f'(x) = -x^{-2}$$

Derivative of Power Functions

Derivative of Power Functions

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

Derivative of Power Functions

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^{\boxed{2}}$$

$$f(x) = x^{\boxed{3}}$$

$$f(x) = x^{\boxed{-1}}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$

$$f'(x) = 2x^1$$

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f(x) = x^{-1}$$

$$f'(x) = (-1)x^{-2}$$



Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$

$$f(x) = x^3$$
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Derivative of Power Functions

Can you spot the pattern?

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Derivative of Power Functions

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Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$

-1

$$f(x) = x^3$$
$$f'(x) = 3x^2$$

-1

$$f(x) = x^{-1}$$
$$f'(x) = (-1)x^{-2}$$

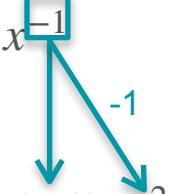
-1

Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$


$$f(x) = x^3$$
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Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$


$$f(x) = x^3$$
$$f'(x) = 3x^2$$


$$f(x) = x^{-1}$$
$$f'(x) = (-1)x^{-2}$$


$$f(x) = x^n \quad \rightarrow \quad f'(x) = \frac{d}{dx} f(x) = nx^{n-1}$$



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Derivatives and Optimization

The inverse function and its derivative

Inverse Function

What's an inverse?

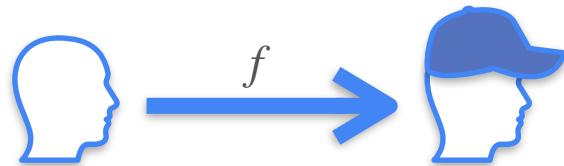
Inverse Function

What's an inverse?



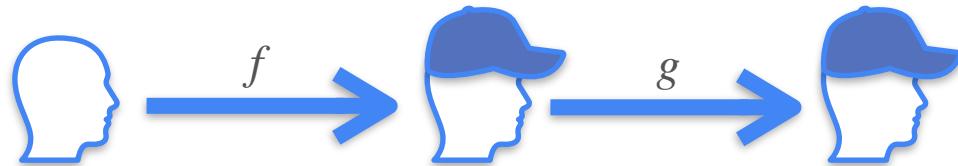
Inverse Function

What's an inverse?



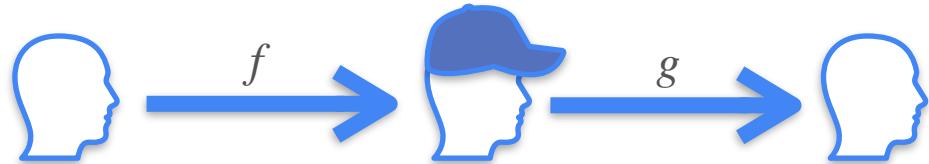
Inverse Function

What's an inverse?



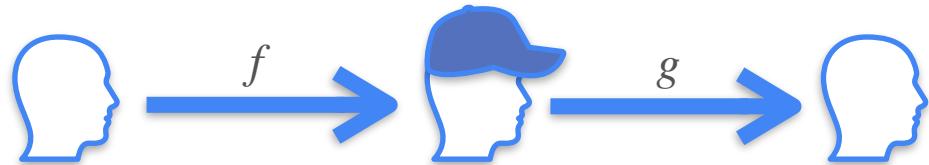
Inverse Function

What's an inverse?



Inverse Function

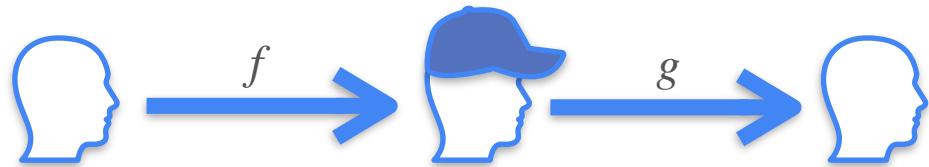
What's an inverse?



x

Inverse Function

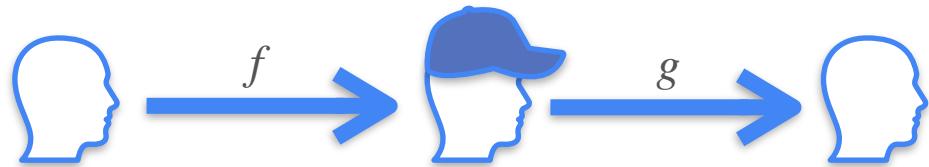
What's an inverse?



$$x \xrightarrow{f} x^2$$

Inverse Function

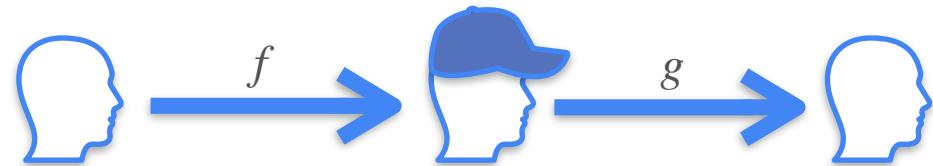
What's an inverse?



$$x \xrightarrow{f} x^2 \xrightarrow{g} x$$

Inverse Function

What's an inverse?

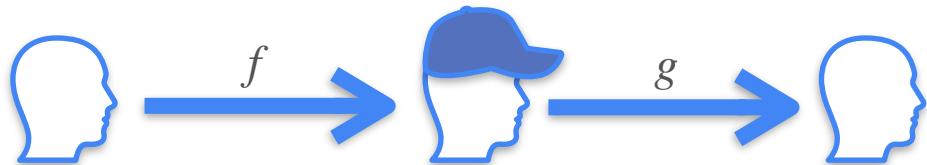


$g(x)$ and $f(x)$ are
inverses

$$x \xrightarrow{f} x^2 \xrightarrow{g} x$$

Inverse Function

What's an inverse?



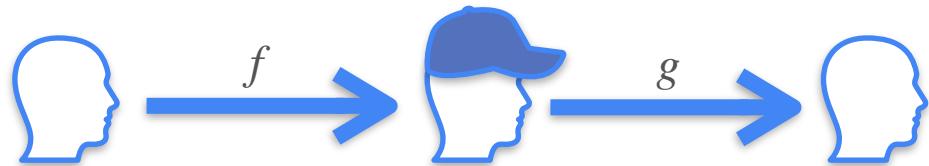
$g(x)$ and $f(x)$ are
inverses

$$g(x) = f^{-1}(x)$$



Inverse Function

What's an inverse?



$g(x)$ and $f(x)$ are
inverses

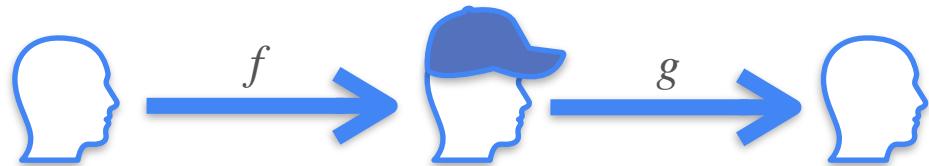
$$g(x) = f^{-1}(x)$$

$$g(f(x)) = x$$



Inverse Function

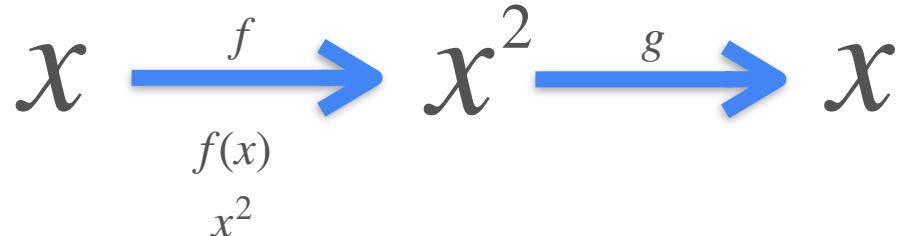
What's an inverse?



$g(x)$ and $f(x)$ are inverses

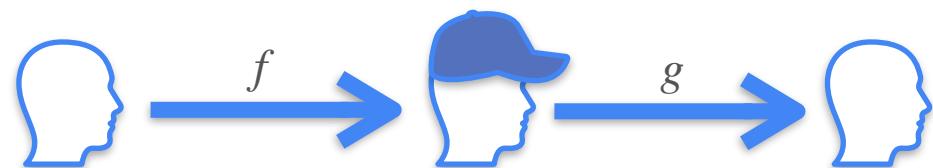
$$g(x) = f^{-1}(x)$$

$$g(f(x)) = x$$



Inverse Function

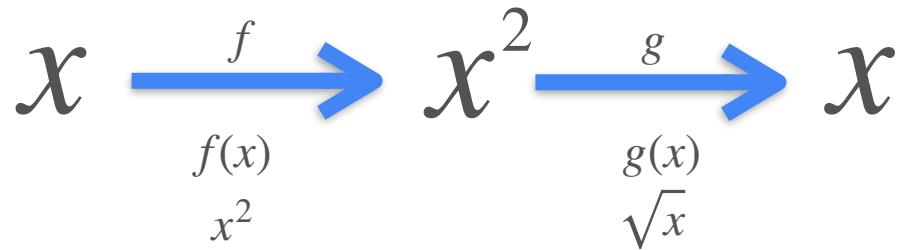
What's an inverse?



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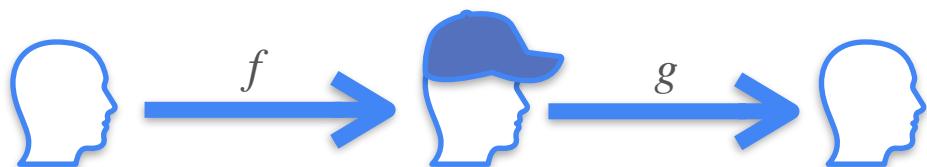
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Inverse Function

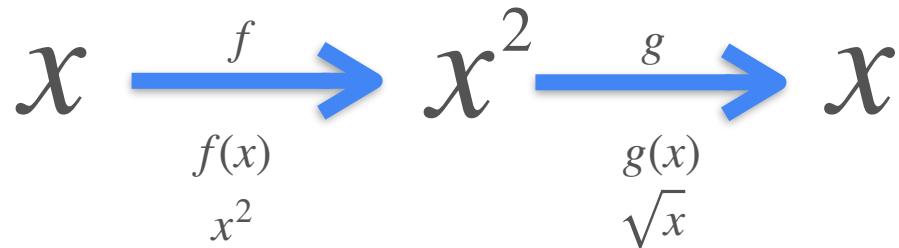
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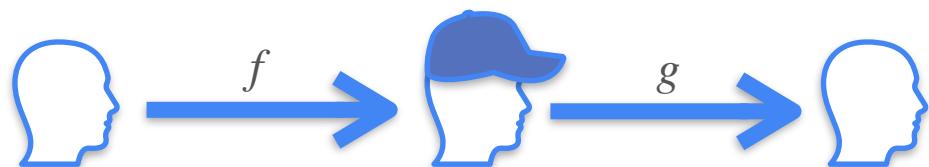
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$$\sqrt{x^2}$$

Inverse Function

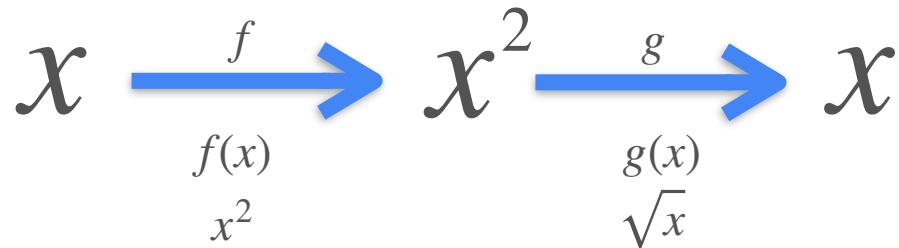
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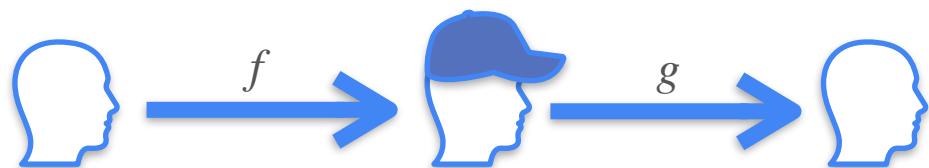
$$g(f(x)) = x$$



$$\sqrt{x^2} = x$$

Inverse Function

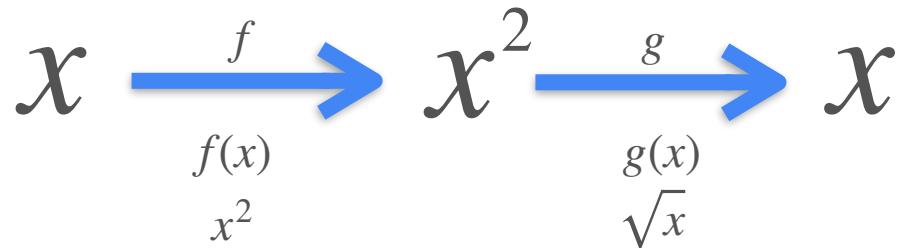
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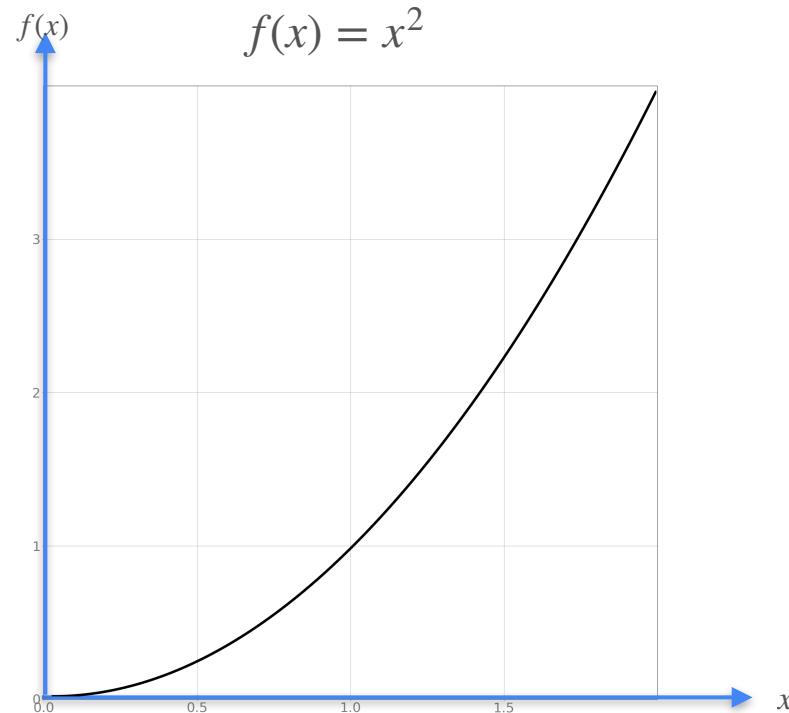
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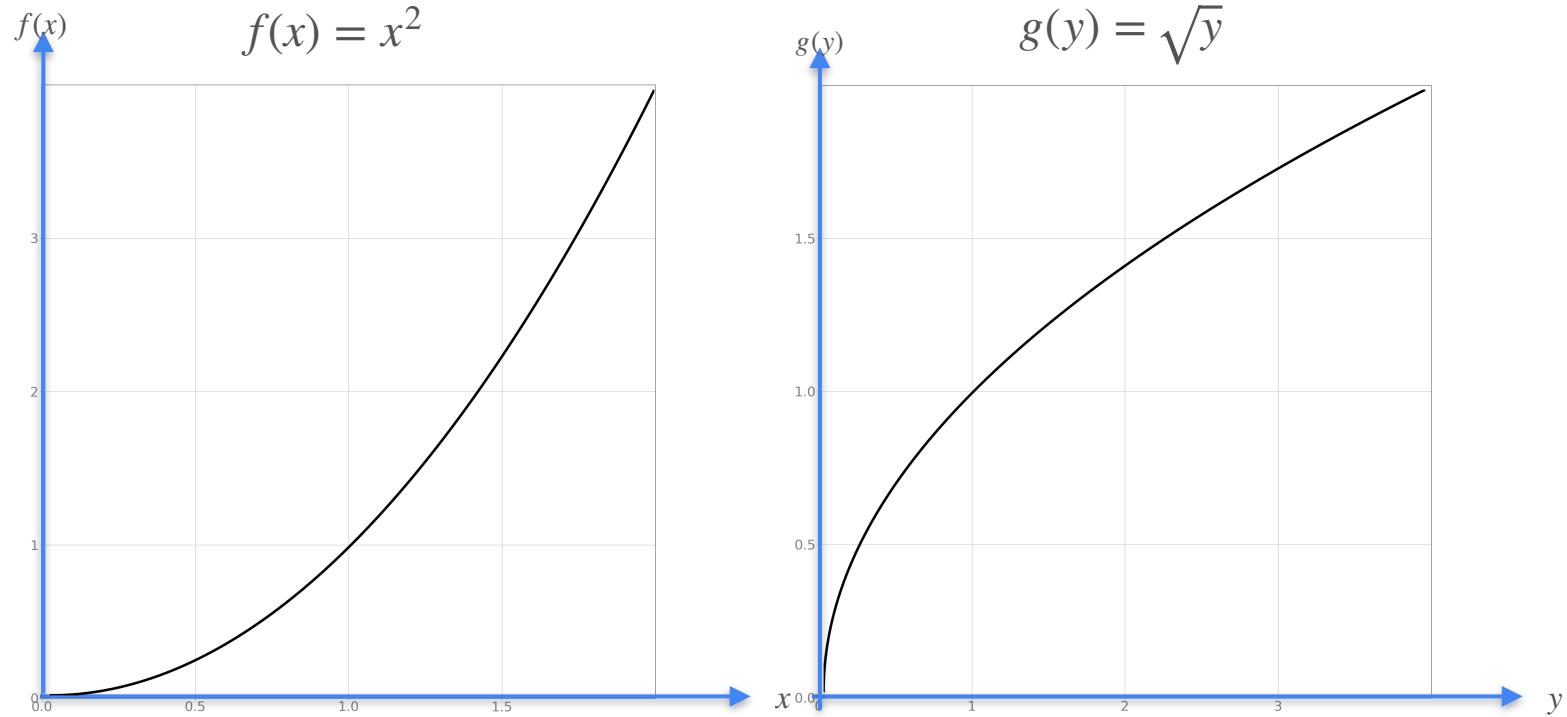
$$\sqrt{x^2} = x \quad \text{for } x > 0$$

Derivative of the Inverse

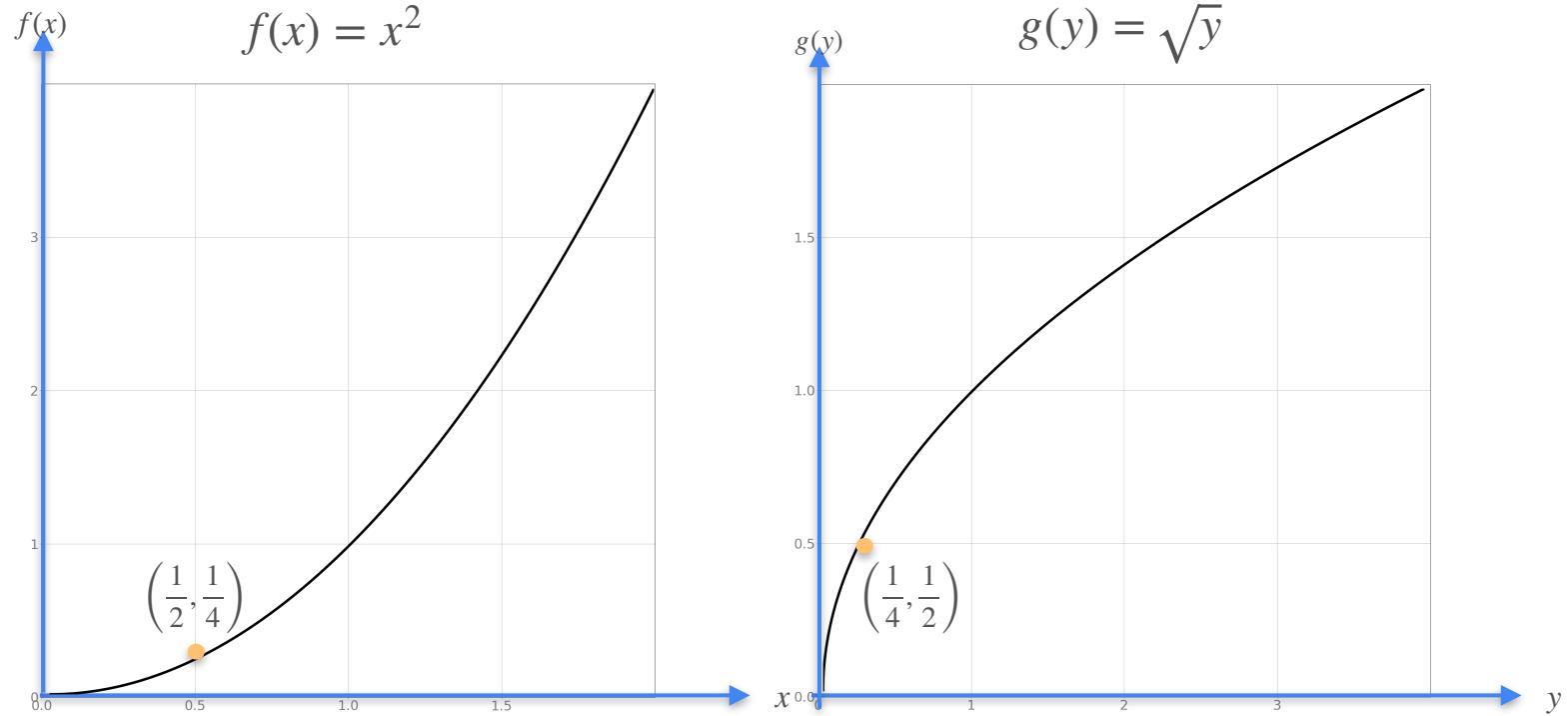
Derivative of the Inverse



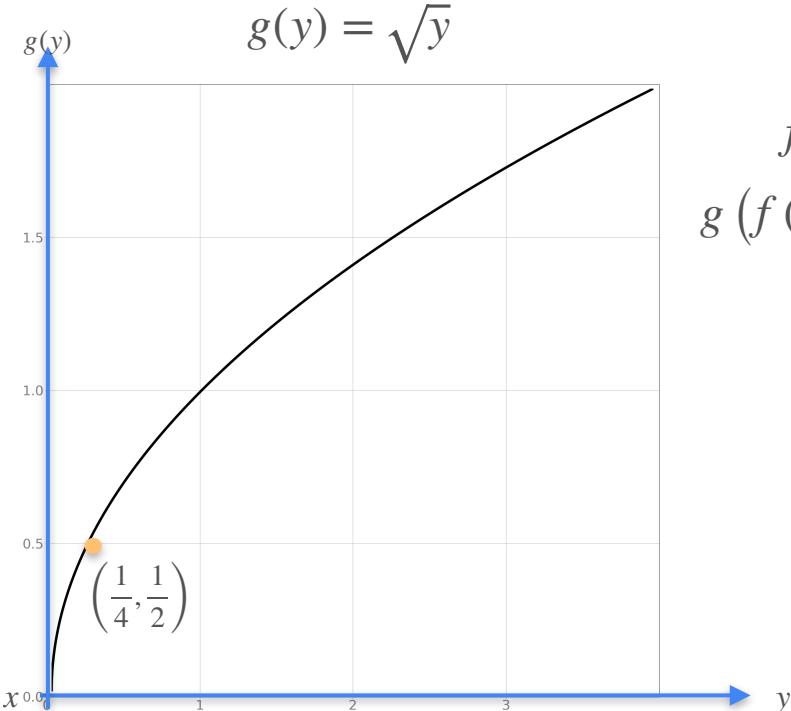
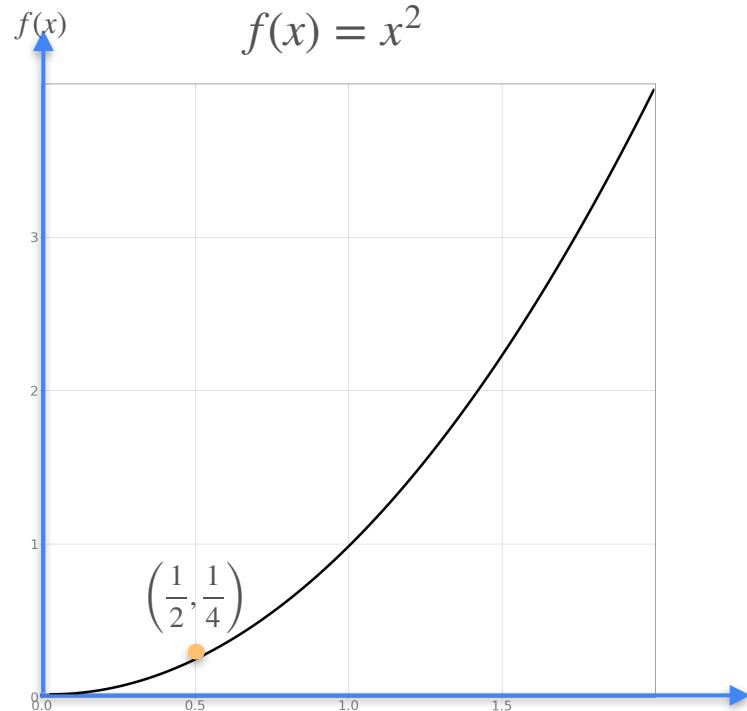
Derivative of the Inverse



Derivative of the Inverse



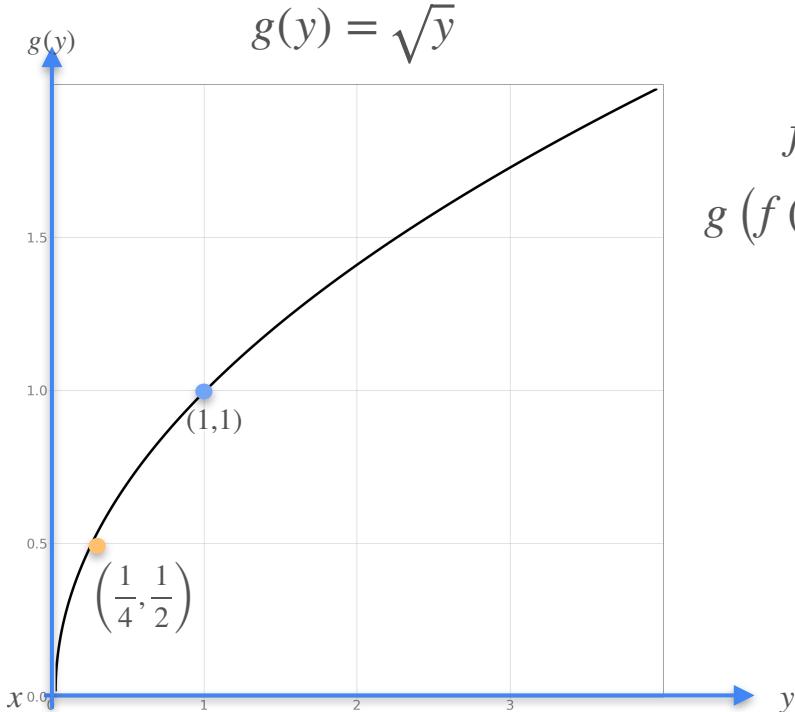
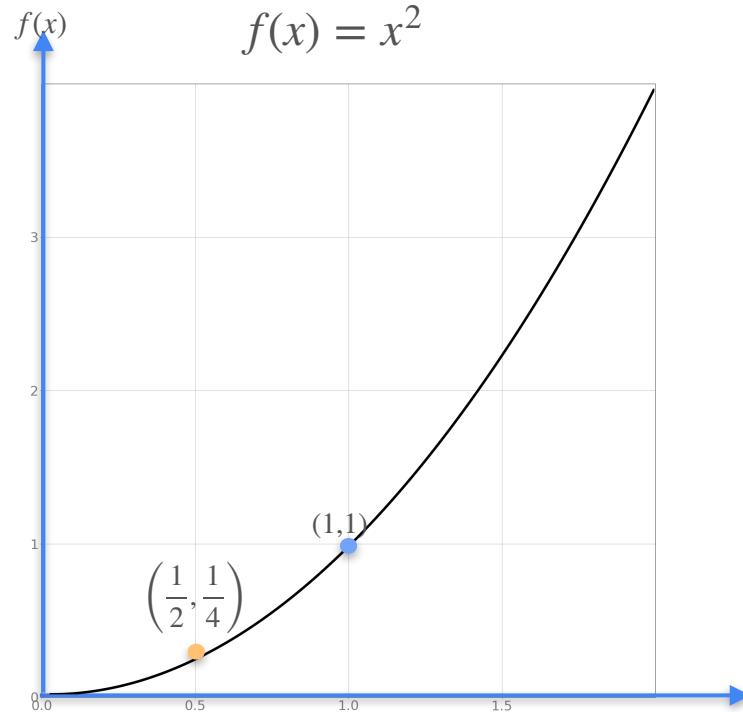
Derivative of the Inverse



$$f(1/2) = 1/4$$

$$\begin{aligned}g(f(1/2)) &= g(1/4) \\&= 1/2\end{aligned}$$

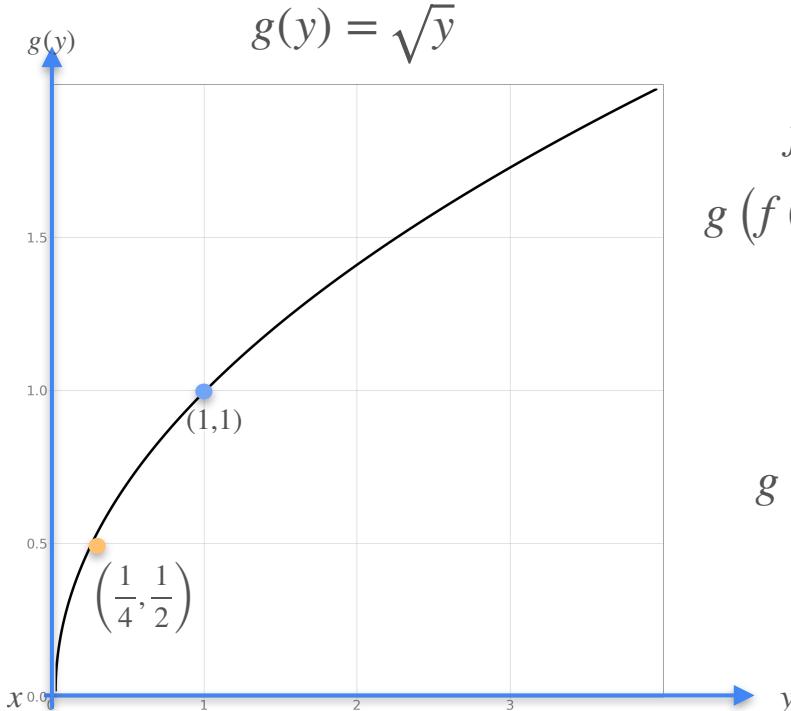
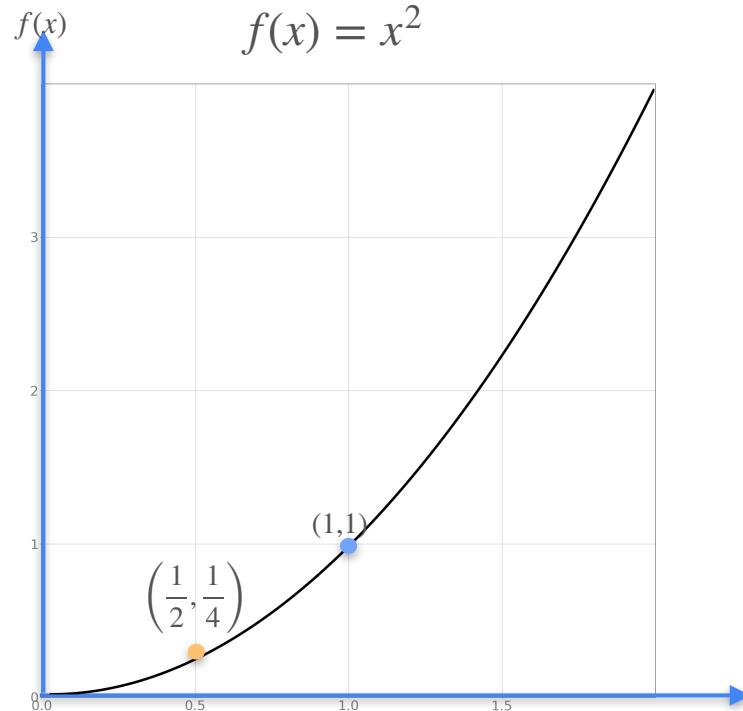
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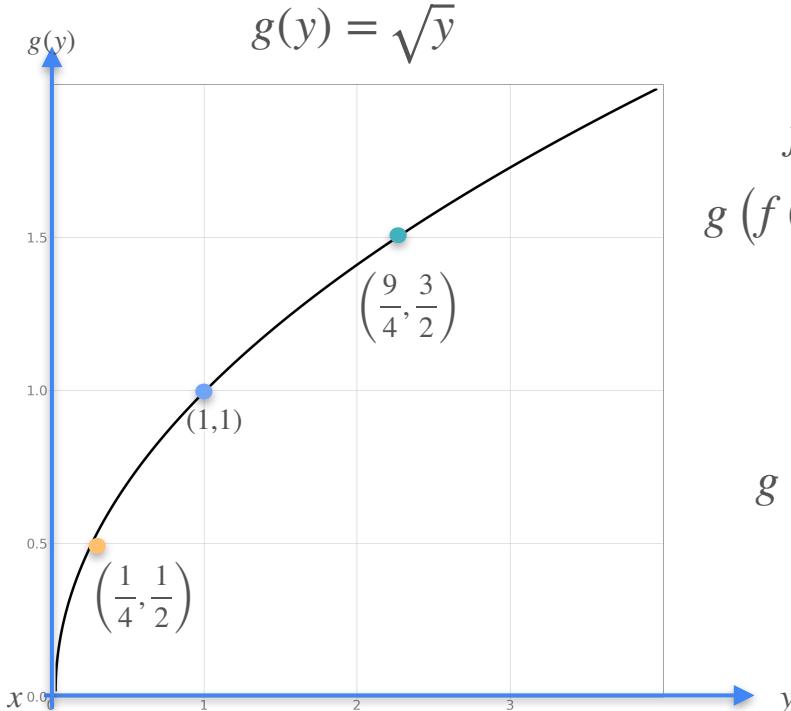
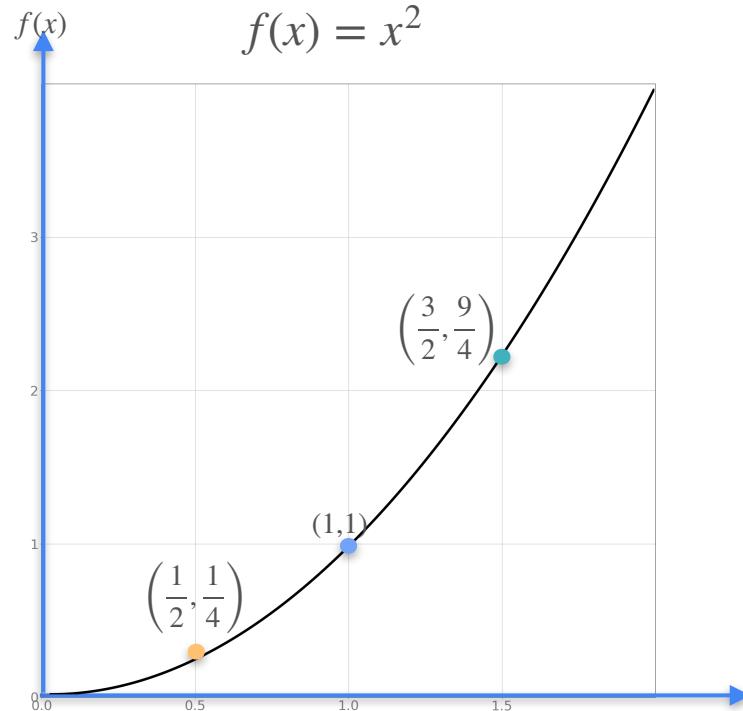
$$f(1/2) = 1/4$$

$$\begin{aligned} g(f(1/2)) &= g(1/4) \\ &= 1/2 \end{aligned}$$

$$f(1) = 1$$

$$\begin{aligned} g(f(1)) &= g(1) \\ &= 1 \end{aligned}$$

Derivative of the Inverse



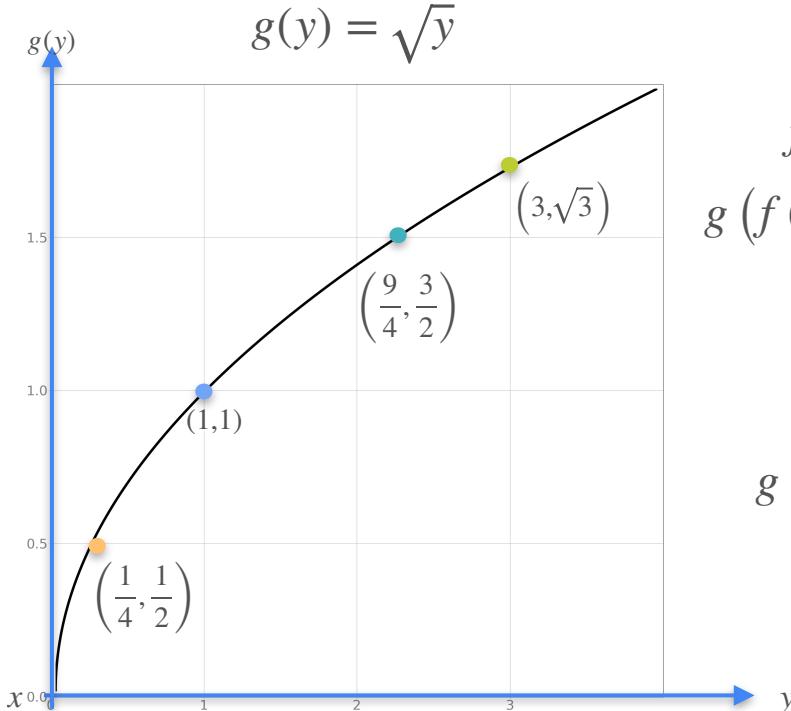
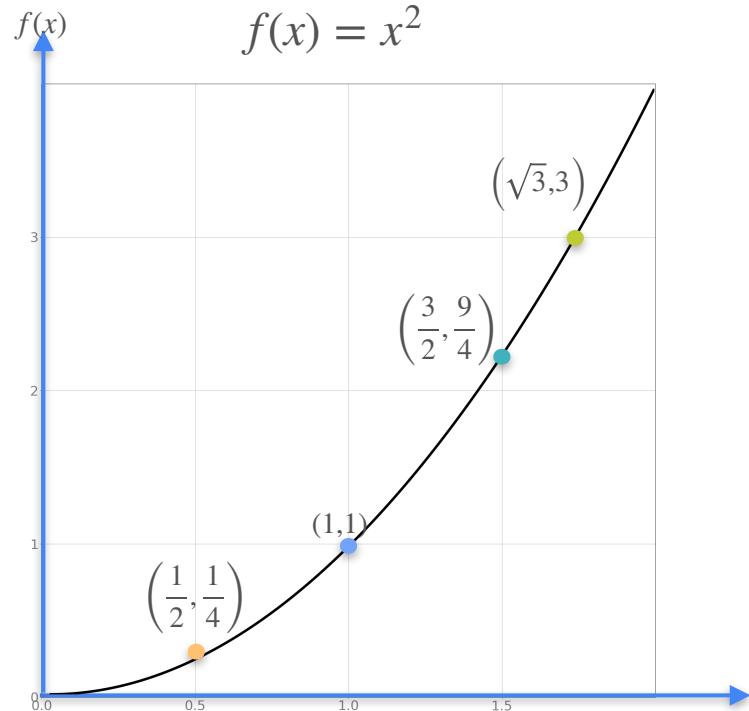
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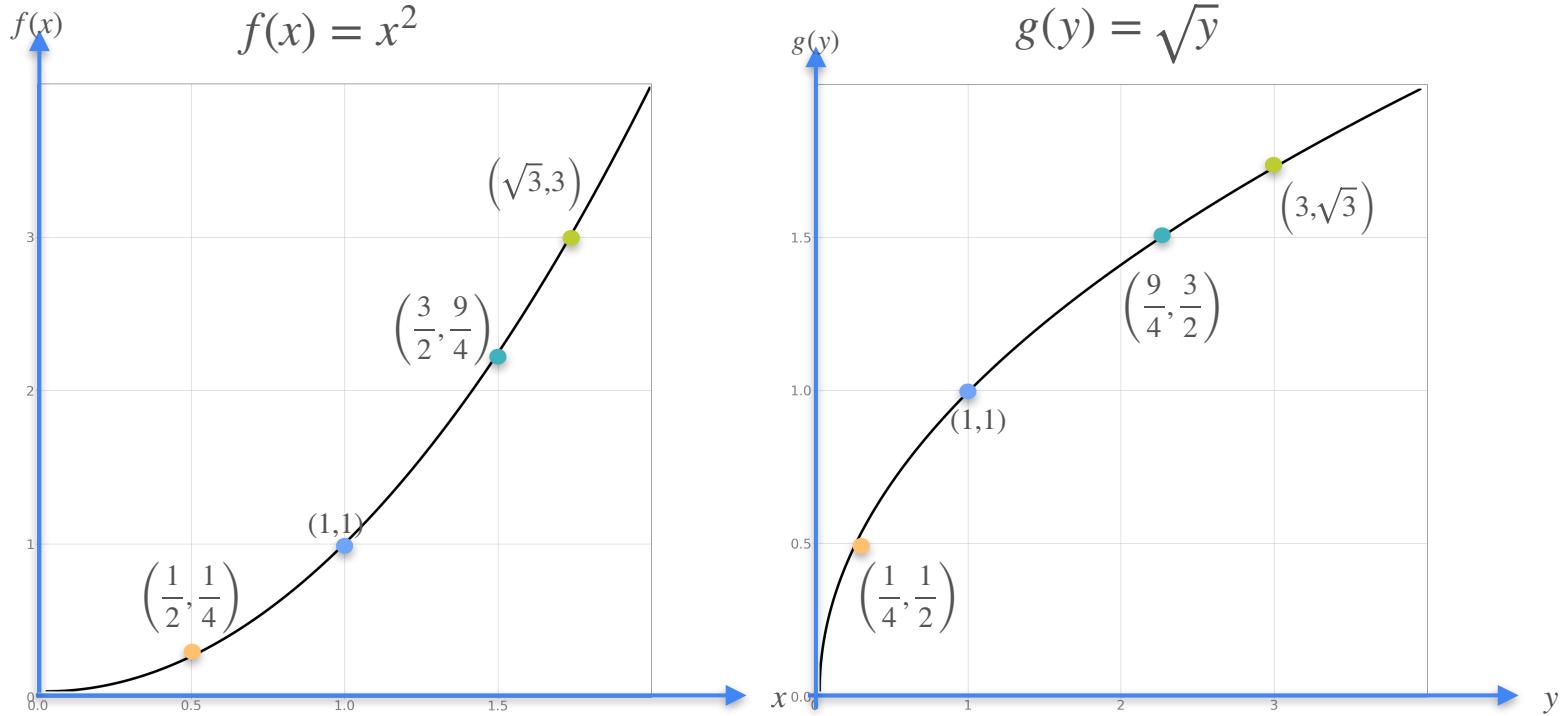
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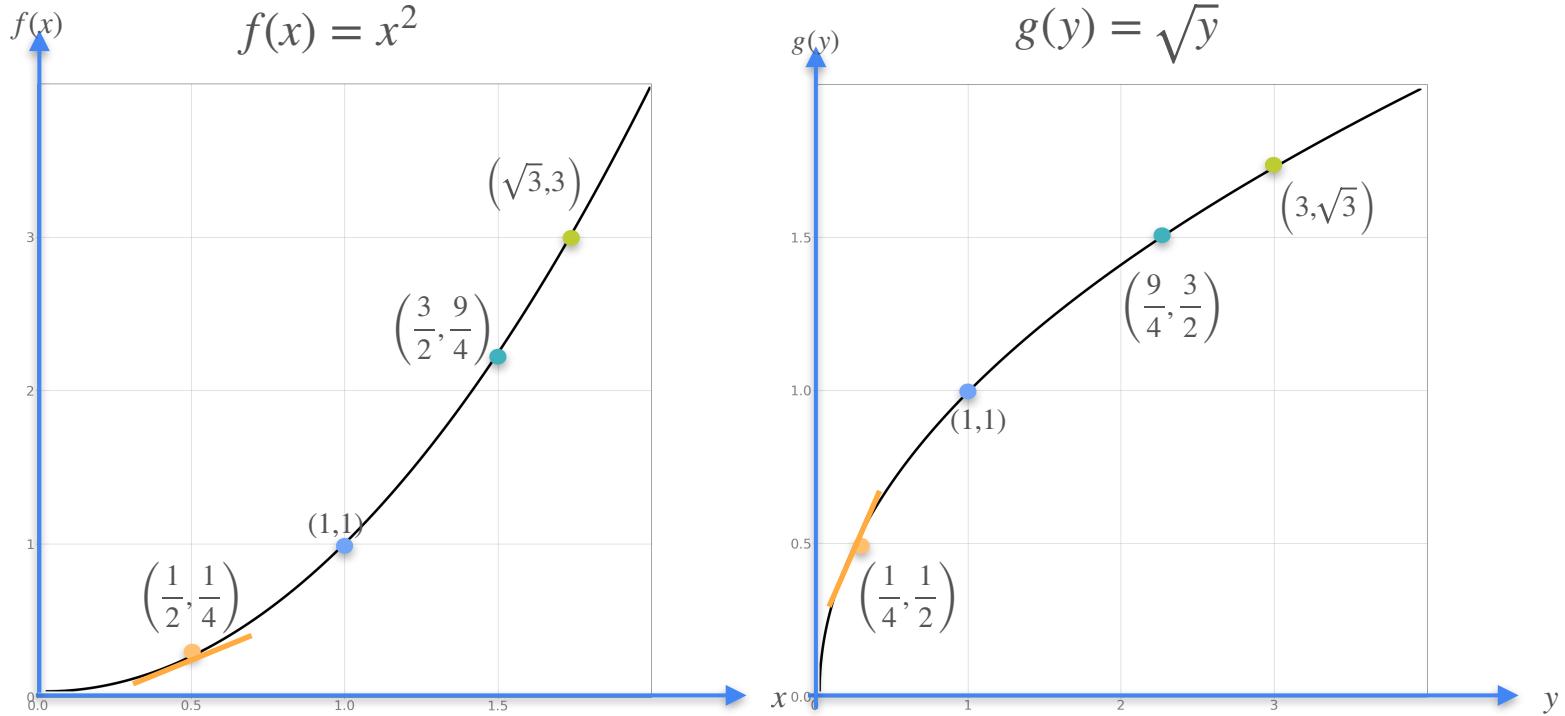
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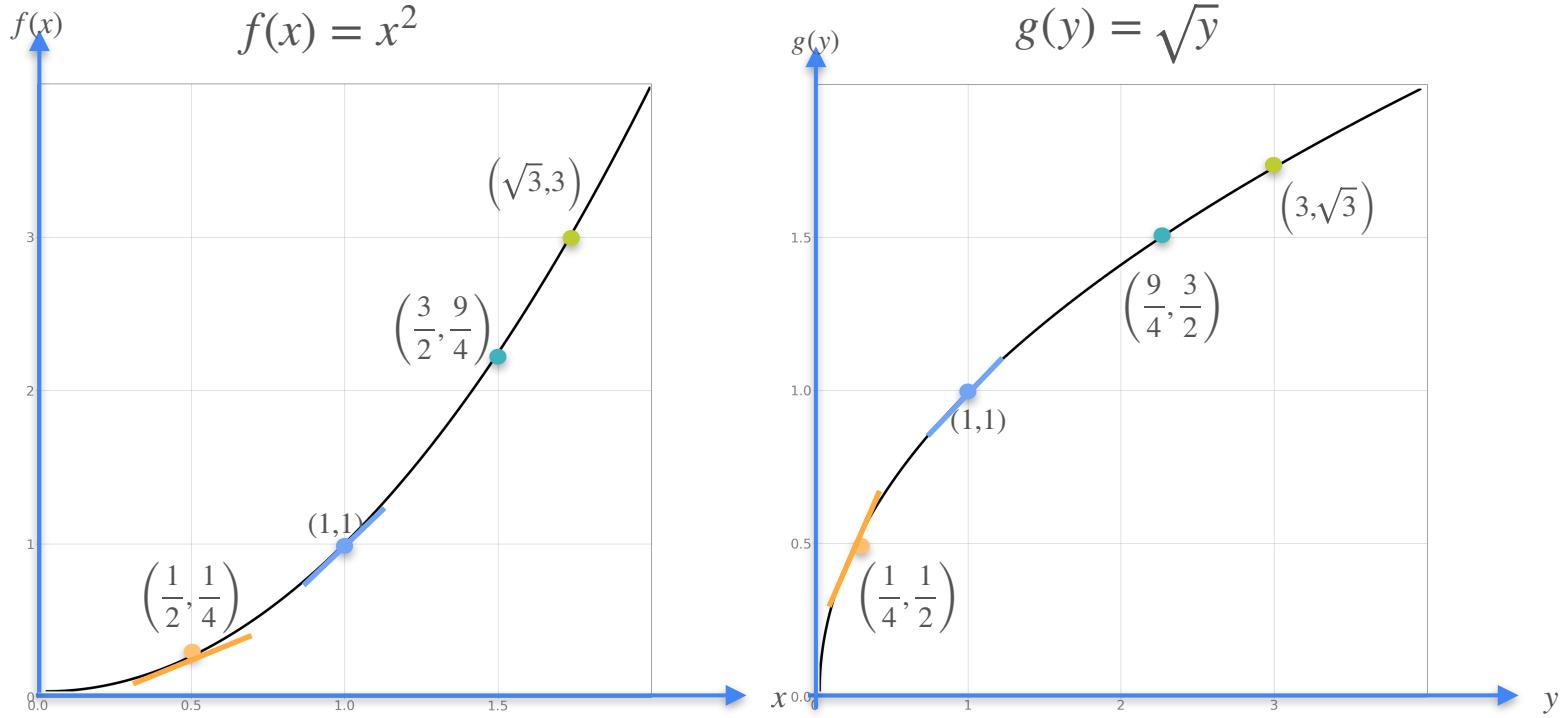
Derivative of the Inverse



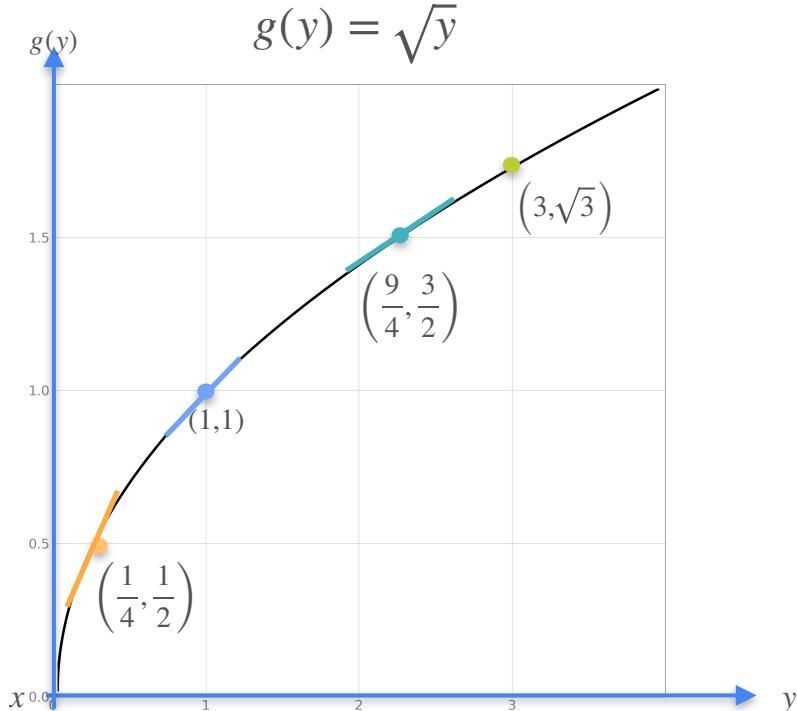
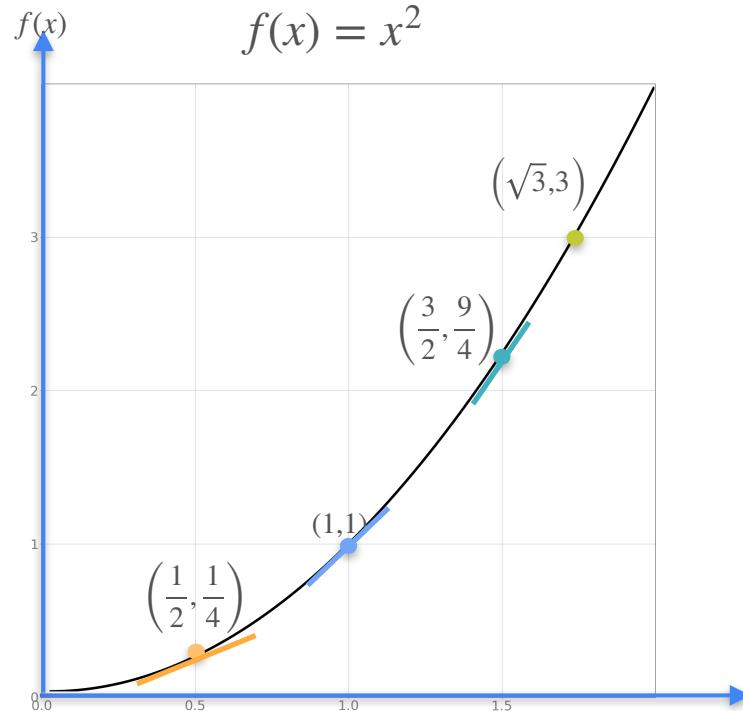
Derivative of the Inverse



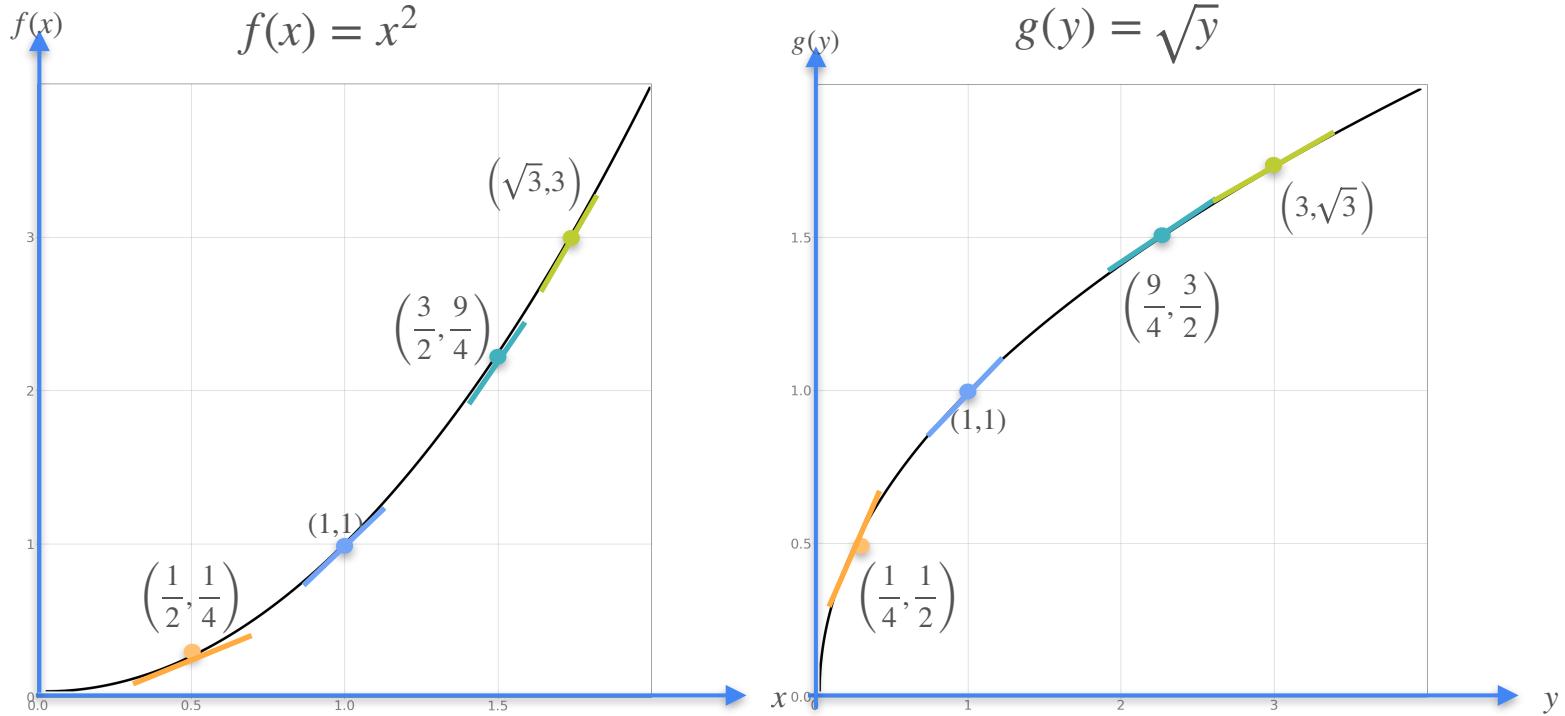
Derivative of the Inverse



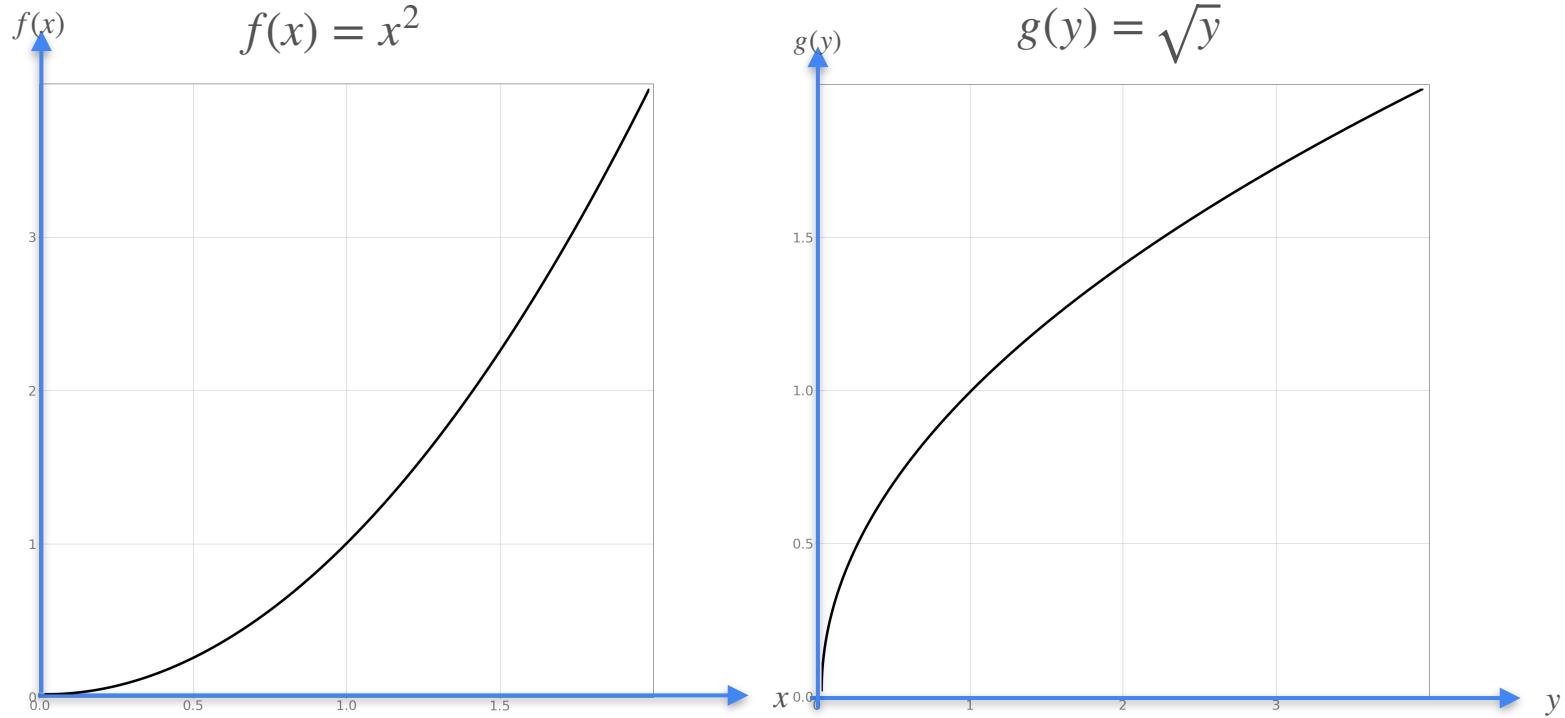
Derivative of the Inverse



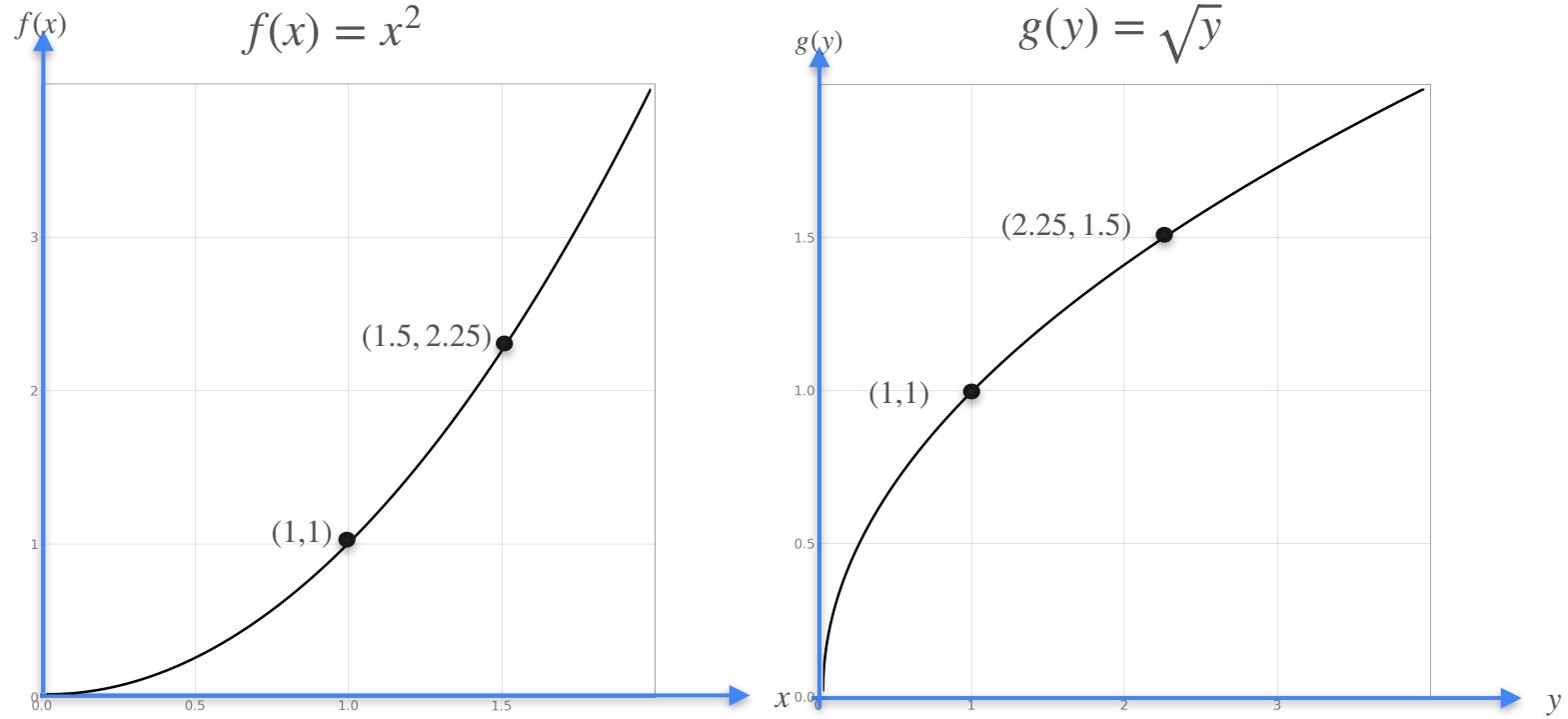
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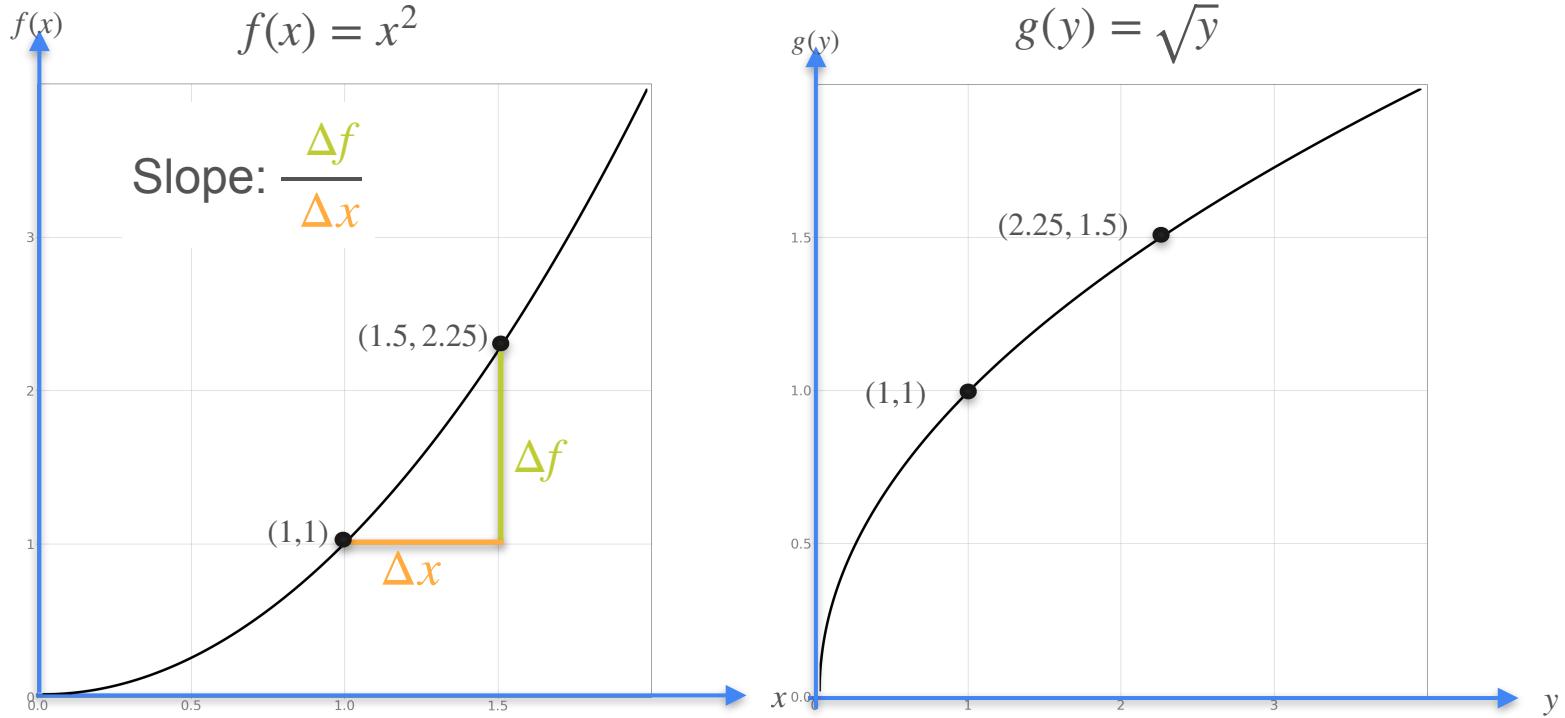
Derivative of the Inverse



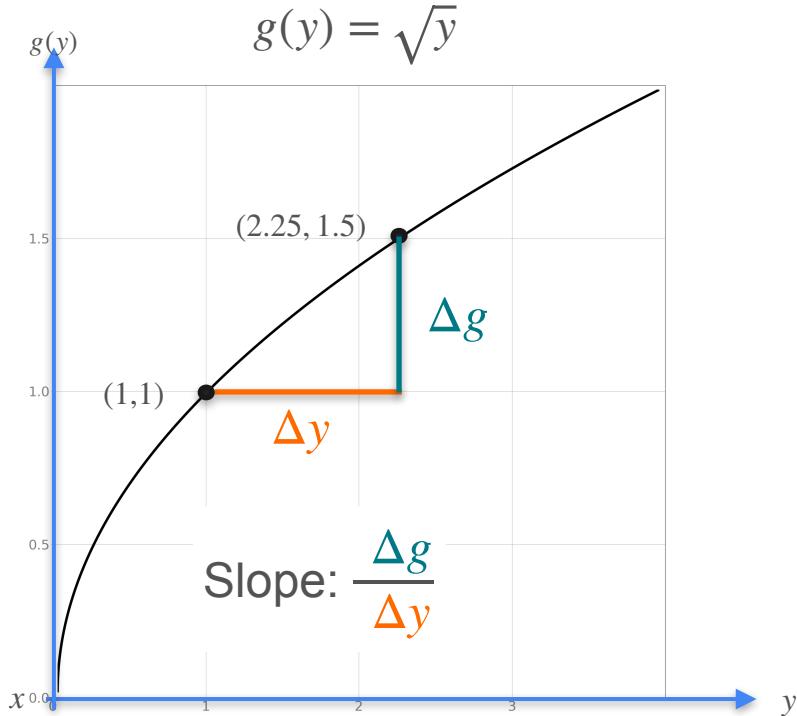
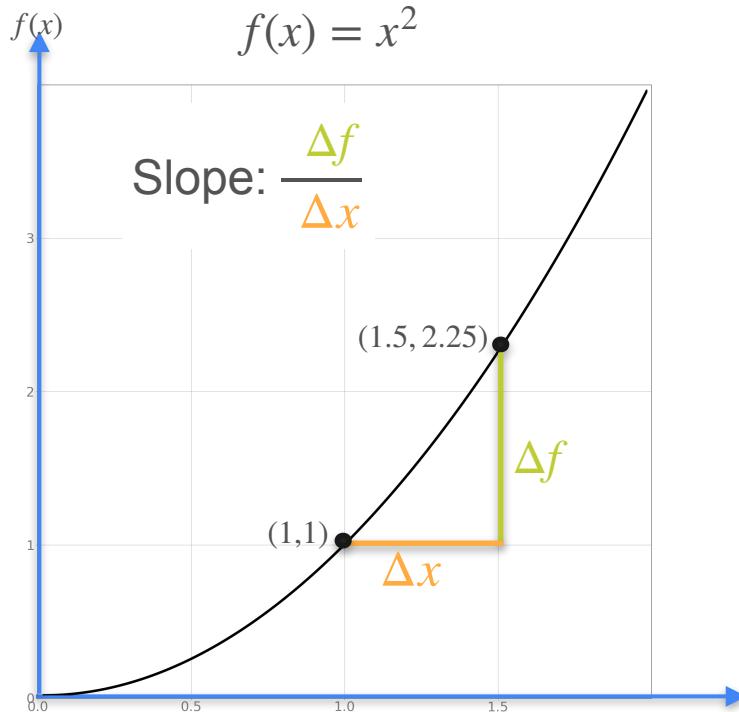
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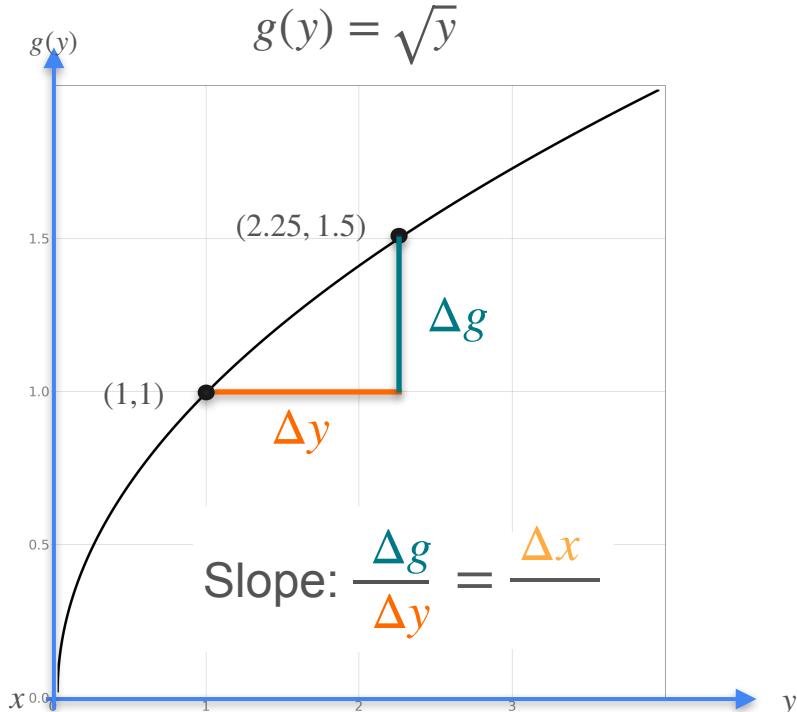
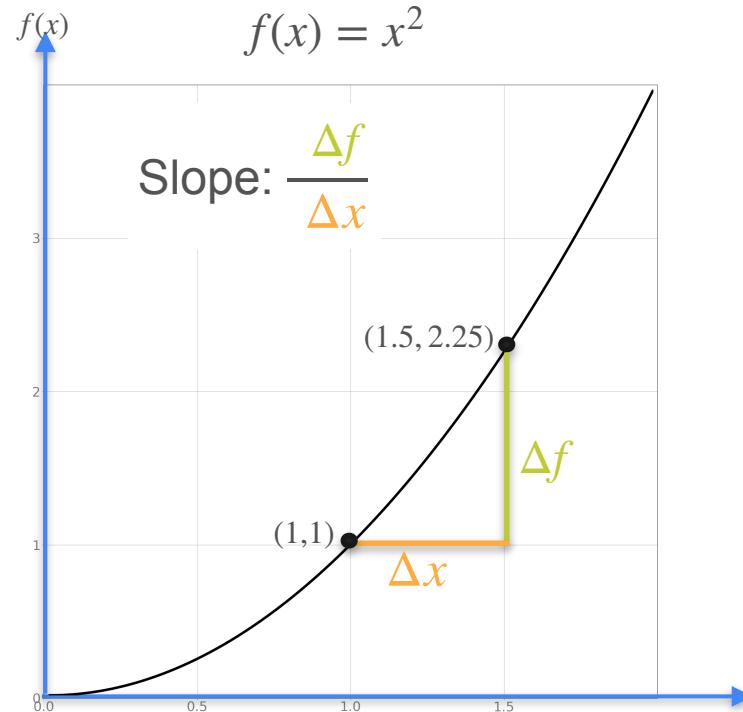
Derivative of the Inverse



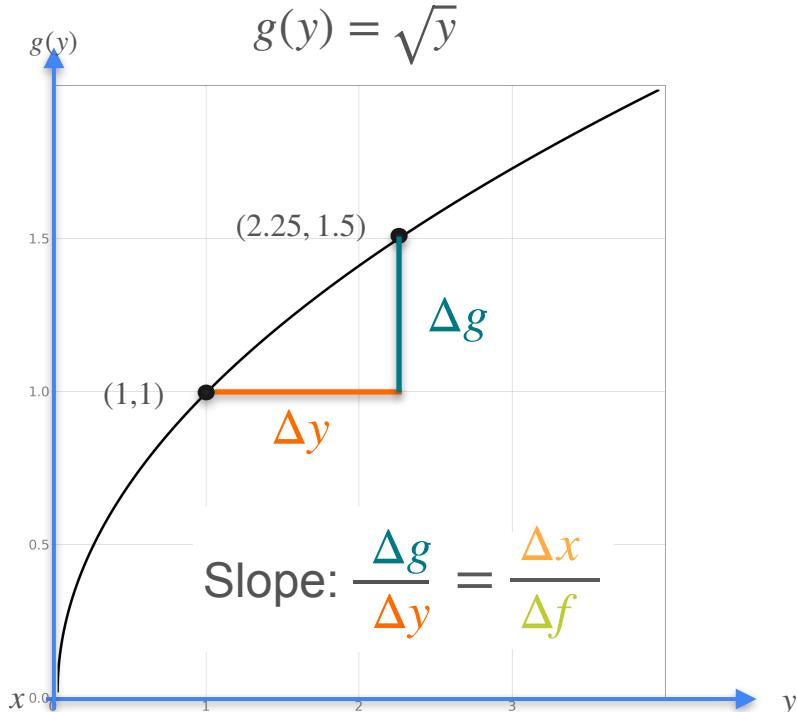
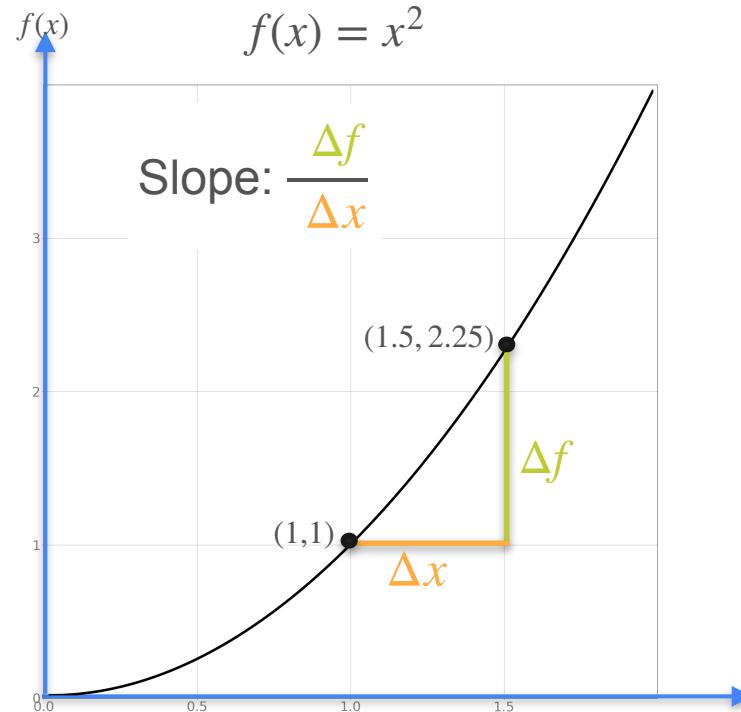
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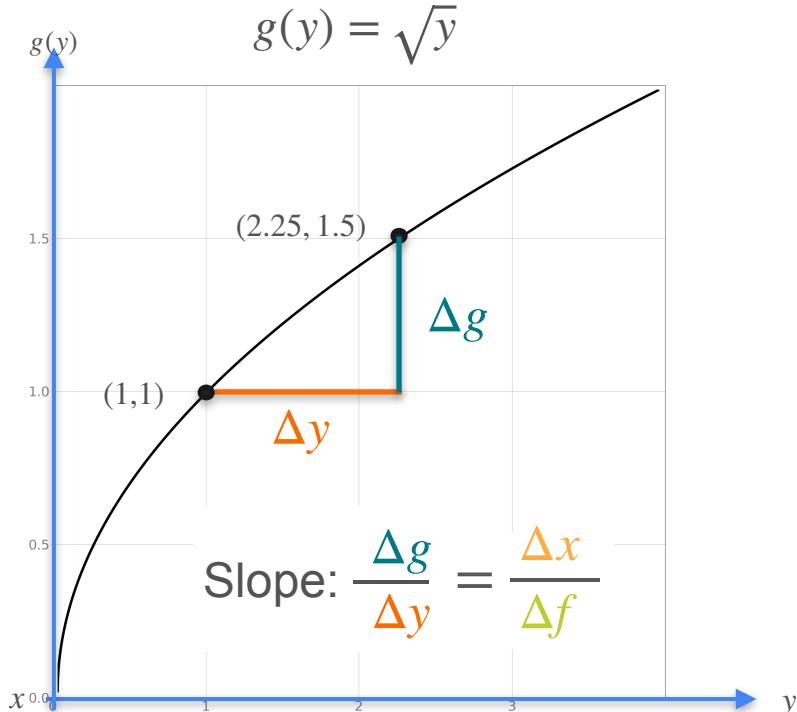
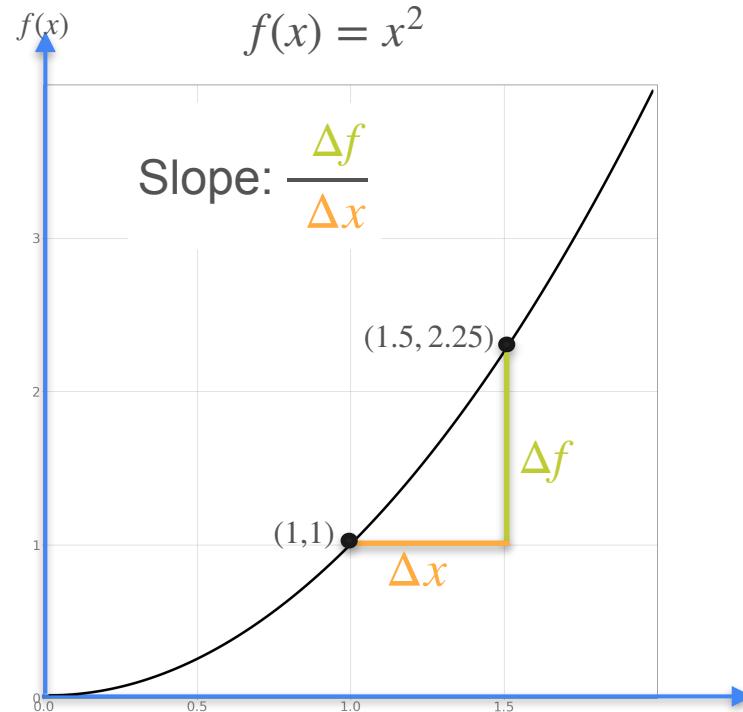
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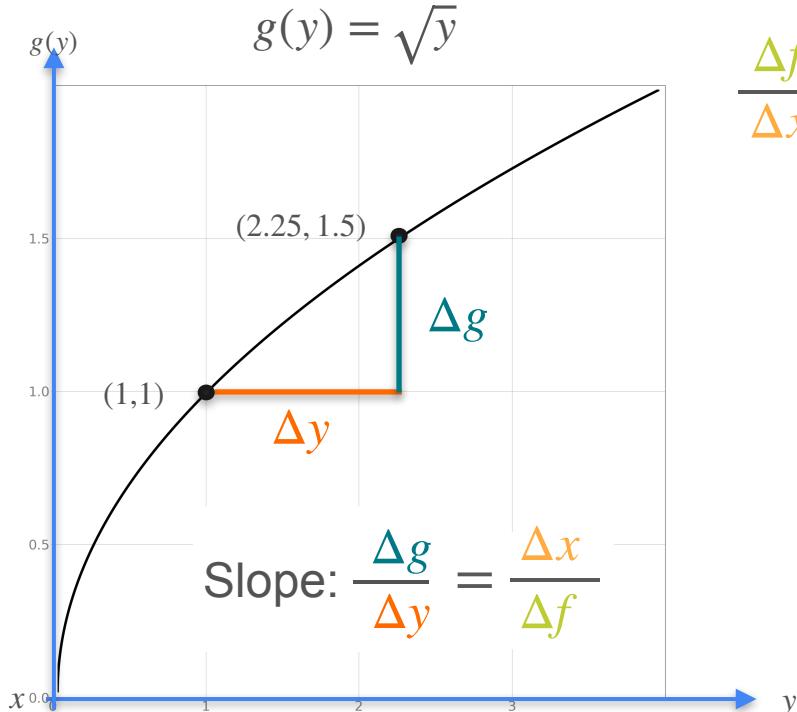
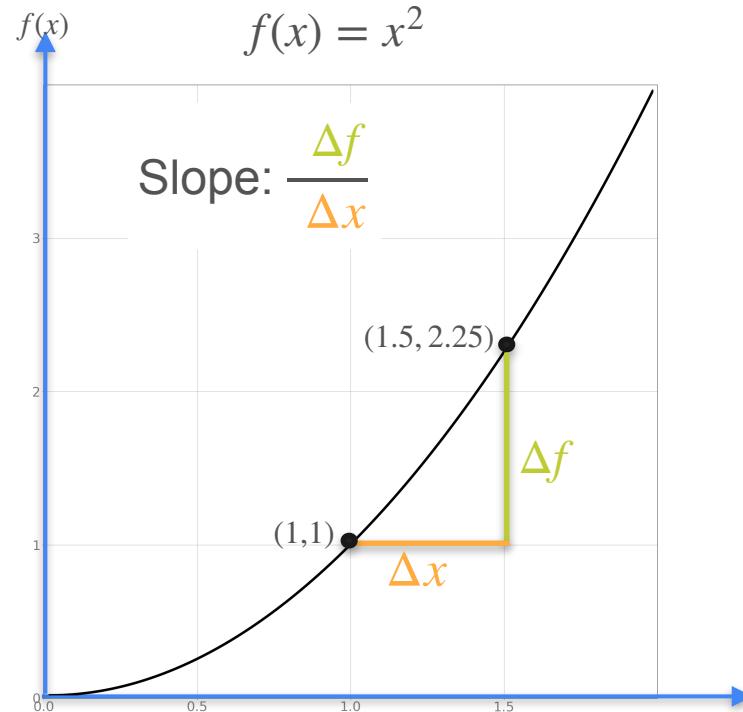
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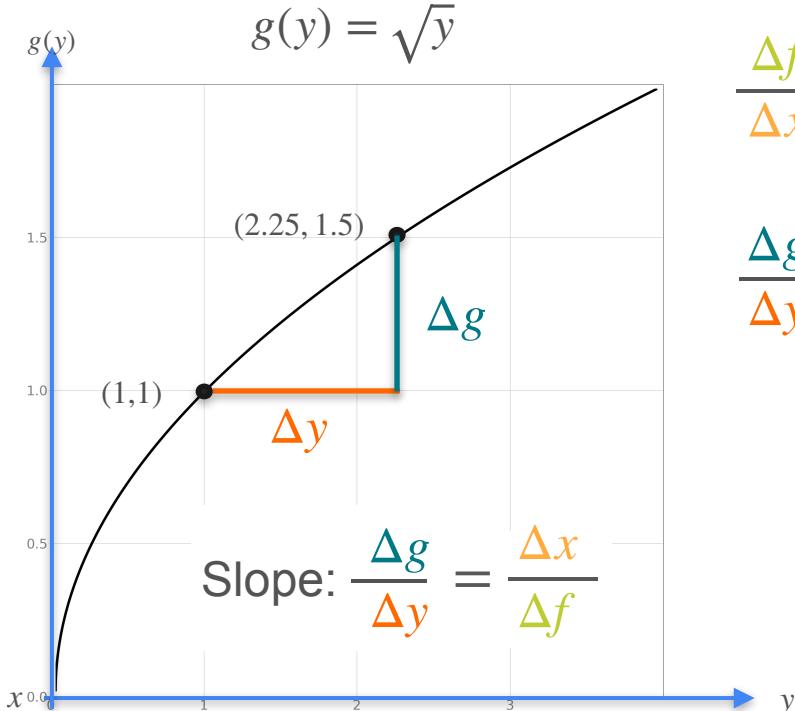
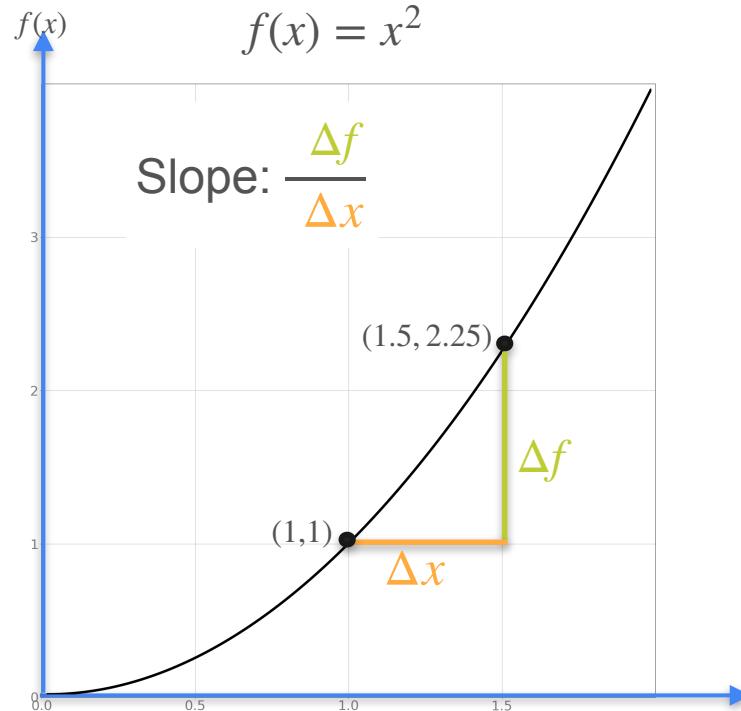
Derivative of the Inverse



$$\frac{\Delta f}{\Delta x} = f'(x)$$

$$\text{Slope: } \frac{\Delta g}{\Delta y} = \frac{\Delta x}{\Delta f}$$

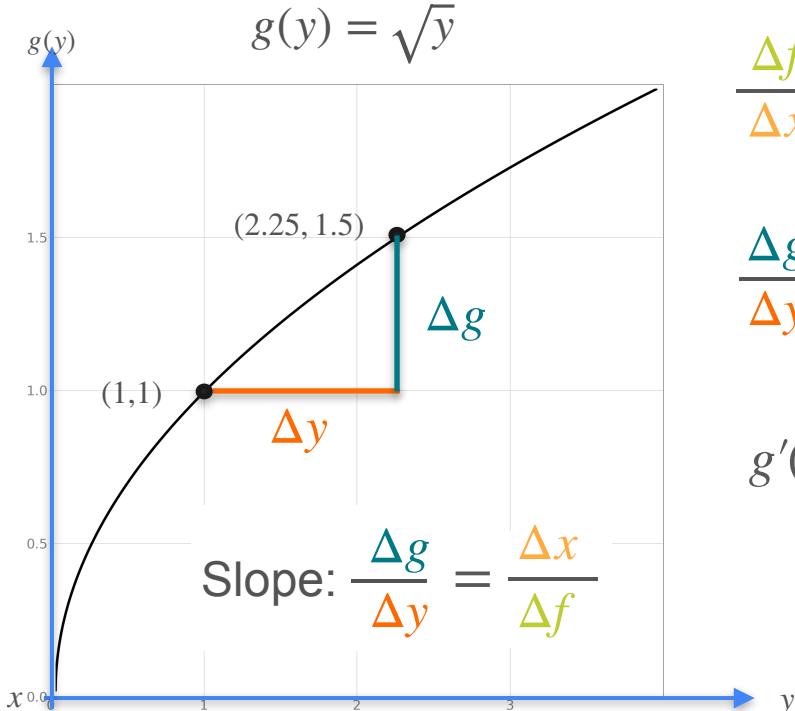
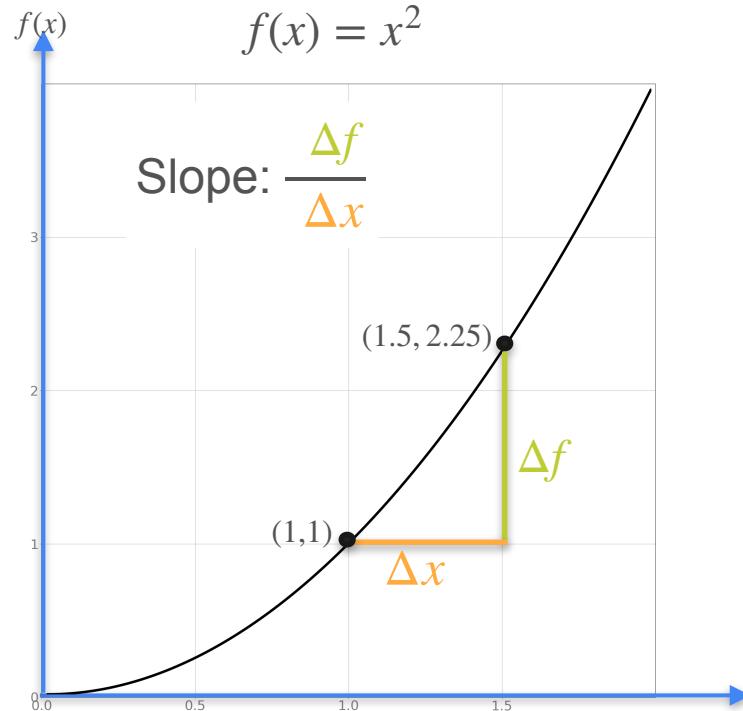
Derivative of the Inverse



$$\frac{\Delta f}{\Delta x} = f'(x)$$

$$\frac{\Delta g}{\Delta y} = g'(y)$$

Derivative of the Inverse

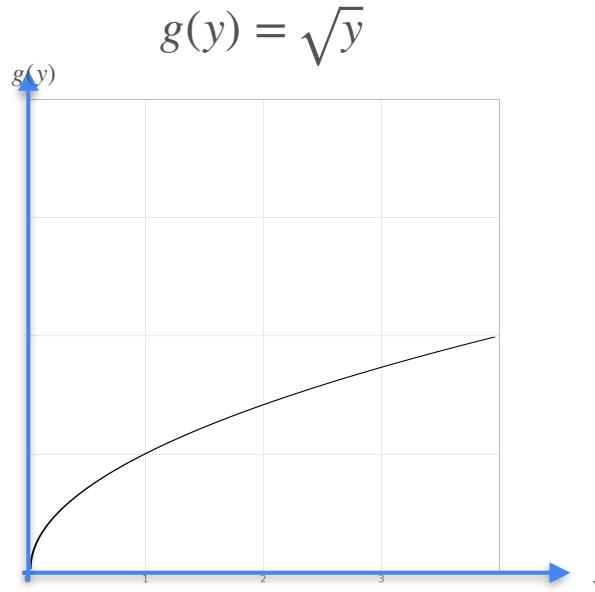
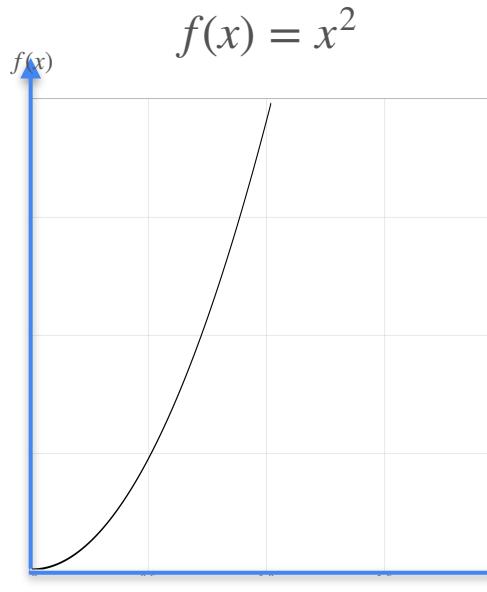


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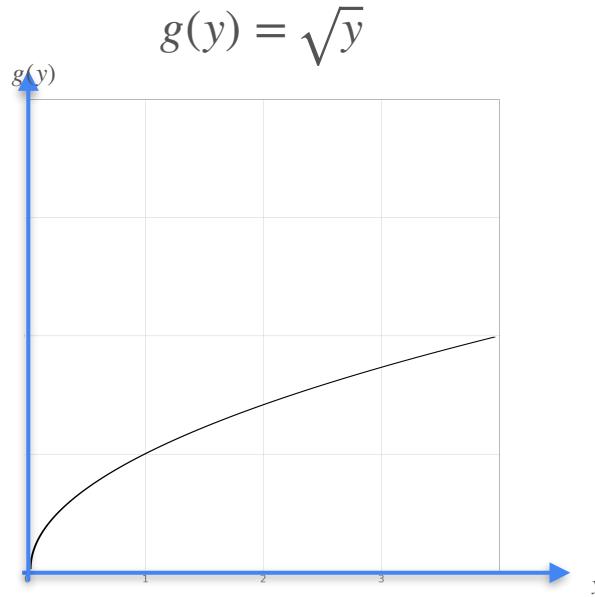
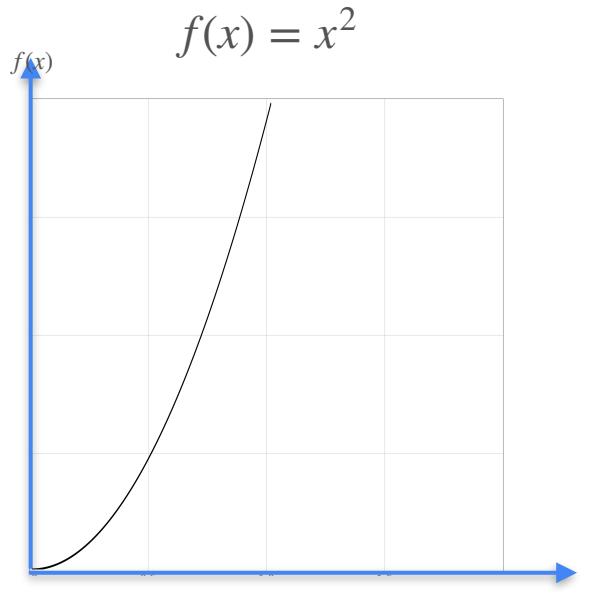
$$g'(y) = \frac{1}{f'(x)}$$

Derivative of the Inverse



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Derivative of the Inverse

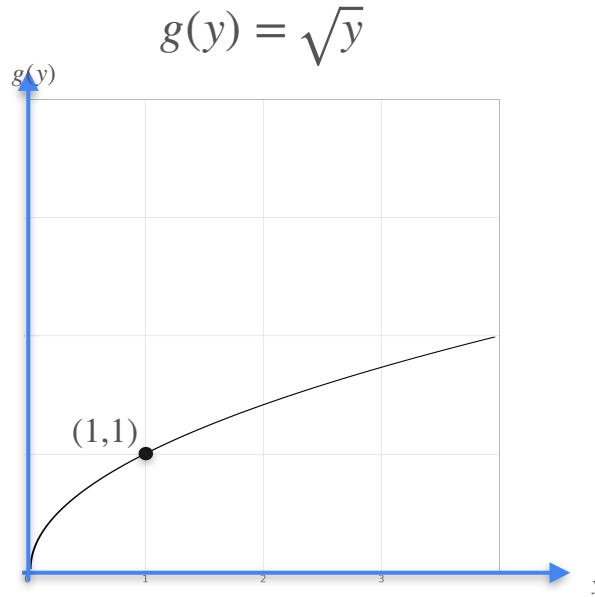
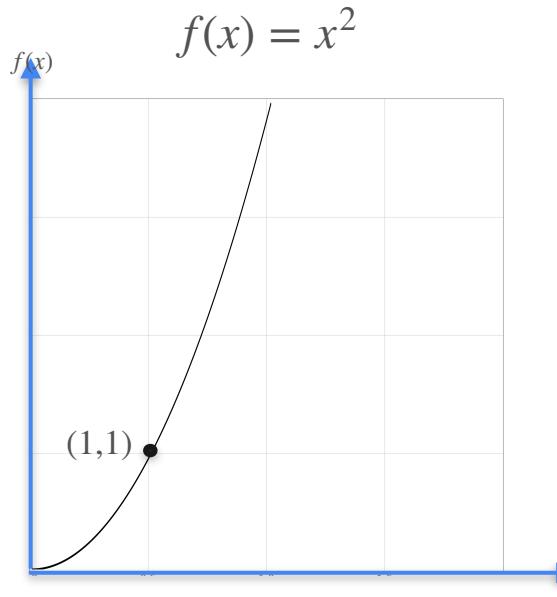


$$g'(y) = \frac{1}{f'(x)}$$

at the point (1,1)

$$f(1) = 1 \quad g(1) = 1$$

Derivative of the Inverse

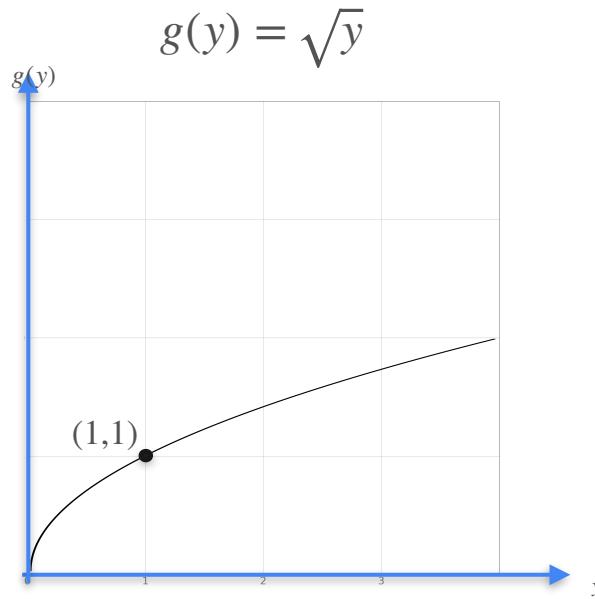
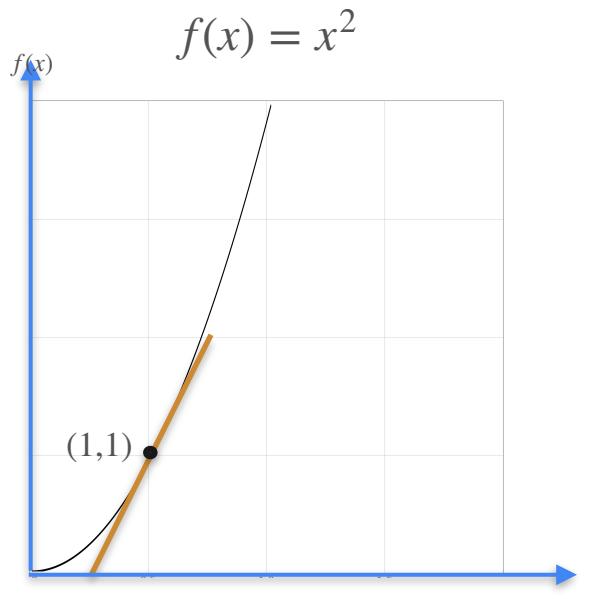


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Derivative of the Inverse

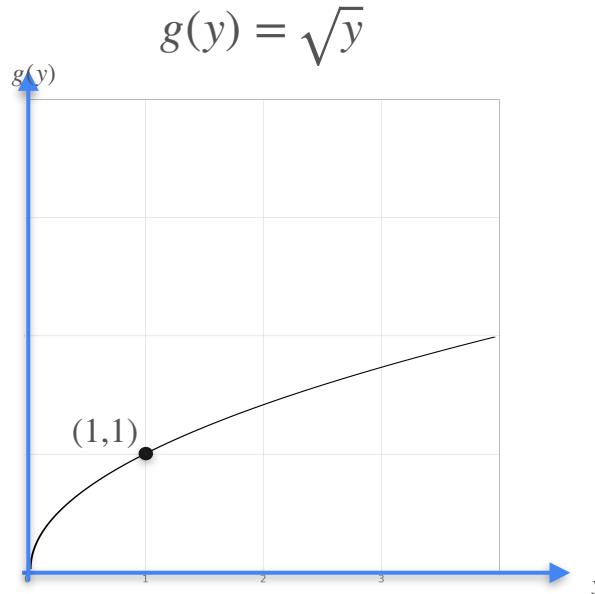
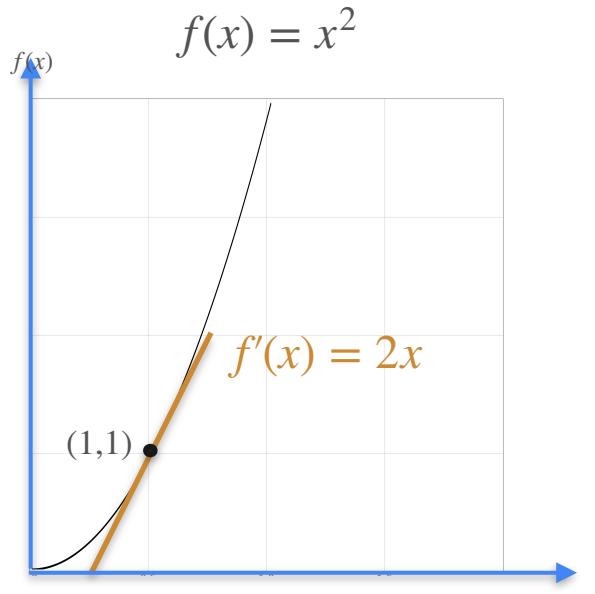


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Derivative of the Inverse

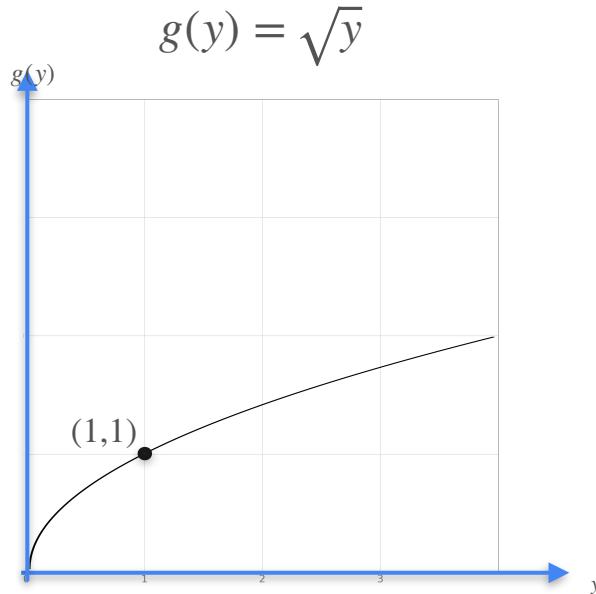
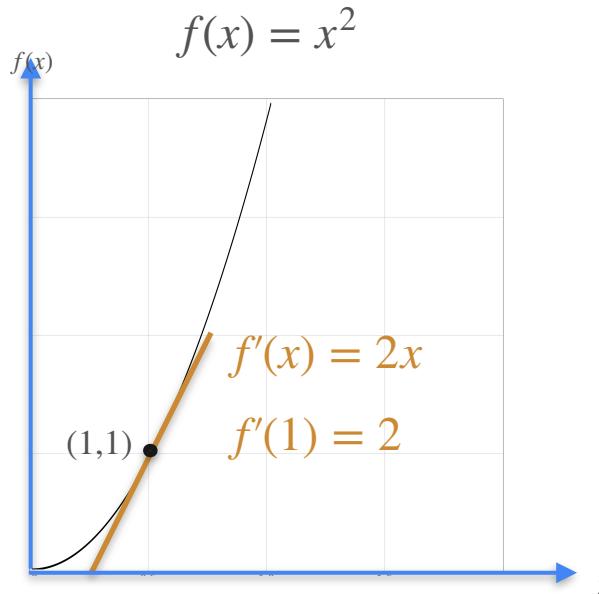


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Derivative of the Inverse



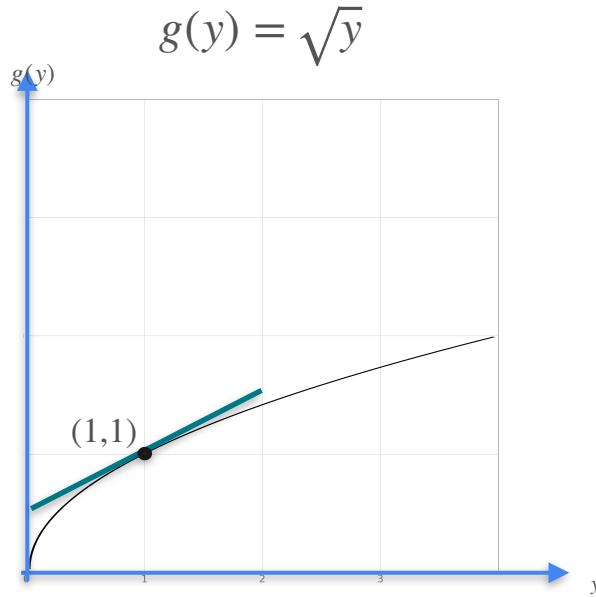
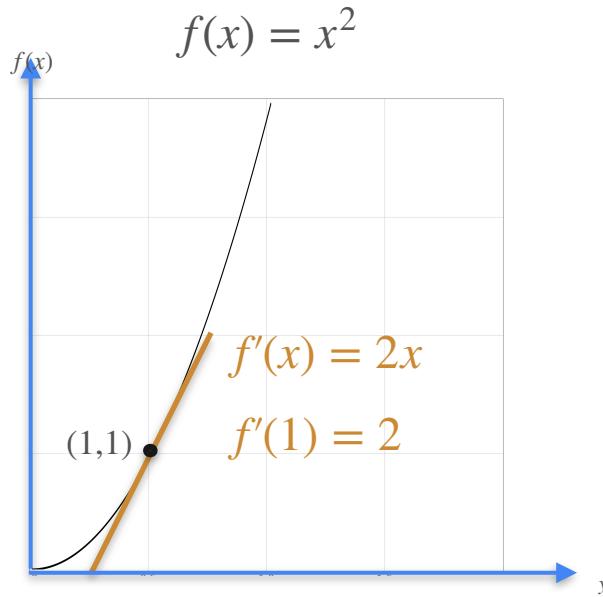
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at the point $(1,1)$

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$$g(1) = 1$$

Derivative of the Inverse

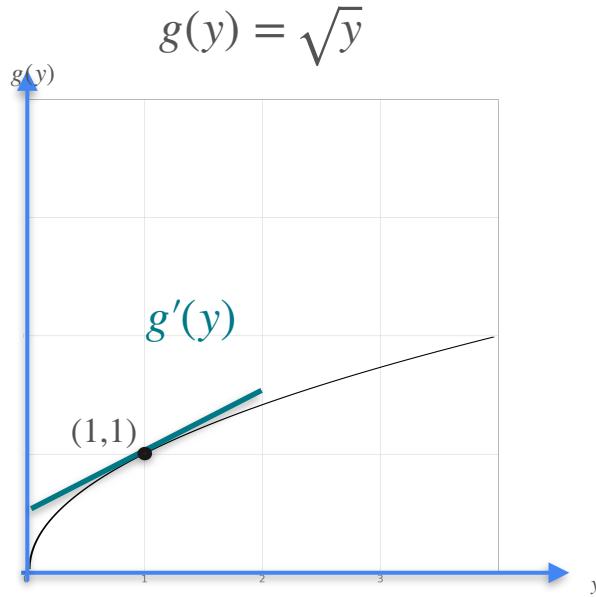
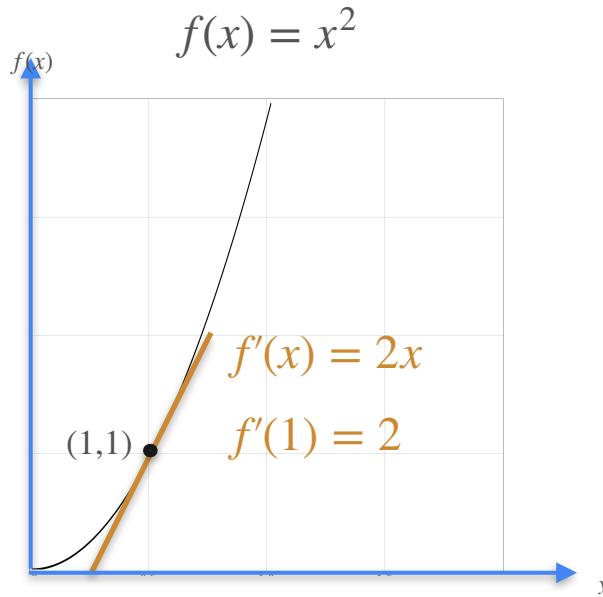


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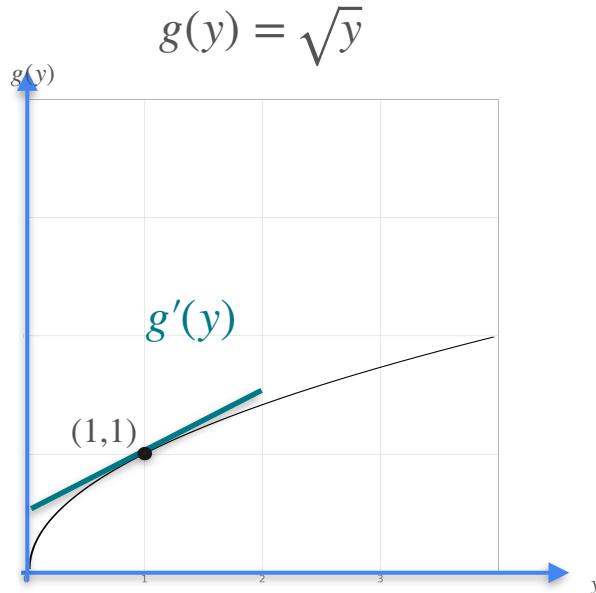
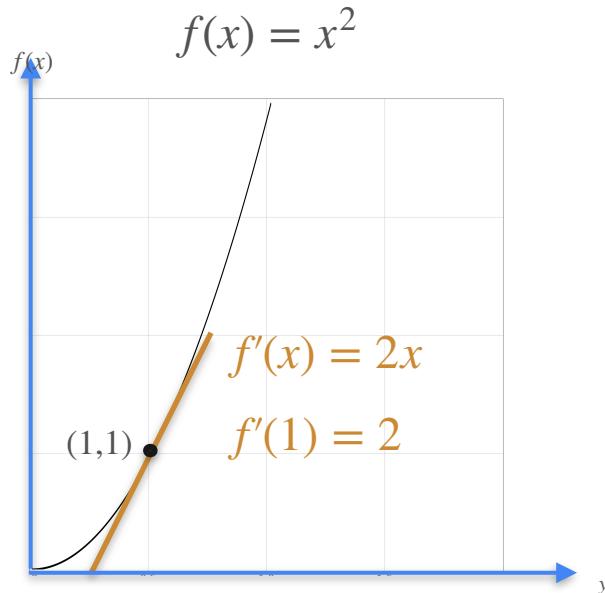


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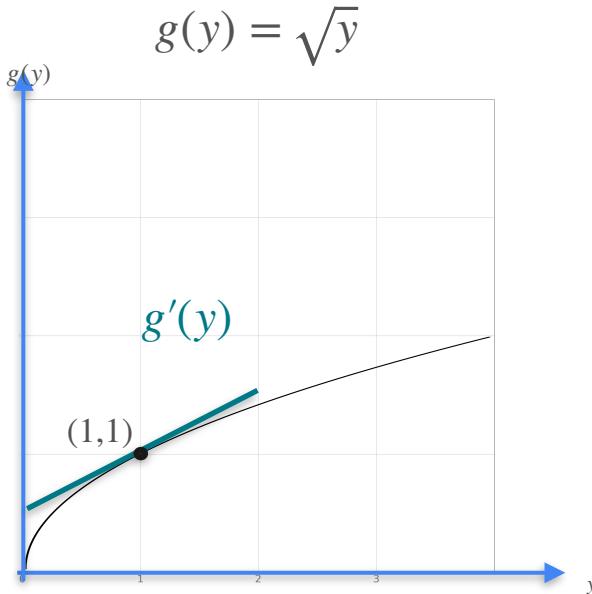
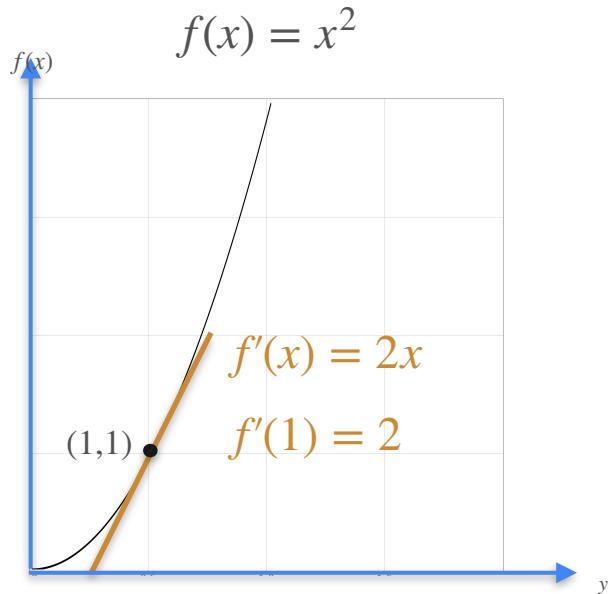
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Derivative of the Inverse



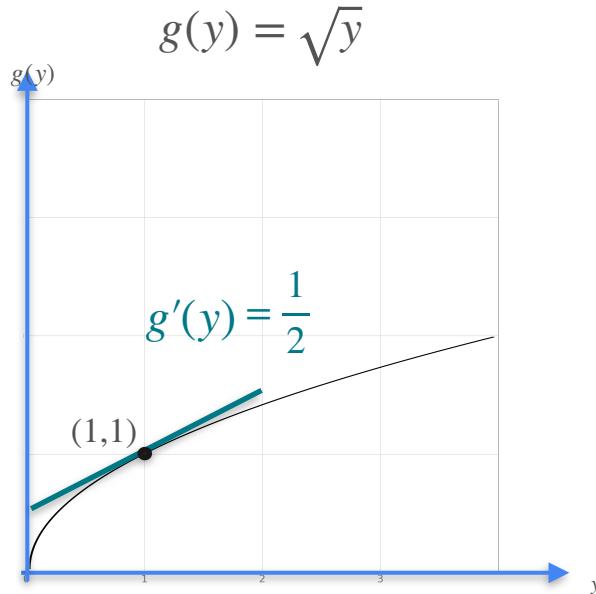
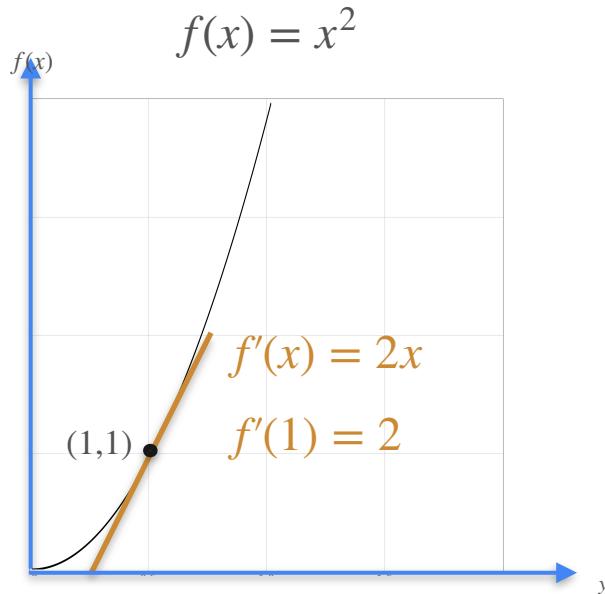
$$g'(y) = \frac{1}{f'(x)}$$

at the point $(1,1)$

$$f(1) = 1 \qquad g(1) = 1$$

$$g'(1) = \frac{1}{f'(1)} = \frac{1}{2}$$

Derivative of the Inverse



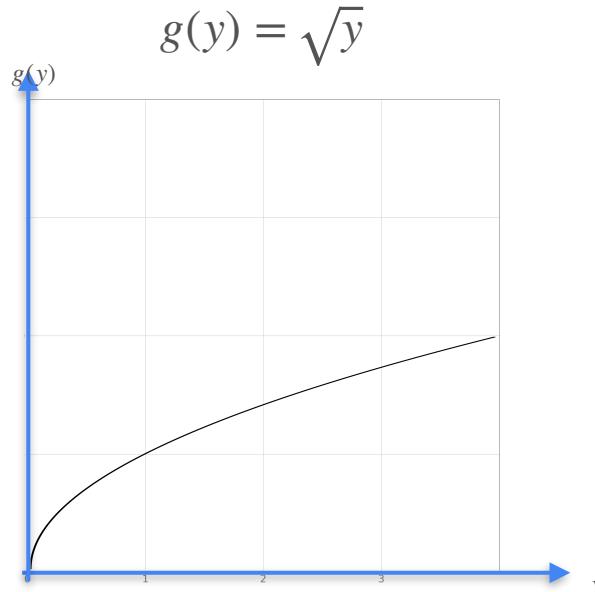
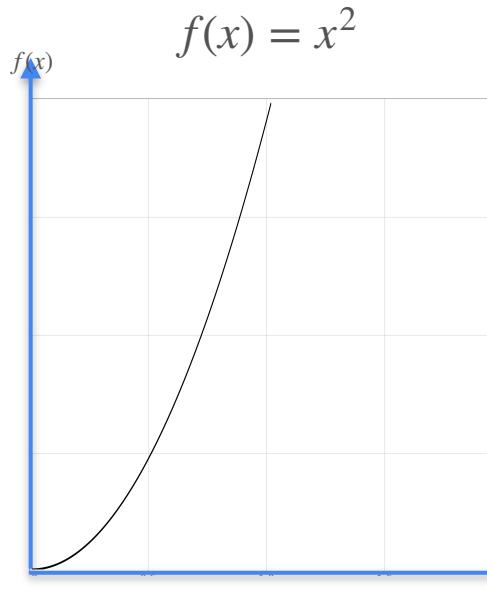
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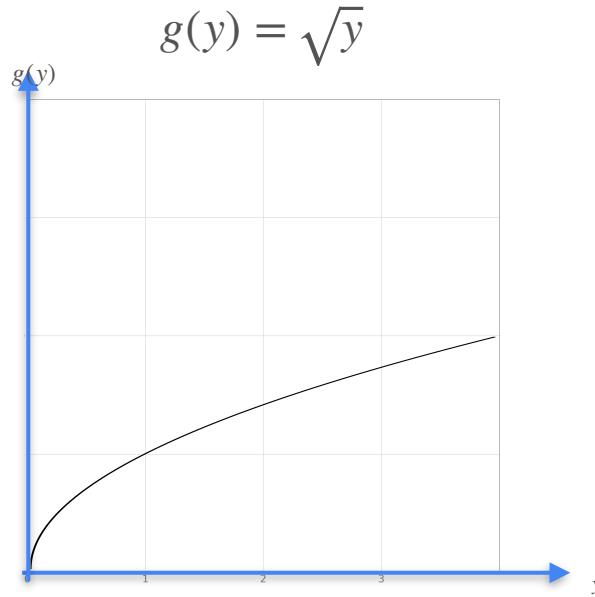
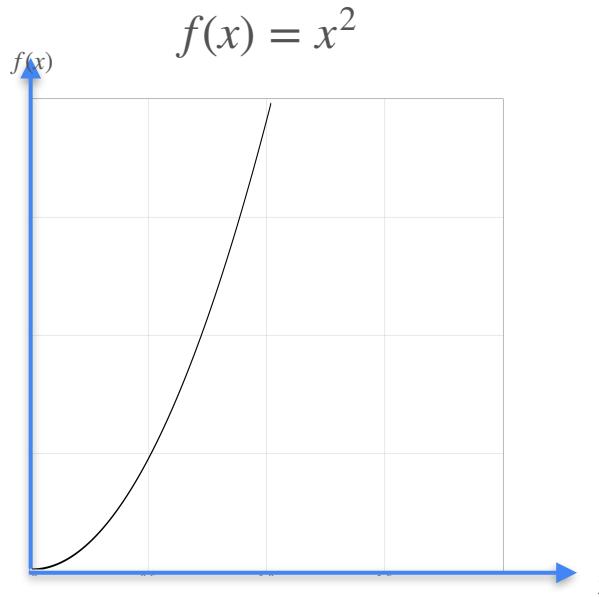
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Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

Derivative of the Inverse



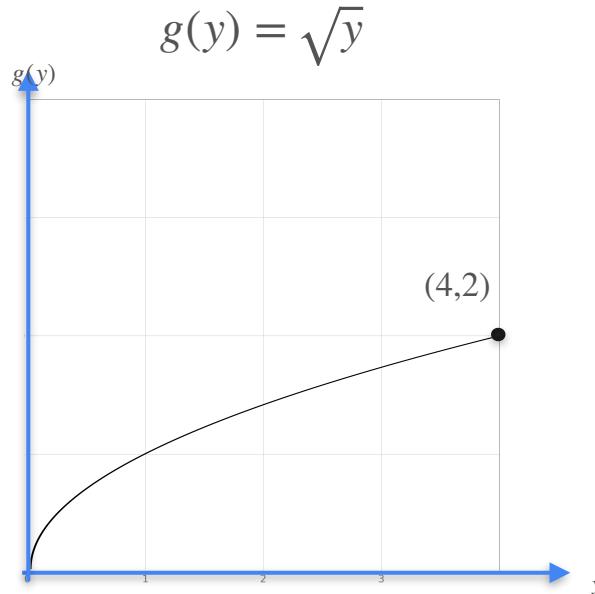
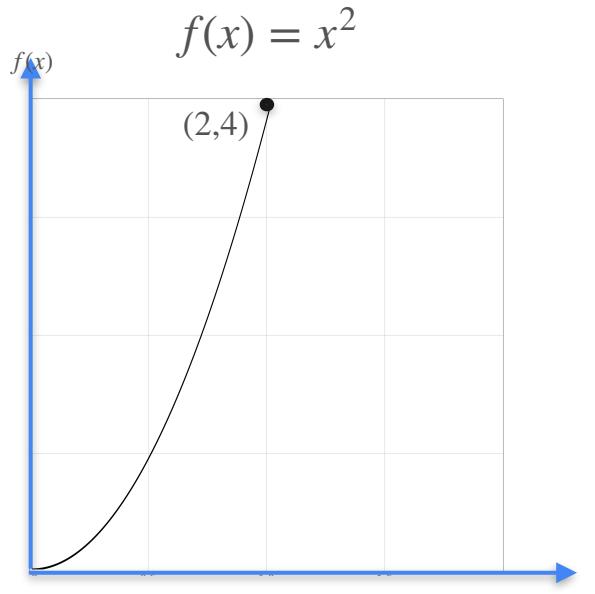
$$g'(y) = \frac{1}{f'(x)}$$

at the point (2,4)

$$f(2) = 4$$

$$g(4) = 2$$

Derivative of the Inverse



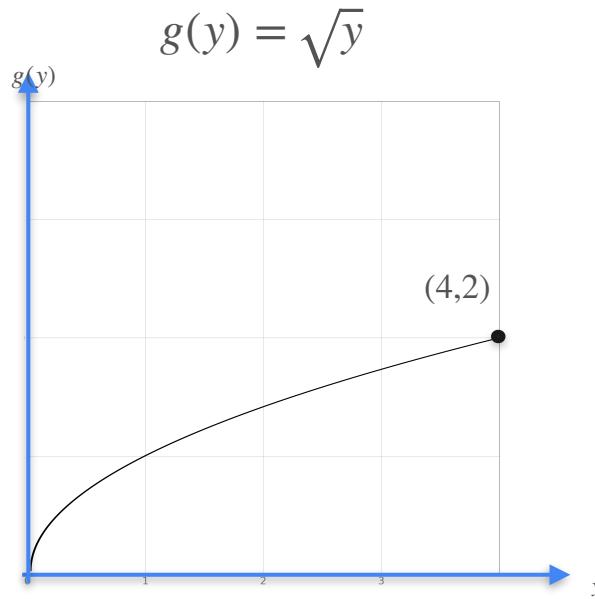
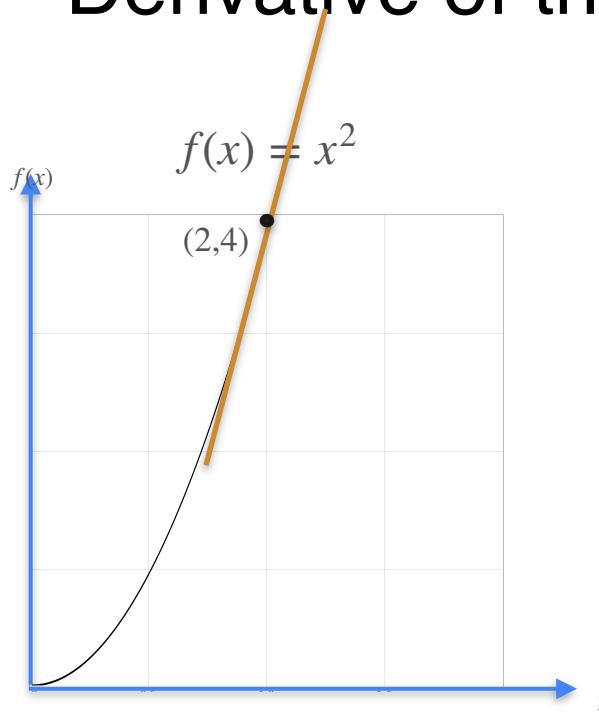
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Derivative of the Inverse



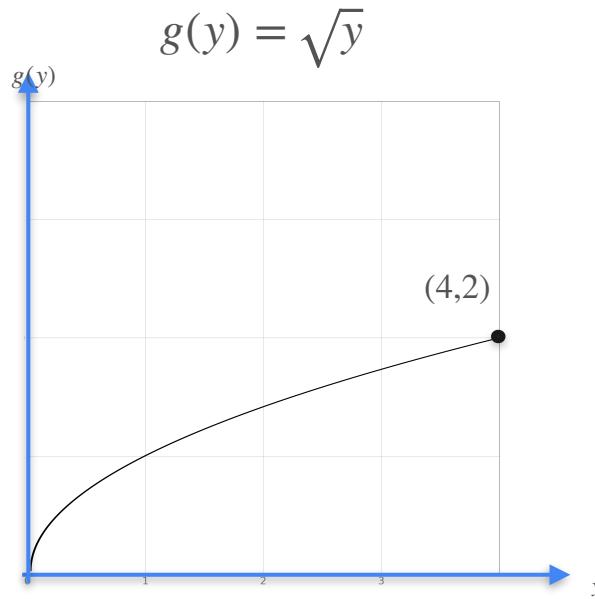
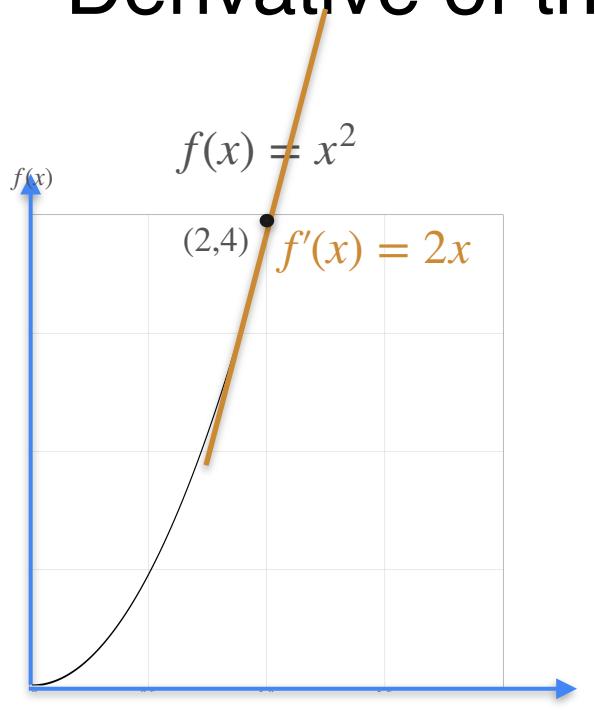
$$g'(y) = \frac{1}{f'(x)}$$

at the point $(2,4)$

$$f(2) = 4$$

$$g(4) = 2$$

Derivative of the Inverse

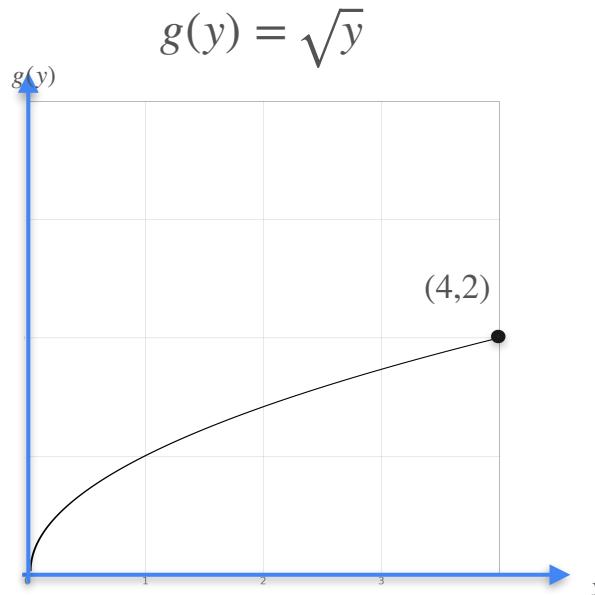
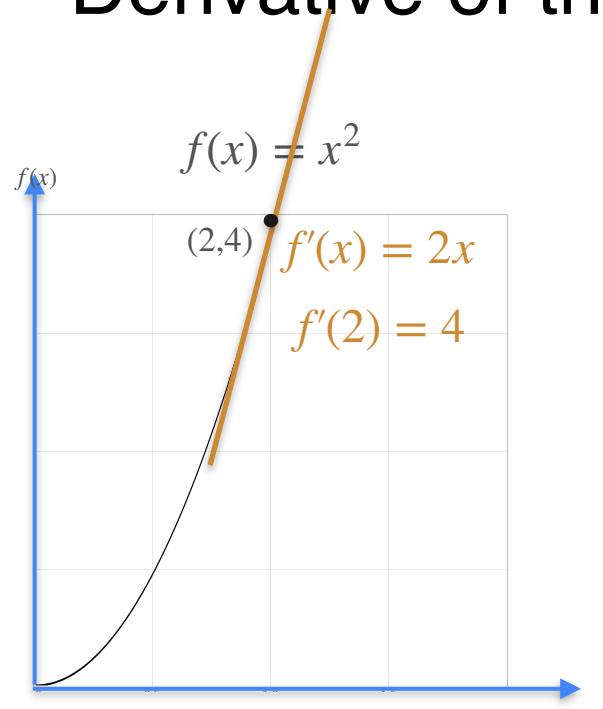


$$g'(y) = \frac{1}{f'(x)}$$

at the point $(2,4)$

$$f(2) = 4 \qquad g(4) = 2$$

Derivative of the Inverse



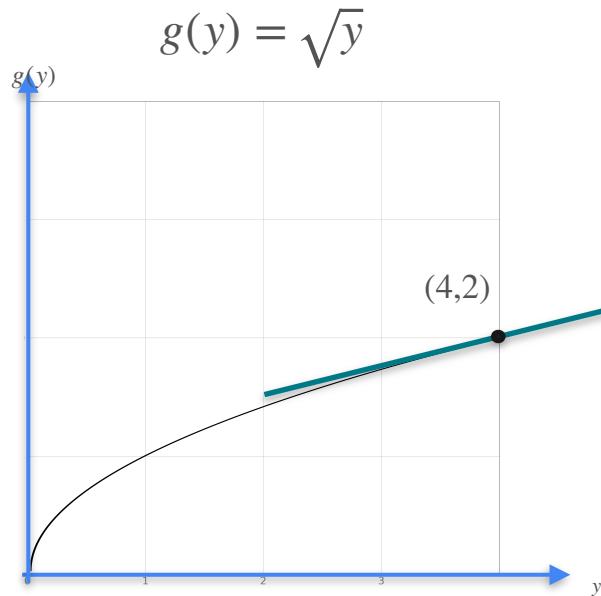
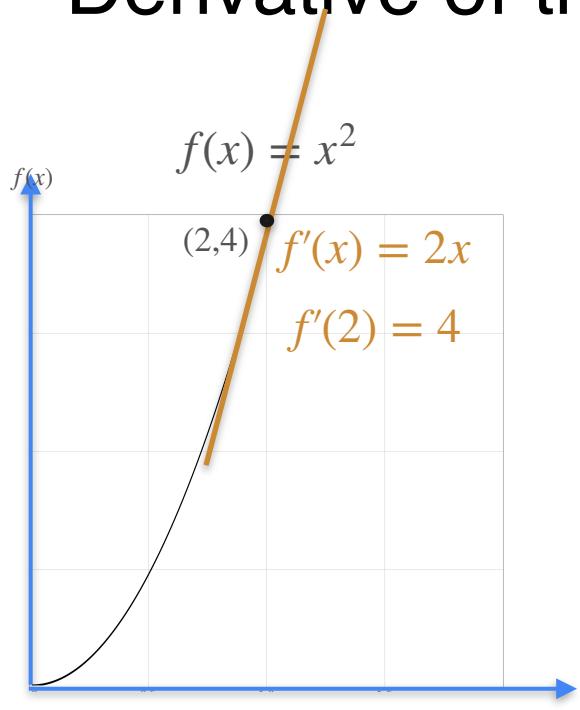
$$g'(y) = \frac{1}{f'(x)}$$

at the point $(2,4)$

$$f(2) = 4$$

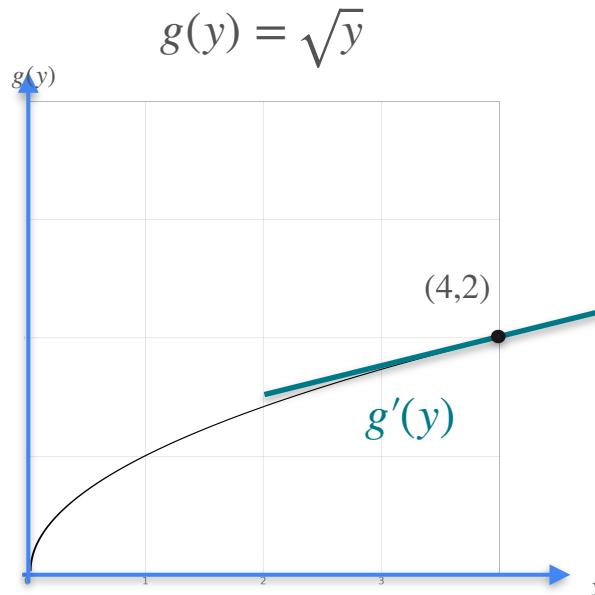
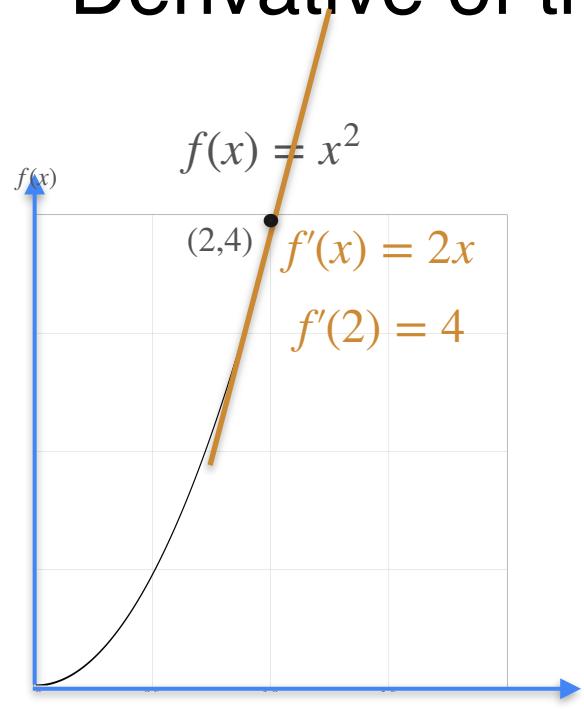
$$g(4) = 2$$

Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

Derivative of the Inverse



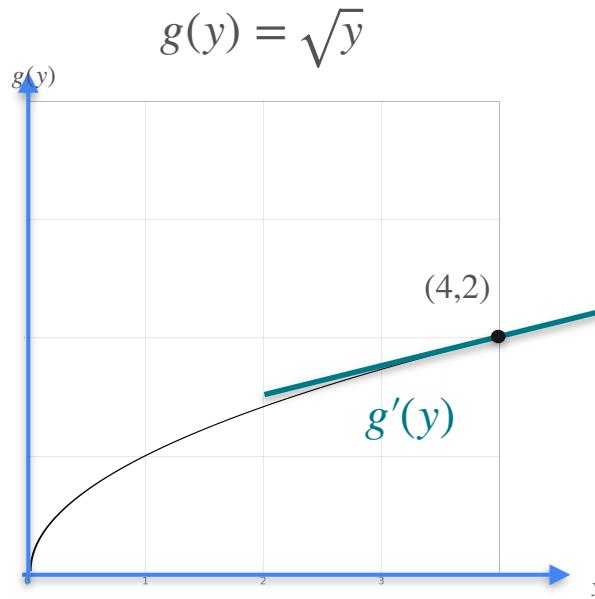
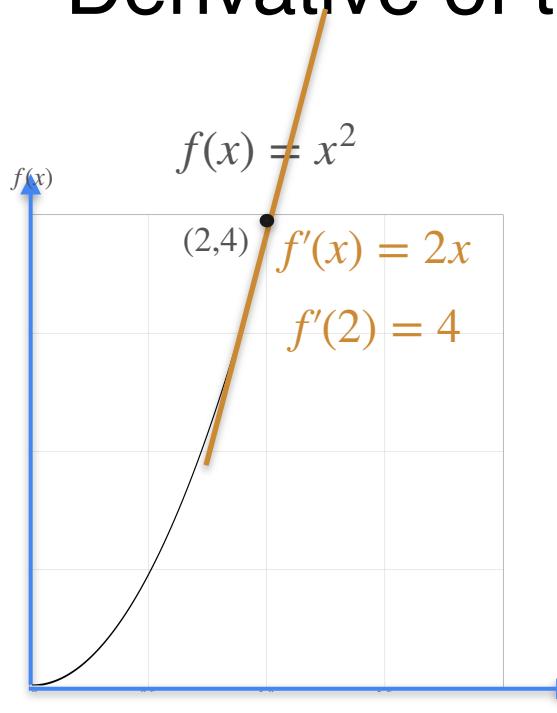
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at the point $(2,4)$

$$f(2) = 4$$

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Derivative of the Inverse

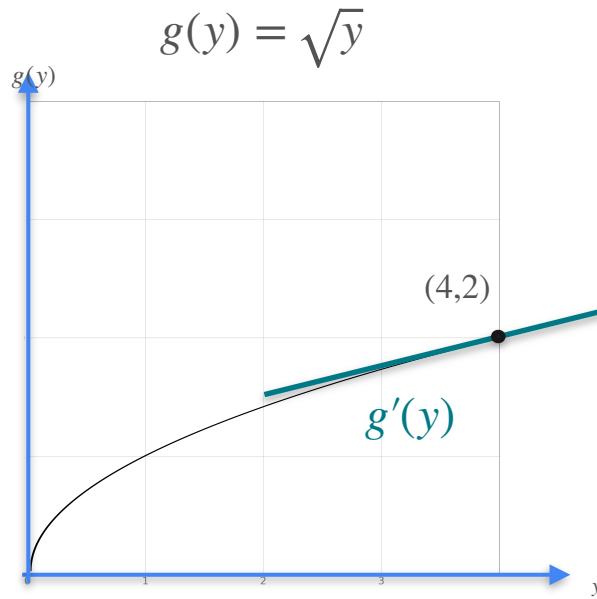
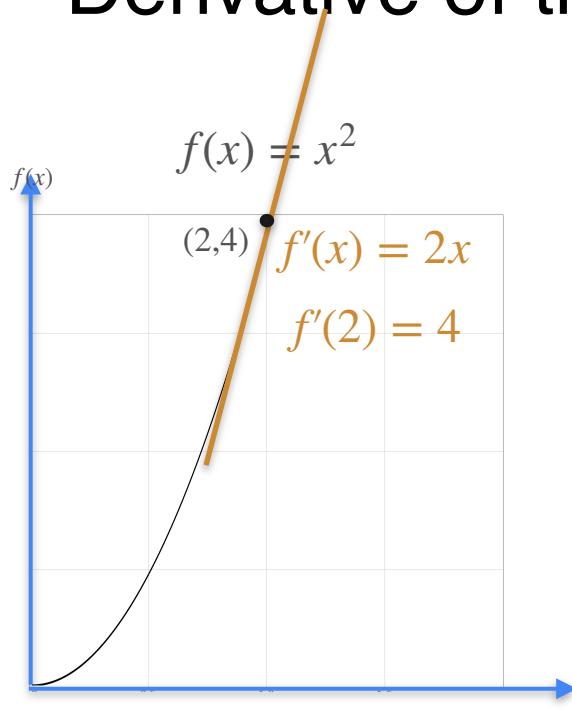


$$g'(y) = \frac{1}{f'(x)}$$

at the point $(2,4)$

$$f(2) = 4 \quad g(4) = 2$$
$$g'(4) = \frac{1}{f'(2)}$$

Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

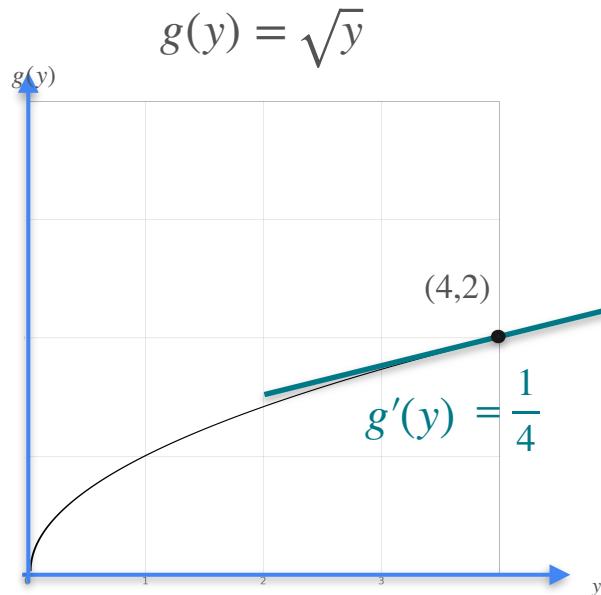
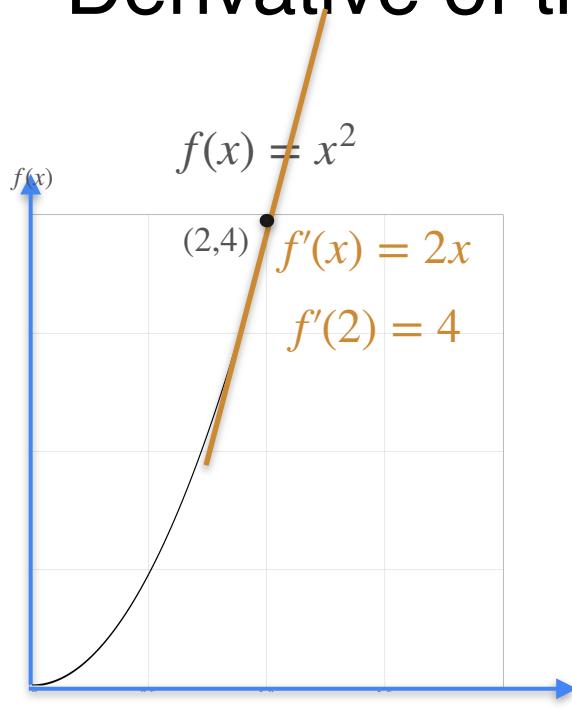
at the point $(2,4)$

$$f(2) = 4$$

$$g(4) = 2$$

$$g'(4) = \frac{1}{f'(2)} = \frac{1}{4}$$

Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

at the point $(2,4)$

$$f(2) = 4$$

$$g(4) = 2$$

$$g'(4) = \frac{1}{f'(2)}$$



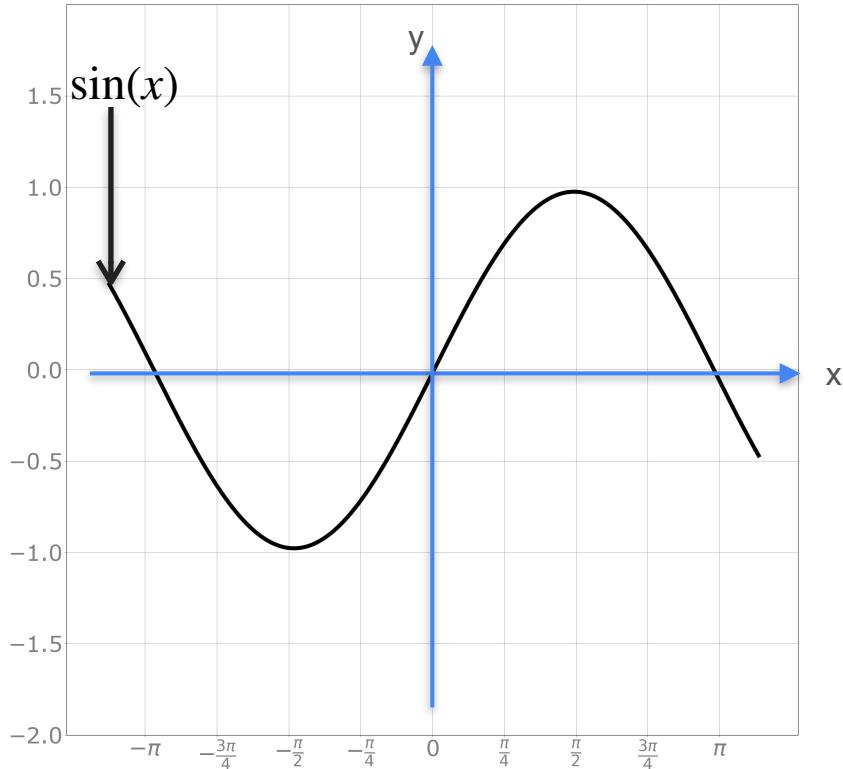
DeepLearning.AI

Derivatives and Optimization

Derivative of trigonometric functions

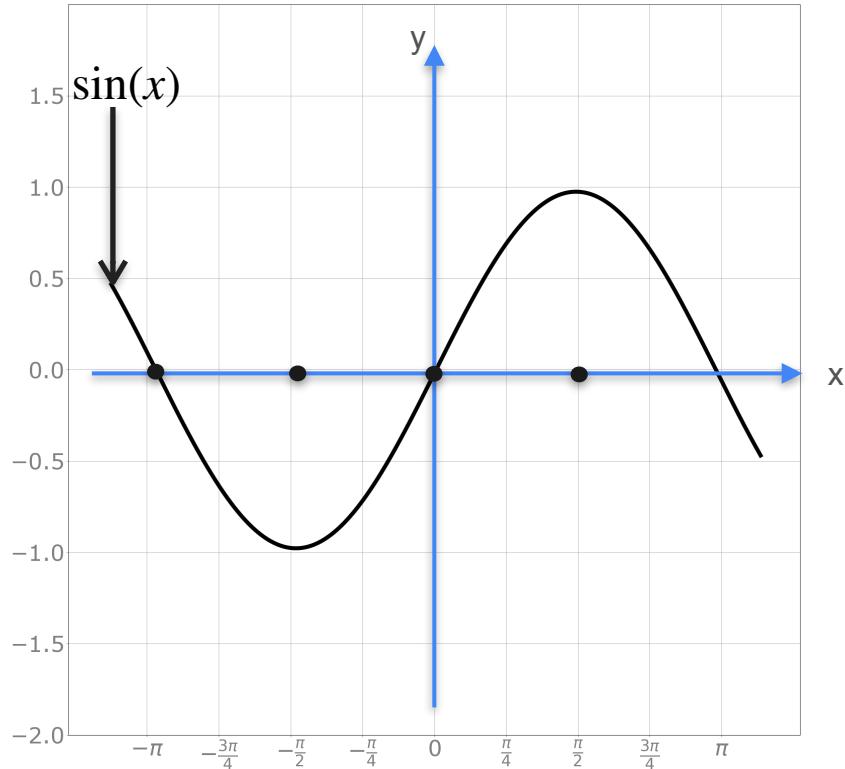
Derivative of Trigonometric Functions

Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

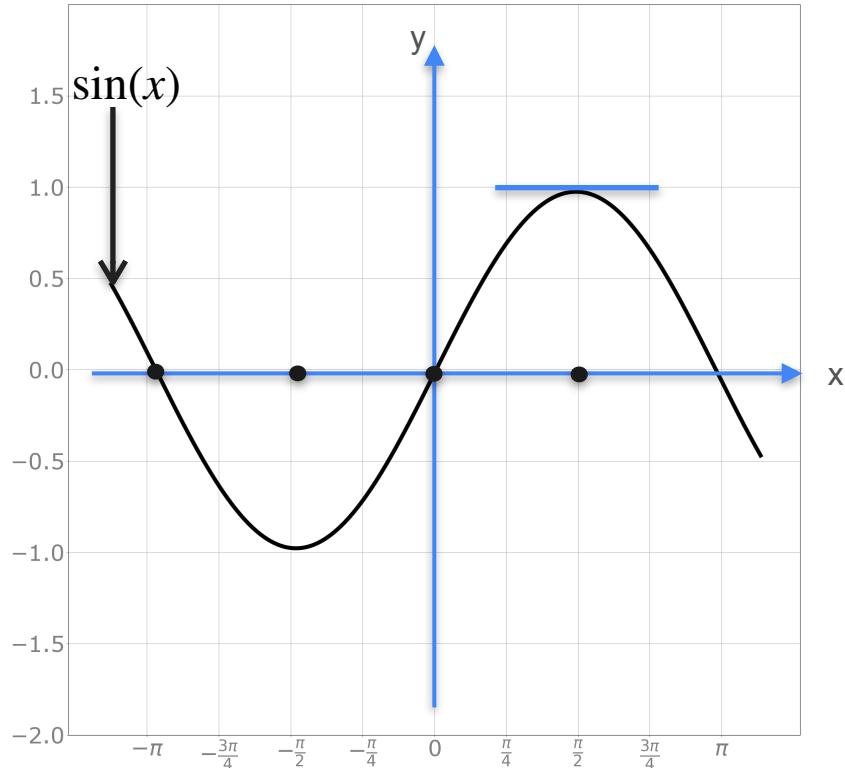
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
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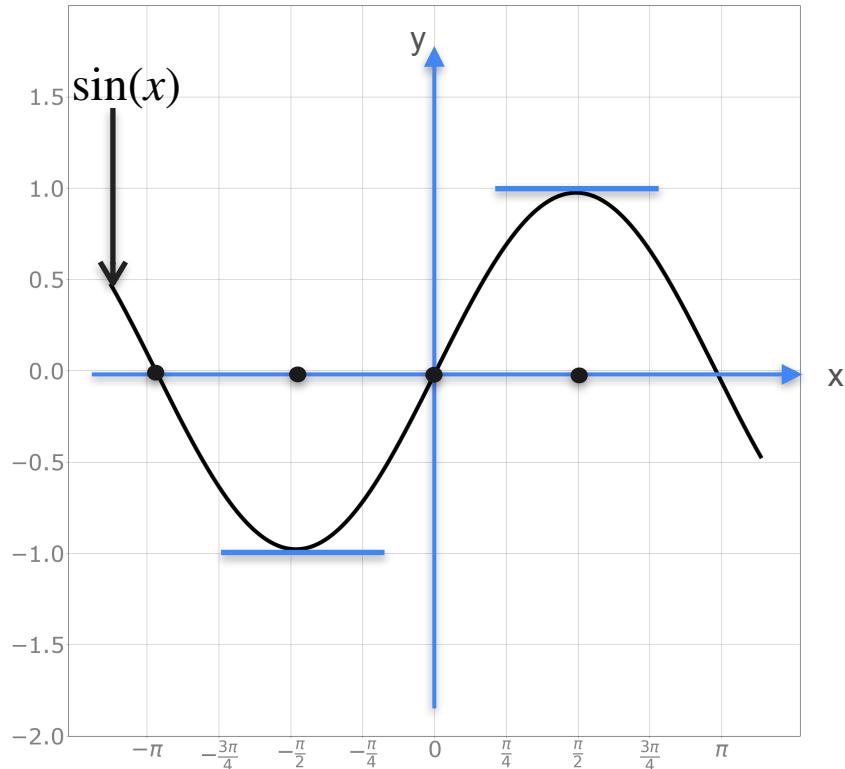
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0			

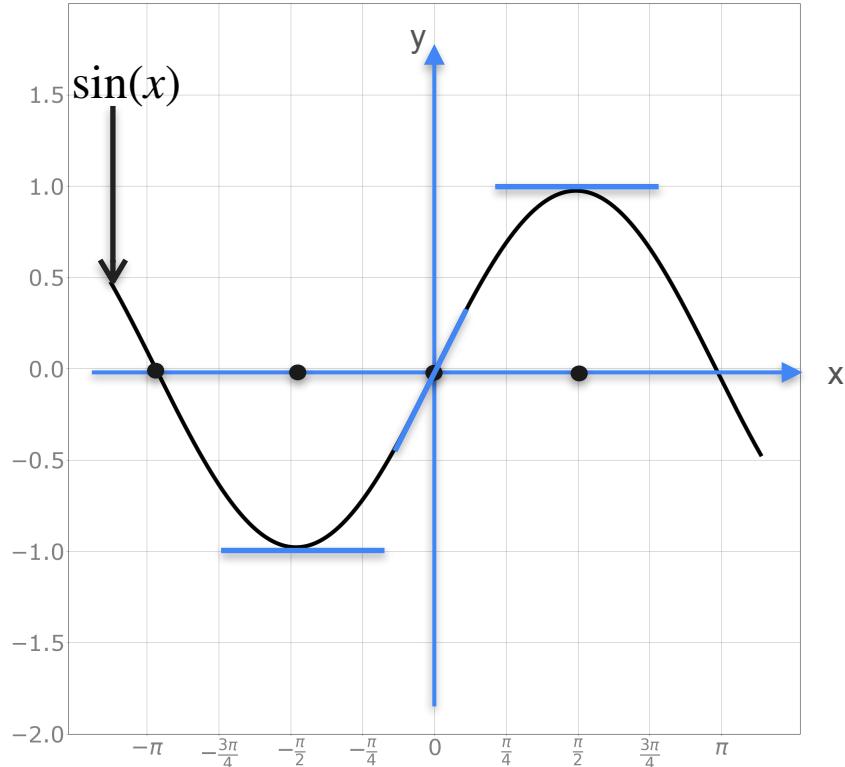
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0		

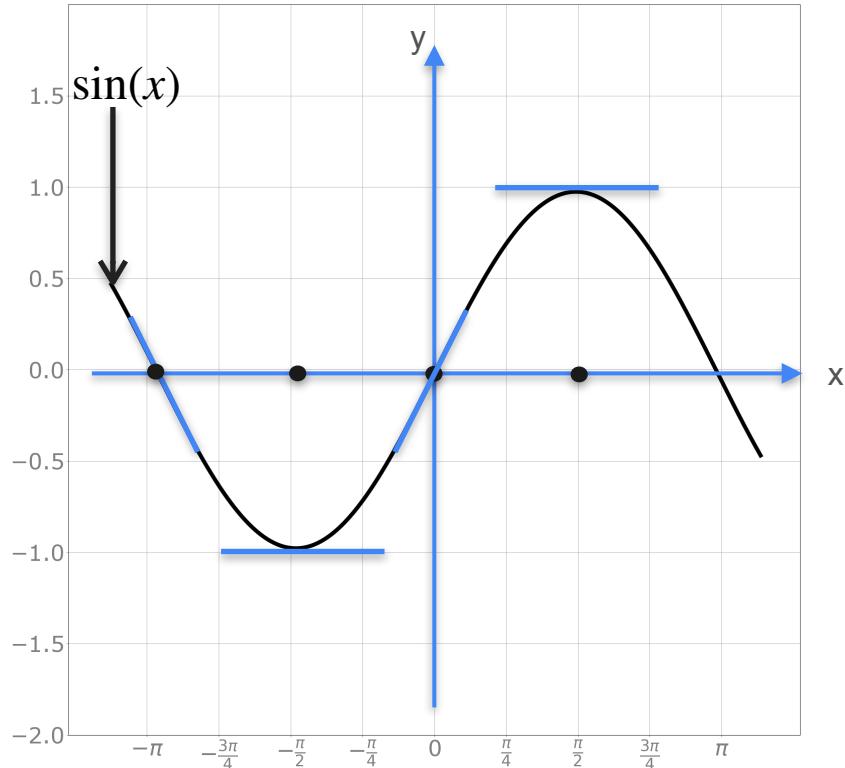
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	

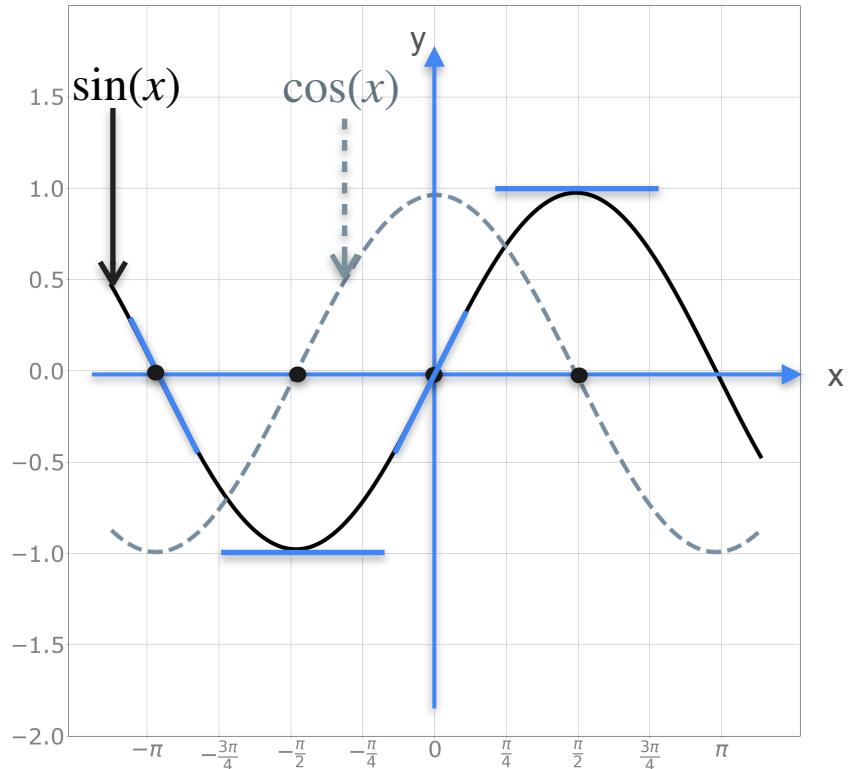
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

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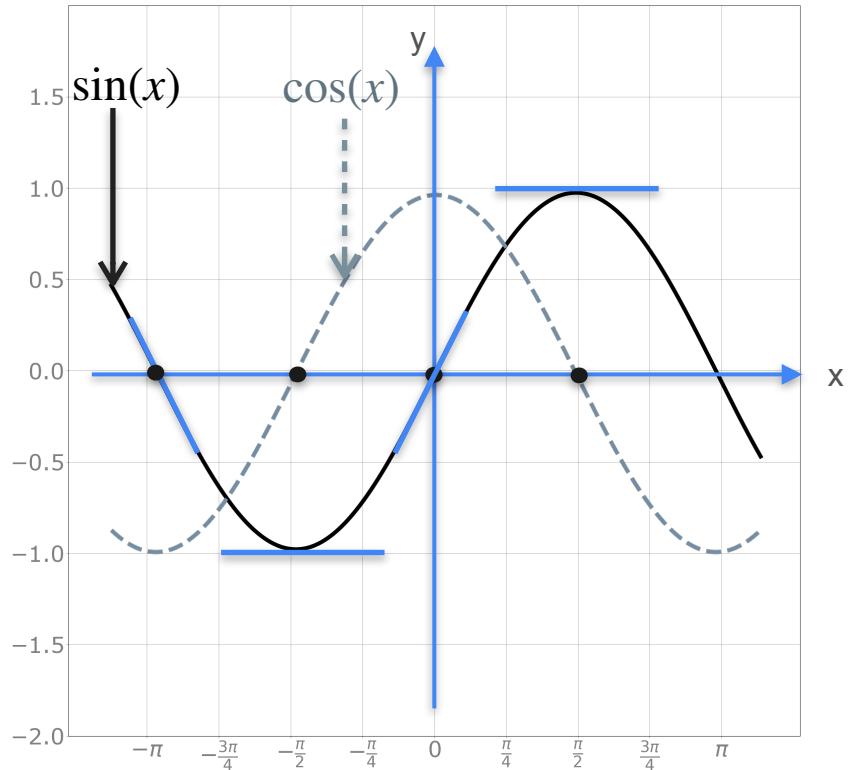
Derivative of Trigonometric Functions



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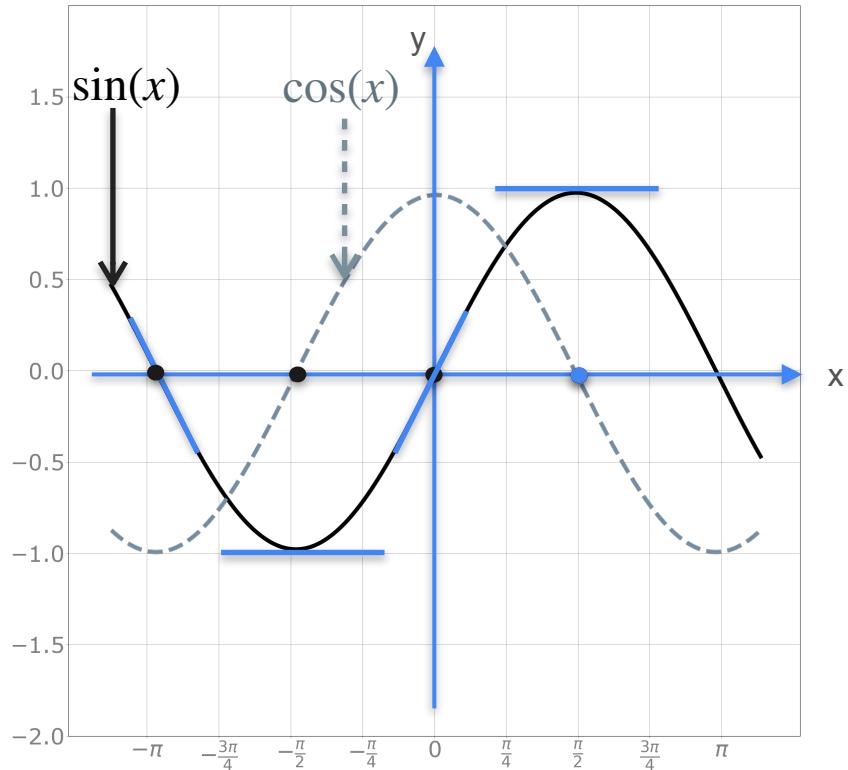
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
$\cos(x)$				

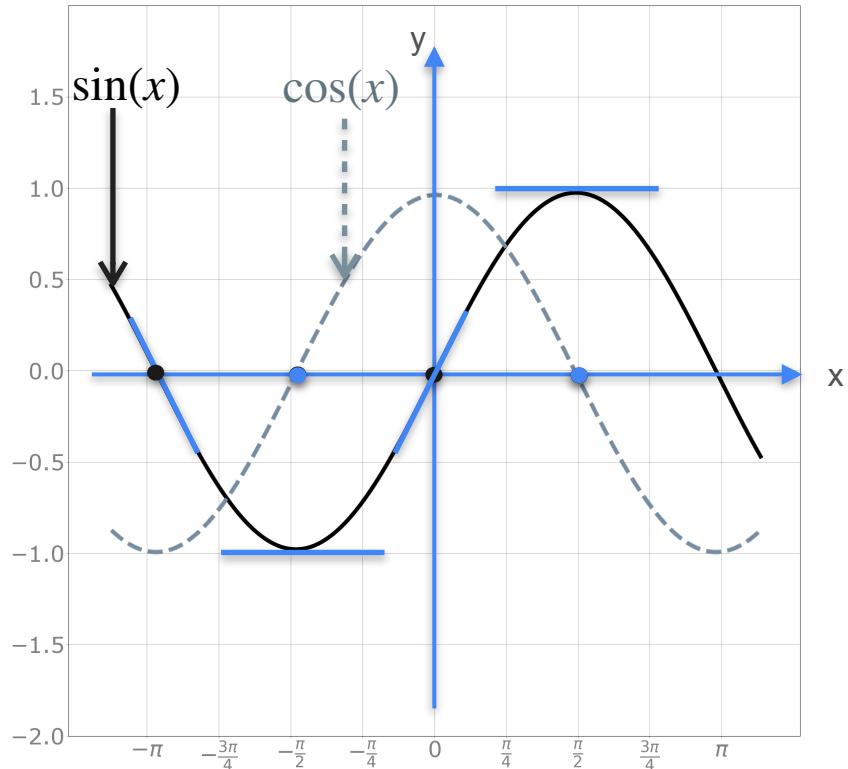
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
$\cos(x)$	0			

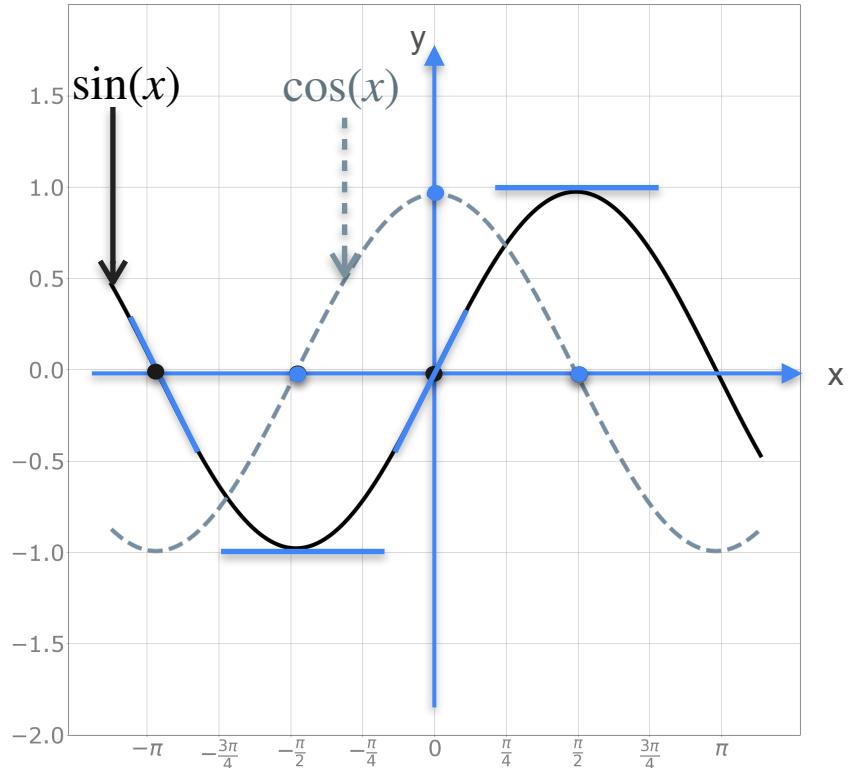
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

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Slope	0	0	1	-1
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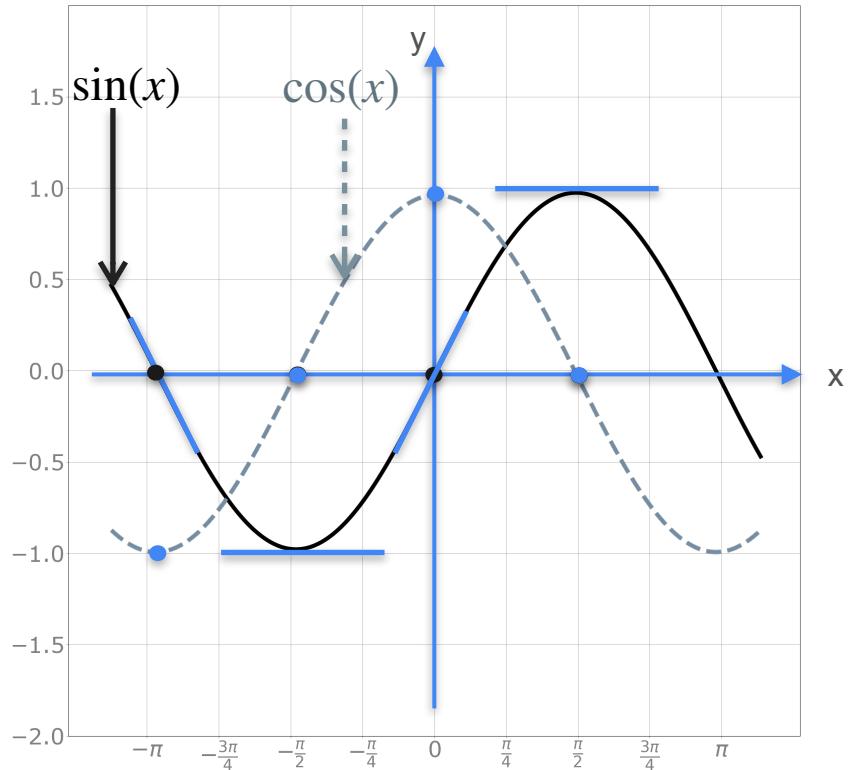
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

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Slope	0	0	1	-1
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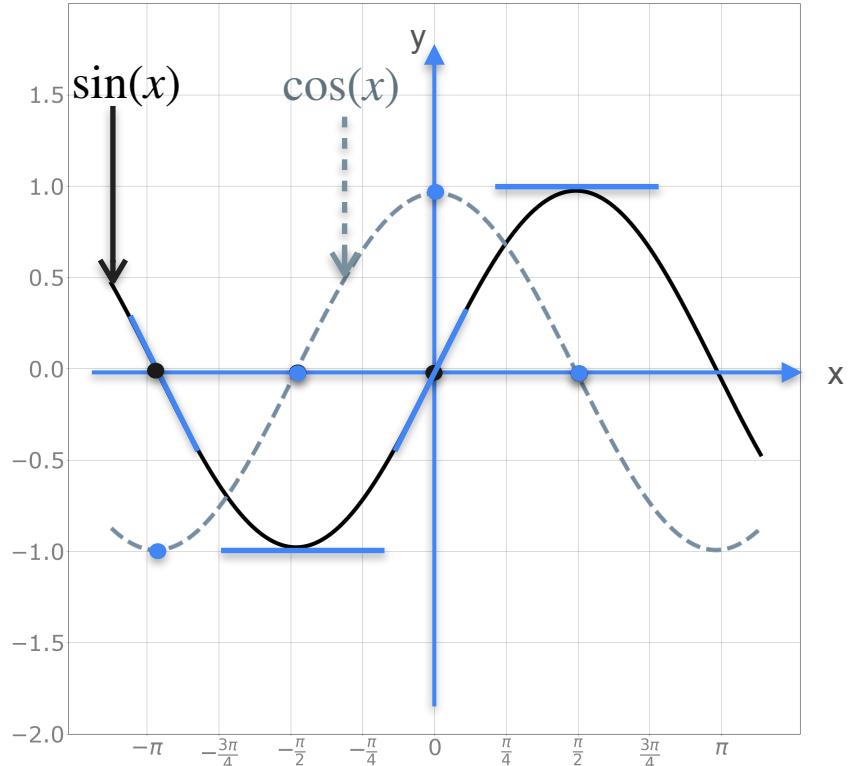
Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
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Derivative of Trigonometric Functions

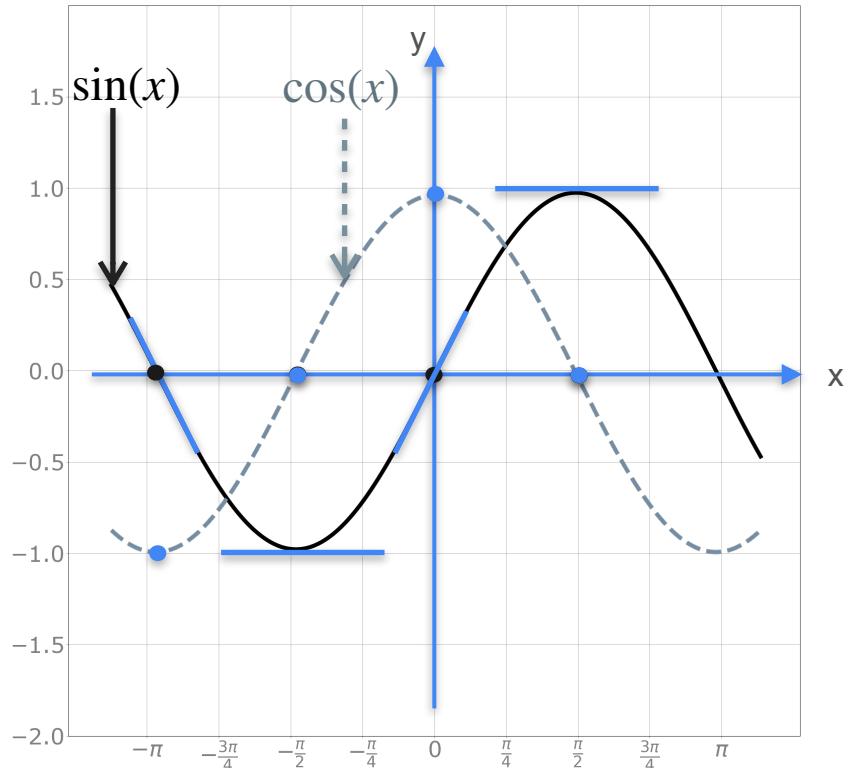


Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
$\cos(x)$	0	0	1	-1

$$f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$

Derivative of Trigonometric Functions



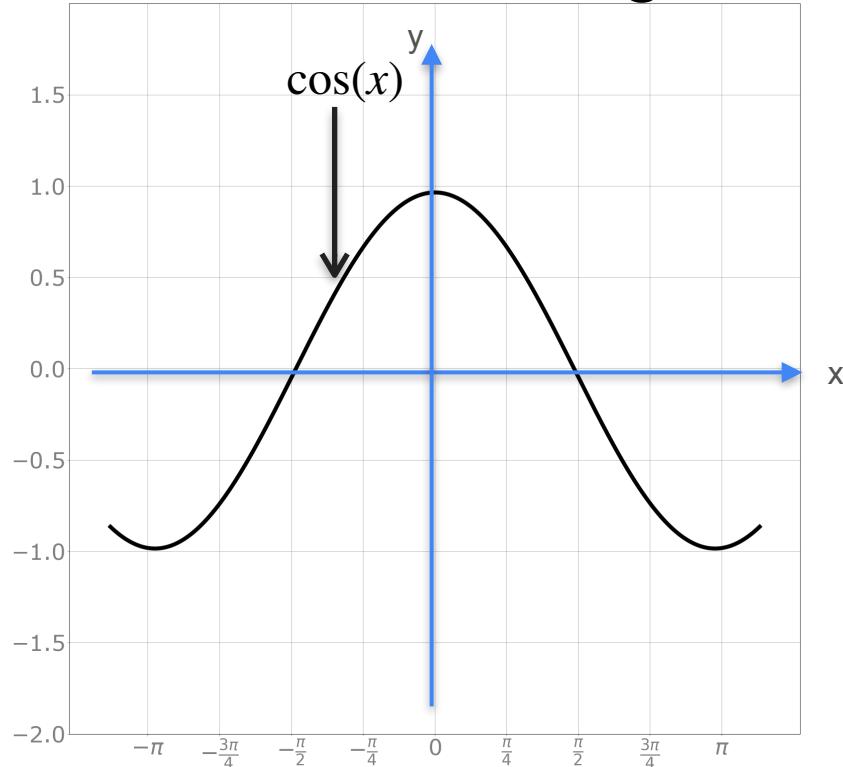
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Slope	0	0	1	-1
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$f(x) = \sin(x) \xrightarrow{??} f'(x) = \cos(x)$

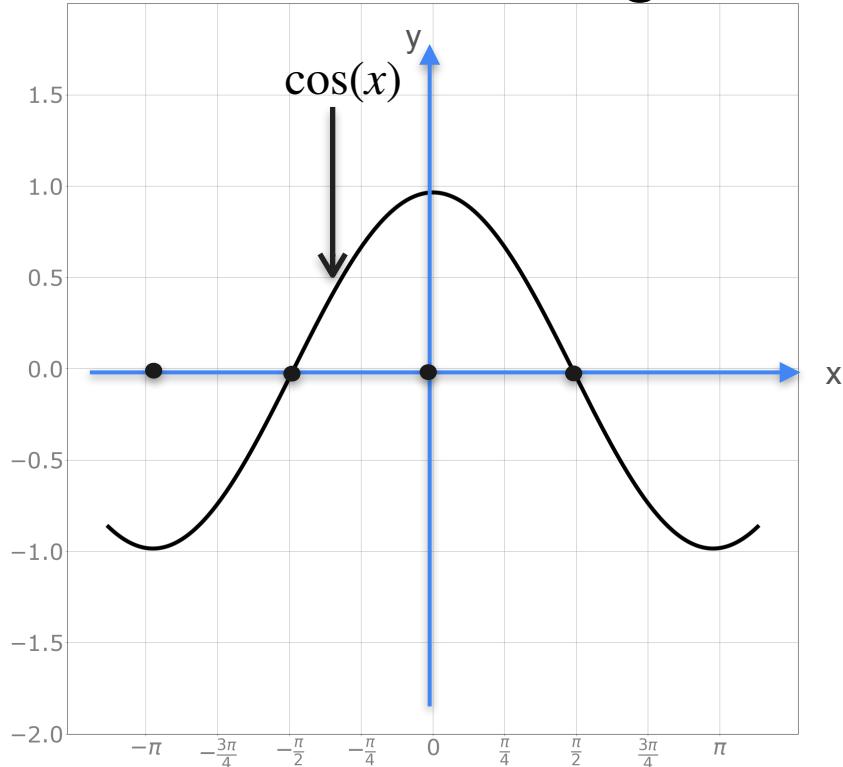
Derivative of Trigonometric Functions

Derivative of Trigonometric Functions



Cosine $y = f(x) = \cos(x)$

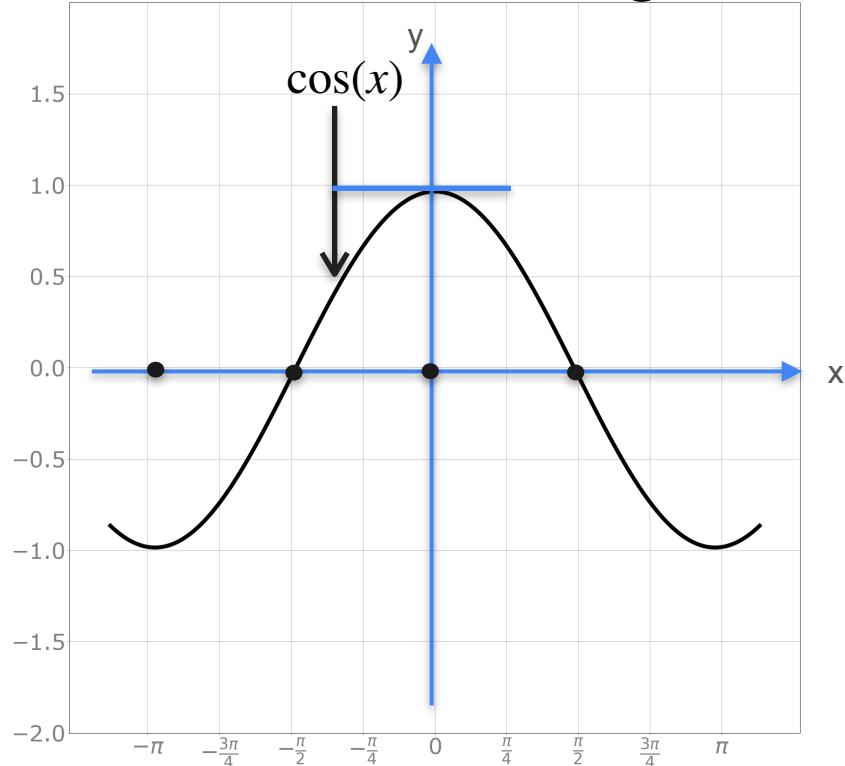
Derivative of Trigonometric Functions



Cosine $y = f(x) = \cos(x)$

x	0	$-\pi$	$\pi/2$	$-\pi/2$
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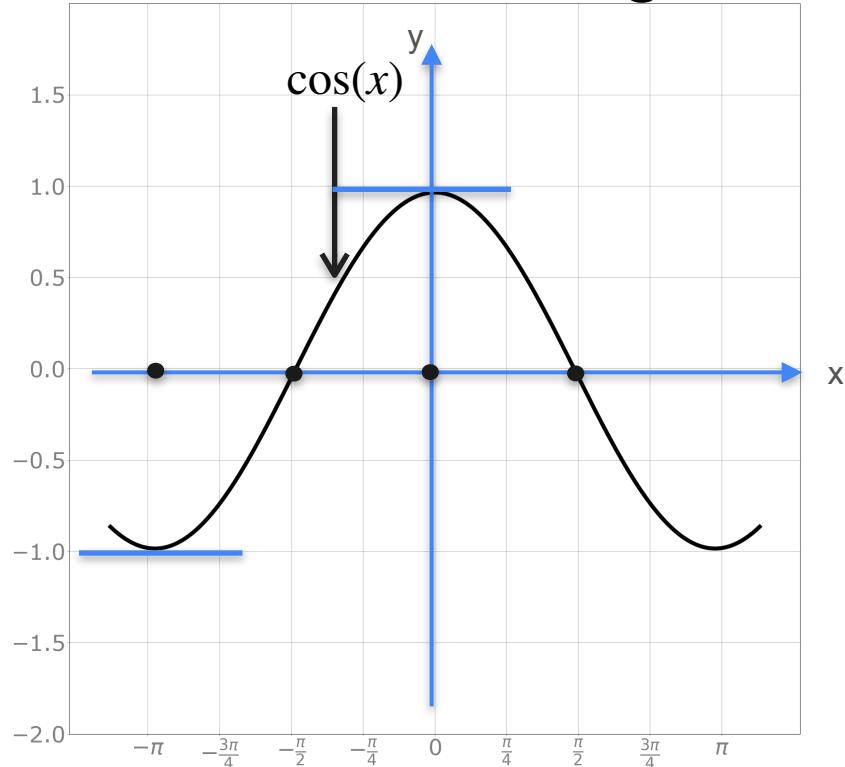
Derivative of Trigonometric Functions



Cosine $y = f(x) = \cos(x)$

x	0	$-\pi$	$\pi/2$	$-\pi/2$
Slope	0			

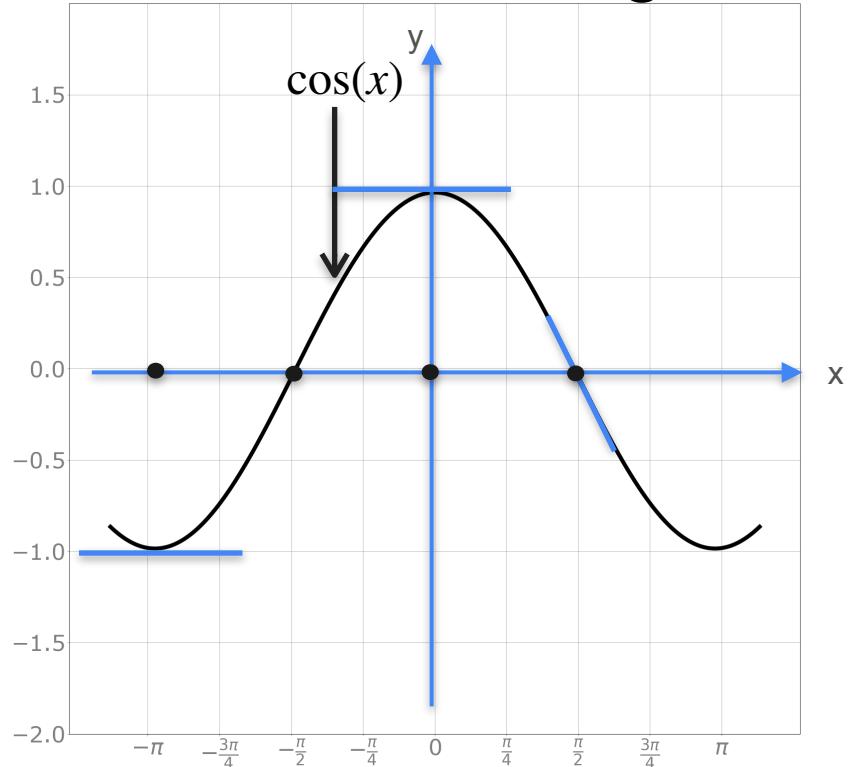
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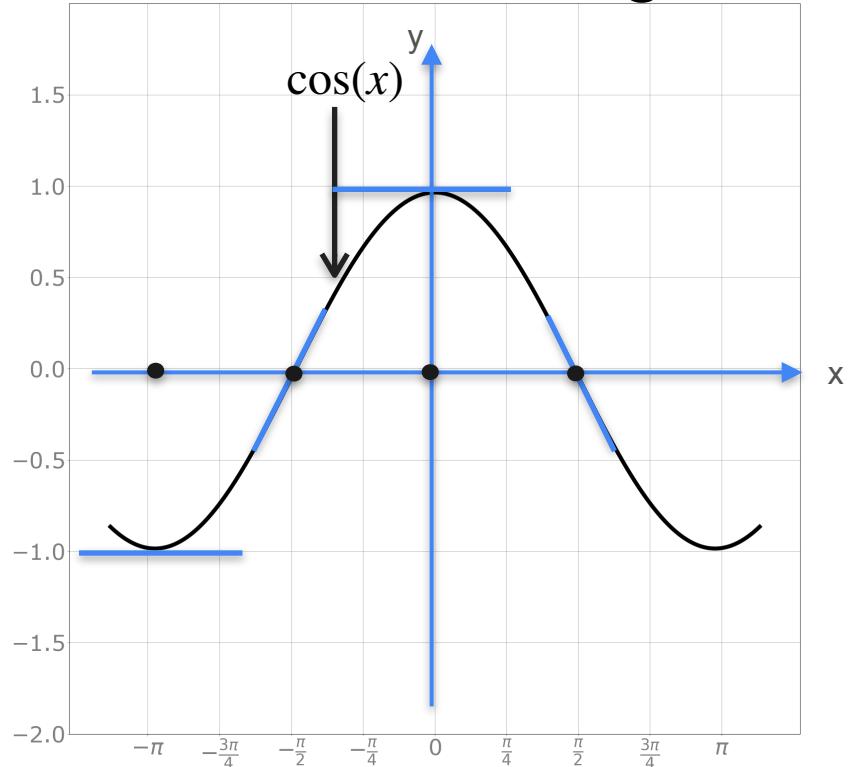
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Cosine $y = f(x) = \cos(x)$

x	0	$-\pi$	$\pi/2$	$-\pi/2$
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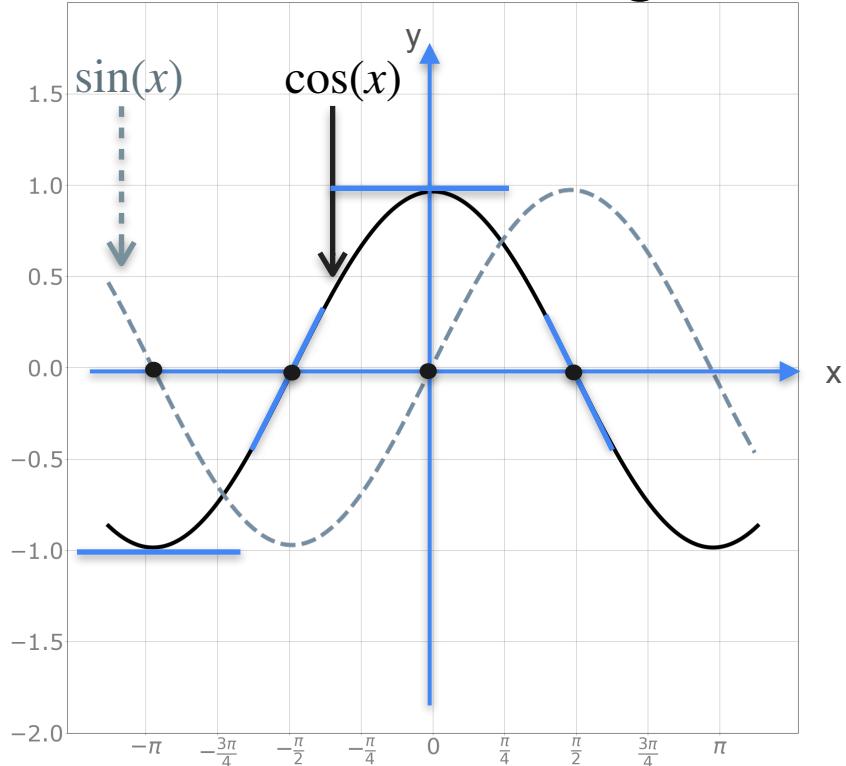
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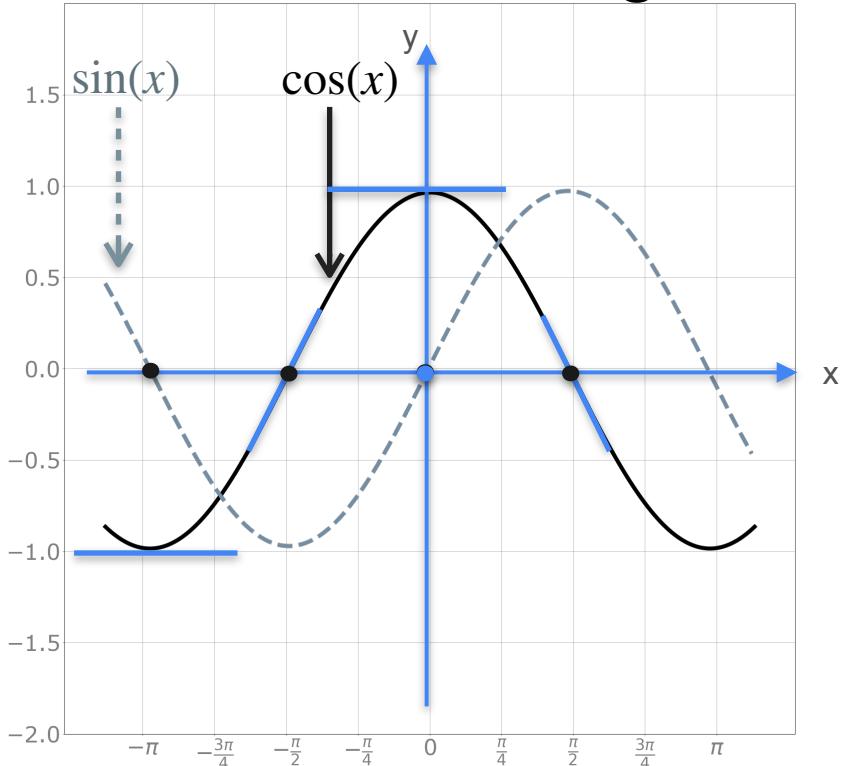
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Slope	0	0	-1	1
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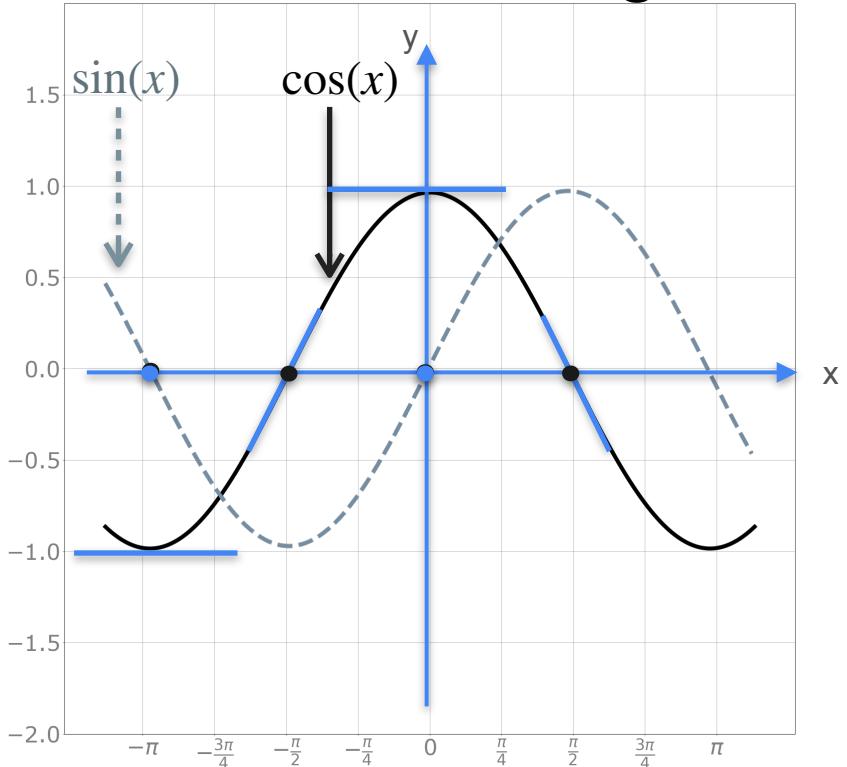
Derivative of Trigonometric Functions



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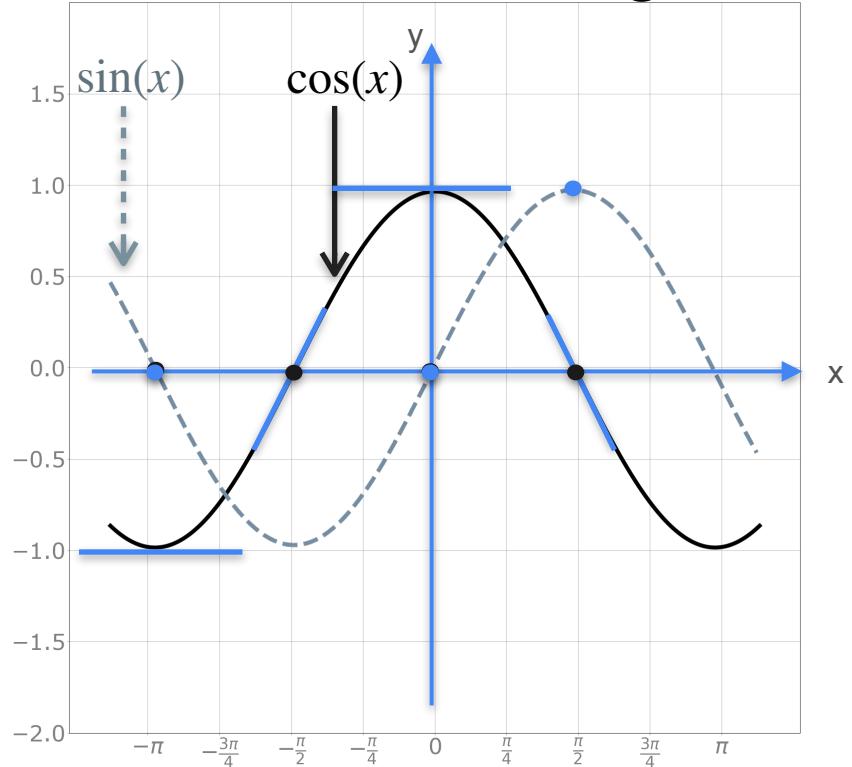
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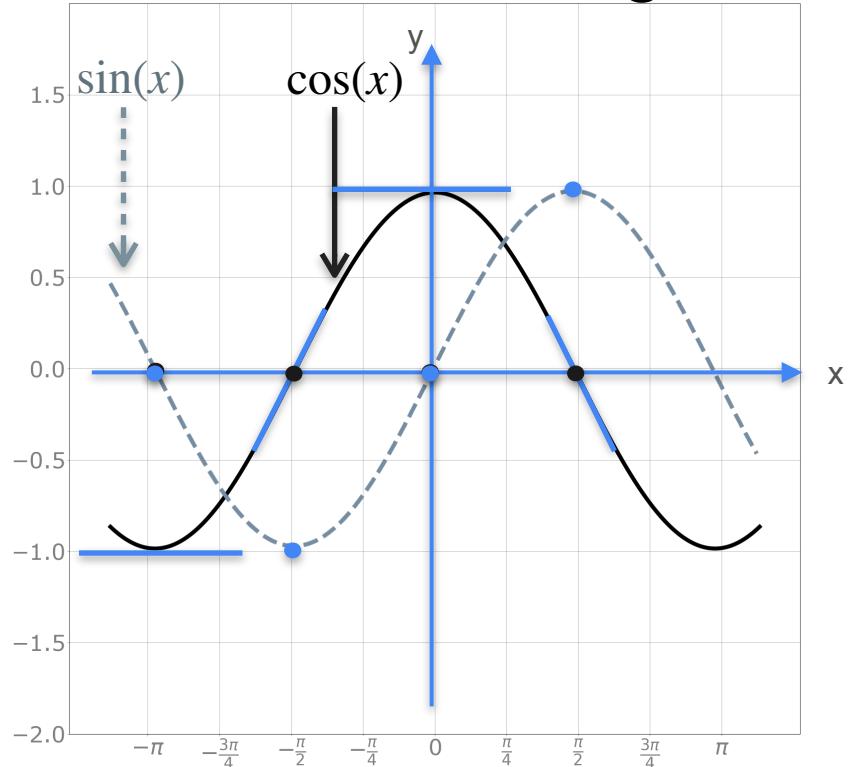
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Slope	0	0	-1	1
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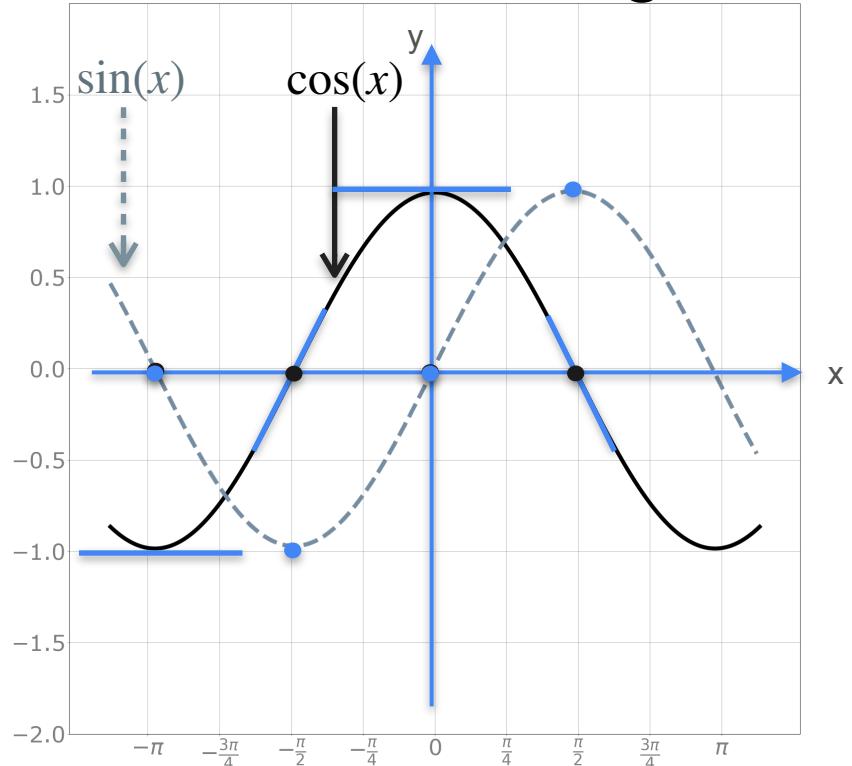
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Cosine $y = f(x) = \cos(x)$

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Derivative of Trigonometric Functions

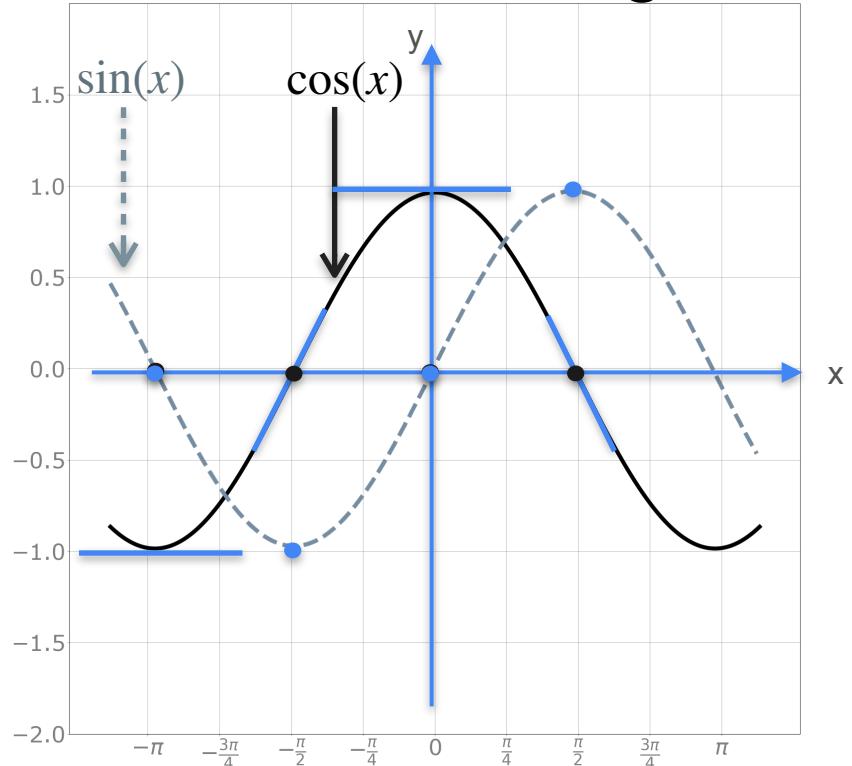


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Slope	0	0	-1	1
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$$f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$$

Derivative of Trigonometric Functions



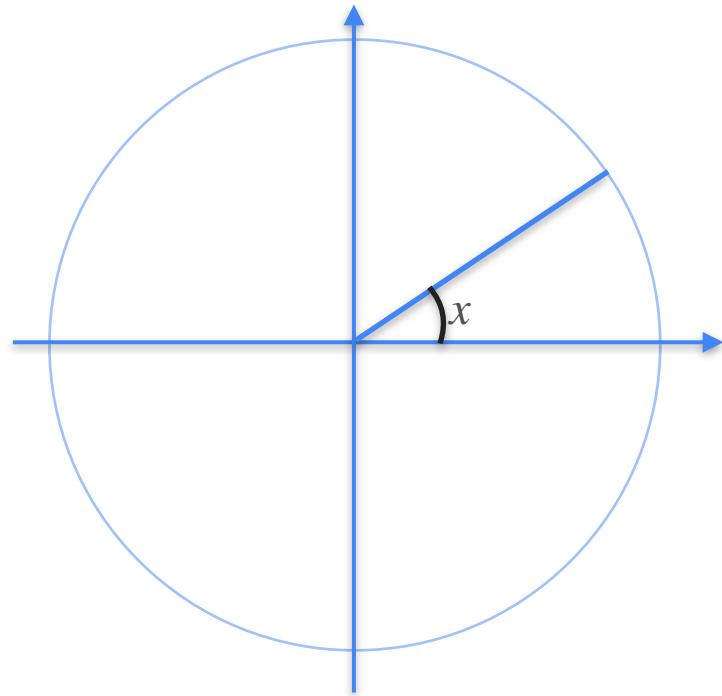
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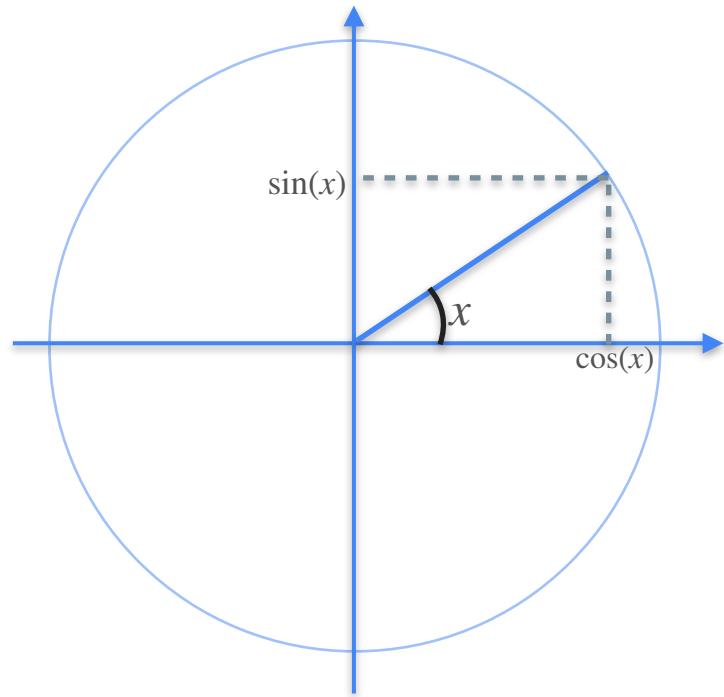
$f(x) = \cos(x) \xrightarrow{??} f'(x) = -\sin(x)$

Derivative of Trigonometric Functions

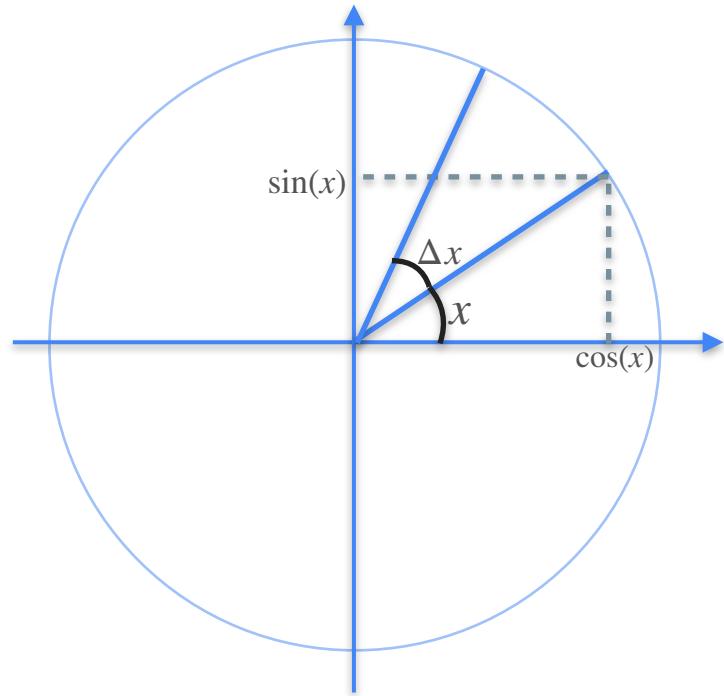
Derivative of Trigonometric Functions



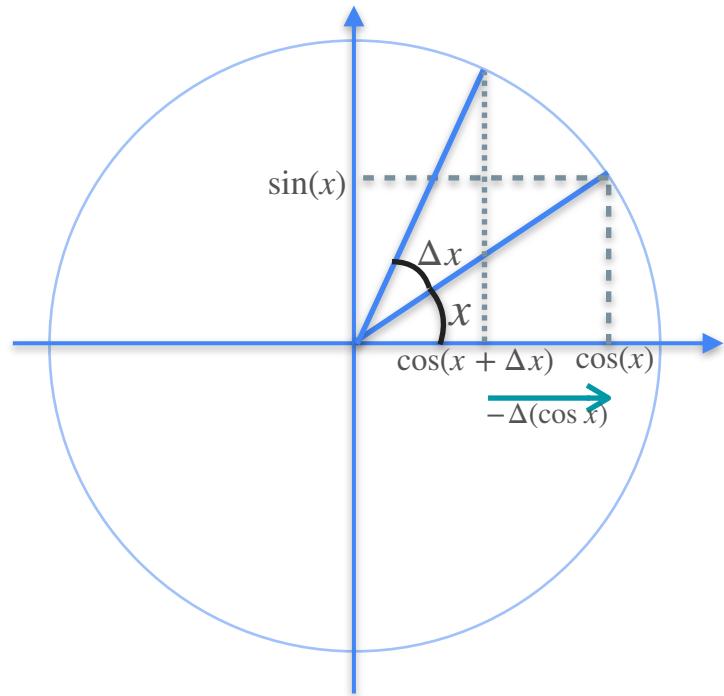
Derivative of Trigonometric Functions



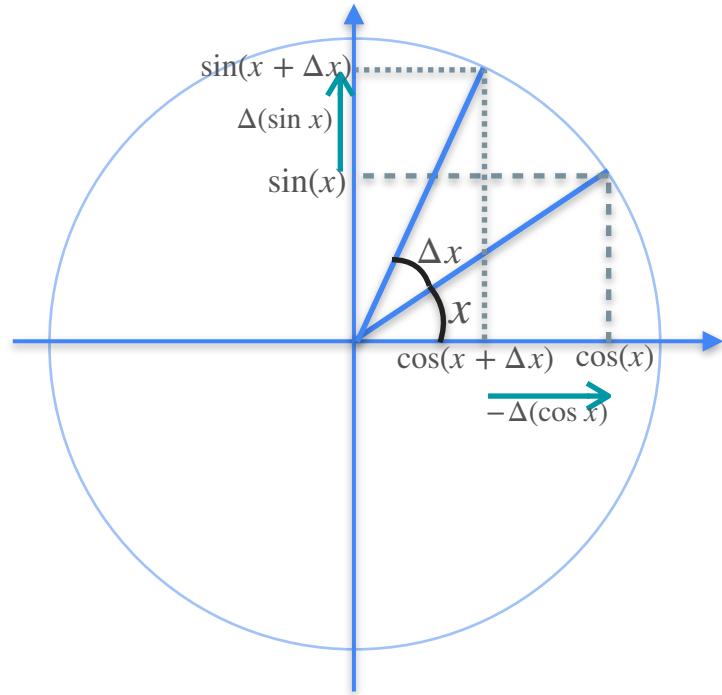
Derivative of Trigonometric Functions



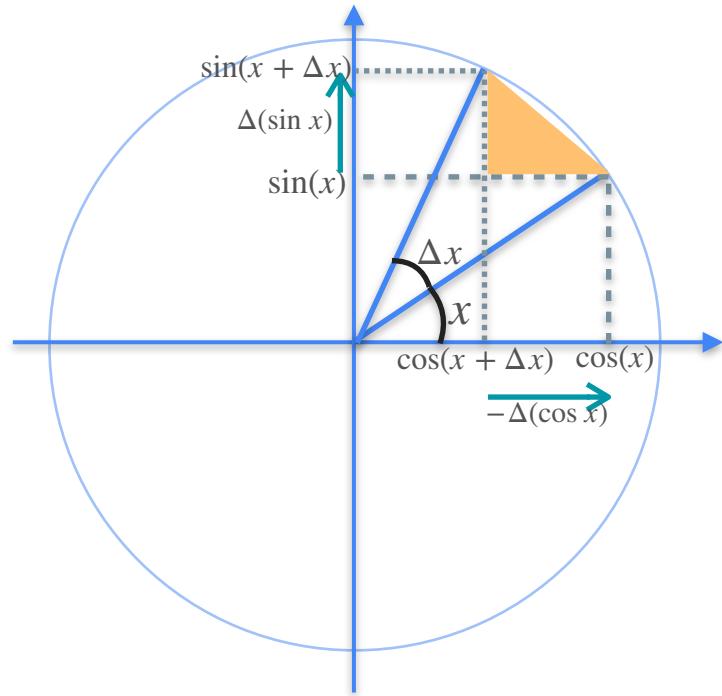
Derivative of Trigonometric Functions



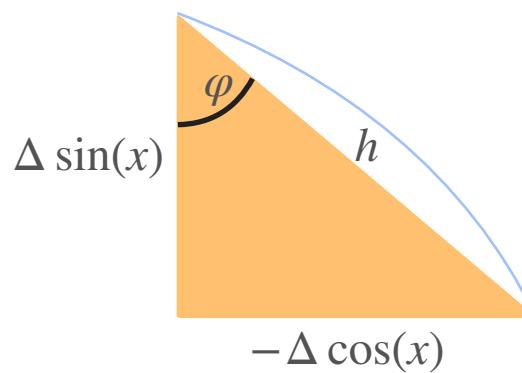
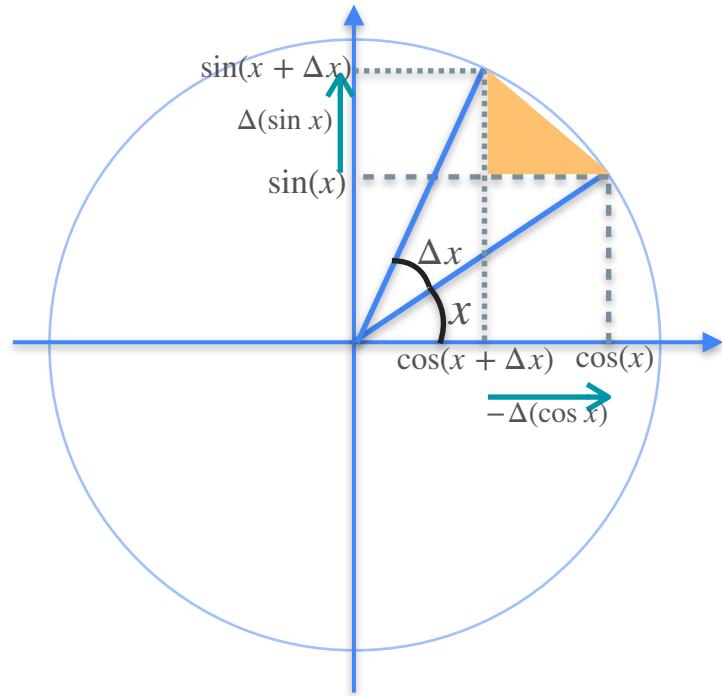
Derivative of Trigonometric Functions



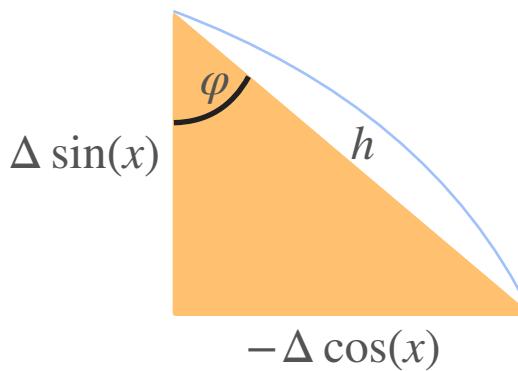
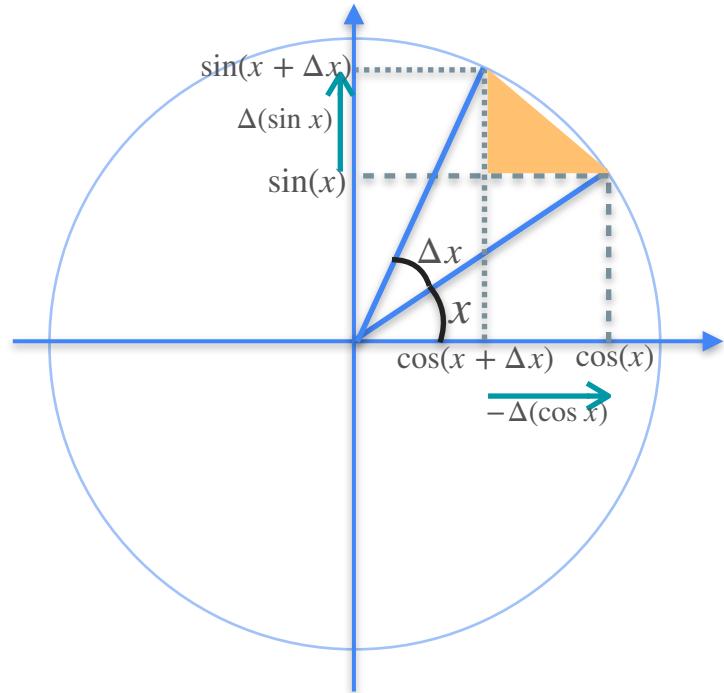
Derivative of Trigonometric Functions



Derivative of Trigonometric Functions

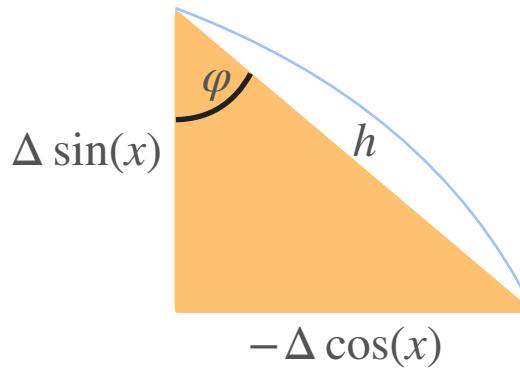
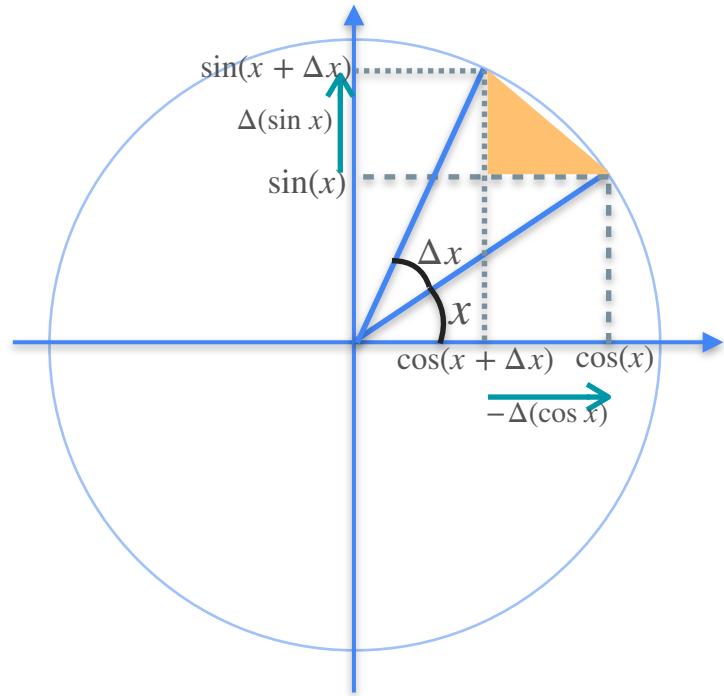


Derivative of Trigonometric Functions



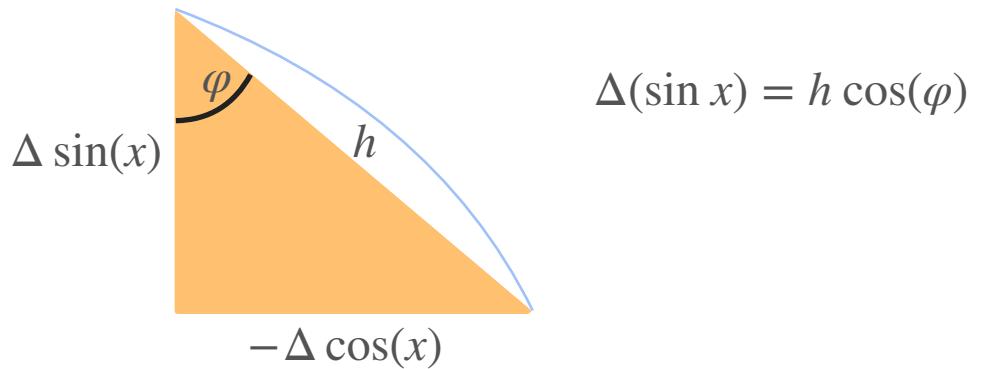
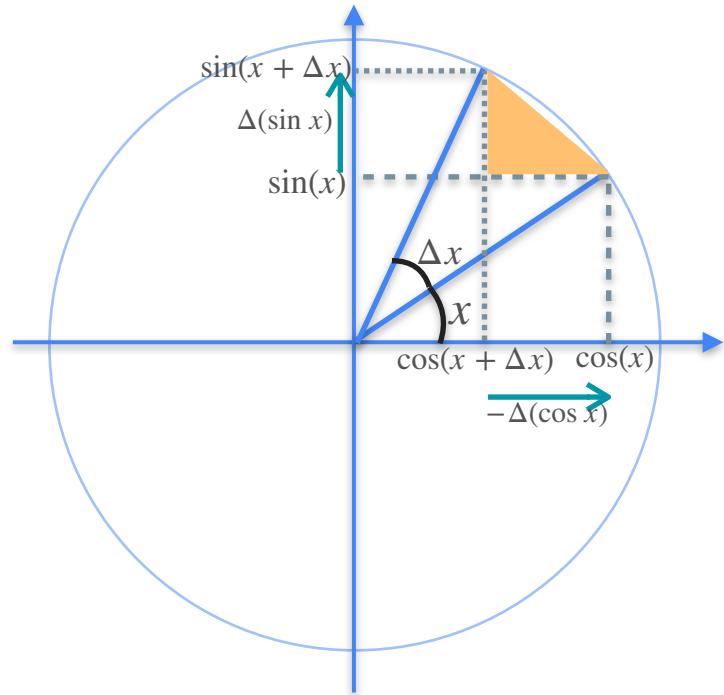
$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}}$$

Derivative of Trigonometric Functions



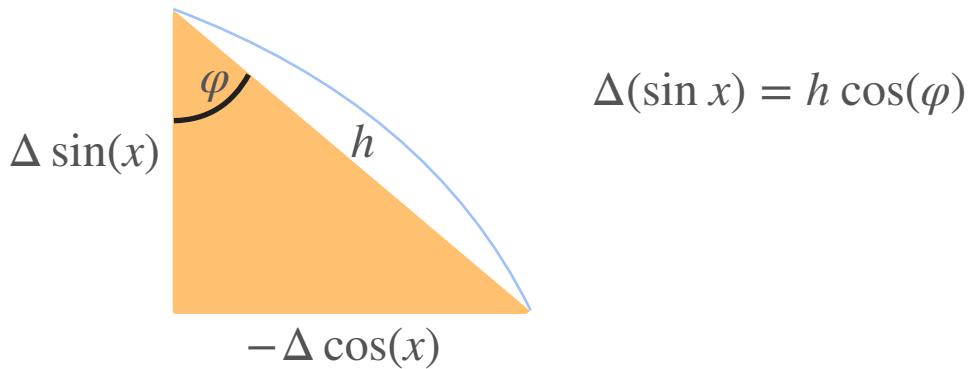
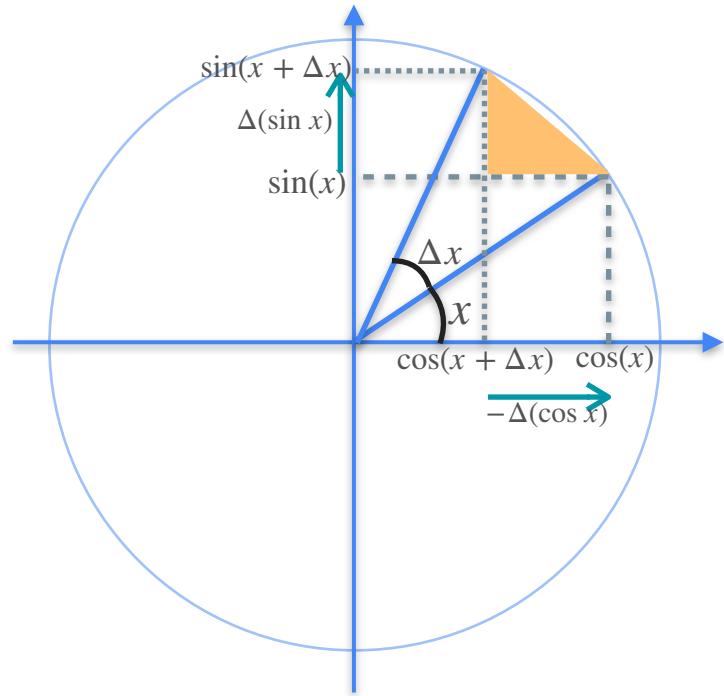
$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$

Derivative of Trigonometric Functions



$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$

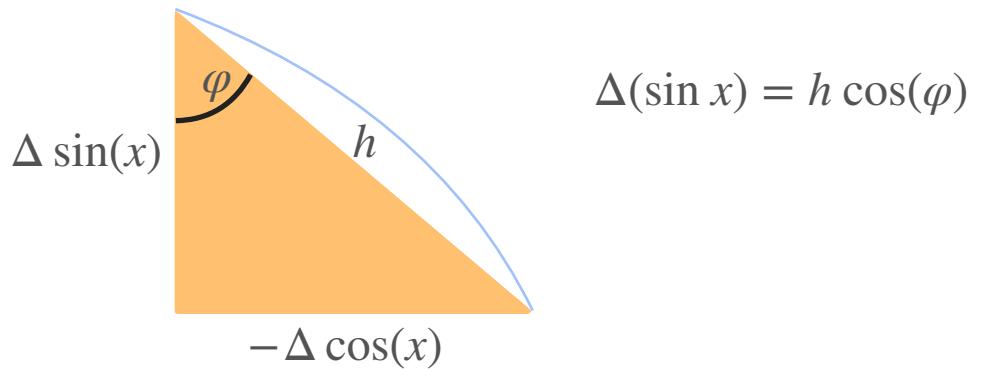
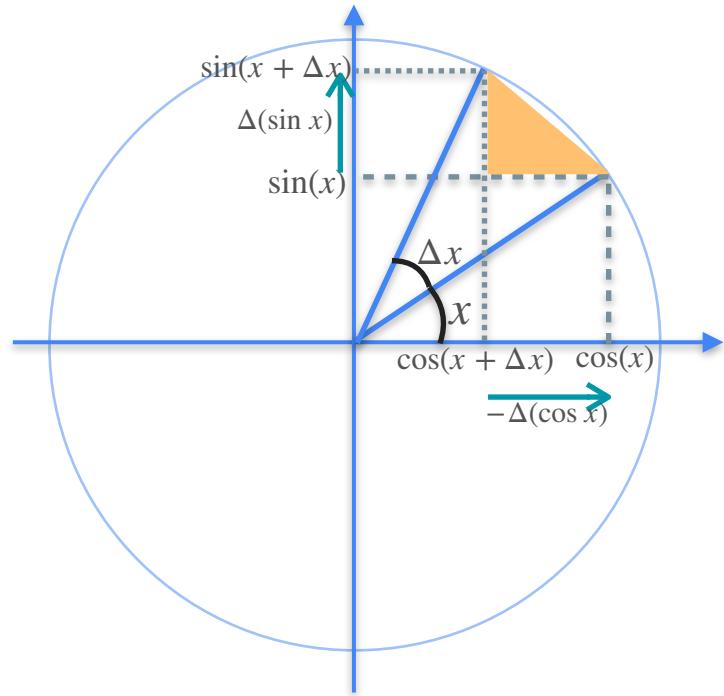
Derivative of Trigonometric Functions



$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$

$$\sin(\varphi) = \frac{\text{opp}}{\text{hyp}}$$

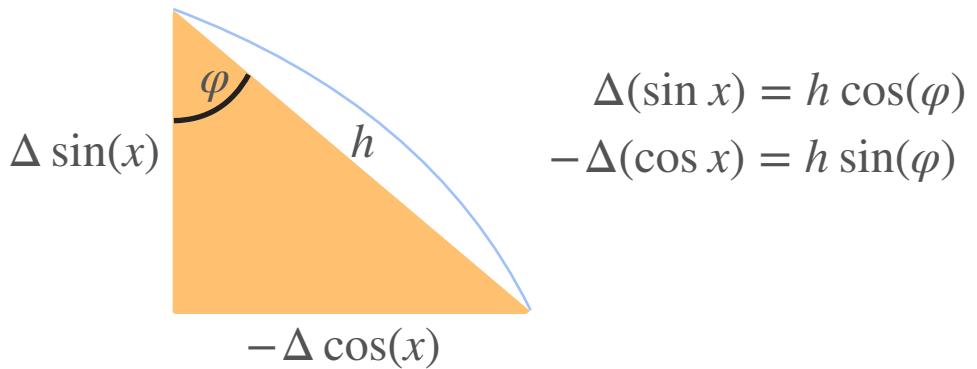
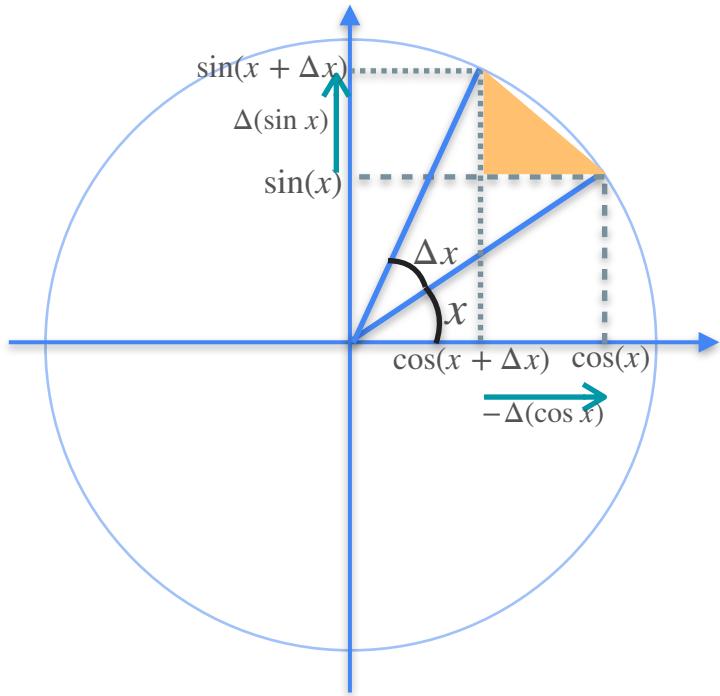
Derivative of Trigonometric Functions



$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$

$$\sin(\varphi) = \frac{\text{opp}}{\text{hyp}} = \frac{-\Delta(\cos x)}{h}$$

Derivative of Trigonometric Functions

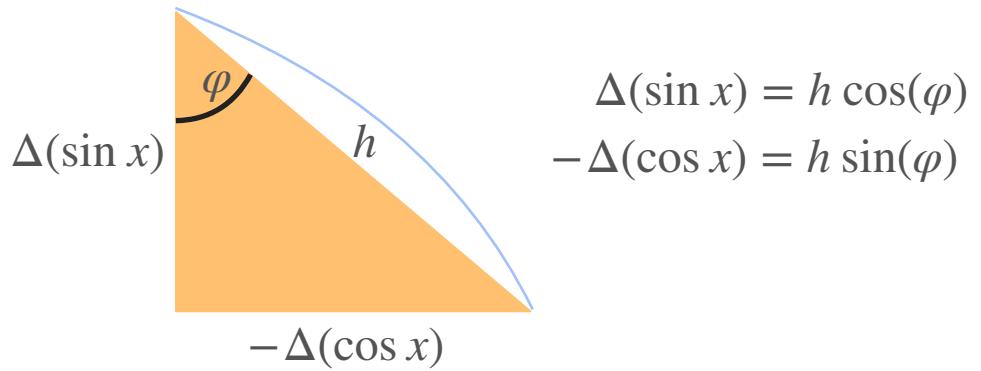


$$\cos(\varphi) = \frac{adj}{hyp} = \frac{\Delta(\sin x)}{h}$$

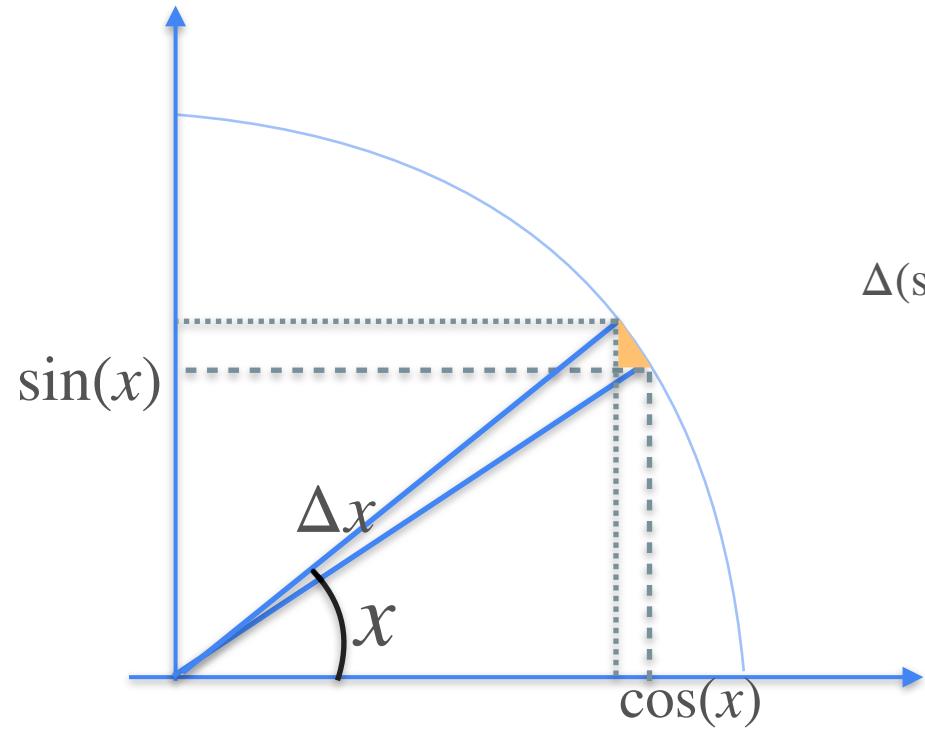
$$\sin(\varphi) = \frac{opp}{hyp} = \frac{-\Delta(\cos x)}{h}$$

$$\begin{aligned}\Delta(\sin x) &= h \cos(\varphi) \\ -\Delta(\cos x) &= h \sin(\varphi)\end{aligned}$$

Derivative of Trigonometric Functions



Derivative of Trigonometric Functions



$$\Delta(\sin x)$$

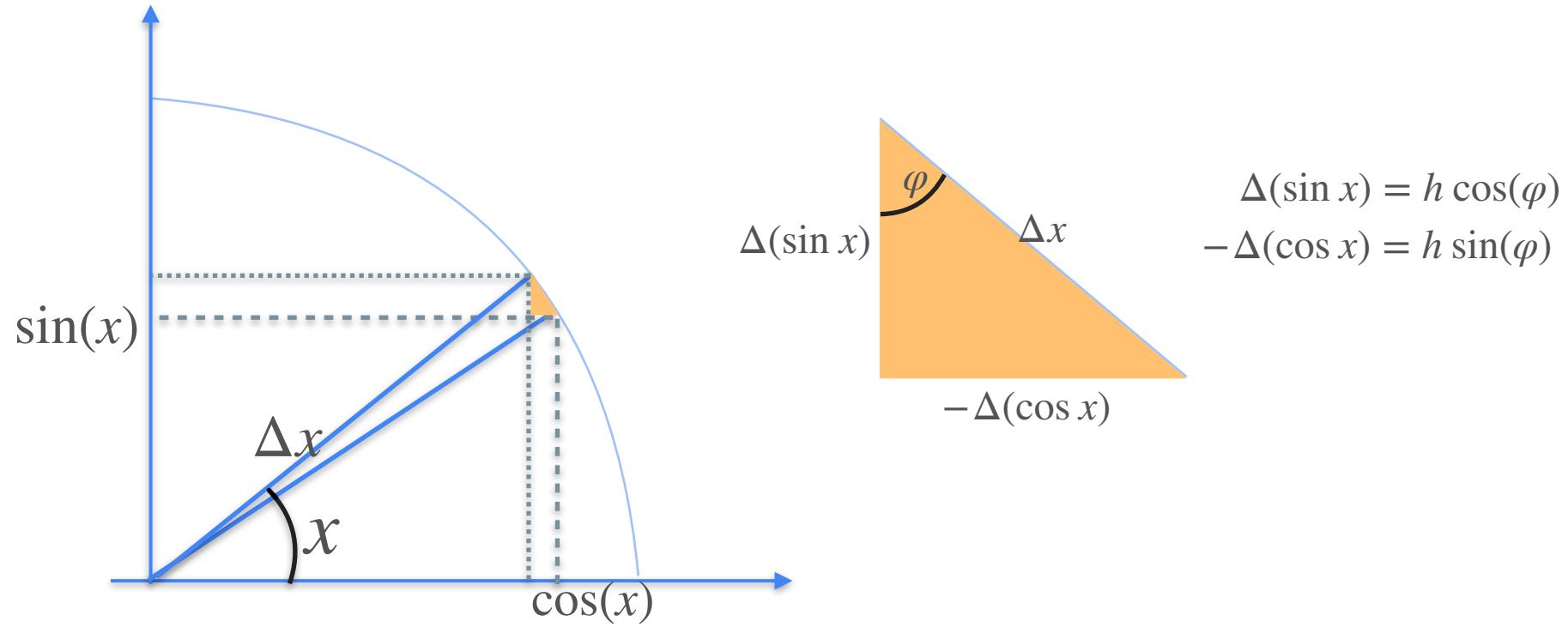
$$h$$

$$-\Delta(\cos x)$$

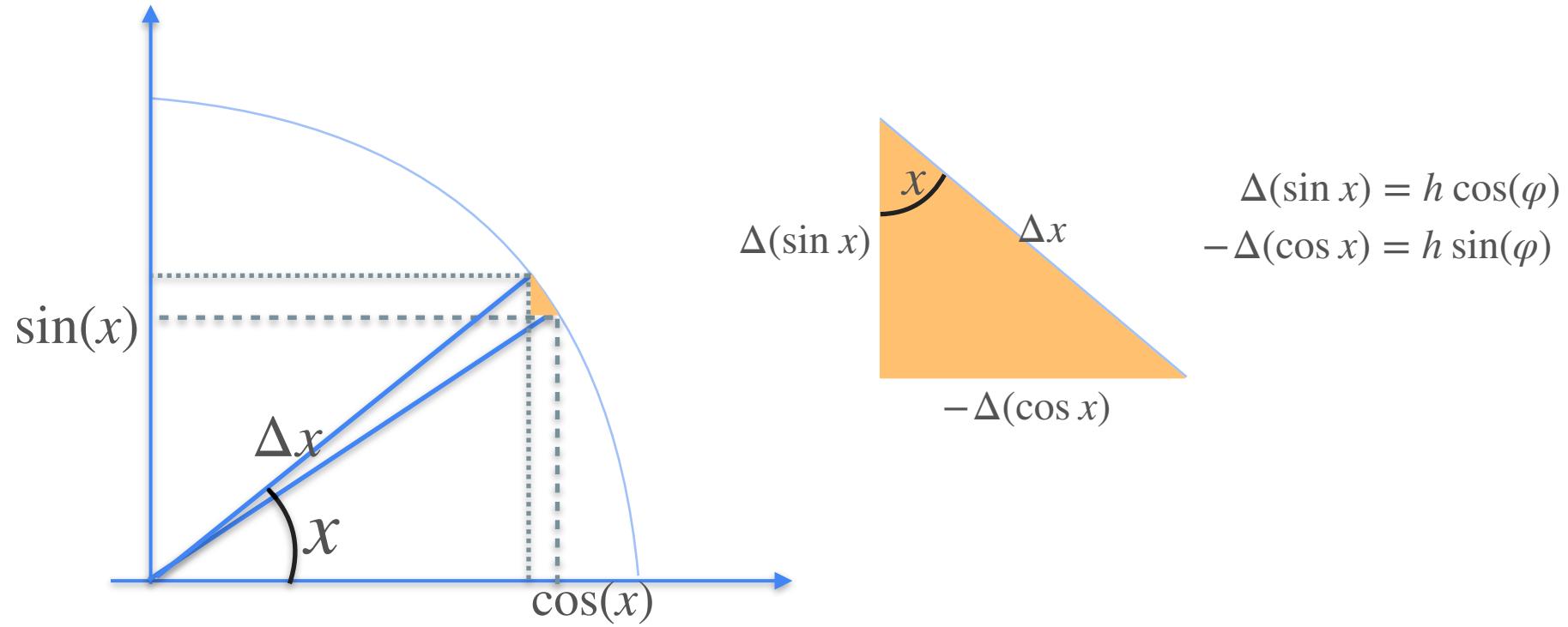
$$\Delta(\sin x) = h \cos(\varphi)$$

$$-\Delta(\cos x) = h \sin(\varphi)$$

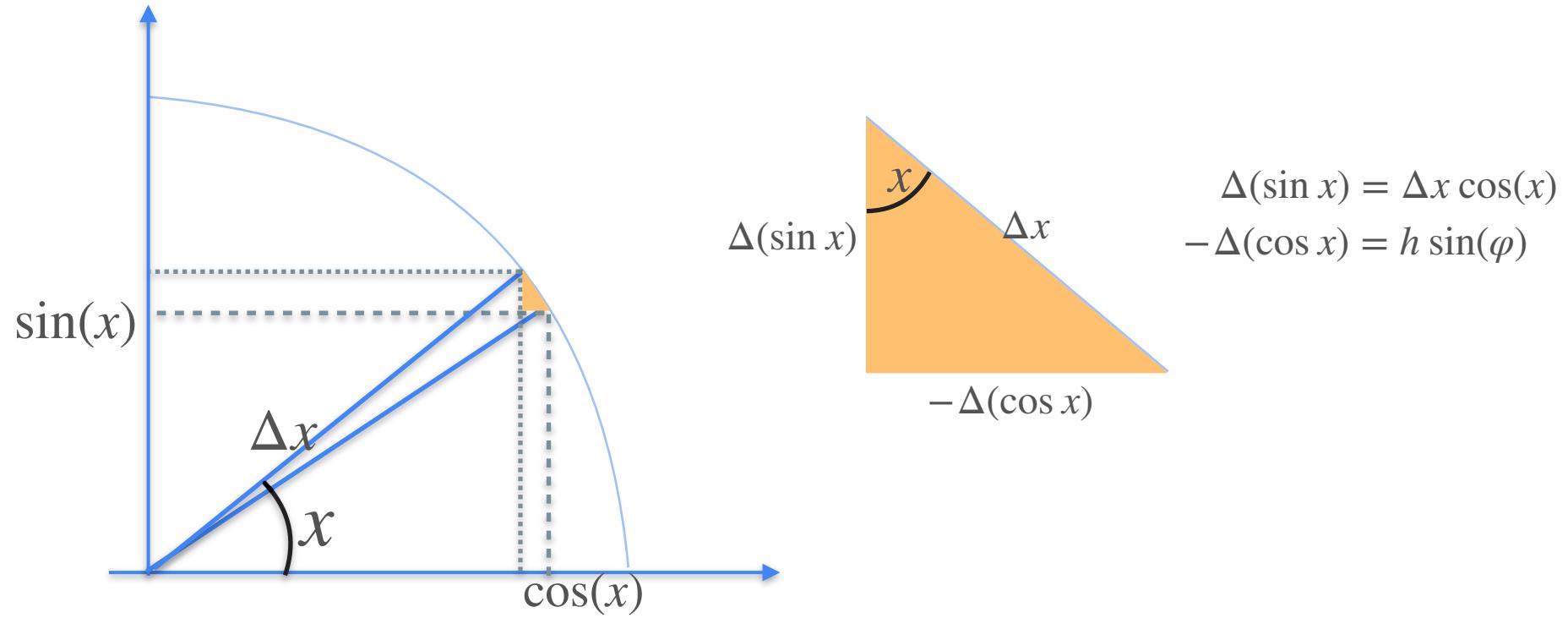
Derivative of Trigonometric Functions



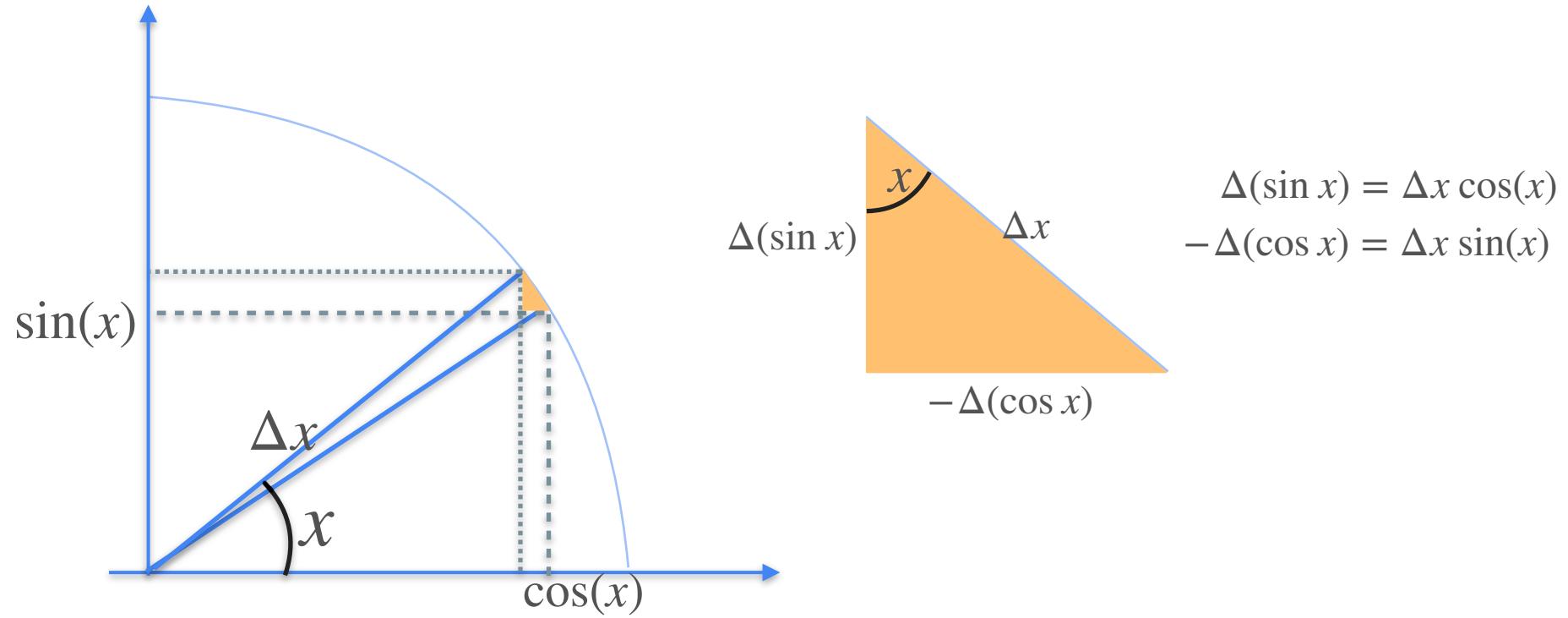
Derivative of Trigonometric Functions



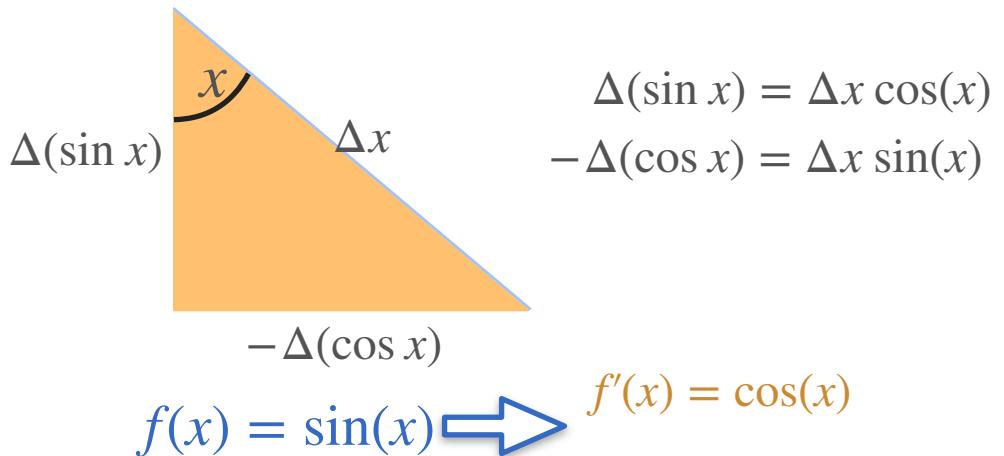
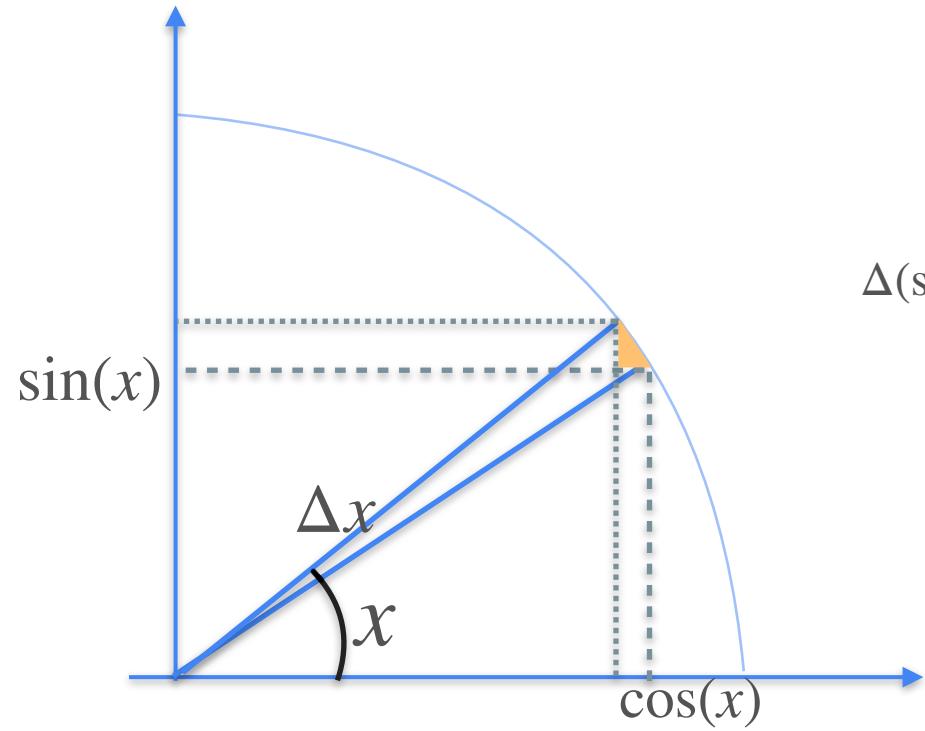
Derivative of Trigonometric Functions



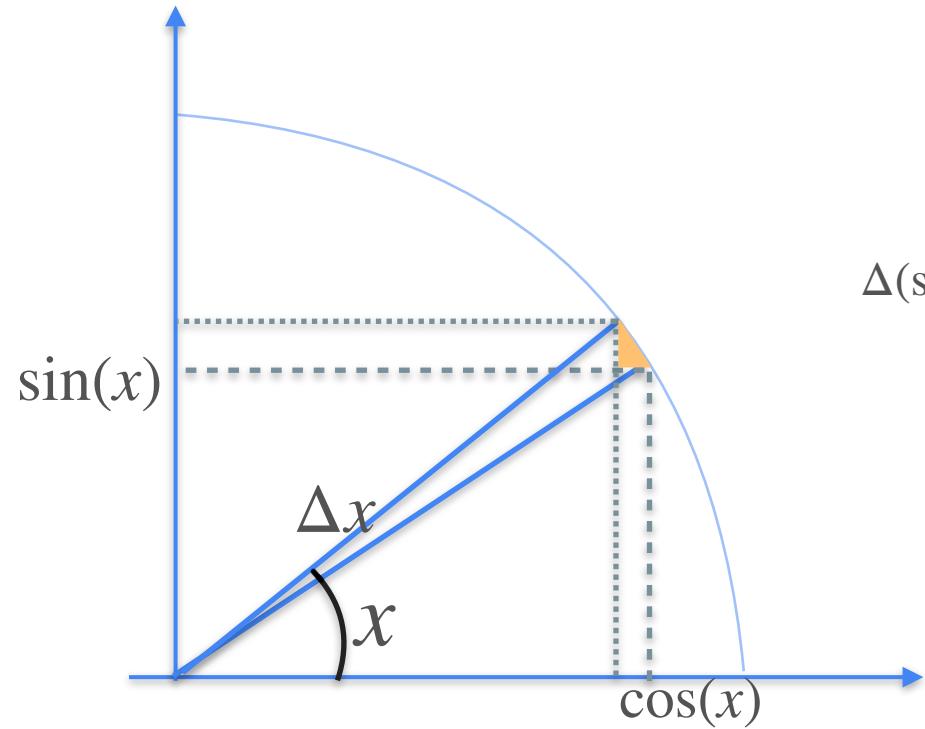
Derivative of Trigonometric Functions



Derivative of Trigonometric Functions



Derivative of Trigonometric Functions



$$\Delta(\sin x)$$

$$\Delta x$$

$$-\Delta(\cos x)$$

$$f(x) = \sin(x) \rightarrow$$

$$f'(x) = \cos(x)$$

$$g(x) = \cos(x) \rightarrow$$

$$g'(x) = -\sin(x)$$



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Derivatives and Optimization

Meaning of the exponential (e)

$e = 2.71828182\dots$

$e = 2.71828182\dots$

$$\begin{array}{c} n \\ \left(1 + \frac{1}{n}\right)^n \end{array}$$

$e = 2.71828182\dots$

n	1
-----	---

$$\left(1 + \frac{1}{n}\right)^n$$

$$\left(1 + \frac{1}{1}\right)^1$$

$e = 2.71828182\dots$

n	1	10
$\left(1 + \frac{1}{n}\right)^n$	2	2.594

$$\left(1 + \frac{1}{10}\right)^{10}$$

$e = 2.71828182\dots$

n	1	10	100
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705

$$\left(1 + \frac{1}{100}\right)^{100}$$

$e = 2.71828182\dots$

n	1	10	100	1000
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717

$$\left(1 + \frac{1}{1000}\right)^{1000}$$

$e = 2.71828182\dots$

n	1	10	100	1000	∞
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	2.718

$e = 2.71828182\dots$

n	1	10	100	1000	∞
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	e

$$f(x) = e^x$$
$$f'(x) = e^x$$

$e = 2.71828182\dots$



n	1	10	100	1000	∞
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	e

A diagram illustrating the exponential function $f(x) = e^x$. It shows two equations in a teal oval: $f(x) = e^x$ and $f'(x) = e^x$. A curved teal arrow points from the oval towards the value e in the table, indicating that the derivative of the exponential function at $x=0$ is equal to the function's value at that point.

Choosing a Bank



Choosing a Bank



Bank 1

Interests

100% every year

Choosing a Bank



Bank 1

Interests

100% every year

(all your money once a year)

Choosing a Bank



Bank 1



Bank 2

Interests

100% every year

(all your money once a year)

Choosing a Bank



Bank 1



Bank 2

Interests

100% every year

(all your money once a year)

Interests

50% every 6 months

Choosing a Bank



Bank 1



Bank 2

Interests

100% every year

(all your money once a year)

Interests

50% every 6 months

(half of your money twice a year)

Choosing a Bank



Bank 1

Interests
100% every year
(all your money once a year)



Bank 2

Interests
50% every 6 months
(half of your money twice a year)



Bank 3

Choosing a Bank



Bank 1

Interests
100% every year
(all your money once a year)



Bank 2

Interests
50% every 6 months
(half of your money twice a year)



Bank 3

Interests
33.3% every 4 months

Choosing a Bank



Interests
100% every year

(all your money once a year)



Interests
50% every 6 months

(half of your money twice a year)



Interests
33.3% every 4 months

(A third of your money three times a year)

Which bank is better?



Bank 1

Interests
100% every year
(all your money once a year)



Bank 2

Interests
50% every 6 months
(half of your money twice a year)



Bank 3

Interests
33.3% every 4 months
(A third of your money three times a year)

Which bank is better?



Bank 1

Interests
100% every year
(all your money once a year)



Bank 2

Interests
50% every 6 months
(half of your money twice a year)



Bank 3

Interests
33.3% every 4 months
(A third of your money three times a year)

Which bank is better?



Bank 1

Interests
100% every year

(all your money once a year)



Bank 2

Interests
50% every 6 months

(half of your money twice a year)



Bank 3



Interests
33.3% every 4 months

(A third of your money three times a year)

You have \$1



Bank 1

Interests
100% every year

(all your money once a year)



Bank 2

Interests
50% every 6 months

(half of your money twice a year)



Bank 3

Interests
33.3% every 4 months

(A third of your money three times a year)

You have \$1



Bank 1



Bank 2



Bank 3



Interests
100% every year

(all your money once a year)

Interests
50% every 6 months

(half of your money twice a year)

Interests
33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	US\$2

You have \$1



Bank 1



Bank 2



Bank 3



Interests
100% every year

(all your money once a year)

Interests
50% every 6 months

(half of your money twice a year)

Interests
33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	US\$2

+100%

You have \$1



Bank 1

Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)



You have \$1



Bank 1

Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

You have \$1



Bank 1

Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?

?



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

You have \$1



Bank 1

Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

How Much Do I Have After 1 Year?



Bank 1

Interests

100% every year



Bank 2

Interests

50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



Interests
100% every year



Bank 2

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



Interests
100% every year

1



Bank 2

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



Interests
100% every year

In 1 year



Bank 2

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests

100% every year

In 1 year



Bank 2

Interests

50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests

100% every year

In 1 year



2



Bank 2

Interests

50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests
100% every year

In 1 year



²
 $(1 + 1)^1$



Bank 2

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



Interests
100% every year

1

In 1 year



2
 $(1 + 1)^1$



Bank 2

Now



Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests
100% every year

In 1 year



²
 $(1 + 1)^1$



Bank 2

Now



1

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Now



Bank 1

Interests
100% every year

1

In 1 year



²
 $(1 + 1)^1$



Now

In 6 months



1

Bank 2

Interests
50% every 6 months

How Much Do I Have After 1 Year?



Now



Bank 1

Interests
100% every year

1

In 1 year



²
 $(1 + 1)^1$



Now

In 6 months



1



Interests
50% every 6 months

How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



2
 $(1 + 1)^1$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



2
 $(1 + 1)^1$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$1 + \frac{1}{2}$

How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



2
 $(1 + 1)^1$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$1 + \frac{1}{2}$

In 1 year



How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2
Interests
50% every 6 months

Now



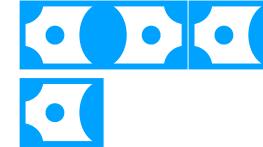
1

In 6 months



$$1.5 \\ 1 + \frac{1}{2}$$

In 1 year



How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



2
 $(1 + 1)^1$



Bank 2
Interests
50% every 6 months

Now



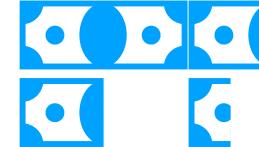
1

In 6 months



1.5
 $1 + \frac{1}{2}$

In 1 year



How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



2
 $(1 + 1)^1$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

$$\left(1 + \frac{1}{2}\right)^2$$

How Much Do I Have After 1 Year?



Bank 1
Interests
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2
Interests
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

$$\left(1 + \frac{1}{2}\right)^2$$

Compound interest



Bank 1
Interests
100% every year

Now



1



Bank 2
Interests
50% every 6 months

Now



1

In 1 year



$$(1 + 1)^1$$

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



$$\left(1 + \frac{1}{2}\right)^2$$

How Much Do I Have After 1 Year?



Now

In 4 months

In 8 months

In 1 year

Interests

33.3% every 4 months

How Much Do I Have After 1 Year?



Interests
33.3% every 4 months

Now



1

In 4 months

In 8 months

In 1 year

How Much Do I Have After 1 Year?



Interests
33.3% every 4 months

	Now	In 4 months	In 8 months	In 1 year
Bank 3	1	2	3	4
Interest	33.3% every 4 months			

How Much Do I Have After 1 Year?



Now



In 4 months



In 8 months

In 1 year

Interests

33.3% every 4 months

1

How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3				
Interests 33.3% every 4 months	1	1.33		

How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3				
Interests	1	1.33		
33.3% every 4 months				

How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3	1	1.33	1.67	2.00
Interests	1	1.33	1.67	2.00
33.3% every 4 months				

How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3	1	1.33	1.77	2.25
Interests	1	1.33	1.77	2.25

33.3% every 4 months

How Much Do I Have After 1 Year?



Now



Bank 3

In 4 months



In 8 months



In 1 year



Interests

1

1.33

1.77

33.3% every 4 months

How Much Do I Have After 1 Year?



Now



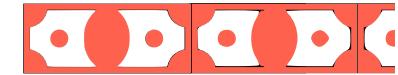
In 4 months



In 8 months



In 1 year



Interests

1

1.33

1.77

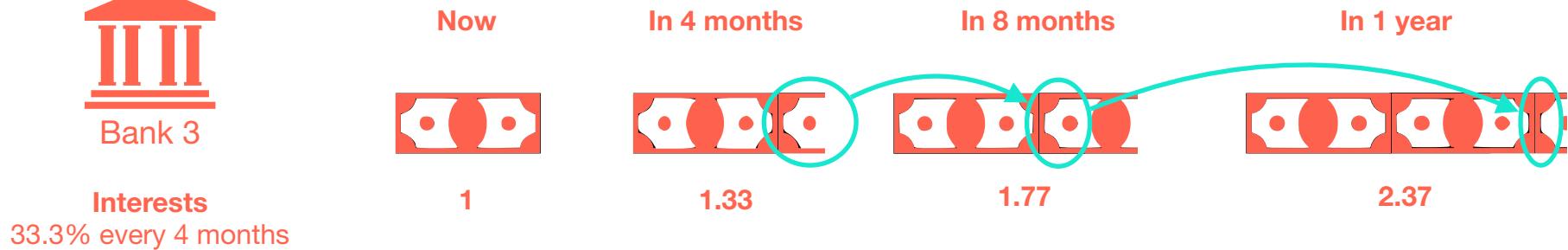
33.3% every 4 months

How Much Do I Have After 1 Year?

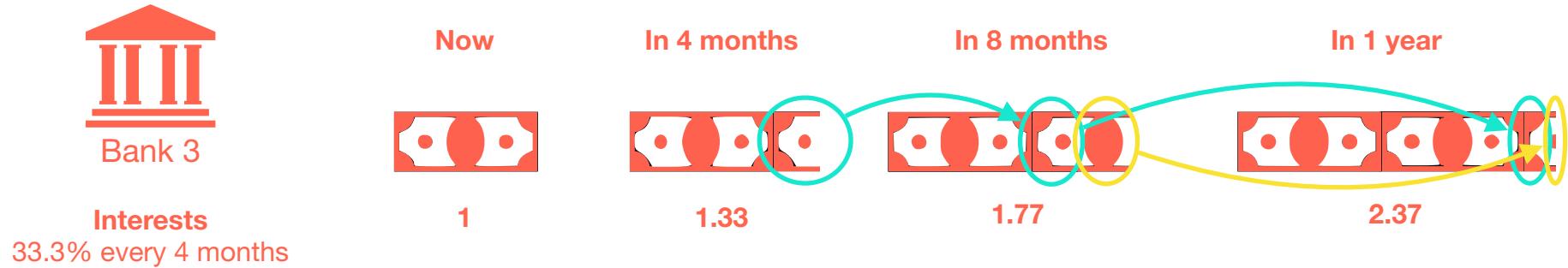


	Now	In 4 months	In 8 months	In 1 year
Bank 3				
Interests	1	1.33	1.77	2.37
33.3% every 4 months				

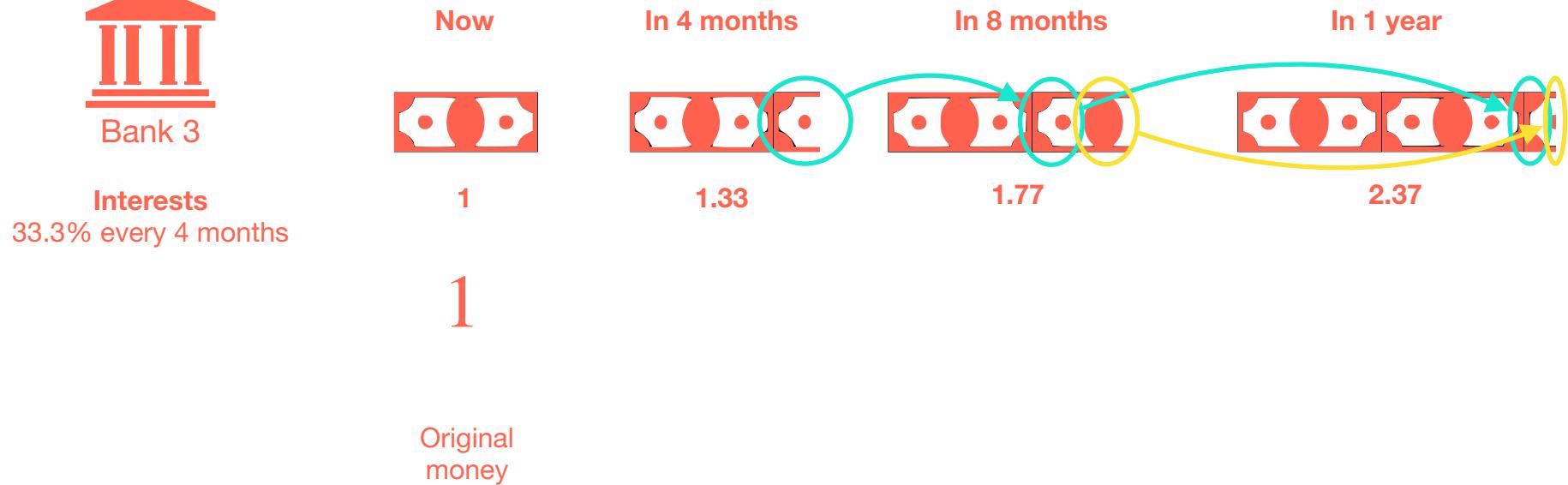
How Much Do I Have After 1 Year?



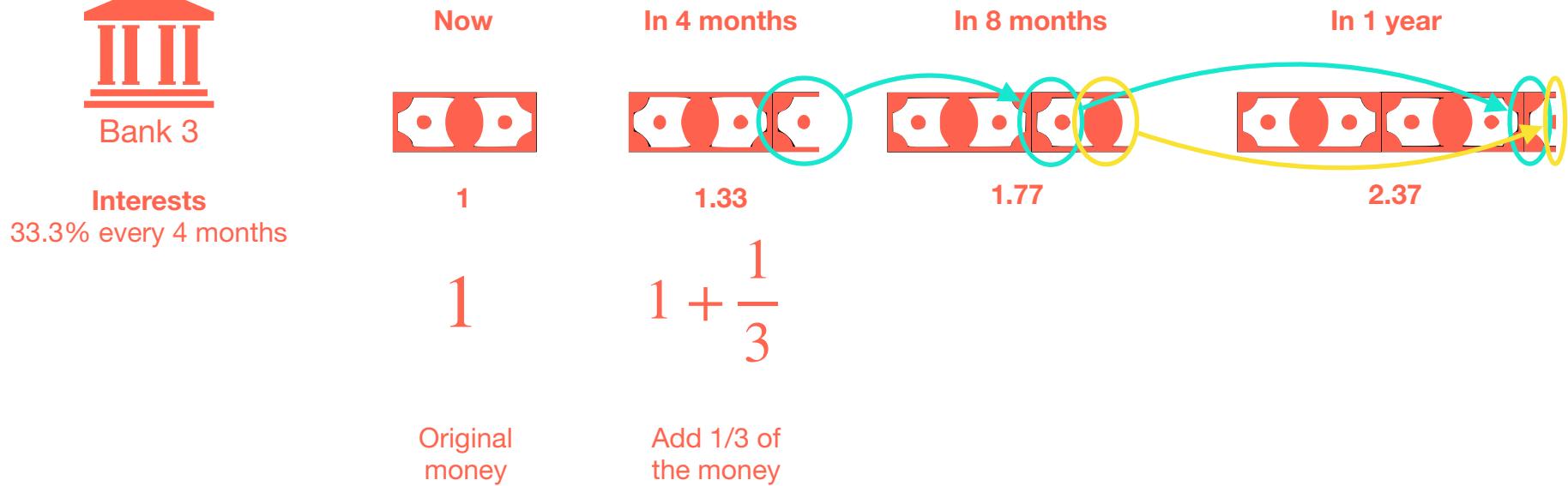
How Much Do I Have After 1 Year?



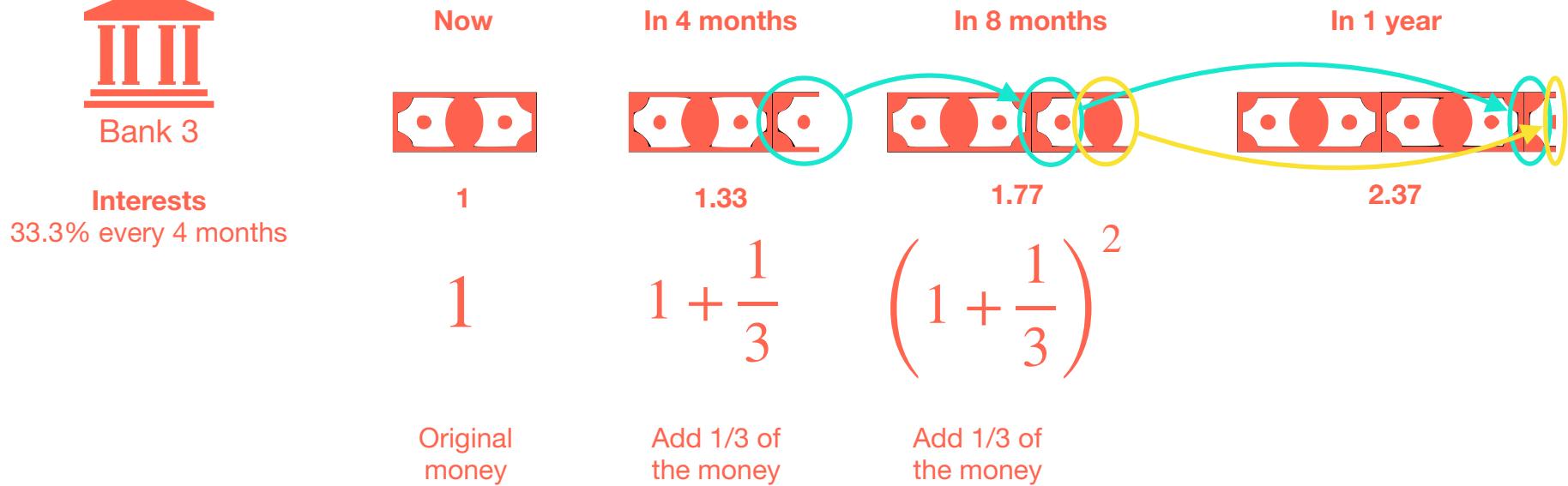
How Much Do I Have After 1 Year?



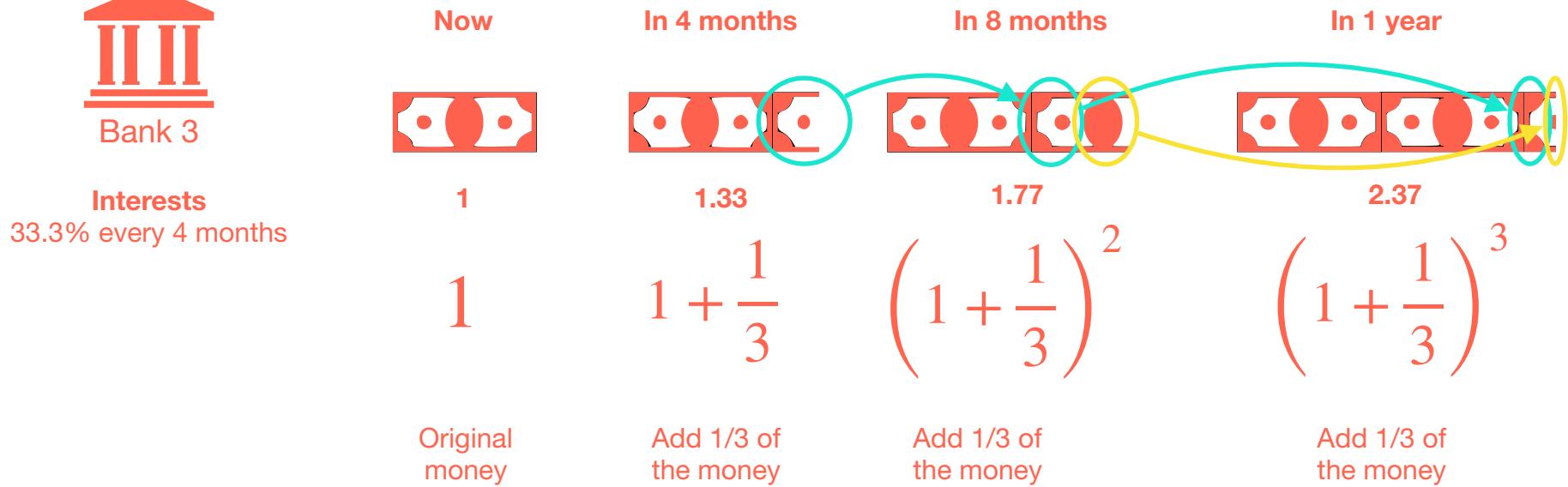
How Much Do I Have After 1 Year?



How Much Do I Have After 1 Year?



How Much Do I Have After 1 Year?



Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)



Bank 2

Interests

50% every 6 months

(half of your money twice a year)



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

Interests

50% every 6 months

(half of your money twice a year)



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2

↗ +100%



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%
+100%
+100%
+100%



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

Curved arrows point from the original \$1 to each subsequent value, labeled +100%.



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25

A curved arrow points from the original \$1 to the final value \$2.25, labeled +125%.



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%
+100%
+100%
+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%
+125%
+125%
+125%



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

Which Bank Is Better?



Bank 1

Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%
+100%
+100%
+100%



Bank 2

Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%
+125%
+125%
+125%



Bank 3

Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

+137%

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%
+100%
+100%
+100%



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%
+125%
+125%
+125%



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37
In 2	\$5.62
In 3	\$13.32
In 4	\$31.57

+137%
+137%
+137%
+137%

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
...	...



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
...	...



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

$$(1 + 1)^1 = 2$$

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
...	...

$$(1 + 1)^1 = 2$$



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...

$$\left(1 + \frac{1}{2}\right)^2 = 2.25$$



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

Which Bank Is Better?



Interests
100% every year

(all your money once a year)

Now	\$1
...	...

$$(1 + 1)^1 = 2$$



Interests
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...

$$\left(1 + \frac{1}{2}\right)^2 = 2.25$$



Interests
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

$$\left(1 + \frac{1}{3}\right)^3 = 2.37$$

How Much Do I Have After 1 Year?



Now

In 1 month

In 2 months

...

In 12 months

Interests
1/12 every month

How Much Do I Have After 1 Year?



Interests
1/12 every month

Now

In 1 month

In 2 months

...

In 12 months

1

Original
money

How Much Do I Have After 1 Year?



	Now	In 1 month	In 2 months	...	In 12 months
	1	$1 + \frac{1}{12}$			
	Original money	Add 1/12 of the money			

How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$		
Original money	Add 1/12 of the money	Add 1/12 of the money		

How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$...	
Original money	Add 1/12 of the money	Add 1/12 of the money		

How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$...	$\left(1 + \frac{1}{12}\right)^{12}$
Original money	Add 1/12 of the money	Add 1/12 of the money		Add 1/12 of the money

At the end of the year:

How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$...	$\left(1 + \frac{1}{12}\right)^{12}$
Original money	Add 1/12 of the money	Add 1/12 of the money		Add 1/12 of the money

At the end of the year: **2.61**

In General



Bank n

Interests
 $1/n$ every interval

In General



Bank n

1	2	3	k	$n-2$	$n-1$	n
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-------	-------	-----

Interests

$1/n$ every interval

In General



Interests
1/n every interval



In General



Interests
1/n every interval



Now

1

In General



Interests
1/n every interval

Now In 1 interval

$$1 \quad 1 + \frac{1}{n}$$

In General



Interests
1/n every interval

Now In 1 interval In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

In General



Interests
1/n every interval

Now In 1 interval In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

\downarrow

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

In General



A horizontal timeline axis with a vertical tick mark labeled "1 year". Below the axis, a sequence of numbers is listed: 1, 2, 3, ., ., ., ., ., k, ., ., ., ., ., ., n-2, n-1, n. The number "k" is highlighted in red.

Interests
1/n every interval

Now In 1 interval In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$



$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money

In General



Interests
 $1/n$ every interval

Now In 1 interval In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

\downarrow

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money Interest

In General



Interests
1/n every interval

Now In 1 interval In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

\downarrow \nearrow

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money Interest

In General



Interests
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$
	$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$		
	Money	Interest	

In General



Interests
 $1/n$ every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$	
	$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$			
	Money	Interest		

In General



A horizontal timeline diagram representing a year-long process. The timeline is a blue line with tick marks. Above the line, the text "1 year" is written in blue. Below the line, there are 15 numbered boxes labeled 1, 2, 3, ., ., ., ., k, ., ., ., ., ., ., n-2, n-1, n. The first four boxes (1, 2, 3, .) are in red, while the remaining boxes (. through n) are in black.

Interests
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In k intervals
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$		
	$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$ <p style="text-align: center;">↓ ↑</p> <p>Money Interest</p>				$\left(1 + \frac{1}{n}\right)^k$

In General



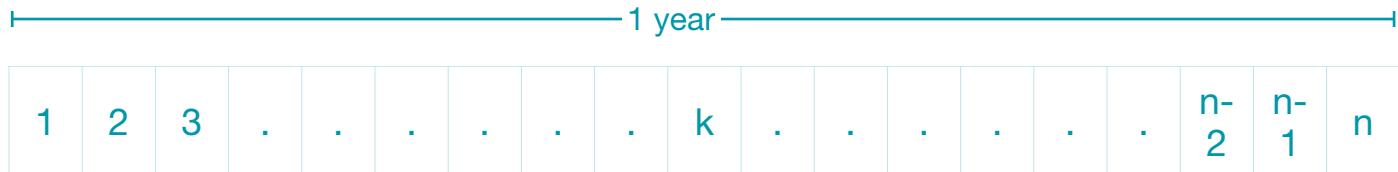
Interests
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In k intervals	...
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$...	$\left(1 + \frac{1}{n}\right)^k$...

$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$

Money Interest

In General



Interests
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In k intervals	...	In 1 year (n intervals)
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$		$\left(1 + \frac{1}{n}\right)^k$		$\left(1 + \frac{1}{n}\right)^n$

$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$

Money Interest

In General



Interests
 $1/n$ every interval

In General



Bank n

Interests
1/n every interval

In 1 year

$$\left(1 + \frac{1}{n}\right)^n$$

In General



Bank n

Interests
 $1/n$ every interval

In 1 year

$$\left(1 + \frac{1}{n} \right)^n$$

↑
1/n of your money

In General



Interests
 $1/n$ every interval

$$\left(1 + \frac{1}{n} \right)$$

In 1 year

n times

1/n of your money

The diagram shows the mathematical expression for compound interest. A large teal bracket groups the term $1 + \frac{1}{n}$. Above this bracket, a pink circle contains the letter n , with a pink arrow pointing to it from the text "n times". A pink arrow also points upwards from the bottom of the bracket to the text "1/n of your money" at the bottom, indicating the fraction of the principal being added each period.

A Lot of Banks

A Lot of Banks



Bank 1



All your money
1 time a year

A Lot of Banks



Bank 1



Bank 2

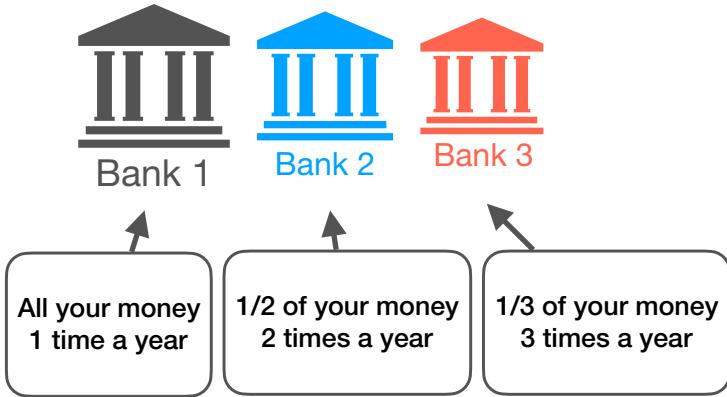


All your money

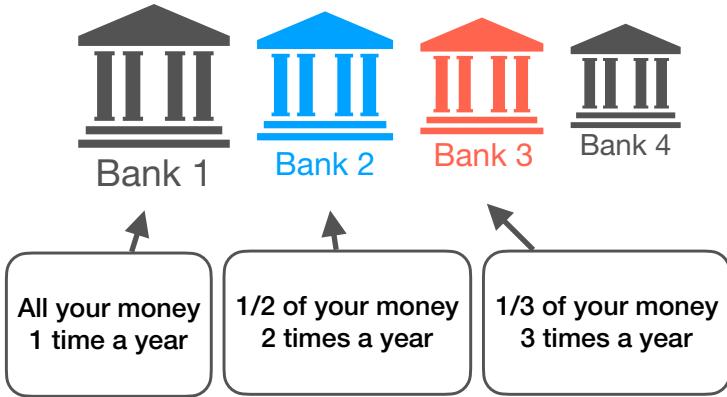
1 time a year

1/2 of your money
2 times a year

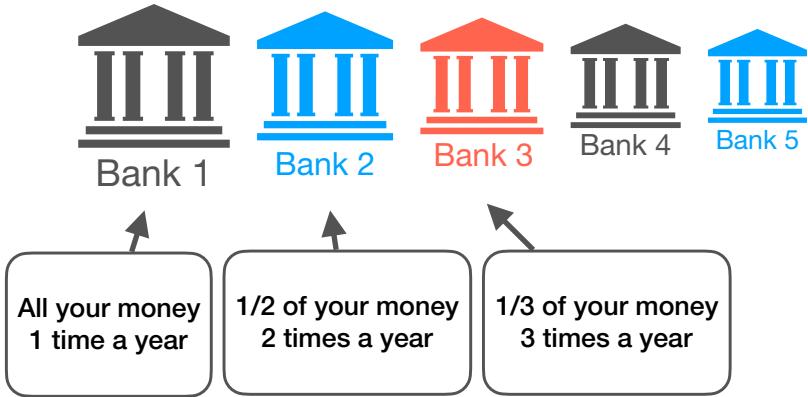
A Lot of Banks



A Lot of Banks



A Lot of Banks



A Lot of Banks



Bank 1



Bank 2



Bank 3



Bank 4



Bank 5



Bank 6



Bank 7



Bank 8



Bank 9



Bank 10



Bank 11



Bank 12



Bank 13

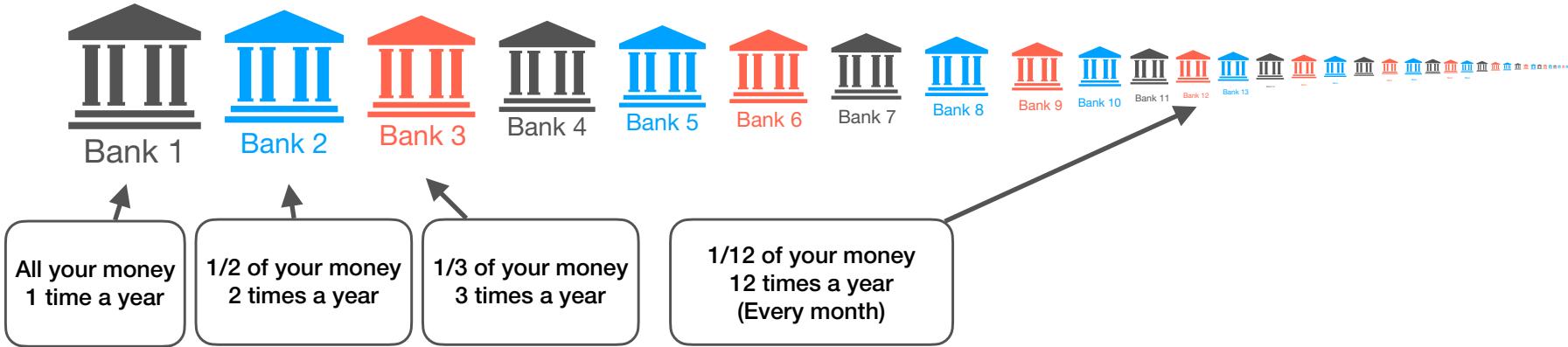
...

All your money
1 time a year

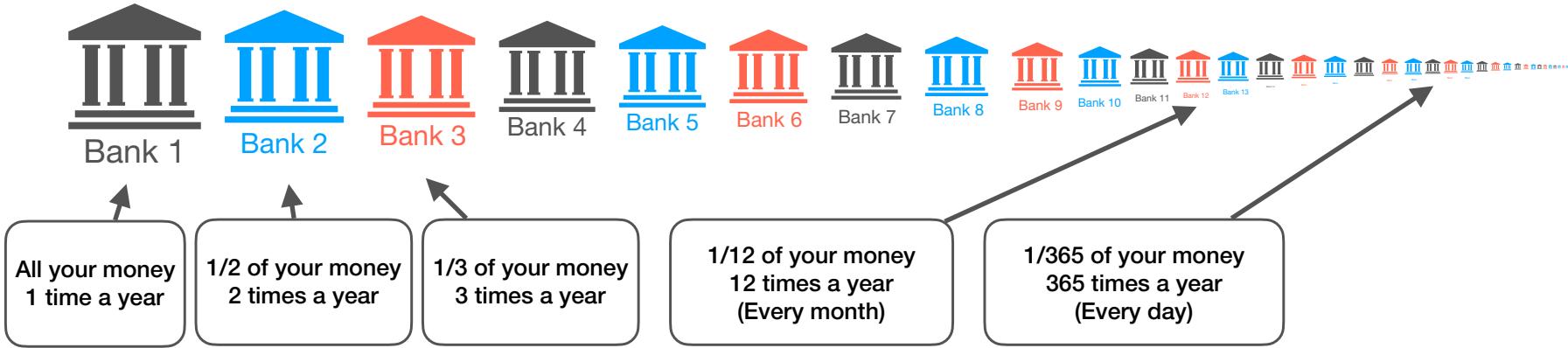
1/2 of your money
2 times a year

1/3 of your money
3 times a year

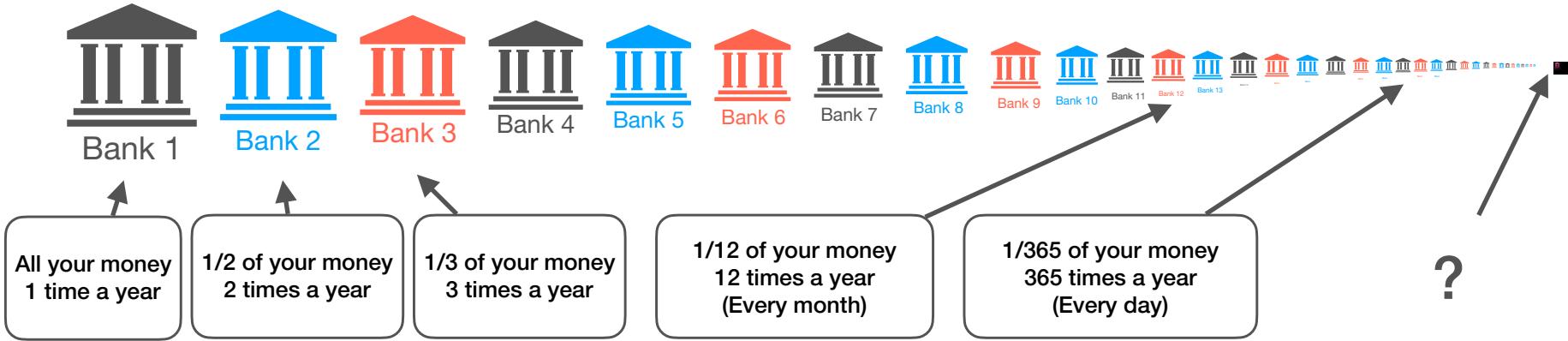
A Lot of Banks



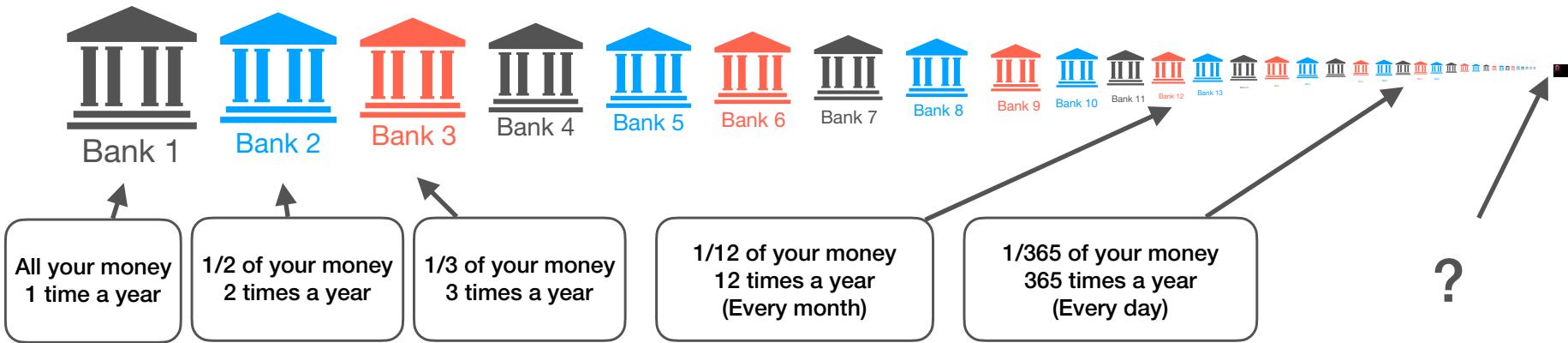
A Lot of Banks



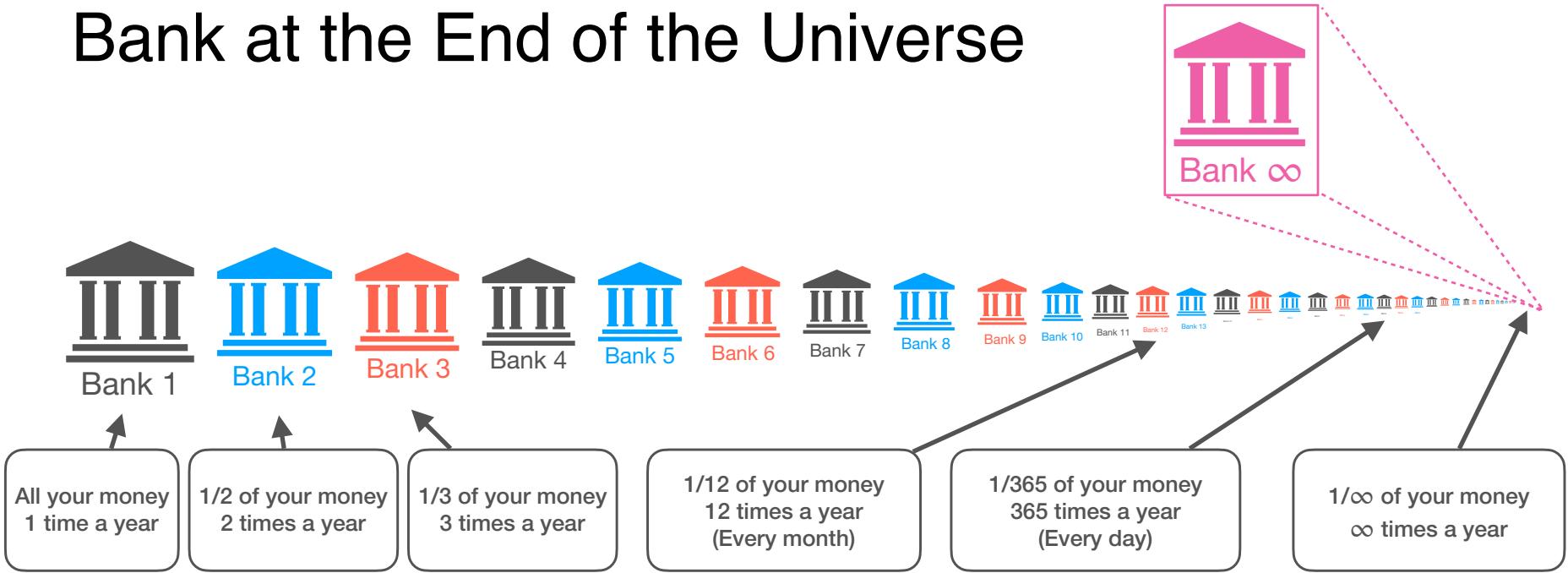
A Lot of Banks



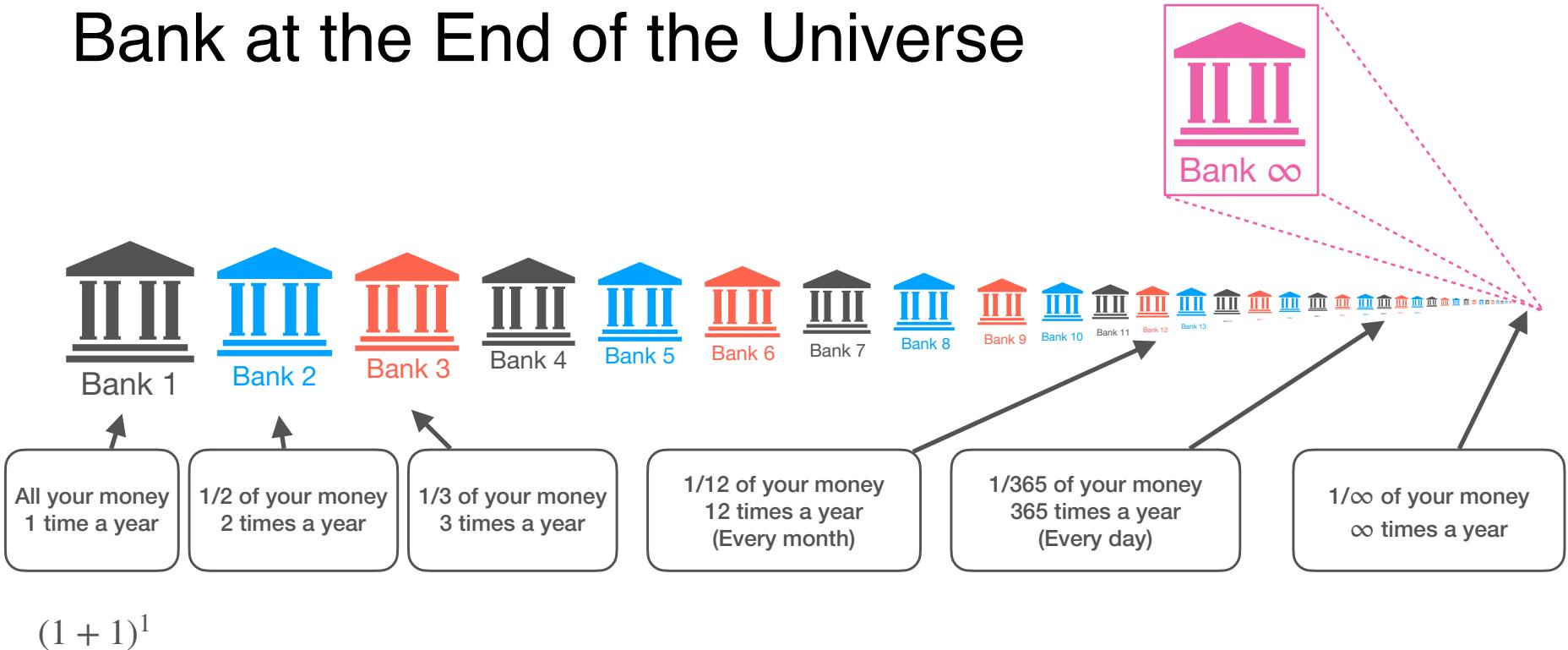
Bank at the end of the universe



Bank at the End of the Universe



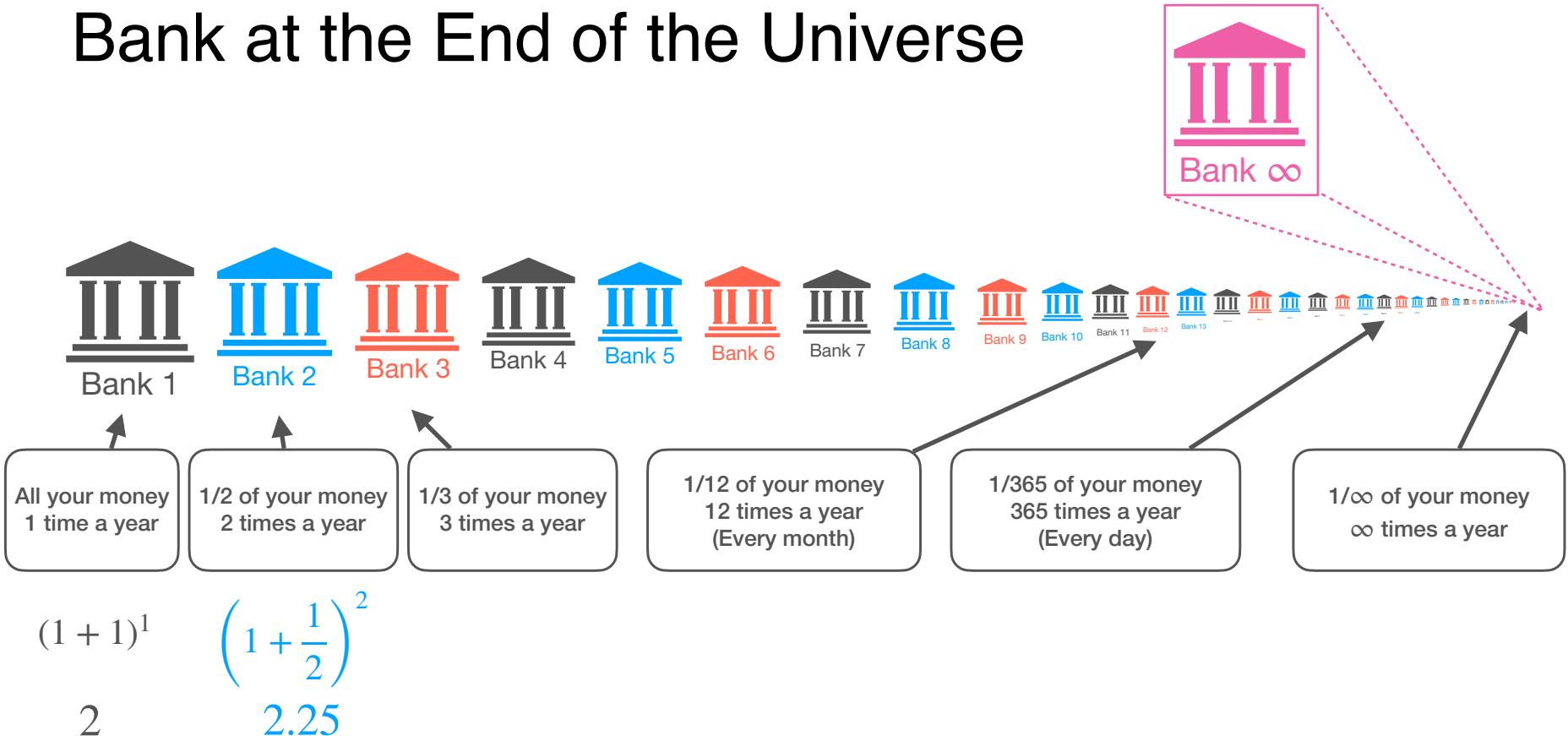
Bank at the End of the Universe



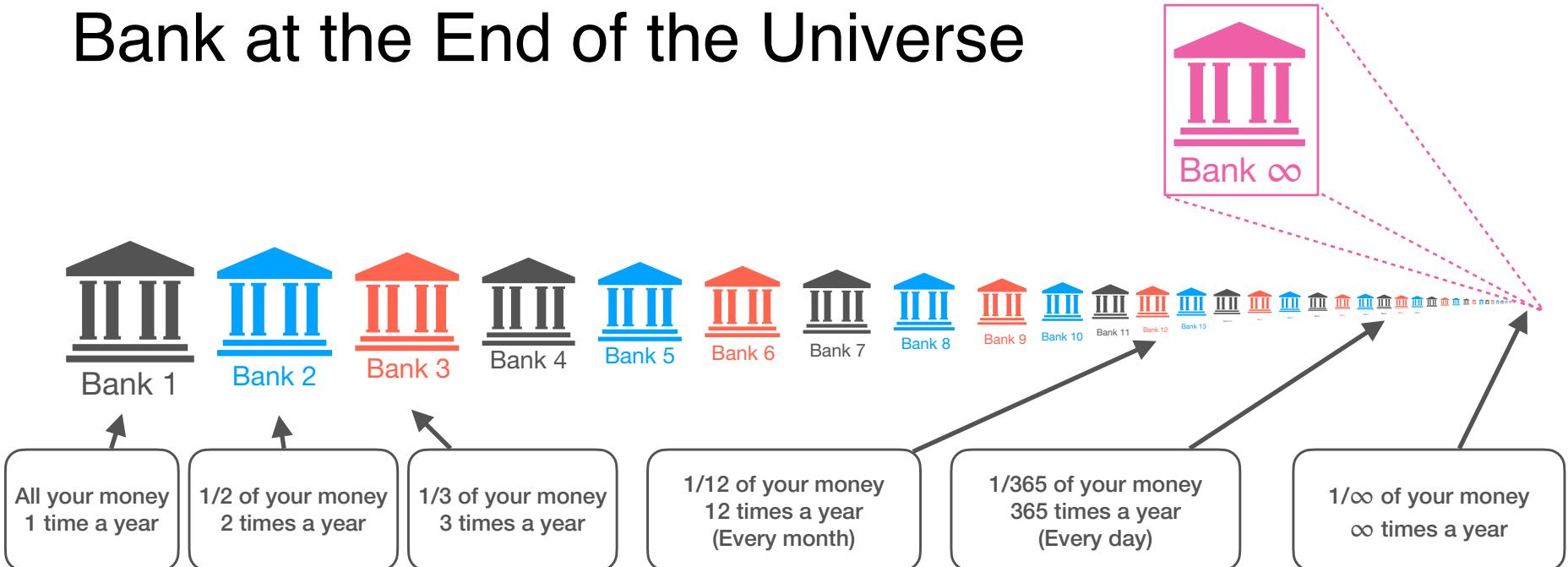
$$(1 + 1)^1$$

2

Bank at the End of the Universe



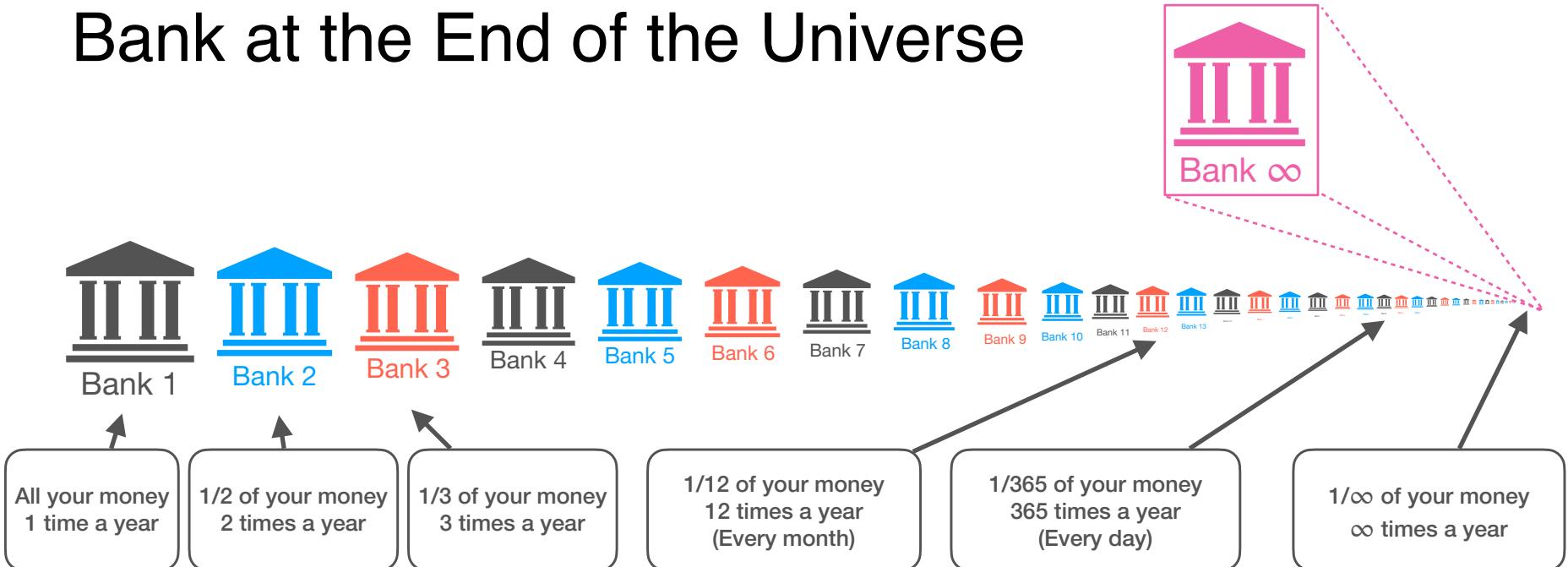
Bank at the End of the Universe



$$(1 + 1)^1 \quad \left(1 + \frac{1}{2}\right)^2 \quad \left(1 + \frac{1}{3}\right)^3$$

2 2.25 2.37

Bank at the End of the Universe



$$(1 + 1)^1 \quad 2$$

$$\left(1 + \frac{1}{2}\right)^2 \quad 2.25$$

$$\left(1 + \frac{1}{3}\right)^3 \quad 2.37$$

$$\left(1 + \frac{1}{12}\right)^{12} \quad 2.613$$

Bank at the End of the Universe



Bank 1



Bank 2



Bank 3



Bank 4



Bank 5



Bank 6



Bank 7



Bank 8



Bank 9



Bank 10



Bank 11



Bank 12



Bank 13

...

All your money
1 time a year

1/2 of your money
2 times a year

1/3 of your money
3 times a year

1/12 of your money
12 times a year
(Every month)

1/365 of your money
365 times a year
(Every day)

1/∞ of your money
∞ times a year

$$(1 + 1)^1$$

2

$$\left(1 + \frac{1}{2}\right)^2$$

2.25

$$\left(1 + \frac{1}{3}\right)^3$$

2.37

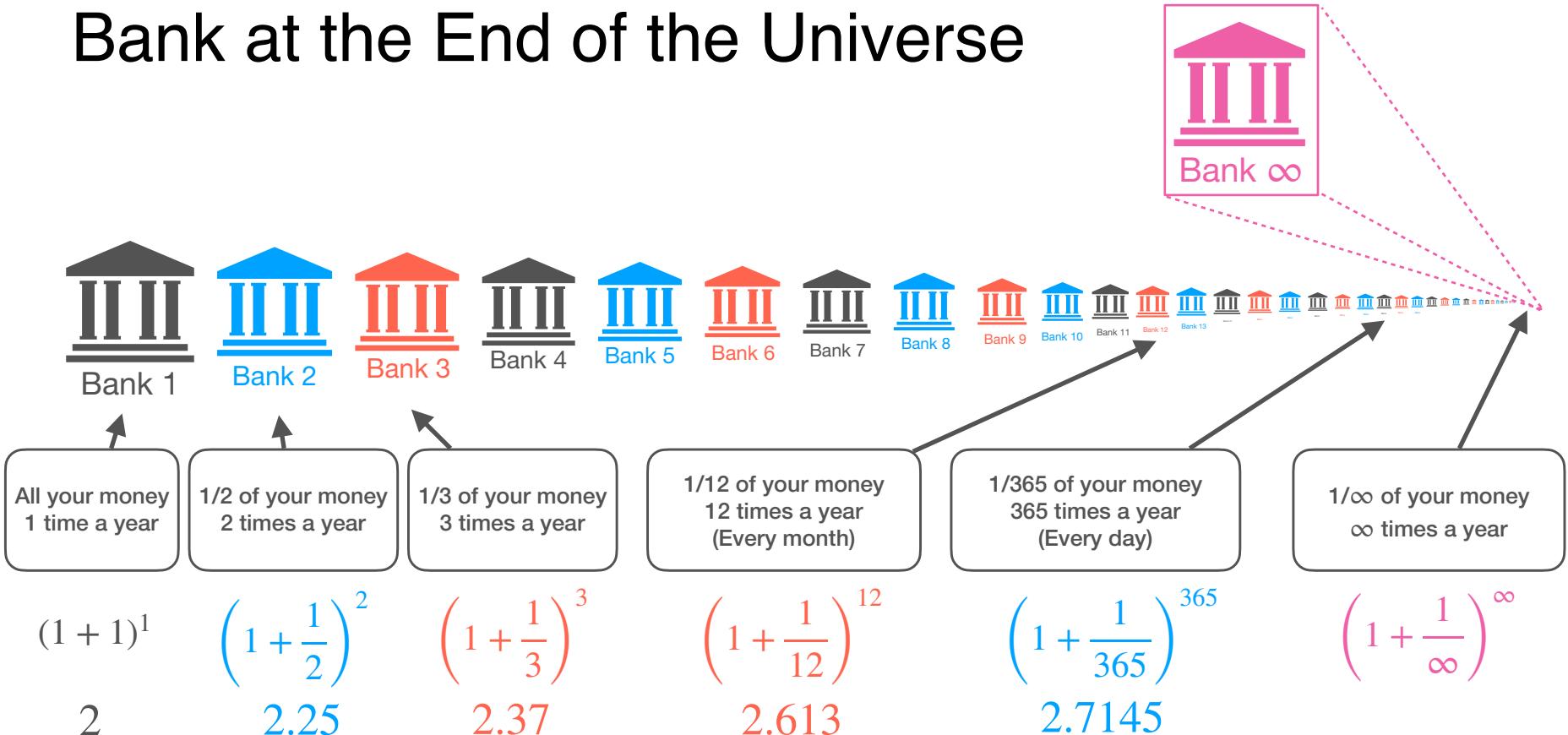
$$\left(1 + \frac{1}{12}\right)^{12}$$

2.613

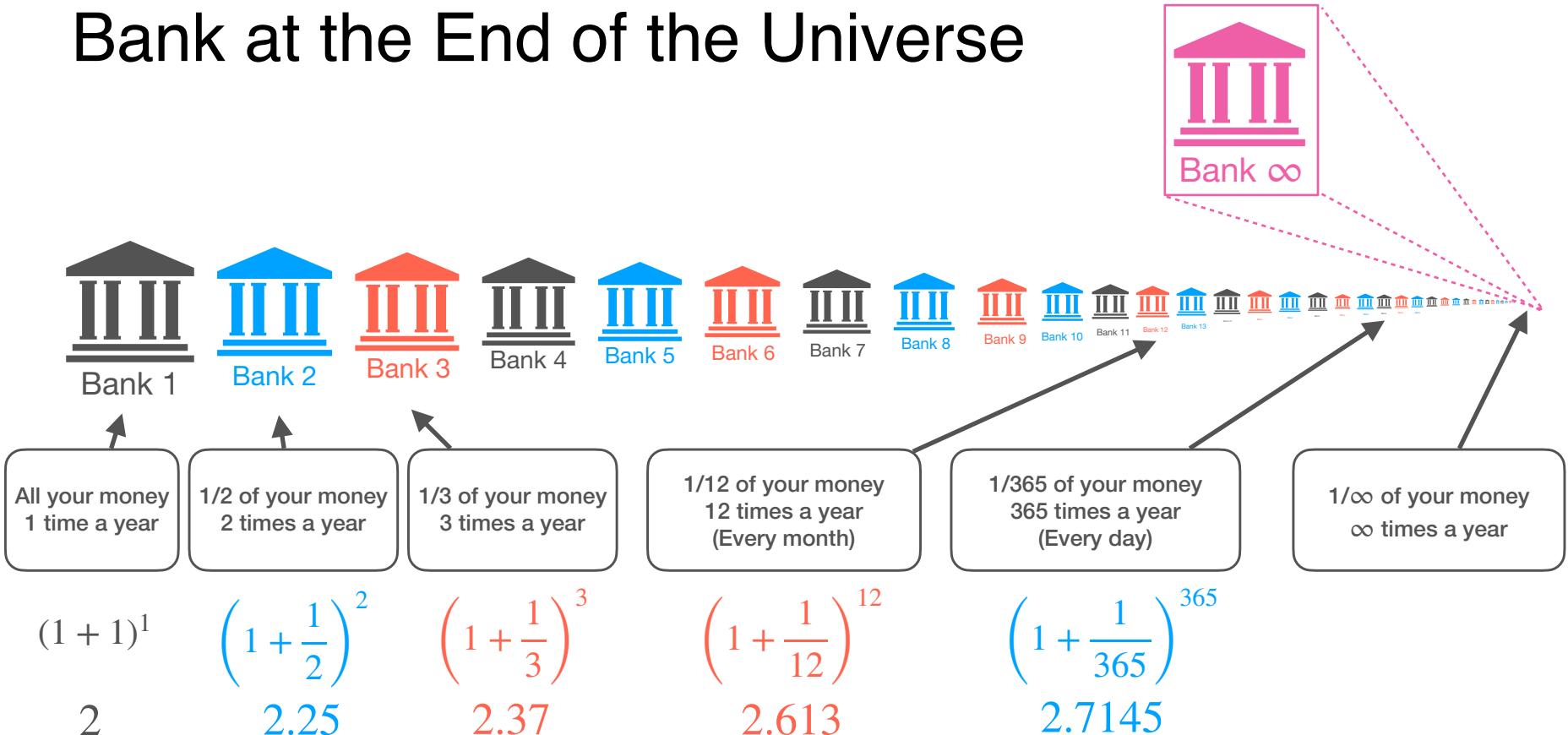
$$\left(1 + \frac{1}{365}\right)^{365}$$

2.7145

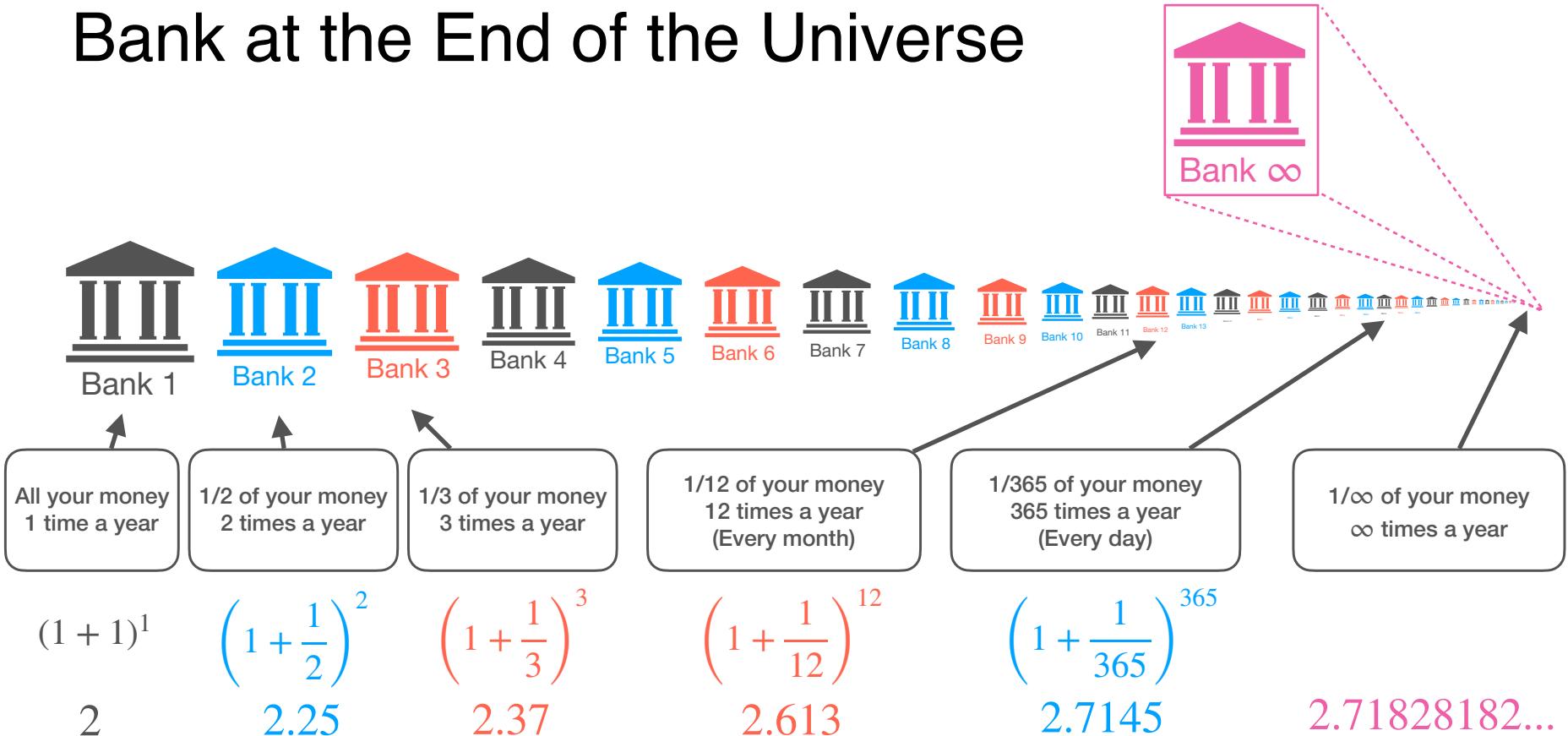
Bank at the End of the Universe



Bank at the End of the Universe



Bank at the End of the Universe



Bank at the End of the Universe



Bank 1

Bank 2

Bank 3

Bank 4

Bank 5

Bank 6

Bank 7

Bank 8

Bank 9

Bank 10

Bank 11

Bank 12

Bank 13

All your money
1 time a year

1/2 of your money
2 times a year

1/3 of your money
3 times a year

1/12 of your money
12 times a year
(Every month)

1/365 of your money
365 times a year
(Every day)

1/∞ of your money
∞ times a year

$$(1 + 1)^1$$

2

$$\left(1 + \frac{1}{2}\right)^2$$

2.25

$$\left(1 + \frac{1}{3}\right)^3$$

2.37

$$\left(1 + \frac{1}{12}\right)^{12}$$

2.613

$$\left(1 + \frac{1}{365}\right)^{365}$$

2.7145

$$e = 2.71828182...$$

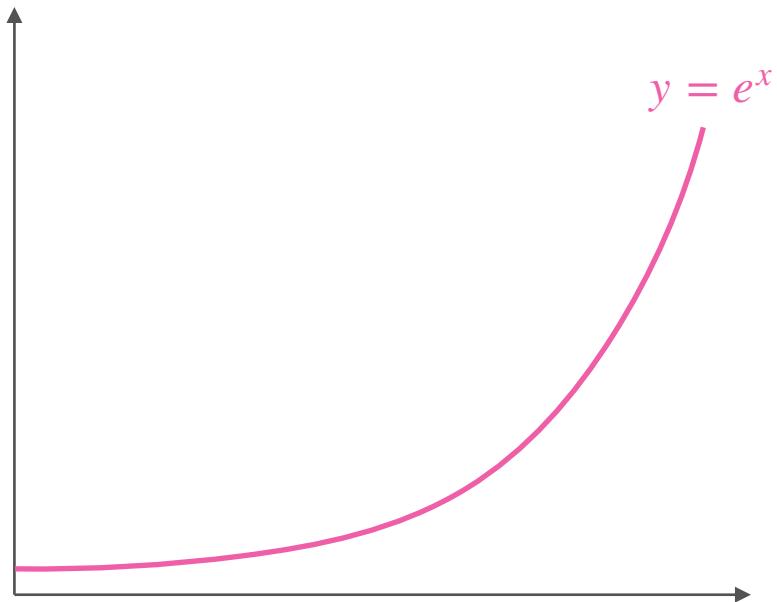


DeepLearning.AI

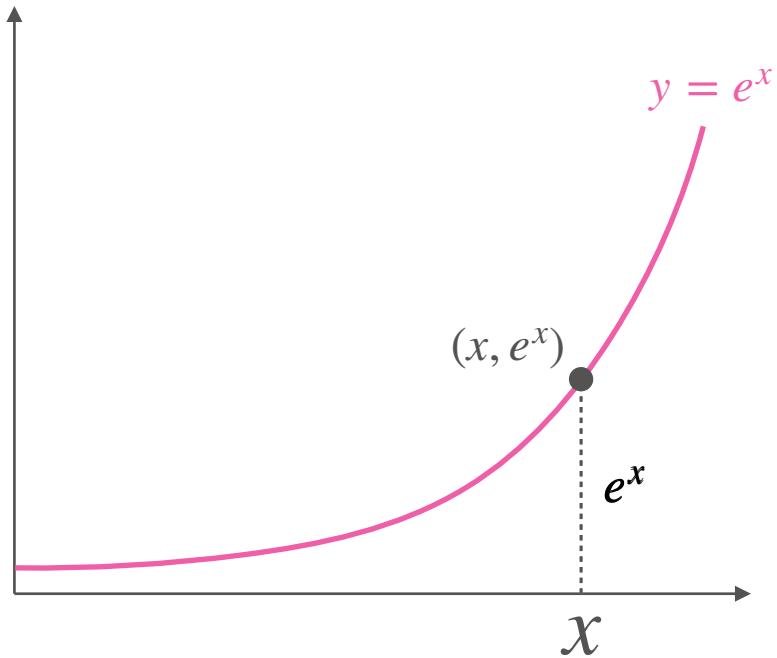
Derivatives and Optimization

The derivative of e^x

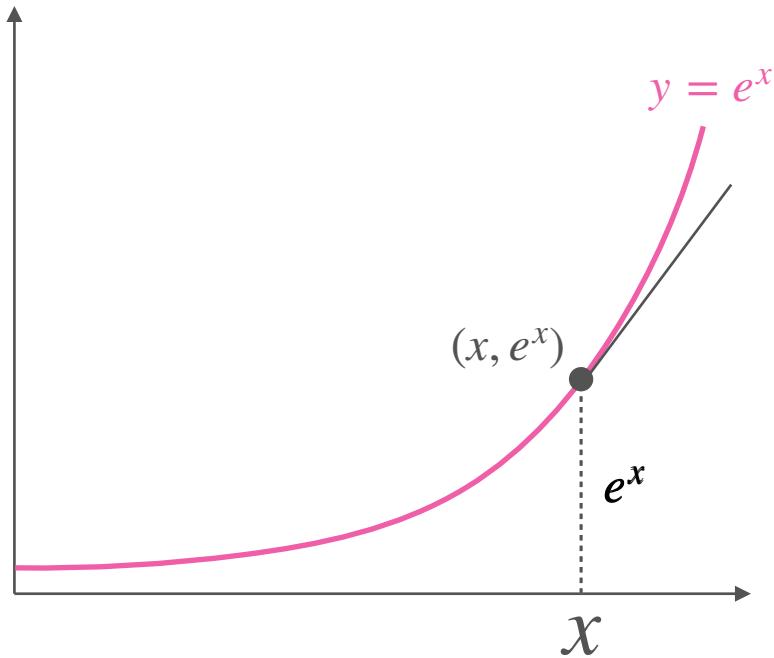
The Derivative of e^x Is Also e^x



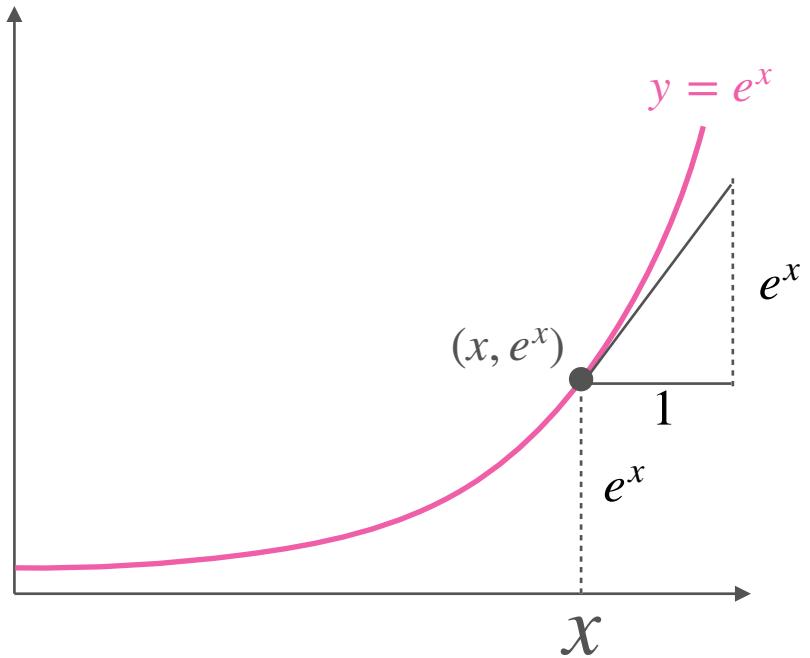
The Derivative of e^x Is Also e^x



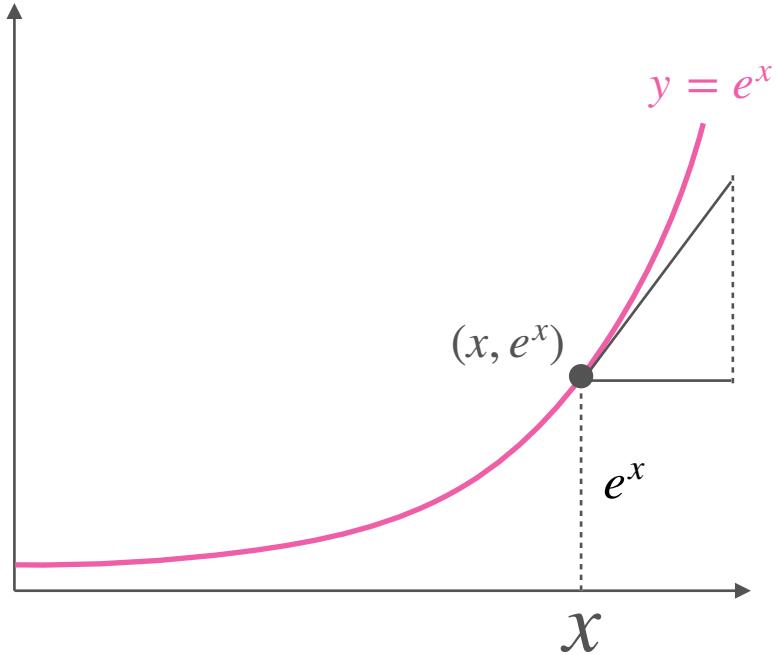
The Derivative of e^x Is Also e^x



The Derivative of e^x Is Also e^x

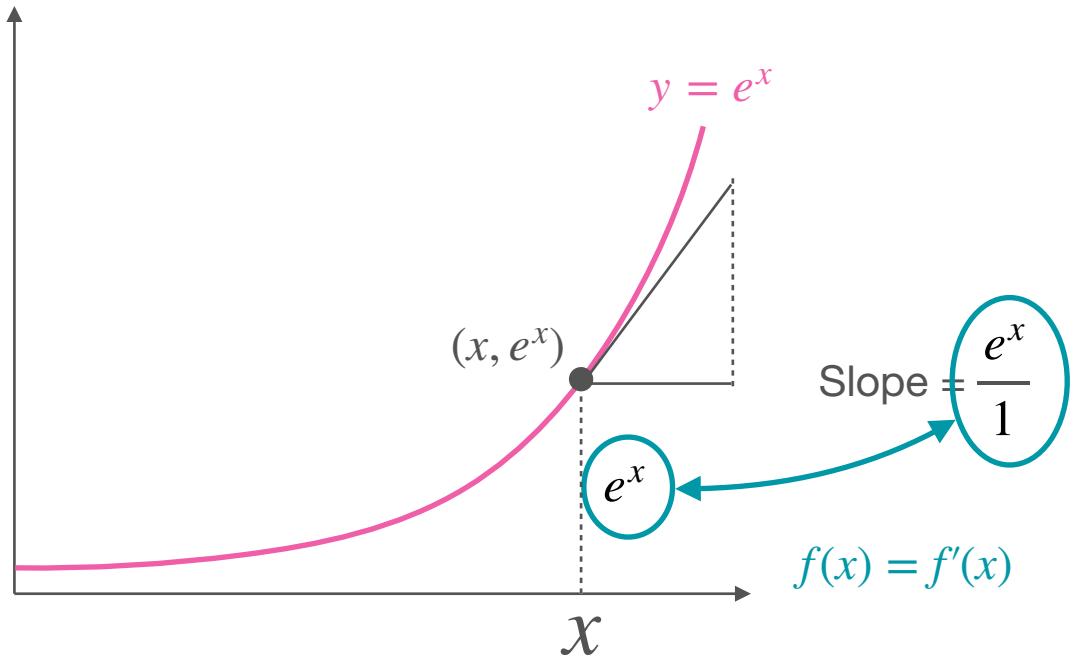


The Derivative of e^x Is Also e^x



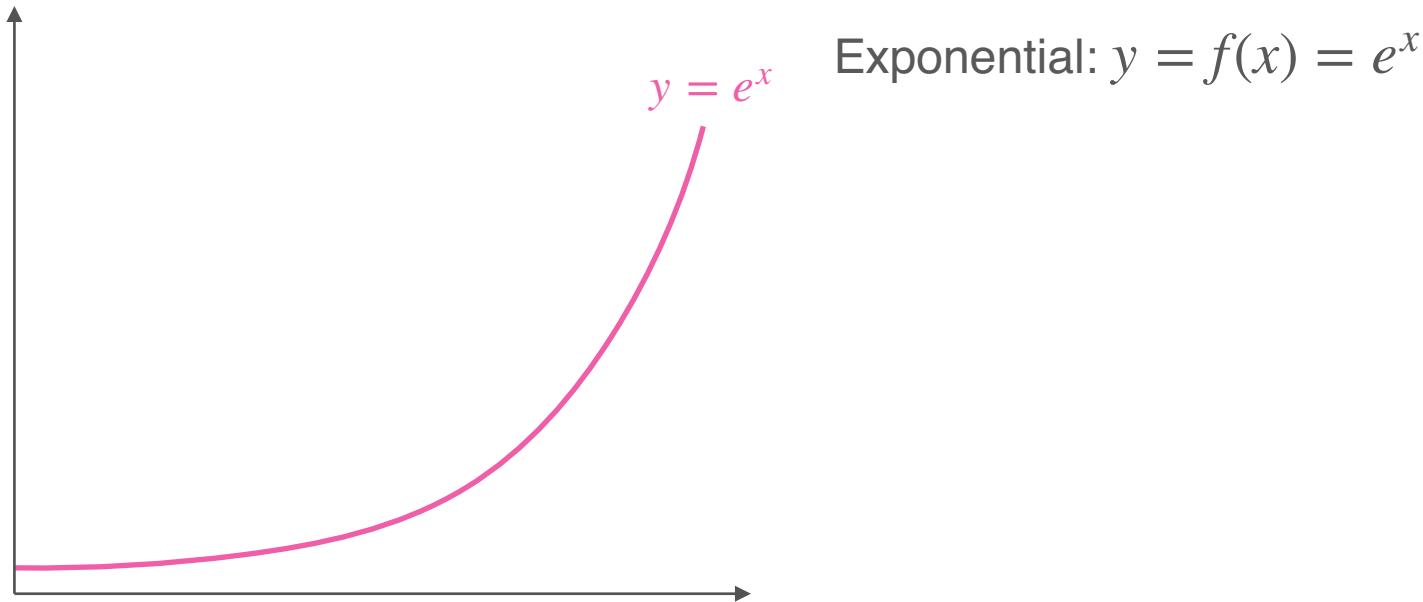
$$\text{Slope} = \frac{e^x}{1}$$

The Derivative of e^x Is Also e^x

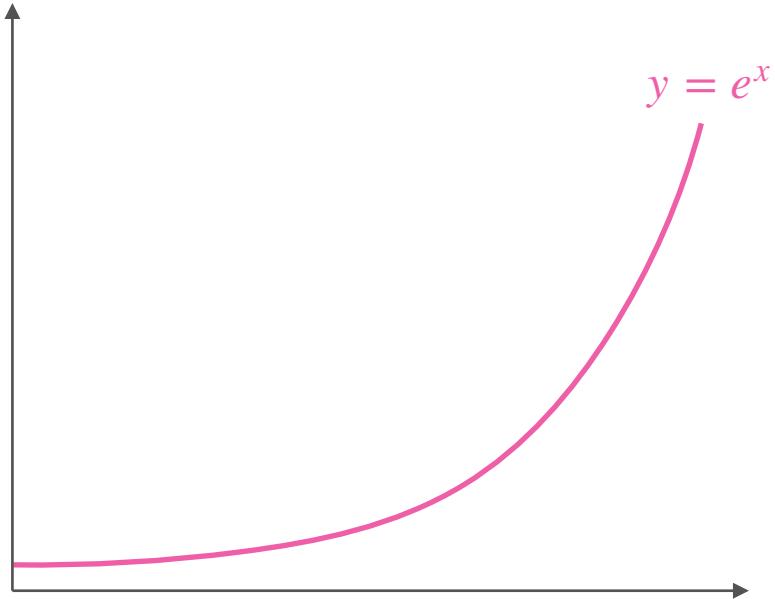


Derivative of e^x

Derivative of e^x



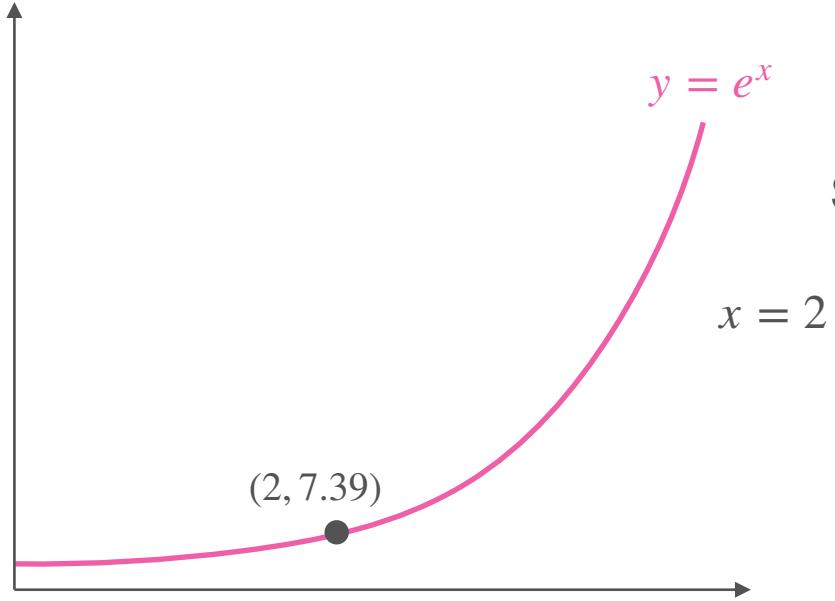
Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

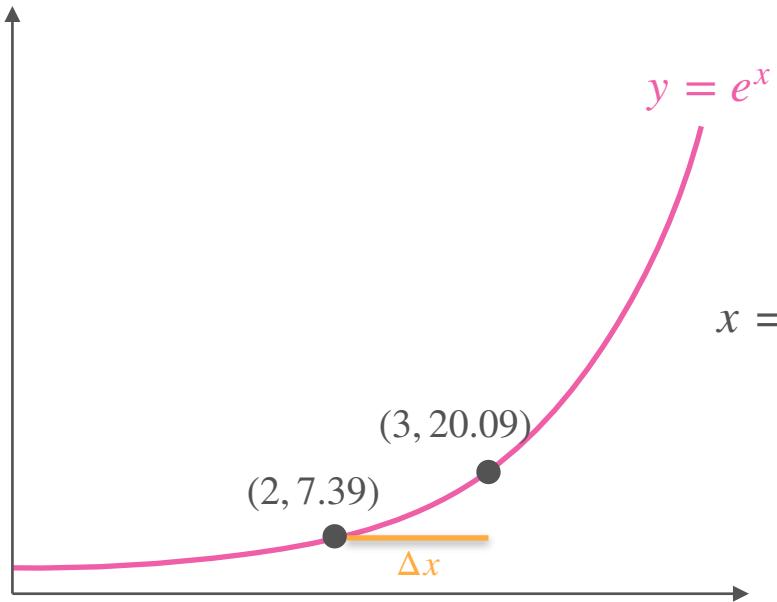
Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Derivative of e^x

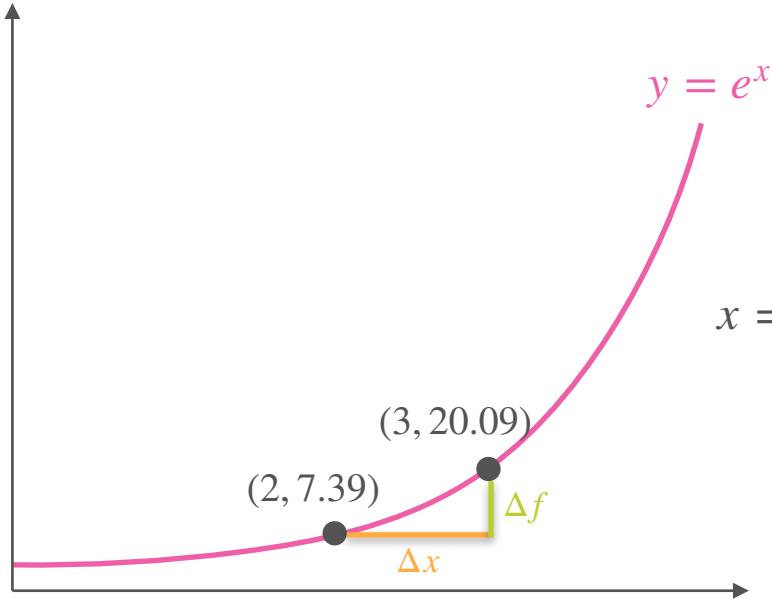


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$$\begin{array}{c|c} \Delta x & 1.0 \end{array}$$

Derivative of e^x



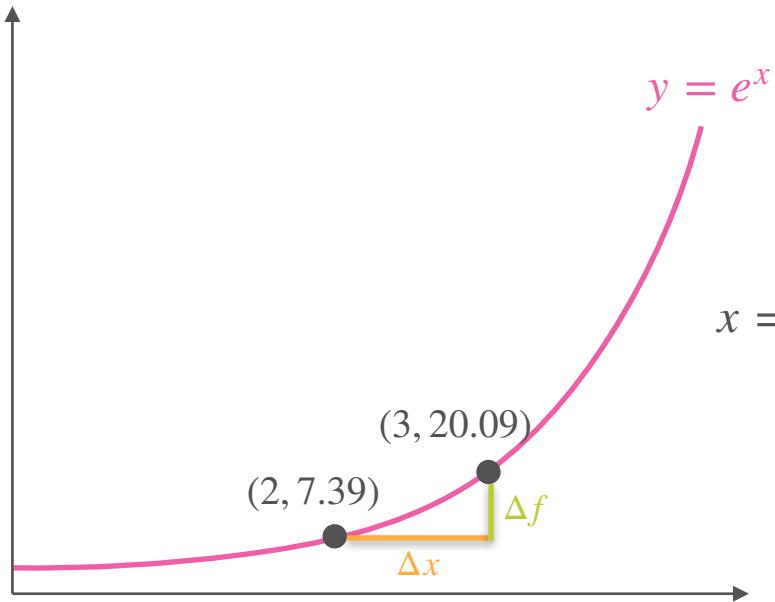
Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0
Δf	12.70

$$e^{2+1} - e^2 = 20.09 - 7.39$$

Derivative of e^x



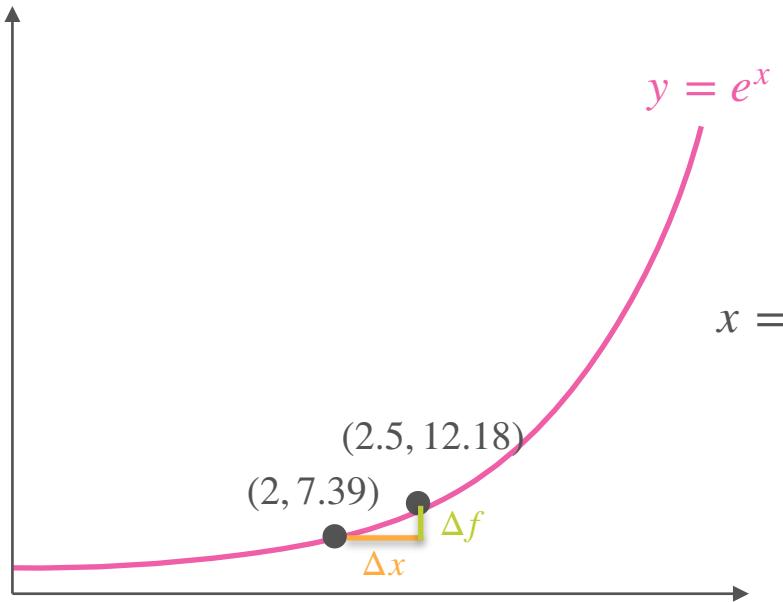
Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0
Δf	12.70
Slope	12.70

$$\frac{e^{2+1} - e^2}{1} = \frac{20.09 - 7.39}{1}$$

Derivative of e^x

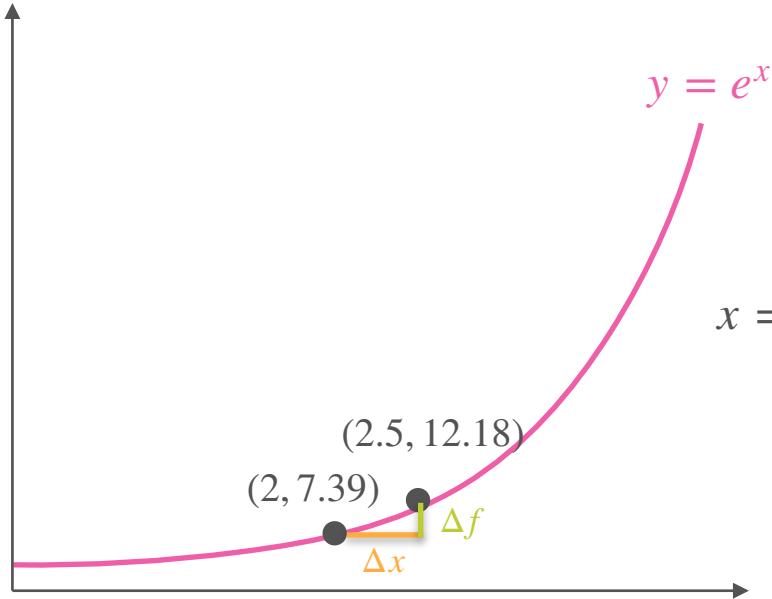


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2
Δf	12.70	
Slope	12.70	

Derivative of e^x



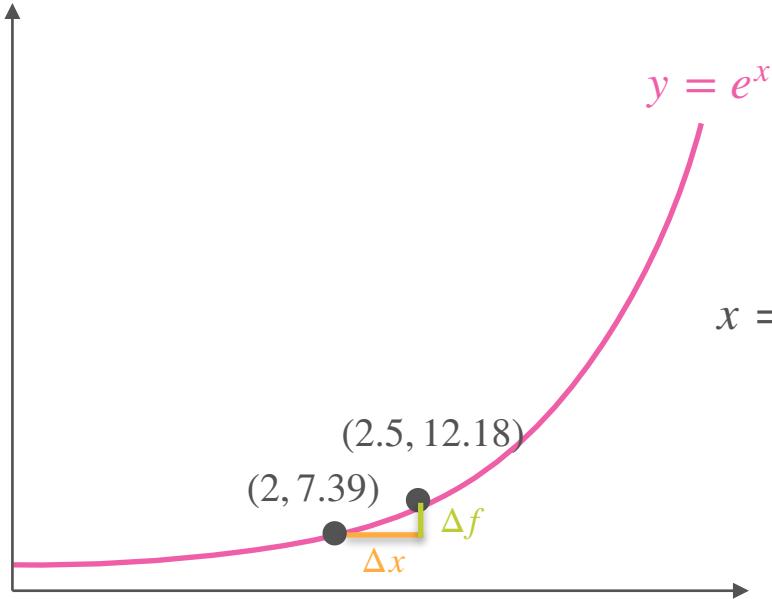
Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2
Δf	12.70	4.79
Slope	12.70	

$$e^{2+0.5} - e^2 = 12.18 - 7.39$$

Derivative of e^x



Exponential: $y = f(x) = e^x$

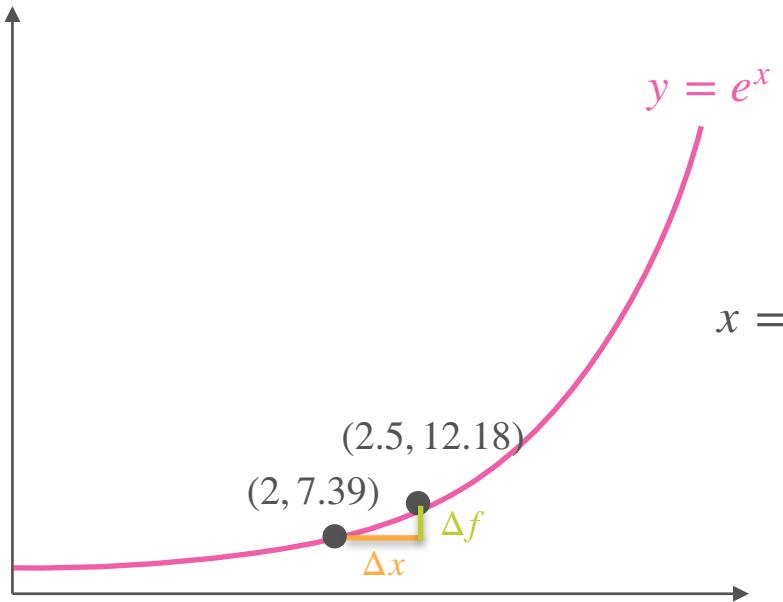
Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2
Δf	12.70	4.79
Slope	12.70	9.59

$$e^{2+0.5} - e^2 = 12.18 - 7.39$$

$$\frac{4.79}{0.5}$$

Derivative of e^x

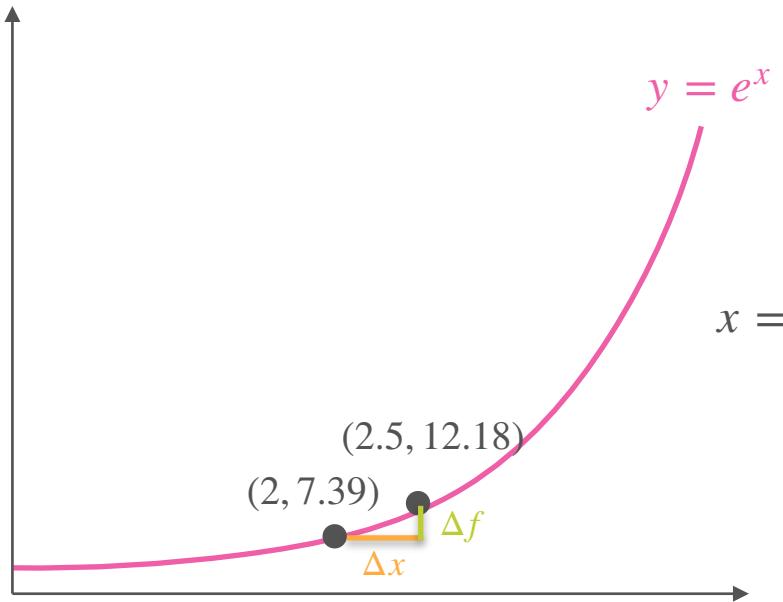


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	12.70	4.79	
Slope	12.70	9.59	

Derivative of e^x

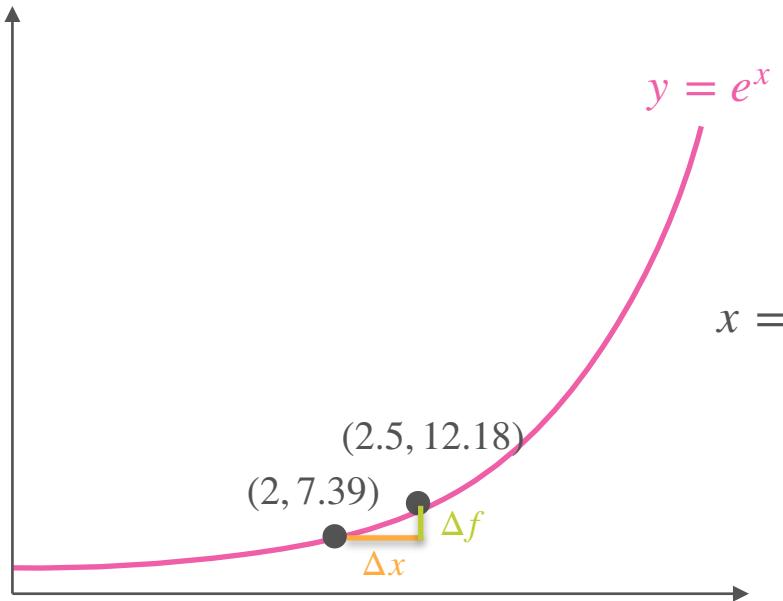


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	12.70	4.79	2.10
Slope	12.70	9.59	

Derivative of e^x

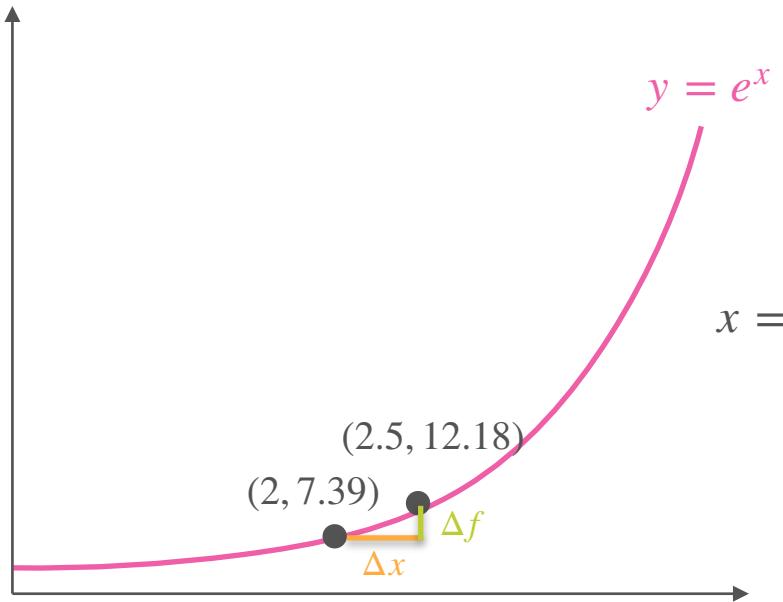


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	12.70	4.79	2.10
Slope	12.70	9.59	8.39

Derivative of e^x

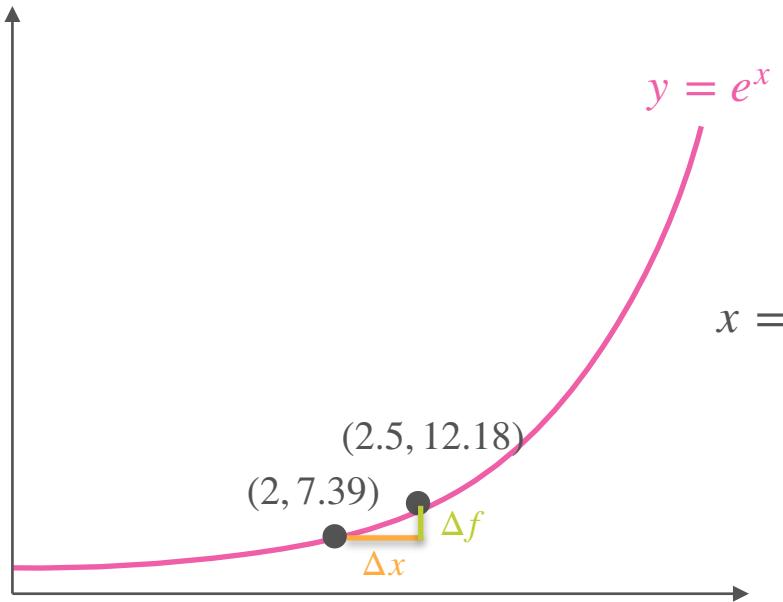


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8
Δf	12.70	4.79	2.10	
Slope	12.70	9.59	8.39	

Derivative of e^x

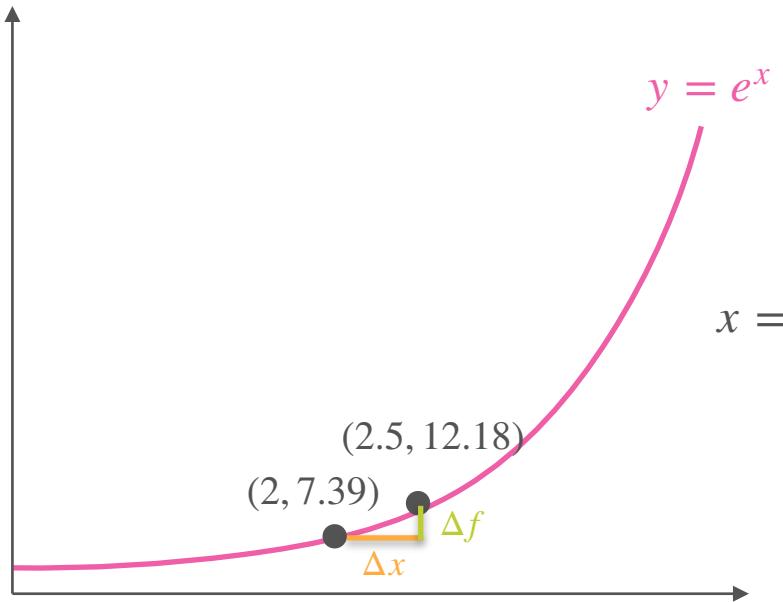


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8
Δf	12.70	4.79	2.10	0.98
Slope	12.70	9.59	8.39	

Derivative of e^x

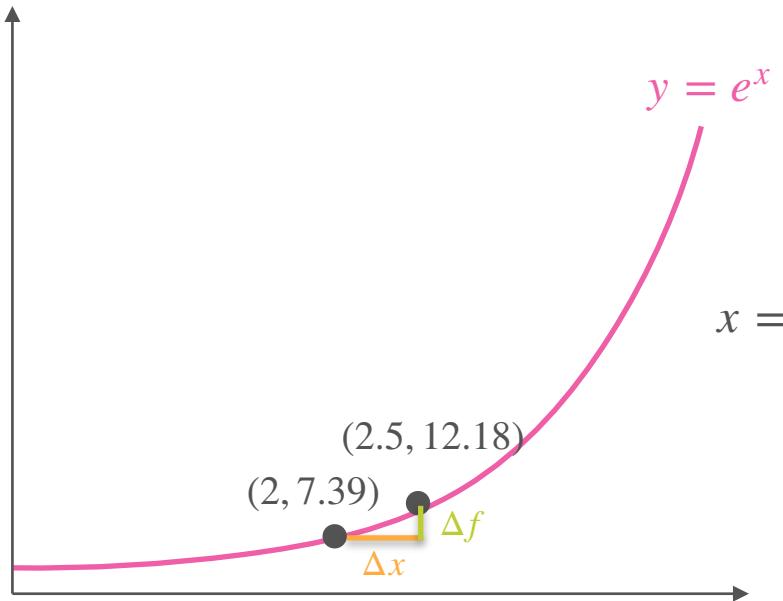


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8
Δf	12.70	4.79	2.10	0.98
Slope	12.70	9.59	8.39	7.87

Derivative of e^x

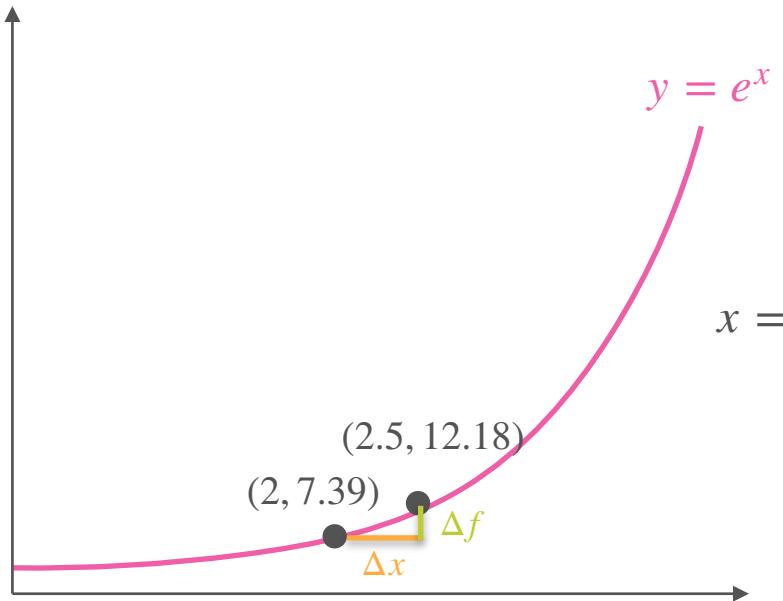


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	12.70	4.79	2.10	0.98	
Slope	12.70	9.59	8.39	7.87	

Derivative of e^x

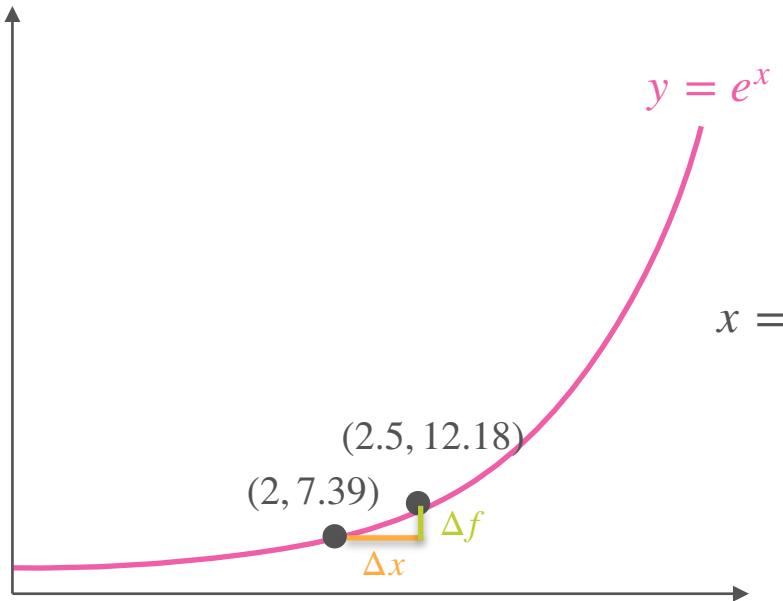


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	12.70	4.79	2.10	0.98	0.48
Slope	12.70	9.59	8.39	7.87	

Derivative of e^x

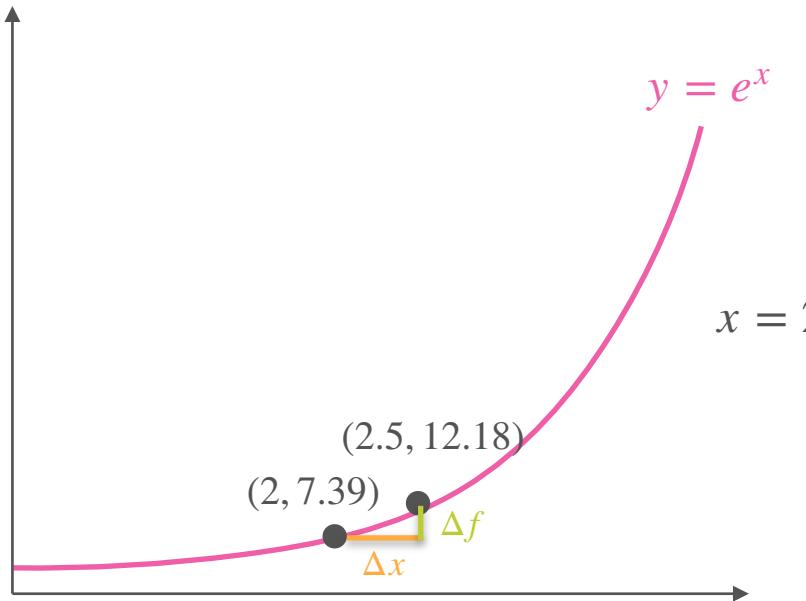


Exponential: $y = f(x) = e^x$

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Derivative of e^x

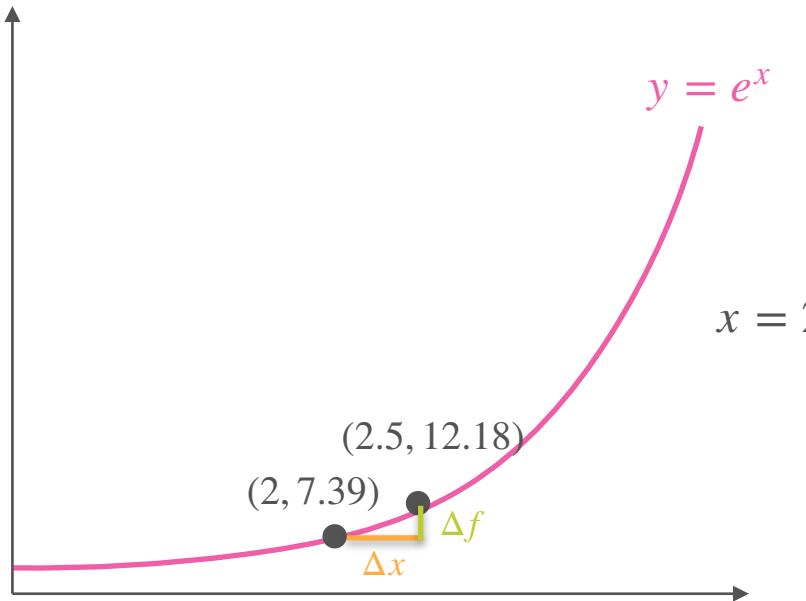


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	12.70	4.79	2.10	0.98	0.48	
Slope	12.70	9.59	8.39	7.87	7.62	

Derivative of e^x

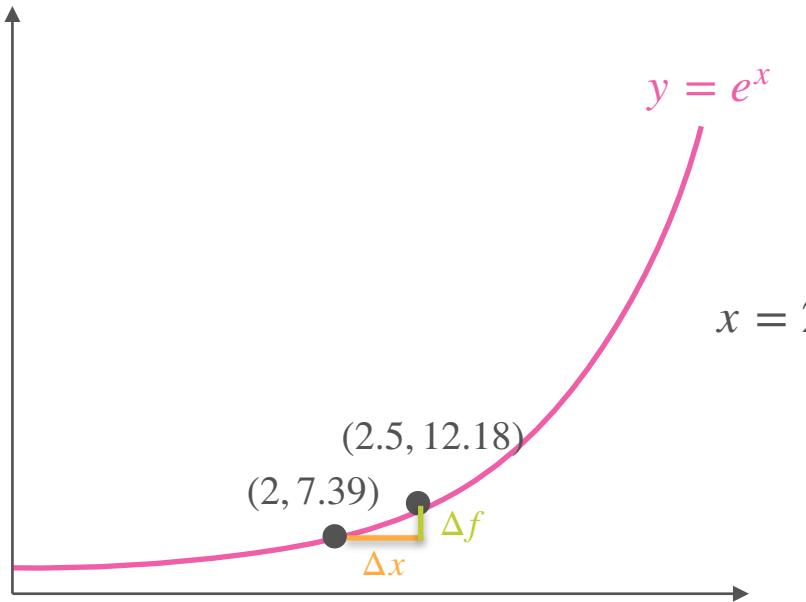


Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	

Derivative of e^x

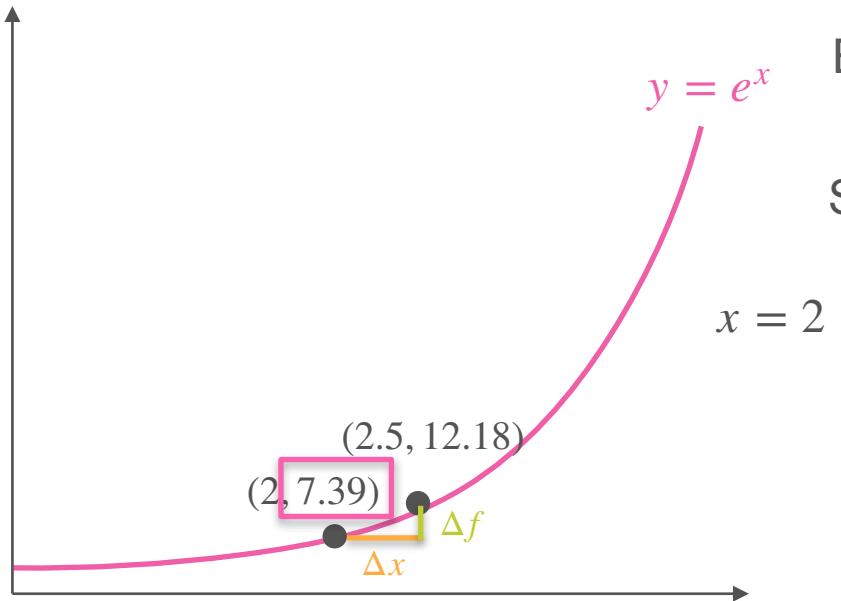


Exponential: $y = f(x) = e^x$

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Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	7.39

Derivative of e^x

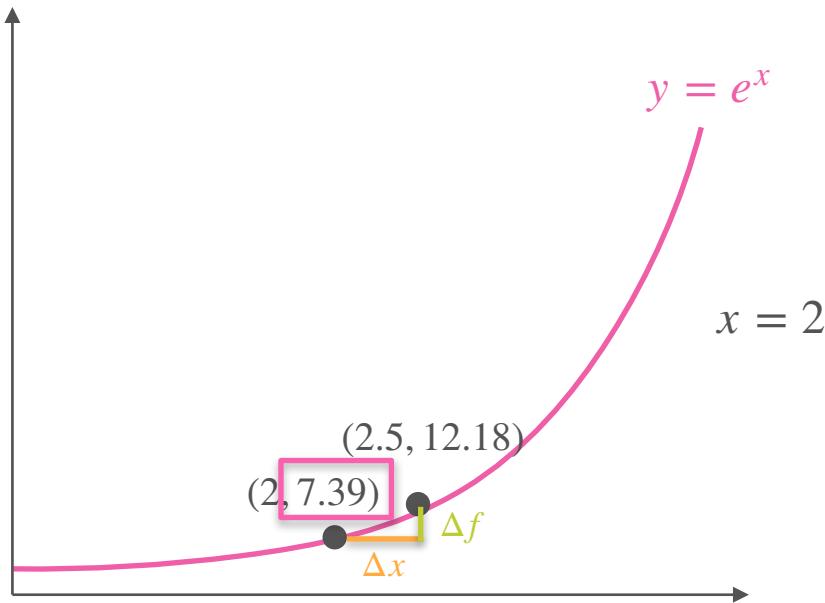


Exponential: $y = f(x) = e^x$

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Slope	12.70	9.59	8.39	7.87	7.62	7.39

Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	7.39

$$e^2$$

(Reading Item) Derivative of a^x

$$a^x = e^{x \log(a)}$$

$$\frac{d}{dx} a^x = \log(a) e^{x \log(a)} = \log(a) a^x$$



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Derivatives and Optimization

The derivative of $\log(x)$

Logarithm

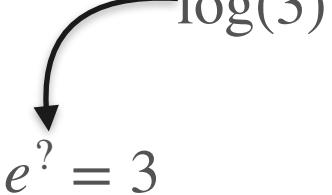
Logarithm

$$e^? = 3$$

Logarithm

$$e^? = 3$$

$\log(3)$



Logarithm

$$e^? = 3 \qquad \qquad \log(3) \qquad e^? = x$$


Logarithm

$$\begin{array}{ll} \log(3) & \log(x) \\ \downarrow & \downarrow \\ e^? = 3 & e^? = x \end{array}$$

Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x$$

Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x$$

$$f^{-1}(y) = \log(y)$$

Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x \quad f^{-1}(y) = \log(y)$$

$$e^{\log(x)} = x$$

Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x \quad f^{-1}(y) = \log(y)$$

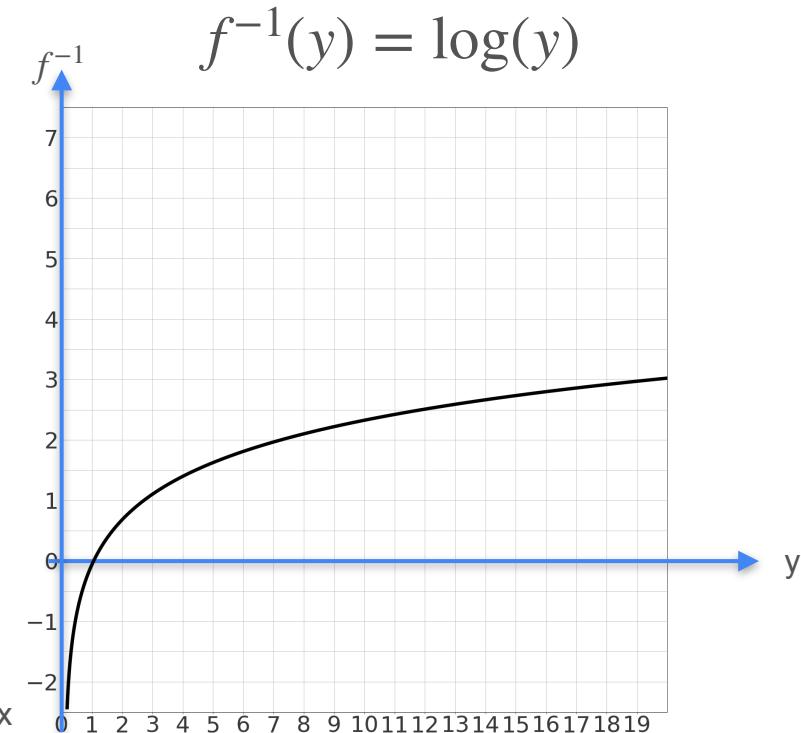
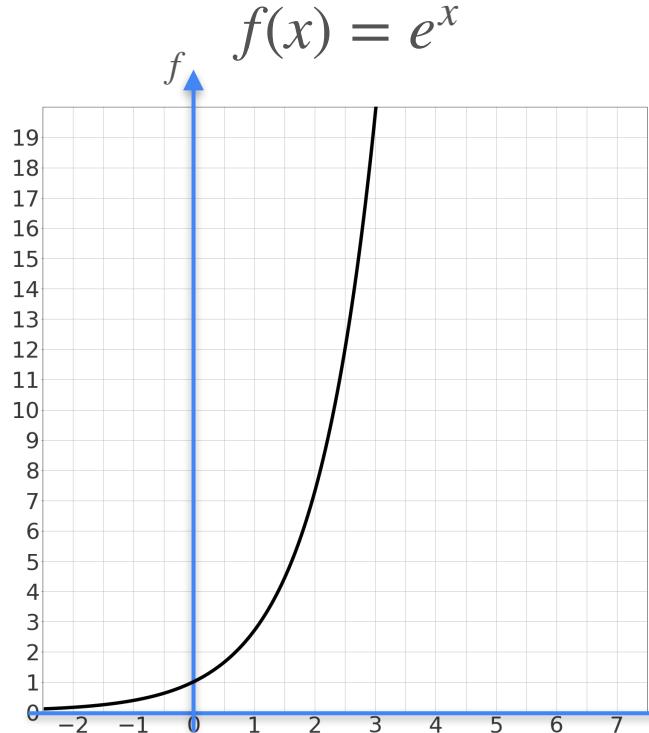
$$e^{\log(x)} = x \quad \log(e^y) = y$$

Logarithm

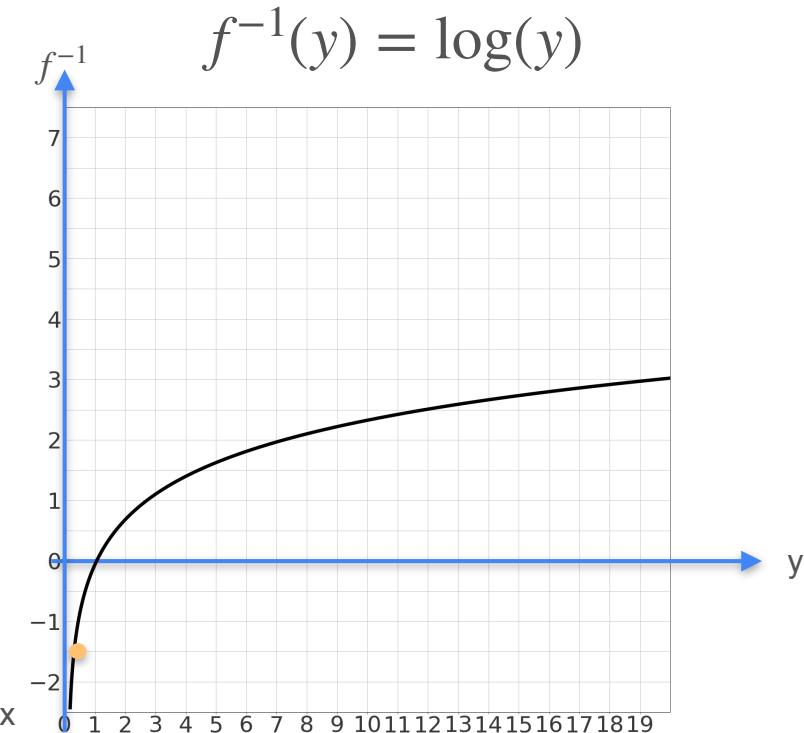
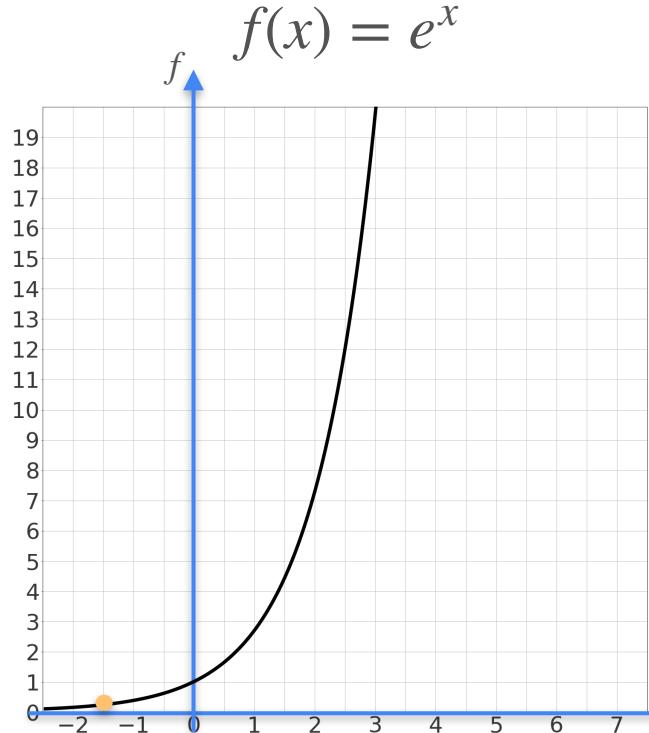
$$f(x) = e^x$$

$$f^{-1}(y) = \log(y)$$

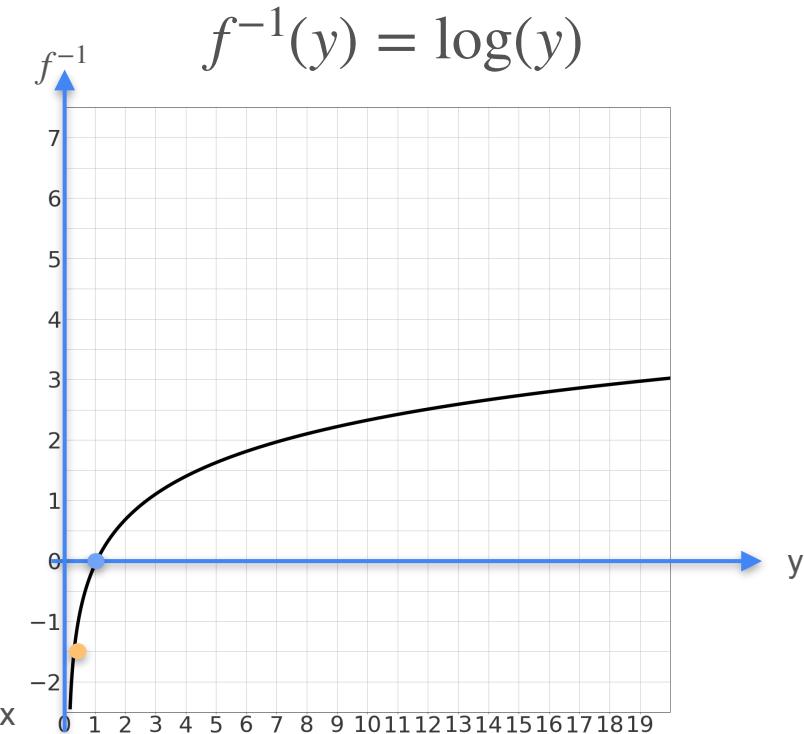
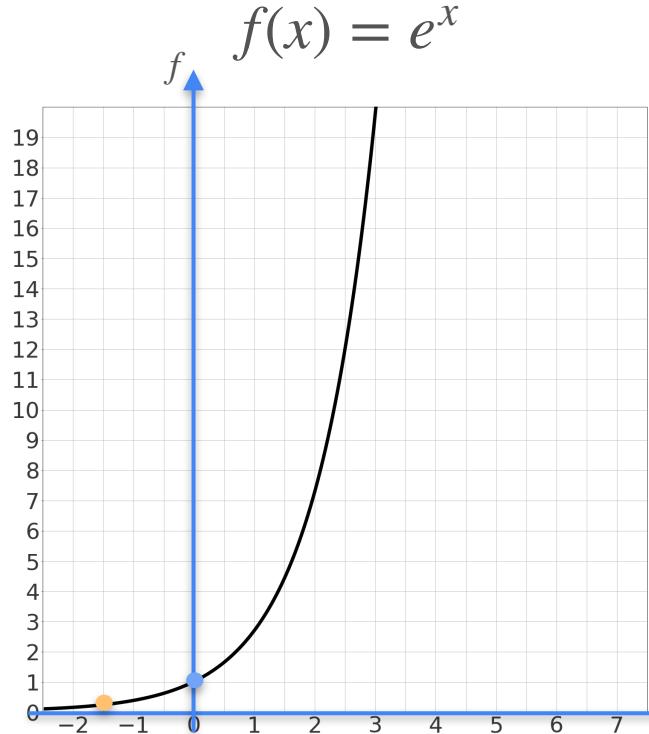
Logarithm



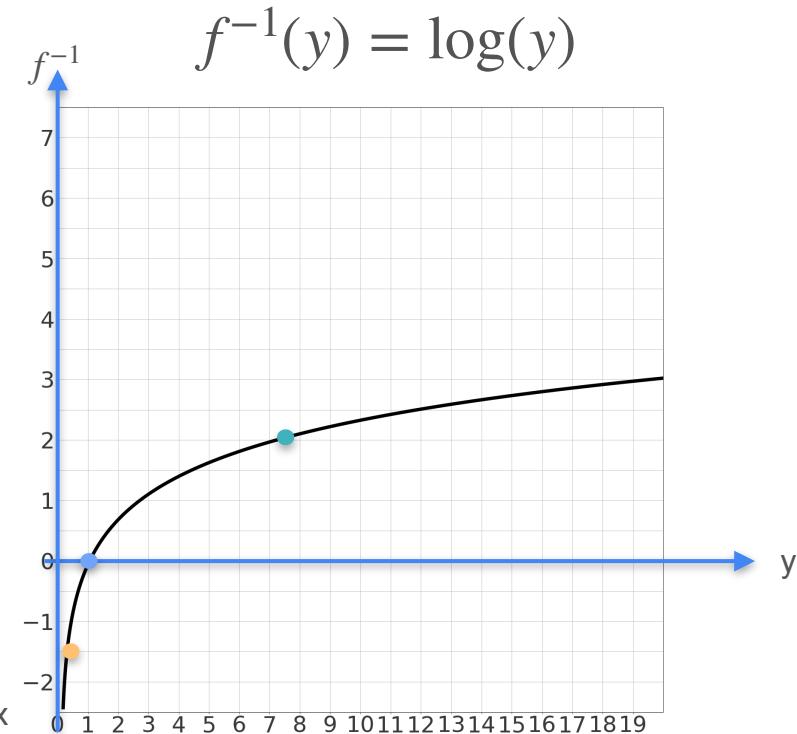
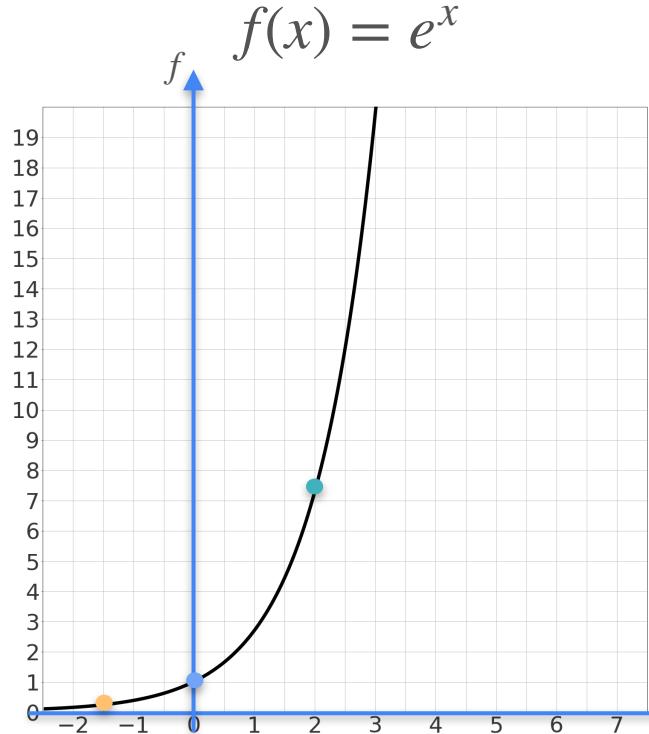
Logarithm



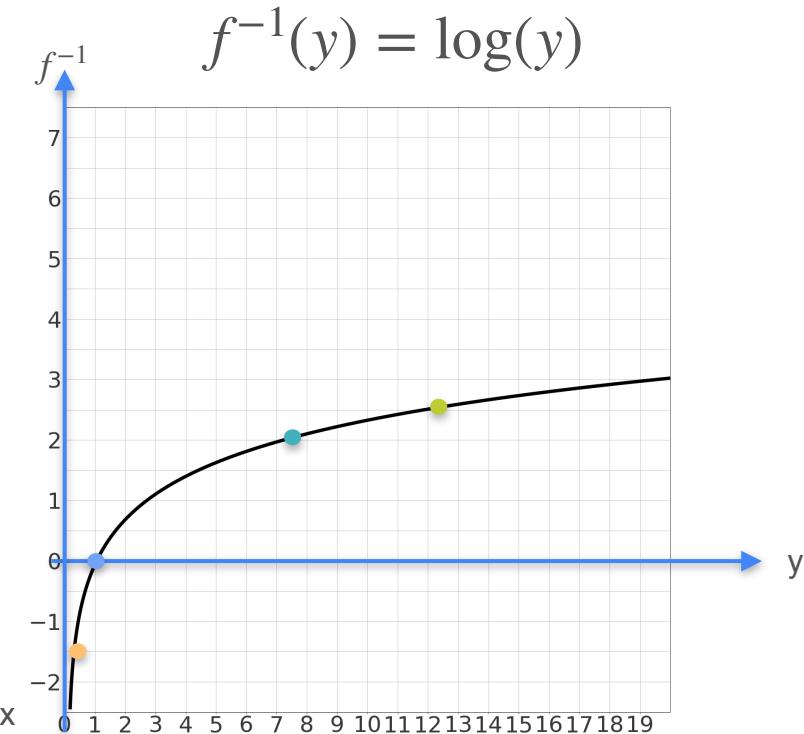
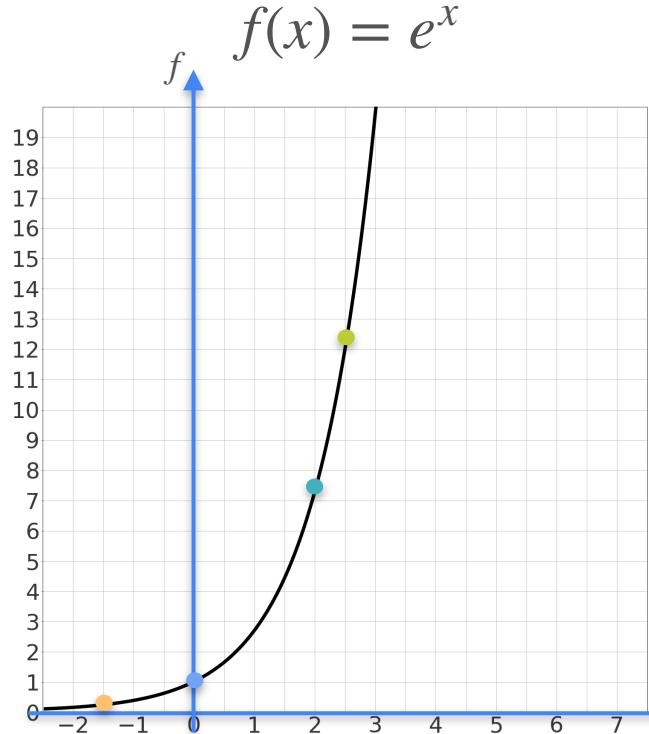
Logarithm



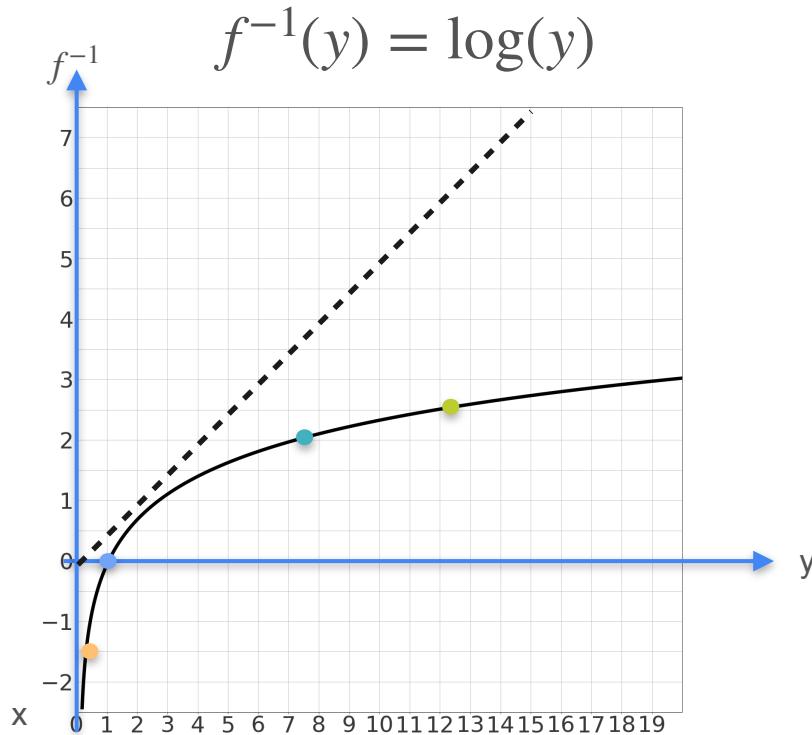
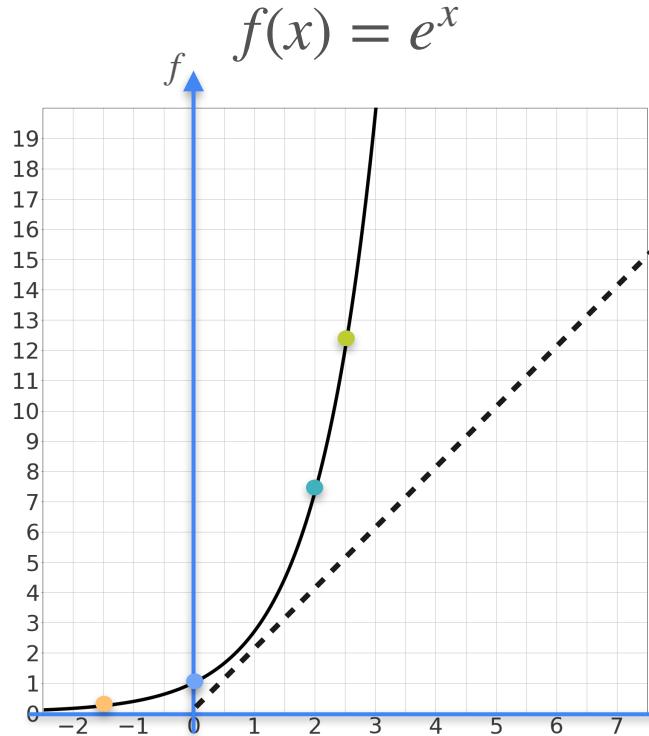
Logarithm



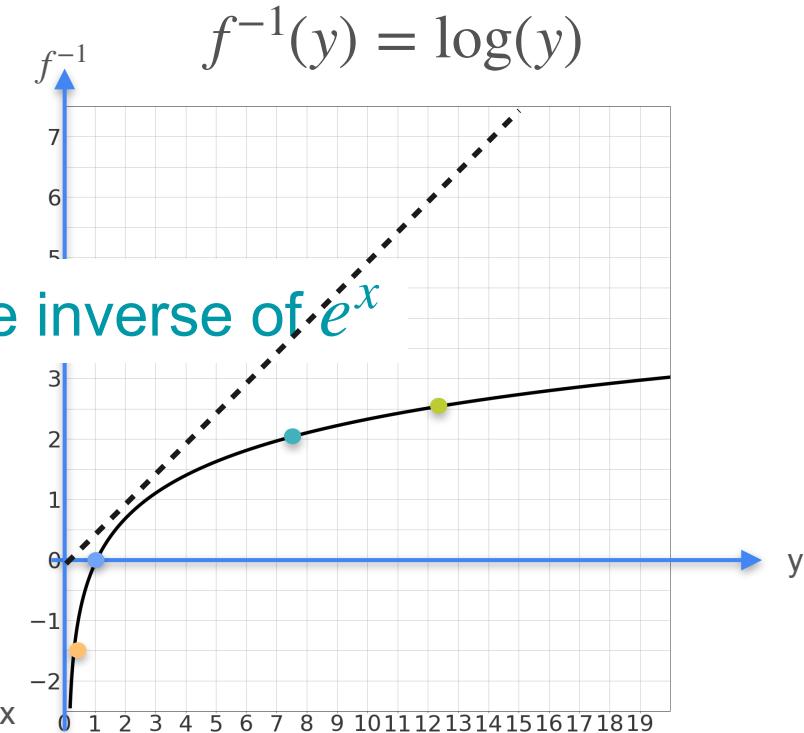
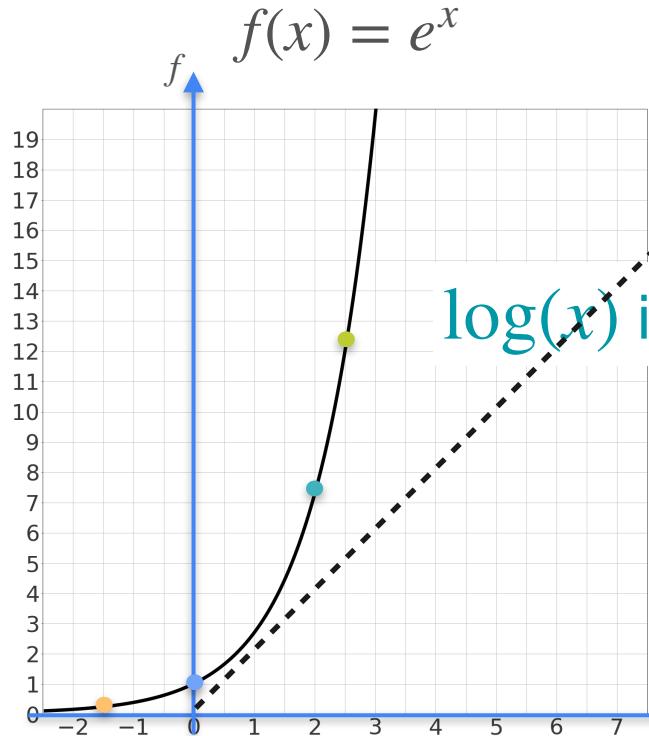
Logarithm



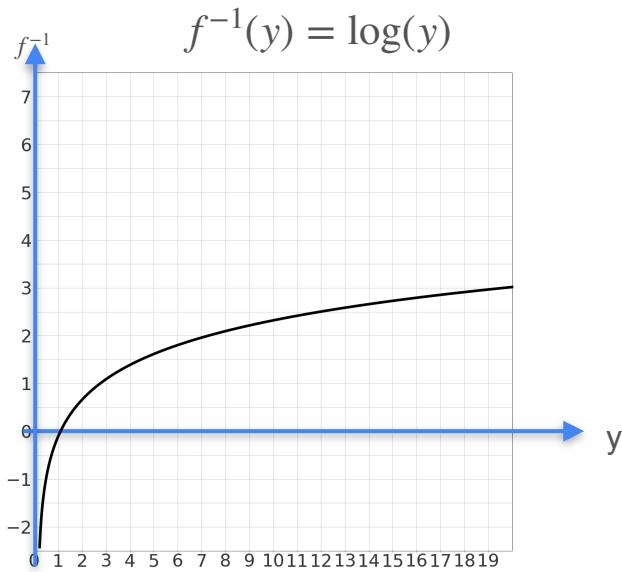
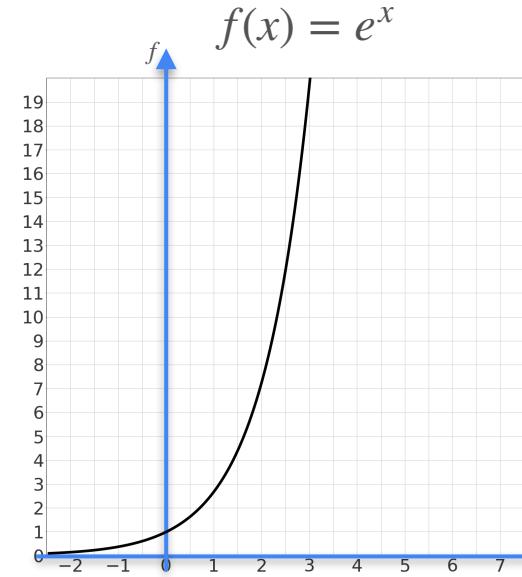
Logarithm



Logarithm

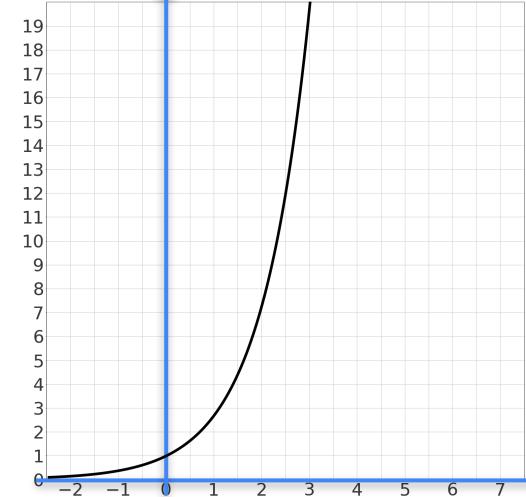


Logarithm

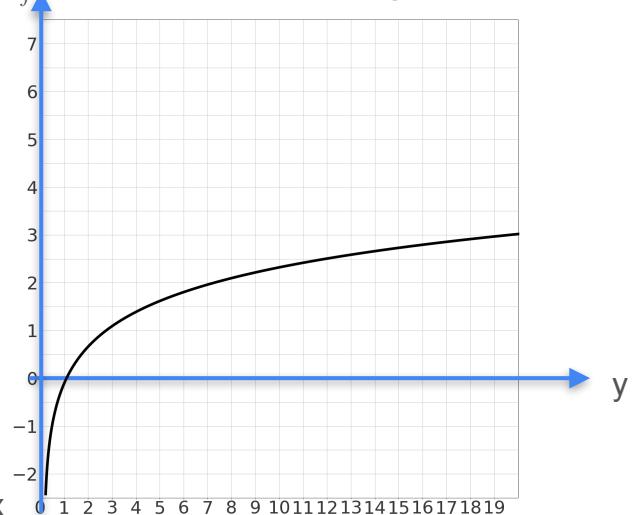


Logarithm

$$f(x) = e^x$$



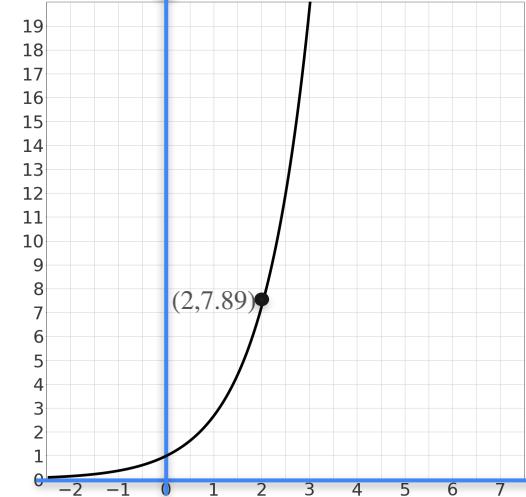
$$f^{-1}(y) = \log(y)$$



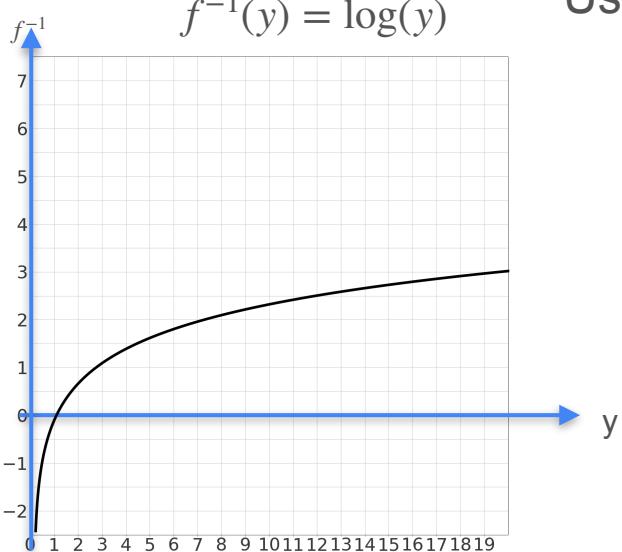
Using the result for inverses

Logarithm

$$f(x) = e^x$$



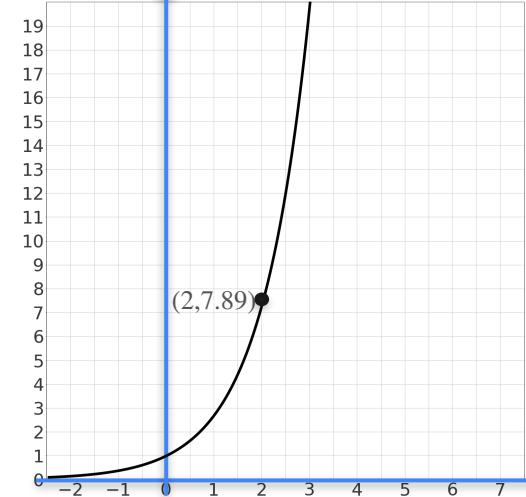
$$f^{-1}(y) = \log(y)$$



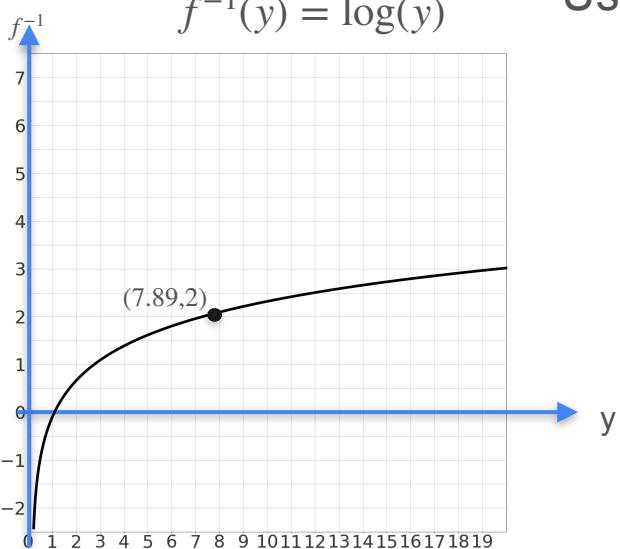
Using the result for inverses

Logarithm

$$f(x) = e^x$$



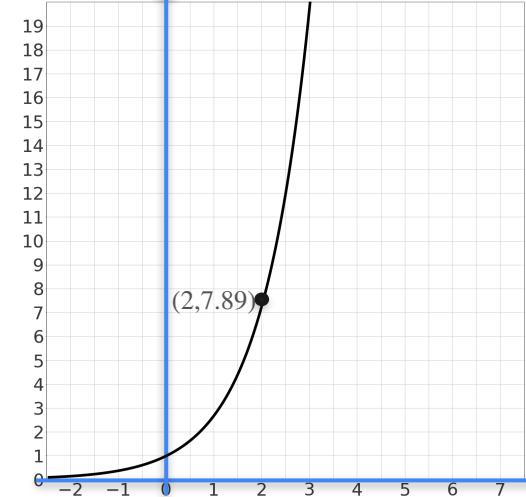
$$f^{-1}(y) = \log(y)$$



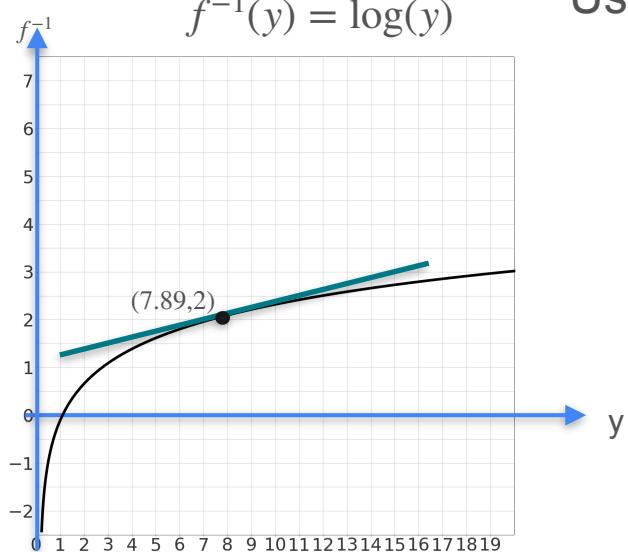
Using the result for inverses

Logarithm

$$f(x) = e^x$$



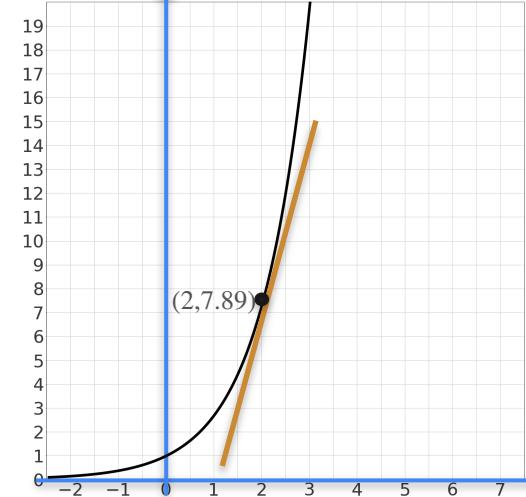
$$f^{-1}(y) = \log(y)$$



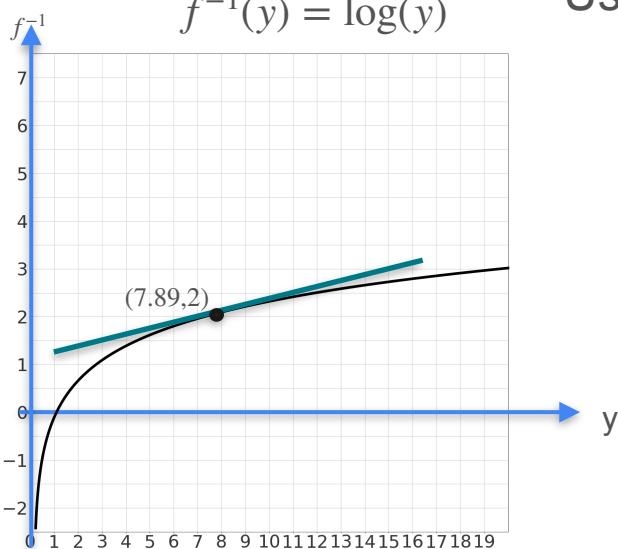
Using the result for inverses

Logarithm

$$f(x) = e^x$$



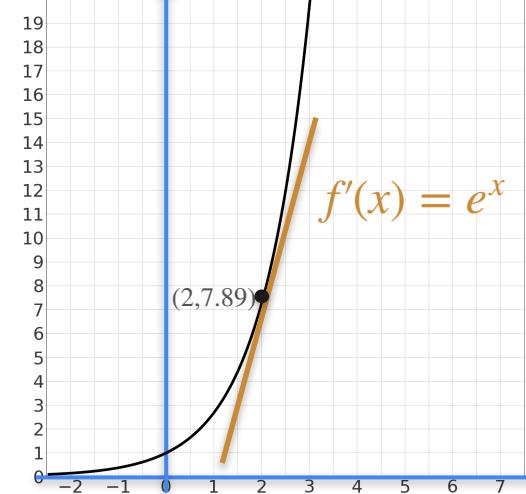
$$f^{-1}(y) = \log(y)$$



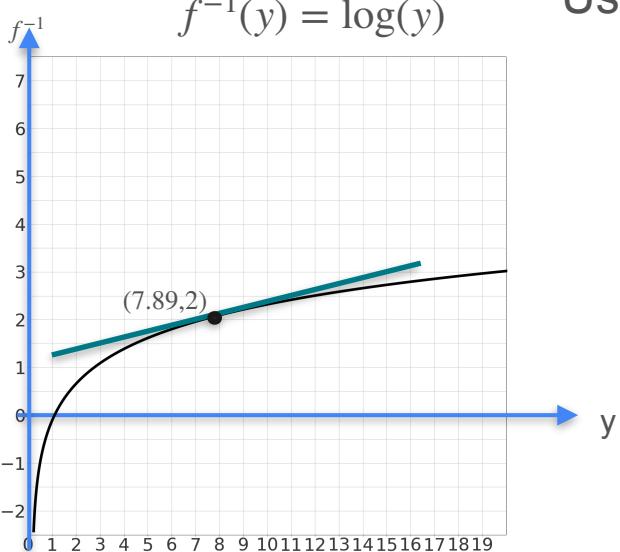
Using the result for inverses

Logarithm

$$f(x) = e^x$$

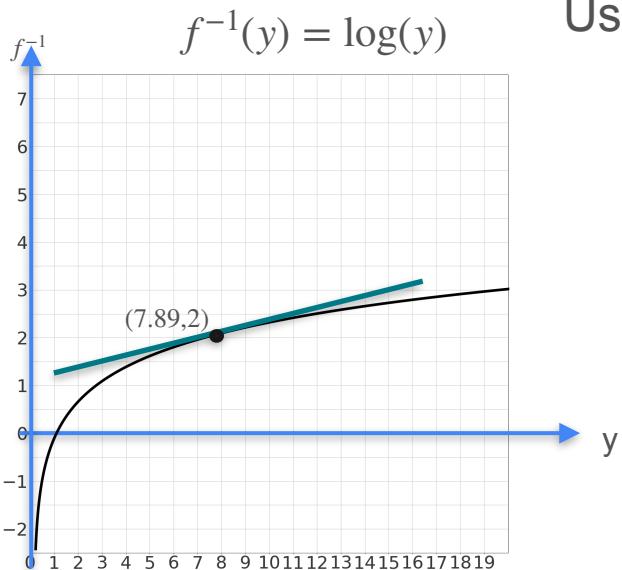
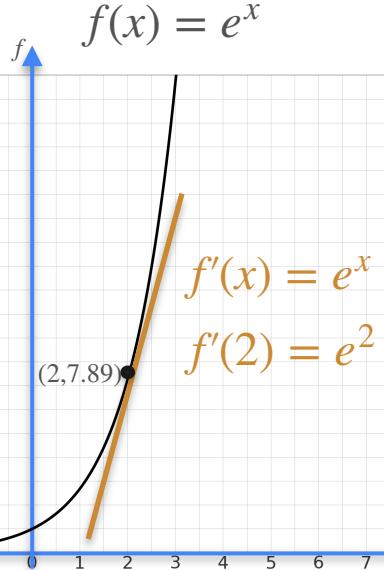


$$f^{-1}(y) = \log(y)$$



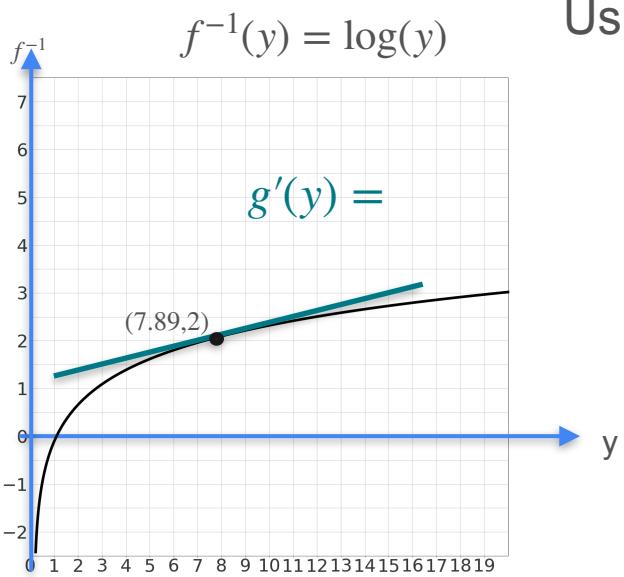
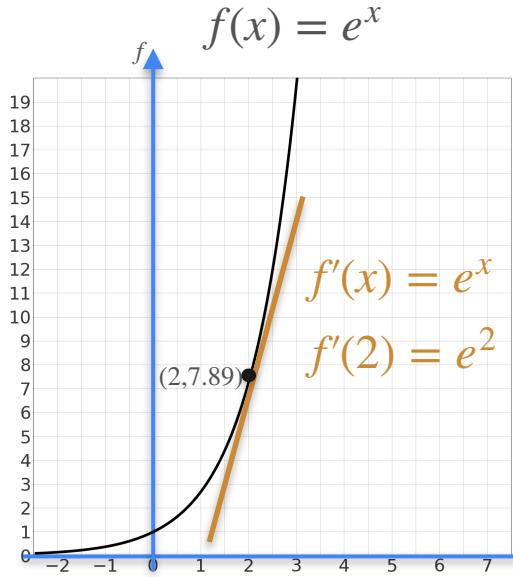
Using the result for inverses

Logarithm



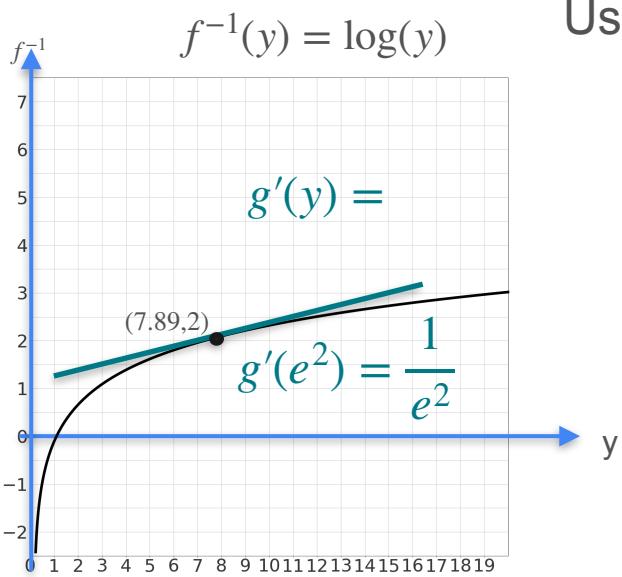
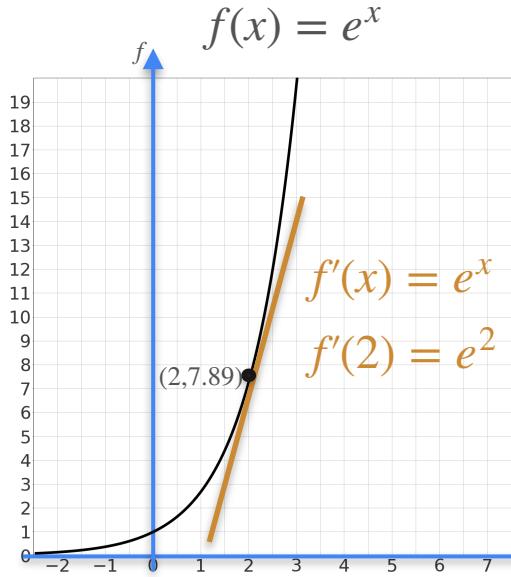
Using the result for inverses

Logarithm



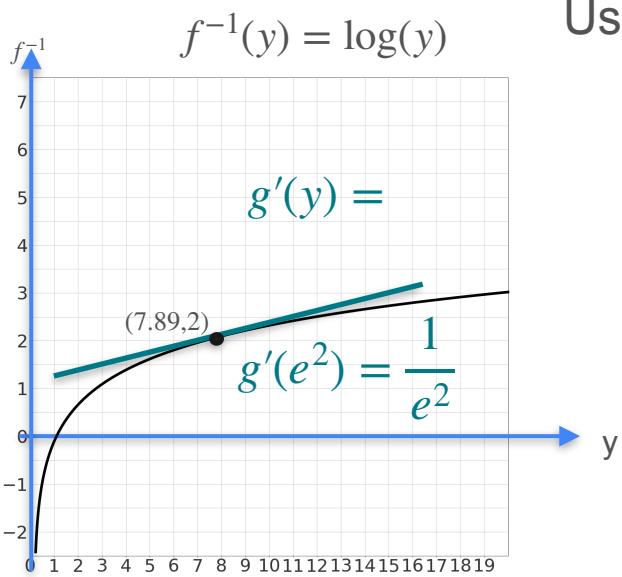
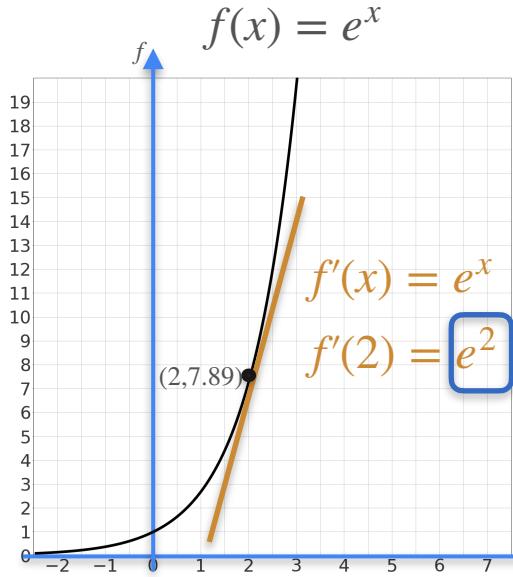
Using the result for inverses

Logarithm



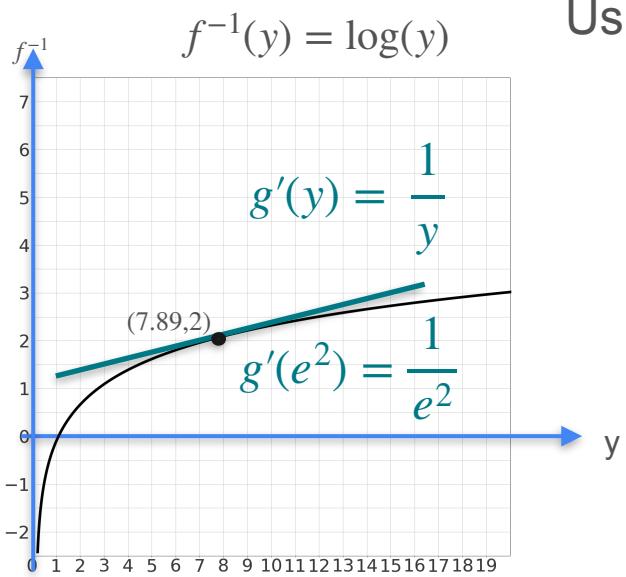
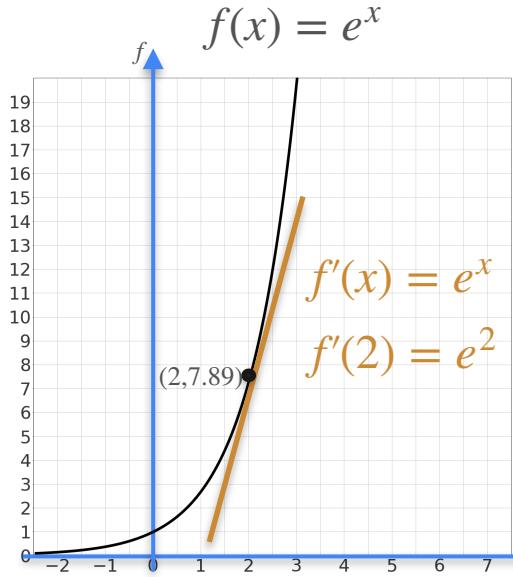
Using the result for inverses

Logarithm



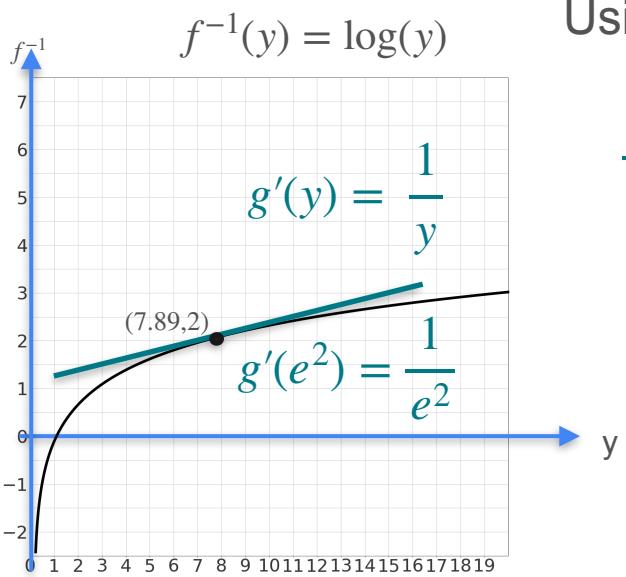
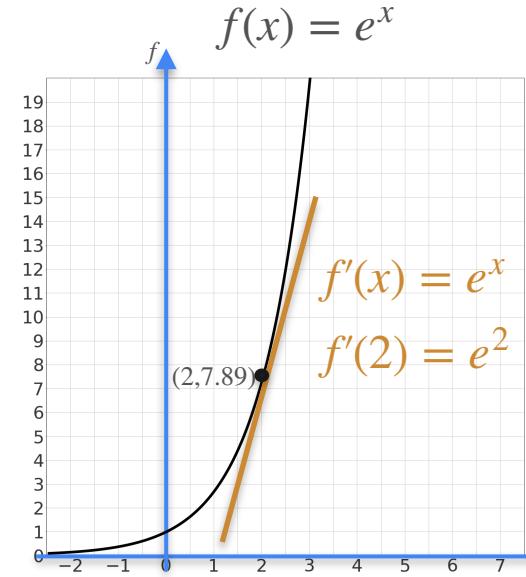
Using the result for inverses

Logarithm



Using the result for inverses

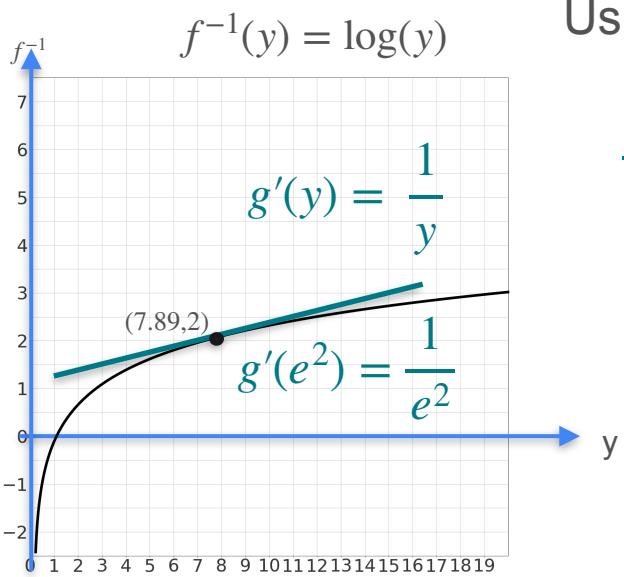
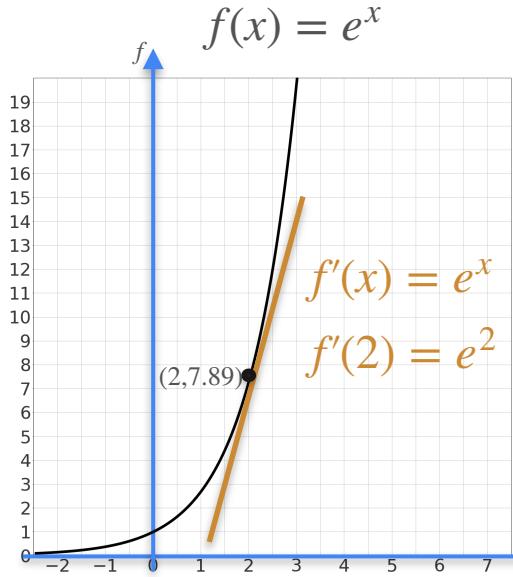
Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(x)}$$

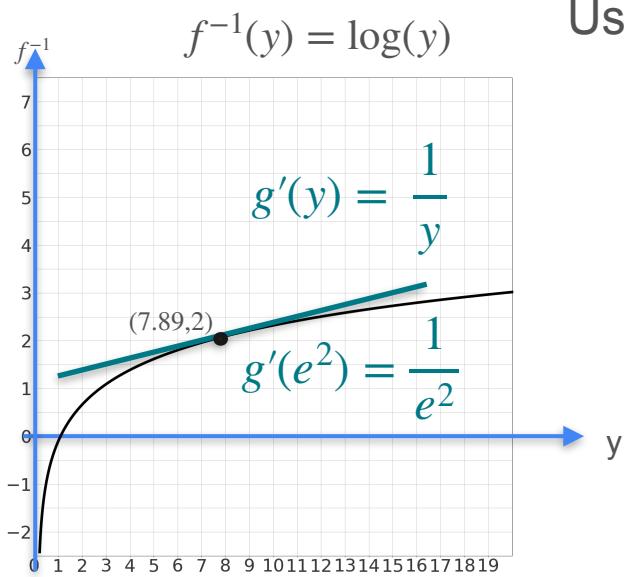
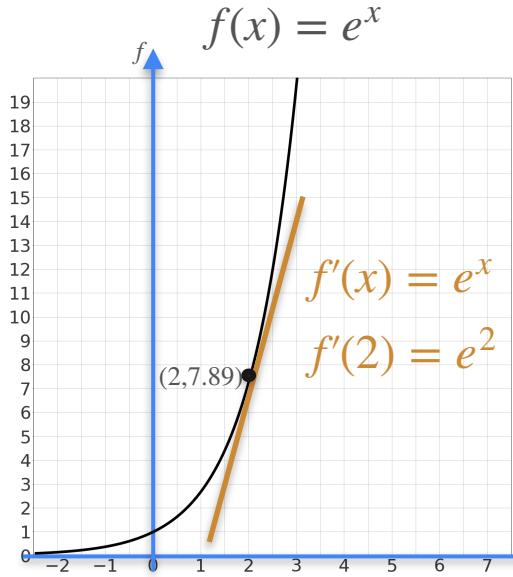
Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

Logarithm

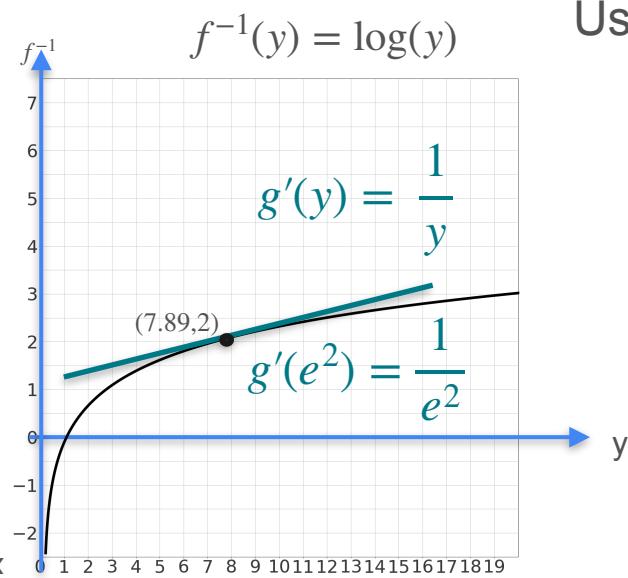
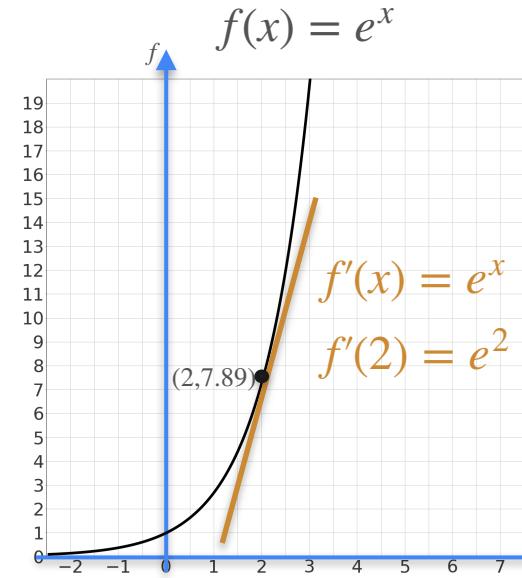


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$\log(y)$

Logarithm

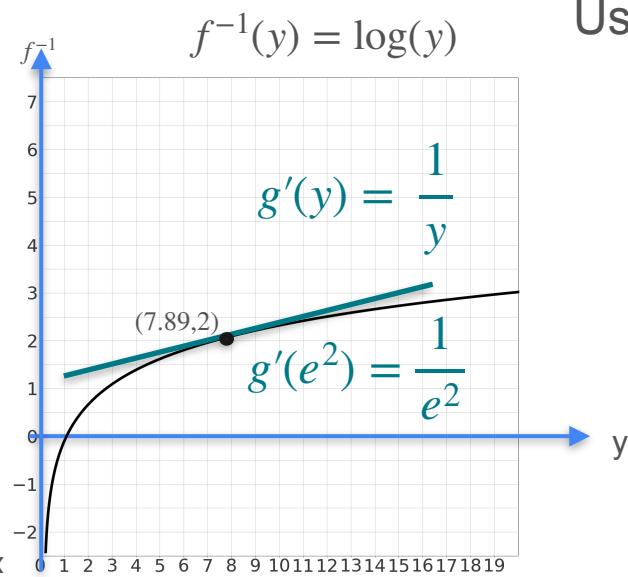
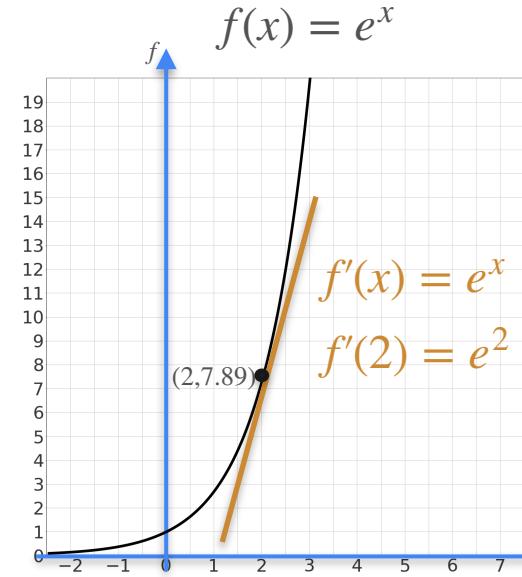


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y)$$

Logarithm

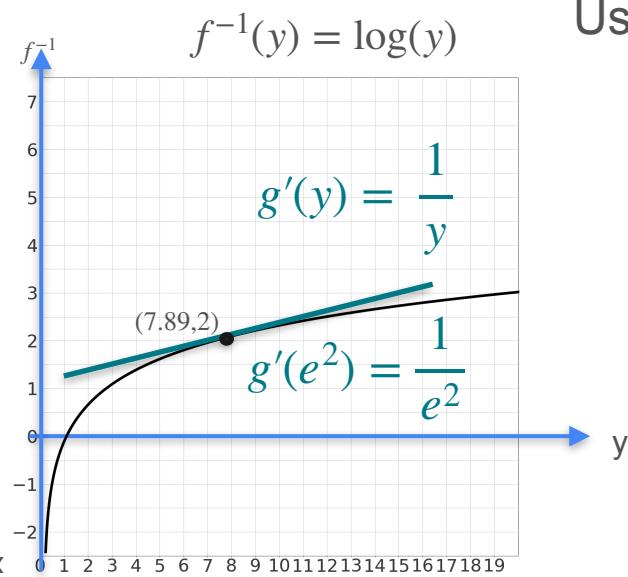
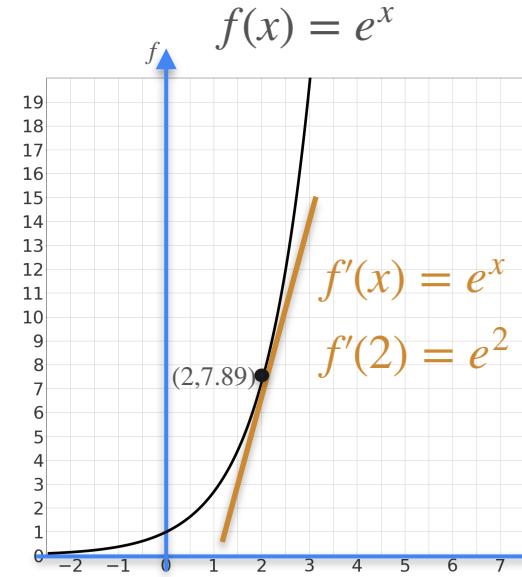


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y) = \frac{1}{y}$$

Logarithm

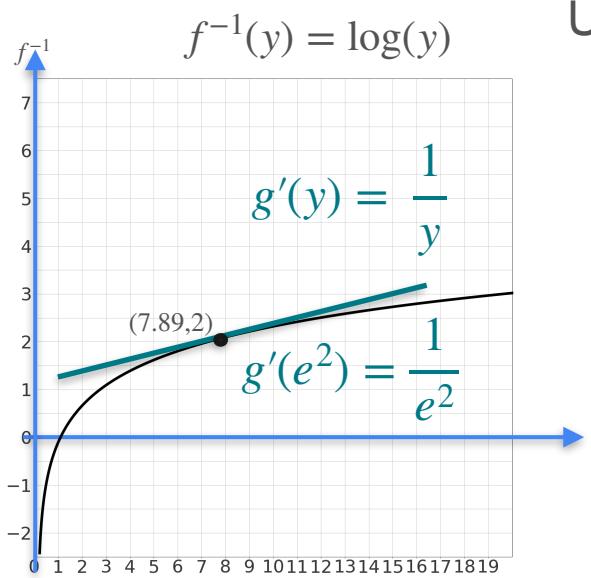
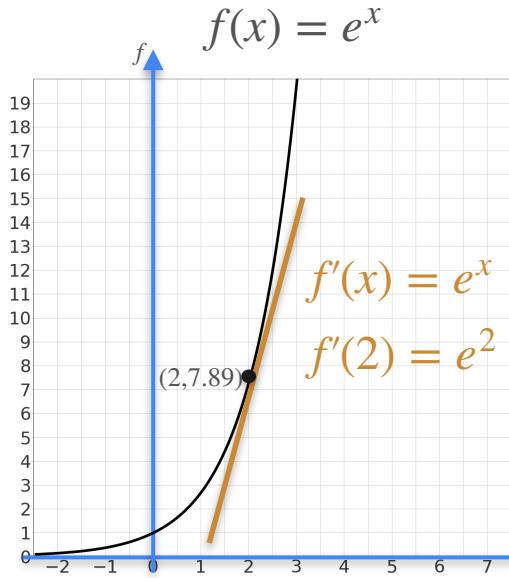


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y) = \frac{1}{e^{\log(y)}}$$

Logarithm

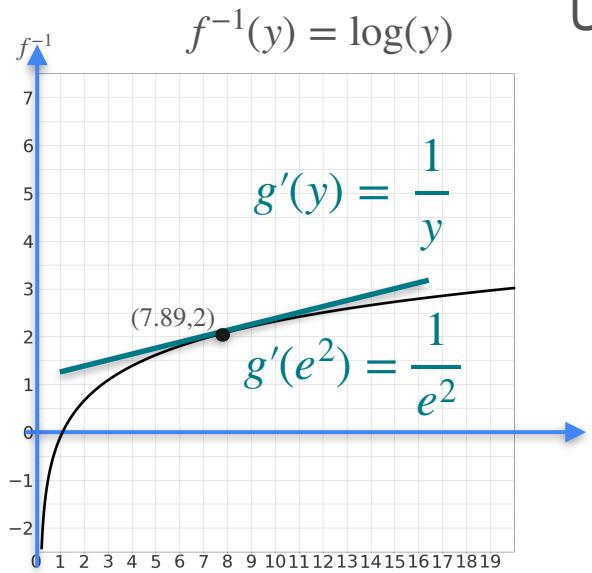
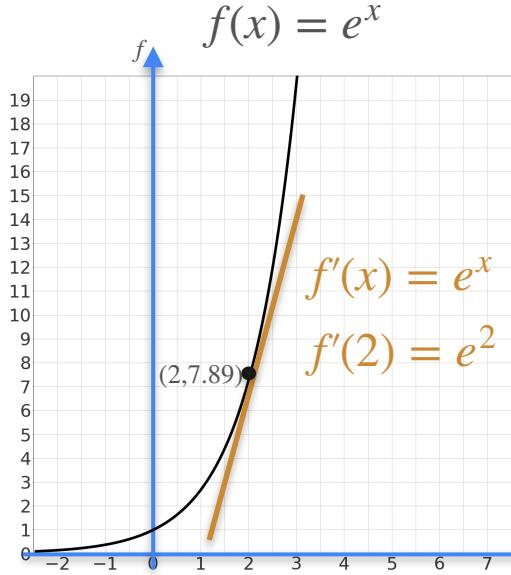


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\begin{aligned}\frac{d}{dy} \log(y) &= \frac{1}{e^{\log(y)}} \\ &= \frac{1}{y}\end{aligned}$$

Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\begin{aligned}\frac{d}{dy} \log(y) &= \frac{1}{e^{\log(y)}} \\ &= \frac{1}{y}\end{aligned}$$

$$\frac{d}{dy} \log(y) = \frac{1}{y}$$



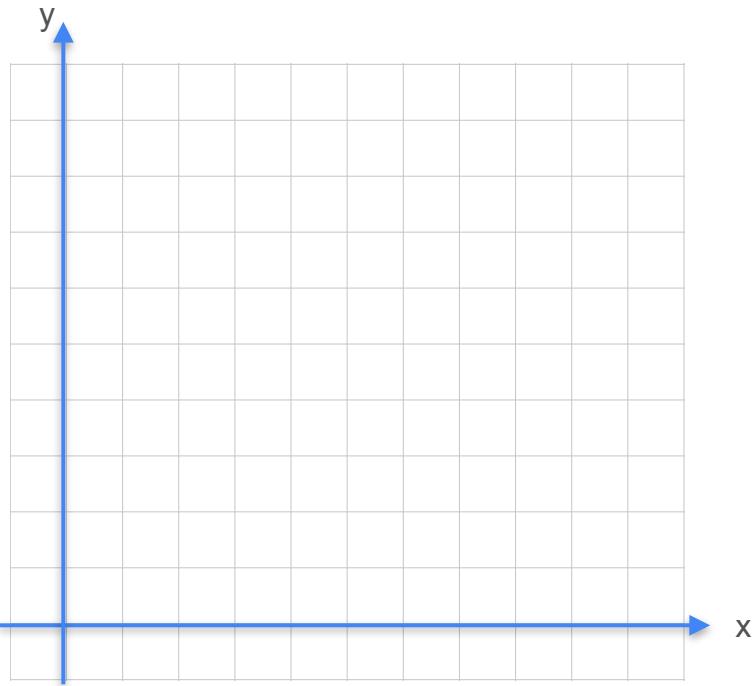
DeepLearning.AI

Derivatives and Optimization

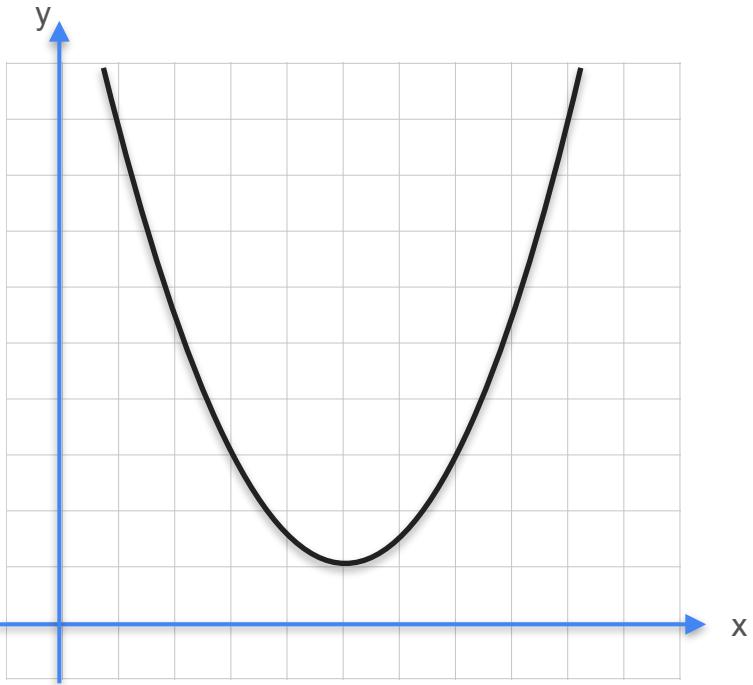
Existence of the derivative

Differentiable Function

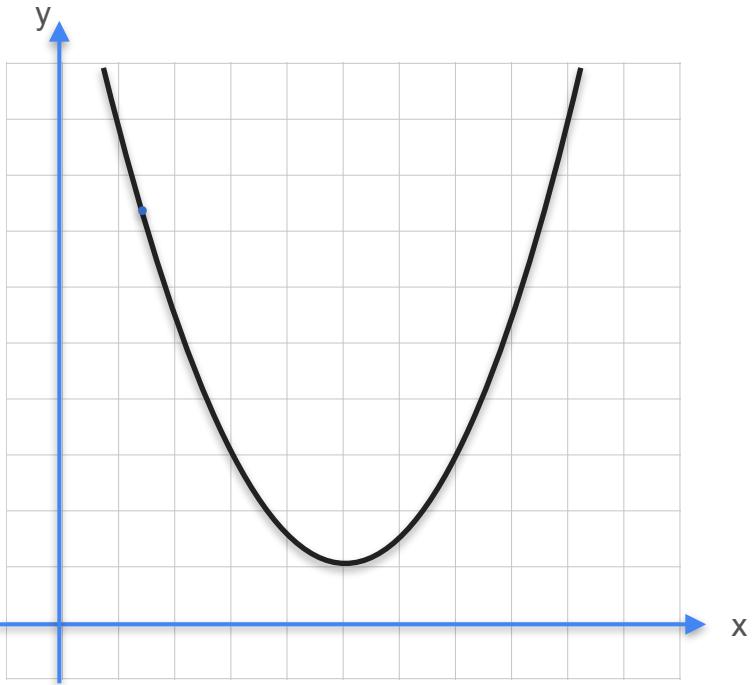
Differentiable Function



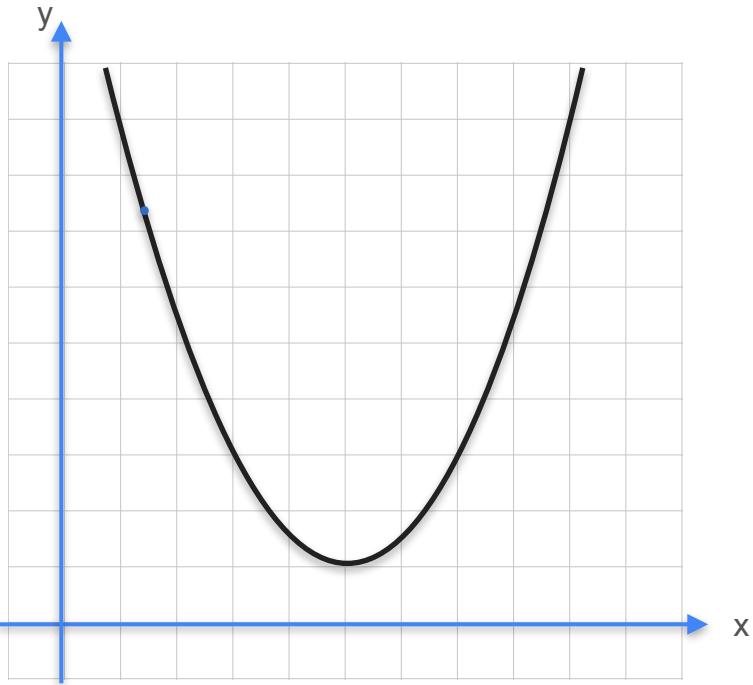
Differentiable Function



Differentiable Function



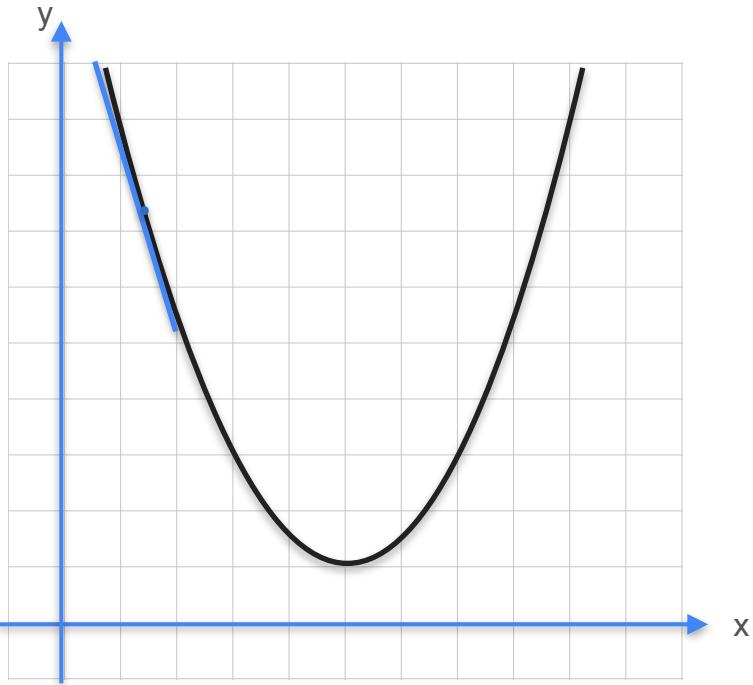
Differentiable Function



For a function to be differentiable at a point:

The derivative has to exist for that point

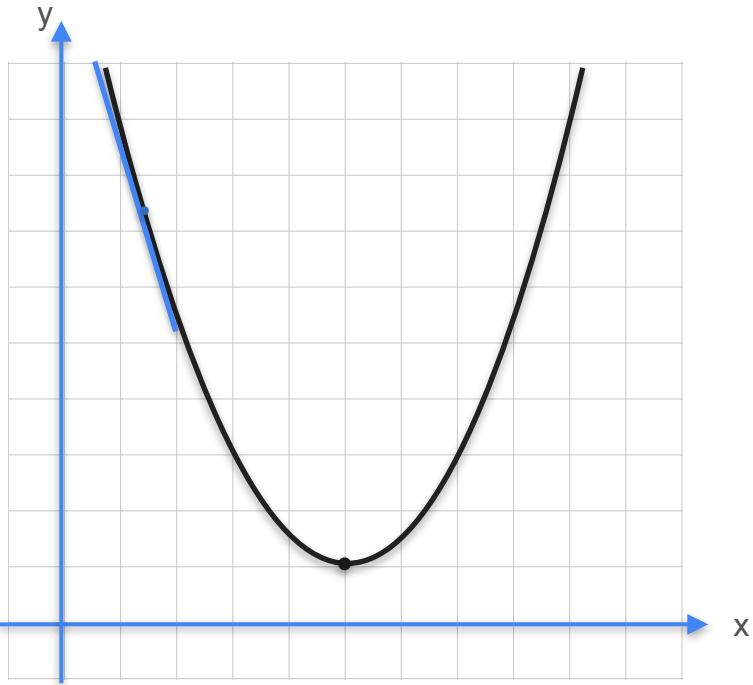
Differentiable Function



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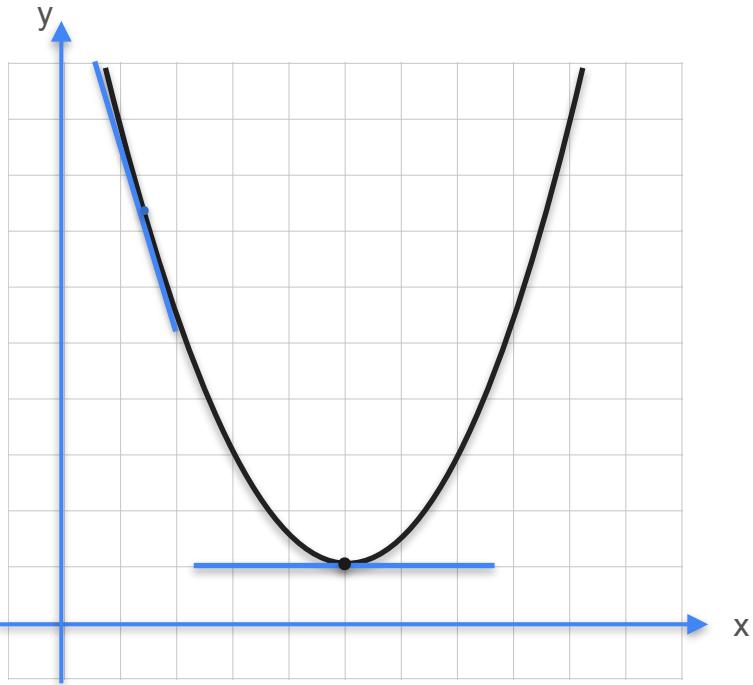
Differentiable Function



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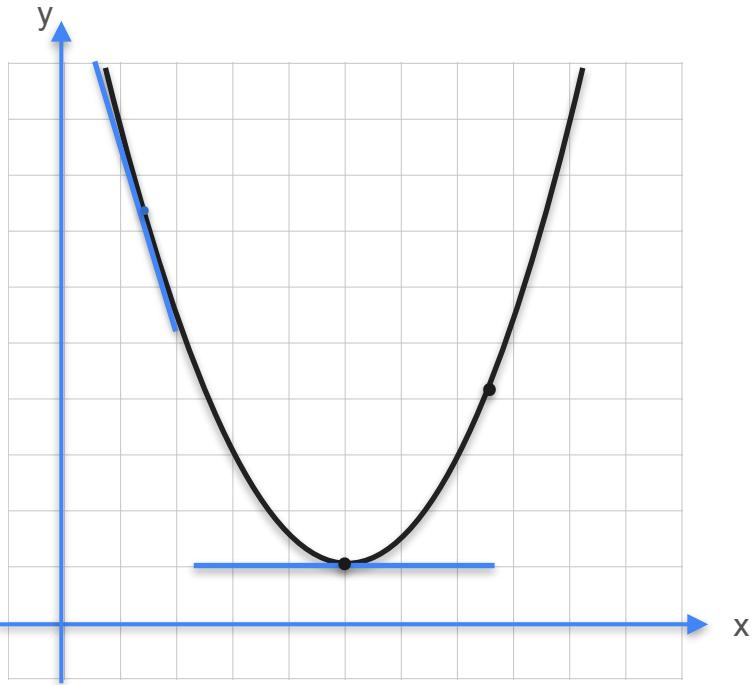
Differentiable Function



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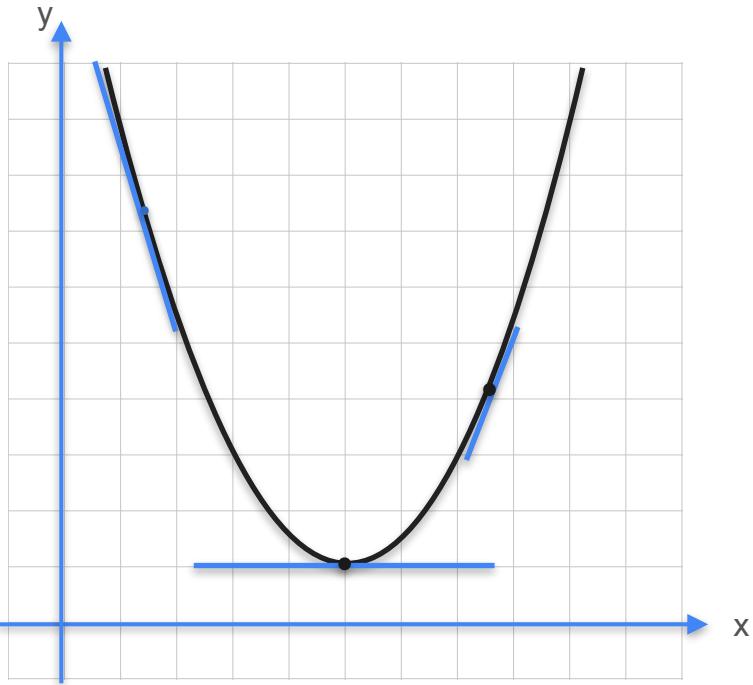
Differentiable Function



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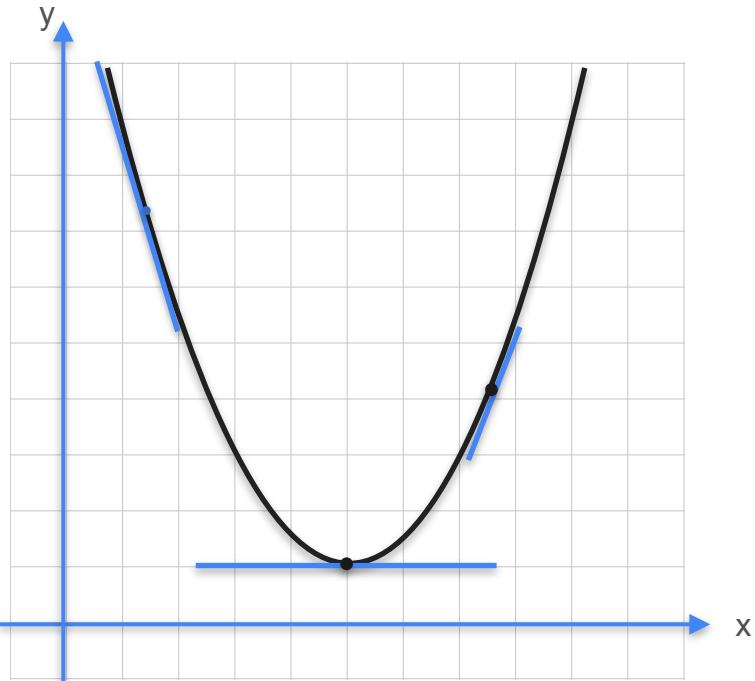
Differentiable Function



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Differentiable Function



For a function to be differentiable at a point:

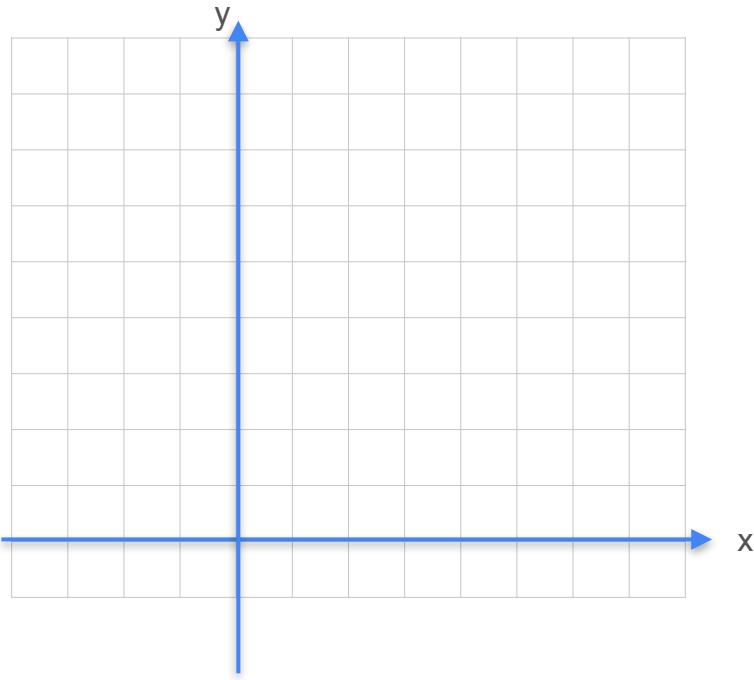
The derivative has to exist for that point

For a function to be differentiable at an interval:

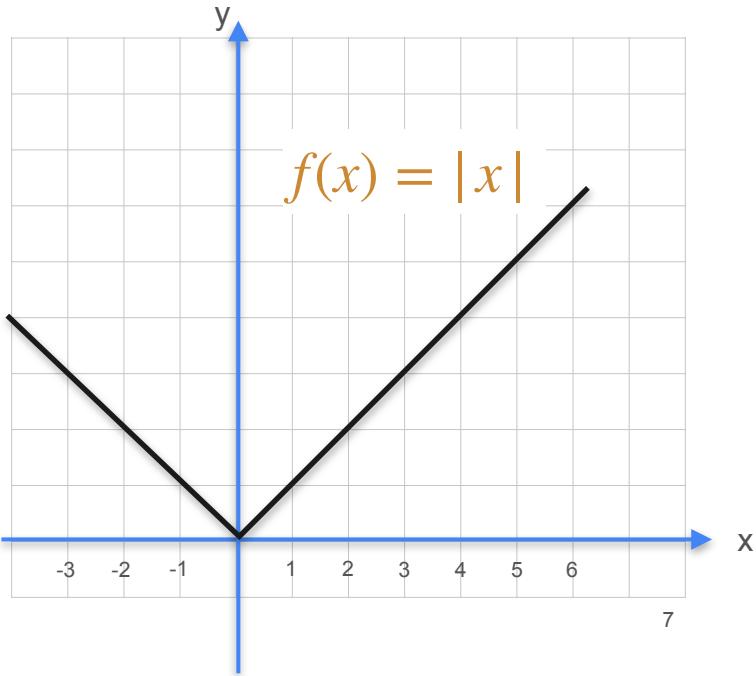
The derivative has to exist for *every* point in the interval

Non Differentiable Functions

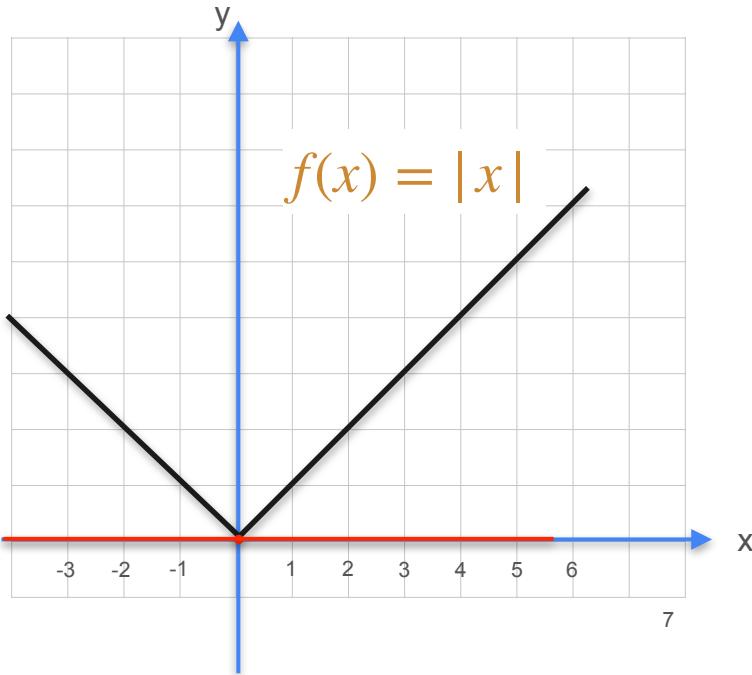
Non Differentiable Functions



Non Differentiable Functions



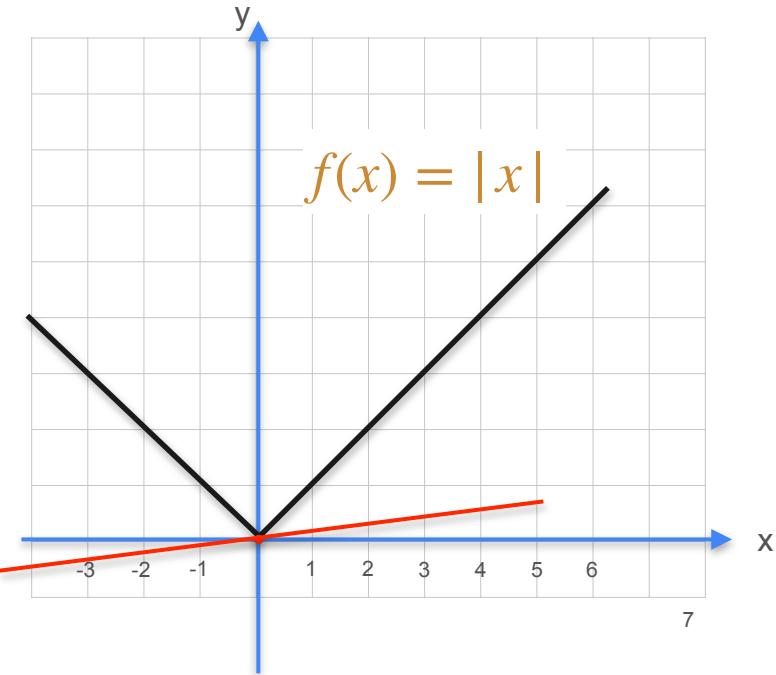
Non Differentiable Functions



The absolute value function.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

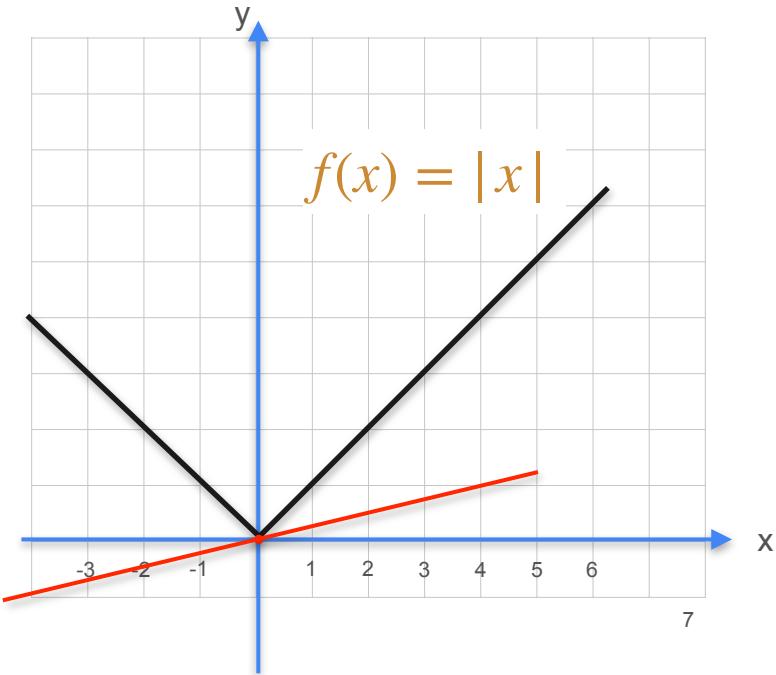
Non Differentiable Functions



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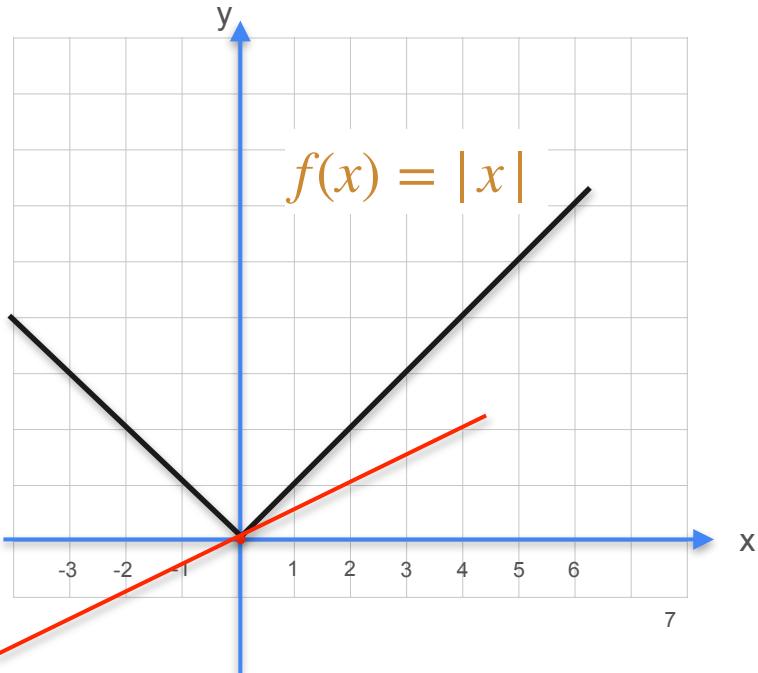
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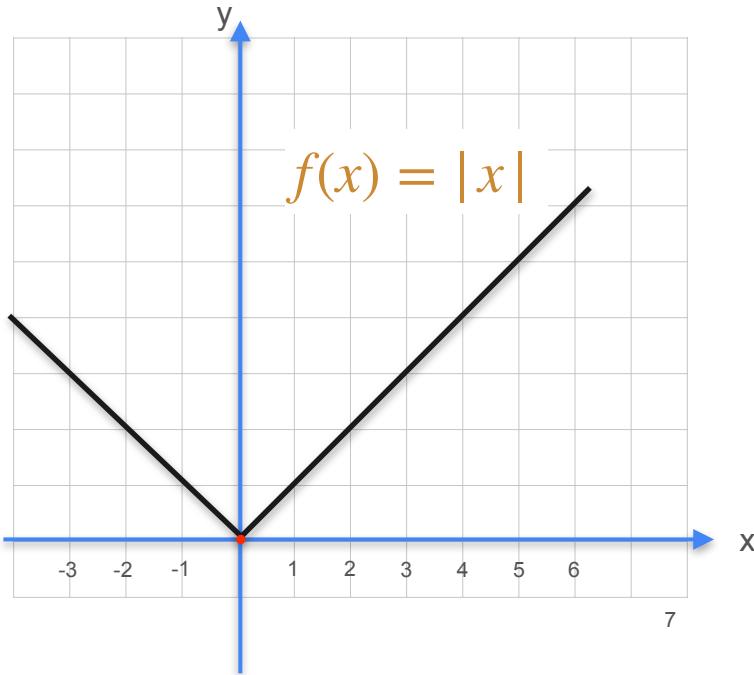
Non Differentiable Functions



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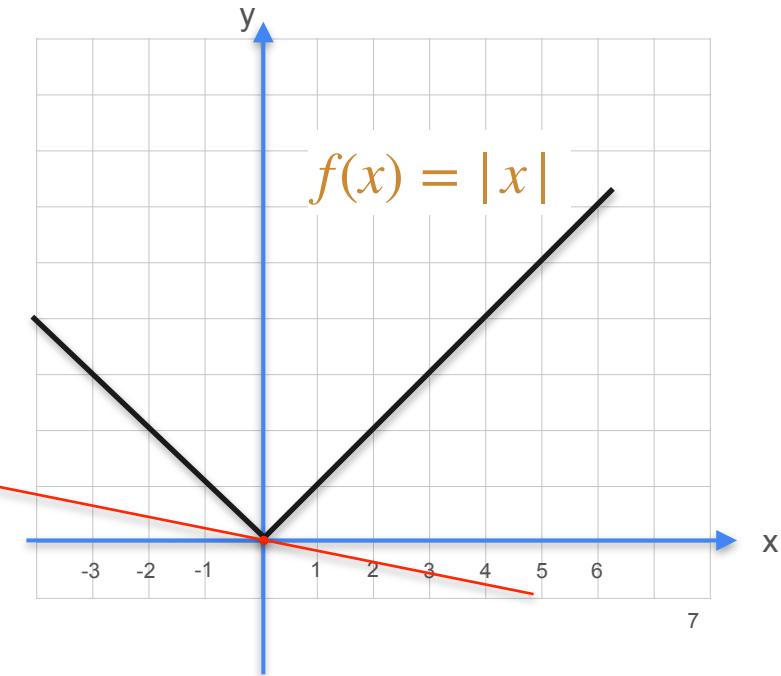
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Non Differentiable Functions



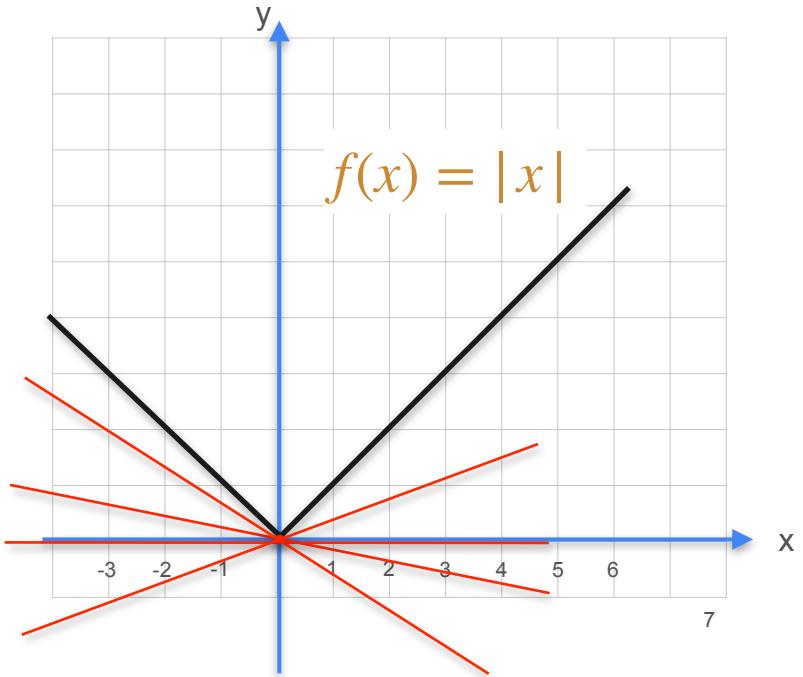
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Non Differentiable Functions



The absolute value function.

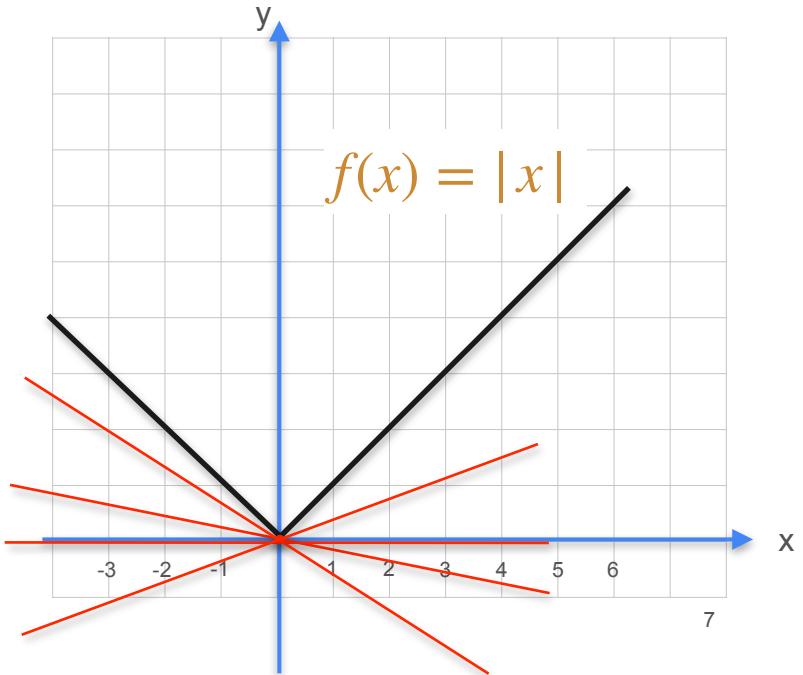
Non Differentiable Functions



The absolute value function.

At $x = 0$, the derivative does not exist

Non Differentiable Functions



The absolute value function.

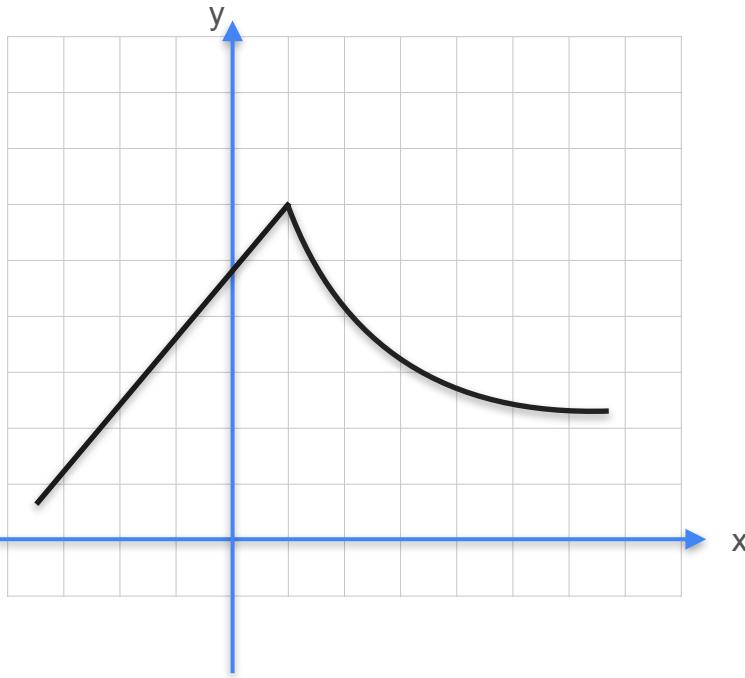
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At $x = 0$, the derivative does not exist

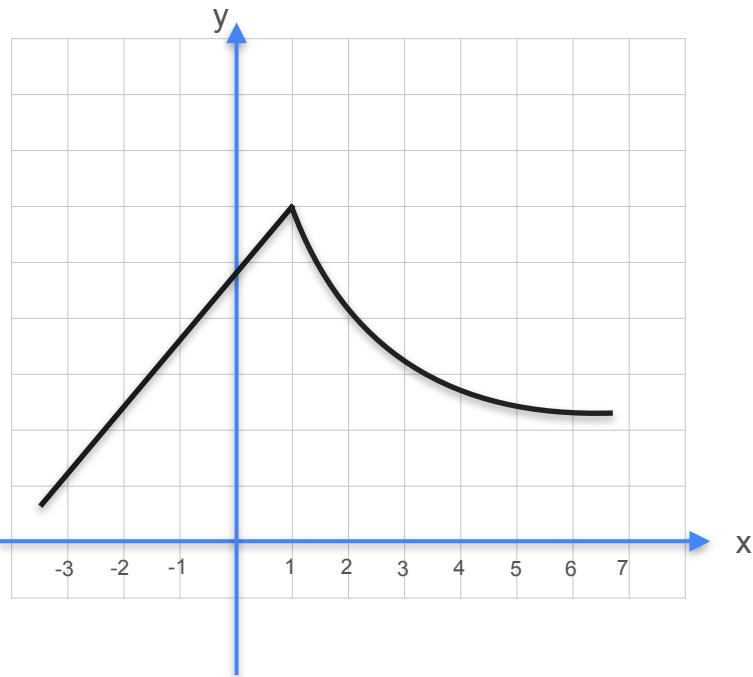
Generally, when a function has a corner or a cusp, the function is not differentiable at that point.

Non Differentiable Functions - Quiz 1

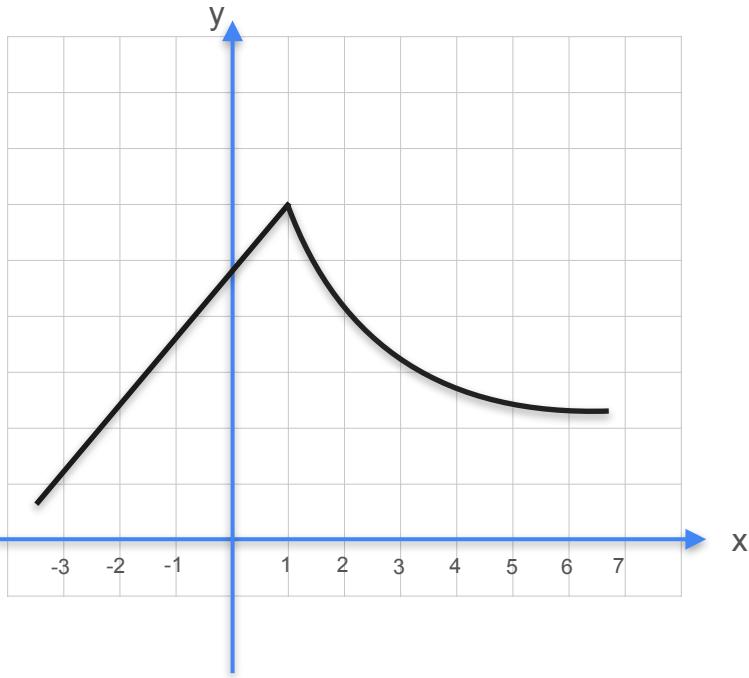
Non Differentiable Functions - Quiz 1



Non Differentiable Functions - Quiz 1

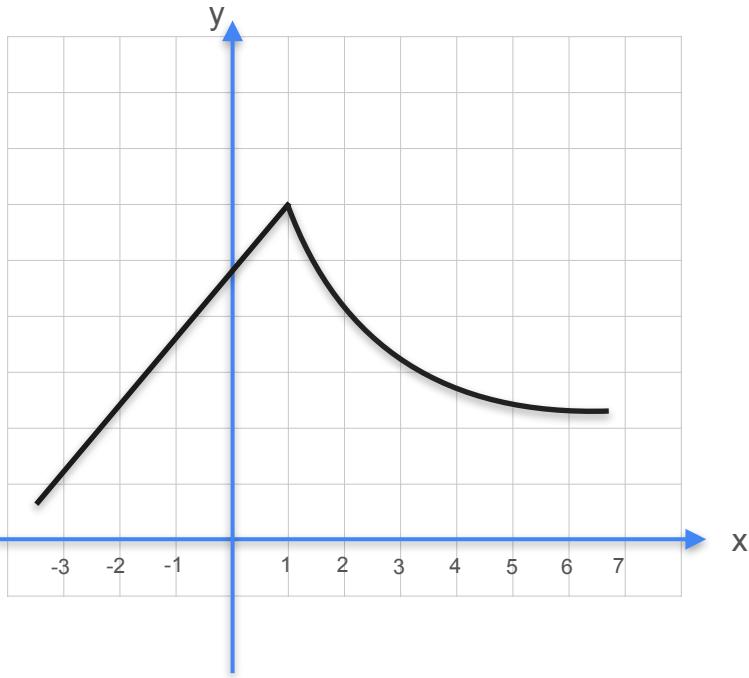


Non Differentiable Functions - Quiz 1



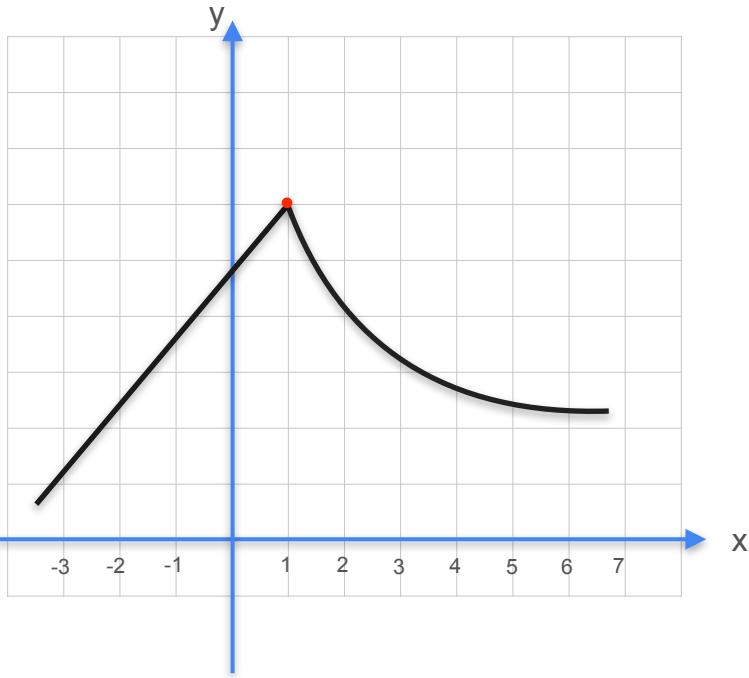
At which point in this function does the derivative not exist?

Non Differentiable Functions - Quiz 1



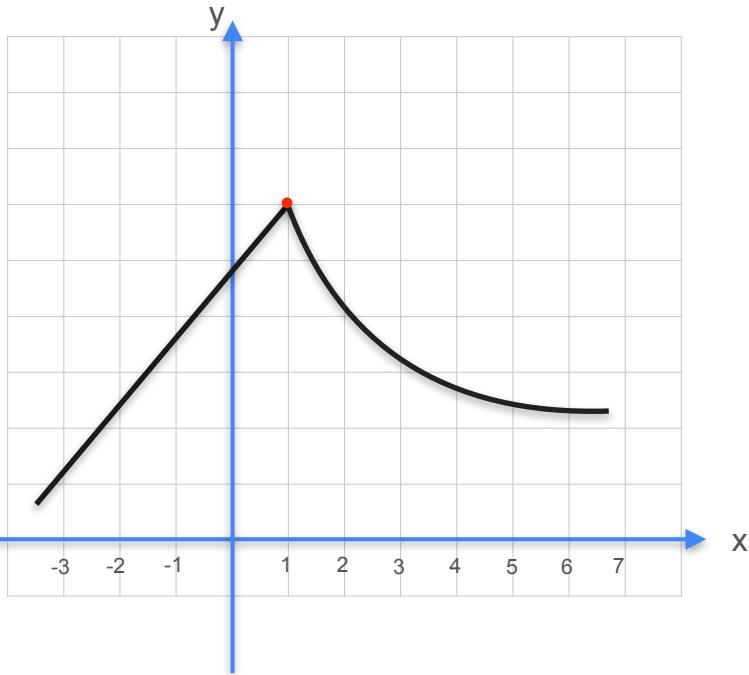
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Non Differentiable Functions - Quiz 1



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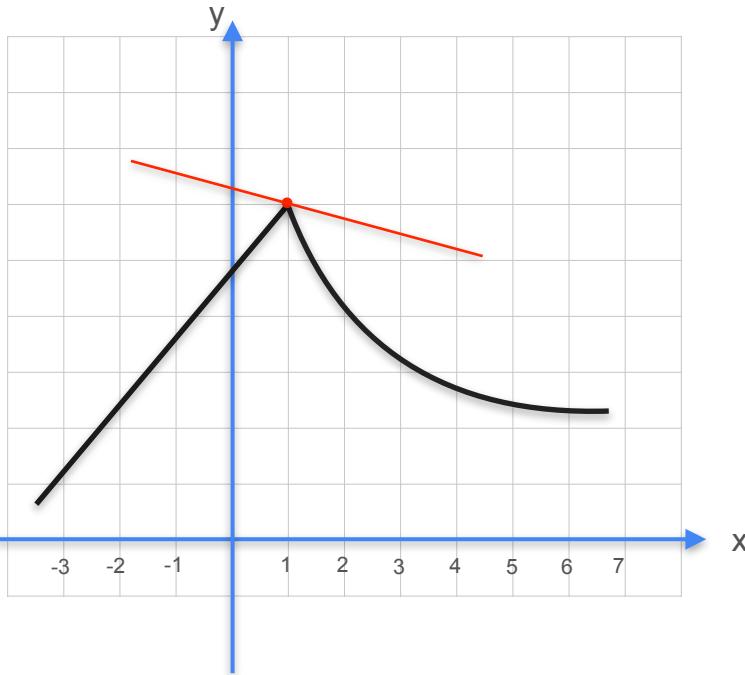
Non Differentiable Functions - Quiz 1



At which point in this function does the derivative not exist?

The entire function is non-differentiable because a derivative does not exist for all points in the domain.

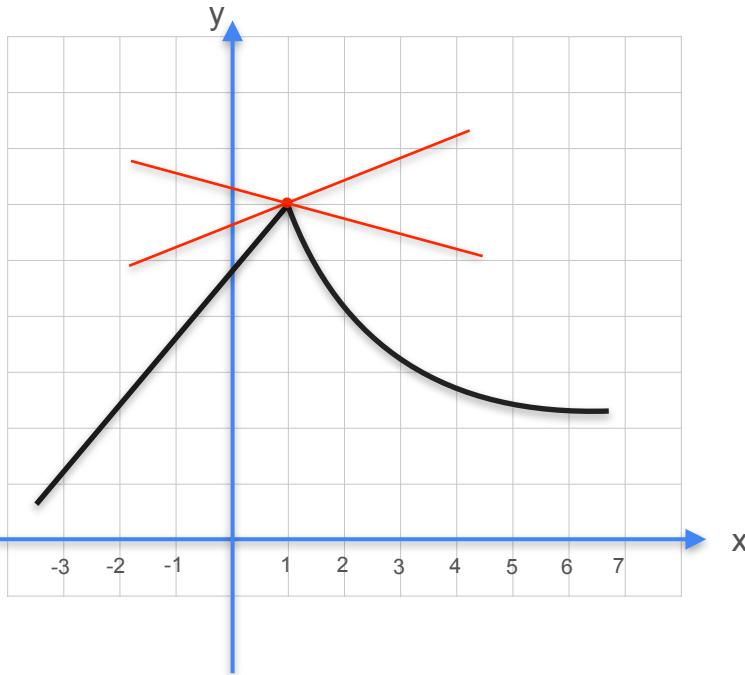
Non Differentiable Functions - Quiz 1



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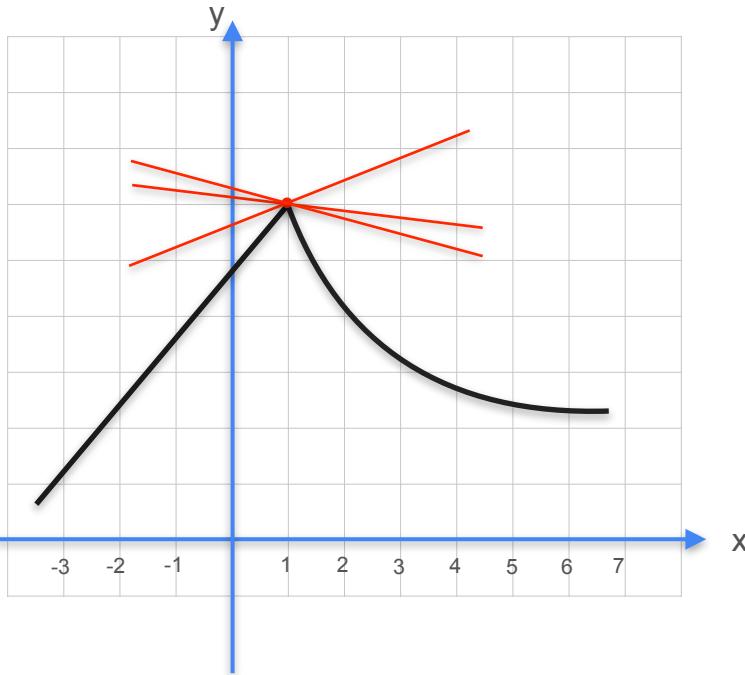
Non Differentiable Functions - Quiz 1



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Non Differentiable Functions - Quiz 1

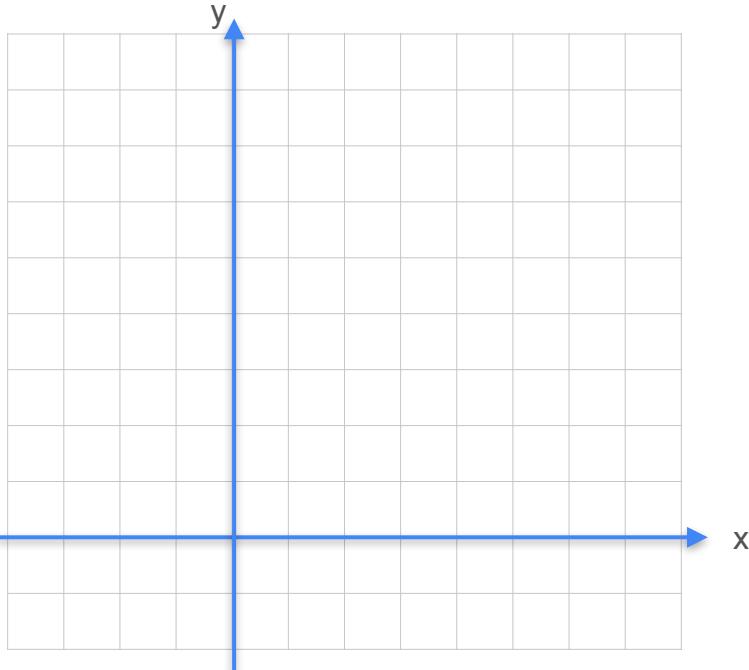


At which point in this function does the derivative not exist?

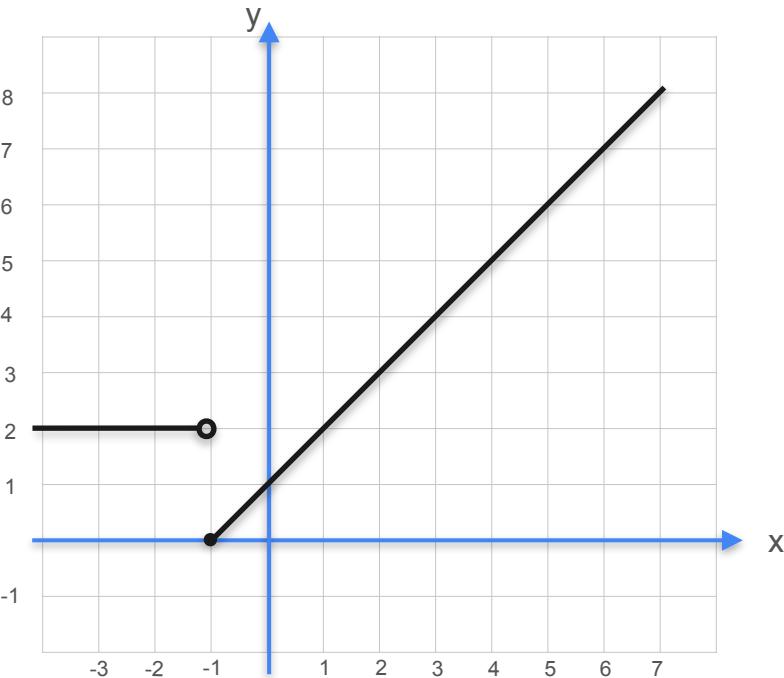
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Non Differentiable Functions - Quiz 2

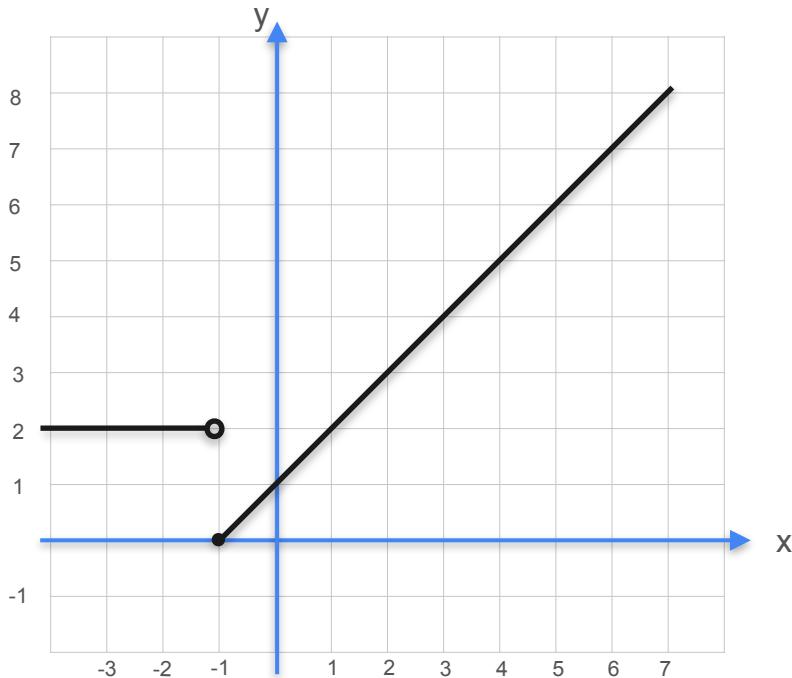
Non Differentiable Functions - Quiz 2



Non Differentiable Functions - Quiz 2

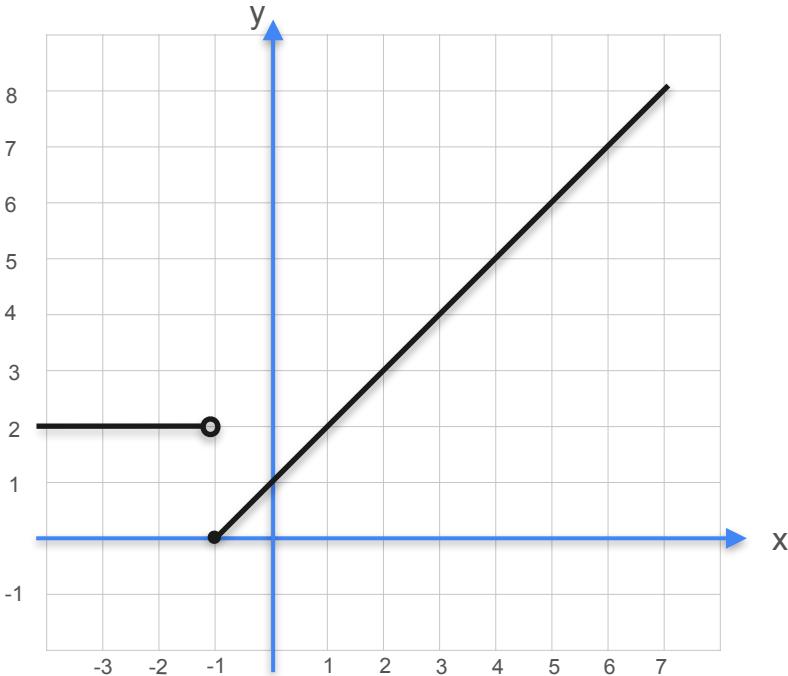


Non Differentiable Functions - Quiz 2

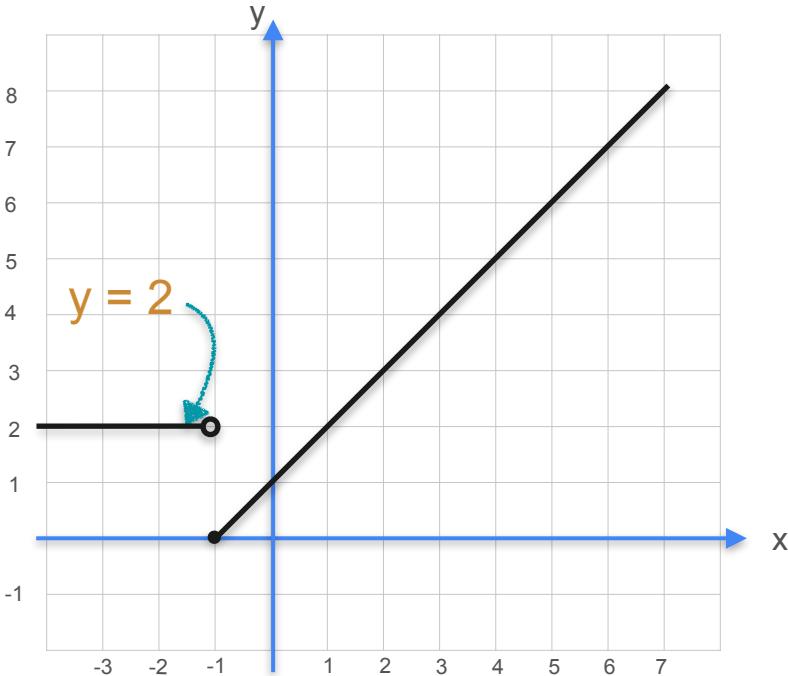


At which point in this function does the derivative not exist?

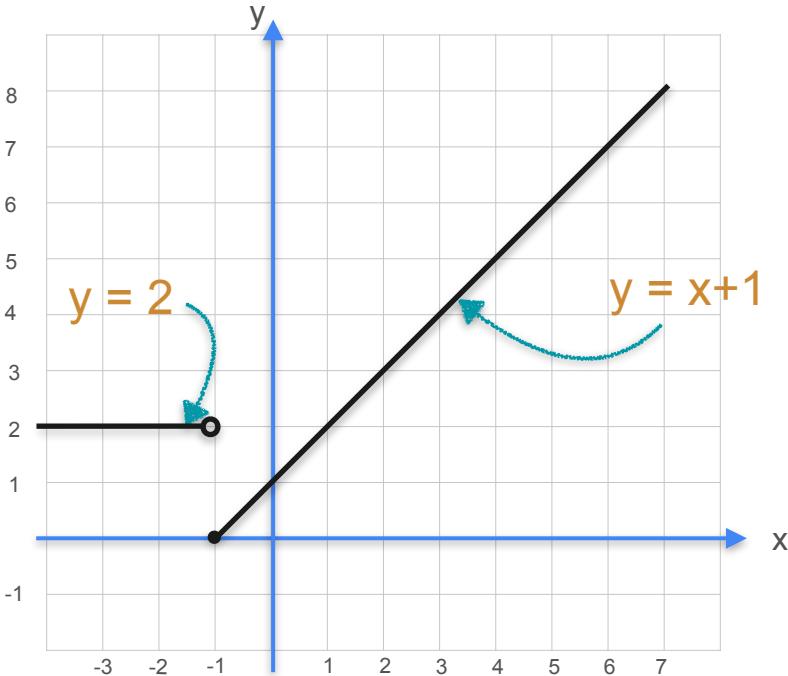
Non Differentiable Functions



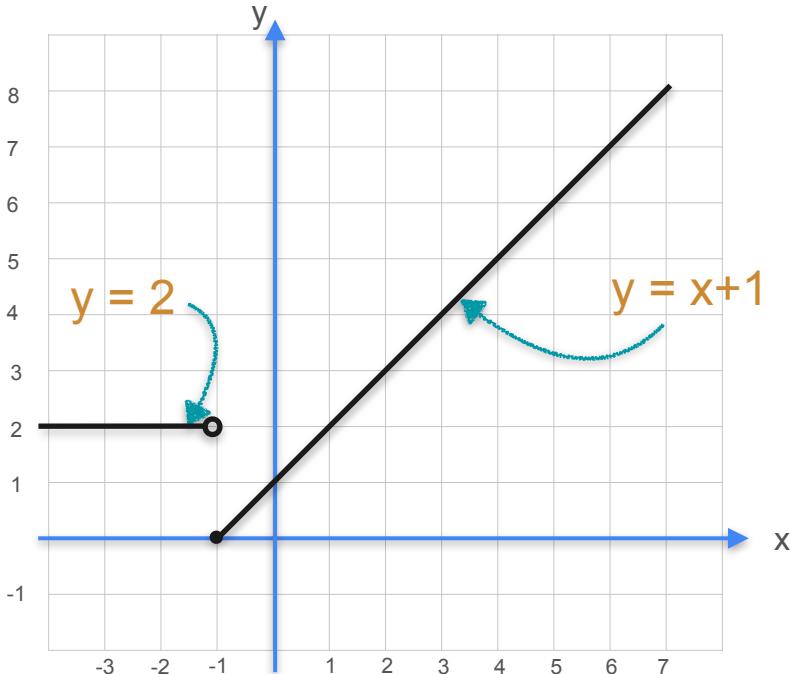
Non Differentiable Functions



Non Differentiable Functions



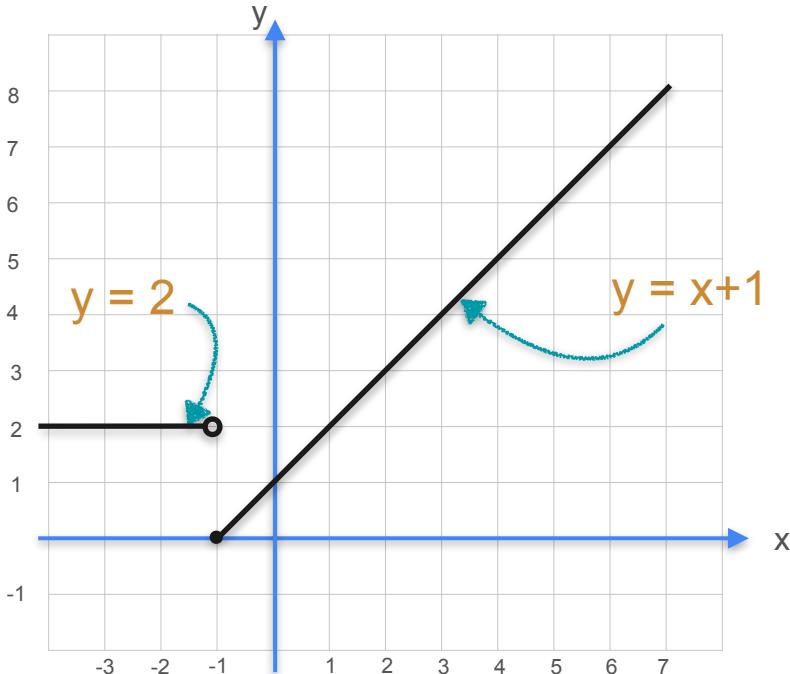
Non Differentiable Functions



This is a piece-wise function.

$$f(x) = \begin{cases} 2, & \text{if } x < -1 \\ x + 1, & \text{if } x \geq -1 \end{cases}$$

Non Differentiable Functions

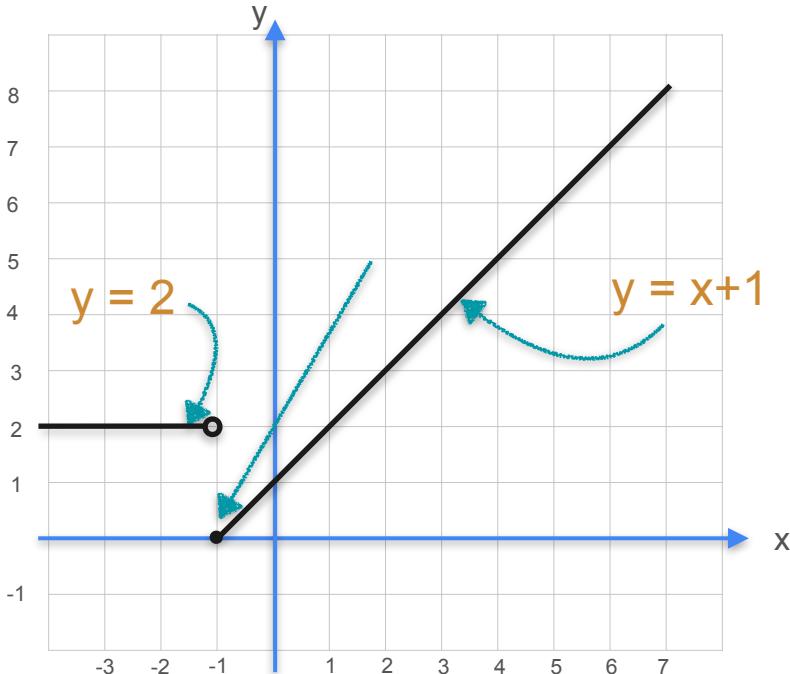


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Jump Discontinuity

Non Differentiable Functions

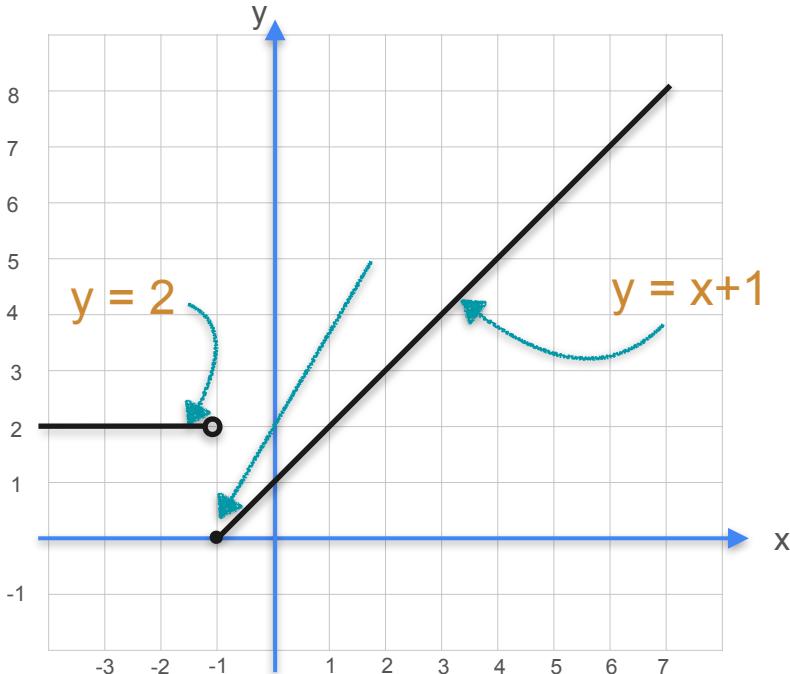


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Non Differentiable Functions



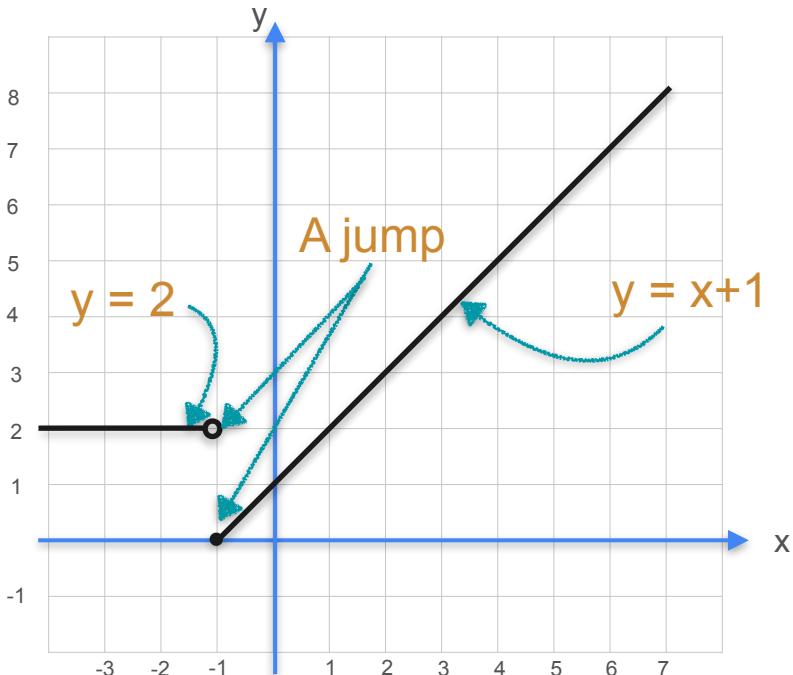
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Jump Discontinuity

The graph of the function does not appear to be continuous

Non Differentiable Functions



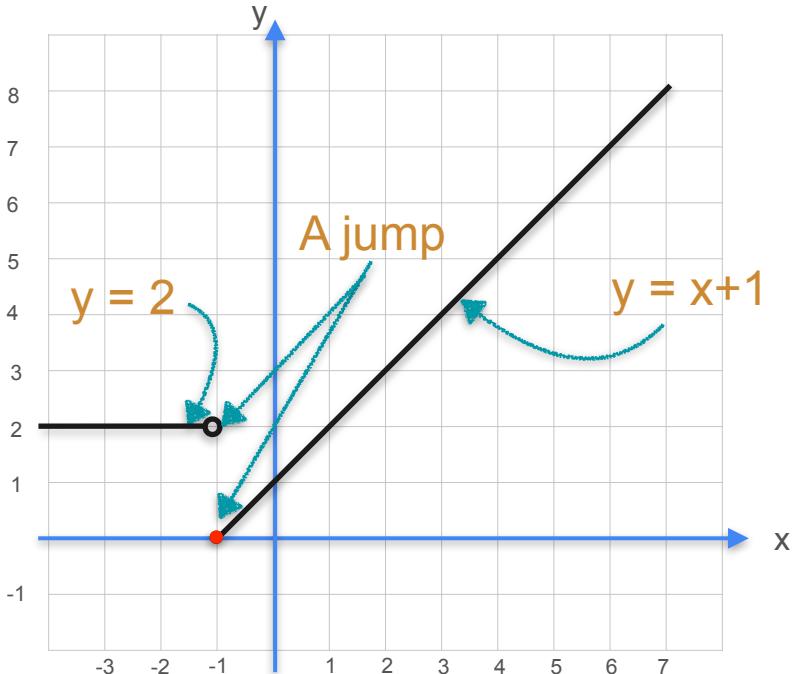
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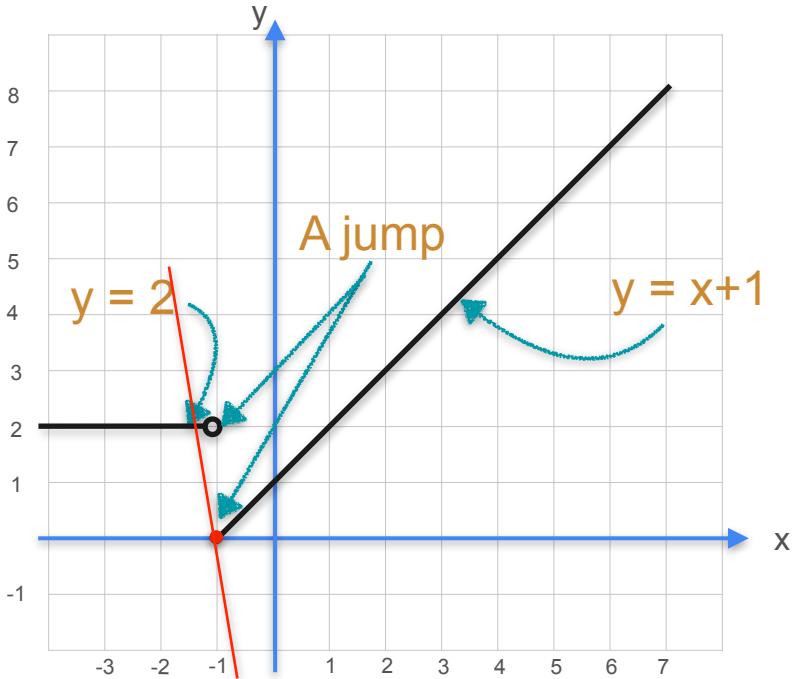
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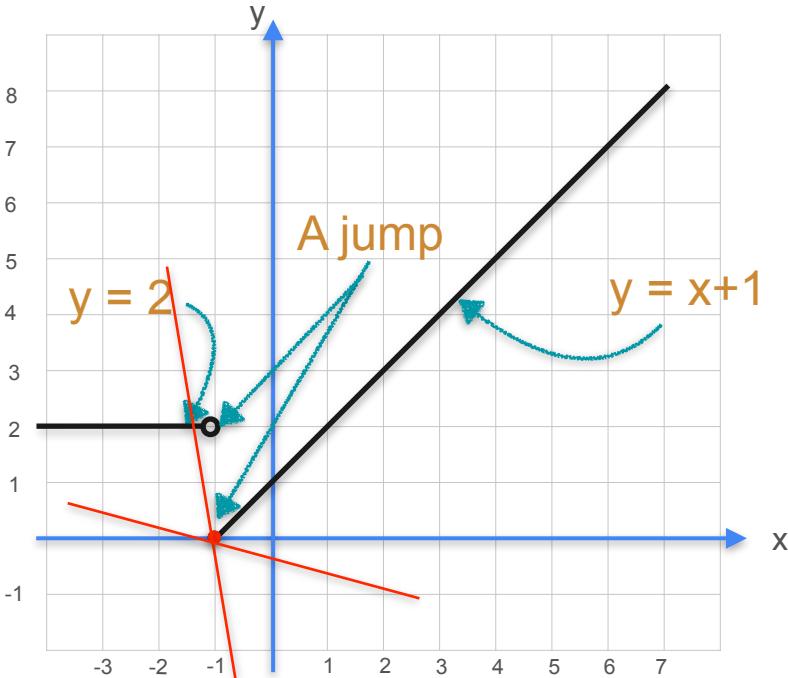
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Non Differentiable Functions



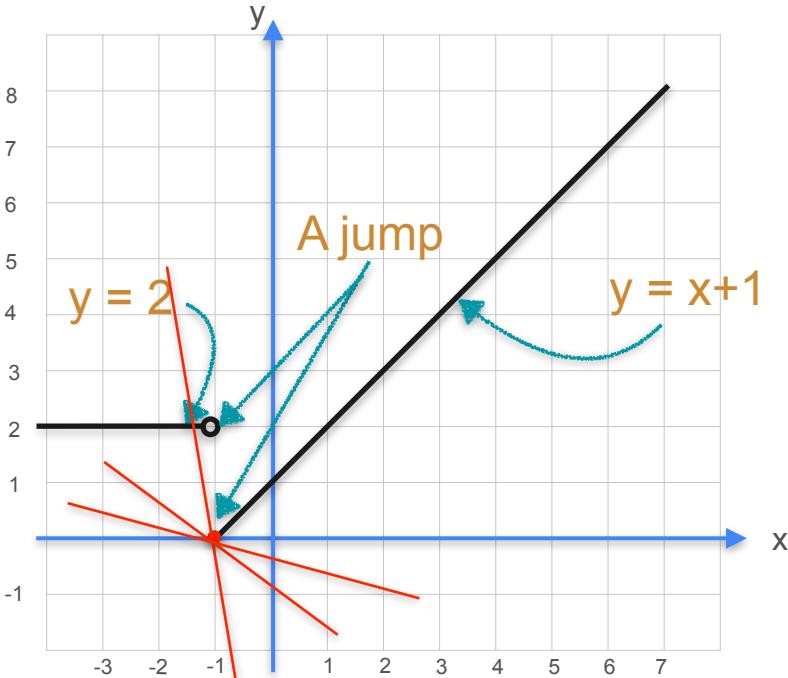
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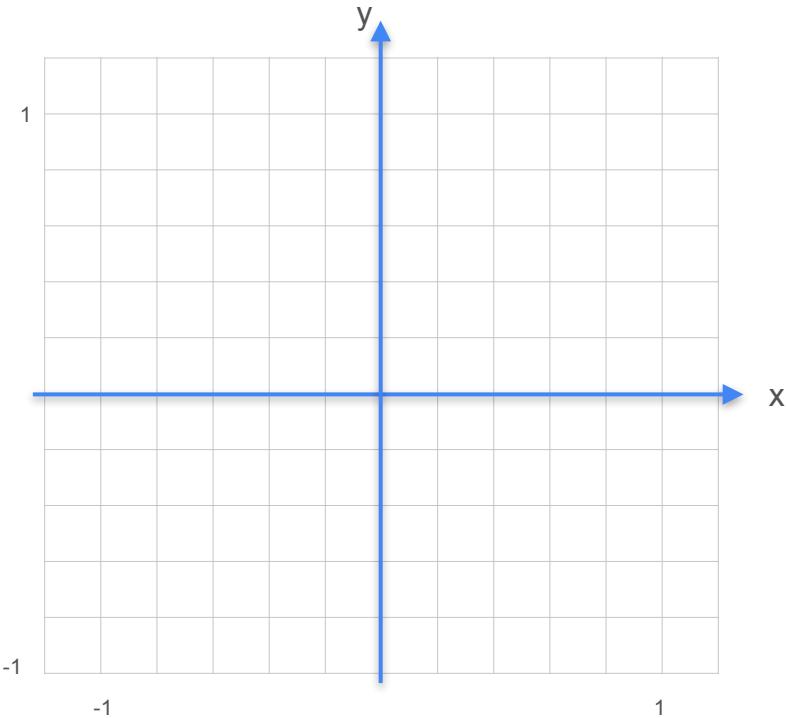
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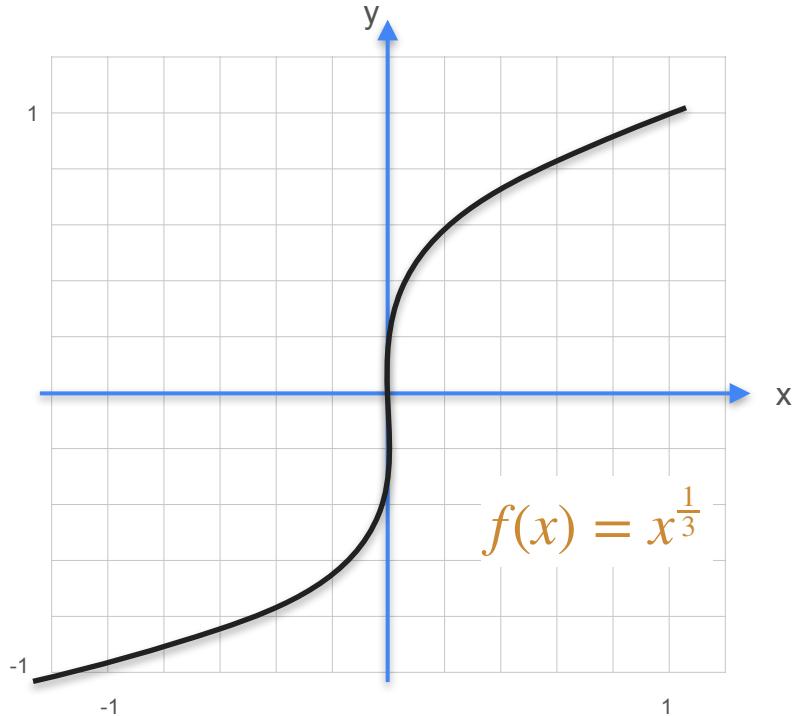
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Non Differentiable Functions

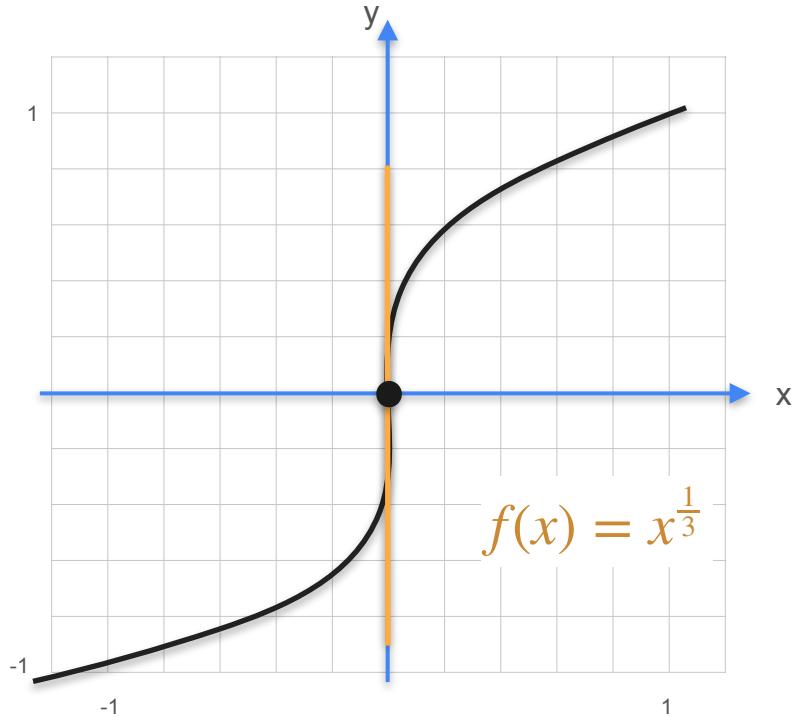
Non Differentiable Functions



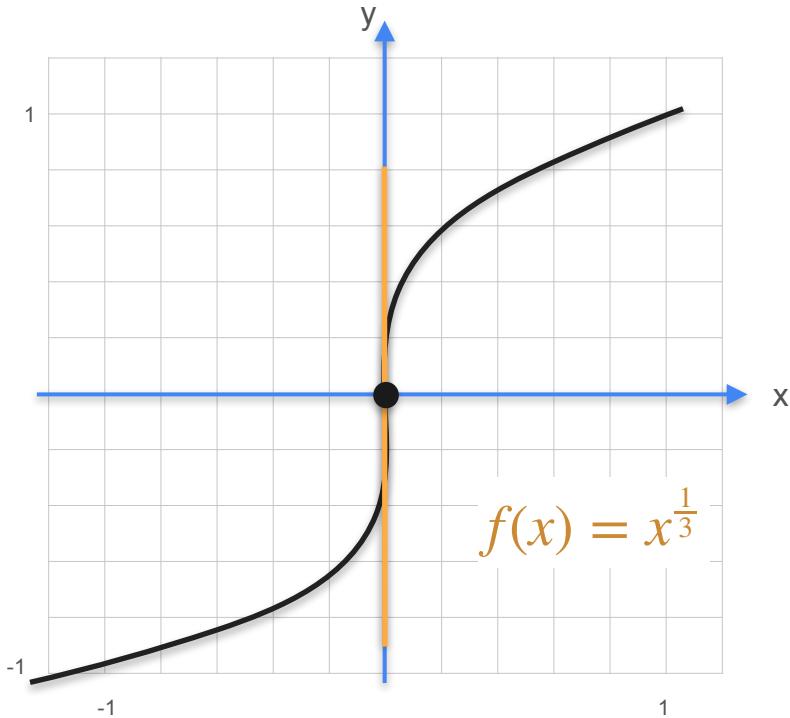
Non Differentiable Functions



Non Differentiable Functions

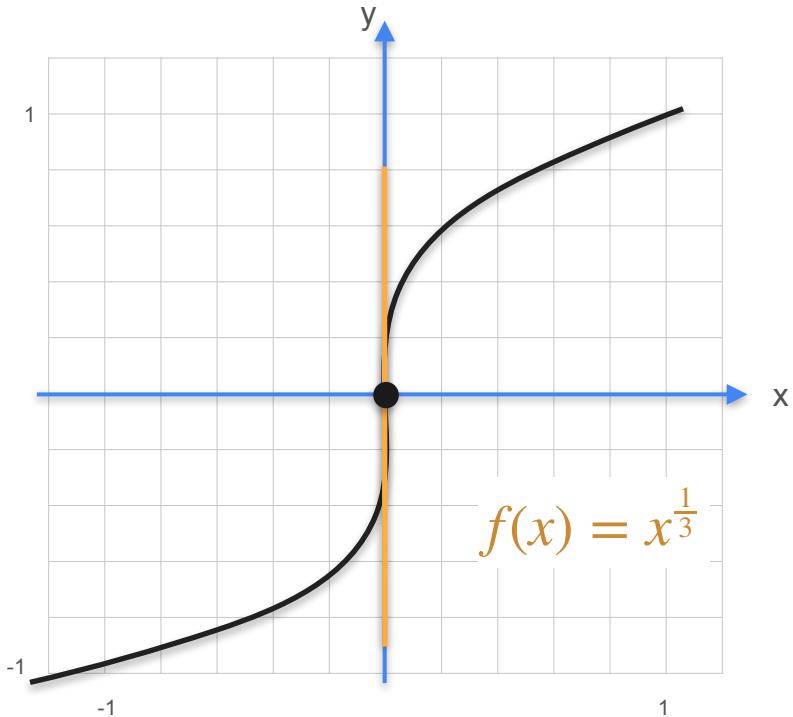


Non Differentiable Functions



Vertical tangents

Non Differentiable Functions

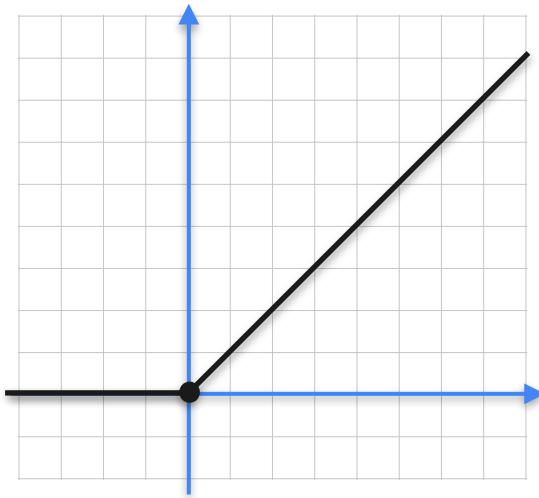


Vertical tangents

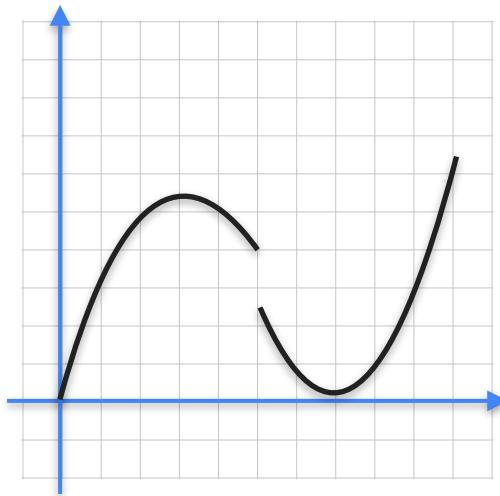
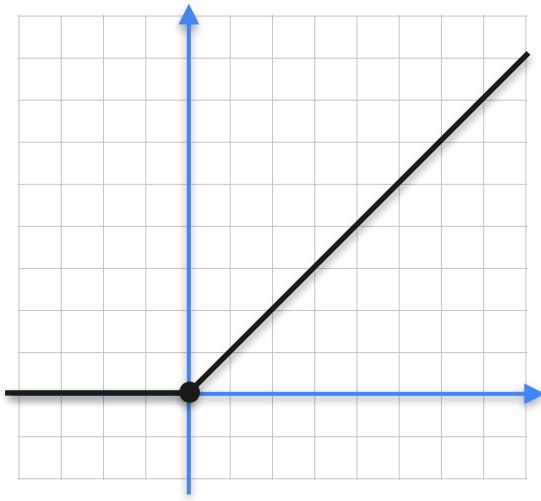
At $x = 0$, this graph has a tangent line that runs straight up parallel to the y-axis

Recap: Non-Differentiable Functions

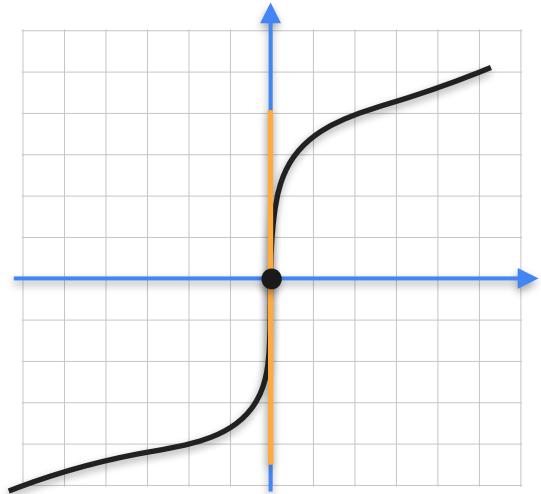
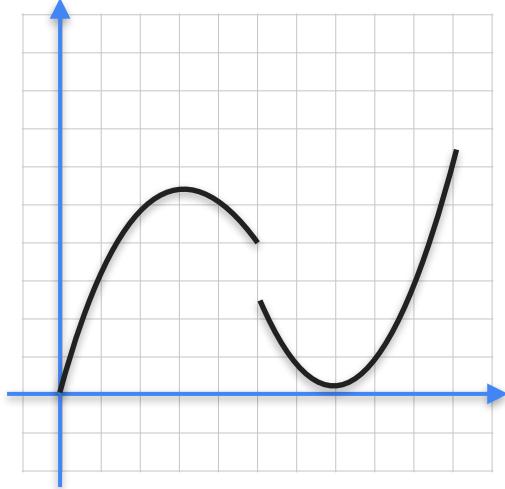
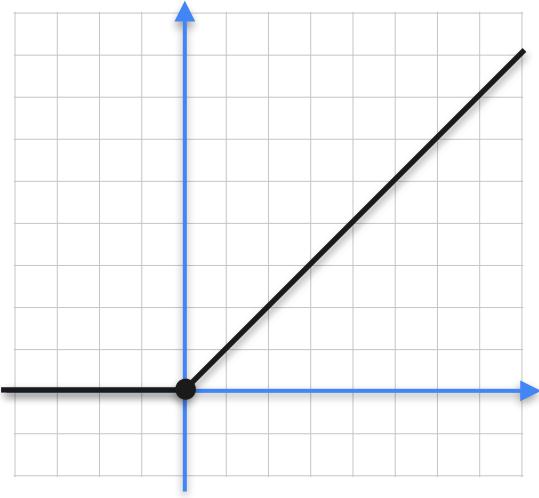
Recap: Non-Differentiable Functions



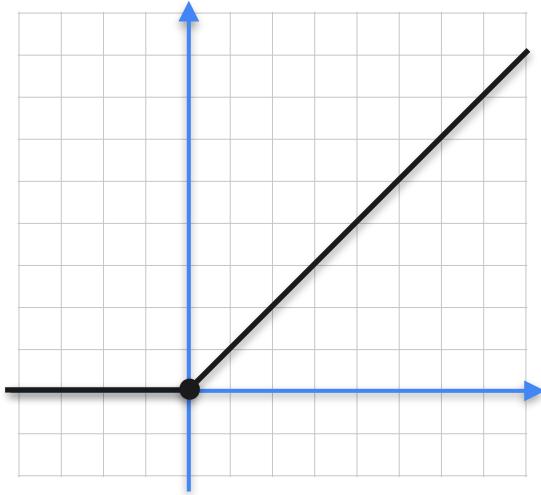
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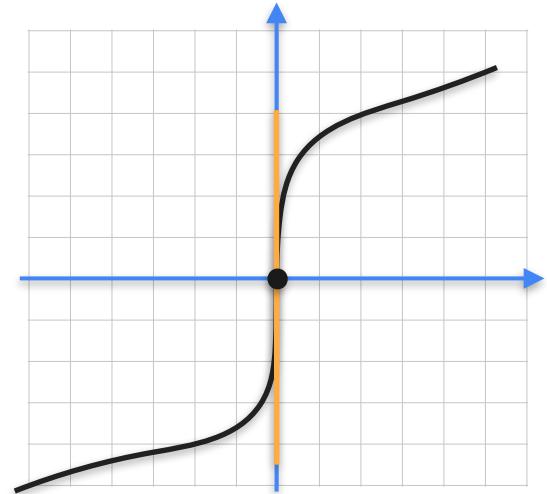
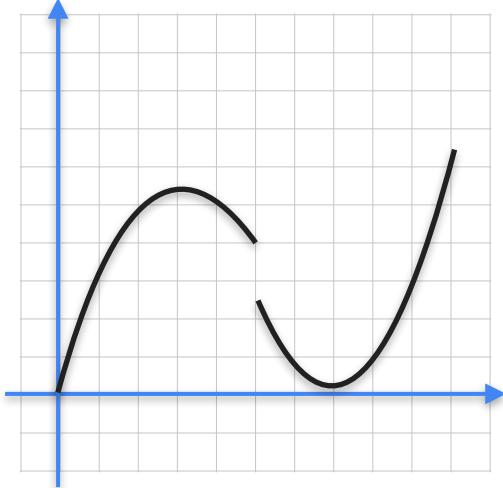
Recap: Non-Differentiable Functions



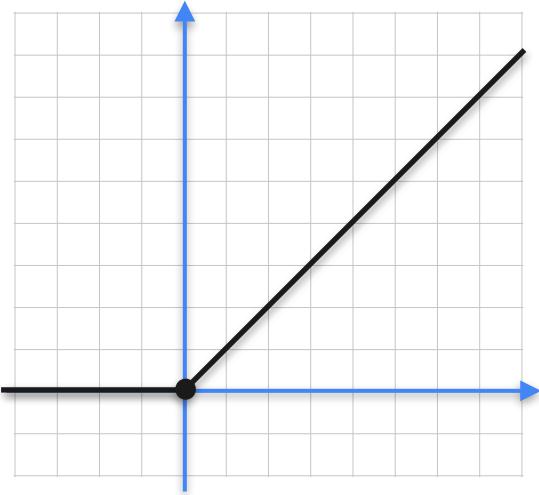
Recap: Non-Differentiable Functions



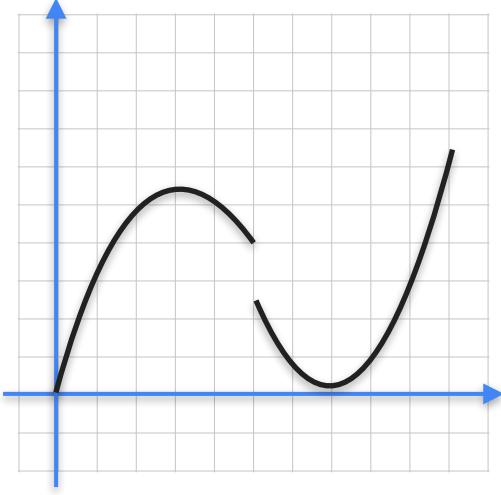
Corners/Cusps



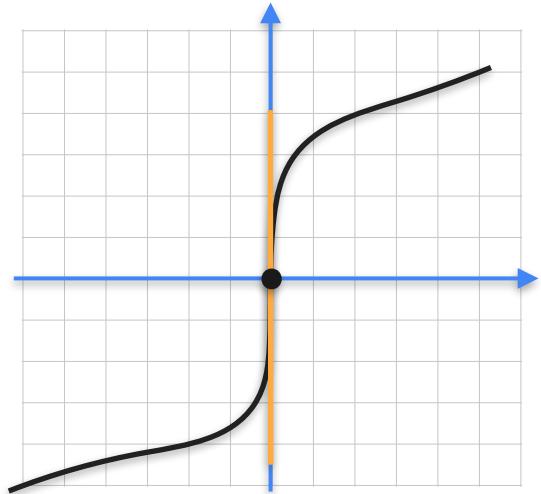
Recap: Non-Differentiable Functions



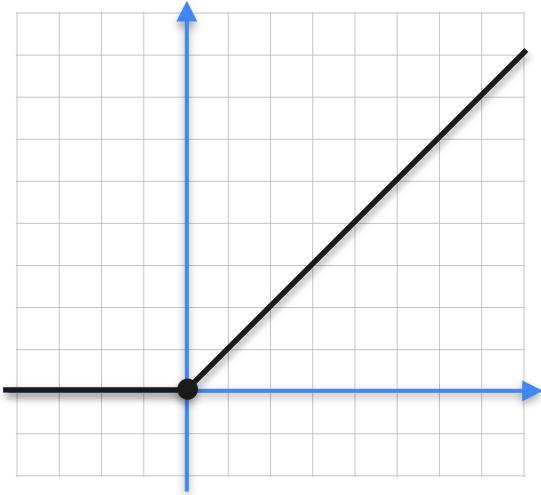
Corners/Cusps



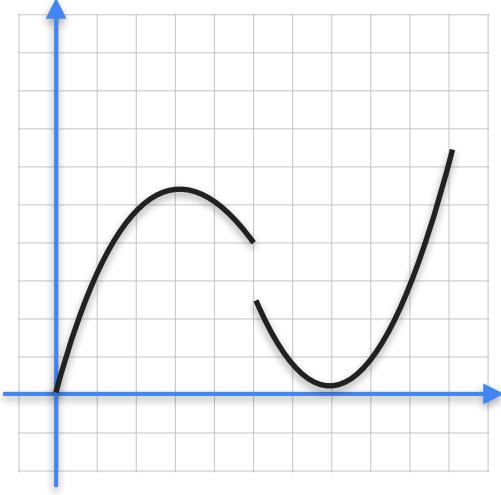
Jump Discontinuity



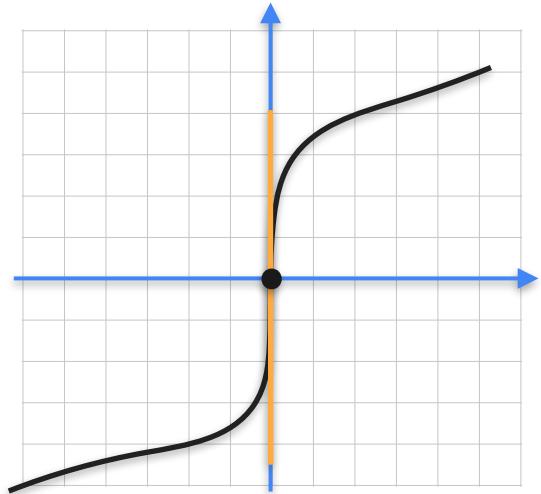
Recap: Non-Differentiable Functions



Corners/Cusps



Jump Discontinuity



Vertical tangents



DeepLearning.AI

Derivatives and Optimization

**Properties of the derivative:
Multiplication by scalars**

The Sum Rule

$$f = 4g$$

The Sum Rule

$$f' = 4g$$

The Sum Rule

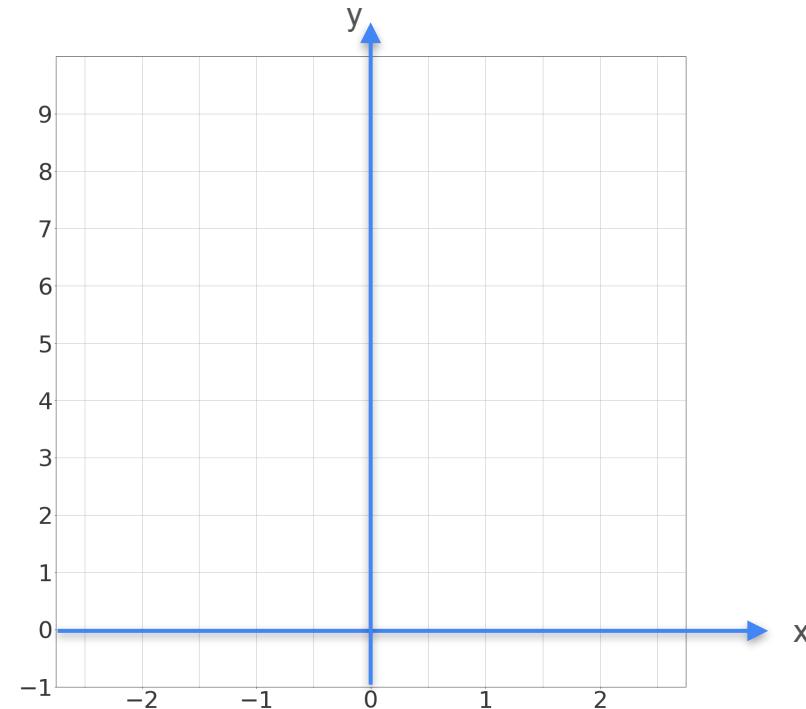
$$f' = 4g'$$

The Sum Rule

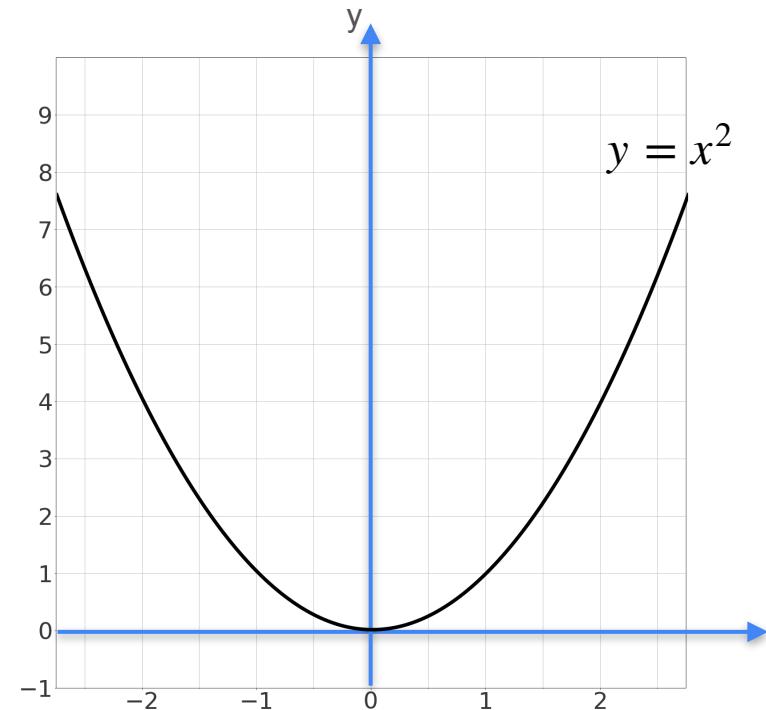
$$f' = c g'$$

Multiplication by a Scalar

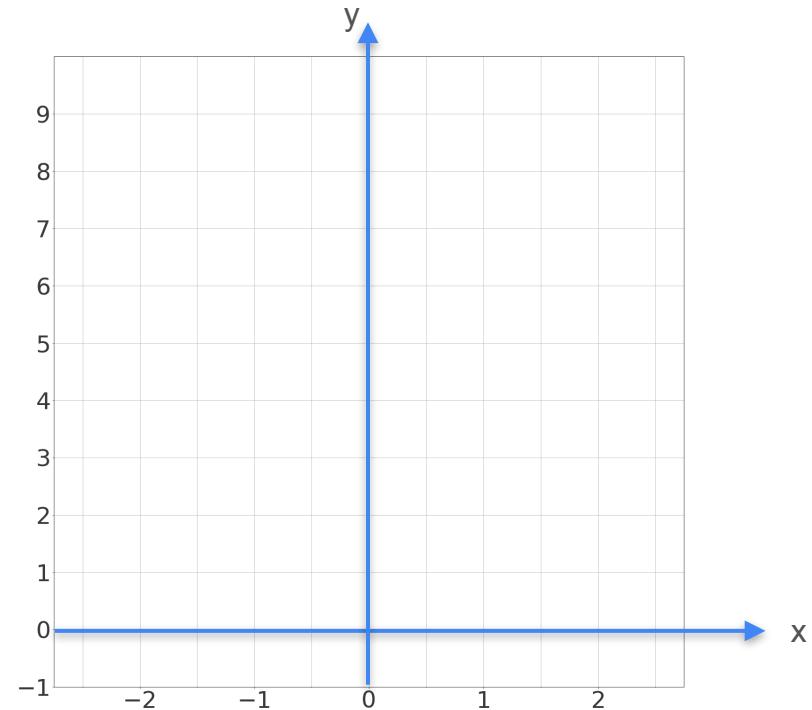
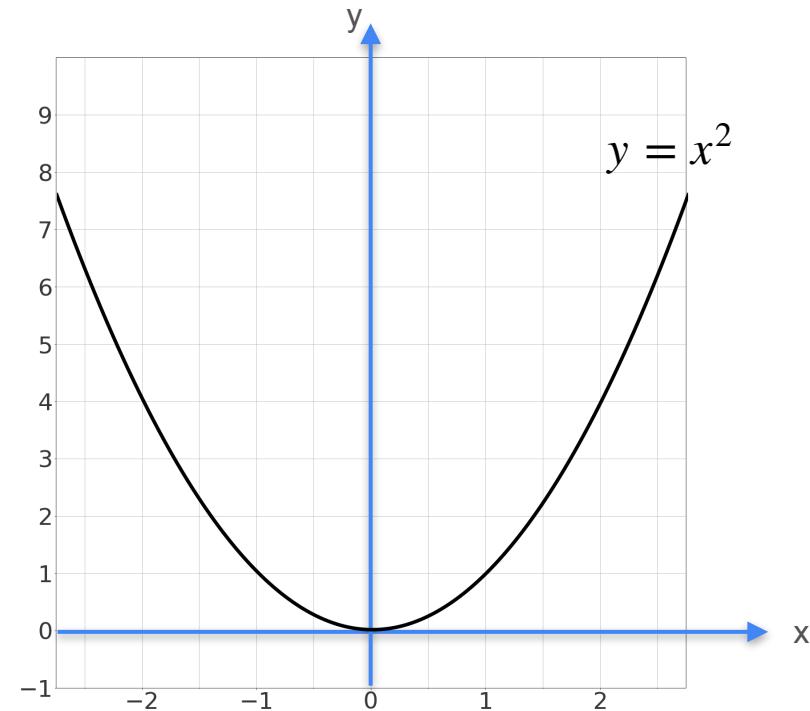
Multiplication by a Scalar



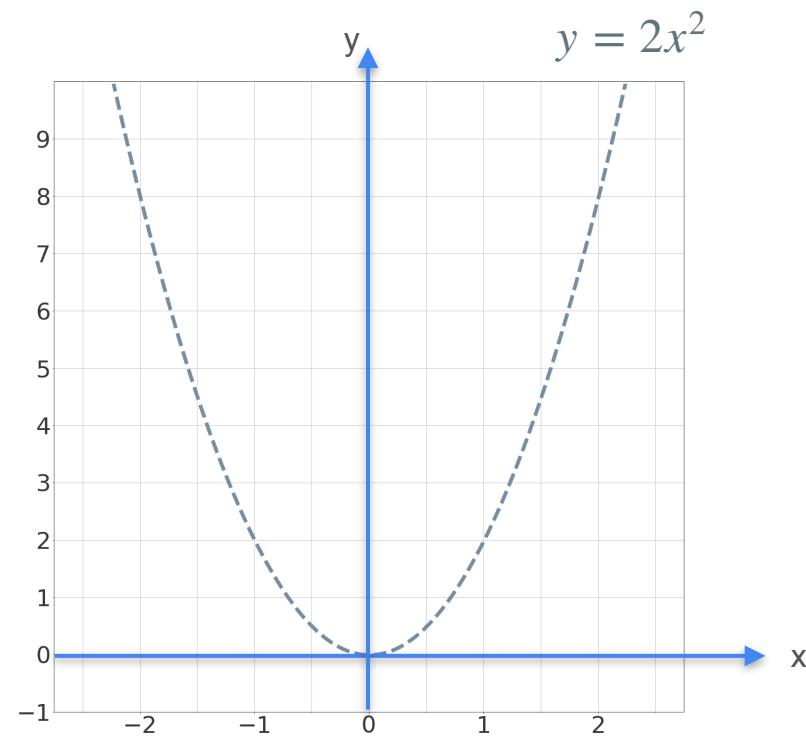
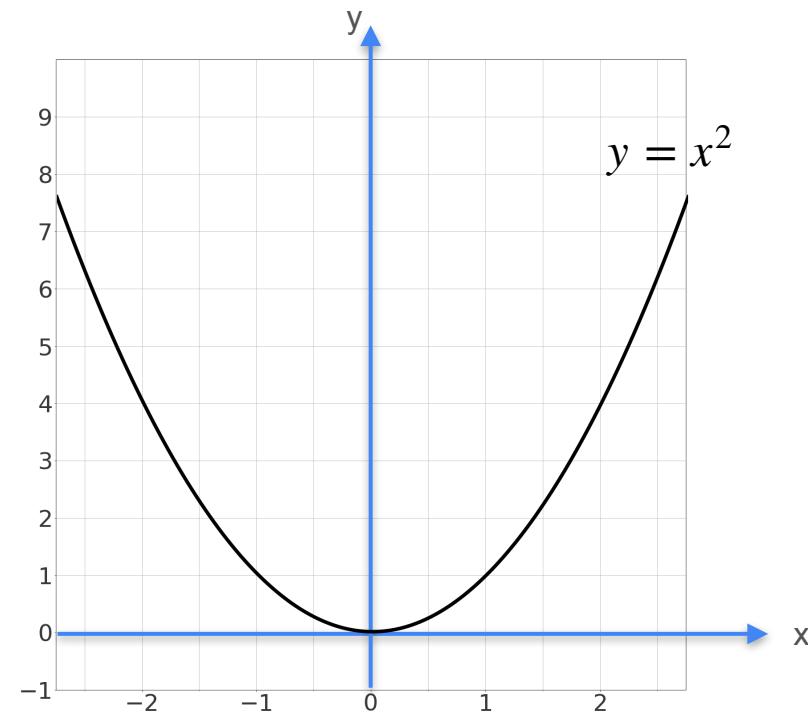
Multiplication by a Scalar



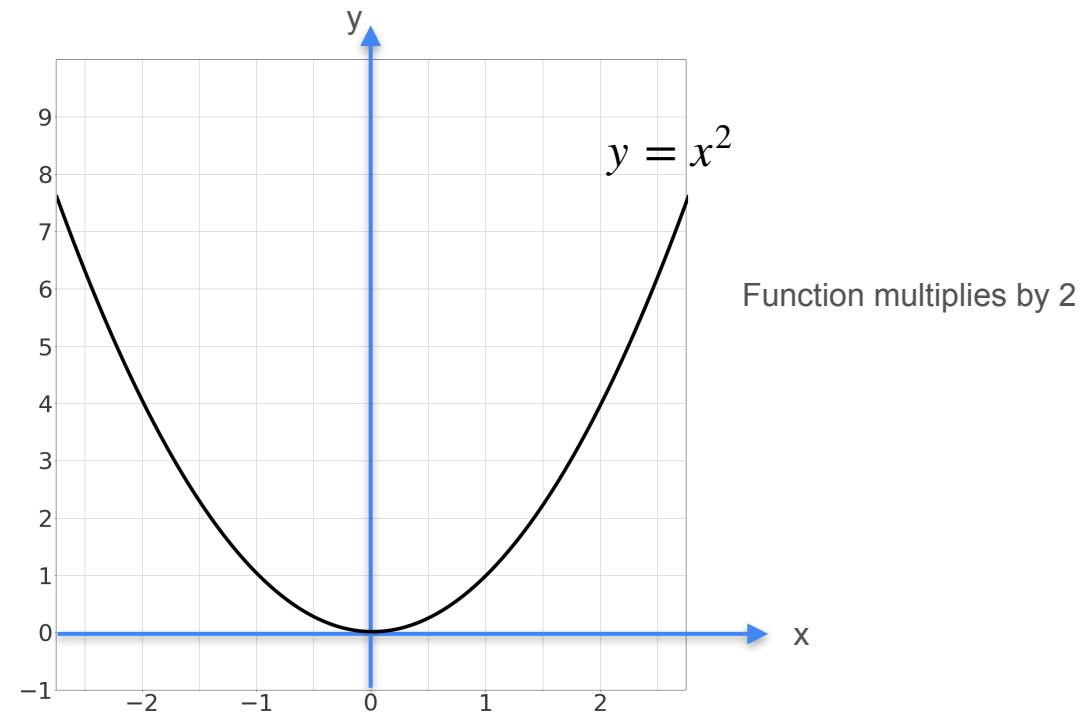
Multiplication by a Scalar



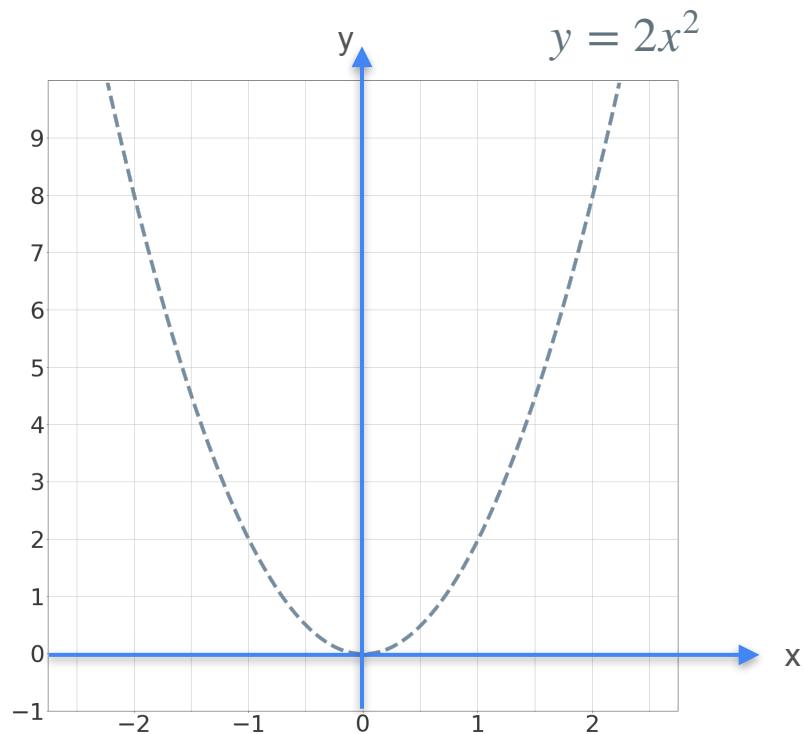
Multiplication by a Scalar



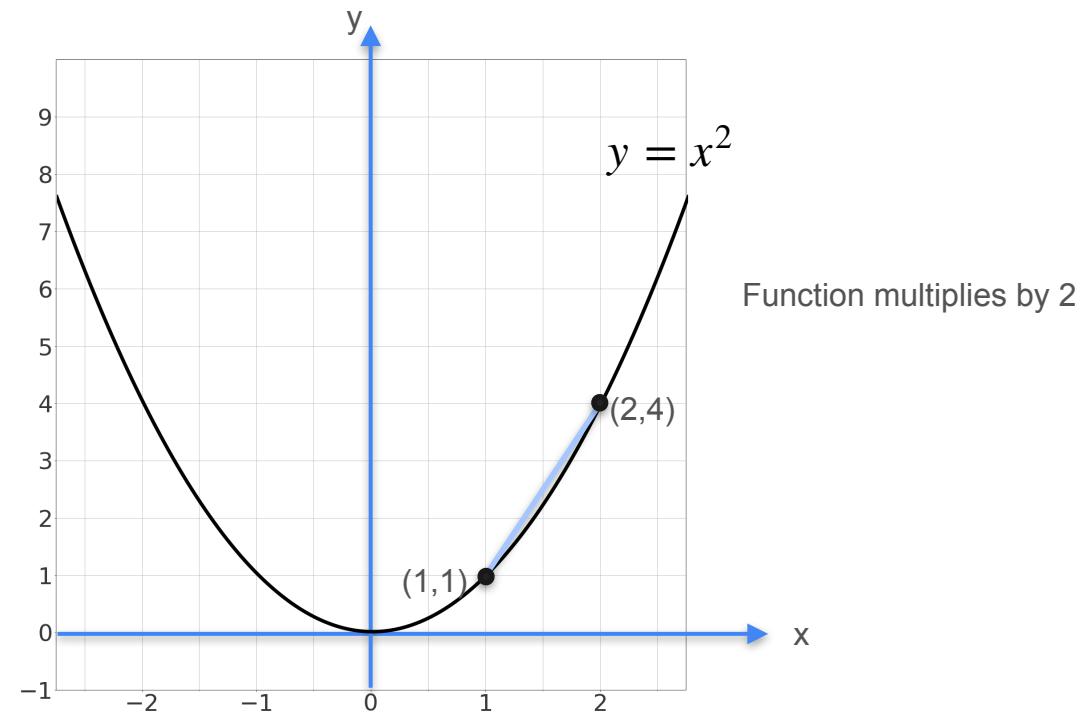
Multiplication by a Scalar



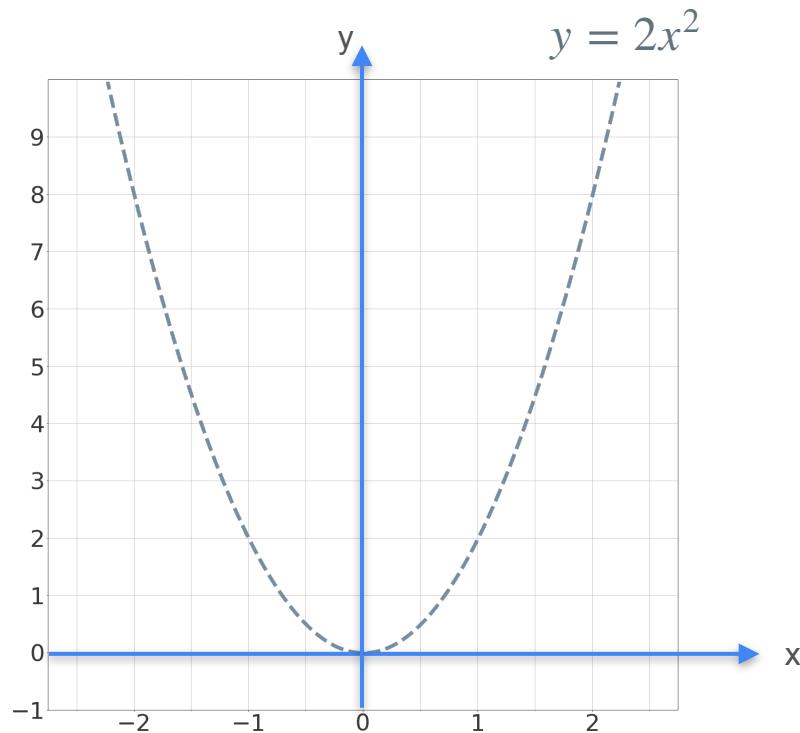
Function multiplies by 2



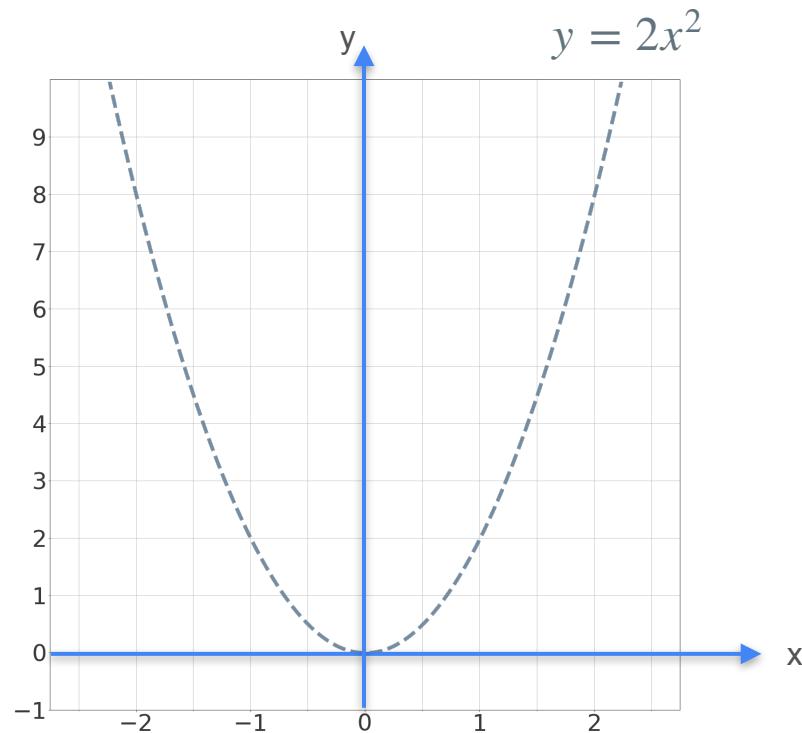
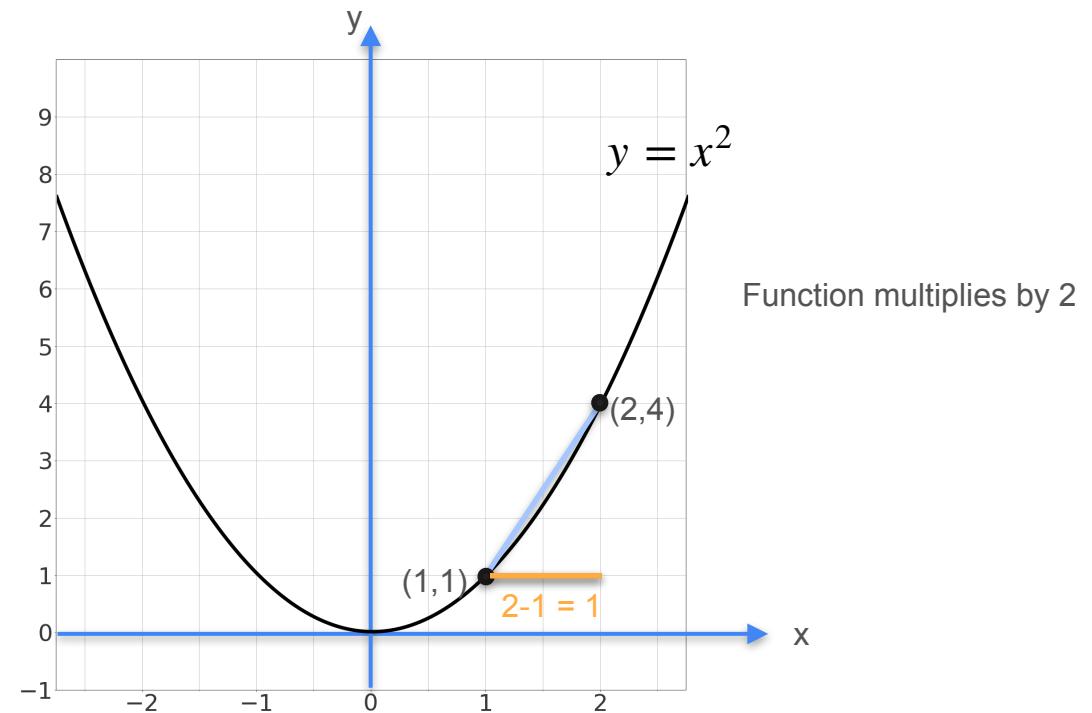
Multiplication by a Scalar



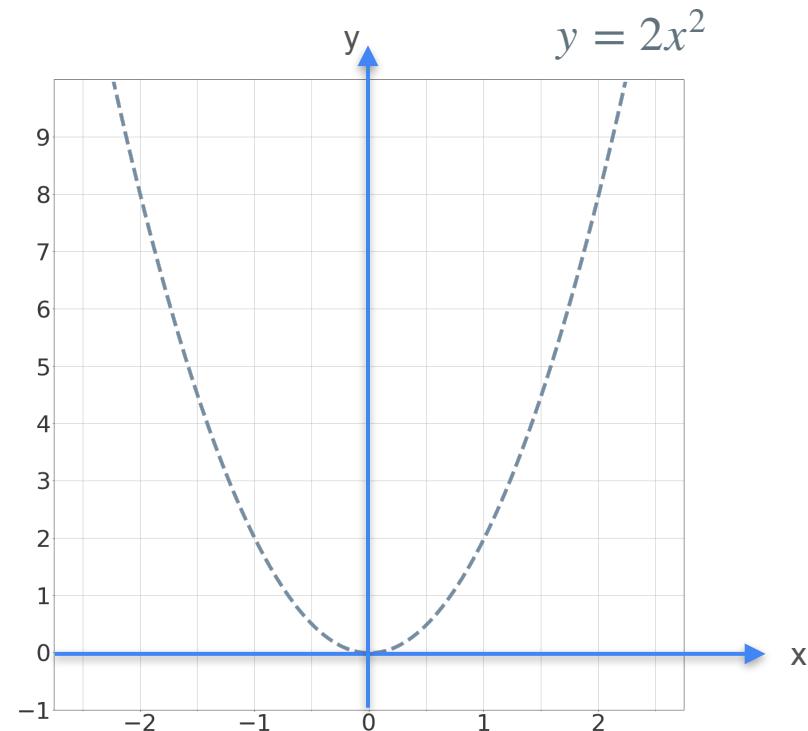
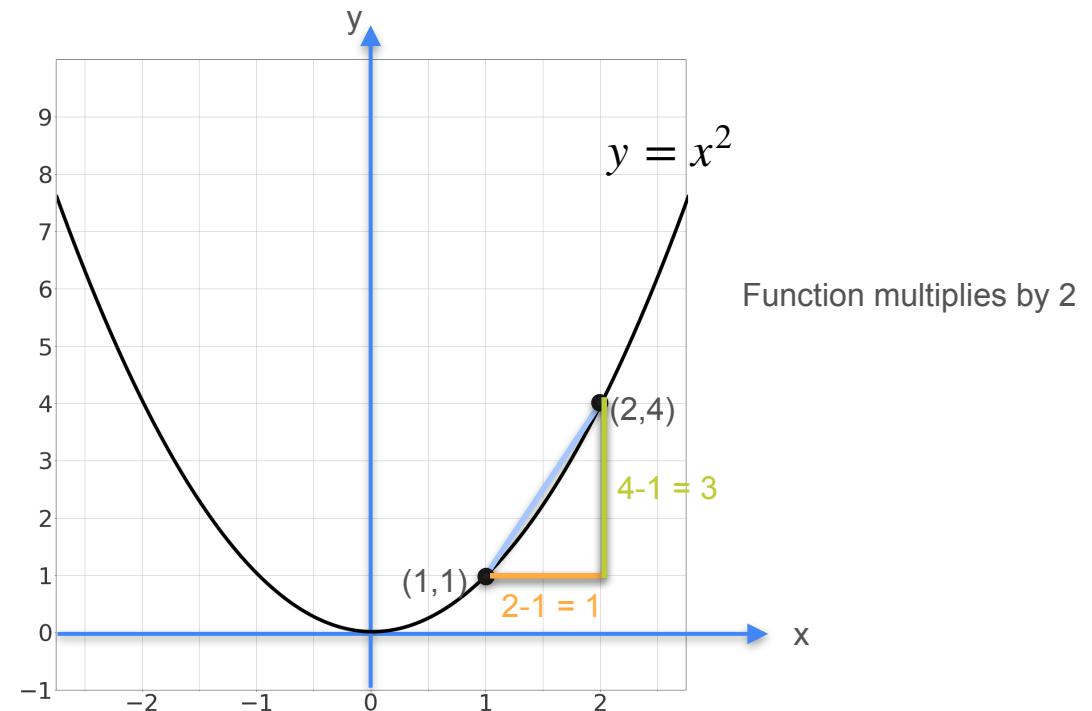
Function multiplies by 2



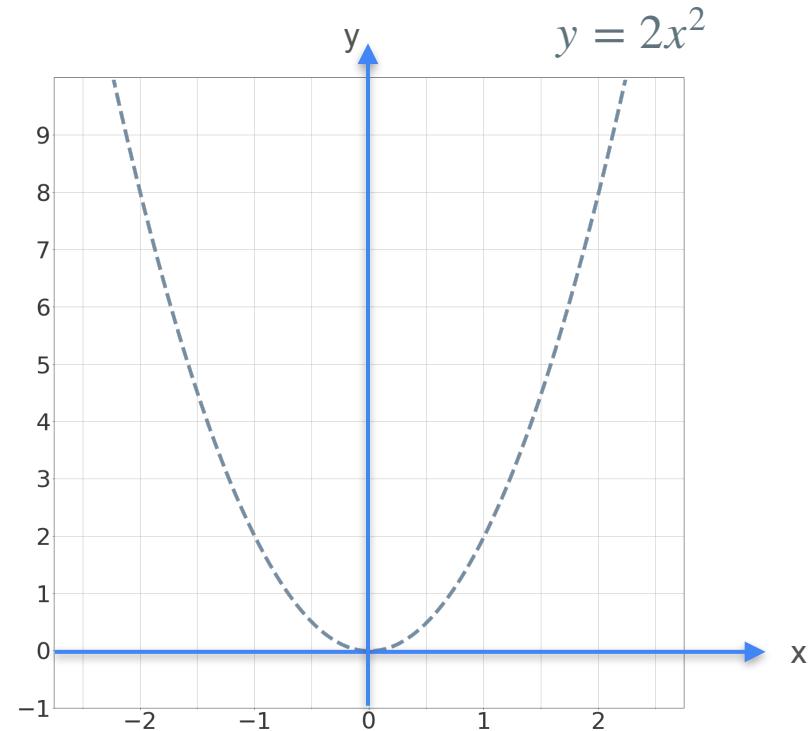
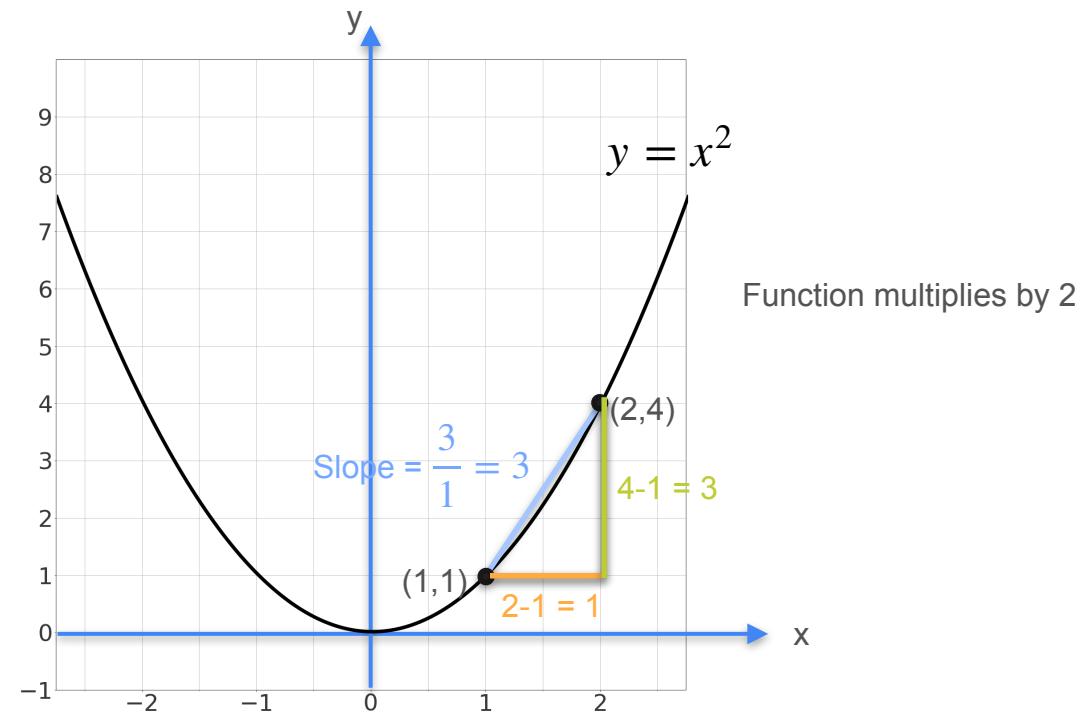
Multiplication by a Scalar



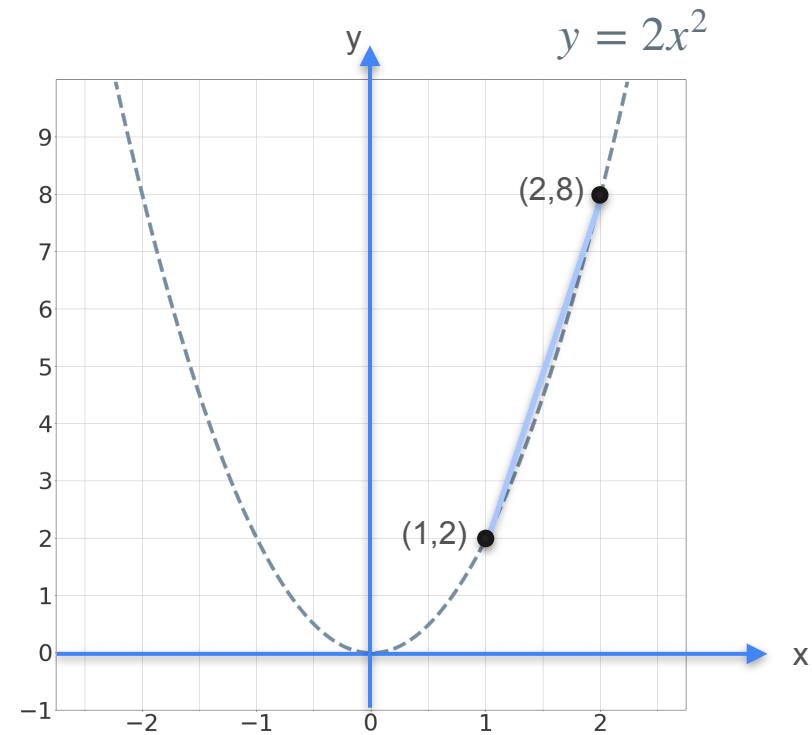
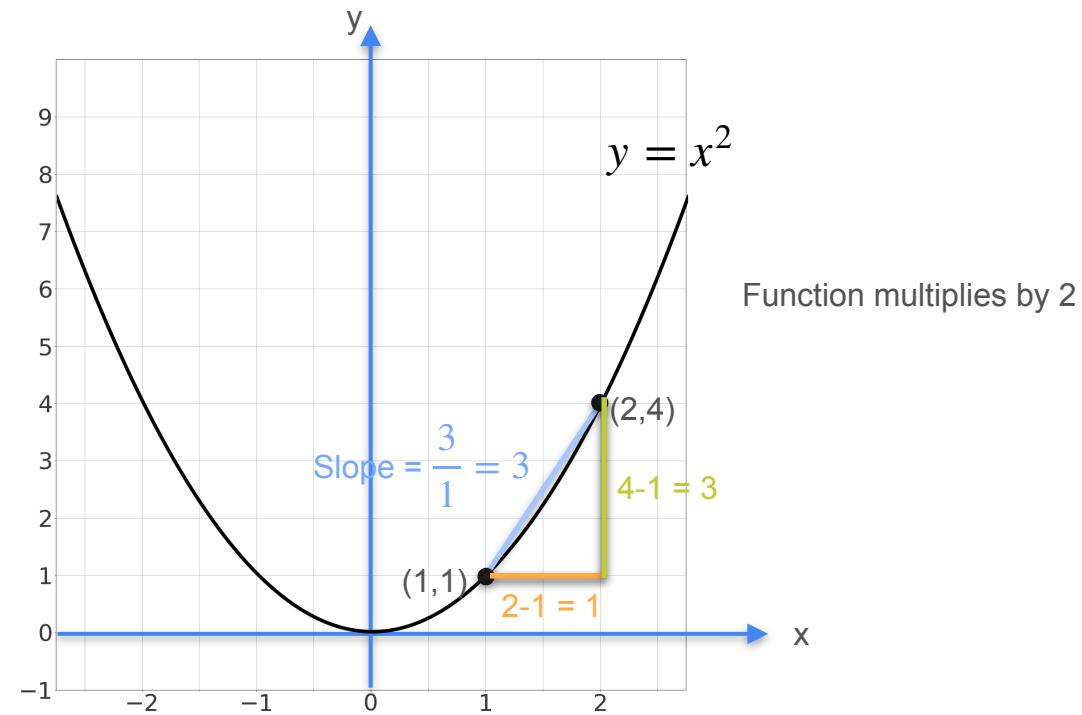
Multiplication by a Scalar



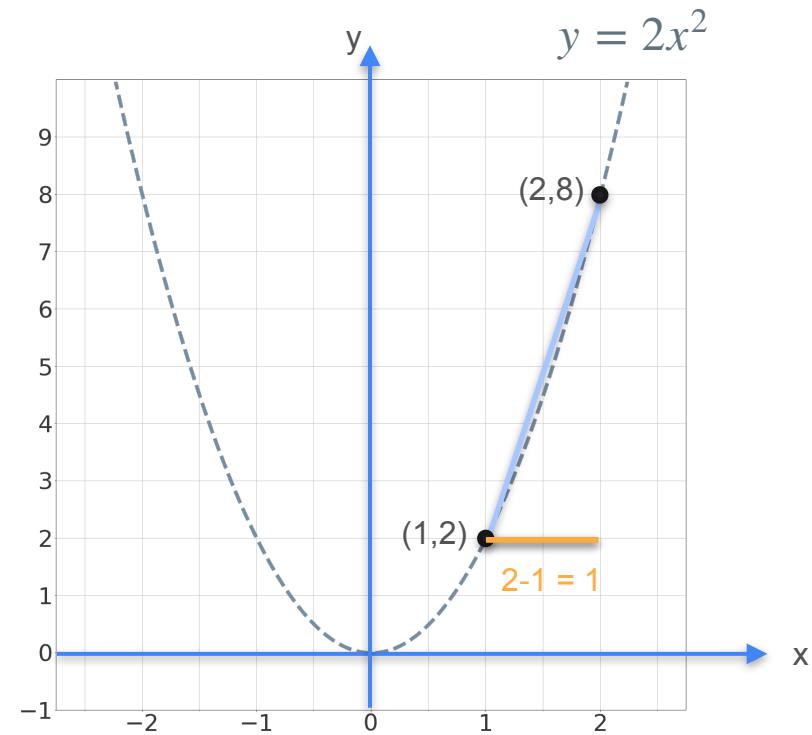
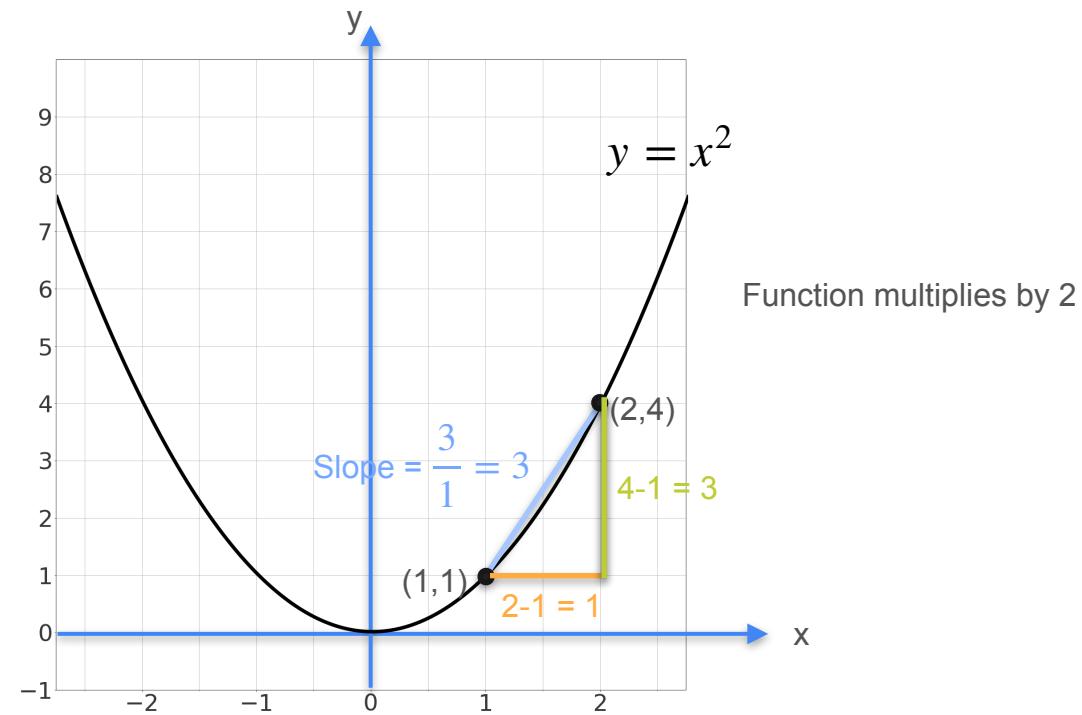
Multiplication by a Scalar



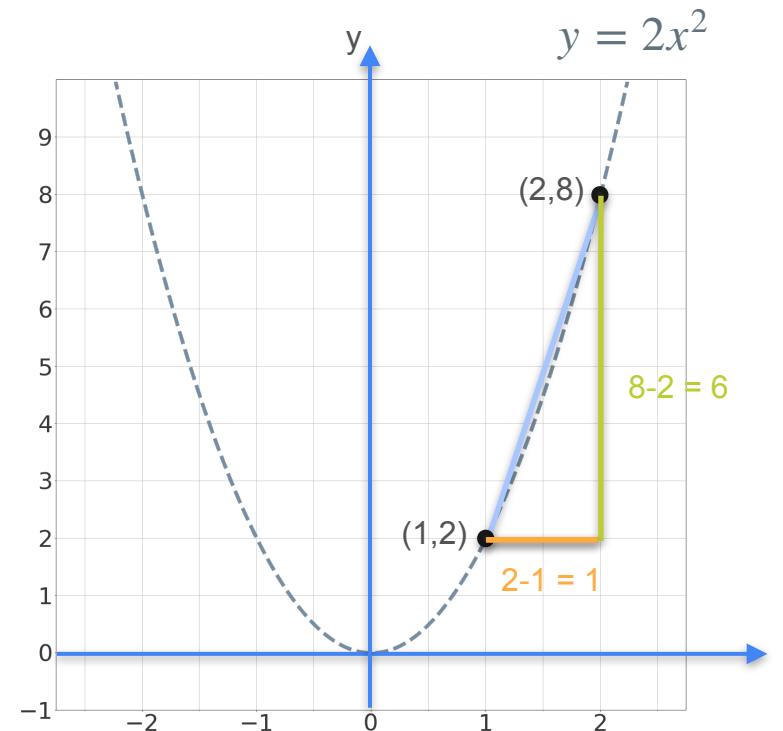
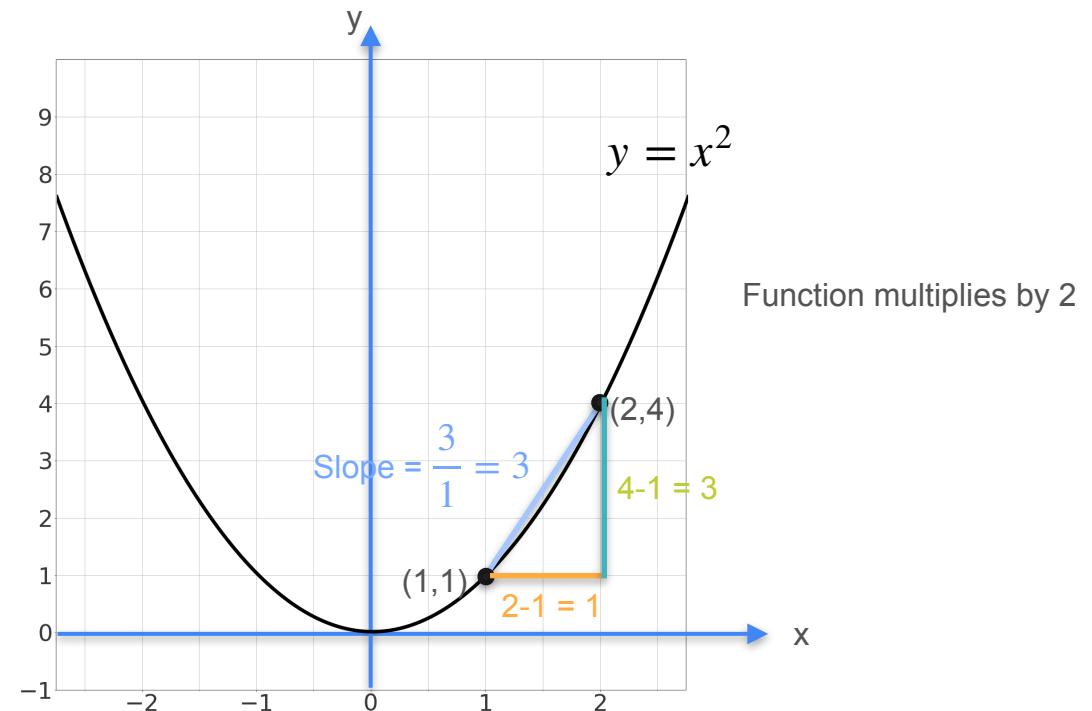
Multiplication by a Scalar



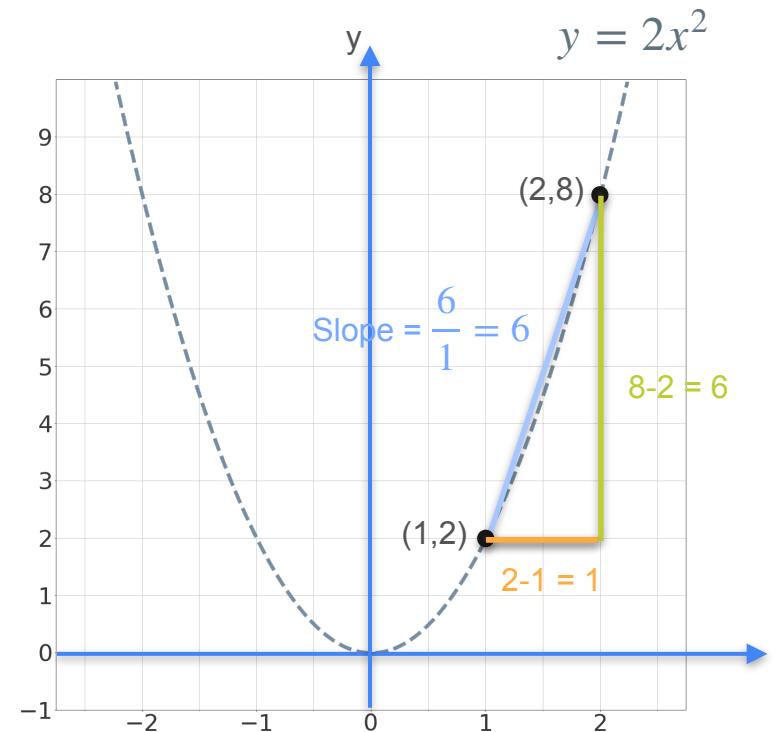
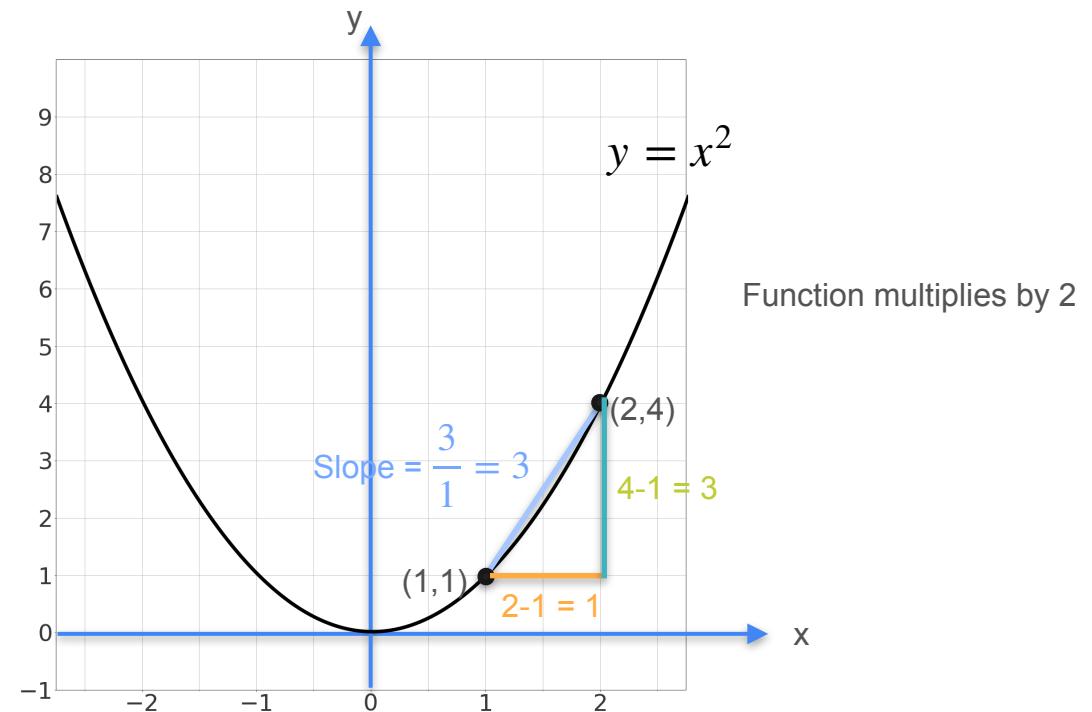
Multiplication by a Scalar



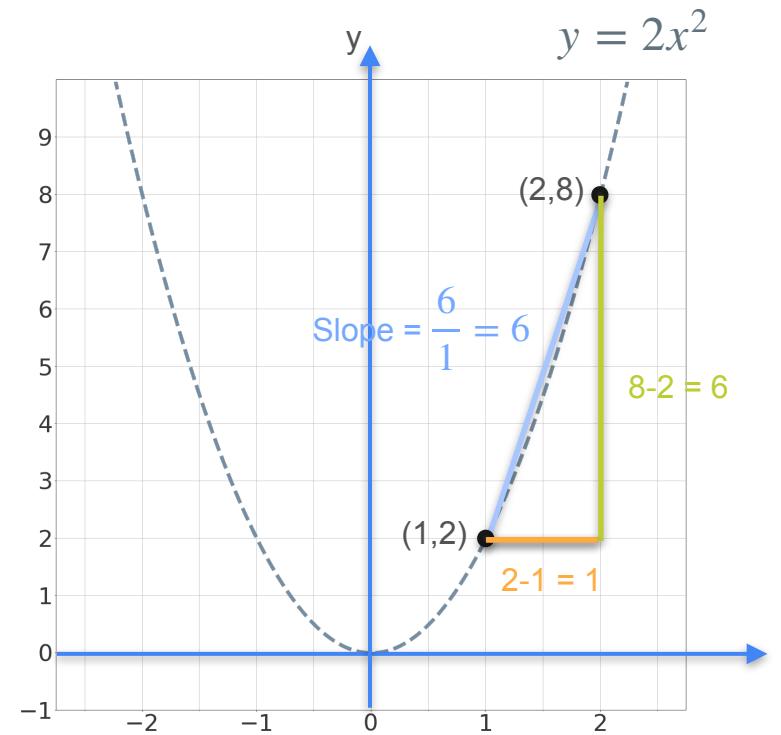
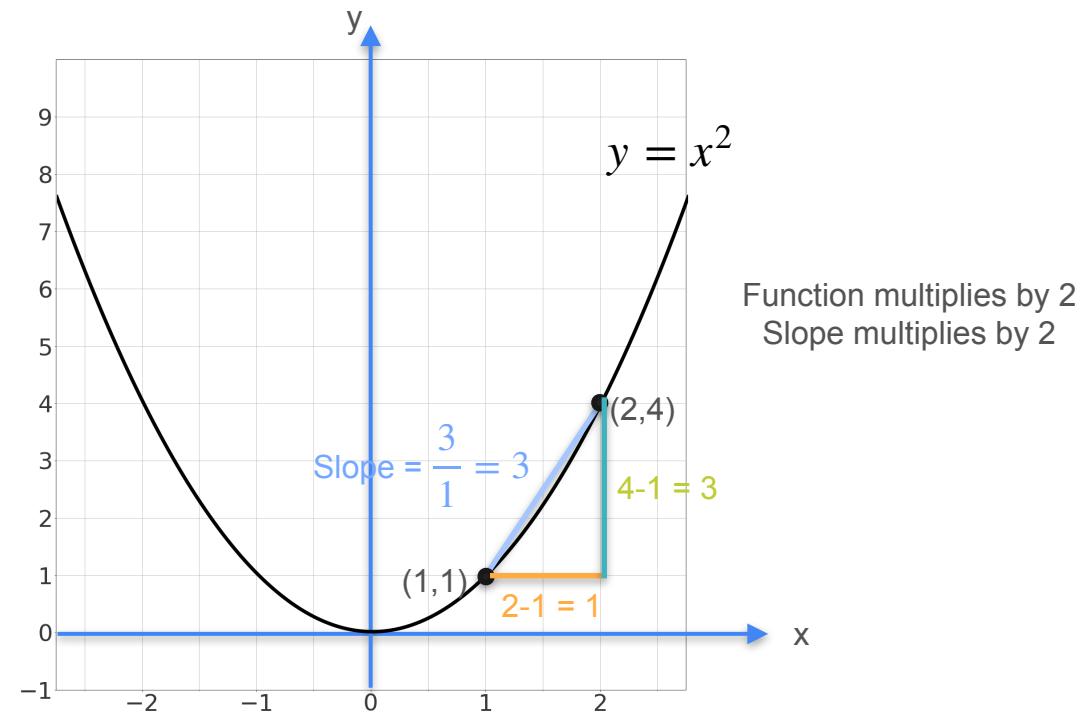
Multiplication by a Scalar



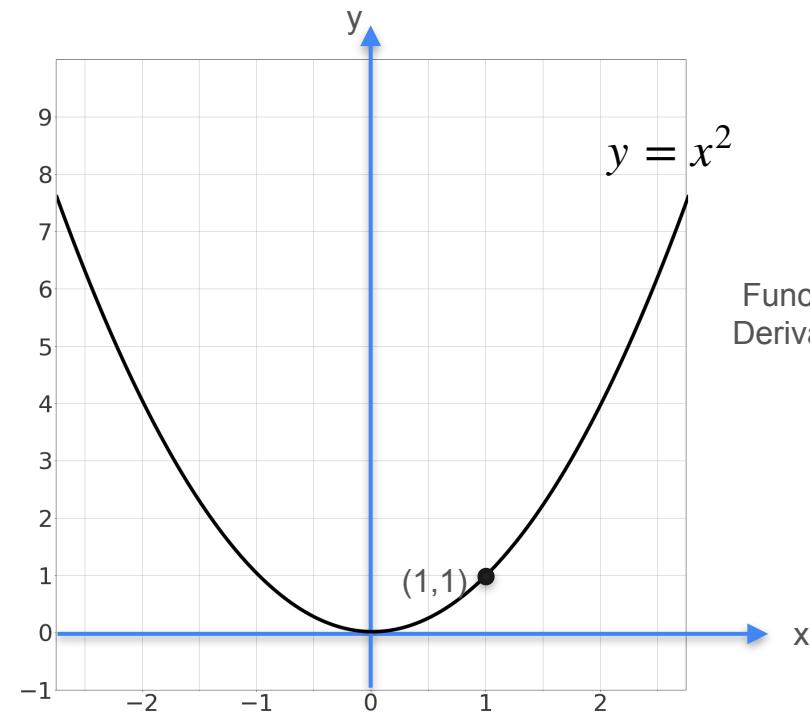
Multiplication by a Scalar



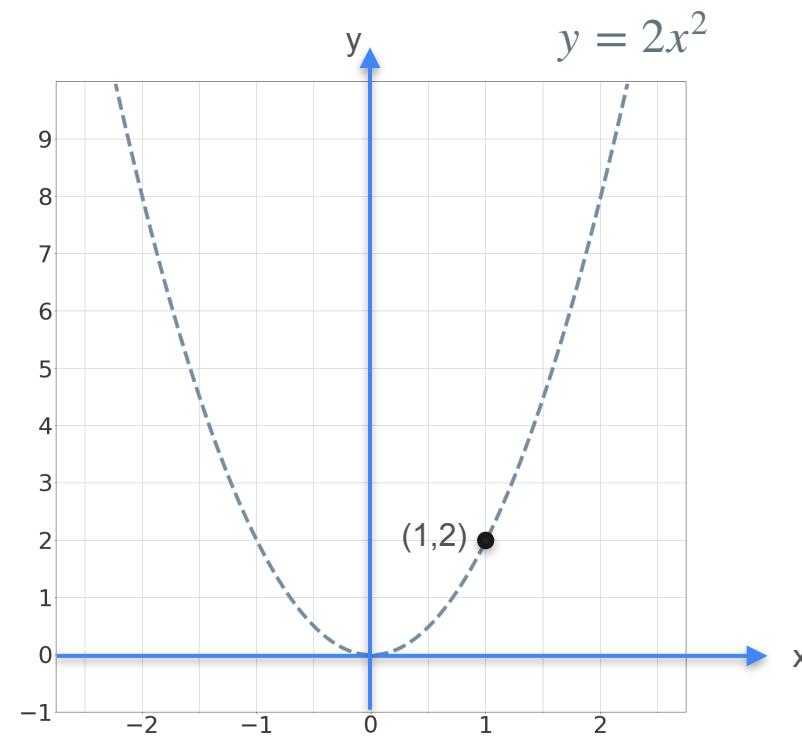
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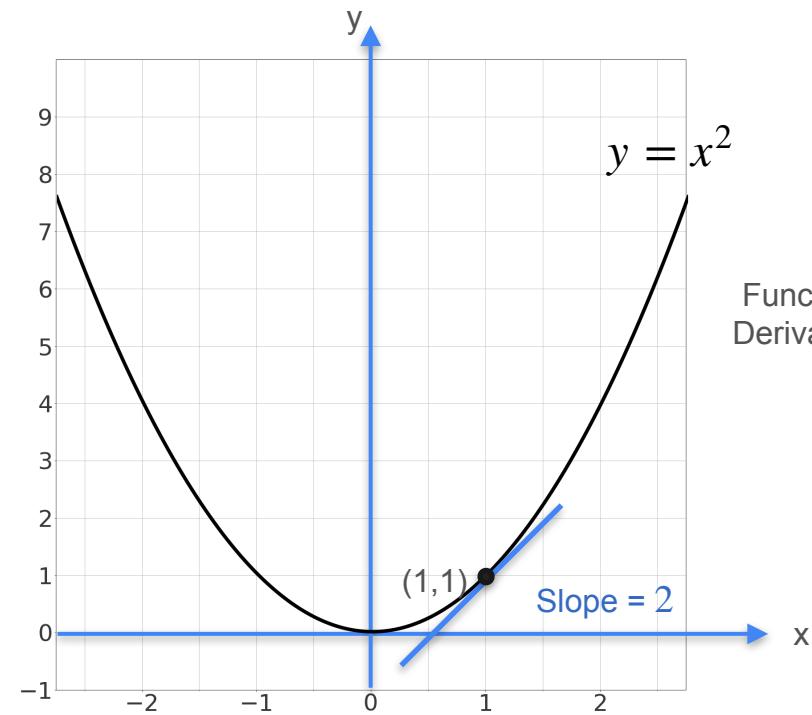
Multiplication by a Scalar



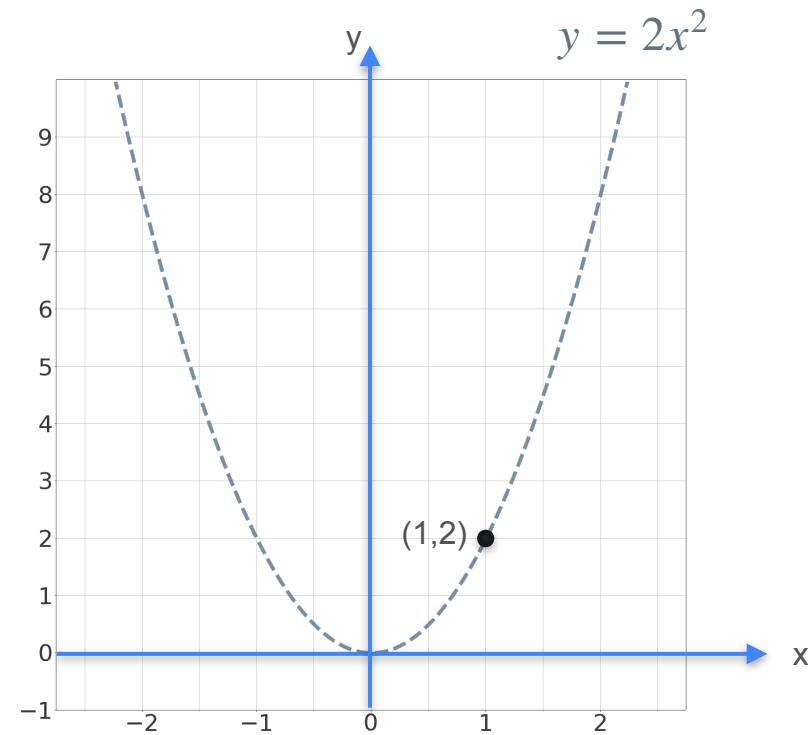
Function multiplies by 2
Derivative multiplies by 2



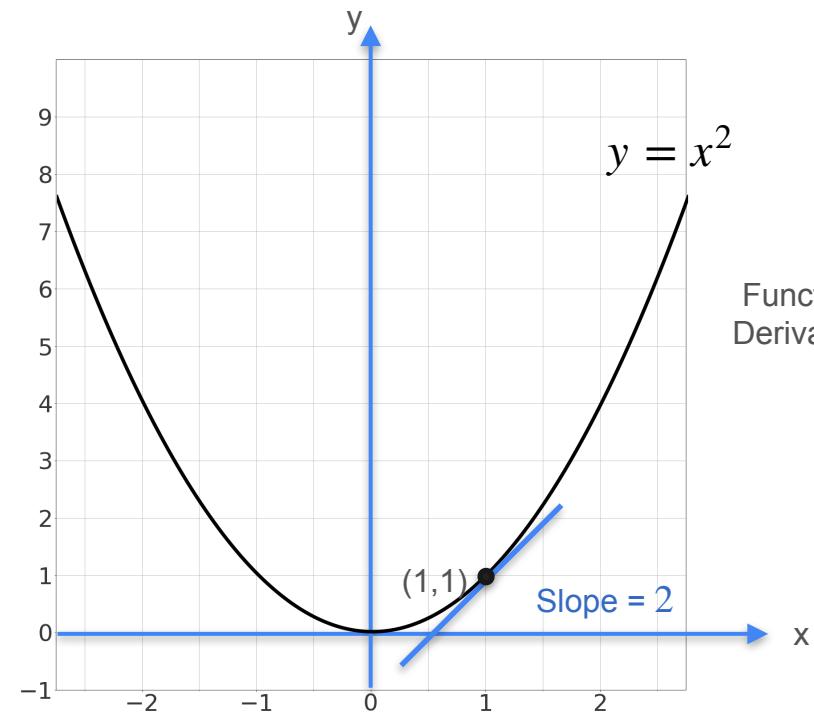
Multiplication by a Scalar



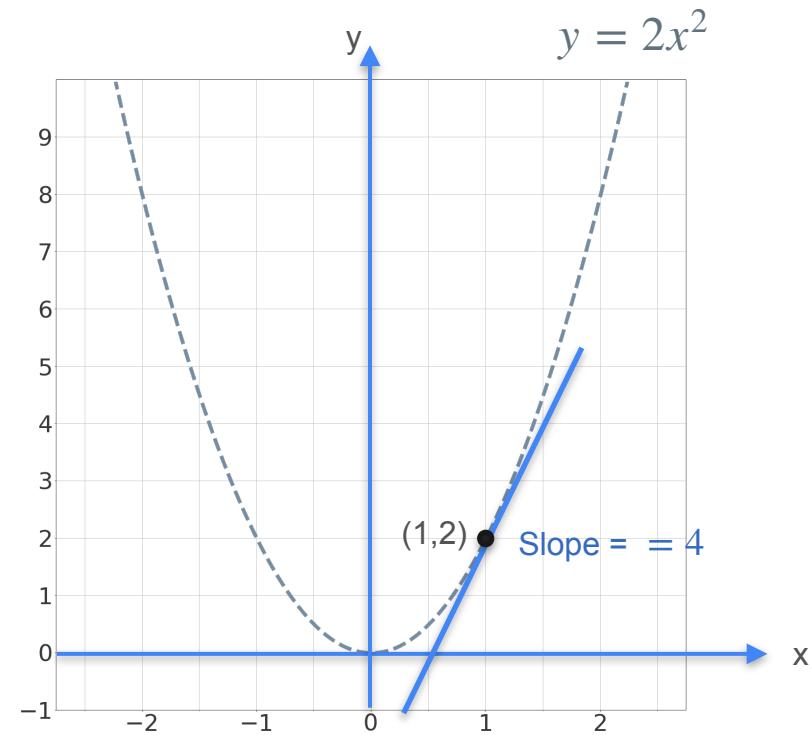
Function multiplies by 2
Derivative multiplies by 2



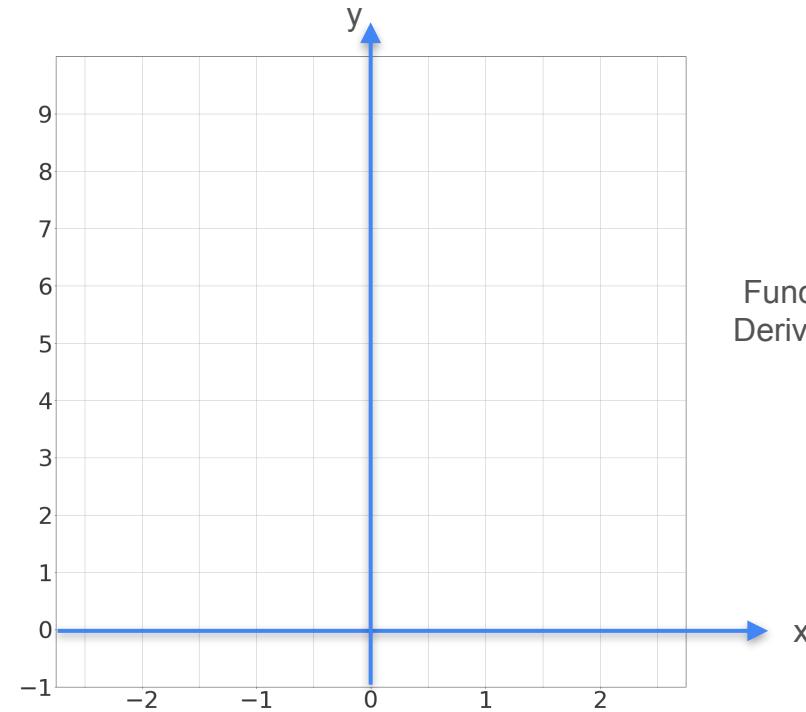
Multiplication by a Scalar



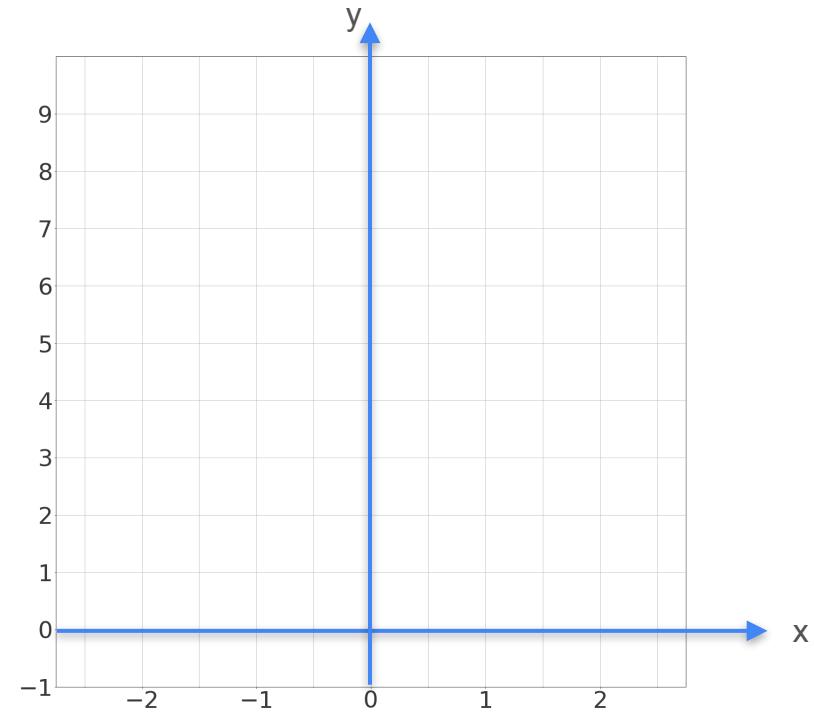
Function multiplies by 2
Derivative multiplies by 2



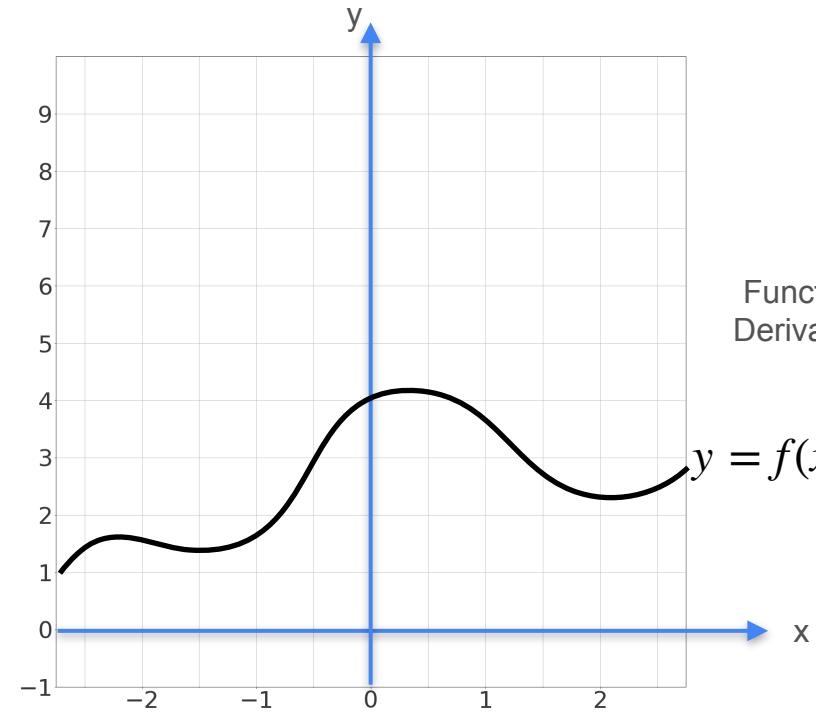
Multiplication by a Scalar



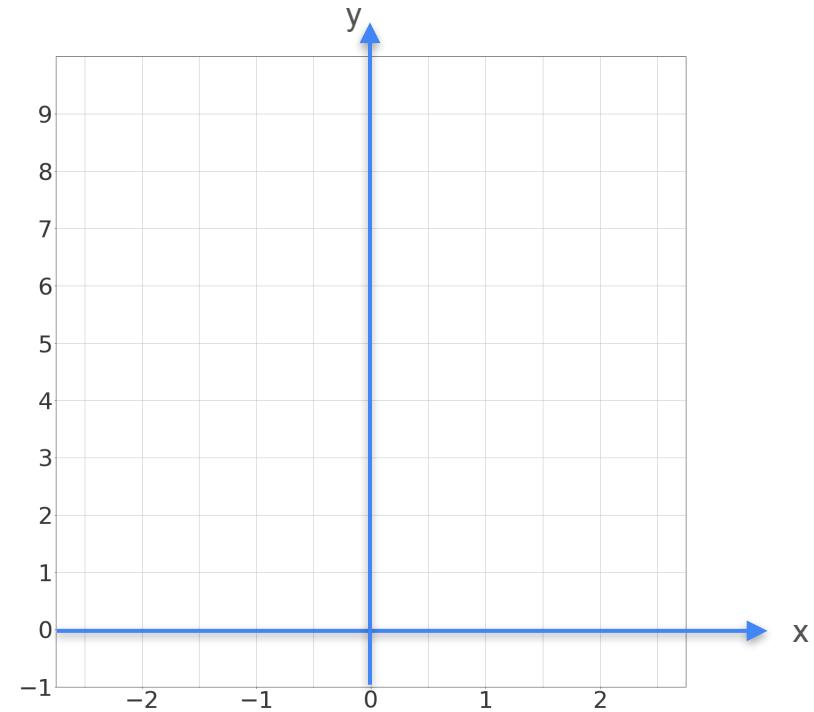
Function multiplies by c
Derivative multiplies by c



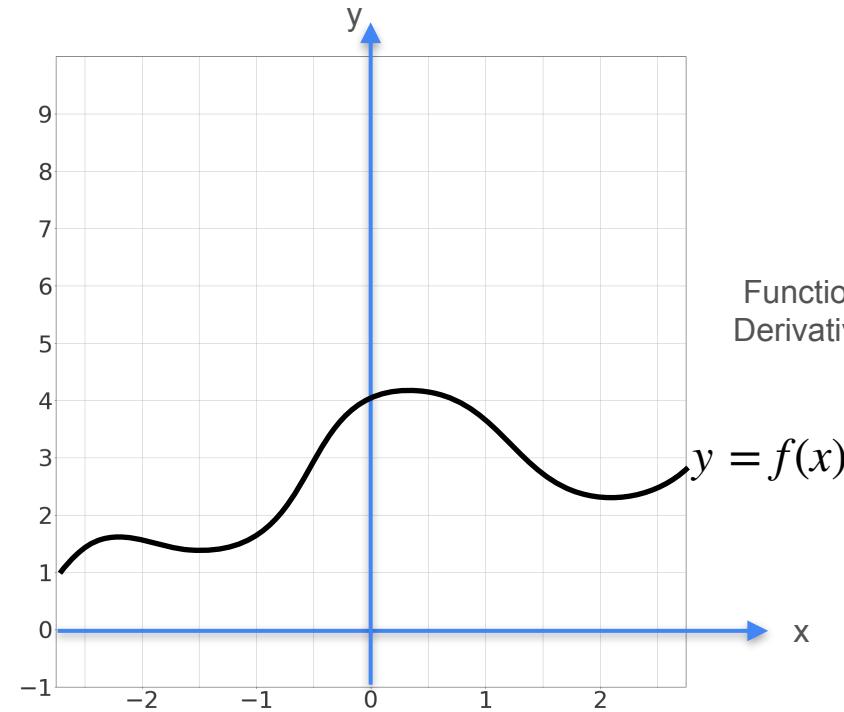
Multiplication by a Scalar



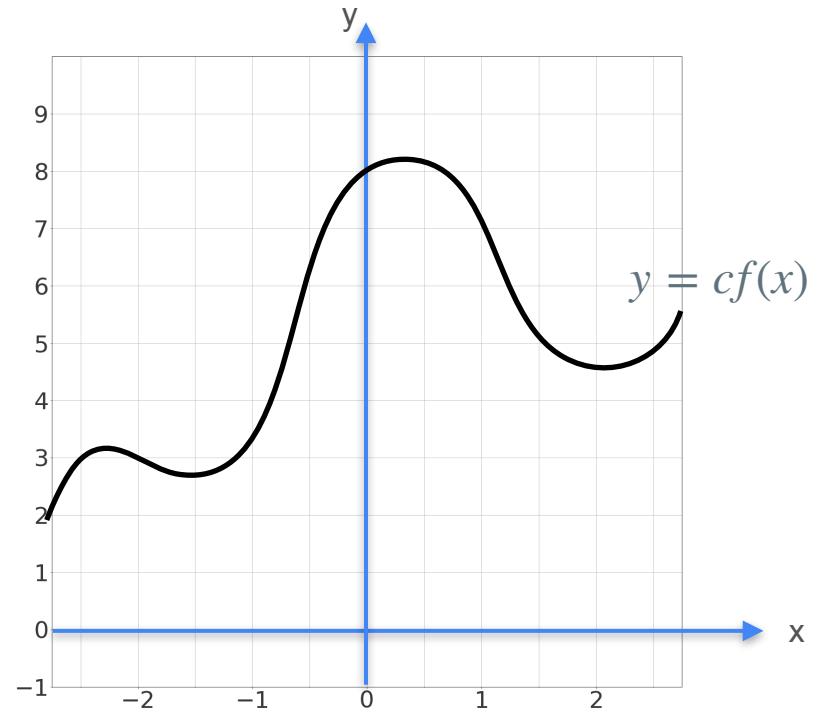
Function multiplies by c
Derivative multiplies by c



Multiplication by a Scalar

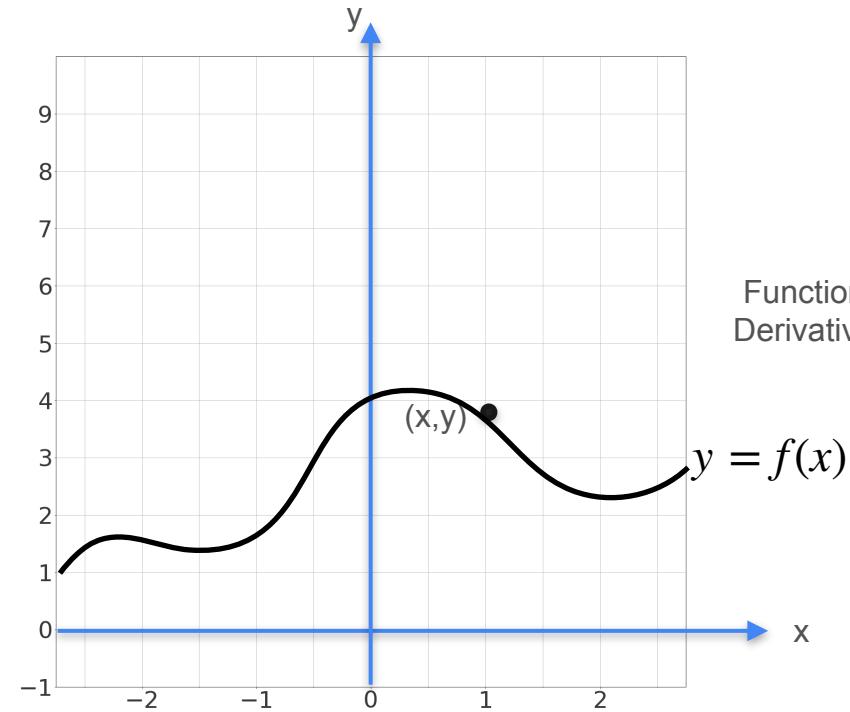


Function multiplies by c
Derivative multiplies by c

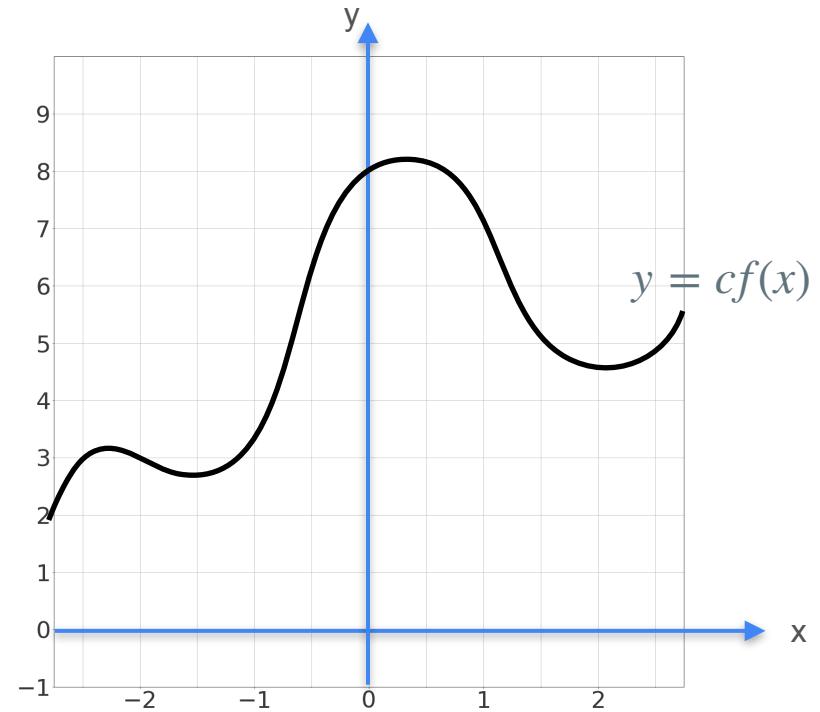


$$y = cf(x)$$

Multiplication by a Scalar

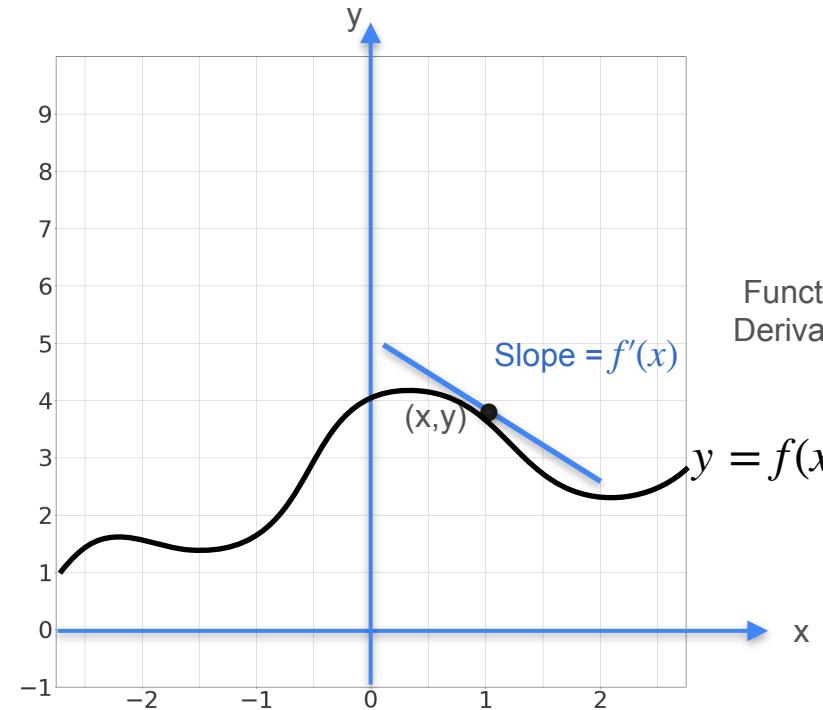


Function multiplies by c
Derivative multiplies by c

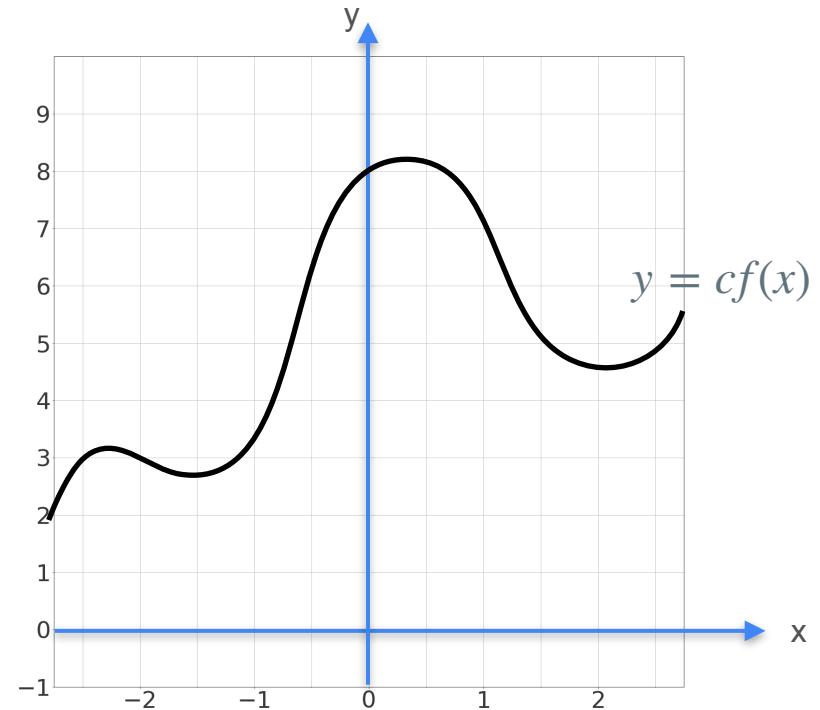


$$y = cf(x)$$

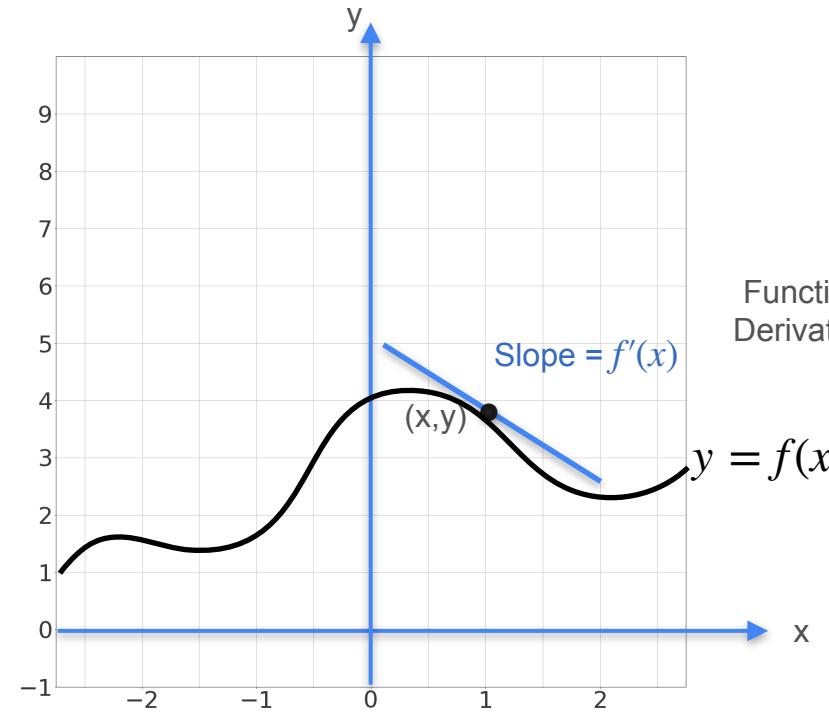
Multiplication by a Scalar



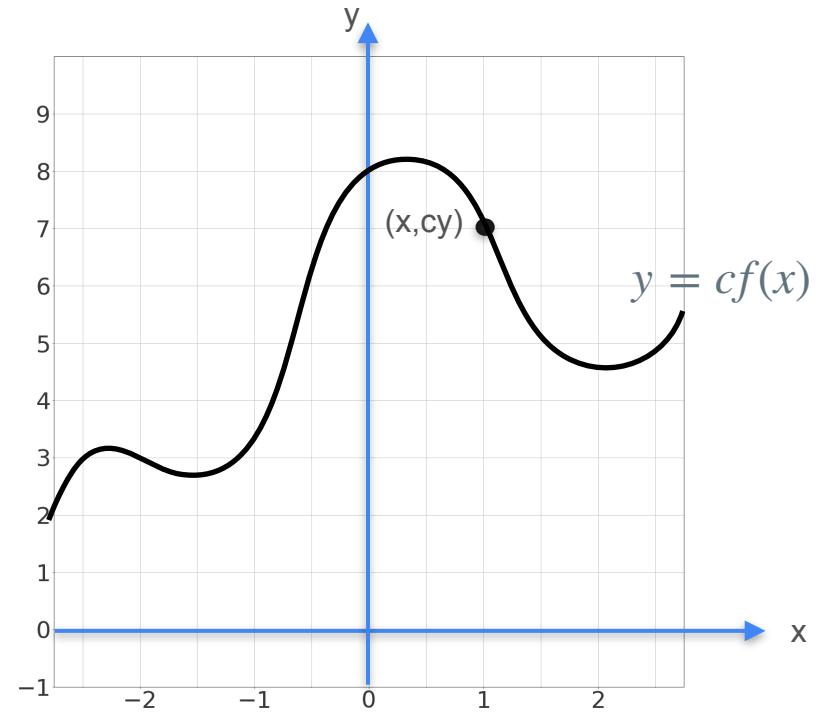
Function multiplies by c
Derivative multiplies by c



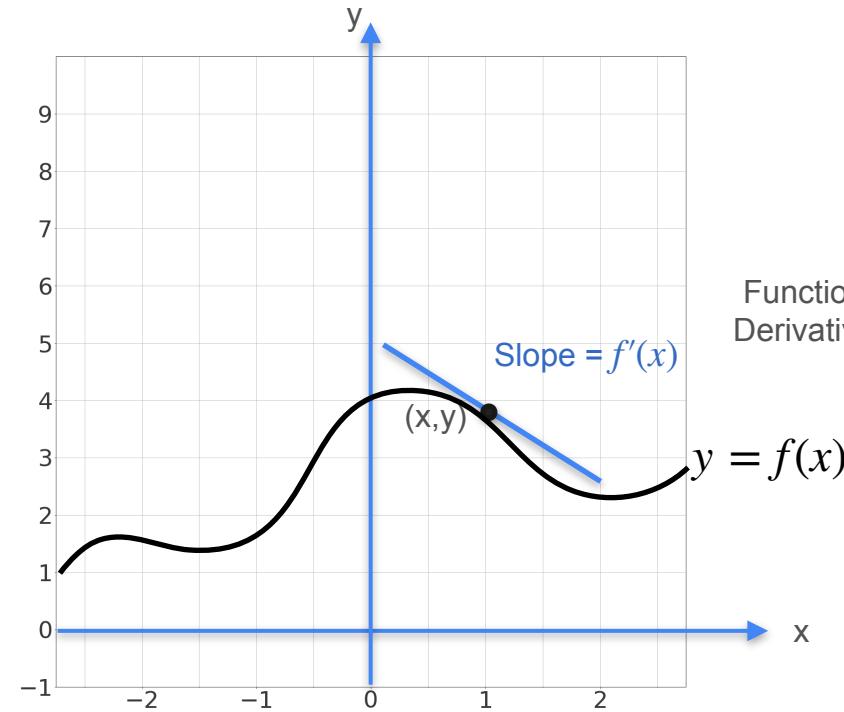
Multiplication by a Scalar



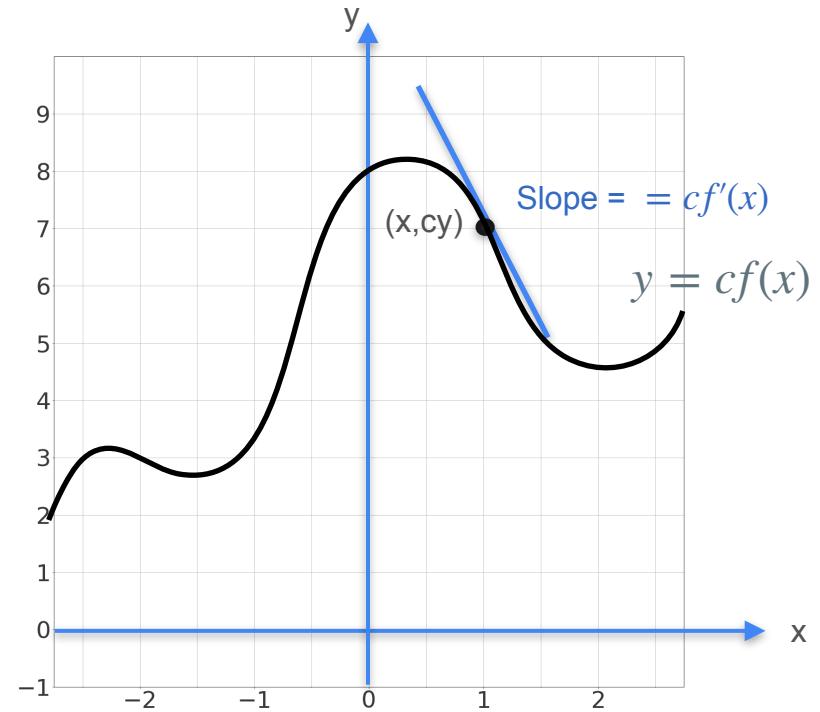
Function multiplies by c
Derivative multiplies by c



Multiplication by a Scalar



Function multiplies by c
Derivative multiplies by c





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Derivatives and Optimization

**Properties of the derivative:
The sum rule**

The Sum Rule

$$f = g + h$$

The Sum Rule

$$f' = g + h$$

The Sum Rule

$$f' = g' + h$$

The Sum Rule

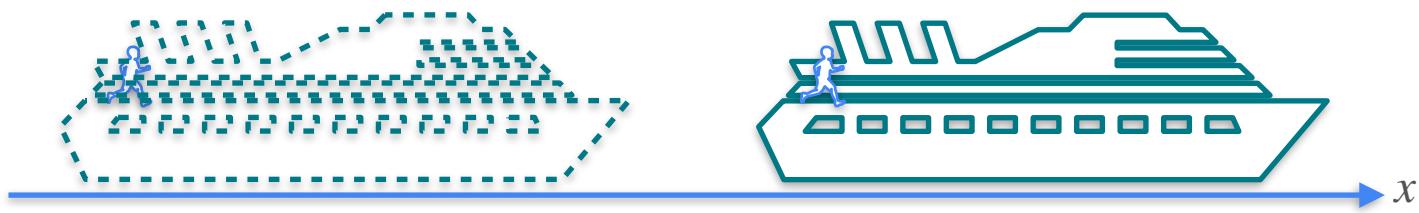
$$f' = g' + h'$$

Sum Rule

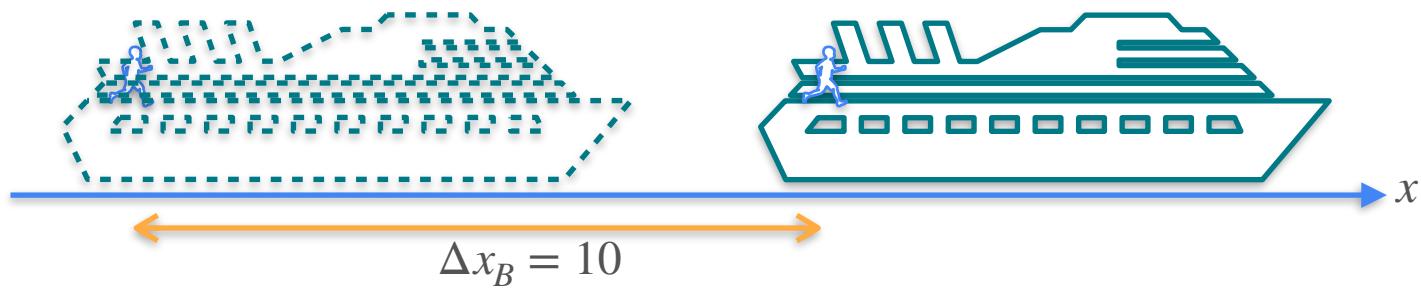
Sum Rule



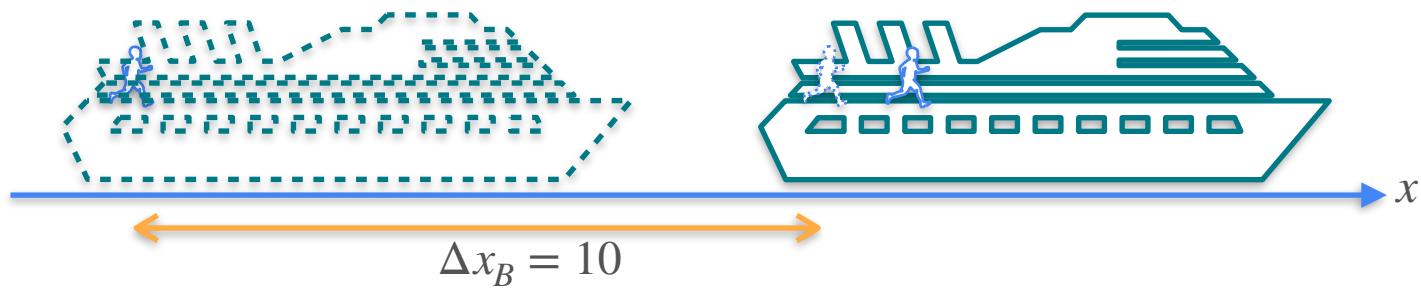
Sum Rule



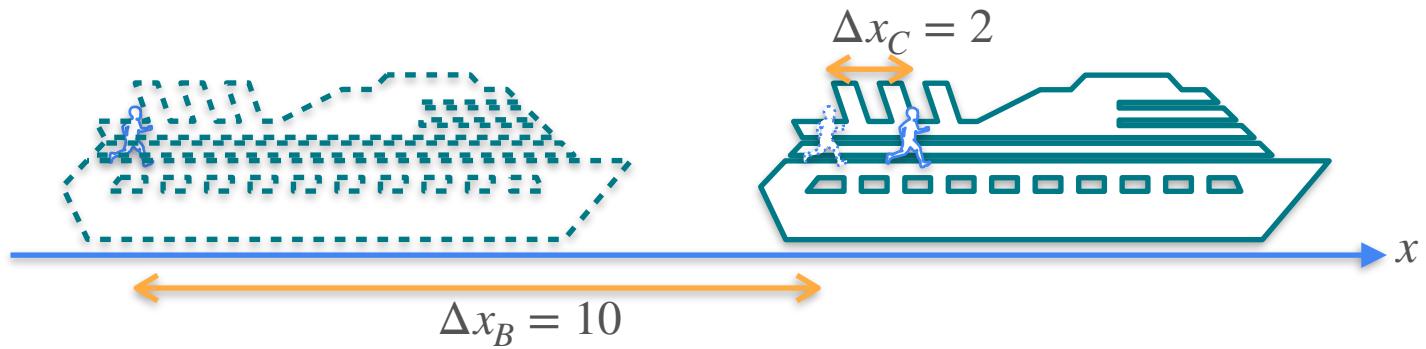
Sum Rule



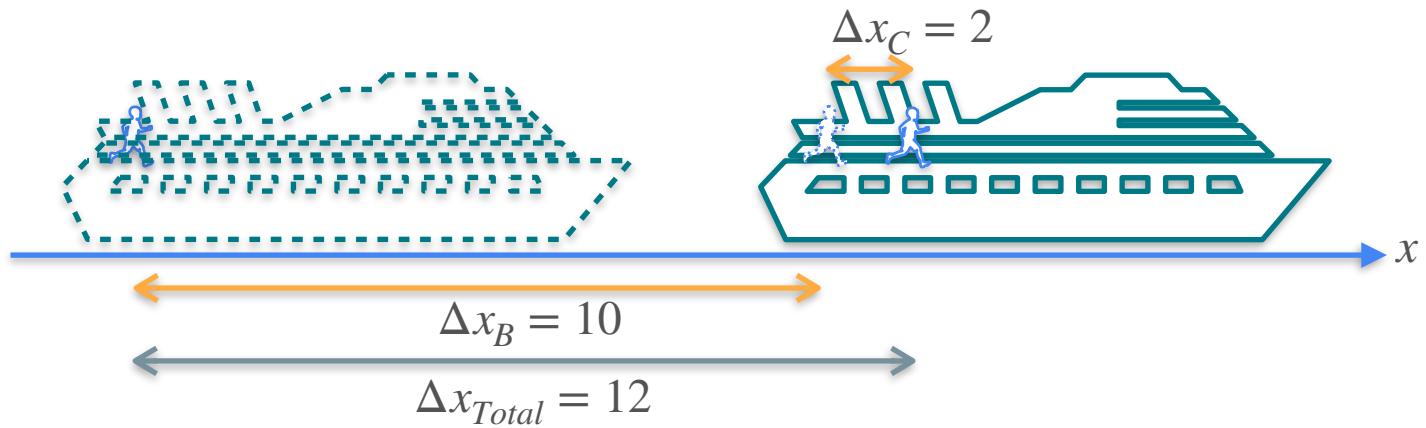
Sum Rule



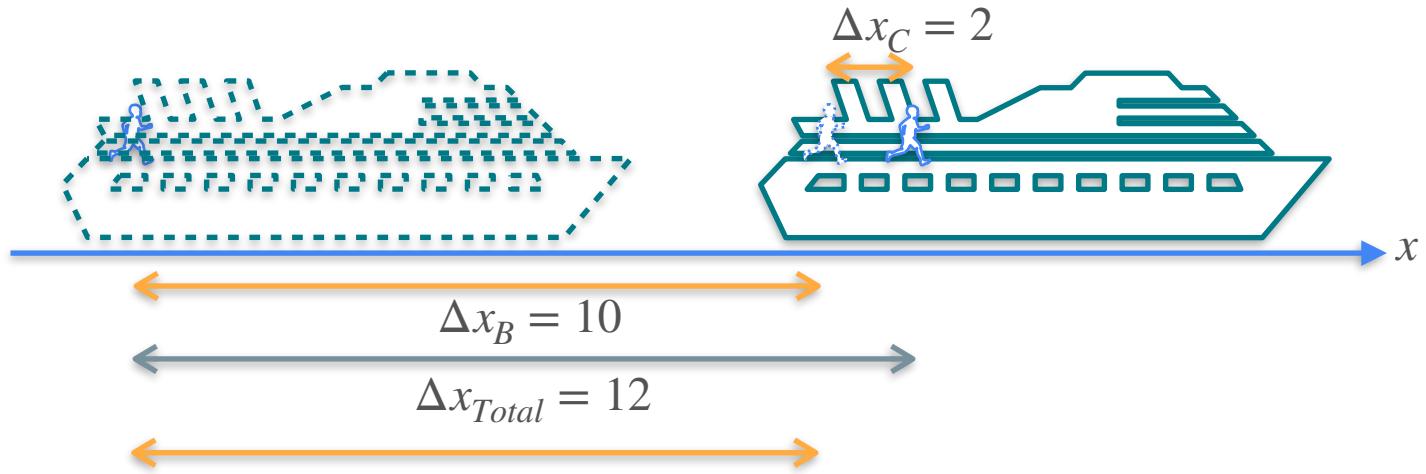
Sum Rule



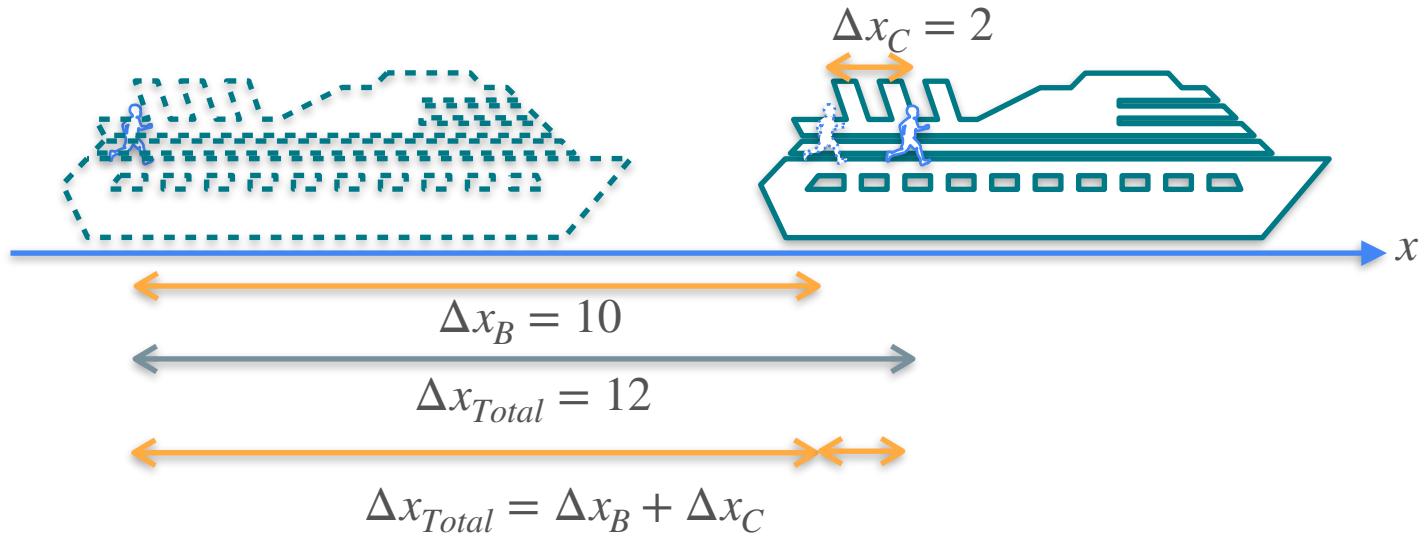
Sum Rule



Sum Rule



Sum Rule



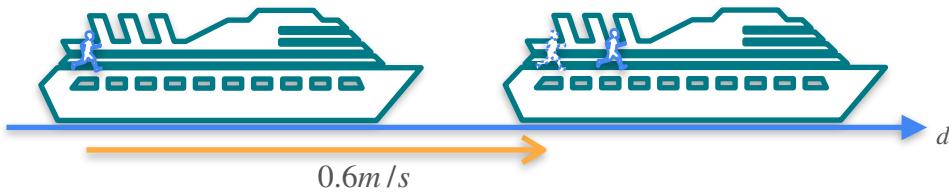
Quiz:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

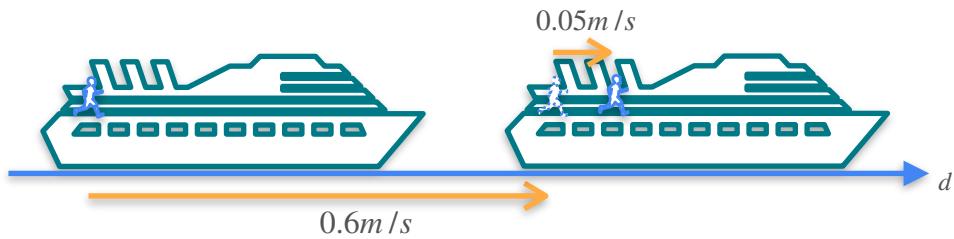
Quiz:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

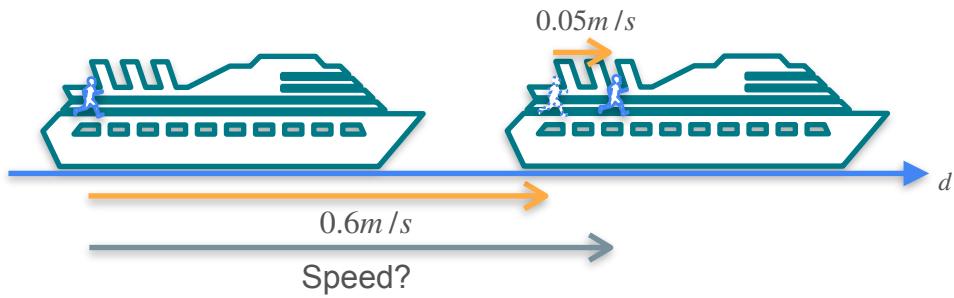
Quiz:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

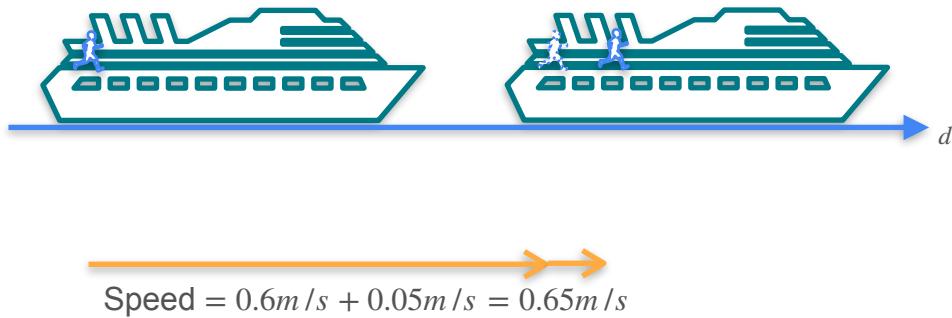
Quiz:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

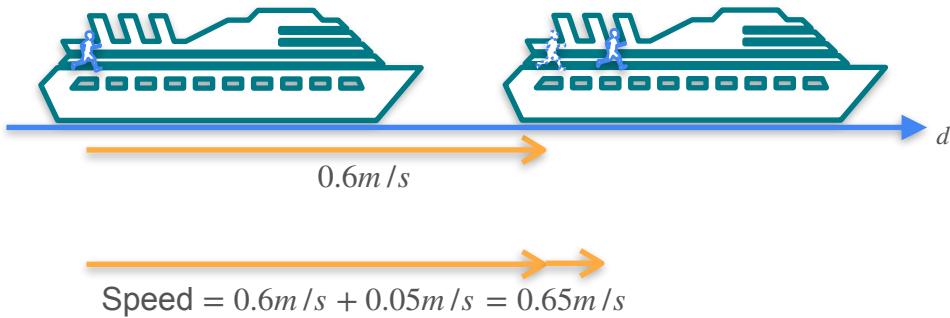
Solution:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

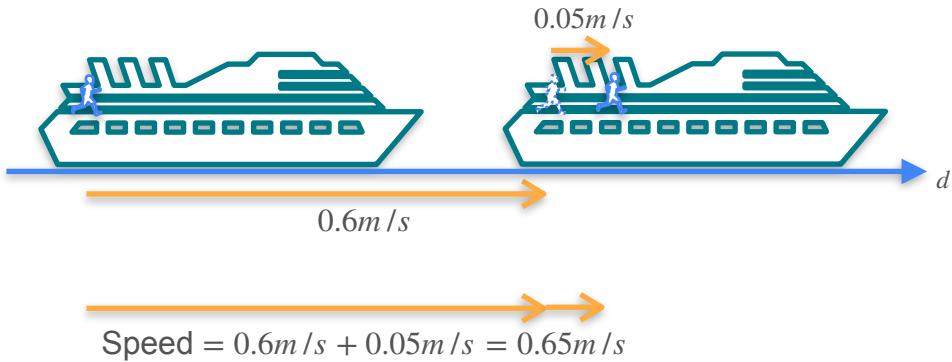
Solution:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

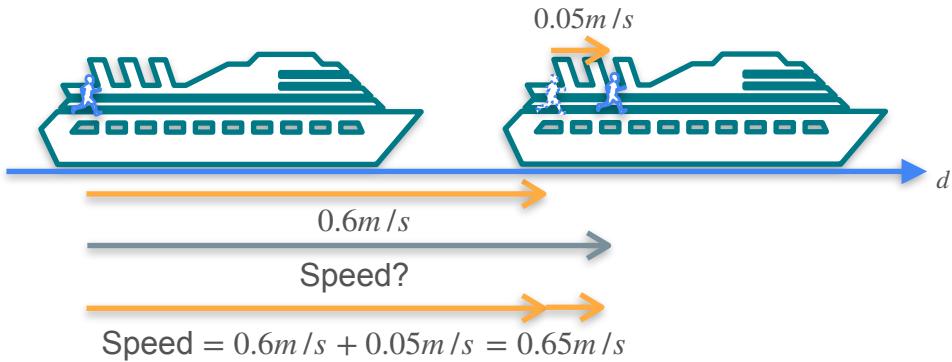
Solution:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

Solution:

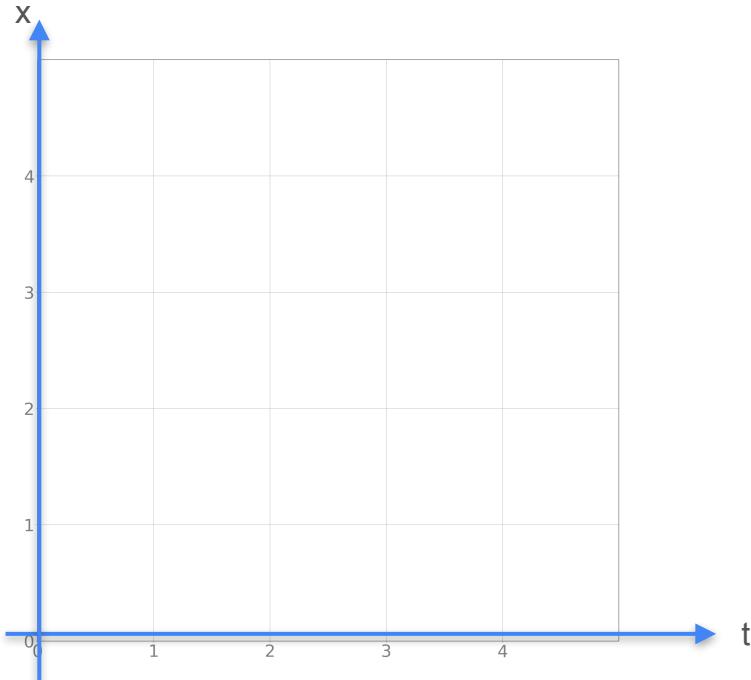


- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

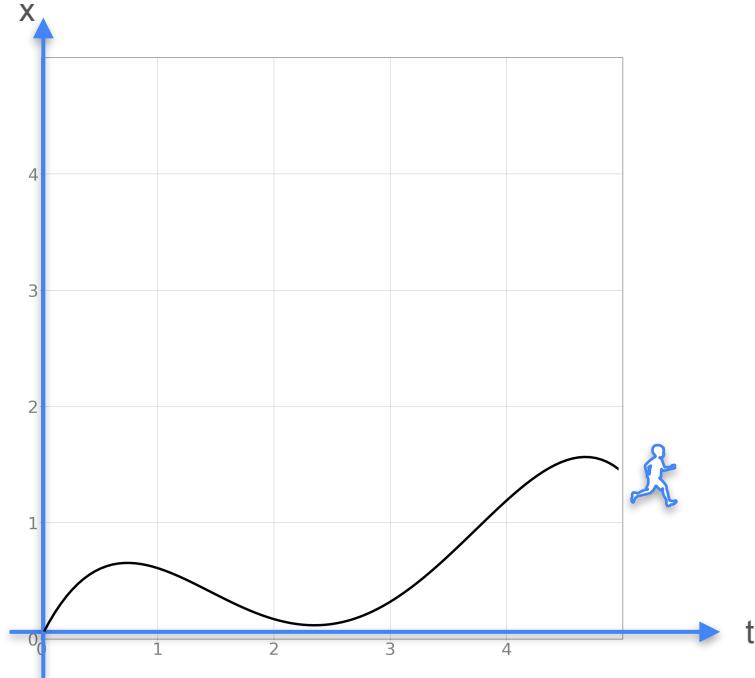
What is the speed of the child with respect to the earth?

Sum Rule

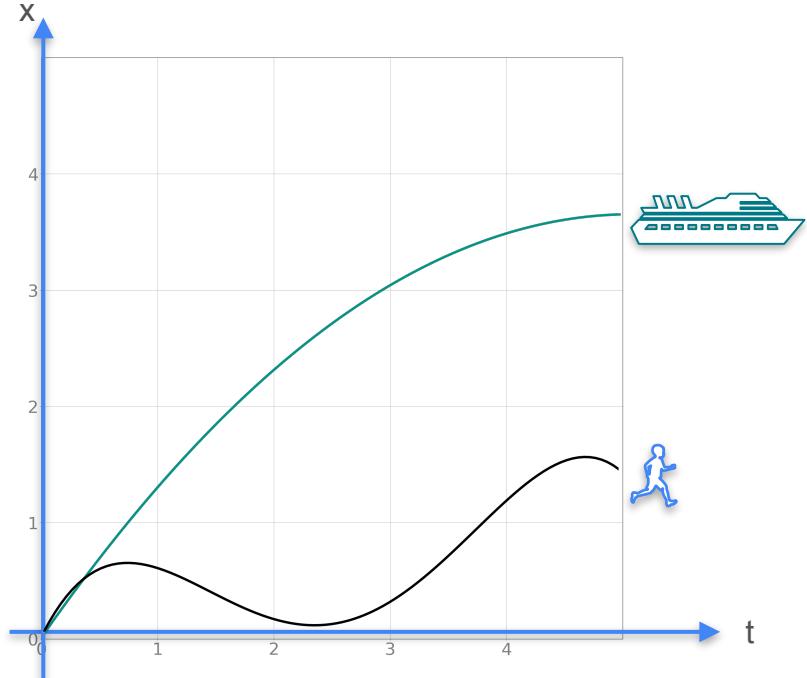
Sum Rule



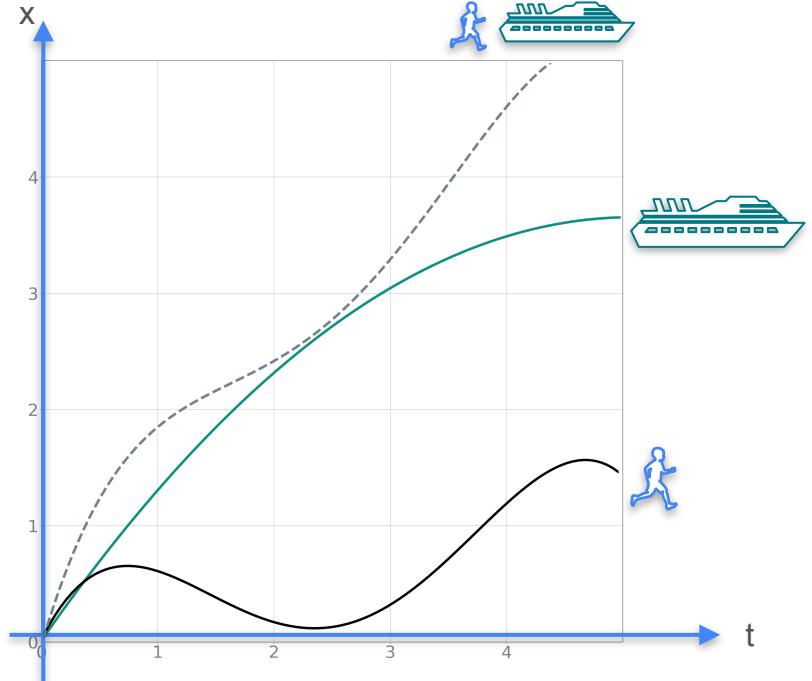
Sum Rule



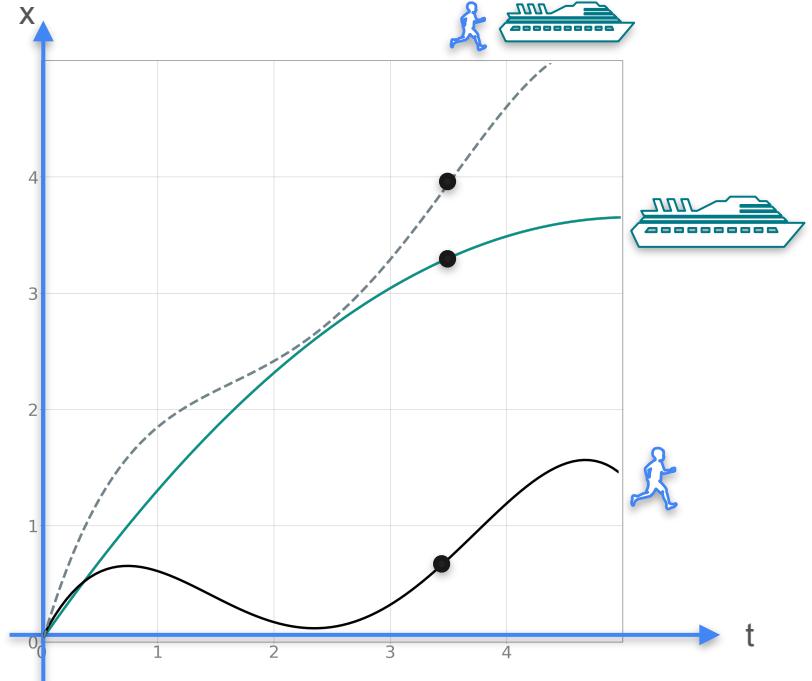
Sum Rule



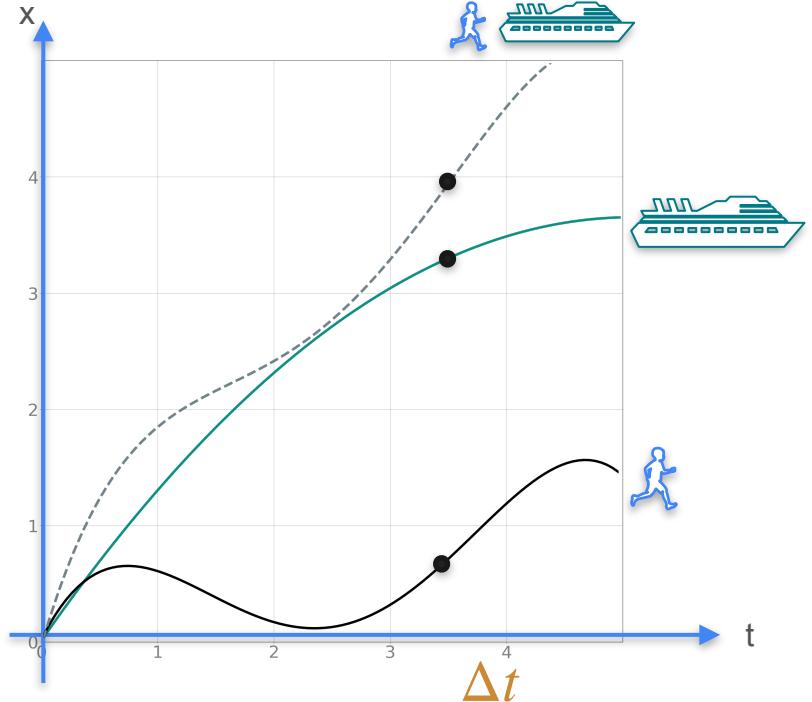
Sum Rule



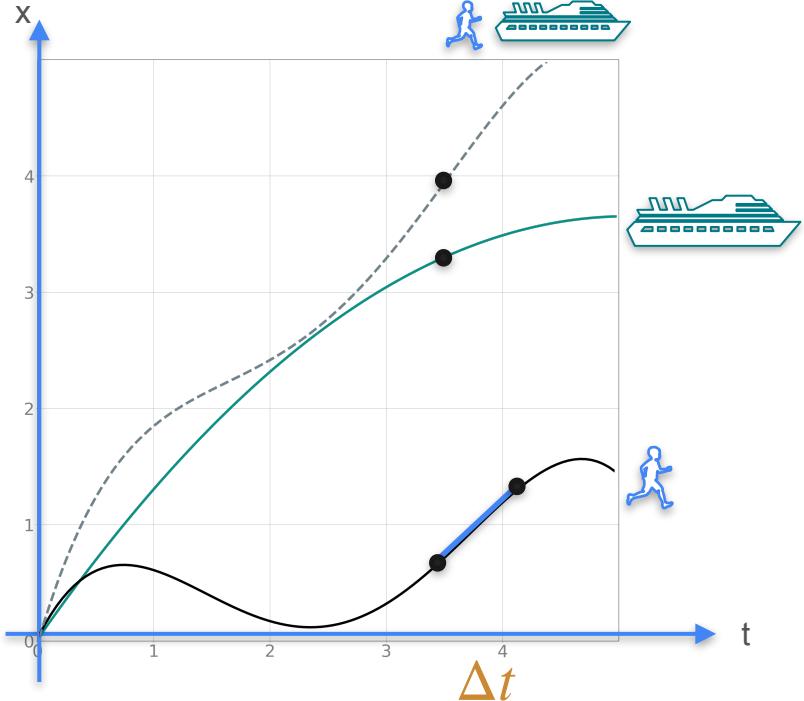
Sum Rule



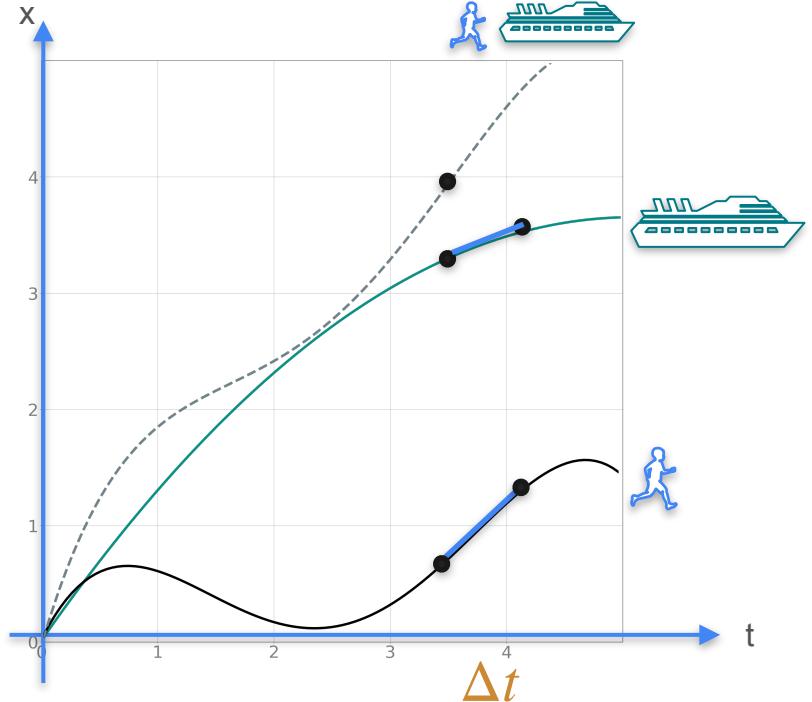
Sum Rule



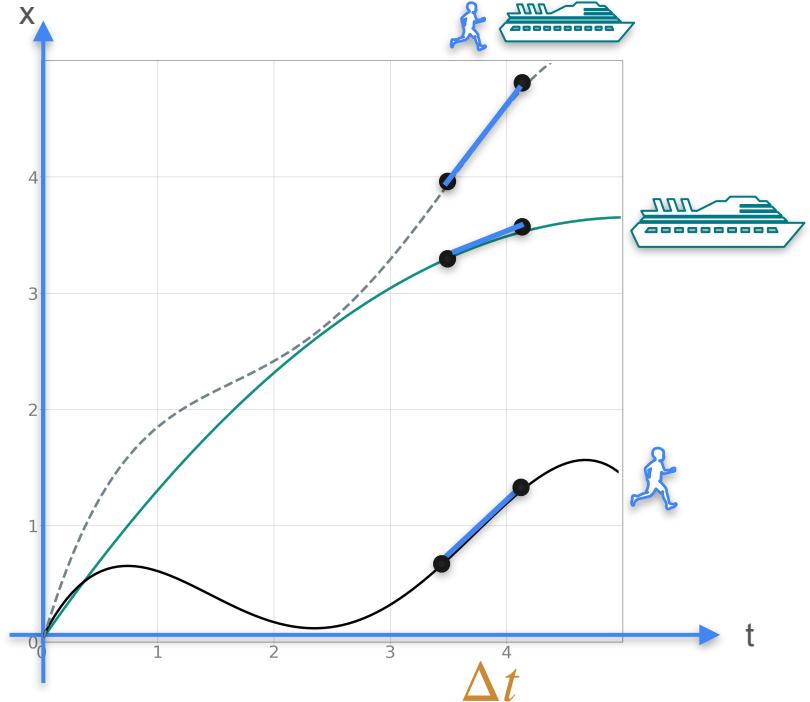
Sum Rule



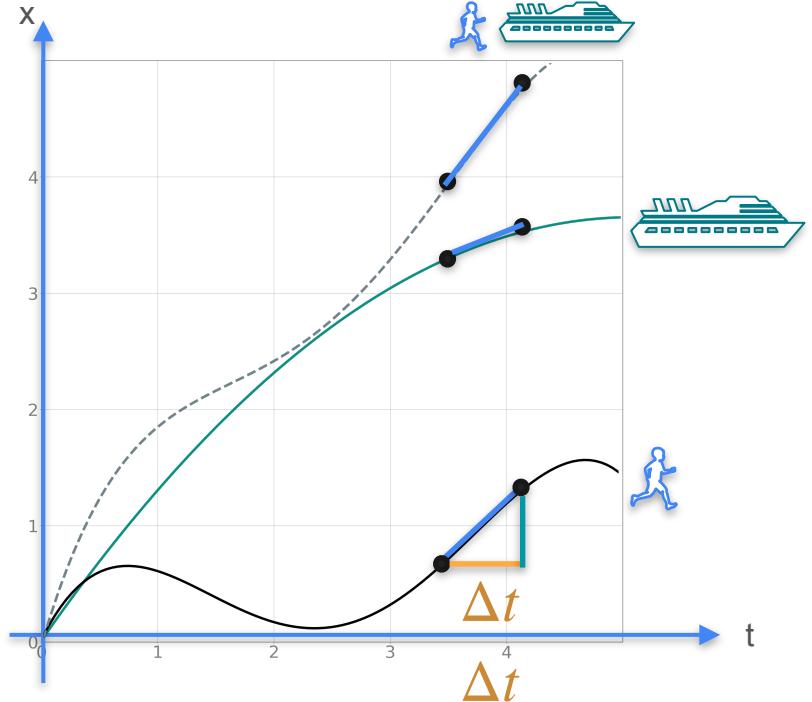
Sum Rule



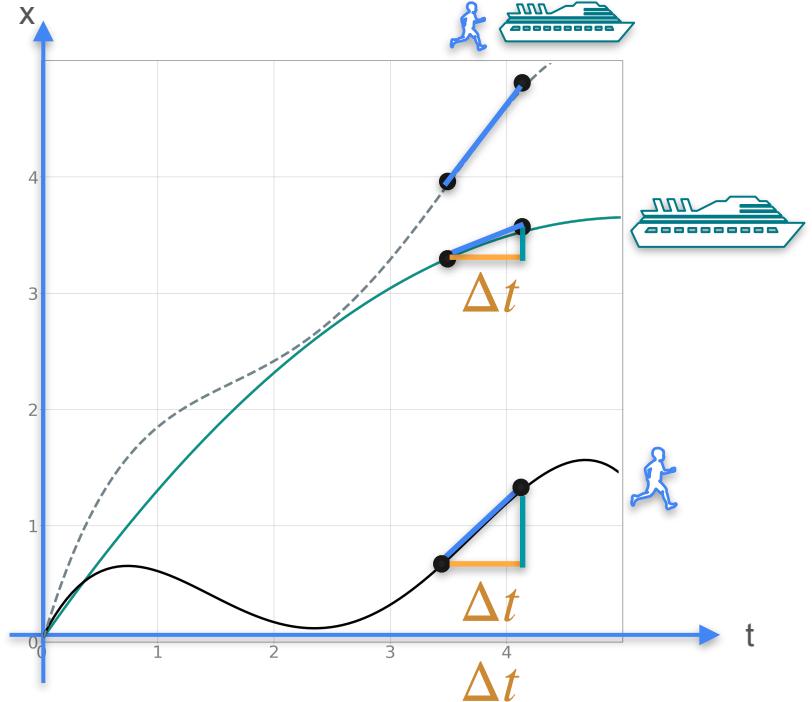
Sum Rule



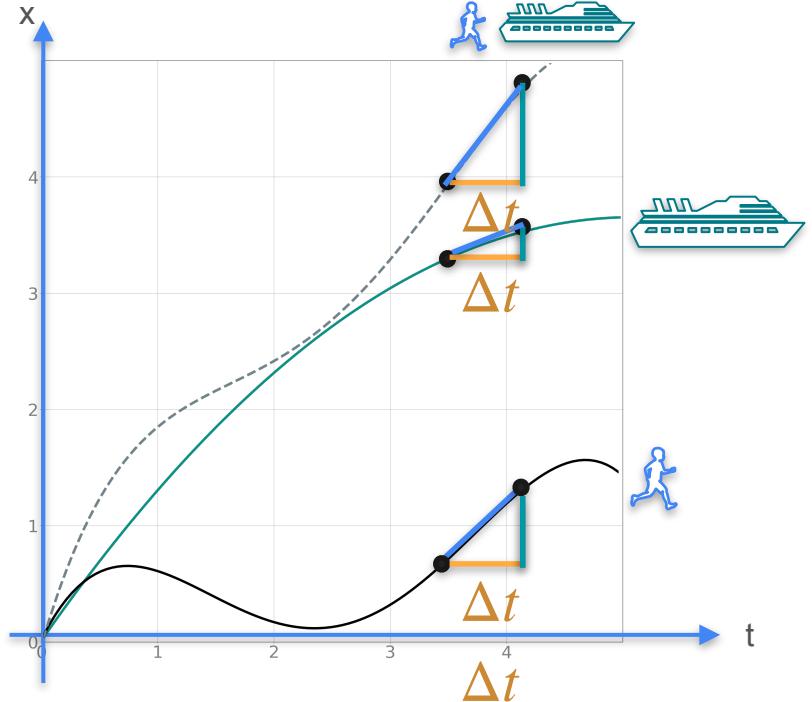
Sum Rule



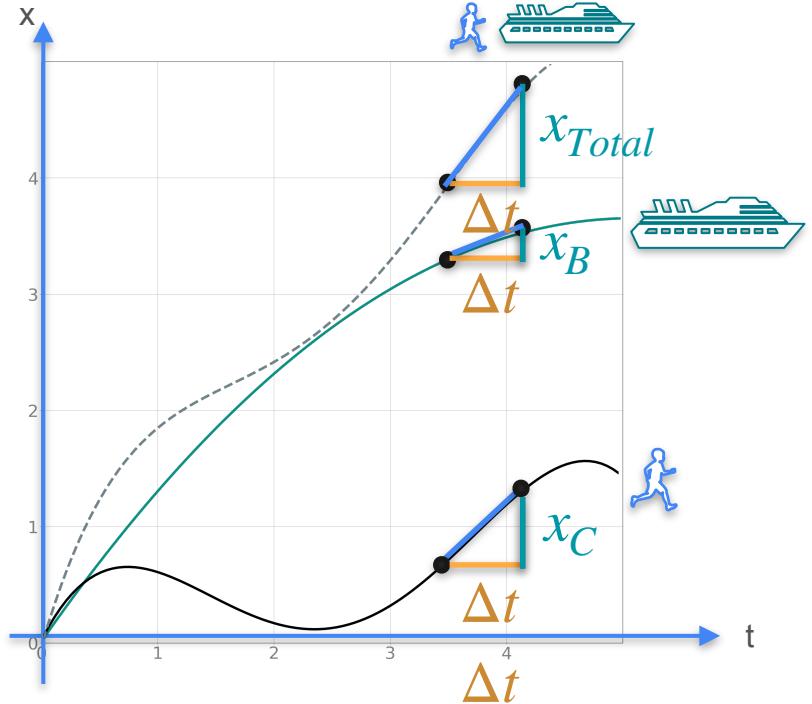
Sum Rule



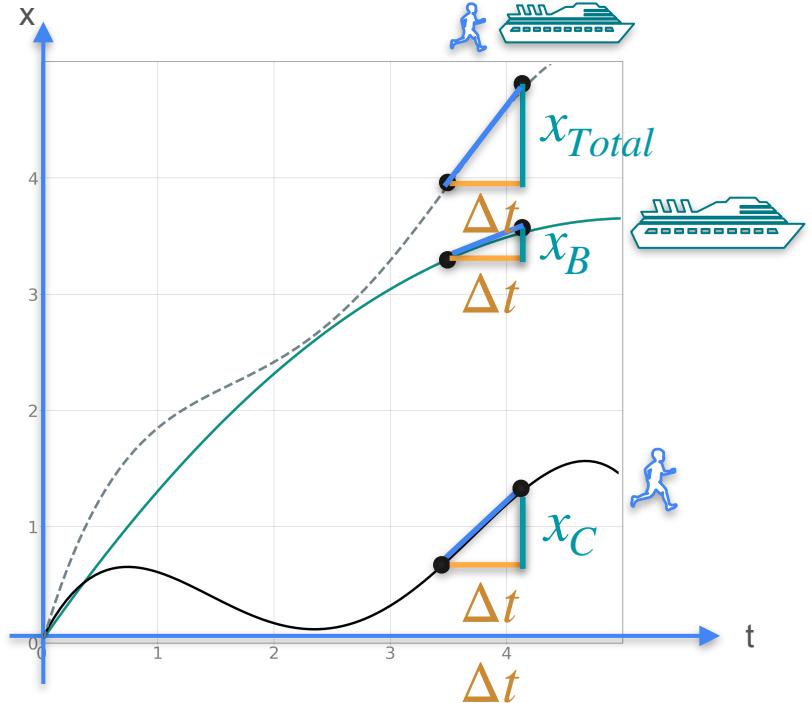
Sum Rule



Sum Rule

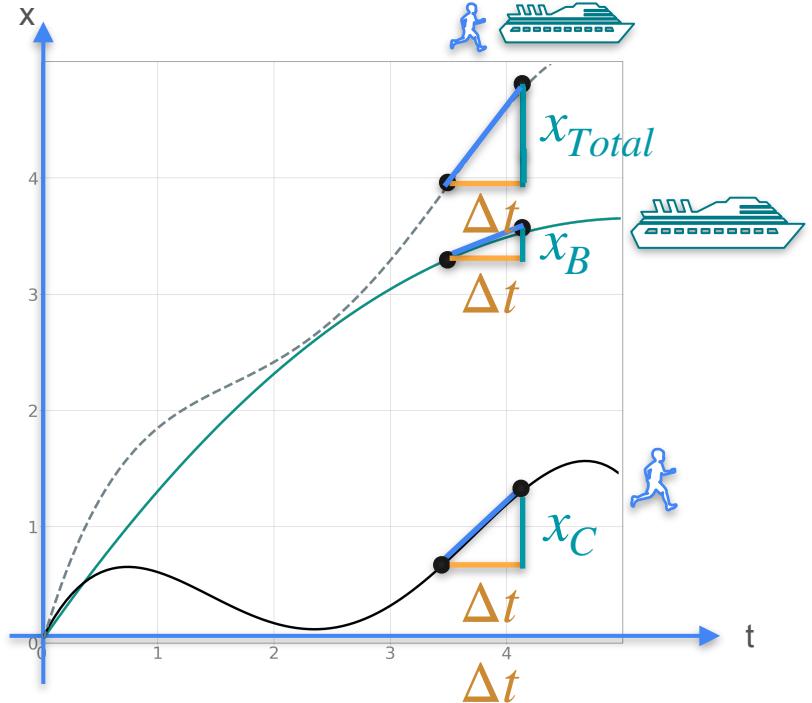


Sum Rule



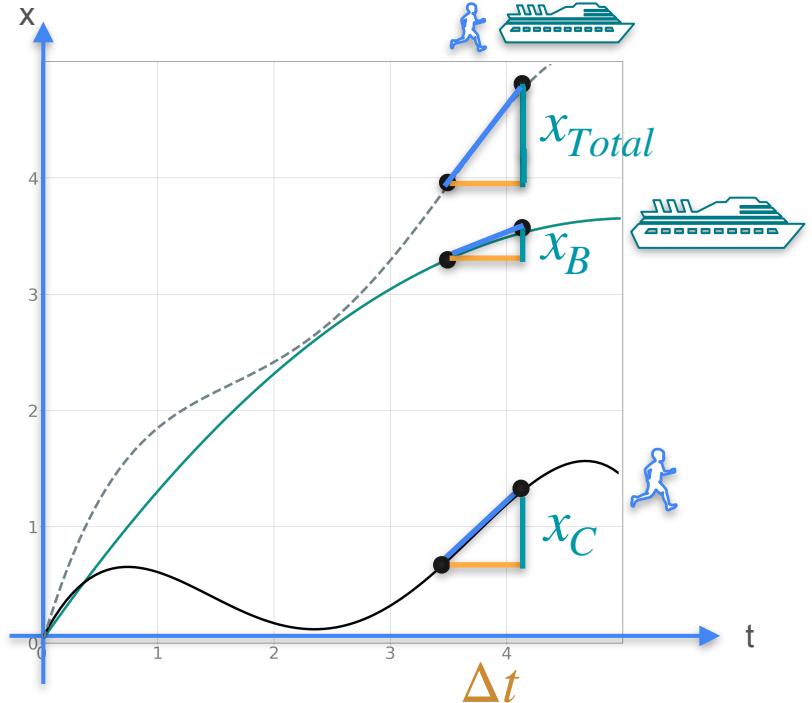
$$x_{Total} = x_B + x_C$$

Sum Rule



$$x_{Total} = x_B + x_C$$

Sum Rule

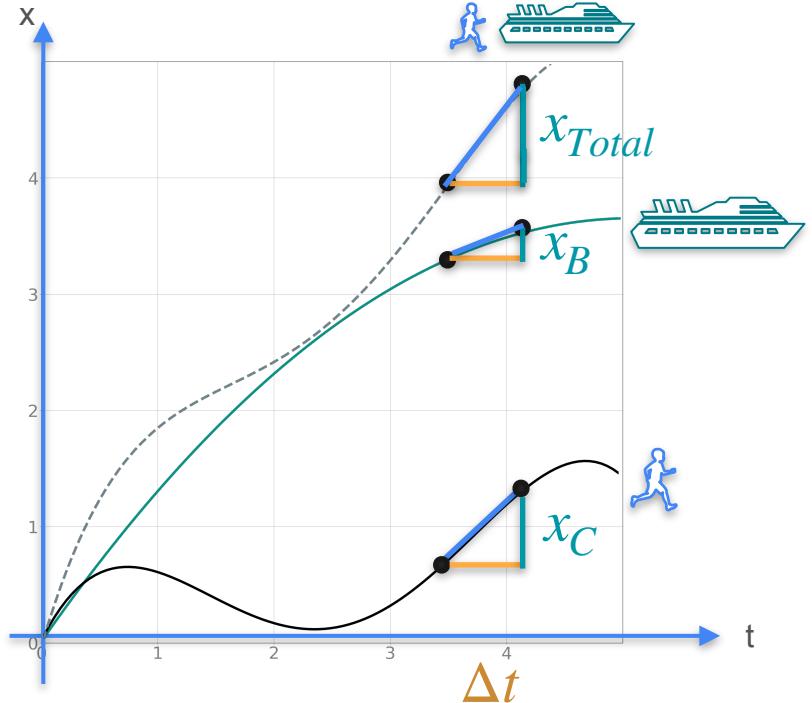


$$x_{Total} = x_B + x_C$$



$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$

Sum Rule



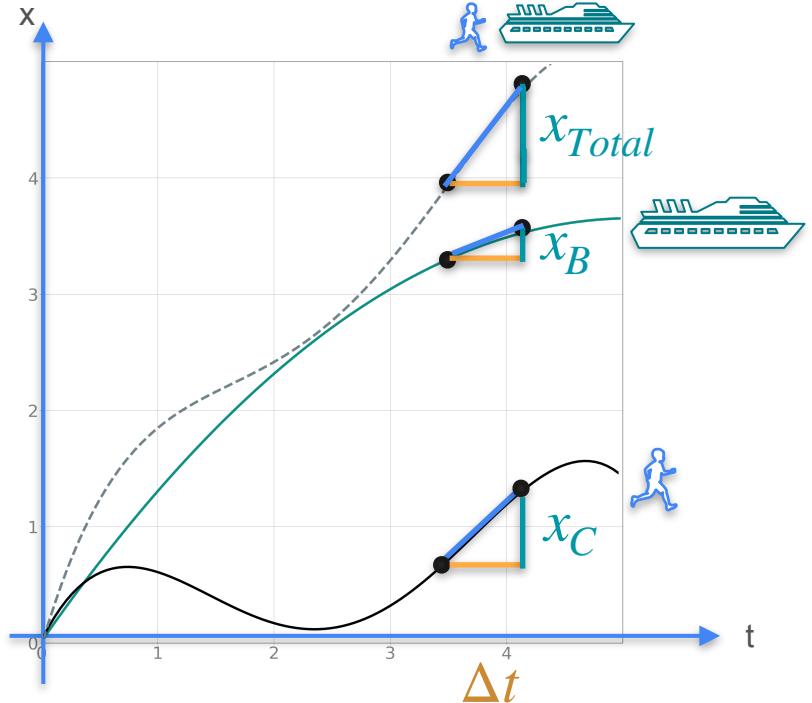
$$x_{Total} = x_B + x_C$$



$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$



Sum Rule



$$x_{Total} = x_B + x_C$$

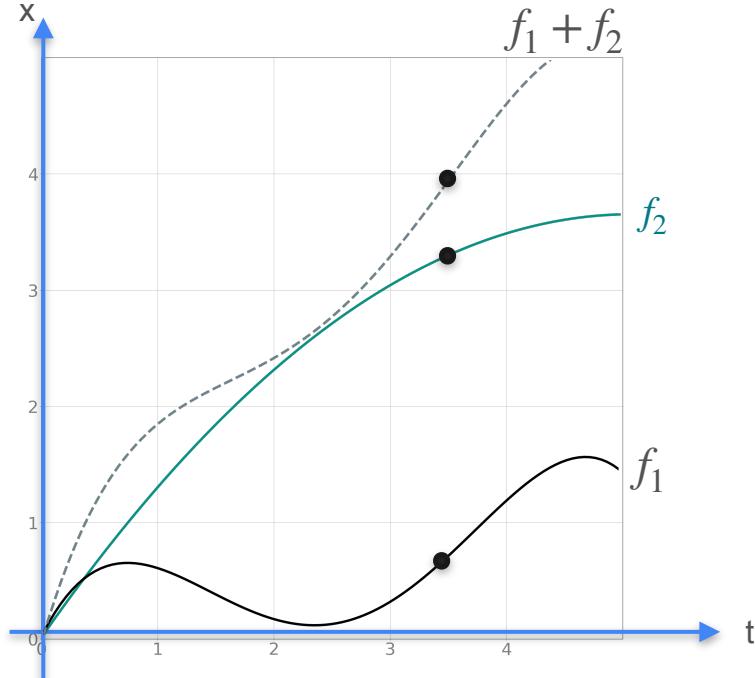


$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$

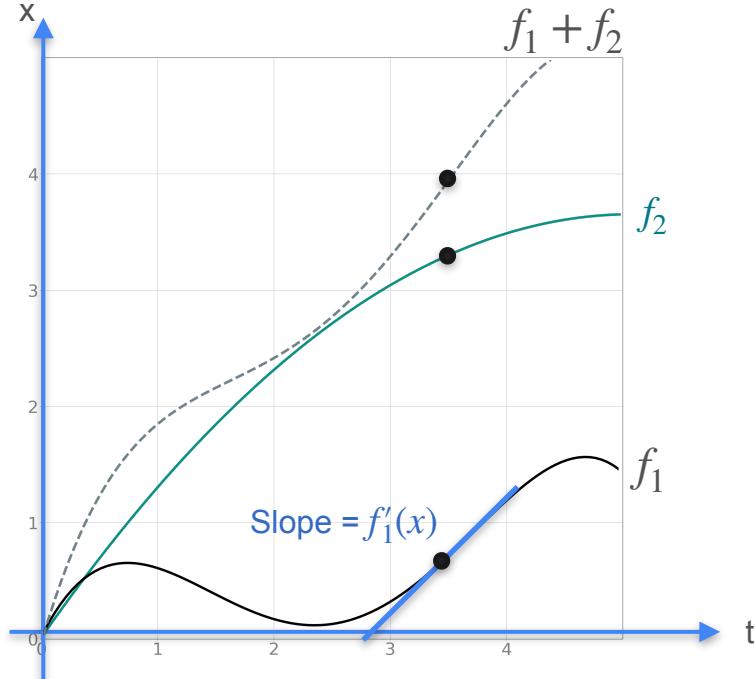


$$v_{Total} = v_B + v_C$$

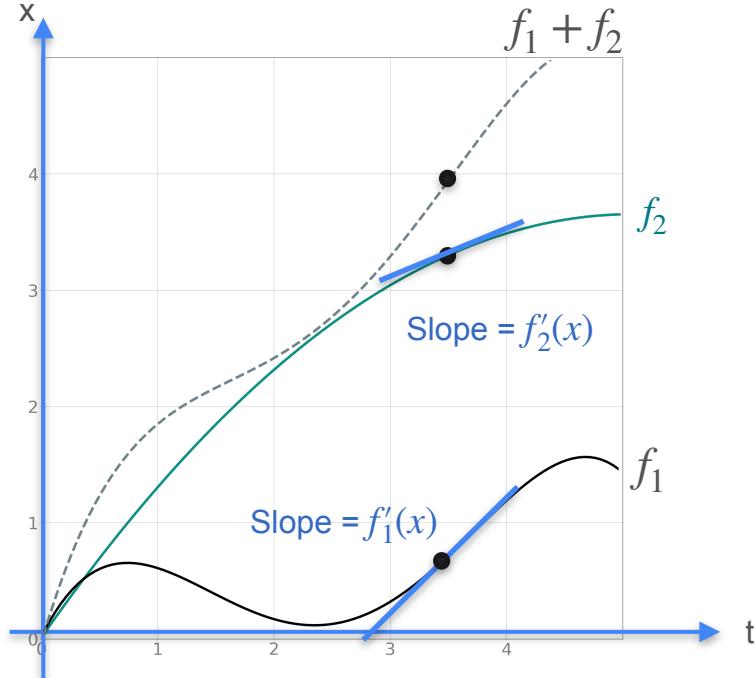
Sum Rule



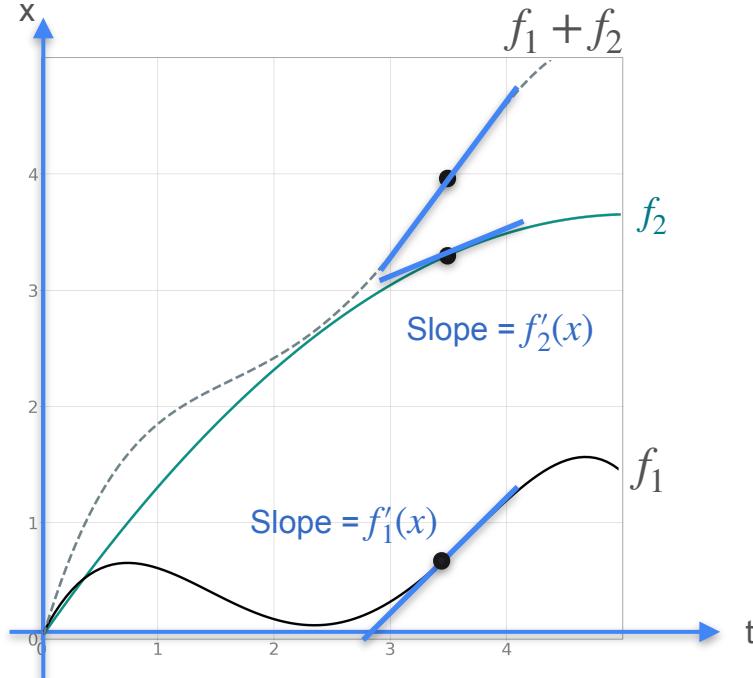
Sum Rule



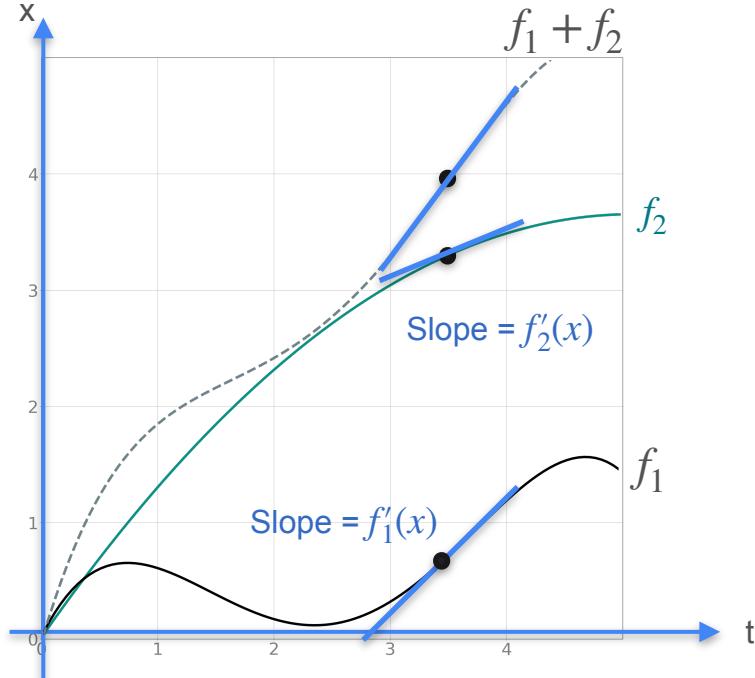
Sum Rule



Sum Rule

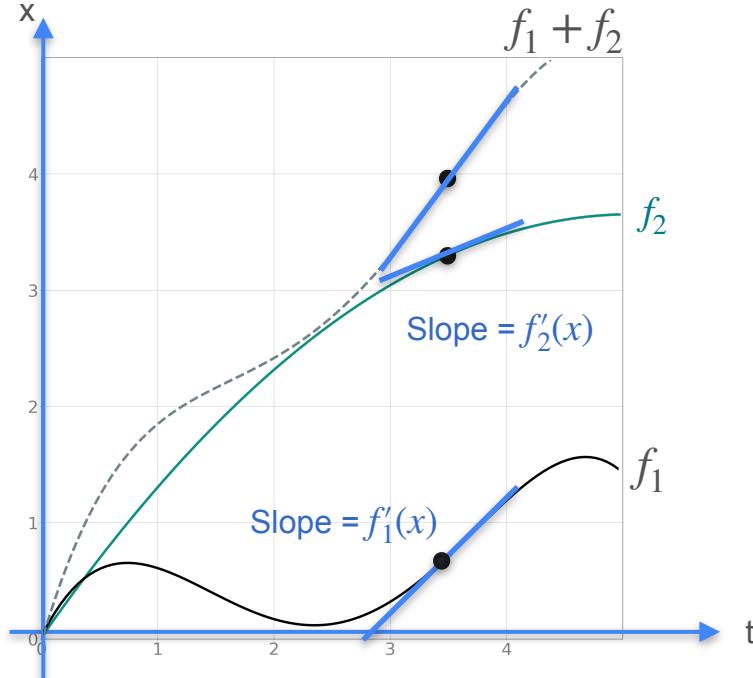


Sum Rule



$$f = f_1 + f_2$$

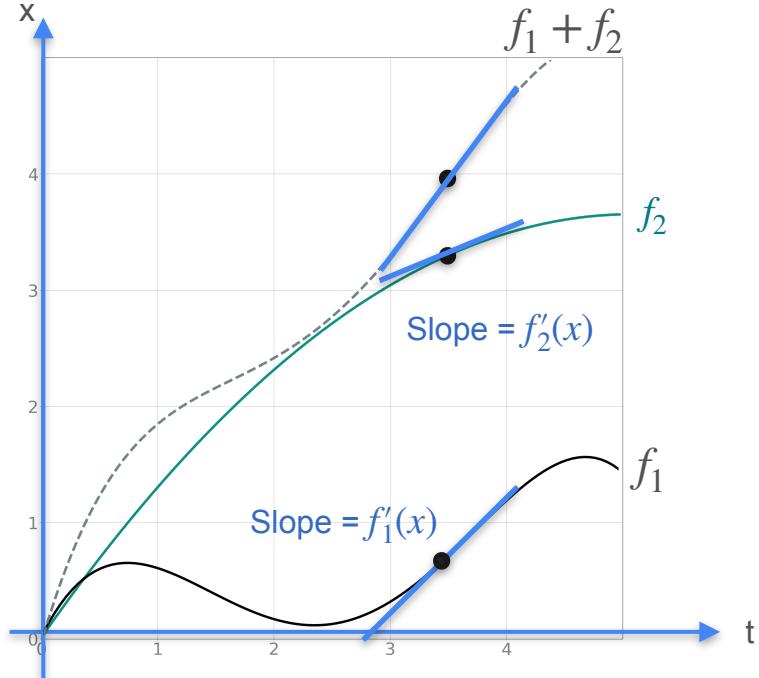
Sum Rule



$$f = f_1 + f_2$$



Sum Rule

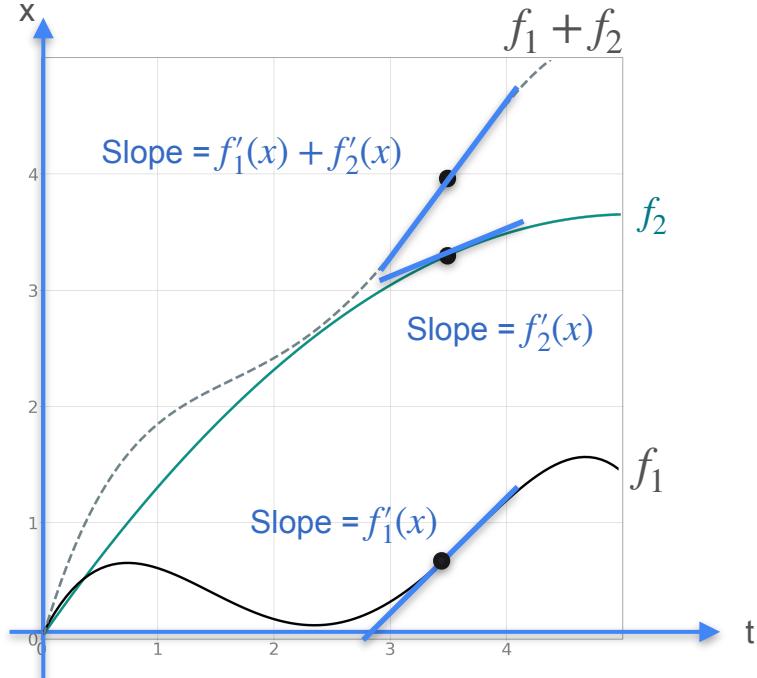


$$f = f_1 + f_2$$



$$f' = f'_1 + f'_2$$

Sum Rule



$$f = f_1 + f_2$$



$$f' = f'_1 + f'_2$$



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Derivatives and Optimization

**Properties of the derivative:
The product rule**

The Product Rule

$$f = gh$$



The Product Rule

$$f = gh$$



$$f' = g'h$$

The Product Rule

$$f = gh$$


$$f' = g'h + gh'$$

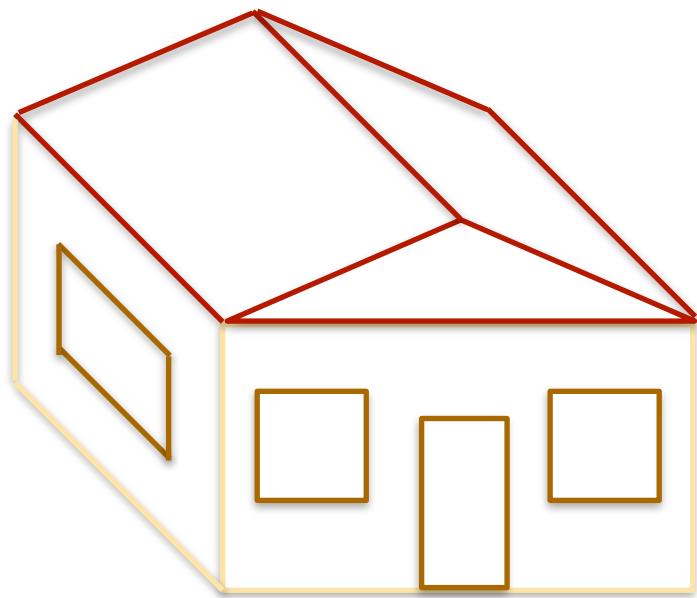
The Product Rule

$$f = gh$$

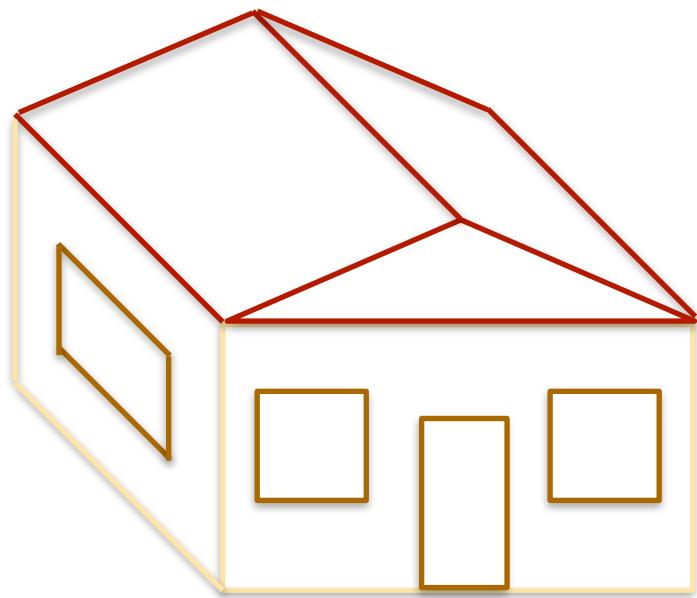
$$f' = g'h + gh'$$

Product Rule

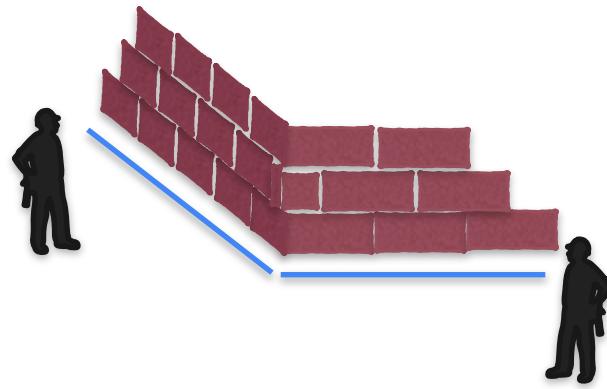
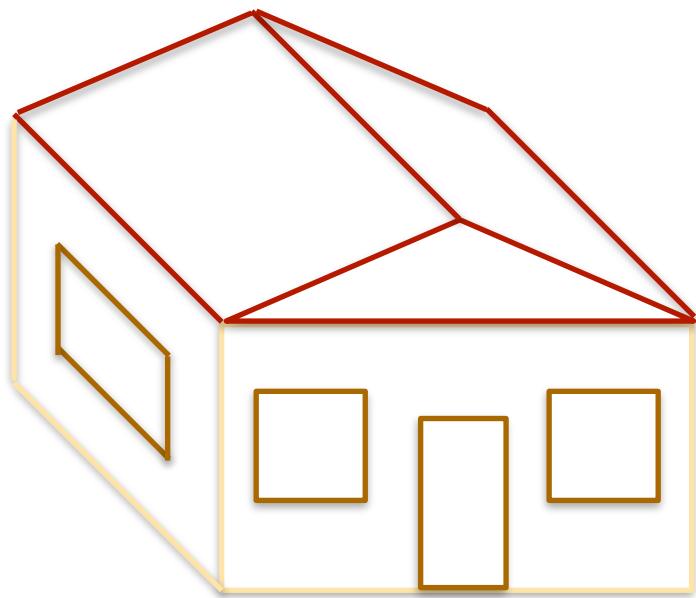
Product Rule



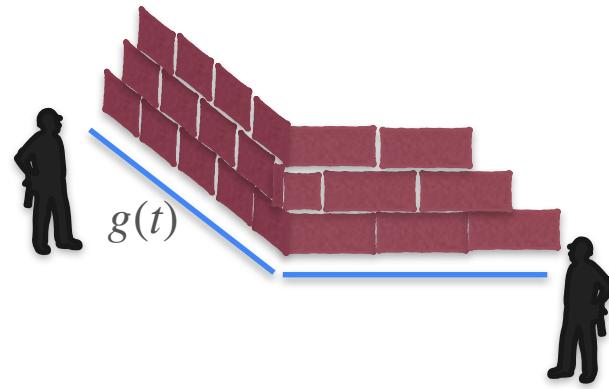
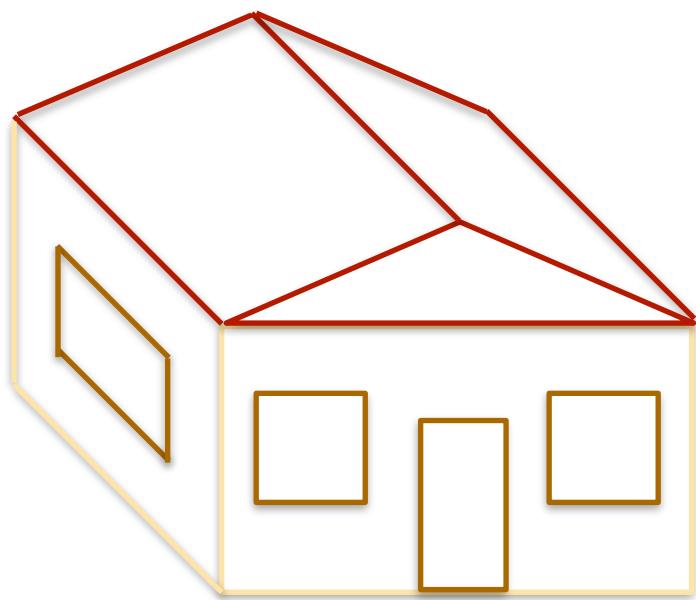
Product Rule



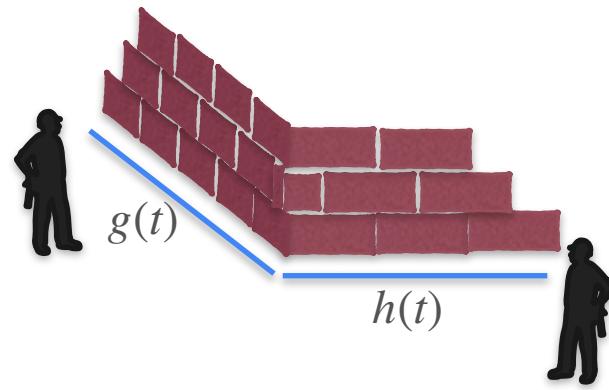
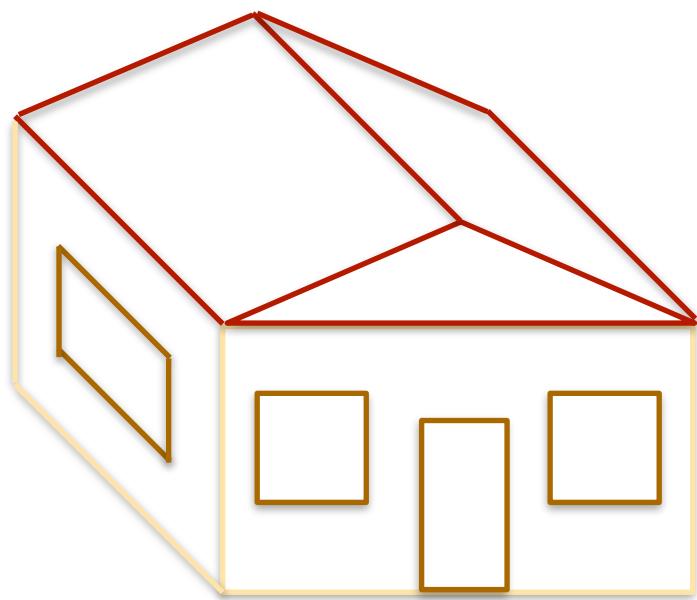
Product Rule



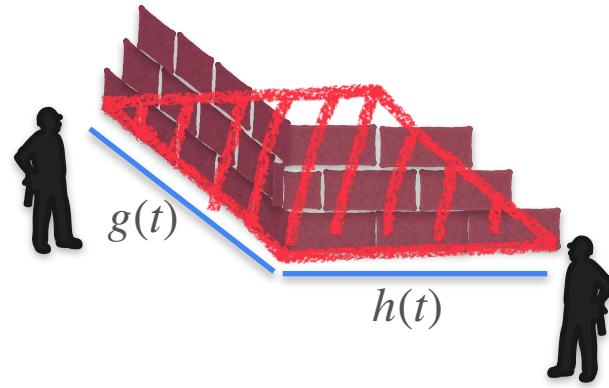
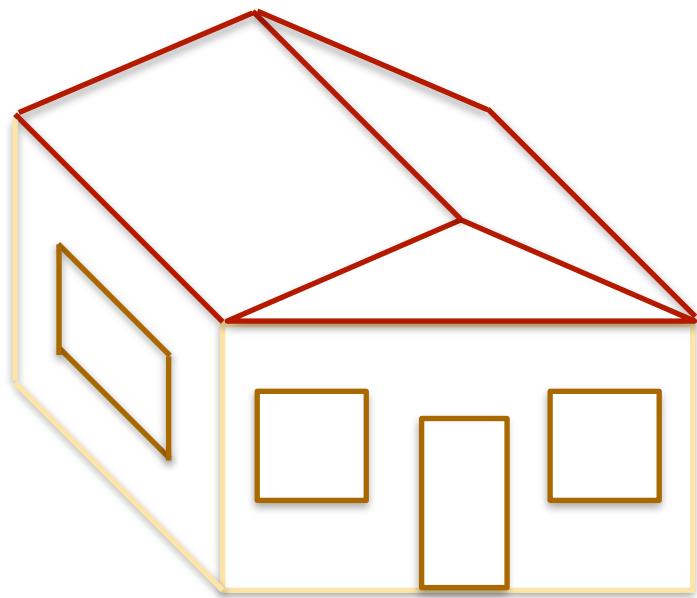
Product Rule



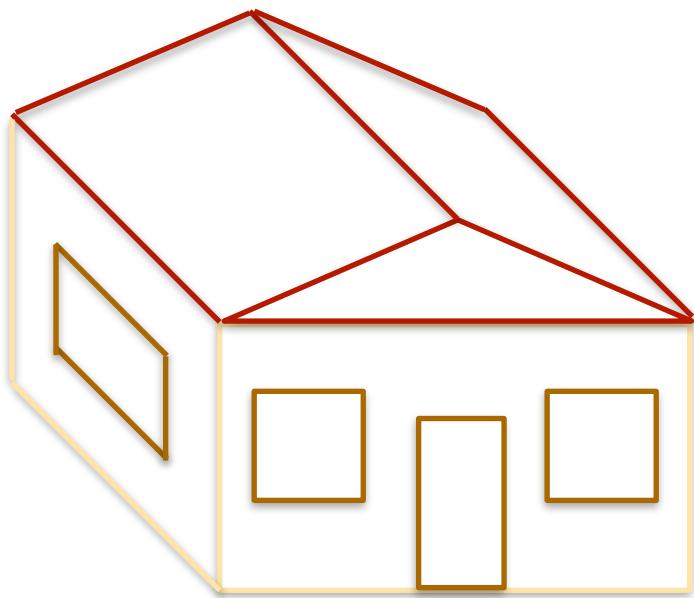
Product Rule



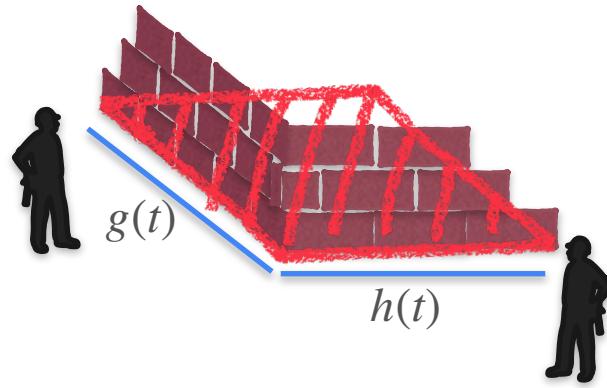
Product Rule



Product Rule



$$f(t) = \underline{g(t)h(t)}$$



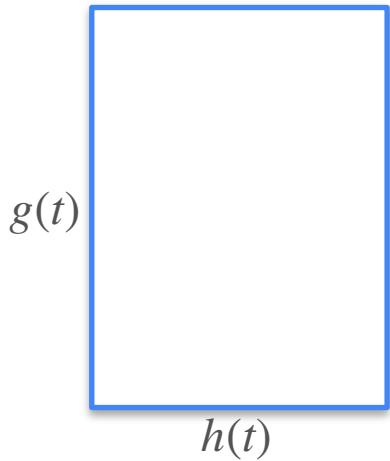
Product Rule

Product Rule

$$y = f(t) = g(t)h(t)$$

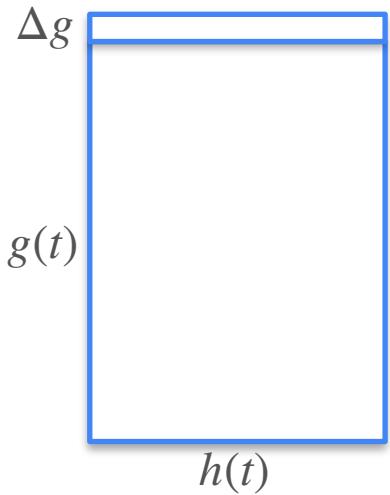
Product Rule

$$y = f(t) = g(t)h(t)$$



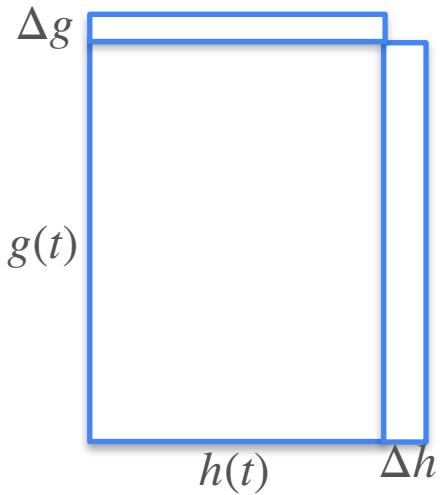
Product Rule

$$y = f(t) = g(t)h(t)$$



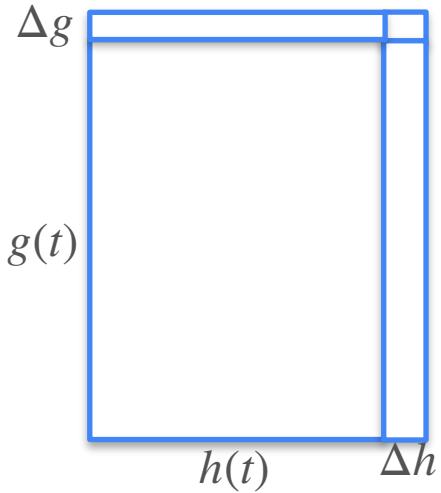
Product Rule

$$y = f(t) = g(t)h(t)$$



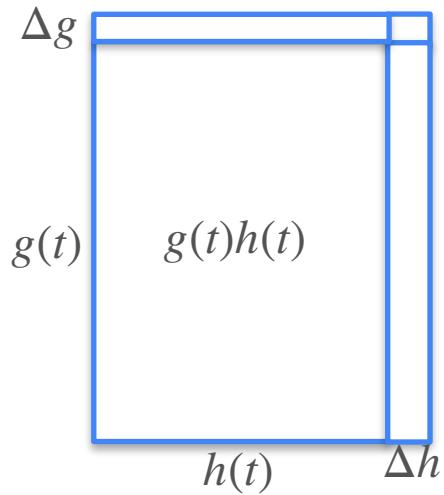
Product Rule

$$y = f(t) = g(t)h(t)$$



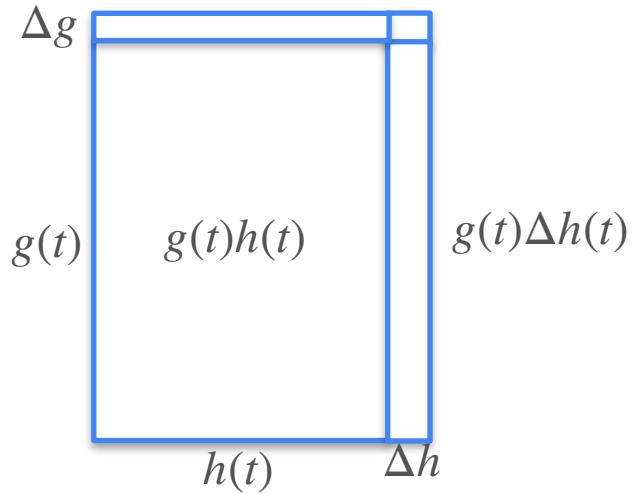
Product Rule

$$y = f(t) = g(t)h(t)$$



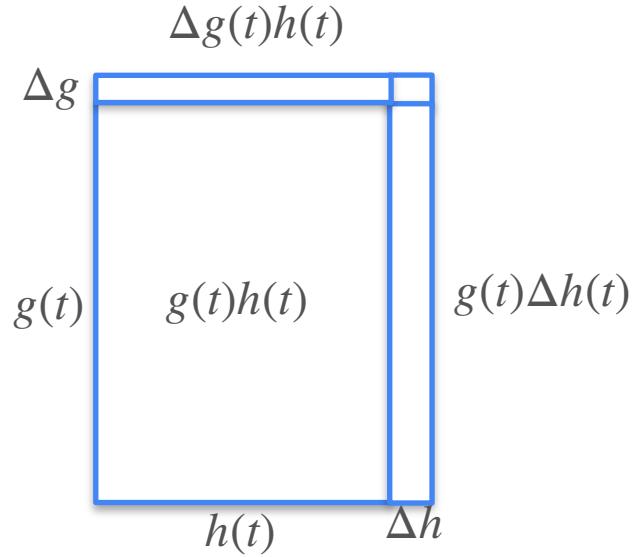
Product Rule

$$y = f(t) = g(t)h(t)$$

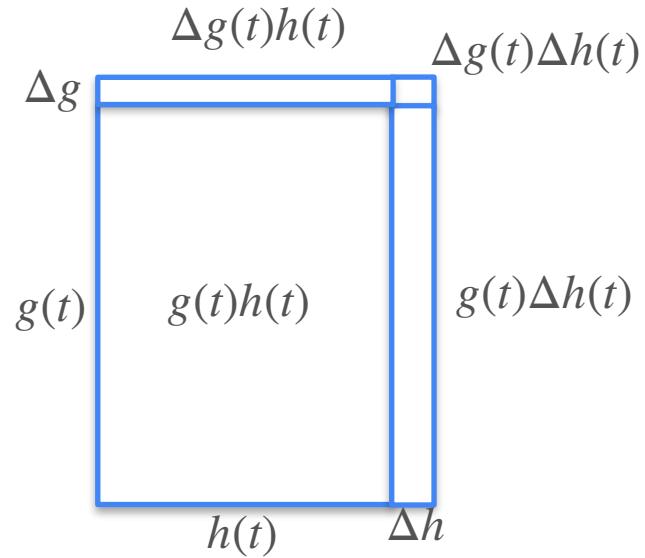


Product Rule

$$y = f(t) = g(t)h(t)$$

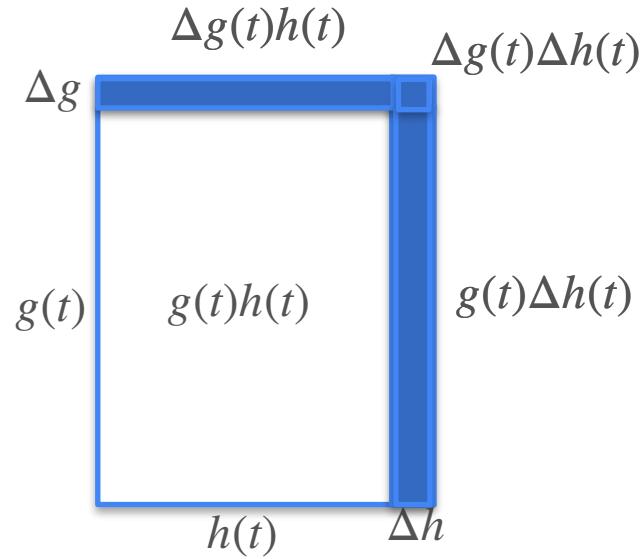


Product Rule



$$y = f(t) = g(t)h(t)$$

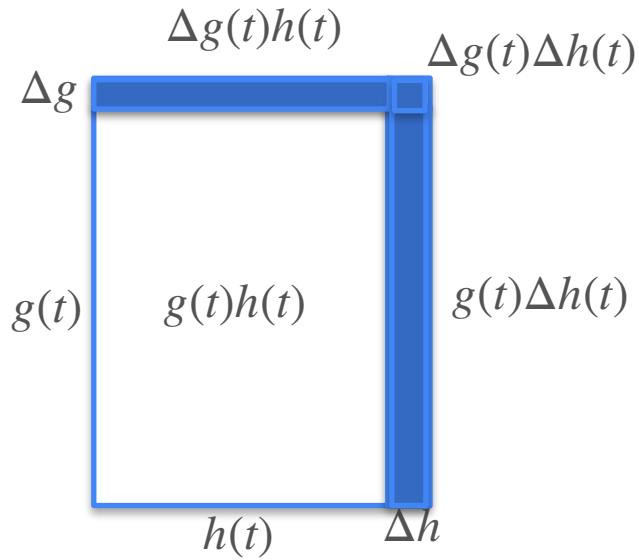
Product Rule



$$y = f(t) = g(t)h(t)$$

$$\frac{\Delta f(t)}{\Delta t}$$

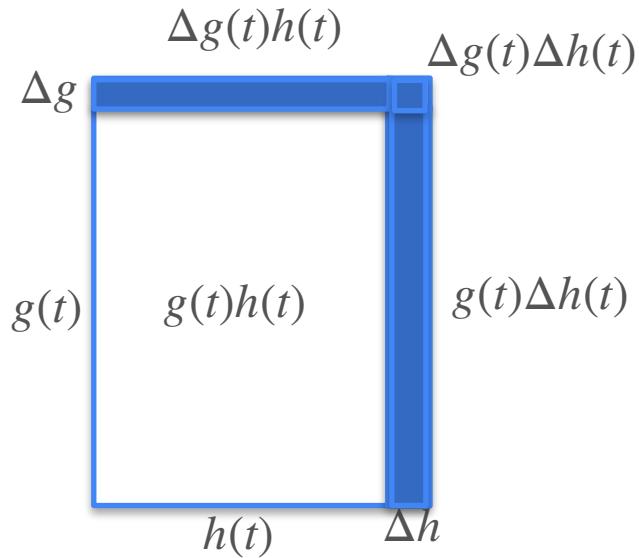
Product Rule



$$y = f(t) = g(t)h(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

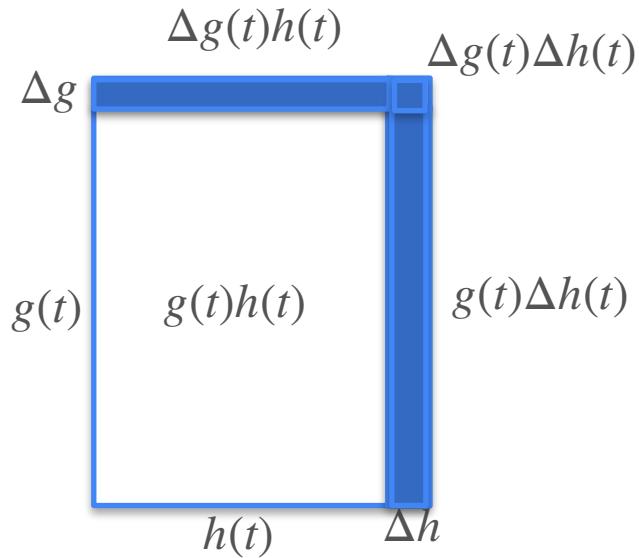
Product Rule



$$y = f(t) = g(t)h(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

Product Rule

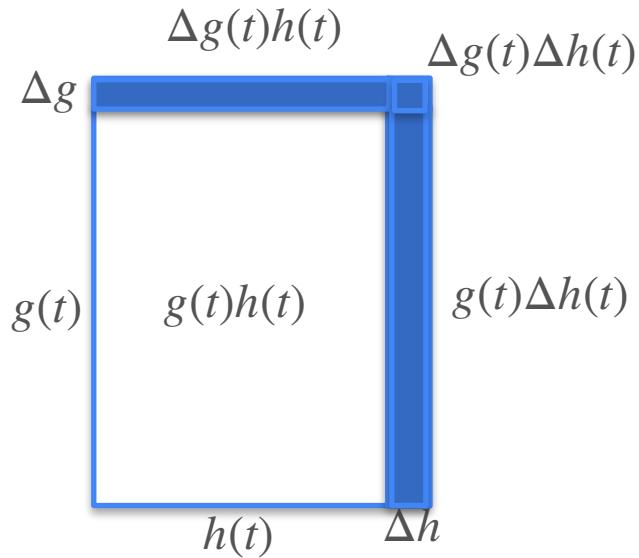


$$y = f(t) = g(t)h(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

as $\Delta t \rightarrow 0$

Product Rule



$$y = f(t) = g(t)h(t)$$

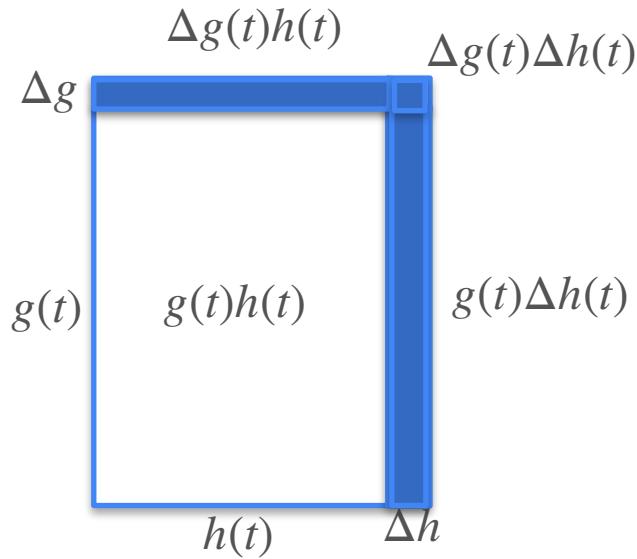
$$f'(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

$$= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}$$

as $\Delta t \rightarrow 0$

Product Rule



$$y = f(t) = g(t)h(t)$$

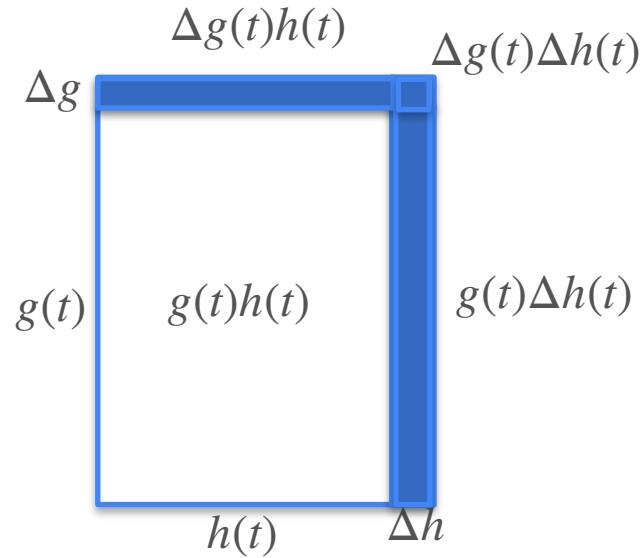
$$f'(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

$$= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}$$

as $\Delta t \rightarrow 0$

Product Rule



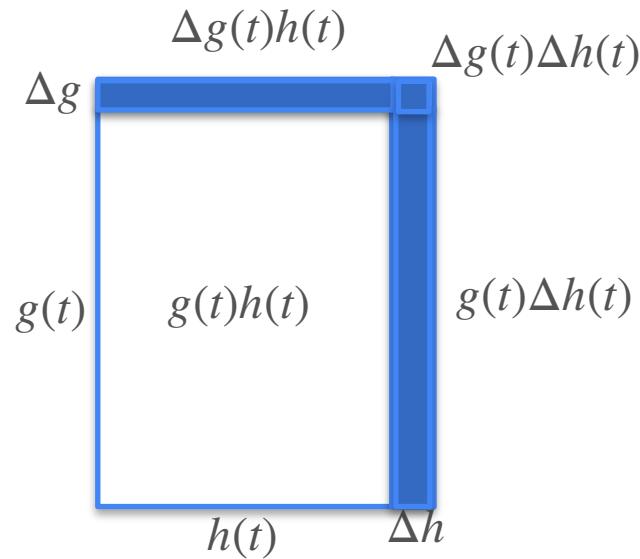
$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

as $\Delta t \rightarrow 0$

Product Rule



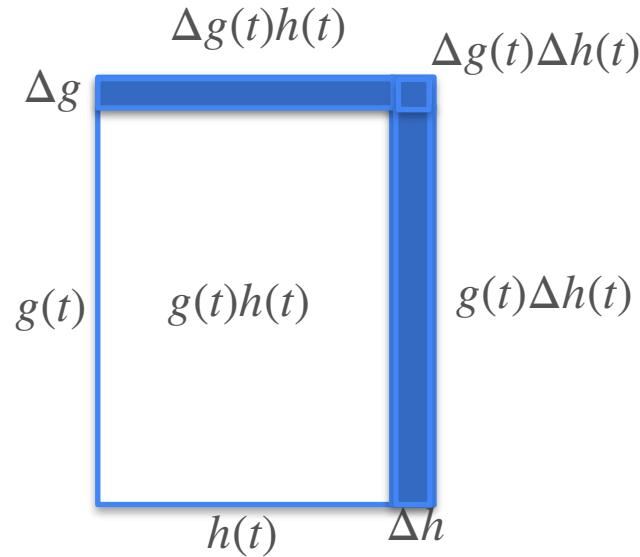
$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{g'(t)h(t)}{\Delta t} + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

as $\Delta t \rightarrow 0$

Product Rule



$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{g'(t)h(t)}{\Delta t} + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t} \\ &\quad \text{as } \Delta t \rightarrow 0\end{aligned}$$

$$f'(t) = g'(t)h(t) + g(t)h'(t)$$



DeepLearning.AI

Derivatives and Optimization

**Properties of the derivative:
The chain rule**

The Chain Rule

The Chain Rule

$$h(t)$$

The Chain Rule

$$g(h(t))$$

The Chain Rule

$$\frac{d}{dt} g(h(t))$$

The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dh}{dt}$$

The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \frac{dh}{dt}$$

The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) \\ = \frac{dg}{dh} \cdot \frac{dh}{dt}\end{aligned}$$

The Chain Rule

$$g(h(t))$$

$$\frac{dg}{dh} \frac{dh}{dt}$$

The Chain Rule

$$f(g(h(t)))$$

$$\frac{dg}{dh} \quad \frac{dh}{dt}$$

The Chain Rule

$$\frac{d}{dt} f(g(h(t)))$$

$$\frac{dg}{dh} \frac{dh}{dt}$$

The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \frac{dg}{dh} \frac{dh}{dt}$$

The Chain Rule

$$\begin{aligned} & \frac{d}{dt} f(g(h(t))) \\ &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \end{aligned}$$

The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \cdot \frac{dh}{dt}$$

The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \cdot \frac{dh}{dt}$$

=

The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) &= \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= h'(t)\end{aligned}$$

The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) &= \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= g'(h(t)) \cdot h'(t)\end{aligned}$$

The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

=

The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

= $h'(t)$

The Chain Rule

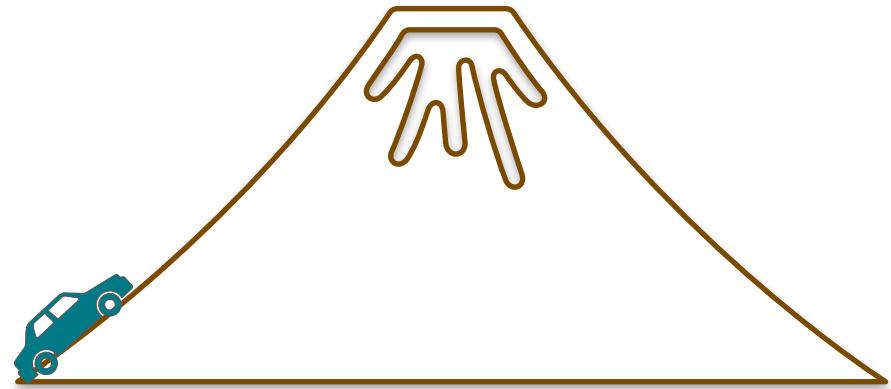
$$\begin{aligned}\frac{d}{dt} f(g(h(t))) &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= g'(h(t)) \cdot h'(t)\end{aligned}$$

The Chain Rule

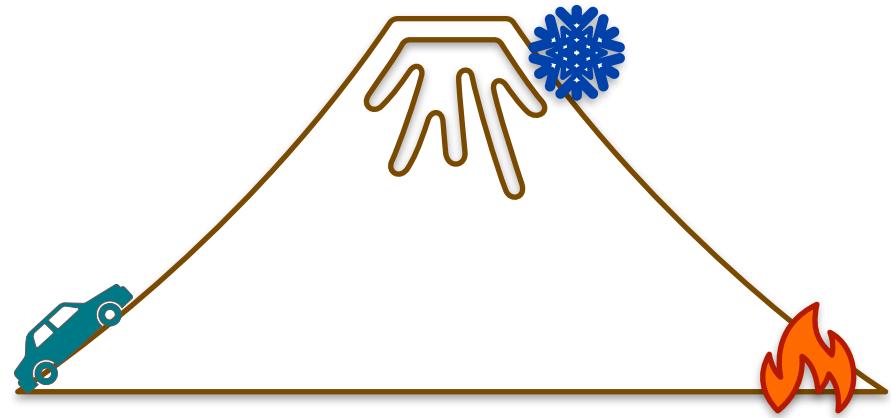
$$\begin{aligned}\frac{d}{dt} f(g(h(t))) &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= f'(g(h(t))) \cdot g'(h(t)) \cdot h'(t)\end{aligned}$$

Idea of Chain Rule

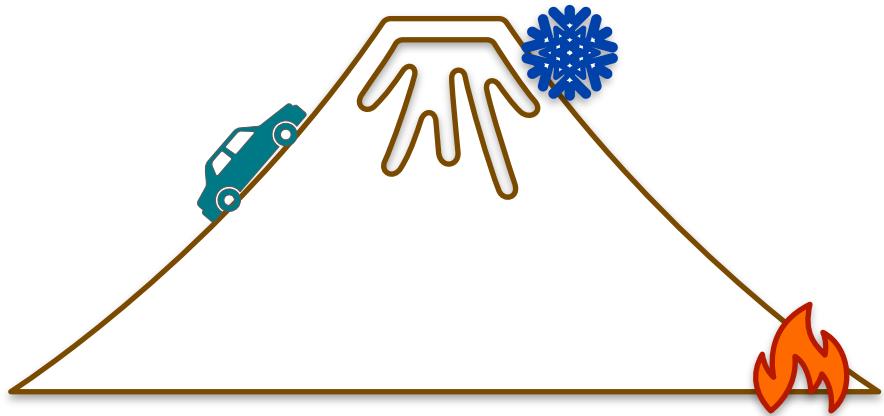
Idea of Chain Rule



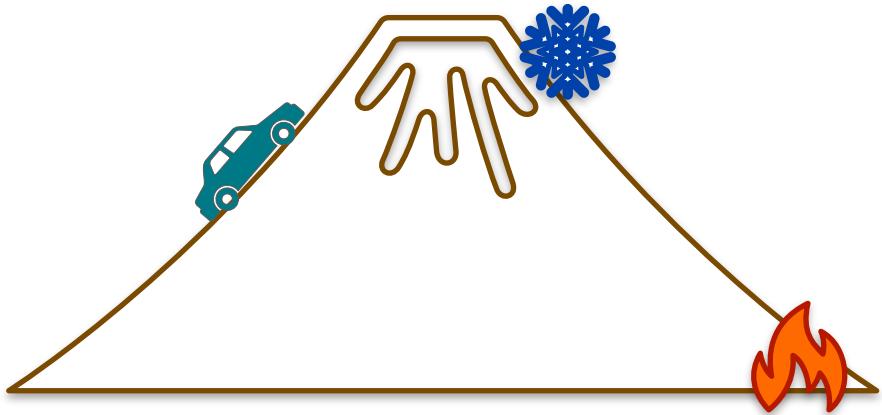
Idea of Chain Rule



Idea of Chain Rule



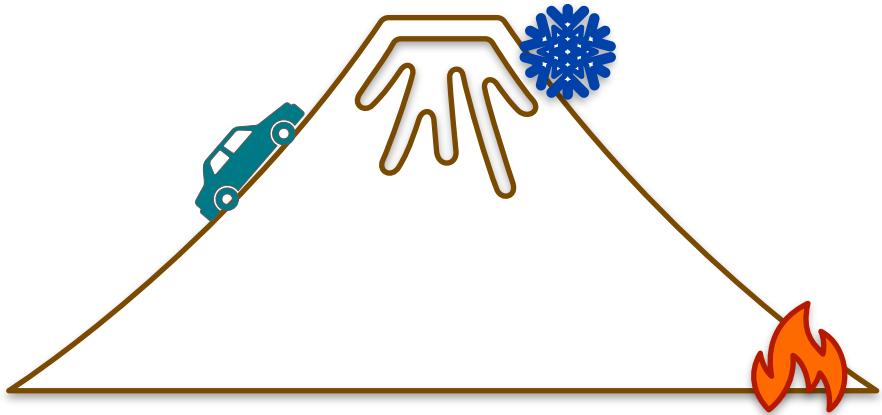
Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

Idea of Chain Rule



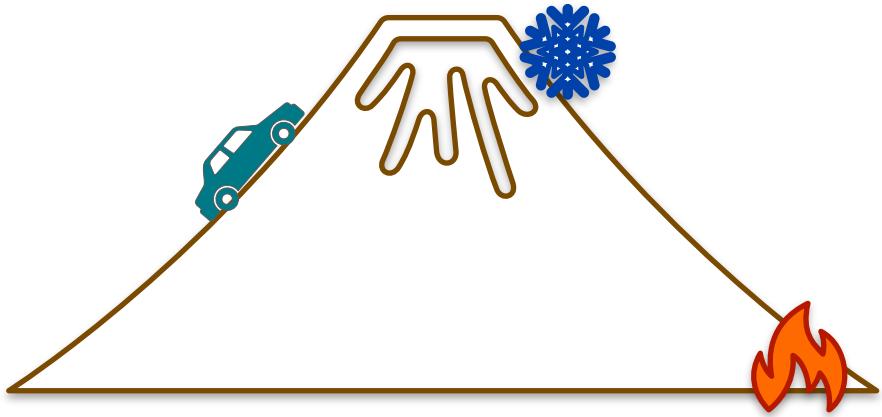
Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

$$\frac{dh}{dt}$$

Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

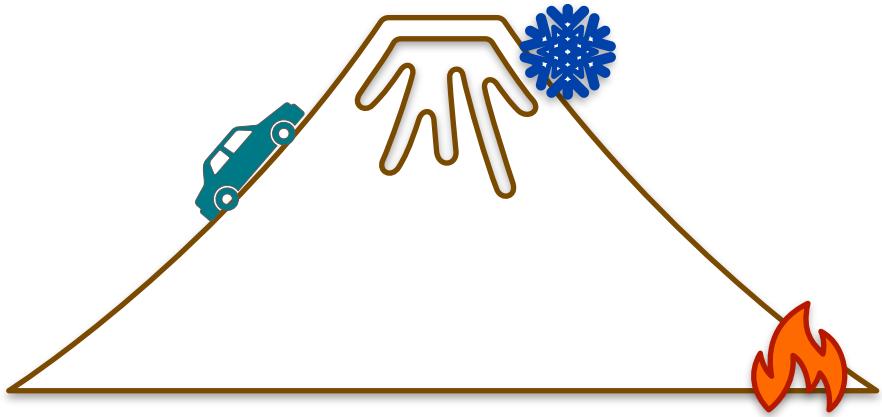
$$\frac{dh}{dt}$$



Temperature changes w.r.t. time

$$\frac{dT}{dt}$$

Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

$$\frac{dh}{dt}$$



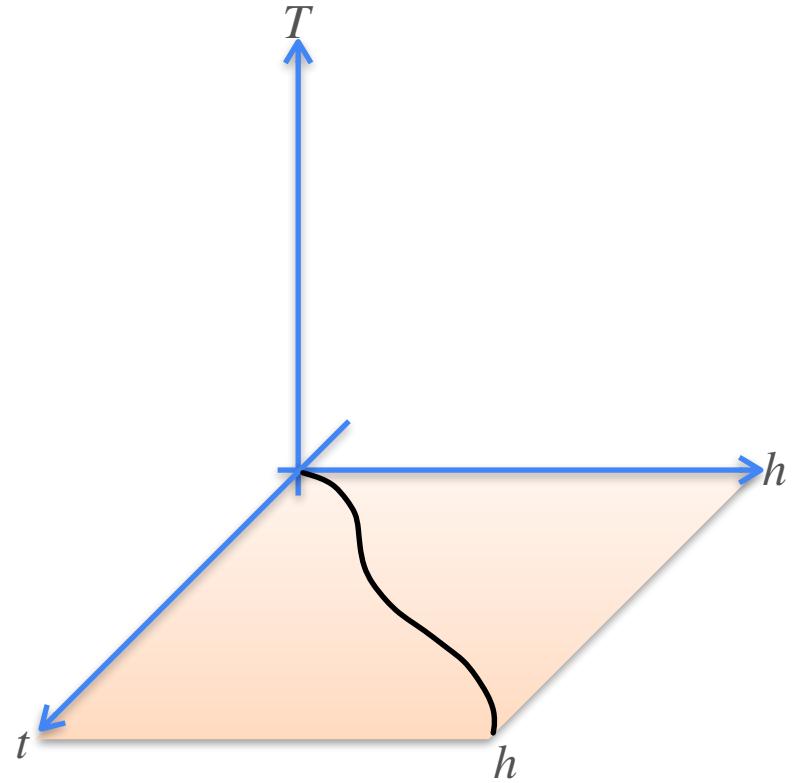
Temperature changes w.r.t. time

$$\frac{dT}{dt}$$

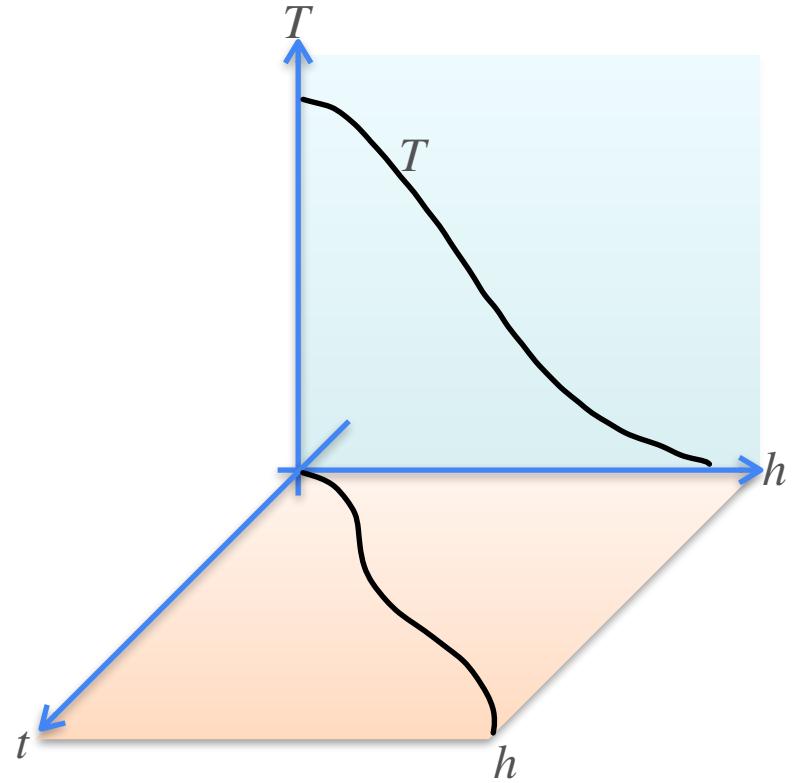
$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$

Chain Rule

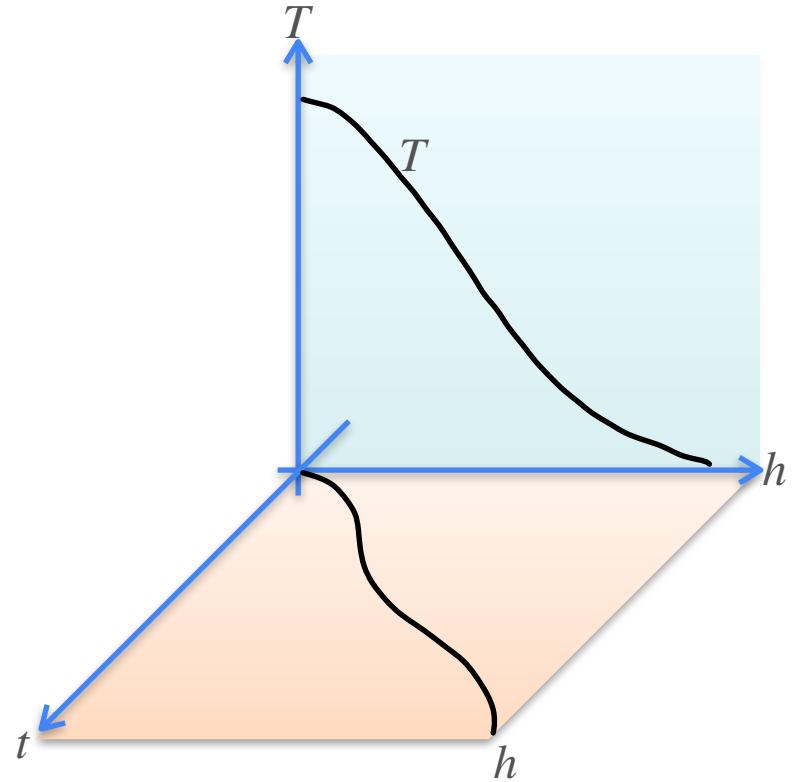
Chain Rule



Chain Rule

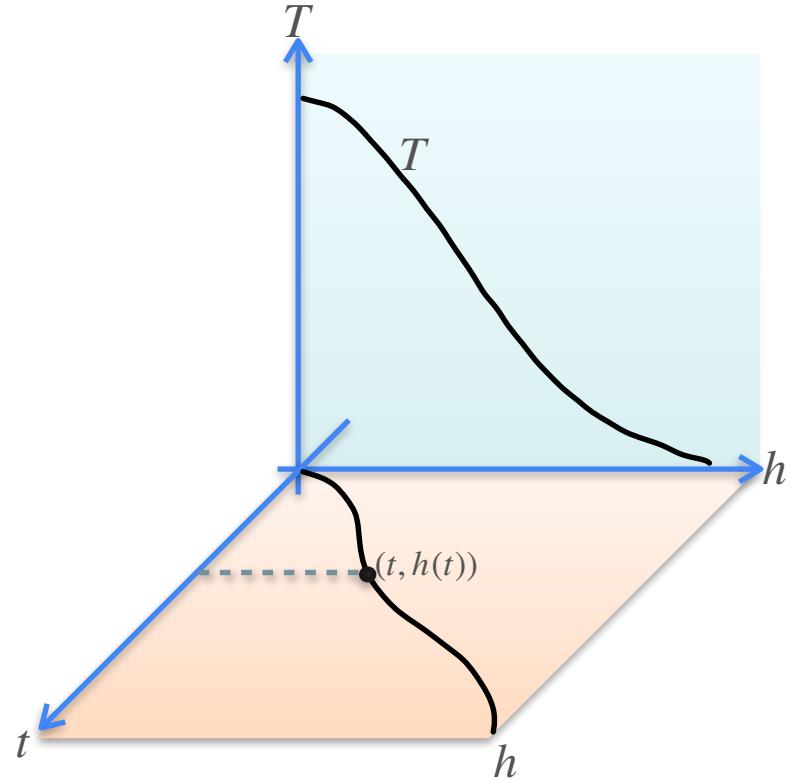


Chain Rule



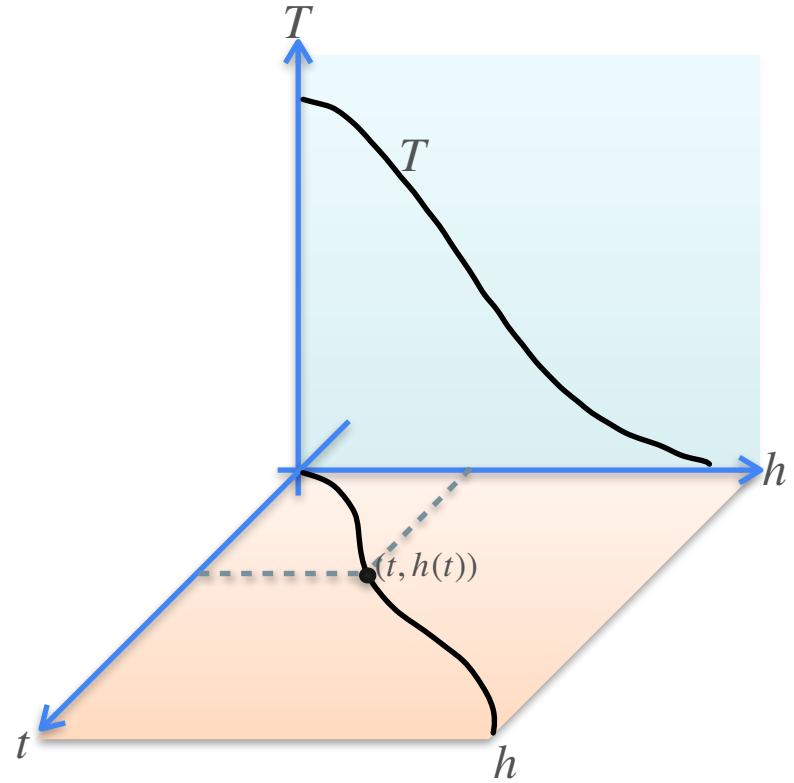
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



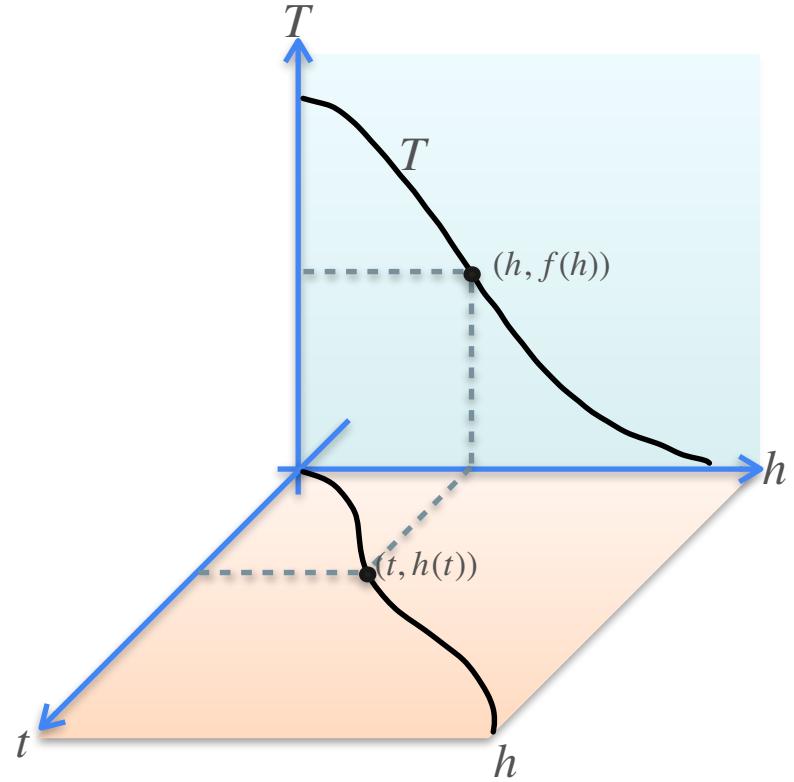
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



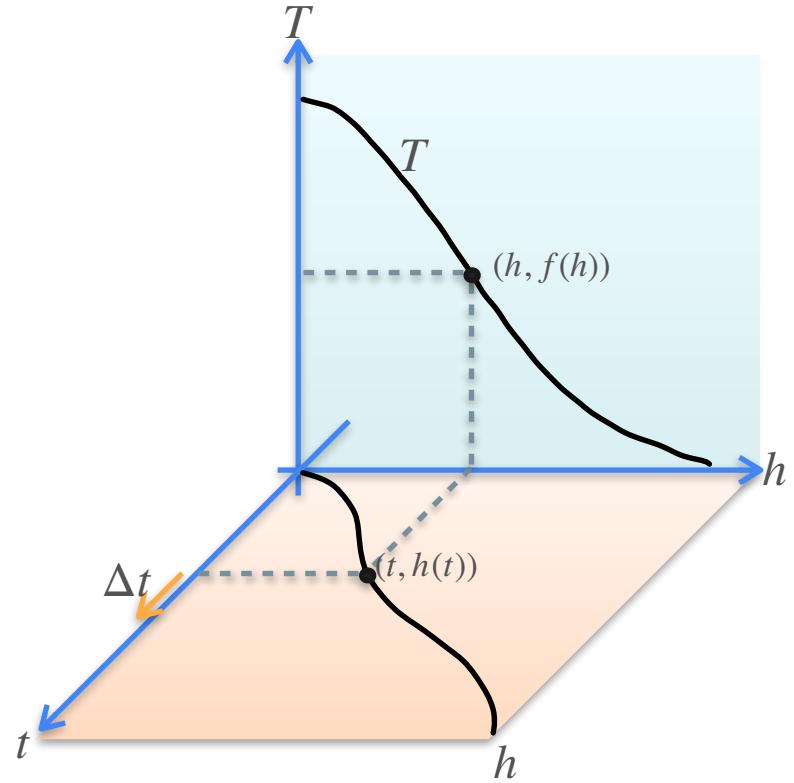
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



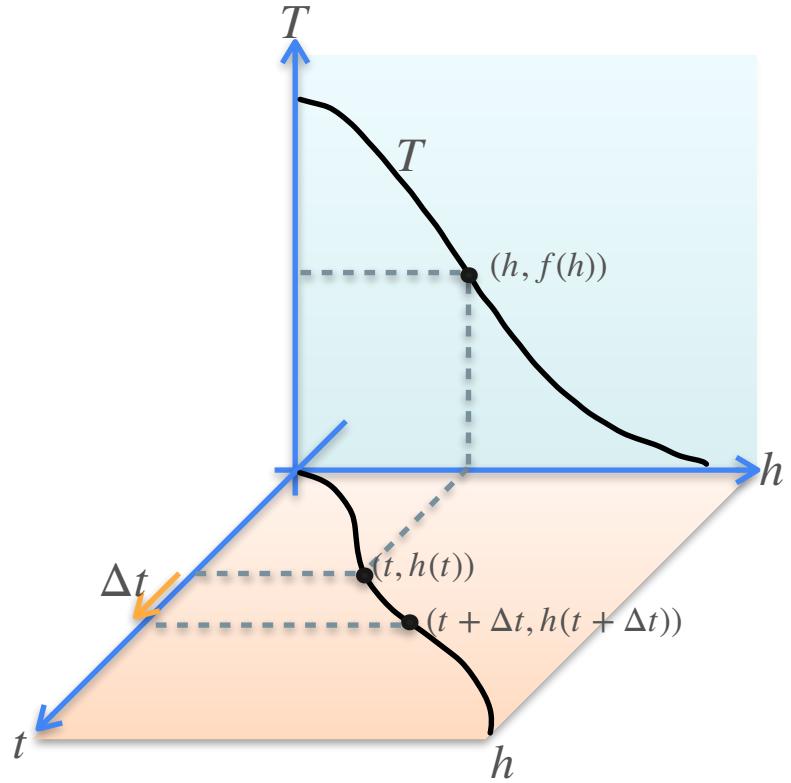
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



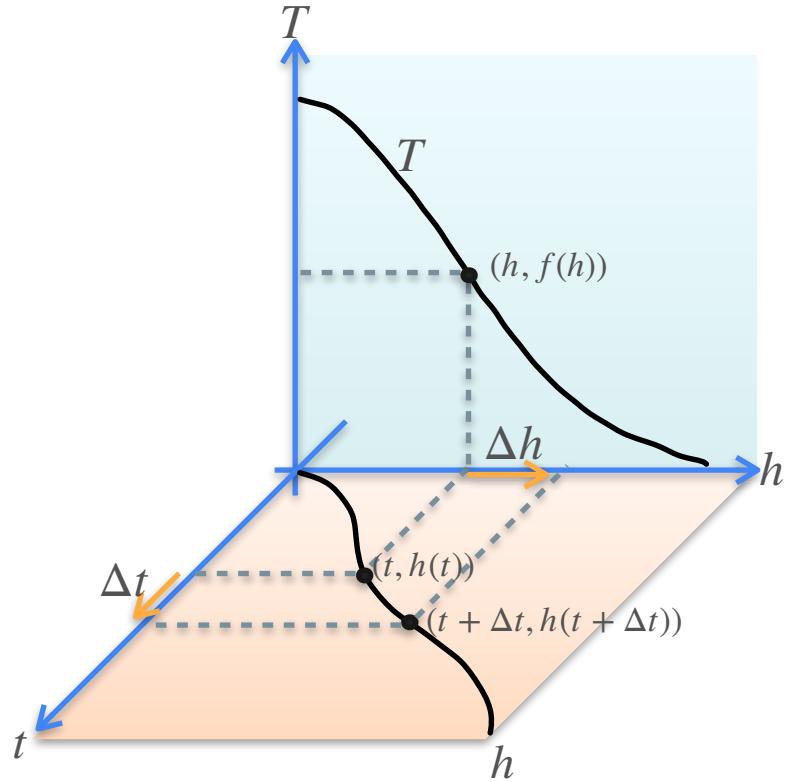
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



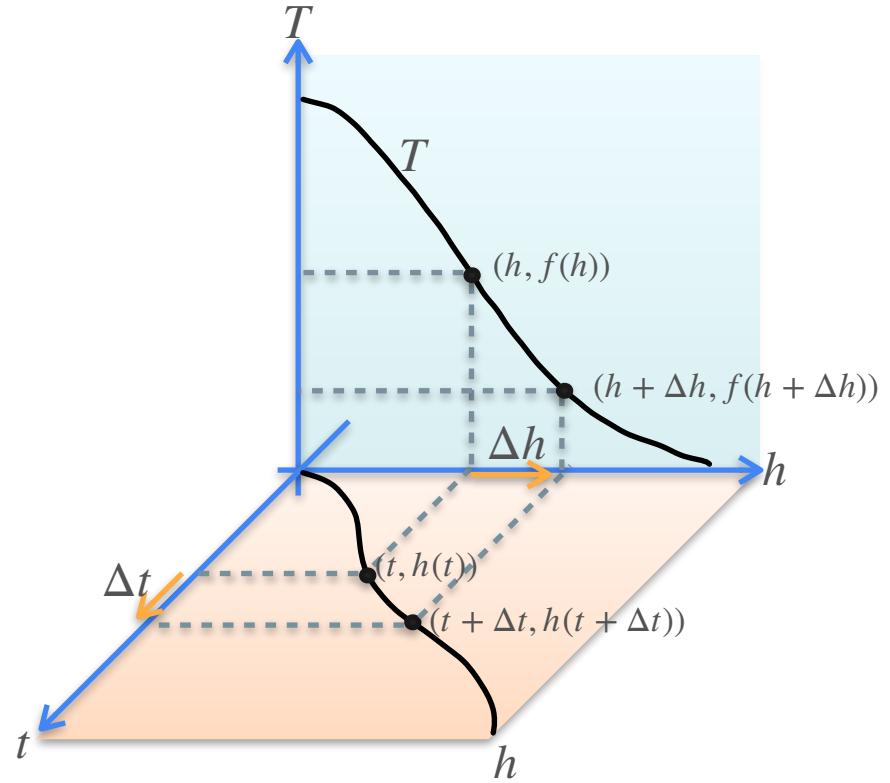
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



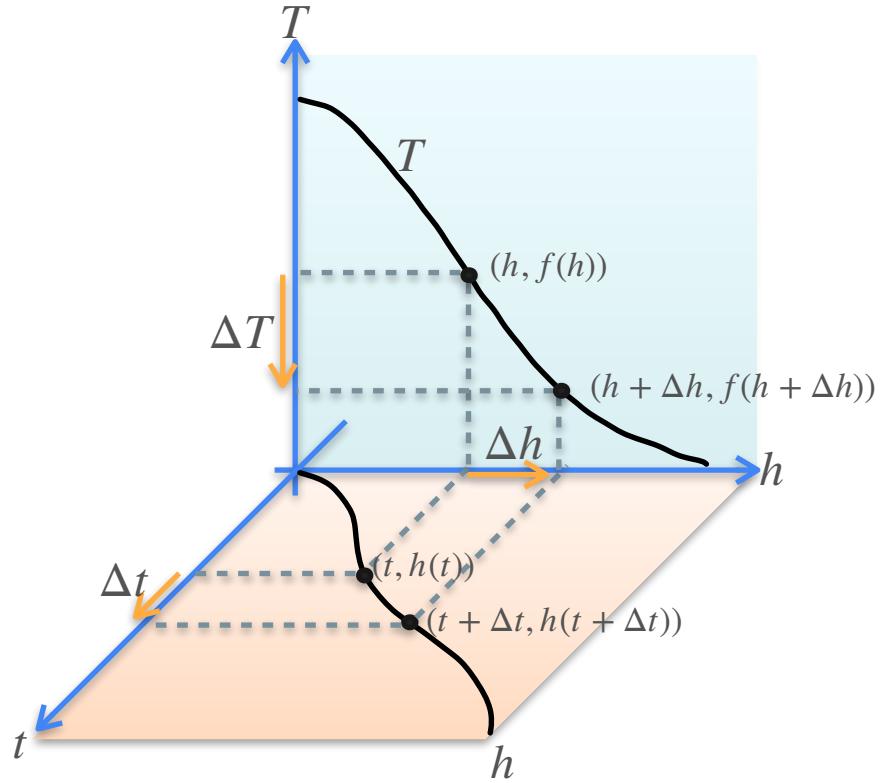
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



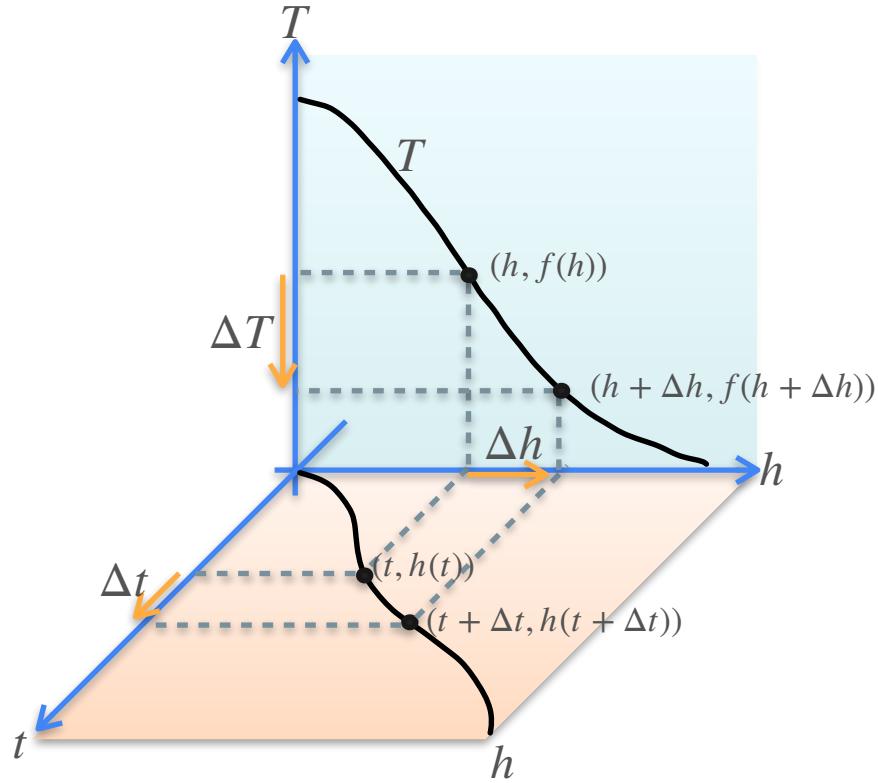
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

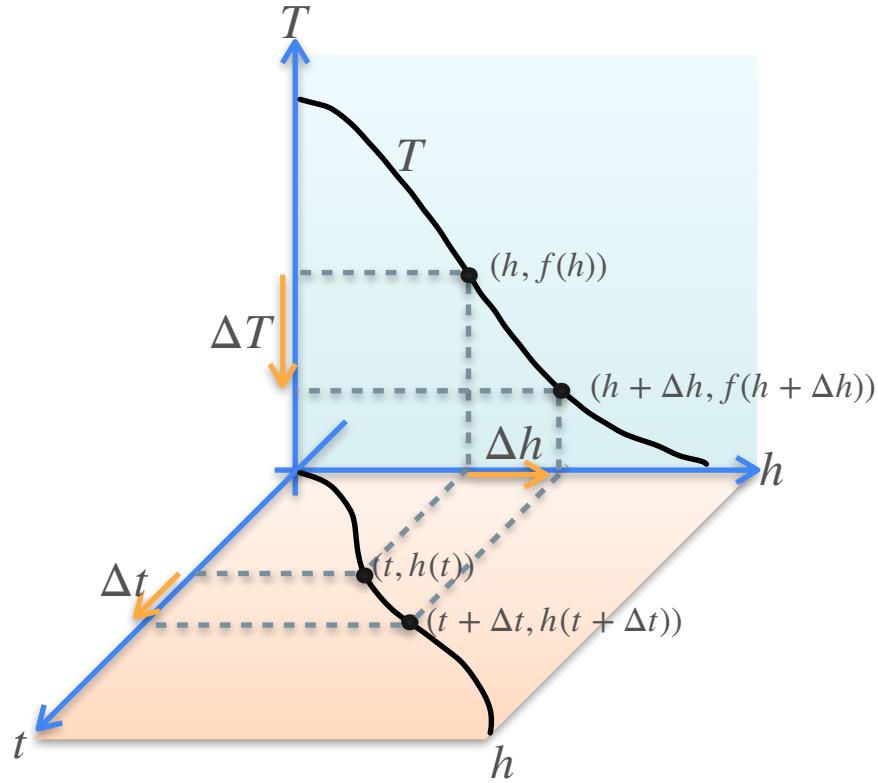
Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

$$\frac{\Delta T}{\Delta t} =$$

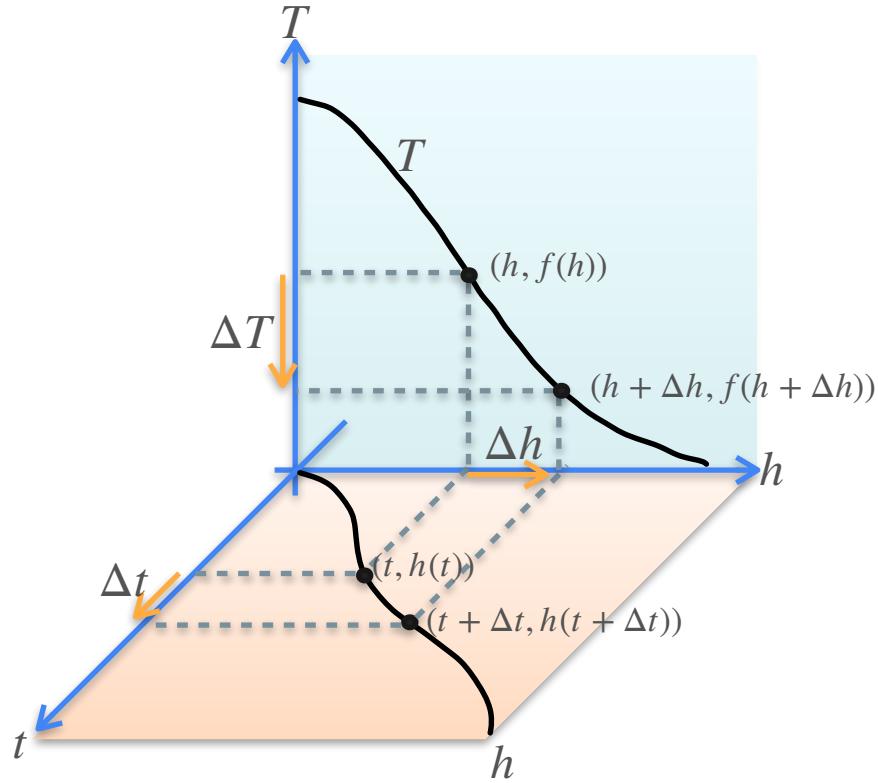
Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta h}{\Delta t}$$

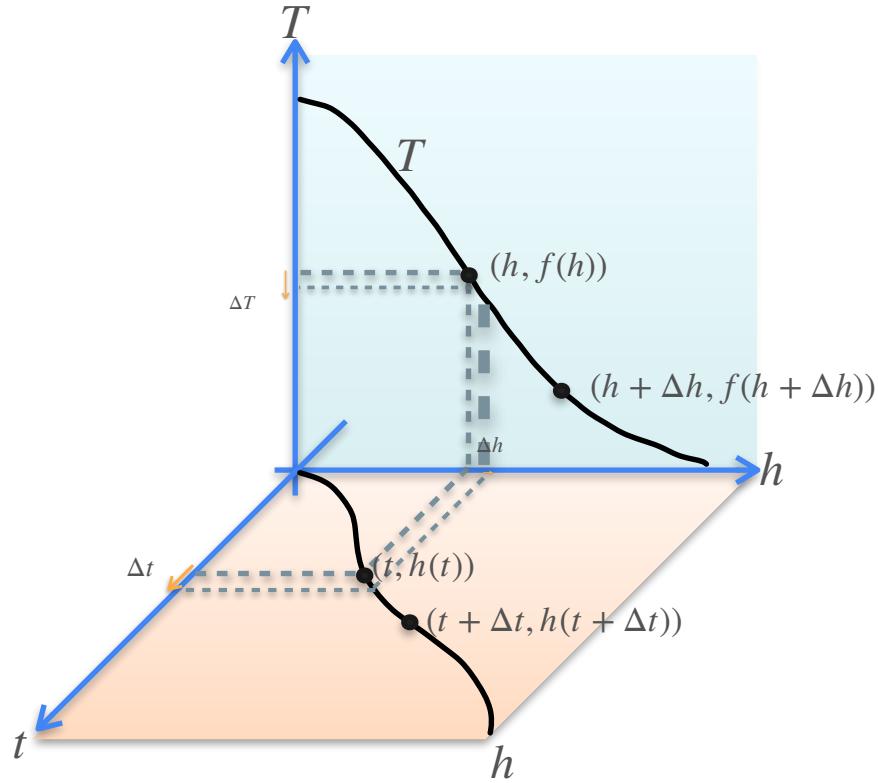
Chain Rule



$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

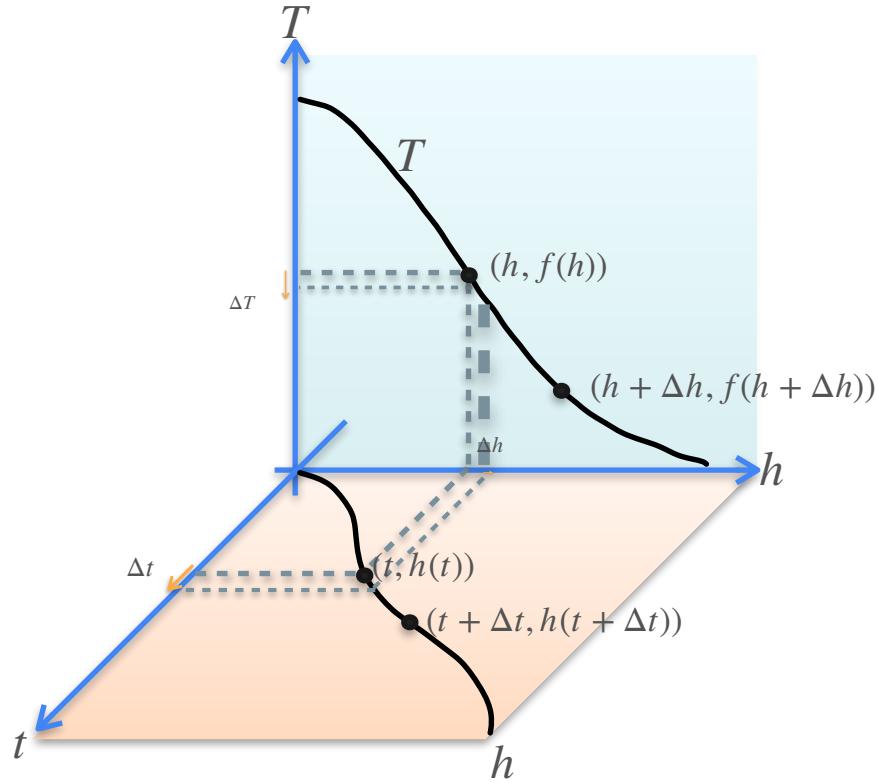
Chain Rule



$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

Chain Rule

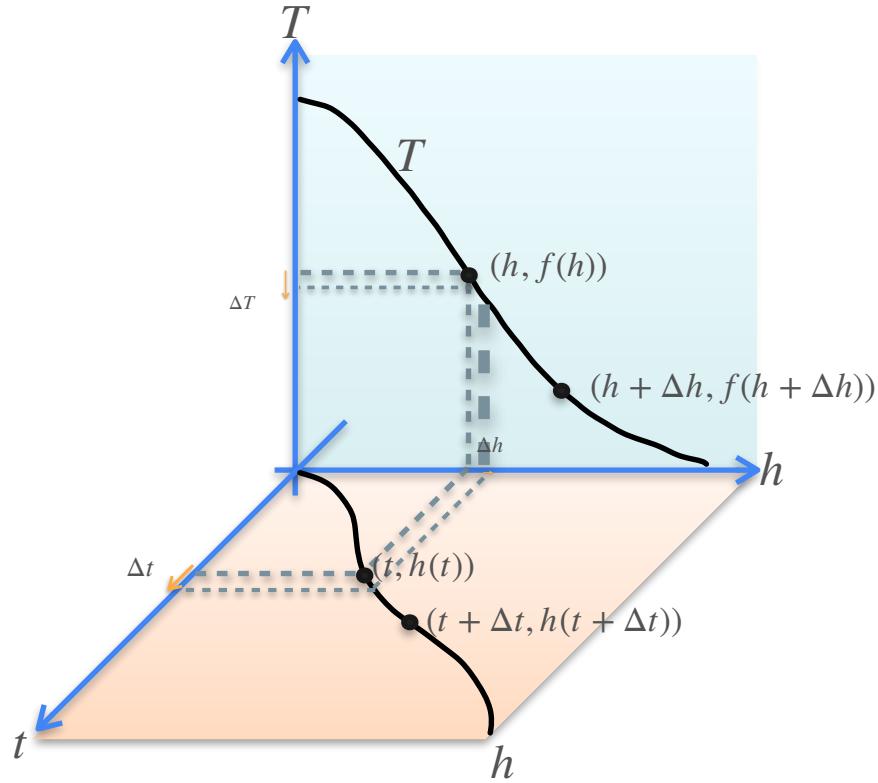


$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt}$$

Chain Rule

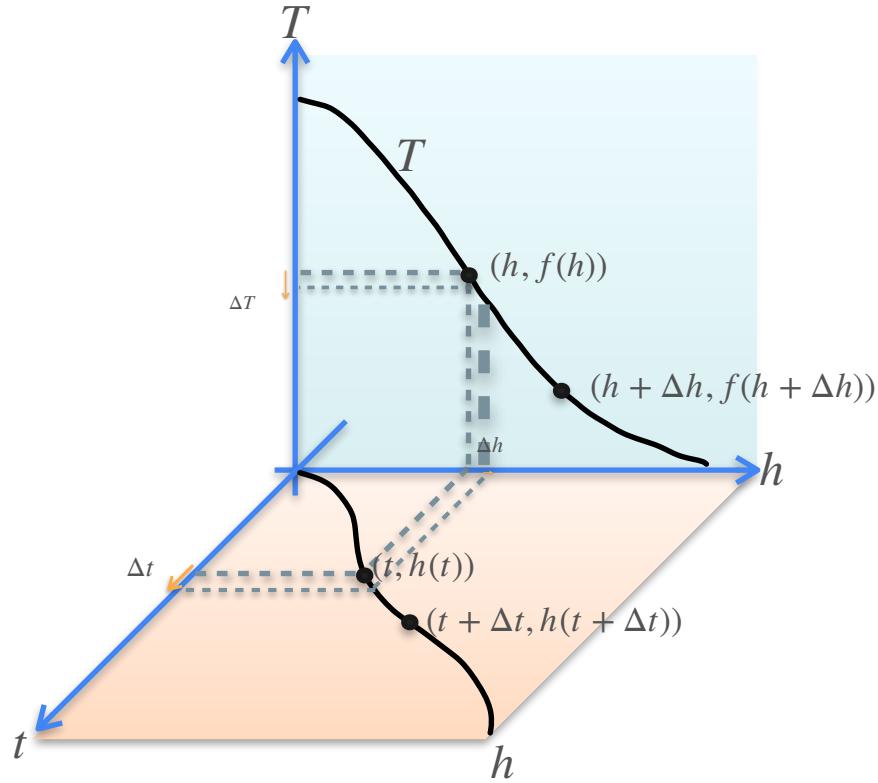


$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$

Chain Rule



$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$



DeepLearning.AI

Derivatives and Optimization

Introduction to optimization

Motivation for Optimization

Motivation for Optimization



Motivation for Optimization



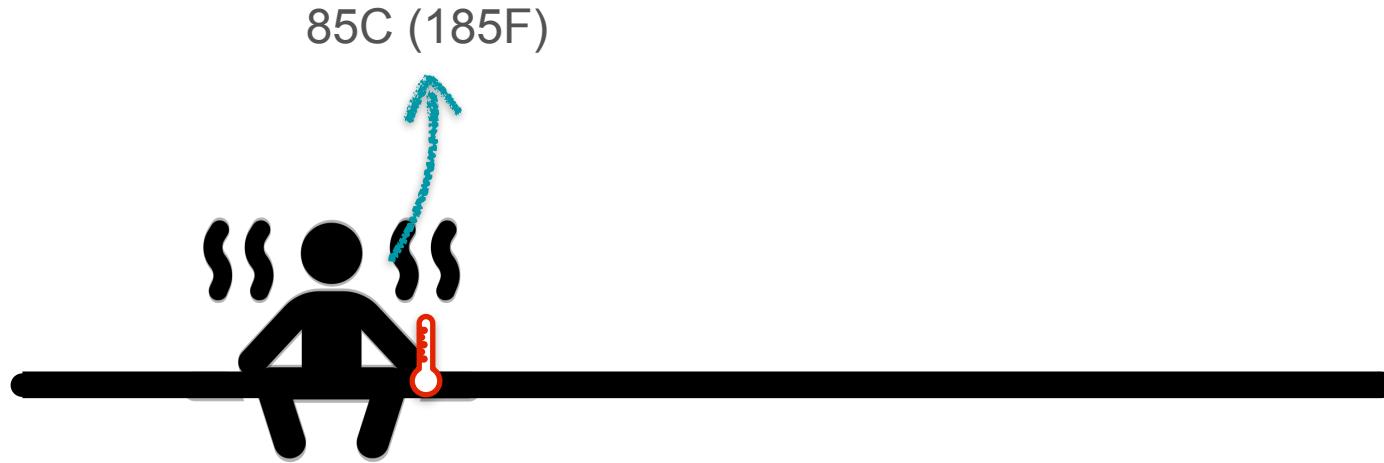
Motivation for Optimization



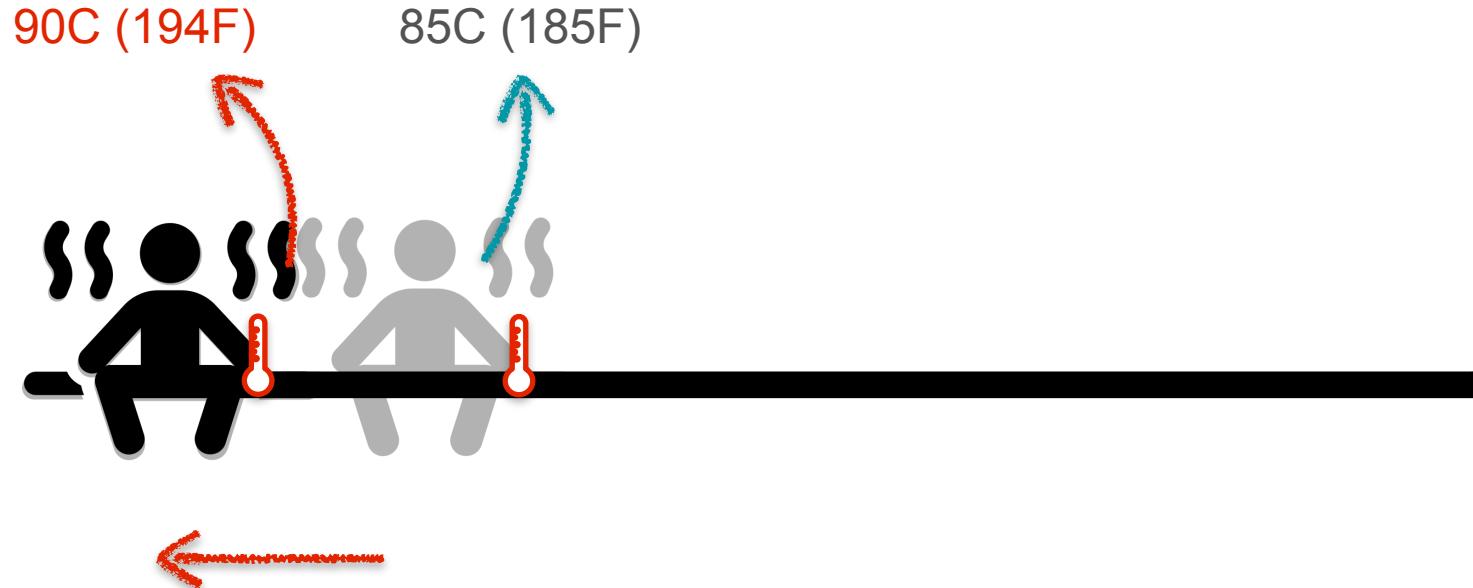
Motivation for Optimization



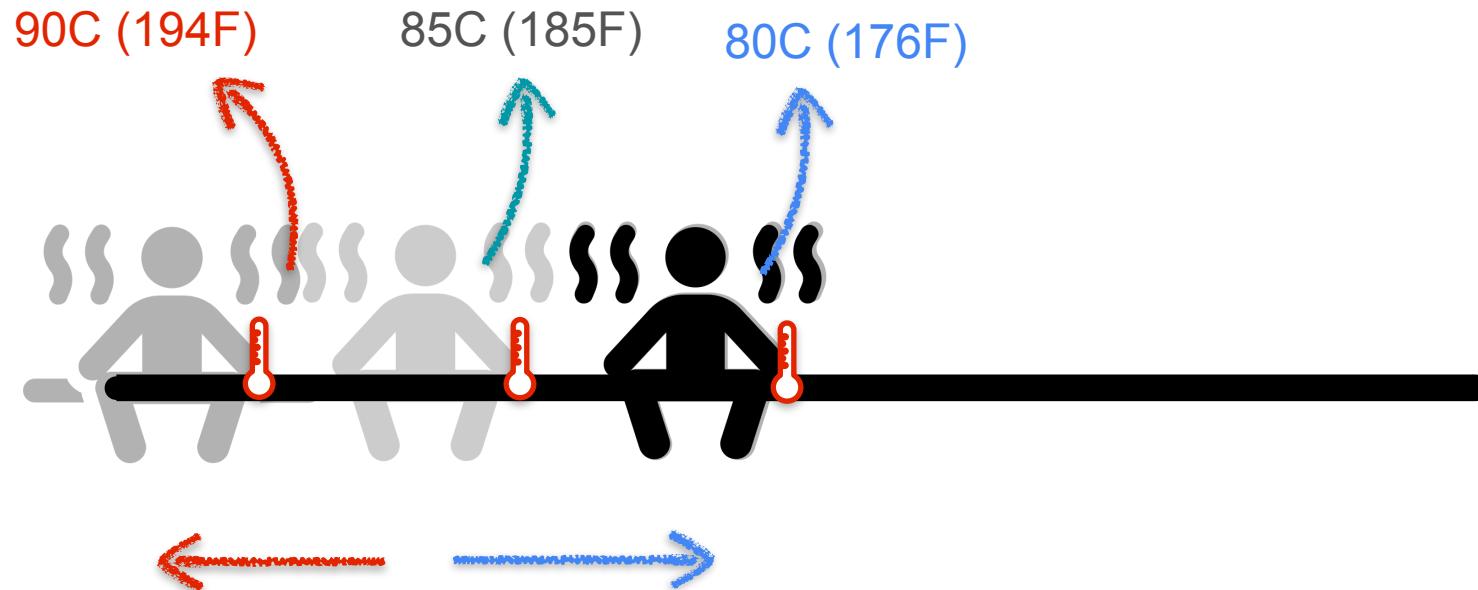
Motivation for Optimization



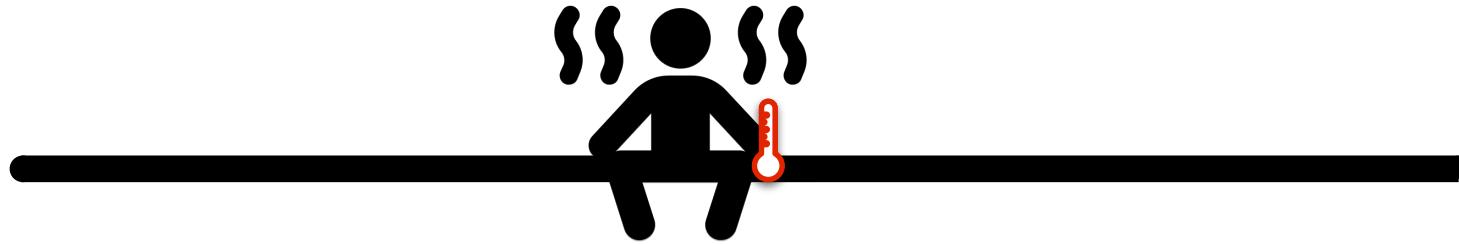
Motivation for Optimization



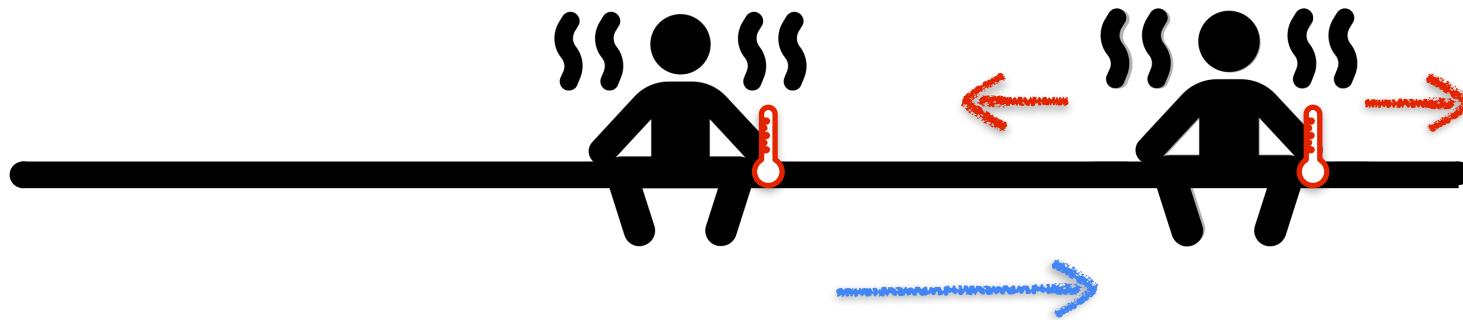
Motivation for Optimization



Motivation for Optimization



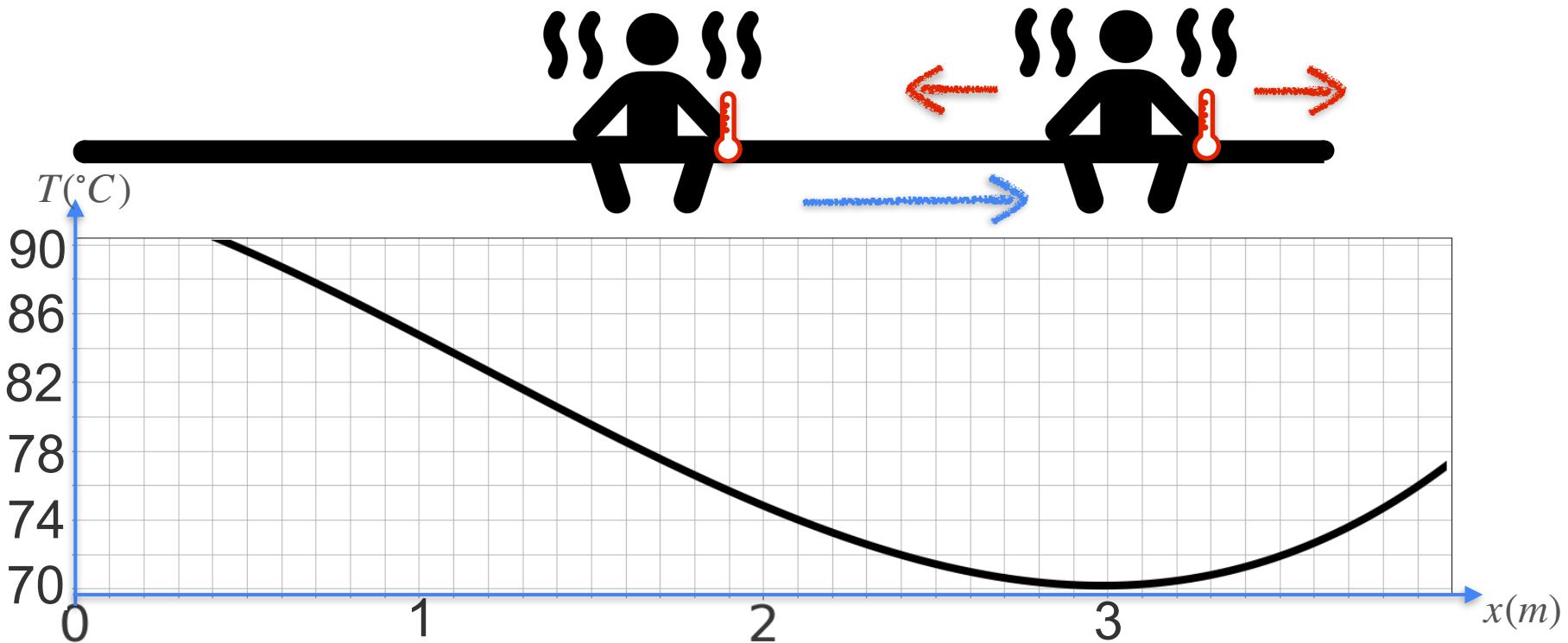
Motivation for Optimization



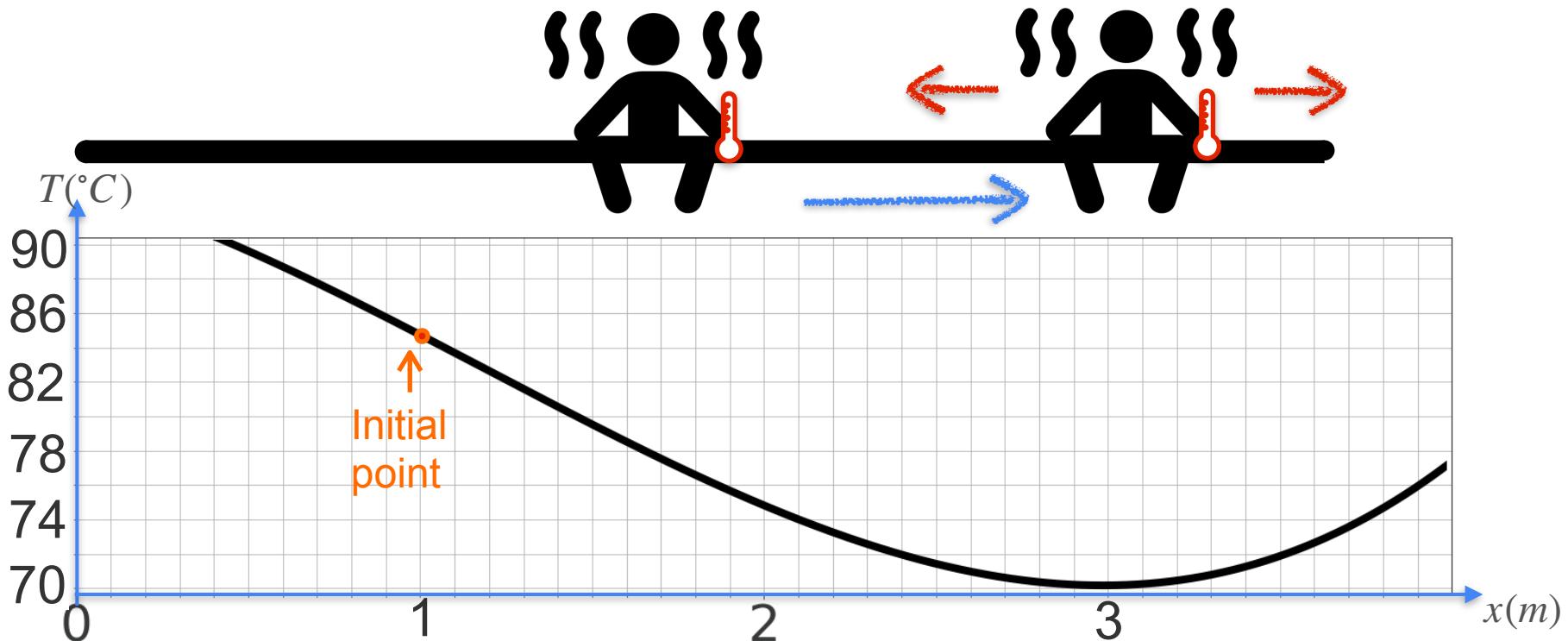
Motivation for Optimization



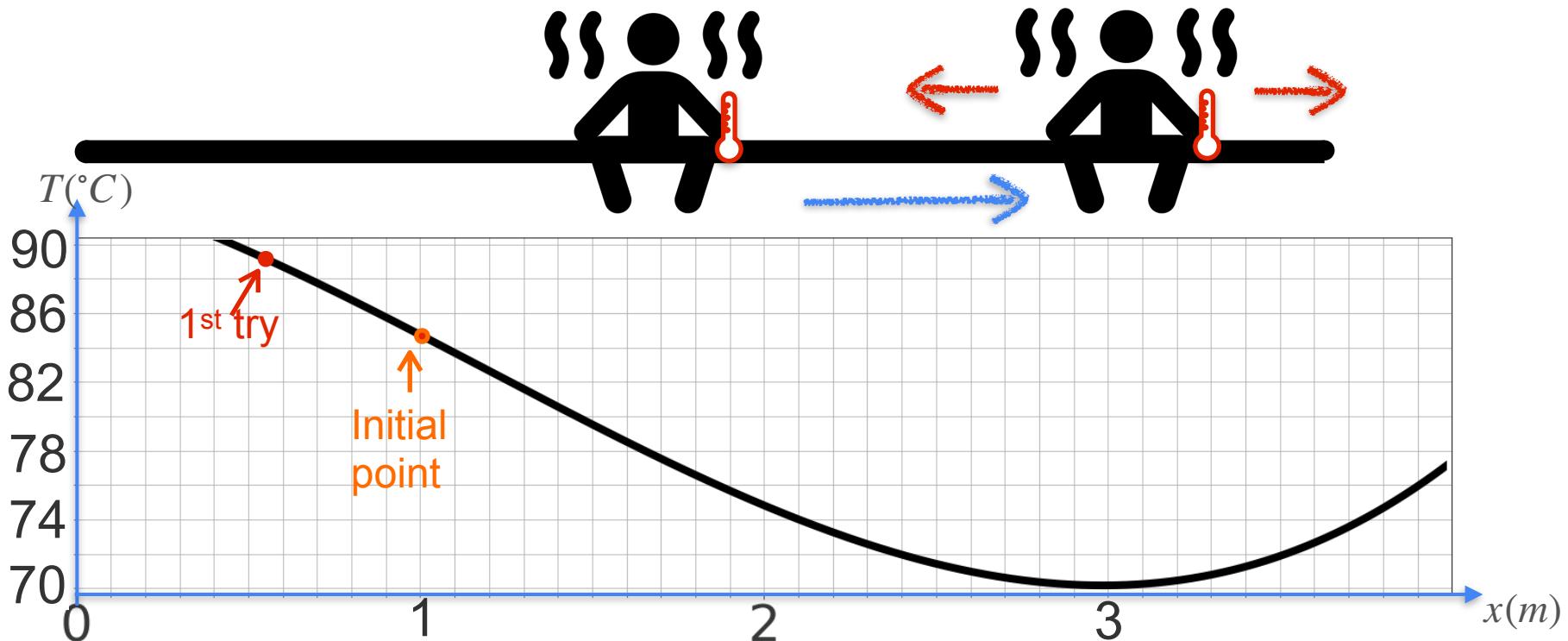
Motivation for Optimization



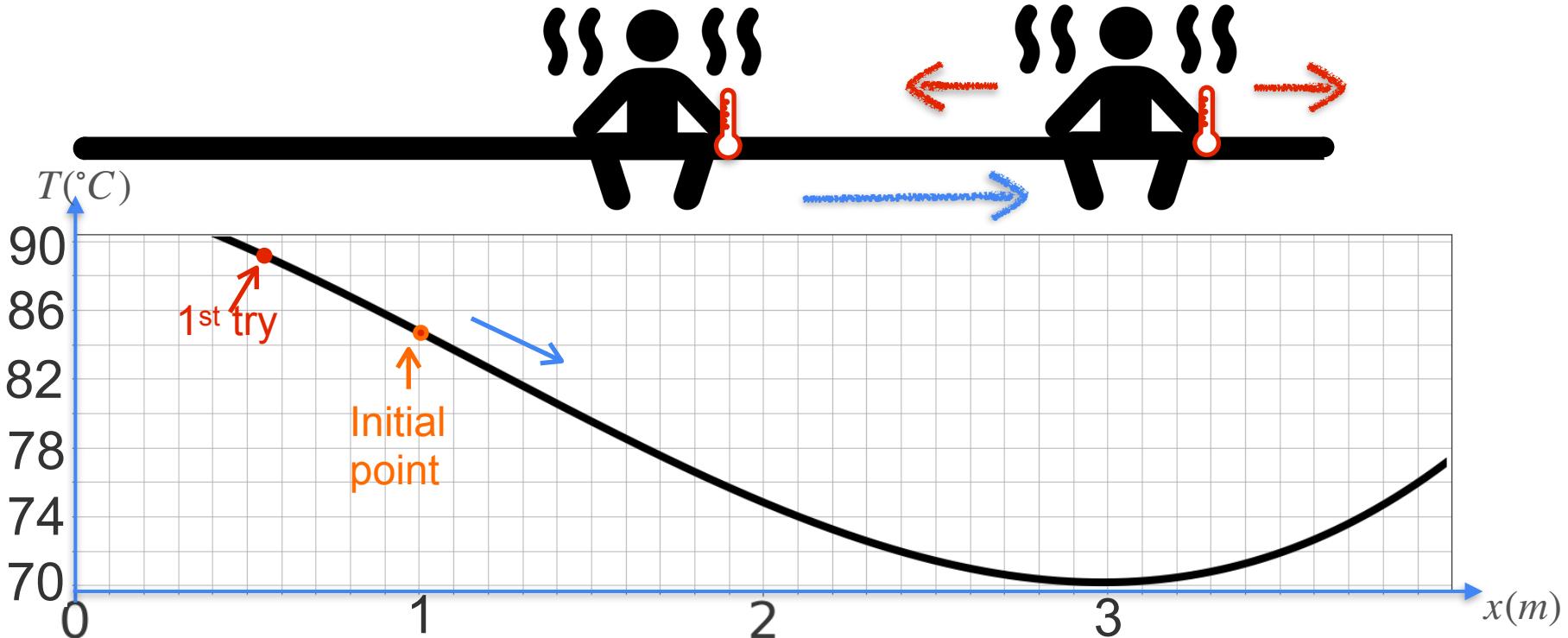
Motivation for Optimization



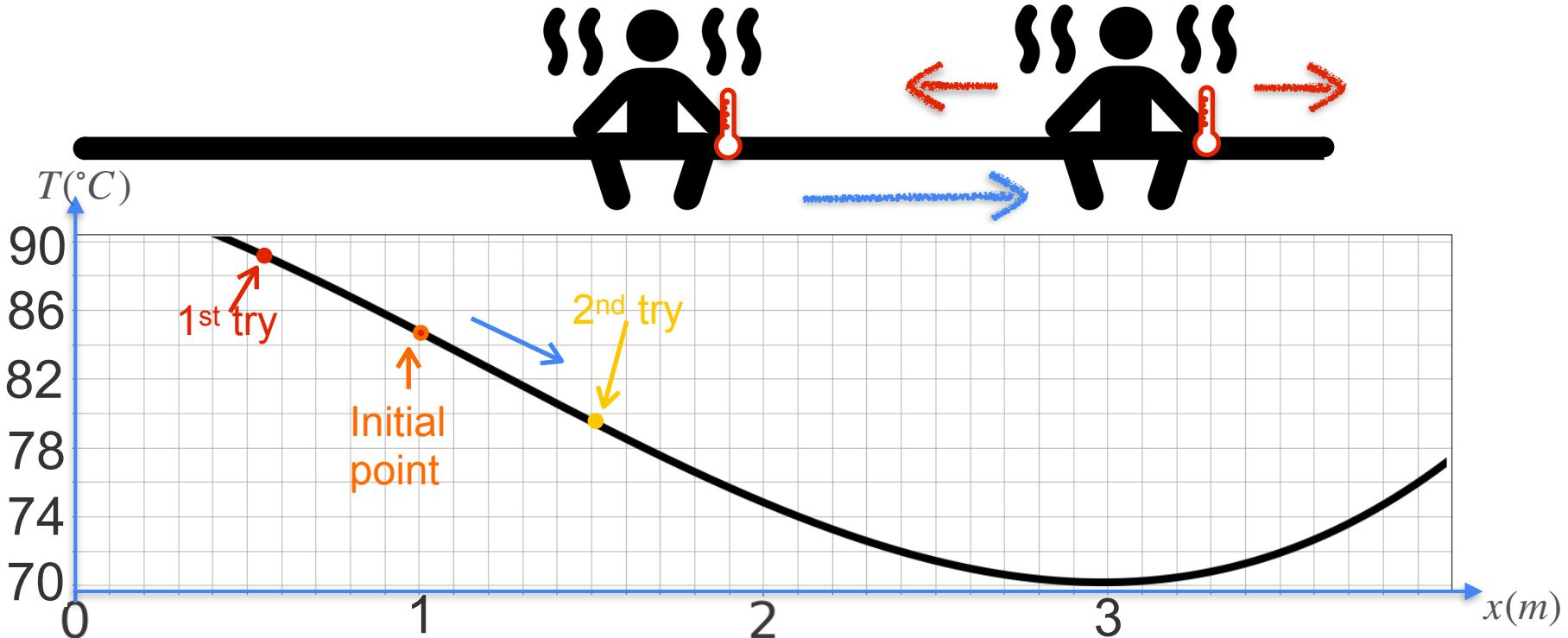
Motivation for Optimization



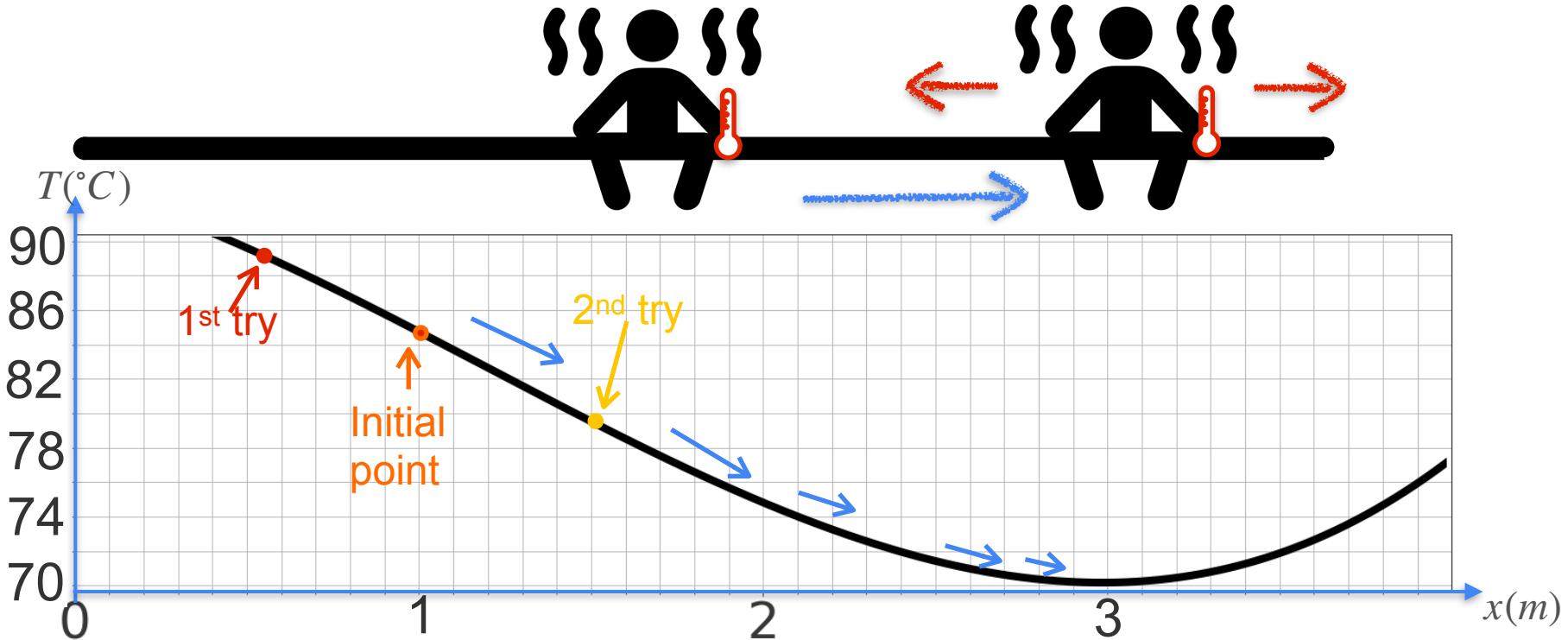
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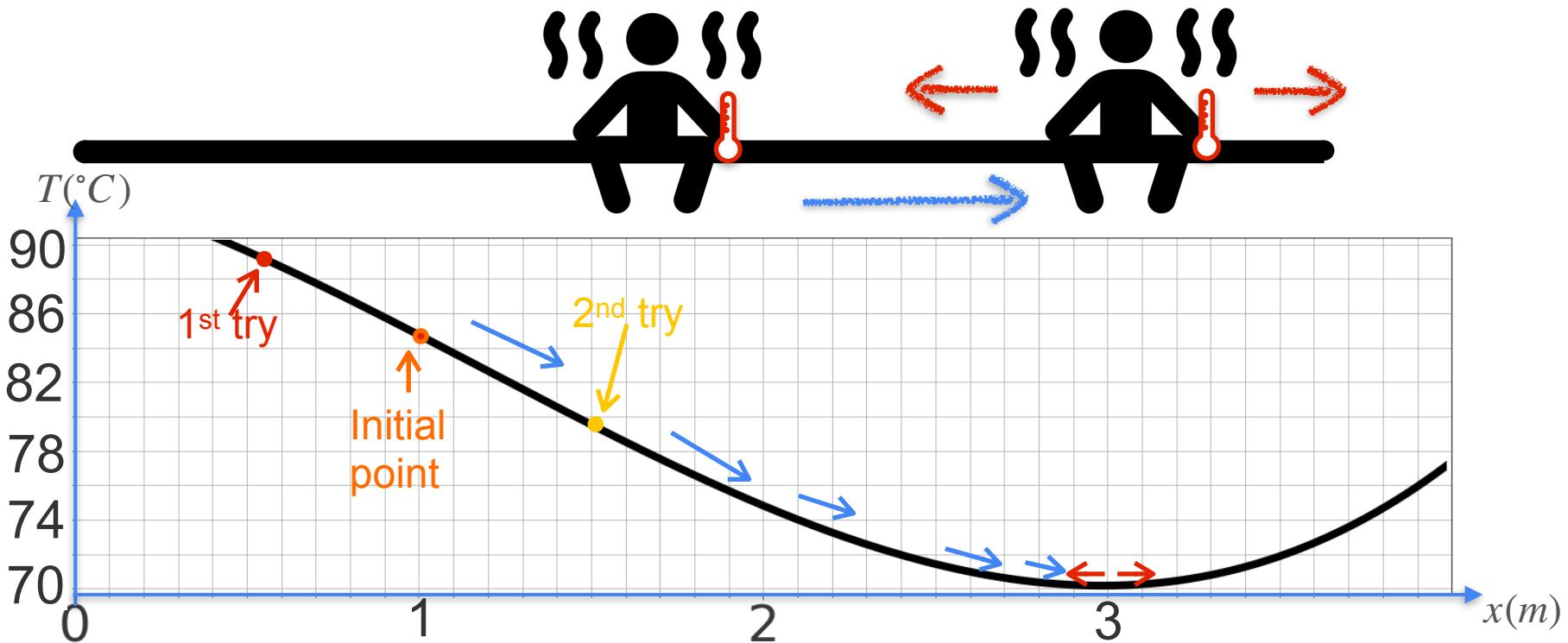
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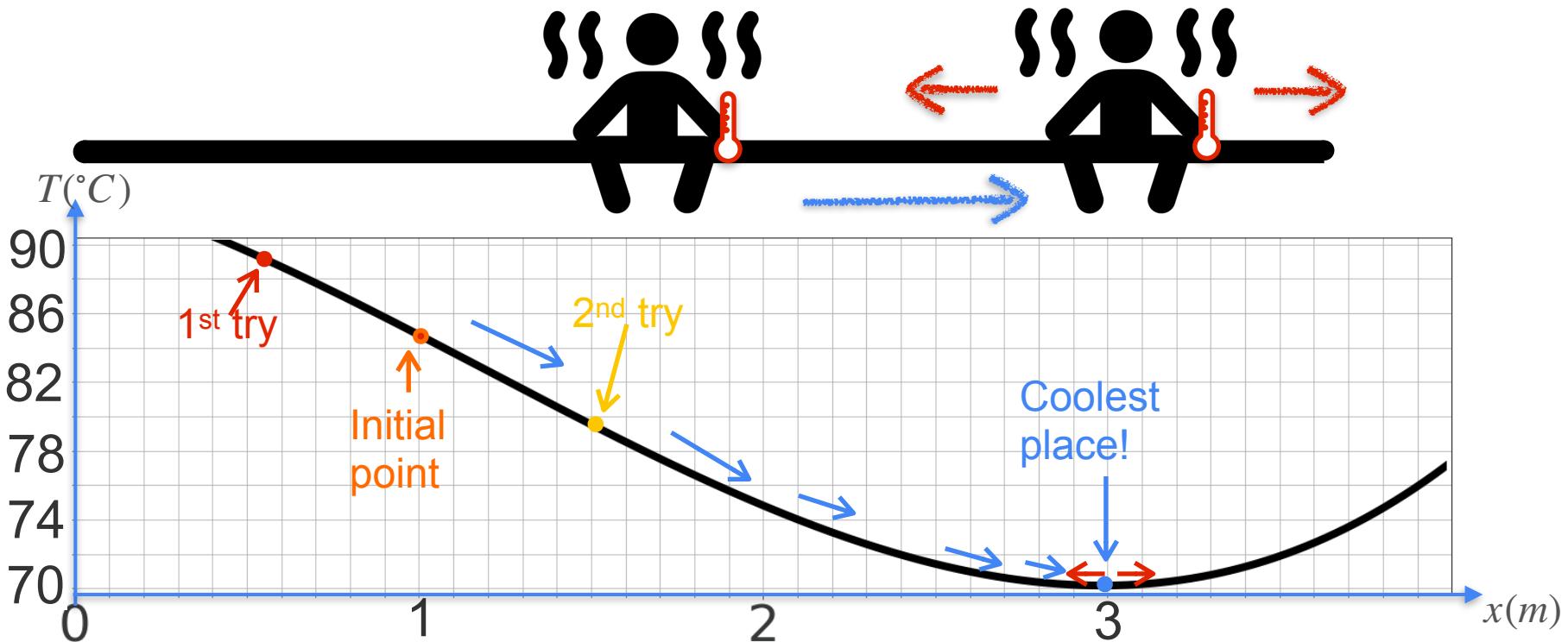
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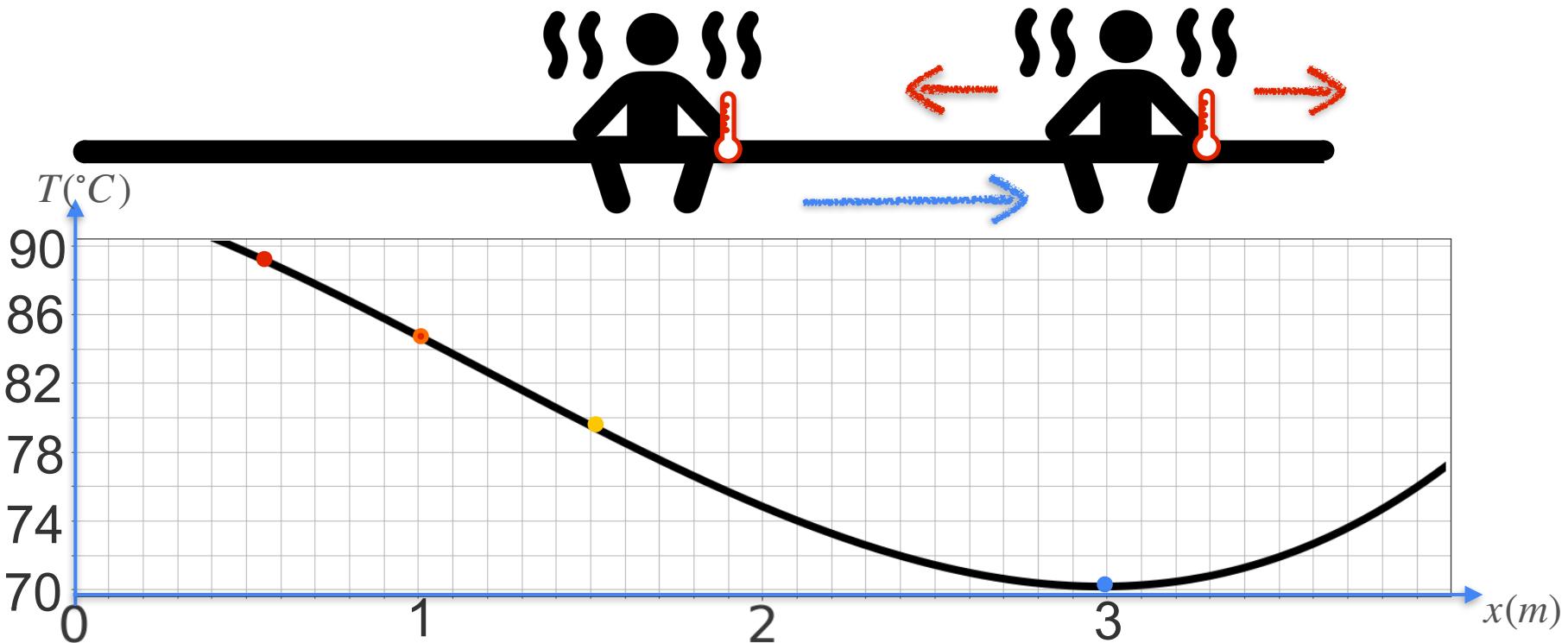
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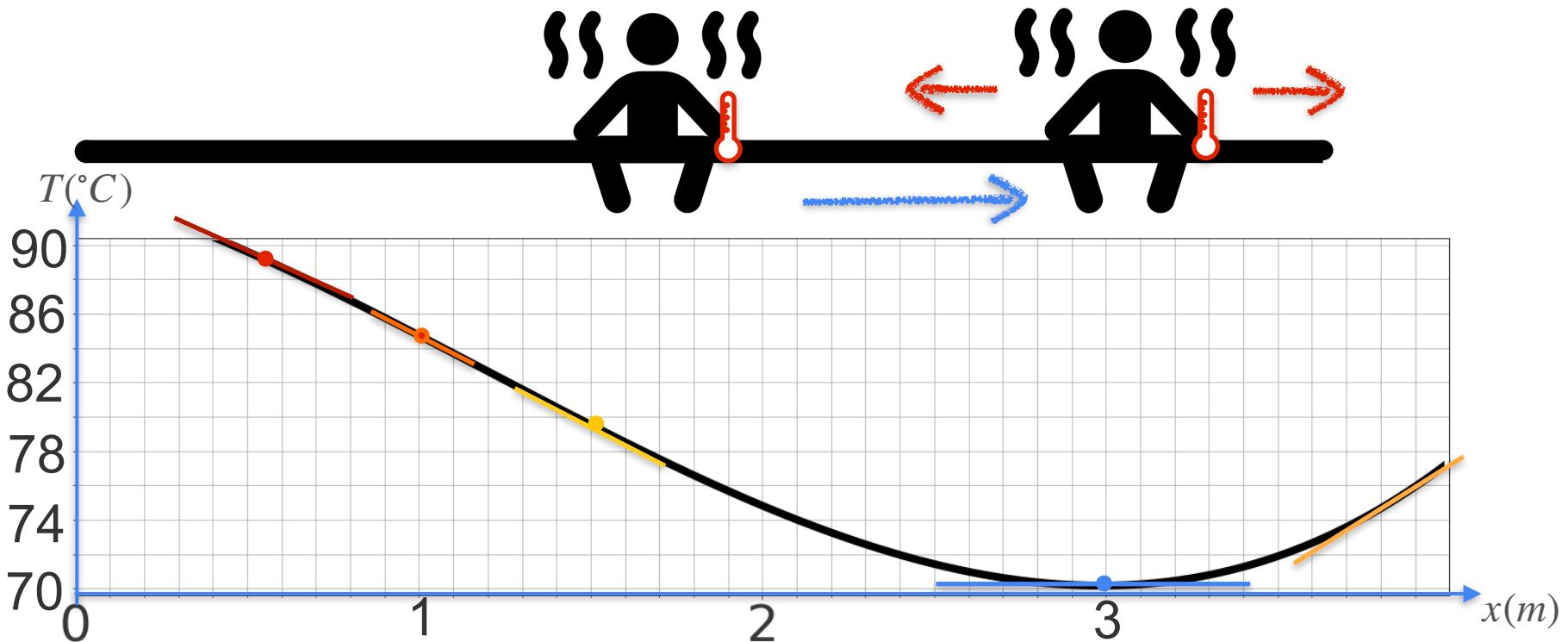
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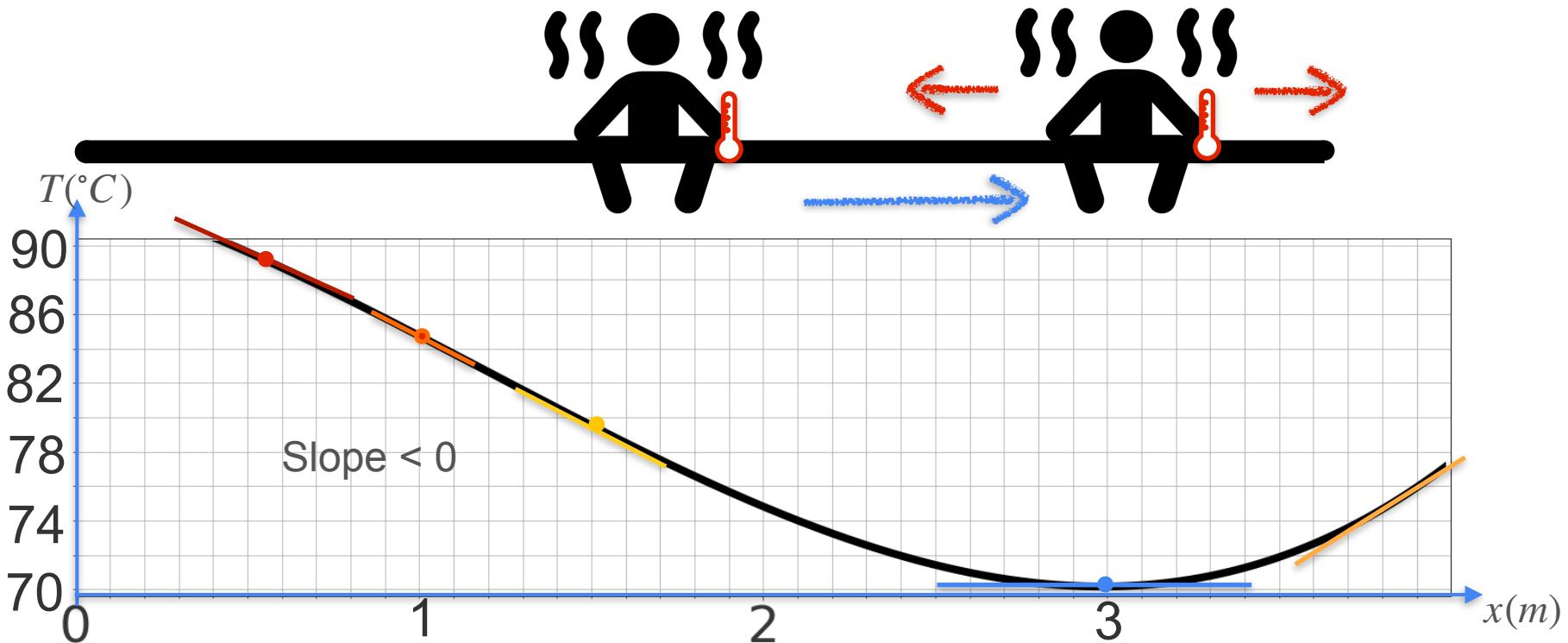
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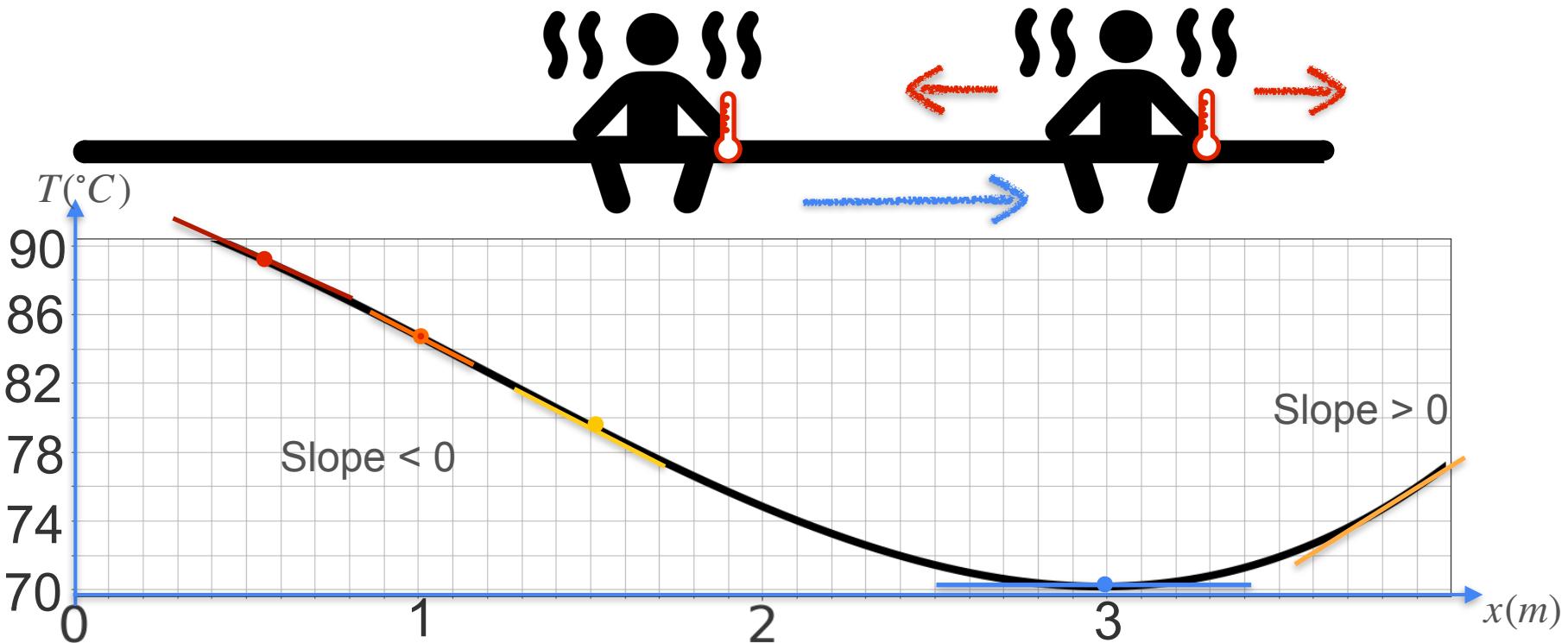
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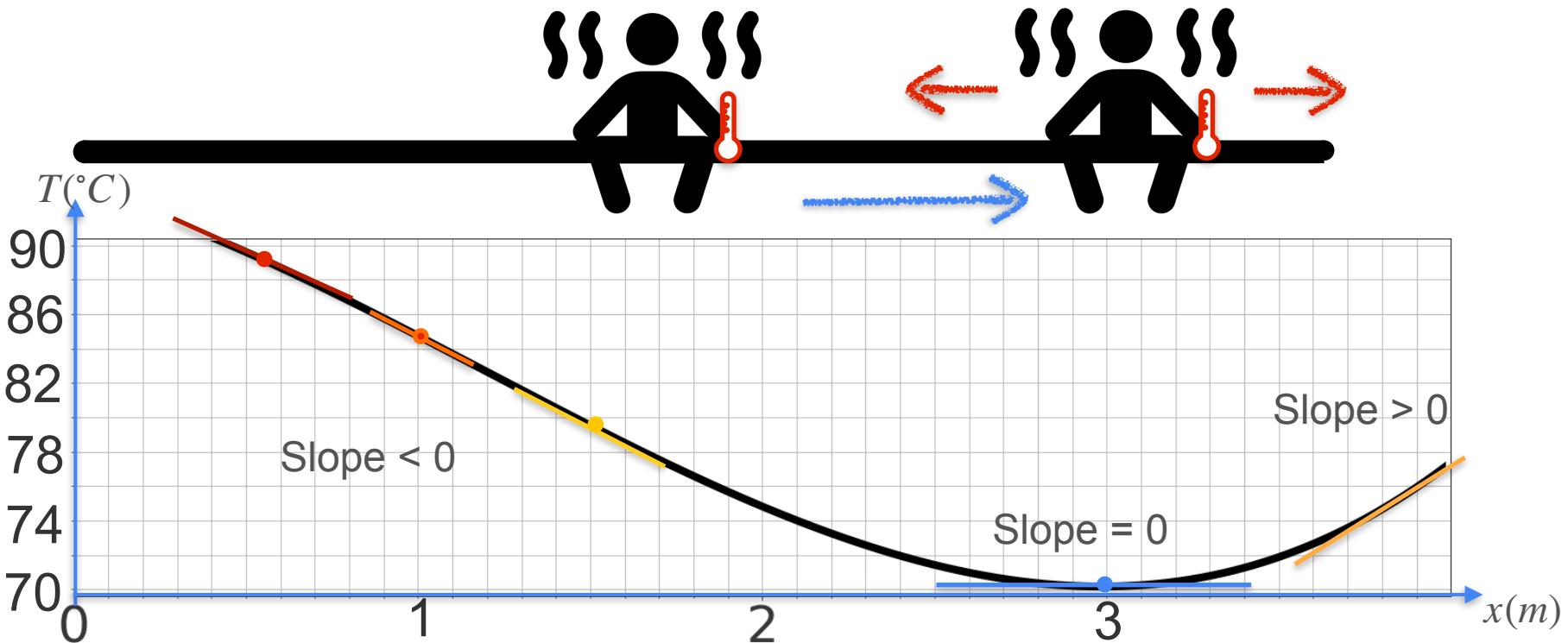
Motivation for Optimization



Motivation for Optimization

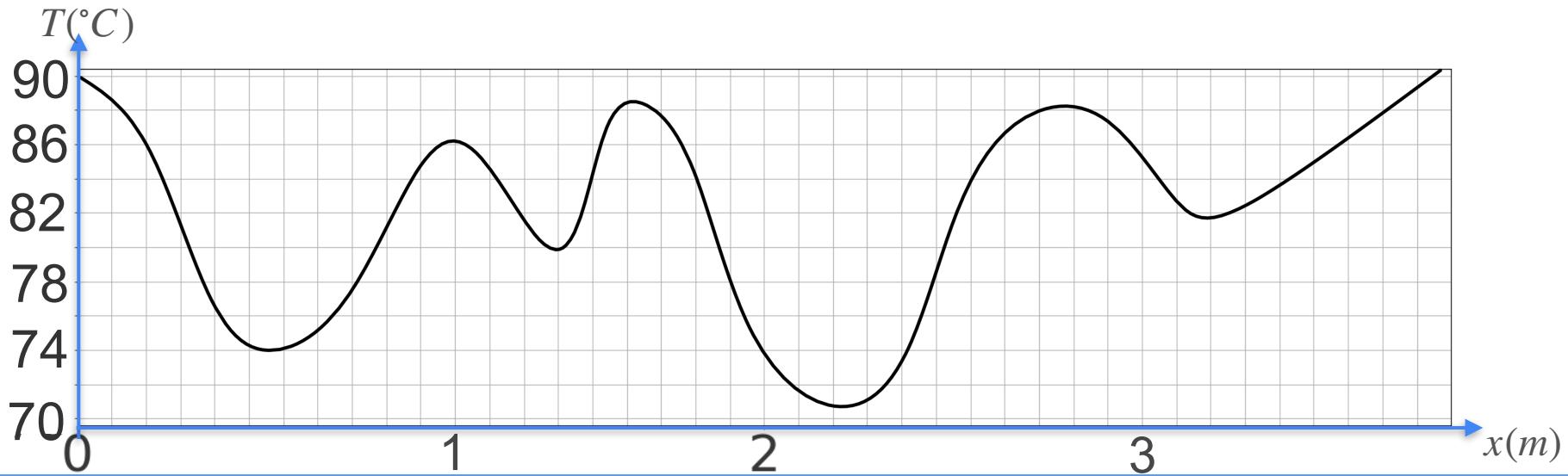


Motivation for Optimization

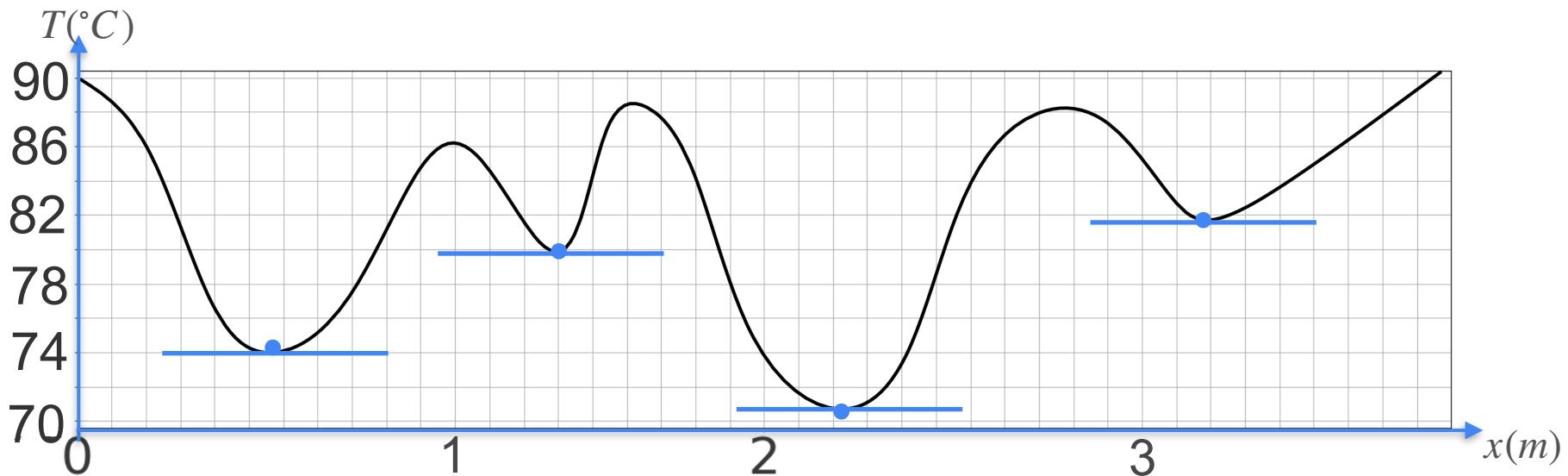


Multiple Minima

Multiple Minima

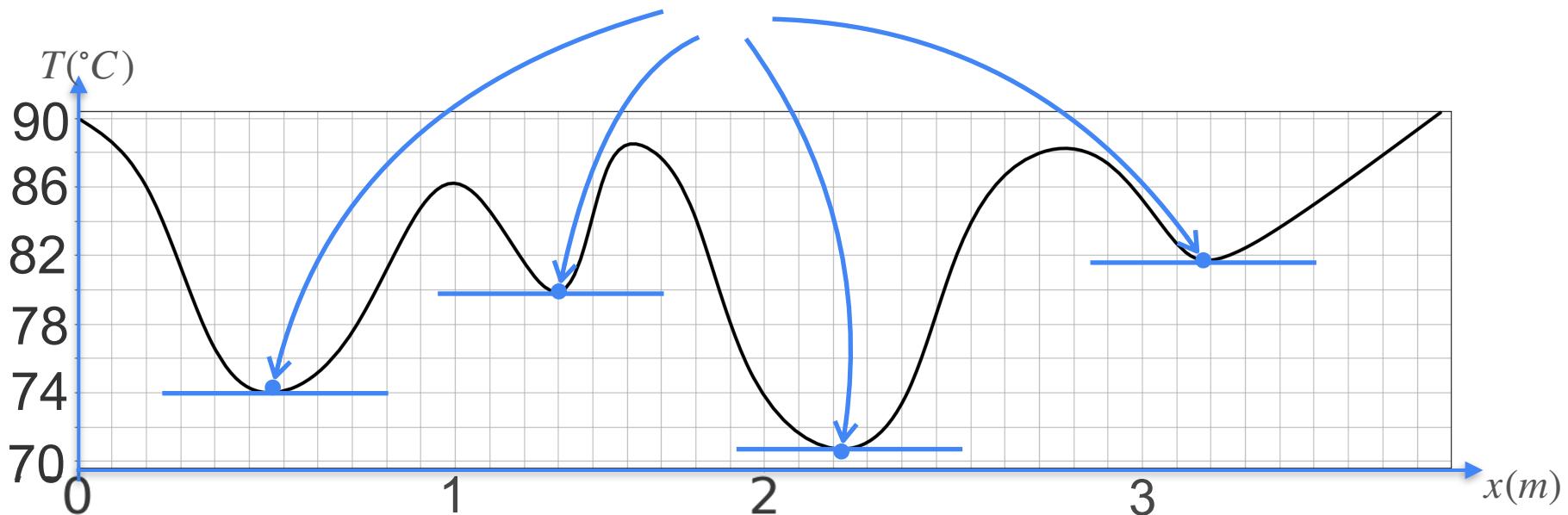


Multiple Minima



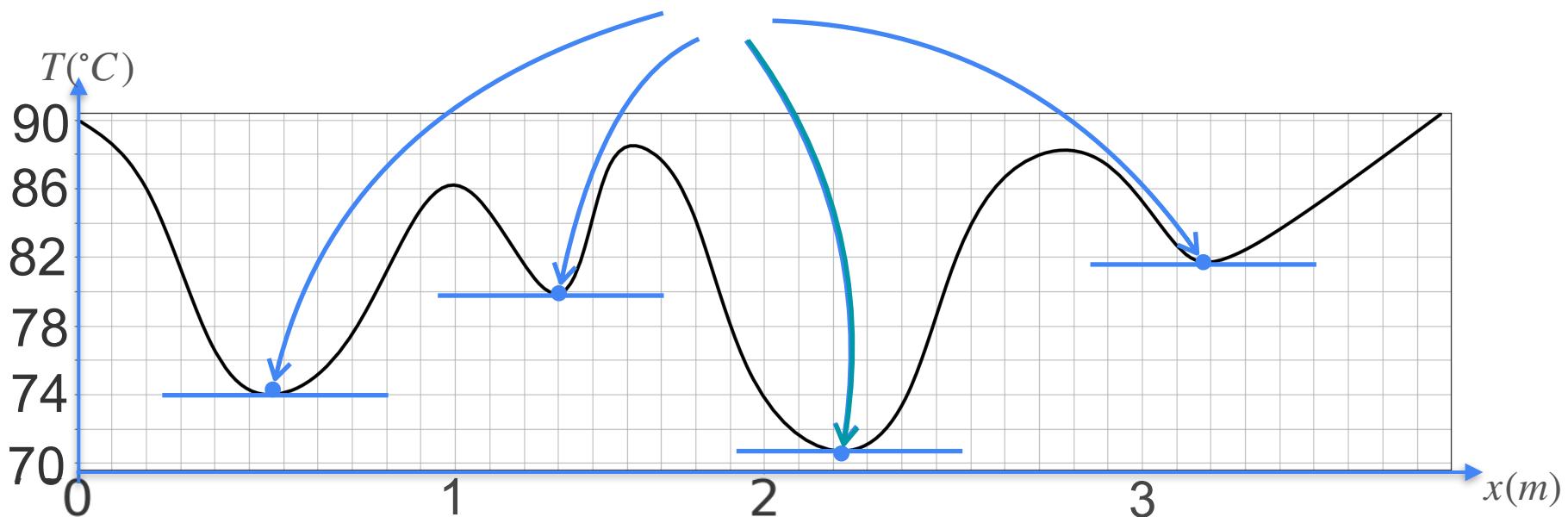
Multiple Minima

Candidates for the minimum are at the points of zero slope



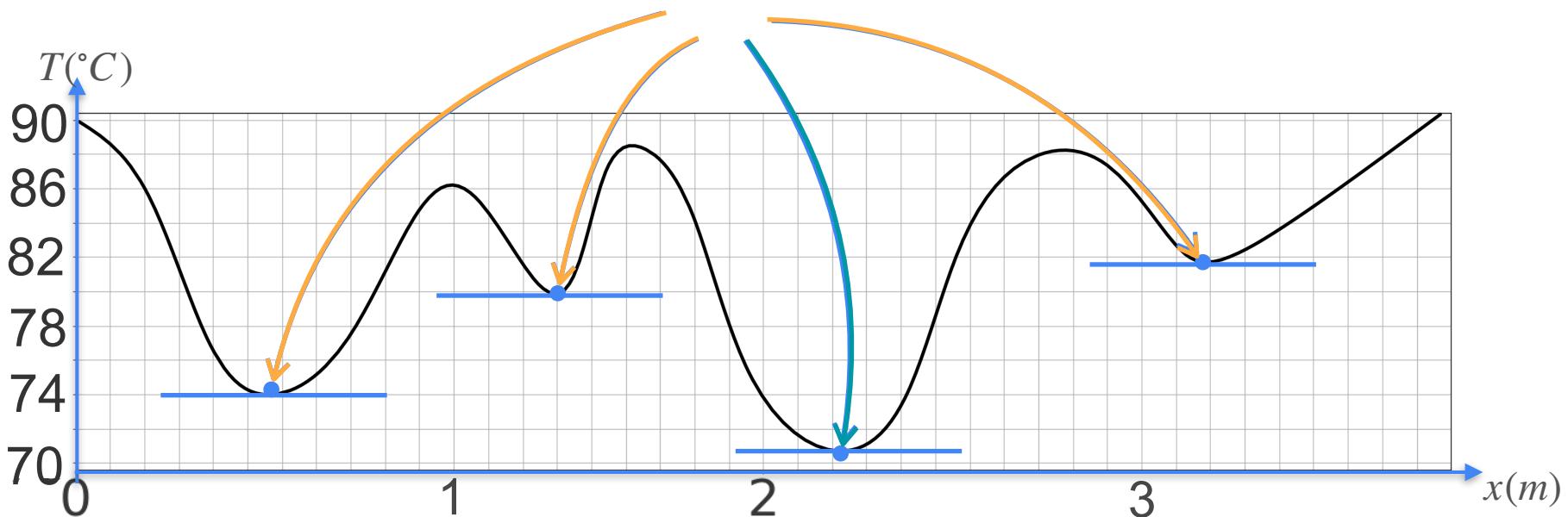
Multiple Minima

Candidates for the minimum are at the points of zero slope



Multiple Minima

Candidates for the minimum are at the points of zero slope





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Derivatives and Optimization

**Optimization of squared loss:
The one powerline problem**

Cost Function Motivation

Cost Function Motivation

Cost Function Motivation



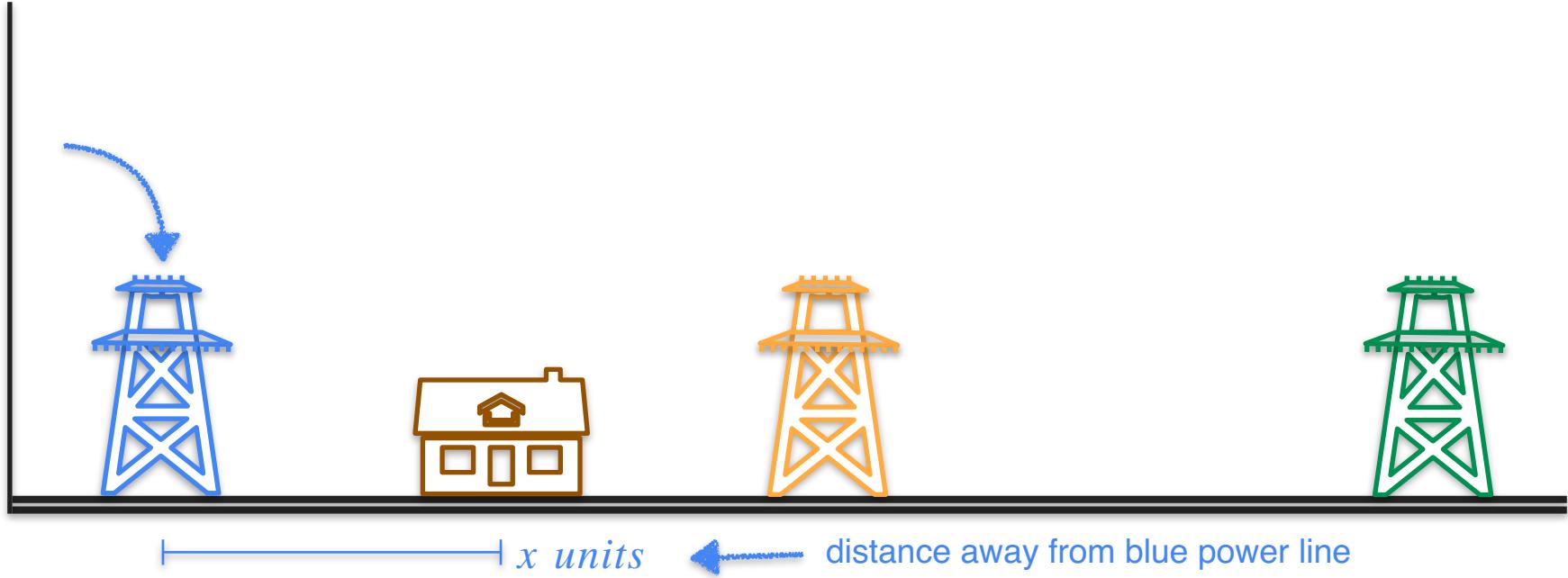
Cost Function Motivation



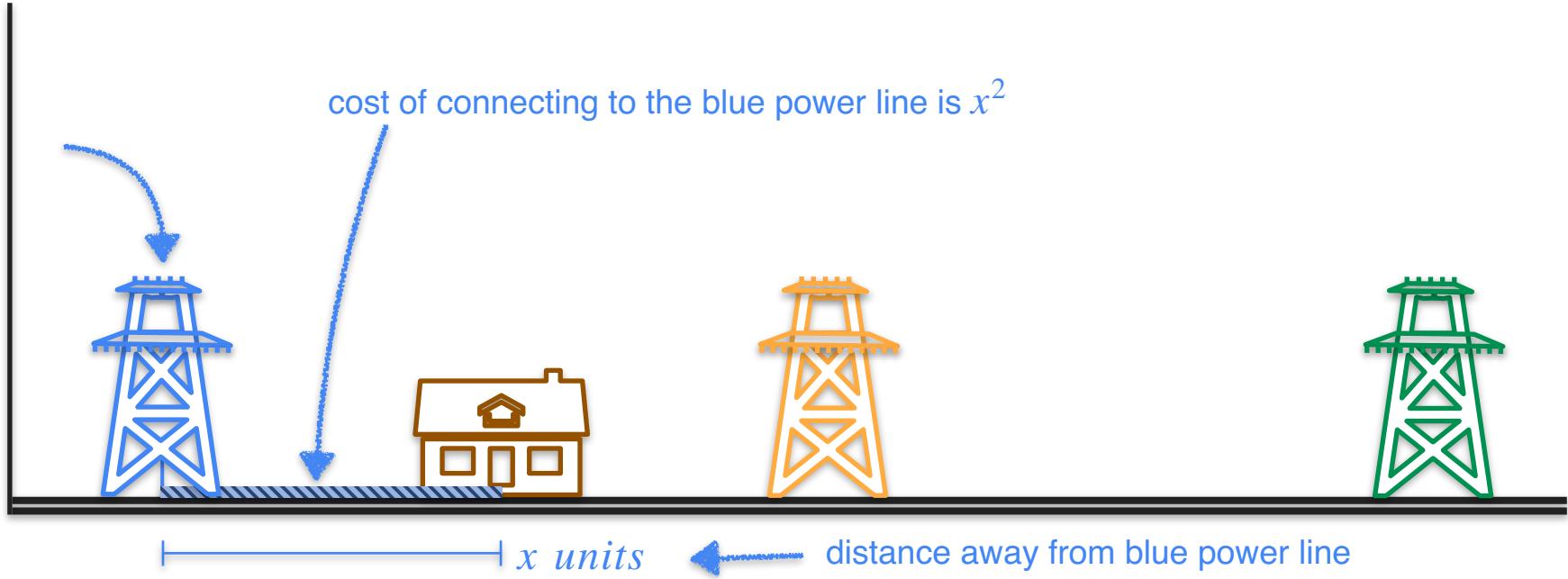
Cost Function Motivation



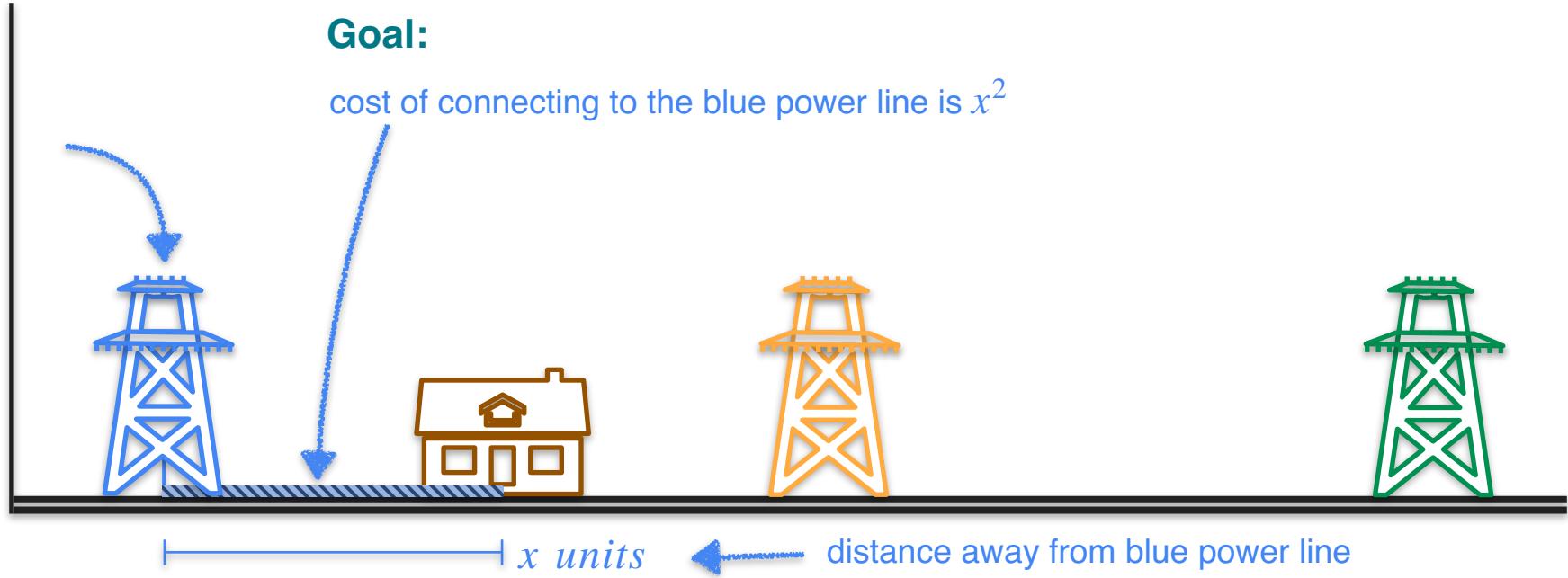
Cost Function Motivation



Cost Function Motivation



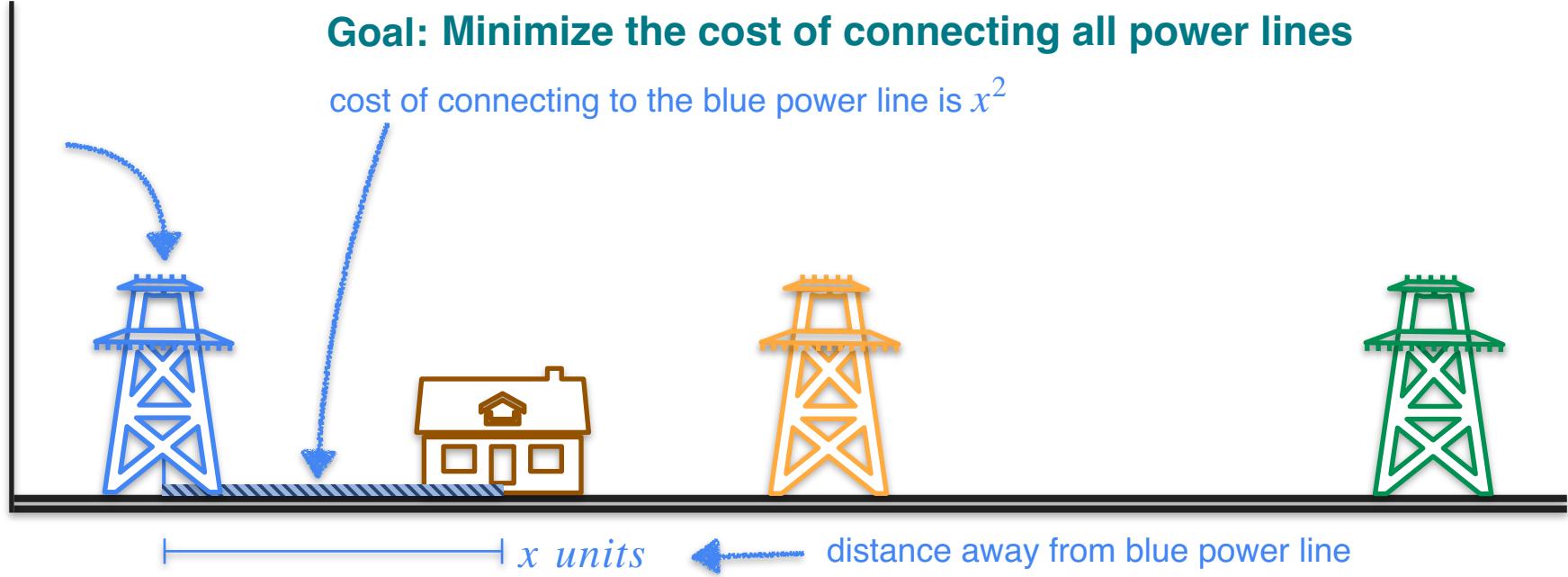
Cost Function Motivation



Cost Function Motivation

Goal: Minimize the cost of connecting all power lines

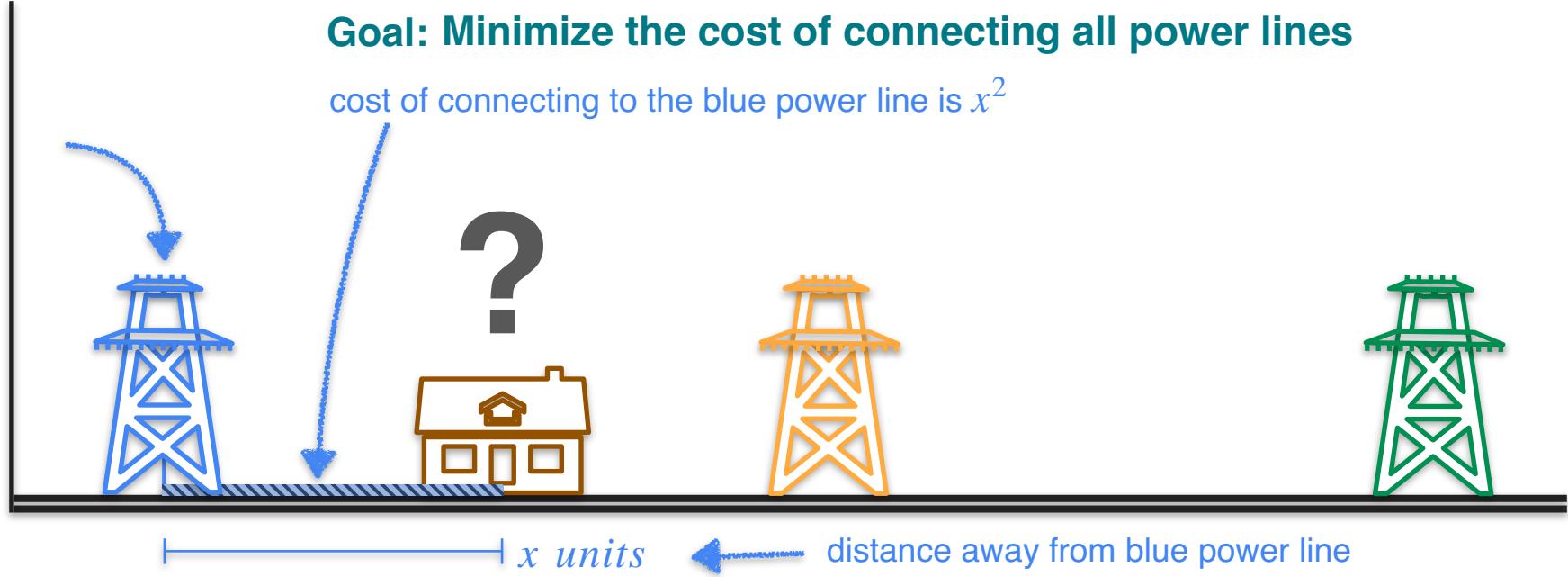
cost of connecting to the blue power line is x^2



Cost Function Motivation

Goal: Minimize the cost of connecting all power lines

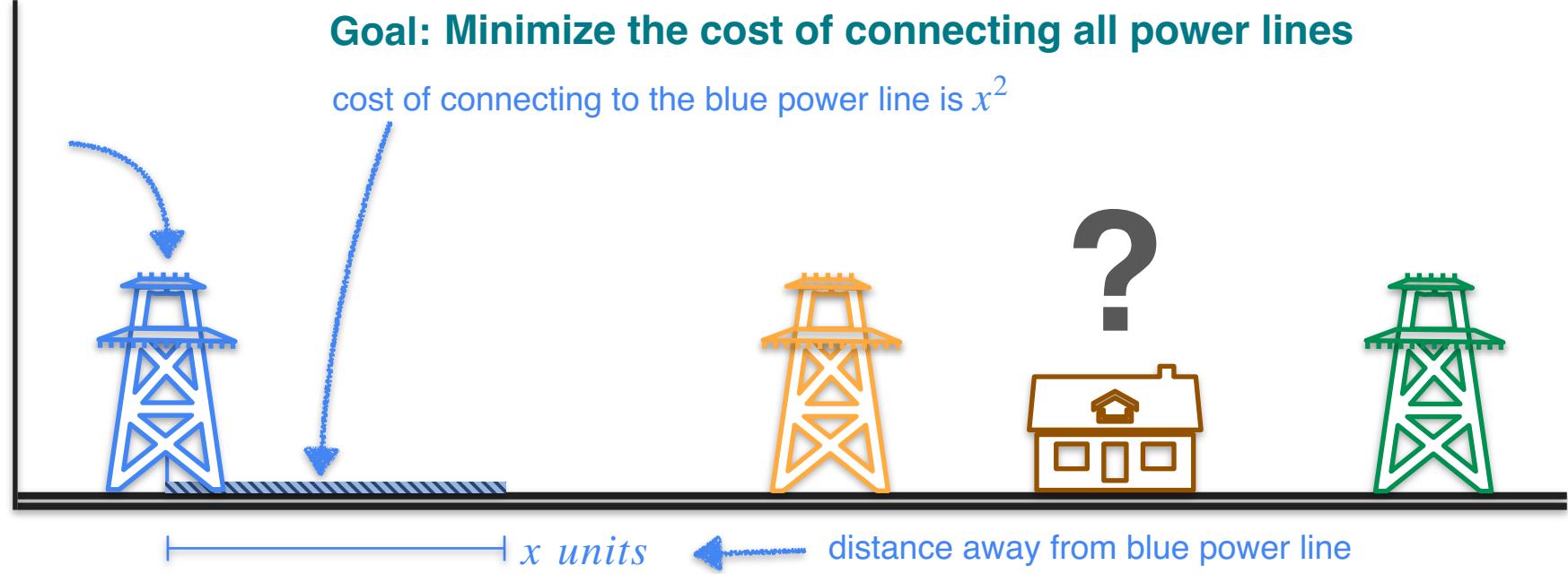
cost of connecting to the blue power line is x^2



Cost Function Motivation

Goal: Minimize the cost of connecting all power lines

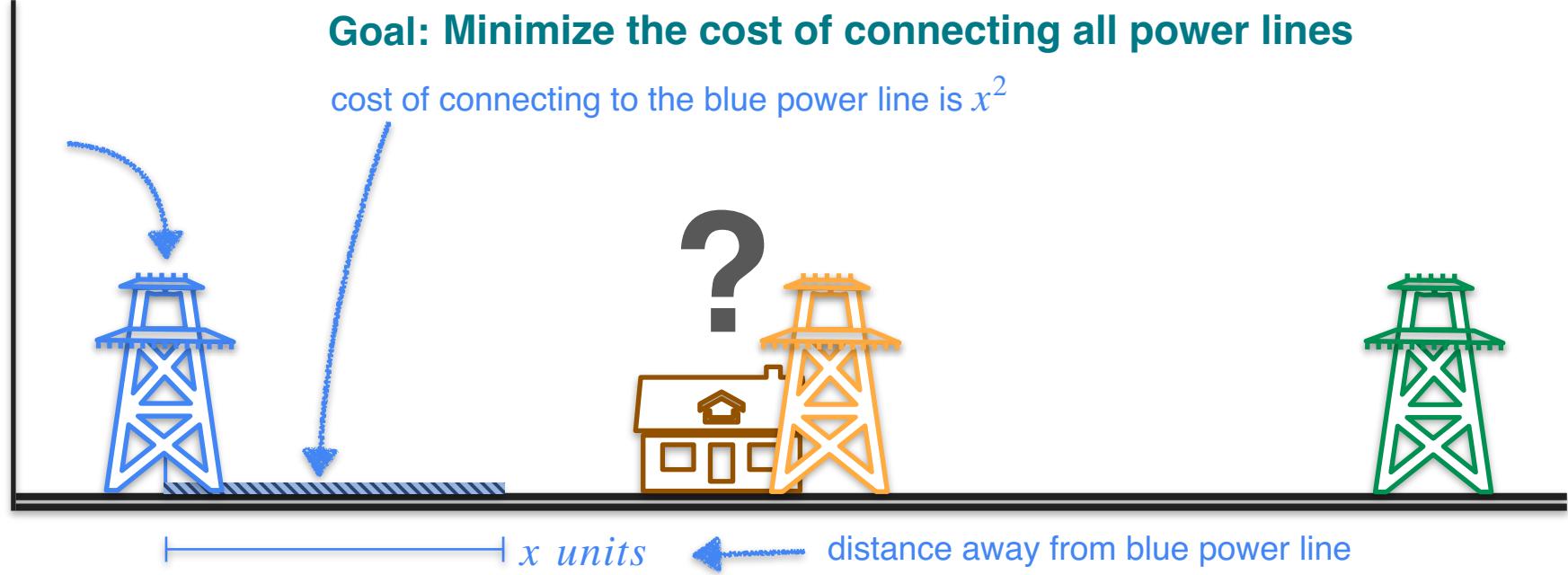
cost of connecting to the blue power line is x^2



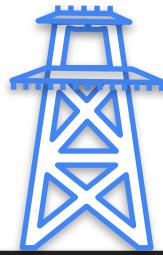
Cost Function Motivation

Goal: Minimize the cost of connecting all power lines

cost of connecting to the blue power line is x^2



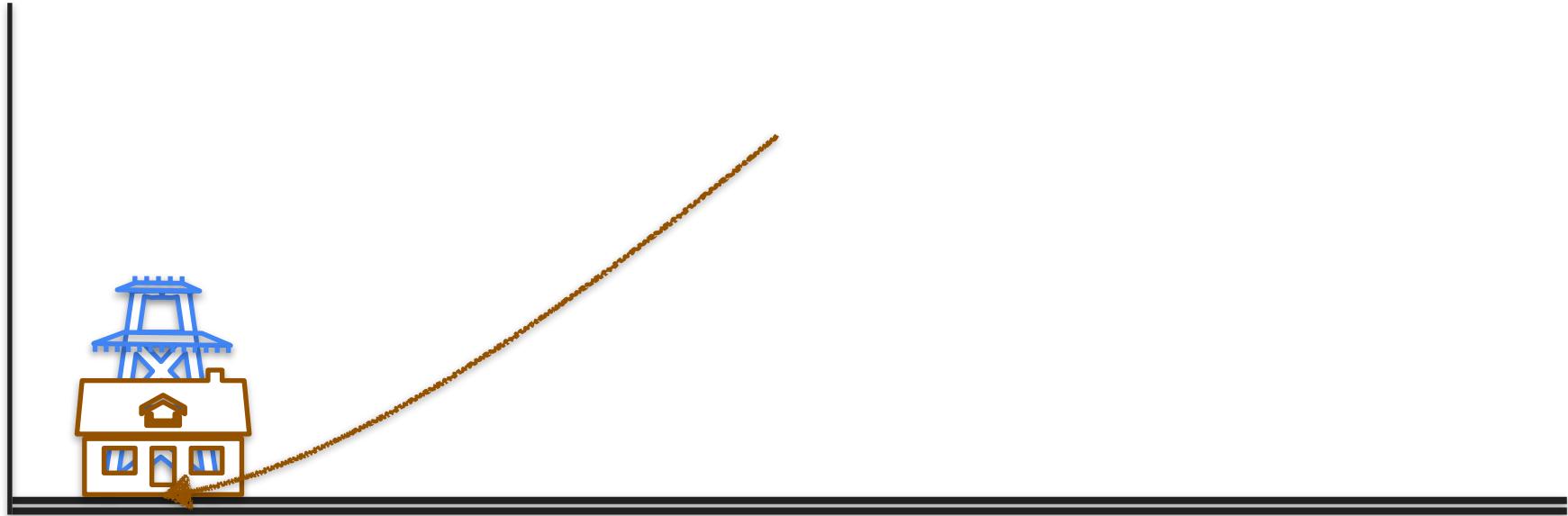
One Power Line Problem



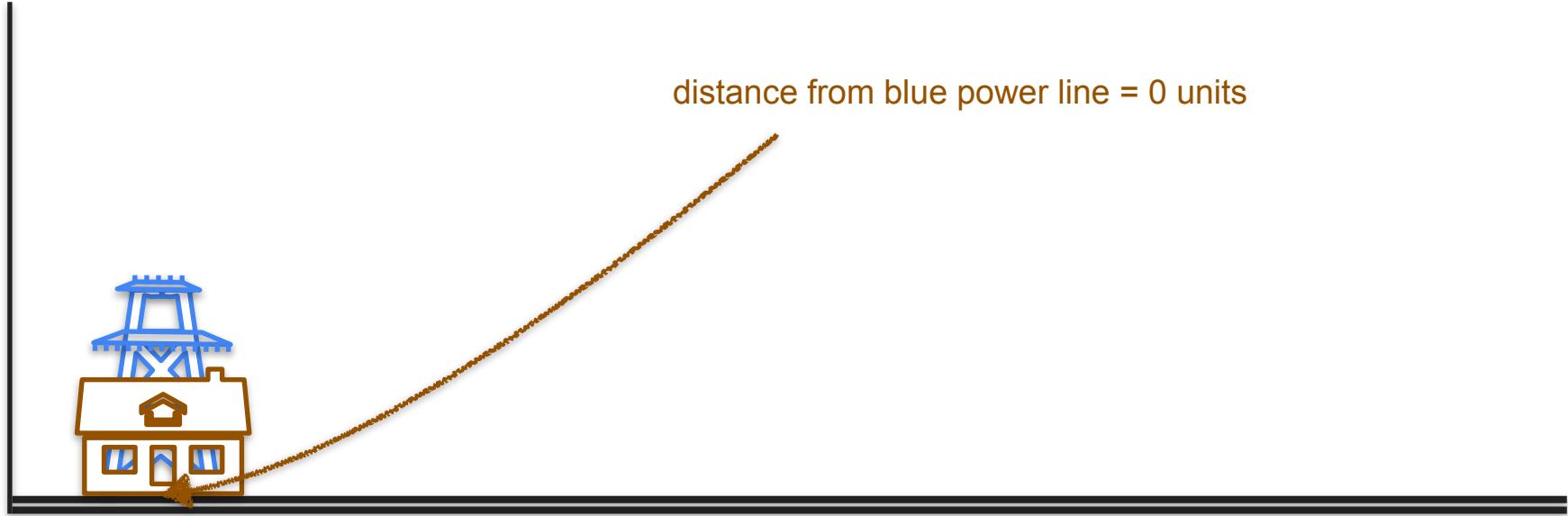
One Power Line Problem



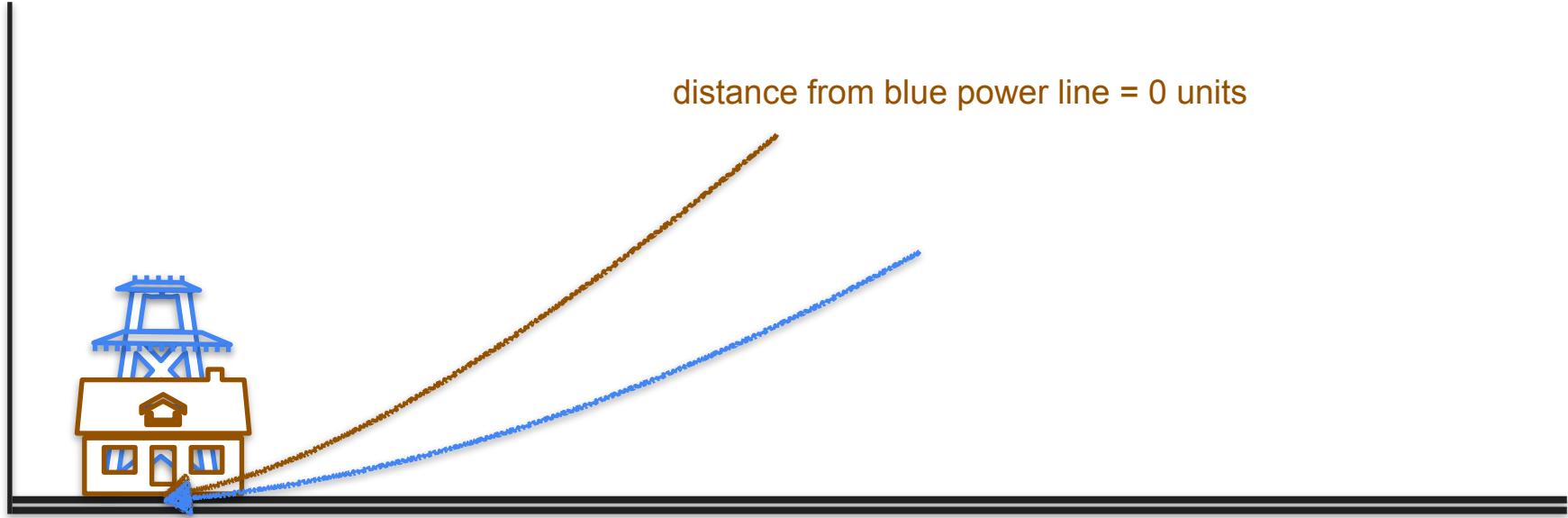
One Power Line Problem



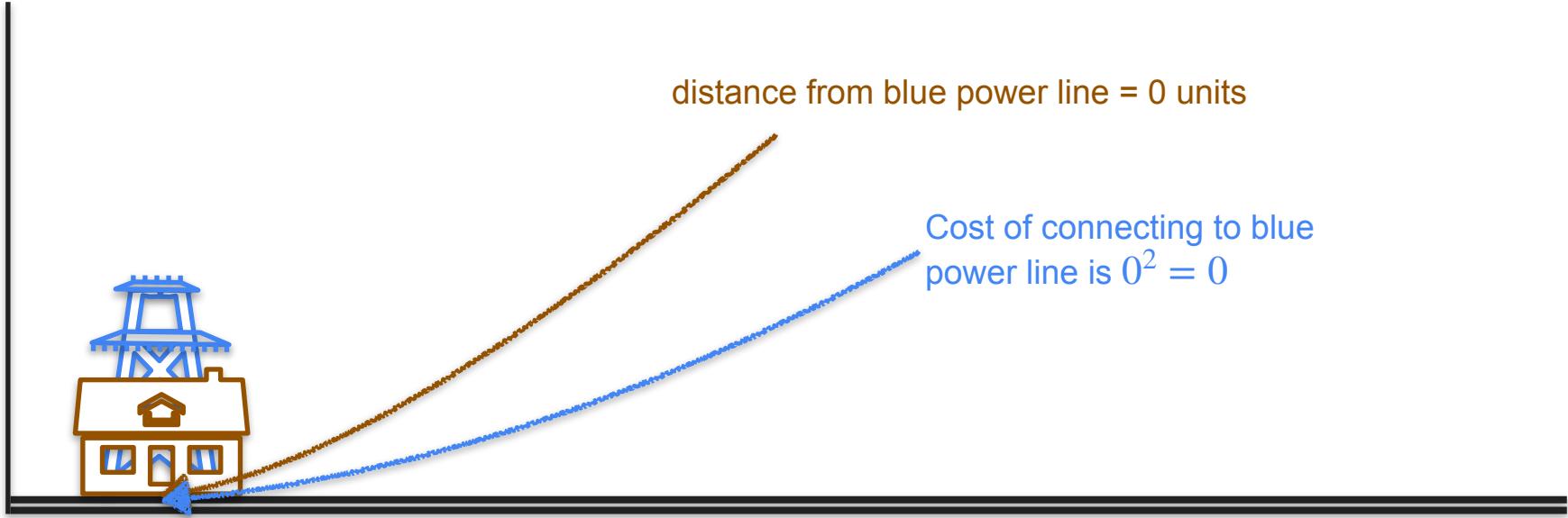
One Power Line Problem



One Power Line Problem



One Power Line Problem





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Derivatives and Optimization

**Optimization of squared loss:
The two powerline problem**

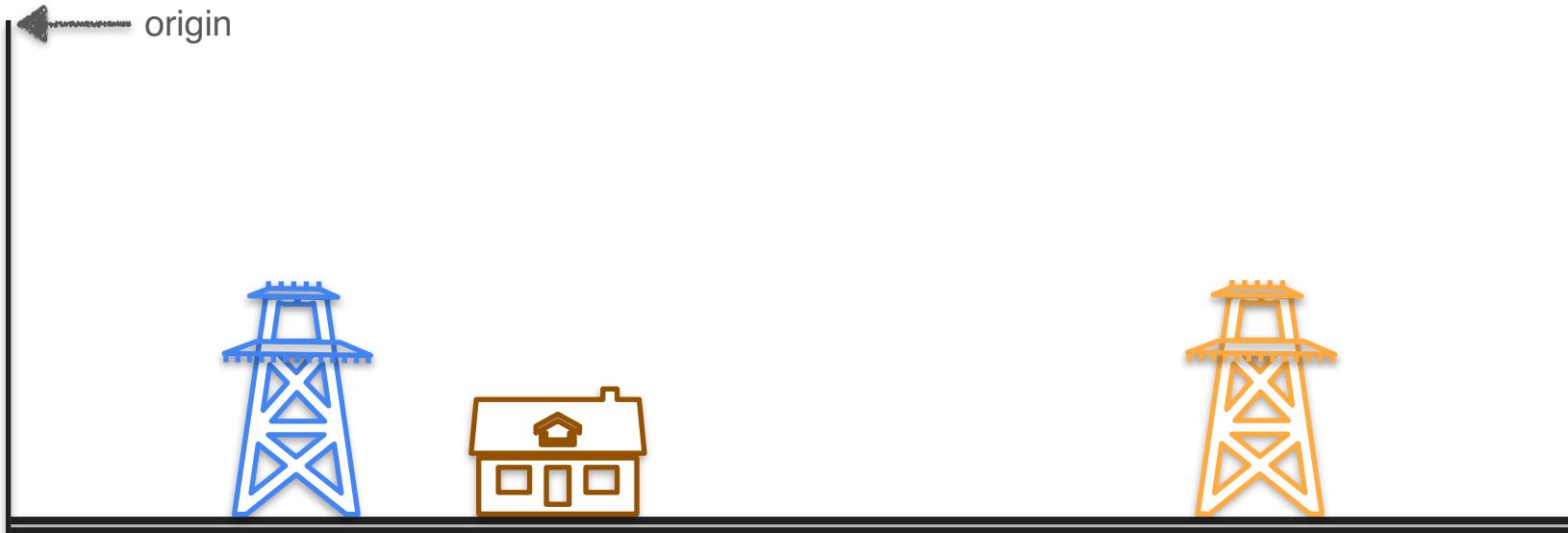
Two Power Line Problem



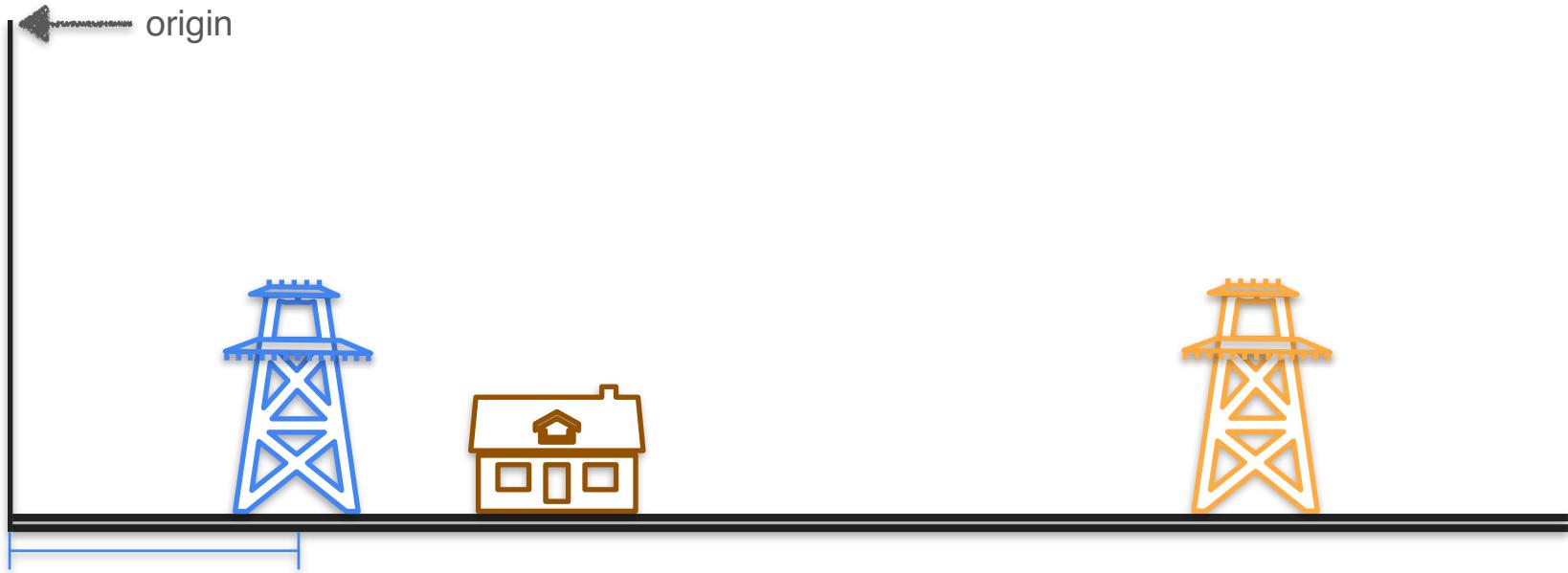
Two Power Line Problem



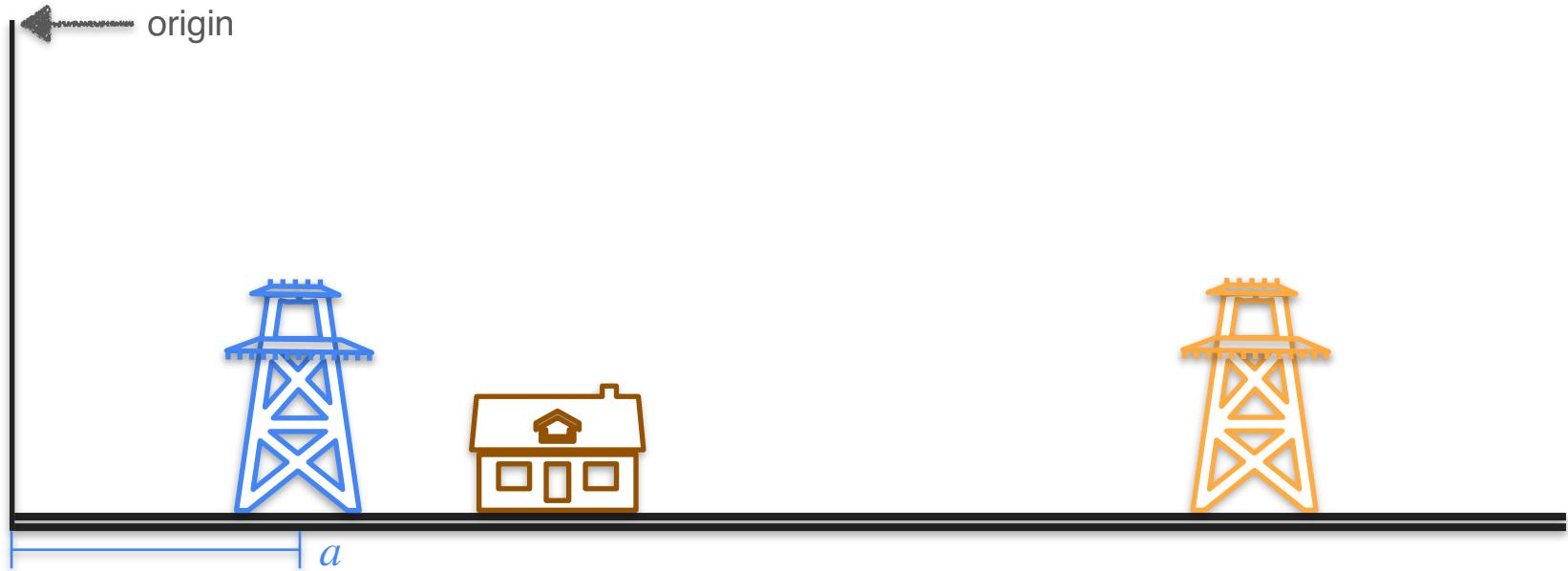
Two Power Line Problem



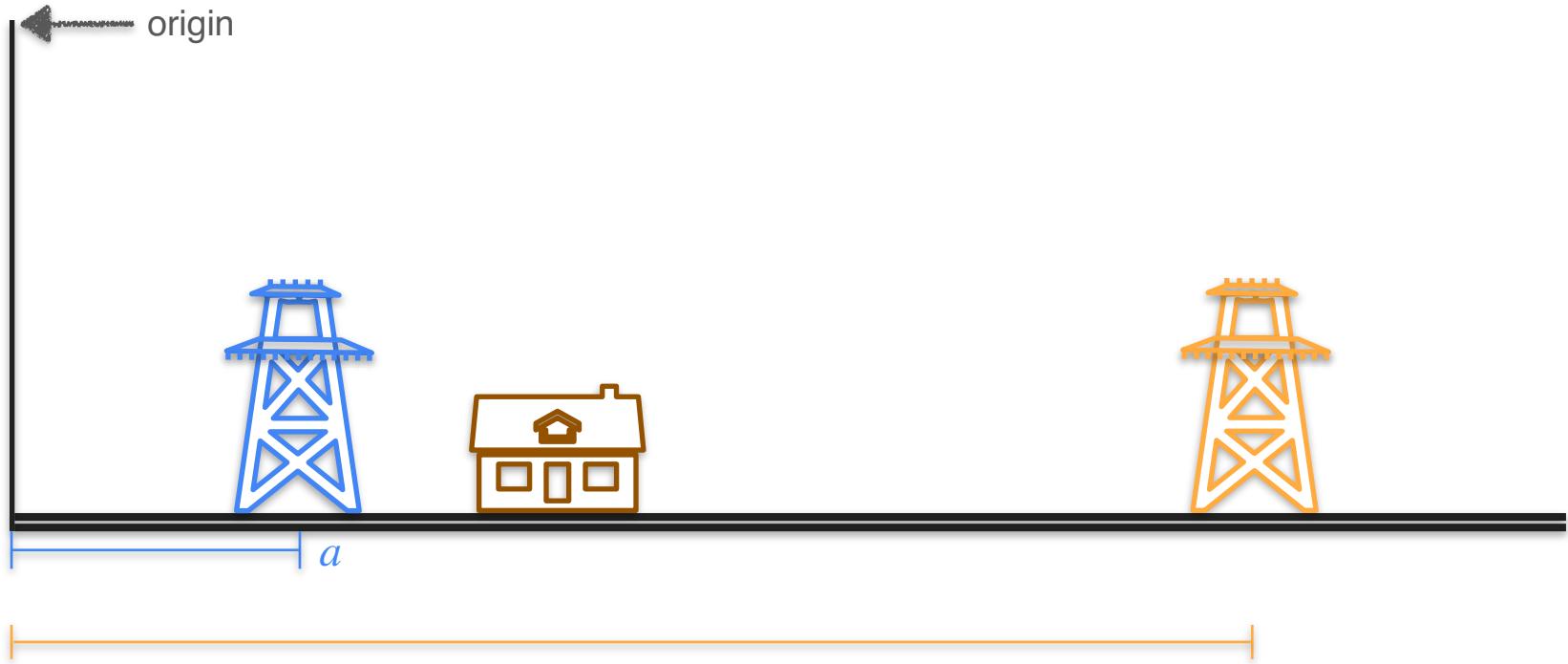
Two Power Line Problem



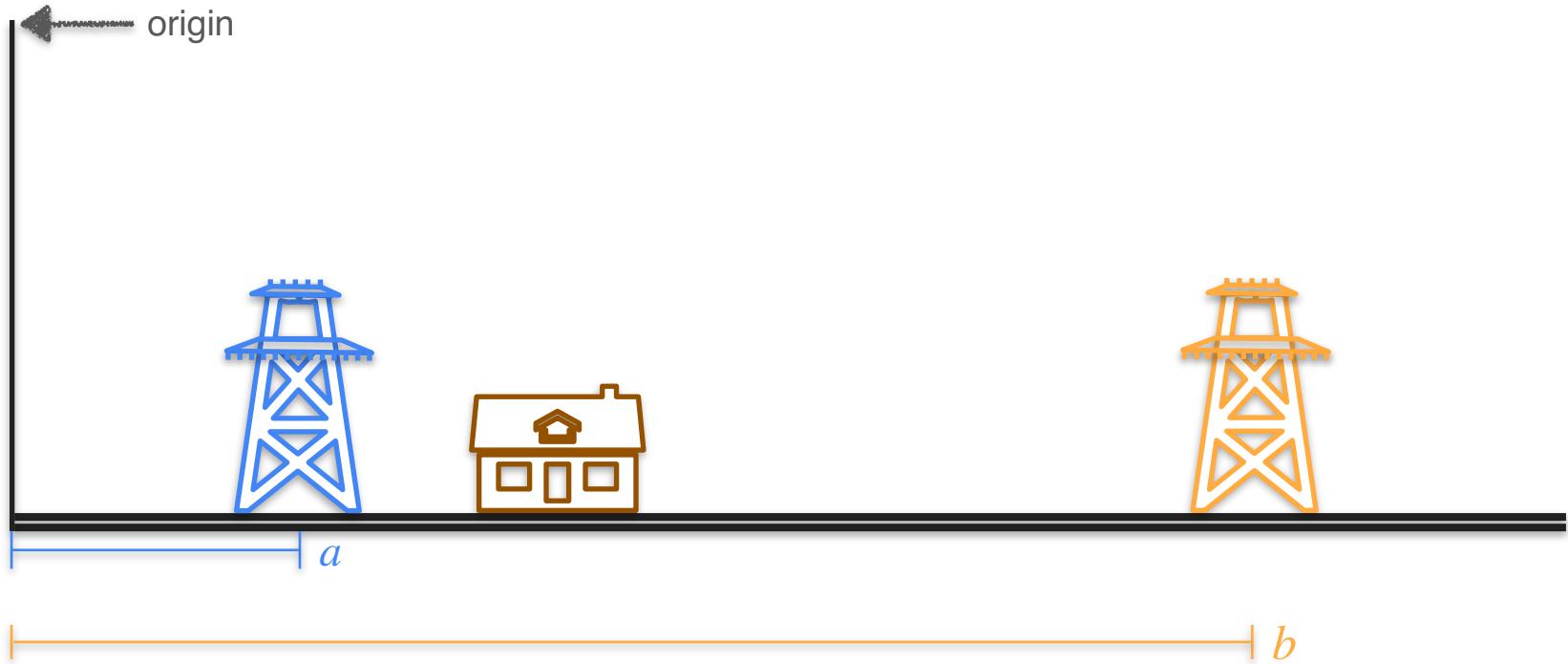
Two Power Line Problem



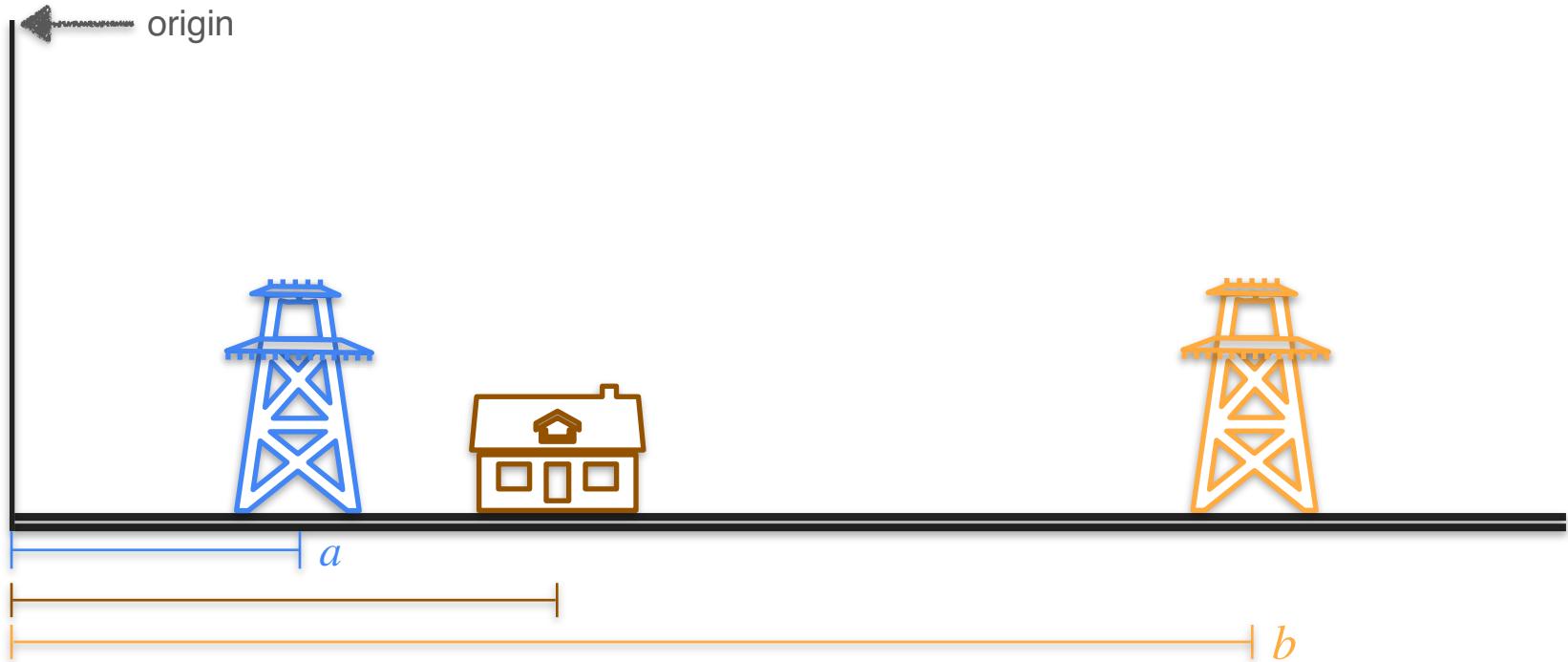
Two Power Line Problem



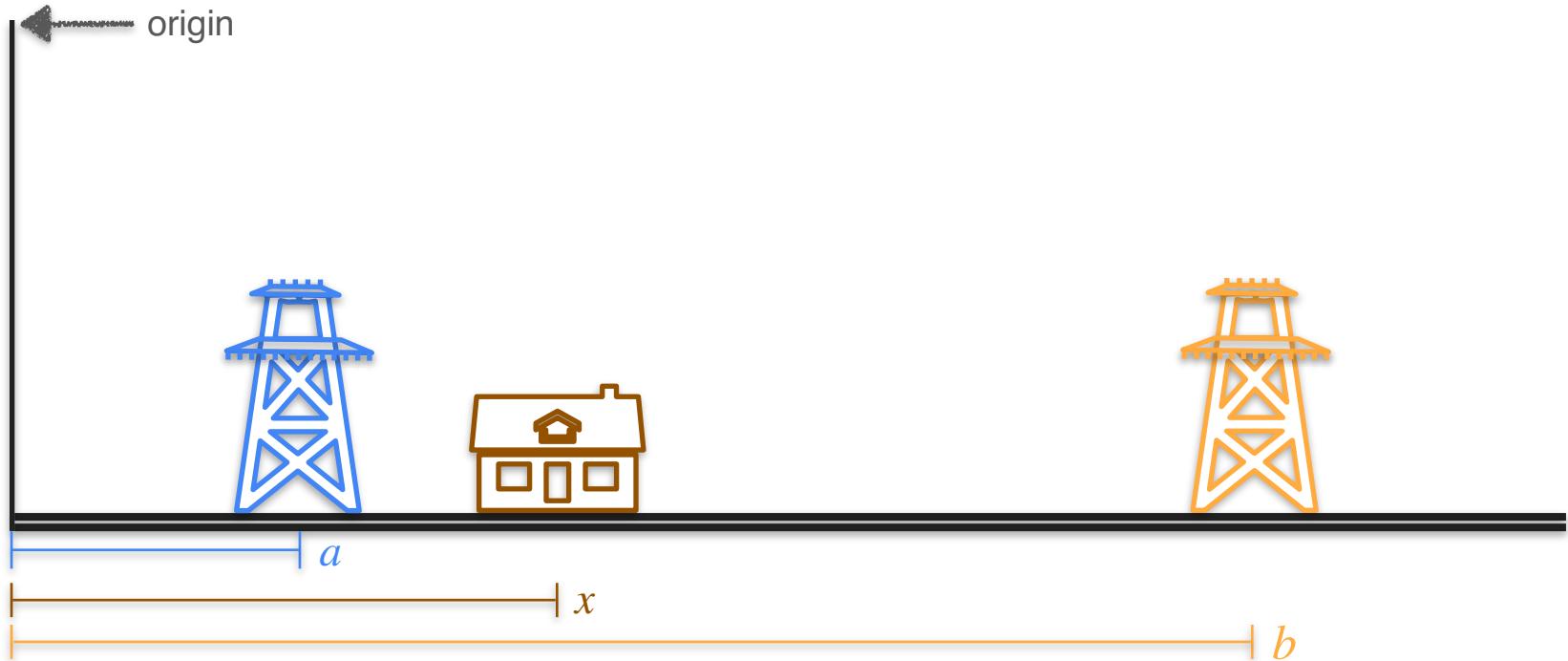
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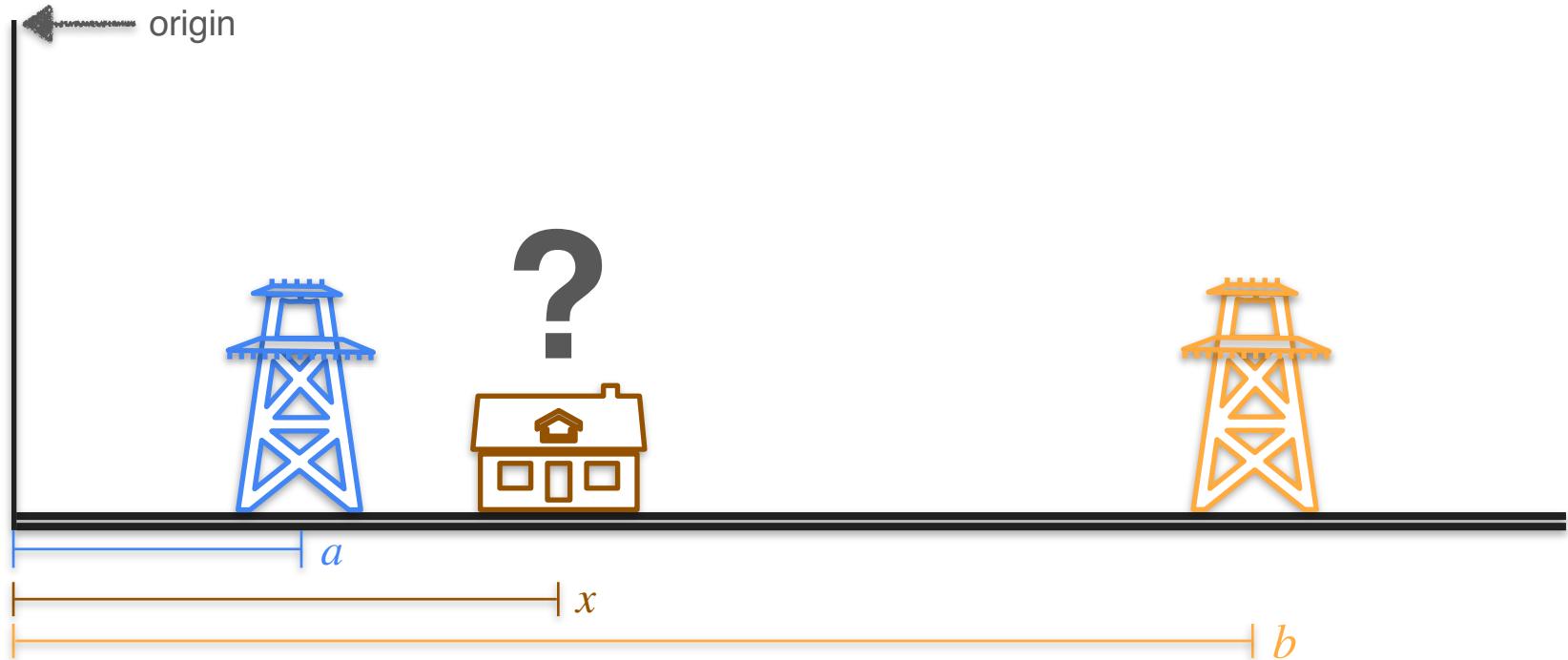
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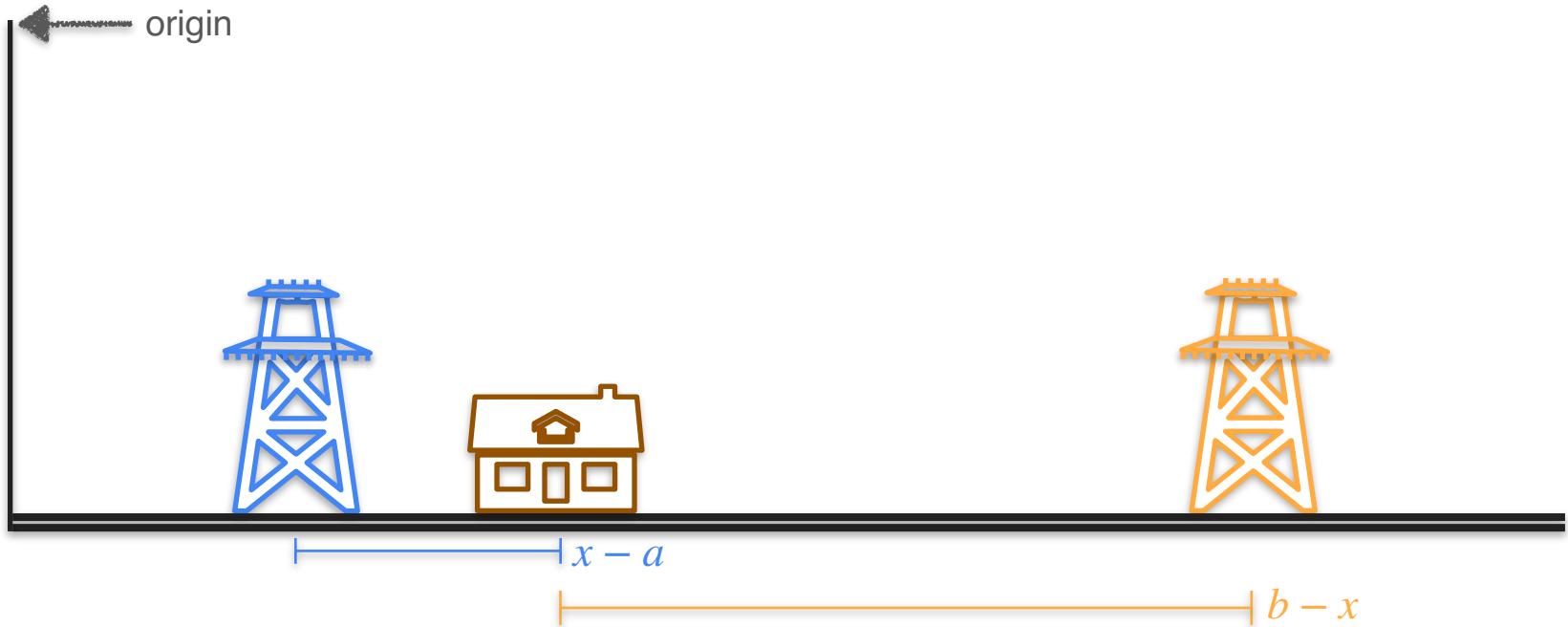
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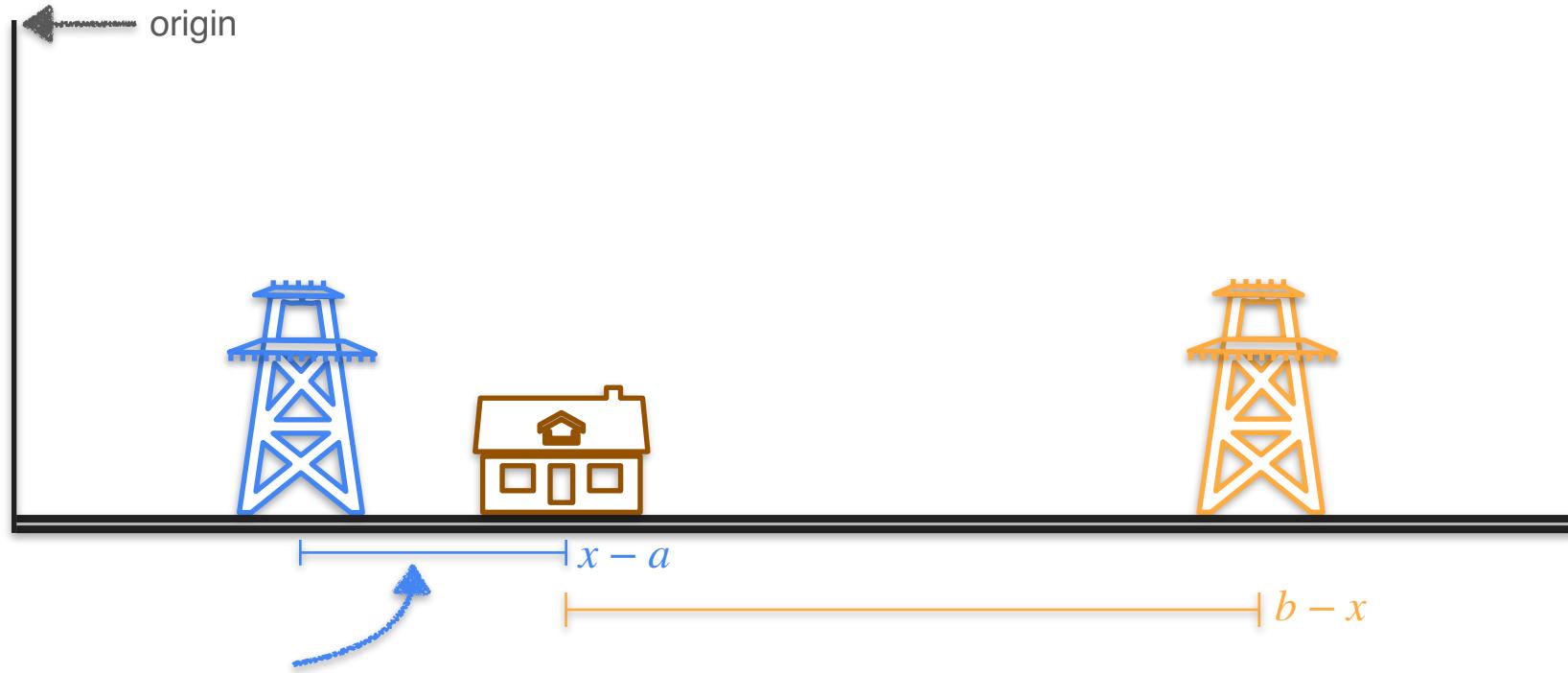
Two Power Line Problem



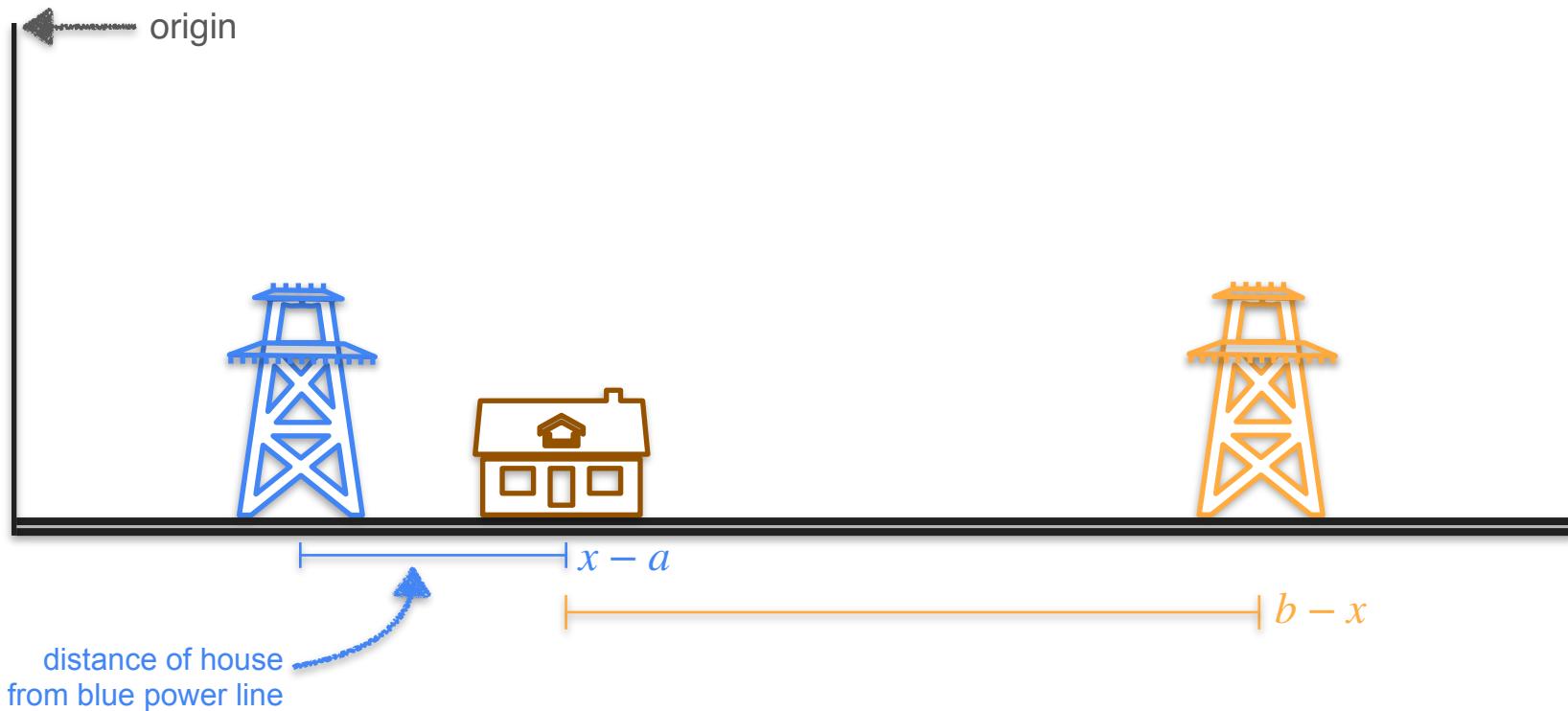
Two Power Line Problem



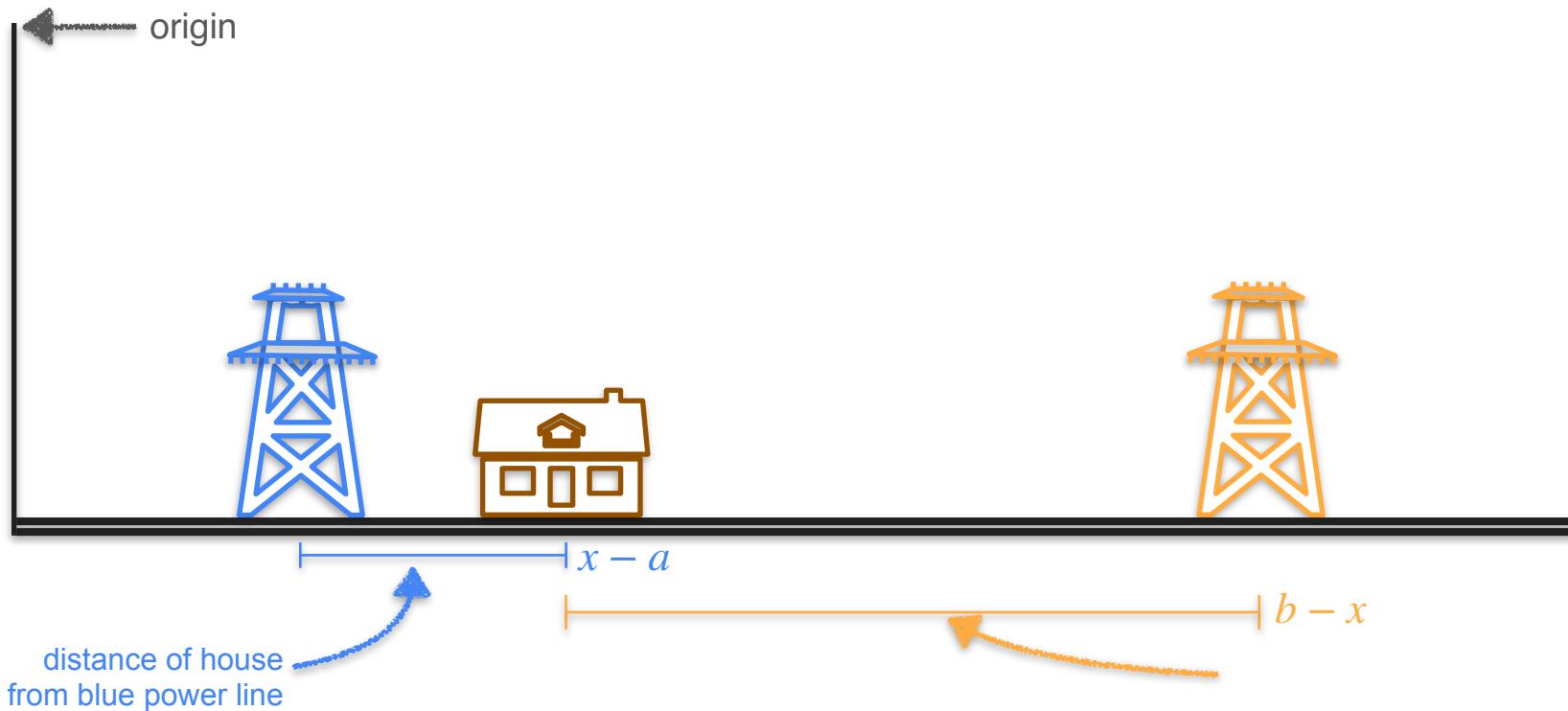
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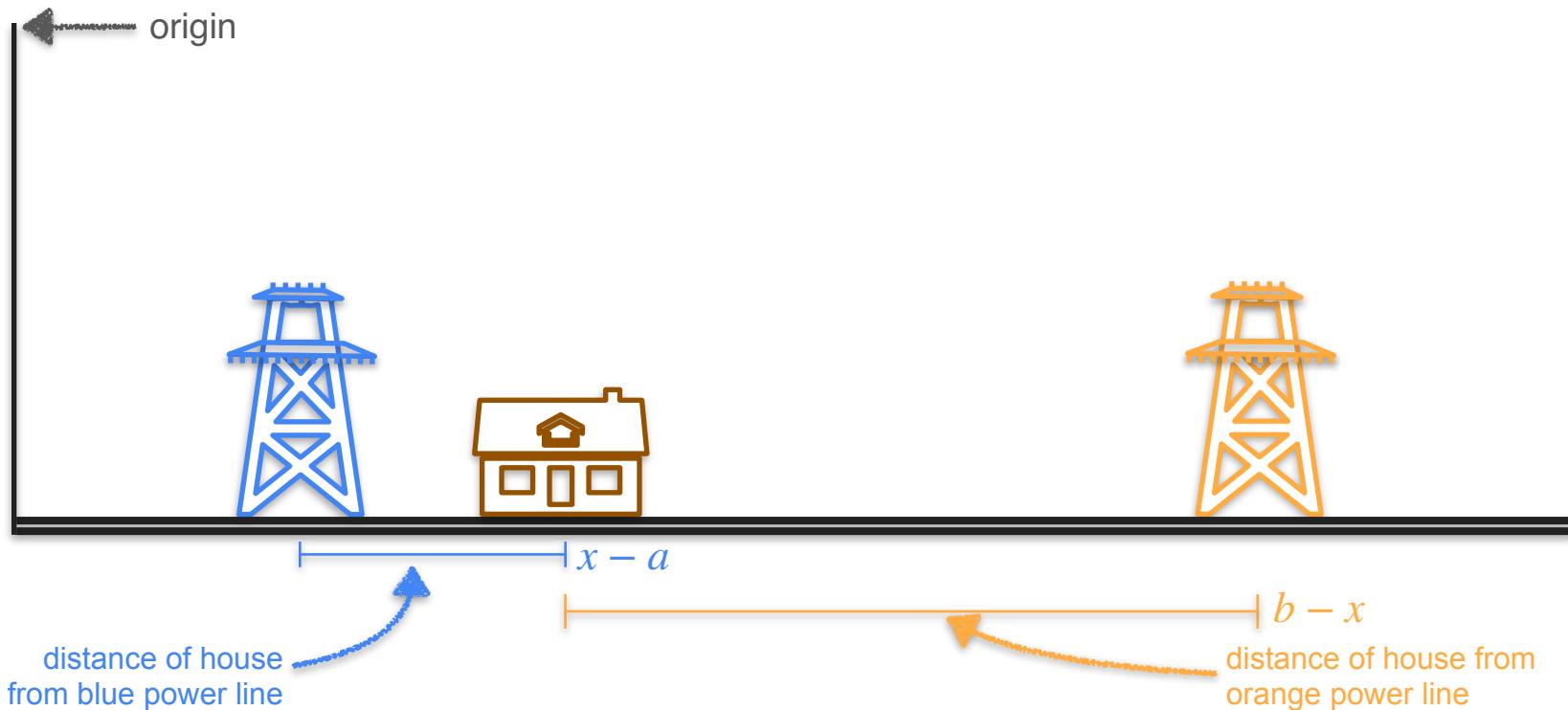
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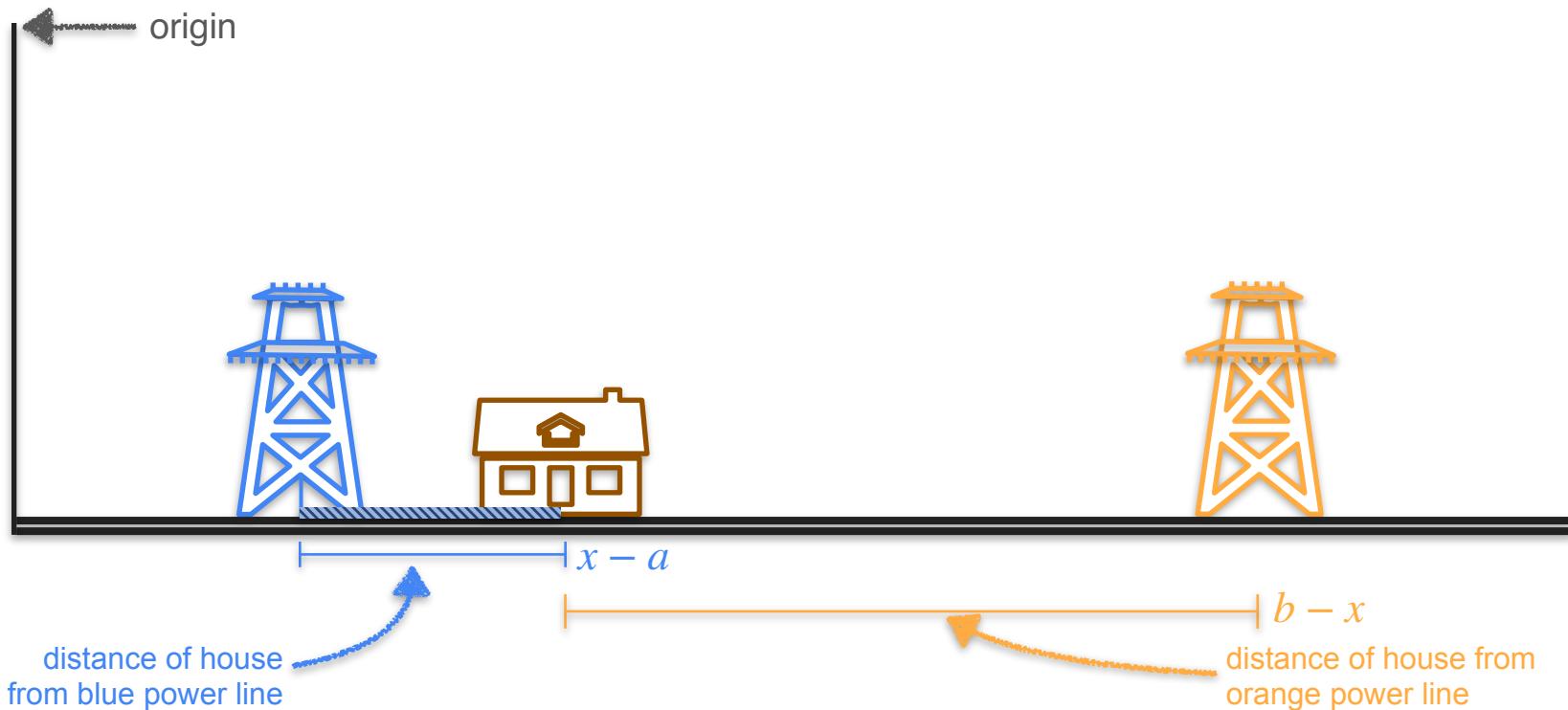
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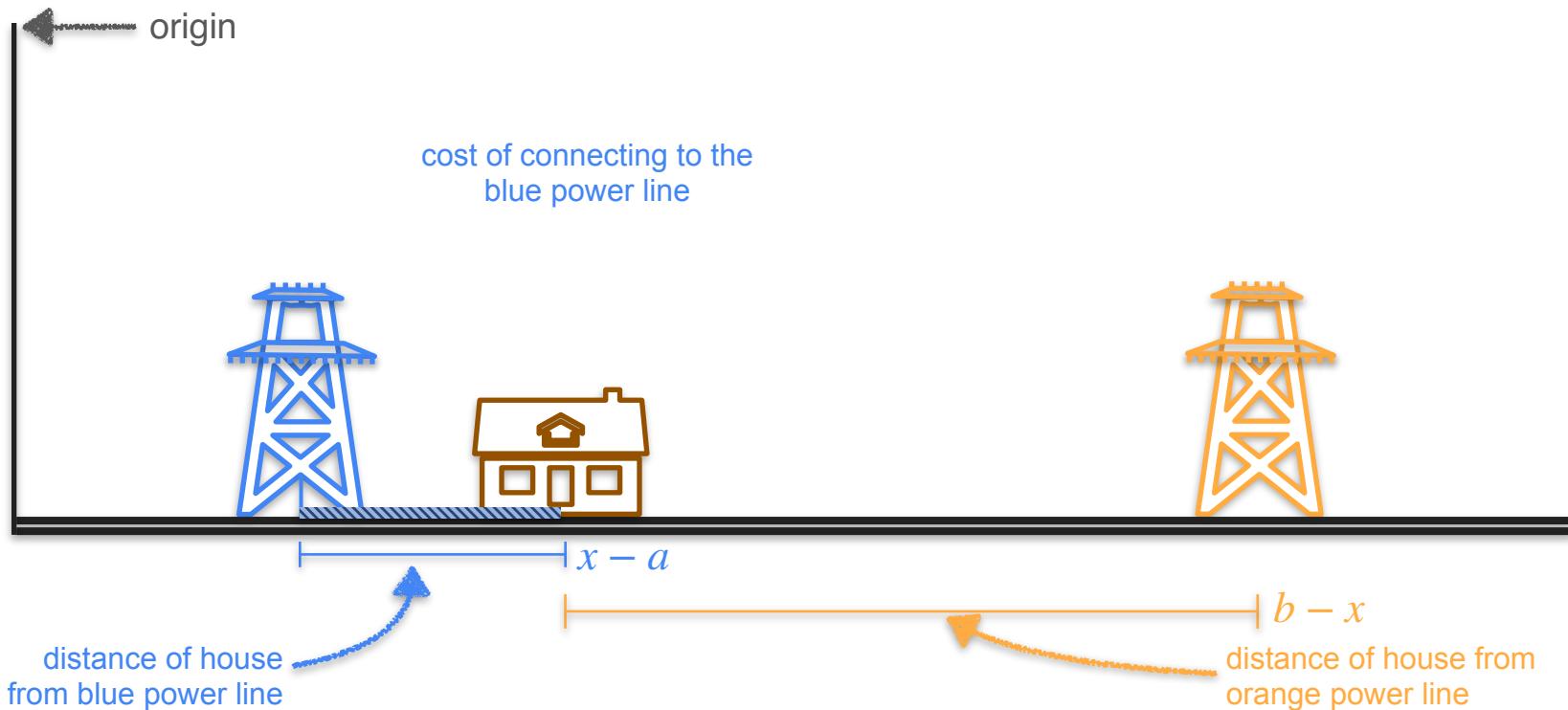
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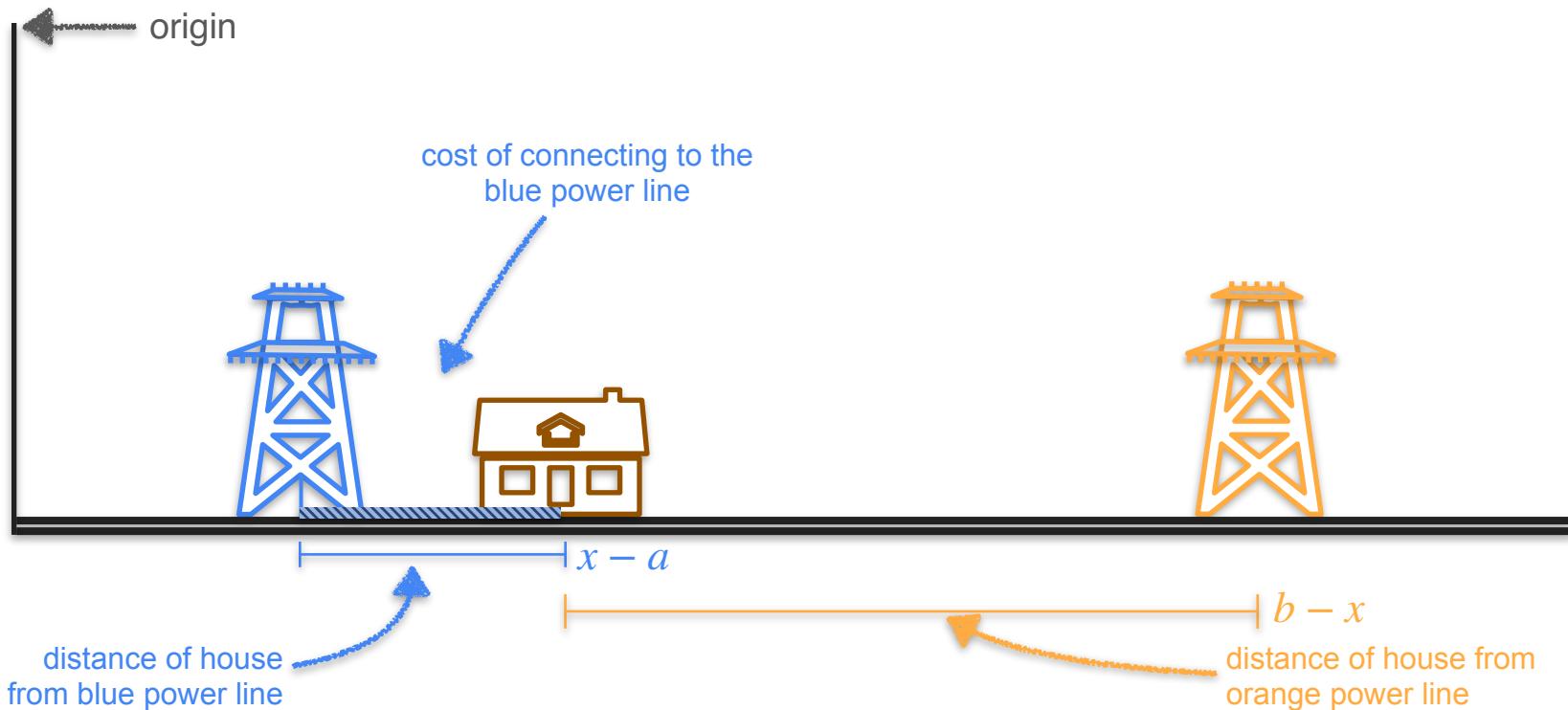
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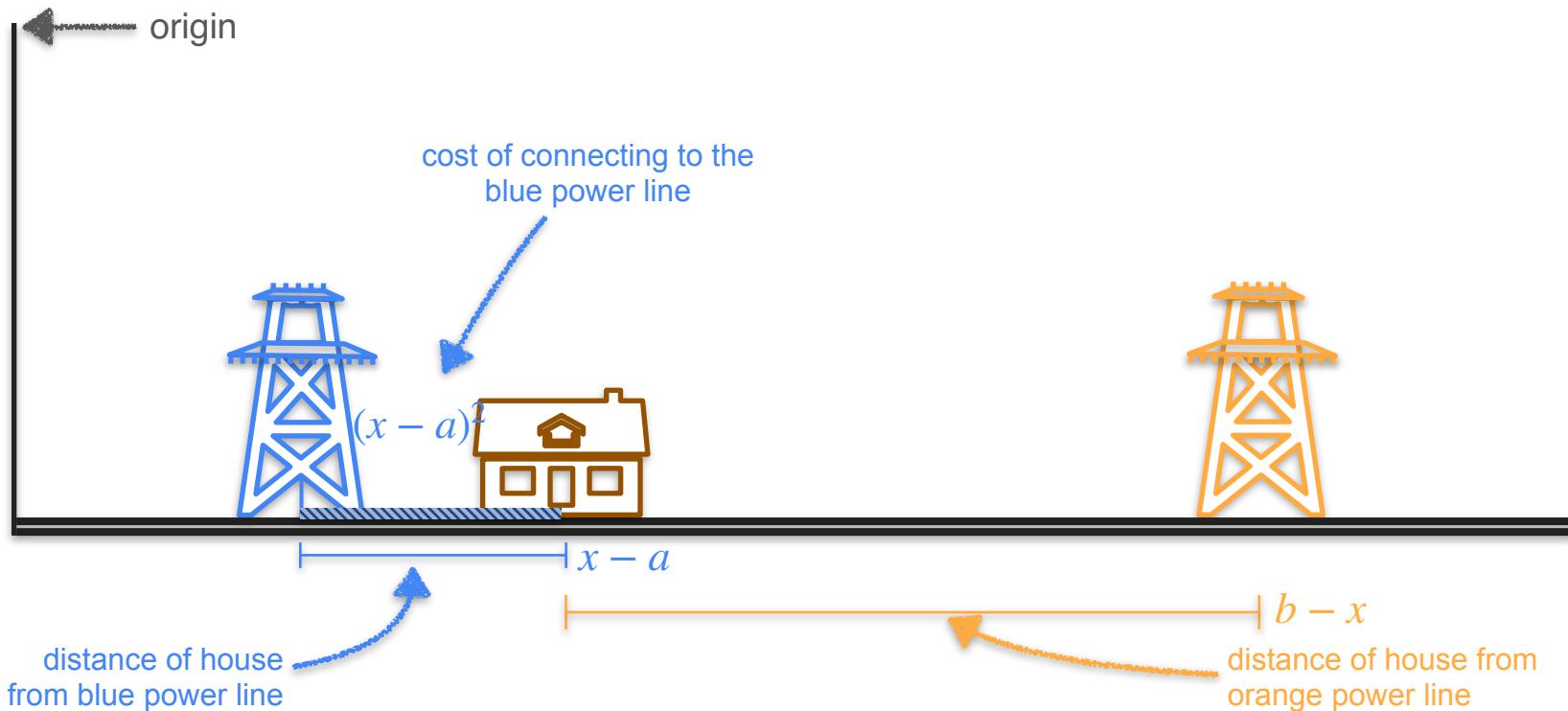
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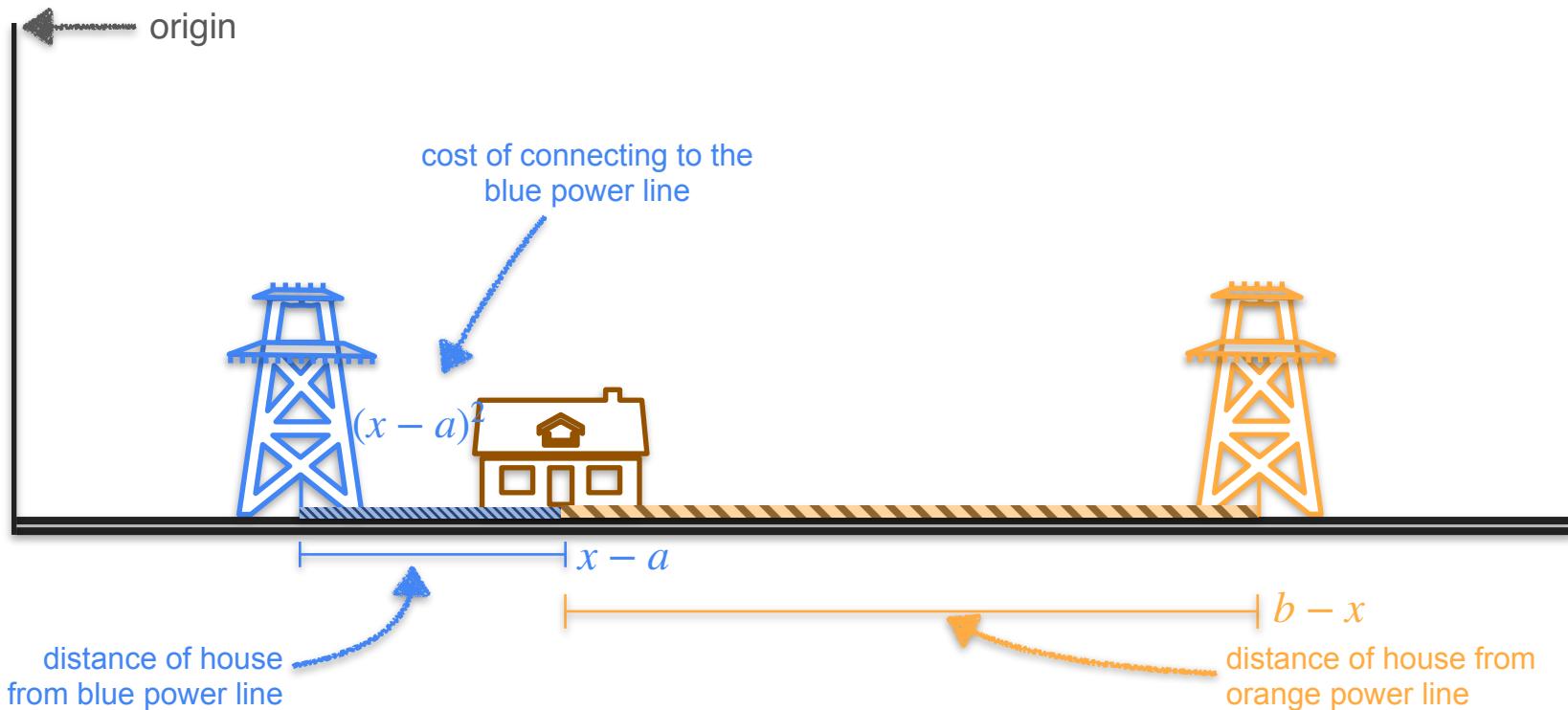
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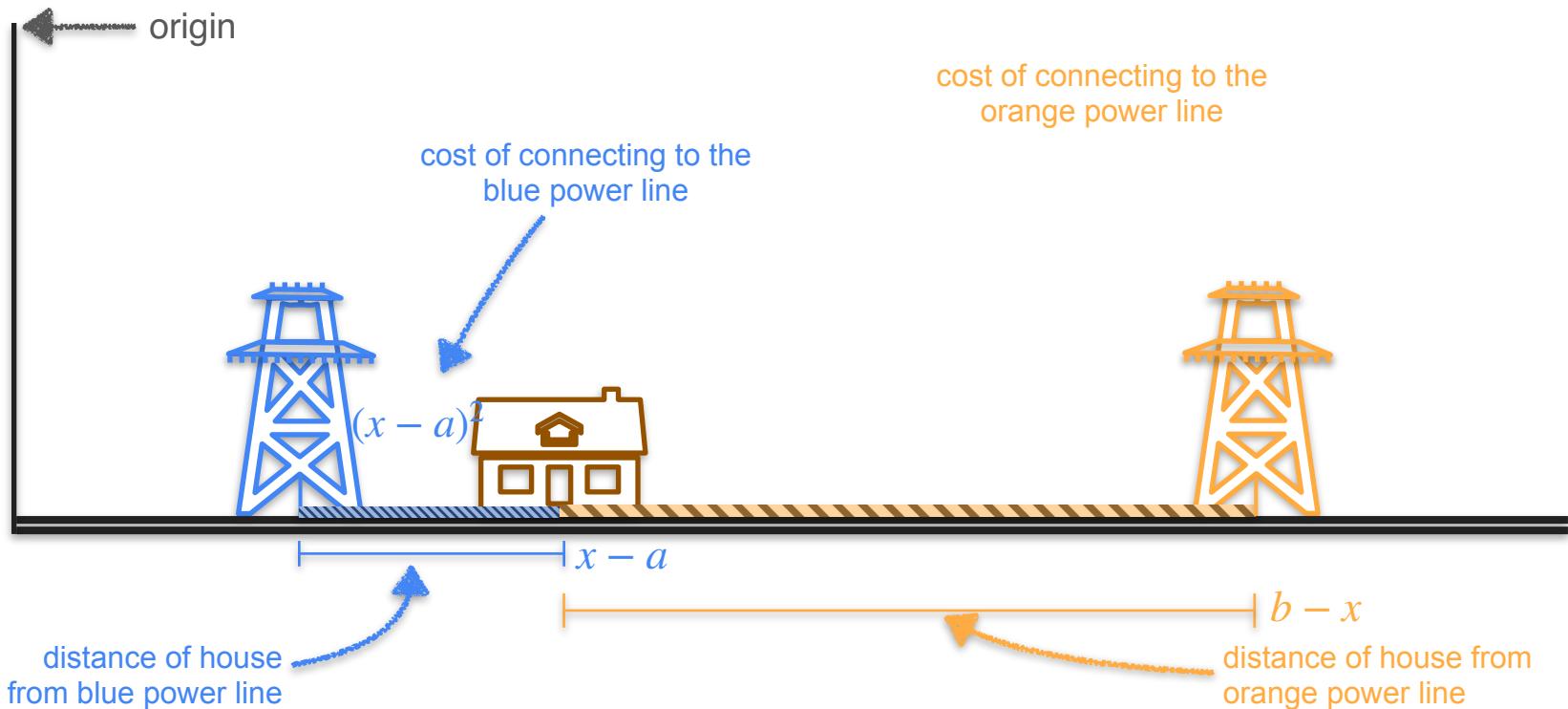
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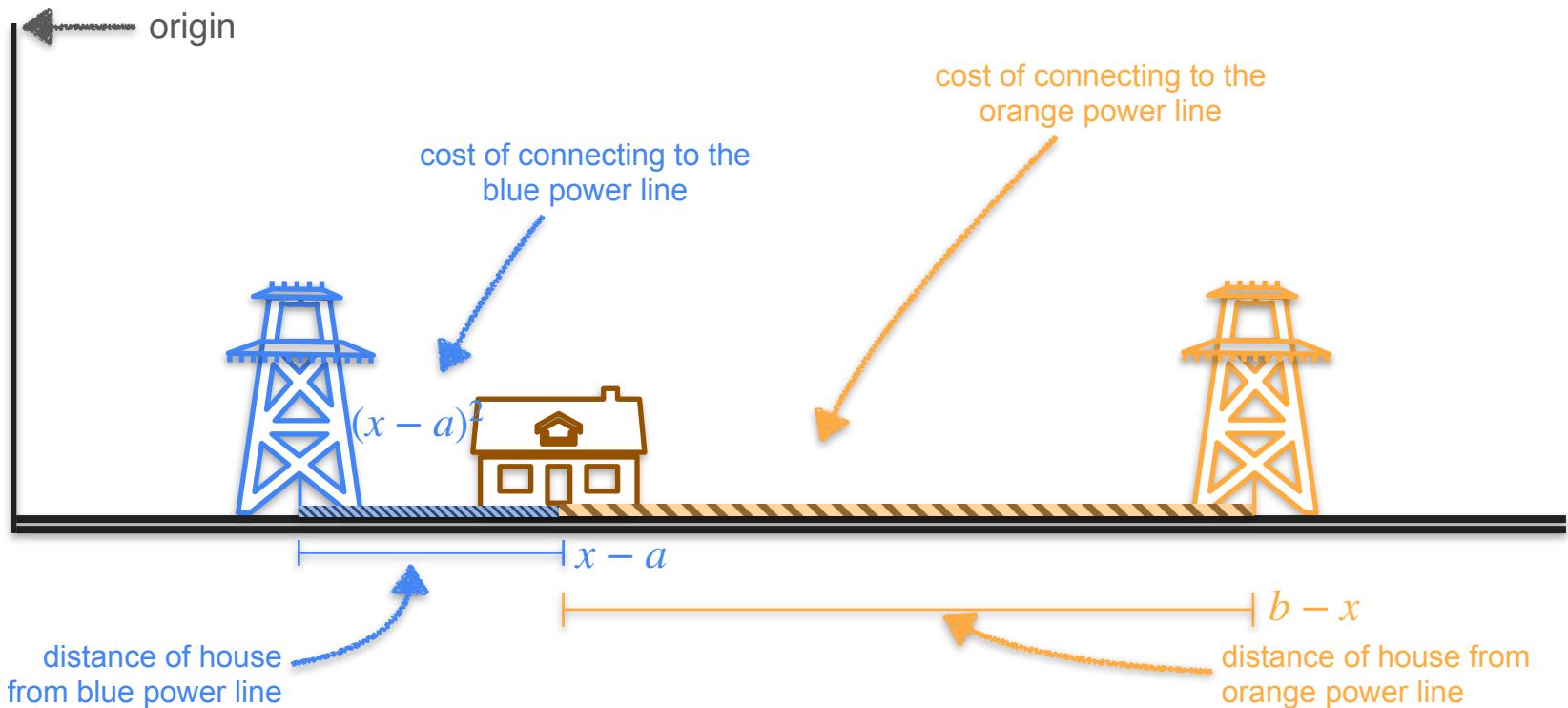
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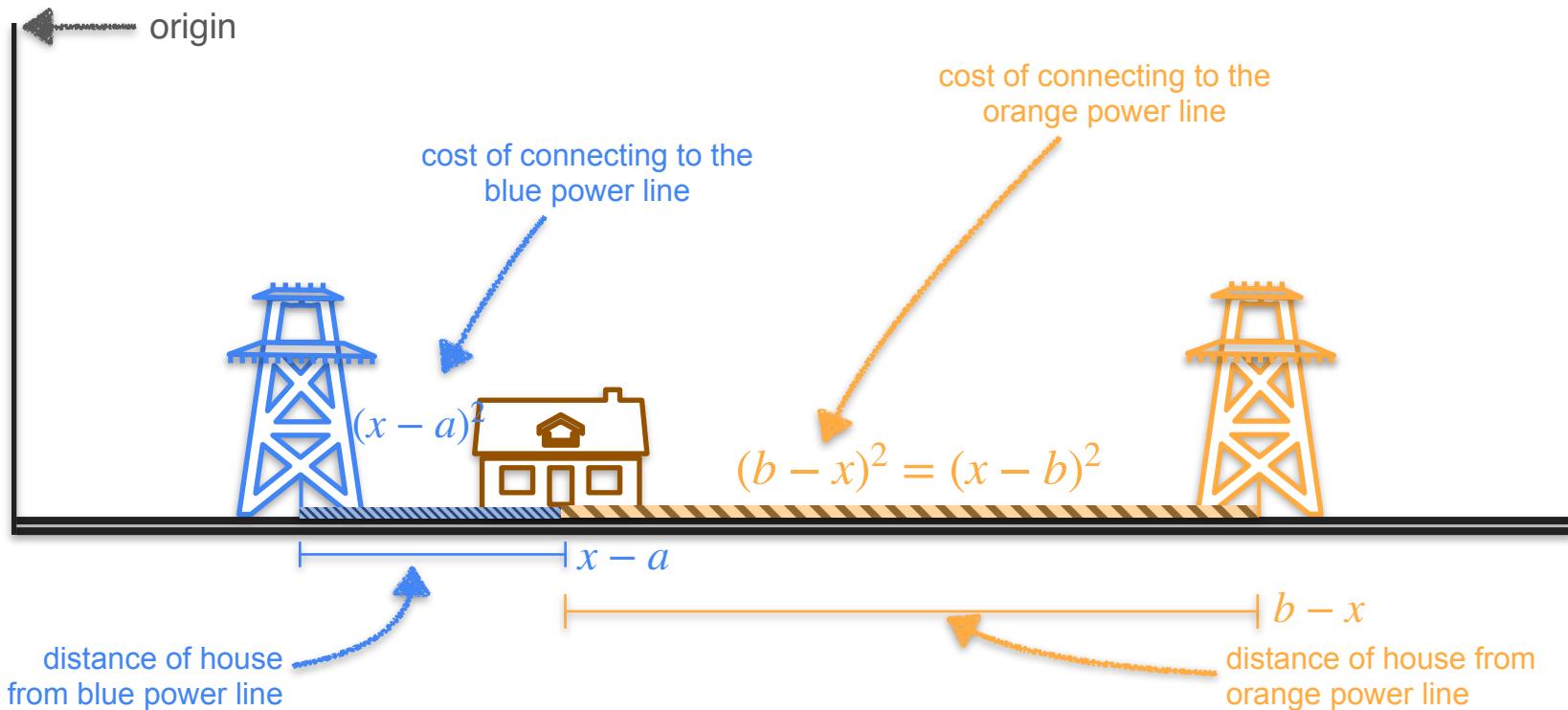
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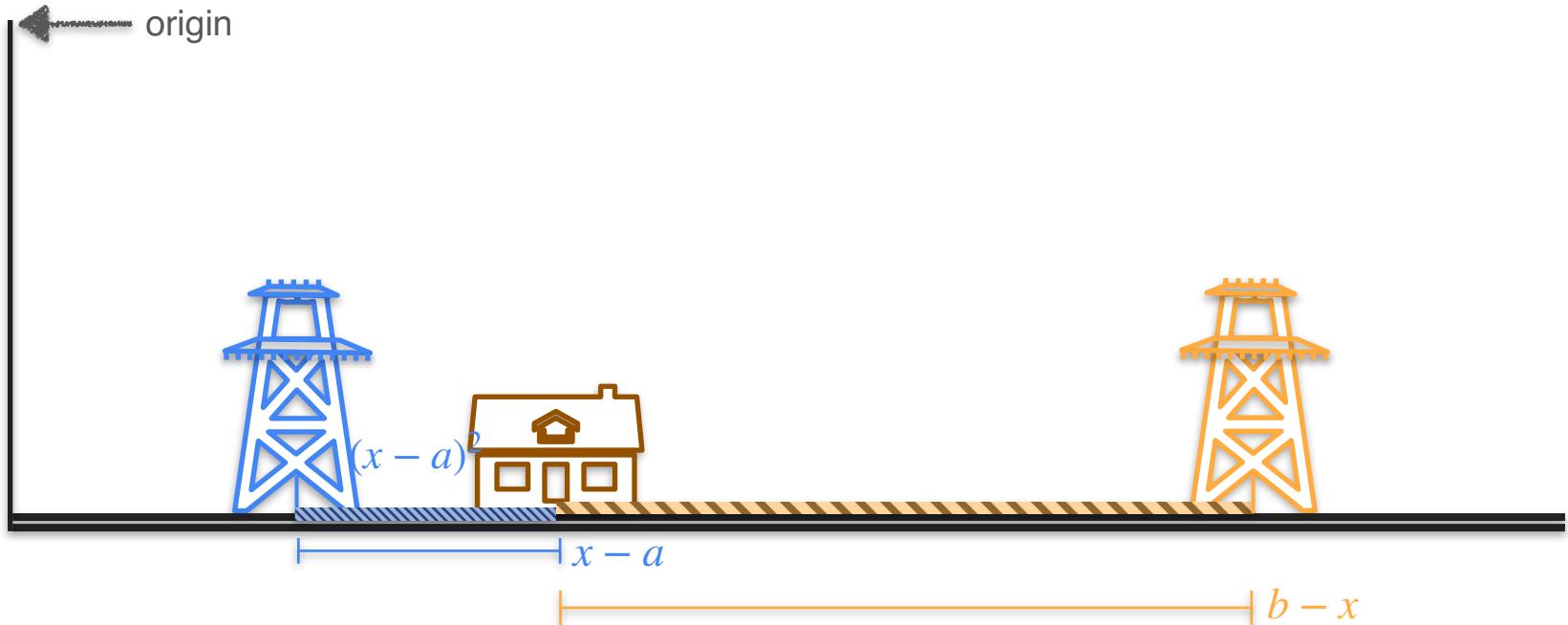
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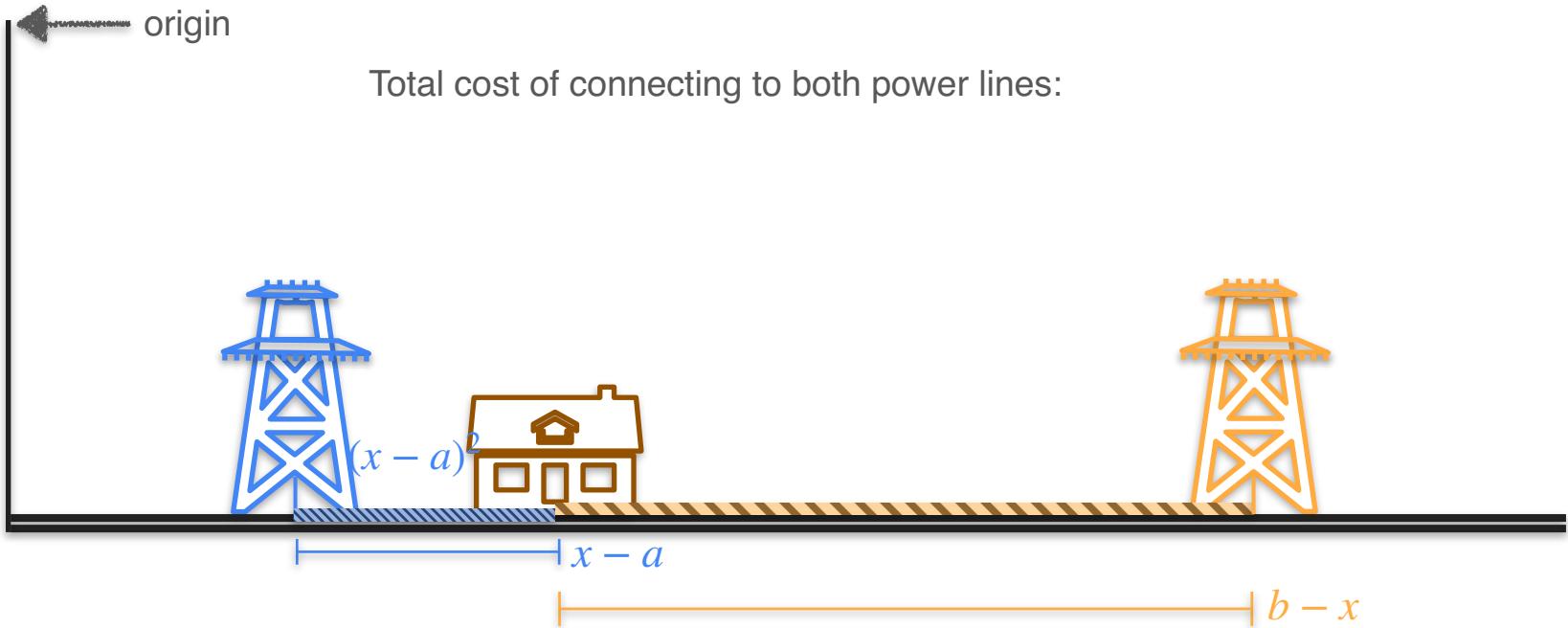
Two Power Line Problem



Two Power Line Problem



Two Power Line Problem

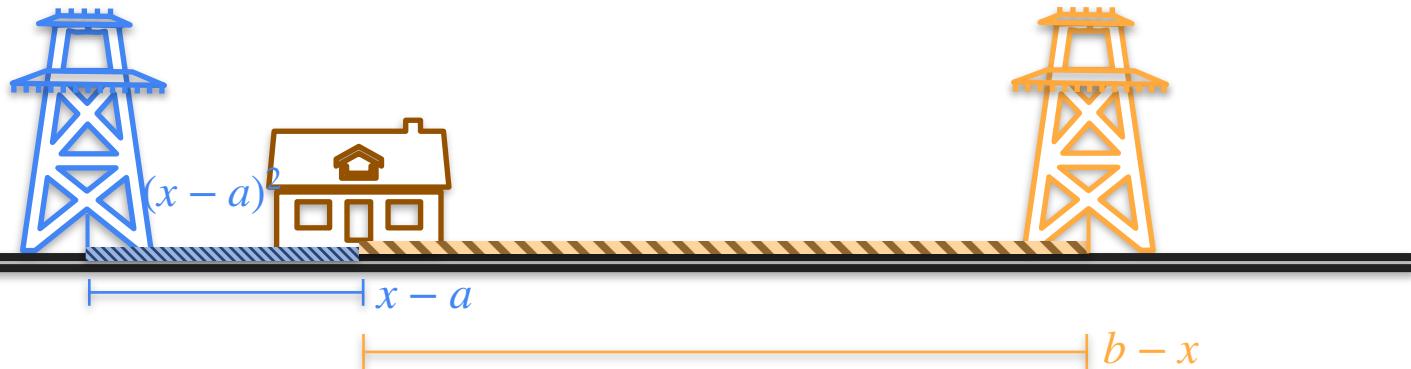


Two Power Line Problem

origin

Total cost of connecting to both power lines:

$$(x - a)^2 + (x - b)^2$$

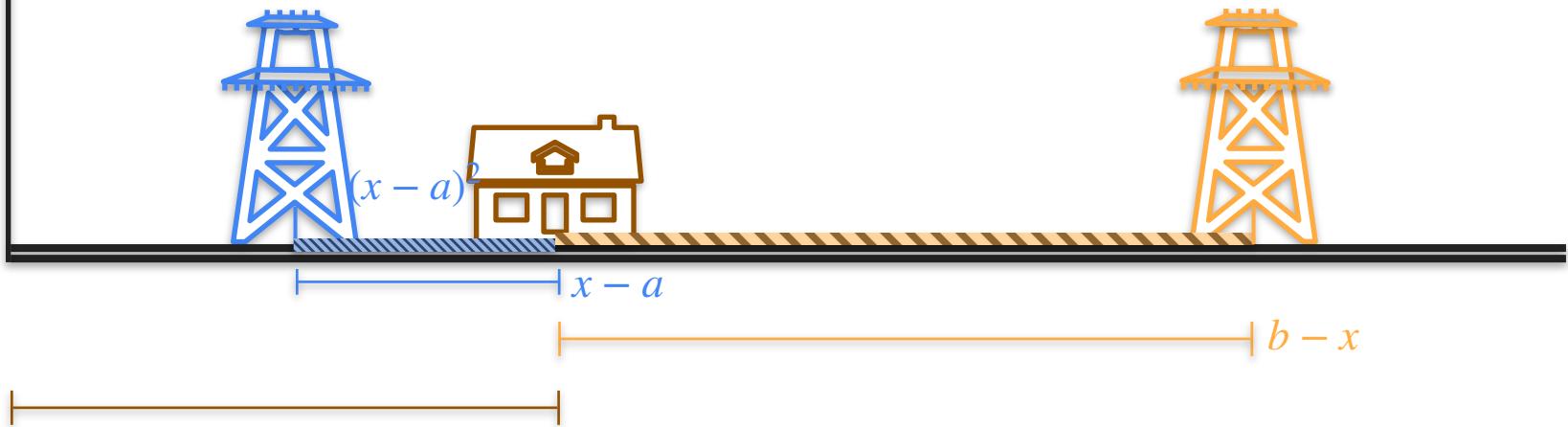


Two Power Line Problem

origin

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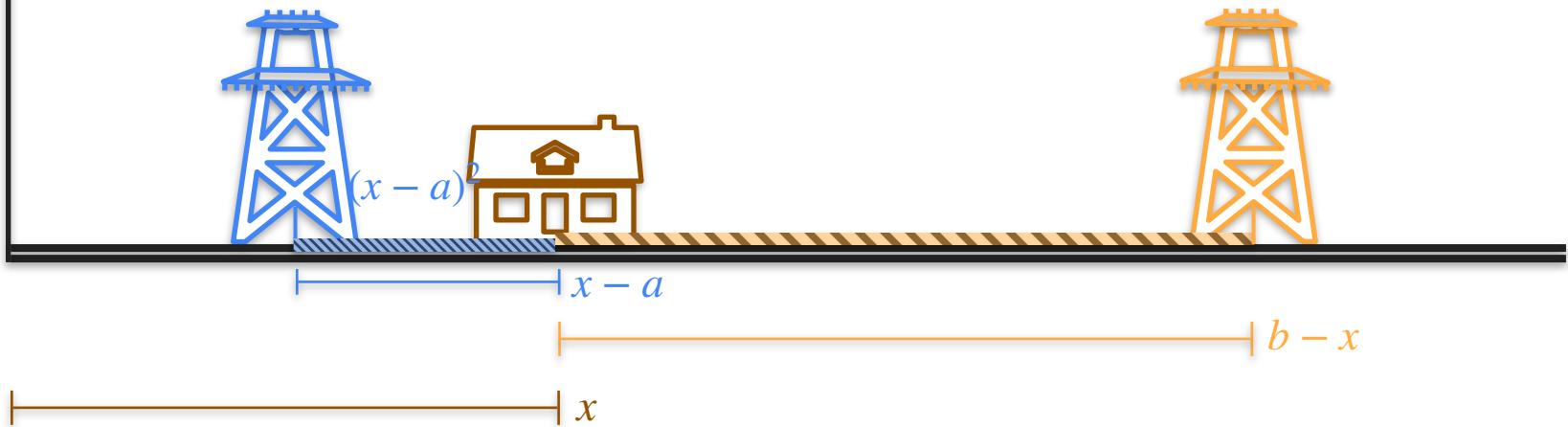


Two Power Line Problem

origin

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$$(x - a)^2 + (x - b)^2$$

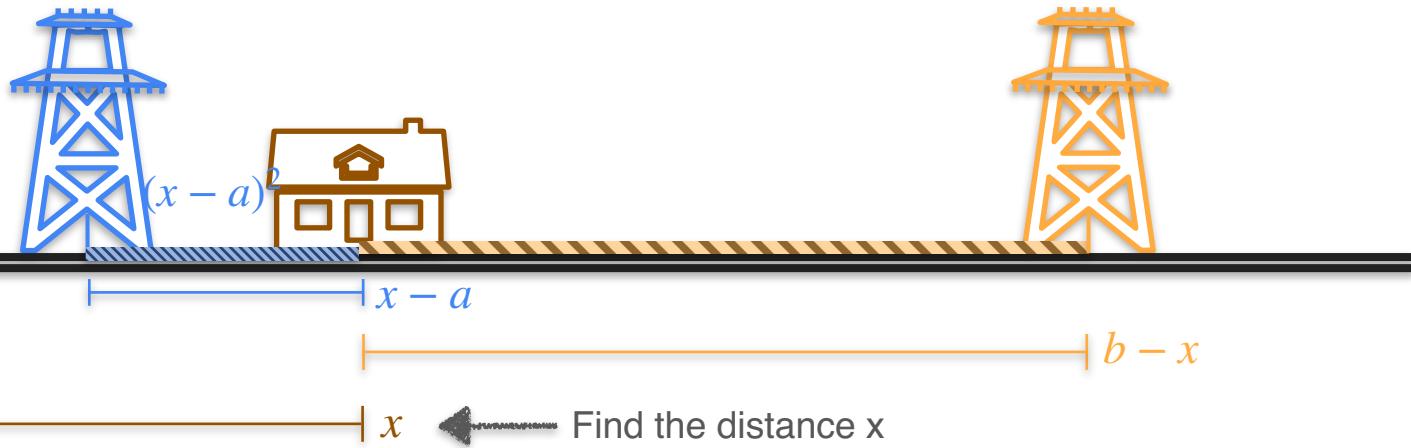


Two Power Line Problem

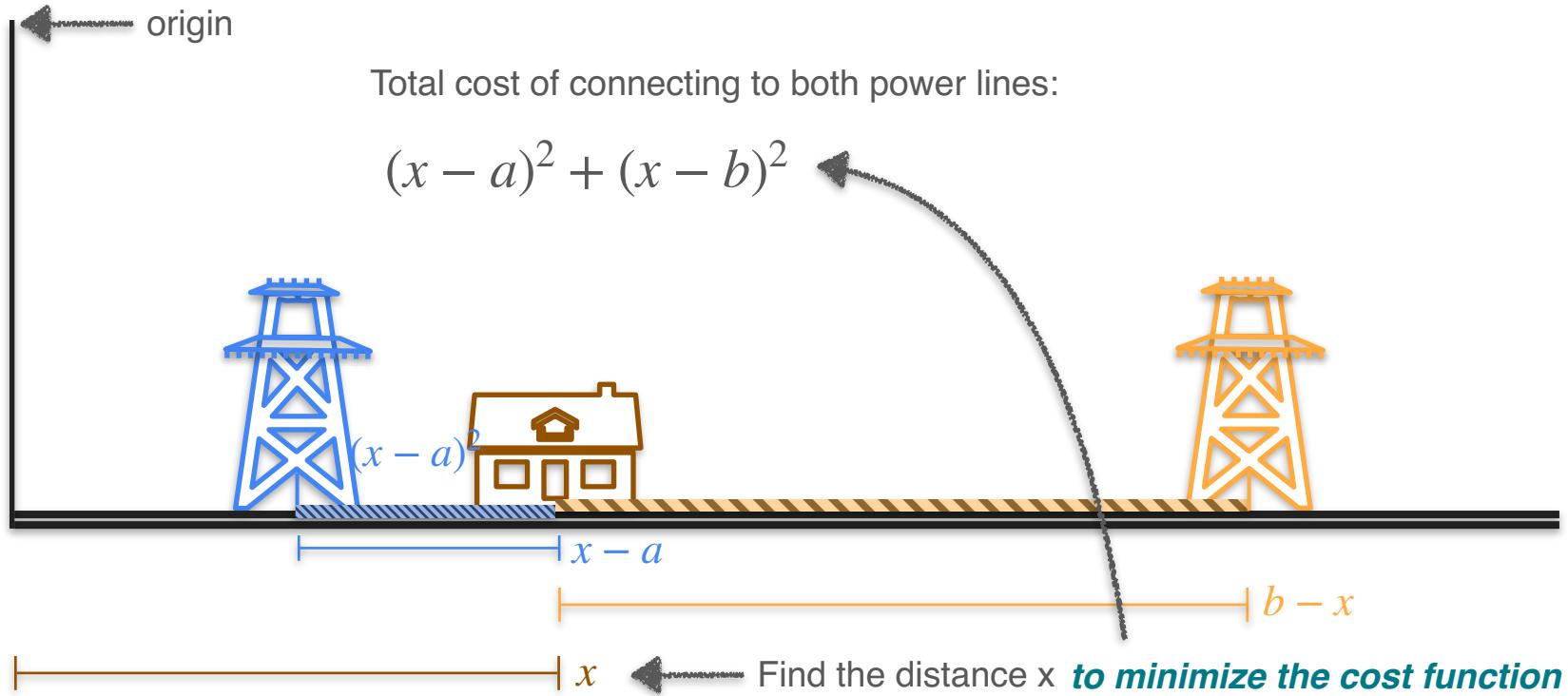
origin

Total cost of connecting to both power lines:

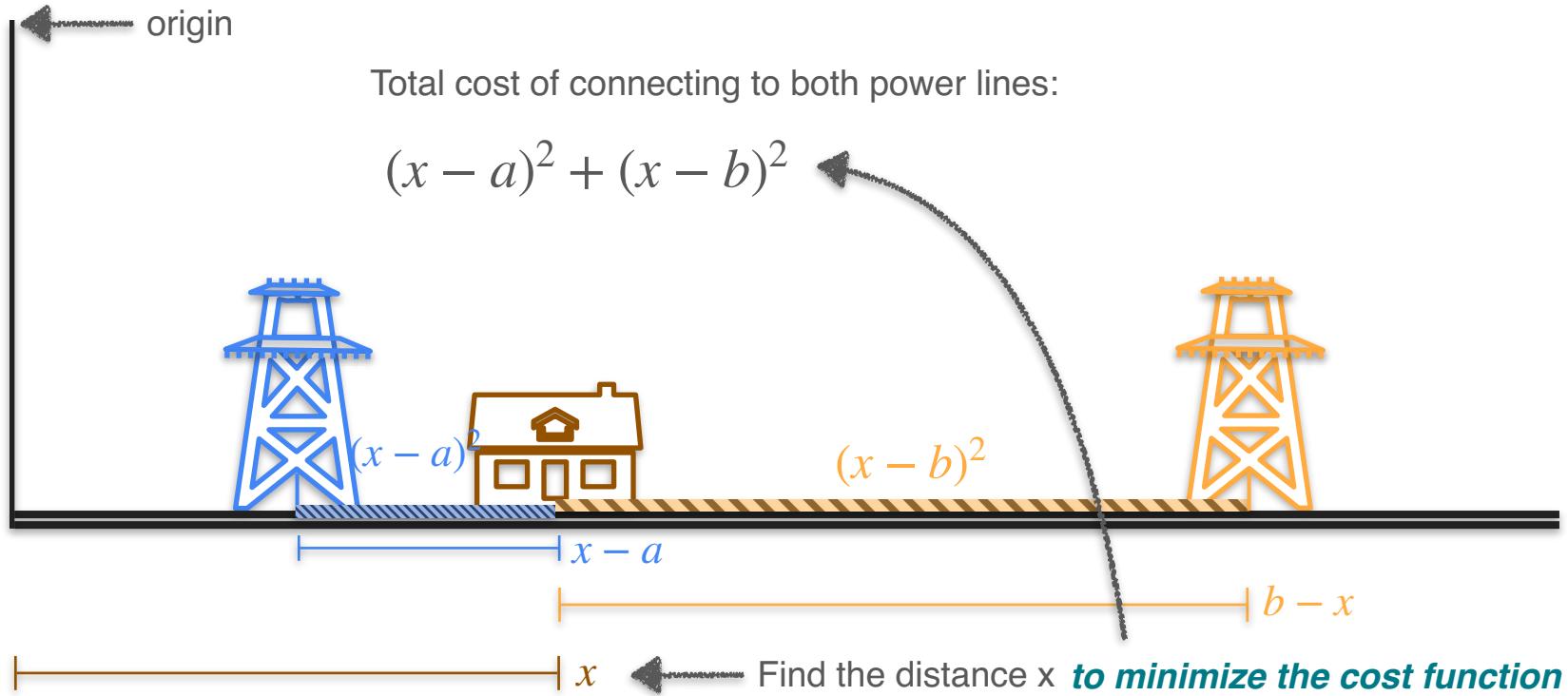
$$(x - a)^2 + (x - b)^2$$



Two Power Line Problem



Two Power Line Problem

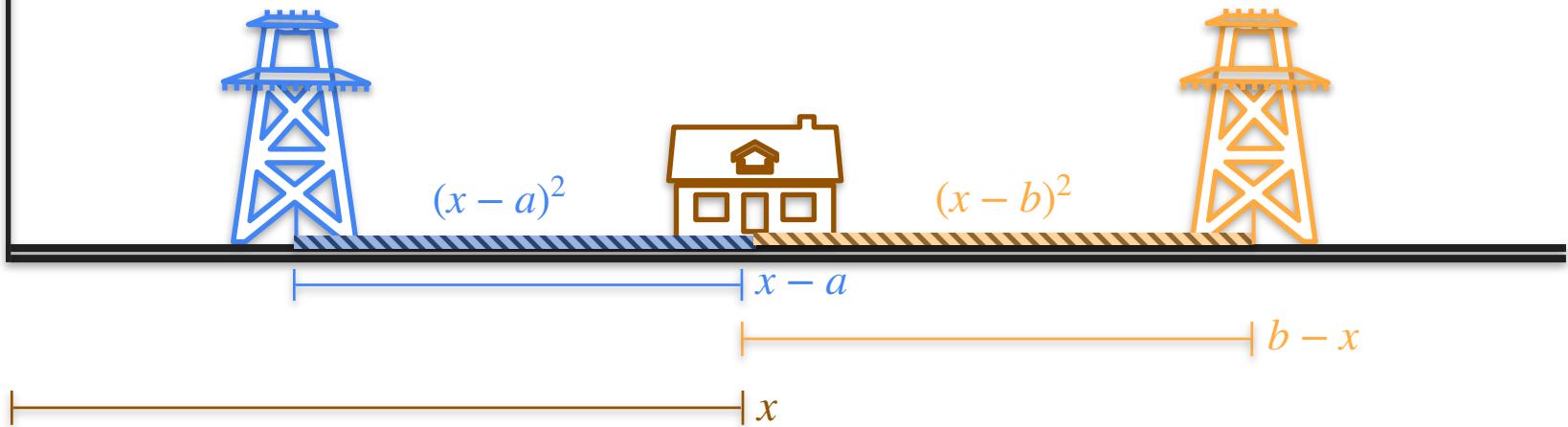


Two Power Line Problem

origin

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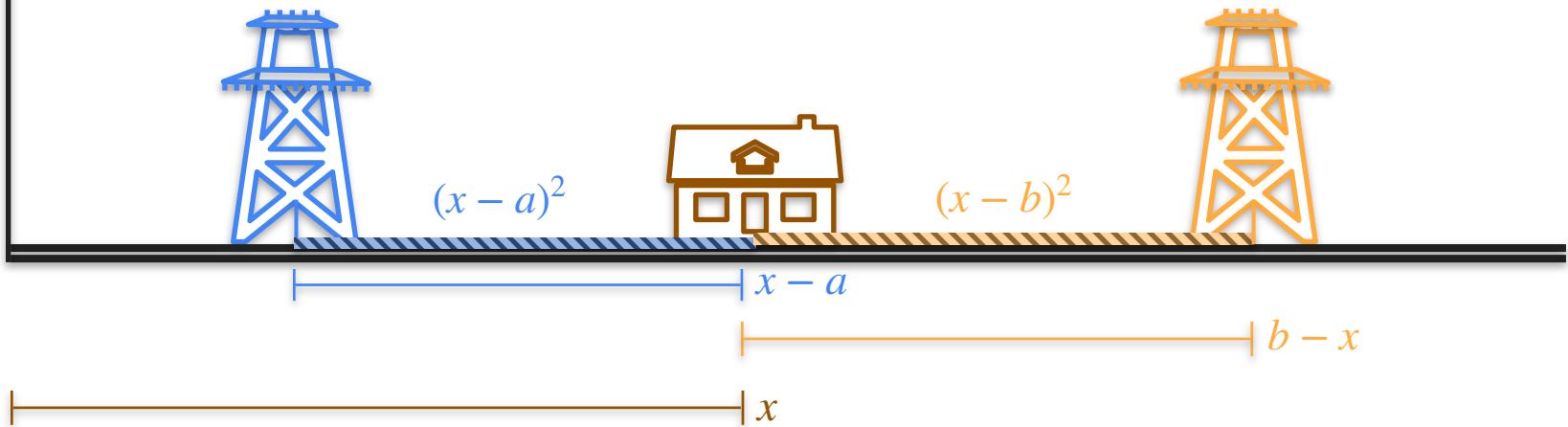


Two Power Line Problem

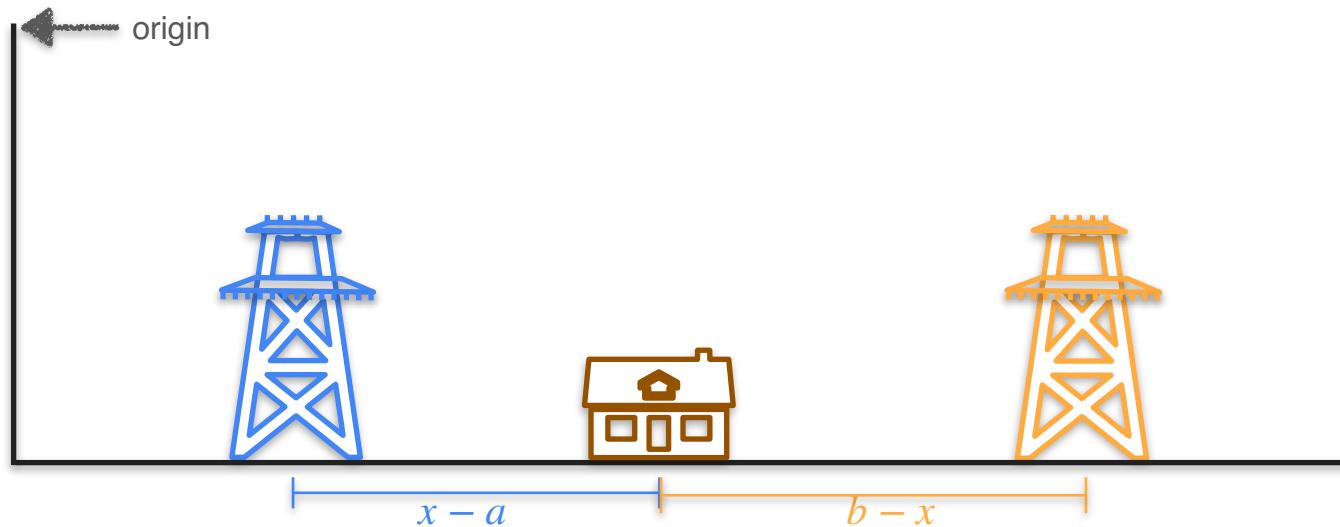
origin

Total cost of connecting to both power lines:

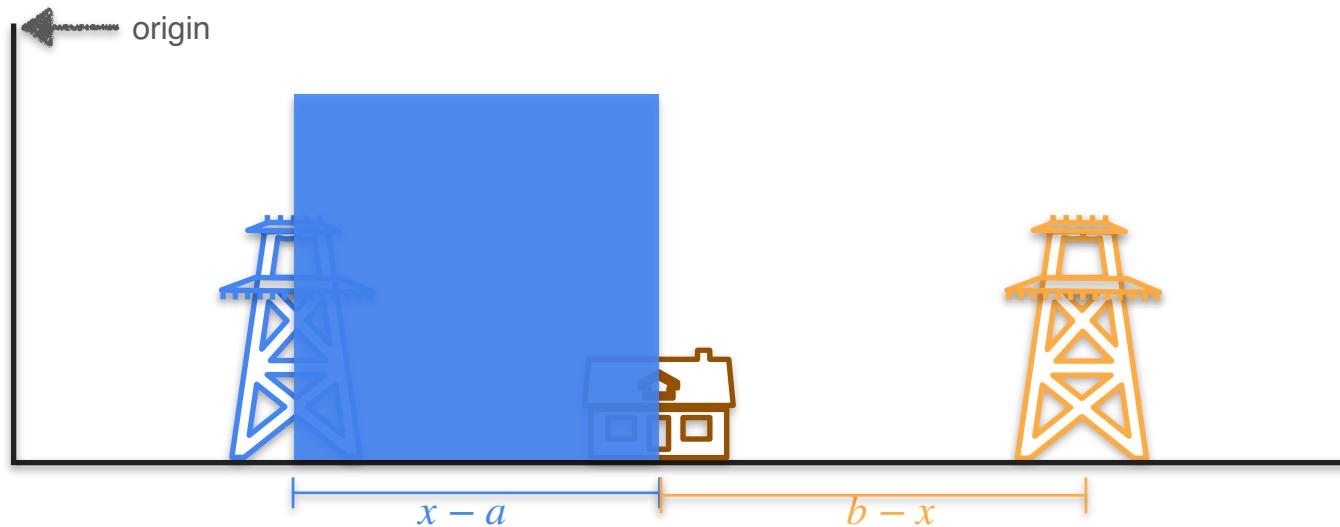
$$(x - a)^2 + (x - b)^2$$



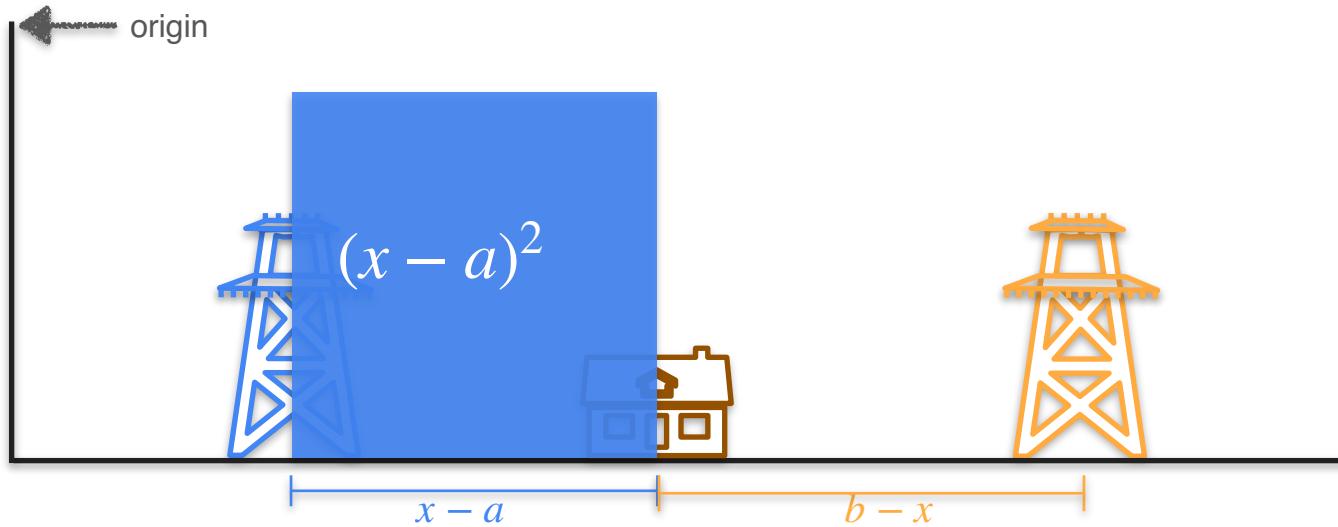
Two Power Line Problem - Square Analogy



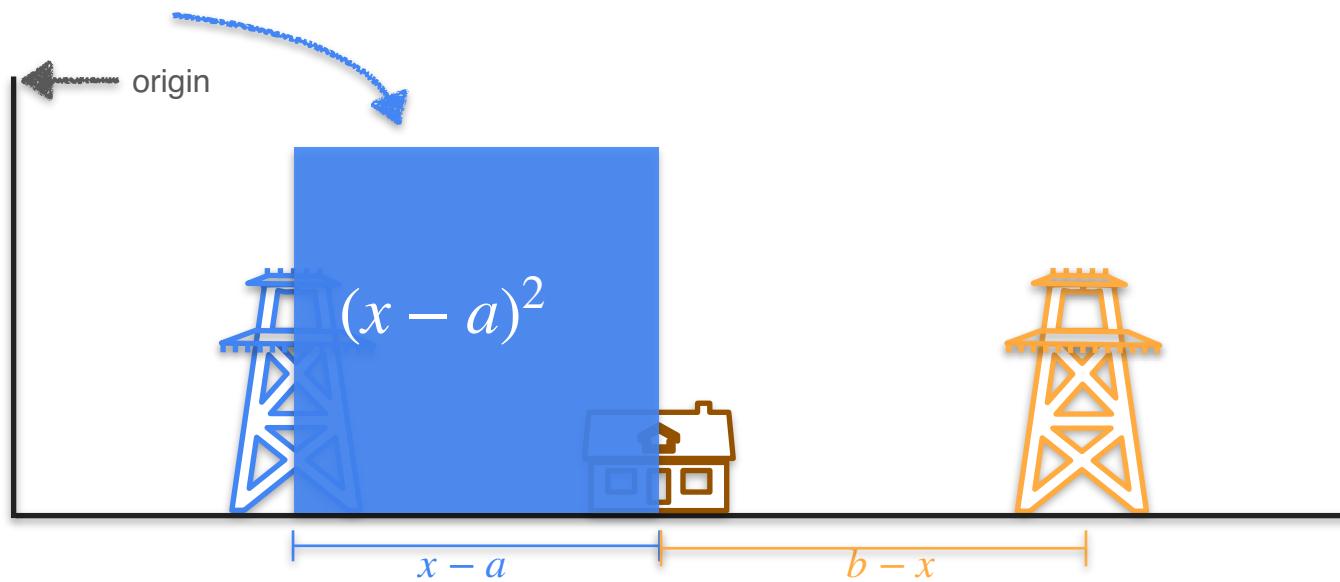
Two Power Line Problem - Square Analogy



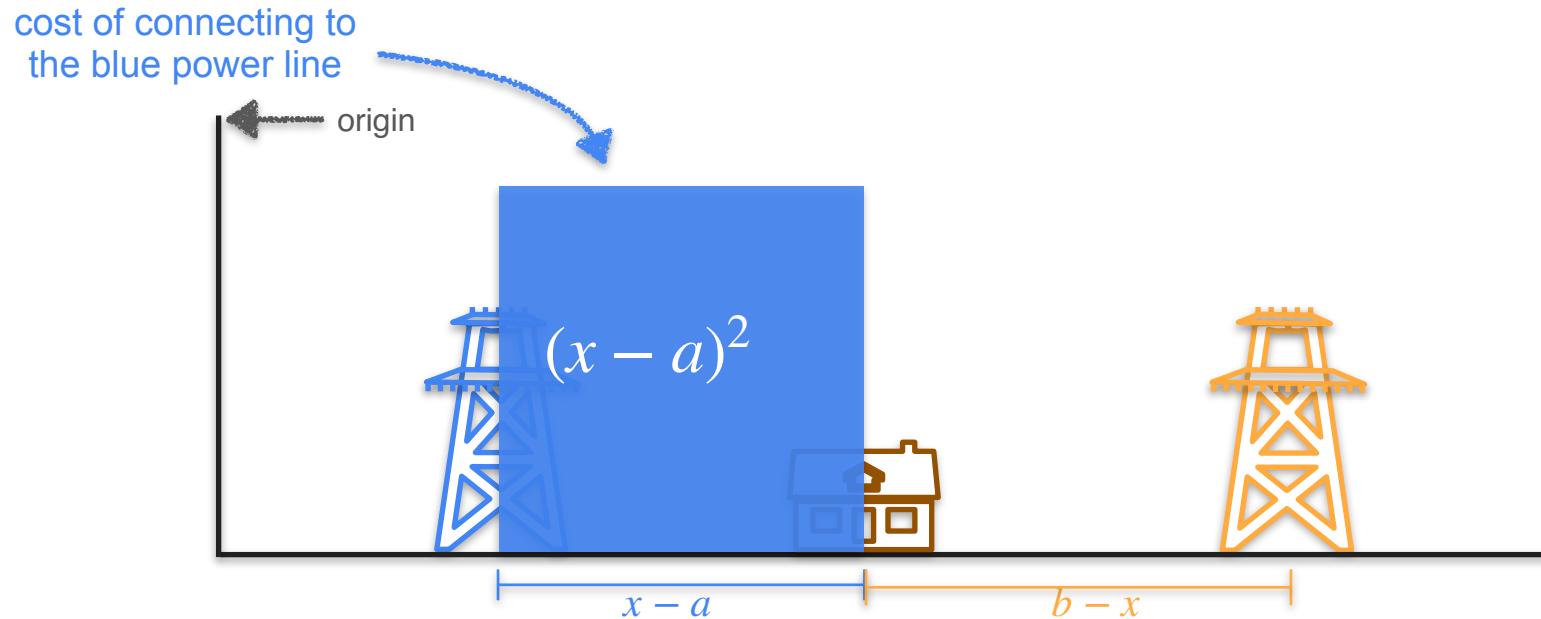
Two Power Line Problem - Square Analogy



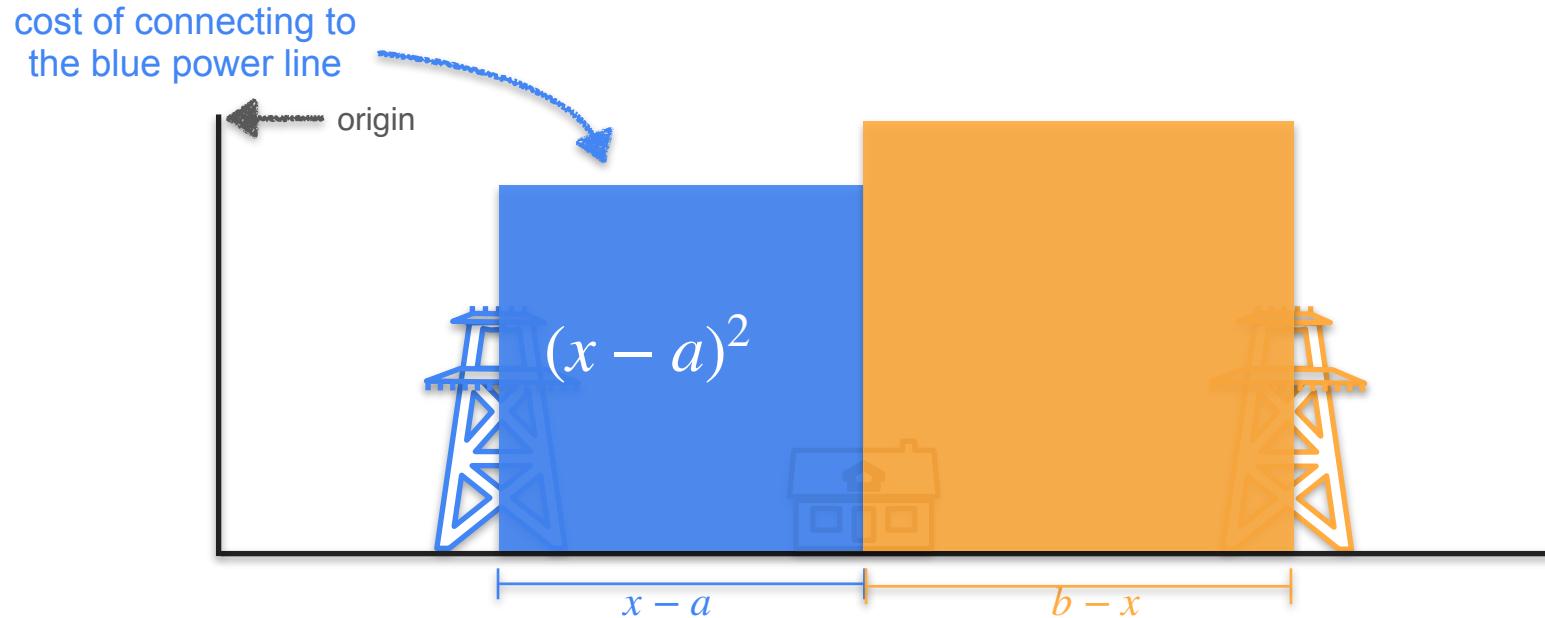
Two Power Line Problem - Square Analogy



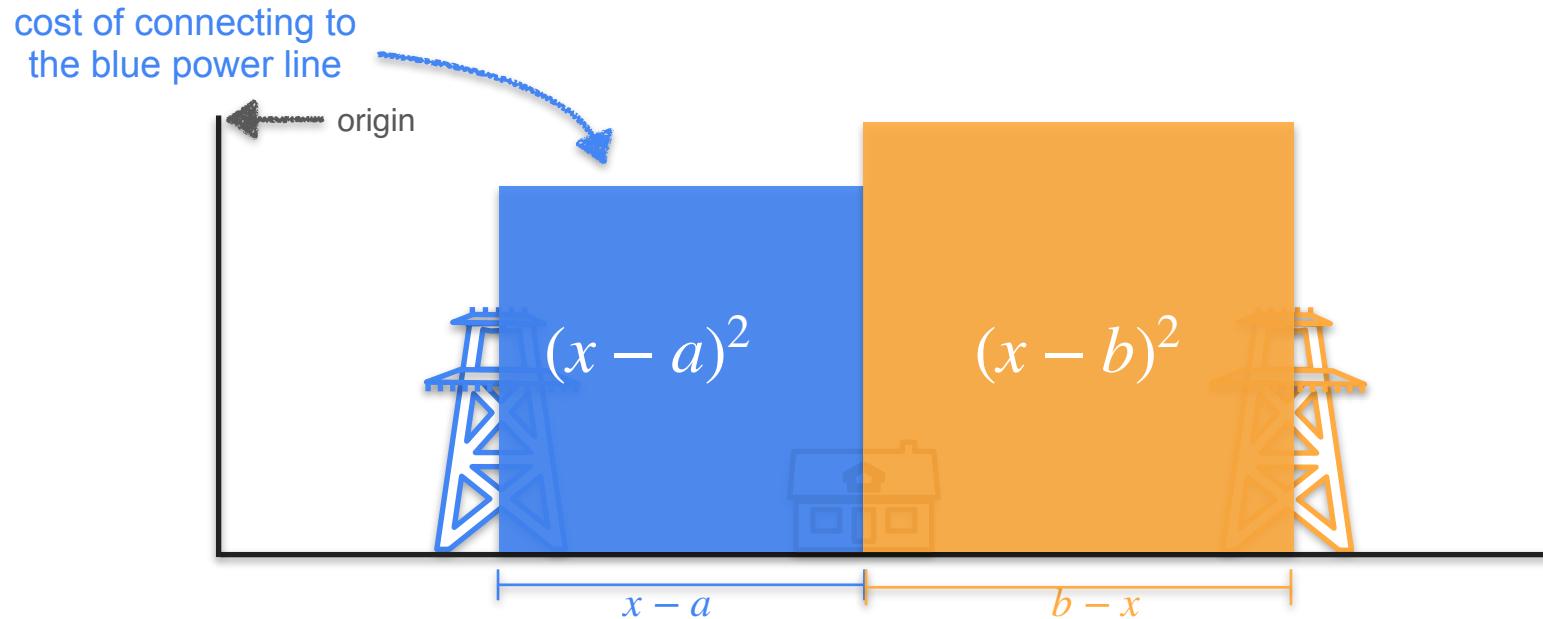
Two Power Line Problem - Square Analogy



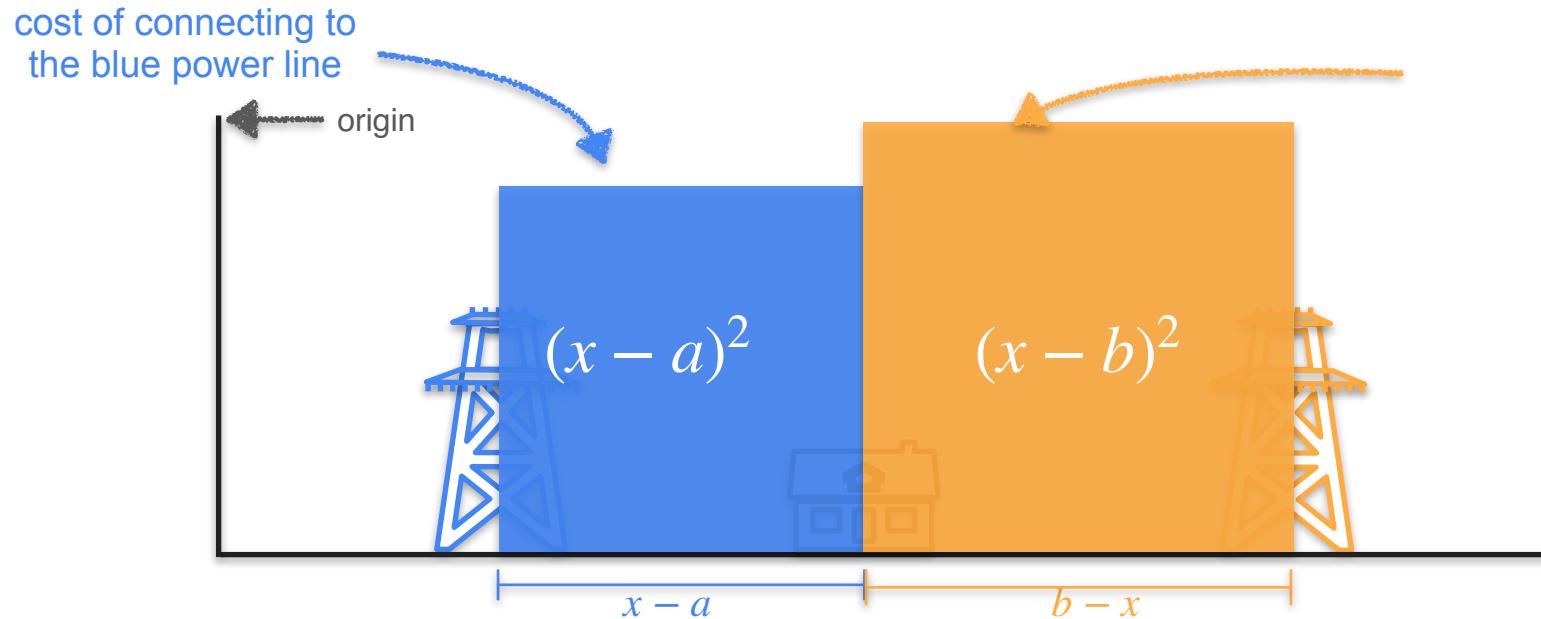
Two Power Line Problem - Square Analogy



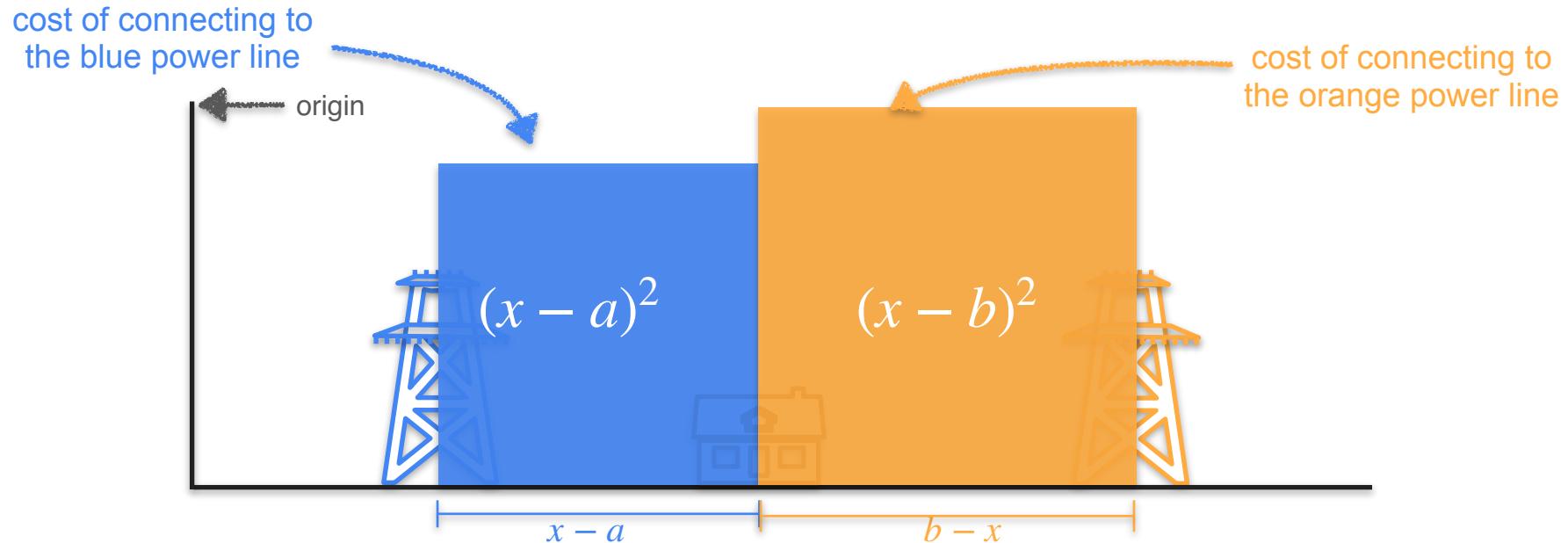
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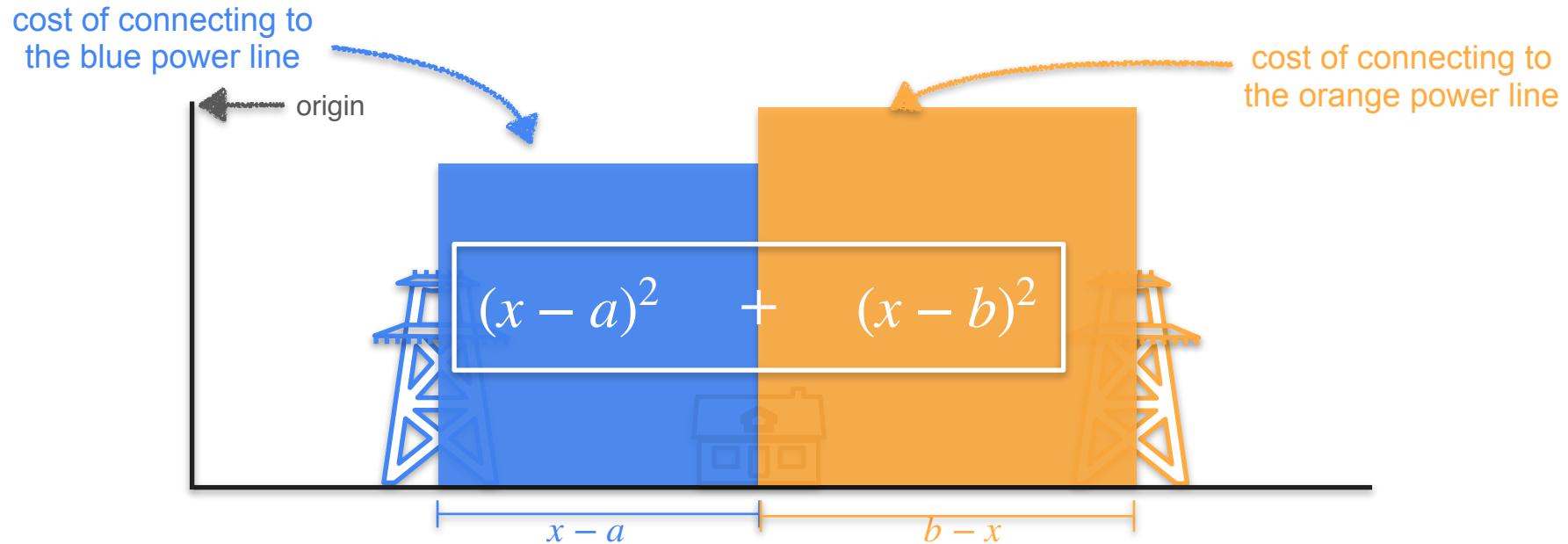
Two Power Line Problem - Square Analogy



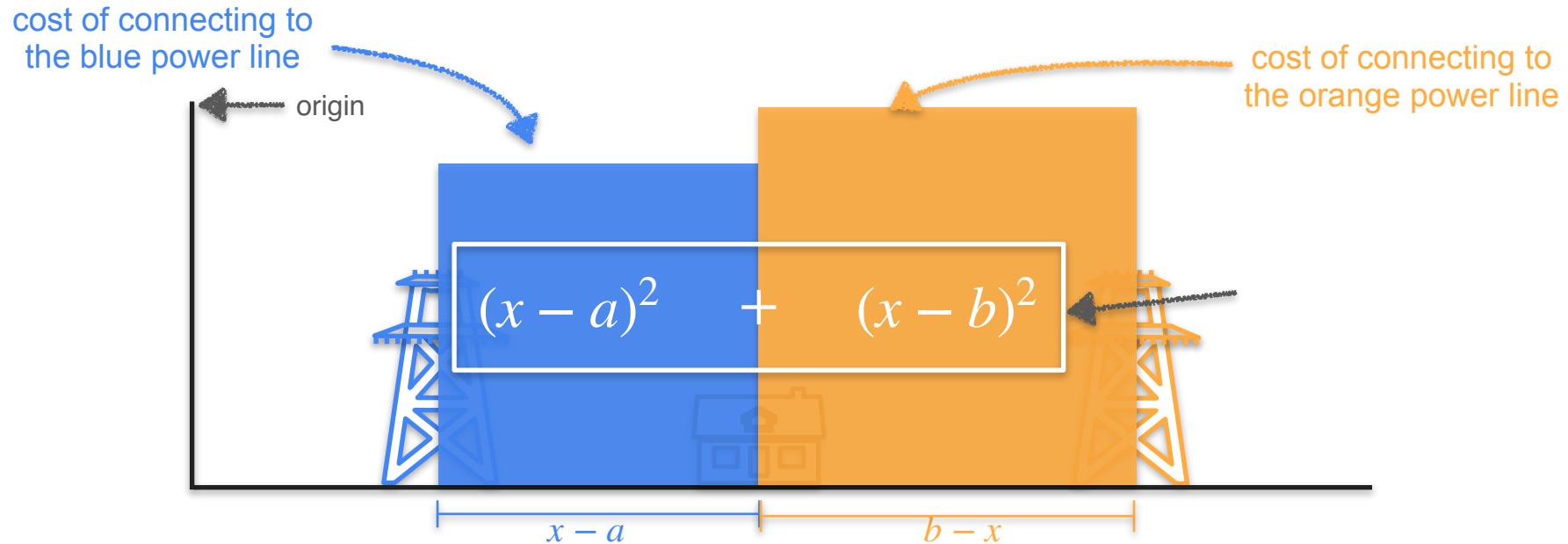
Two Power Line Problem - Square Analogy



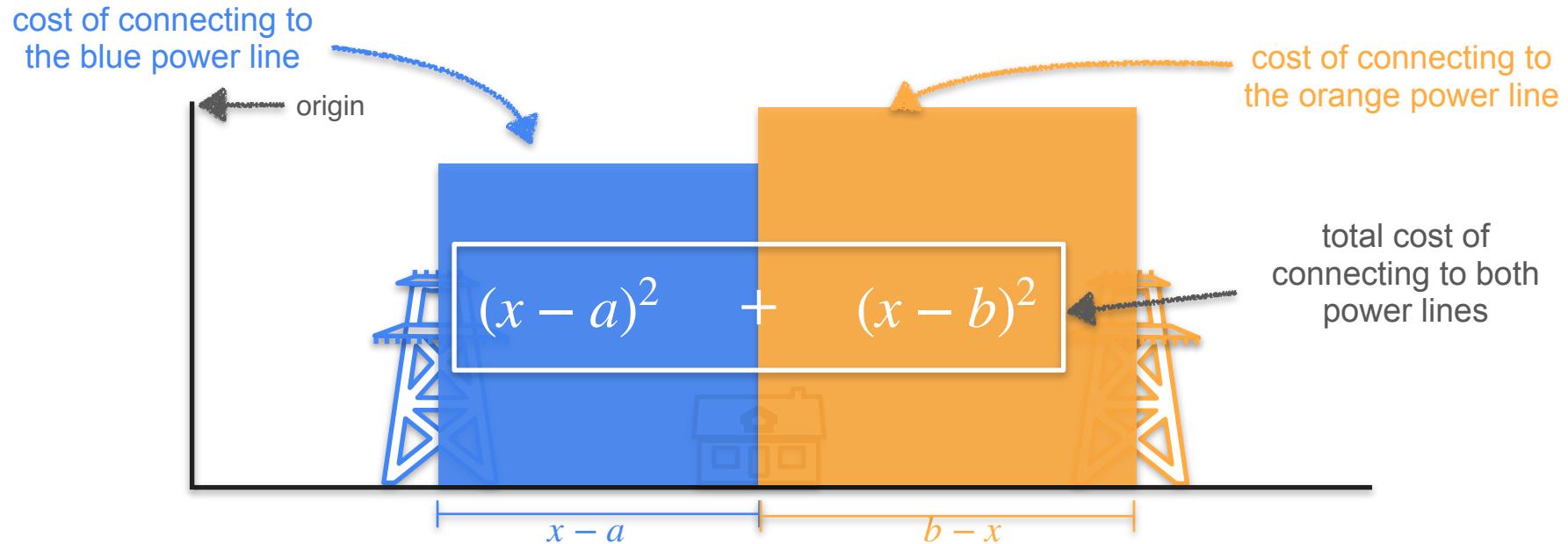
Two Power Line Problem - Square Analogy



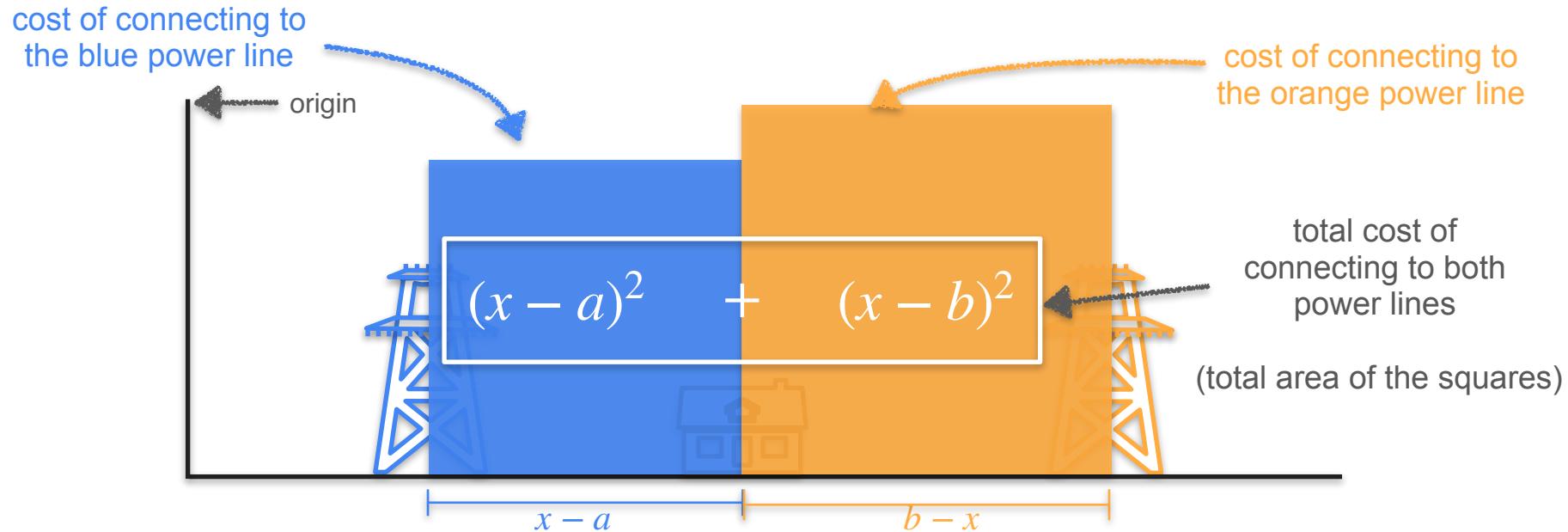
Two Power Line Problem - Square Analogy



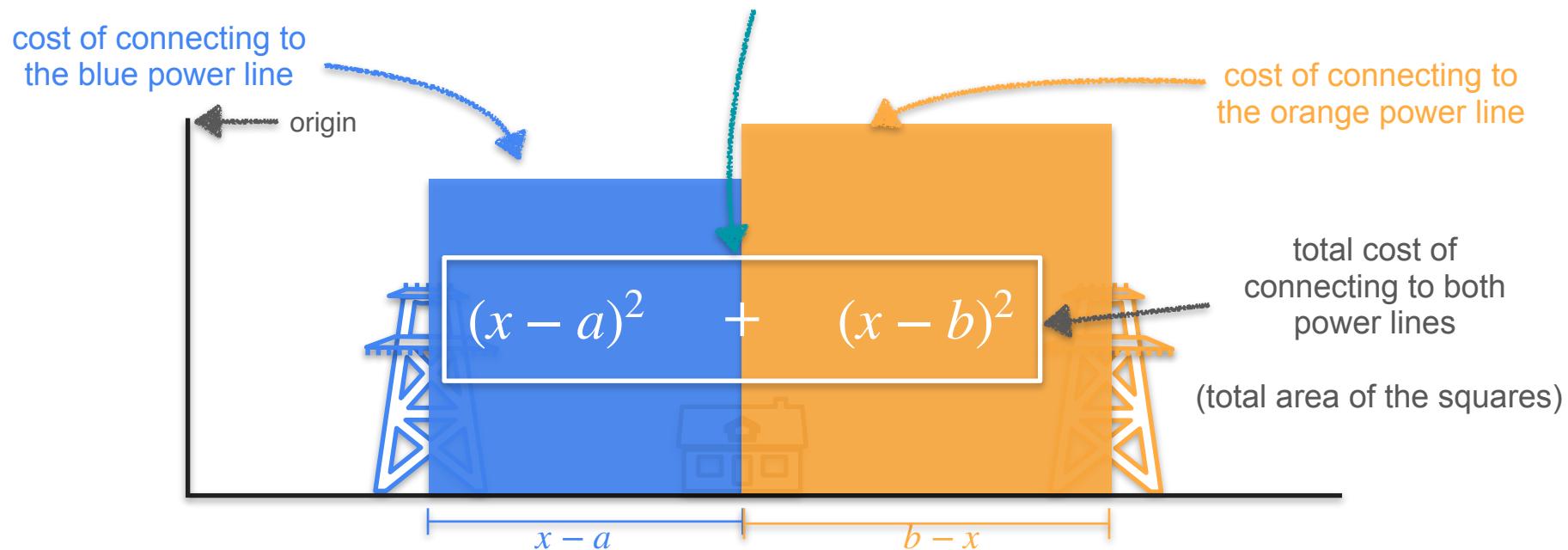
Two Power Line Problem - Square Analogy



Two Power Line Problem - Square Analogy

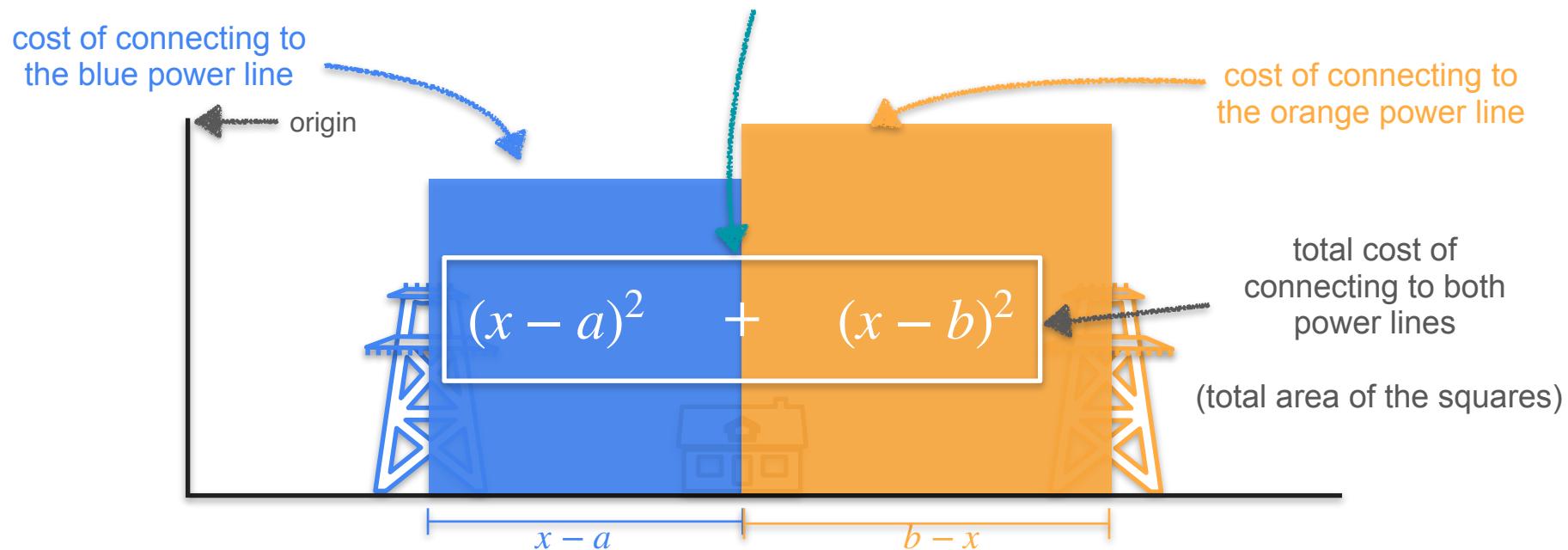


Two Power Line Problem - Square Analogy

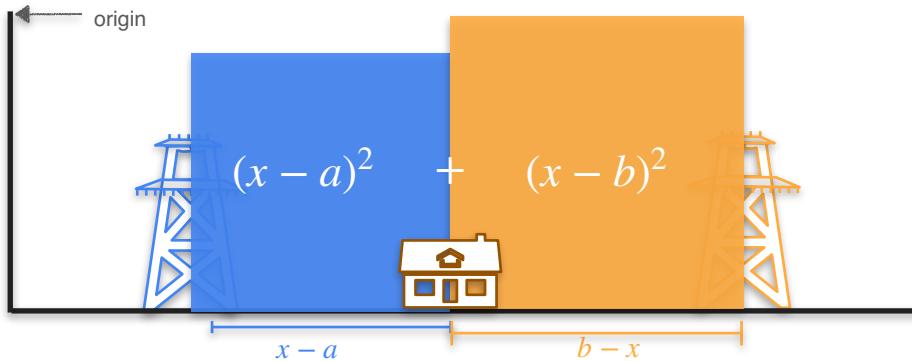


Two Power Line Problem - Square Analogy

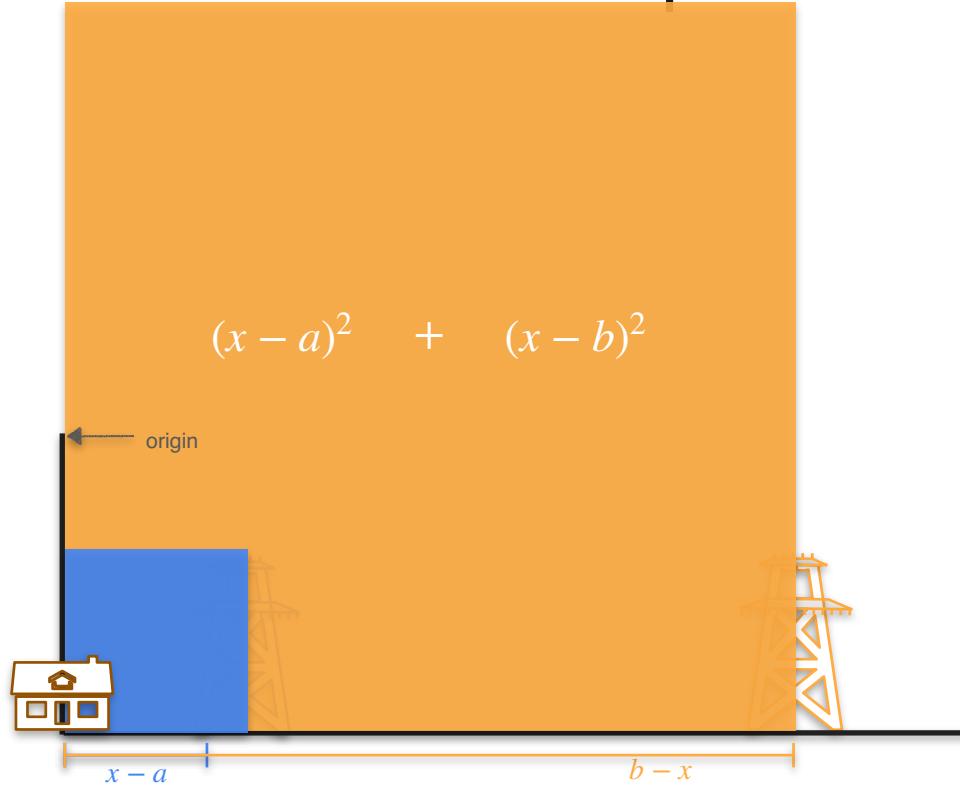
Goal: Minimize the total area of the squares



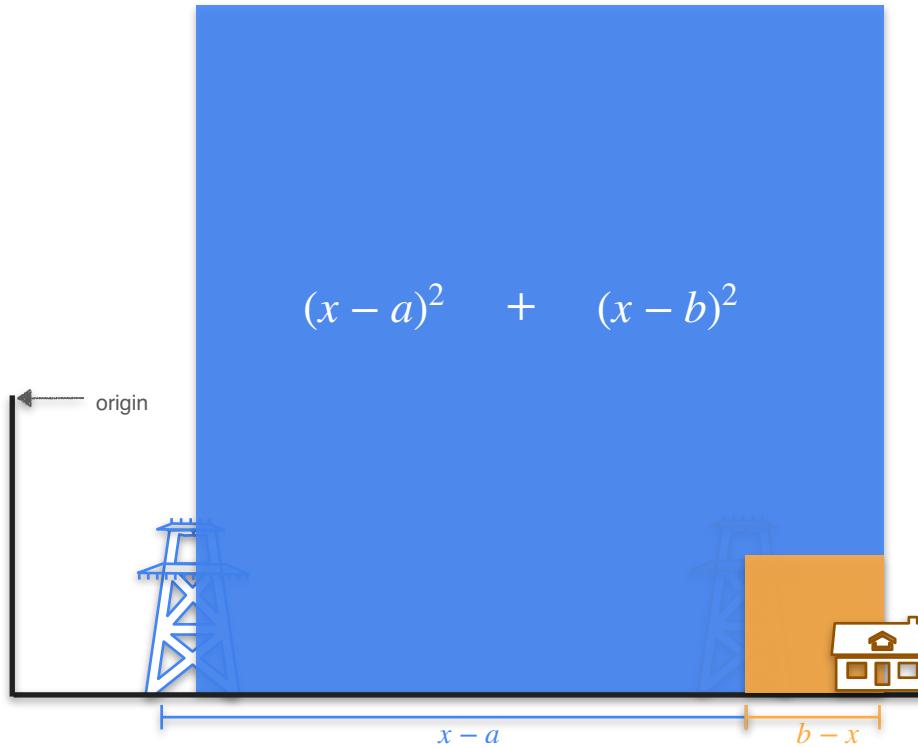
Minimize the Total Area of Squares



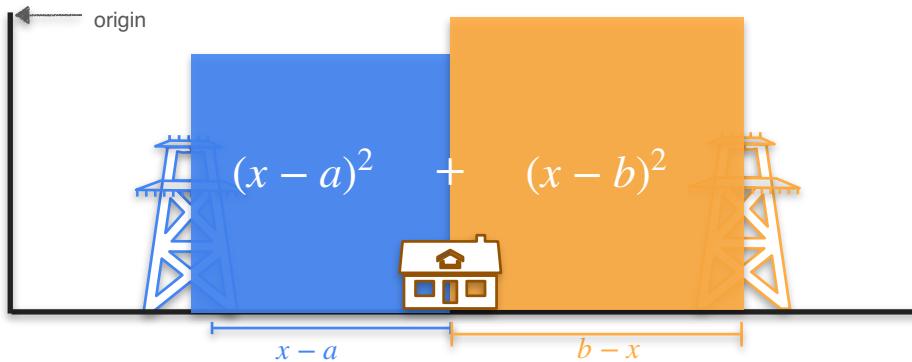
Minimize the Total Area of Squares



Minimize the Total Area of Squares



Minimize the Total Area of Squares



Two Power Line Problem

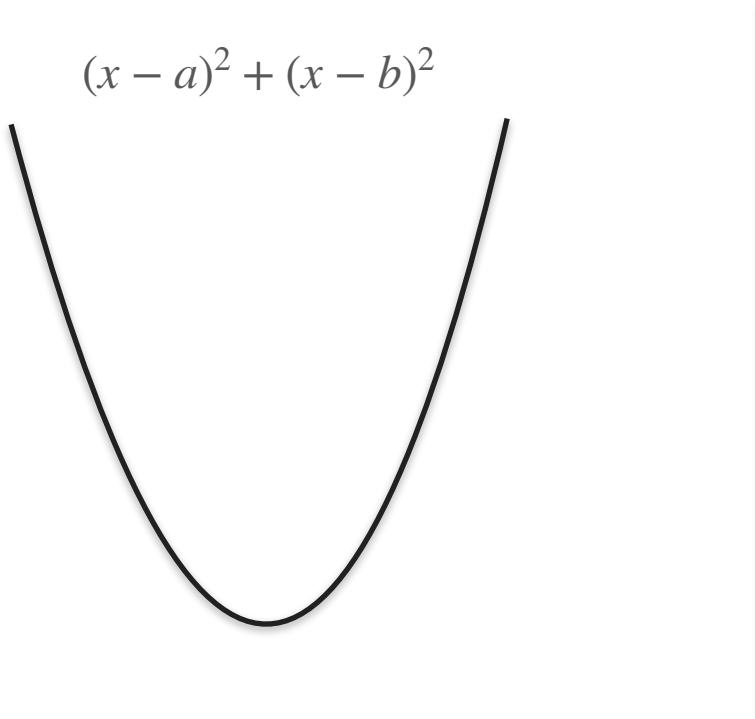


Two Power Line Problem

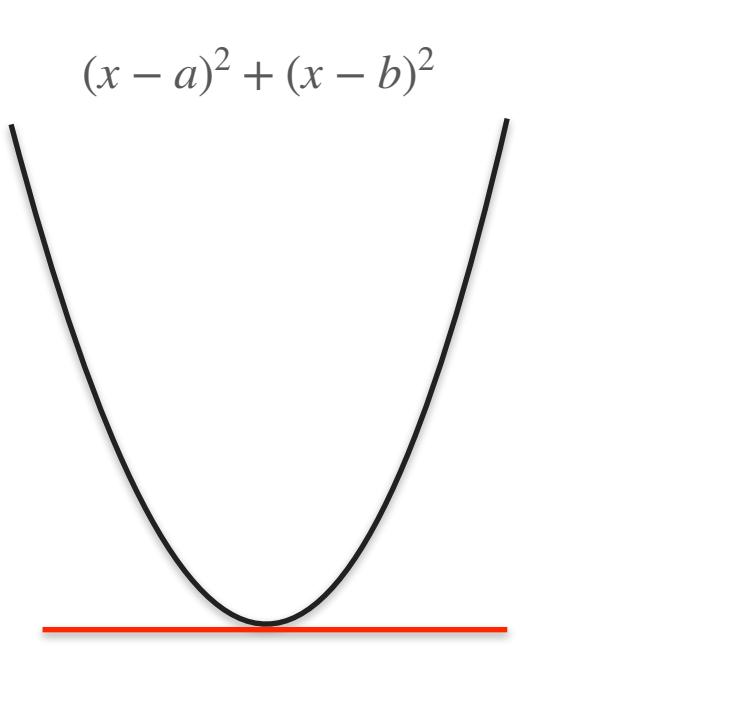
$$(x - a)^2 + (x - b)^2$$



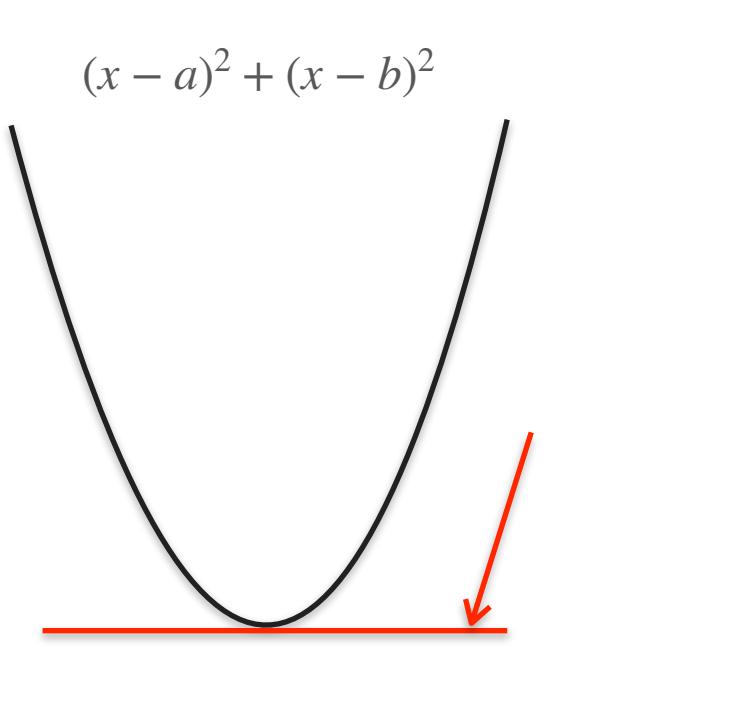
Two Power Line Problem



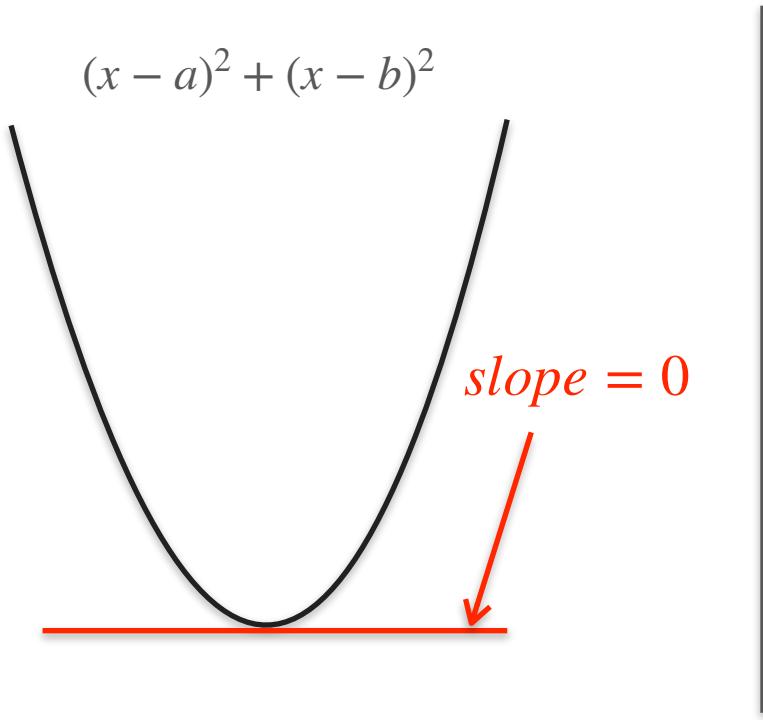
Two Power Line Problem



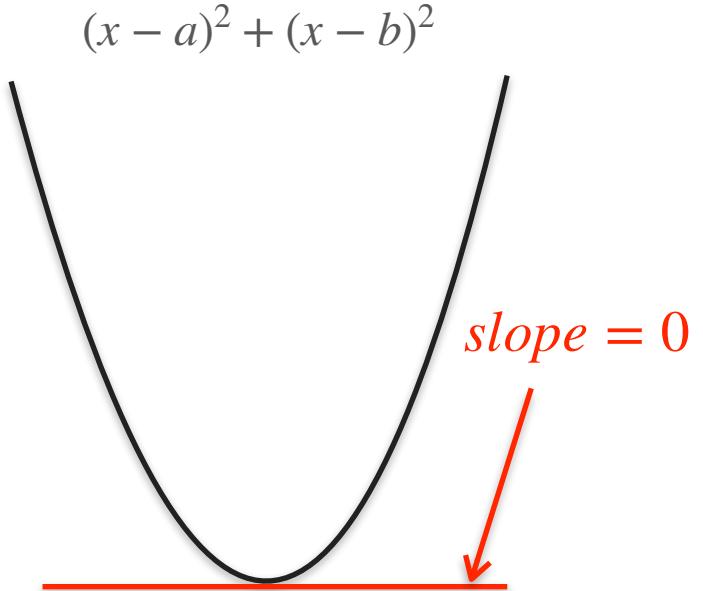
Two Power Line Problem



Two Power Line Problem

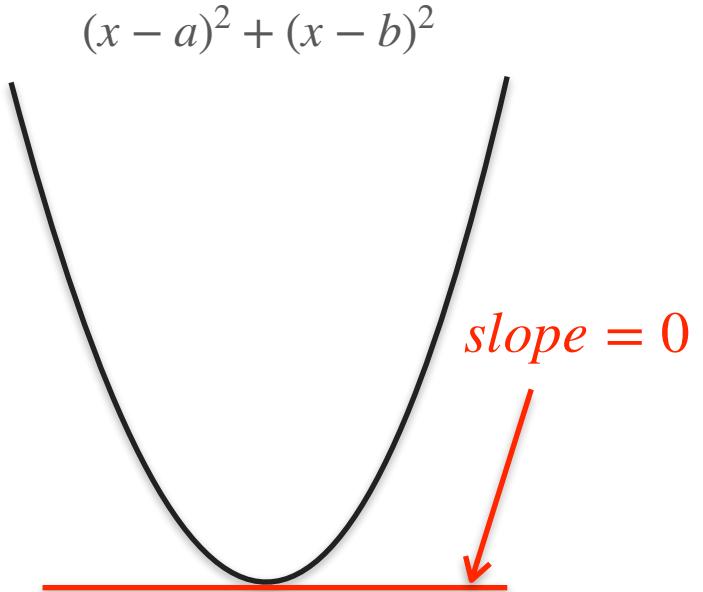


Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

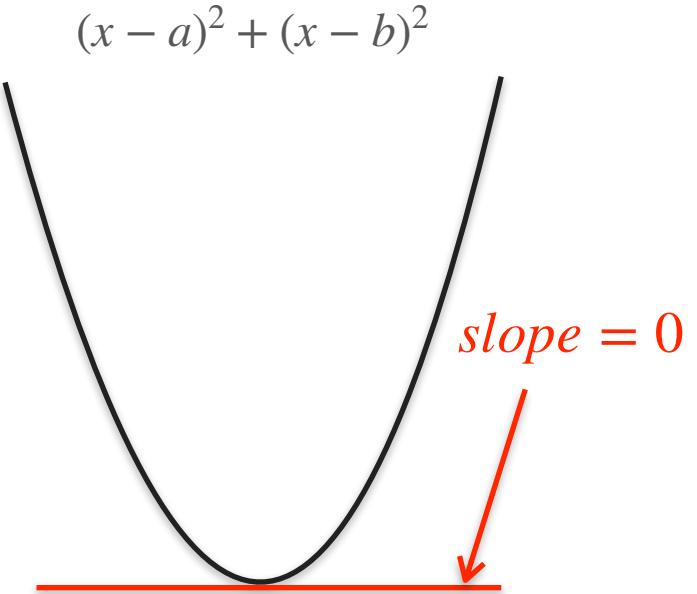
Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

Two Power Line Problem

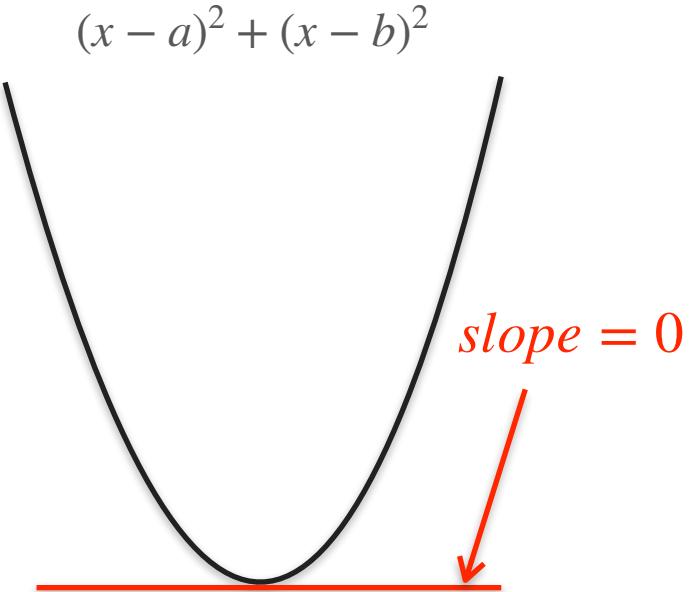


$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

Two Power Line Problem



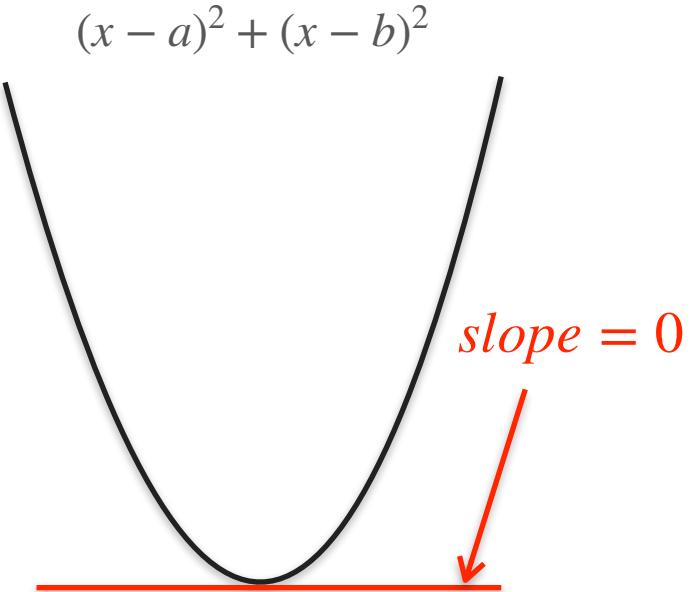
$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

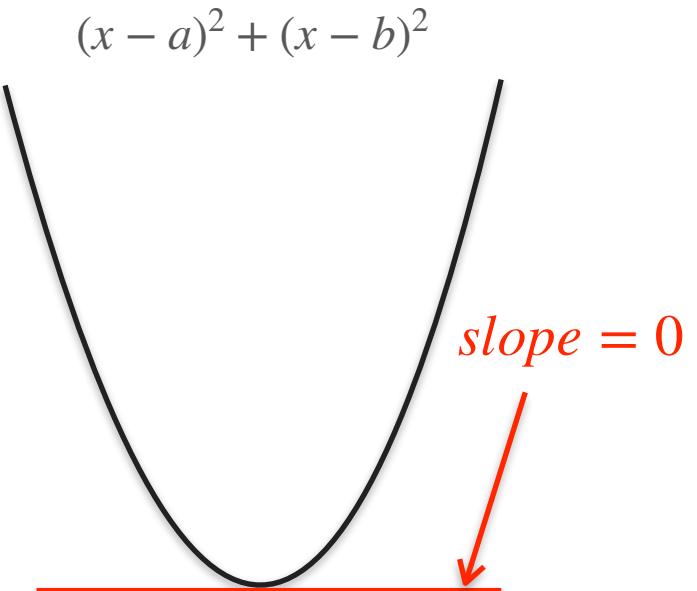
$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

$$2x = a + b$$

Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

$$2x = a + b$$

$$x = \frac{a + b}{2}$$

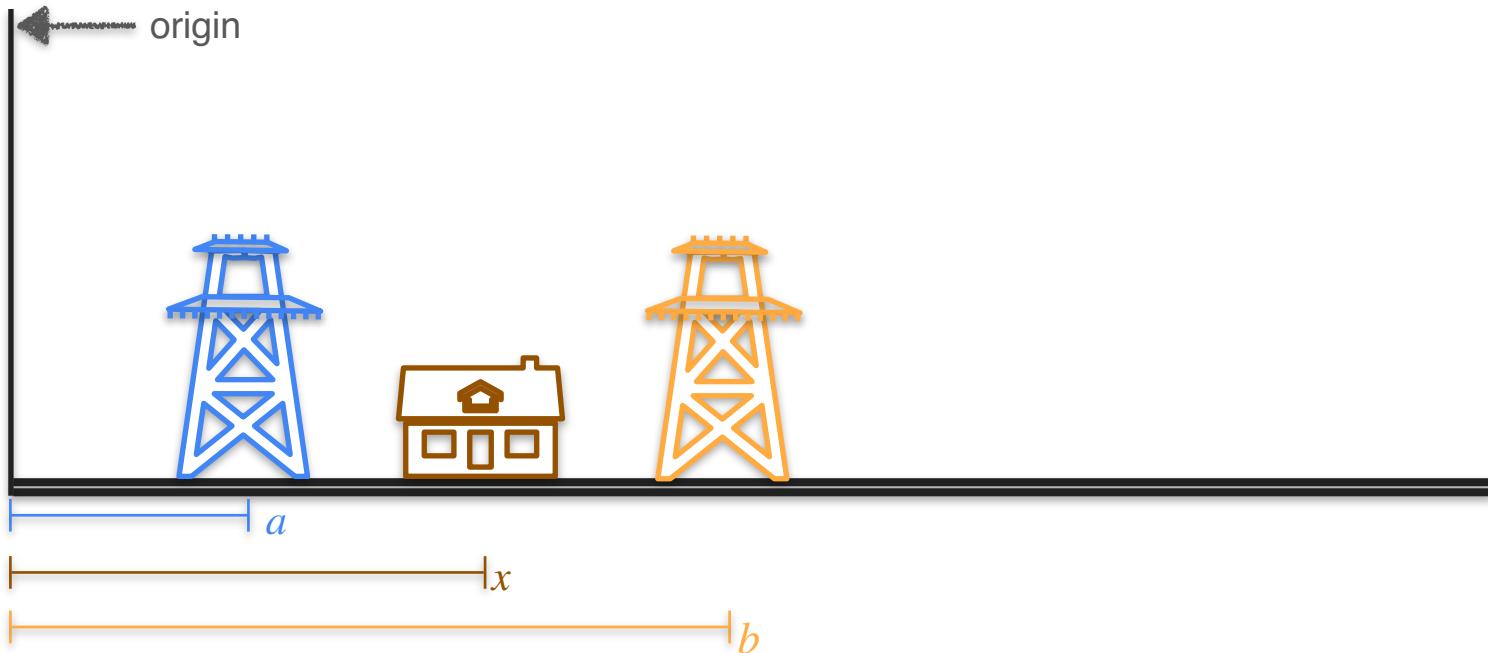


DeepLearning.AI

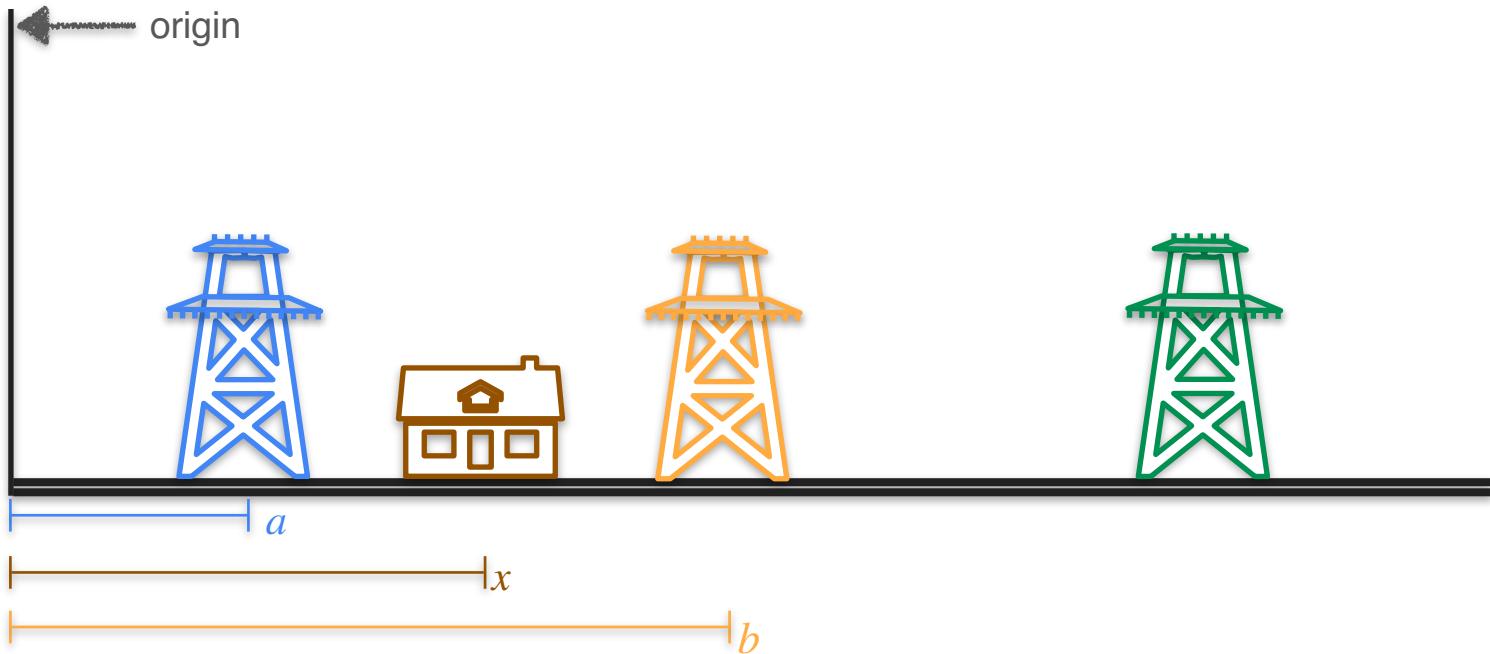
Derivatives and Optimization

**Optimization of squared loss:
The three powerline problem**

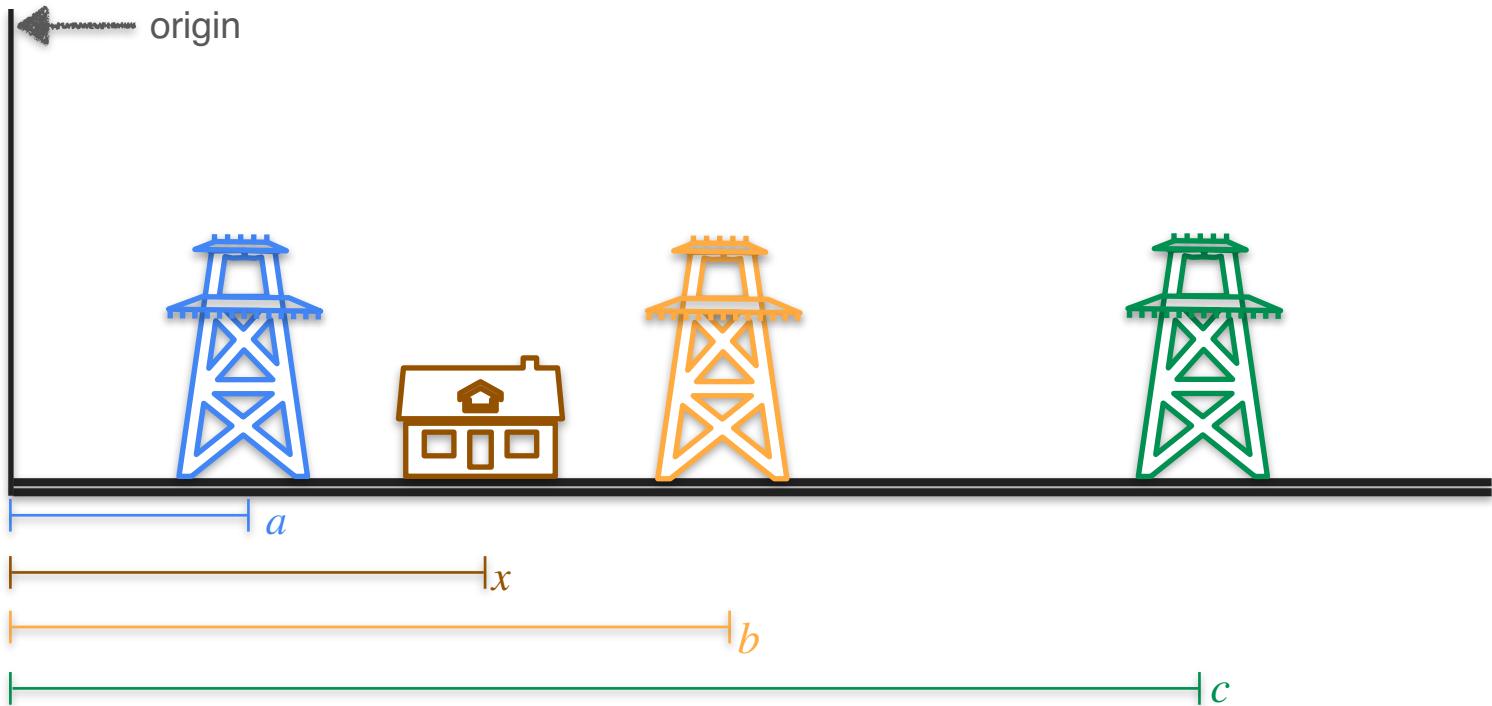
Three Power Line Problem



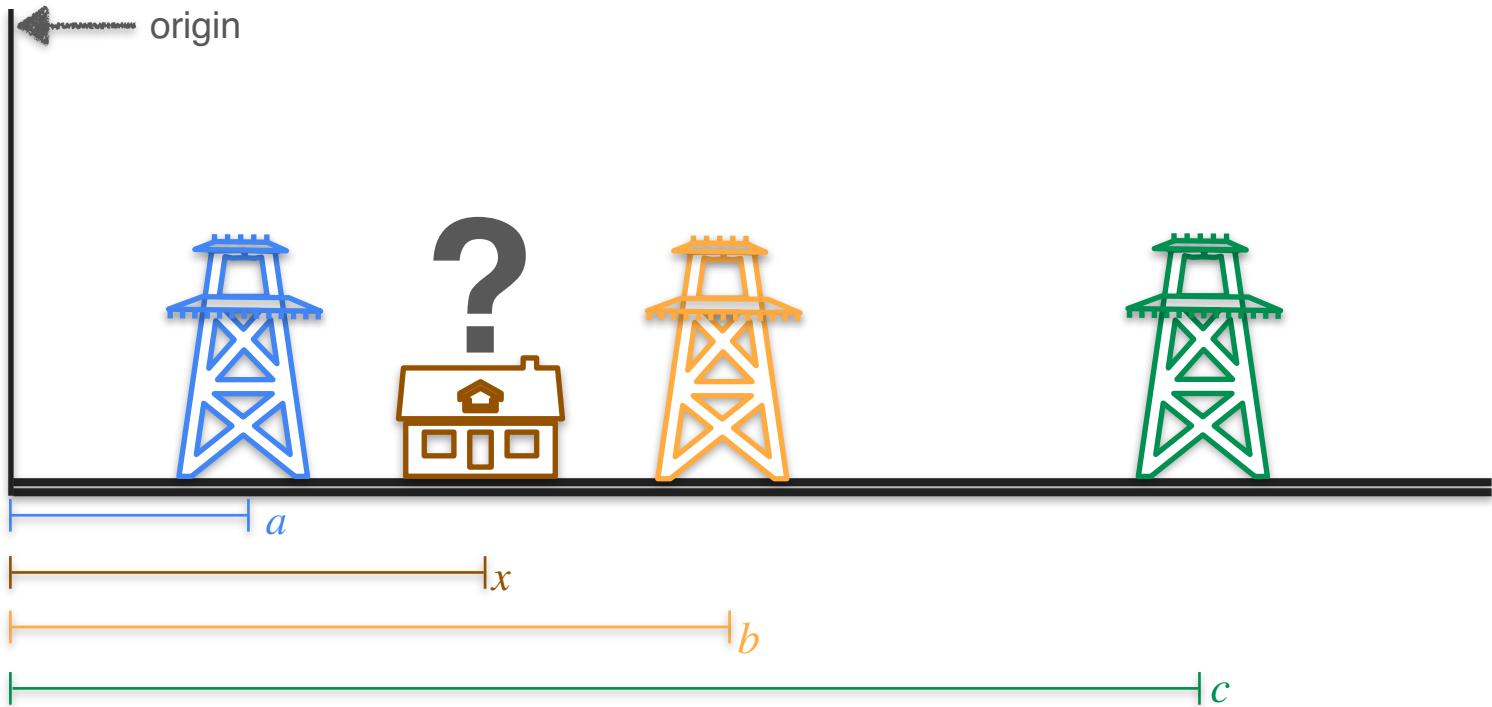
Three Power Line Problem



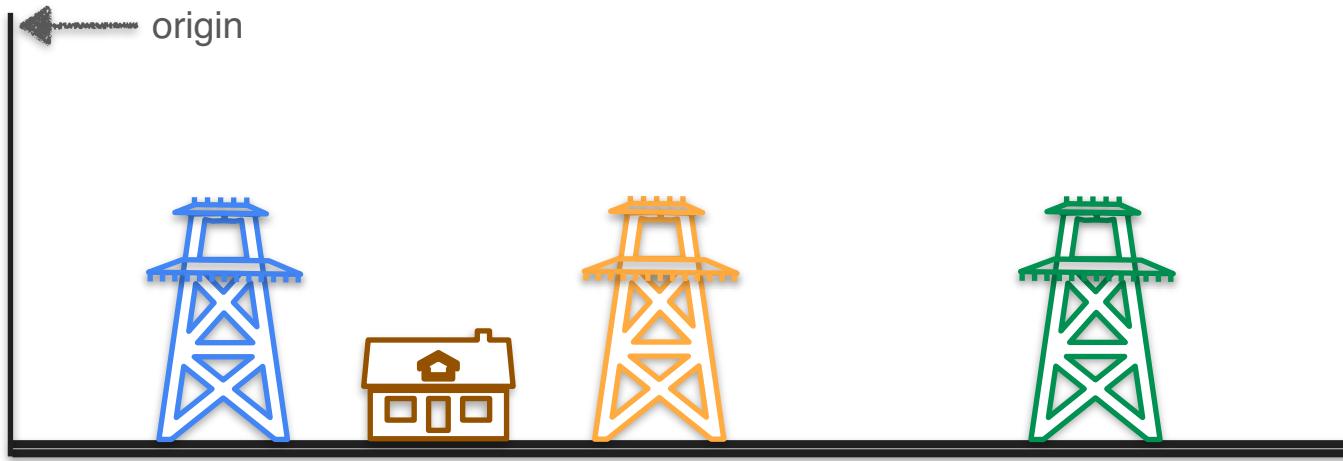
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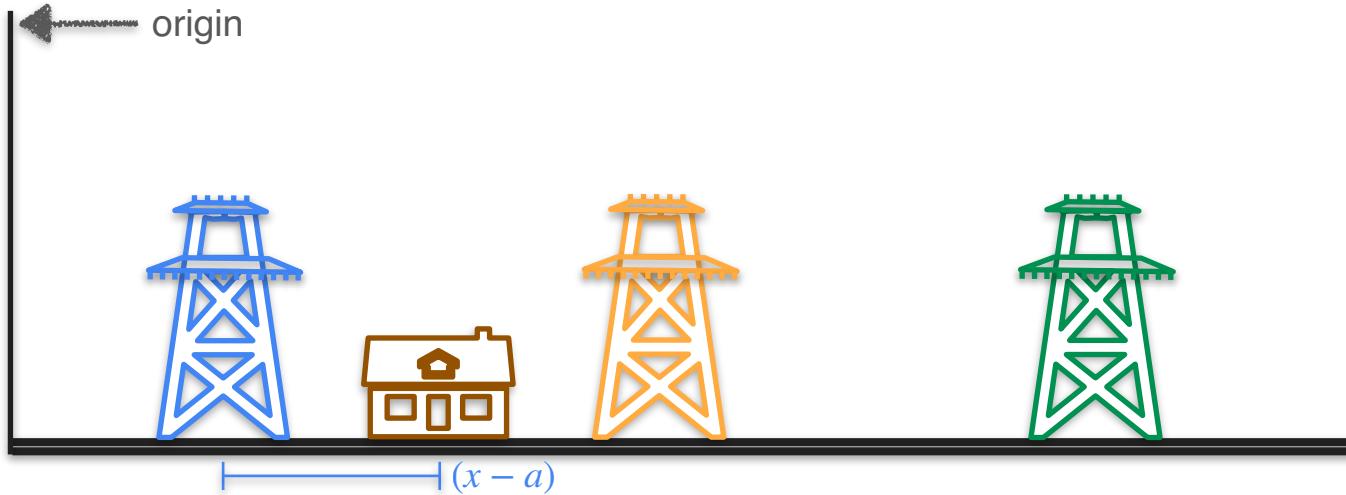
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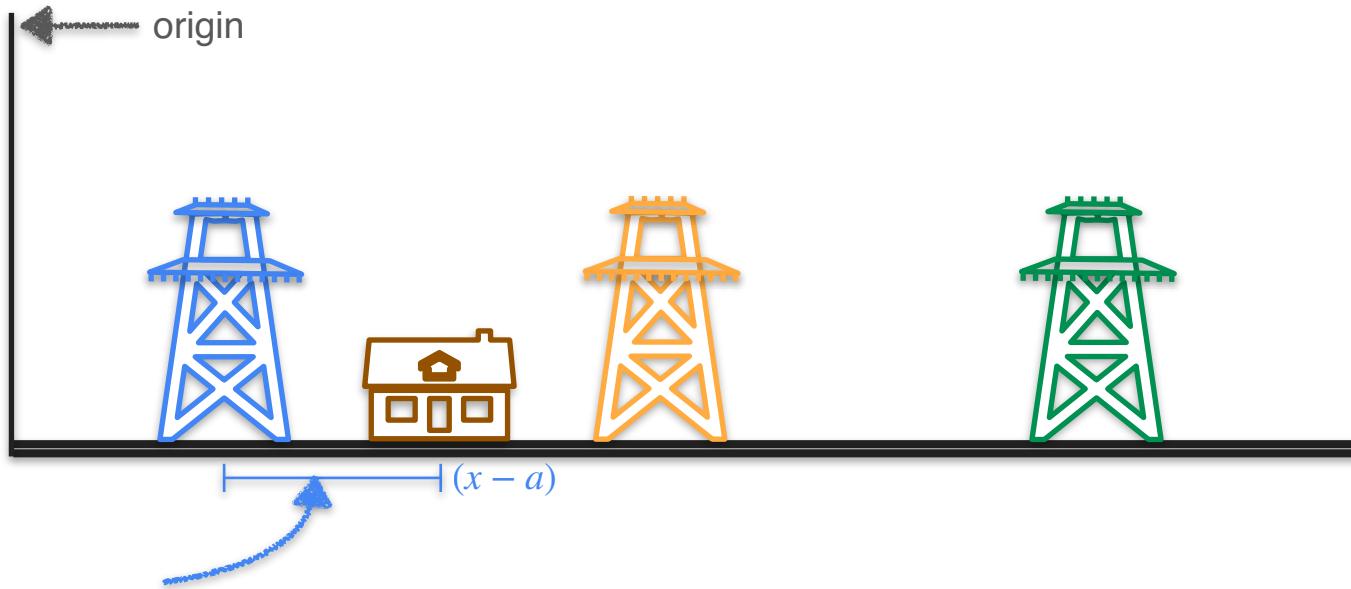
Three Power Line Problem



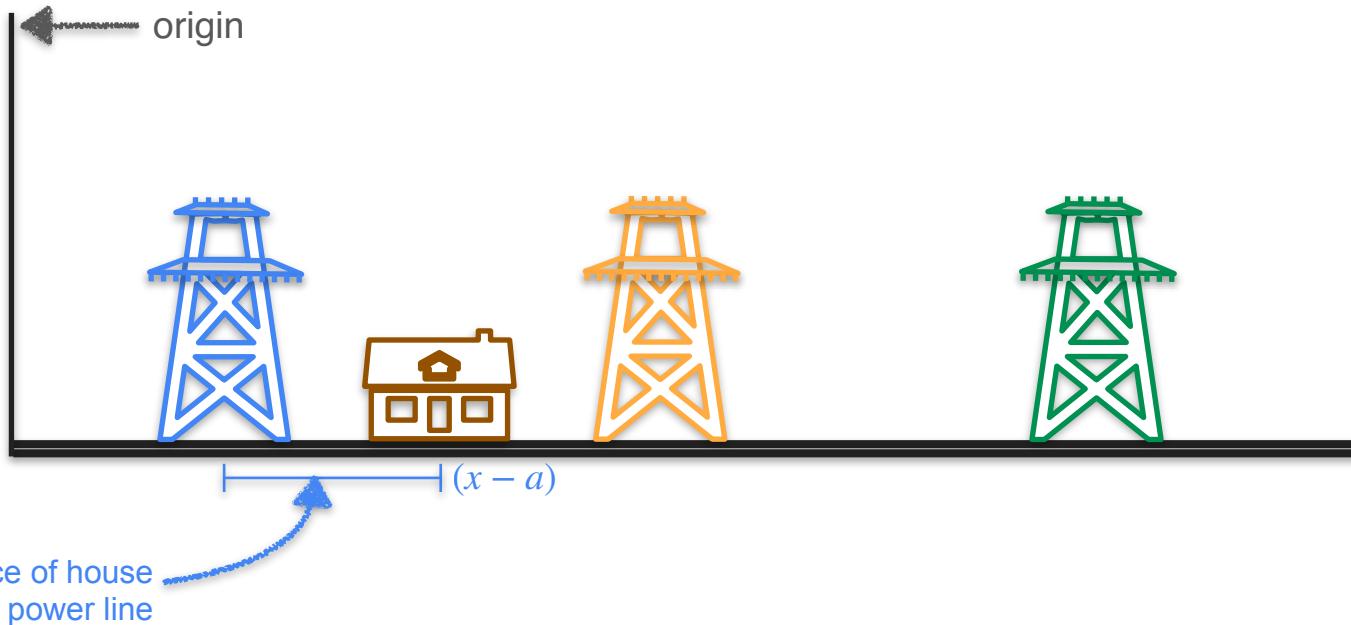
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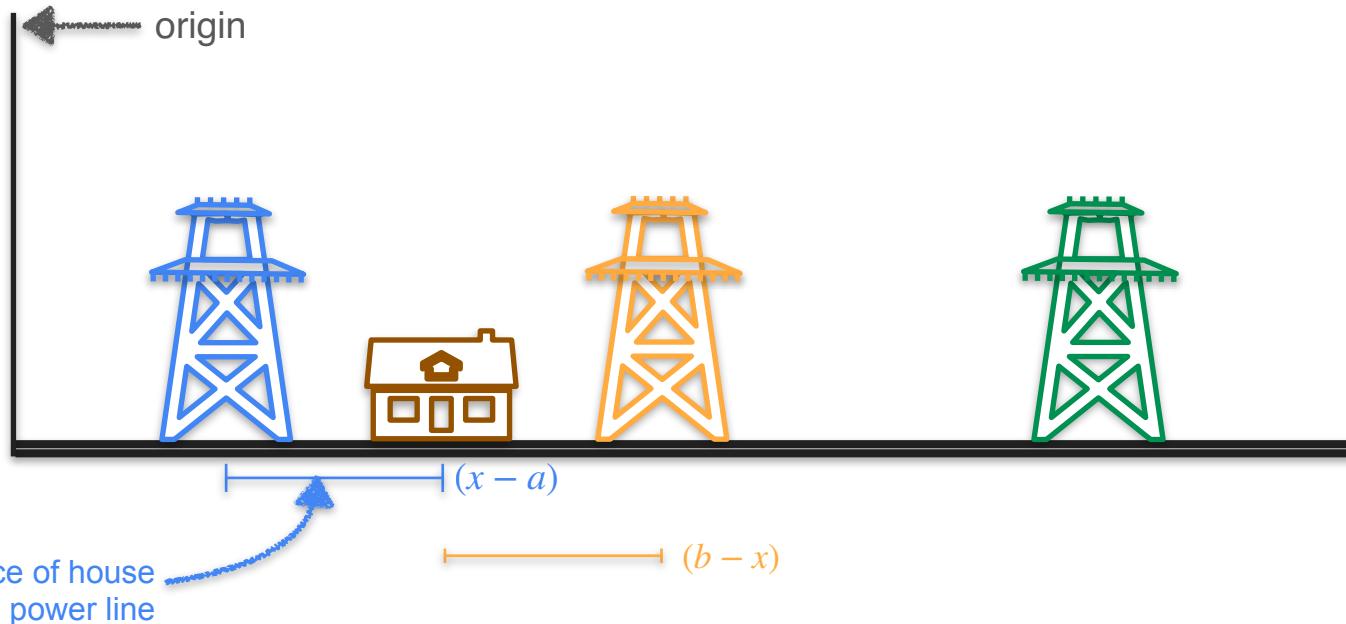
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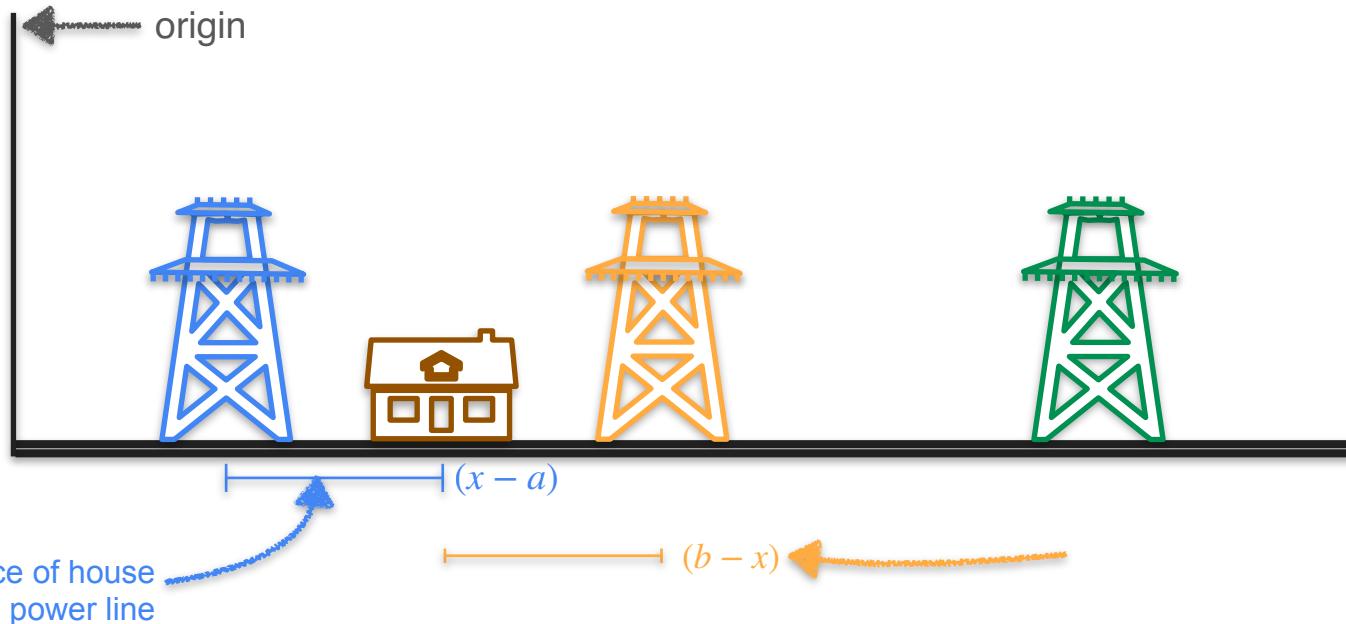
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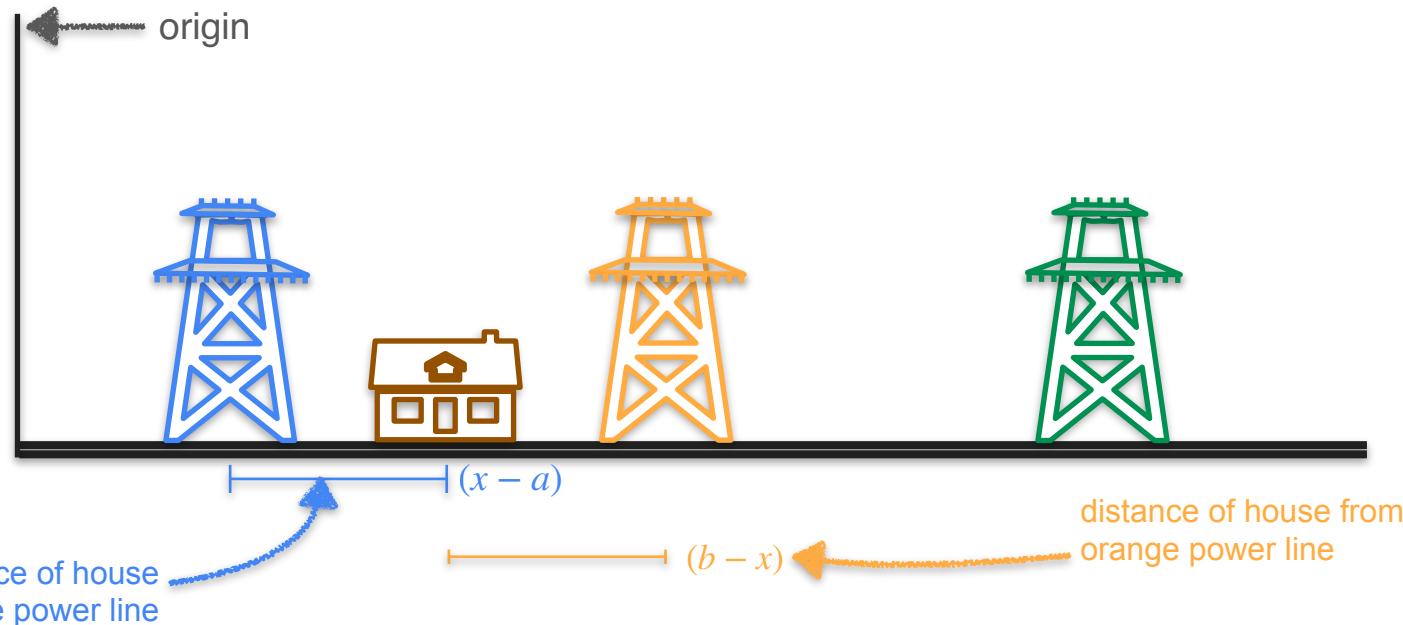
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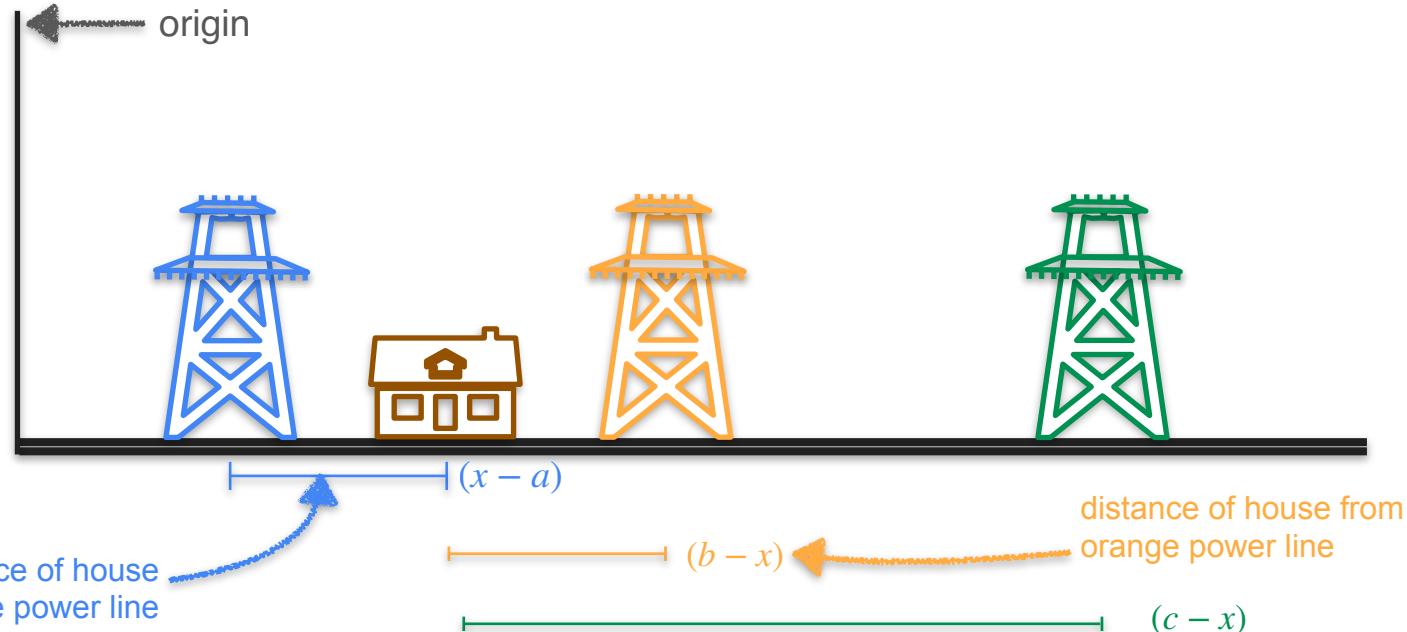
Three Power Line Problem



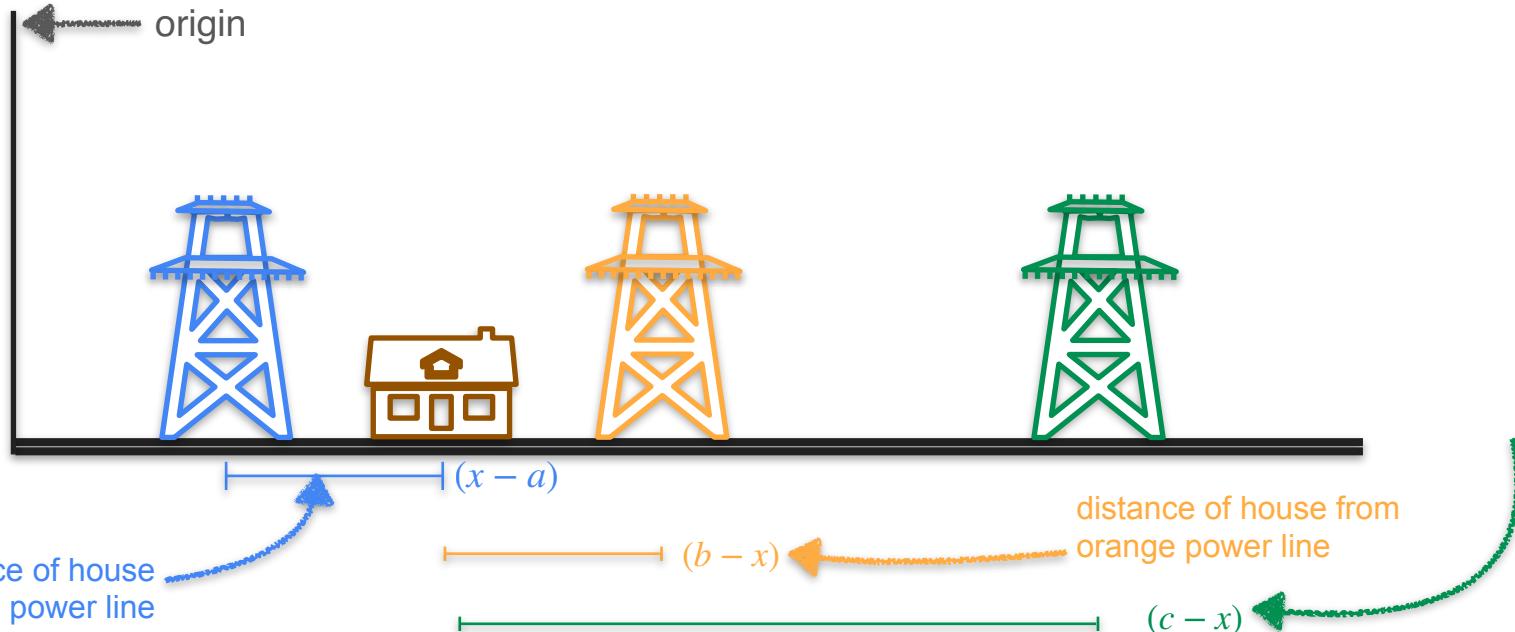
Three Power Line Problem



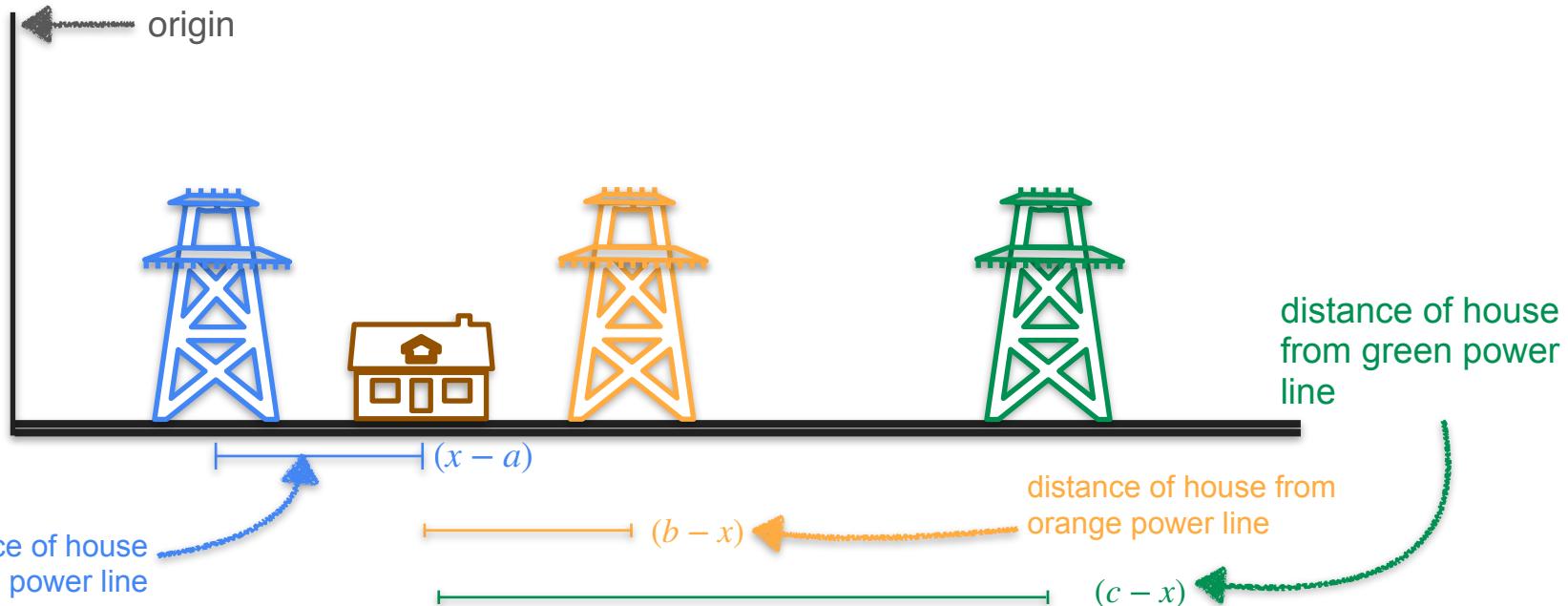
Three Power Line Problem



Three Power Line Problem



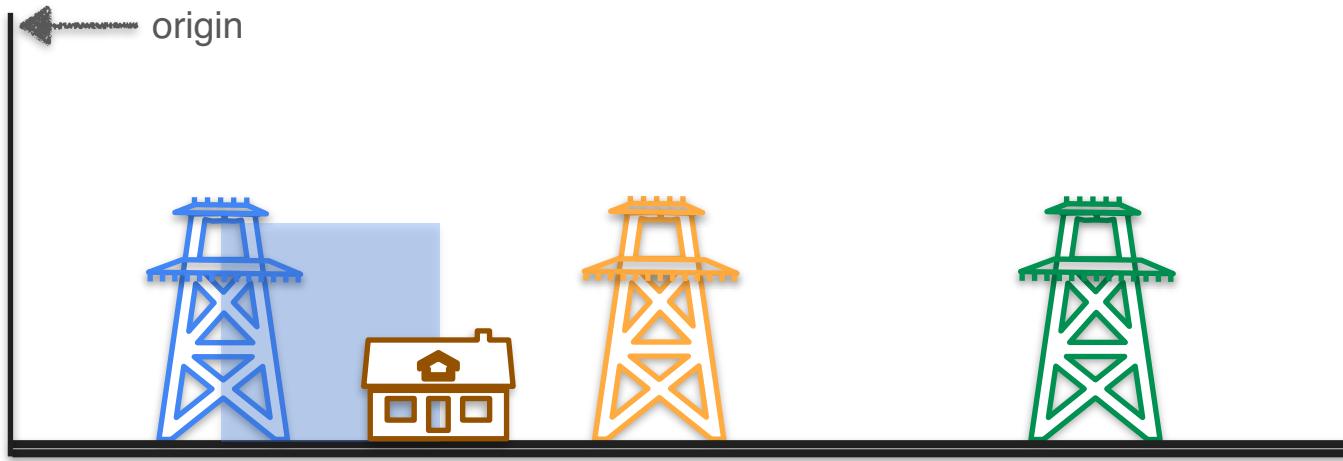
Three Power Line Problem



Three Power Line Problem



Three Power Line Problem



Three Power Line Problem



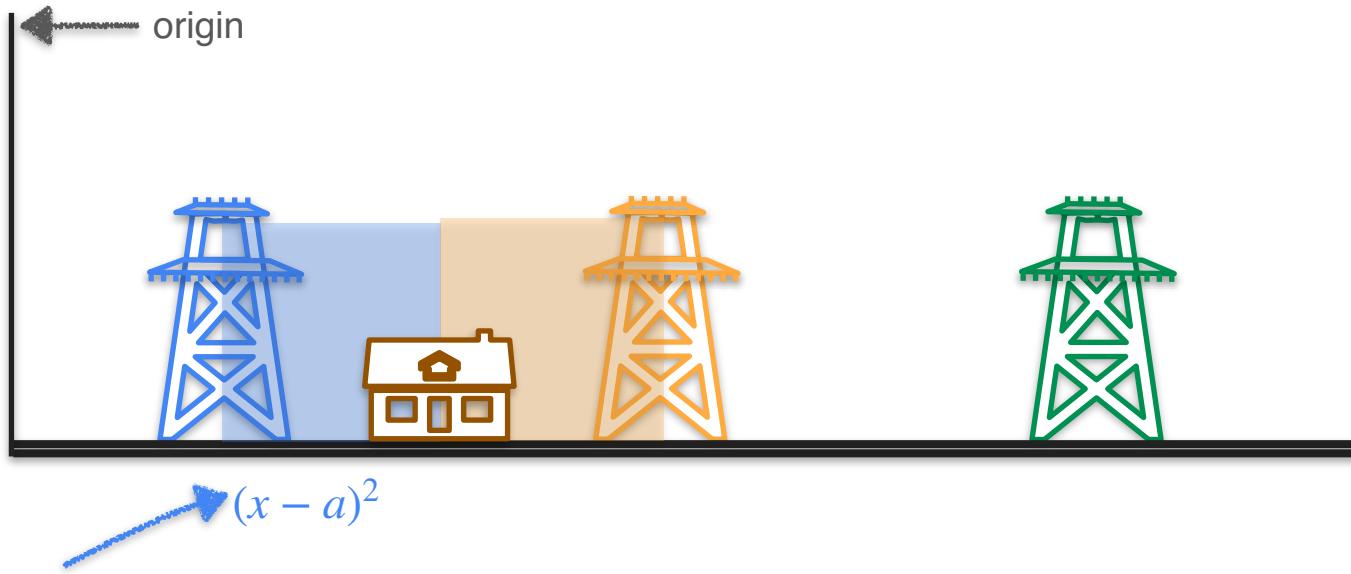
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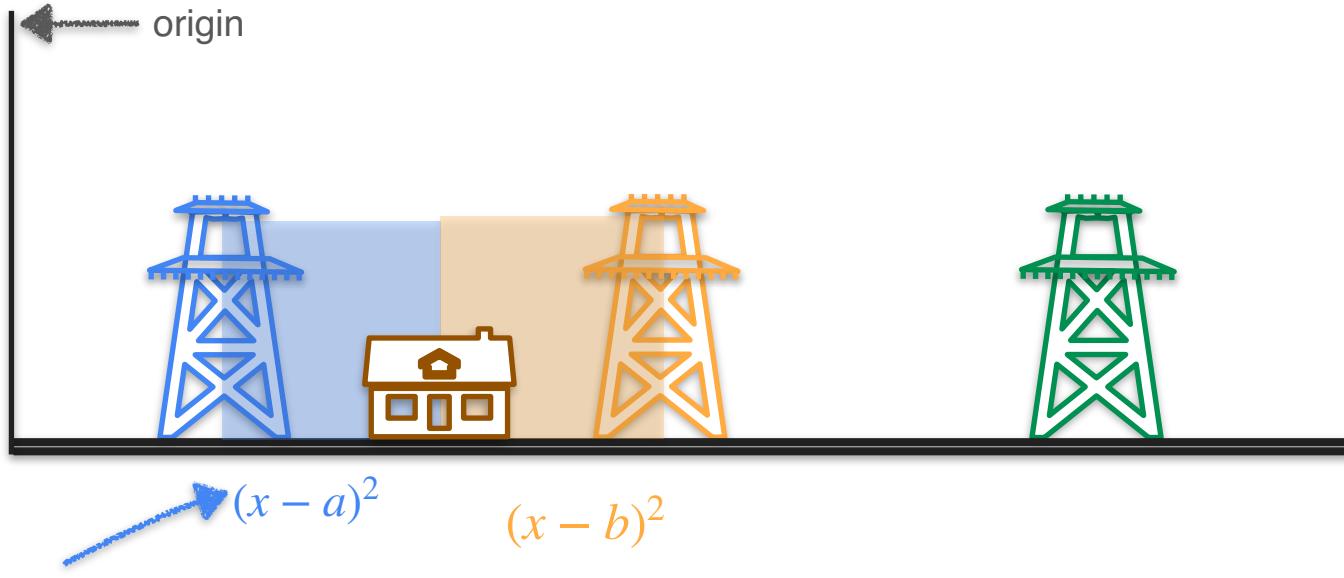
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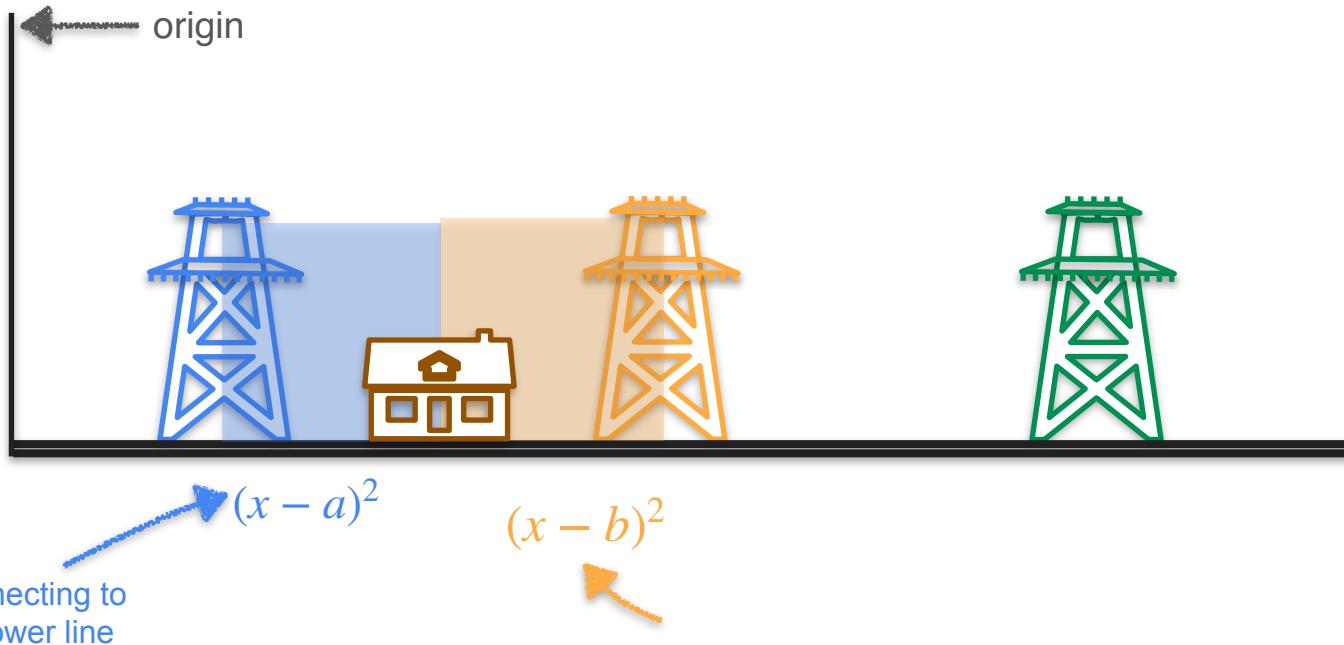
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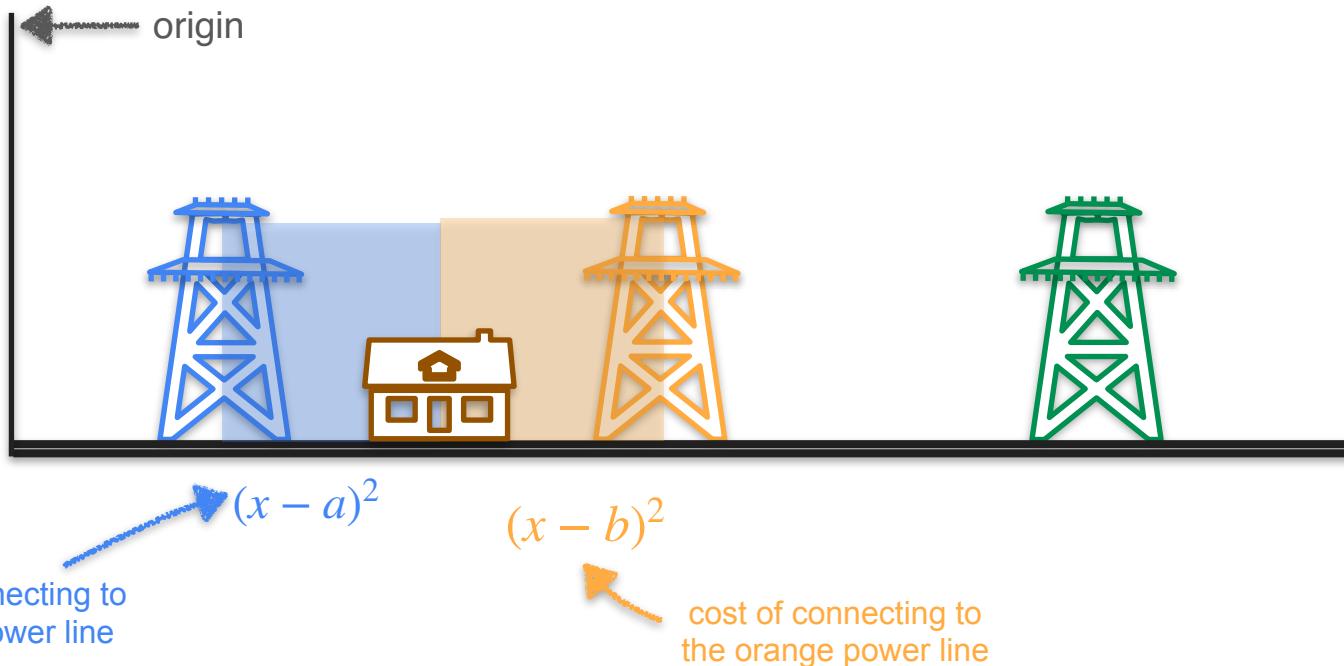
Three Power Line Problem



Three Power Line Problem



Three Power Line Problem

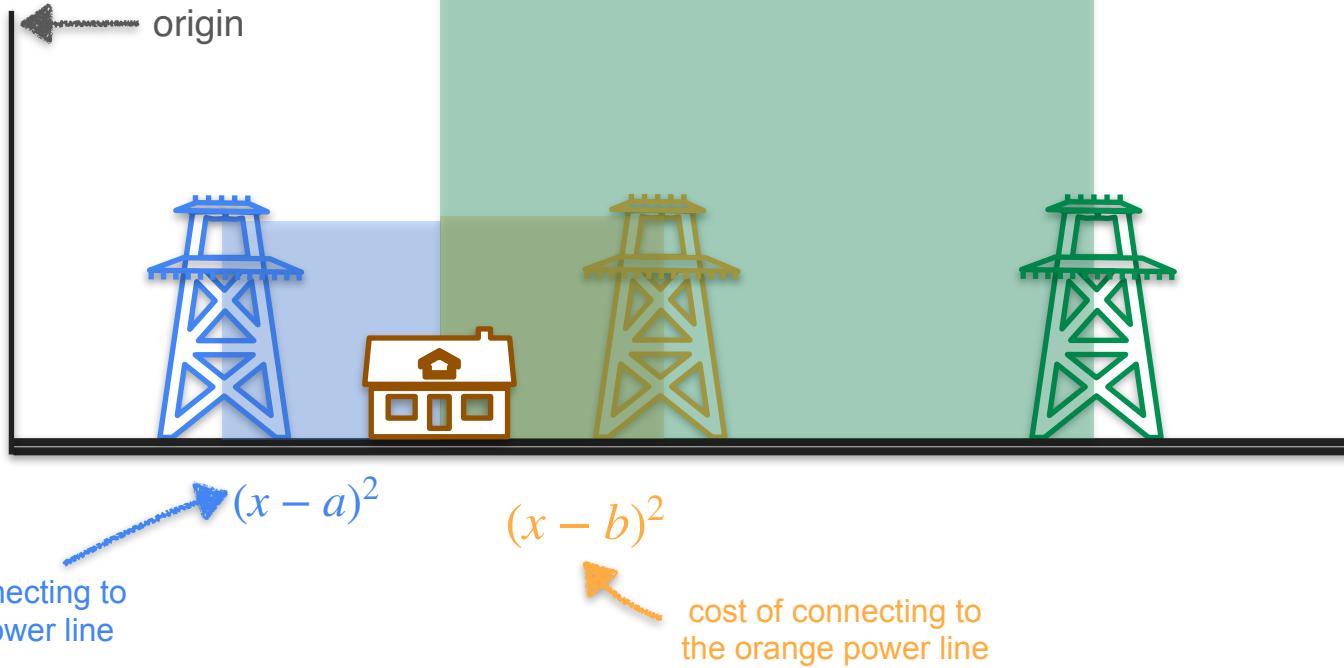


cost of connecting to
the blue power line

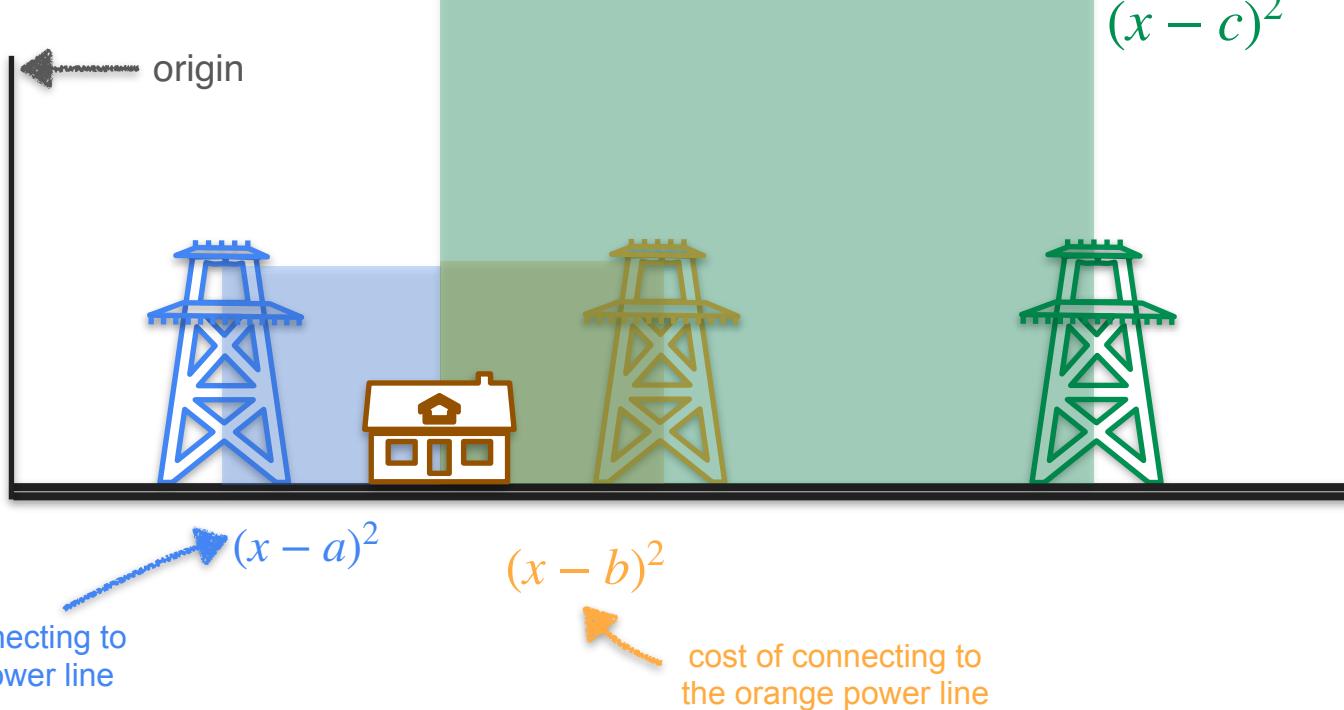
$(x - b)^2$

cost of connecting to
the orange power line

Three Power Line Problem



Three Power Line Problem

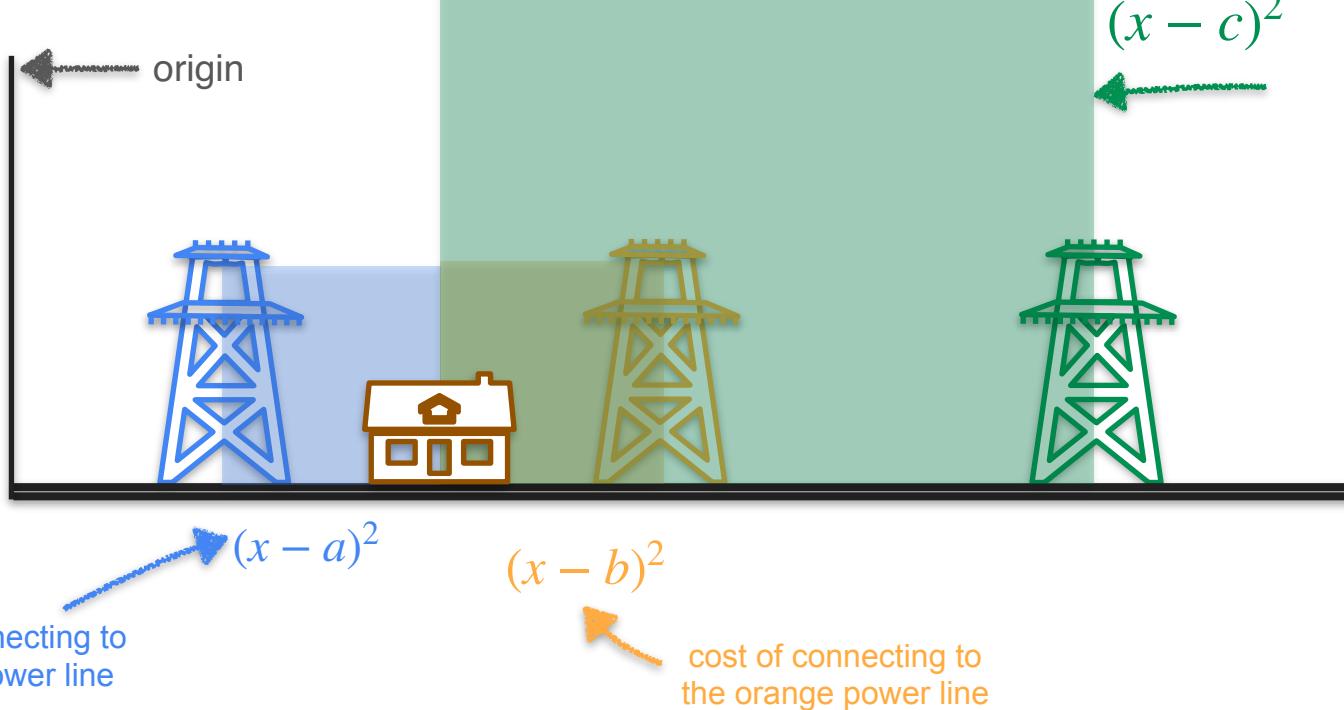


cost of connecting to
the blue power line

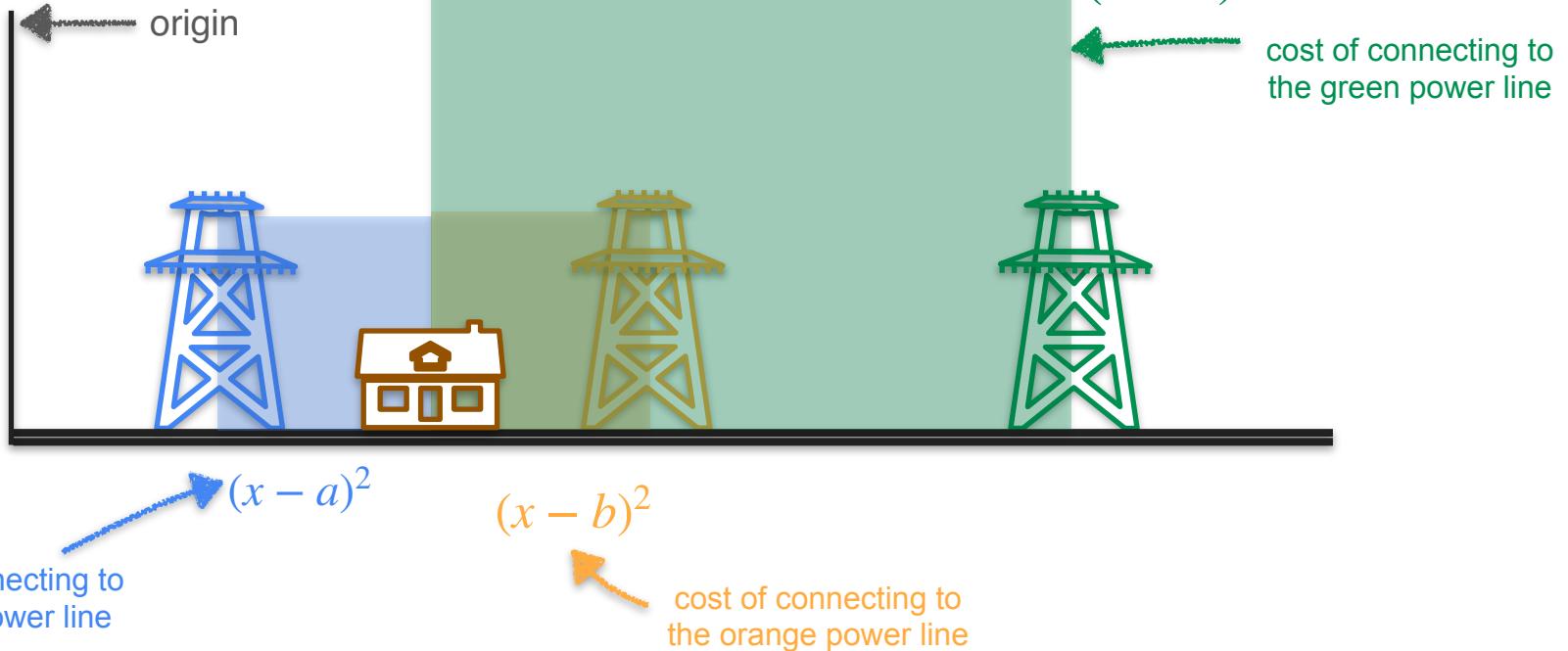
$(x - b)^2$

cost of connecting to
the orange power line

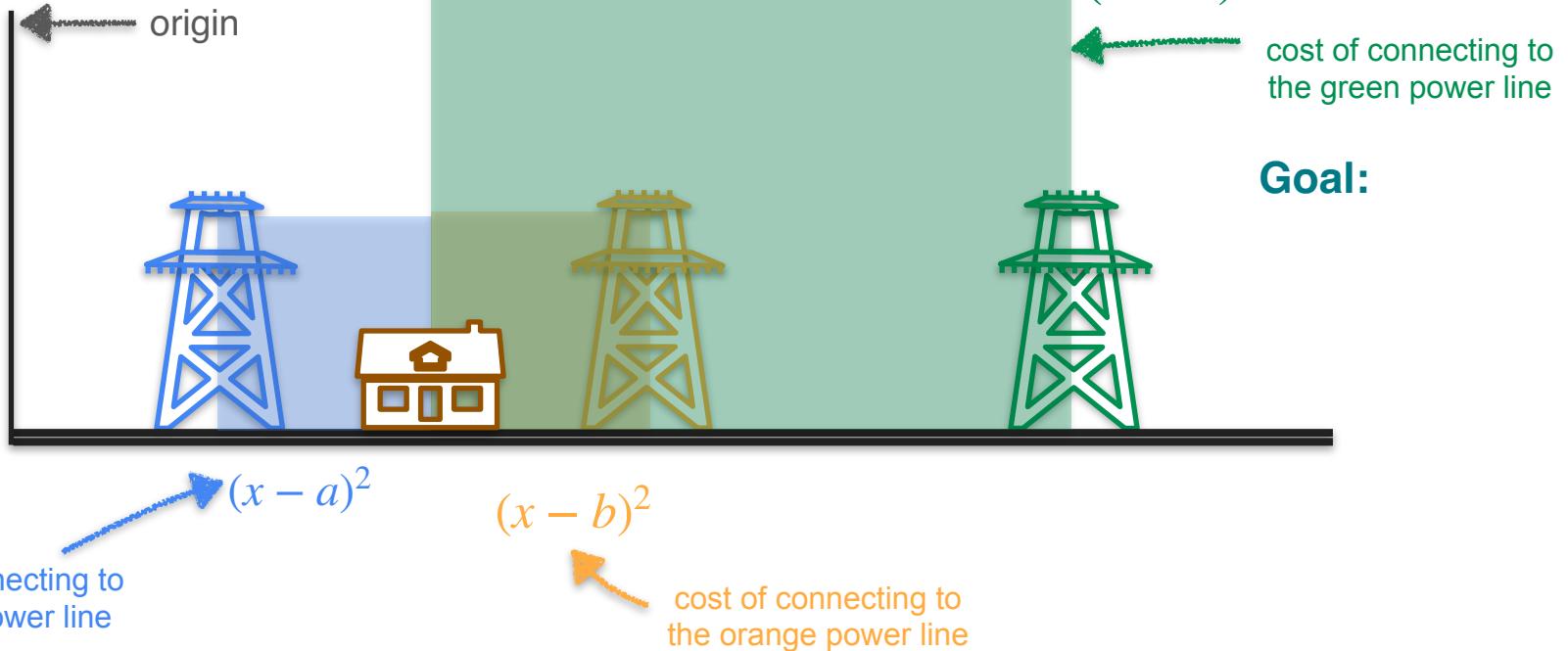
Three Power Line Problem



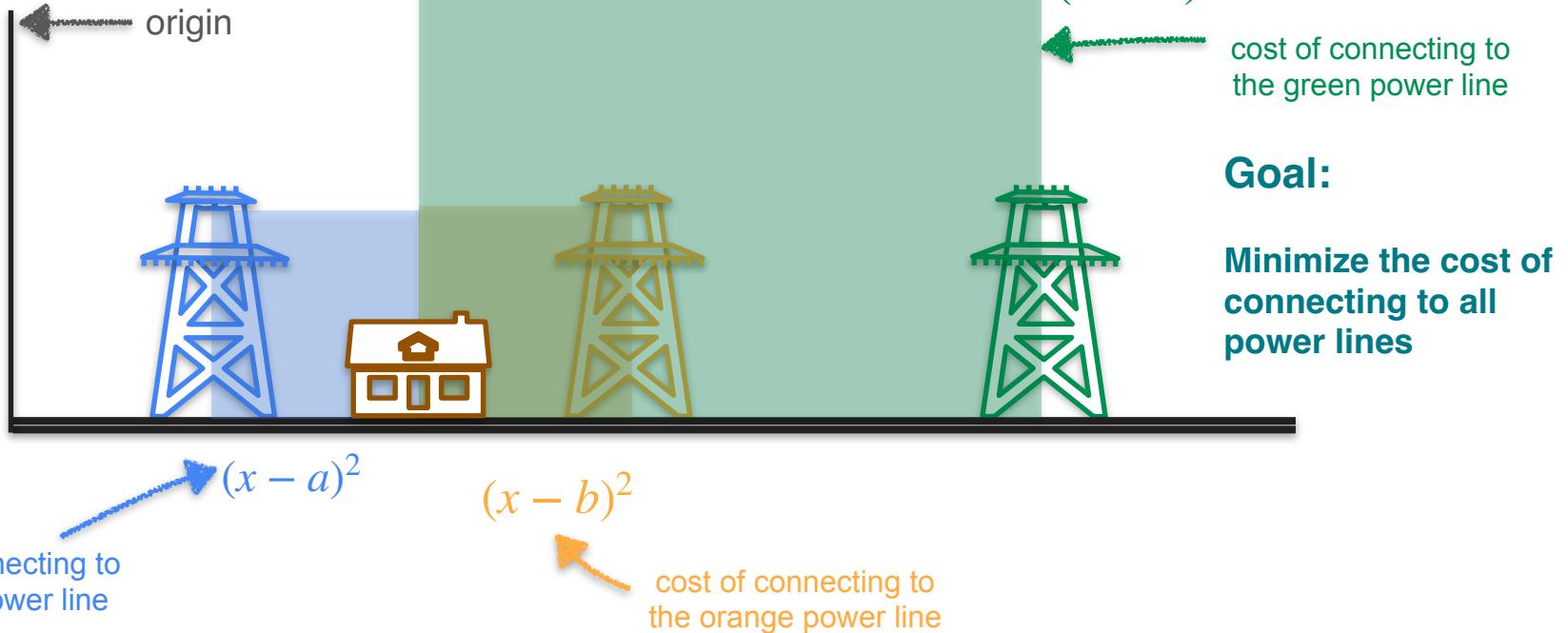
Three Power Line Problem



Three Power Line Problem



Three Power Line Problem



Three Power Line Problem



cost of connecting to
the blue power line

$(x - b)^2$

cost of connecting to
the orange power line

What is the cost function of
this problem?

$(x - c)^2$

cost of connecting to
the green power line

Goal:

**Minimize the cost of
connecting to all
power lines**

Three Power Line Problem



$$(x - a)^2$$

$$(x - b)^2$$

$$(x - c)^2$$

Goal:

Minimize the cost of
connecting all power
lines

Three Power Line Problem



Goal:

Minimize the cost of
connecting all power
lines

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$

Three Power Line Problem



$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Three Power Line Problem



Goal:

**Minimize the cost of
connecting all power
lines**

$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

Three Power Line Problem



Goal:

**Minimize the cost of
connecting all power
lines**

$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

Three Power Line Problem

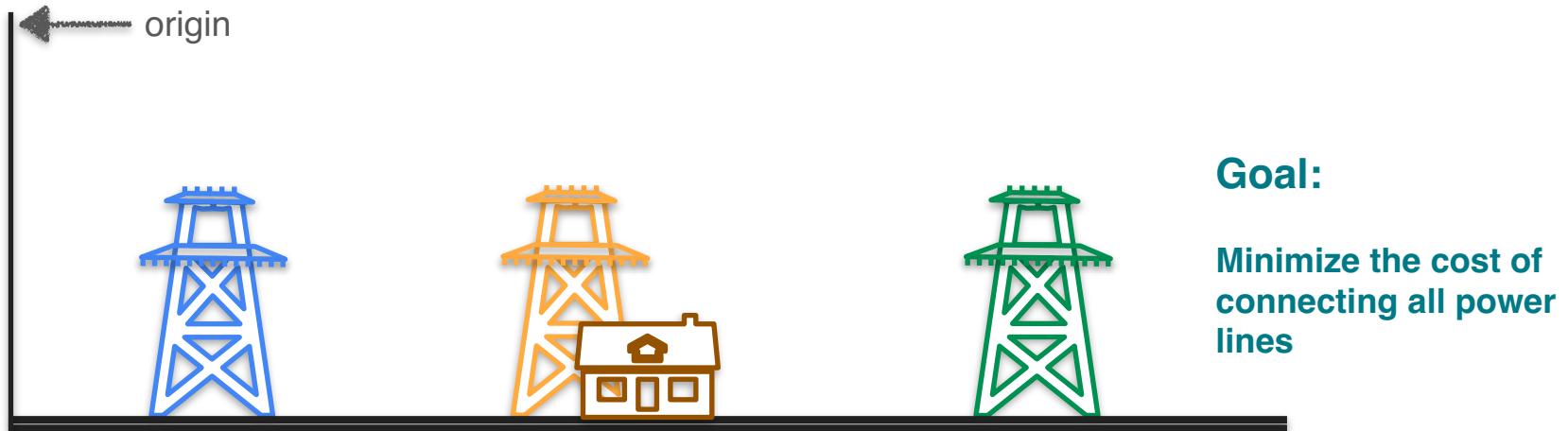


$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

Where should you put the house to minimize the cost?

Three Power Line Problem



$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Minimize this cost function

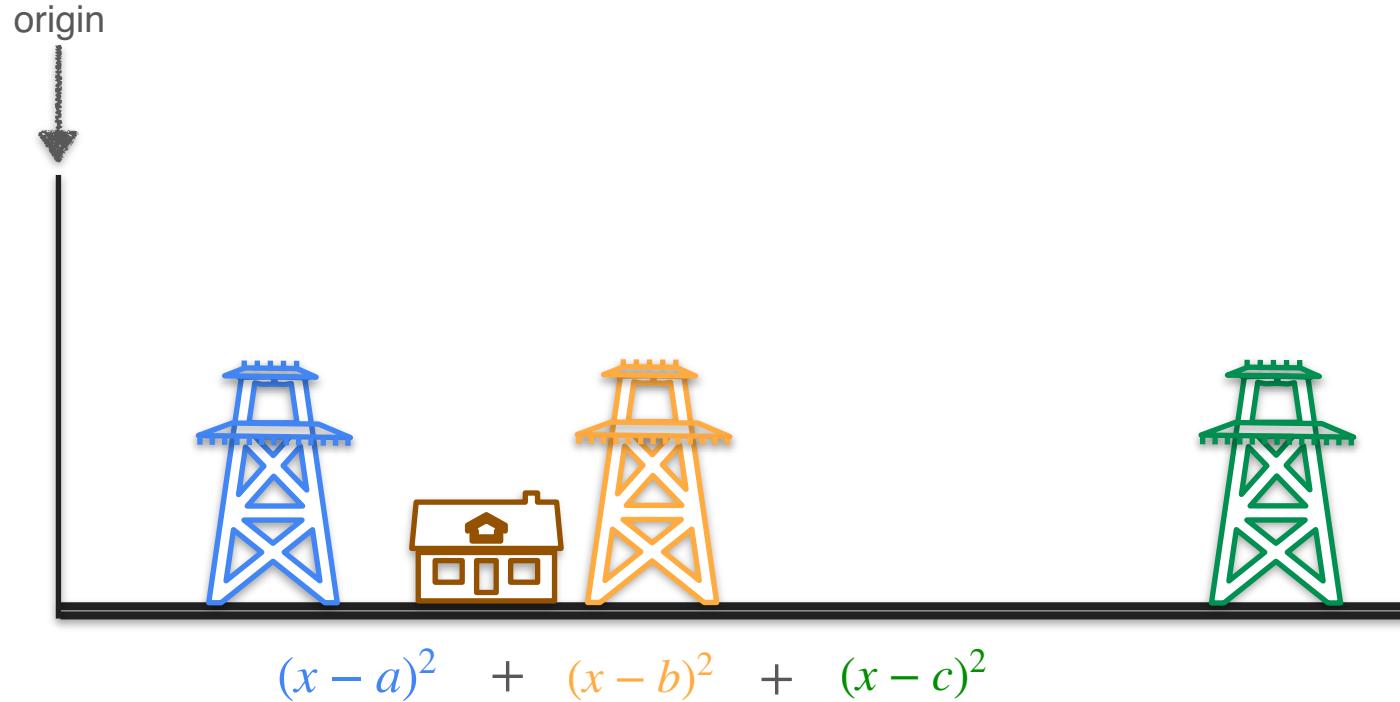
Solution:

Three Power Line Problem

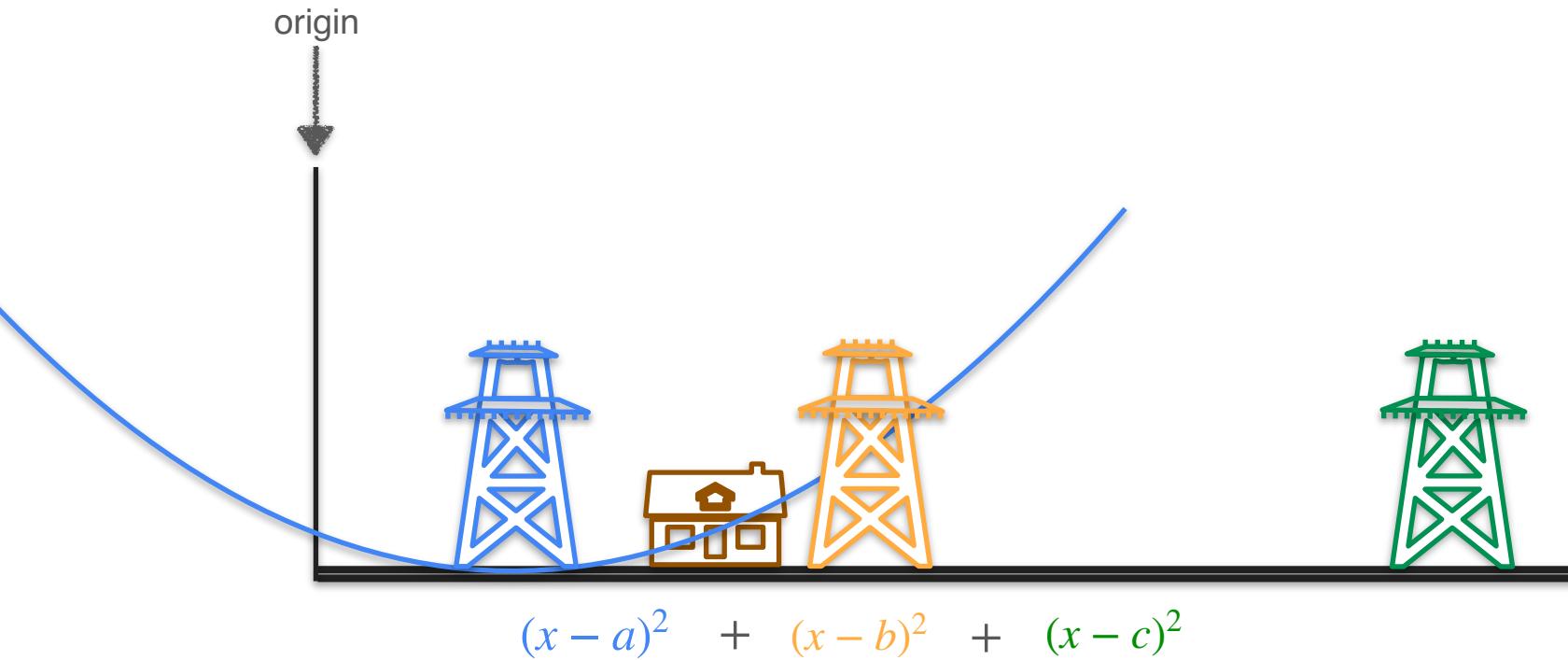


$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2 \quad \leftarrow \text{Minimize this cost function}$$
$$\text{Solution: } x = \frac{a + b + c}{3}$$

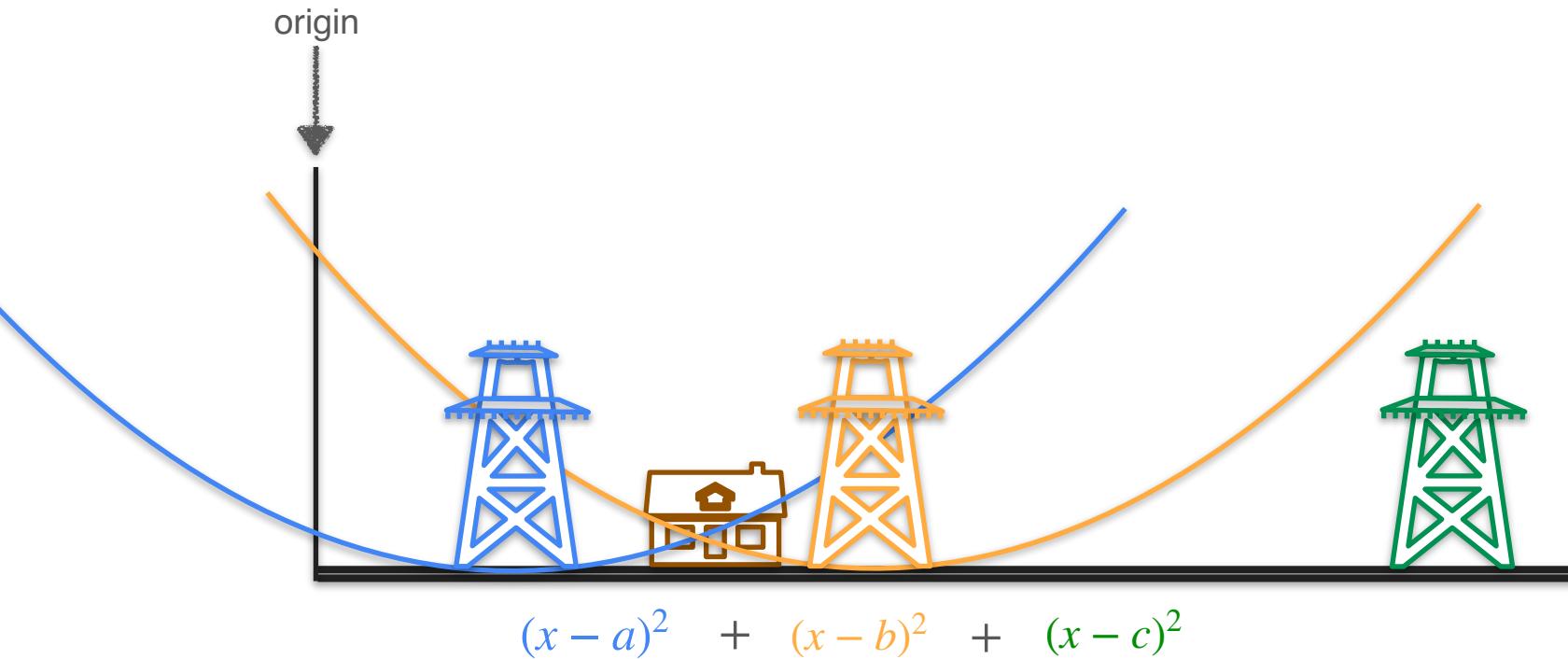
Three Power Line Problem



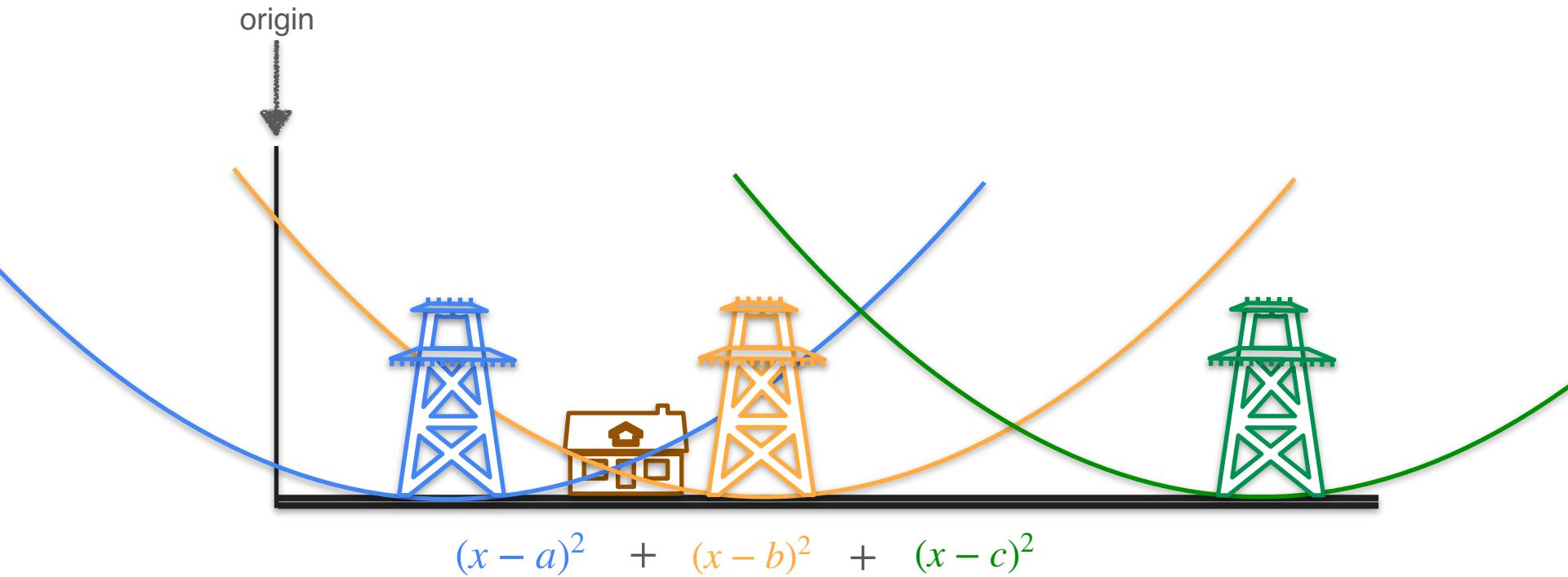
Three Power Line Problem



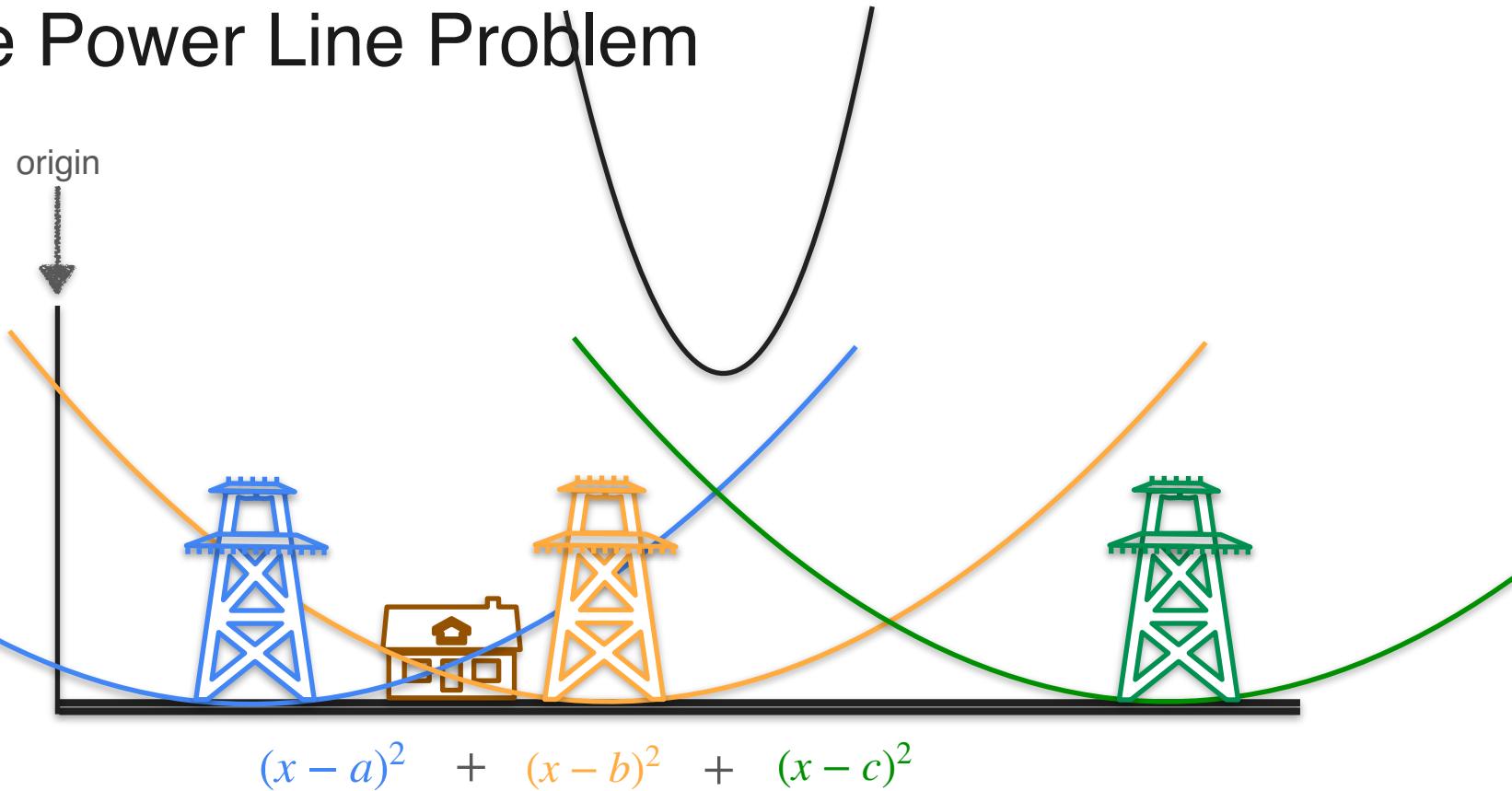
Three Power Line Problem



Three Power Line Problem



Three Power Line Problem



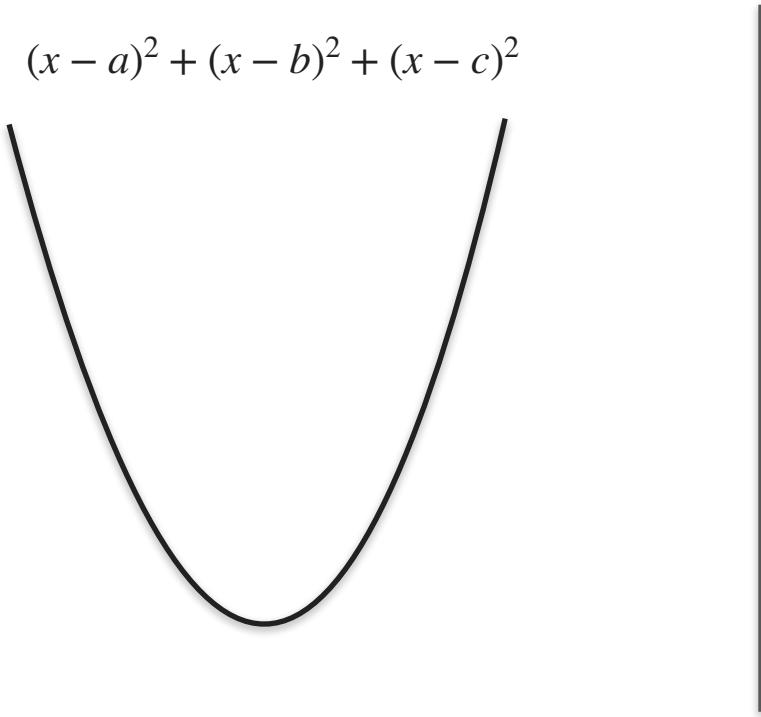
Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



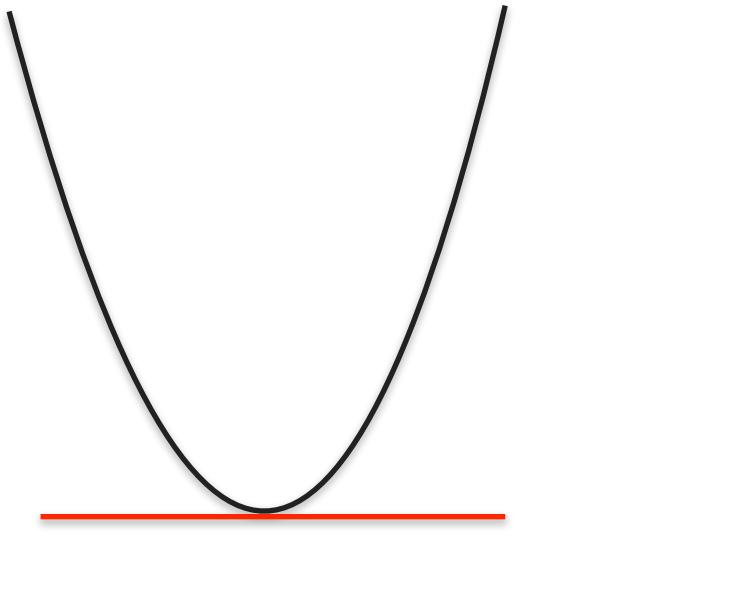
Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



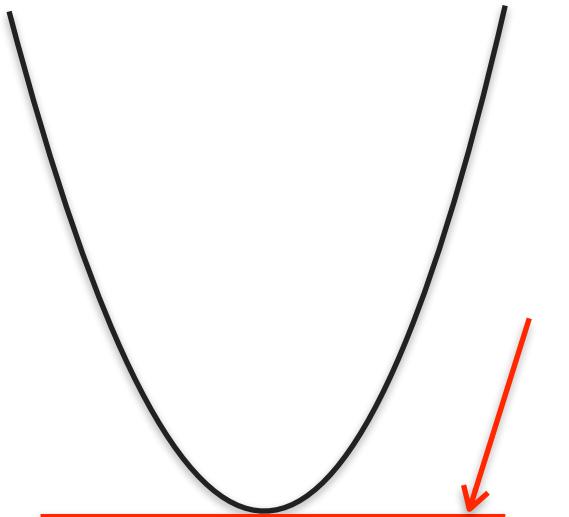
Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



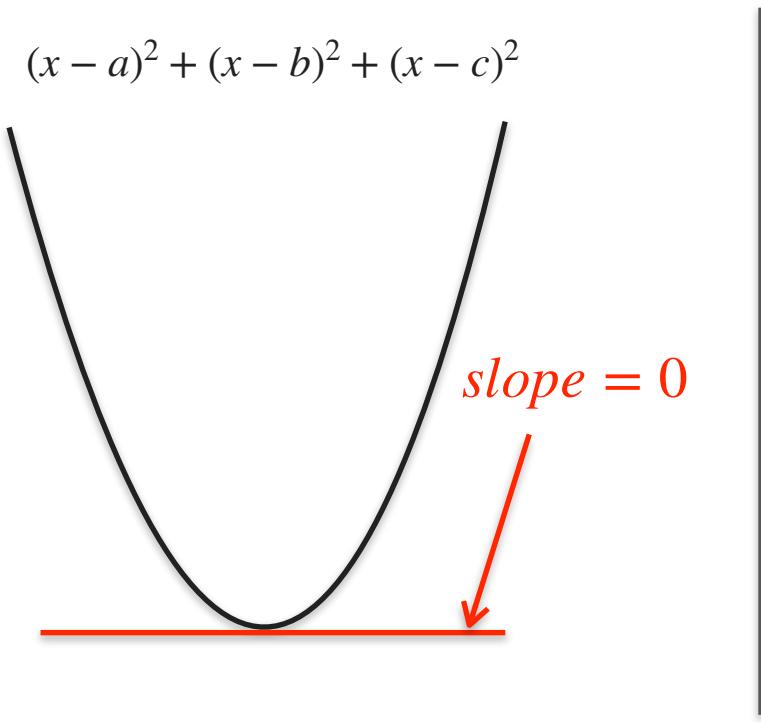
Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



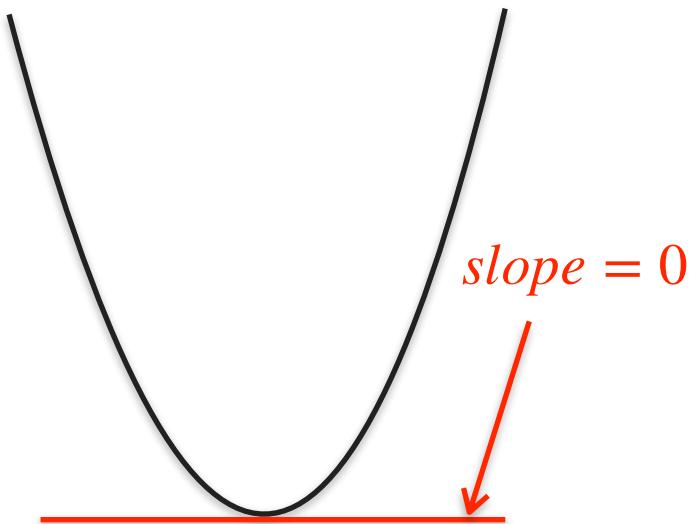
Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



Three Power Line Problem

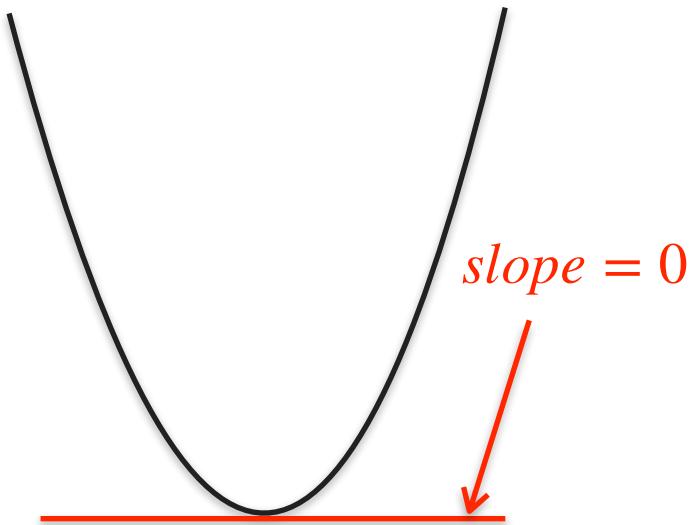
$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$

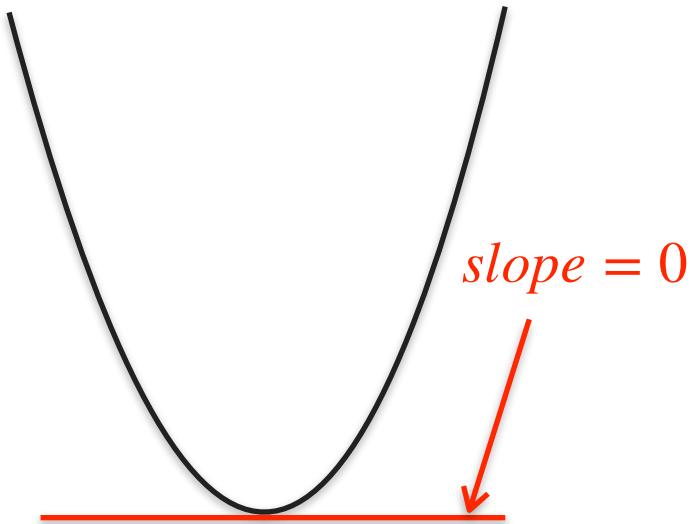


$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



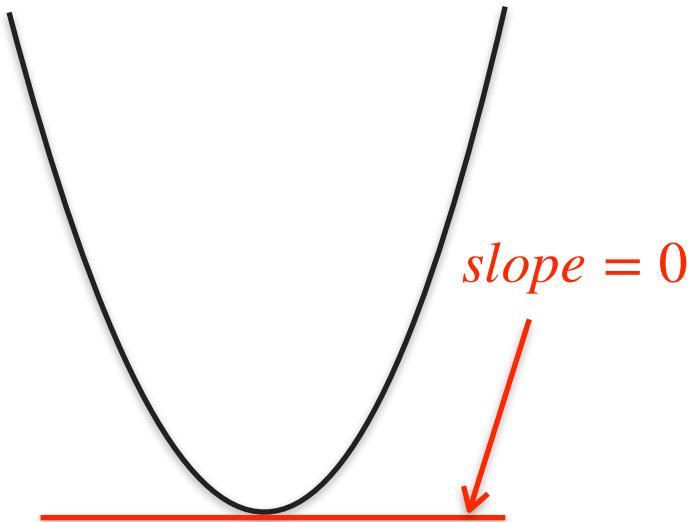
$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

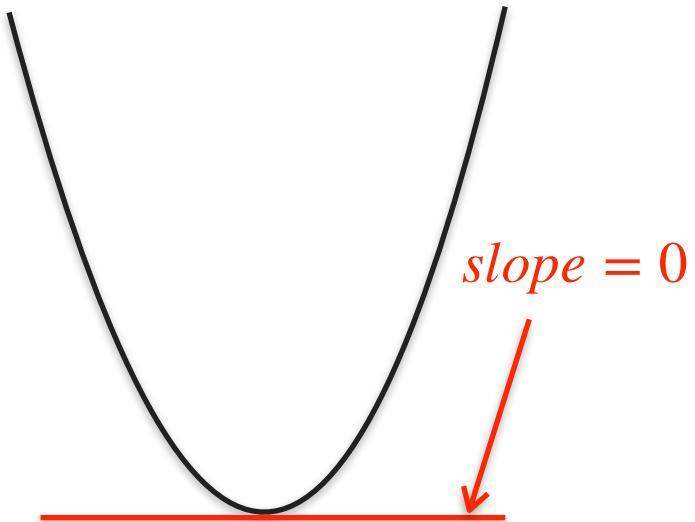
$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

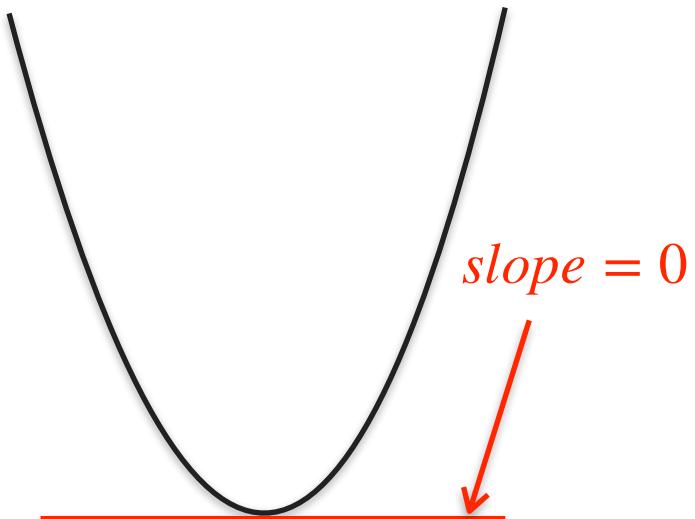
$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

$$3x = a + b + c$$

Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

$$3x = a + b + c$$

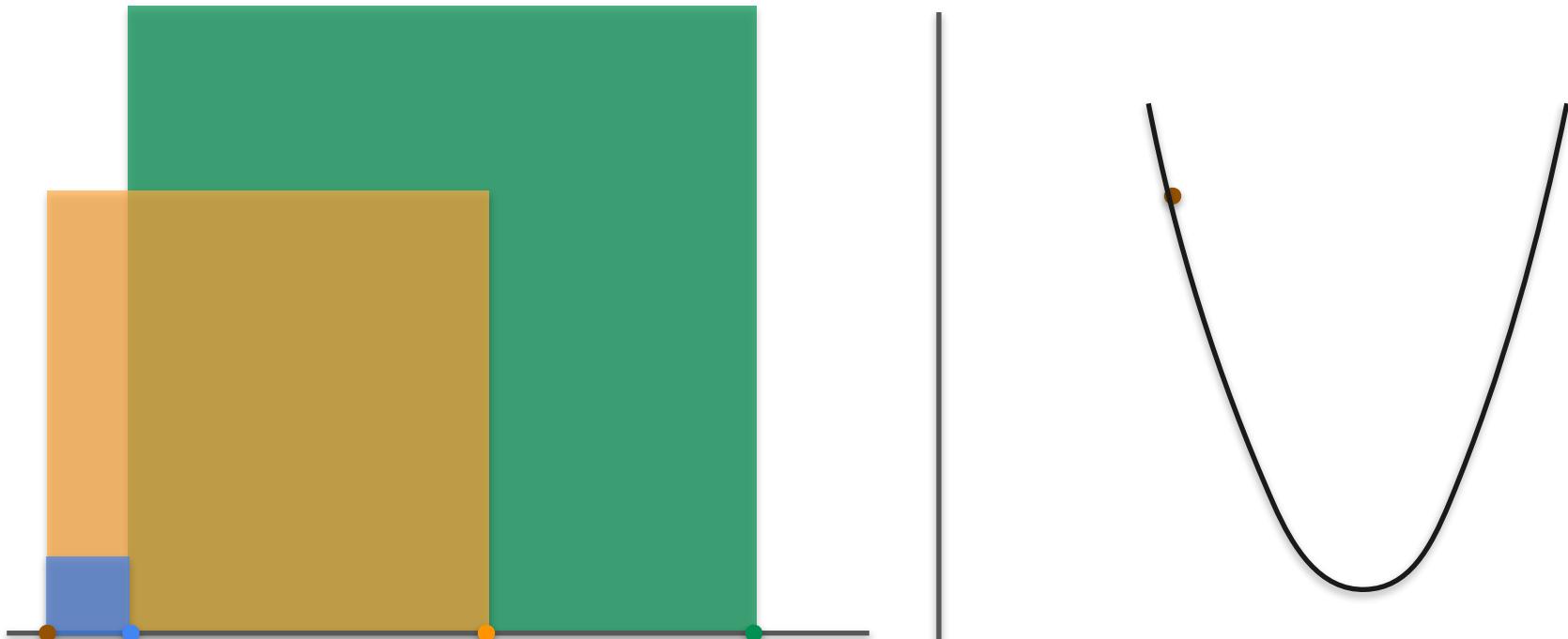
$$x = \frac{a + b + c}{3}$$

Three Power Line Problem: Square Analogy



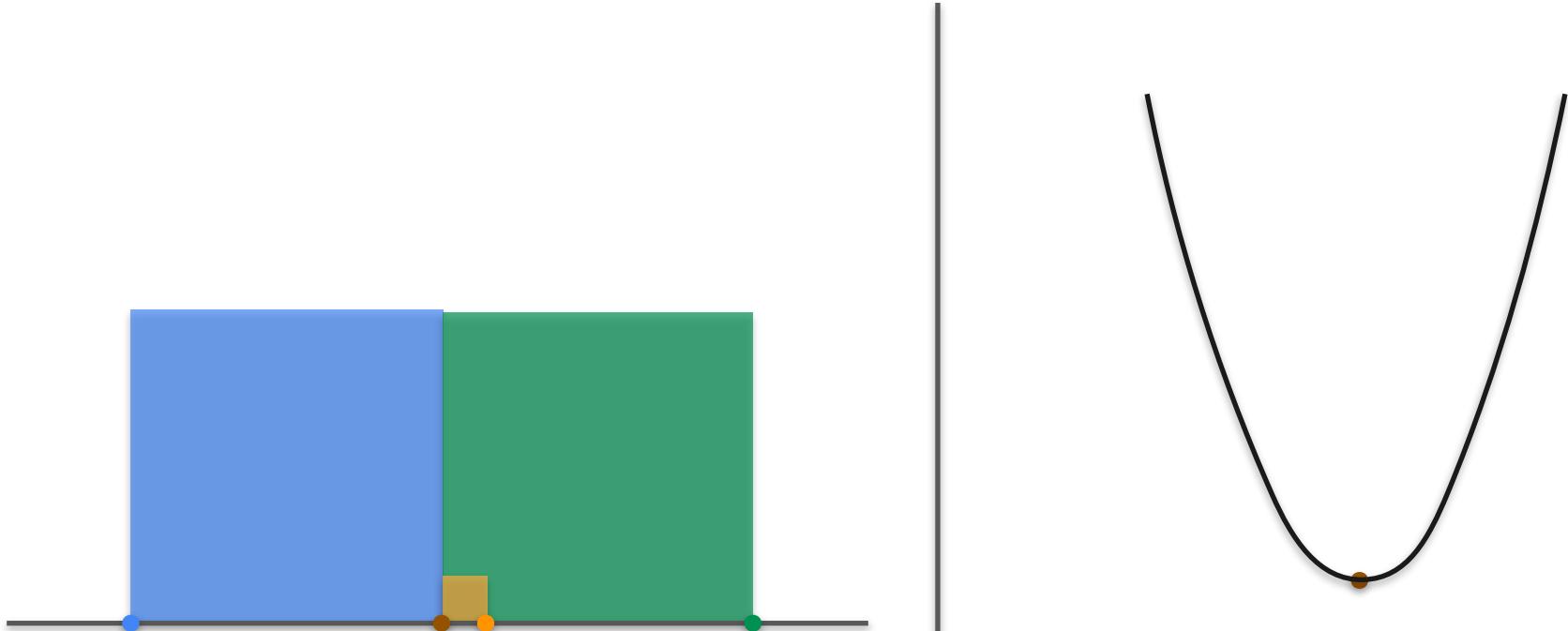
Problem: Minimize total area of the squares

Three Power Line Problem: Square Analogy



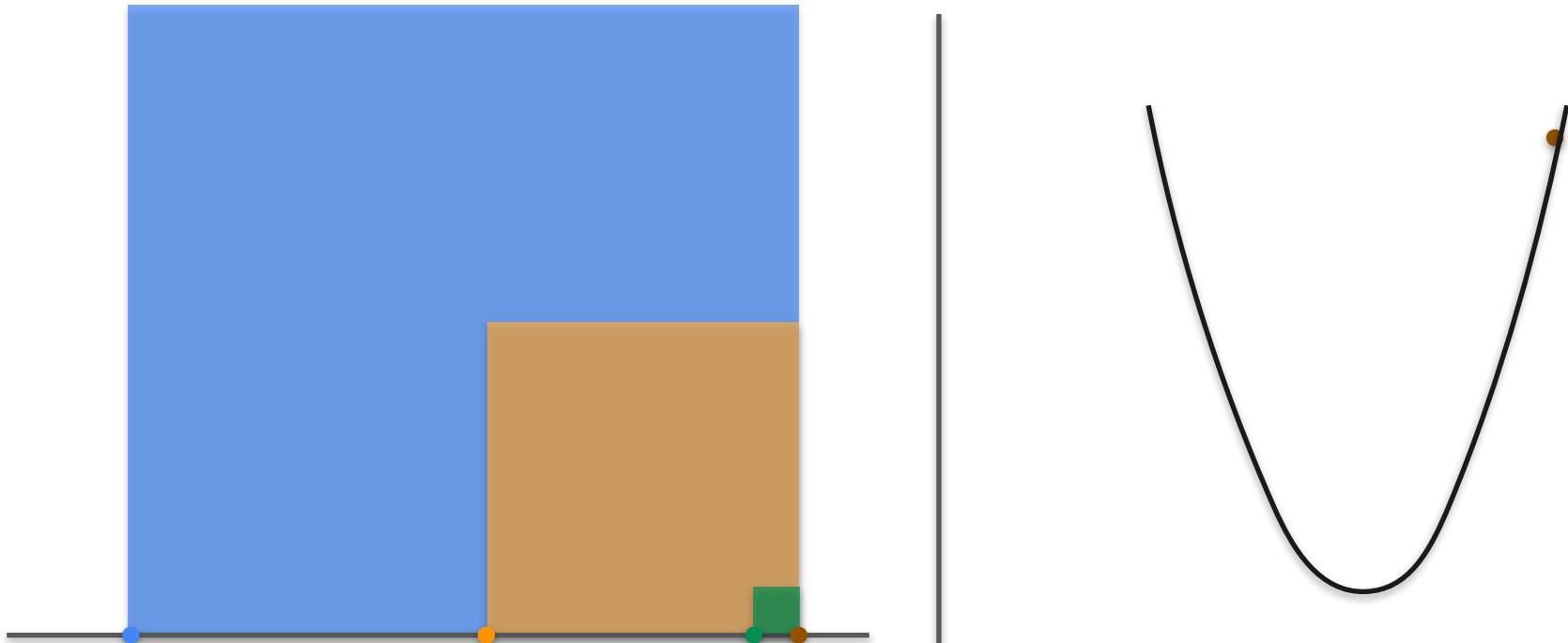
Problem: Minimize total area of the squares

Three Power Line Problem: Square Analogy



Problem: Minimize total area of the squares

Three Power Line Problem: Square Analogy



Problem: Minimize total area of the squares

The Square Loss

Minimize $(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2$

The Square Loss

Minimize $(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2$

Solution: $x = \frac{a_1 + a_2 + \cdots + a_n}{n}$



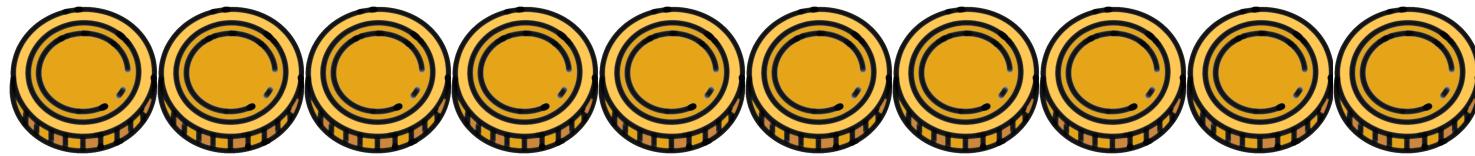
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Derivatives and Optimization

Optimization of log-loss Part 1

Coin Toss

Coin Toss



Coin Toss



Coin Toss



Coin 1



70% 30%

Coin Toss



Coin 1



70% 30%

Coin 2



50% 50%

Coin Toss



Coin 1



70% 30%

Coin 2



50% 50%

Coin 3



30% 70%

Quiz

- Which of the three coins would you choose to maximize your chances of winning?

Coin 1



70% 30%

Coin 2



50% 50%

Coin 3



30% 70%

Coins Toss



Coins Toss



$$\text{Coin 1} \quad 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3 \\ = 0.00222$$

Coins Toss



$$\text{Coin 1} \quad 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3 \\ = 0.00222$$

$$\text{Coin 2} \quad 0.5 \times 0.5 = 0.5^7 0.5^3 \\ = 0.00097$$

Coins Toss



$$\text{Coin 1} \quad 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3 \\ = 0.00222$$

$$\text{Coin 2} \quad 0.5 \times 0.5 = 0.5^7 0.5^3 \\ = 0.00097$$

$$\text{Coin 3} \quad 0.3 \times 0.7 \times 0.7 \times 0.7 = 0.3^7 0.7^3 \\ = 0.00008$$

Coins Toss



$$\text{Coin 1} \quad 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3 \\ = 0.00222$$

$$\text{Coin 2} \quad 0.5 \times 0.5 = 0.5^7 0.5^3 \\ = 0.00097$$

$$\text{Coin 3} \quad 0.3 \times 0.7 \times 0.7 \times 0.7 = 0.3^7 0.7^3 \\ = 0.00008$$

Coin Toss

Coin Toss



p

Coin Toss



p $(1 - p)$

Coin Toss



p $(1 - p)$

Chances of winning: $p^7(1 - p)^3 = g(p)$

Coin Toss



p $(1 - p)$

Chances of winning: $p^7(1 - p)^3 = g(p)$

Goal: maximize $g(p)$

Coin Toss

Coin Toss

$$\frac{dg}{dp} = \frac{d}{dp}(p^7(1-p)^3)$$

Coin Toss

Product rule

$$\frac{dg}{dp} = \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp}$$

Coin Toss

Product rule

$$\begin{aligned}\frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\ &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1)\end{aligned}$$

Chain rule

Coin Toss

Product rule

$$\begin{aligned}\frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\ &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1) \\ &= p^6(1-p)^2[7(1-p) - 3p]\end{aligned}$$

Chain rule

Coin Toss

Product rule

$$\begin{aligned}\frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\ &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1) \quad \text{Chain rule} \\ &= p^6(1-p)^2[7(1-p) - 3p] \\ &= p^6(1-p)^2(7 - 10p)\end{aligned}$$

Coin Toss

Product rule

$$\begin{aligned}\frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\ &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1) \quad \text{Chain rule} \\ &= p^6(1-p)^2[7(1-p) - 3p] \\ &= p^6(1-p)^2(7 - 10p) = 0\end{aligned}$$

Coin Toss

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Coin Toss

Product rule

$$\begin{aligned}
 \frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \quad \Rightarrow \quad \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\
 &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1) \\
 &= p^6(1-p)^2[7(1-p) - 3p] \\
 &= p^6(1-p)^2(7-10p) = 0
 \end{aligned}$$

\downarrow
 \downarrow
 \downarrow
 \downarrow
 \downarrow

Chain rule

$p = 0$ $\rightarrow p = 1$

Coin Toss

Product rule

$$\begin{aligned}\frac{dg}{dp} &= \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp} \\ &= 7p^6(1-p)^3 + p^7 3(1-p)^2(-1) \quad \text{Chain rule} \\ &= p^6(1-p)^2[7(1-p) - 3p] \\ &= p^6(1-p)^2(7-10p) = 0\end{aligned}$$

\downarrow

$$p = 0$$

\rightarrow

$$p = 1$$

\rightarrow

$$p = 0.7$$

Coin Toss

Product rule

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$p \neq 0$

$\rightarrow p = 1$

$\rightarrow p = 0.7$

Coin Toss

Product rule

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$p \neq 0$

$p \neq 1$

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Coin Toss

Coin Toss

$\log(g(p))$

Coin Toss

$$\log(g(p)) = \log(p^7(1-p)^3)$$

Coin Toss

$$\log(g(p)) = \log(p^7(1-p)^3) = \log(p^7) + \log((1-p)^3)$$

Coin Toss

$$\begin{aligned}\log(g(p)) &= \log(p^7(1-p)^3) = \log(p^7) + \log((1-p)^3) \\ &= 7\log(p) + 3\log(1-p)\end{aligned}$$

Coin Toss

$$\begin{aligned}\log(g(p)) &= \log(p^7(1-p)^3) = \log(p^7) + \log((1-p)^3) \\ &= 7\log(p) + 3\log(1-p) = G(p)\end{aligned}$$

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$$\frac{dG(p)}{dp} = \frac{d}{dp}(7\log(p) + 3\log(1-p))$$

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$$\frac{dG(p)}{dp} = \frac{d}{dp}(7\log(p) + 3\log(1-p)) = 7\frac{1}{p} + 3\frac{1}{1-p}(-1)$$

Coin Toss

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$$\begin{aligned}\frac{dG(p)}{dp} &= \frac{d}{dp}(7\log(p) + 3\log(1-p)) = 7\frac{1}{p} + 3\frac{1}{1-p}(-1) \\ &= \frac{7(1-p) - 3p}{p(1-p)}\end{aligned}$$

Coin Toss

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$$7(1-p) - 3p = 0$$

Coin Toss

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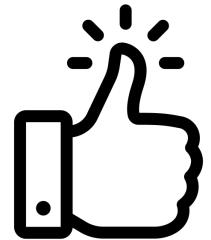
$$7(1-p) - 3p = 0 \quad p = 0.7$$

Coin Toss

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$$7(1-p) - 3p = 0 \quad p = 0.7$$



Coin Toss

$$\log(g(p)) = \log(p^7(1-p)^3) = \log(p^7) + \log((1-p)^3)$$

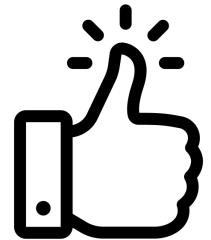
$$= 7\log(p) + 3\log(1-p) = G(p)$$

$-G(p)$ is the logloss

$$\frac{dG(p)}{dp} = \frac{d}{dp}(7\log(p) + 3\log(1-p)) = 7\frac{1}{p} + 3\frac{1}{1-p}(-1)$$

$$= \frac{7(1-p) - 3p}{p(1-p)} = 0$$

$$7(1-p) - 3p = 0 \quad p = 0.7$$





DeepLearning.AI

Derivatives and Optimization

**Optimization of log-loss
Part 2**

Relationship With ML

Relationship With ML



Relationship With ML



Data

Relationship With ML



Data

Relationship With ML

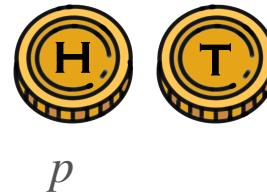


Data

Relationship With ML



Data

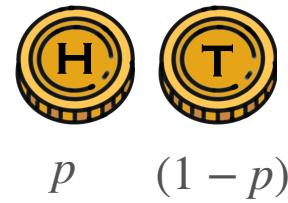


p

Relationship With ML



Data



Relationship With ML

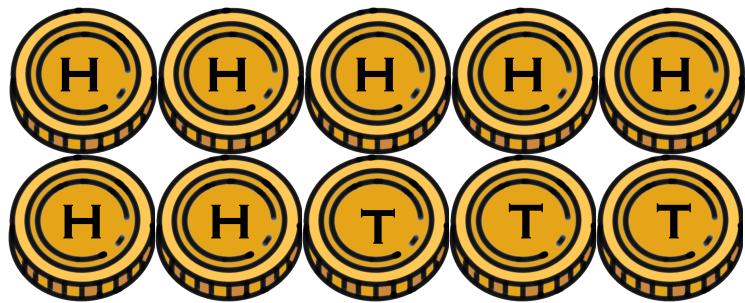


Data



Model

Relationship With ML



Data



Model

Minimized log-loss

Relationship With ML



Data

Minimized log-loss



Model

$$p = 0.7$$

Why the Logarithm?

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1. Derivative of products is hard, derivative of sums is easy

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$$f(p) = p^6(1 - p)^2(3 - p)^9(p - 4)^{13}(10 - p)^{500}$$

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1. Derivative of products is hard, derivative of sums is easy

$$f(p) = p^6(1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500}$$

$$\frac{df}{dp}$$



$$[6p^5](1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500} +$$

$$p^6 [2(1-p)](3-p)^9(p-4)^{13}(10-p)^{500}(-1) +$$

$$p^6(1-p)^2[9(3-p)^8](p-4)^{13}(10-p)^{500}(-1) +$$

$$p^6(1-p)^2(3-p)^9[13(p-4)^{12}](10-p)^{500} +$$

$$p^6(1-p)^2(3-p)^9(p-4)^{13}[500(10-p)^{499}](-1)$$

Why the Logarithm?

1. Derivative of products is hard, derivative of sums is easy

$$f(p) = p^6(1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500}$$

$$\frac{df}{dp}$$



$$[6p^5](1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500} +$$

$$p^6 [2(1-p)](3-p)^9(p-4)^{13}(10-p)^{500}(-1) +$$

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$$p^6(1-p)^2(3-p)^9[13(p-4)^{12}](10-p)^{500} +$$

$$p^6(1-p)^2(3-p)^9(p-4)^{13}[500(10-p)^{499}](-1)$$



$$\frac{d}{dp} \log(f)$$



$$\frac{6}{p} + \frac{2}{1-p}(-1) + \frac{9}{3-p}(-1) +$$

$$\frac{13}{p-4} + \frac{500}{10-p}(-1)$$

Why the Logarithm?

1. Derivative of products is hard, derivative if sums is easy

$$f(p) = p^6(1 - p)^2(3 - p)^9(p - 4)^{13}(10 - p)^{500}$$

$$\frac{df}{dp}$$



$$\frac{d}{dp} \log(f)$$



Why the Logarithm?

1. Derivative of products is hard, derivative if sums is easy

$$f(p) = p^6(1 - p)^2(3 - p)^9(p - 4)^{13}(10 - p)^{500}$$

$$\frac{df}{dp}$$



$$\frac{d}{dp} \log(f)$$



2. Product of lots of tiny things is tiny!



DeepLearning.AI

Derivatives and Optimization

Conclusion