

# Copyright Notice

These slides are distributed under the Creative Commons License.

[DeepLearning.AI](#) makes these slides available for educational purposes. You may not use or distribute these slides for commercial purposes. You may make copies of these slides and use or distribute them for educational purposes as long as you cite [DeepLearning.AI](#) as the source of the slides.

For the rest of the details of the license, see <https://creativecommons.org/licenses/by-sa/2.0/legalcode>



DeepLearning.AI

# Gradients and Gradient Descent

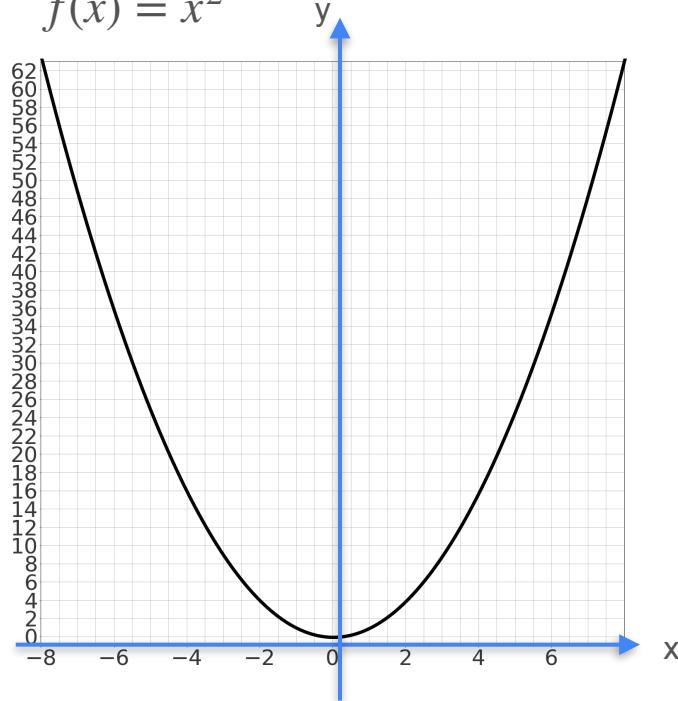
---

## Tangent planes

# Functions of Two Variables

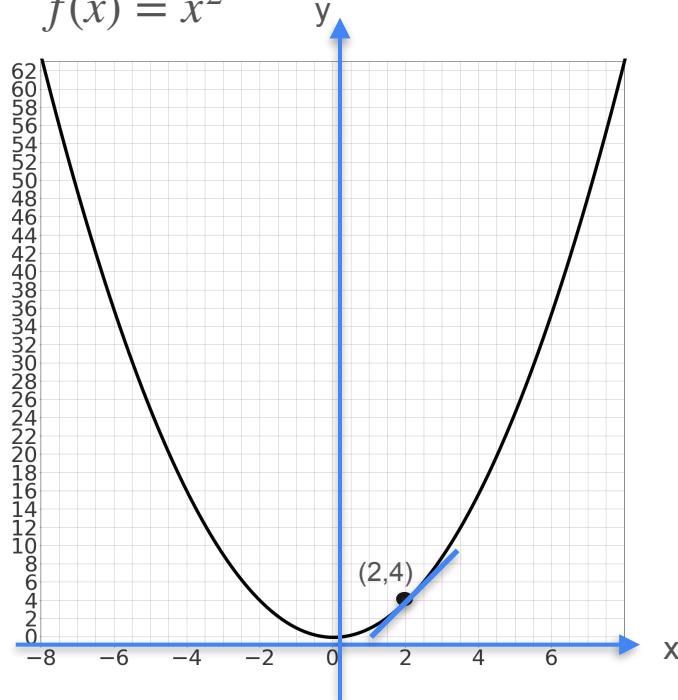
# Functions of Two Variables

$$f(x) = x^2$$



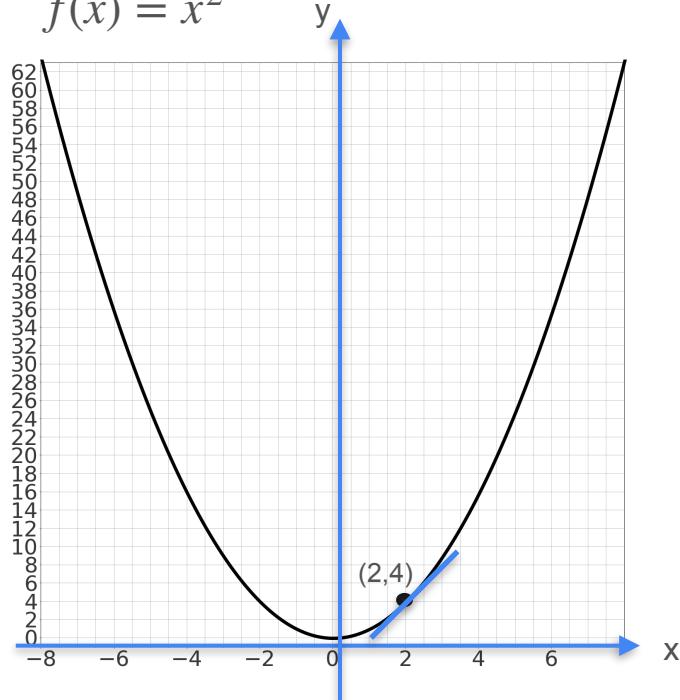
# Functions of Two Variables

$$f(x) = x^2$$

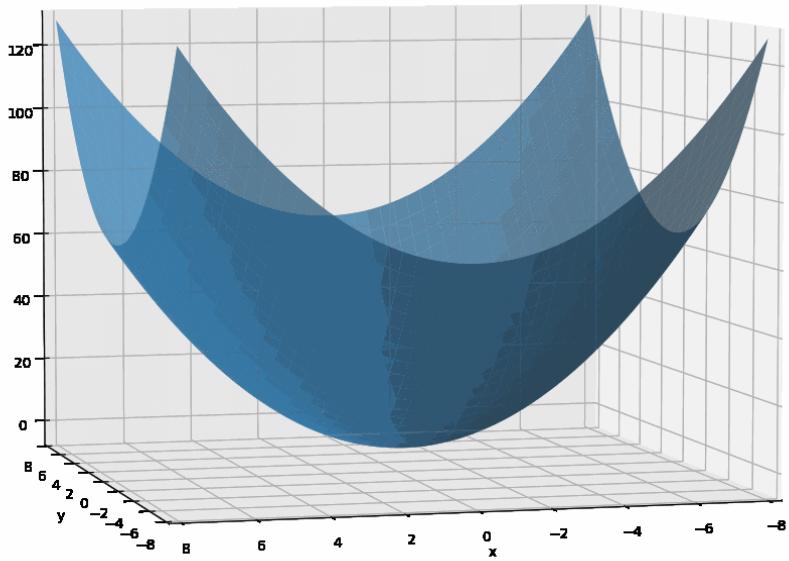


# Functions of Two Variables

$$f(x) = x^2$$

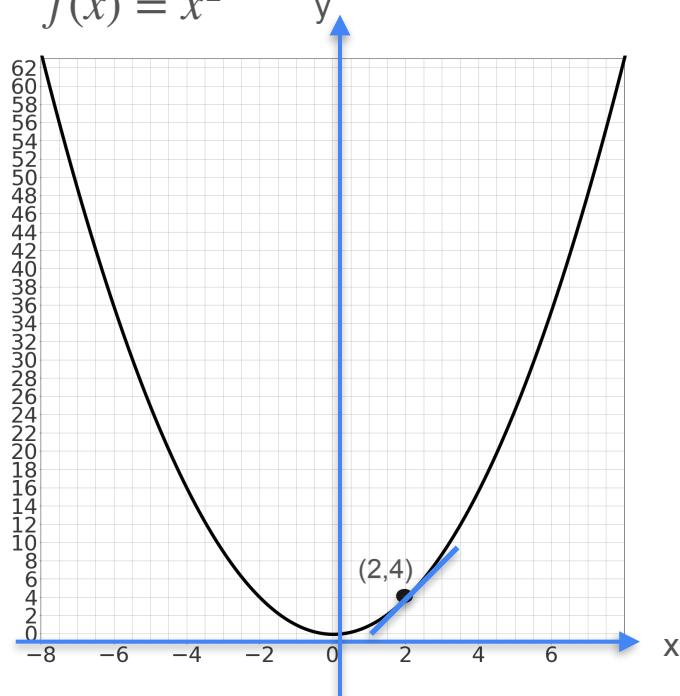


$$f(x, y) = x^2 + y^2$$

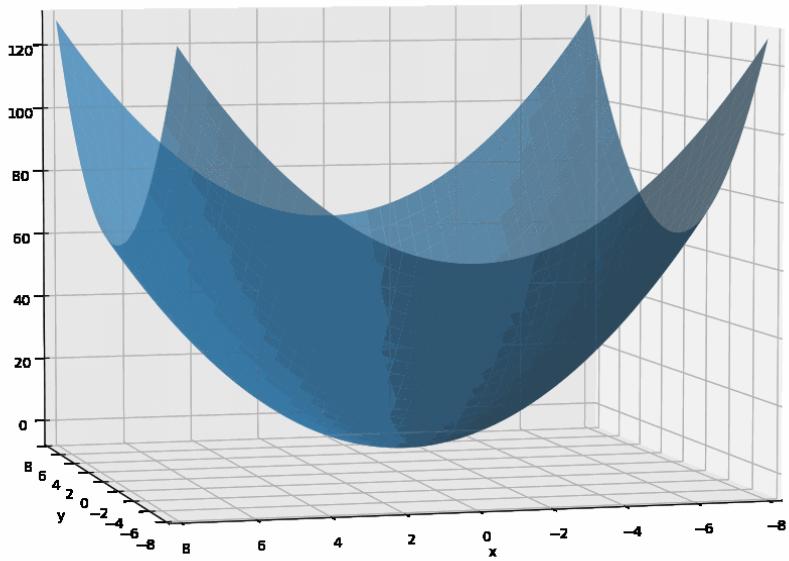


# Functions of Two Variables

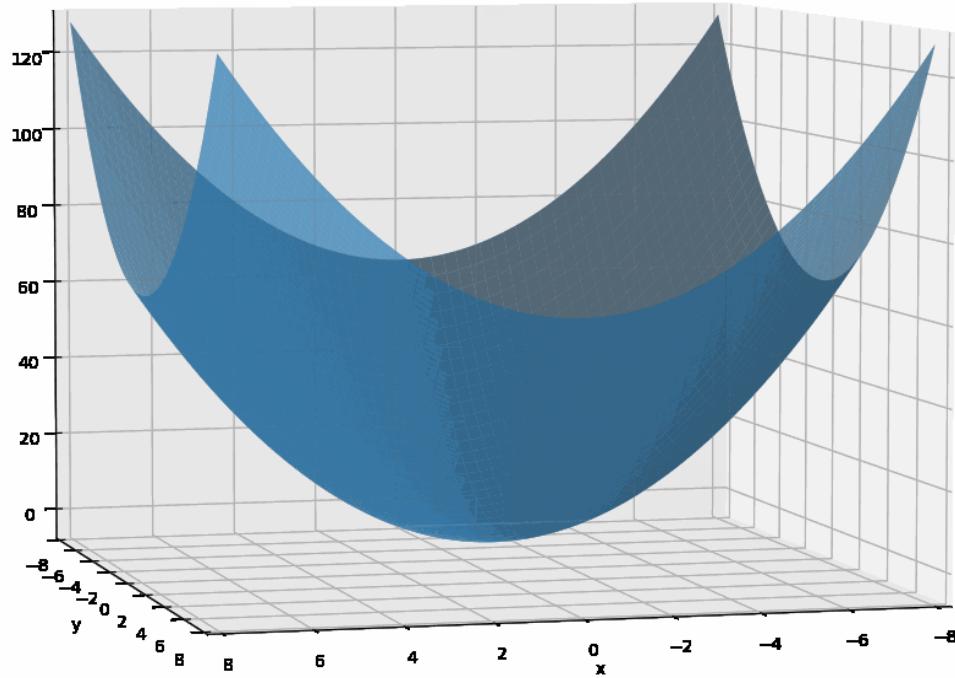
$$f(x) = x^2$$



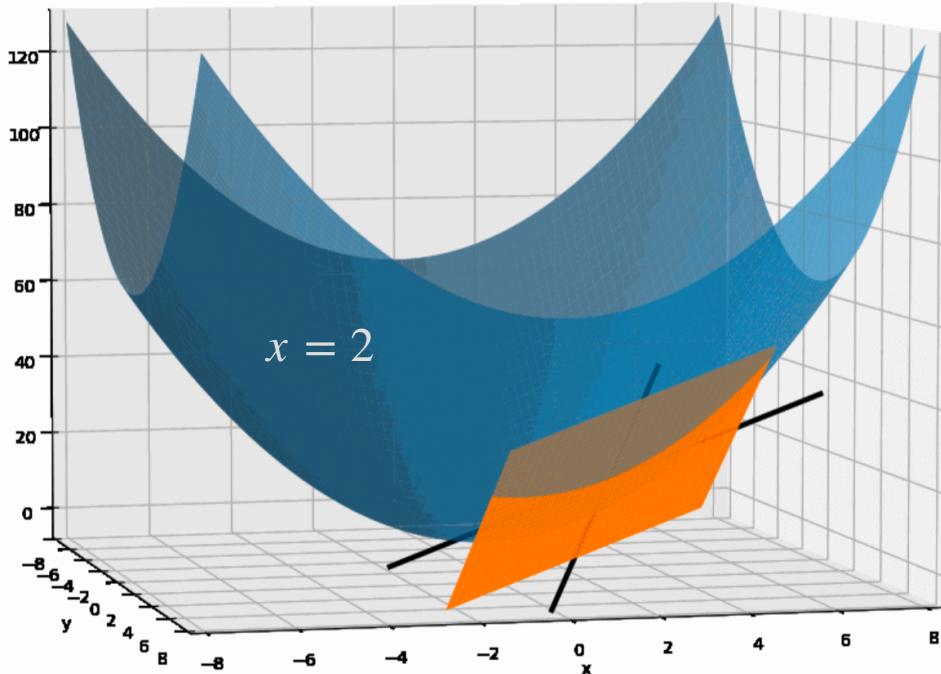
$$f(x, y) = x^2 + y^2$$



# Finding the Tangent Plane



# Finding the Tangent Plane



Fix  $y=4$   $f(x,4) = x^2 + 4^2$

$$\frac{d}{dx} (f(x,4)) = 2x$$

Fix  $x=2$   $f(2,y) = 2^2 + y^2$

$$\frac{d}{dy} (f(2,y)) = 2y$$

The tangent plane contains both tangent lines.

# Video 2: Introduction to Partial Derivatives

Example with the parabola, show tangent plane and slices



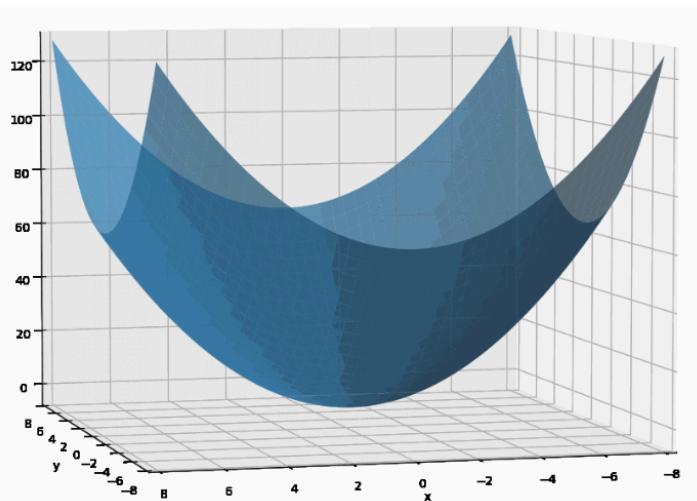
DeepLearning.AI

# Gradients and Gradient Descent

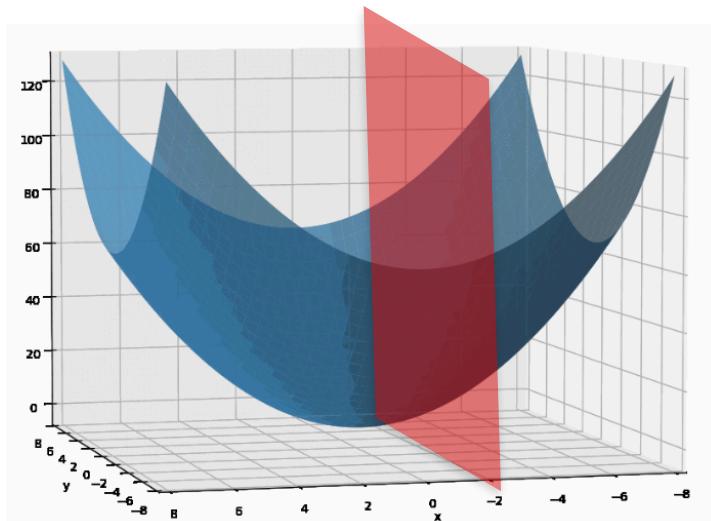
---

## Partial derivatives - Part 1

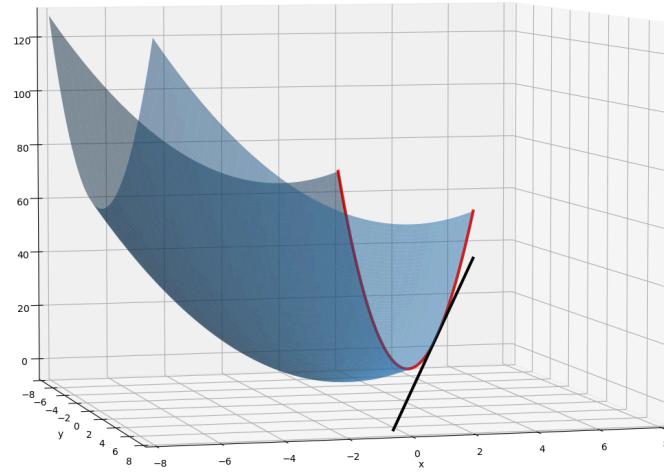
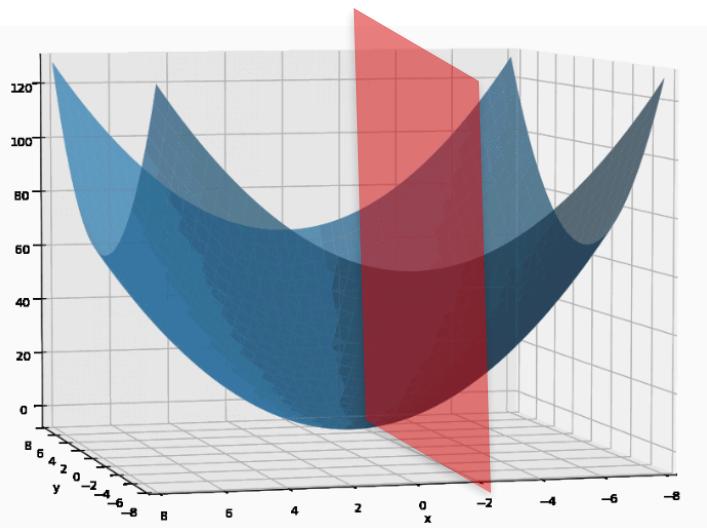
# Slicing the Space



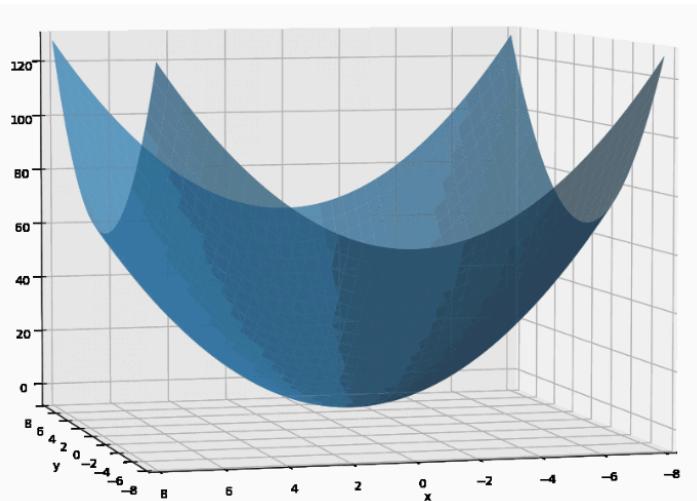
# Slicing the Space



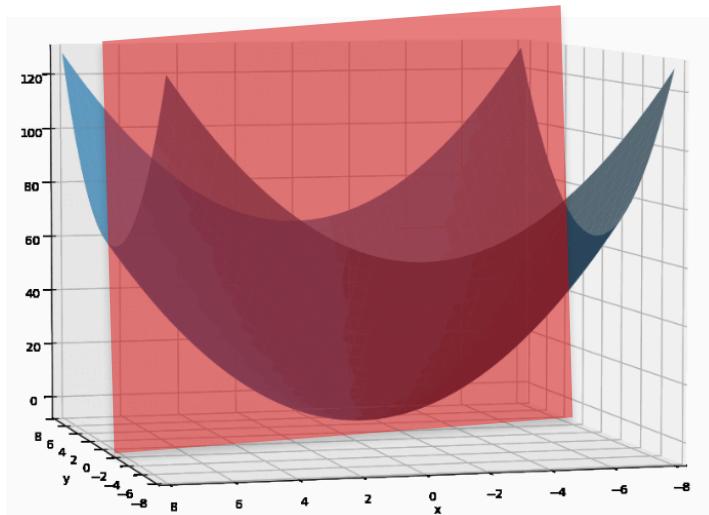
# Slicing the Space



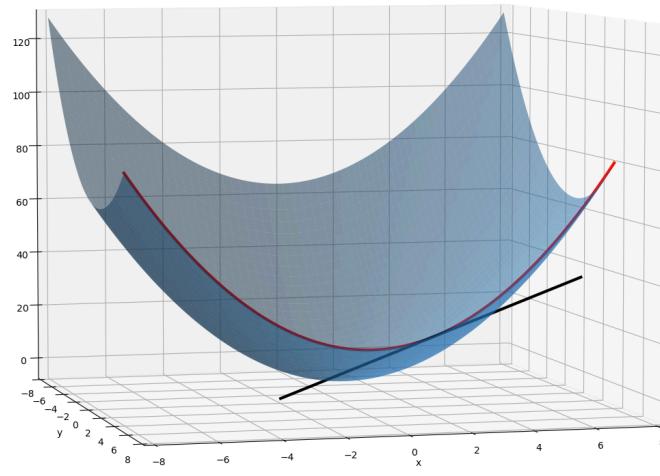
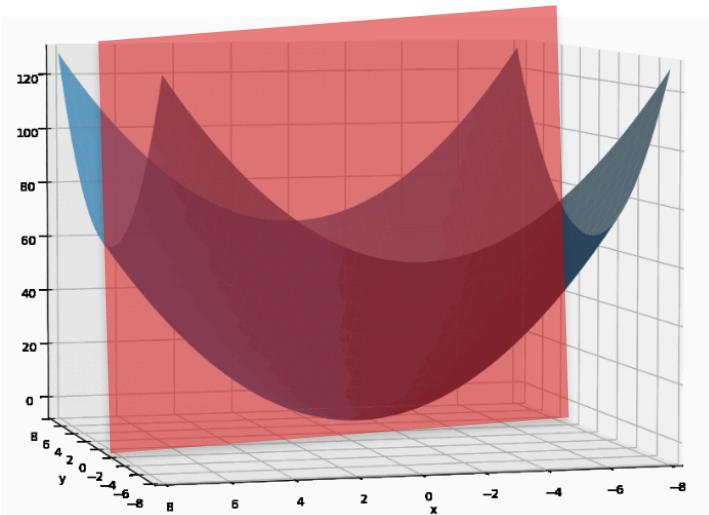
# Slicing the Space



# Slicing the Space

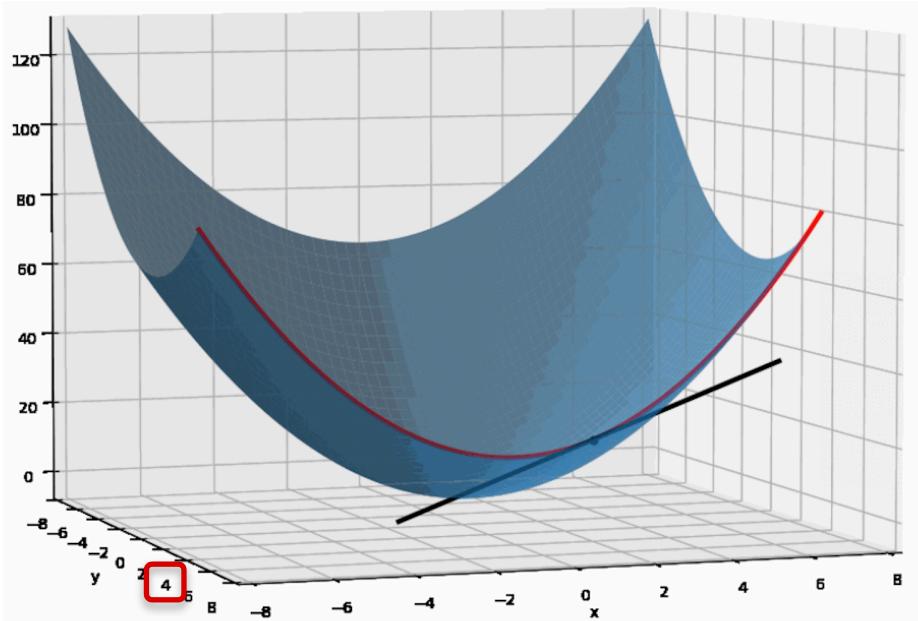


# Slicing the Space



# Partial Derivatives

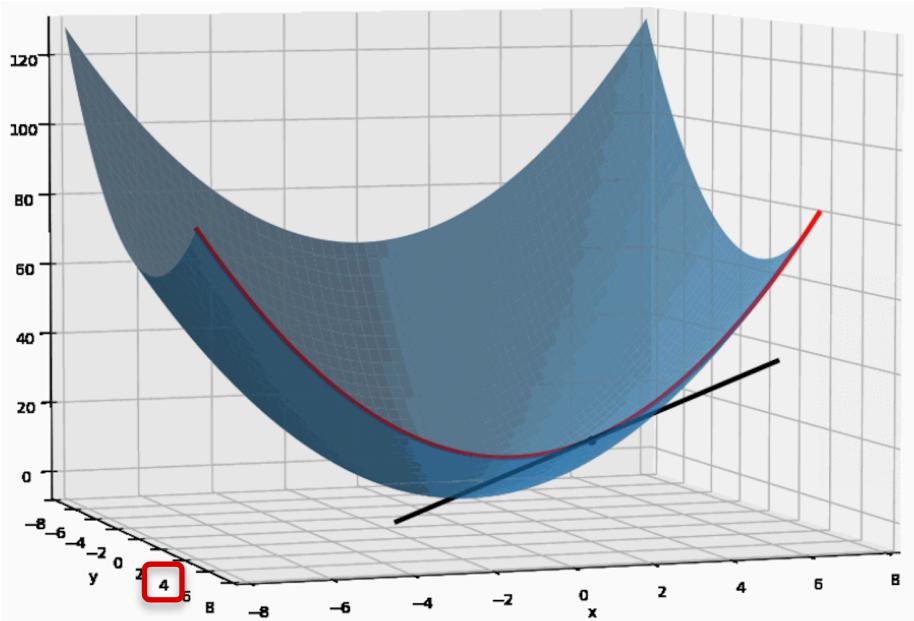
$$f(x, y) = x^2 + y^2$$



# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant

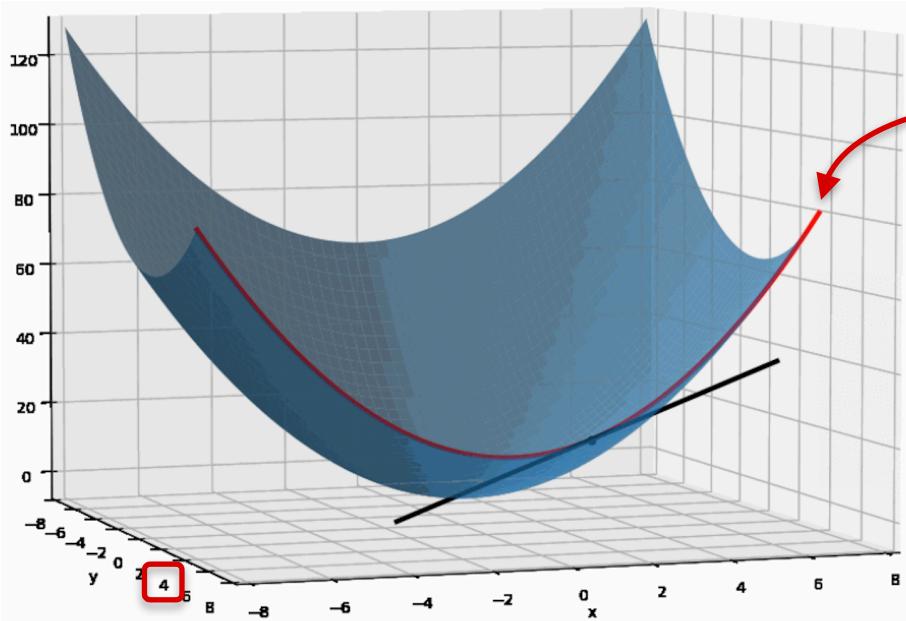


# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant

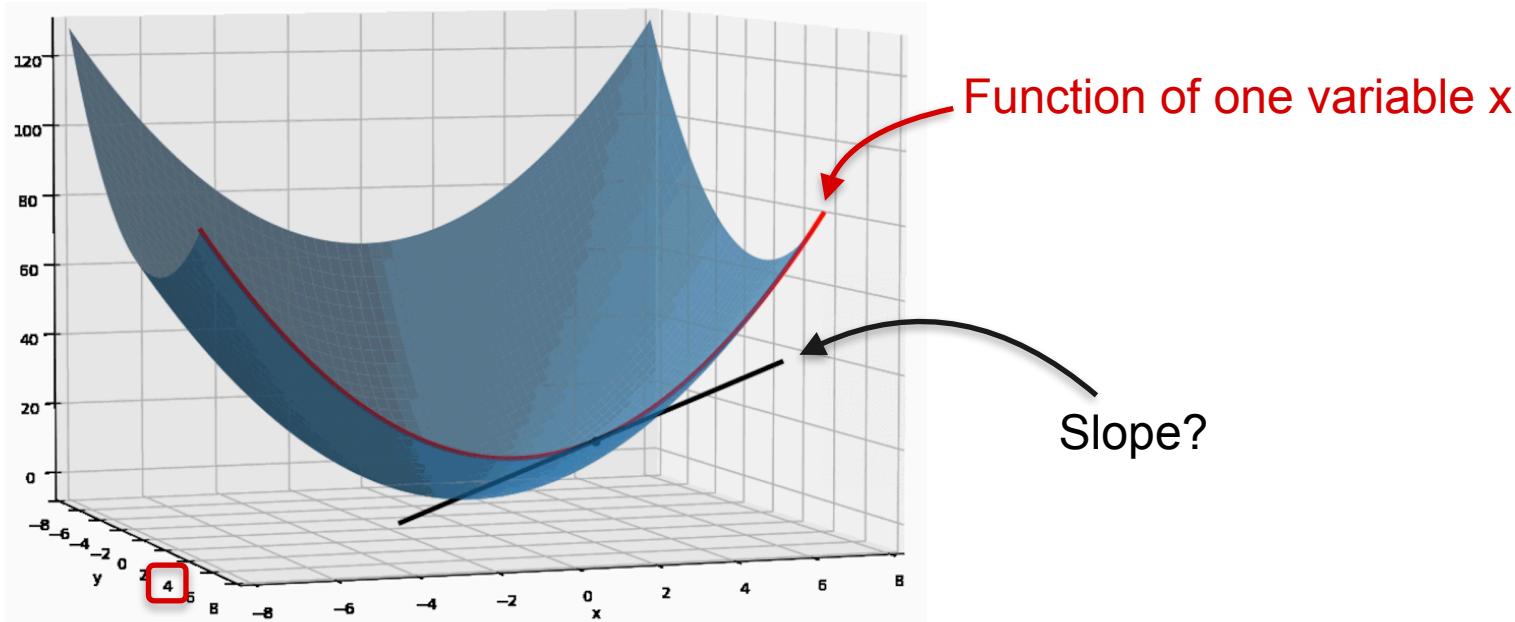
Function of one variable x



# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

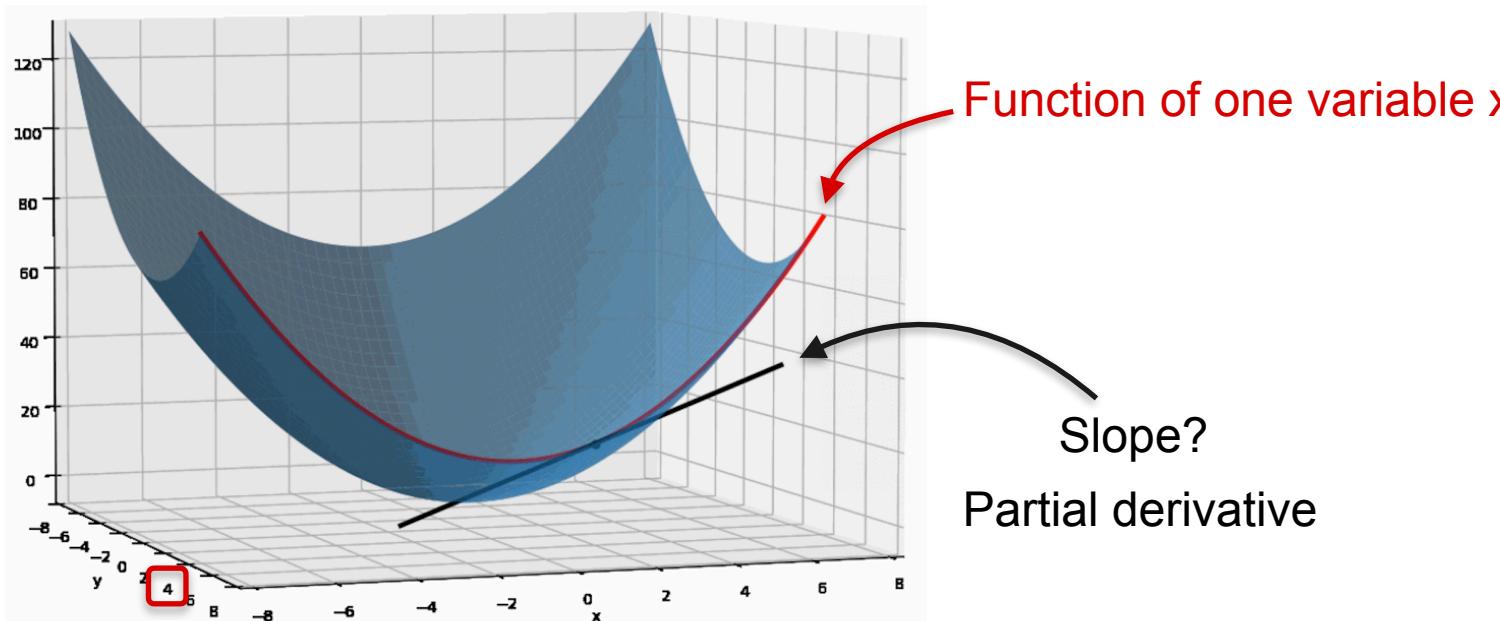
Treat y as a constant



# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

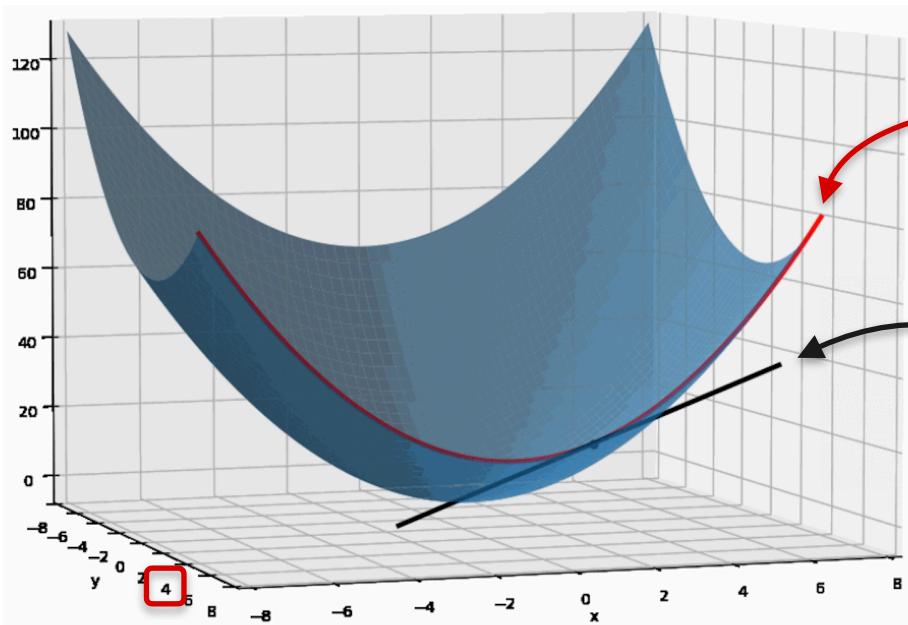
Treat y as a constant



# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

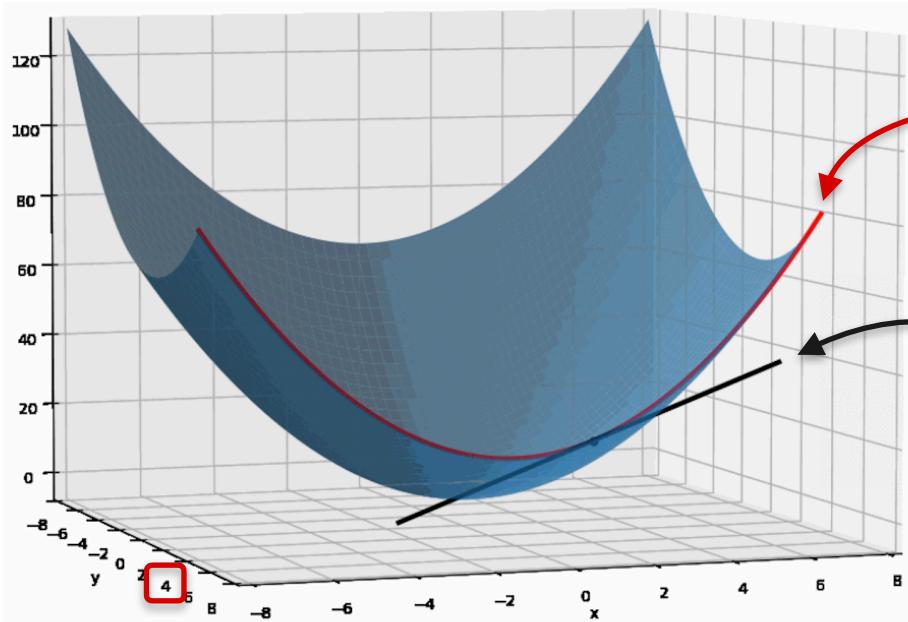
$$f(x, y) = x^2 + y^2$$

Slope?  
Partial derivative

# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Constant

$$f(x, y) = x^2 + \boxed{y^2}$$

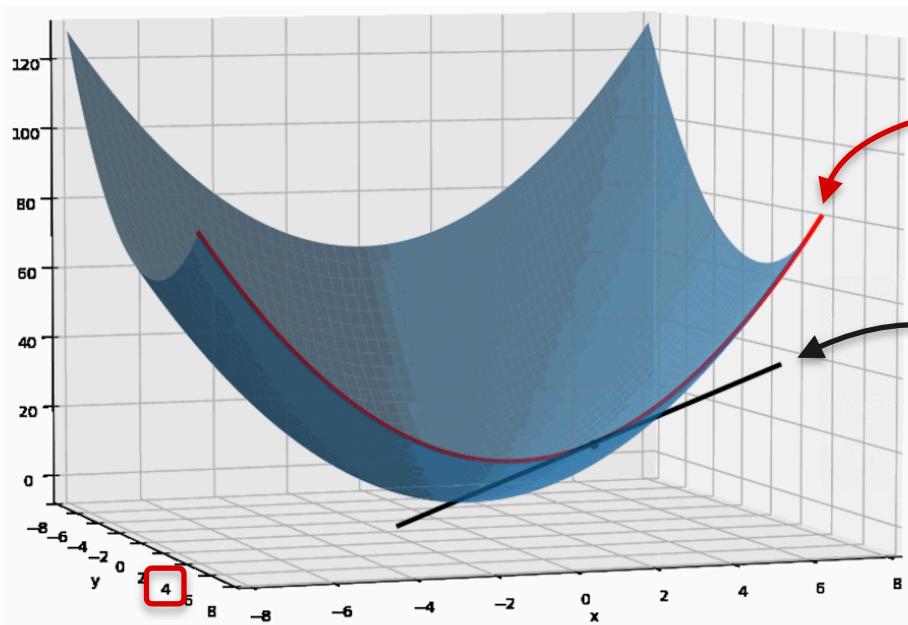
Slope?

Partial derivative

# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Slope?

Partial derivative

Constant

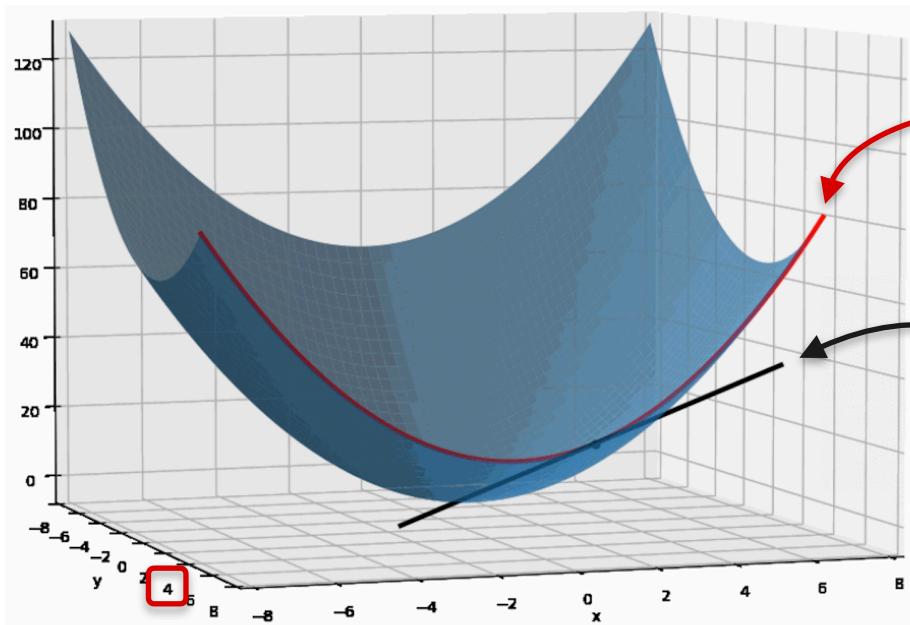
$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + 0$$

# Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Slope?

Partial derivative

Constant

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + \boxed{0}$$

Derivative = 0

# Partial Derivatives

# Partial Derivatives

$$x^2 + y^2$$

# Partial Derivatives

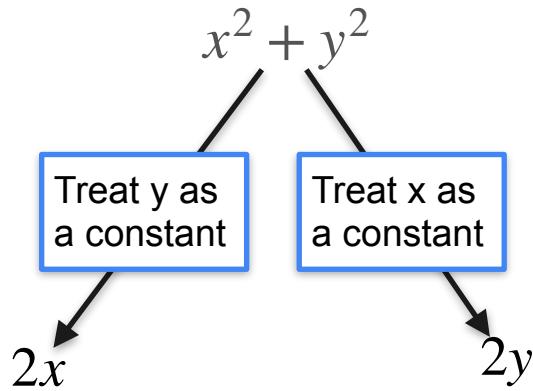
$$x^2 + y^2$$

Treat y as  
a constant

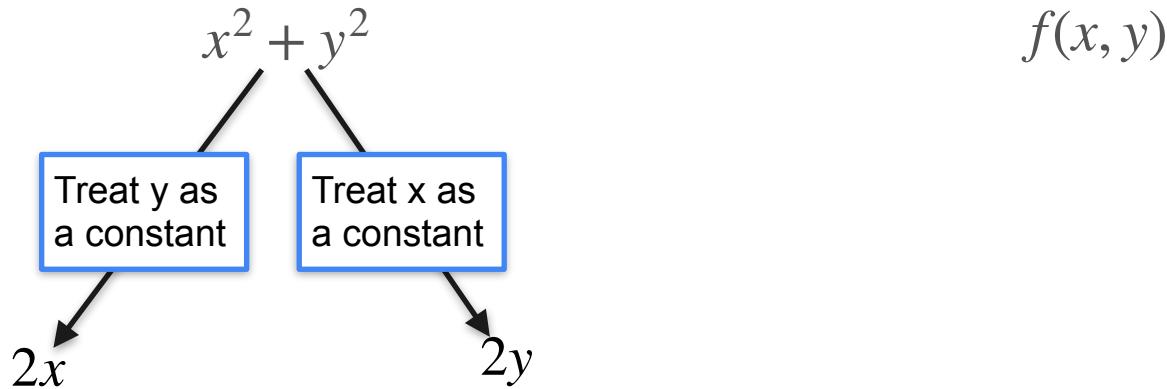
$$2x$$

The diagram illustrates the calculation of the partial derivative of the function  $x^2 + y^2$  with respect to  $x$ . A blue box contains the text "Treat y as a constant". An arrow points from this box to the term  $2x$ , indicating that since  $y$  is treated as a constant, only the term containing  $x$  is differentiated.

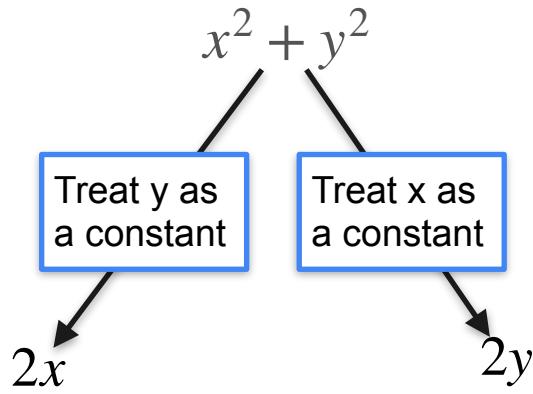
# Partial Derivatives



# Partial Derivatives

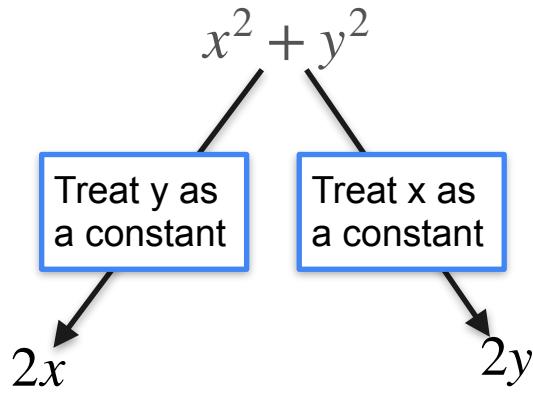


# Partial Derivatives



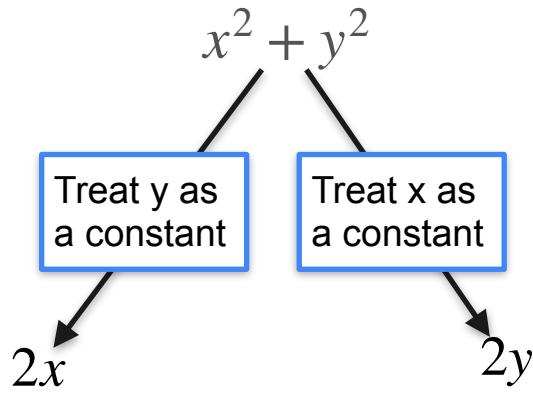
The diagram shows the formula for the partial derivative  $f_x$  of the function  $f(x, y)$ . The function  $f(x, y)$  is at the top, connected by an arrow to the formula  $f_x = \frac{\partial f}{\partial x}$ , which is at the bottom.

# Partial Derivatives



The diagram shows the general form of partial derivatives. At the top, the function  $f(x, y)$  is shown. Two arrows point downwards from this expression to two equations below it. The left equation is  $f_x = \frac{\partial f}{\partial x}$  and the right equation is  $f_y = \frac{\partial f}{\partial y}$ .

# Partial Derivatives

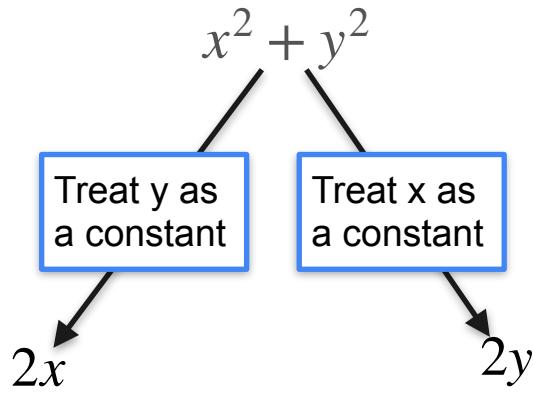


A diagram illustrating the general concept of partial derivatives. At the top, the function  $f(x, y)$  is shown. Two arrows point downwards from it to the partial derivative formulas  $f_x = \frac{\partial f}{\partial x}$  and  $f_y = \frac{\partial f}{\partial y}$ .

$$f_x = \frac{\partial f}{\partial x}$$
$$f_y = \frac{\partial f}{\partial y}$$

Partial derivative of  
 $f$  with respect to  $x$

# Partial Derivatives



A diagram illustrating the general concept of partial derivatives. At the top, the function  $f(x, y)$  is shown. Two arrows point downwards from it to the partial derivative formulas  $f_x = \frac{\partial f}{\partial x}$  and  $f_y = \frac{\partial f}{\partial y}$ , which are labeled as the "Partial derivative of  $f$  with respect to  $x$ " and "Partial derivative of  $f$  with respect to  $y$ ".

# Intro To Partial Derivatives

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

**TASK**

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} =$$

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} =$$

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

$$\frac{\partial f}{\partial y} =$$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

# Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

# Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivatives of  $f$  with respect to  $x$  and  $y$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

**TASK**

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

Step 1:

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:**

# Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$
  
$$\frac{\partial f}{\partial y} = ?$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$


## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$


## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$


## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$


## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

$$\frac{\partial f}{\partial x} = 2x$$

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

$$\frac{\partial f}{\partial y} =$$

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} = 2x$$

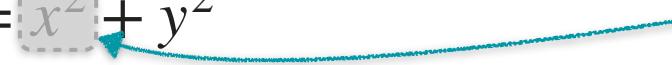
Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

$$\frac{\partial f}{\partial y} =$$

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = \boxed{x^2} + y^2$$


## TASK

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

$$\frac{\partial f}{\partial y} =$$

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

## TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

$$\frac{\partial f}{\partial y} = 2y$$

**Step 2:** Differentiate the function using the normal rules of differentiation.



DeepLearning.AI

# Gradients and Gradient Descent

---

## Partial derivatives -Part 2

# Partial Derivatives (More Examples)

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

## TASK

What is the partial derivate of  $f$  with respect to  $x$ ?

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = ?$$

## TASK

What is the partial derivate of  $f$  with respect to  $x$ ?

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

**TASK**

Find partial derivate of  $f$  with respect to  $x$

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

**TASK**

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$


$$\frac{\partial f}{\partial x} =$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} =$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} = 3$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to  $x$

## TASK

Find partial derivate of  $f$  with respect to  $x$

$$\frac{\partial f}{\partial x} = 3$$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to  $x$

## TASK

Find partial derivate of  $f$  with respect to  $x$

$$\frac{\partial f}{\partial x} = 3(2x)$$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3(2x)$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$



treat as constant coefficient

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$



treat as constant coefficient

## TASK

Find partial derivate of  $f$  with respect to  $x$

$$\frac{\partial f}{\partial x} = 3(2x)$$



**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

$$= 6xy^3$$

## TASK

Find partial derivate of  $f$  with respect to  $x$

**Step 1:** Treat all other variables as a constant. In our case  $y$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} = ?$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

**TASK**

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3 \text{ } \square y^3$$

$$\frac{\partial f}{\partial y} =$$

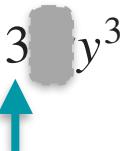
**TASK**

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$


$$\frac{\partial f}{\partial y} = 3$$

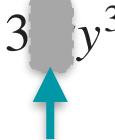
## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$


$$\frac{\partial f}{\partial y} = 3 \quad \text{[grey box]}$$

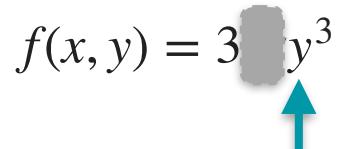
**TASK**

What is the partial derivative of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3 \quad y^3$$


$$\frac{\partial f}{\partial y} = 3 \quad (3y^2)$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$



$$\begin{aligned}\frac{\partial f}{\partial y} &= 3(x^2)(3y^2) \\ &= 9x^2y^2\end{aligned}$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.

# Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= 3(x^2)(3y^2) \\ &= 9x^2y^2\end{aligned}$$

## TASK

What is the partial derivate of  $f$  with respect to  $y$ ?

**Step 1:** Treat all other variables as a constant. In our case  $x$ .

**Step 2:** Differentiate the function using the normal rules of differentiation.



DeepLearning.AI

# Gradients and Gradient Descent

---

## Gradients

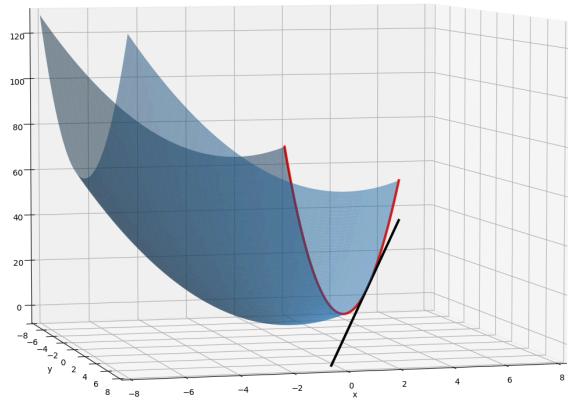
# Gradient

$$f(x, y) = x^2 + y^2$$

# Gradient

$$f(x, y) = x^2 + y^2$$

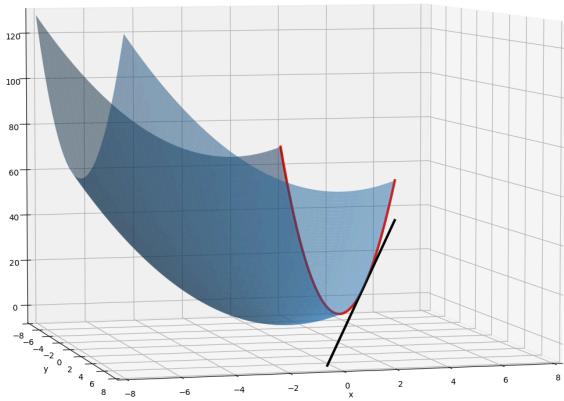
Treat y as a constant



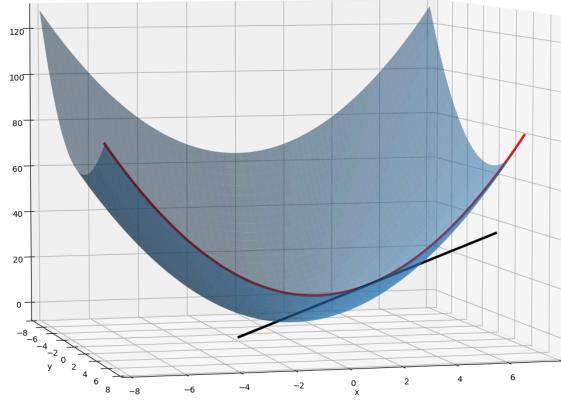
# Gradient

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



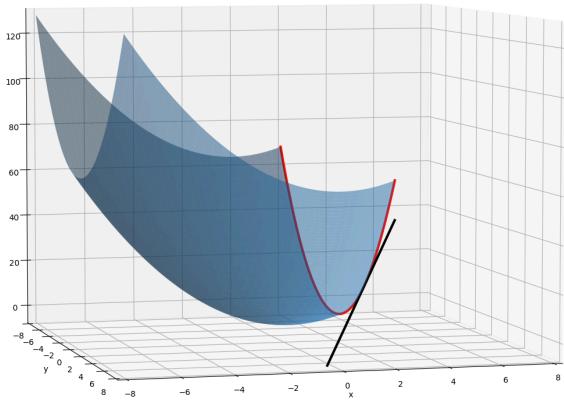
Treat x as a constant



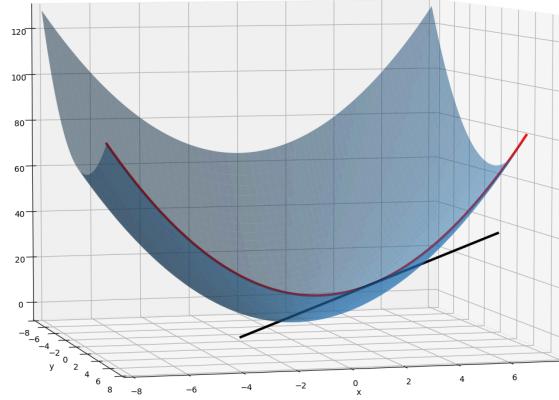
# Gradient

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Treat x as a constant

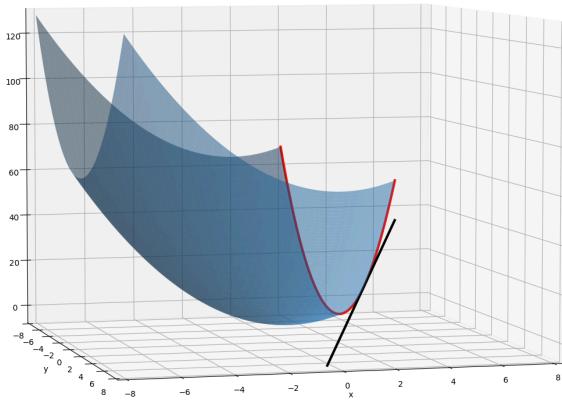


$$\frac{\partial f}{\partial x} = 2x$$

# Gradient

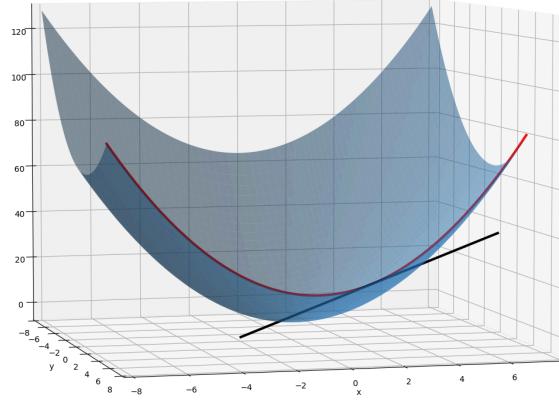
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



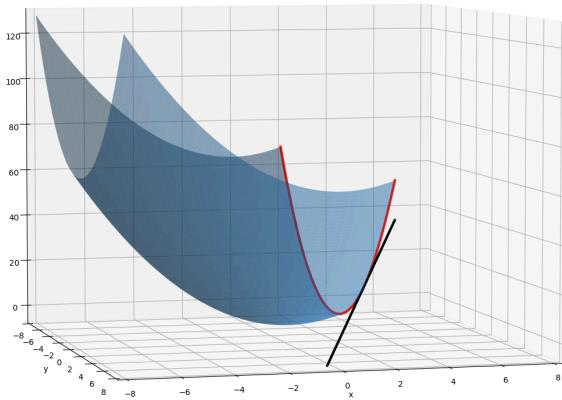
$$\frac{\partial f}{\partial y} = 2y$$

# Gradient

$$f(x, y) = x^2 + y^2$$

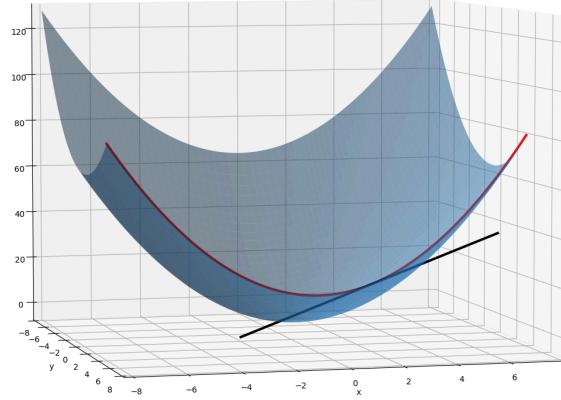
Gradient

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant

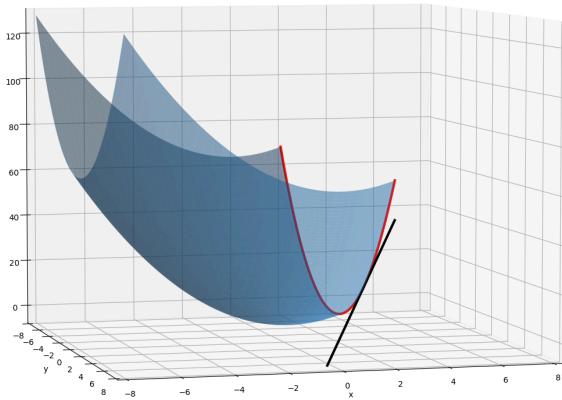


$$\frac{\partial f}{\partial y} = 2y$$

# Gradient

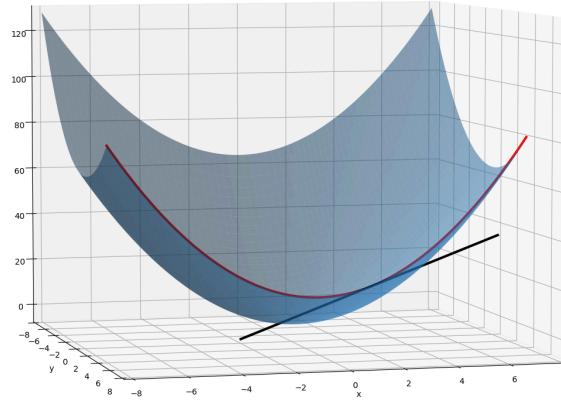
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



$$\frac{\partial f}{\partial y} = 2y$$

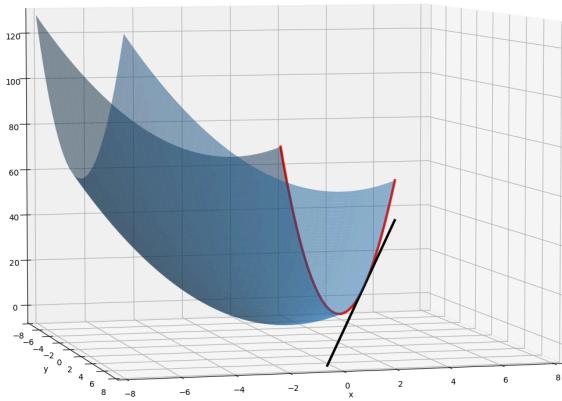
Gradient

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

# Gradient

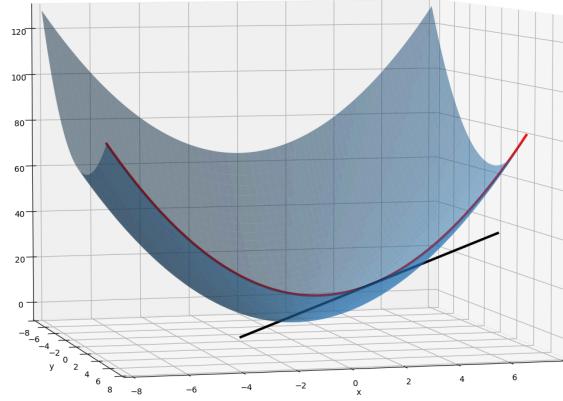
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



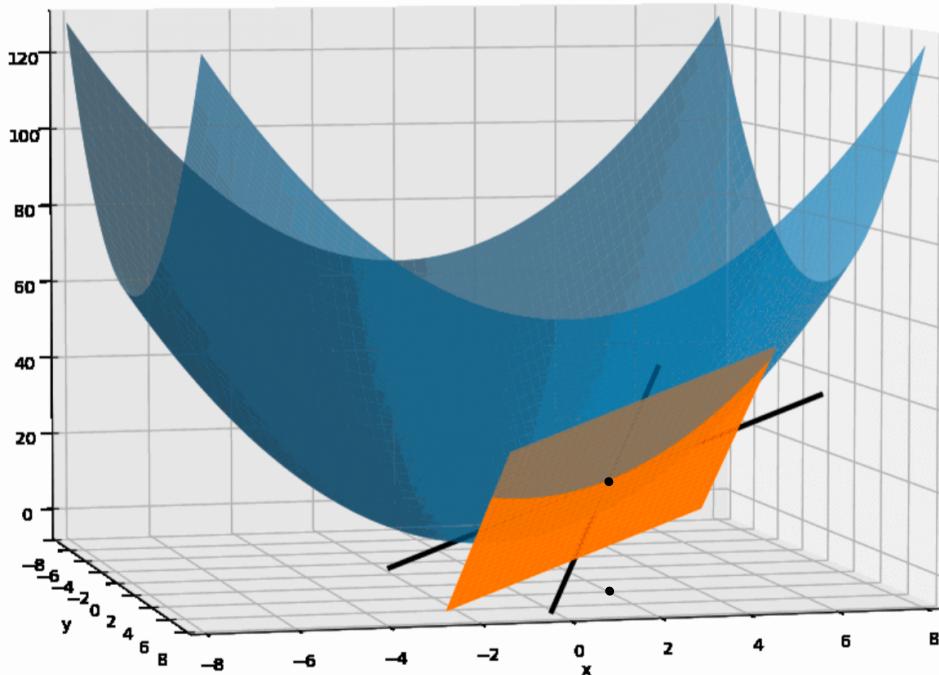
$$\frac{\partial f}{\partial y} = 2y$$

Gradient

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

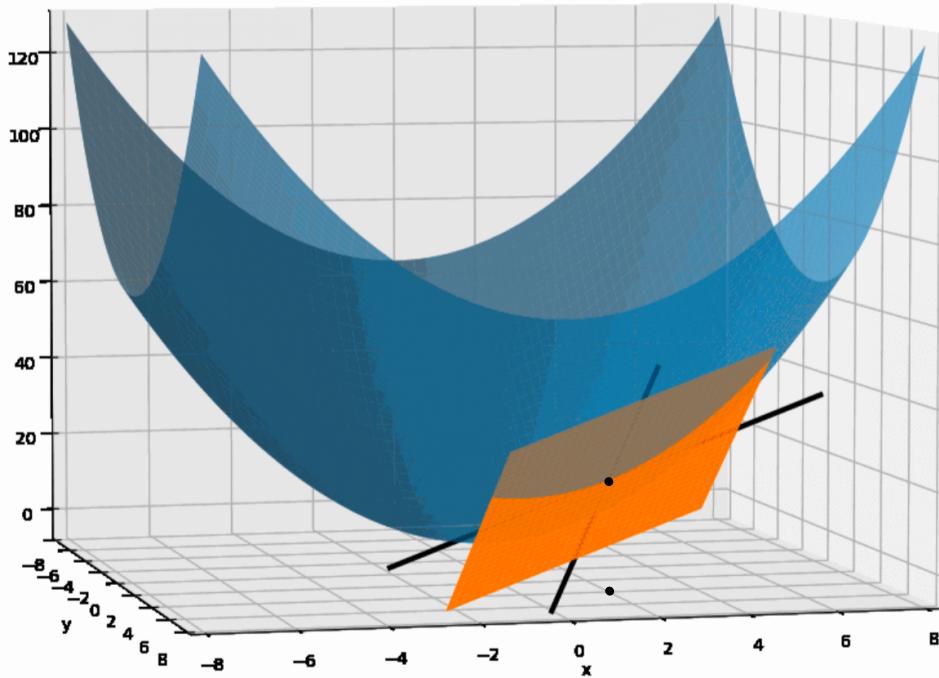
# Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of  $f(x, y)$  is:  $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

# Gradient



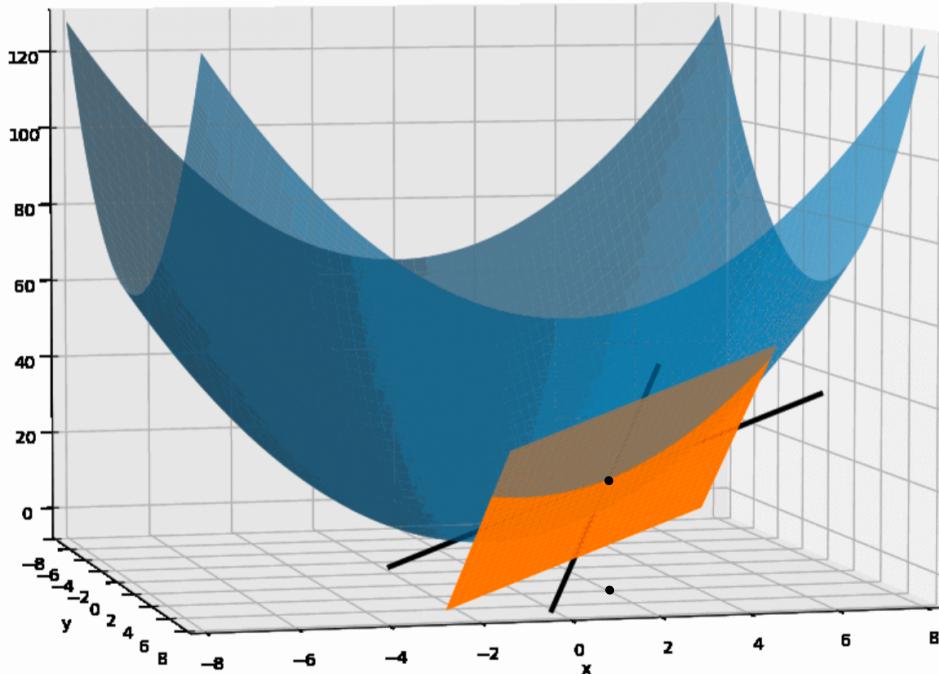
$$f(x, y) = x^2 + y^2$$

The gradient of  $f(x, y)$  is:  $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

## TASK

Find the gradient of  $f(x, y)$  at  $(2, 3)$

# Gradient



$$f(x, y) = x^2 + y^2$$

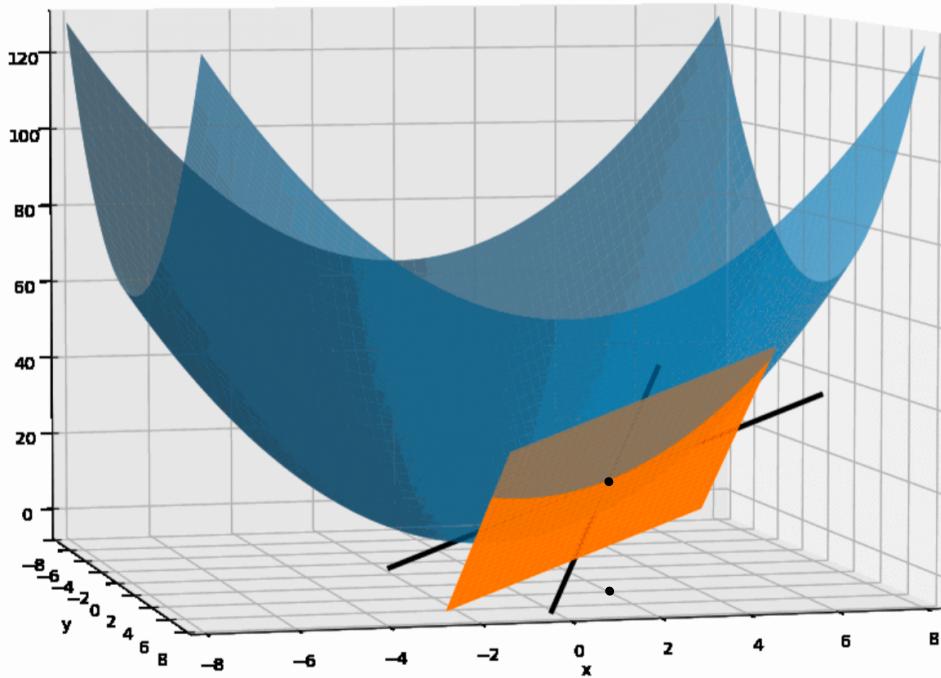
The gradient of  $f(x, y)$  is:  $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

## TASK

Find the gradient of  $f(x, y)$  at  $(2, 3)$

The gradient of  $f(x, y)$  is given as:

# Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of  $f(x, y)$  is:  $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

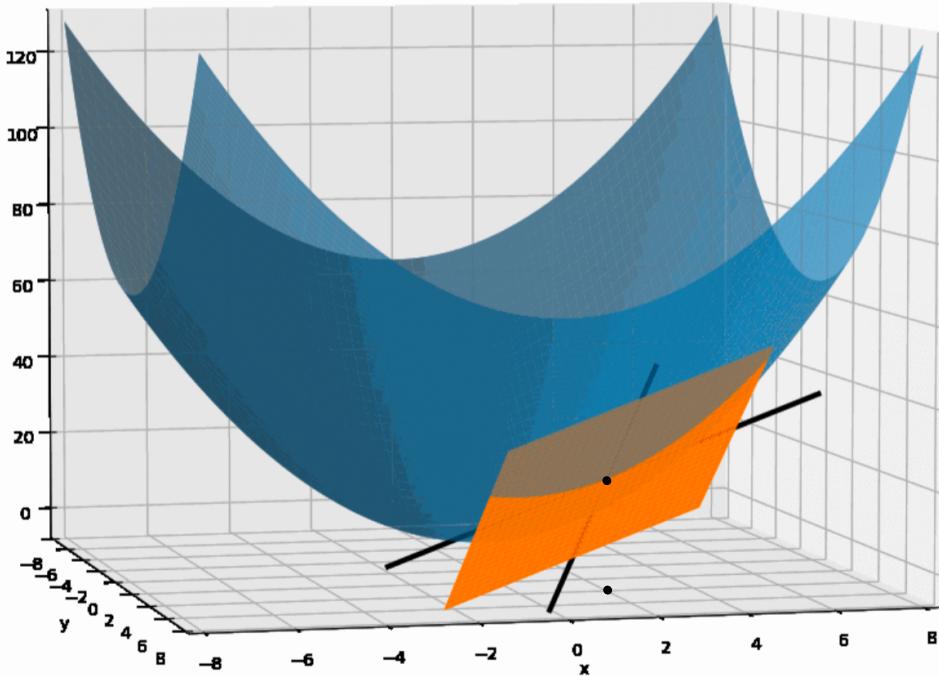
## TASK

Find the gradient of  $f(x, y)$  at  $(2, 3)$

The gradient of  $f(x, y)$  is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix}$$

# Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of  $f(x, y)$  is:  $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

## TASK

Find the gradient of  $f(x, y)$  at  $(2, 3)$

The gradient of  $f(x, y)$  is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$



DeepLearning.AI

# Gradients and Gradient Descent

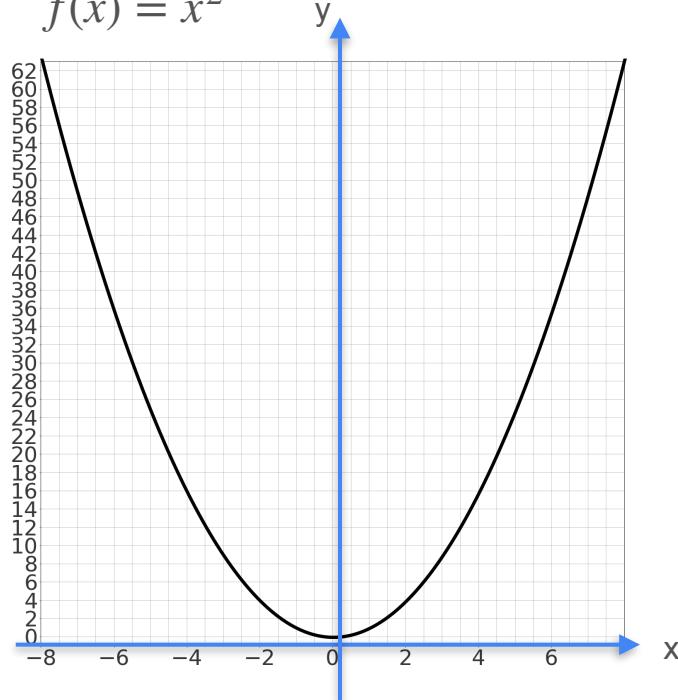
---

**Gradients and maxima/  
minima**

# Functions of Two Variables

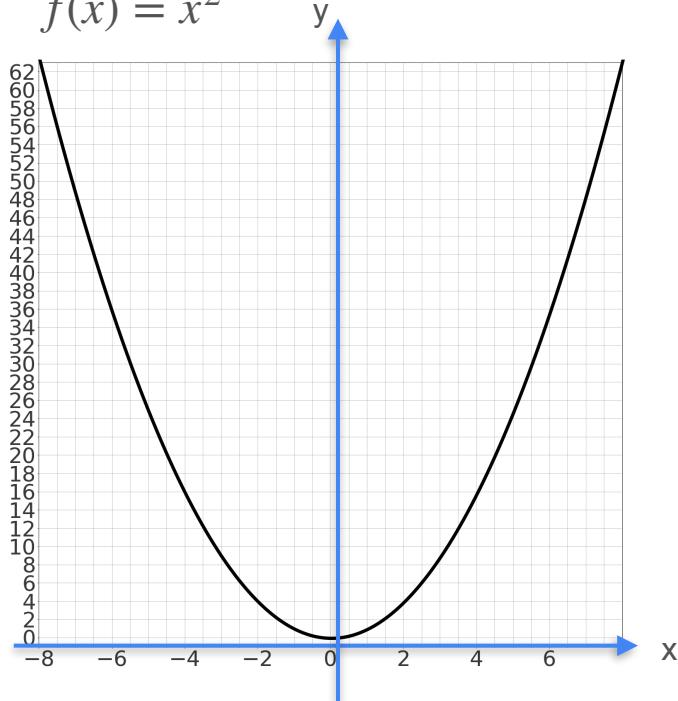
# Functions of Two Variables

$$f(x) = x^2$$

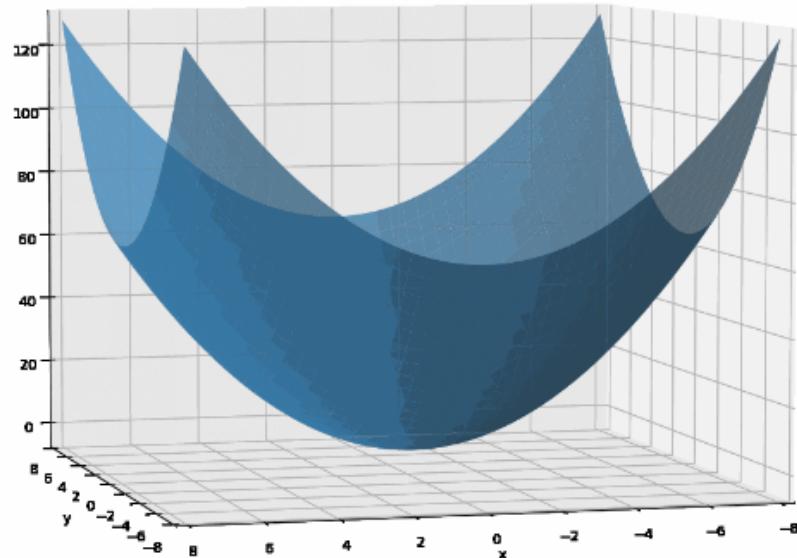


# Functions of Two Variables

$$f(x) = x^2$$

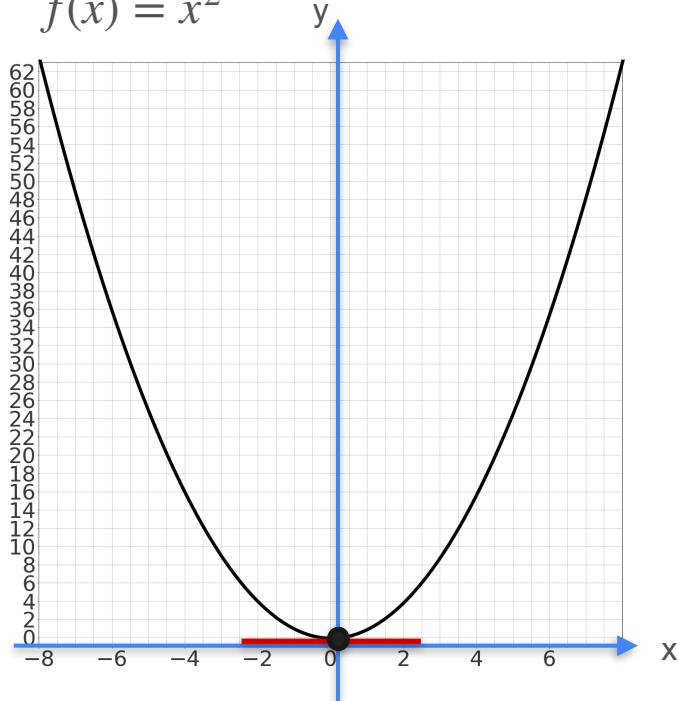


$$f(x, y) = x^2 + y^2$$

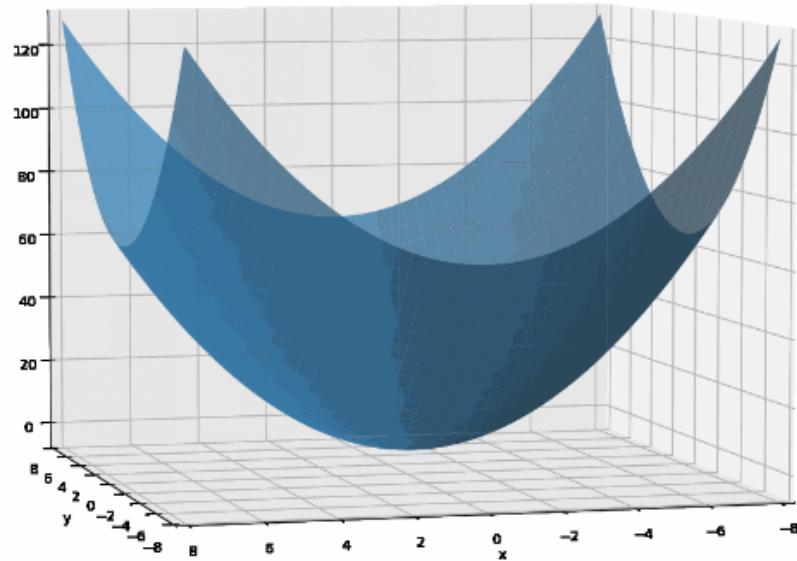


# Functions of Two Variables

$$f(x) = x^2$$

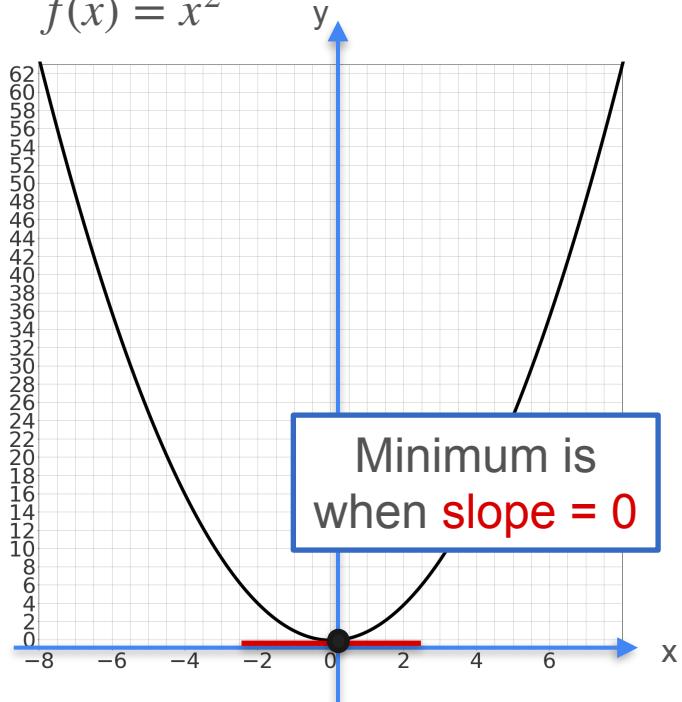


$$f(x, y) = x^2 + y^2$$

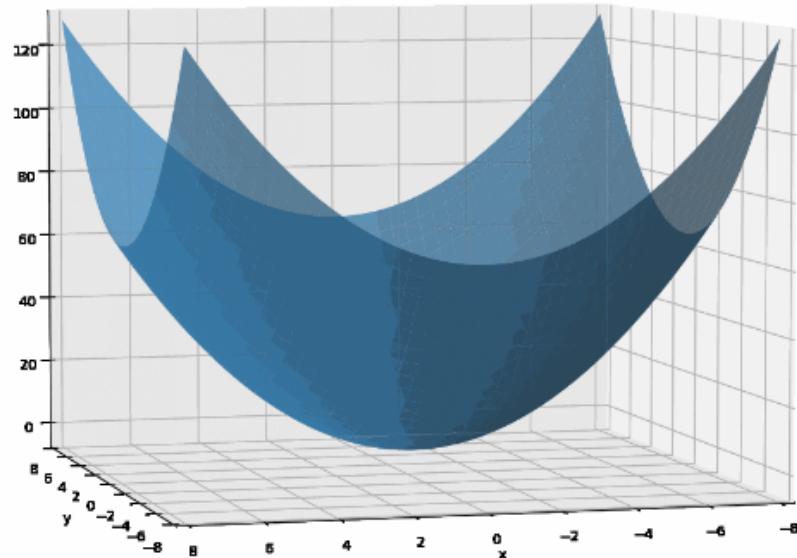


# Functions of Two Variables

$$f(x) = x^2$$

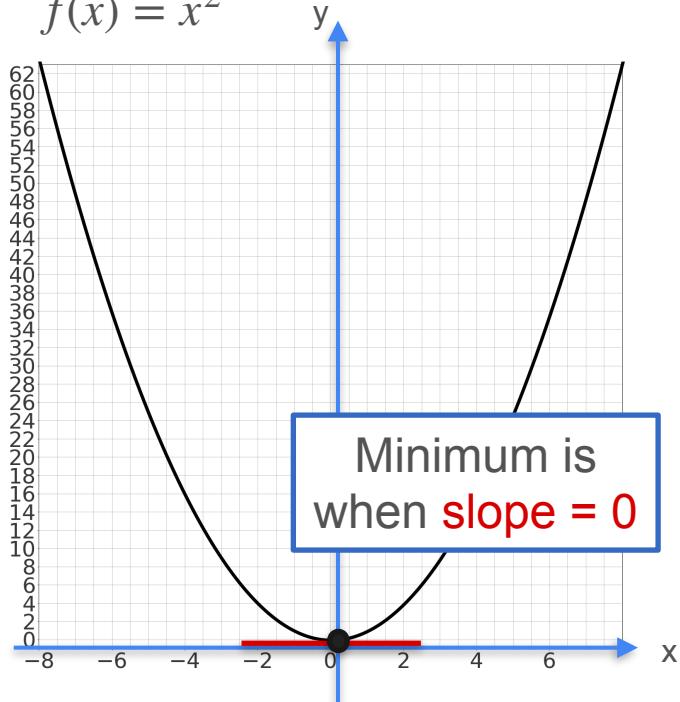


$$f(x, y) = x^2 + y^2$$

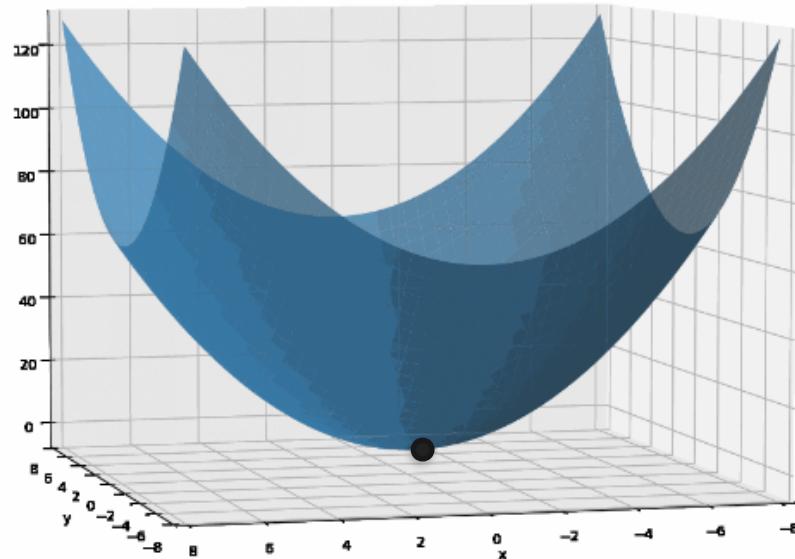


# Functions of Two Variables

$$f(x) = x^2$$

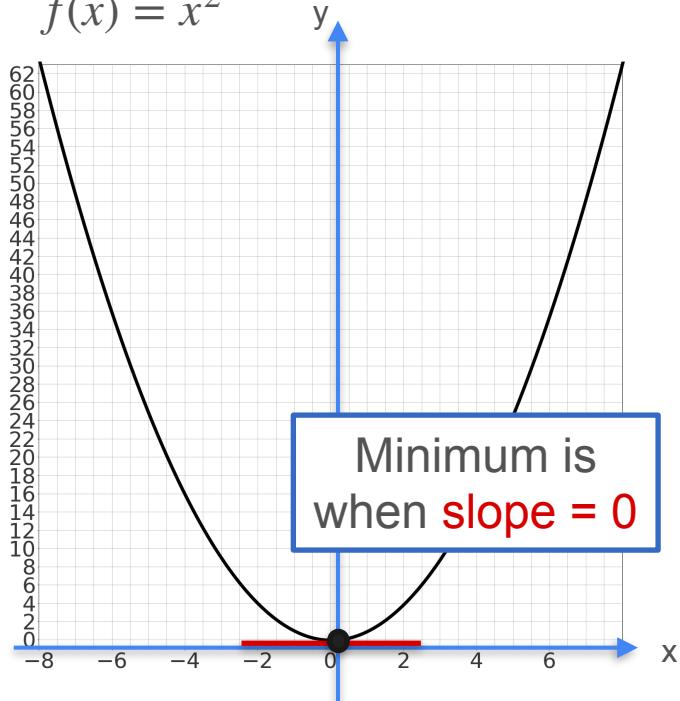


$$f(x, y) = x^2 + y^2$$

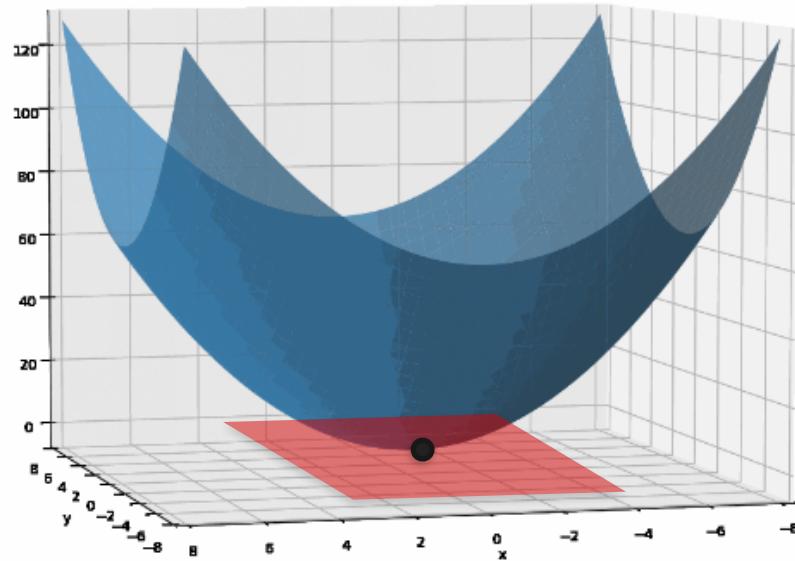


# Functions of Two Variables

$$f(x) = x^2$$

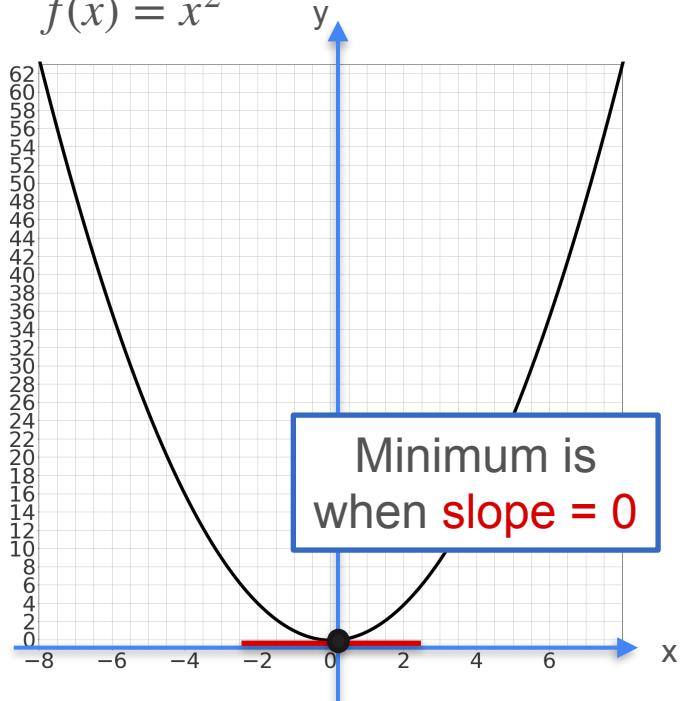


$$f(x, y) = x^2 + y^2$$

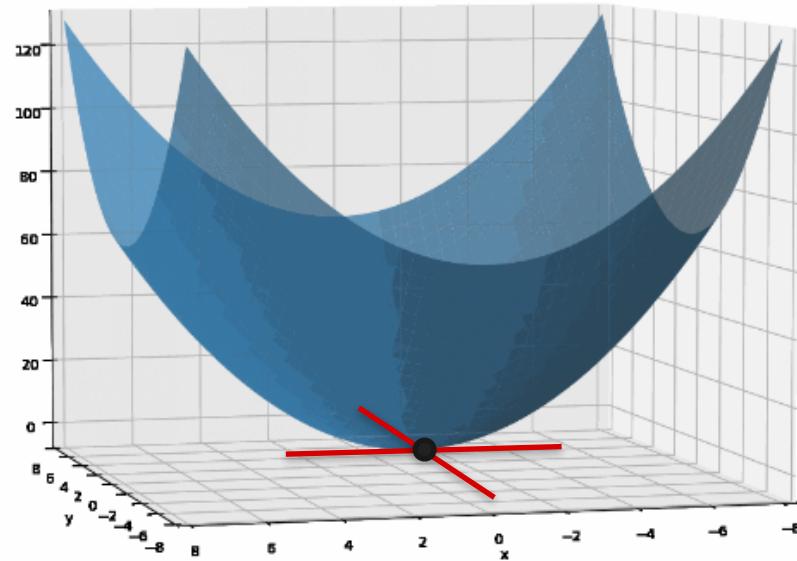


# Functions of Two Variables

$$f(x) = x^2$$

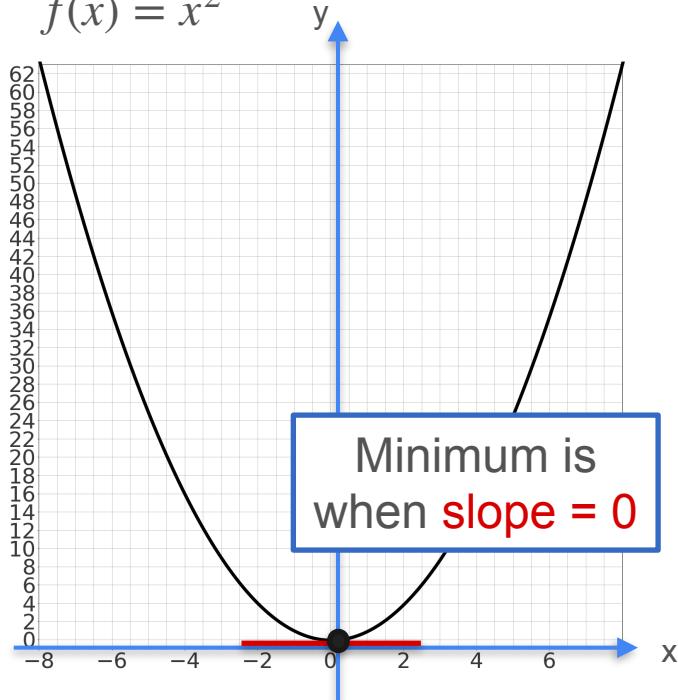


$$f(x, y) = x^2 + y^2$$

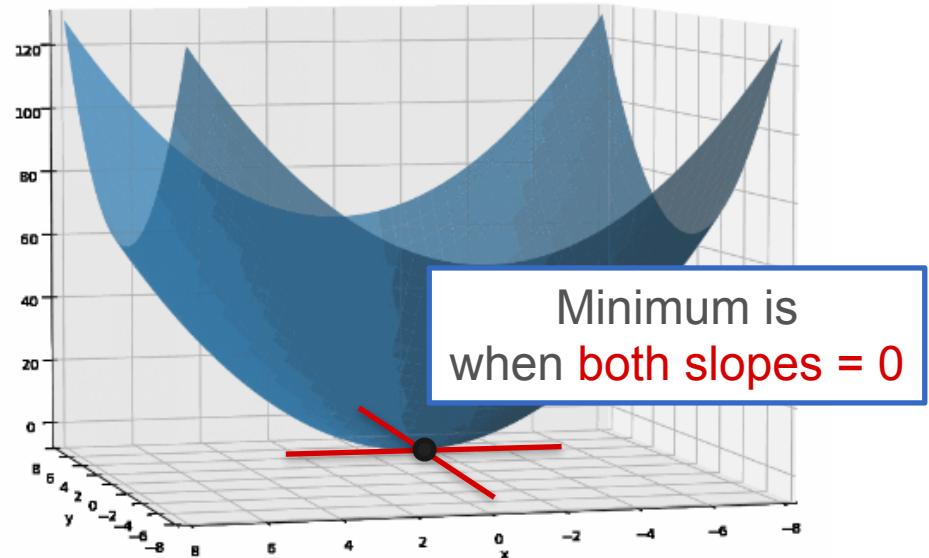


# Functions of Two Variables

$$f(x) = x^2$$

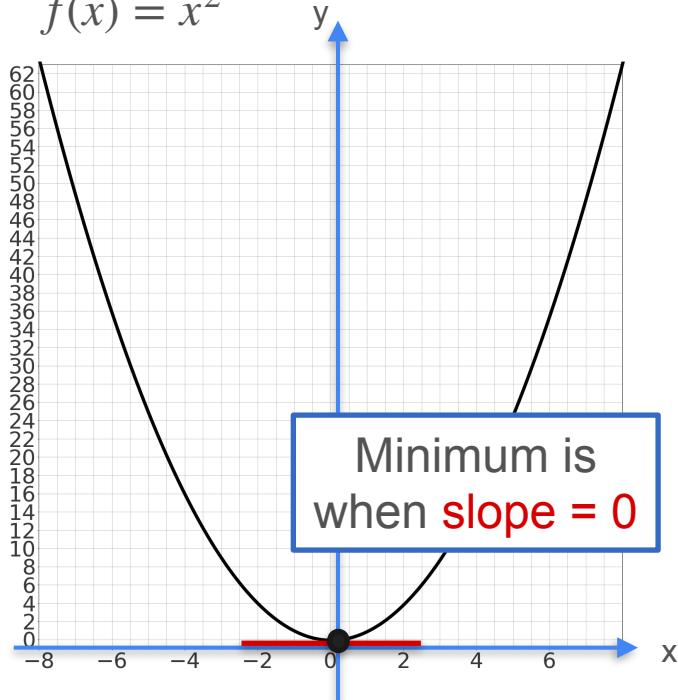


$$f(x, y) = x^2 + y^2$$

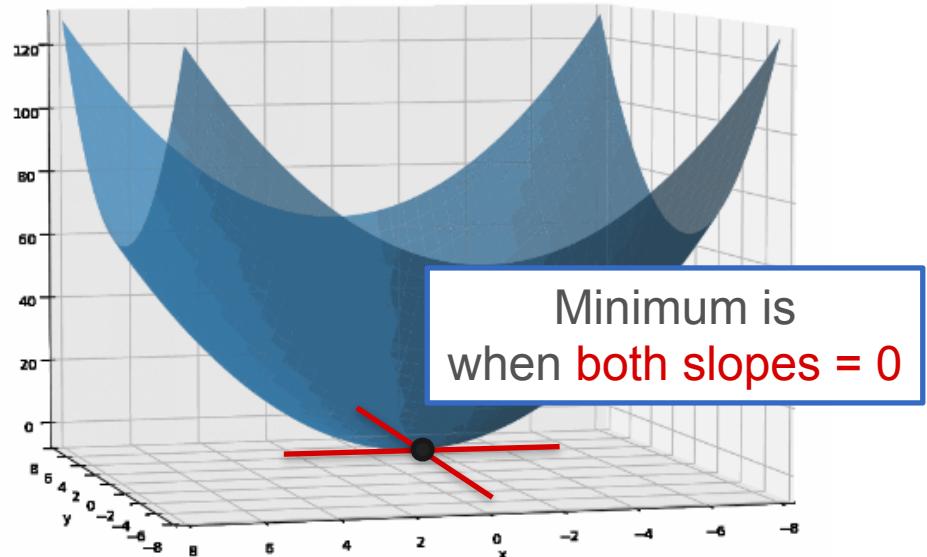


# Functions of Two Variables

$$f(x) = x^2$$

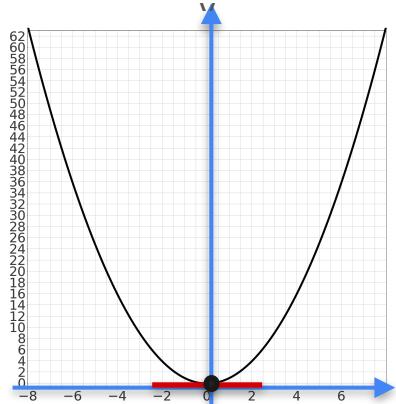


$$f(x, y) = x^2 + y^2$$



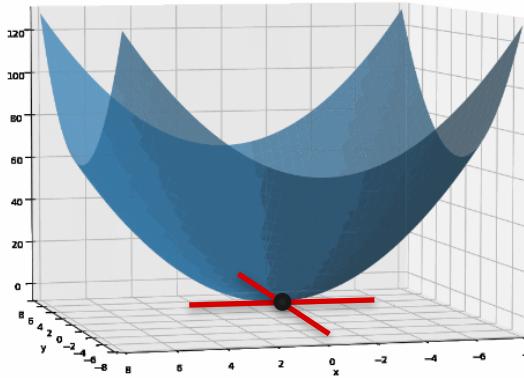
# Functions of Two Variables

$$f(x) = x^2$$



Minimum is  
when **slope = 0**

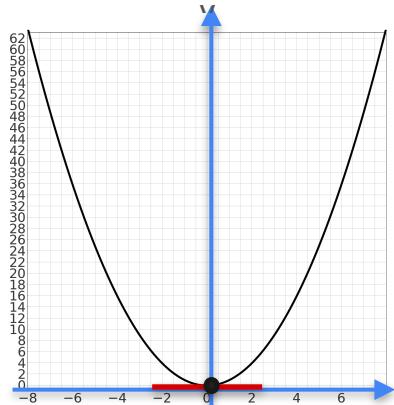
$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

# Functions of Two Variables

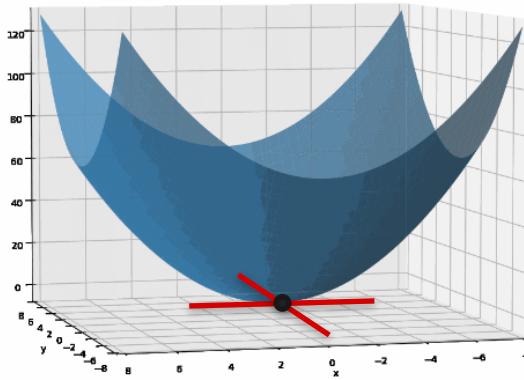
$$f(x) = x^2$$



Minimum is  
when **slope = 0**

$$f'(x) = 0$$

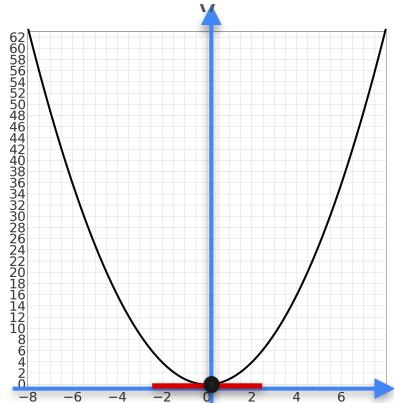
$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

# Functions of Two Variables

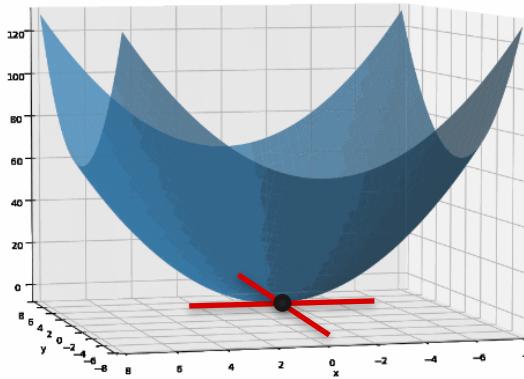
$$f(x) = x^2$$



Minimum is  
when **slope = 0**

$$\begin{aligned}f'(x) &= 0 \\2x &= 0\end{aligned}$$

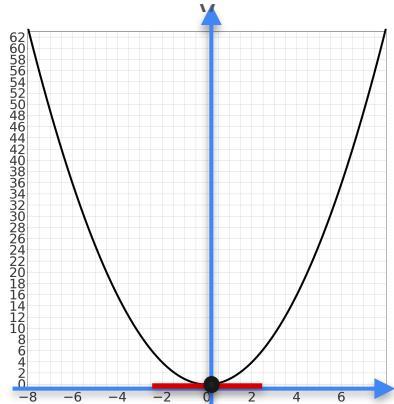
$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

# Functions of Two Variables

$$f(x) = x^2$$



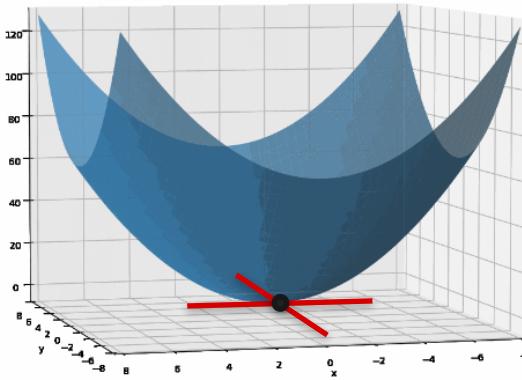
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

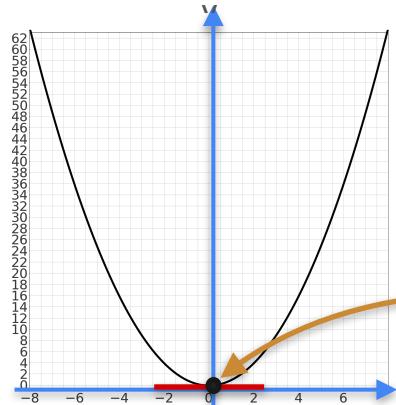
$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

# Functions of Two Variables

$$f(x) = x^2$$



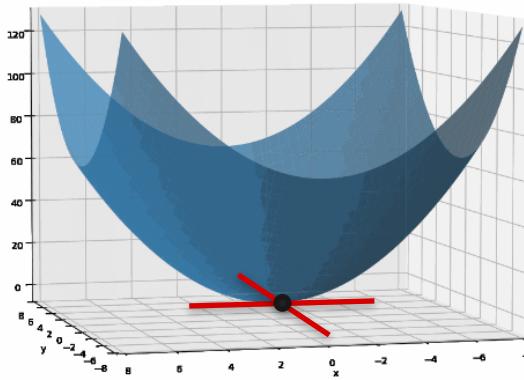
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

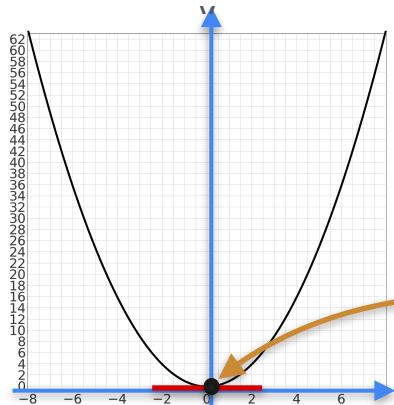
$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

# Functions of Two Variables

$$f(x) = x^2$$



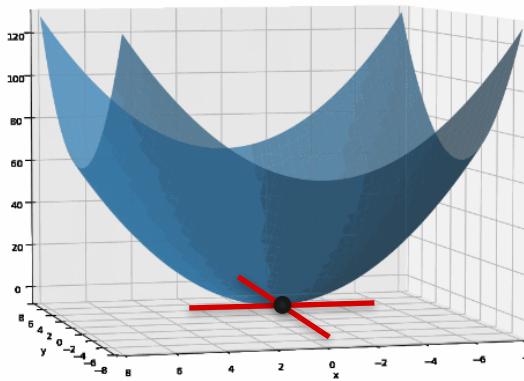
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$

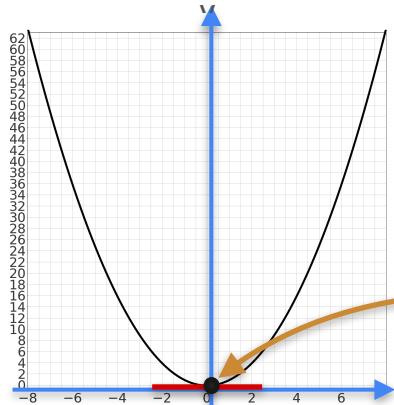


Minimum is  
when **both slopes = 0**

$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

# Functions of Two Variables

$$f(x) = x^2$$



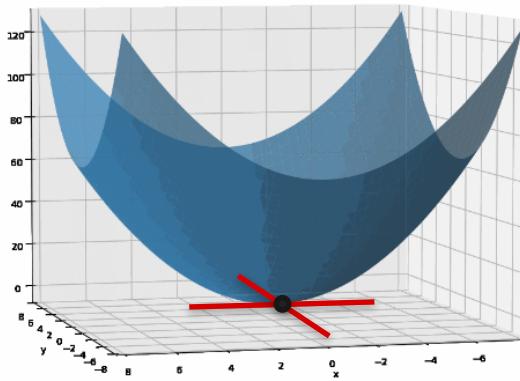
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$



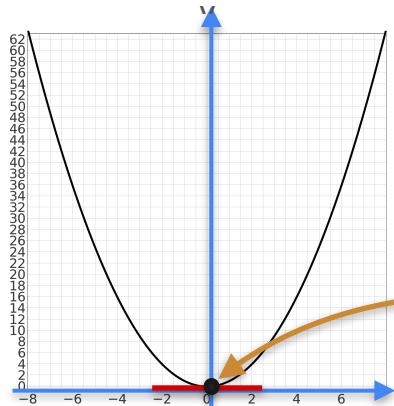
$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \text{ and } 2y = 0$$

Minimum is  
when **both slopes = 0**

# Functions of Two Variables

$$f(x) = x^2$$



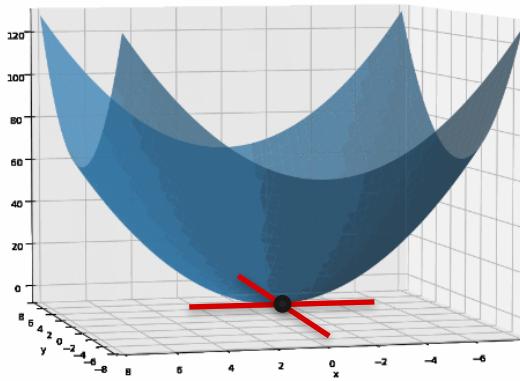
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$



$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

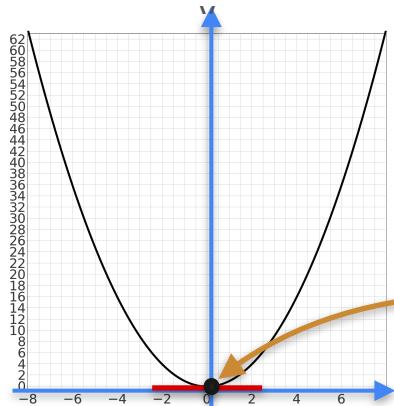
$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$

Minimum is  
when **both slopes = 0**

# Functions of Two Variables

$$f(x) = x^2$$



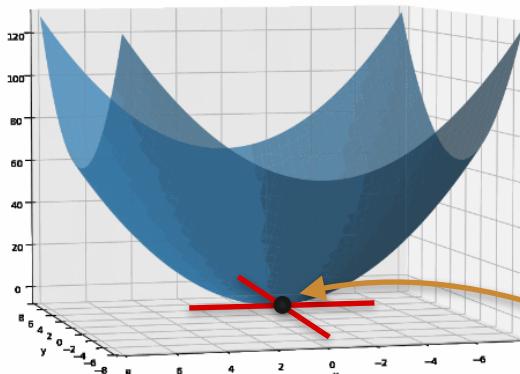
Minimum is  
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$



Minimum is  
when **both slopes = 0**

$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$



DeepLearning.AI

# Gradients and Gradient Descent

---

**Optimization with gradients:  
An example**

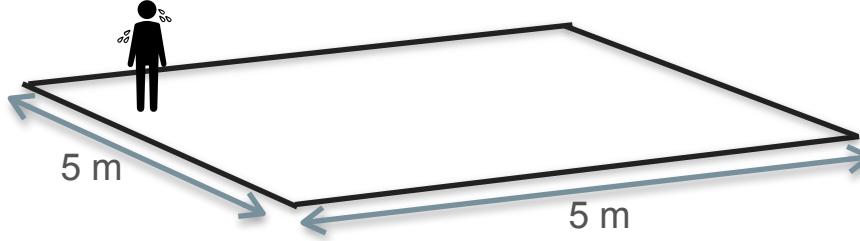
# Motivation for Optimization in Two Variables



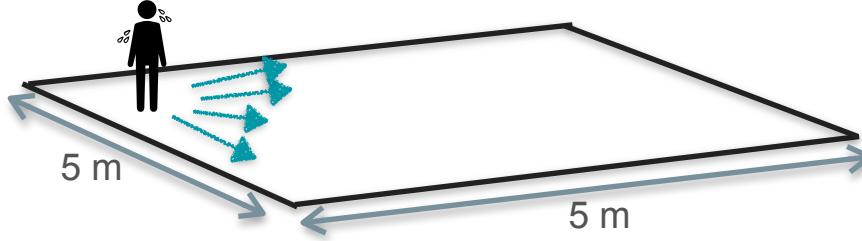
# Motivation for Optimization in Two Variables



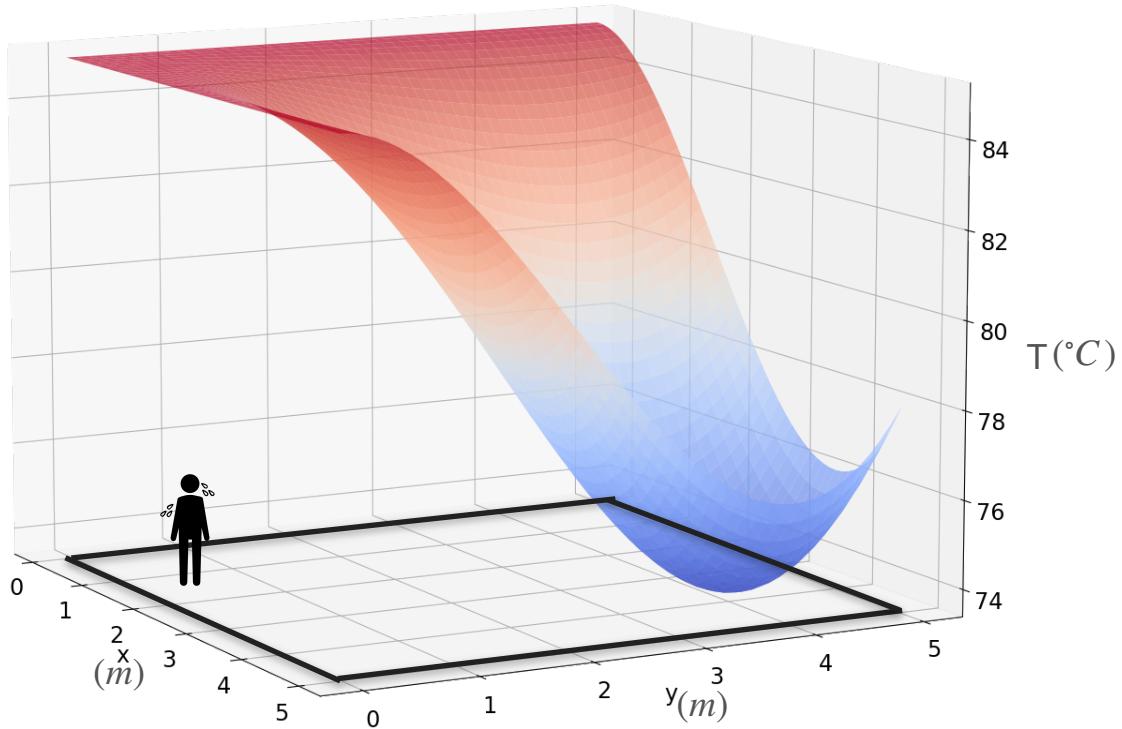
# Motivation for Optimization in Two Variables



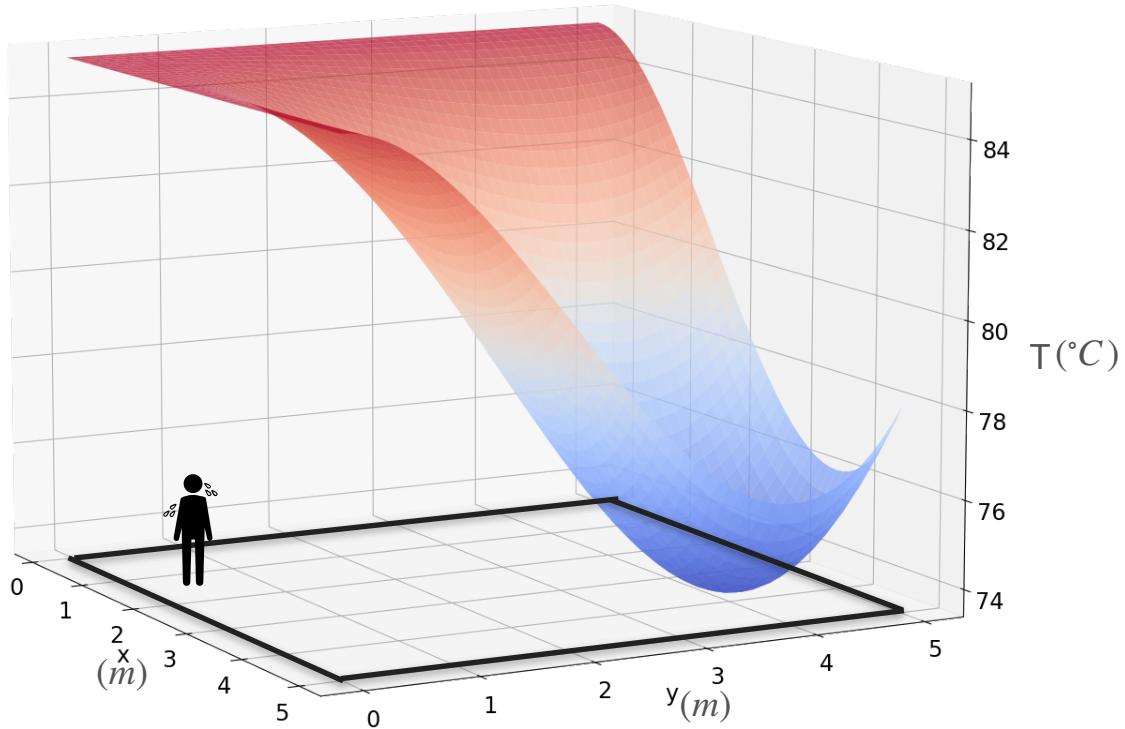
# Motivation for Optimization in Two Variables



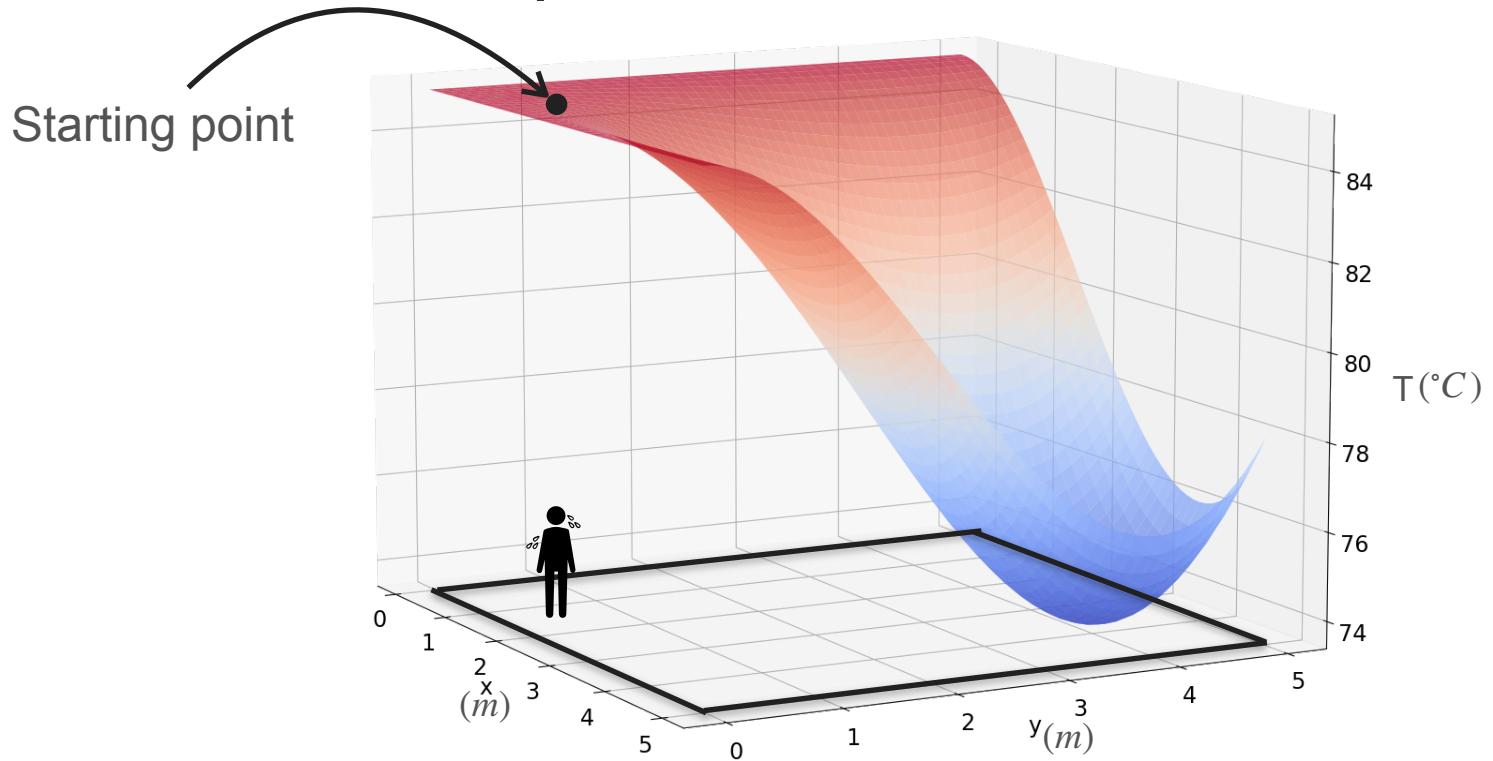
# Motivation for Optimization in Two Variables



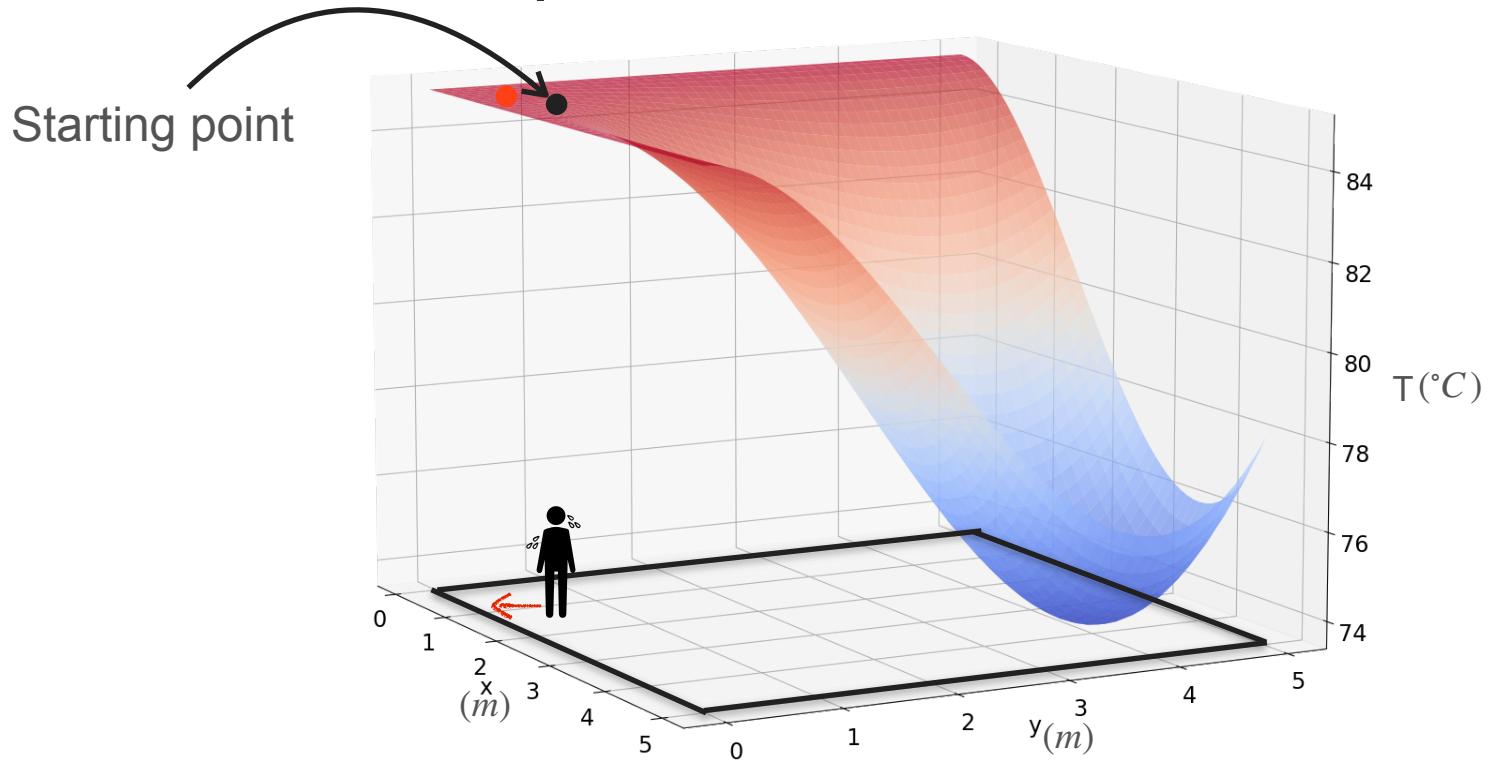
# Motivation for Optimization in Two Variables



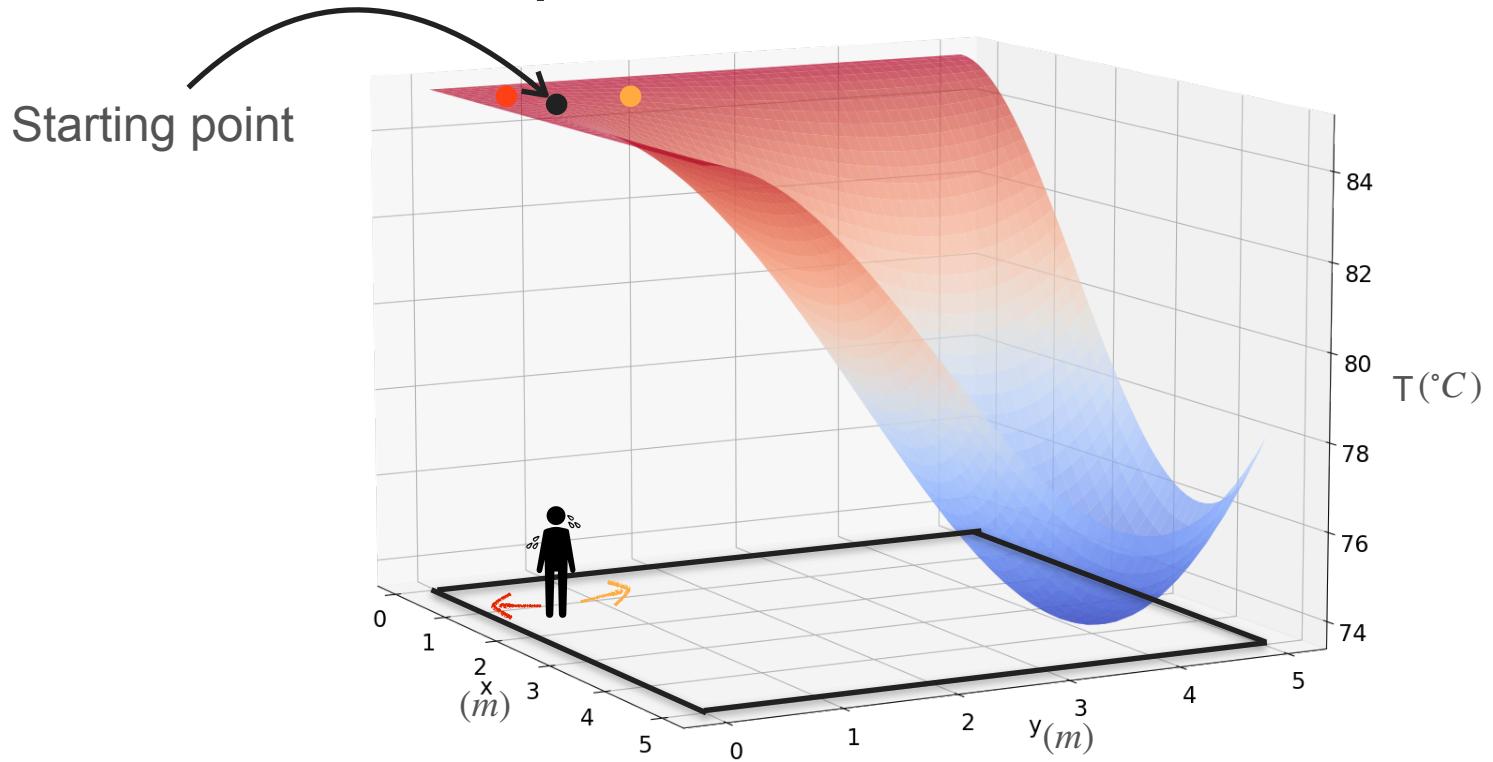
# Motivation for Optimization in Two Variables



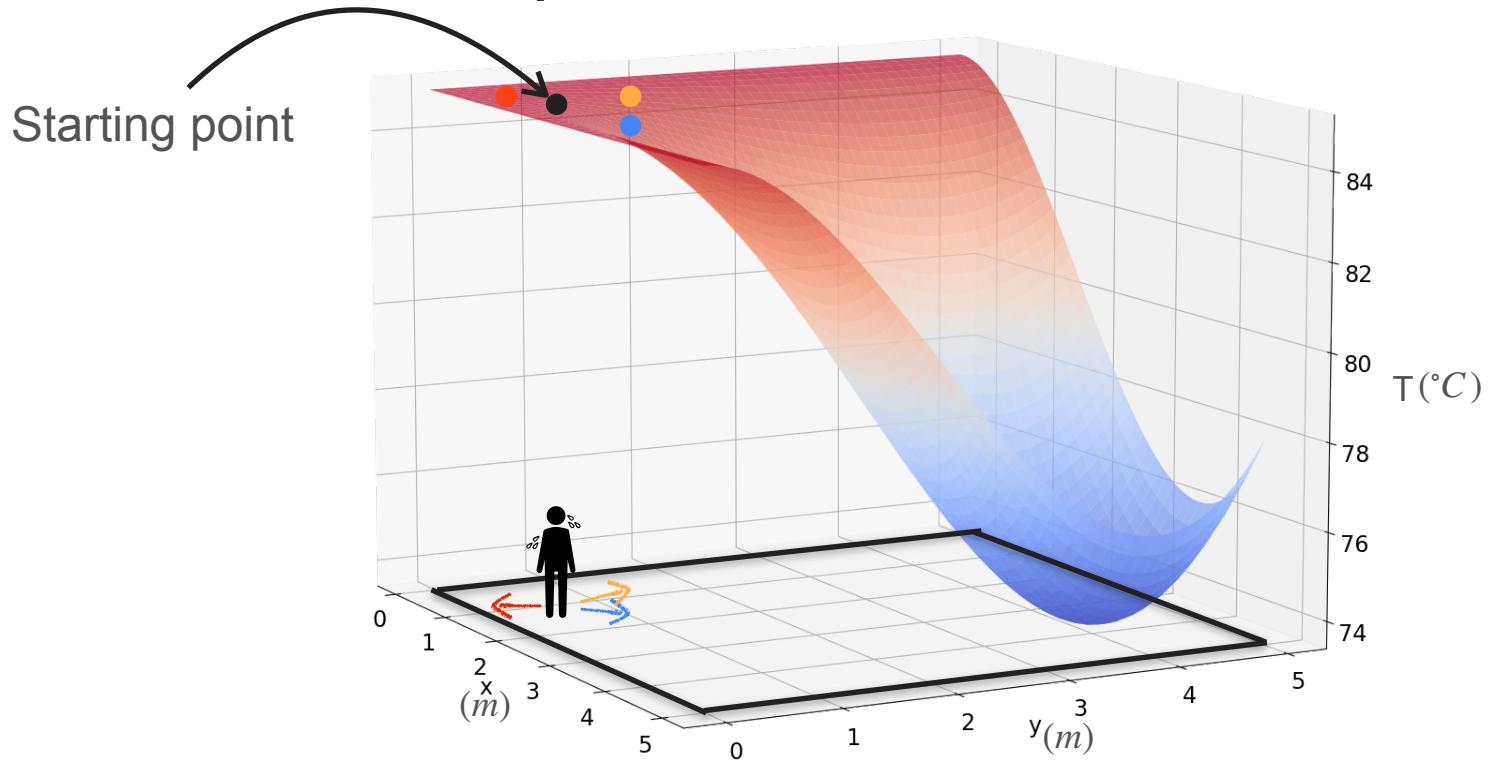
# Motivation for Optimization in Two Variables



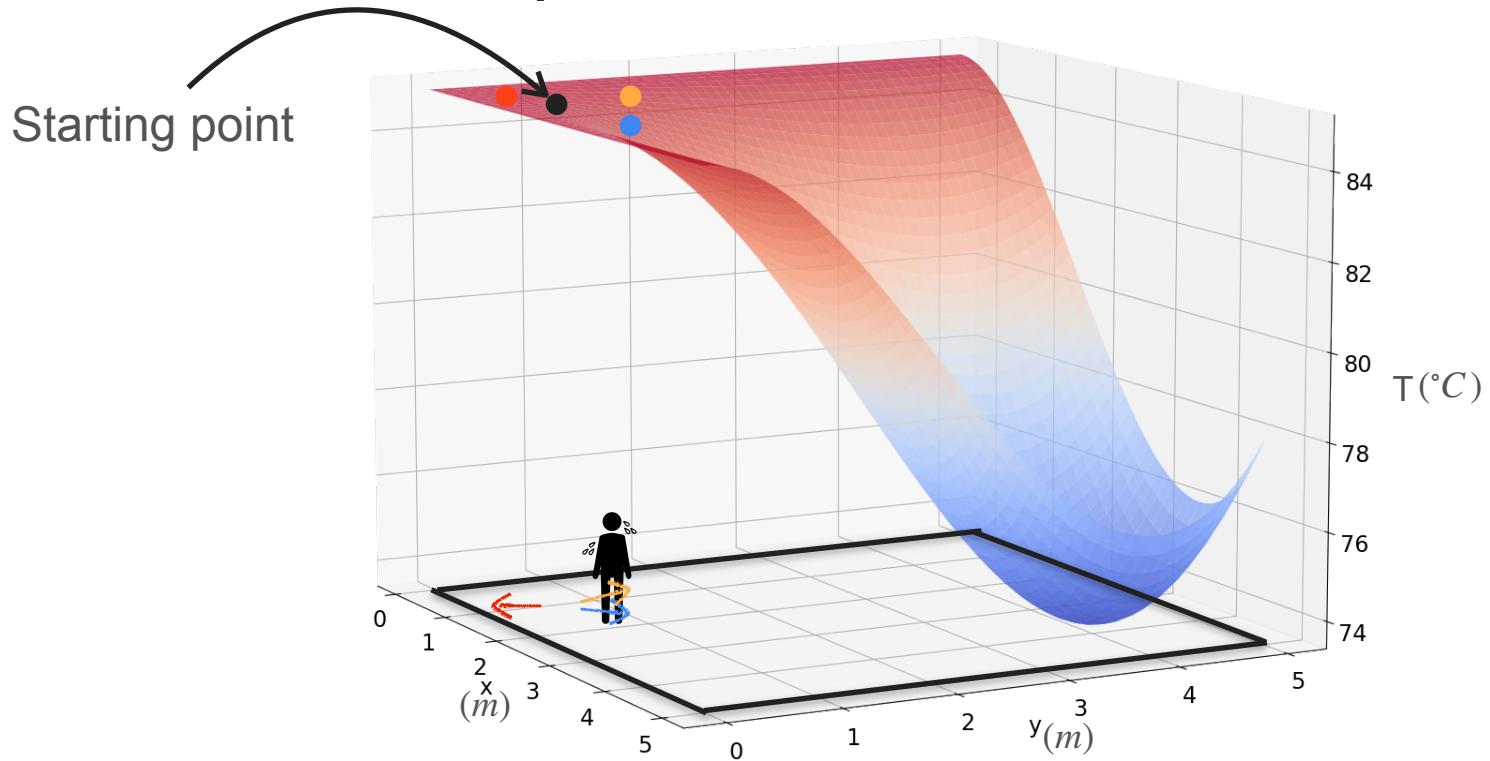
# Motivation for Optimization in Two Variables



# Motivation for Optimization in Two Variables

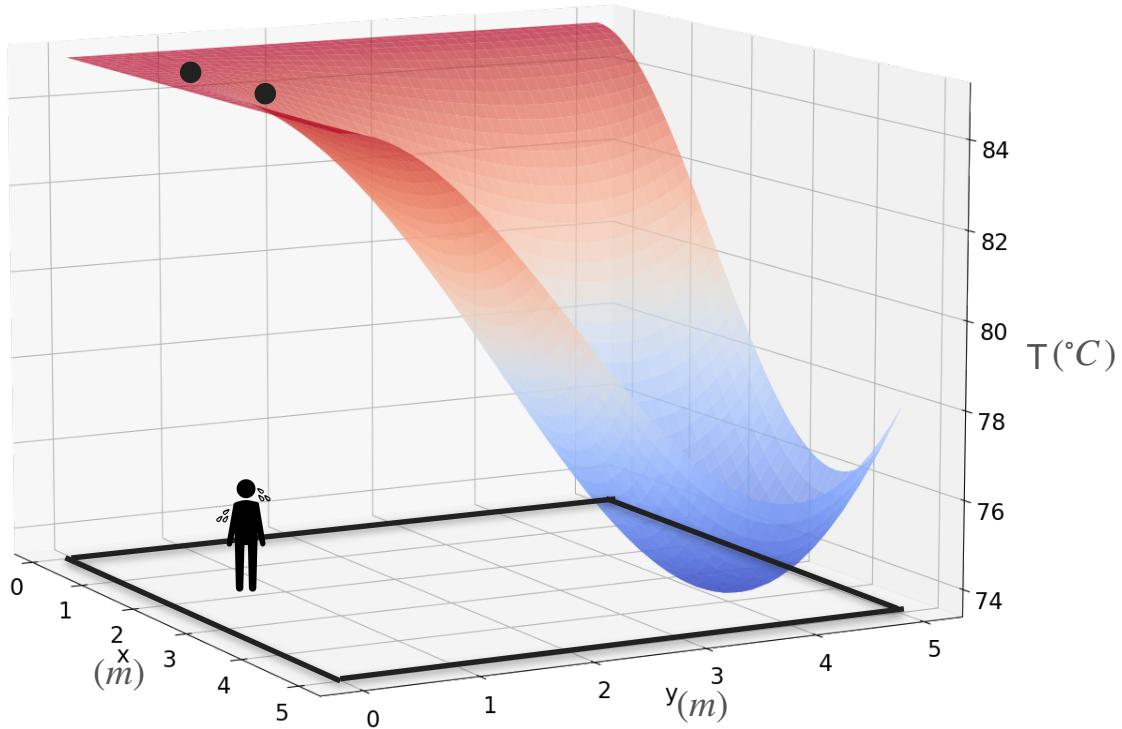


# Motivation for Optimization in Two Variables

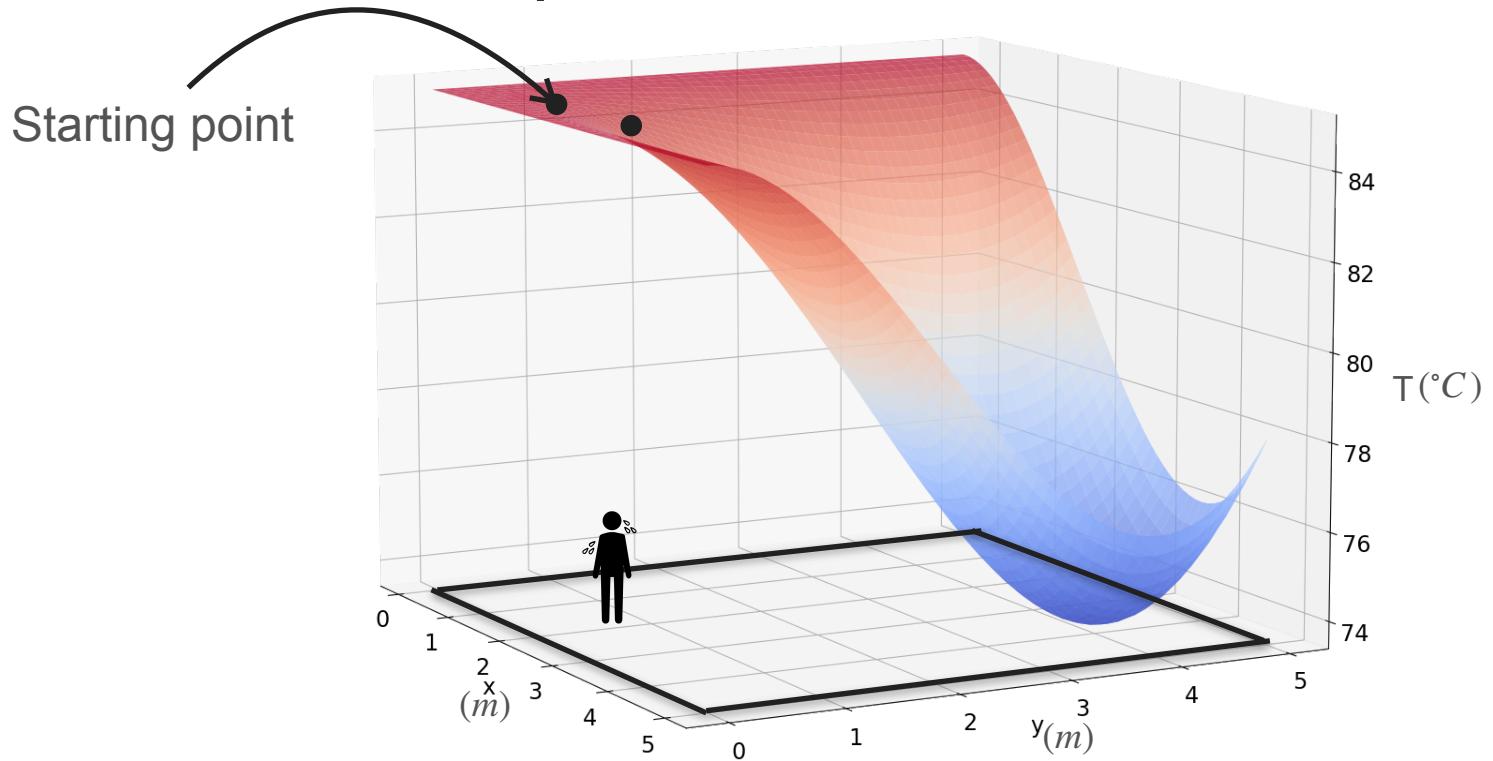


# Motivation for Optimization in Two Variables

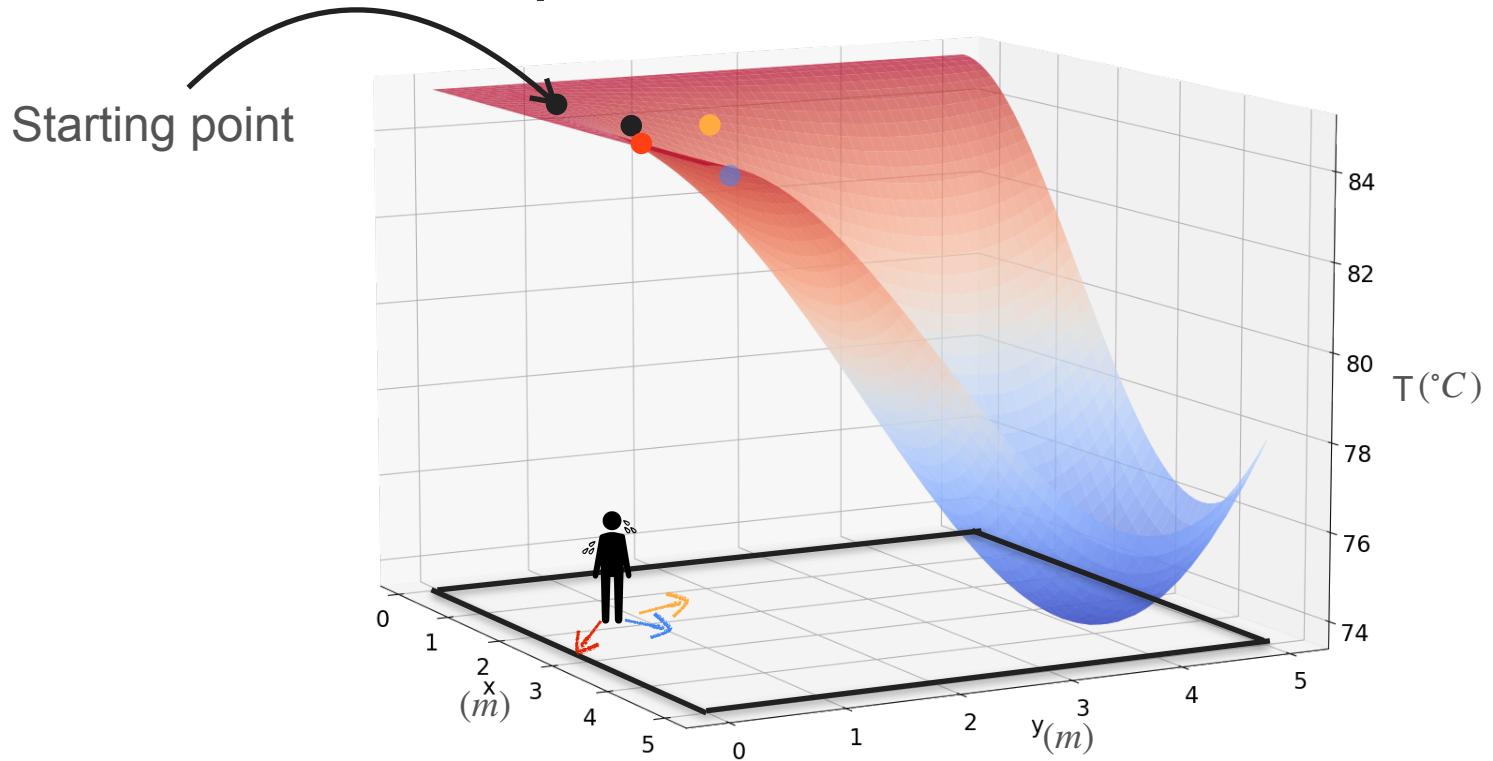
Starting point



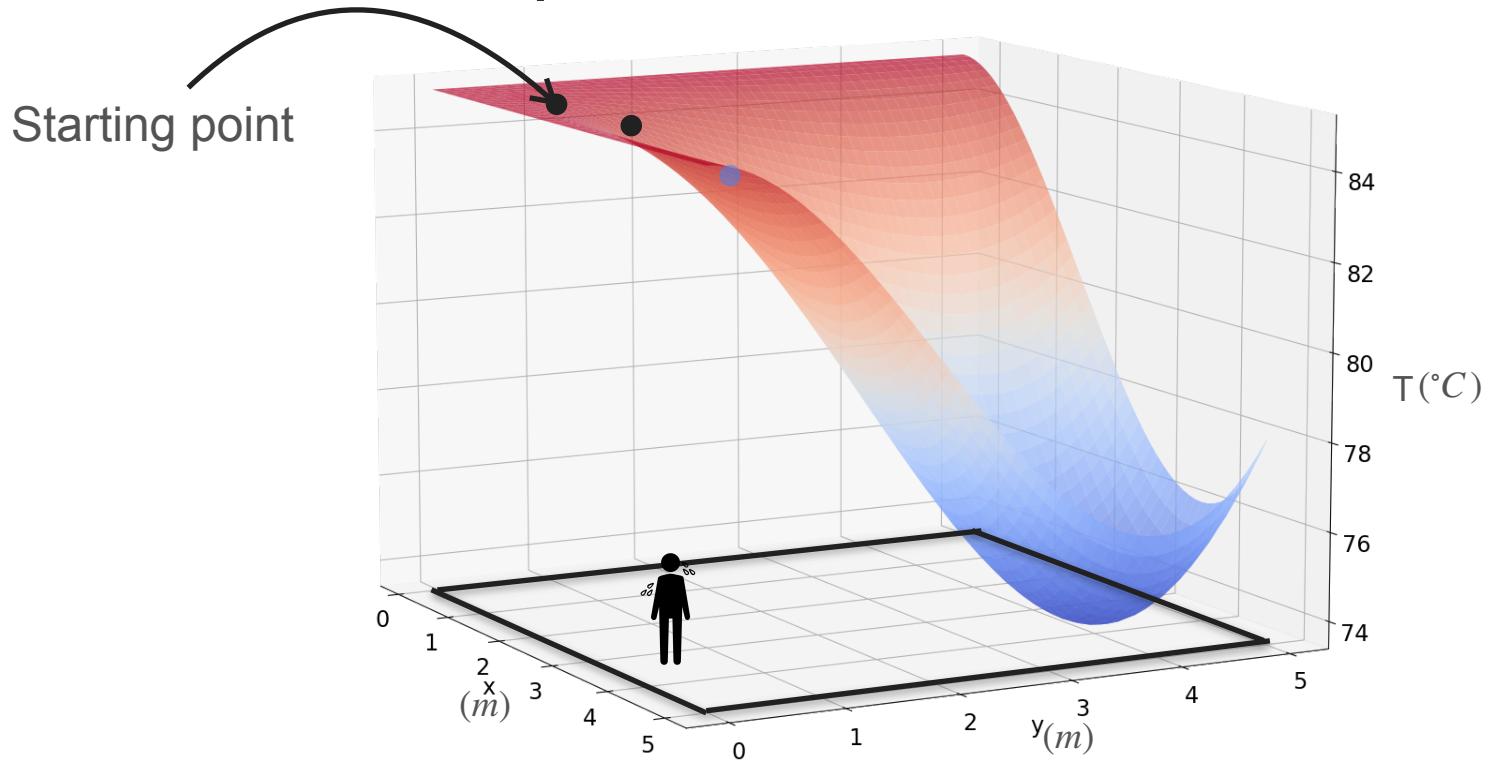
# Motivation for Optimization in Two Variables



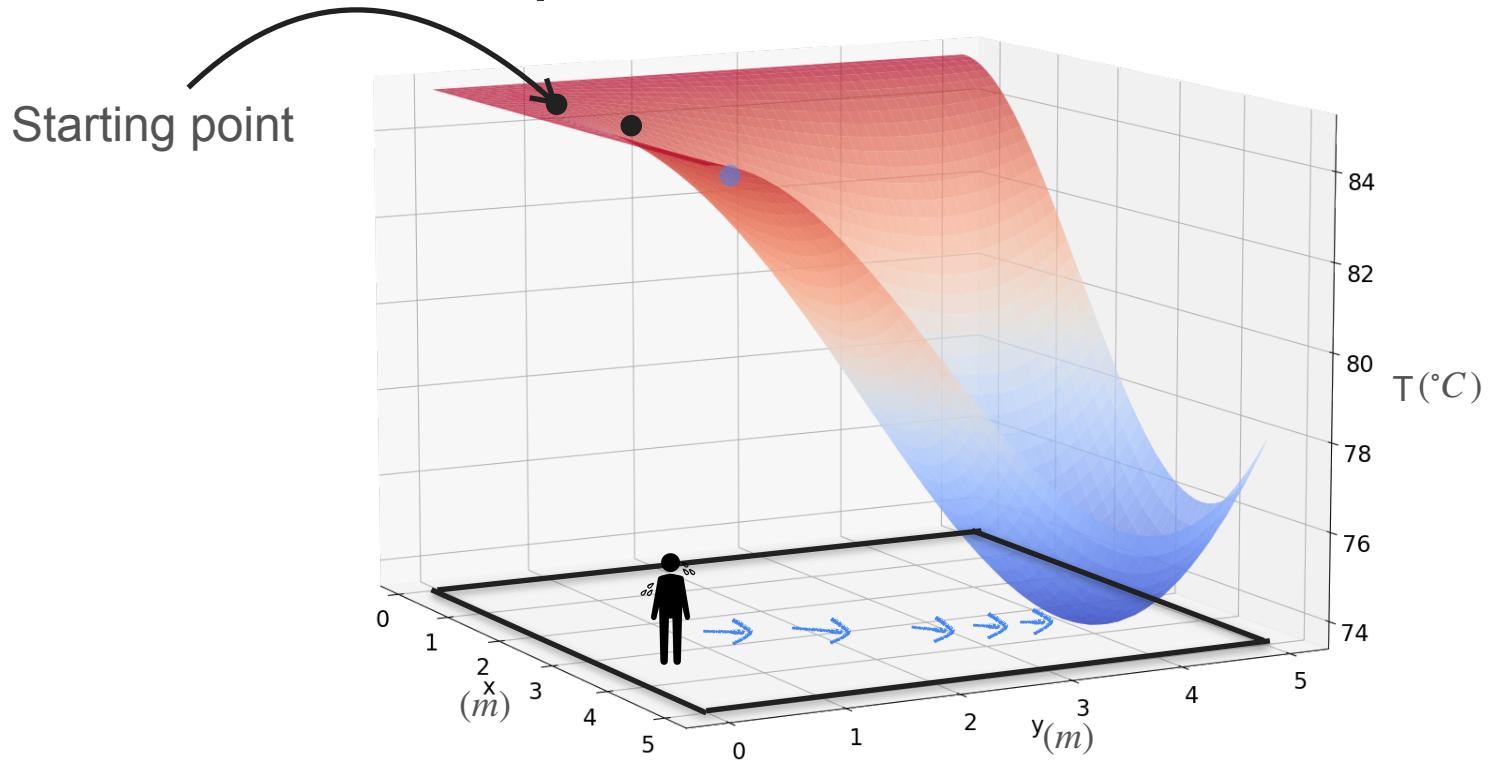
# Motivation for Optimization in Two Variables



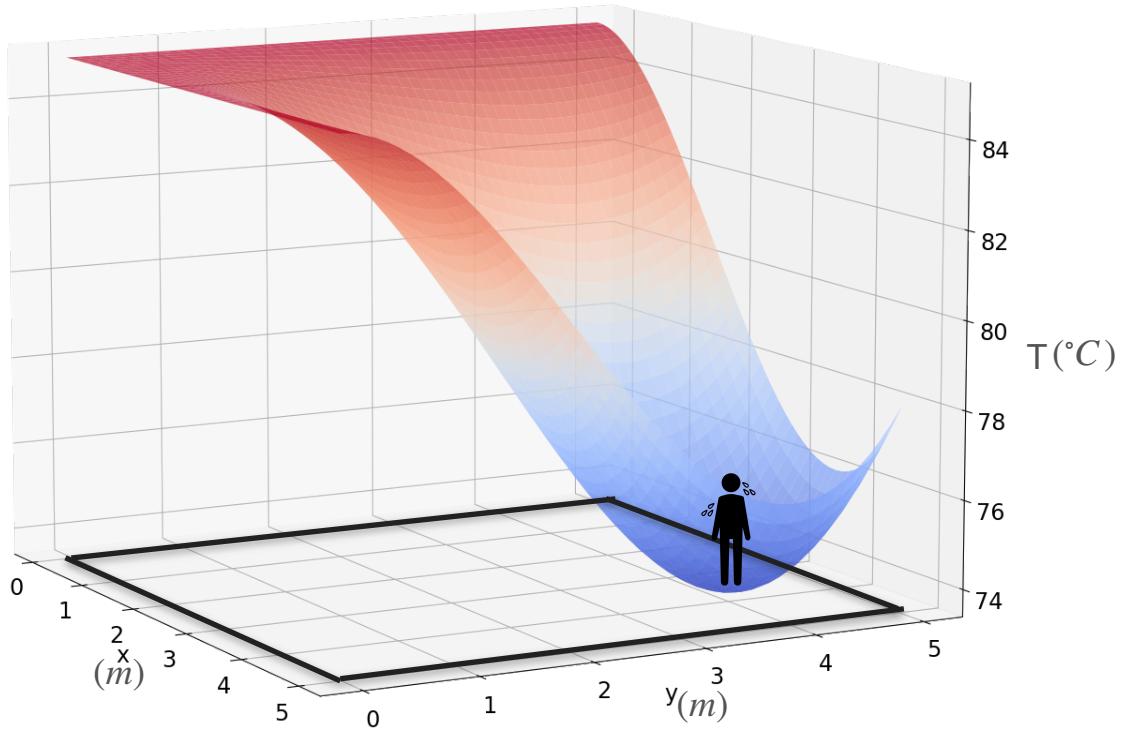
# Motivation for Optimization in Two Variables



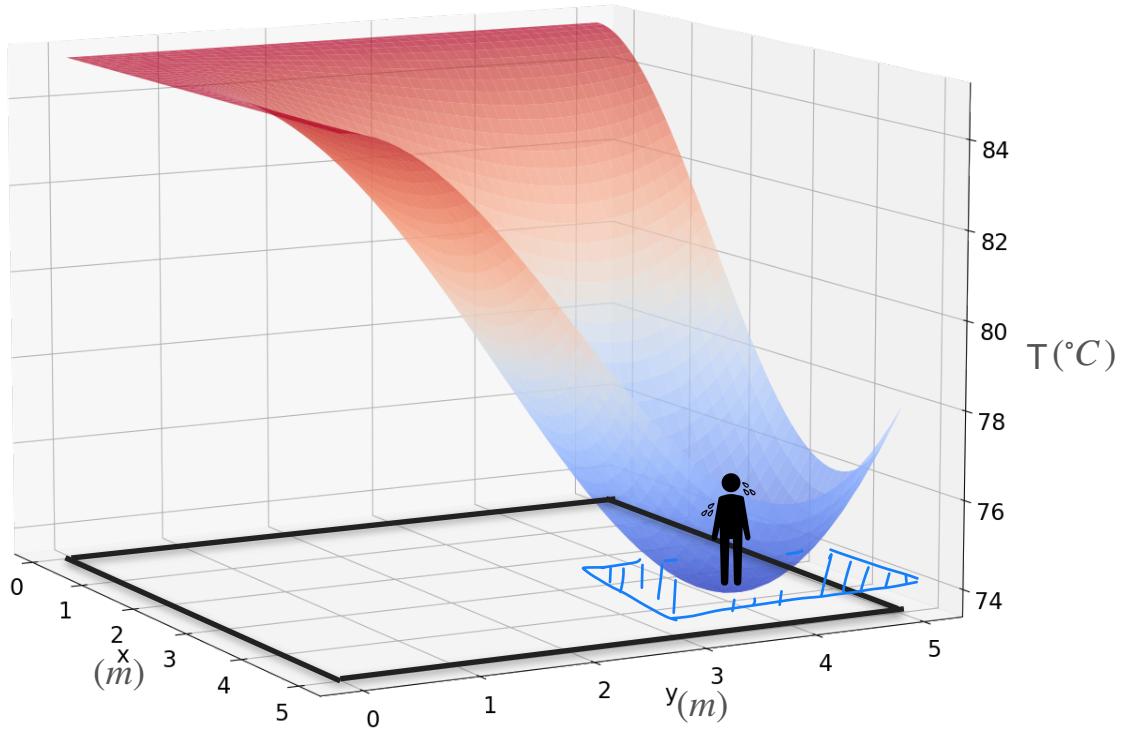
# Motivation for Optimization in Two Variables



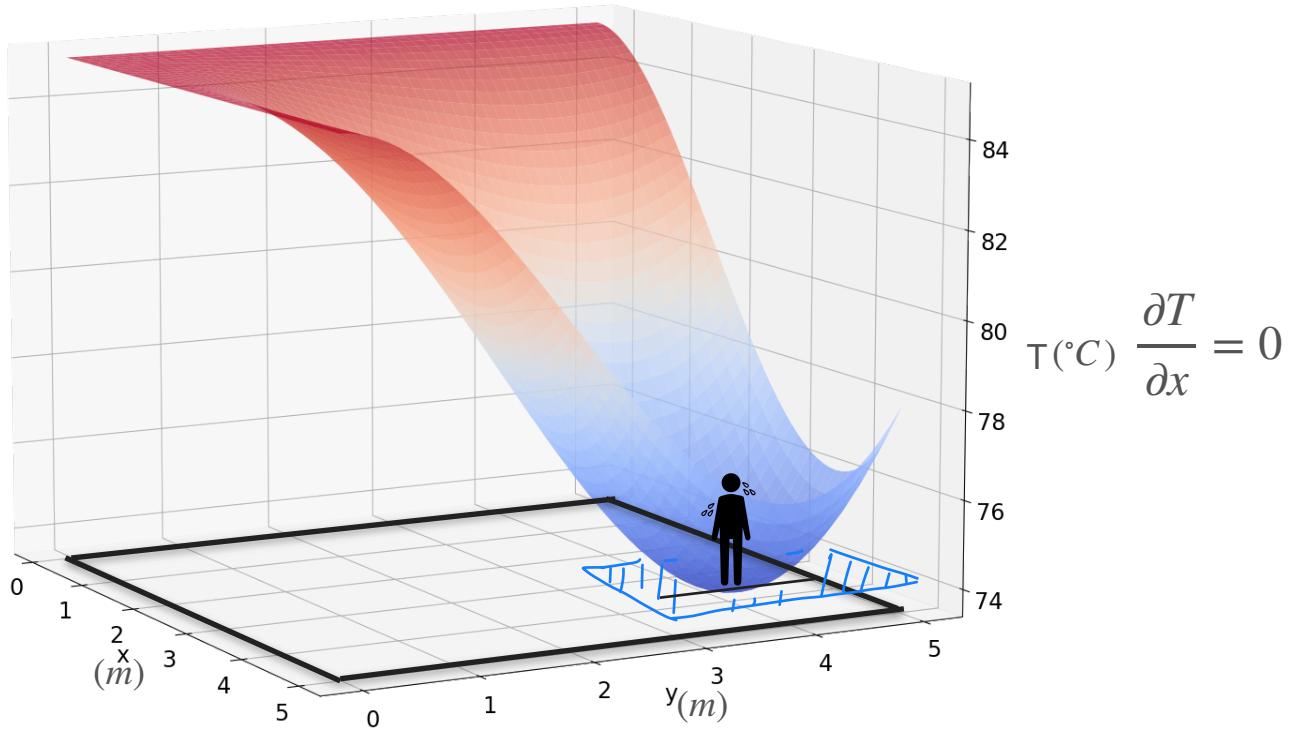
# Motivation for Optimization in Two Variables



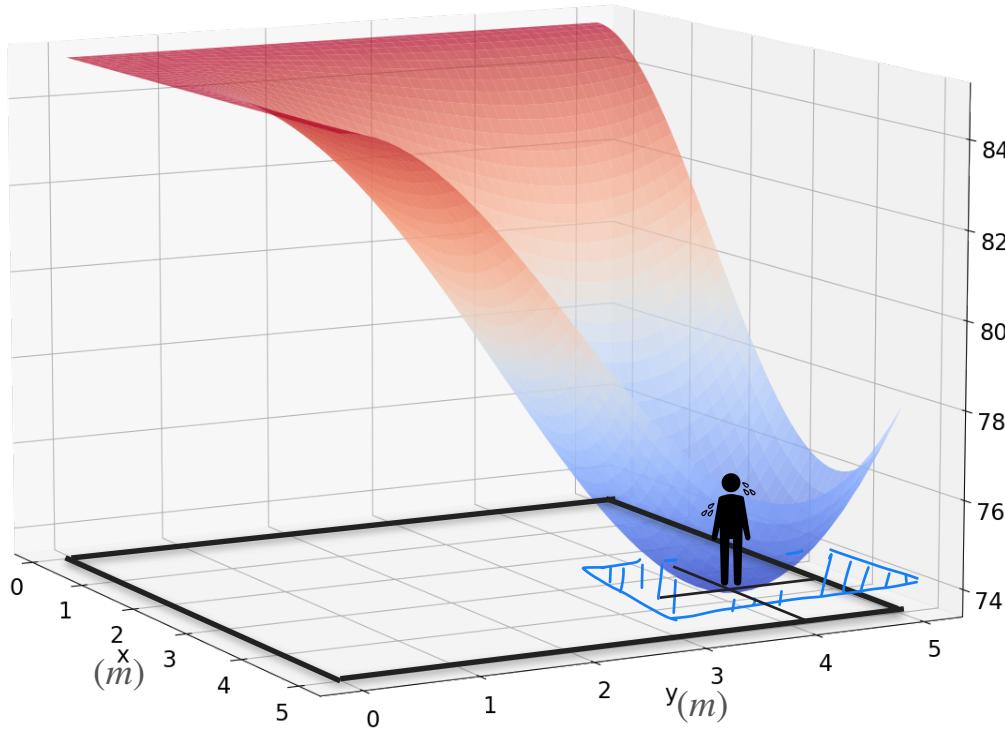
# Motivation for Optimization in Two Variables



# Motivation for Optimization in Two Variables



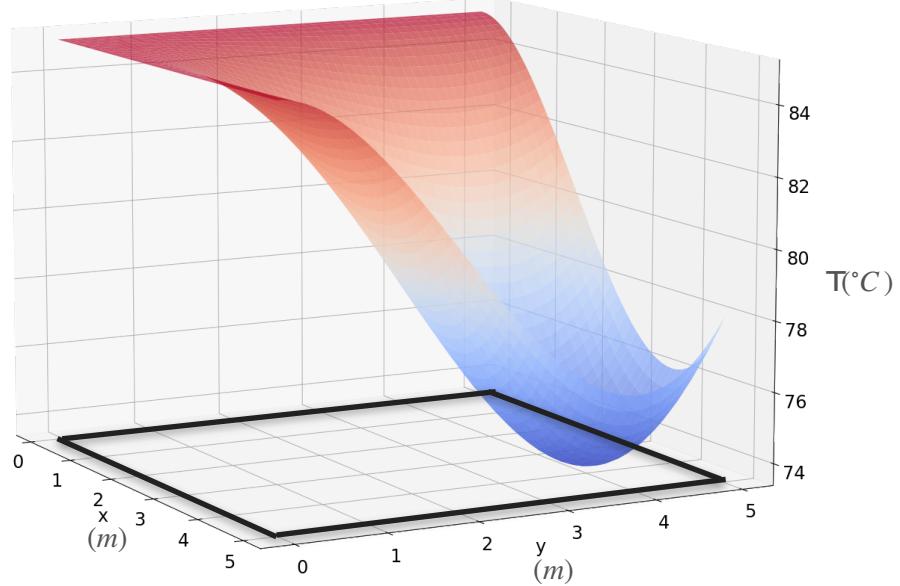
# Motivation for Optimization in Two Variables



$$T(\text{°C}) \frac{\partial T}{\partial x} = 0$$

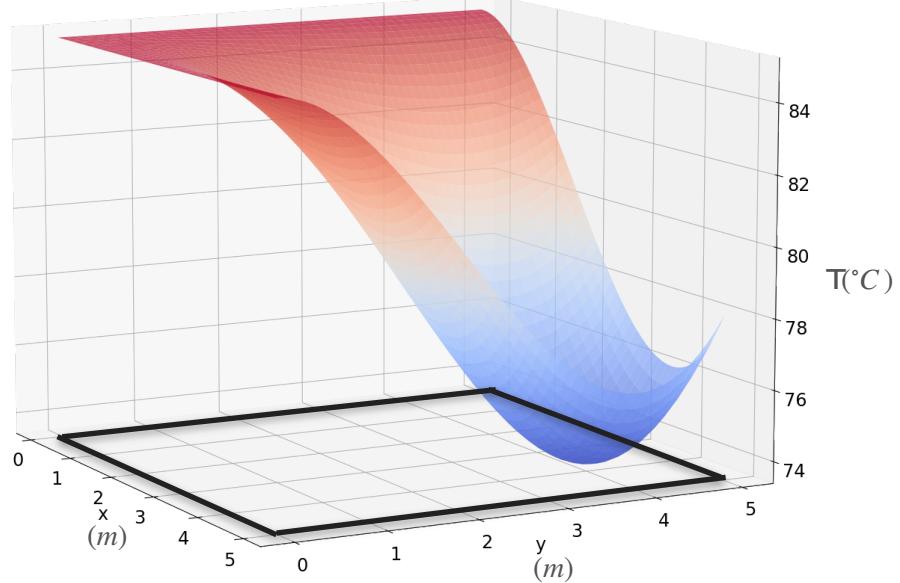
$$\frac{\partial T}{\partial y} = 0$$

# Exercise



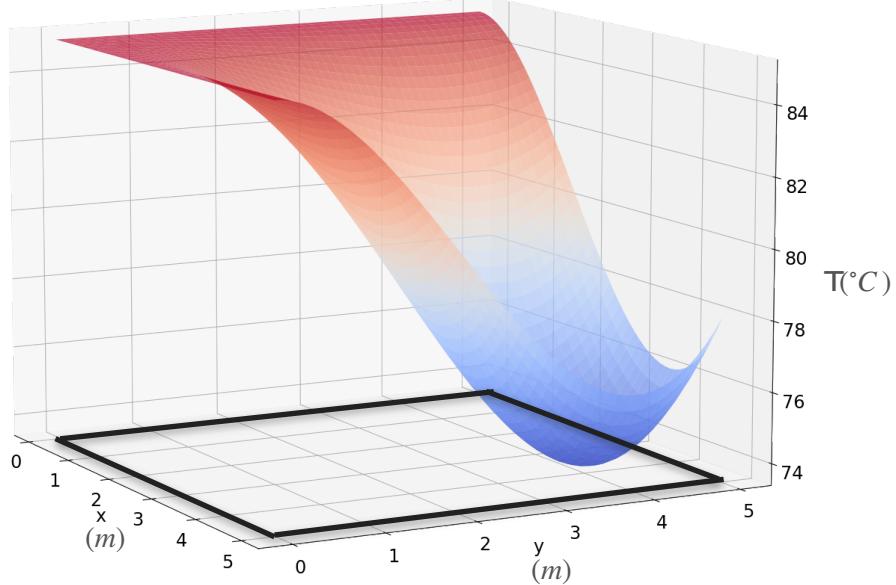
# Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



# Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

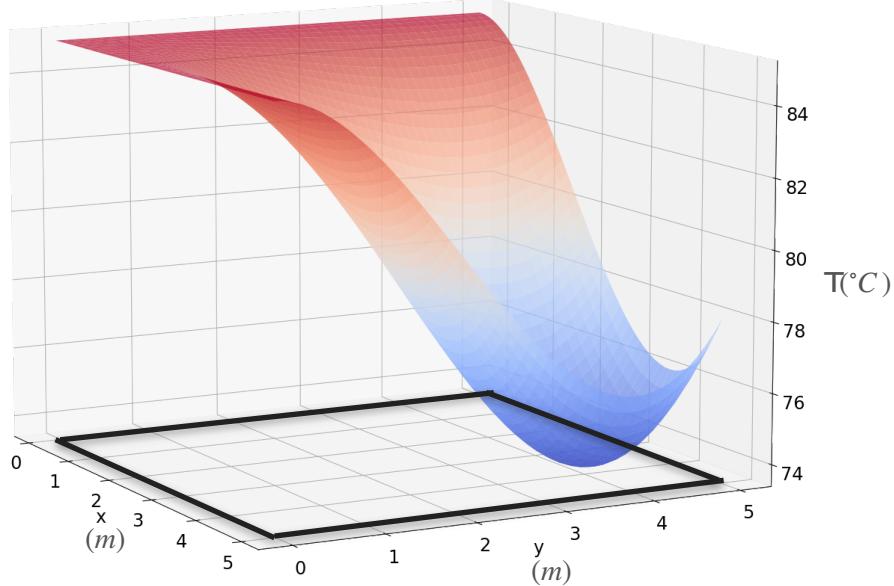


Try and calculate

$$\frac{\partial f}{\partial x}$$

# Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Try and calculate

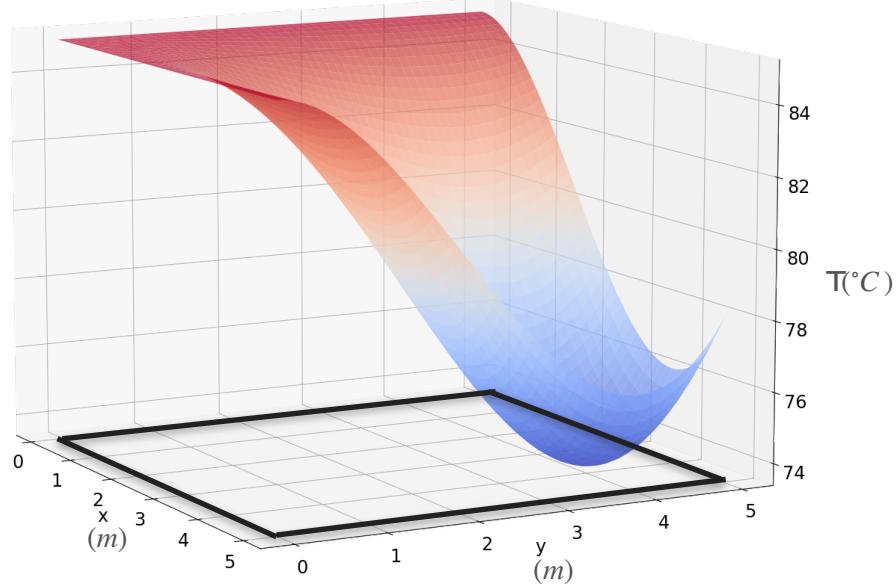
$$\frac{\partial f}{\partial x}$$

and

$$\frac{\partial f}{\partial y}$$

# Motivation for Optimization in Two Variables

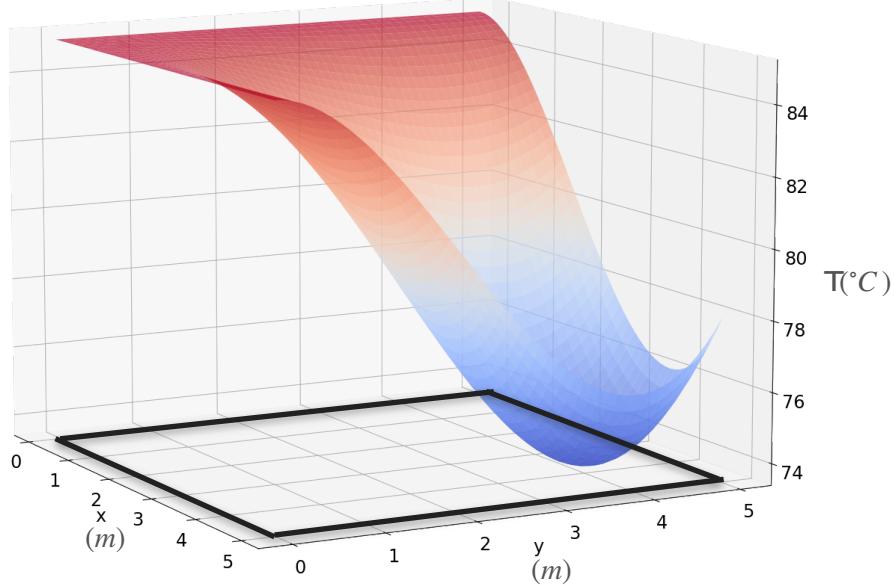
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

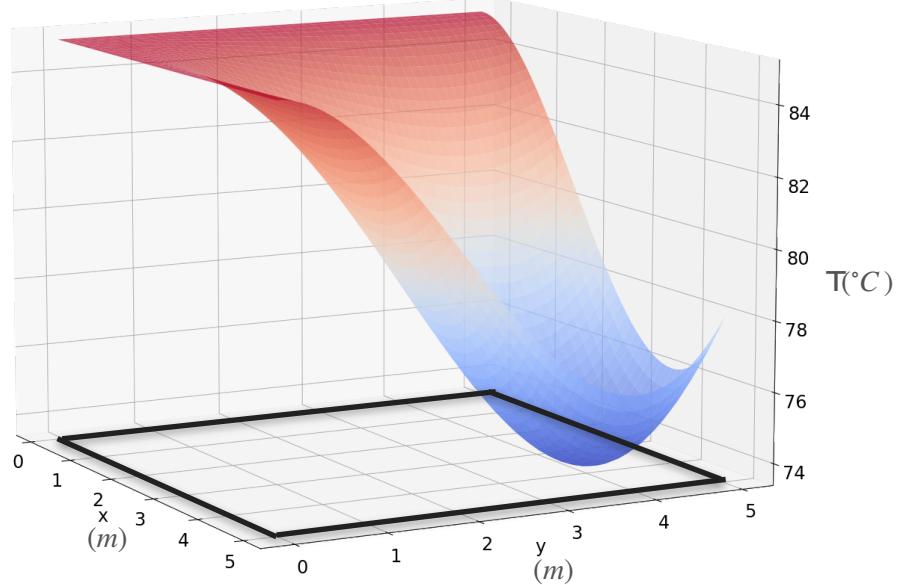
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$



# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$

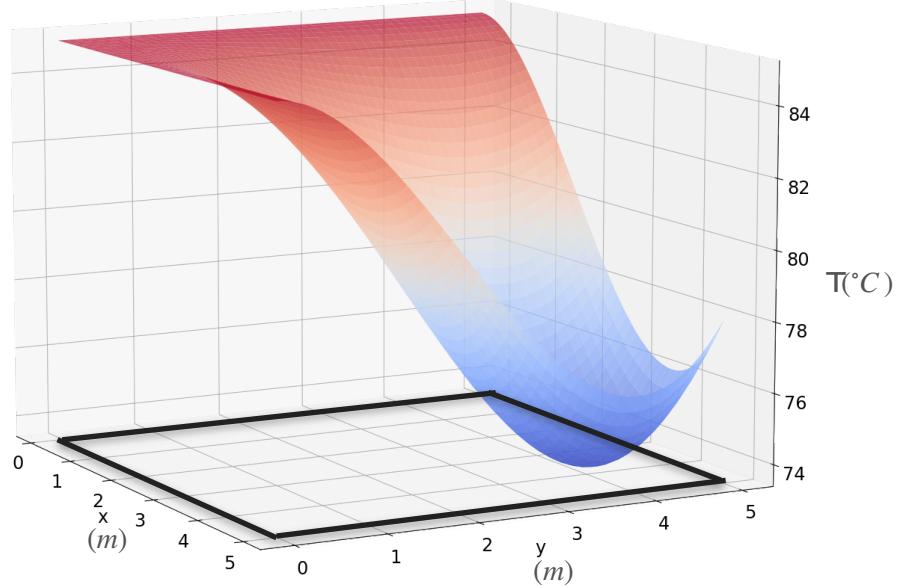


$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12)$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

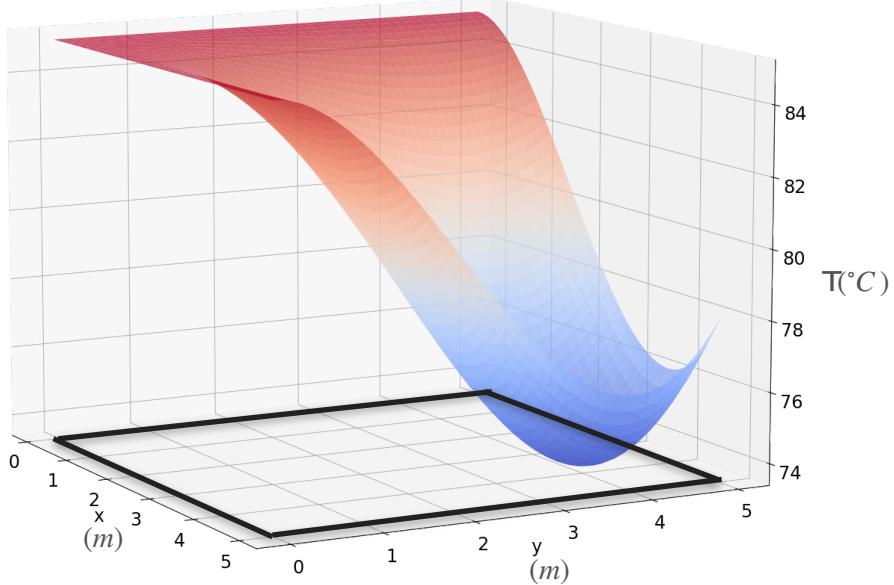
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$



$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

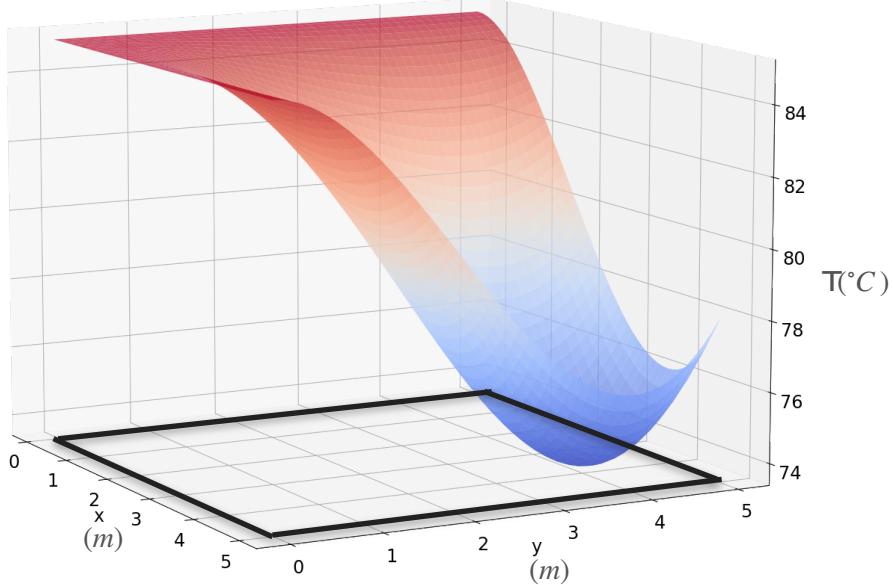


$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



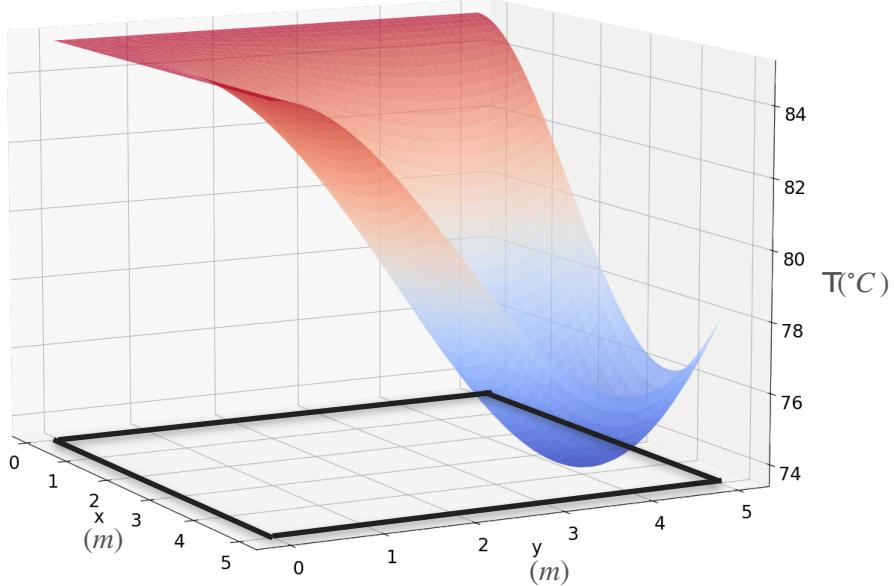
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$x = 0 \quad x = 4$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



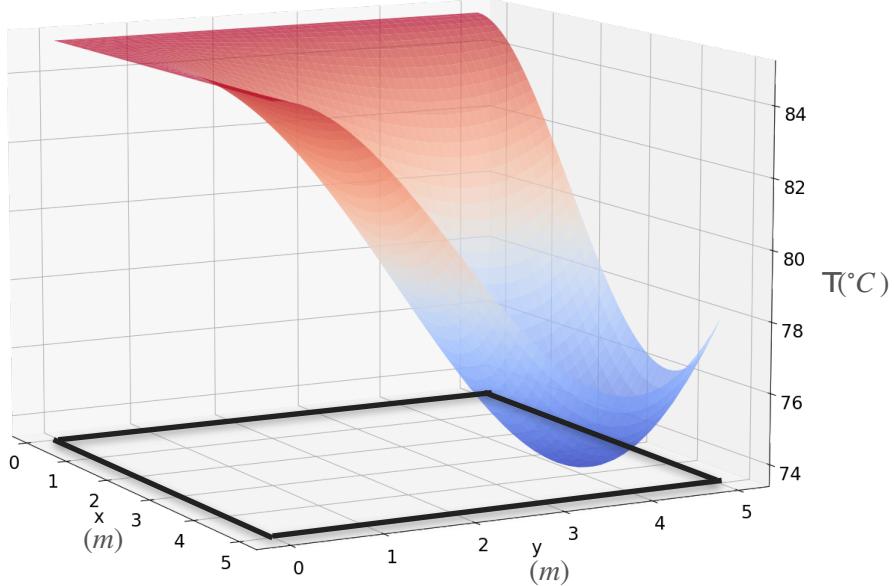
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$x = 0 \quad x = 4 \quad y = 0$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



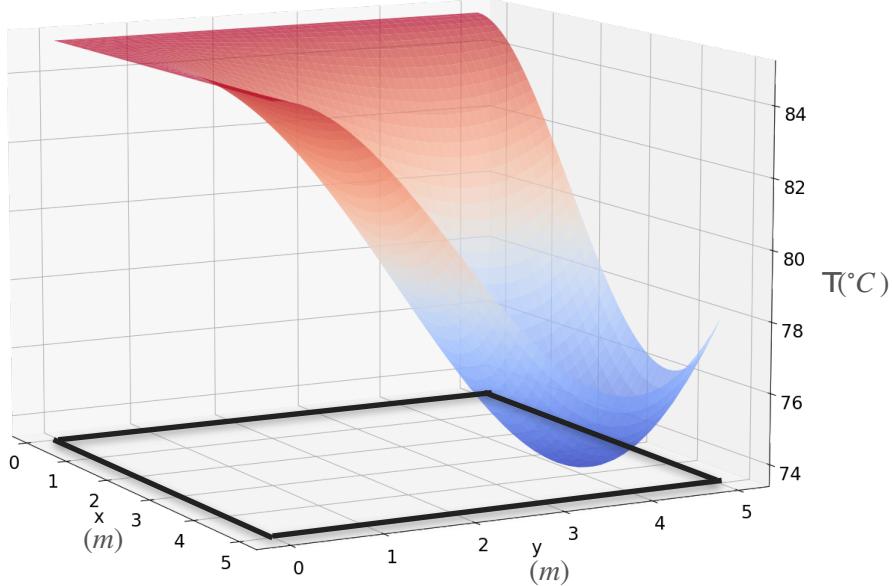
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$x = 0 \quad x = 4 \quad y = 0 \quad y = 6$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

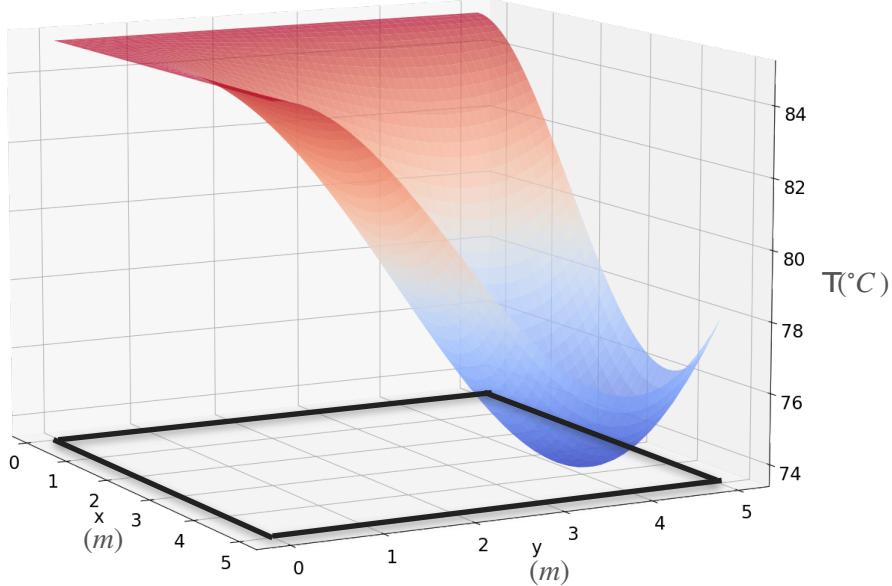
$x = 0$     $x = 4$     $y = 0$     $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

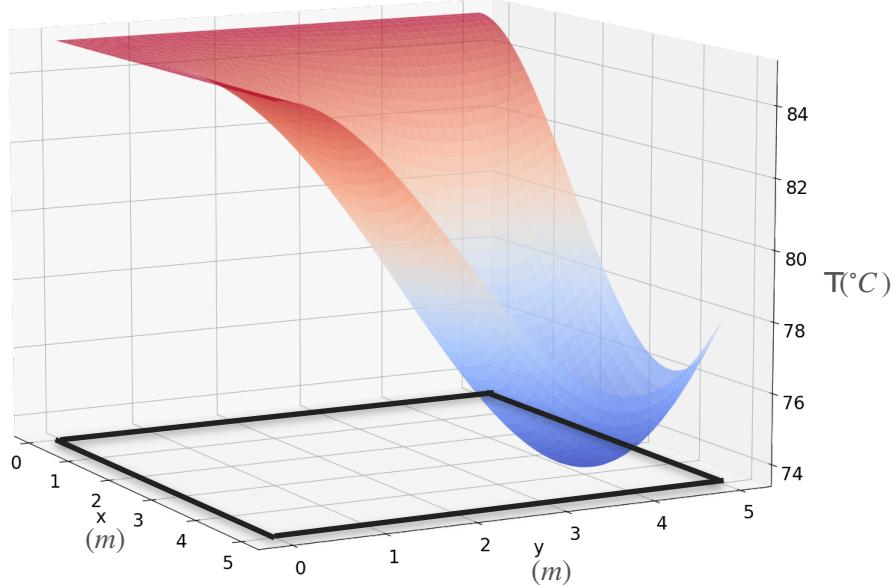
$x = 0$     $x = 4$     $y = 0$     $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$     $x = 6$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

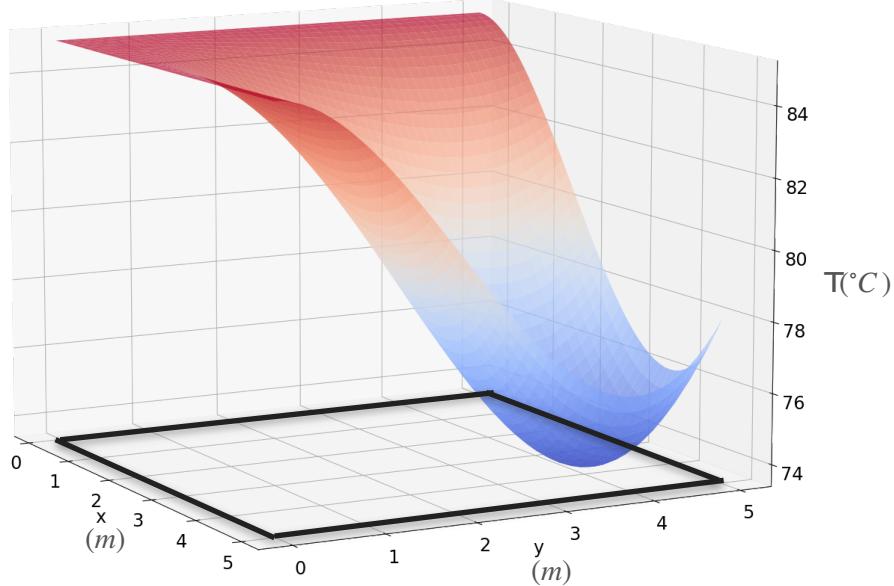
$$x = 0 \quad x = 4 \quad y = 0 \quad y = 6$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$$x = 0 \quad x = 6 \quad y = 0$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

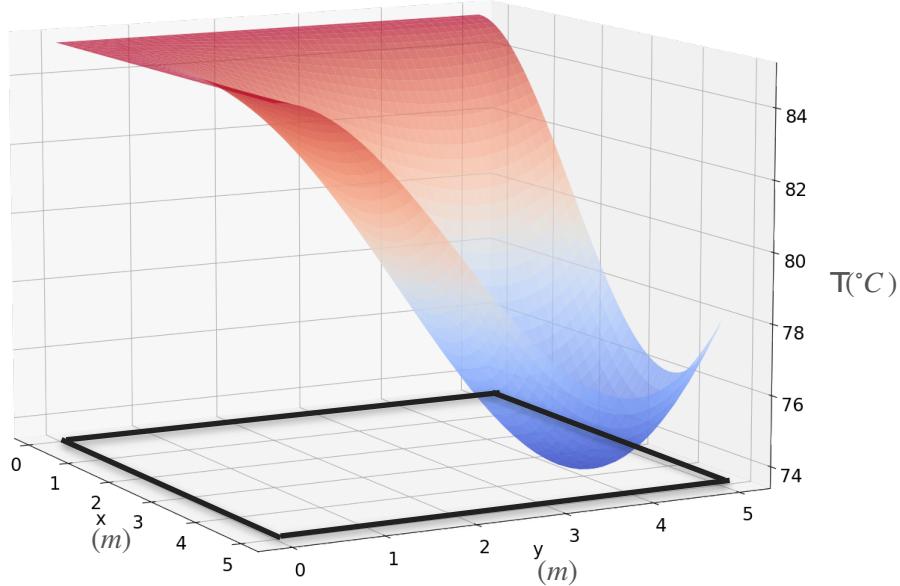
$$\begin{aligned}x &= 0 \\x &= 4 \\y &= 0 \\y &= 6\end{aligned}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$$\begin{aligned}x &= 0 \\x &= 6 \\y &= 0 \\y &= 4\end{aligned}$$

# Motivation for Optimization in Two Variables

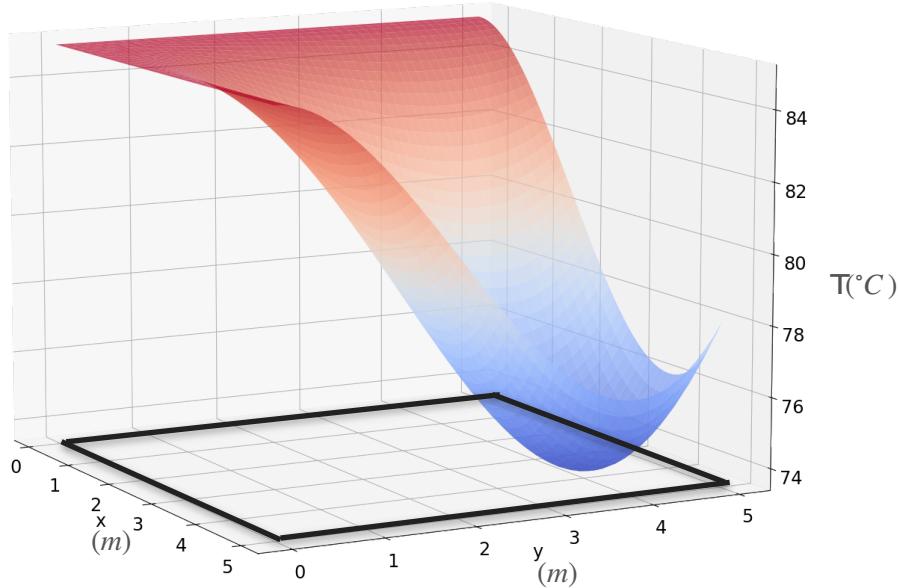
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



# Motivation for Optimization in Two Variables

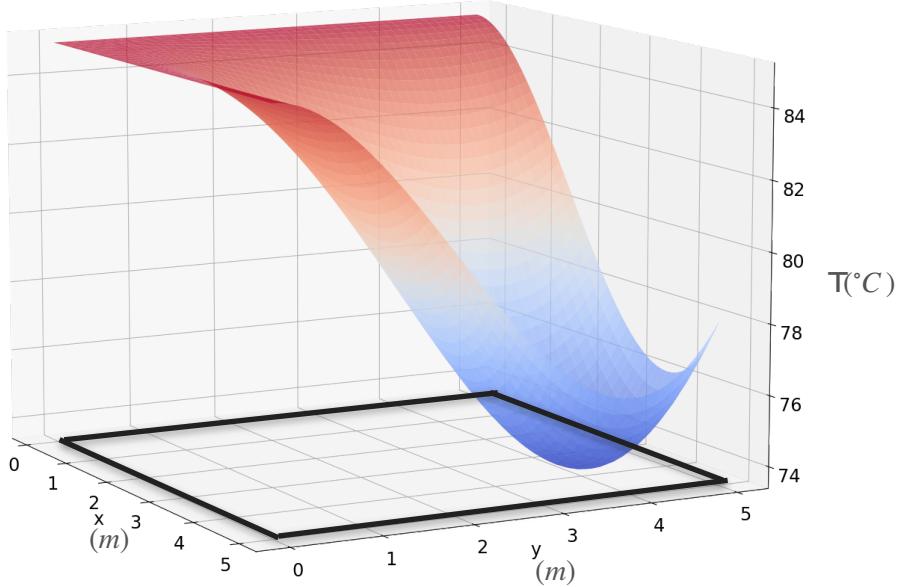
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Candidate points for the minima



# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$x = 0$$

$$y = 0$$

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

$$x = 4, y = 0$$

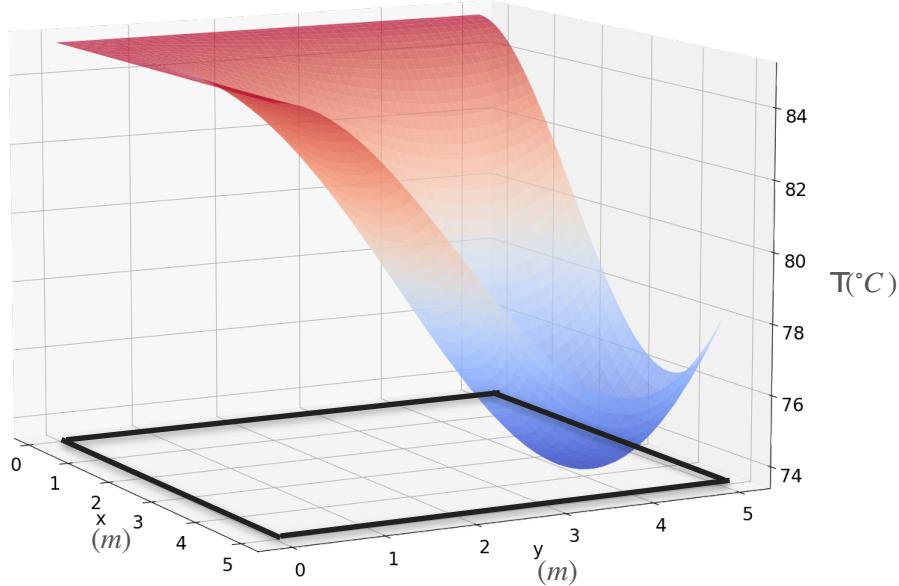
$$x = 4, y = 4$$

$$x = 6, y = 0$$

$$x = 6, y = 6$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$x = 0$$

$$y = 0$$

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

Outside

$$x = 4, y = 0$$

$$x = 4, y = 4$$

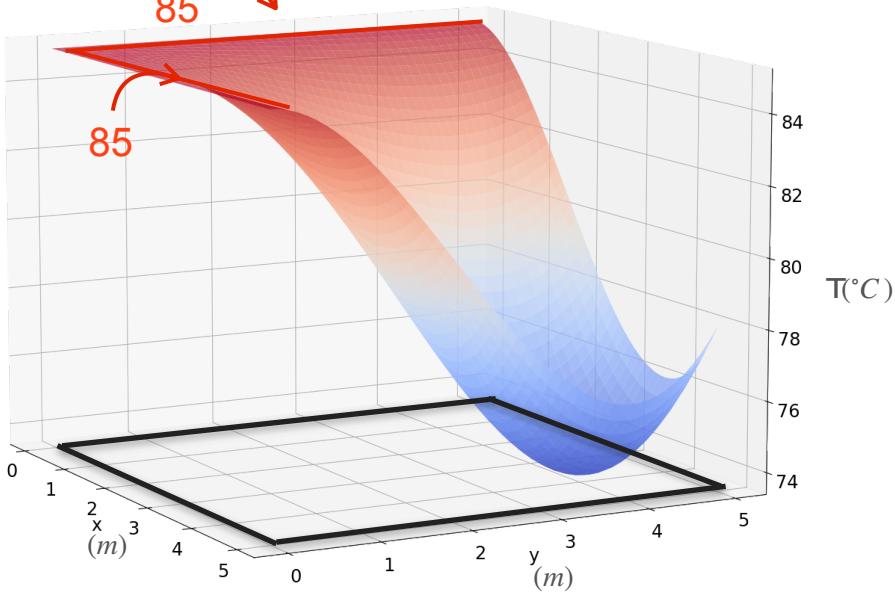
$$x = 6, y = 0$$

Outside

$$x = 6, y = 6$$

# Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

Outside

$$x = 4, y = 0$$

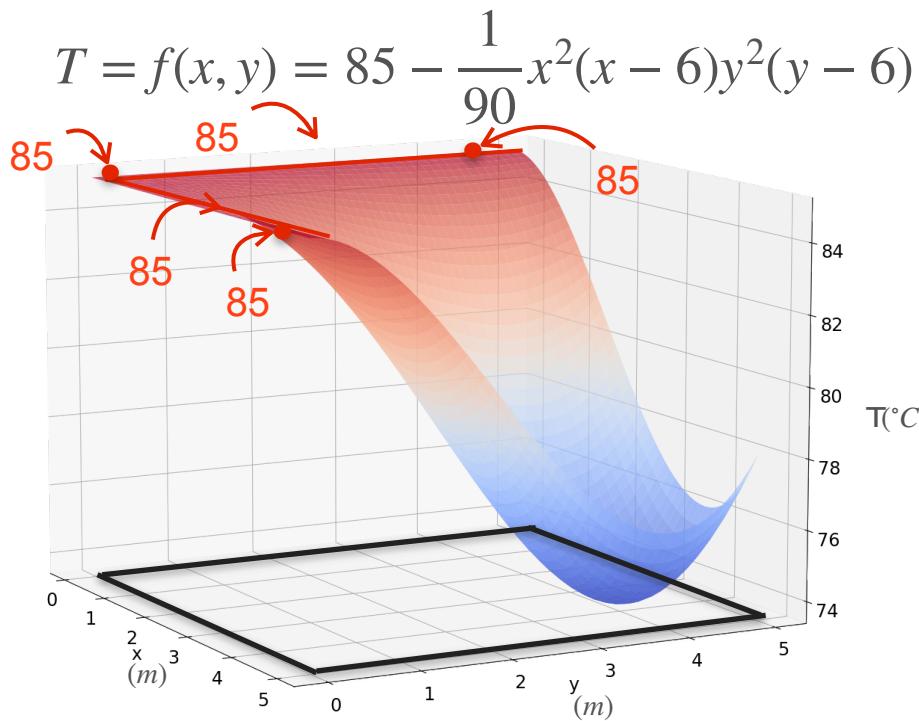
$$x = 4, y = 4$$

$$x = 6, y = 0$$

Outside

$$x = 6, y = 6$$

# Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

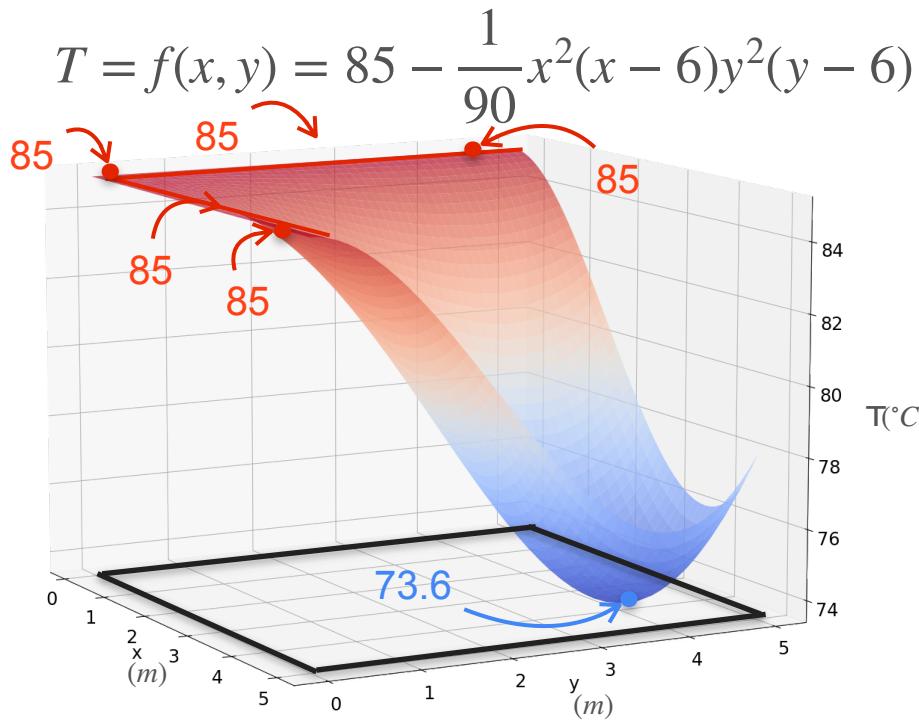
Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside

# Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

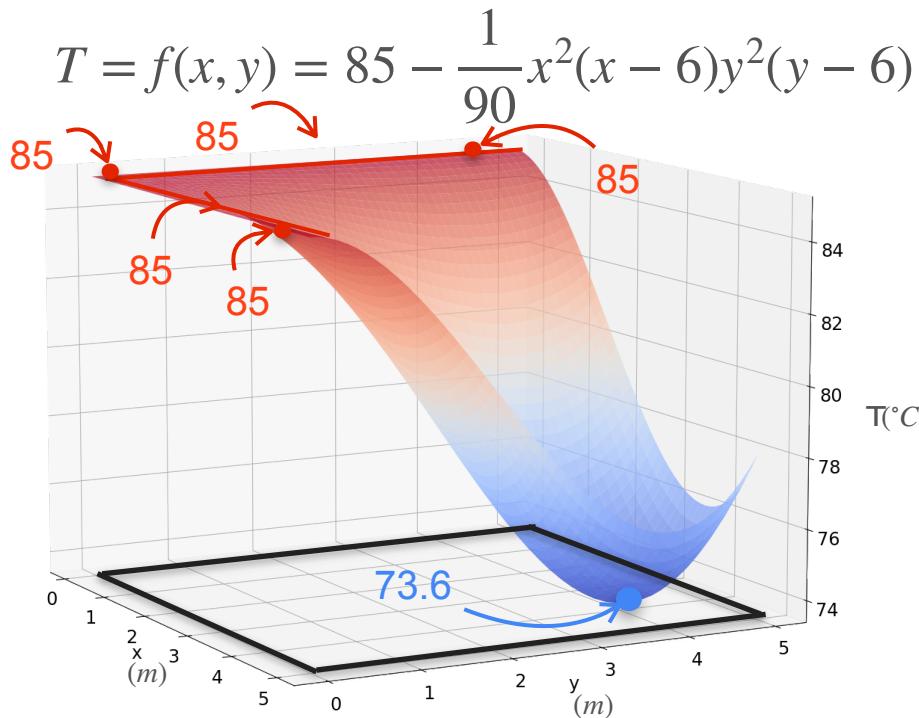
Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside

# Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

Minimum

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside



DeepLearning.AI

# Gradients and Gradient Descent

---

**Optimization using gradients**  
**- Analytical method**

# Linear Regression: Analytical Approach

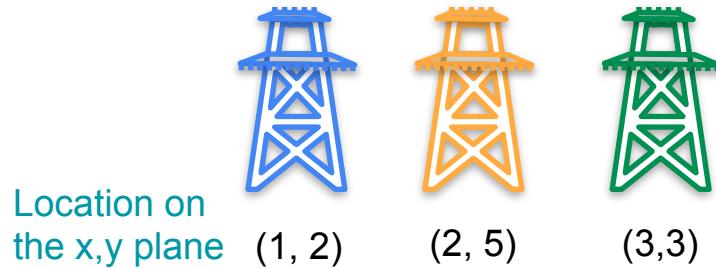
# Linear Regression: Analytical Approach



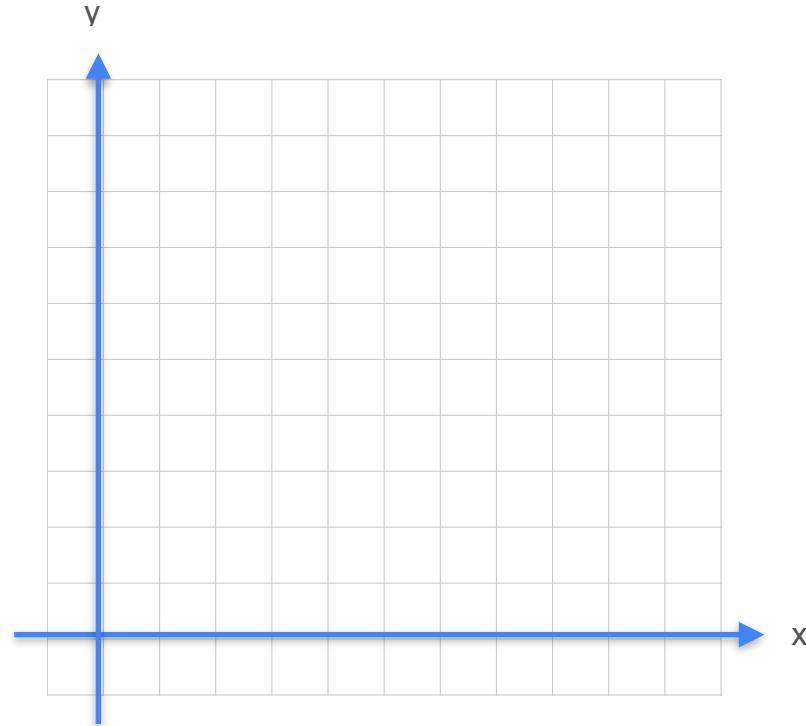
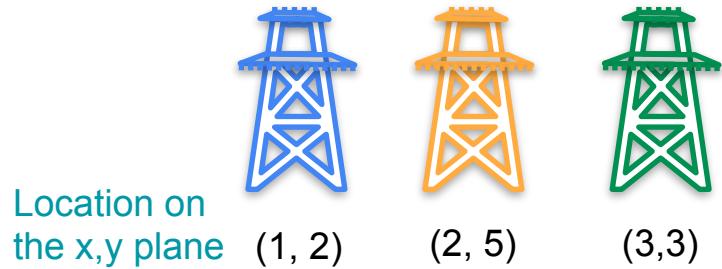
# Linear Regression: Analytical Approach



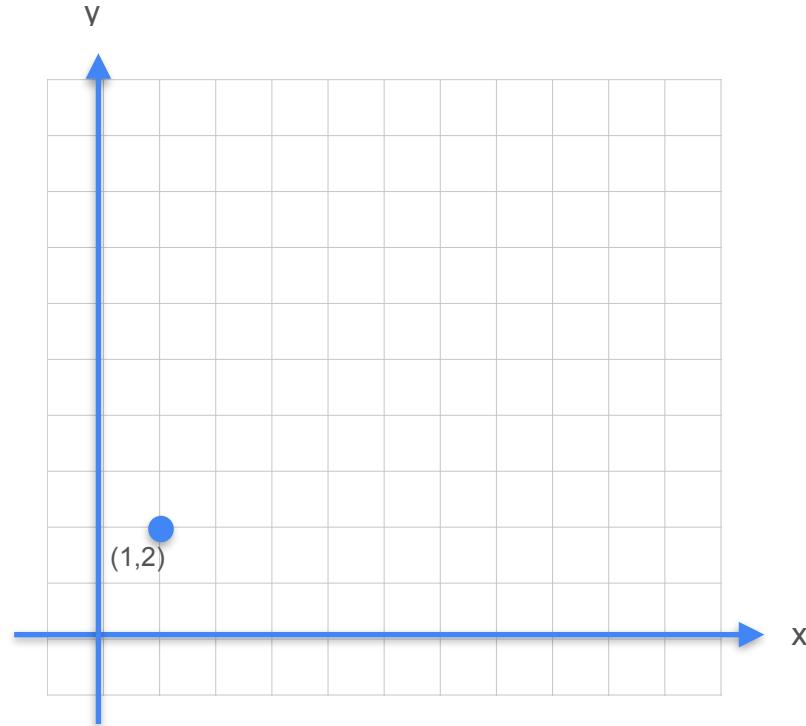
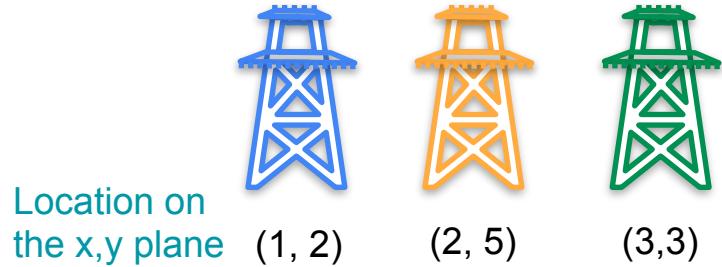
# Linear Regression: Analytical Approach



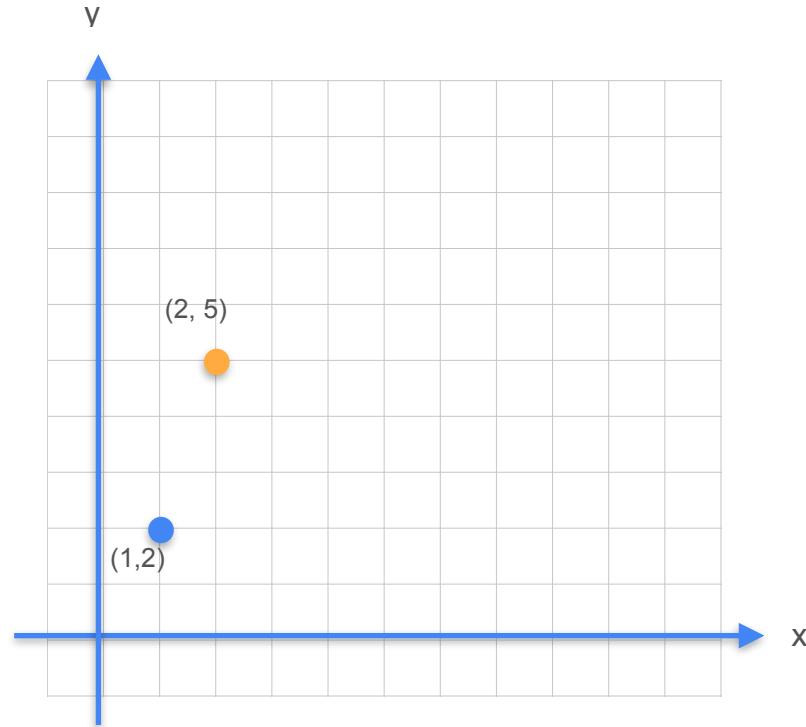
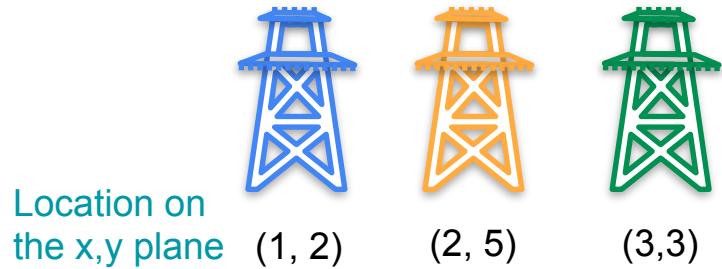
# Linear Regression: Analytical Approach



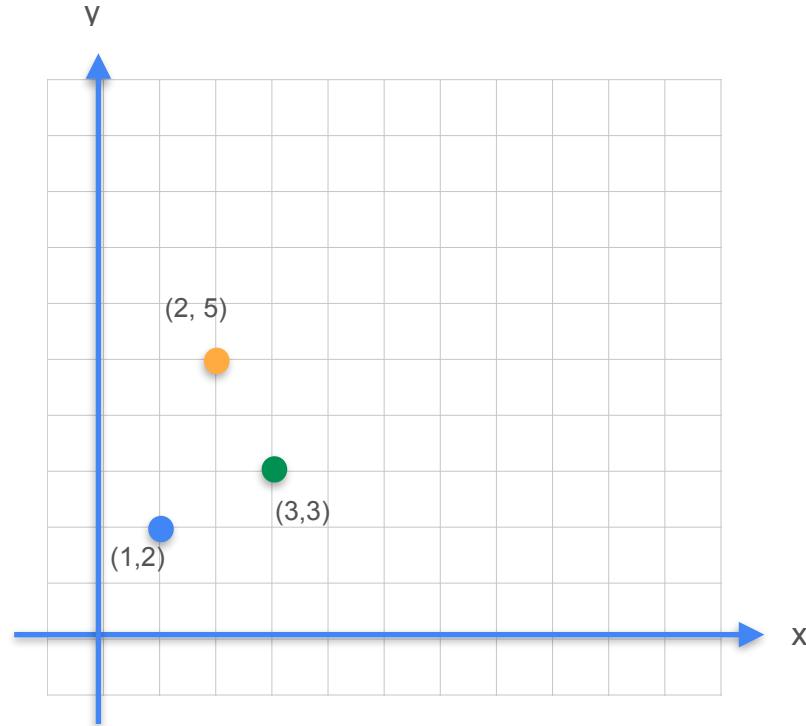
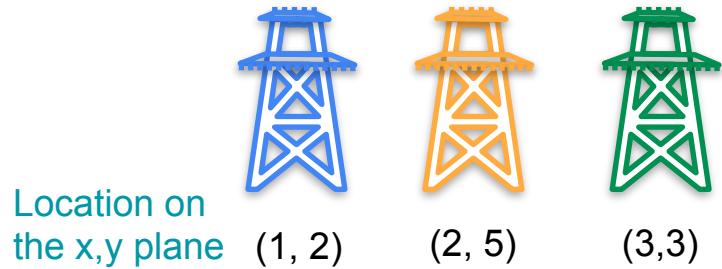
# Linear Regression: Analytical Approach



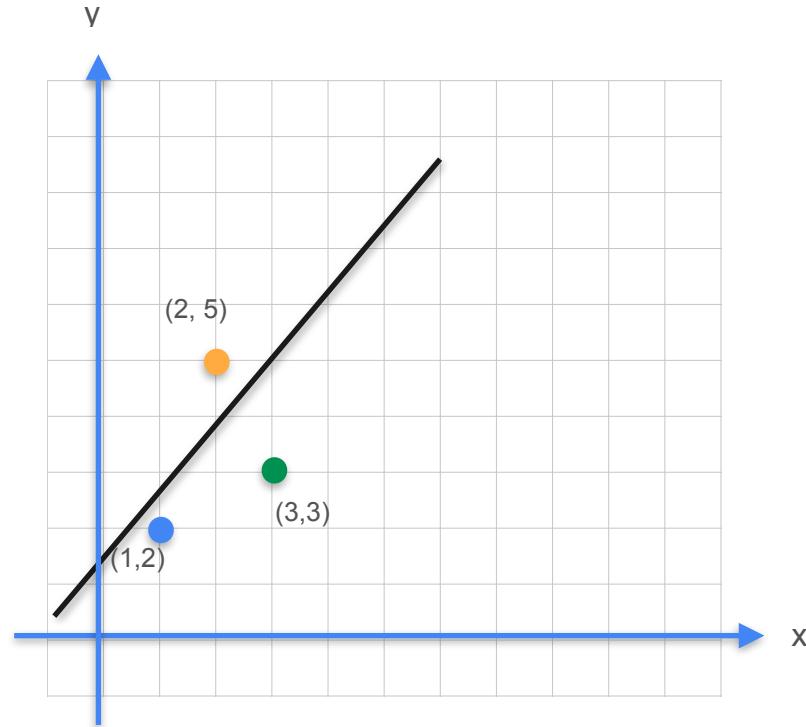
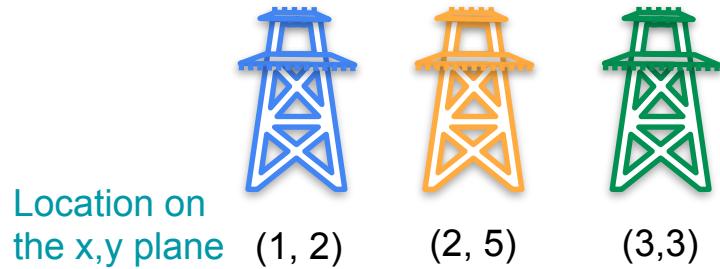
# Linear Regression: Analytical Approach



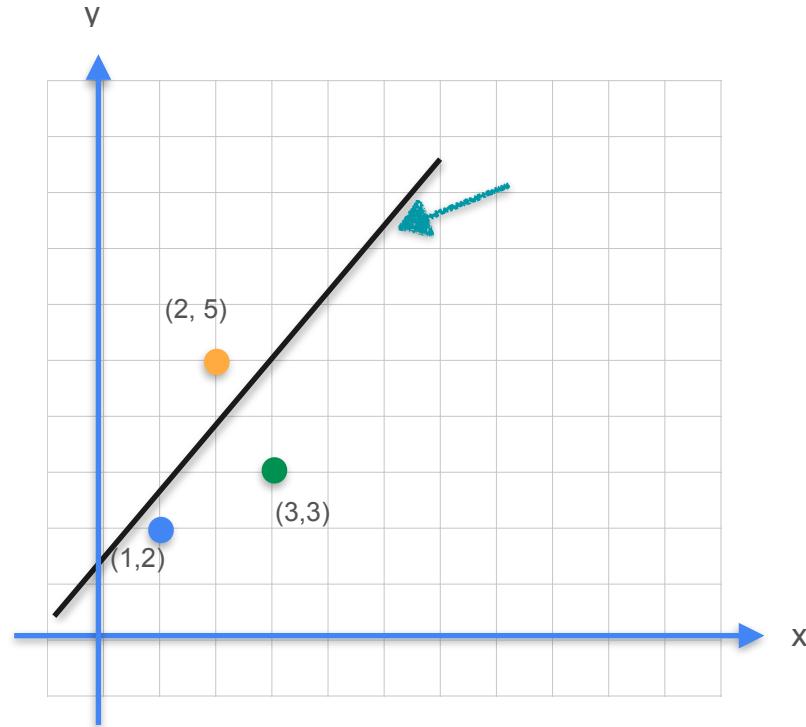
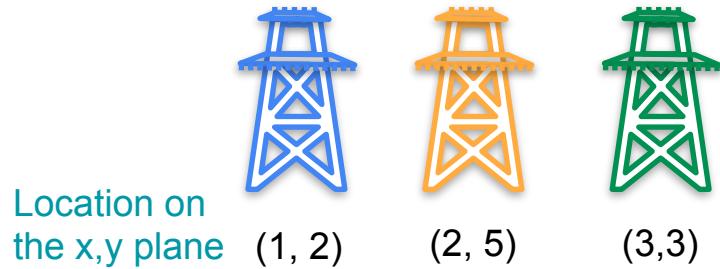
# Linear Regression: Analytical Approach



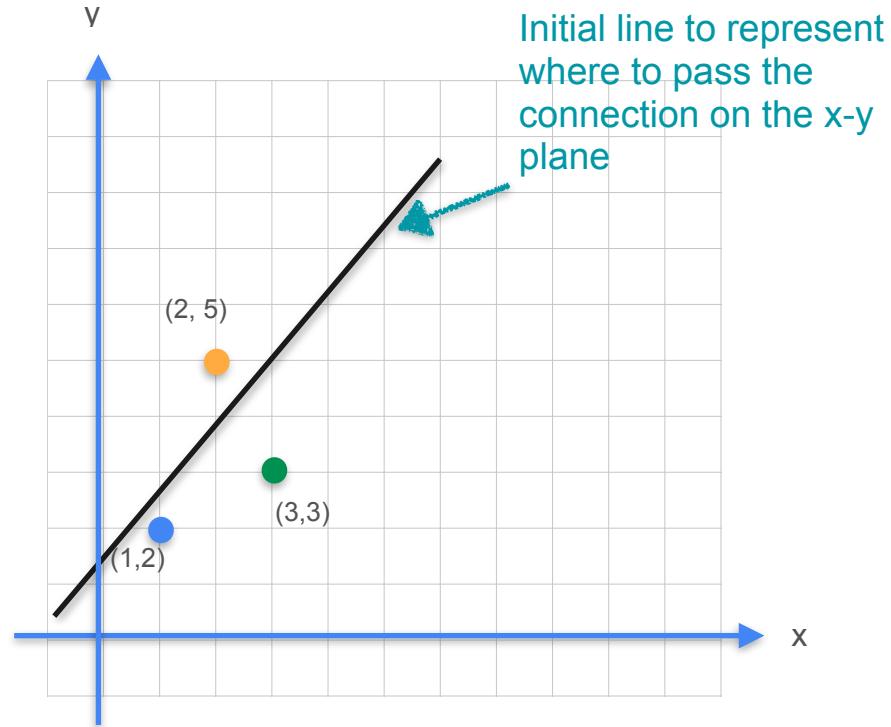
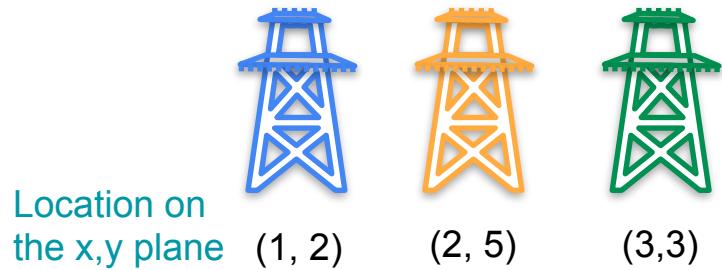
# Linear Regression: Analytical Approach



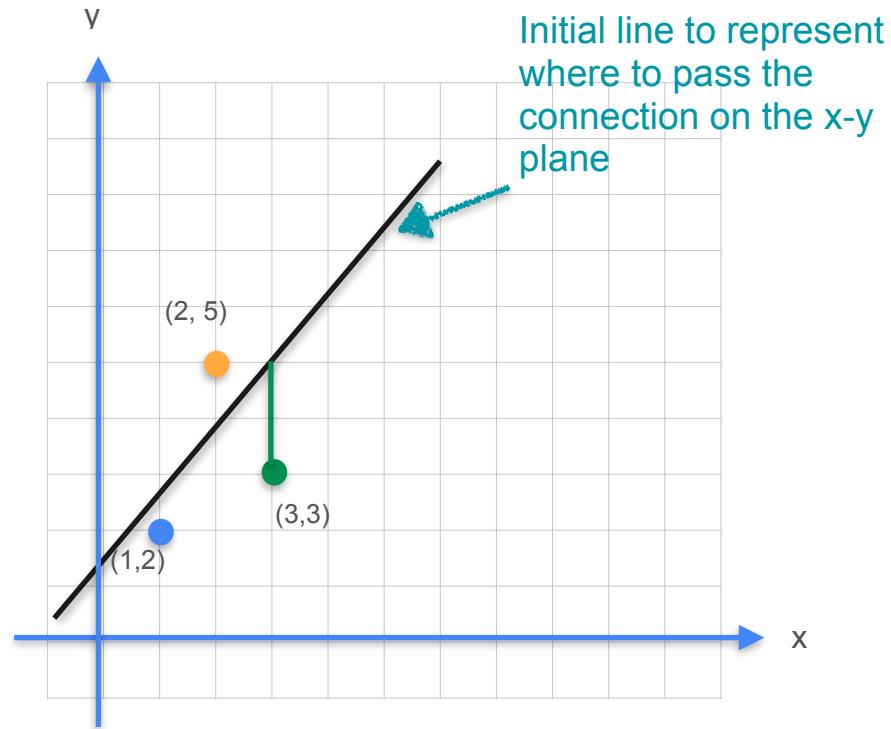
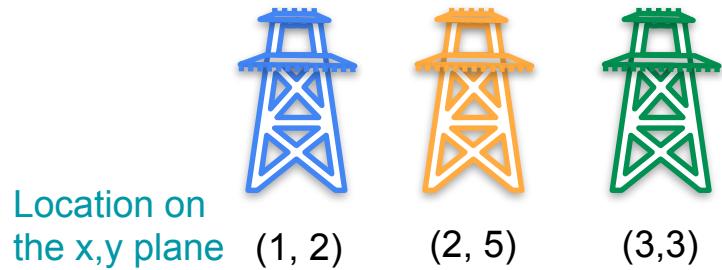
# Linear Regression: Analytical Approach



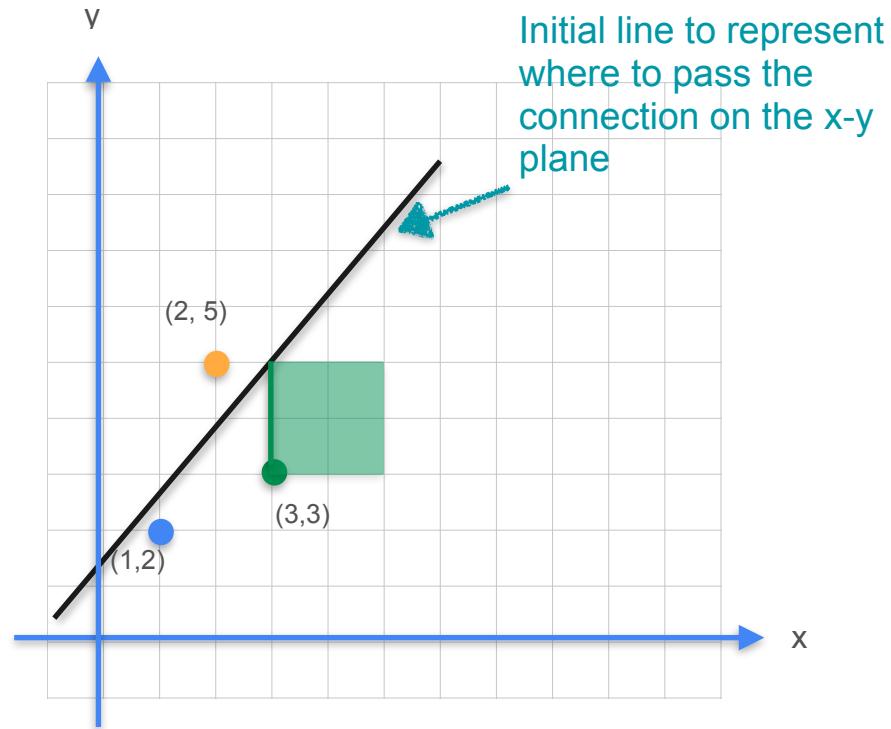
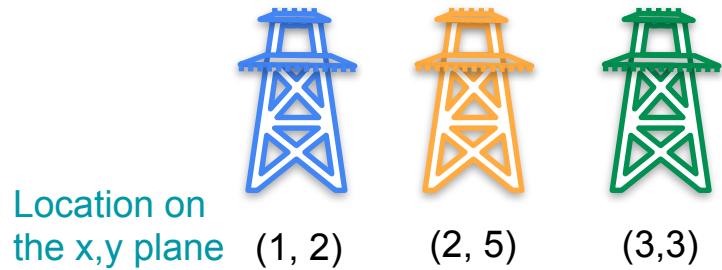
# Linear Regression: Analytical Approach



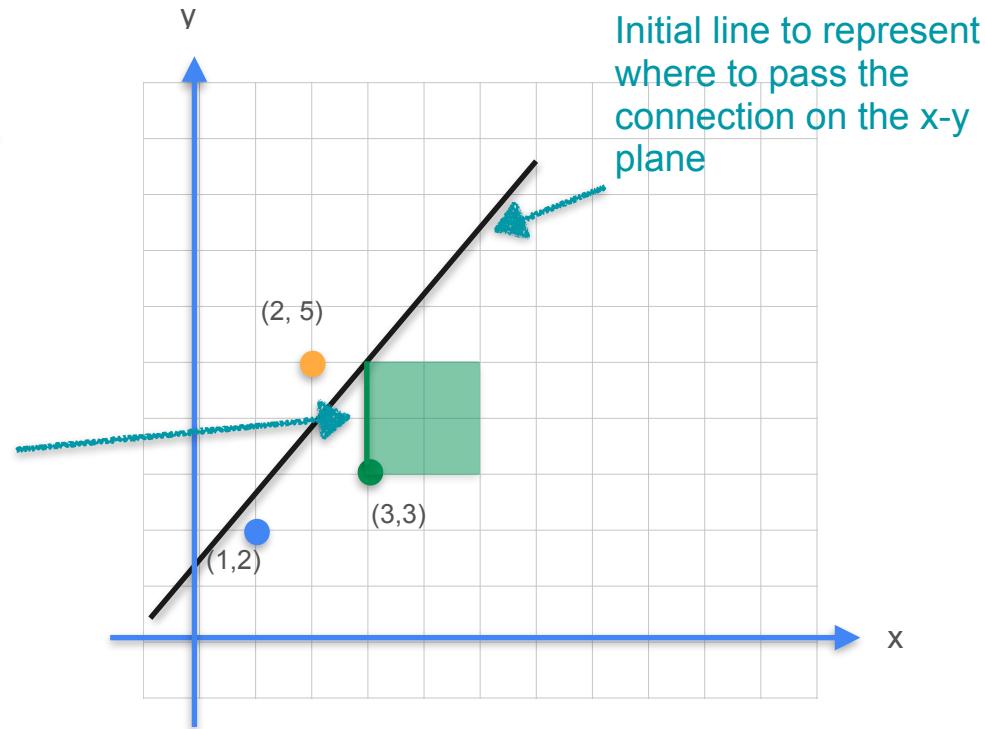
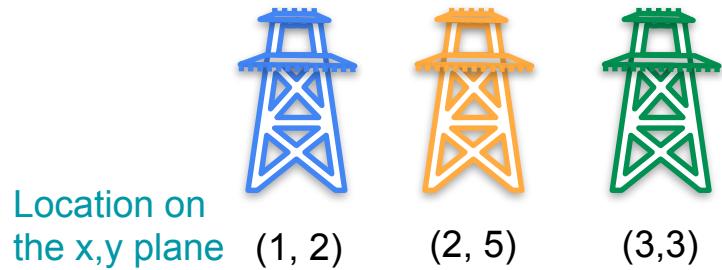
# Linear Regression: Analytical Approach



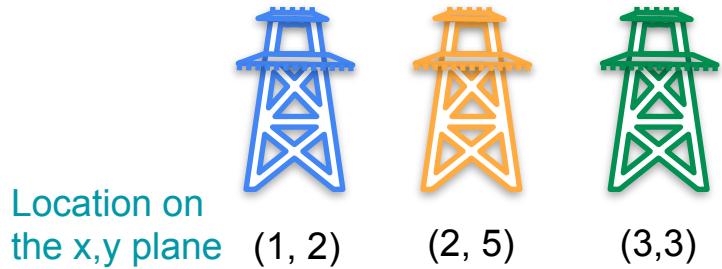
# Linear Regression: Analytical Approach



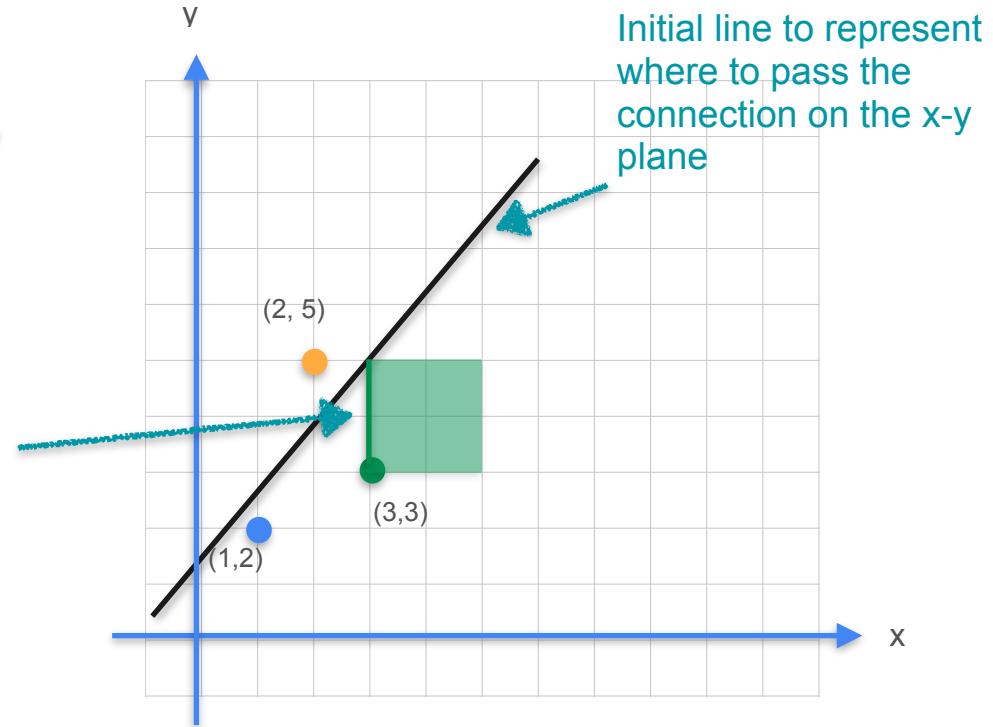
# Linear Regression: Analytical Approach



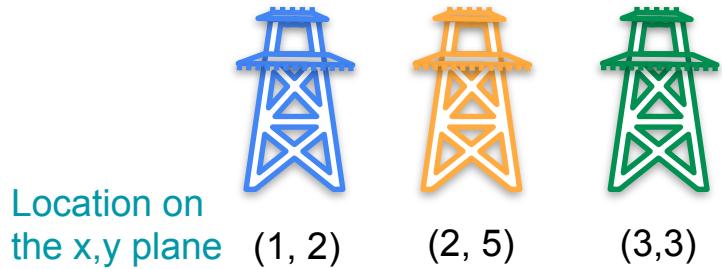
# Linear Regression: Analytical Approach



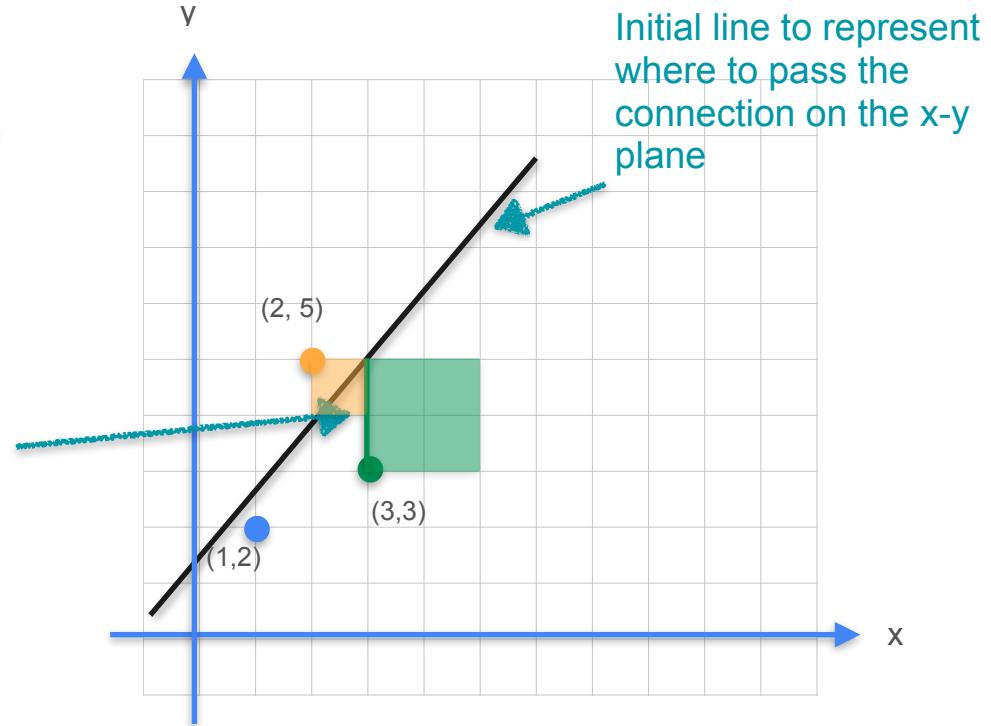
The cost of connecting connection to the powerline



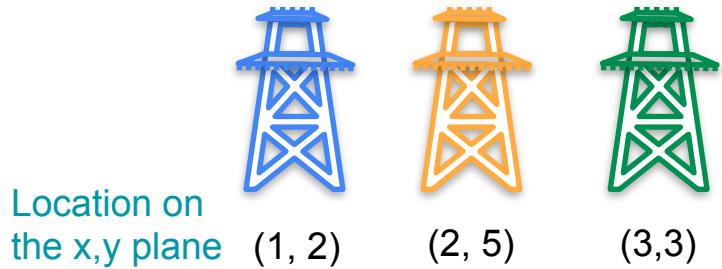
# Linear Regression: Analytical Approach



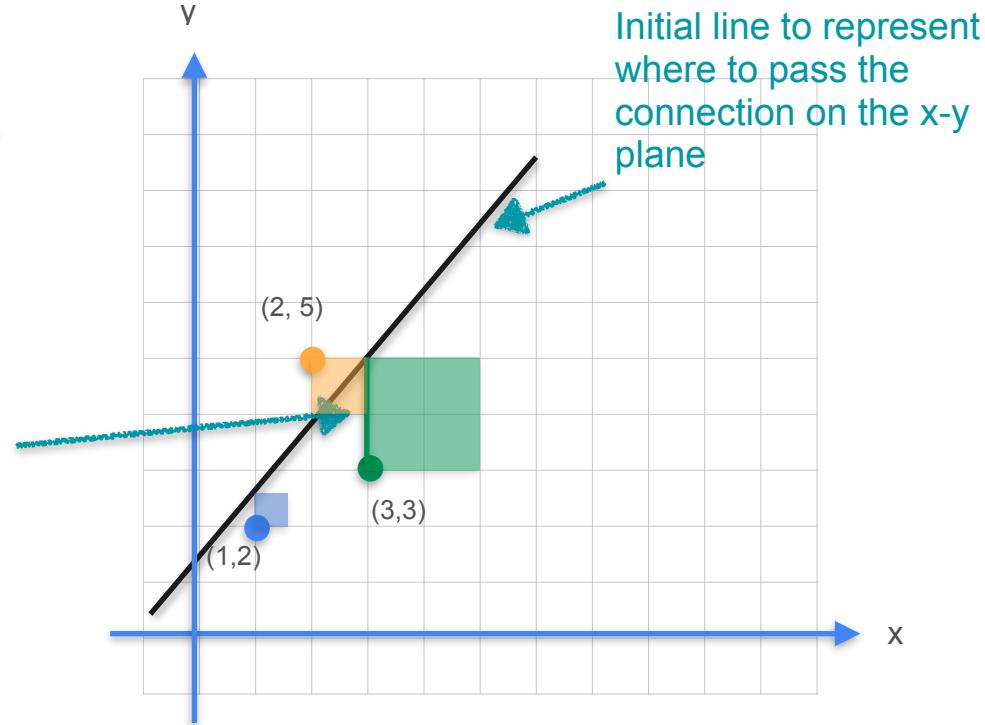
The cost of connecting connection to the powerline



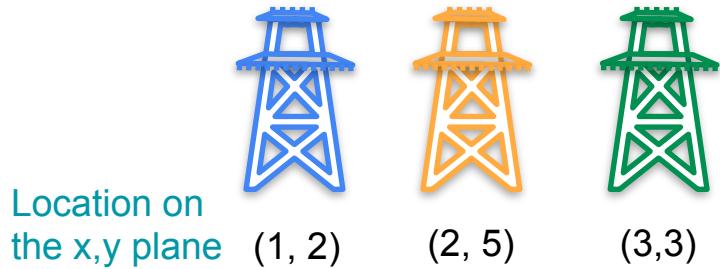
# Linear Regression: Analytical Approach



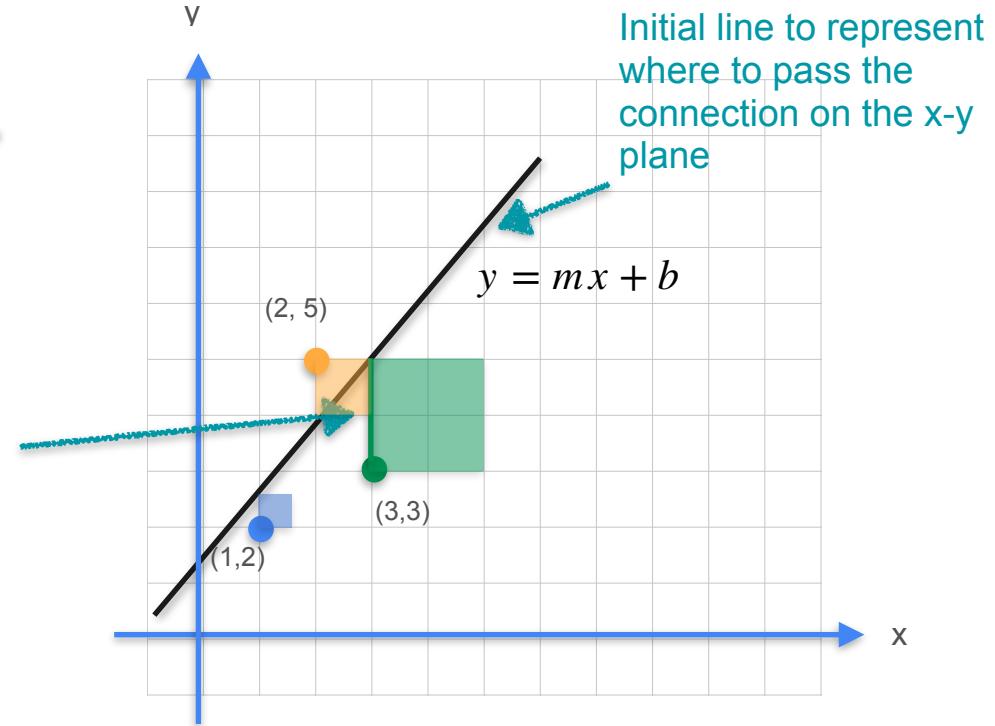
The cost of connecting connection to the powerline



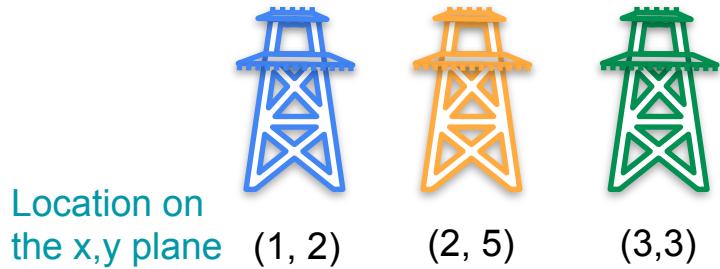
# Linear Regression: Analytical Approach



The cost of connecting connection to the powerline

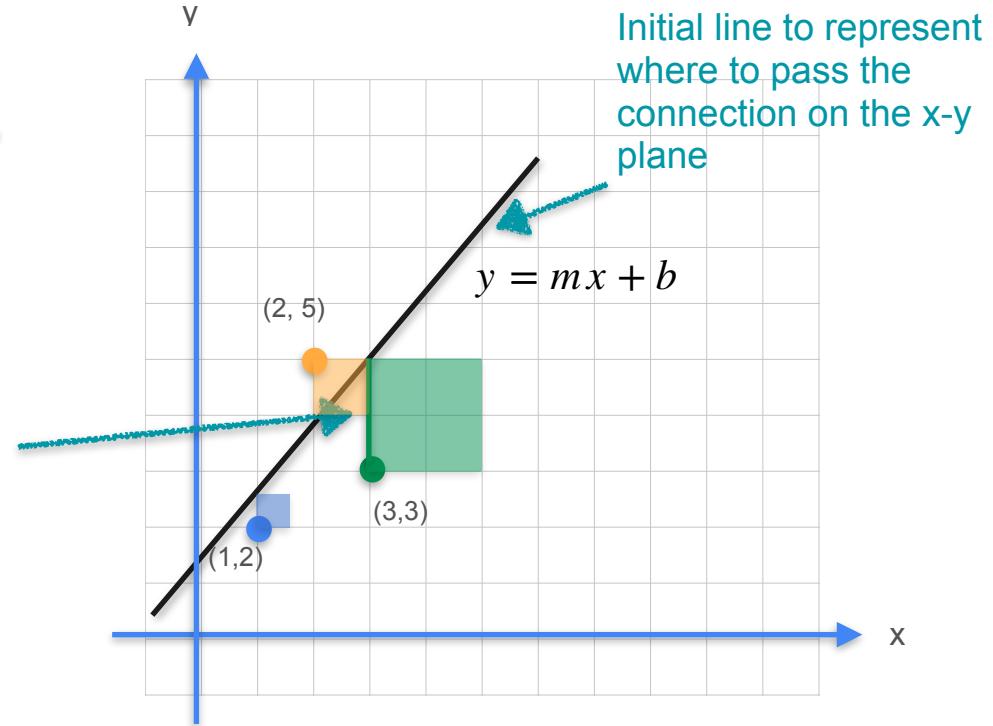


# Linear Regression: Analytical Approach

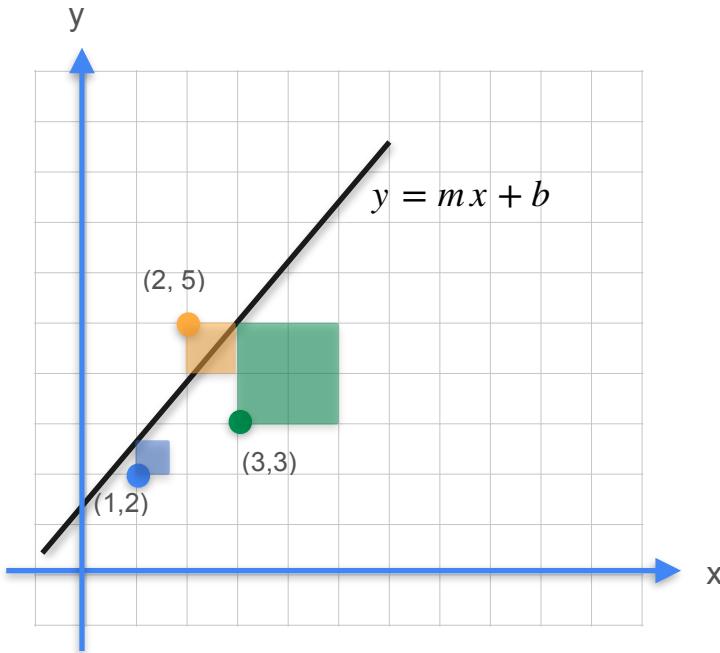


The cost of connecting connection to the powerline

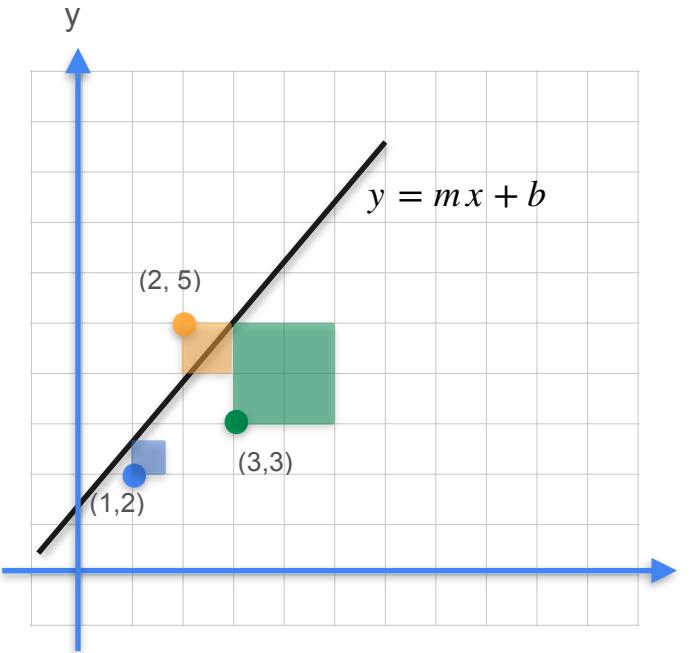
**Goal:** Find  $m, b$  such that you minimize sum of squares cost



# Linear Regression: Analytical Approach

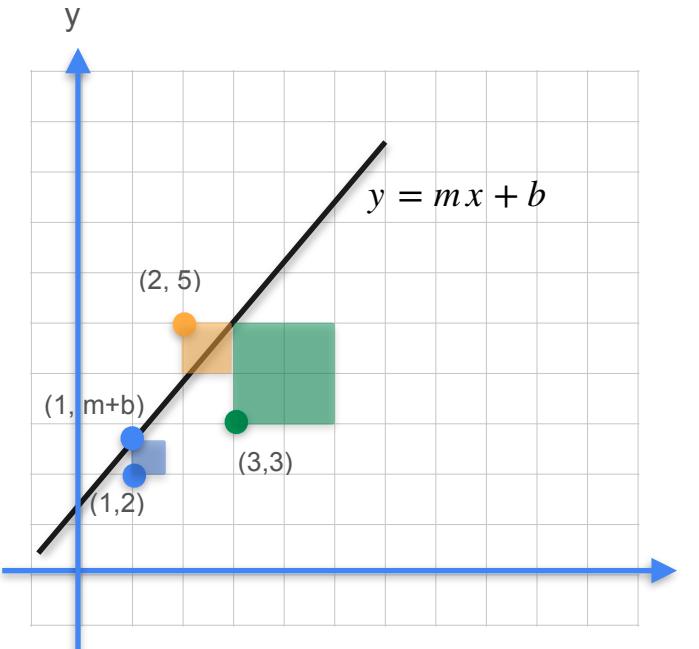


# Linear Regression: Analytical Approach



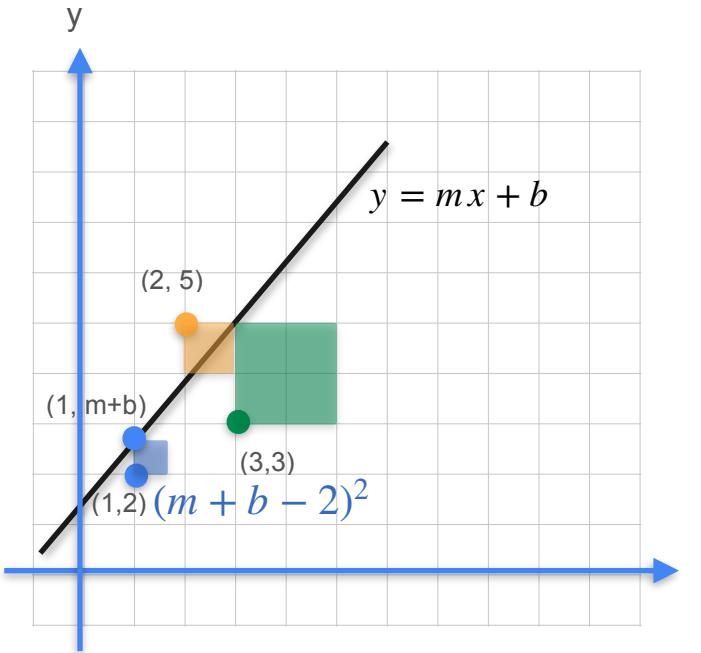
**Goal: Minimize sum of squares cost**

# Linear Regression: Analytical Approach



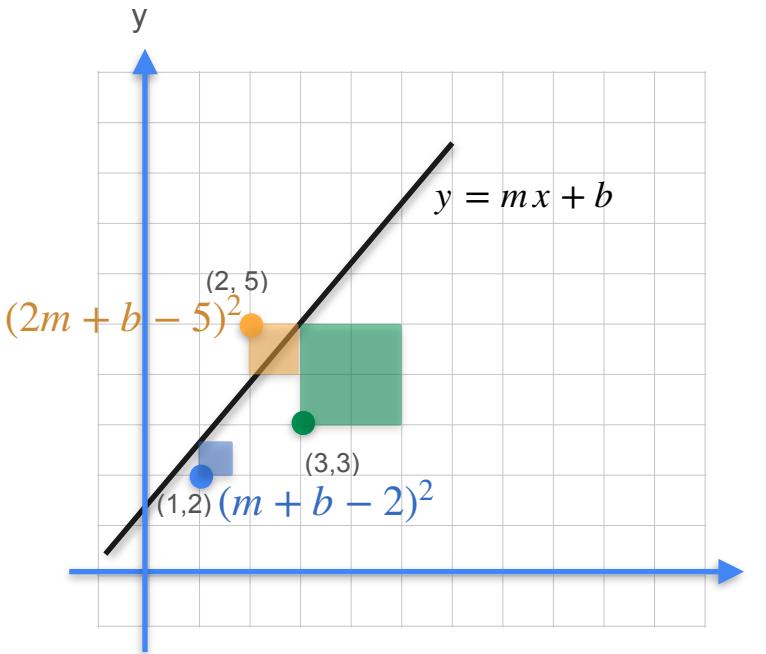
**Goal: Minimize sum of squares cost**

# Linear Regression: Analytical Approach



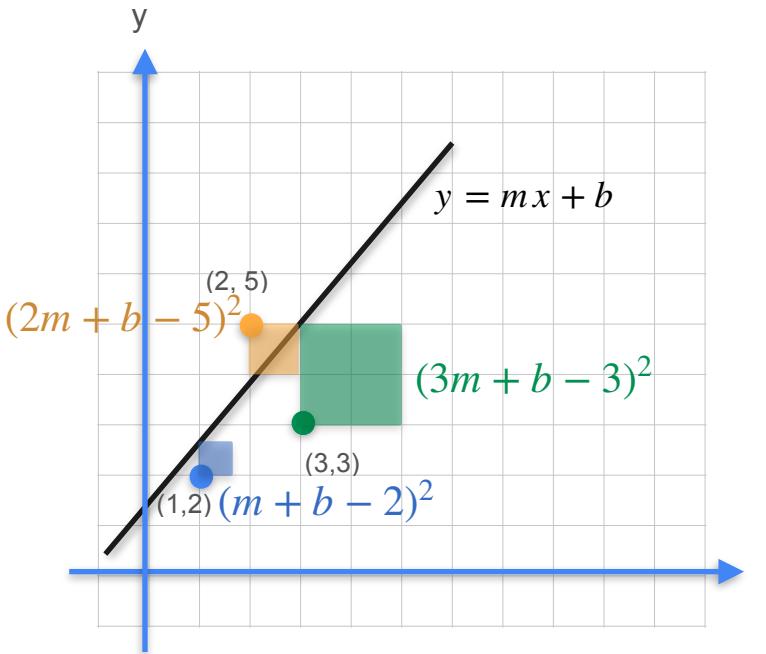
Goal: Minimize sum of squares cost

# Linear Regression: Analytical Approach



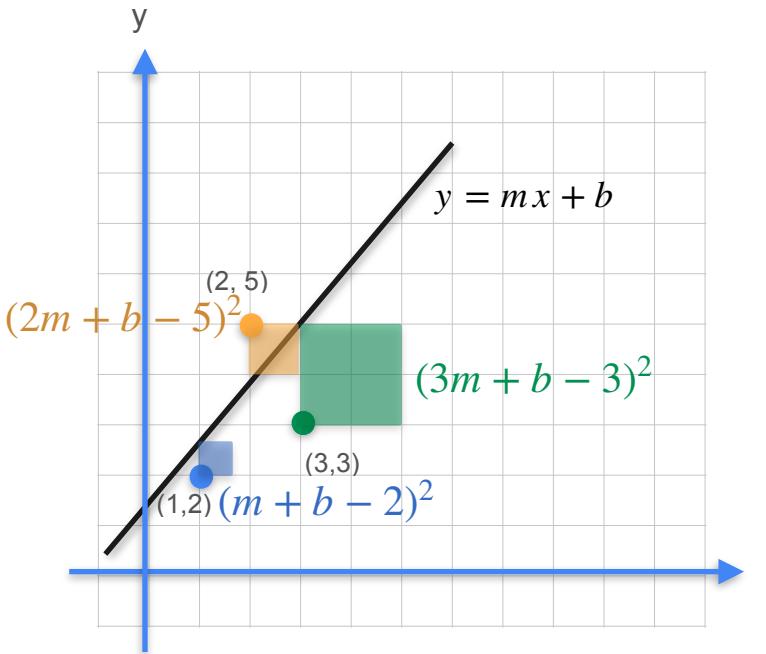
Goal: Minimize sum of squares cost

# Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

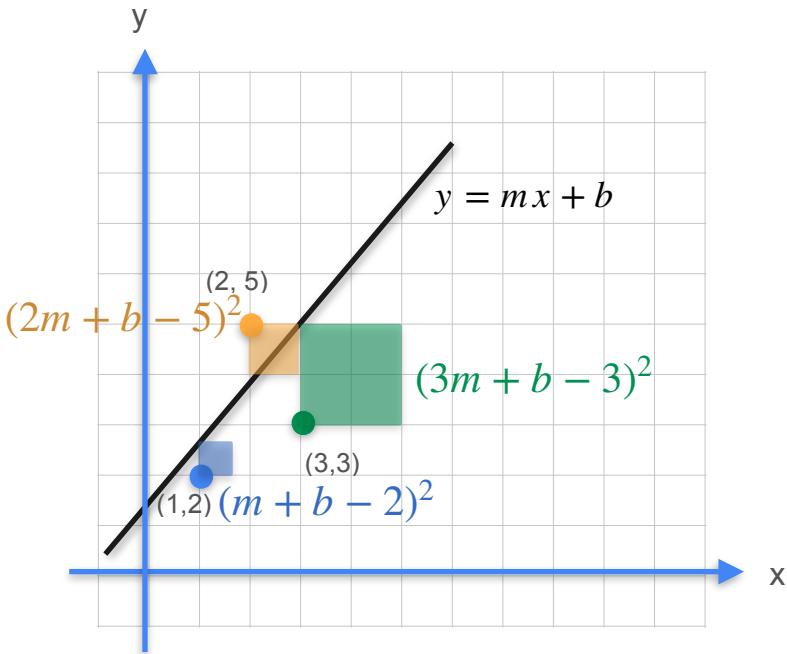
# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

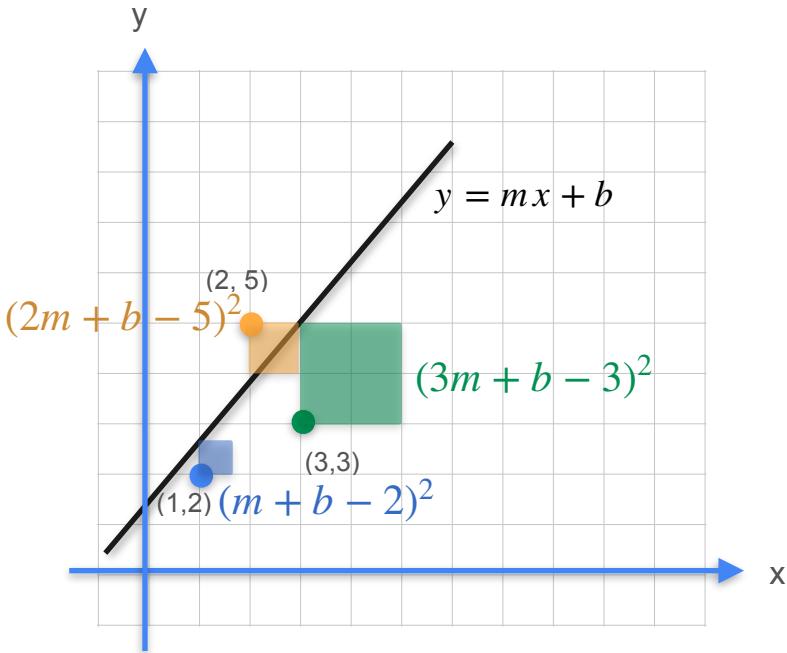
# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$
$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

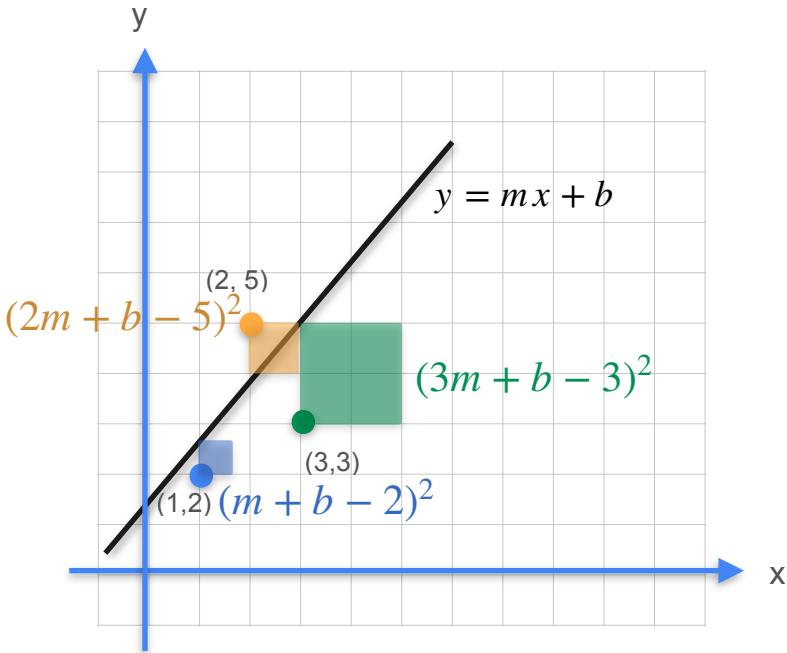
# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$\begin{aligned}(m + b - 2)^2 &+ (2m + b - 5)^2 + (3m + b - 3)^2 \\ m^2 &+ b^2 + 4 + 2mb - 4m - 4b \\ +4m^2 &+ b^2 + 25 + 4mb - 20m - 10b\end{aligned}$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

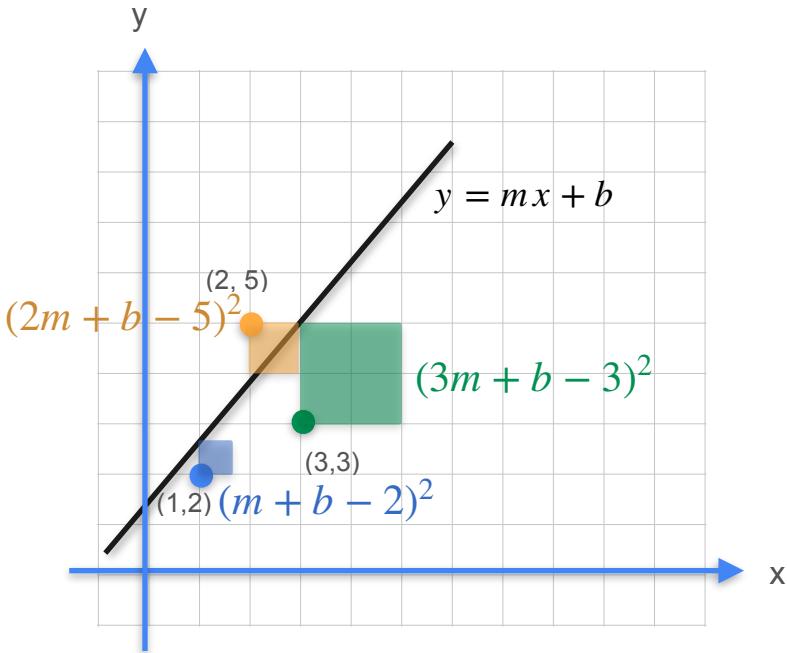
$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$

$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

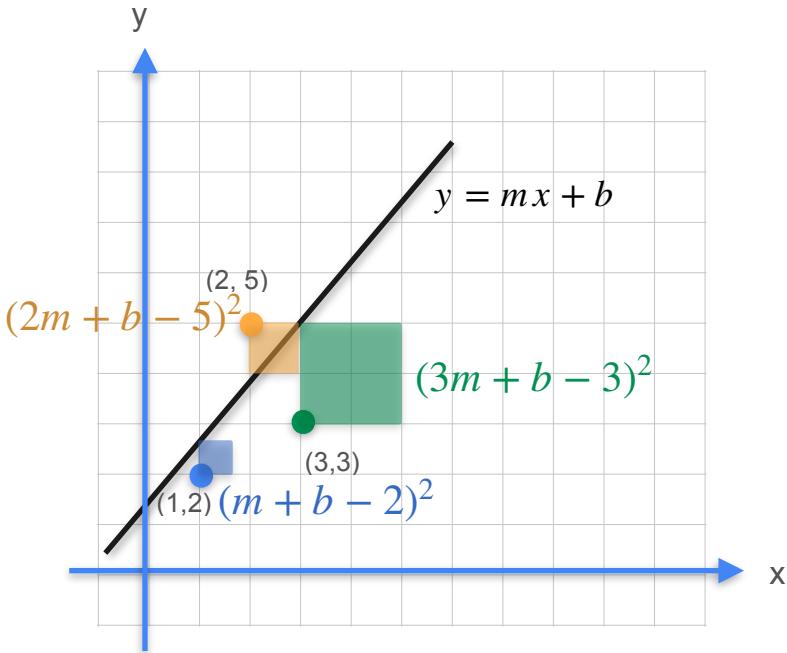
$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$

$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

---

# Linear Regression: Analytical Approach



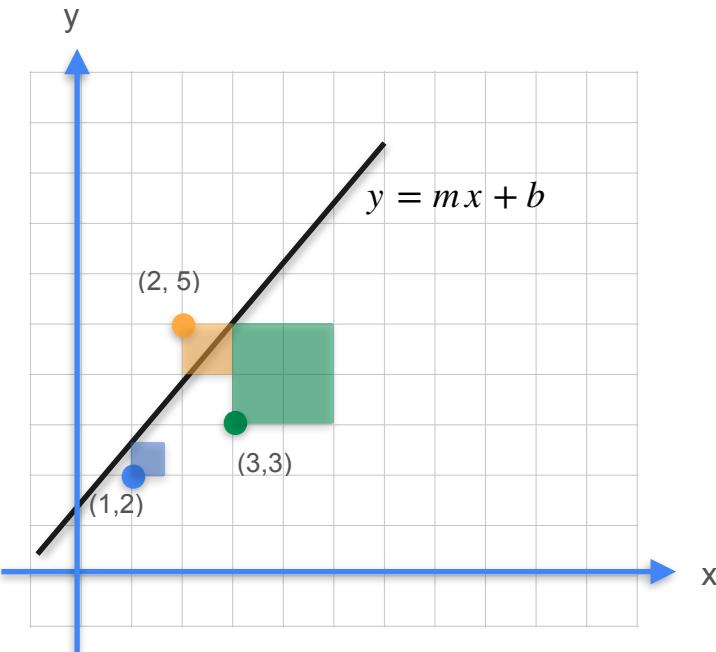
**Goal: Minimize sum of squares cost**

$$\begin{aligned}(m + b - 2)^2 &+ (2m + b - 5)^2 + (3m + b - 3)^2 \\ m^2 &+ b^2 + 4 + 2mb - 4m - 4b \\ &+ 4m^2 + b^2 + 25 + 4mb - 20m - 10b \\ &+ 9m^2 + b^2 + 9 + 6mb - 18m - 6b\end{aligned}$$

---

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

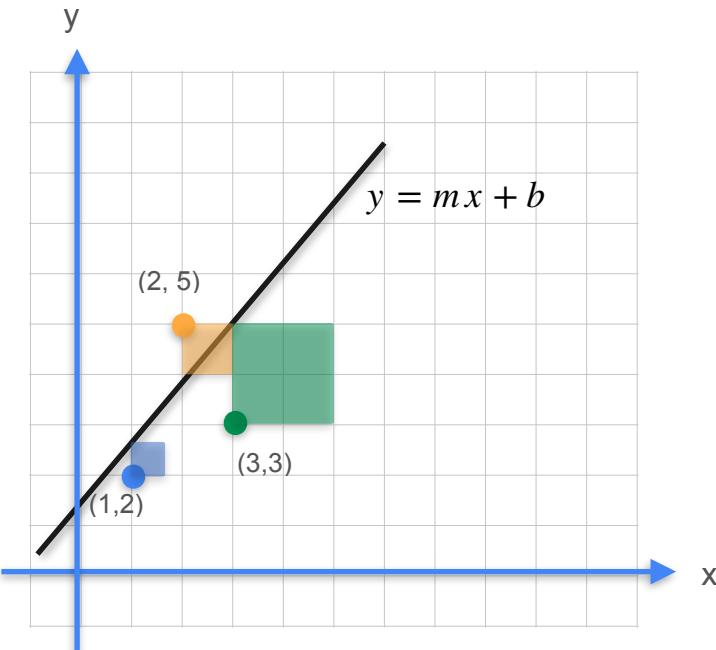
# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

# Linear Regression: Analytical Approach

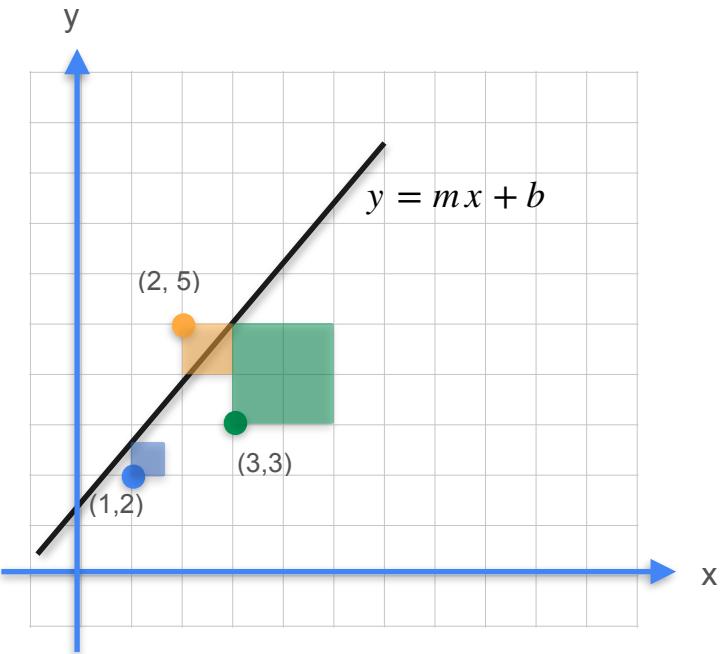


**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 0$$

# Linear Regression: Analytical Approach



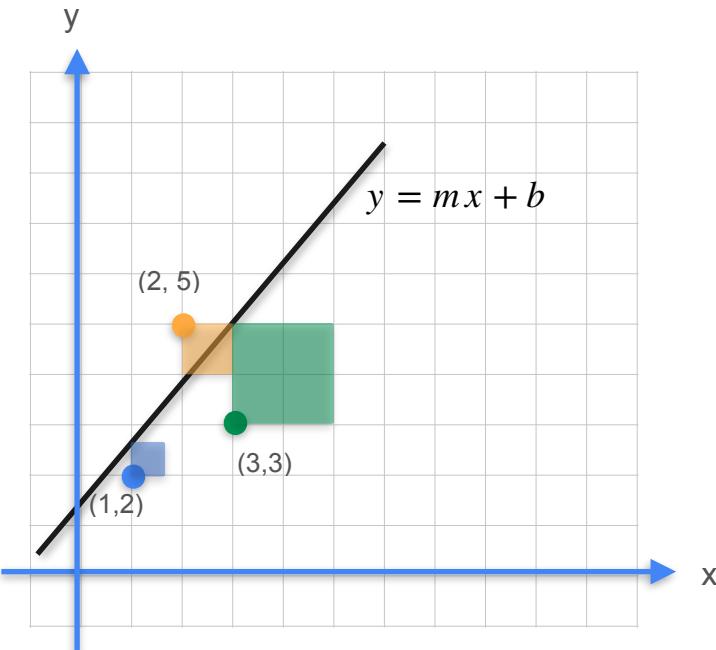
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial E}{\partial b} = 0$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

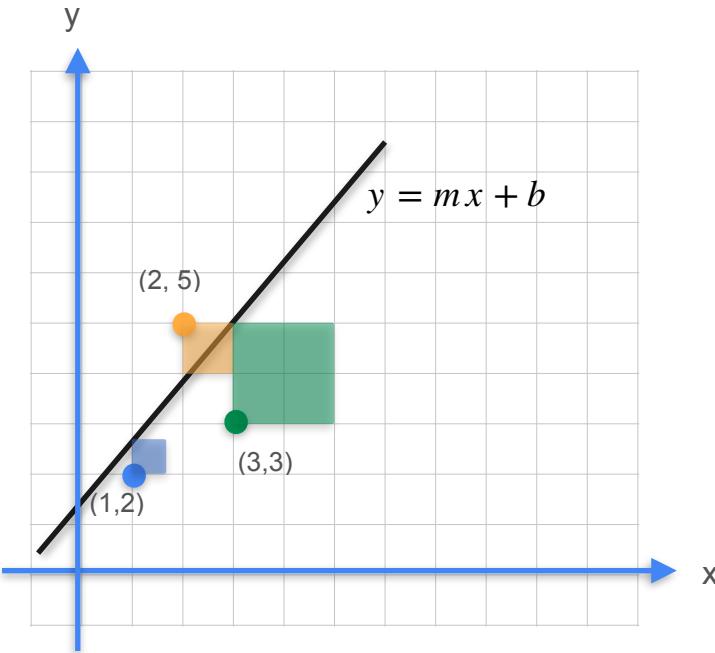
$$\frac{\partial E}{\partial m} = 0$$

**Quiz:**

$$\frac{\partial E}{\partial b} = 0$$

**Find the partial derivative of  $E$  with respect to  $m$**

# Linear Regression: Analytical Approach



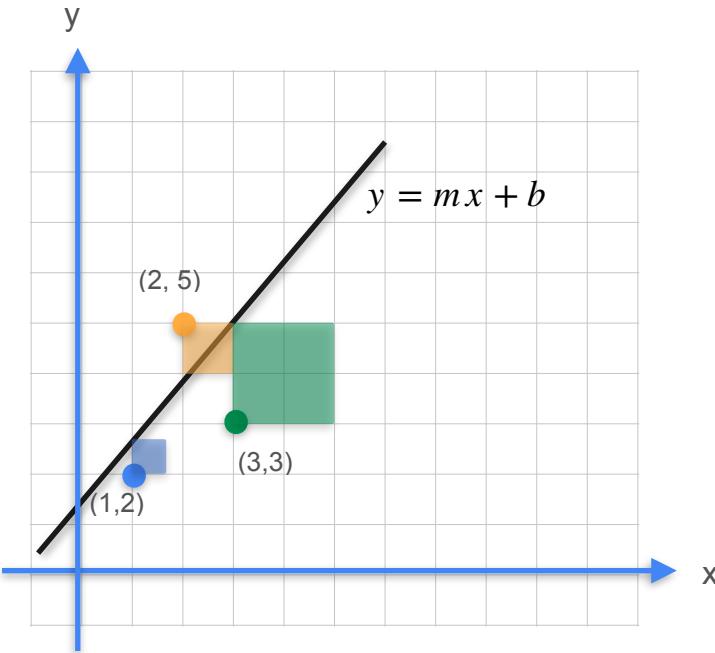
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

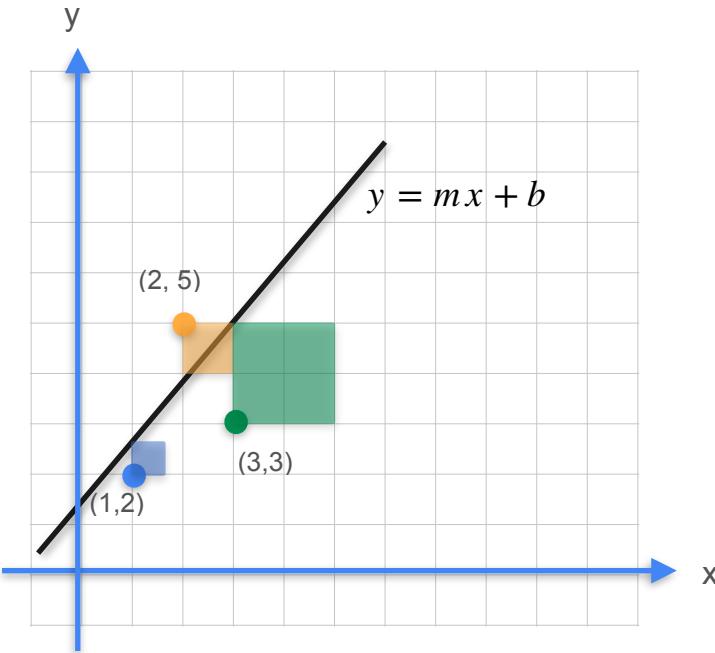
$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

**Quiz:**

**Find the partial derivative of  $E$  with respect to  $b$**

# Linear Regression: Analytical Approach



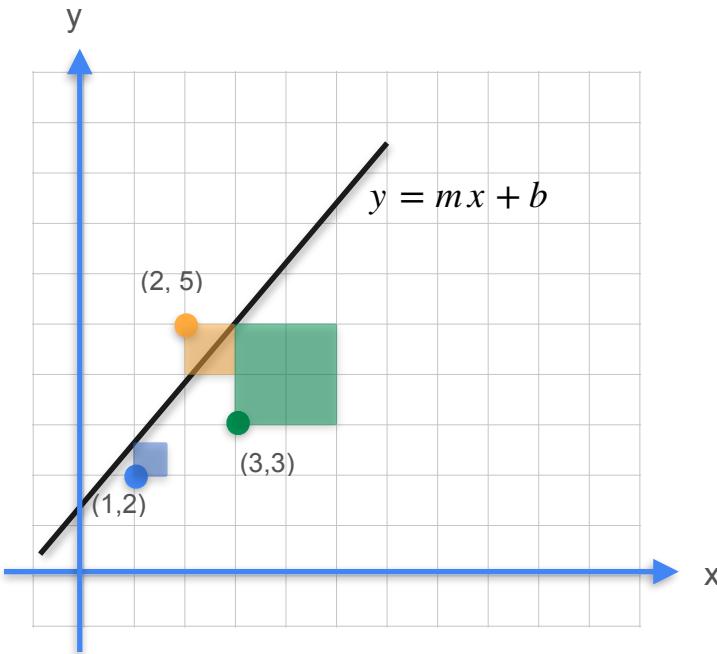
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

# Linear Regression: Analytical Approach



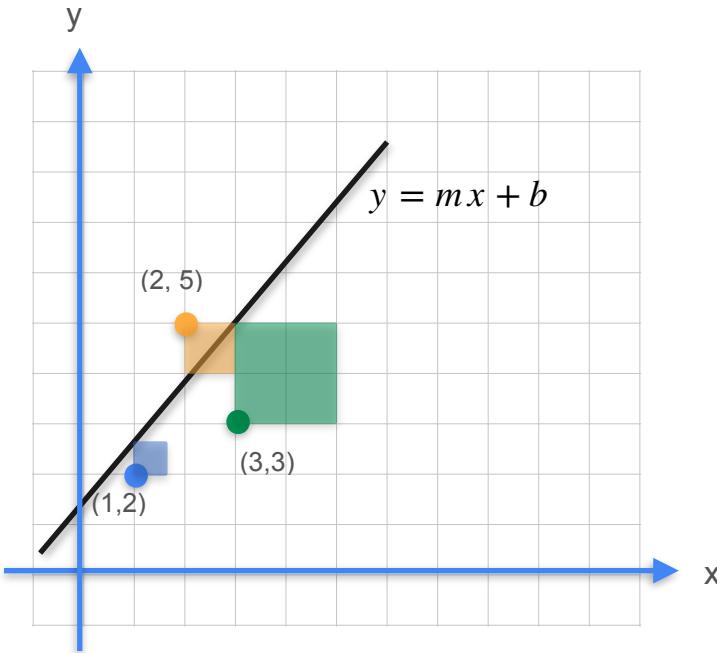
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

# Linear Regression: Analytical Approach



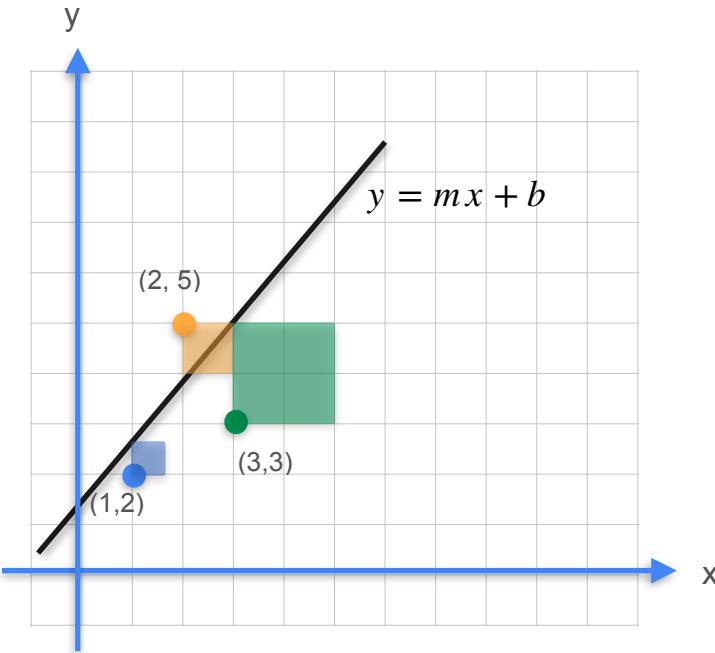
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

# Linear Regression: Analytical Approach



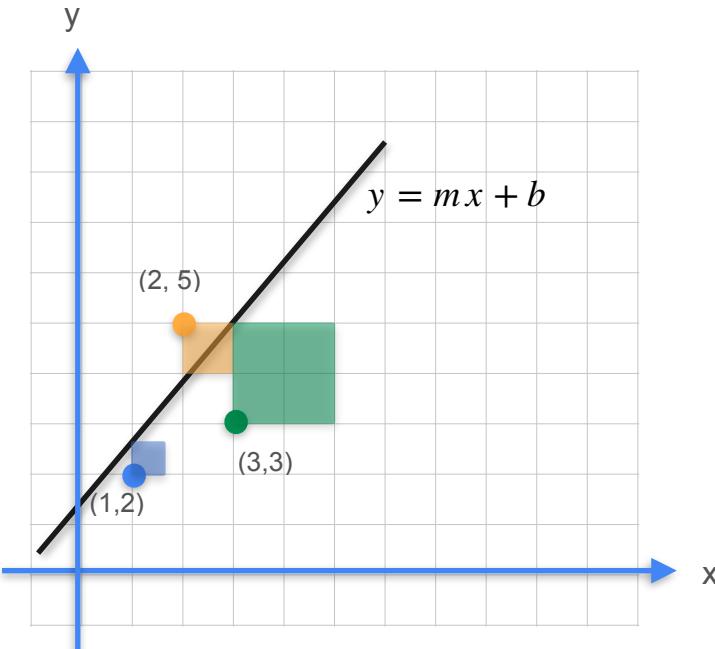
**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

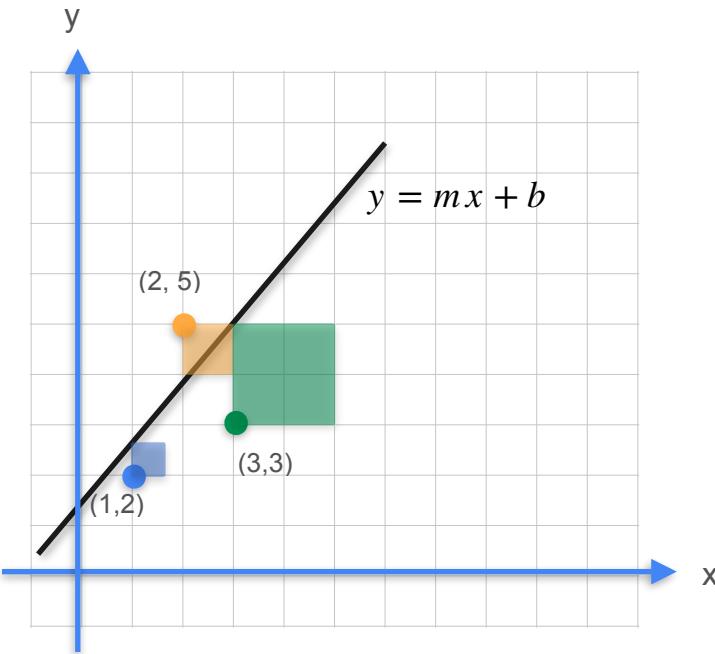
$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$m =$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

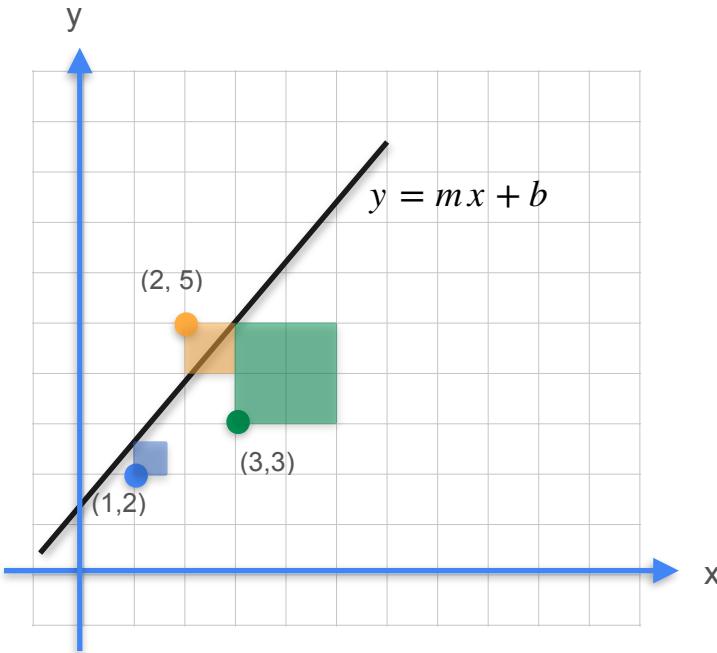
$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$m =$$

$$b =$$

# Linear Regression: Analytical Approach



**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$12b + 24m - 40 = 0$$



# Linear Regression: Analytical Approach

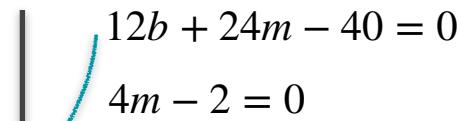
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$\begin{aligned} 12b + 24m - 40 &= 0 \\ 4m - 2 &= 0 \end{aligned}$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$\begin{aligned} 12b + 24m - 40 &= 0 \\ 4m - 2 &= 0 \\ m &= \frac{2}{4} = 0.5 \end{aligned}$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

# Linear Regression: Analytical Approach

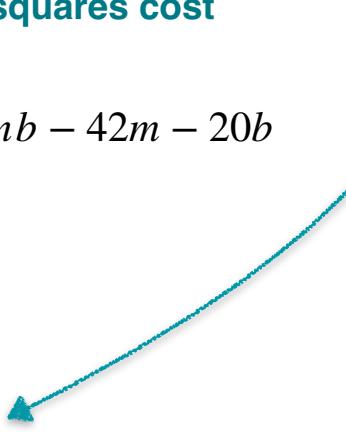
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

$$b = \frac{14}{6} = \frac{7}{3}$$

# Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

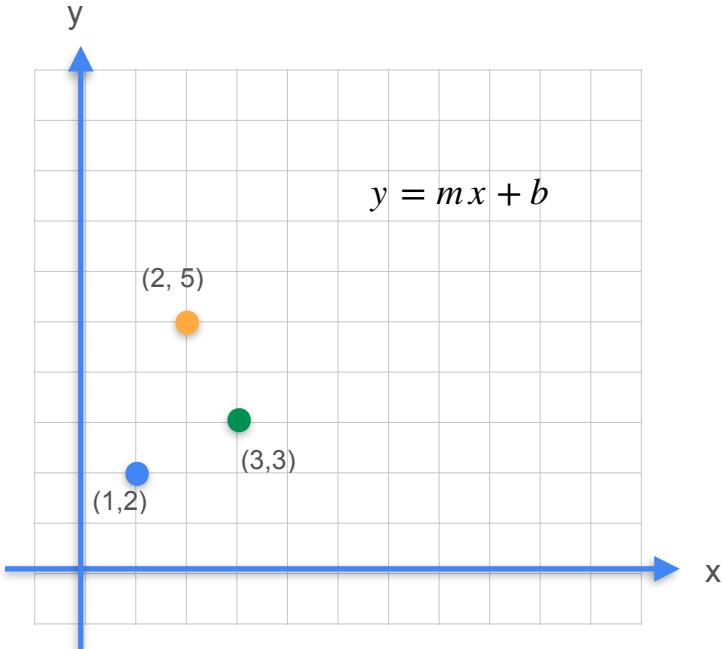
$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

# Linear Regression: Optimal Solution

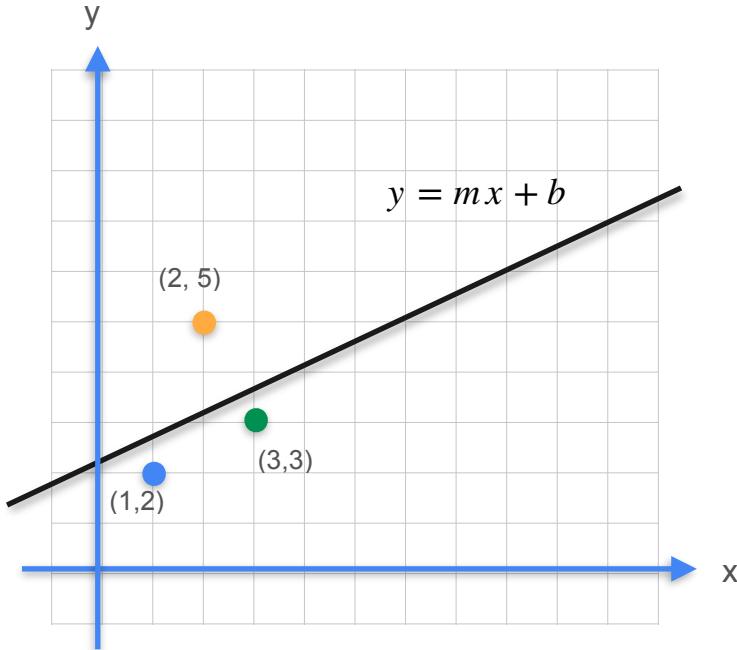


$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

# Linear Regression: Optimal Solution

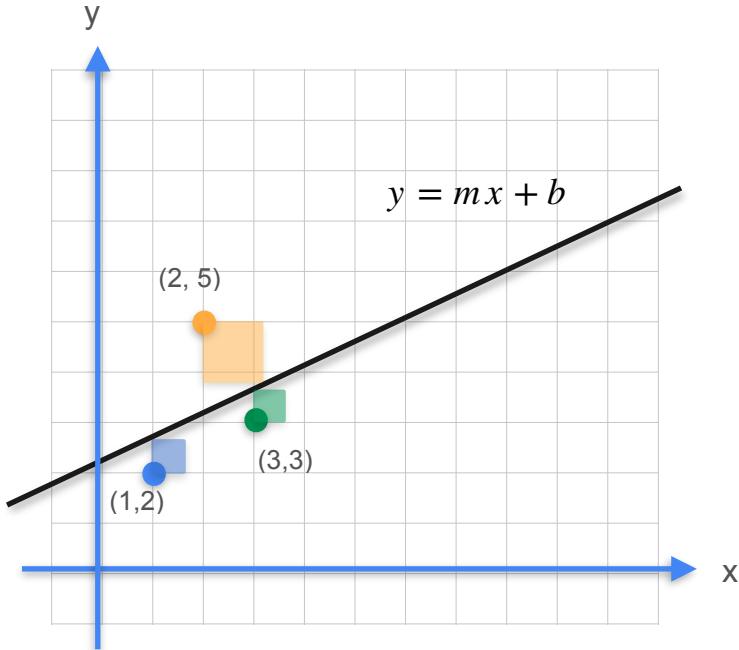


$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

# Linear Regression: Optimal Solution



$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

# Linear Regression: Gradient Descent

**Goal: Minimize sum of squares cost**

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

**Gradient Descent to the rescue**

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



DeepLearning.AI

# Gradients and Gradient Descent

---

**Optimization using Gradient  
Descent in one variable -  
Part 1**

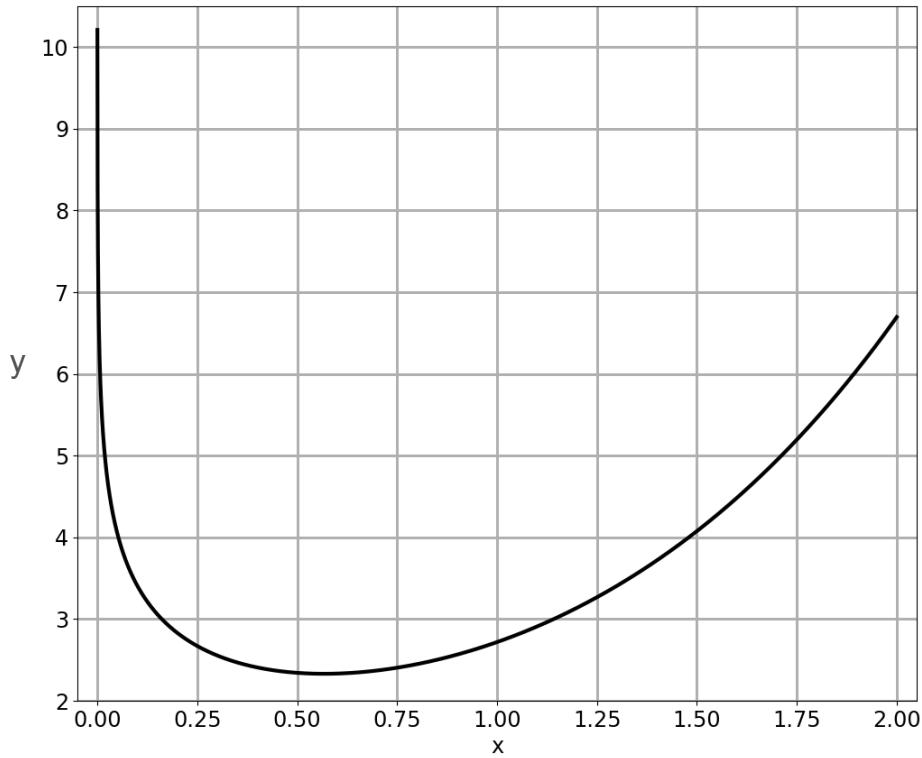
# Hard To Optimize Functions

# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

# Hard To Optimize Functions

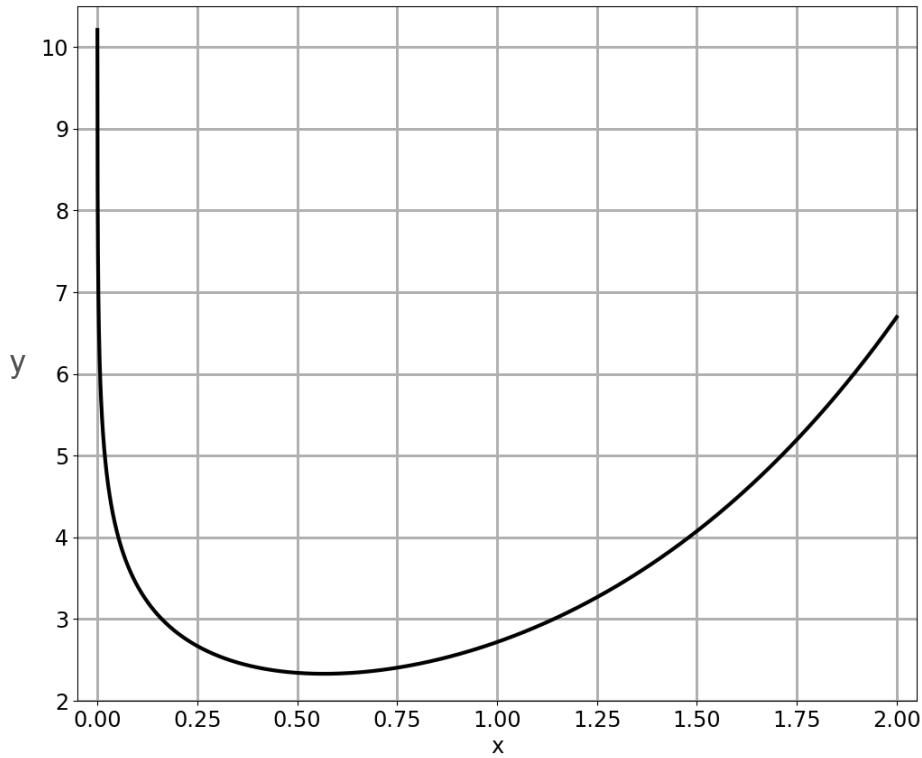
$$f(x) = e^x - \log(x)$$



# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

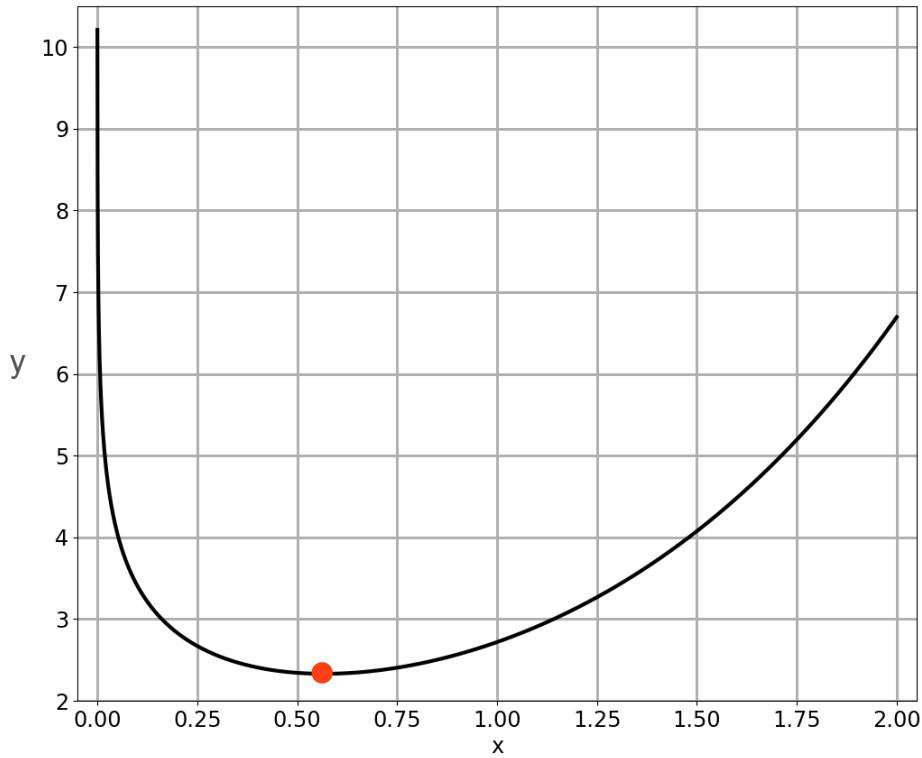
Minimum?



# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

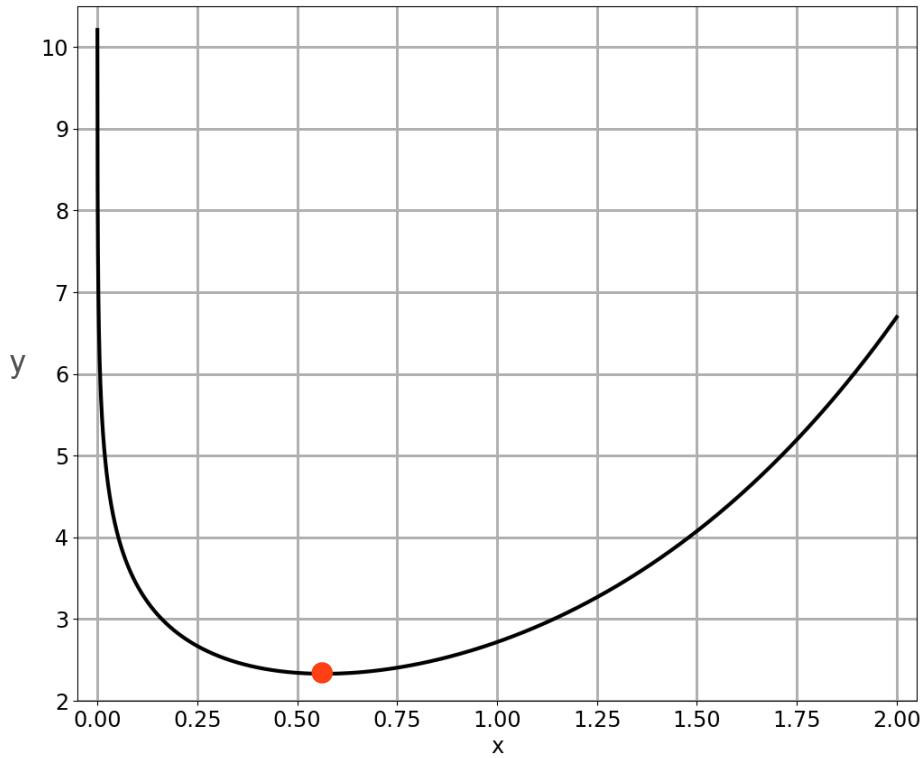


# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x)=0$$

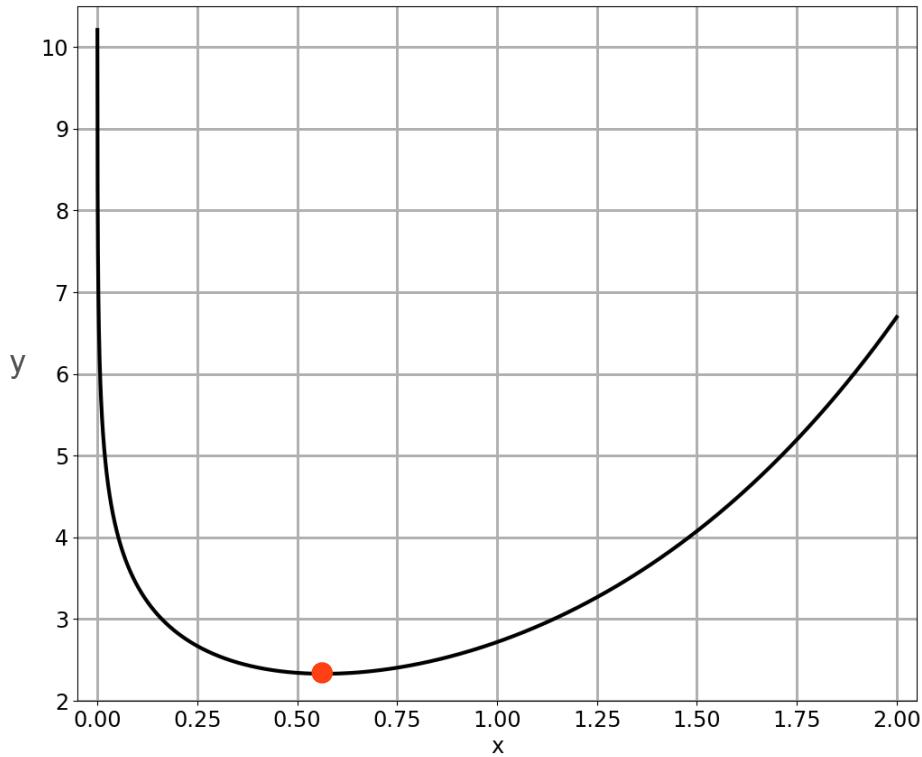


# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

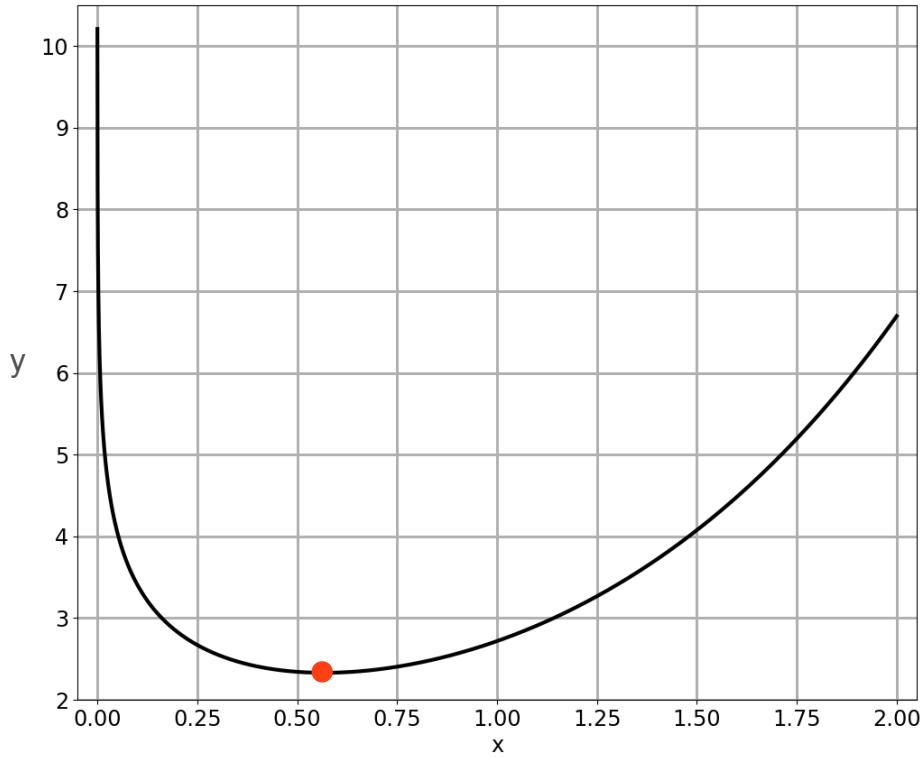


# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

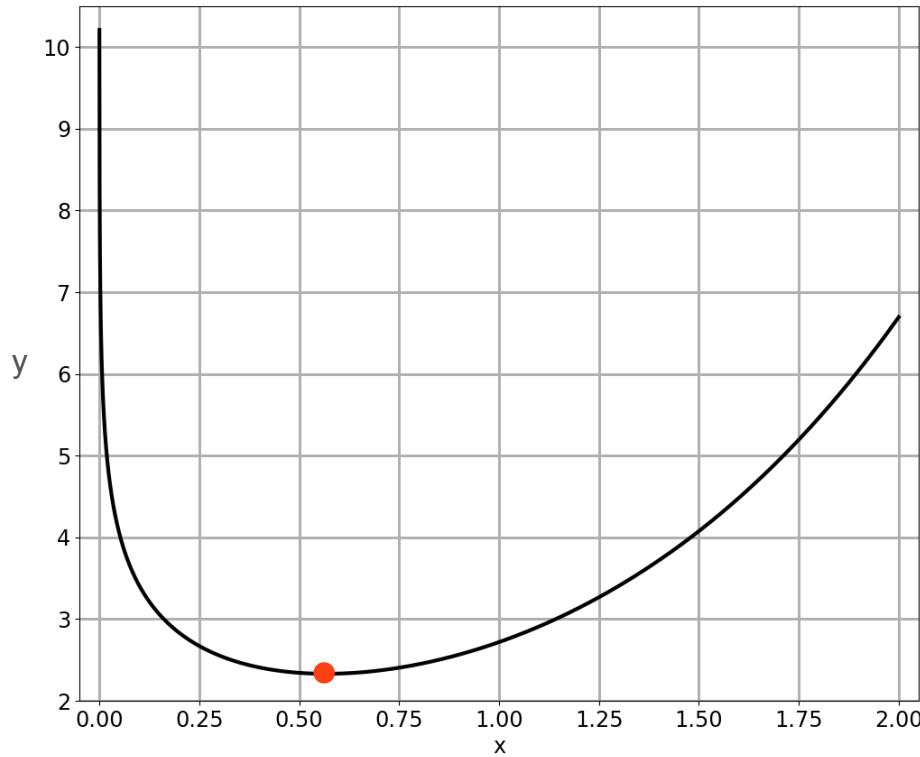


# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

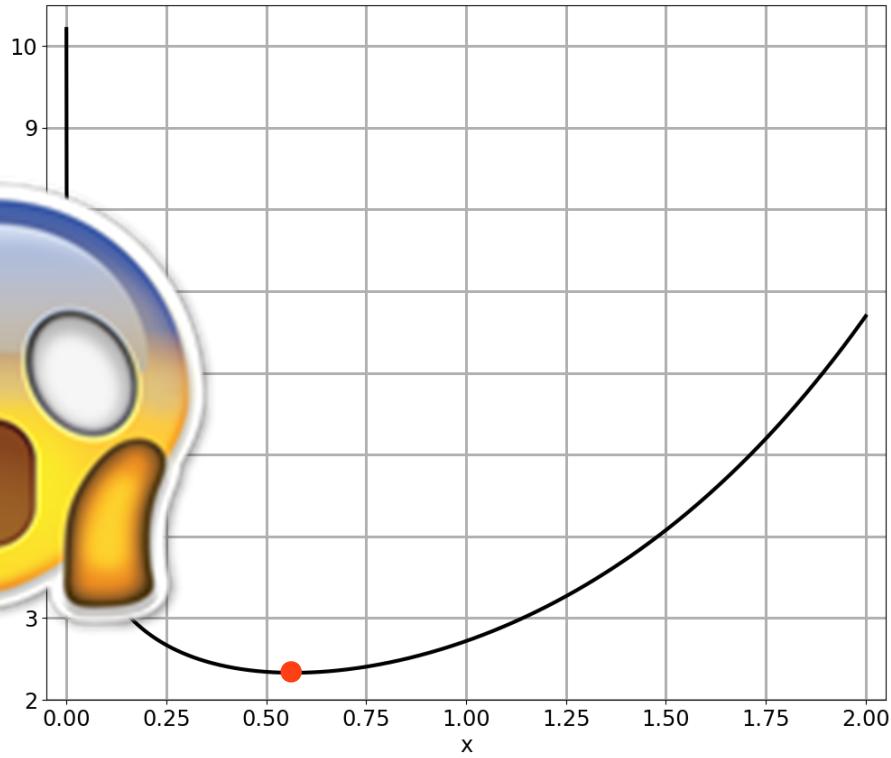


# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

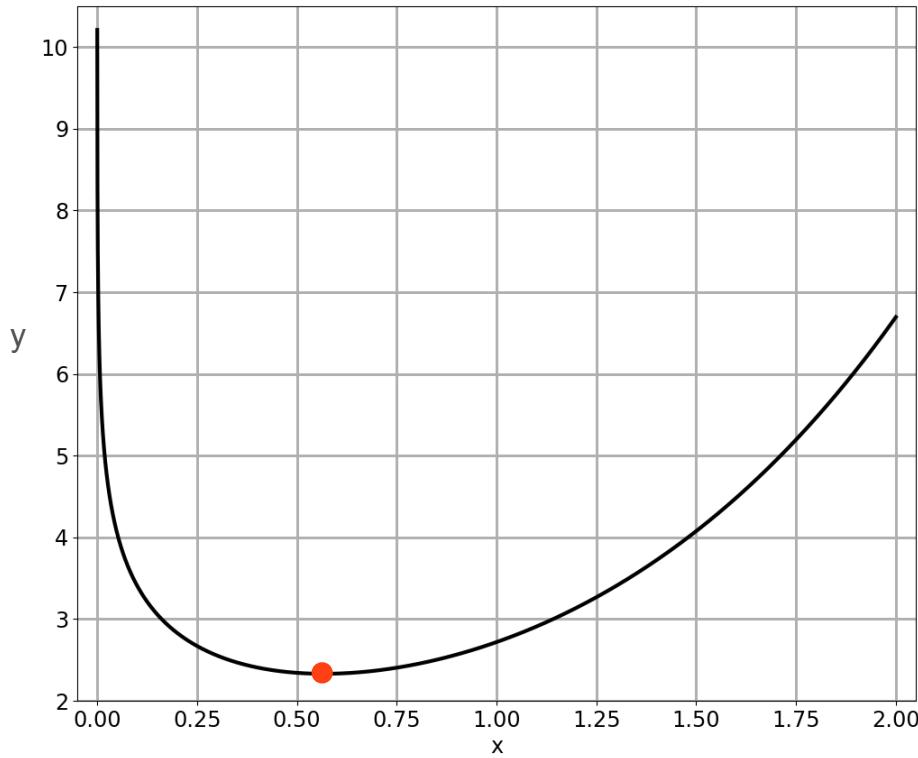
$$f'(x) = e^x - \frac{1}{x} = 0$$



# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

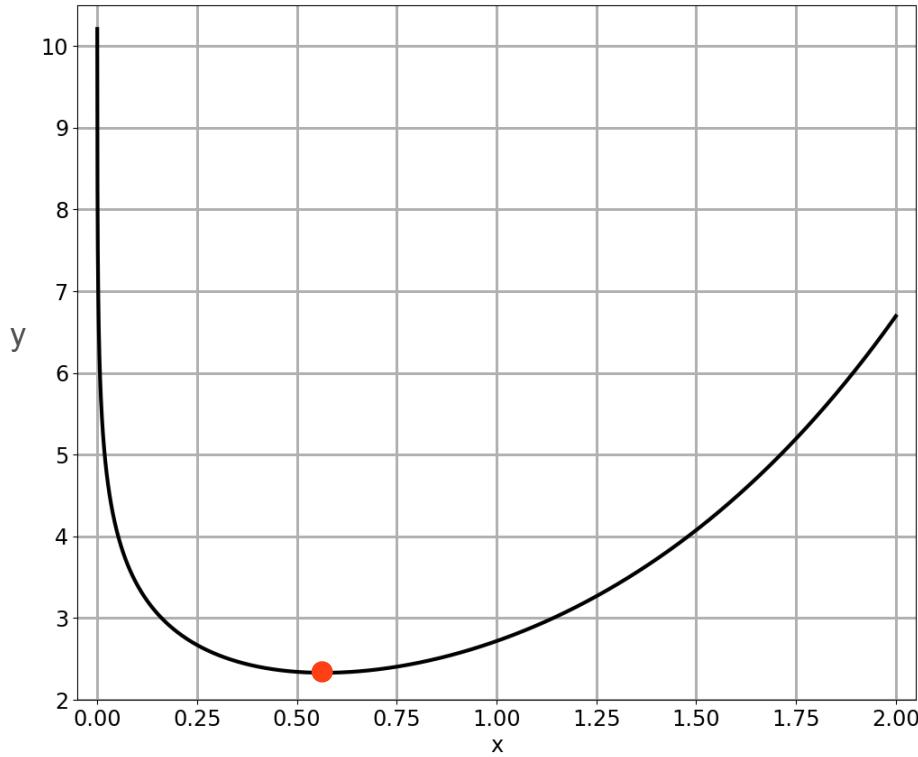
$$f'(x) = e^x - \frac{1}{x}$$



# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$



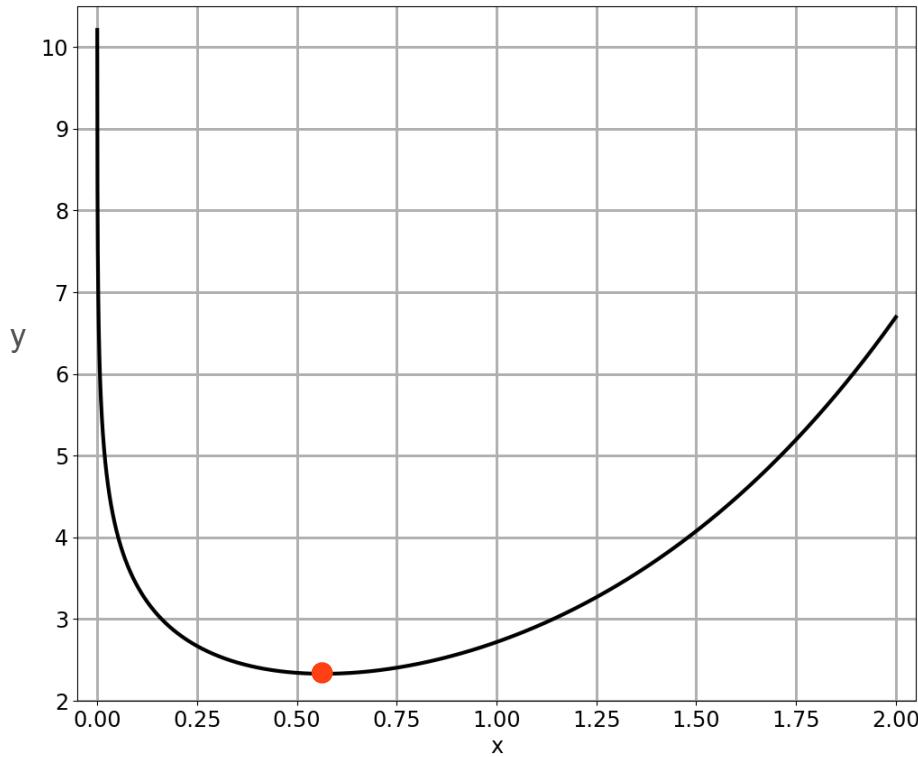
# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$



$$e^x = \frac{1}{x}$$



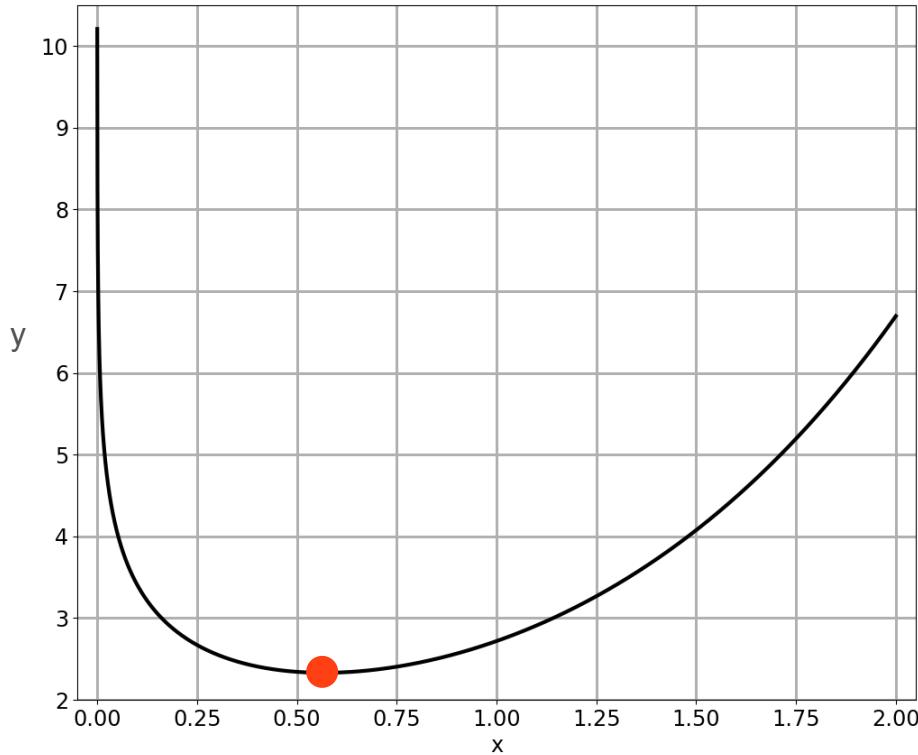
# Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

→  $e^x = \frac{1}{x}$

Solution:  $x = 0.5671\dots$



# Hard To Optimize Functions

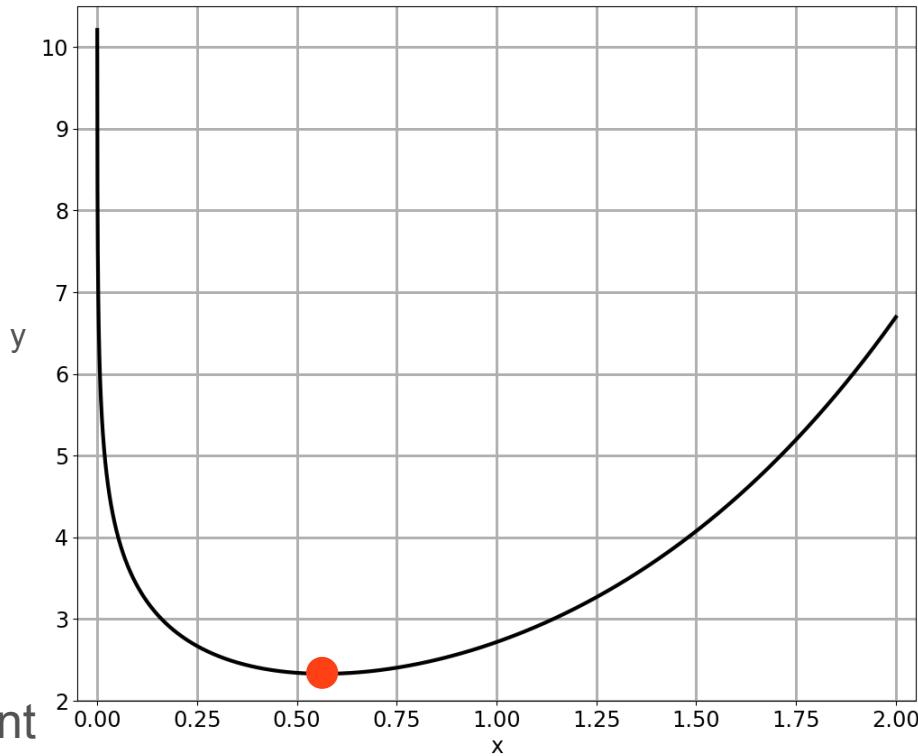
$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

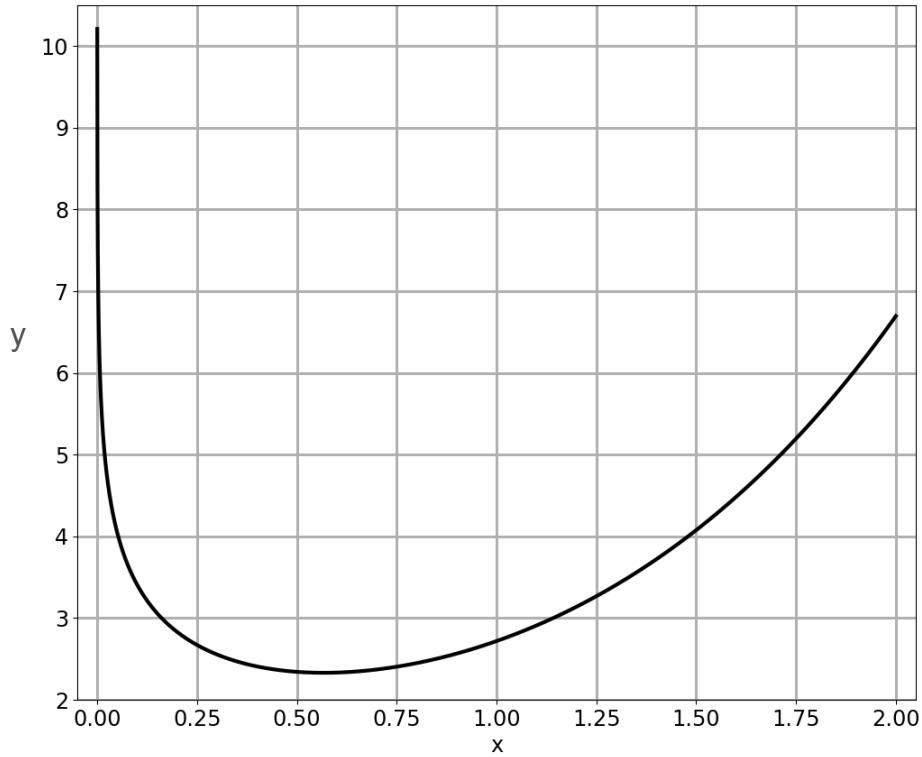
→  $e^x = \frac{1}{x}$

Solution:  $x = 0.5671\dots$

Also known as the Omega constant

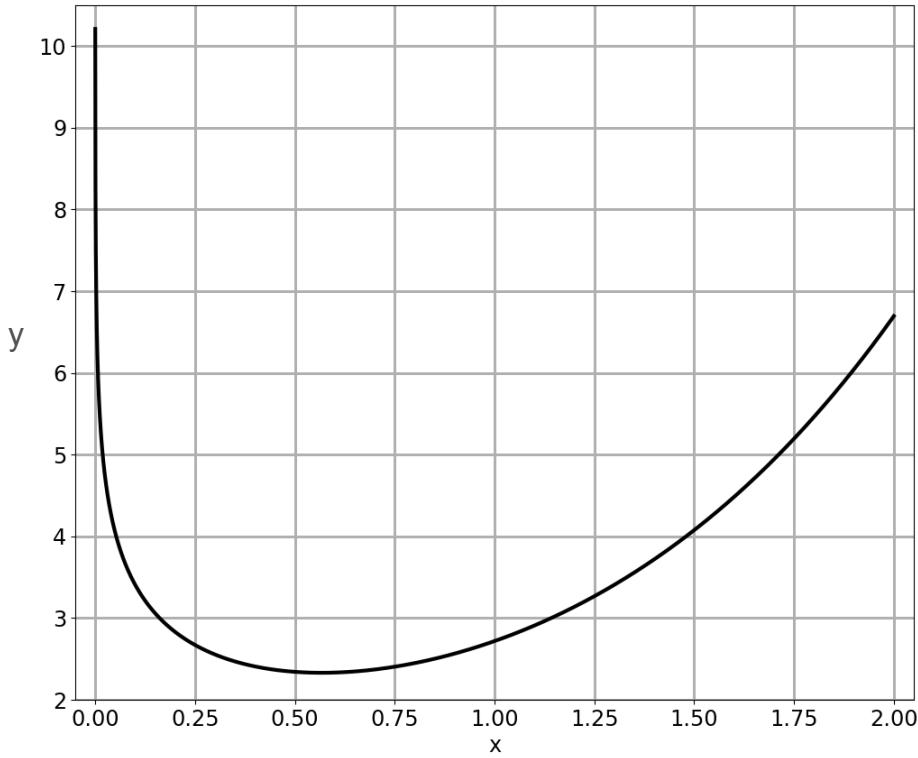


# Method 1: Try Both Directions



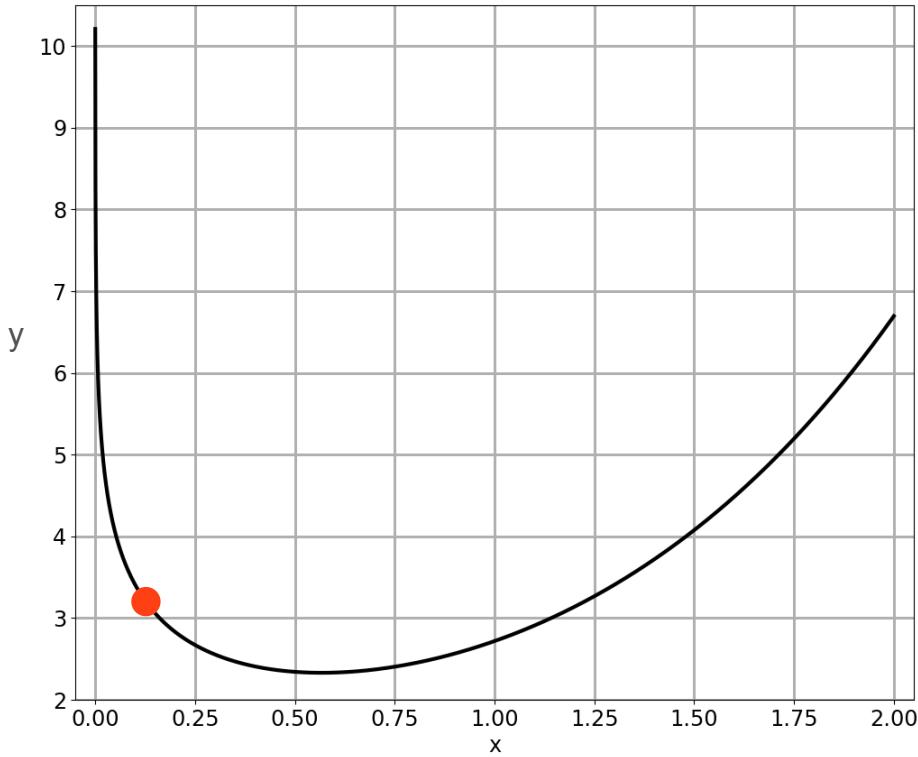
# Method 1: Try Both Directions

Is there any  
other way?



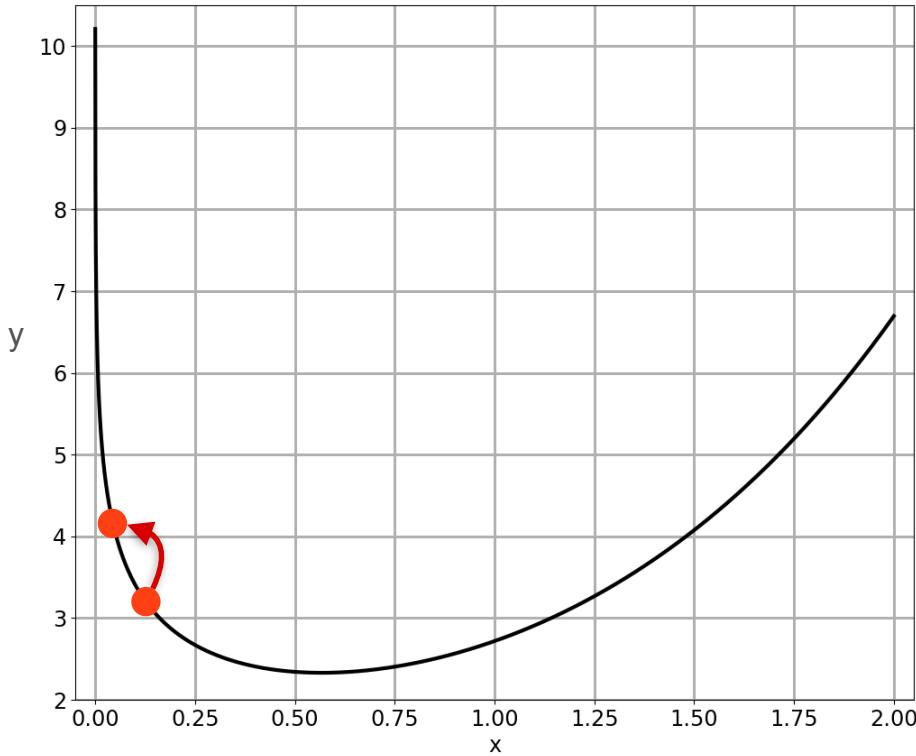
# Method 1: Try Both Directions

Is there any  
other way?



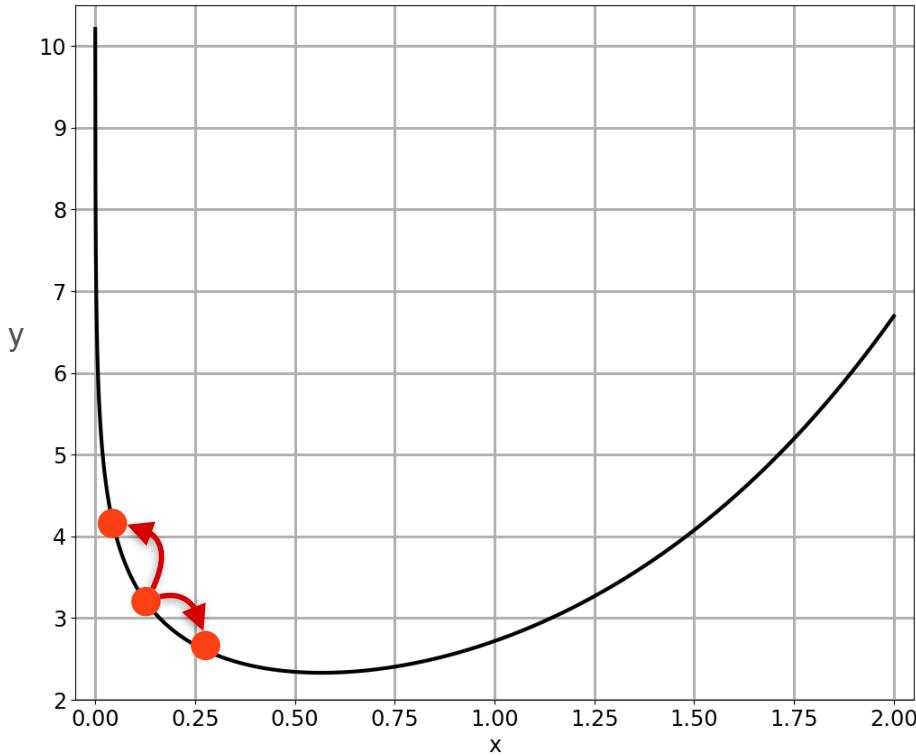
# Method 1: Try Both Directions

Is there any  
other way?



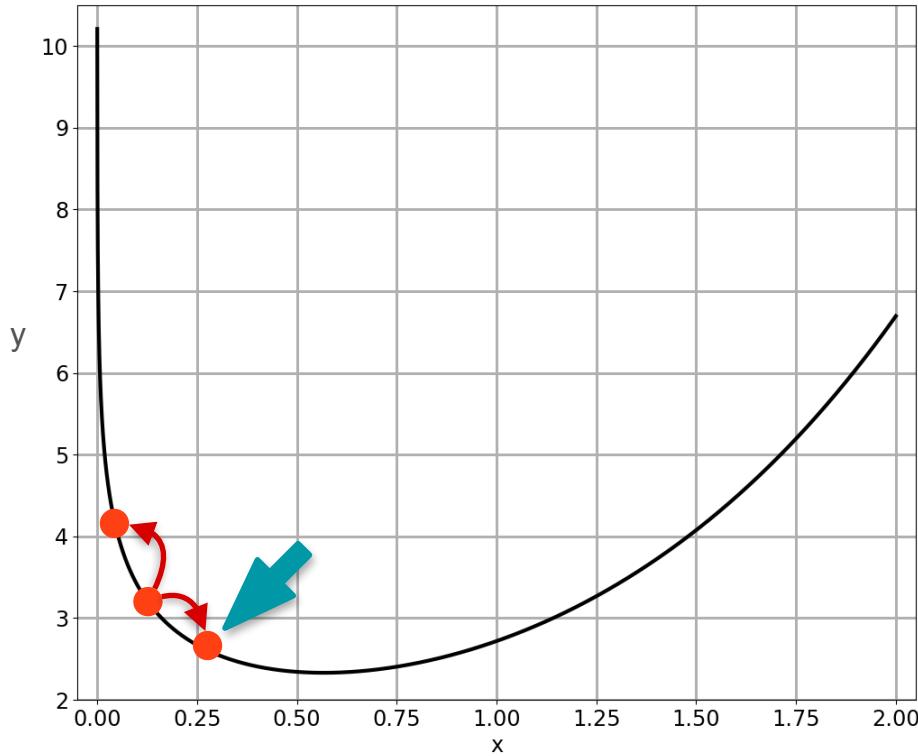
# Method 1: Try Both Directions

Is there any  
other way?



# Method 1: Try Both Directions

Is there any  
other way?

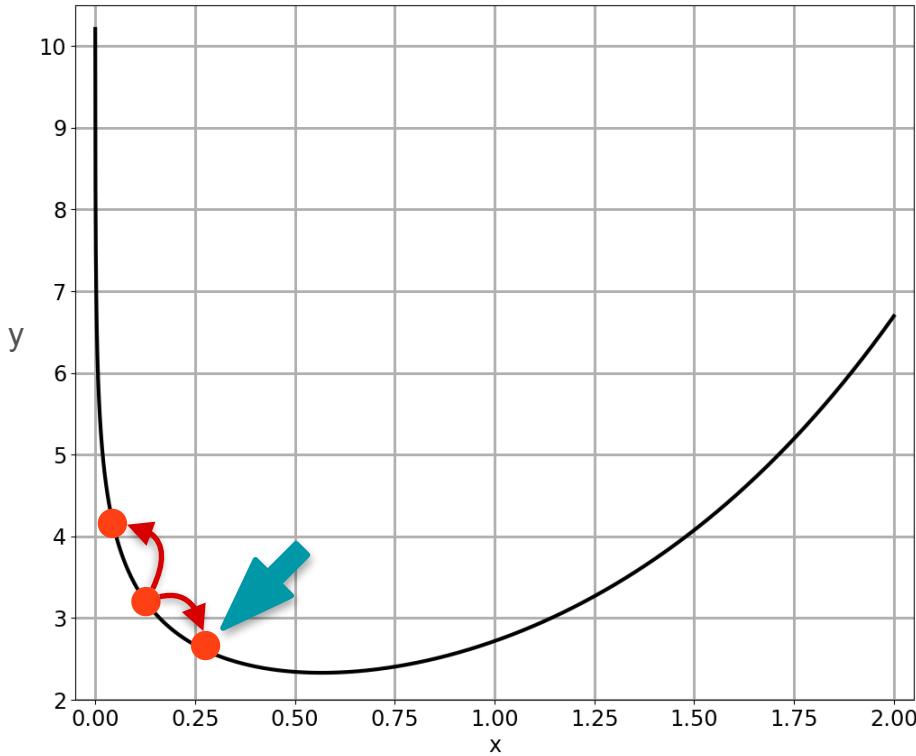


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

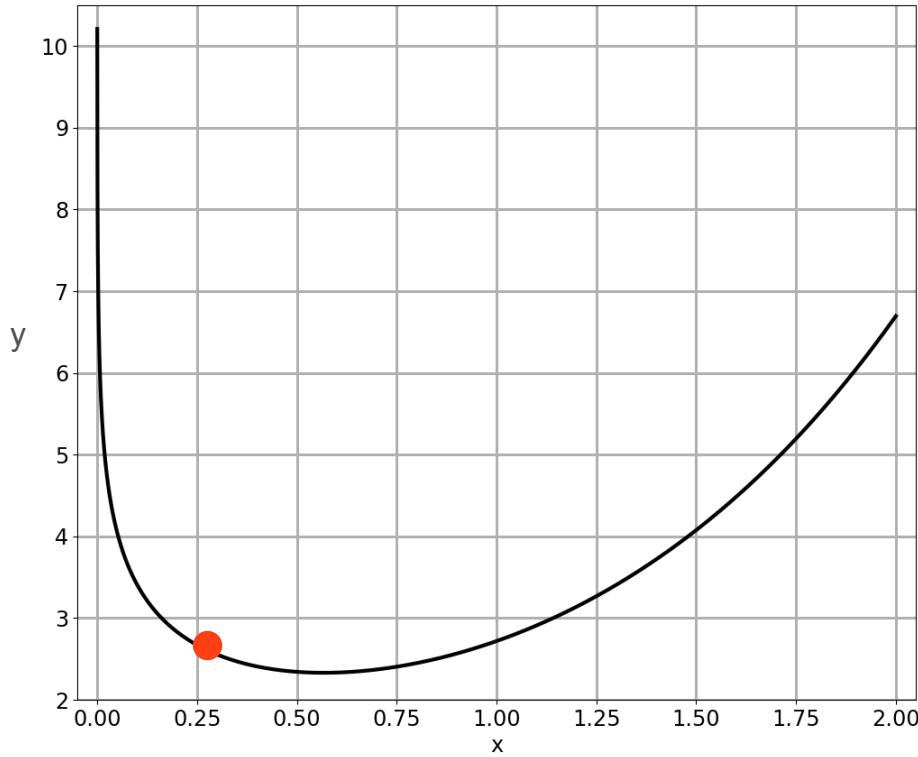


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

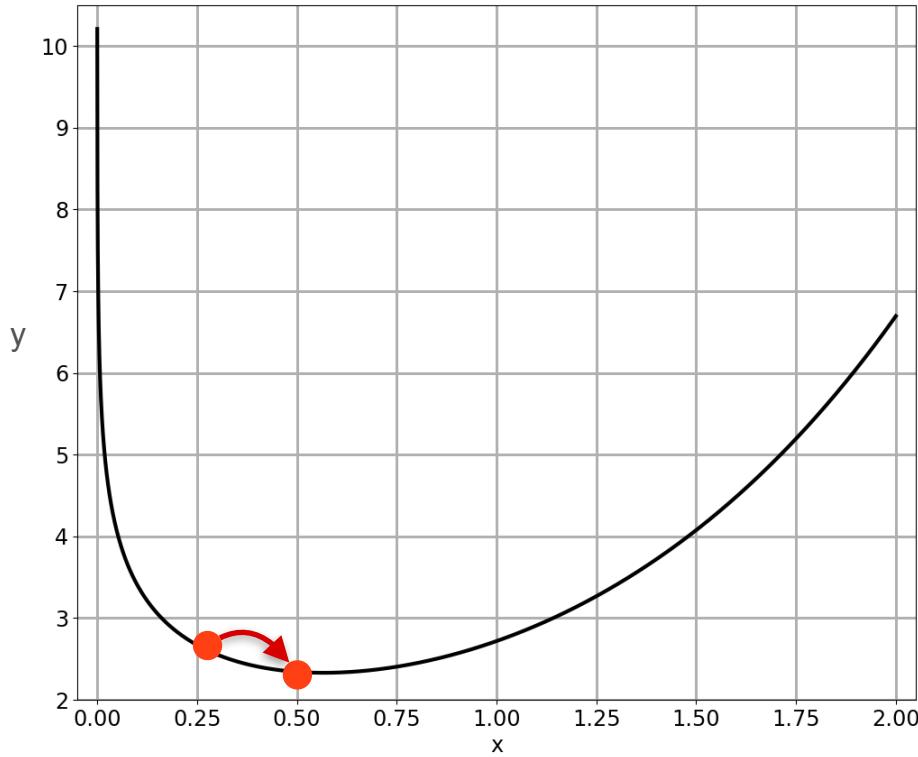


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

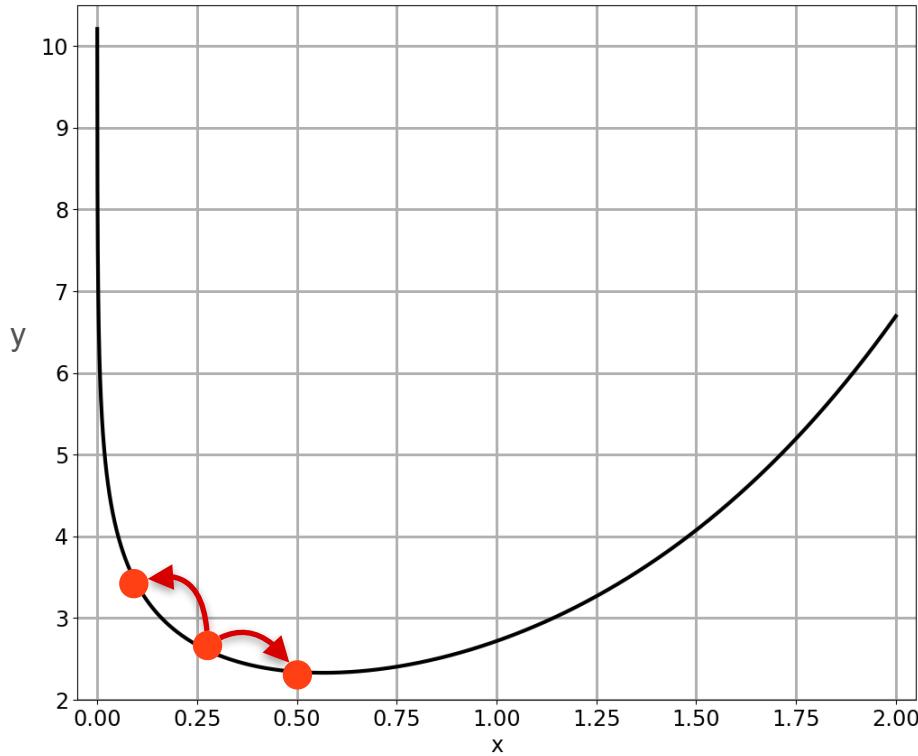


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

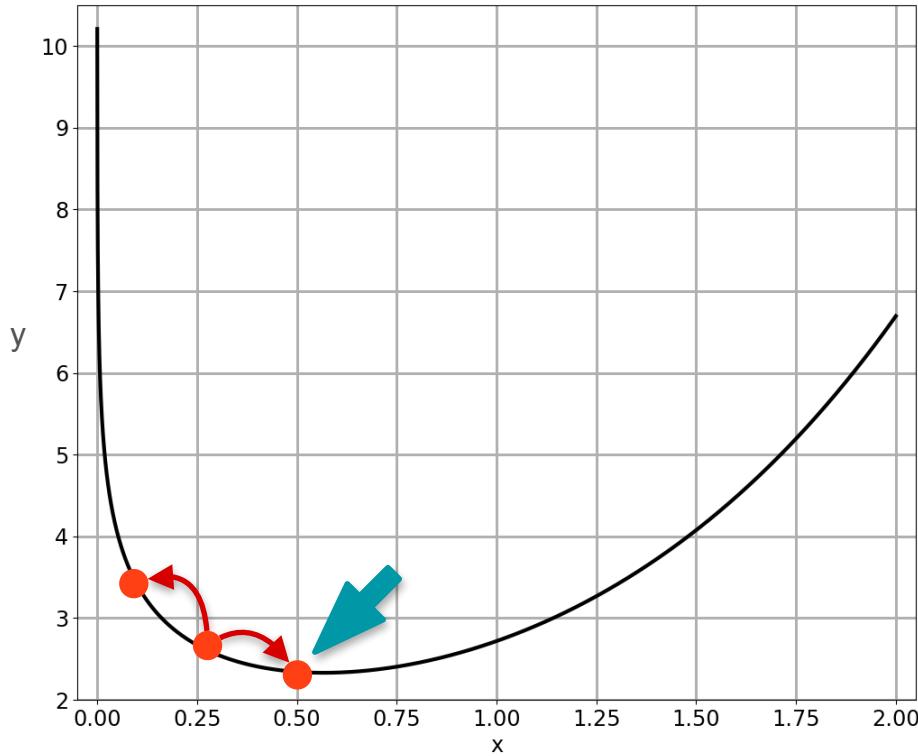


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

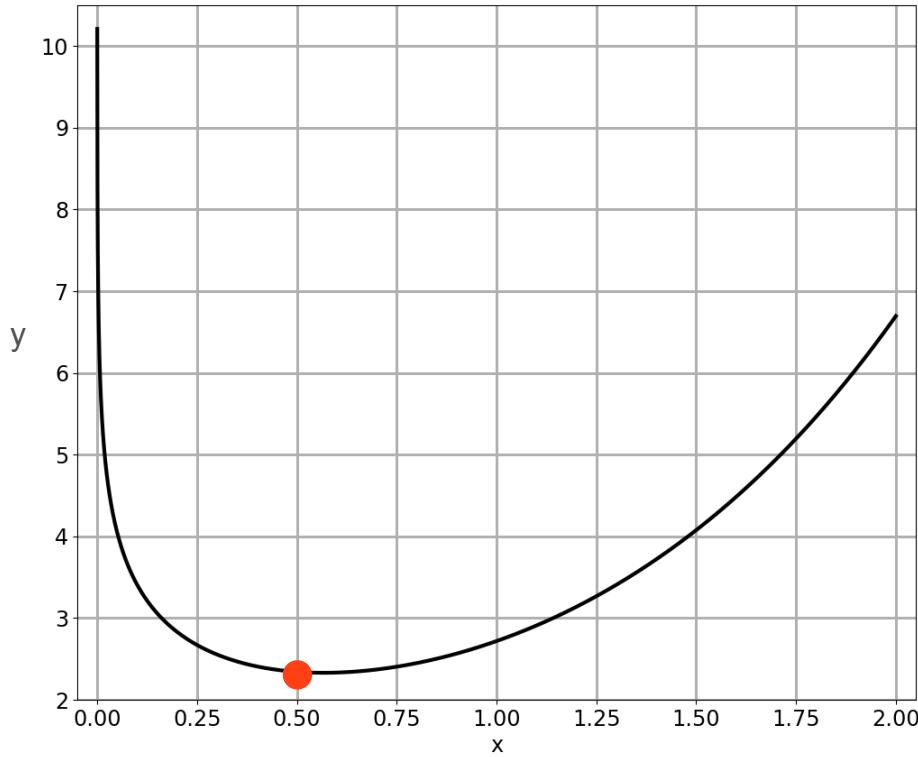


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

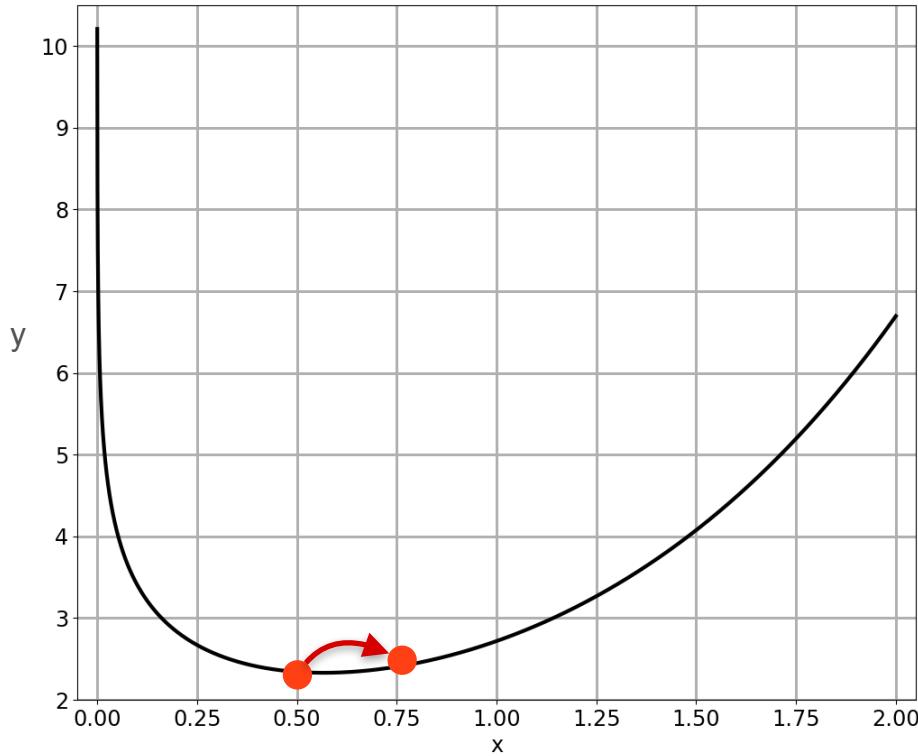


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

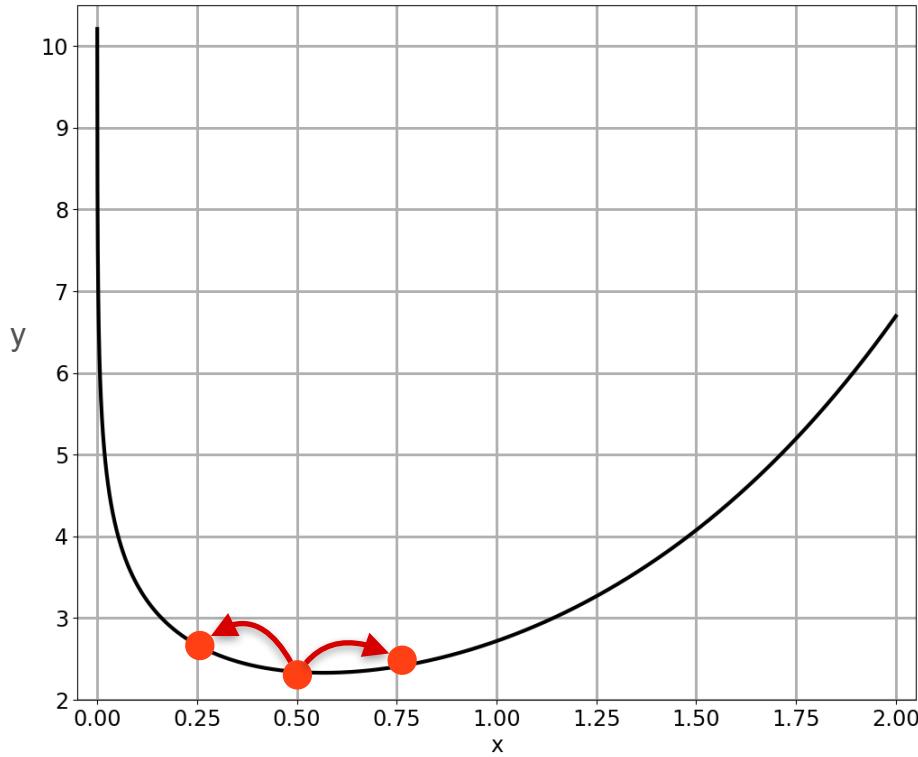


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

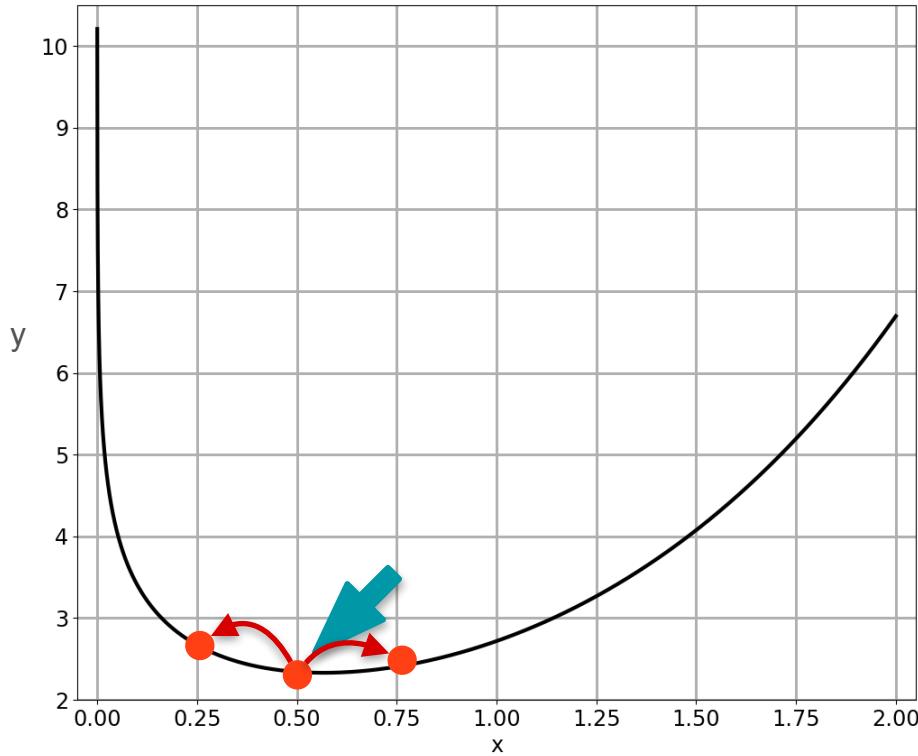


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!

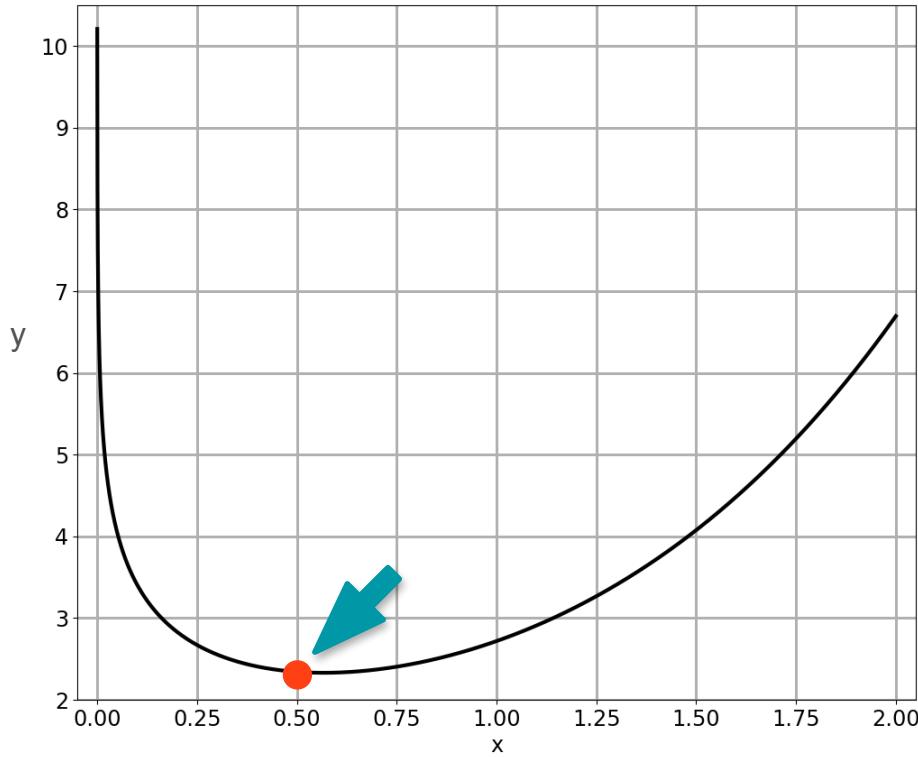


# Method 1: Try Both Directions

Is there any  
other way?



Repeat!





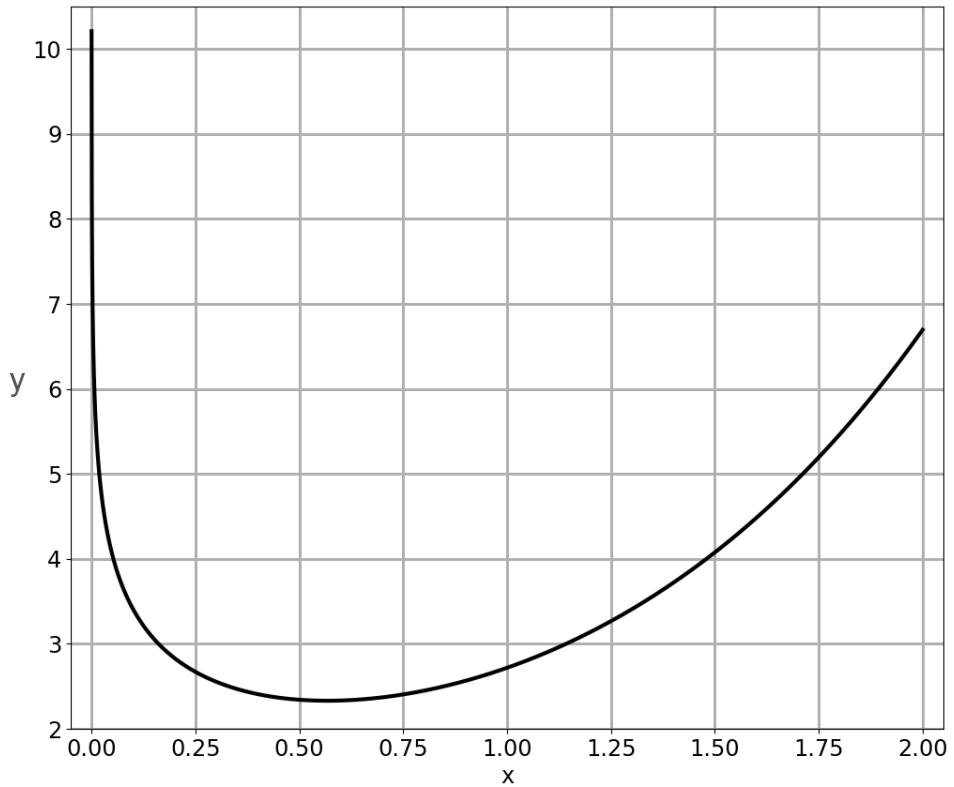
DeepLearning.AI

# Gradients and Gradient Descent

---

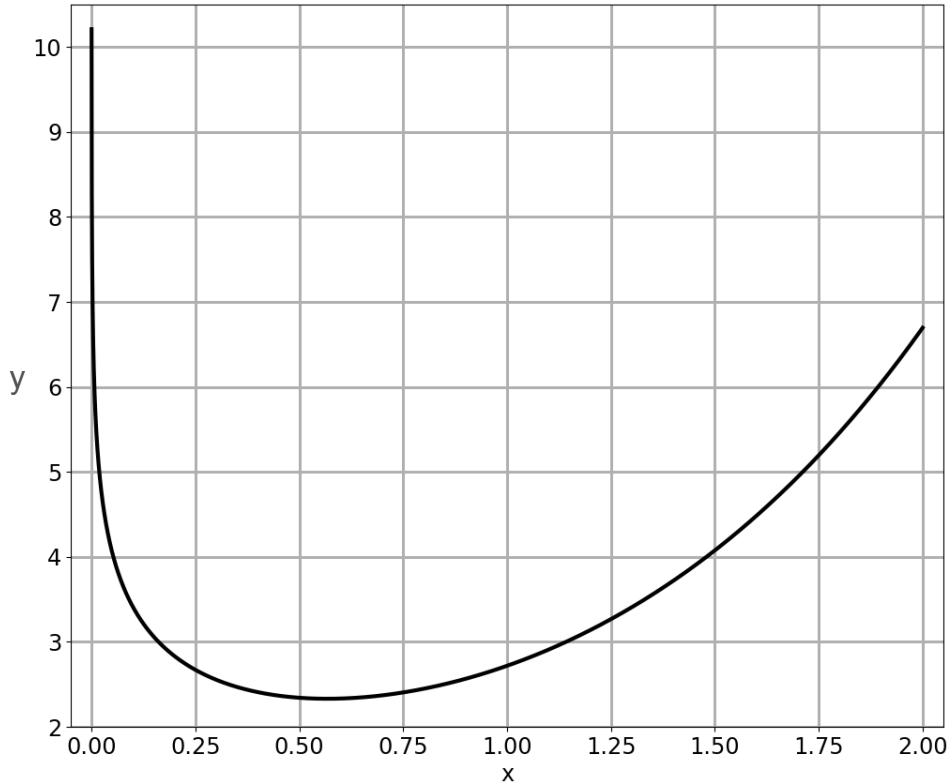
**Optimization using Gradient  
Descent in one variable -  
Part 2**

# Method 2: Be Clever



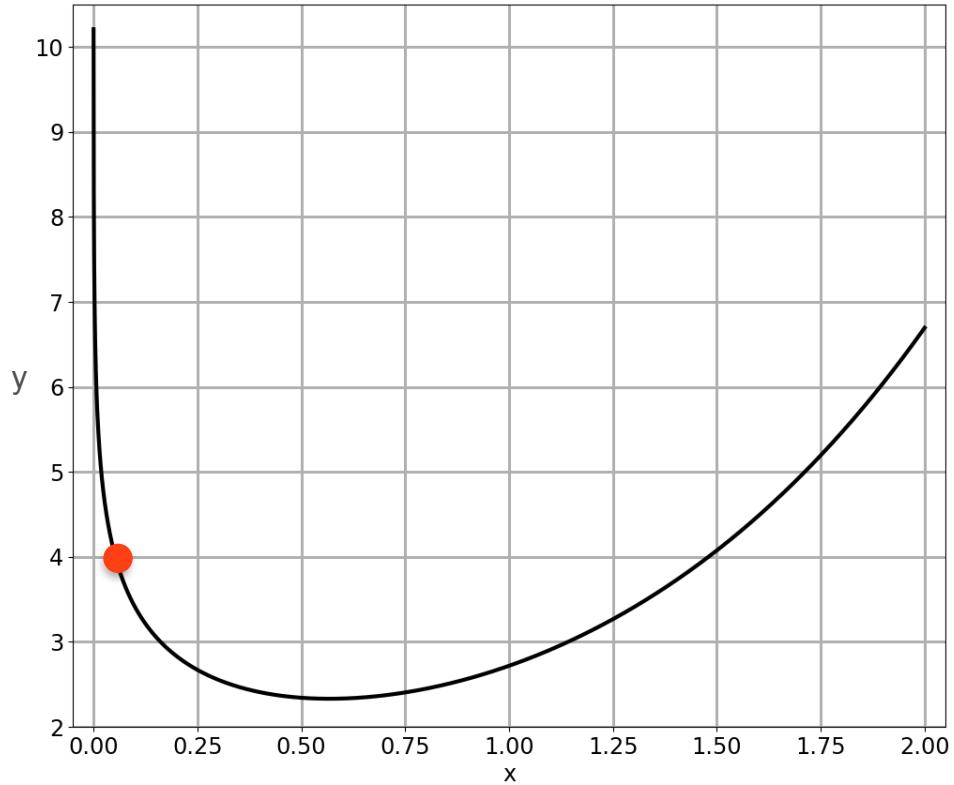
# Method 2: Be Clever

Try something  
smarter...



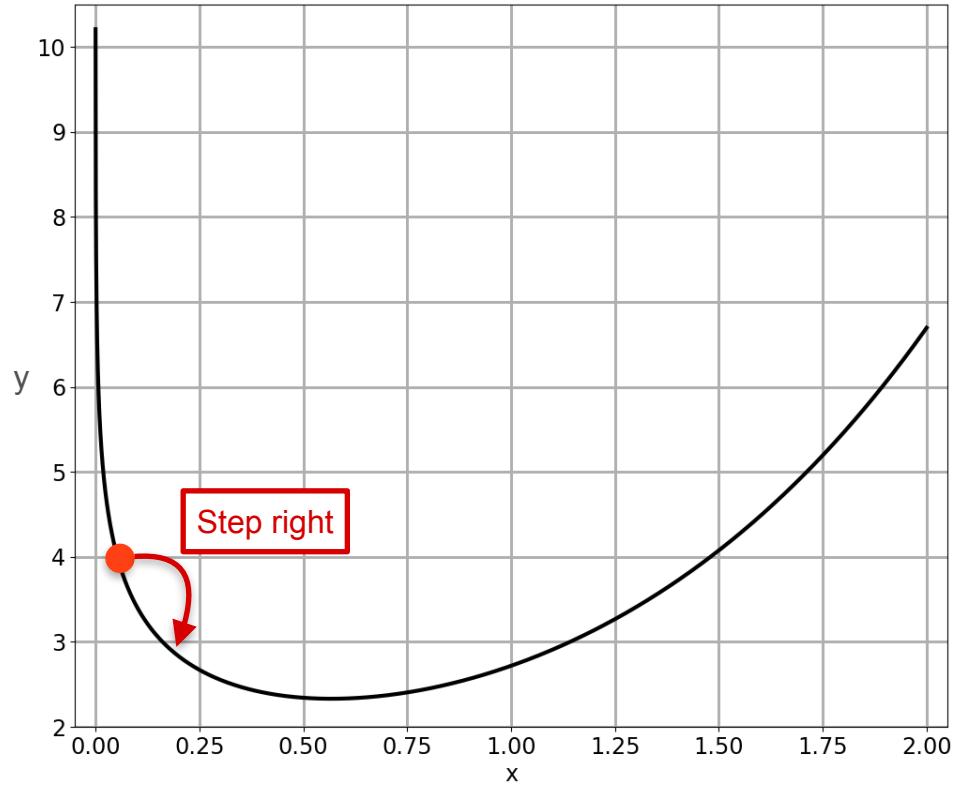
# Method 2: Be Clever

Try something  
smarter...



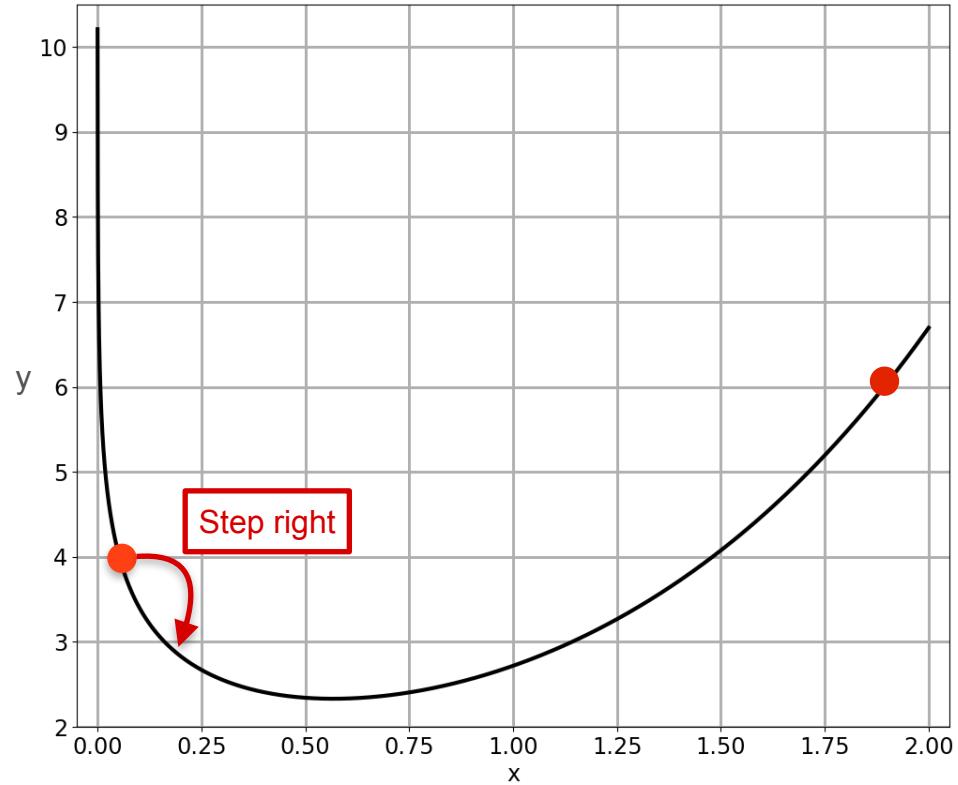
# Method 2: Be Clever

Try something  
smarter...



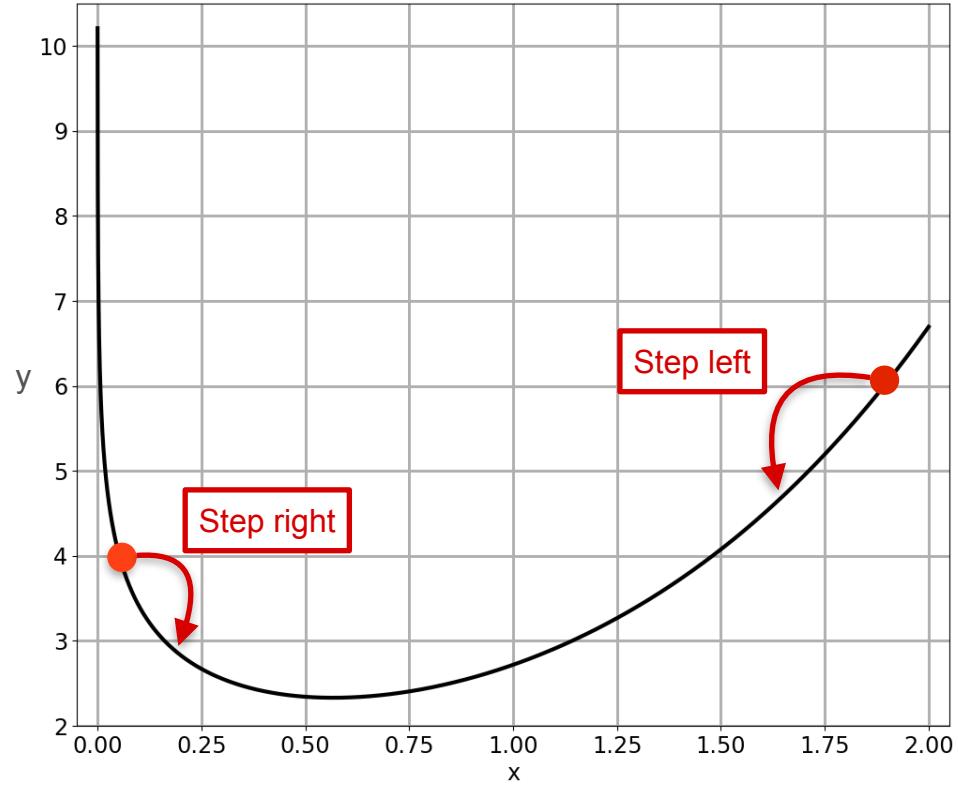
# Method 2: Be Clever

Try something  
smarter...



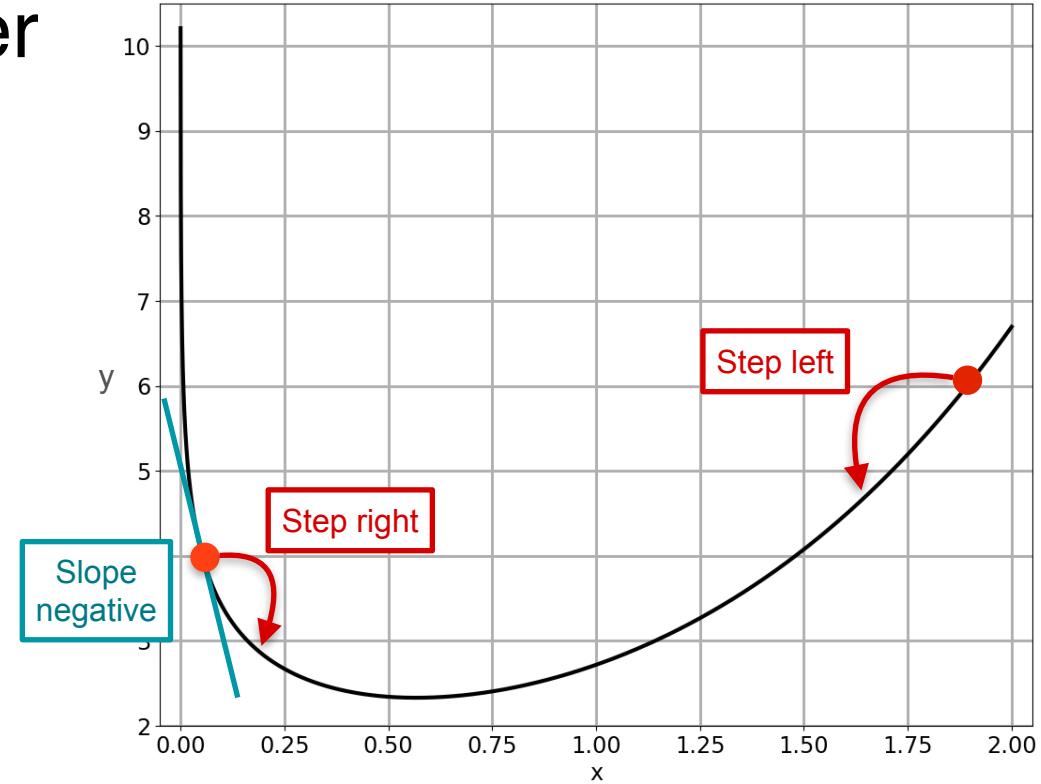
# Method 2: Be Clever

Try something  
smarter...



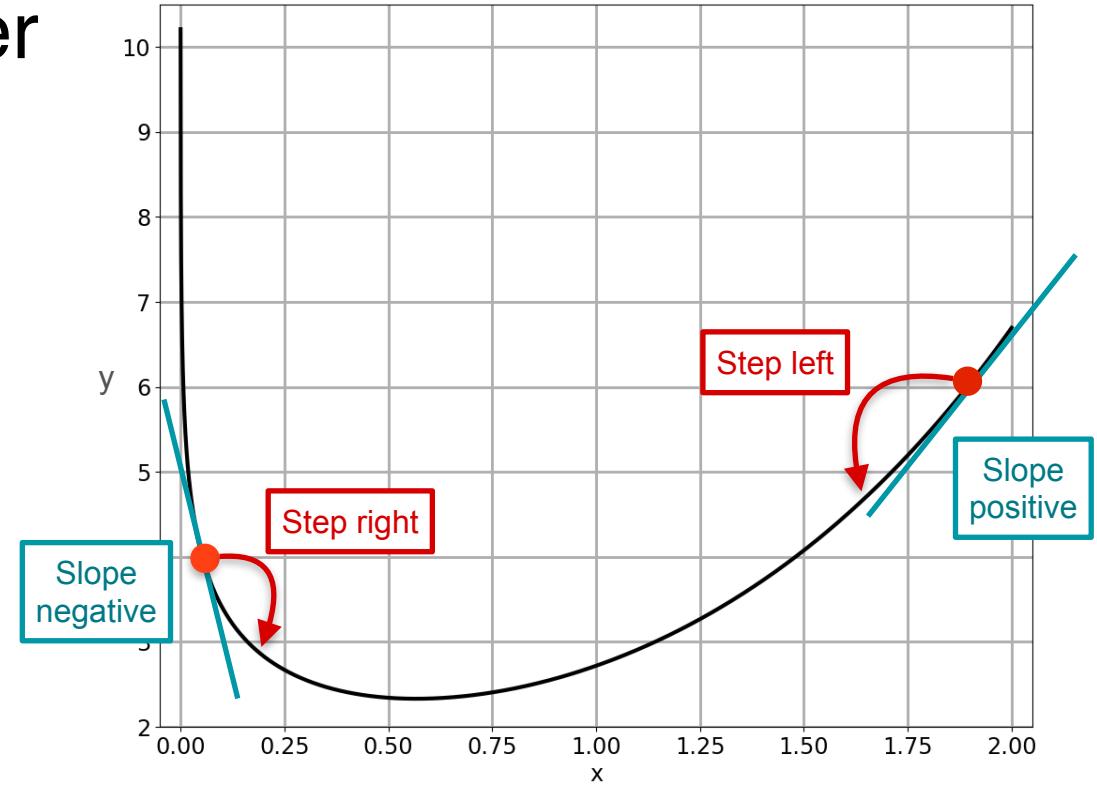
# Method 2: Be Clever

Try something  
smarter...



# Method 2: Be Clever

Try something  
smarter...

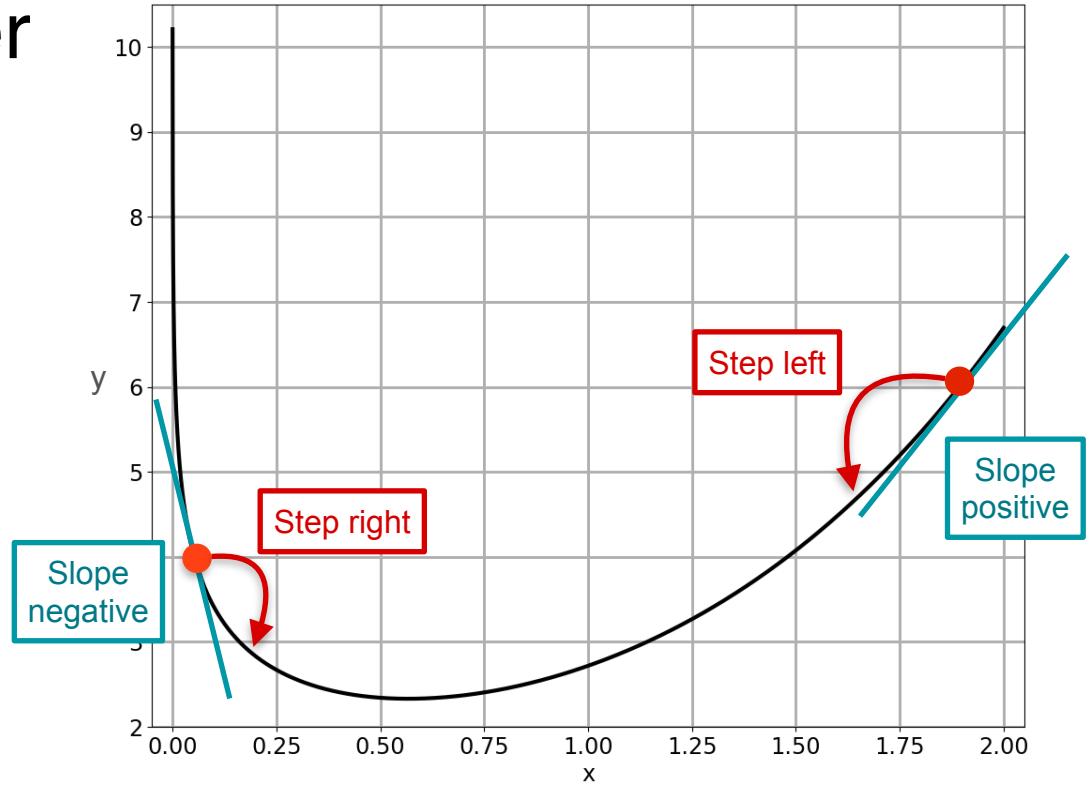


# Method 2: Be Clever

Try something  
smarter...



new point

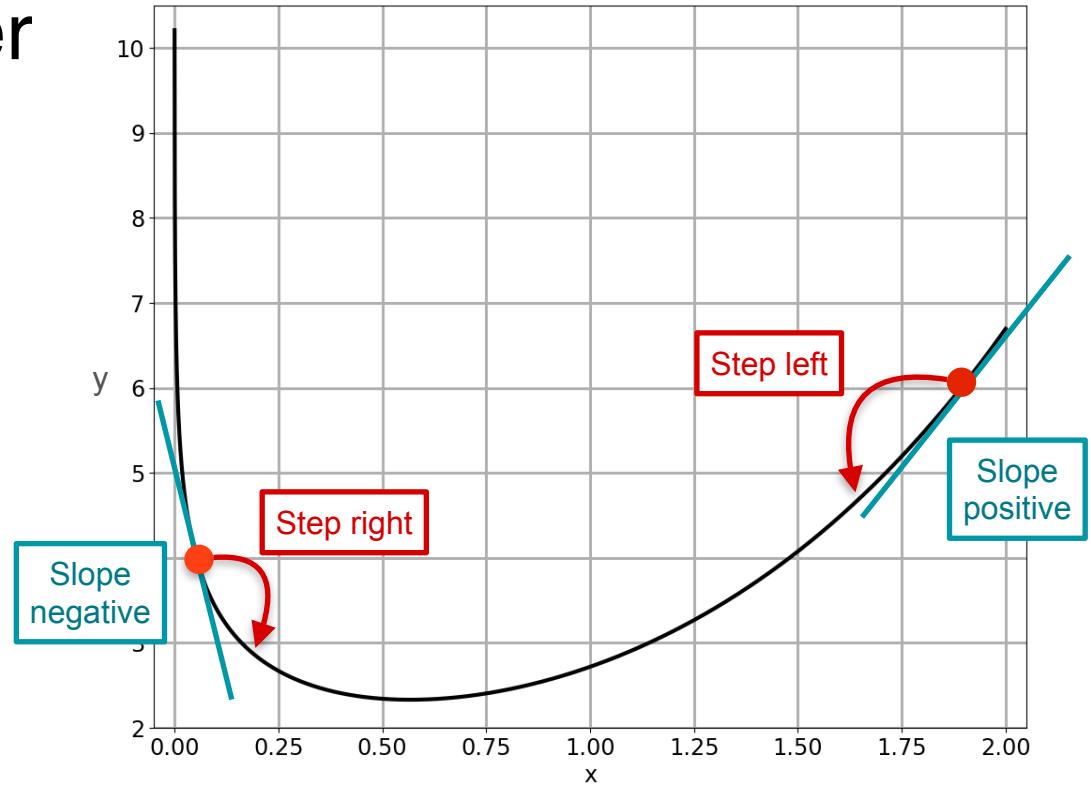


# Method 2: Be Clever

Try something  
smarter...



new point = old point

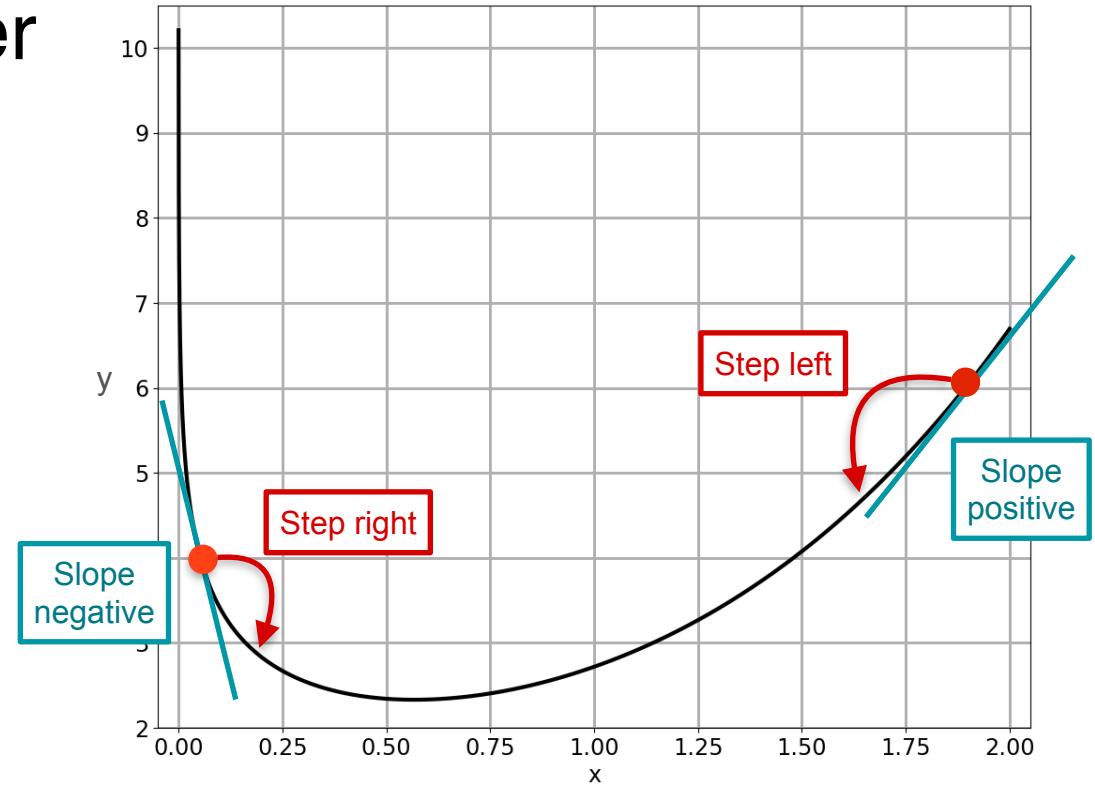


# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope



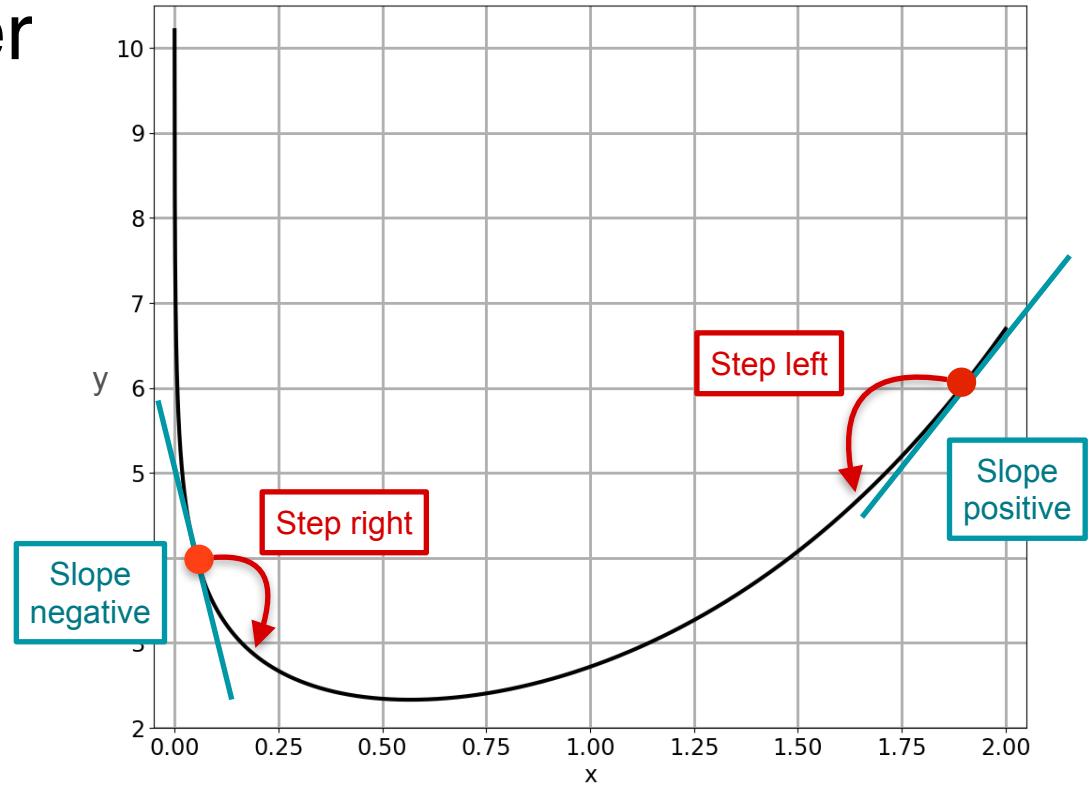
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$x_1$



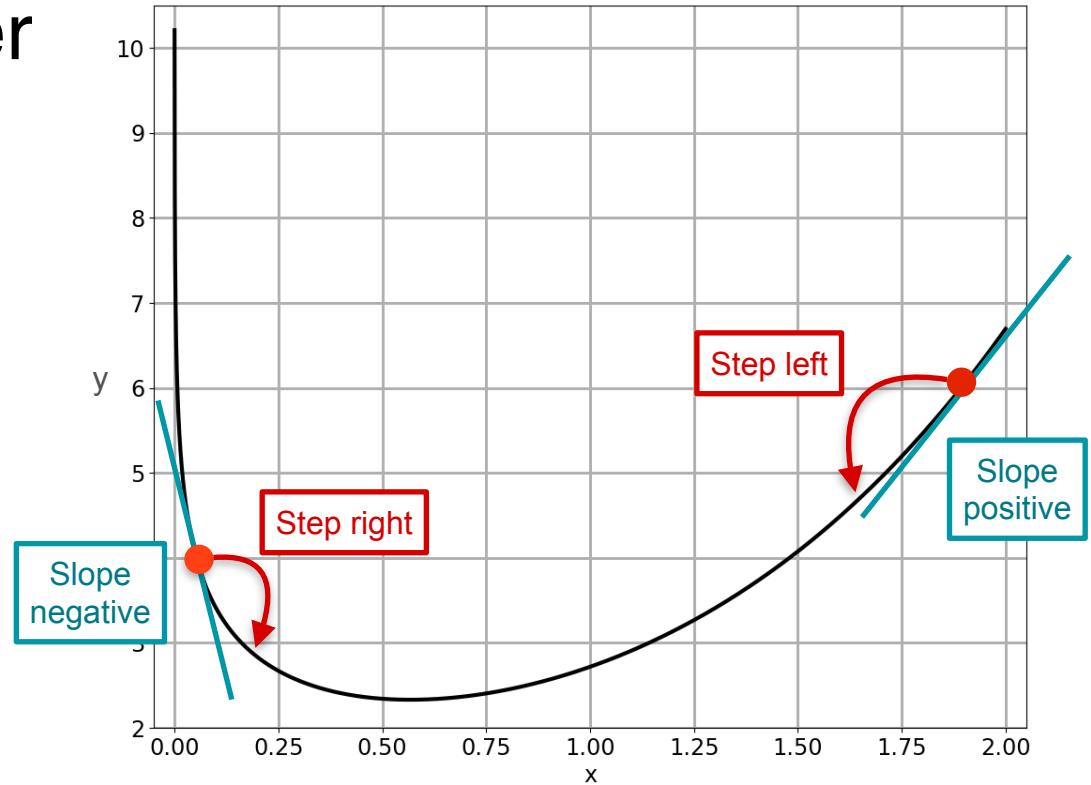
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0$$



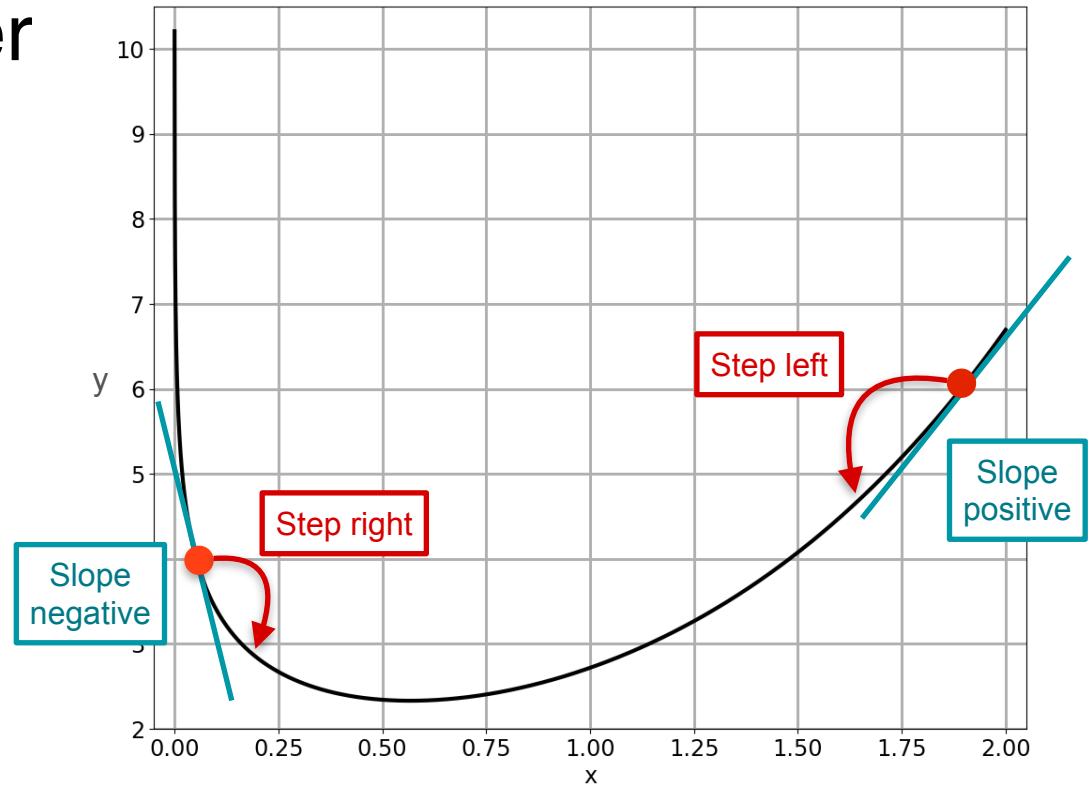
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



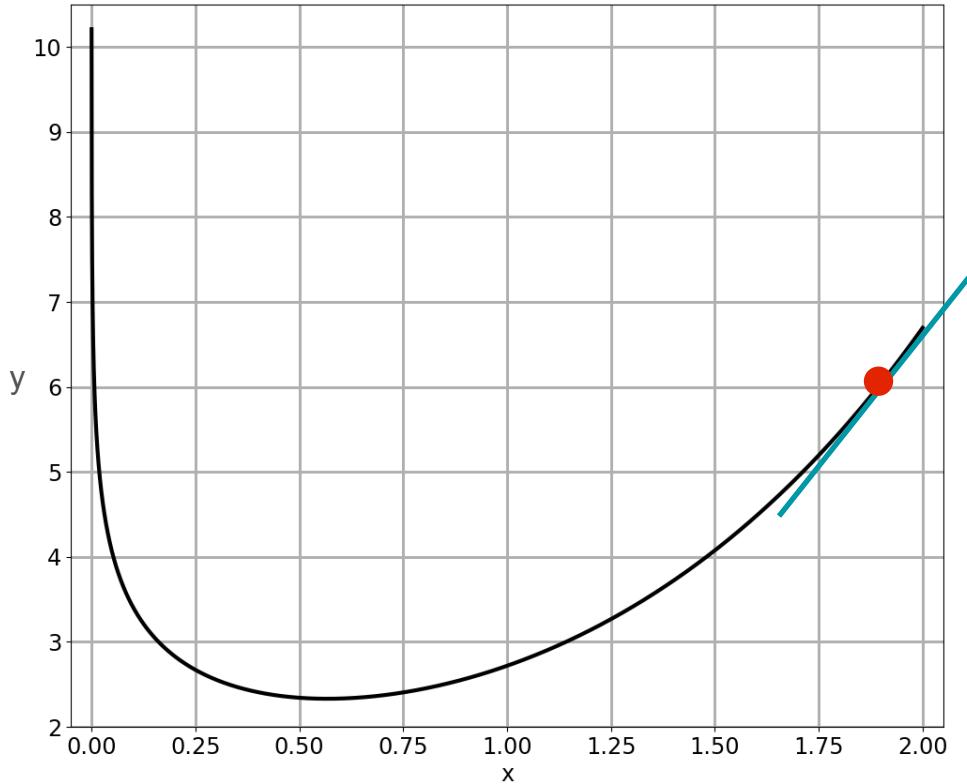
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



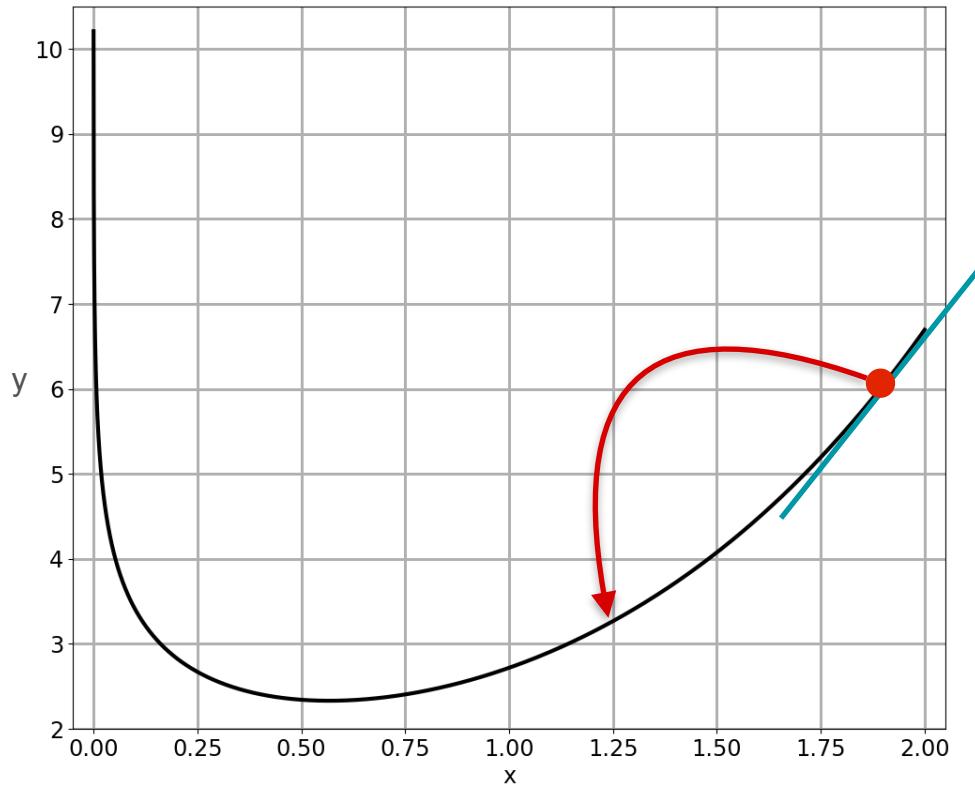
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



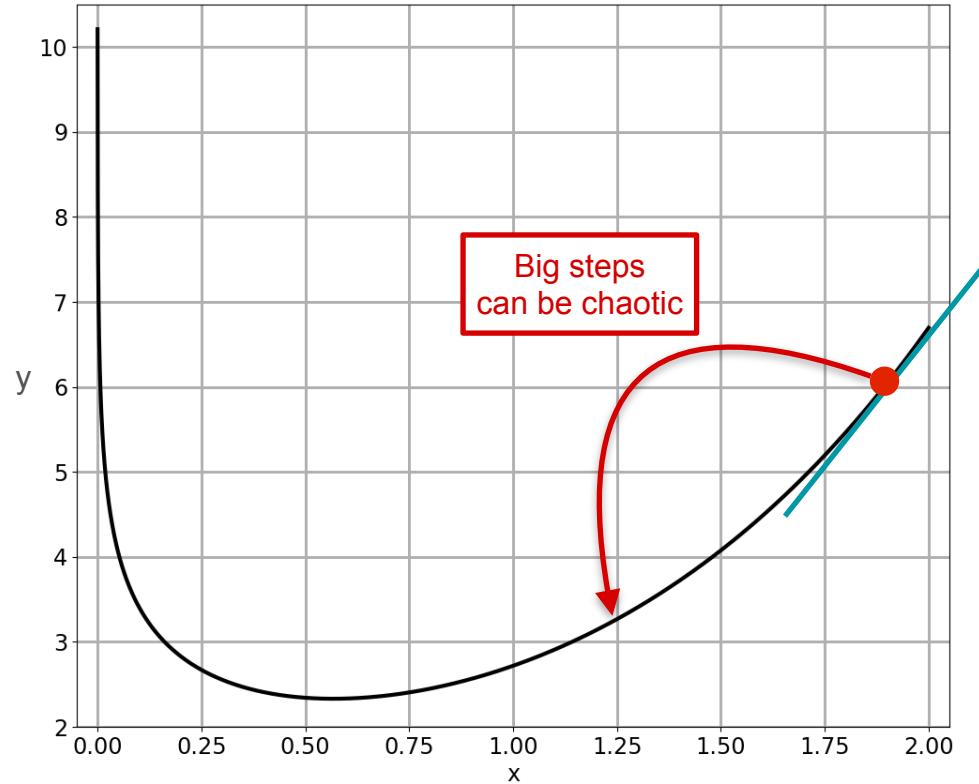
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



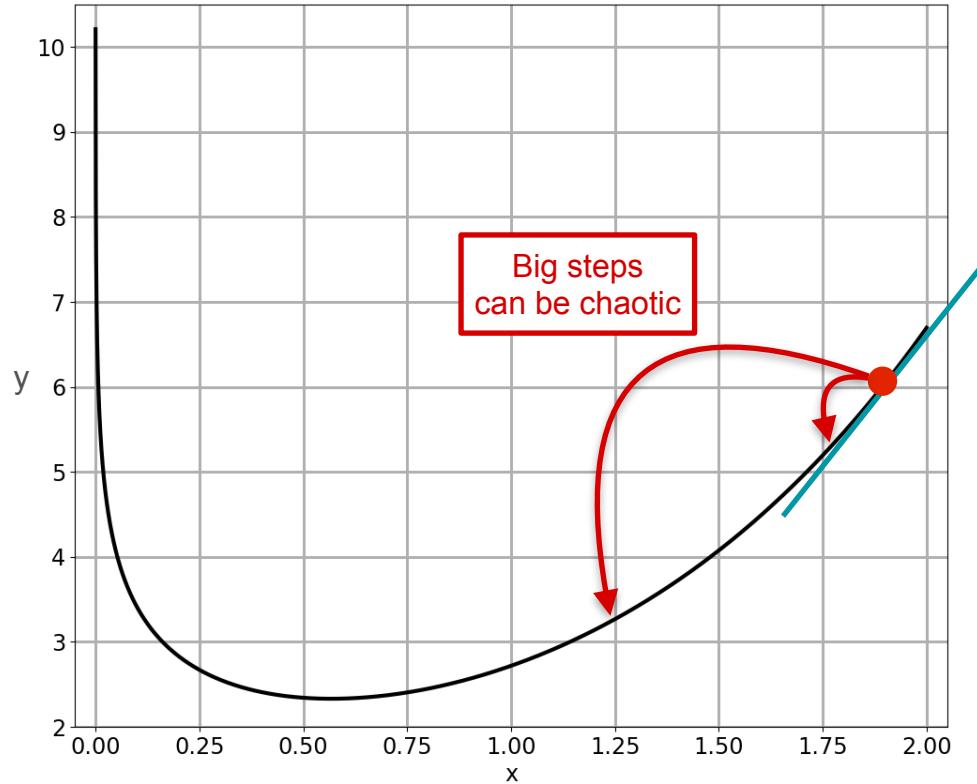
# Method 2: Be Clever

Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



# Method 2: Be Clever

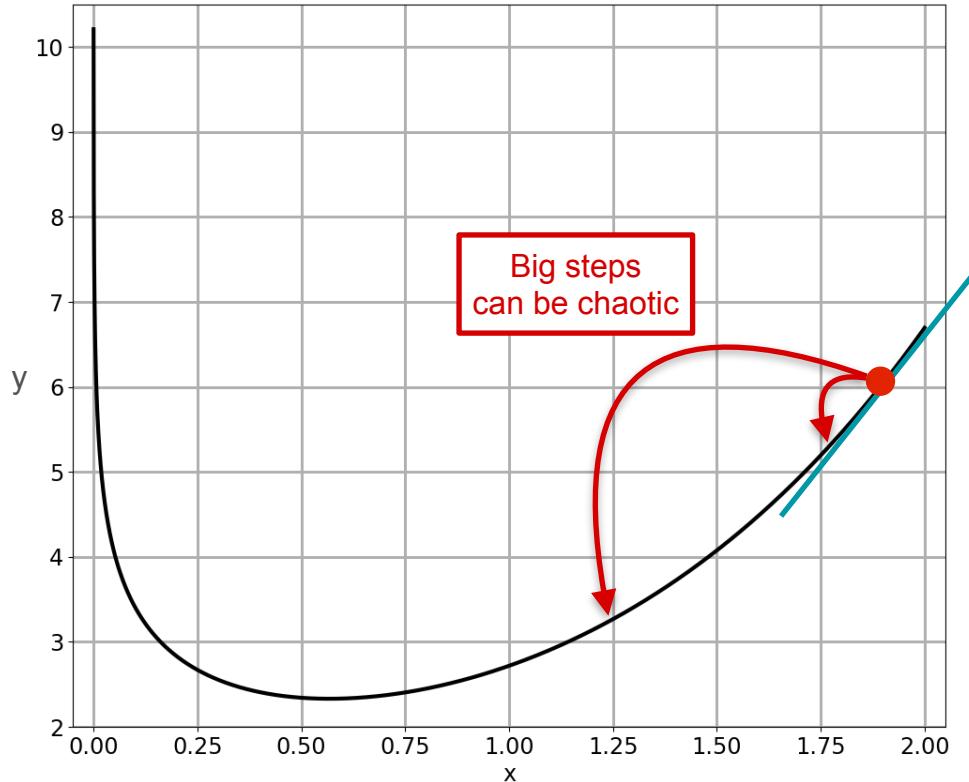
Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - 0.01 f'(x_0)$$



# Method 2: Be Clever

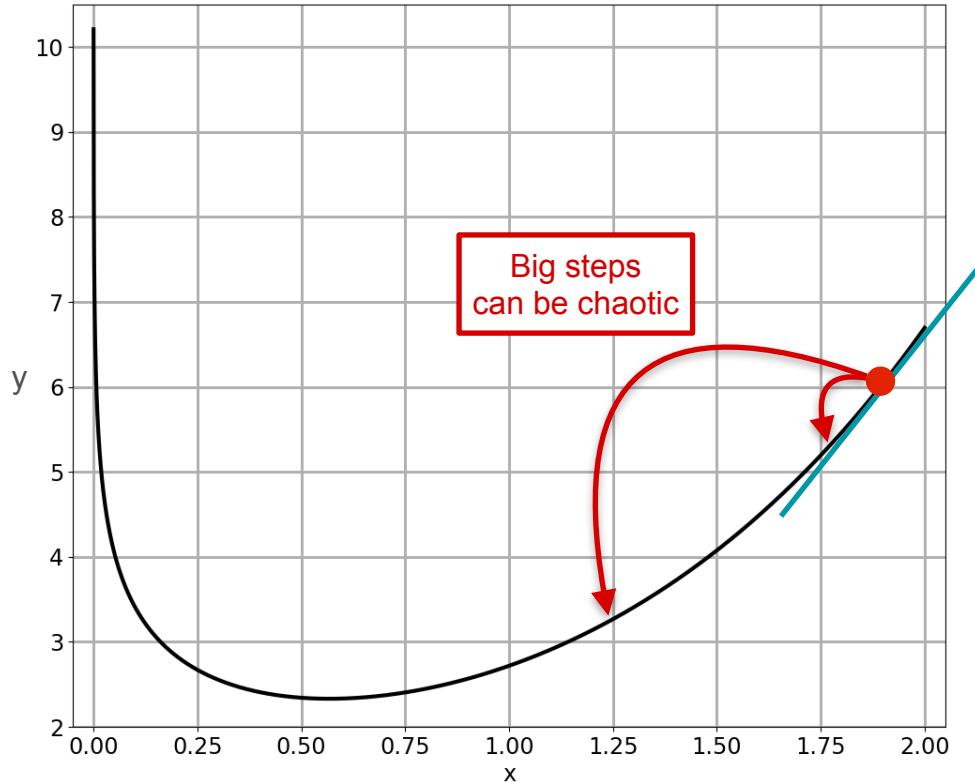
Try something  
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha f'(x_0)$$



# Method 2: Be Clever

Try something  
smarter...



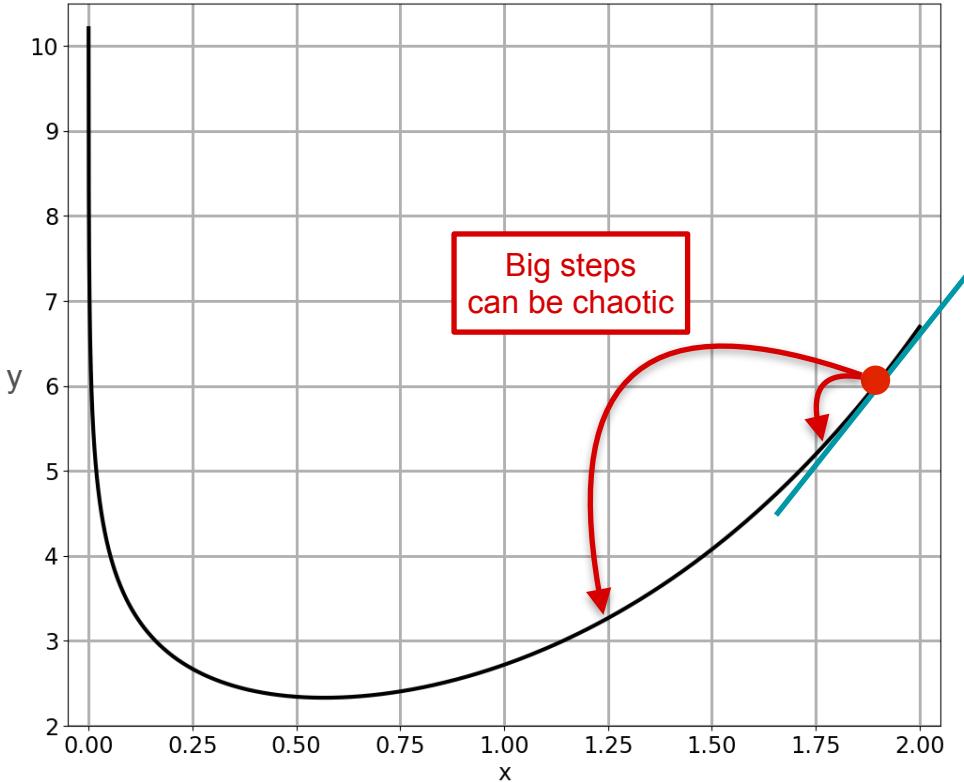
new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha f'(x_0)$$



Learning rate

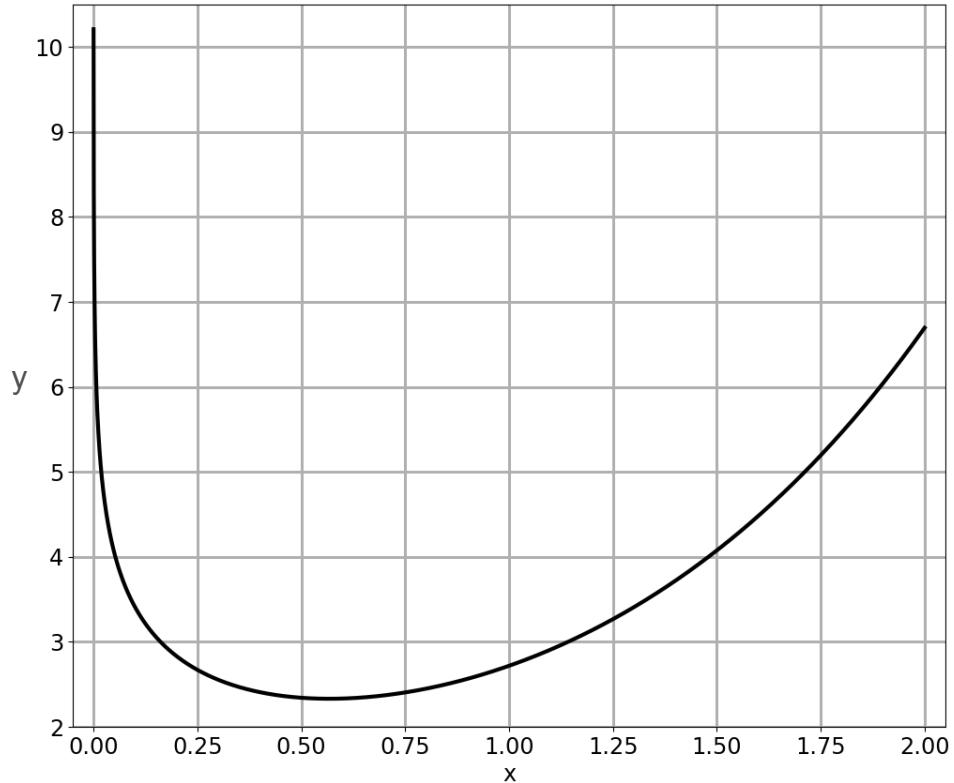


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

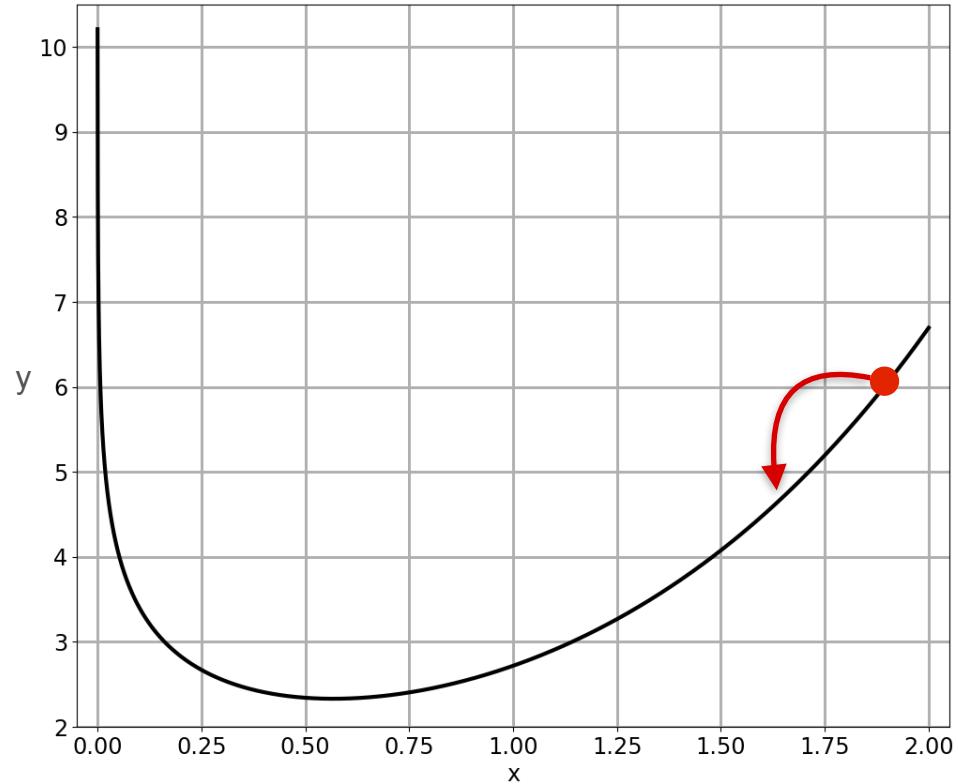


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

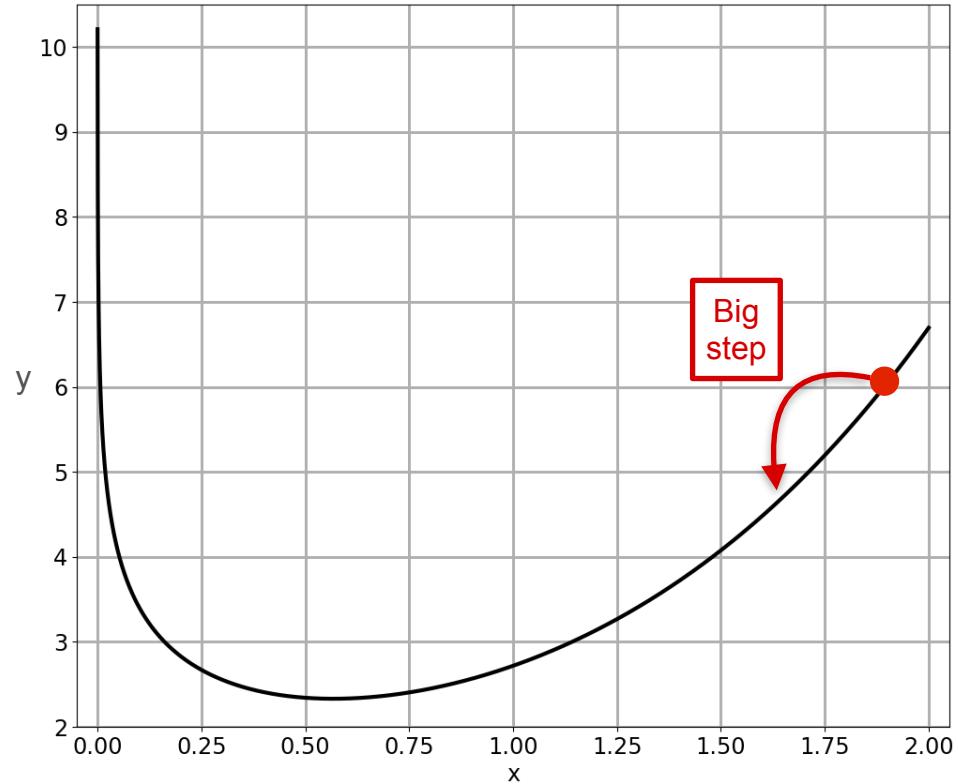


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

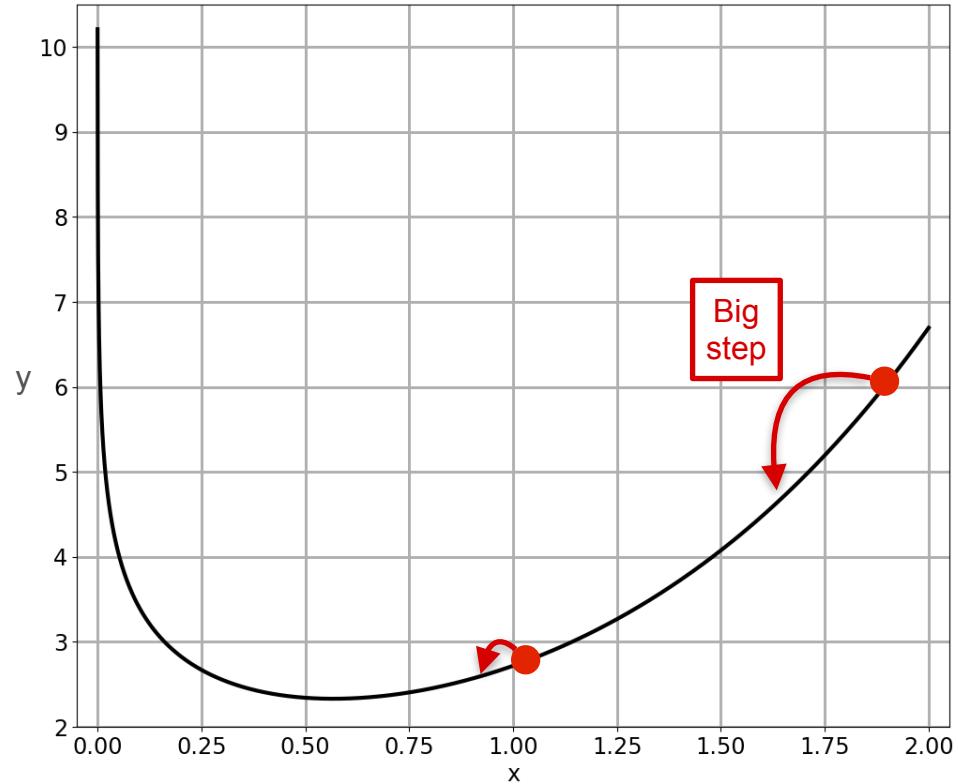


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

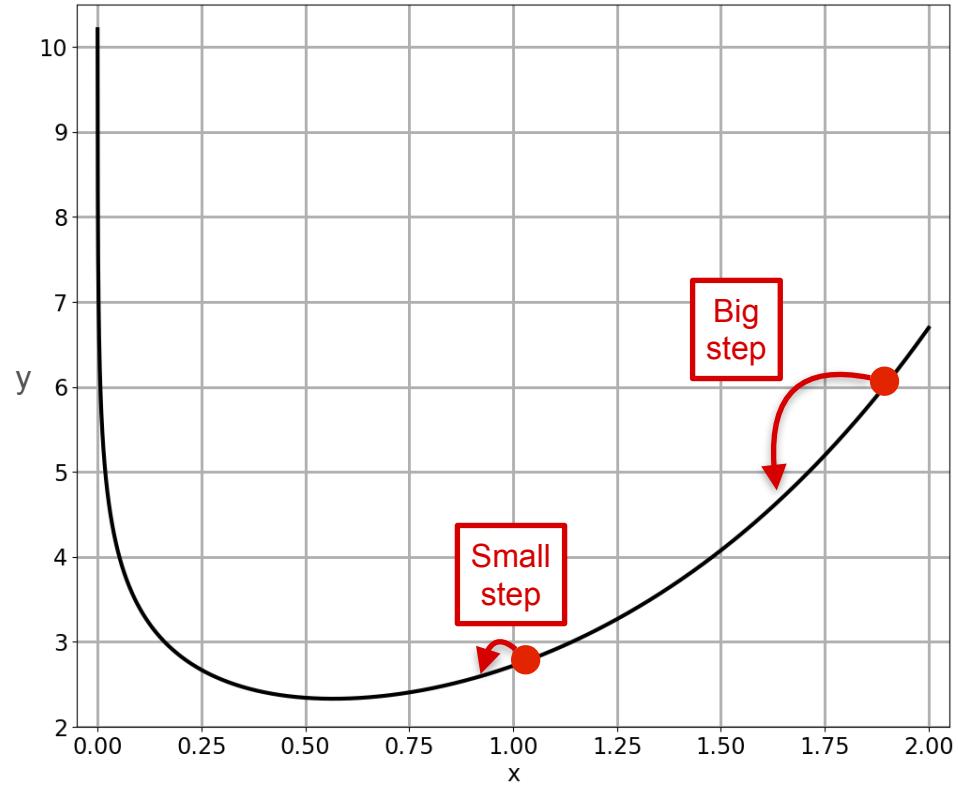


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

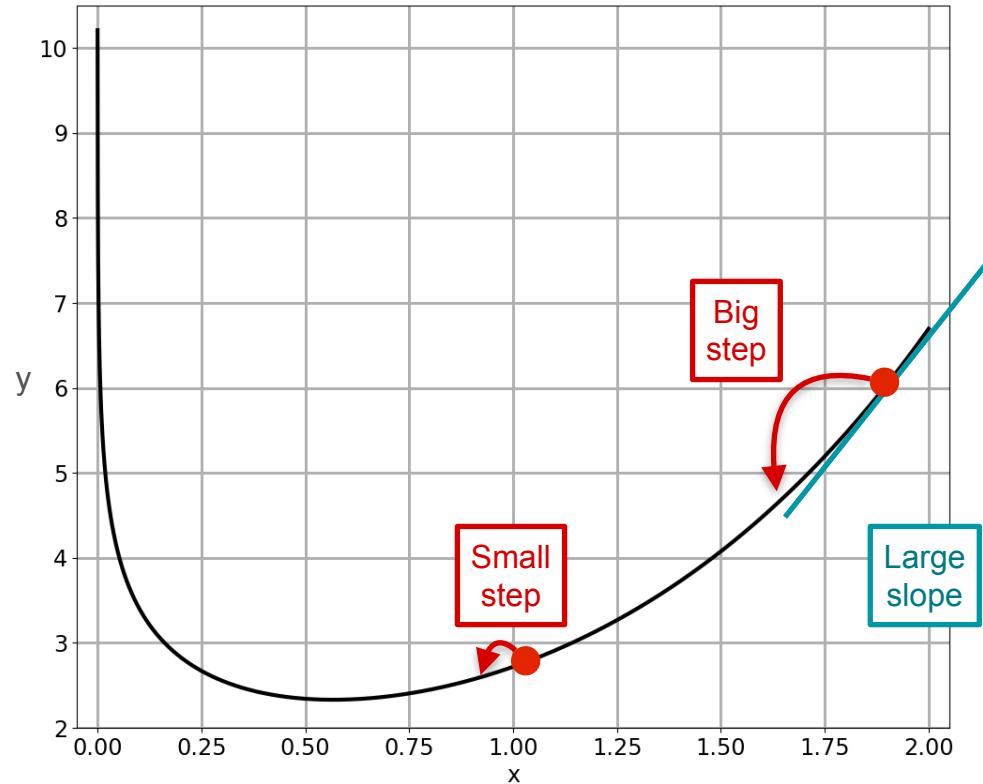


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

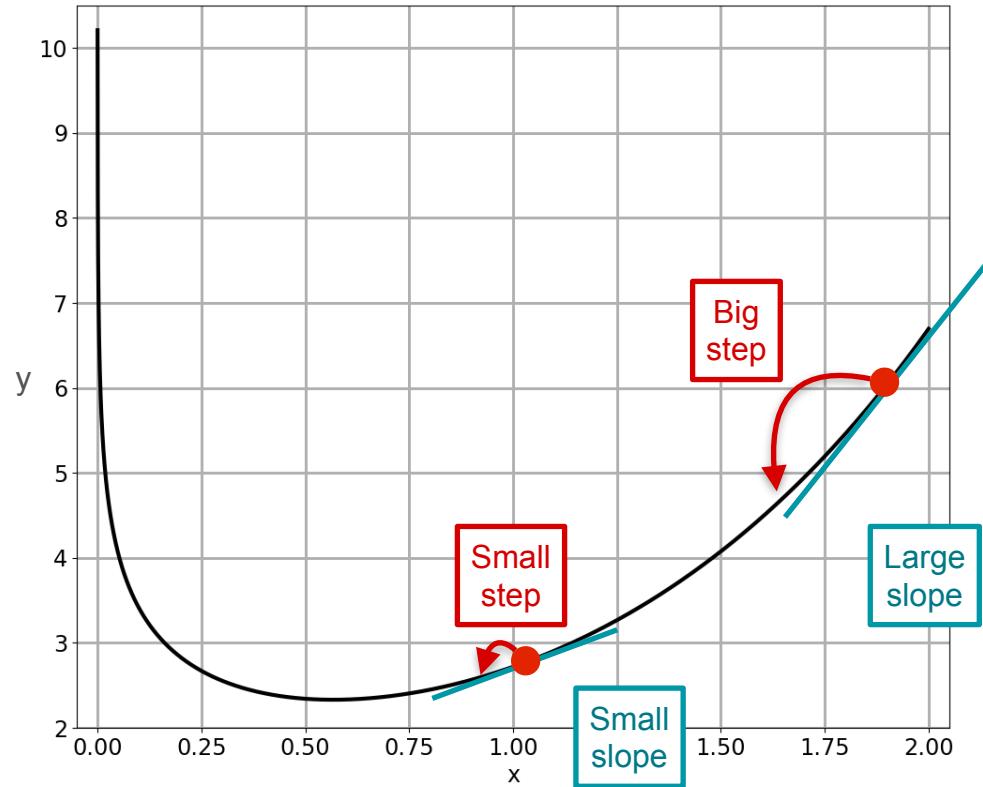


# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$



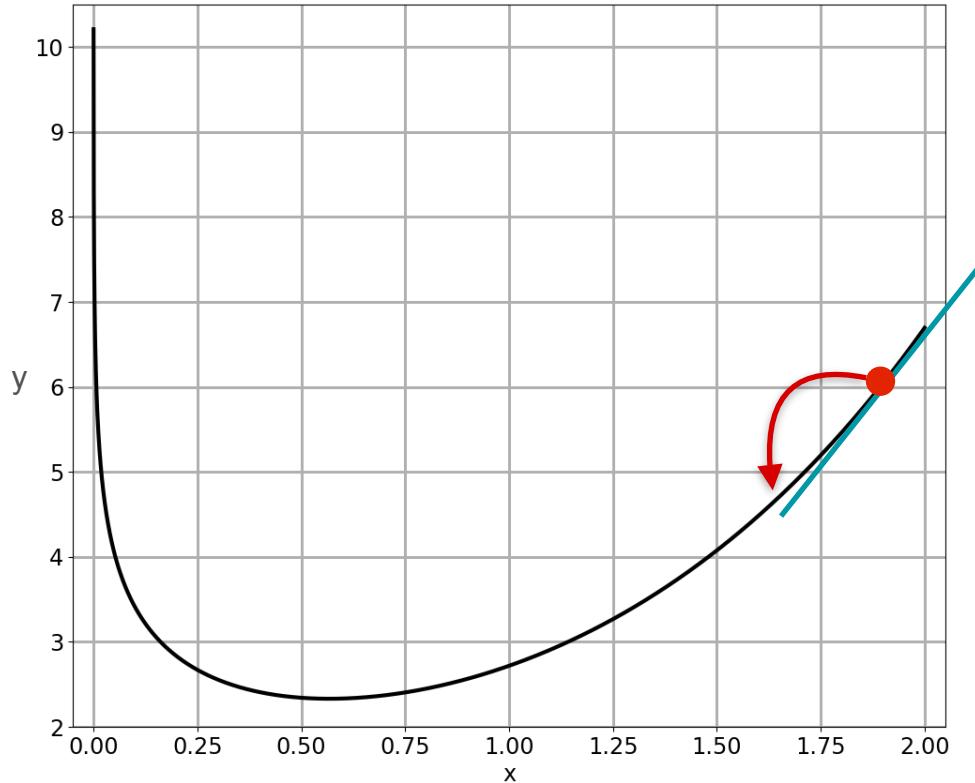
# Method 2: Be Clever

Try something  
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

## Gradient descent



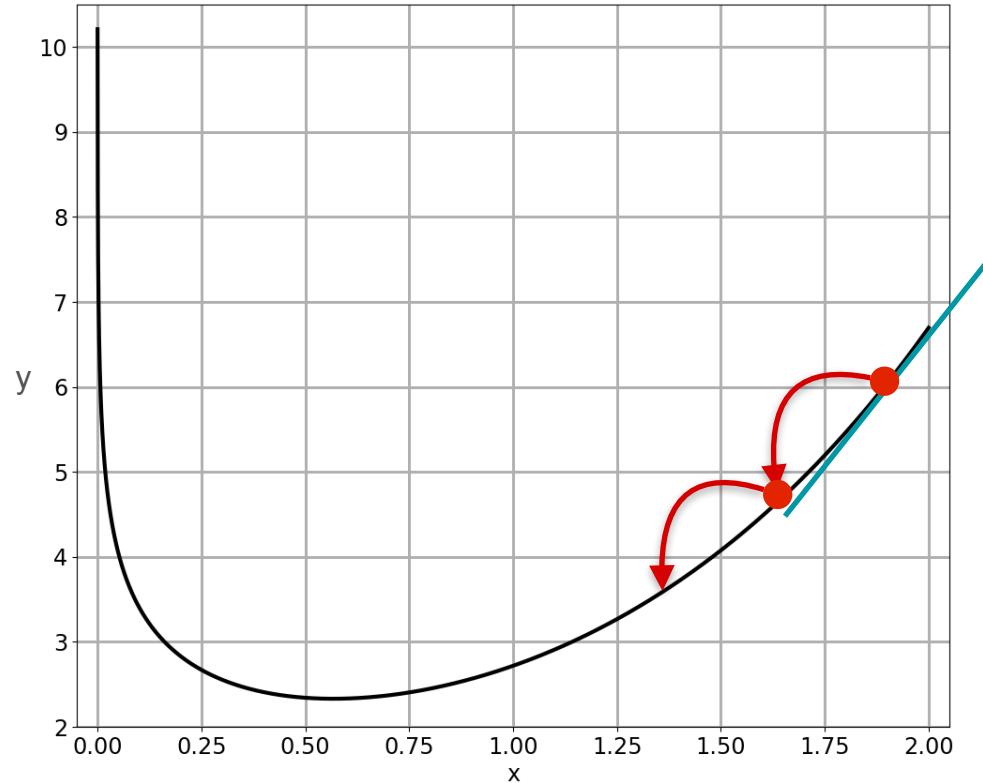
# Method 2: Be Clever

Try something  
smarter...



$$x_2 = x_1 - \alpha f'(x_1)$$

## Gradient descent



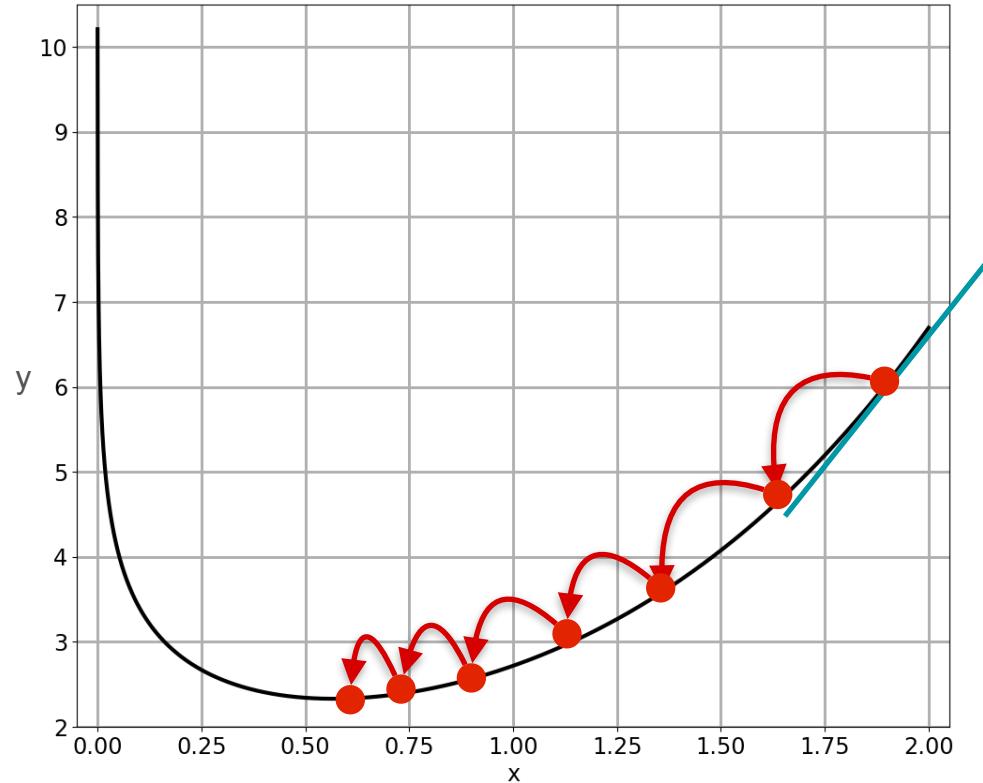
# Method 2: Be Clever

Try something  
smarter...



$$x_{20} = x_{19} - \alpha f'(x_{19})$$

## Gradient descent



# Gradient Descent

Function:  $f(x)$

Goal: find minimum of  $f(x)$

Step 1:

Define a learning rate  $\alpha$

Choose a starting point  $x_0$

Step 2:

Update:  $x_k = x_{k-1} - \alpha f'(x_{k-1})$

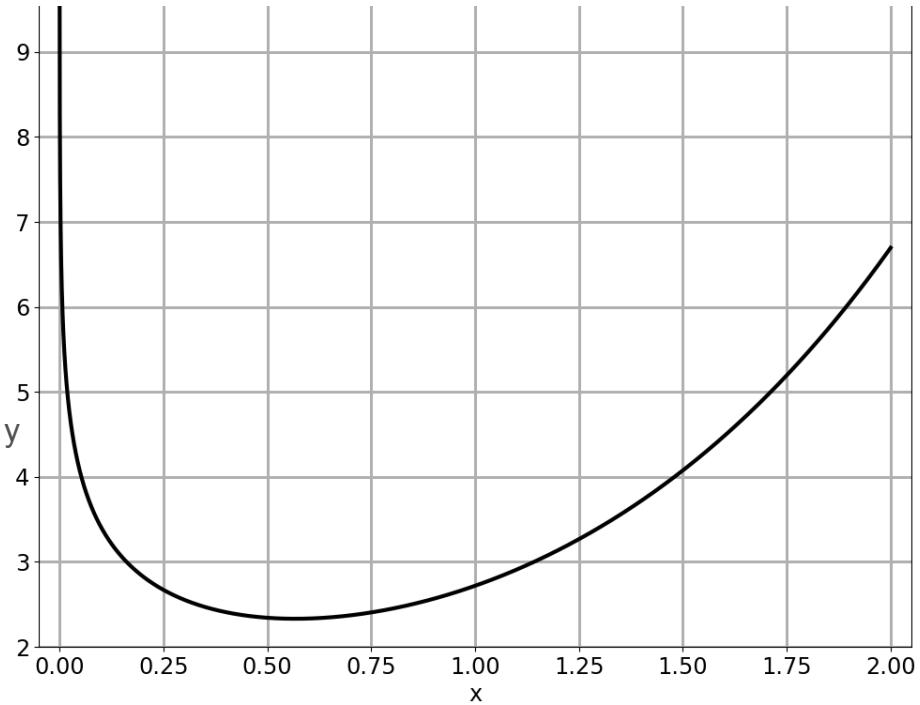
Step 3:

Repeat Step 2 until you are close enough to  
the true minimum  $x^*$

# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$



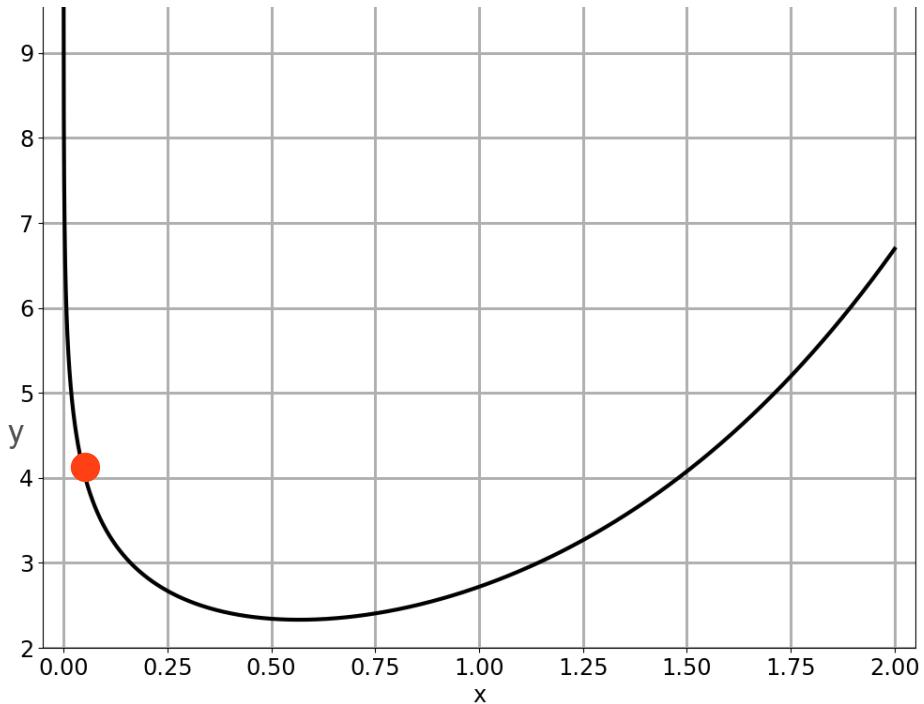
# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start:  $x = 0.05$

Rate:  $\alpha = 0.005$



# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start:  $x = 0.05$

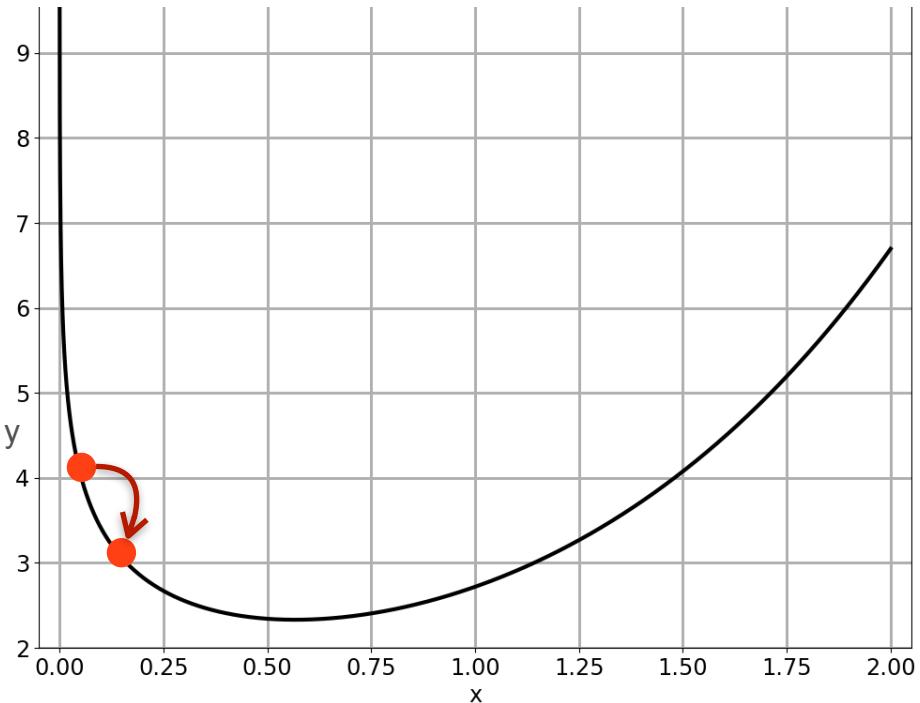
Rate:  $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1447$$



# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start:  $x = 0.05$

Rate:  $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by  $-0.005f'(0.05)$

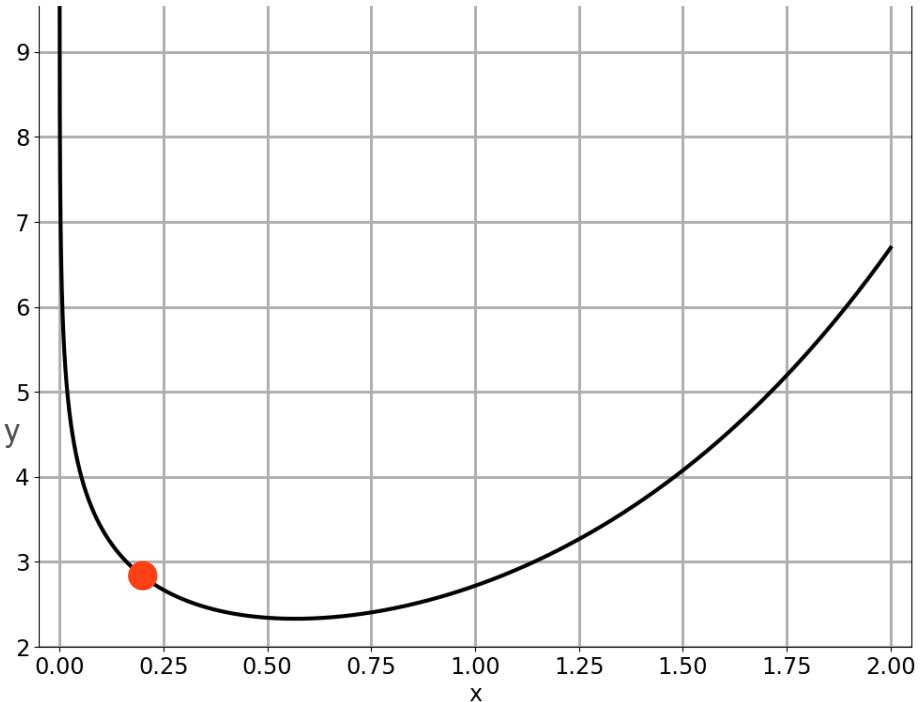
$$x \mapsto 0.1447$$

Find:

$$f'(0.1447) = -5.7552$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1735$$



# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start:  $x = 0.05$

Rate:  $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1447$$

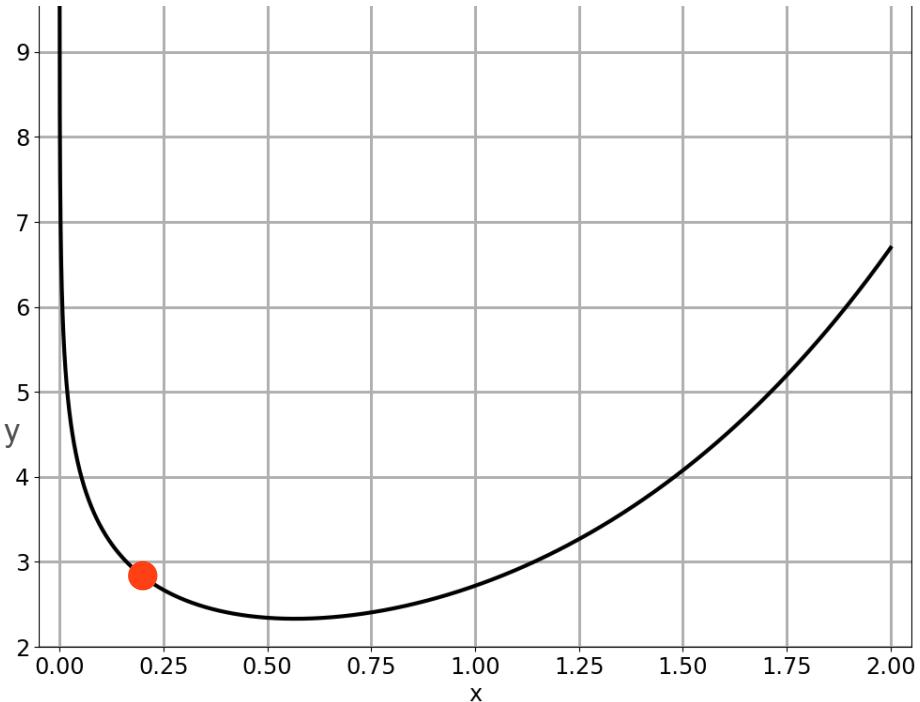
Find:

$$f'(0.1447) = -5.7552$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1735$$

**Repeat!**



# Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start:  $x = 0.05$

Rate:  $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1447$$

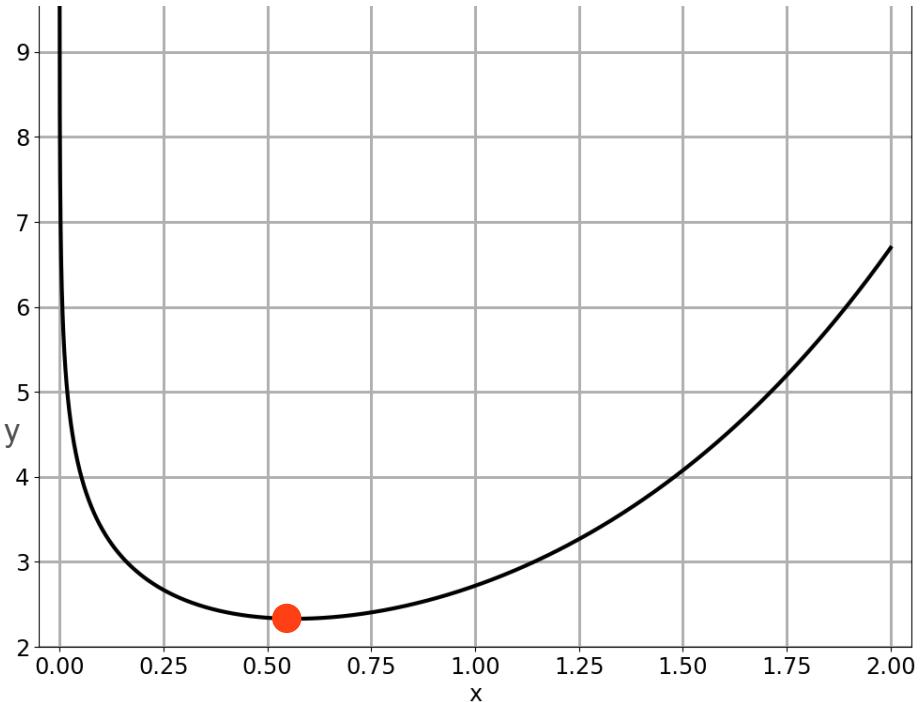
Find:

$$f'(0.1447) = -5.7552$$

Move by  $-0.005f'(0.05)$

$$x \mapsto 0.1735$$

**Repeat!**





DeepLearning.AI

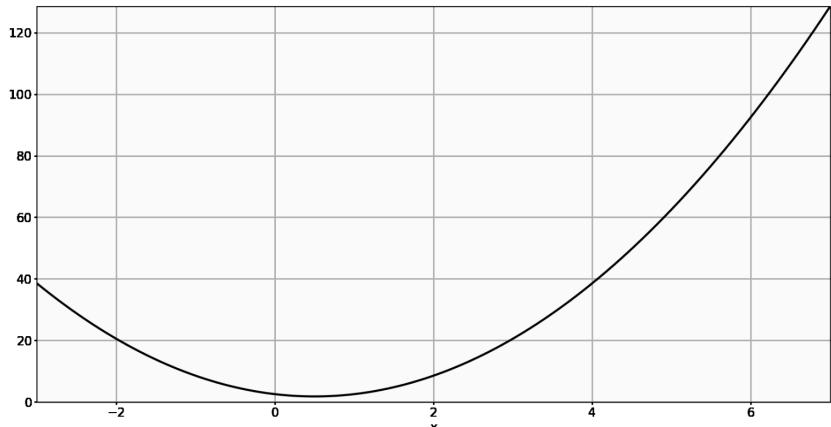
# Gradients and Gradient Descent

---

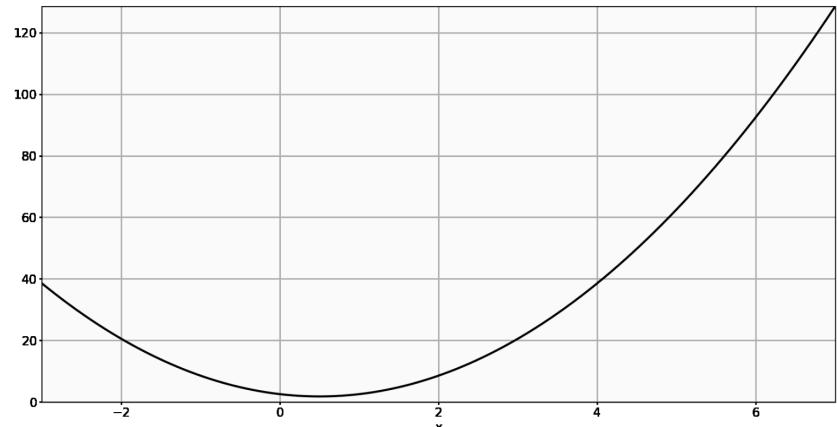
**Optimization using Gradient  
Descent in one variable -  
Part 3**

# What Is a Good Learning Rate?

Too large

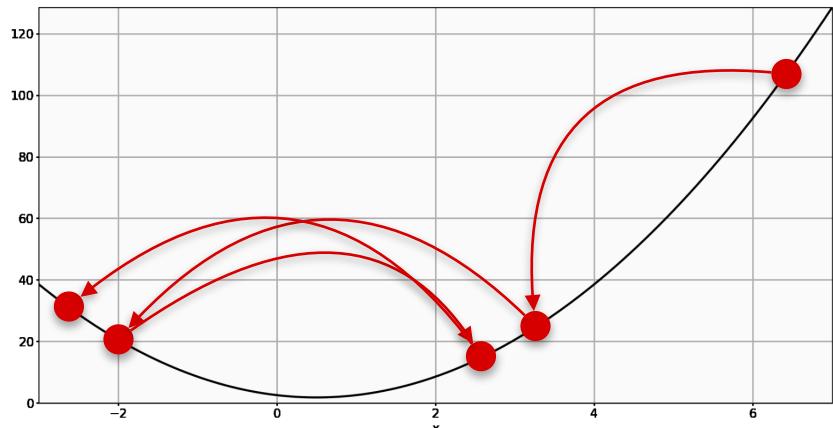


Too small

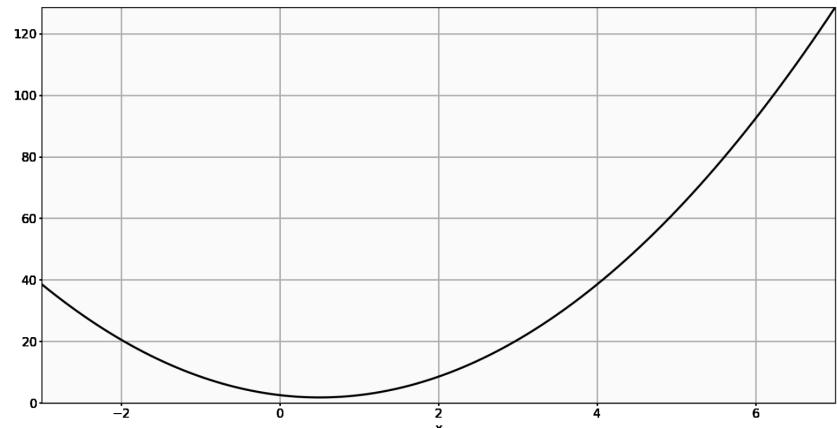


# What Is a Good Learning Rate?

Too large

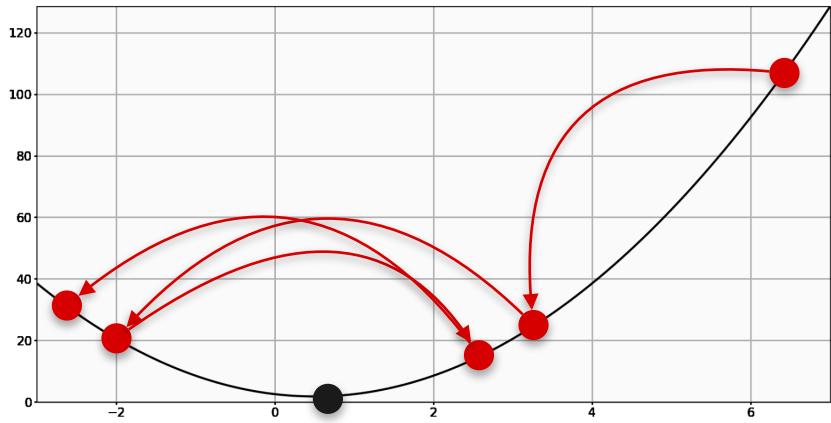


Too small

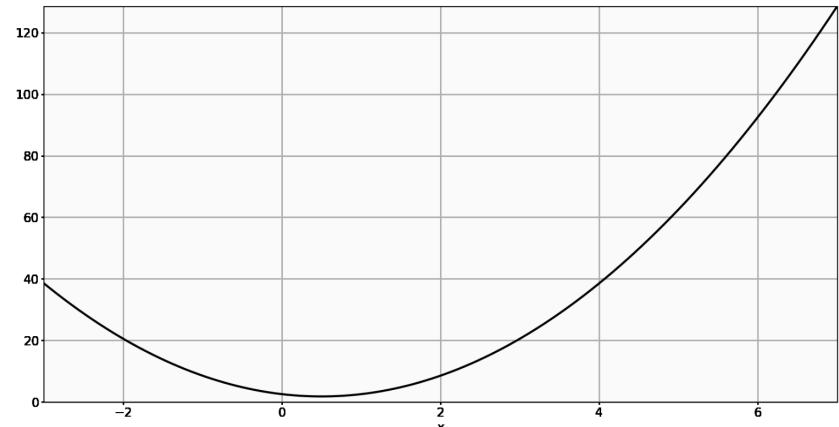


# What Is a Good Learning Rate?

Too large

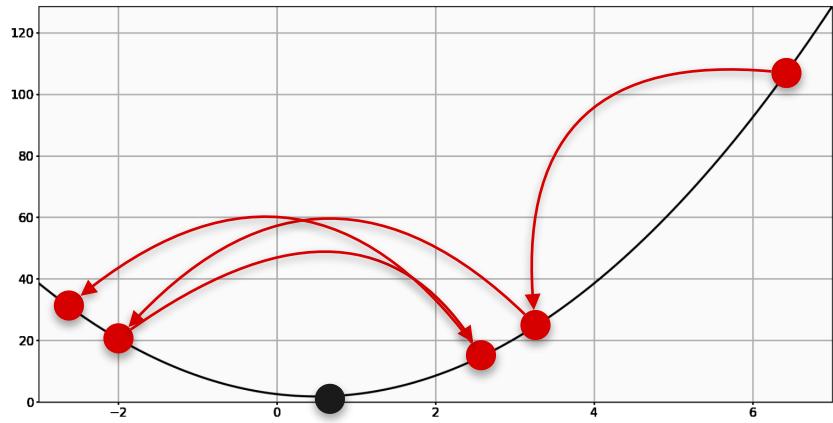


Too small

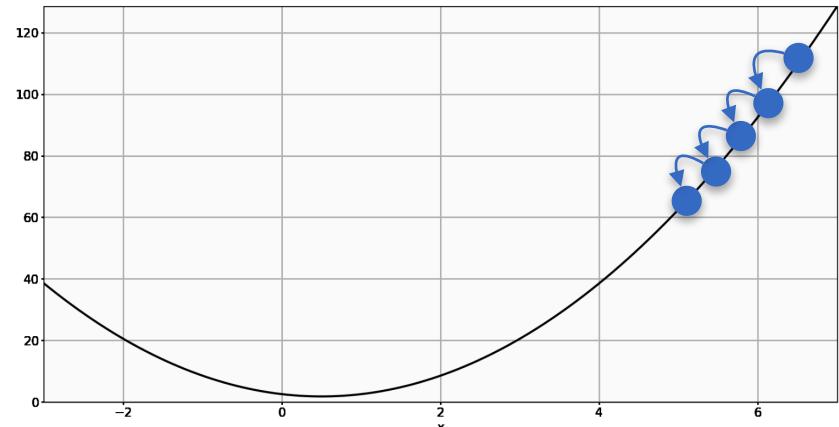


# What Is a Good Learning Rate?

Too large

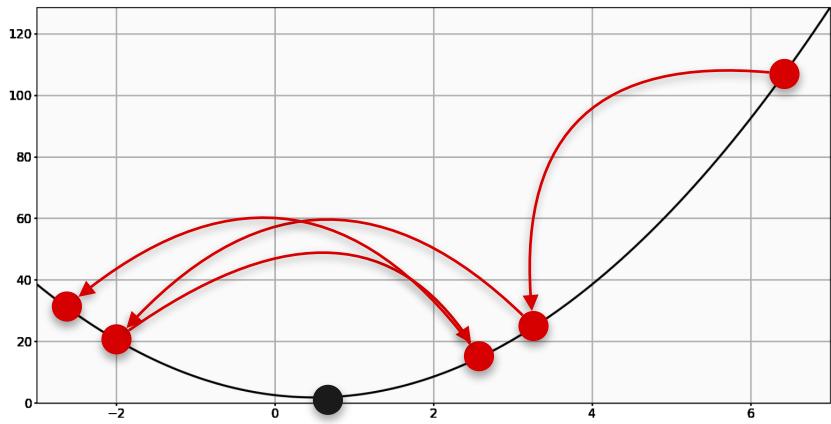


Too small

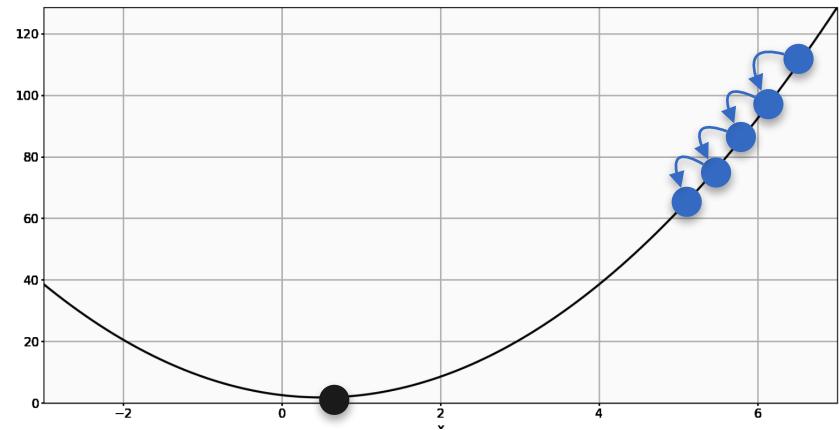


# What Is a Good Learning Rate?

Too large

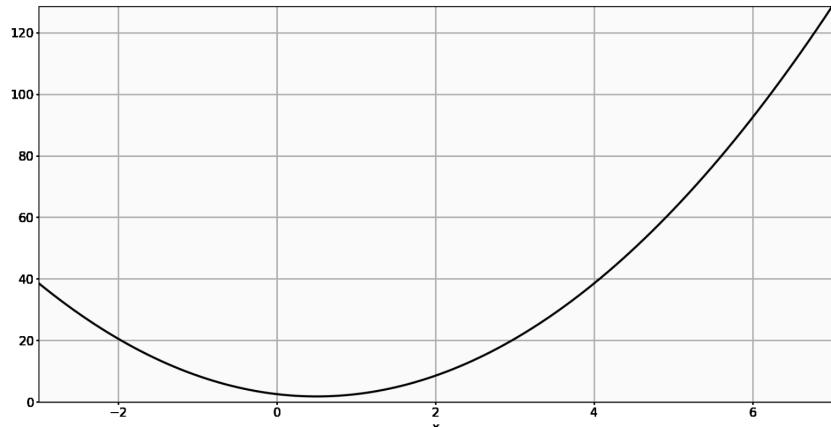


Too small



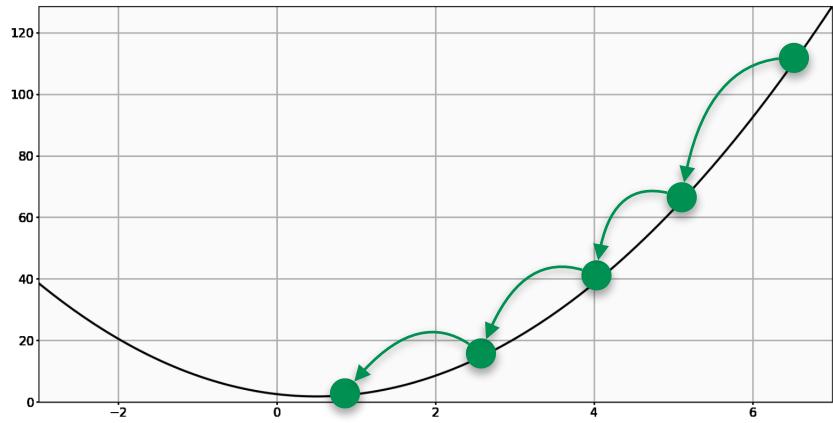
# What Is a Good Learning Rate?

Just right



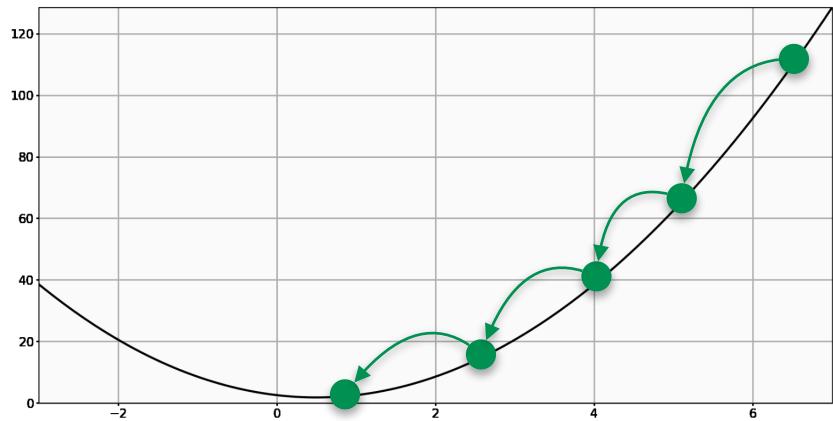
# What Is a Good Learning Rate?

Just right



# What Is a Good Learning Rate?

Just right

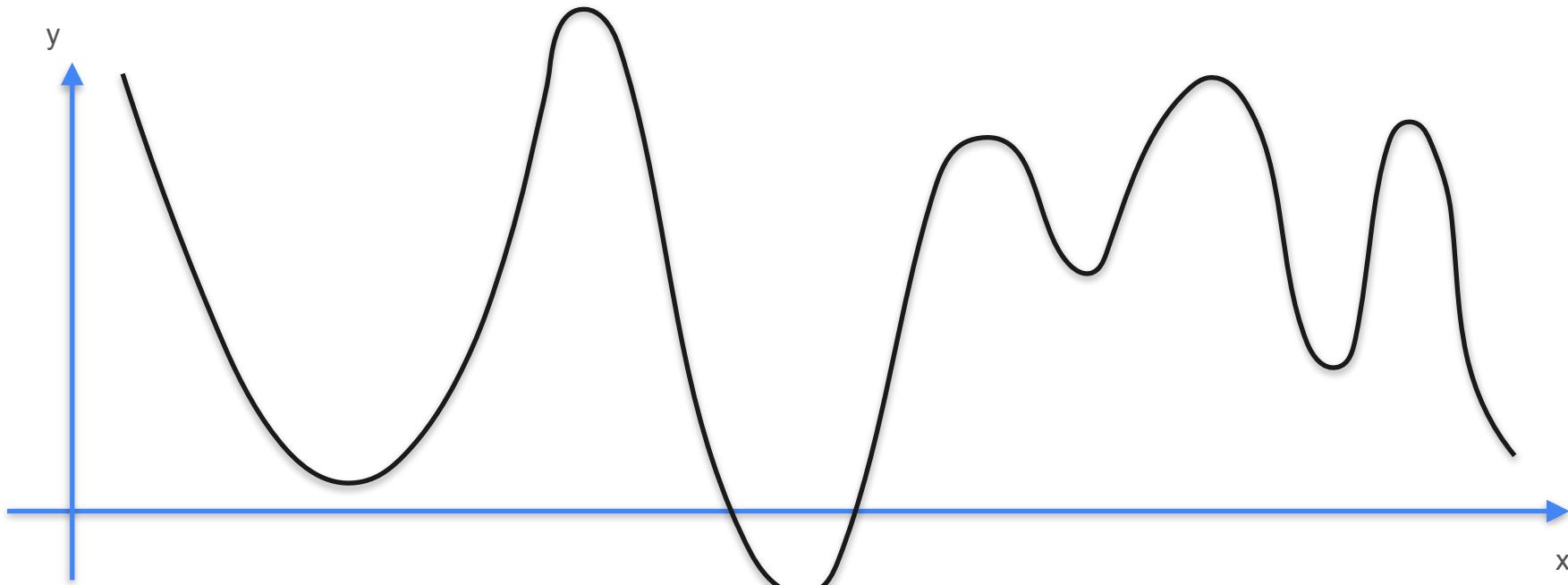


Unfortunately, there is no rule to give the best learning rate  $\alpha$

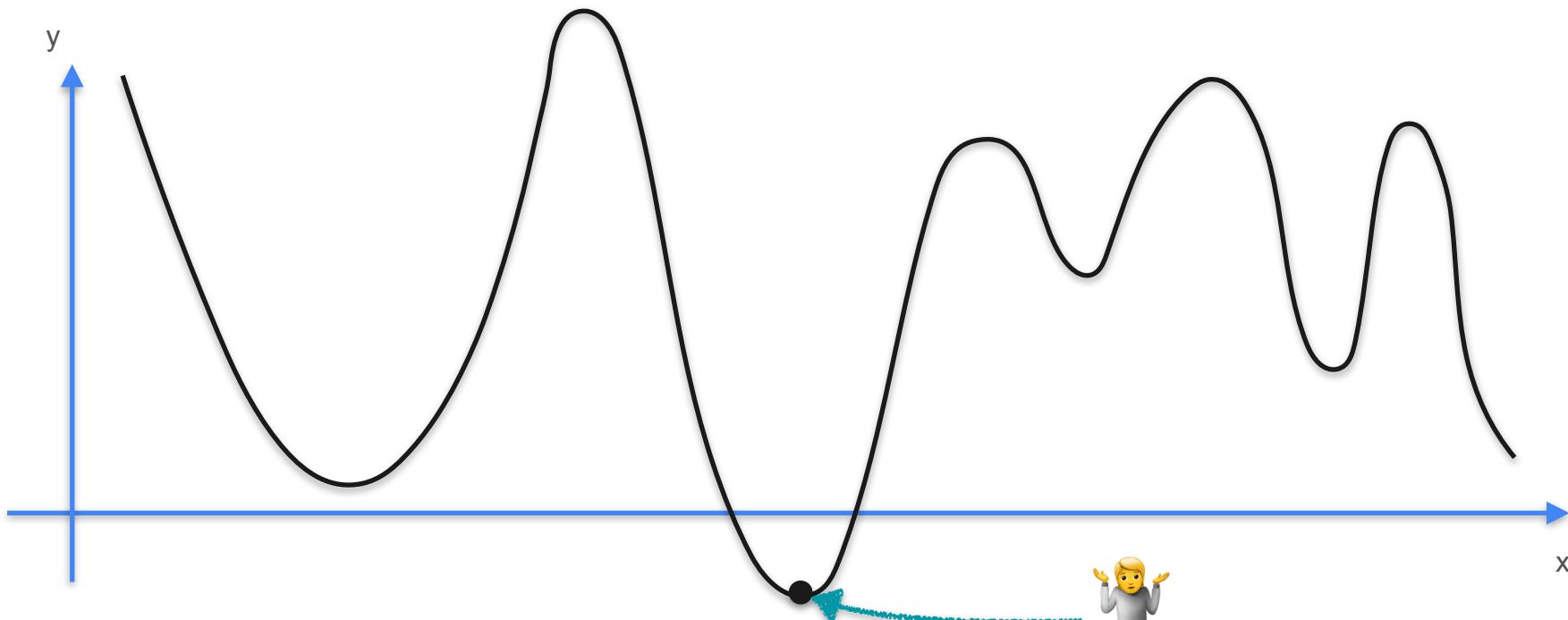


# Drawbacks of Gradient Descent

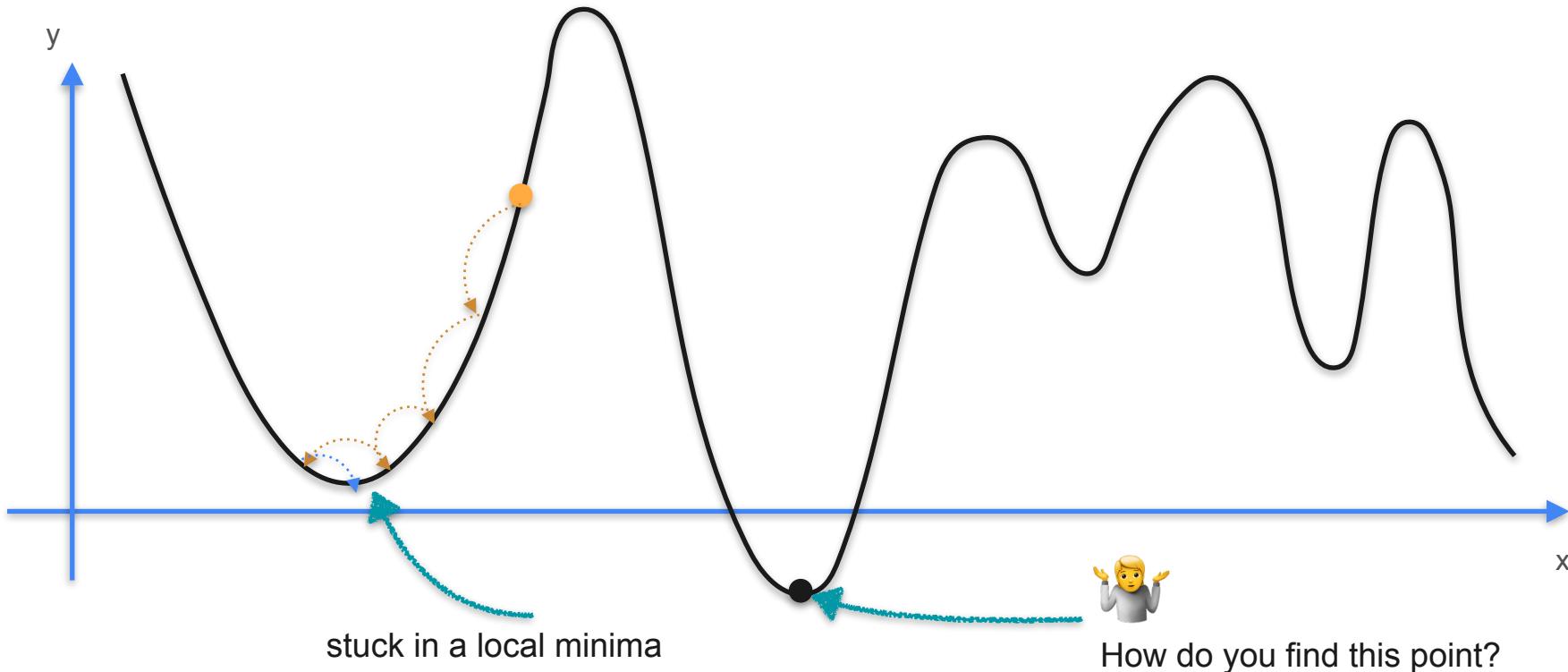
# Drawbacks of Gradient Descent



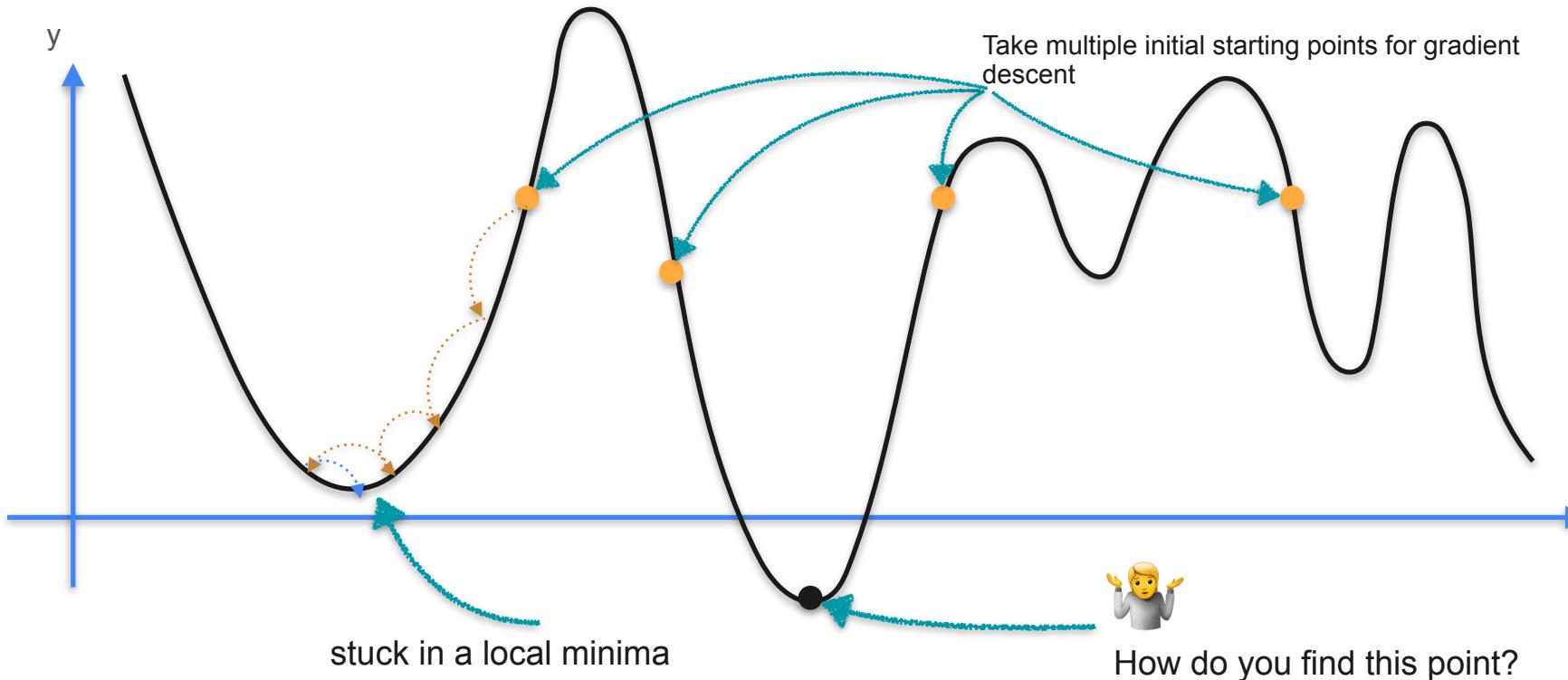
# Drawbacks of Gradient Descent



# Drawbacks of Gradient Descent



# Drawbacks of Gradient Descent





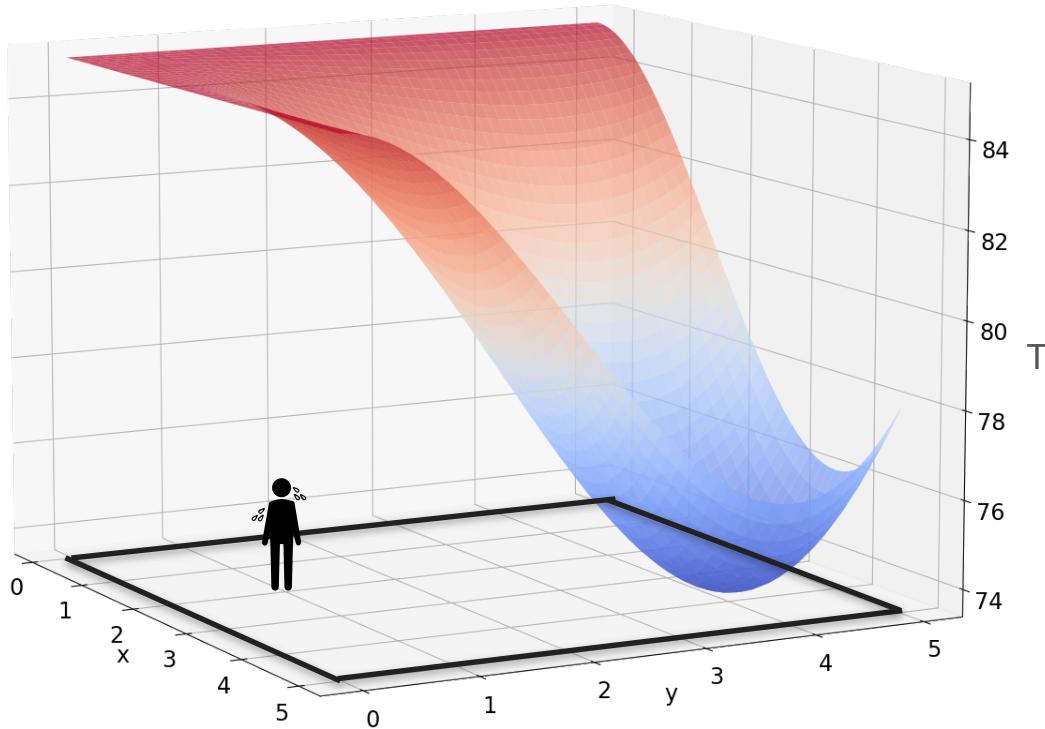
DeepLearning.AI

# Gradients and Gradient Descent

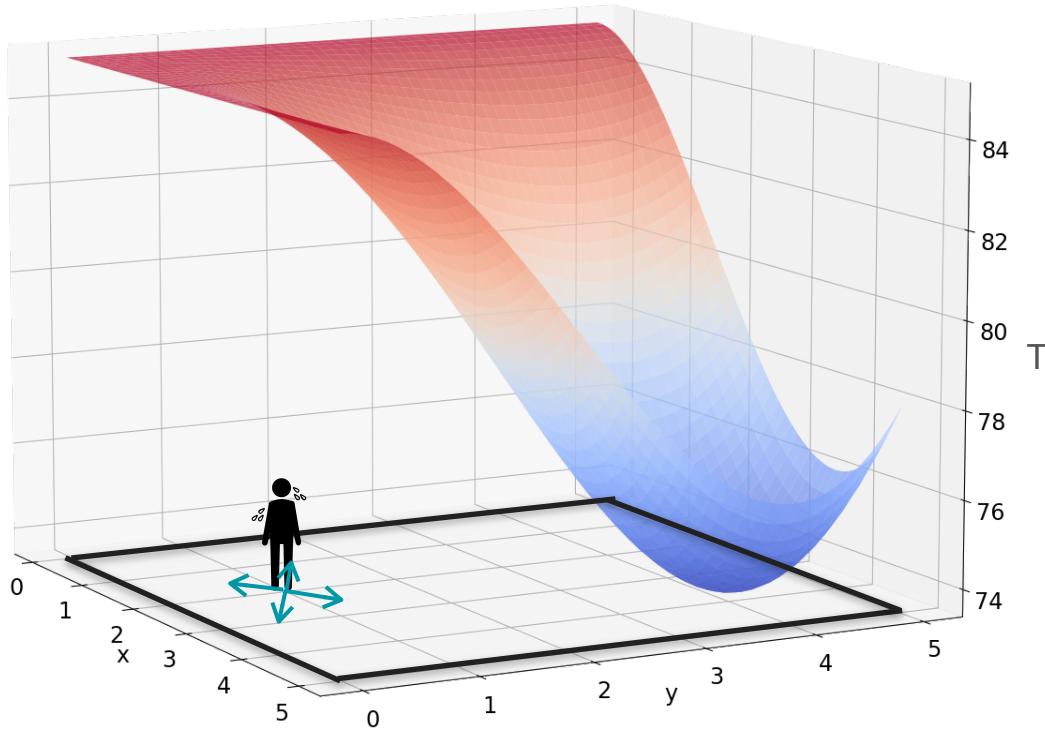
---

**Optimization using Gradient  
Descent in two variables -  
Part 1**

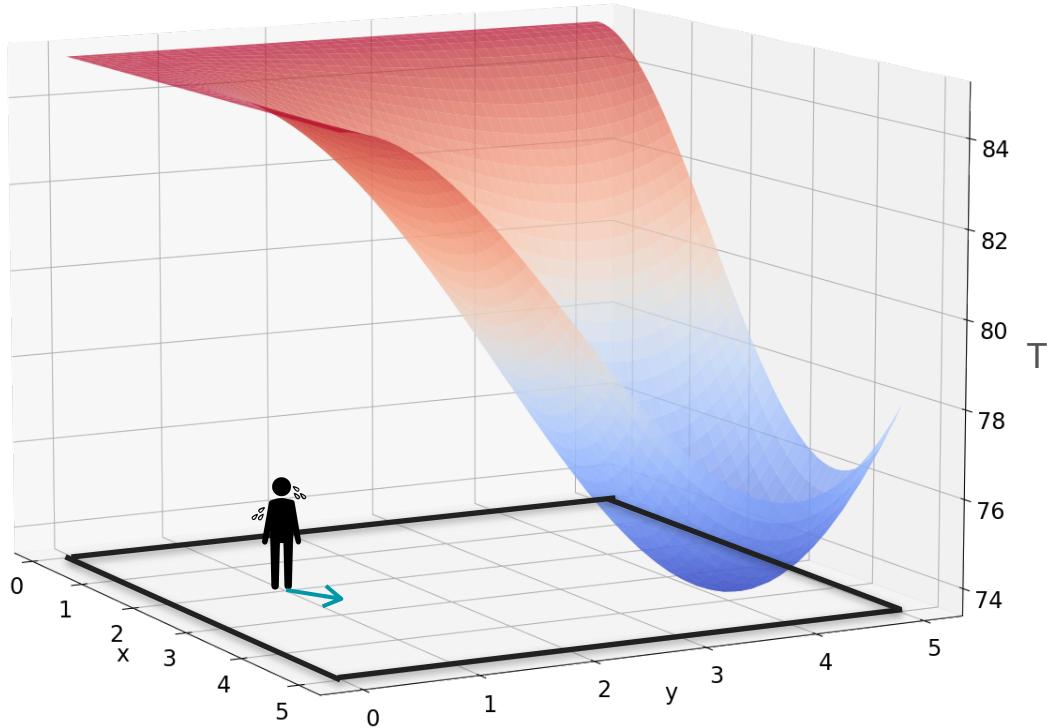
# Gradient Descent With Heat Example



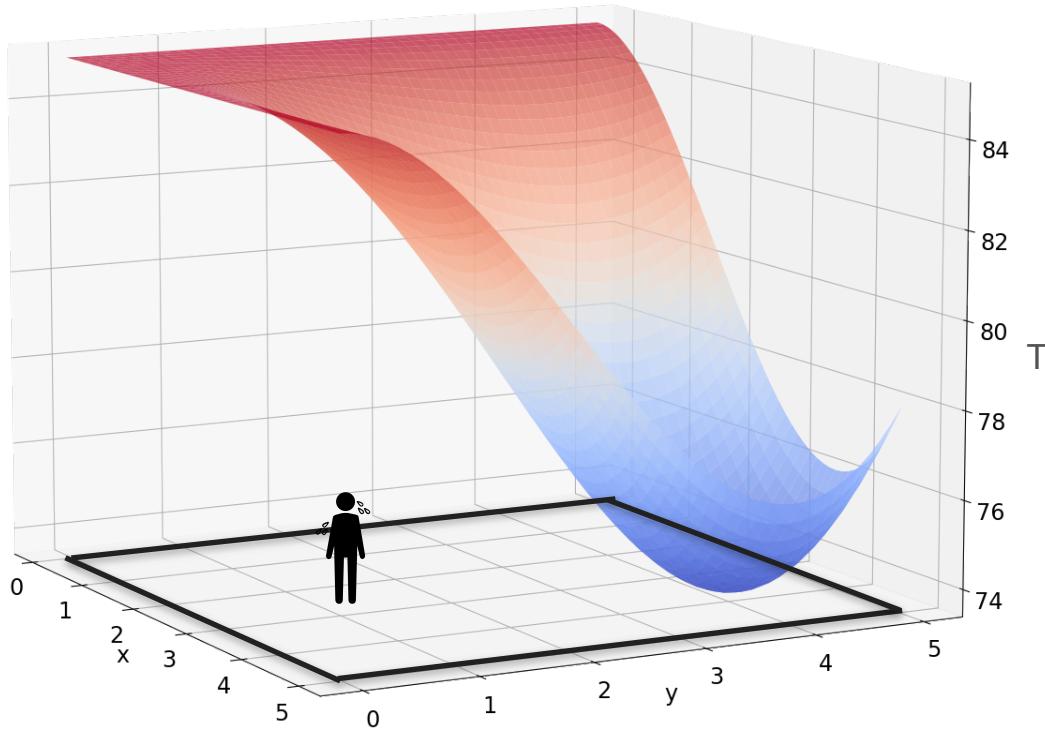
# Gradient Descent With Heat Example



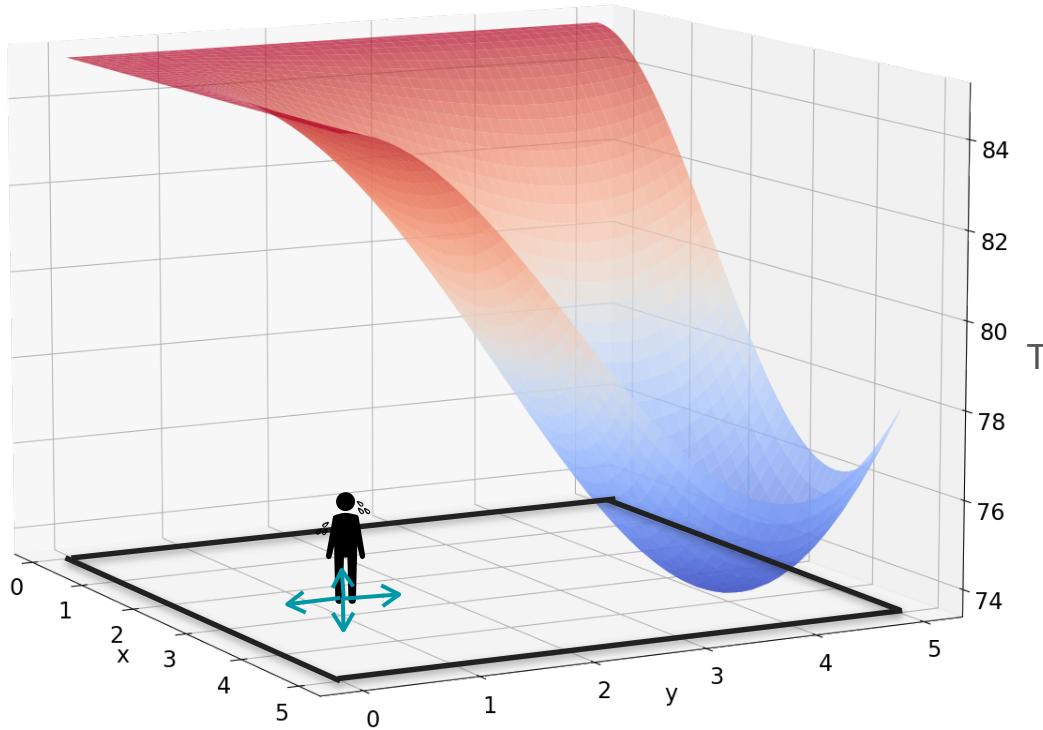
# Gradient Descent With Heat Example



# Gradient Descent With Heat Example

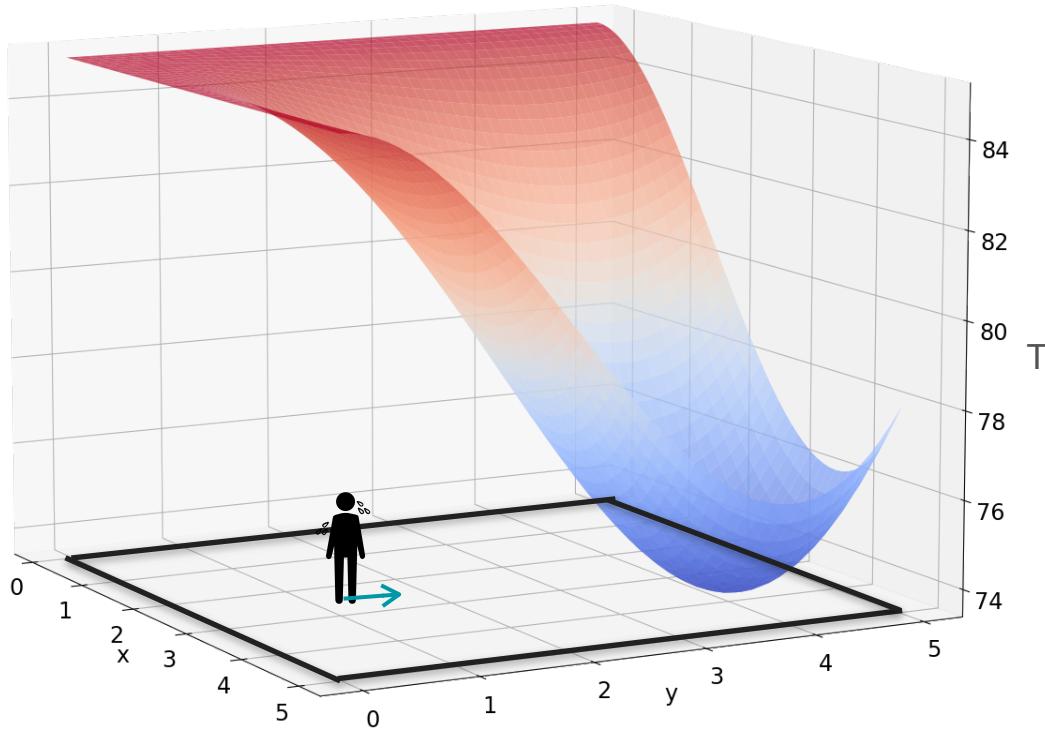


# Gradient Descent With Heat Example

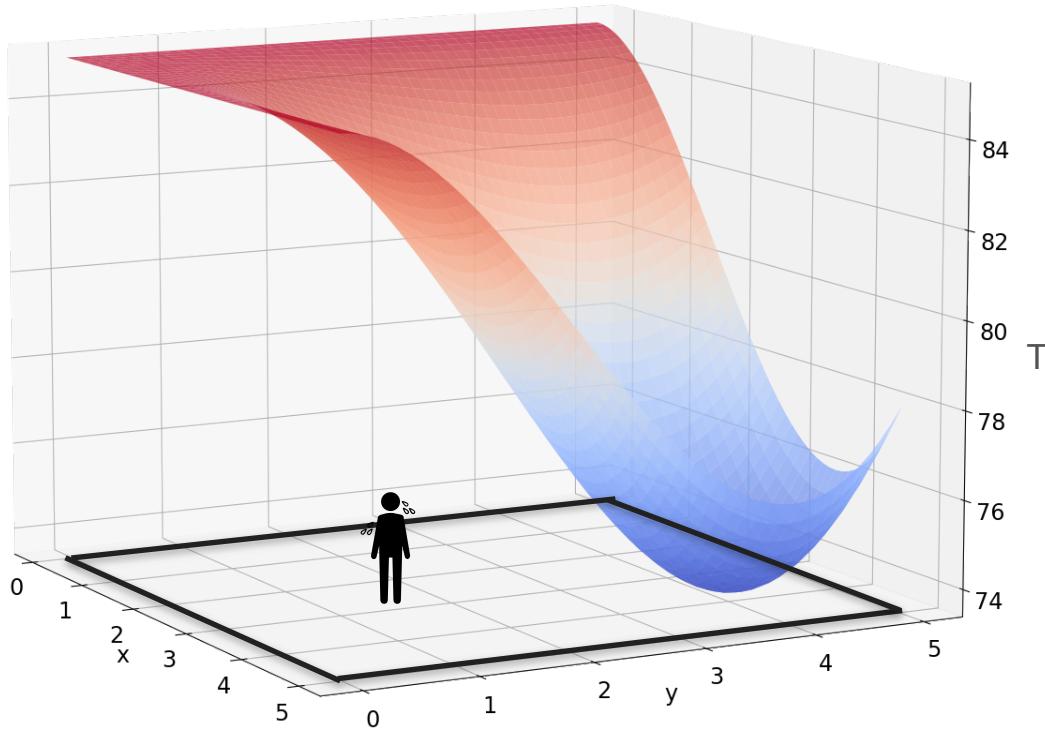


# Gradient Descent With Heat Example

Repeat!

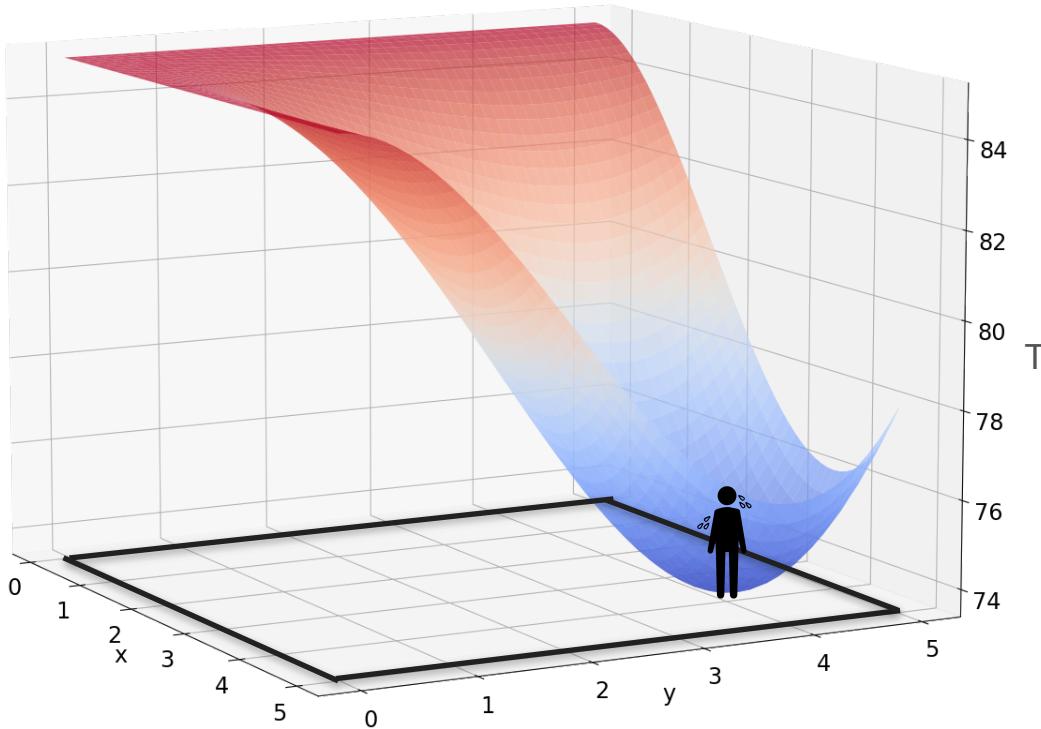


# Gradient Descent With Heat Example



# Gradient Descent With Heat Example

Repeat!





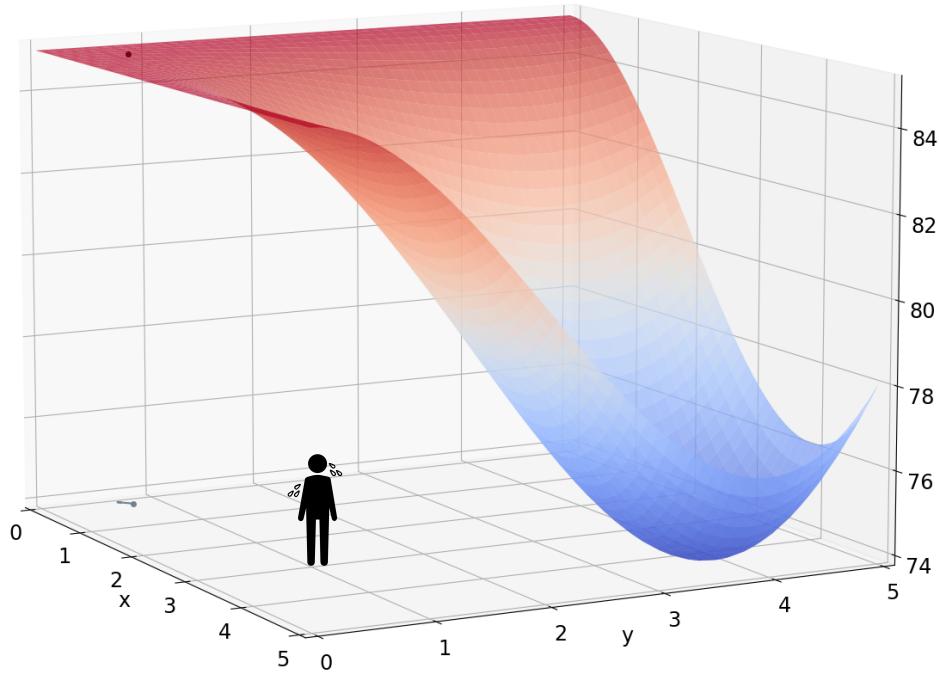
DeepLearning.AI

# Gradients and Gradient Descent

---

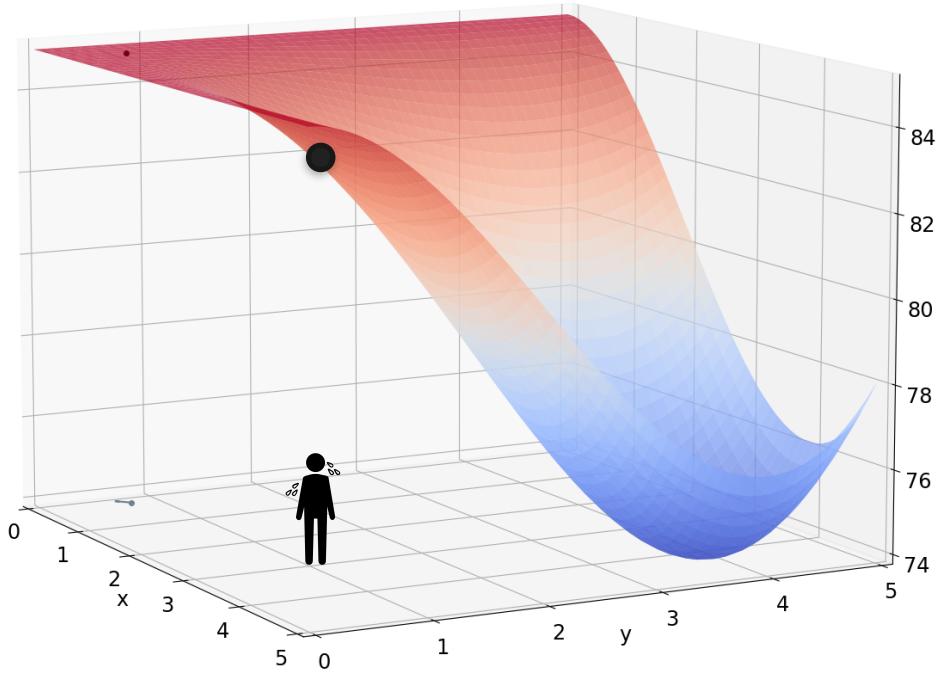
**Optimization using Gradient  
Descent in two variables -  
Part 2**

# Idea for Gradient Descent



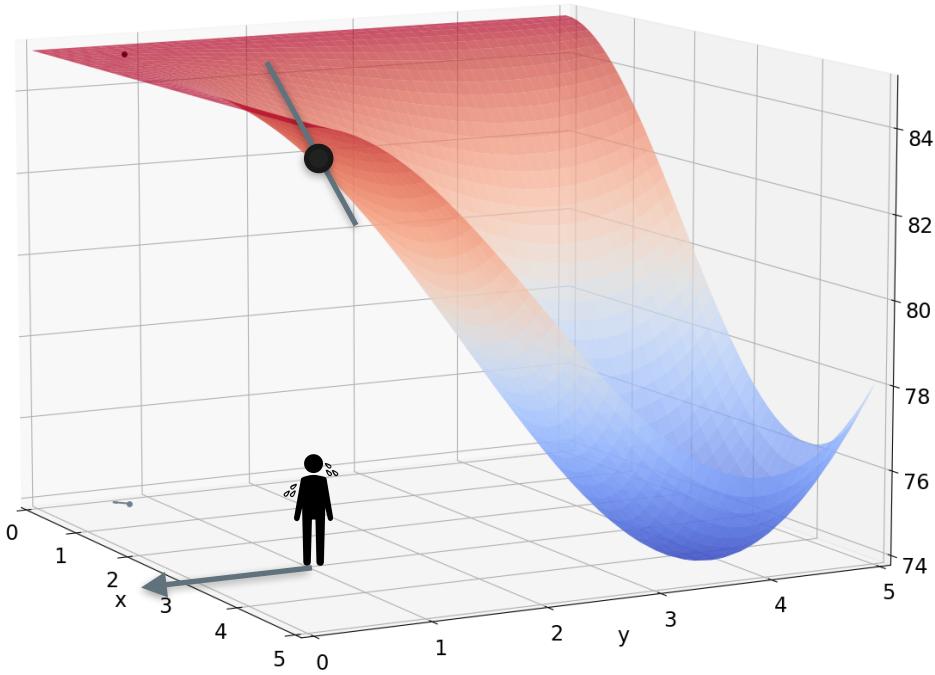
# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$



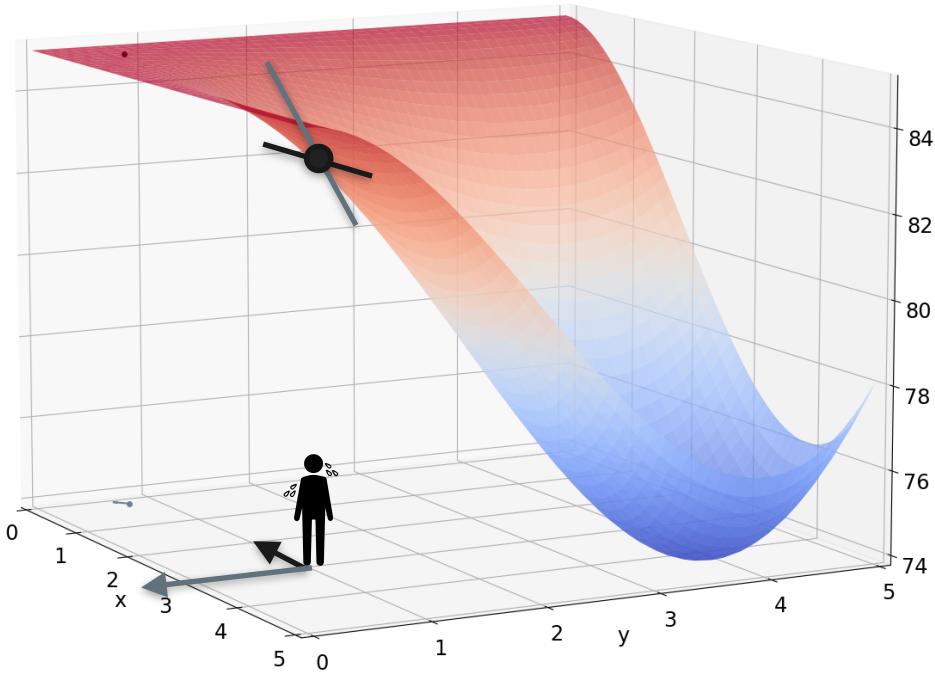
# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$



# Idea for Gradient Descent

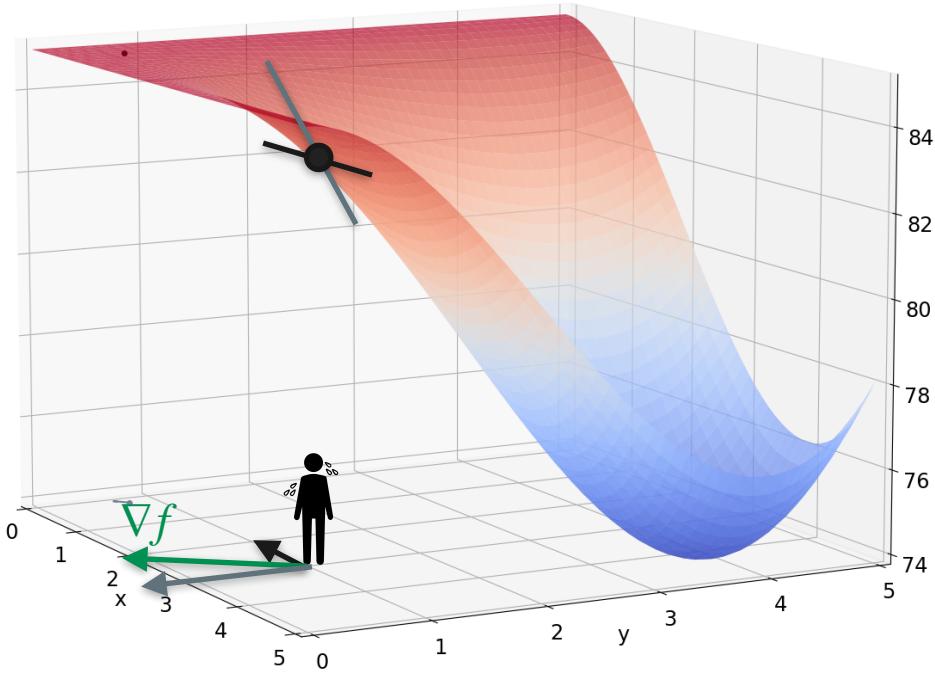
Initial position:  $(x_0, y_0)$



# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

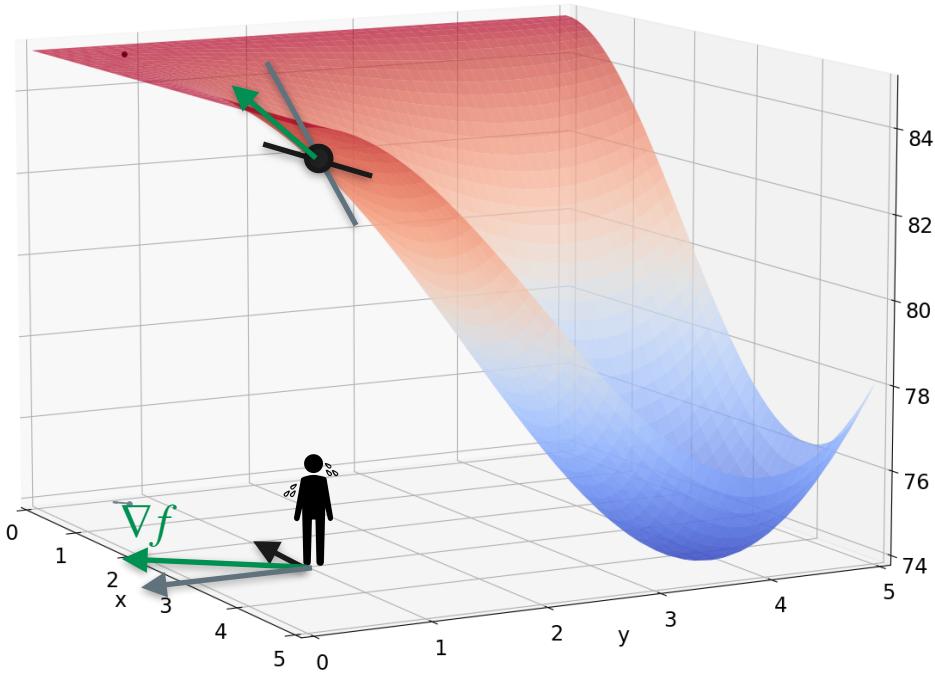
Direction of greatest ascent:  $\nabla f$



# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

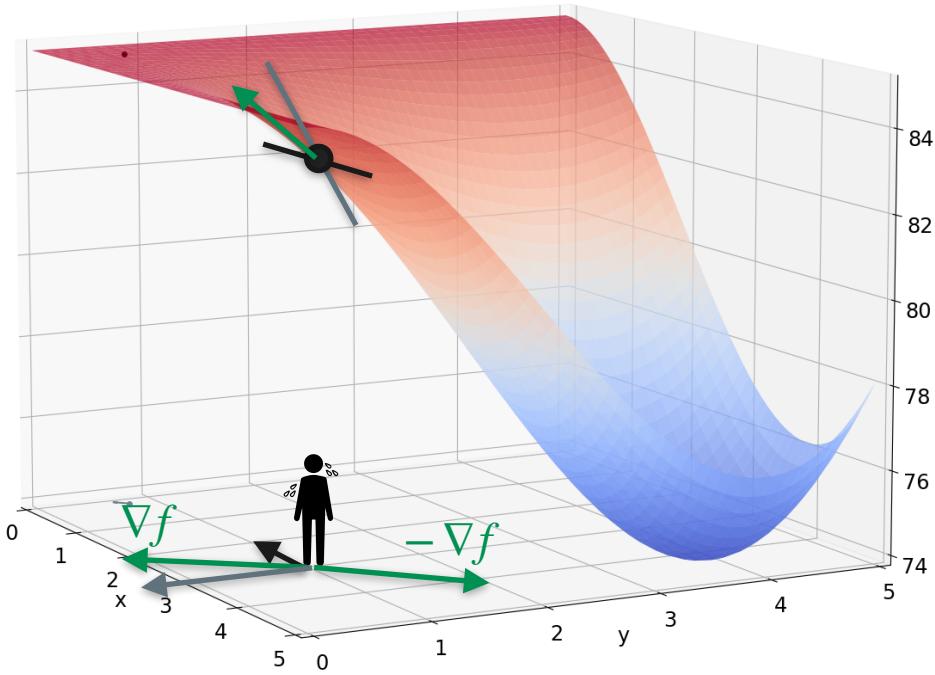


# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$

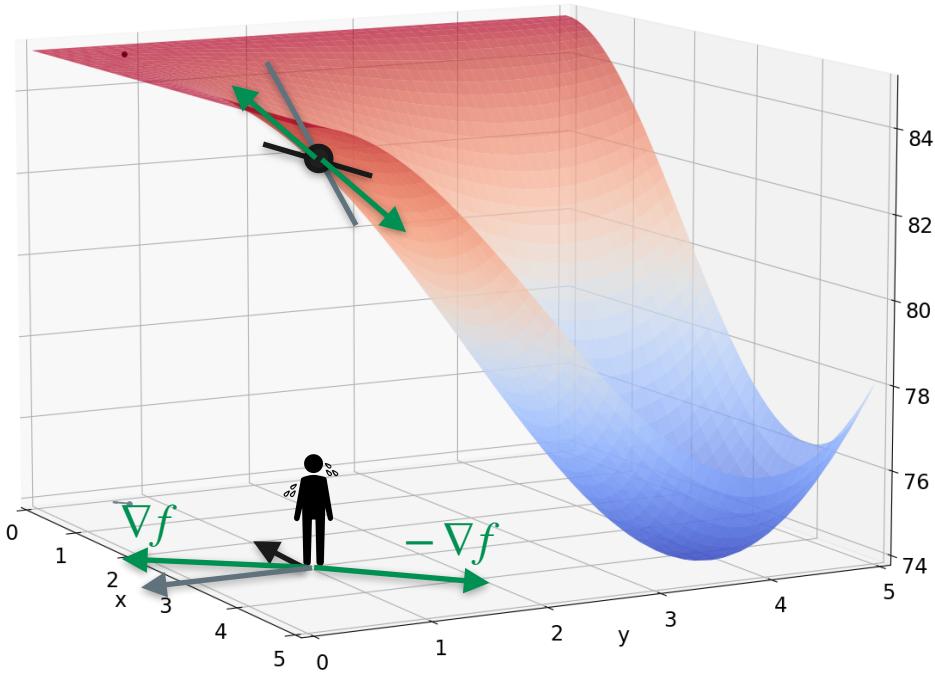


# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$

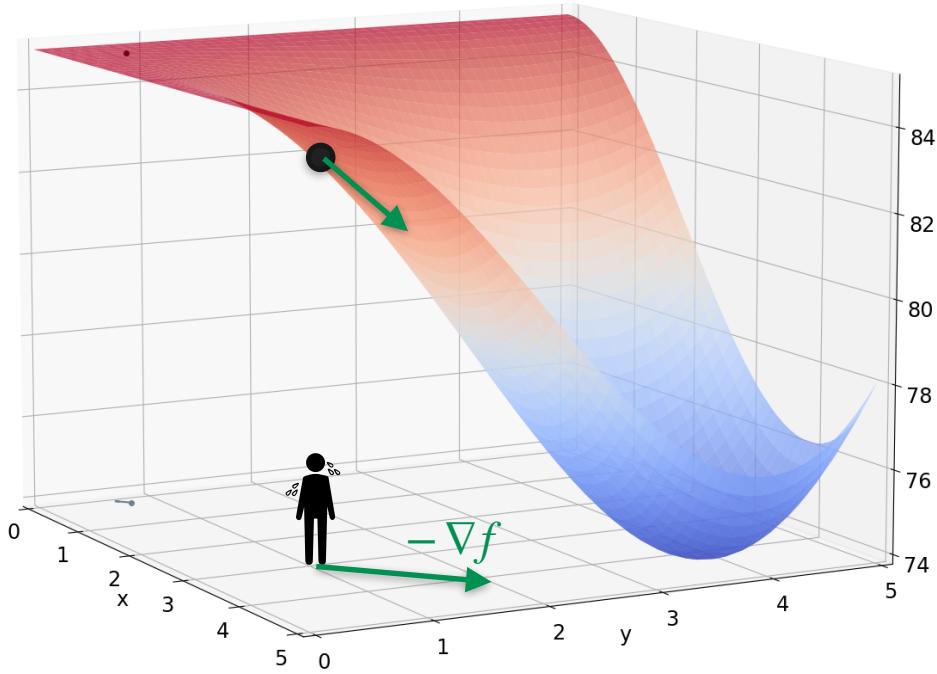


# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$



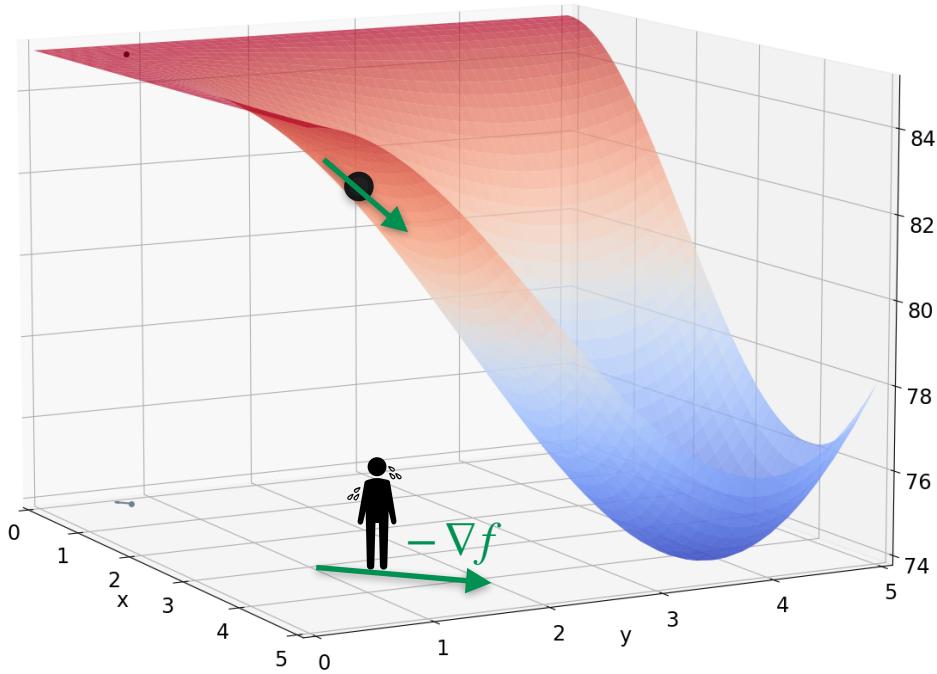
# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$

Updated position:  $(x_0, y_0) - \alpha \nabla f$



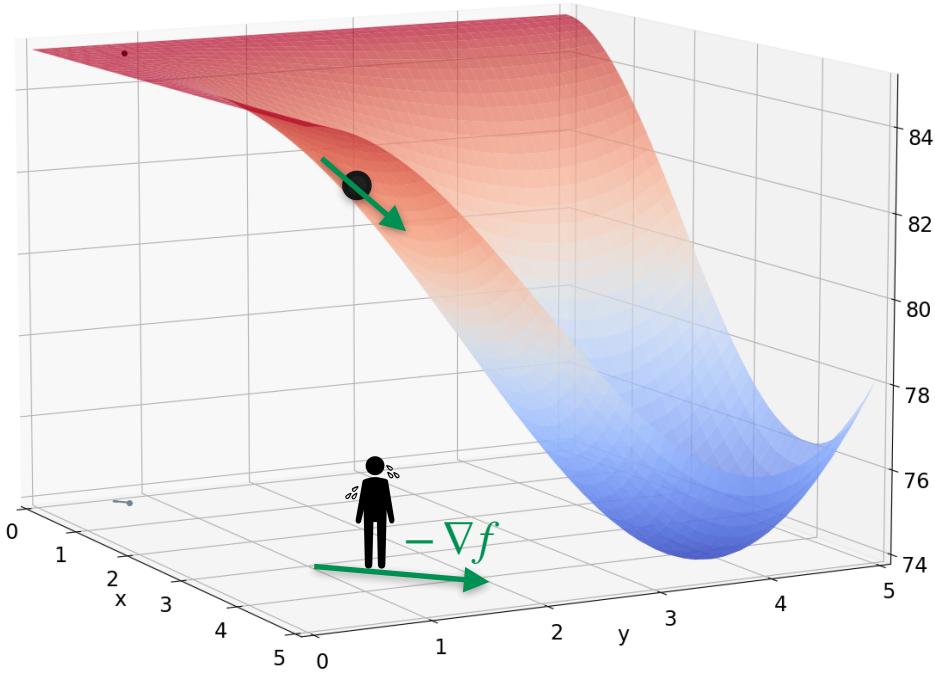
# Idea for Gradient Descent

Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$

Updated position:  $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$



# Idea for Gradient Descent

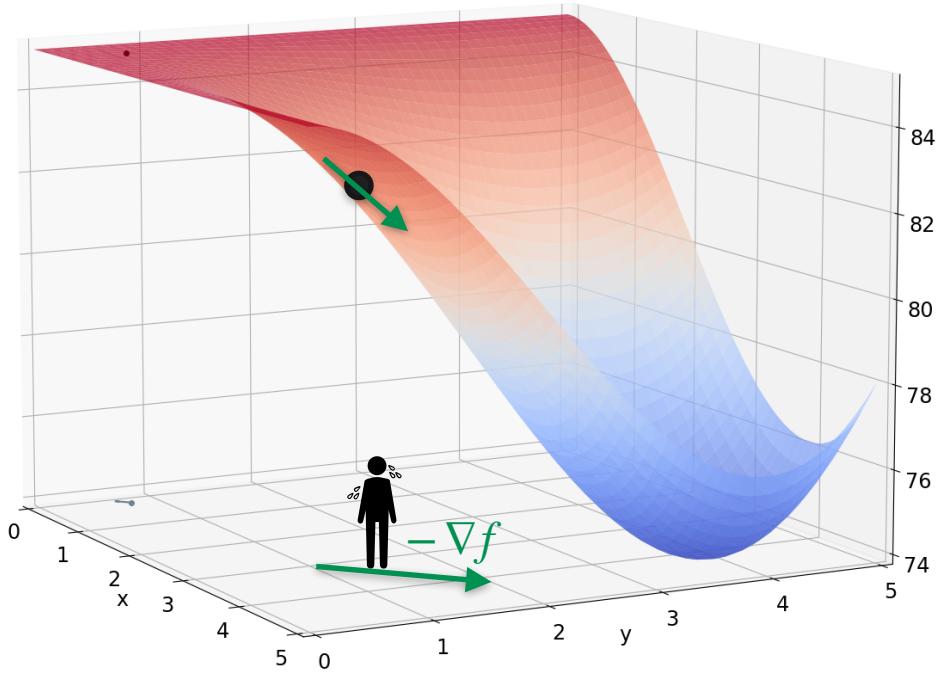
Initial position:  $(x_0, y_0)$

Direction of greatest ascent:  $\nabla f$

Direction of greatest descent:  $-\nabla f$

Updated position:  $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$

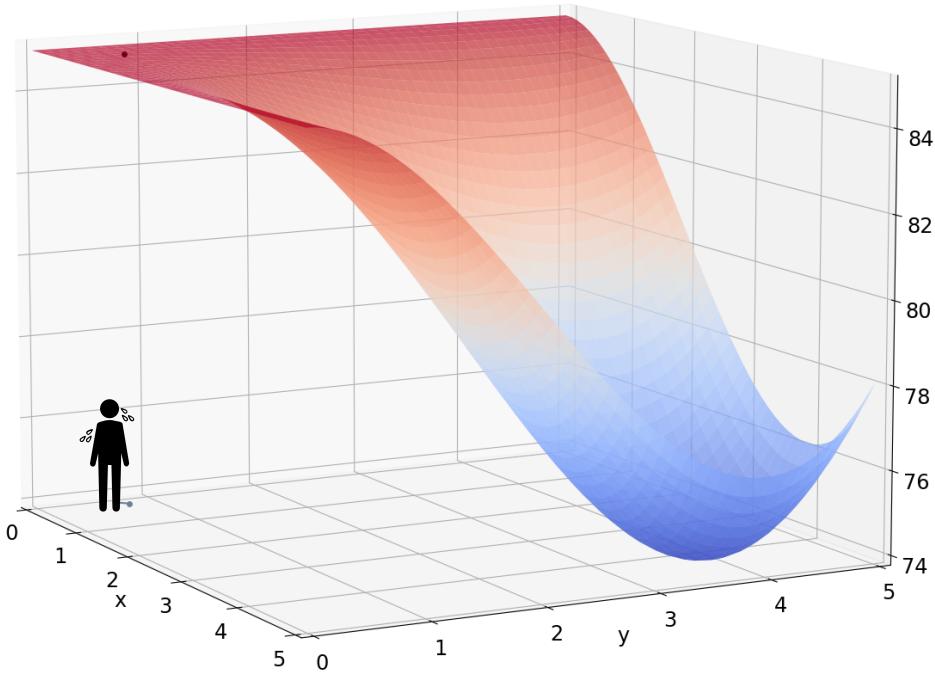
Better point!



# Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start:  $x = 0.5, y = 0.6$

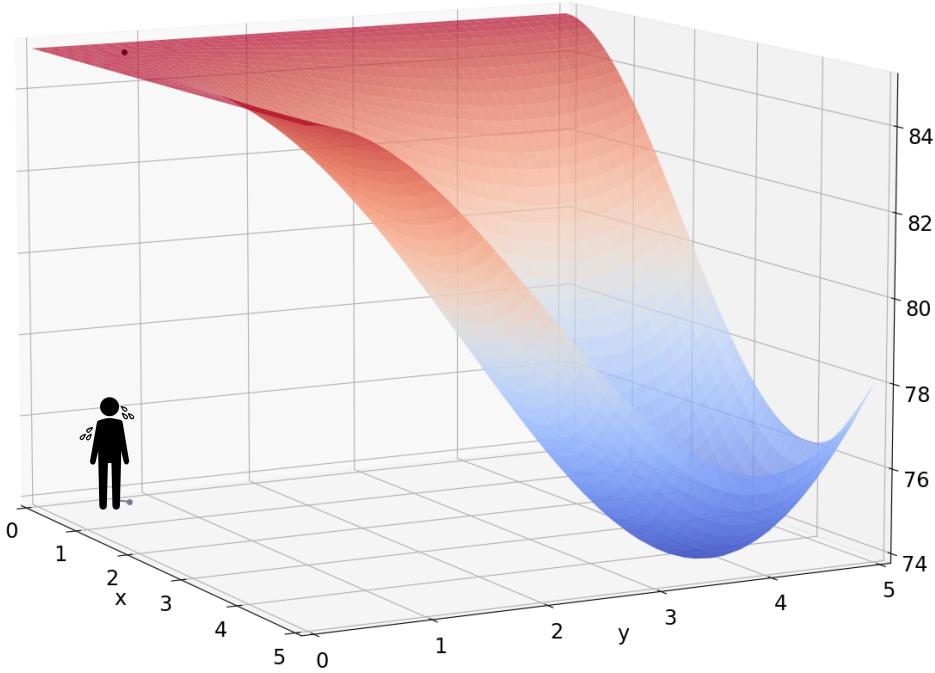


# Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start:  $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



# Method 2: Gradient Descent

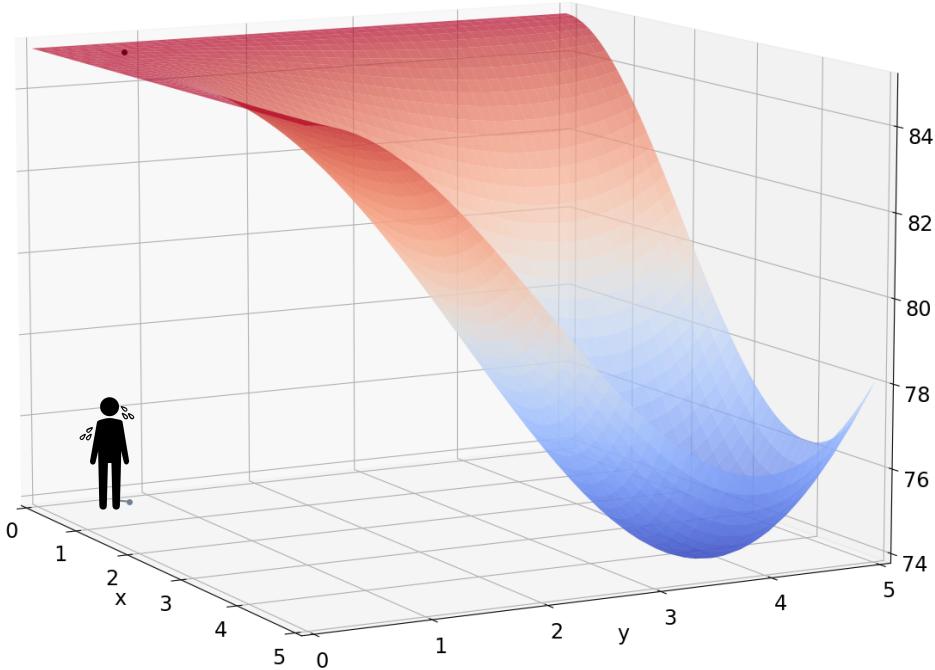
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start:  $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12)$$



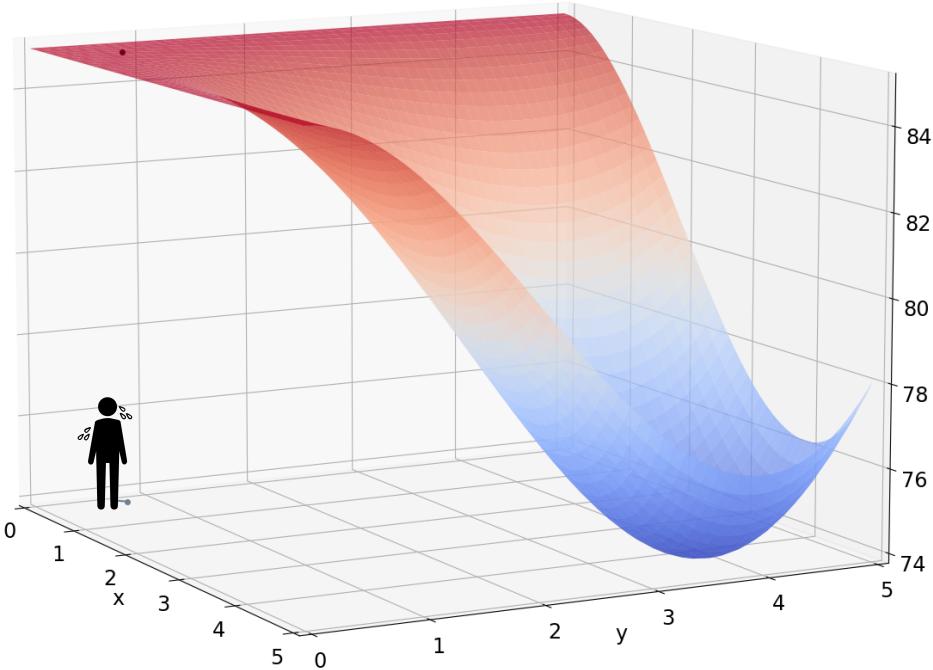
# Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start:  $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} -\frac{1}{90}x(3x - 12)y^2(y - 6) \\ -\frac{1}{90}x^2(x - 6)y(3y - 12) \end{bmatrix}$$



# Method 2: Gradient Descent

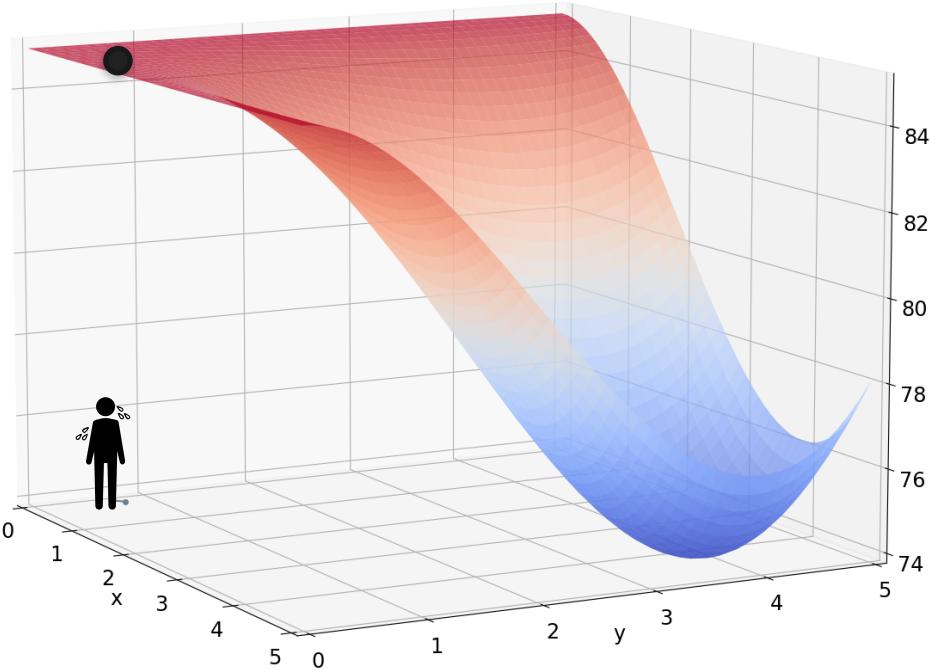
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start:  $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} -\frac{1}{90}x(3x - 12)y^2(y - 6) \\ -\frac{1}{90}x^2(x - 6)y(3y - 12) \end{bmatrix}$$

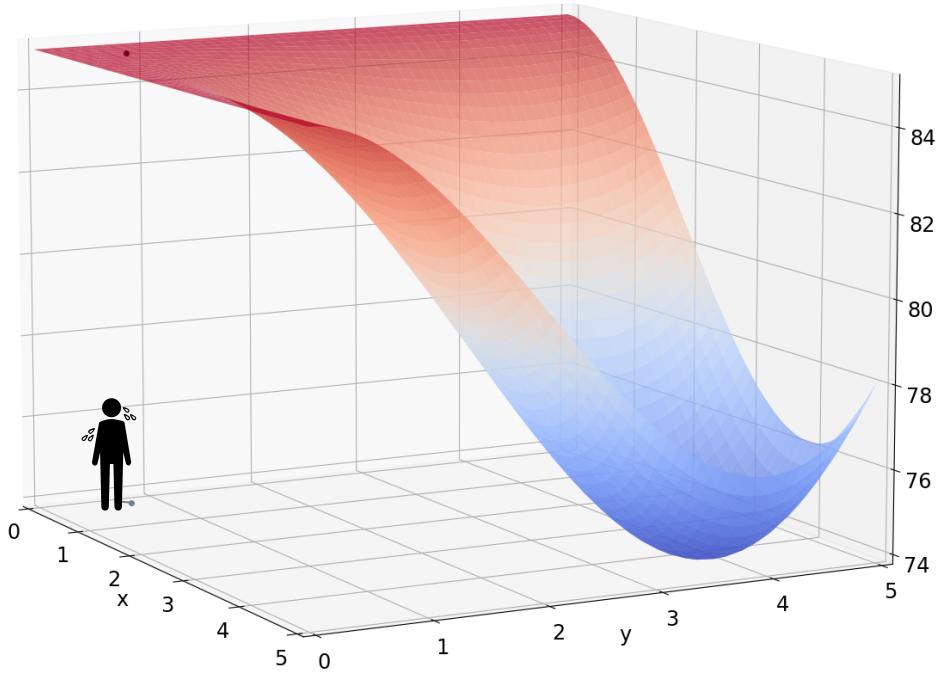
$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$



# Method 2: Gradient Descent

Start:  $x = 0.5, y = 0.6$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

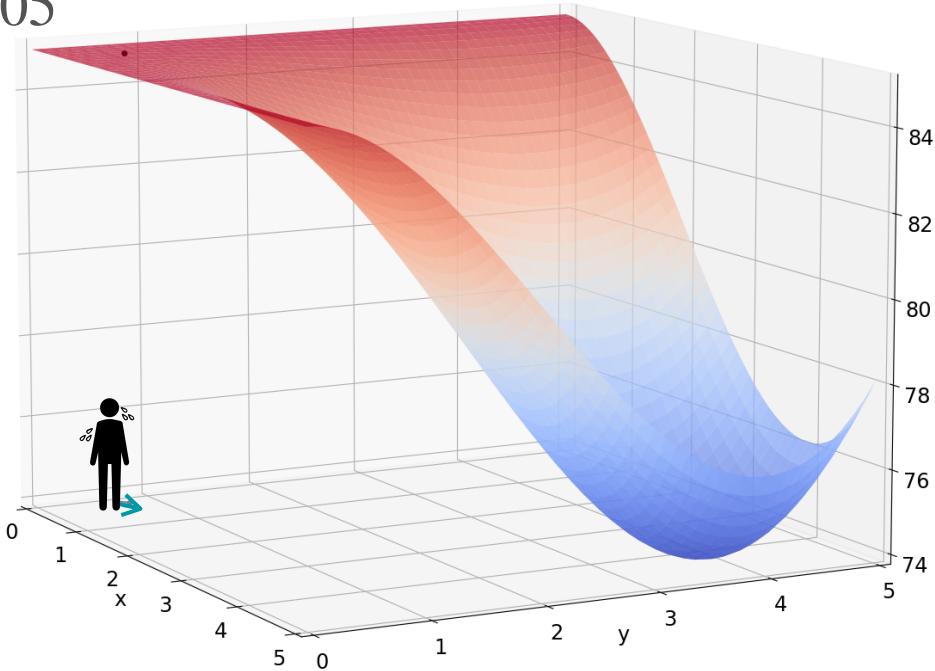


# Method 2: Gradient Descent

Start:  $x = 0.5, y = 0.6$     Rate:  $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5, 0.6)$



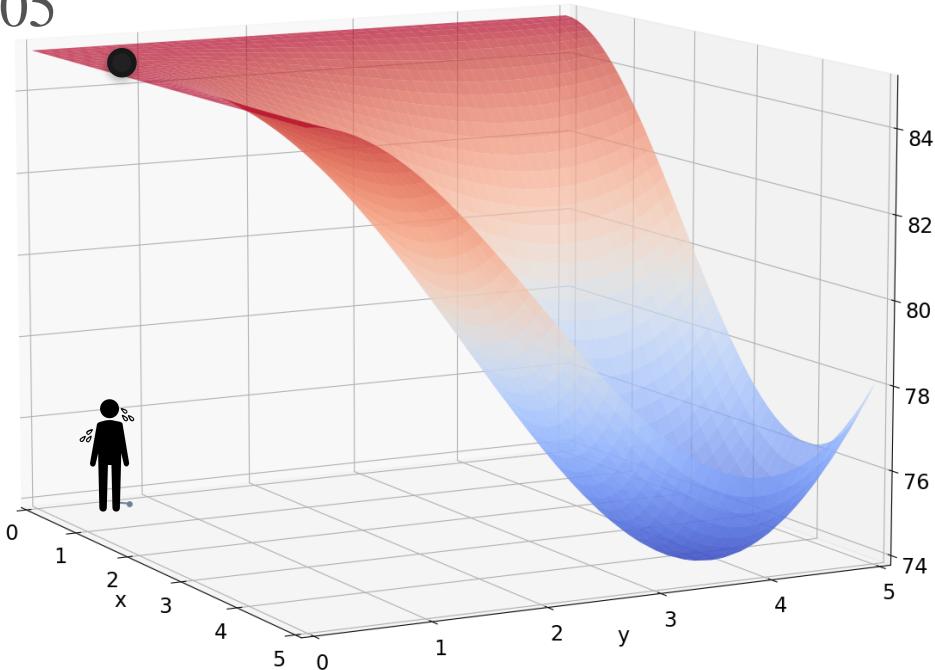
# Method 2: Gradient Descent

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5, 0.6)$

$$\begin{aligned} x &\mapsto 0.5057 \\ y &\mapsto 0.6047 \end{aligned}$$



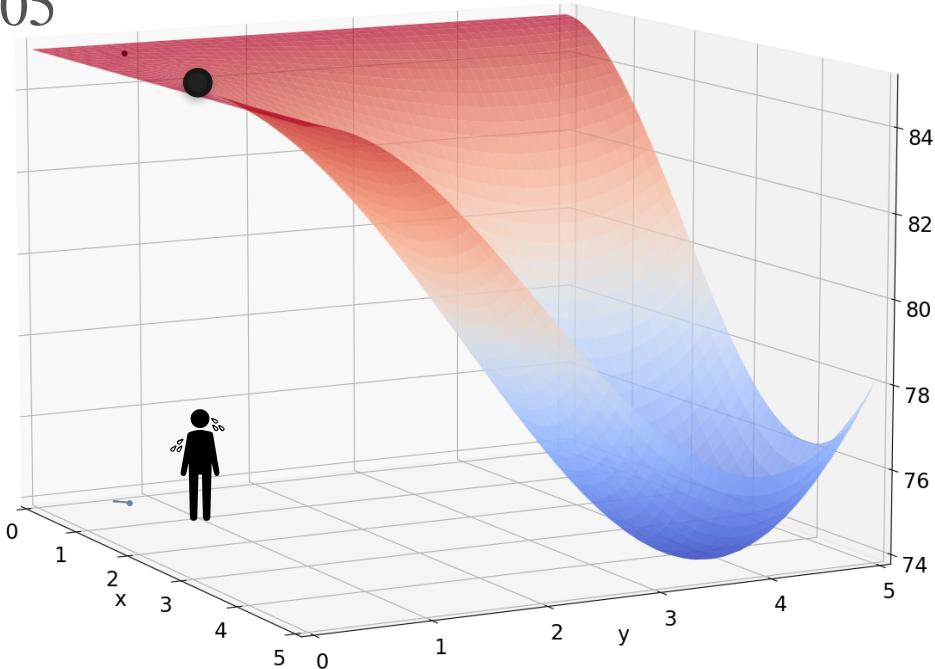
# Method 2: Gradient Descent

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

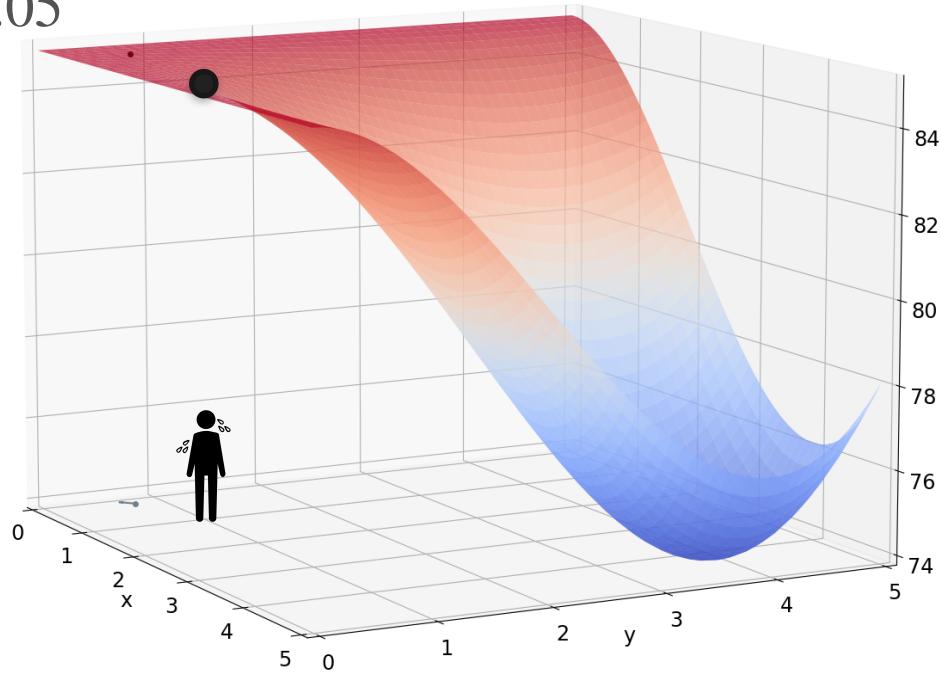
Move by  
 $-0.05 \nabla f(0.5, 0.6)$

$$\begin{aligned} x &\mapsto 0.5057 \\ y &\mapsto 0.6047 \end{aligned}$$



# Method 2

Start:  $x = 0.5, y = 0.6$     Rate:  $\alpha = 0.05$



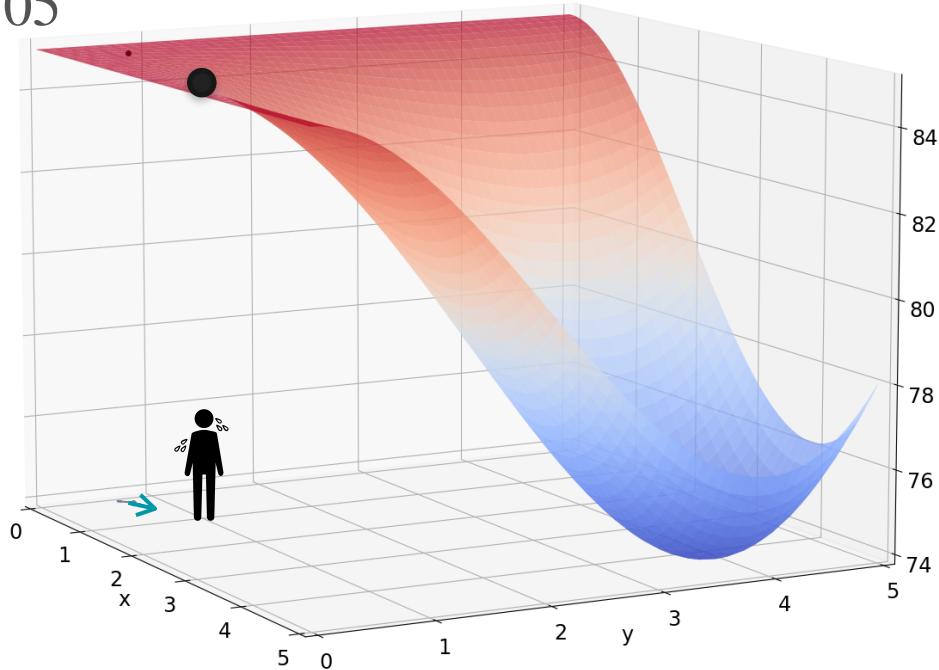
# Method 2

Start:  $x = 0.5, y = 0.6$     Rate:  $\alpha = 0.05$

Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5057, 0.6047)$



# Method 2

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

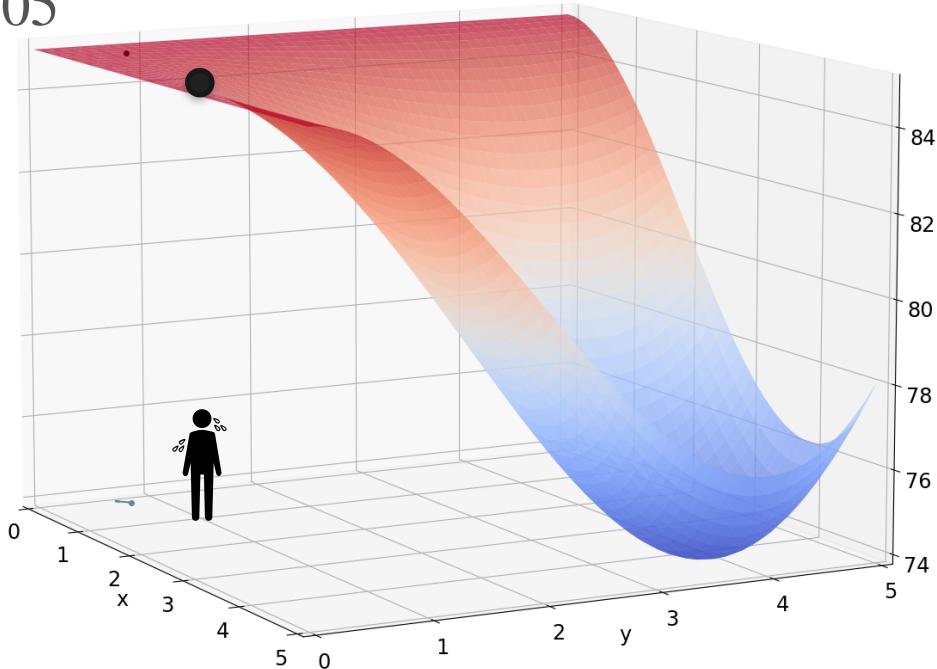
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

**Repeat!**



# Gradient Descent

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

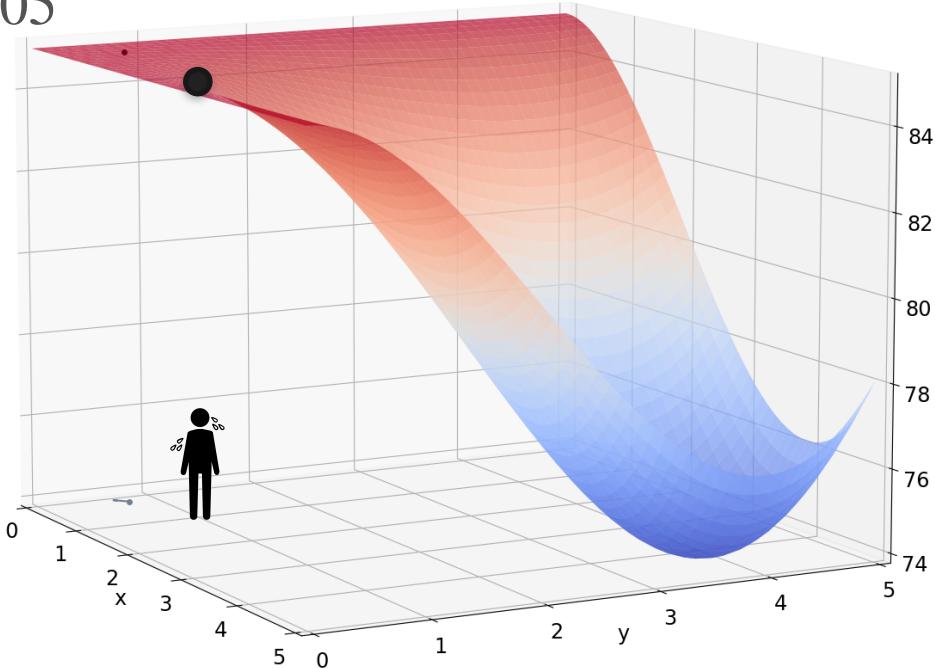
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

**Repeat!**



# Gradient Descent

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

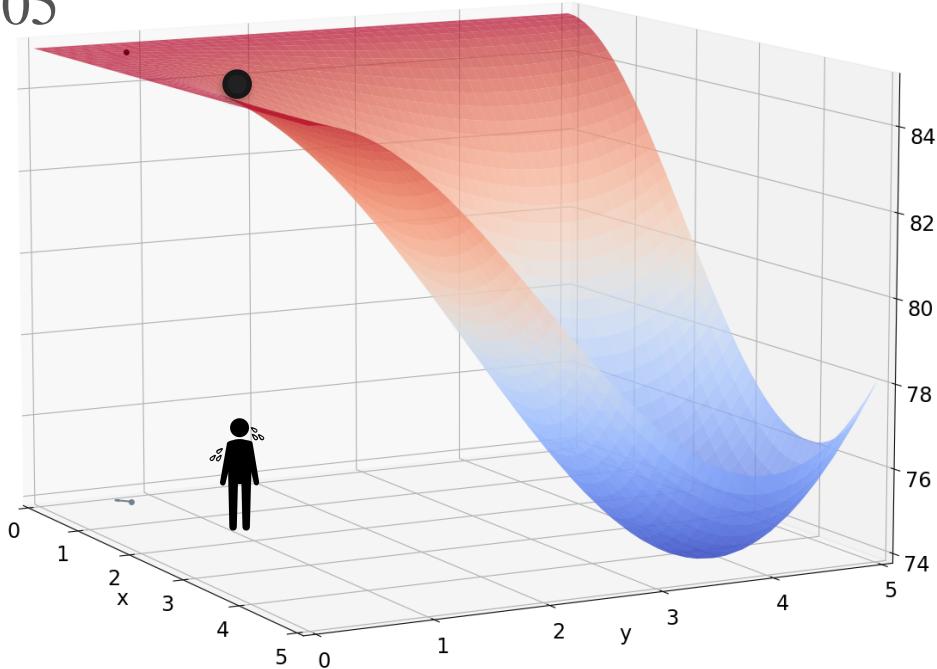
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

**Repeat!**



# Gradient Descent

Start:  $x = 0.5$ ,  $y = 0.6$     Rate:  $\alpha = 0.05$

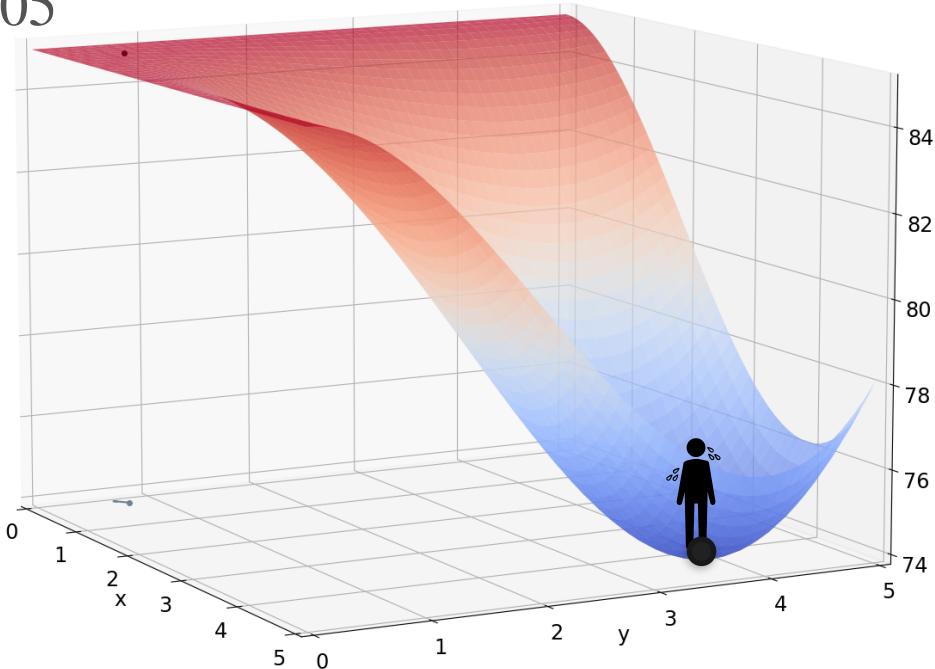
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by  
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

**Repeat!**



# Gradient Descent

# Gradient Descent

Function:  $f(x, y)$

# Gradient Descent

Function:  $f(x, y)$

Goal: find minimum of  $f(x, y)$

# Gradient Descent

Function:  $f(x, y)$

Goal: find minimum of  $f(x, y)$

## Step 1:

Define a learning rate  $\alpha$

Choose a starting point  $(x_0, y_0)$

# Gradient Descent

Function:  $f(x, y)$

Goal: find minimum of  $f(x, y)$

## Step 1:

Define a learning rate  $\alpha$

Choose a starting point  $(x_0, y_0)$

## Step 2:

Update: 
$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$$

# Gradient Descent

Function:  $f(x, y)$

Goal: find minimum of  $f(x, y)$

## Step 1:

Define a learning rate  $\alpha$

Choose a starting point  $(x_0, y_0)$

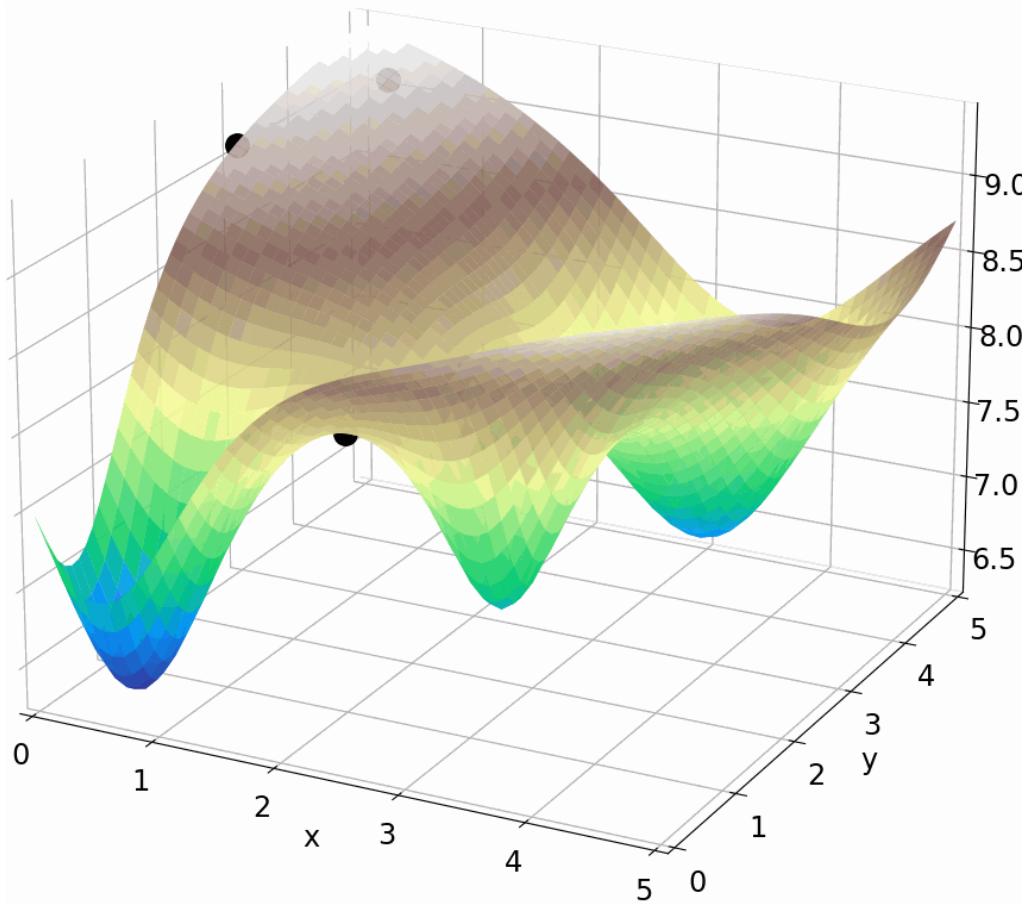
## Step 2:

Update:  $\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$

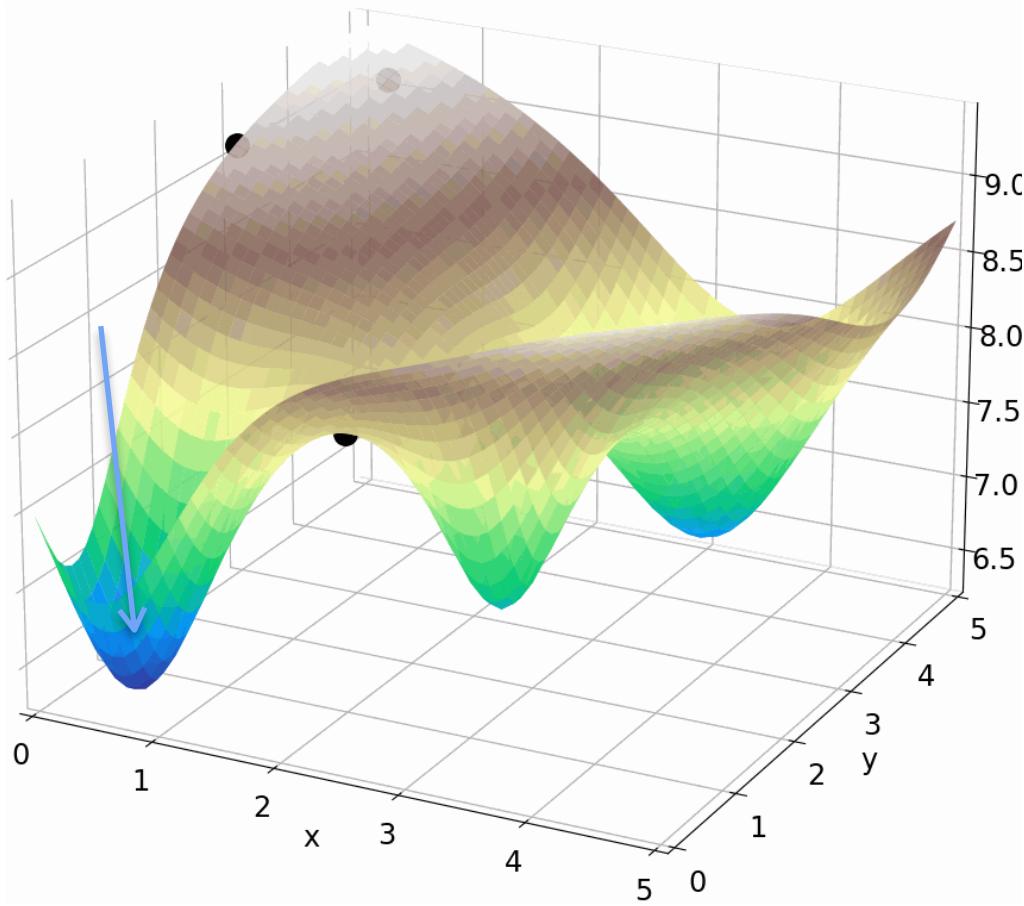
## Step 3:

Repeat Step 2 until you are close enough to  
the true minimum  $(x^*, y^*)$

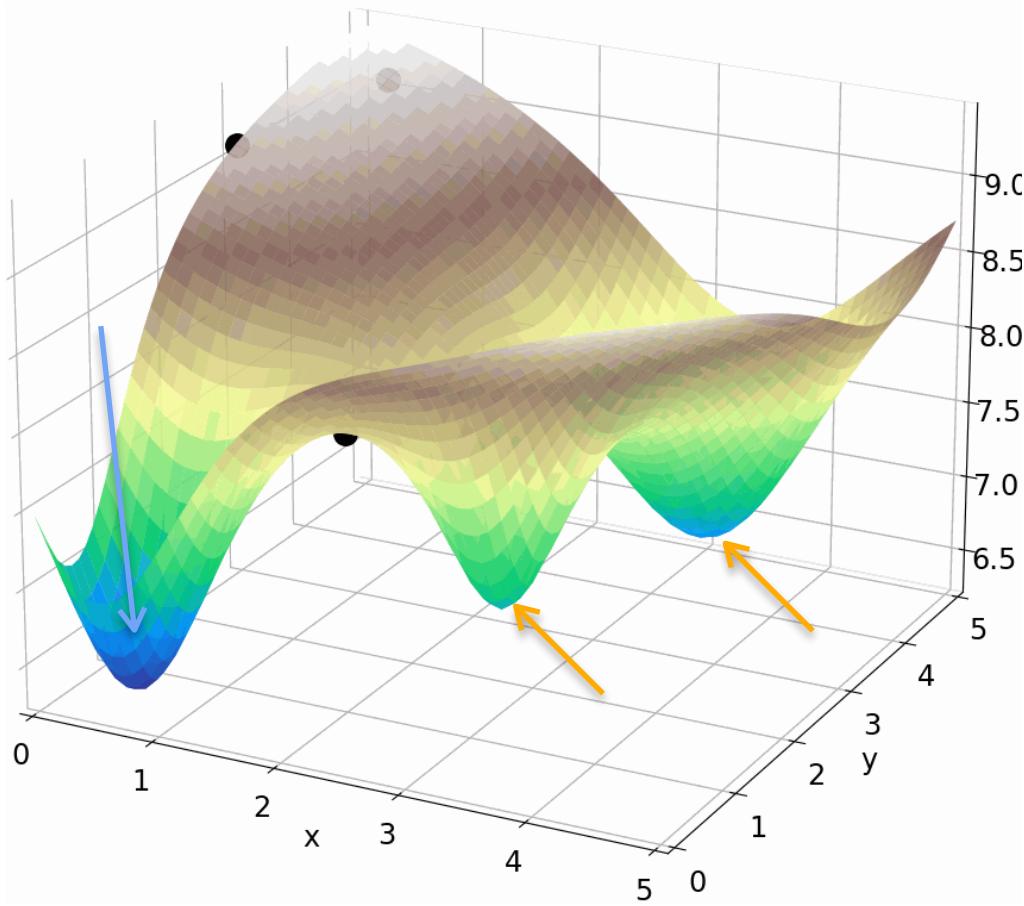
# Gradient Descent With Local Minima



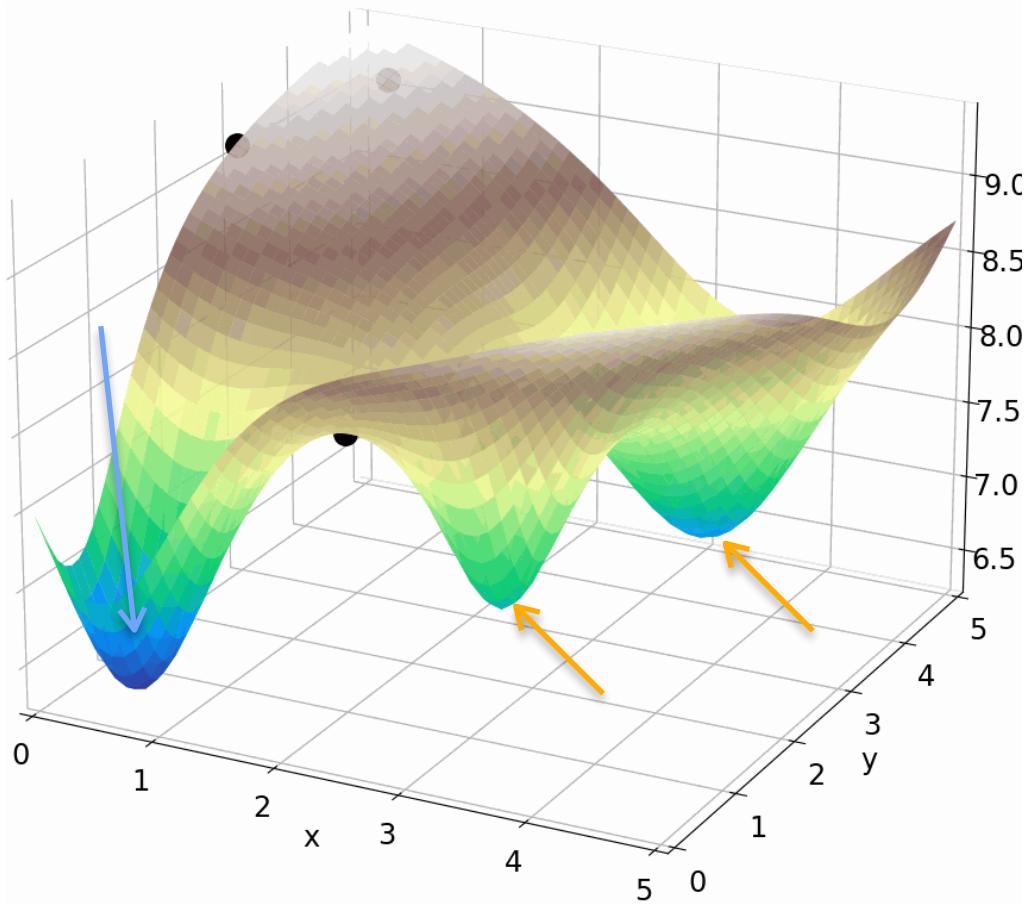
# Gradient Descent With Local Minima



# Gradient Descent With Local Minima



# Gradient Descent With Local Minima





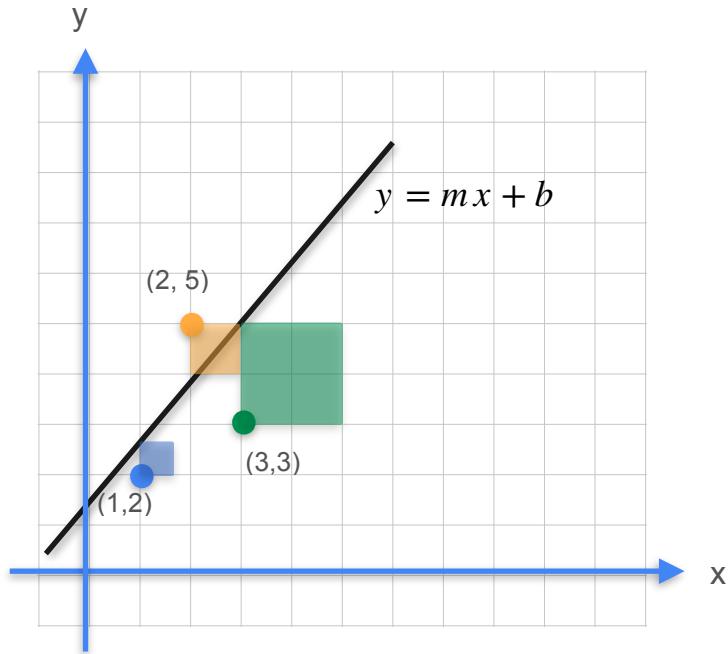
DeepLearning.AI

# Gradients and Gradient Descent

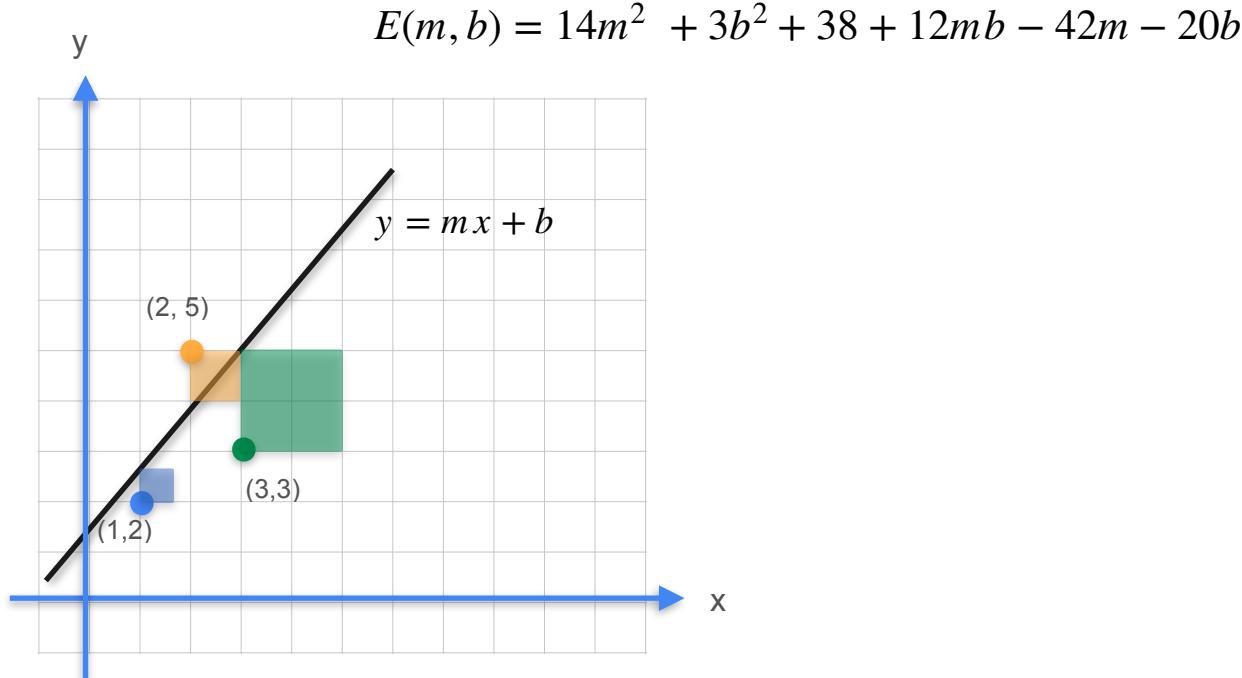
---

**Optimization using Gradient  
Descent - Least squares**

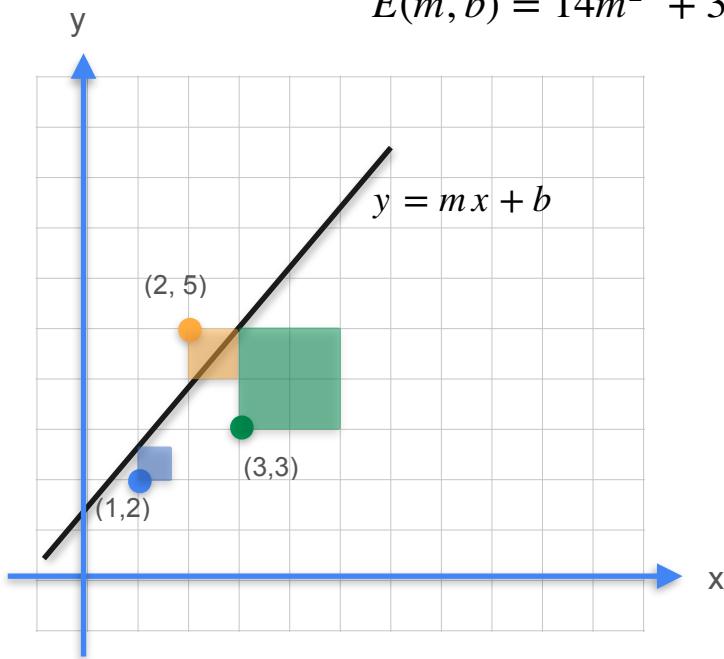
# Gradient Descent With Power Lines Example



# Gradient Descent With Power Lines Example

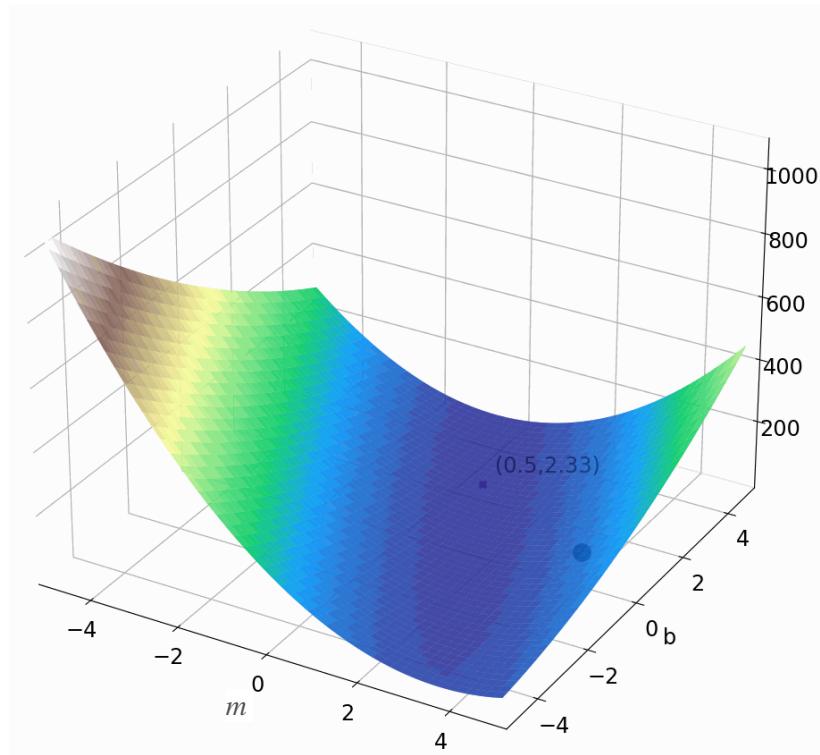


# Gradient Descent With Power Lines Example



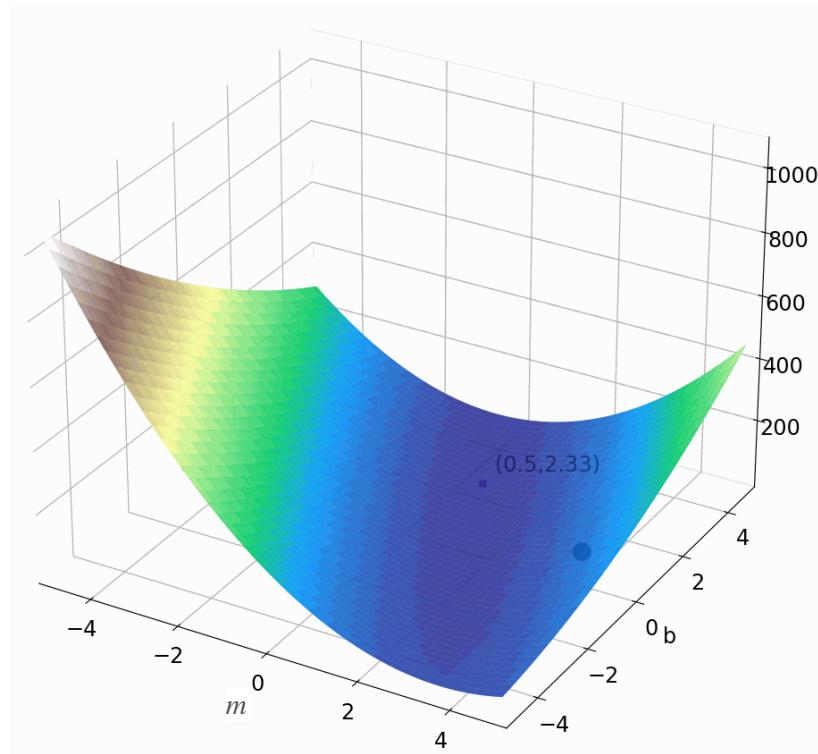
$$E\left(m = \frac{1}{2}, b = \frac{7}{3}\right) \approx 4.167$$

# Linear Regression: Gradient Descent



# Linear Regression: Gradient Descent

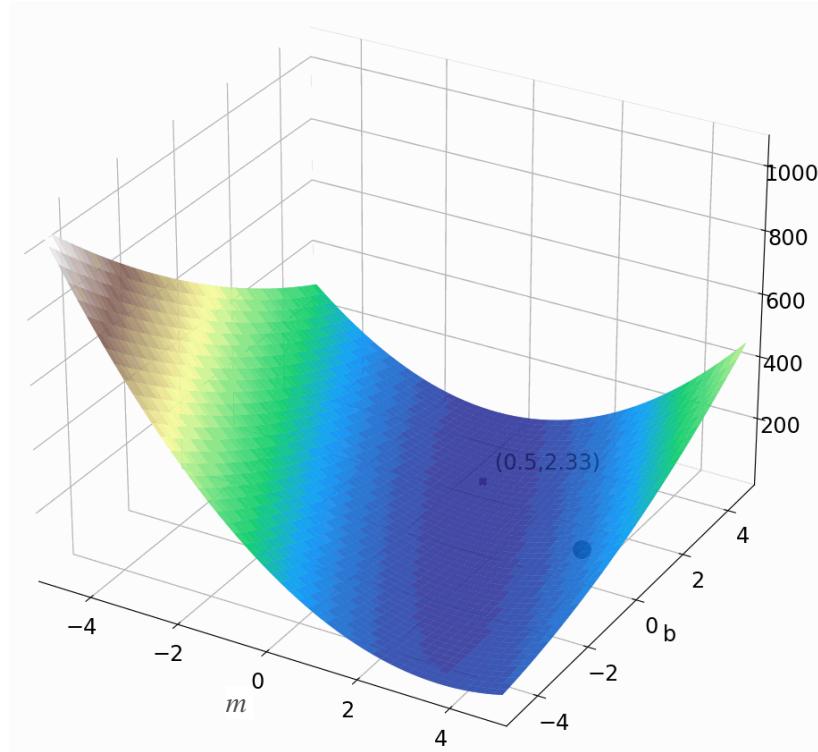
Goal: Minimize sum of squares cost



# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$



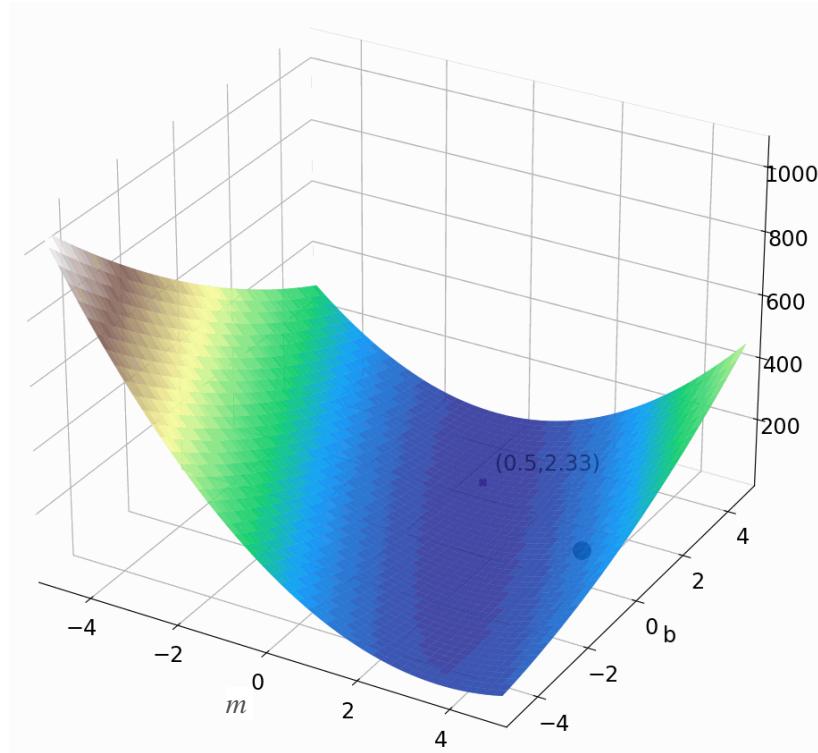
# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$m =$$

$$b =$$

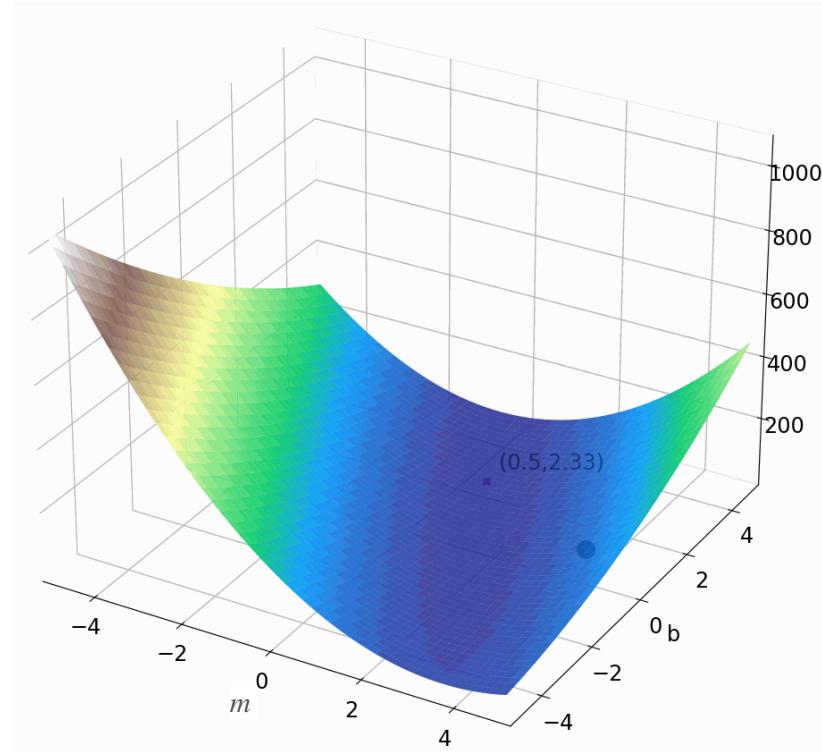


# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

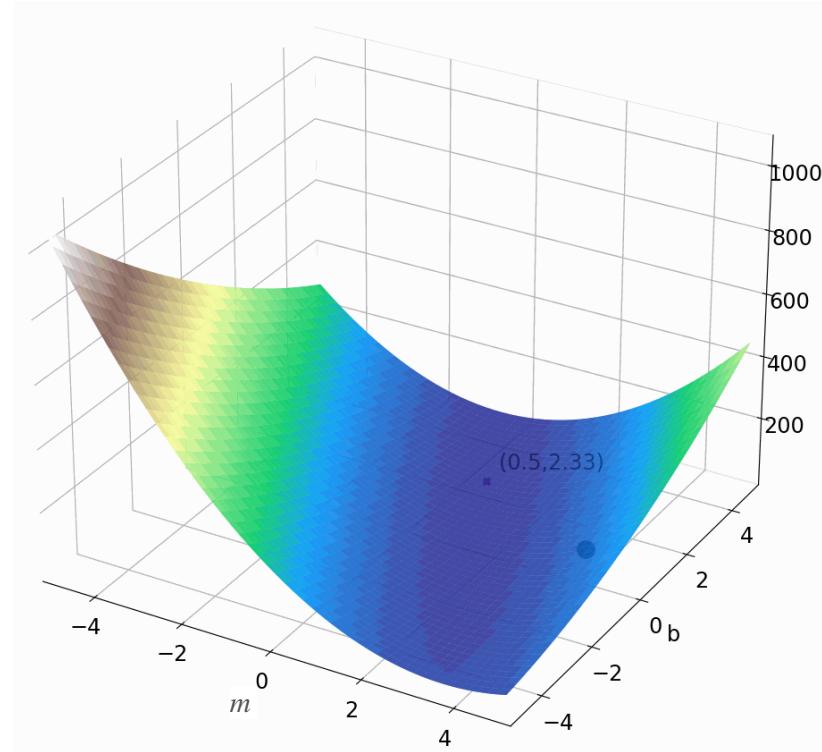


# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

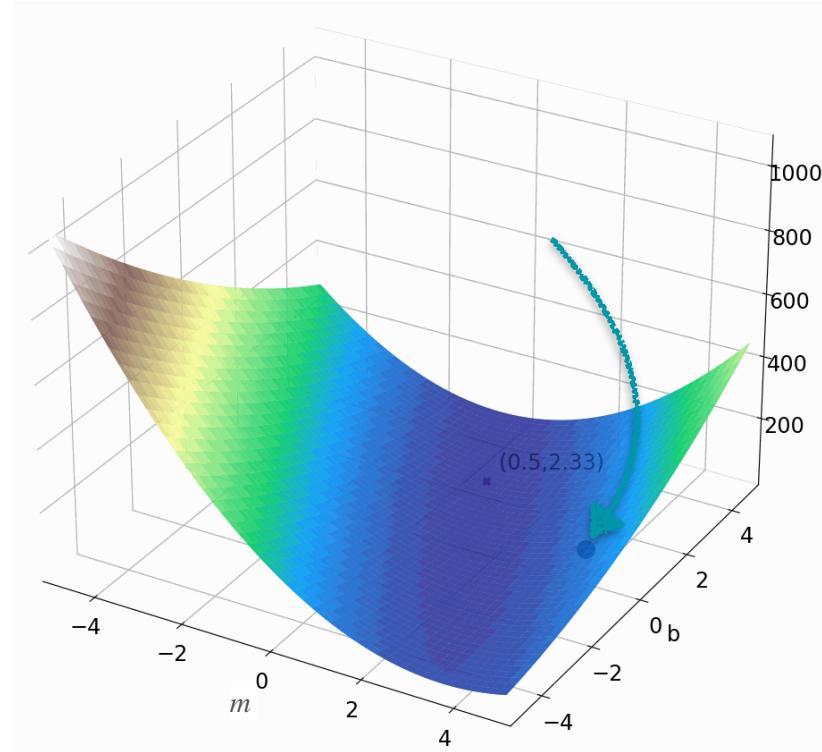


# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

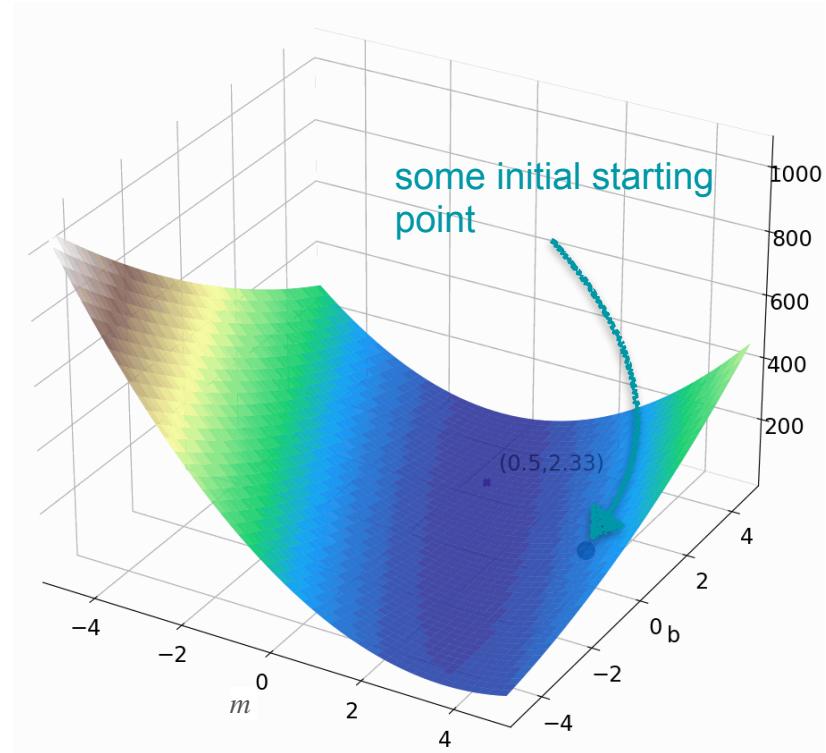


# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

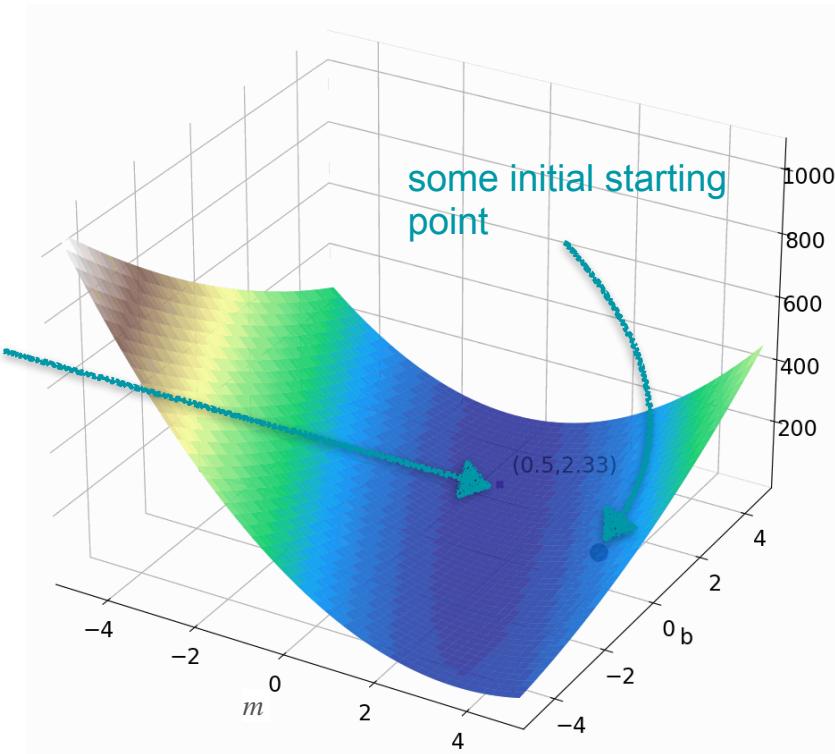


# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



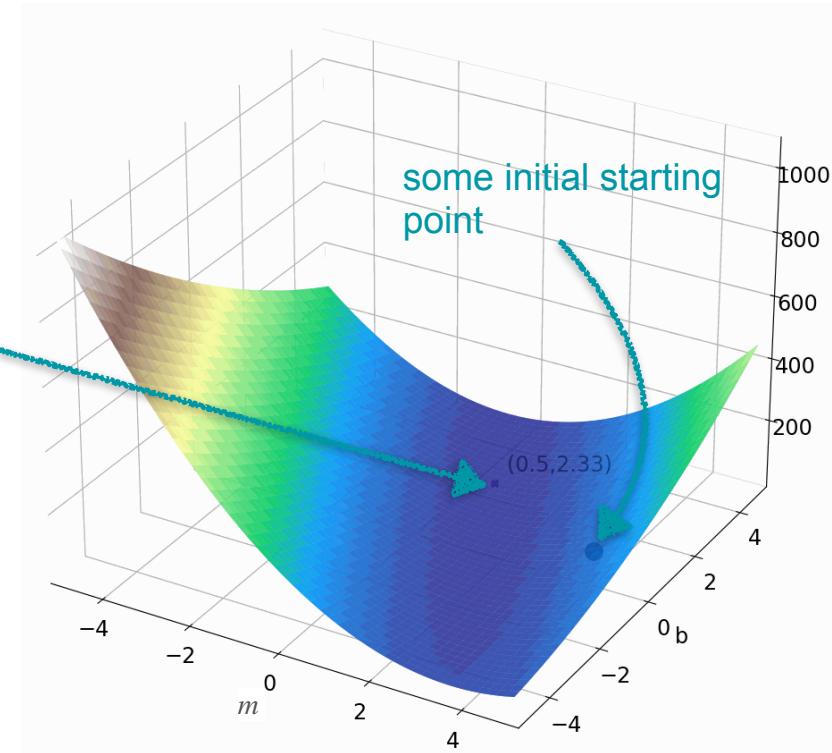
# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points  $m, b$  such that  
the cost is minimum



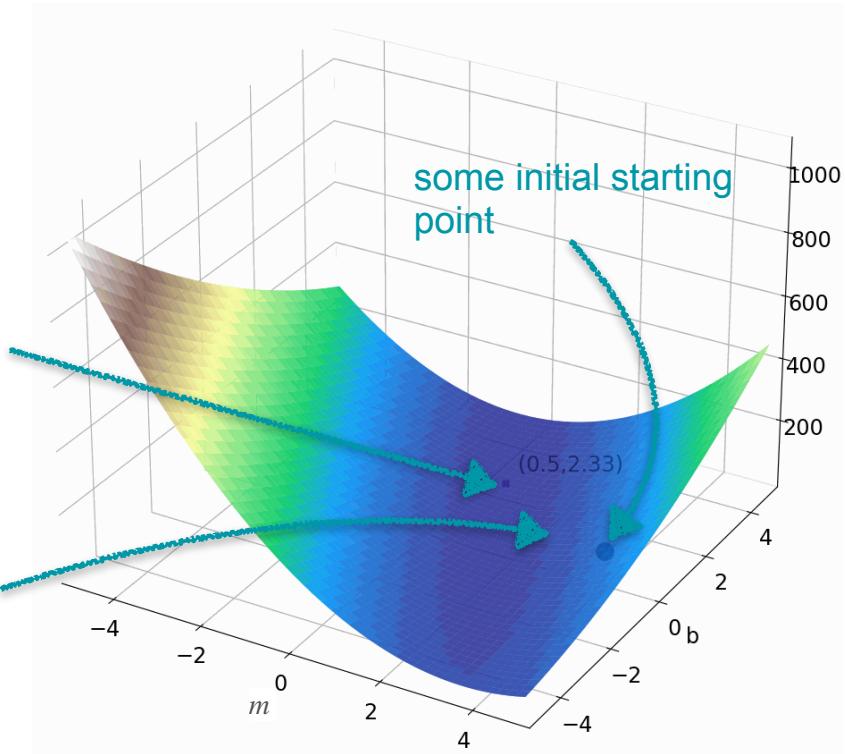
# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points  $m, b$  such that  
the cost is minimum



# Linear Regression: Gradient Descent

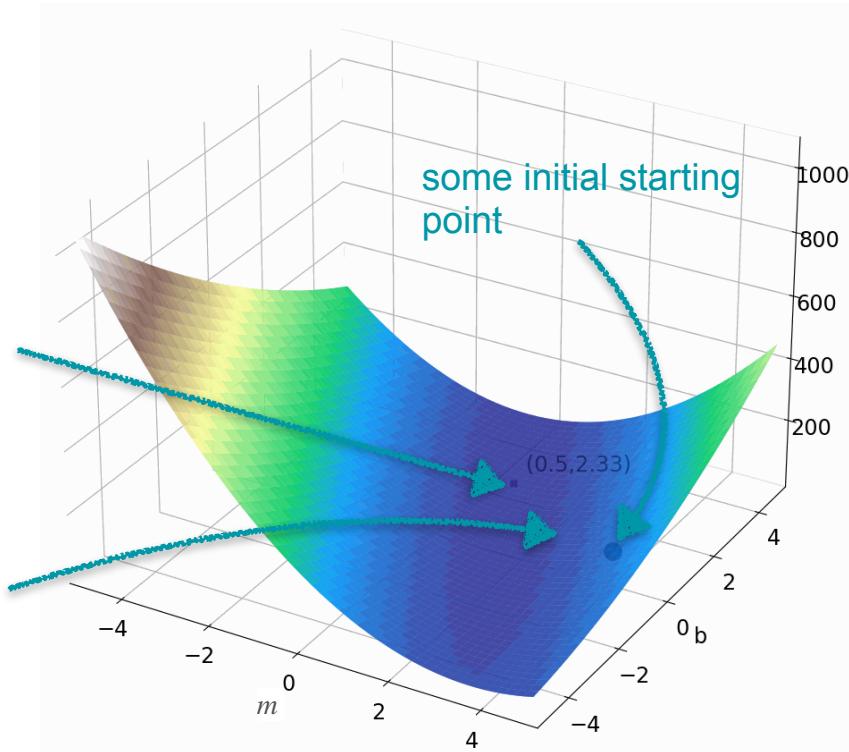
Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points  $m, b$  such that  
the cost is minimum

descend until you  
find the minimum



# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

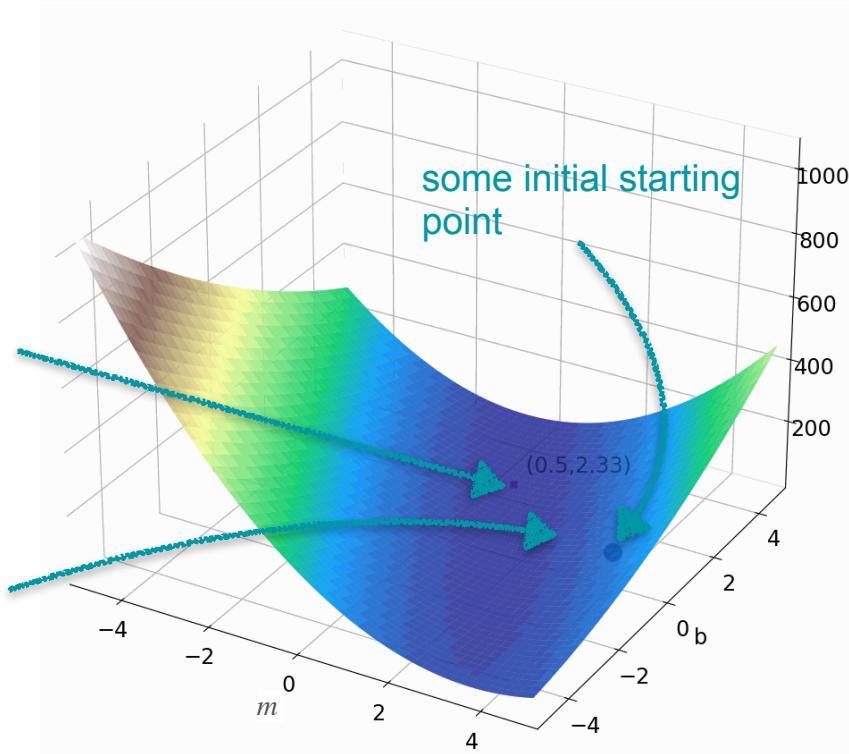
$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Steps:

descend until you  
find the minimum

The points  $m, b$  such that  
the cost is minimum



# Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

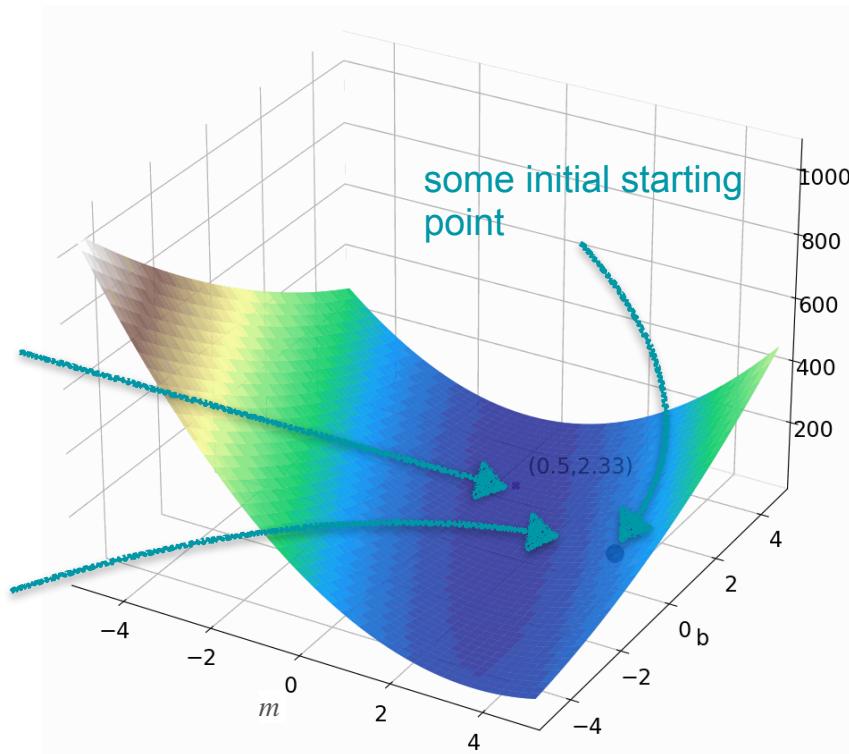
$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Steps:

Start with  $(m_0, b_0)$

descend until you  
find the minimum

The points  $m, b$  such that  
the cost is minimum



# Linear Regression: Gradient Descent

**Goal: Minimize sum of squares cost**

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points  $m, b$  such that  
the cost is minimum

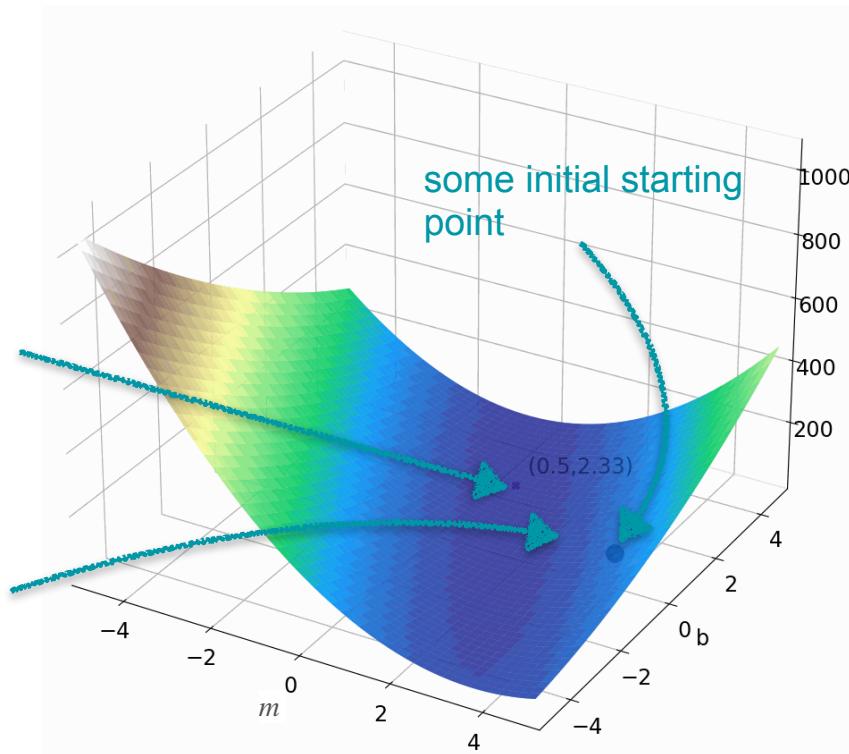
Steps:

Start with  $(m_0, b_0)$

descend until you  
find the minimum

Iterate

$$(m_{k+1}, b_{k+1}) = (m_k, b_k) - \alpha \nabla E(m_k, b_k)$$





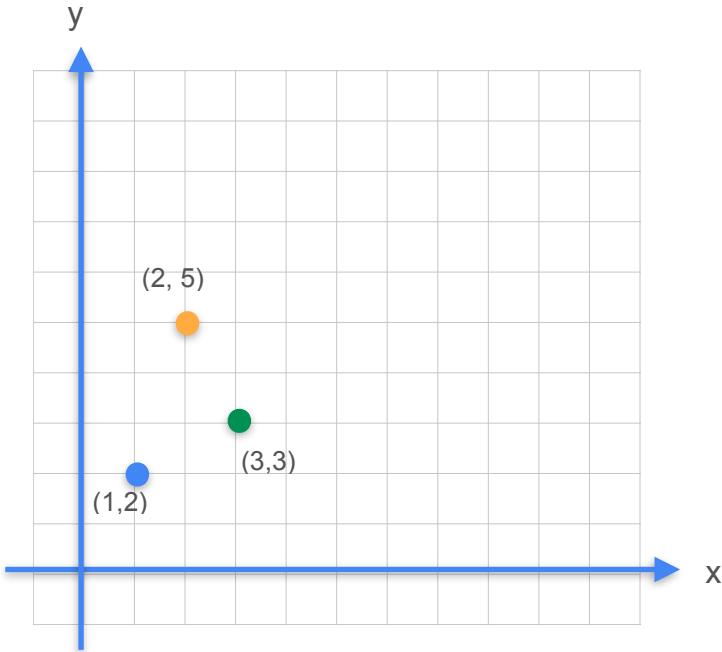
DeepLearning.AI

# Gradients and Gradient Descent

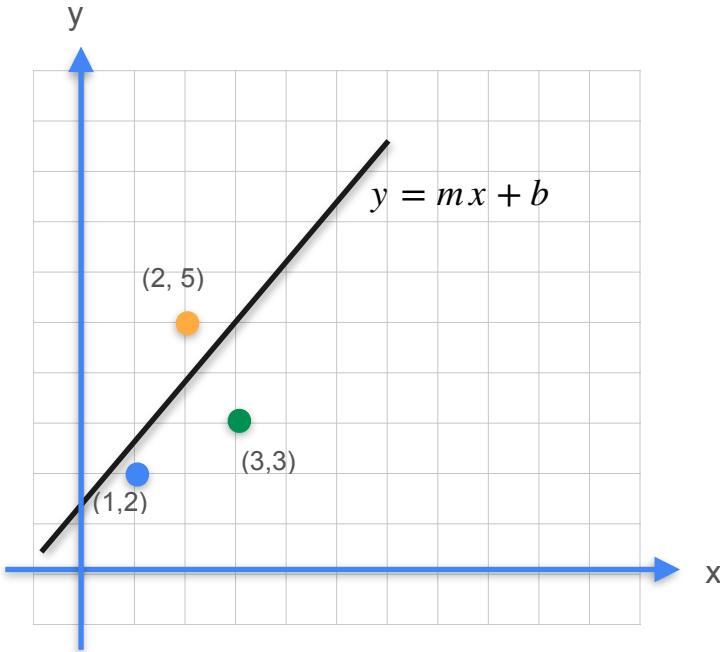
---

**Optimization using Gradient  
Descent - Least squares  
with multiple observations**

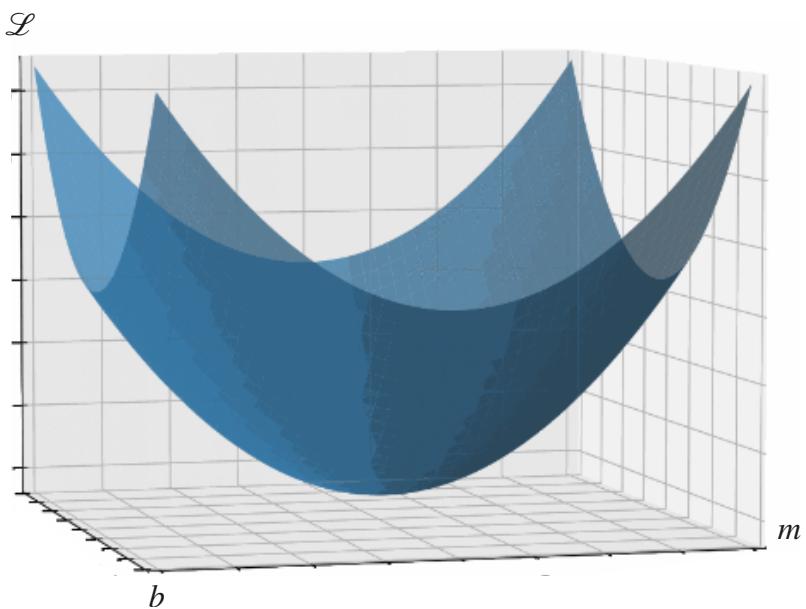
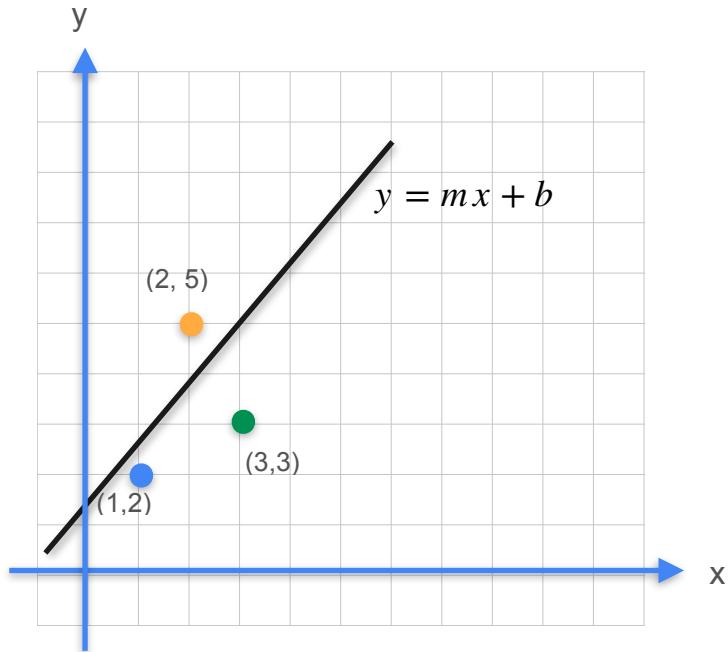
# Gradient Descent



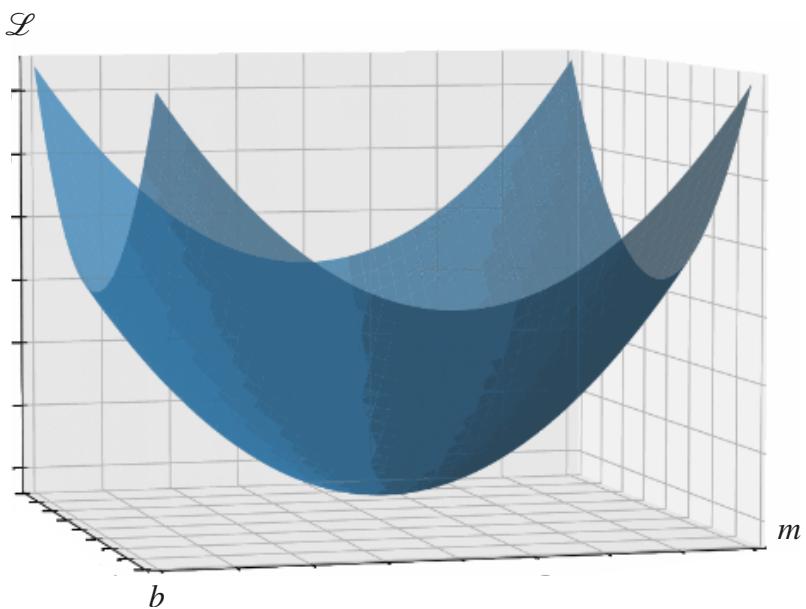
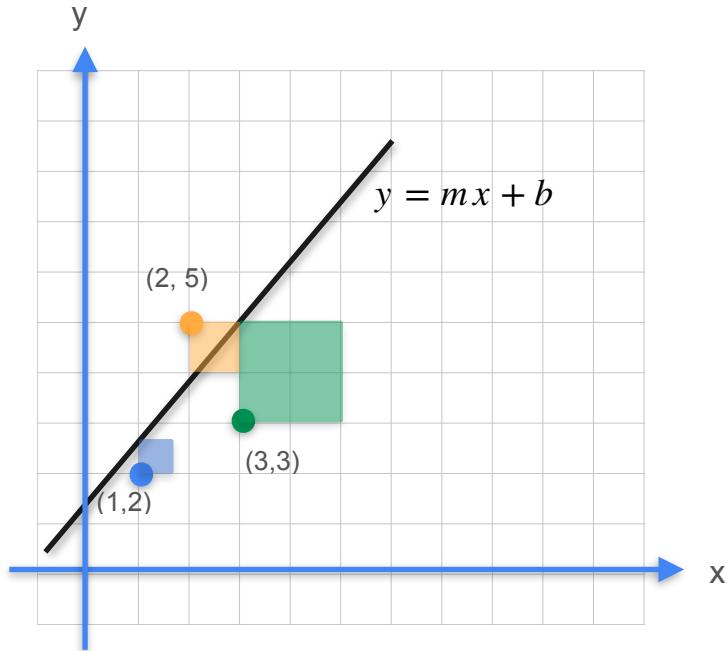
# Gradient Descent



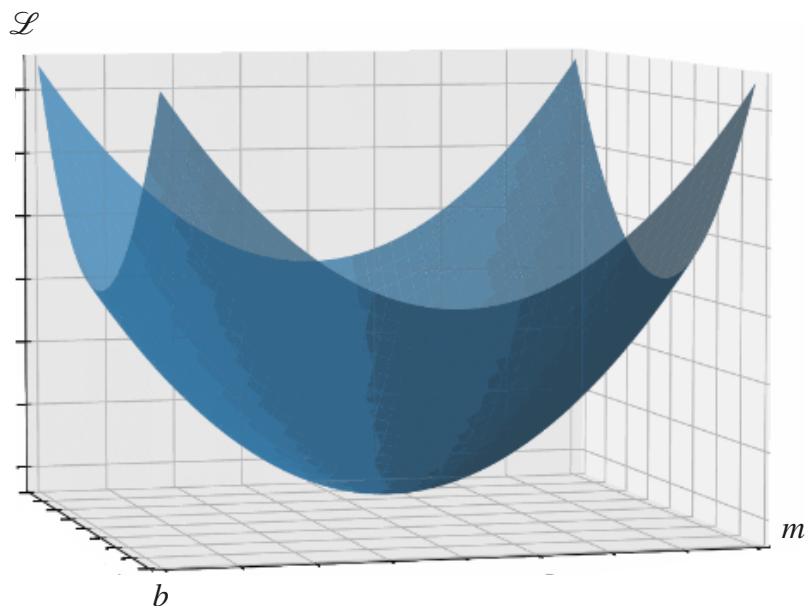
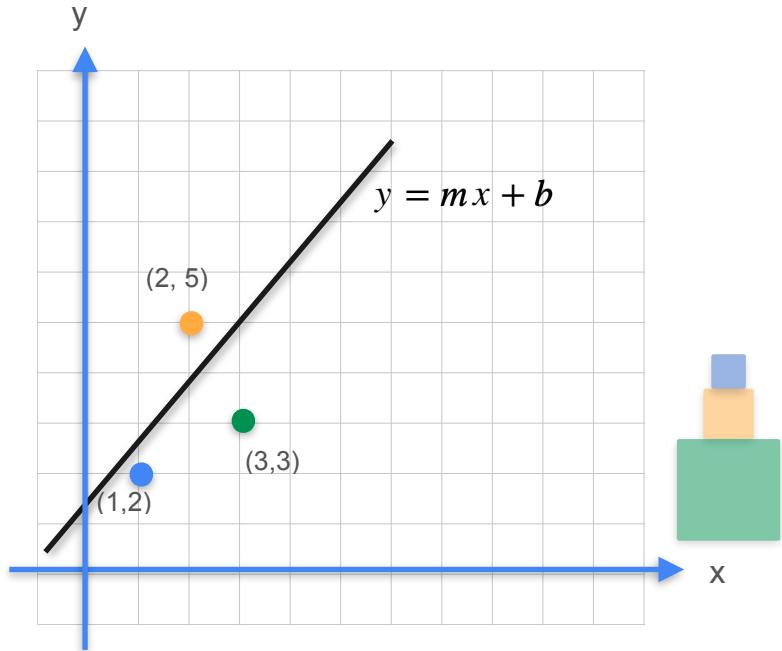
# Gradient Descent



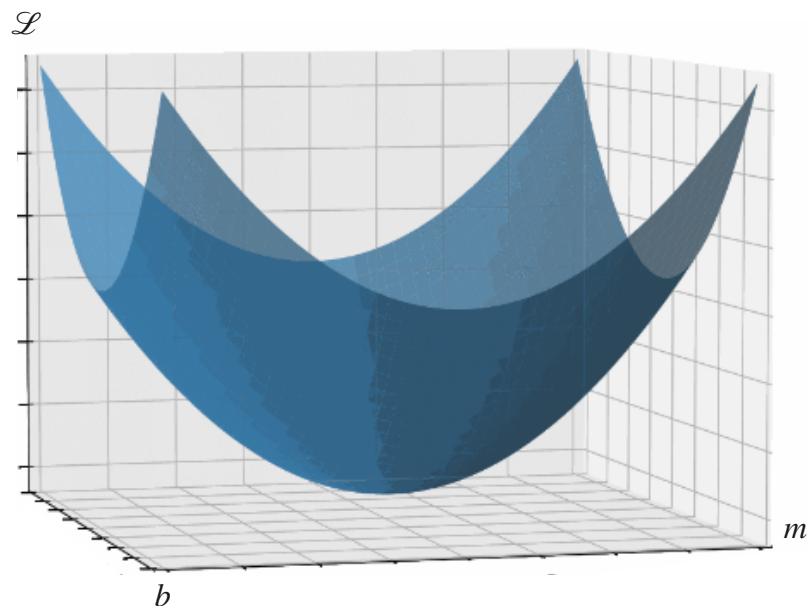
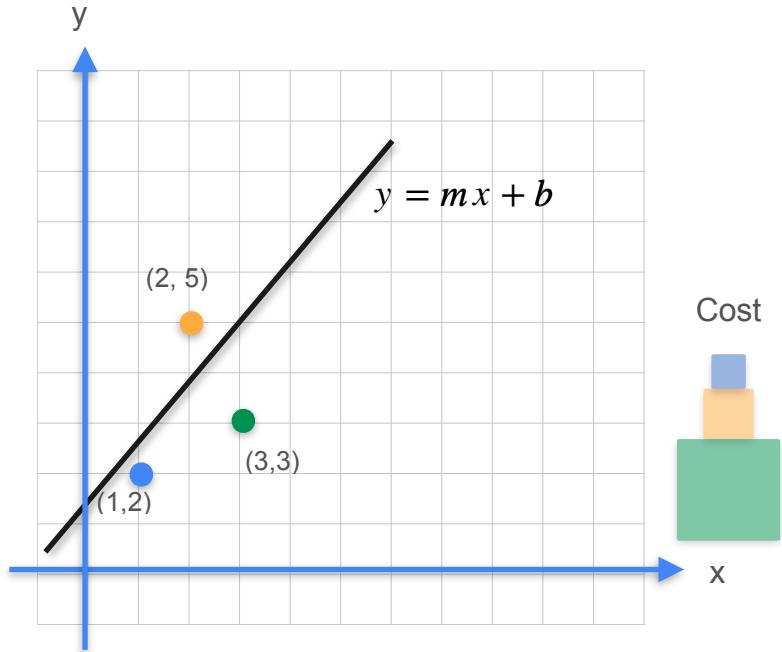
# Gradient Descent



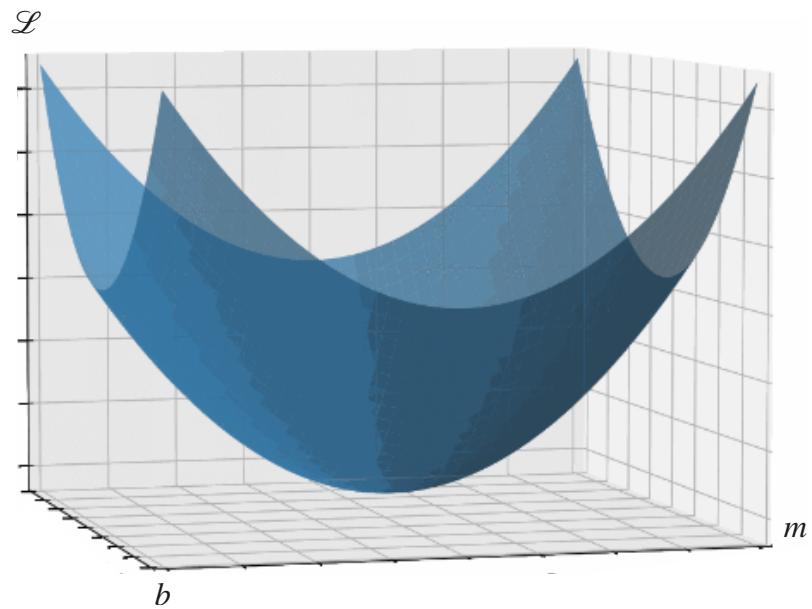
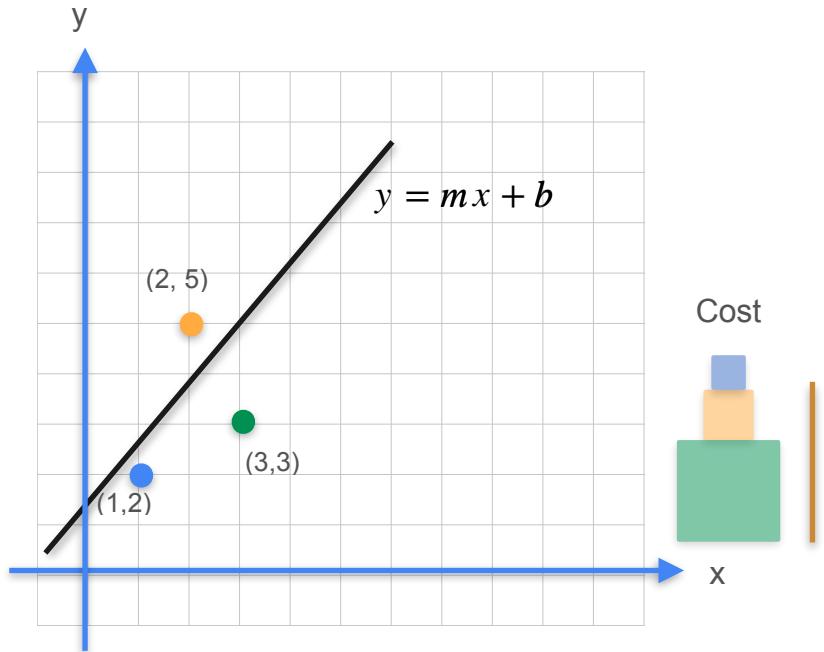
# Gradient Descent



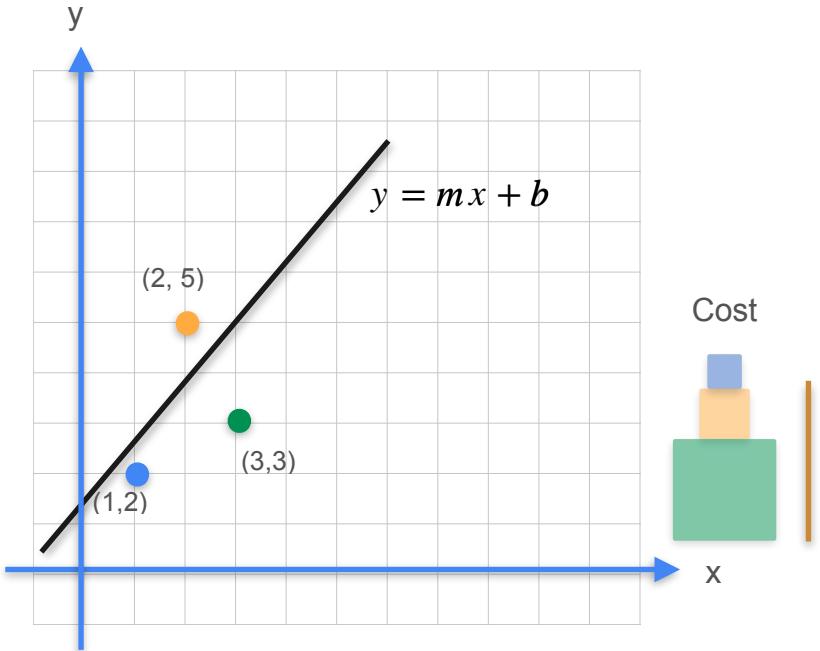
# Gradient Descent



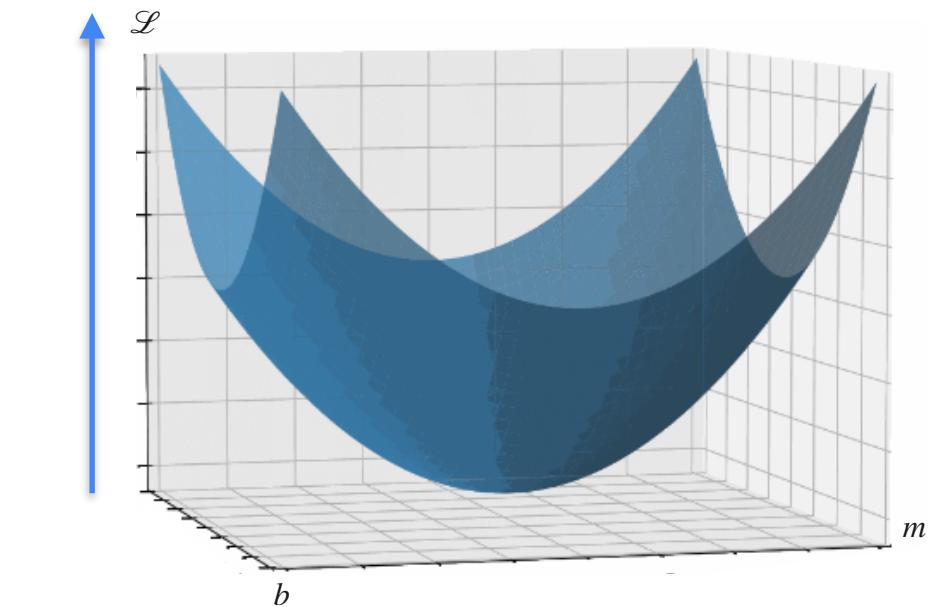
# Gradient Descent



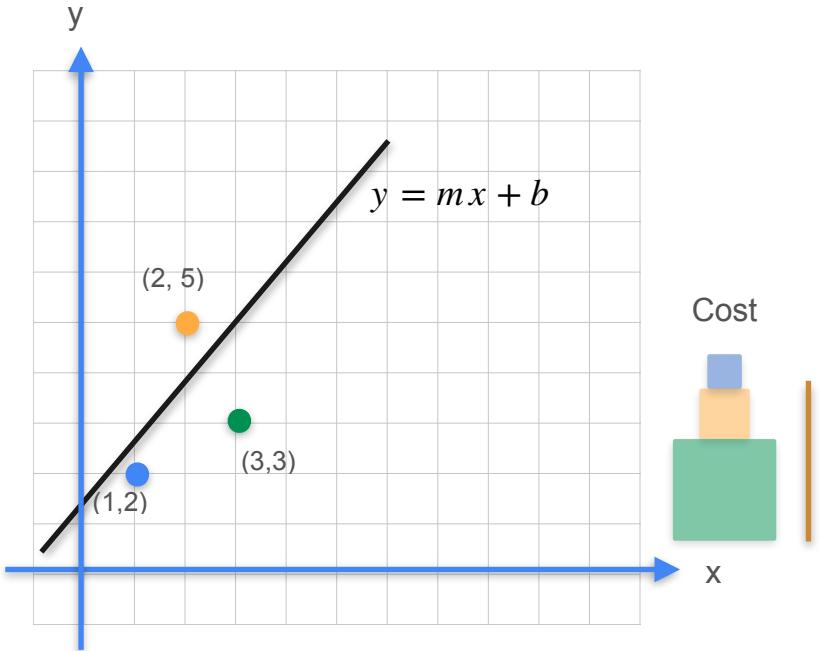
# Gradient Descent



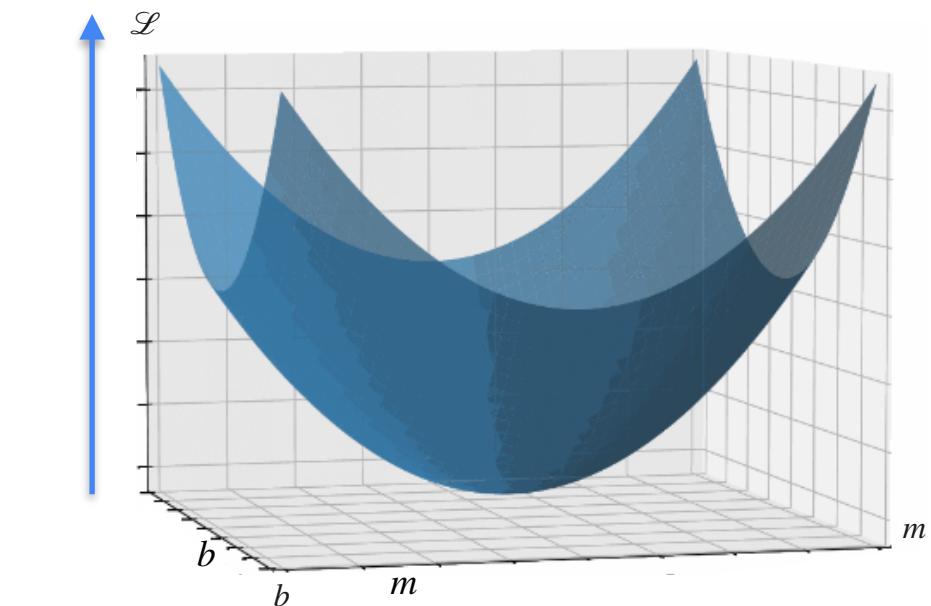
Square loss



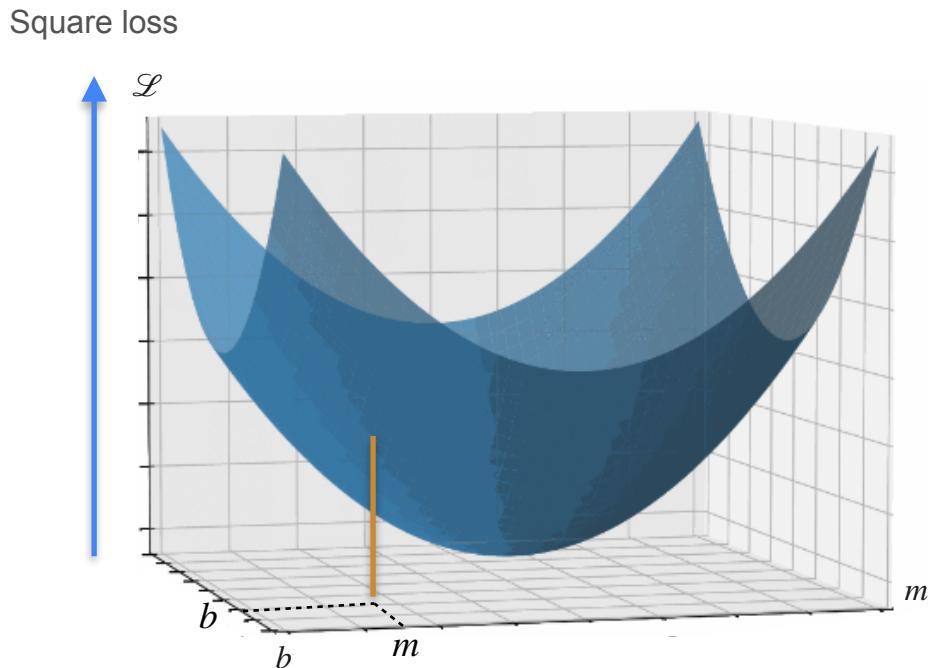
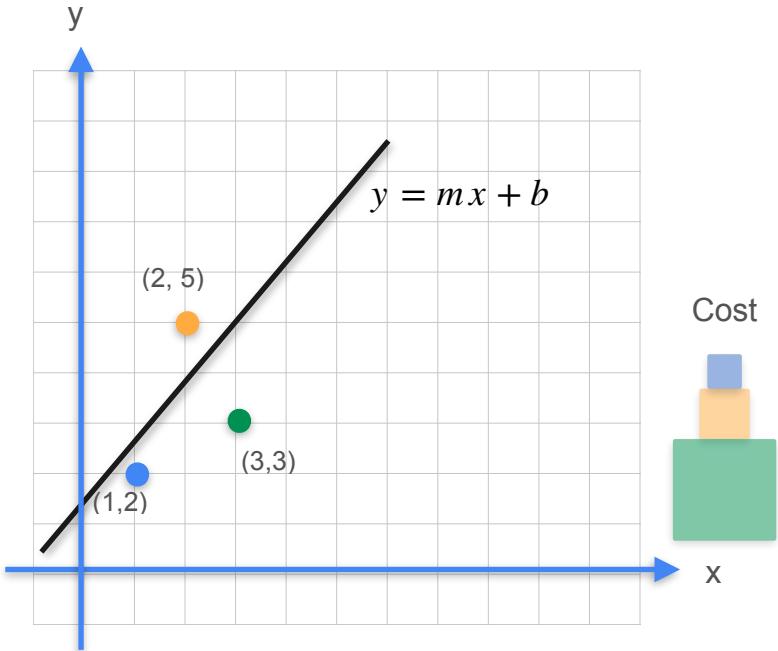
# Gradient Descent



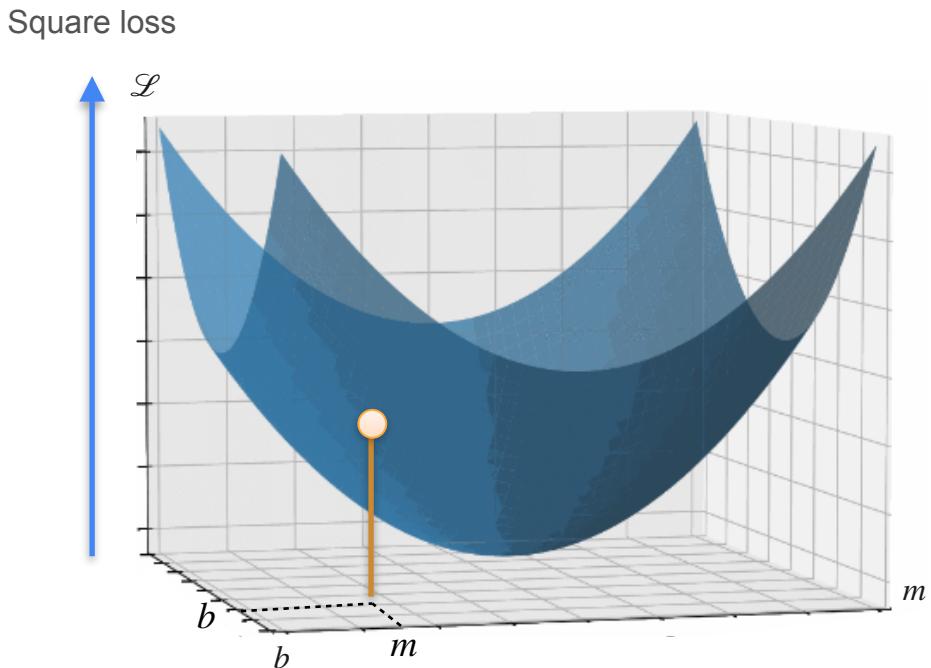
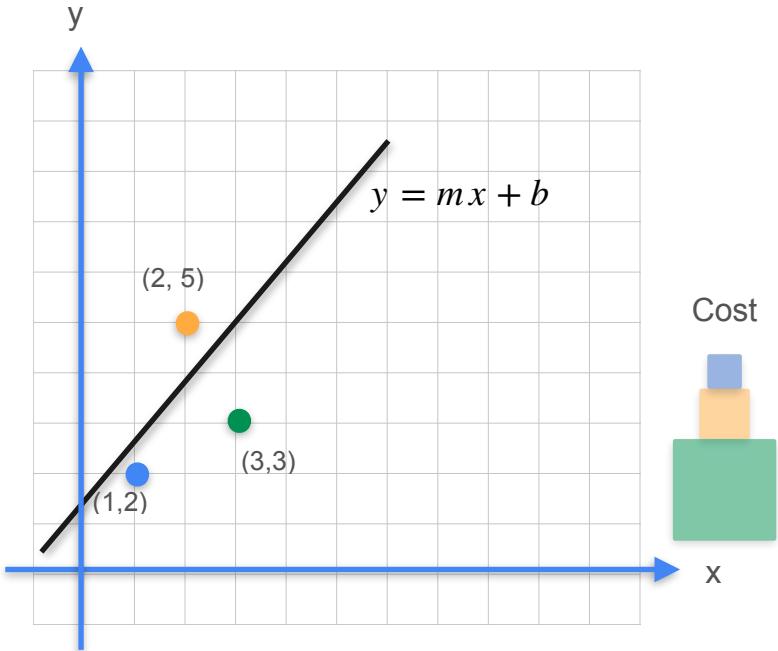
Square loss



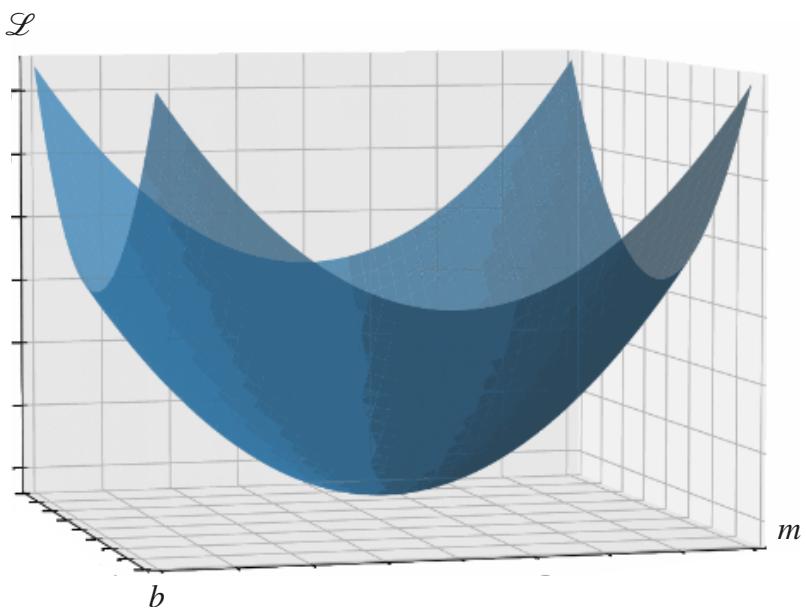
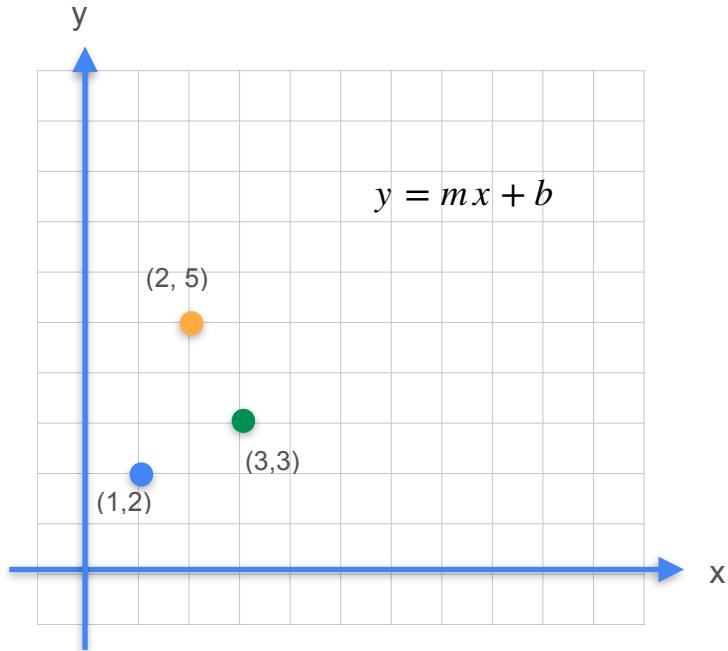
# Gradient Descent



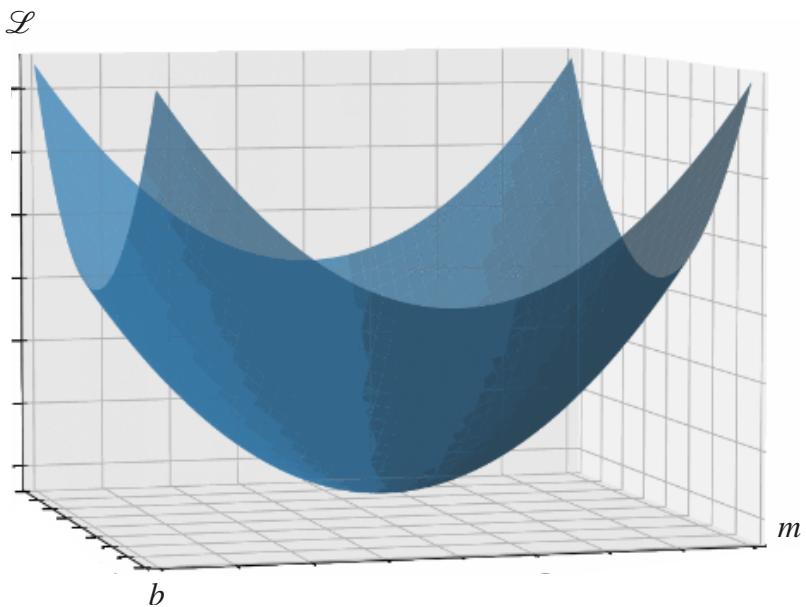
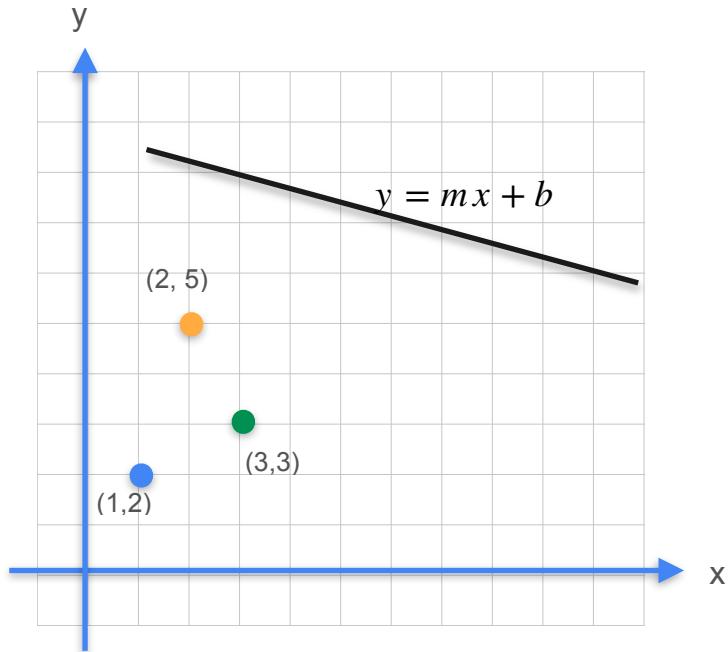
# Gradient Descent



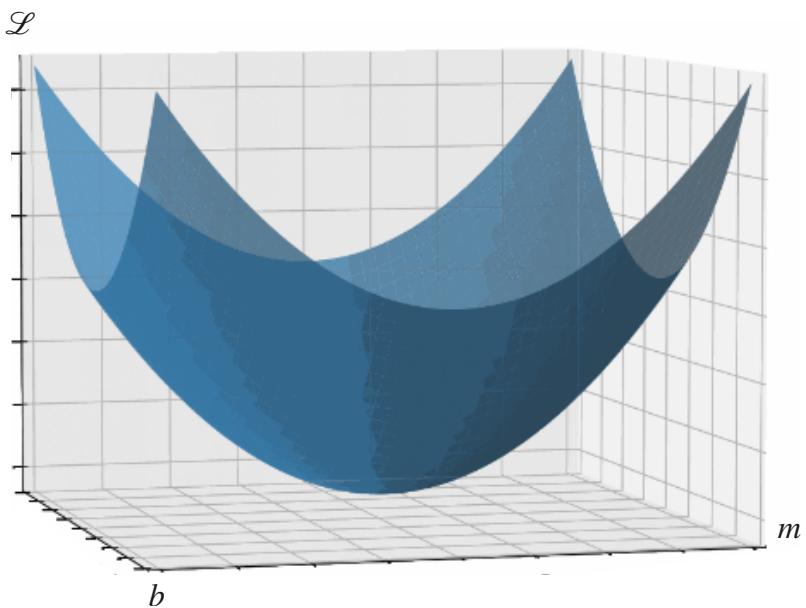
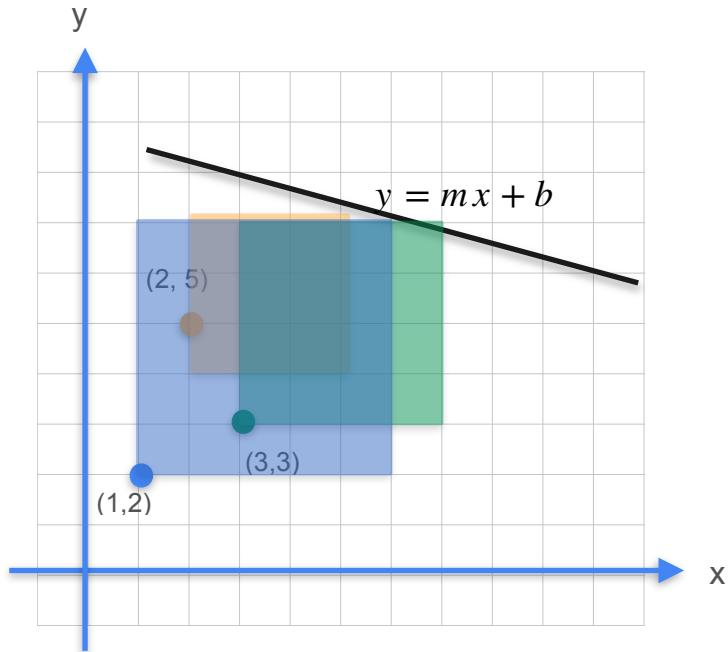
# Gradient Descent



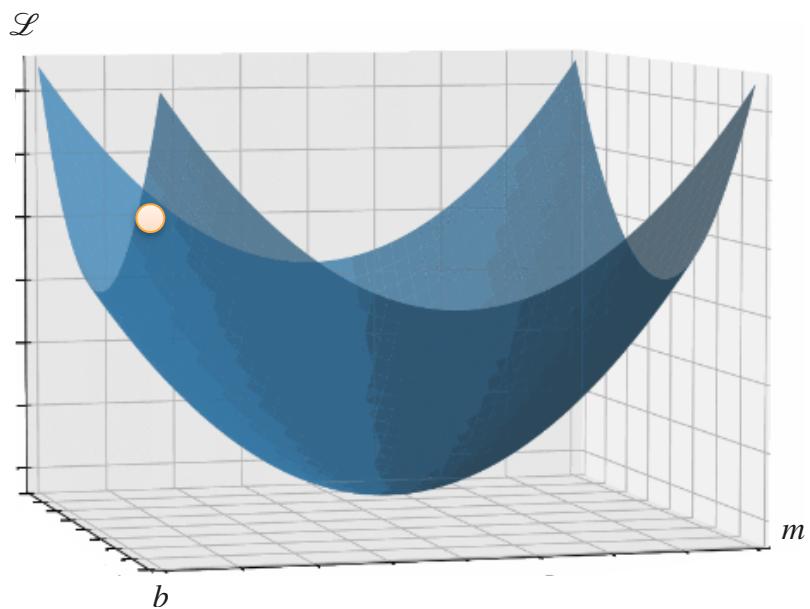
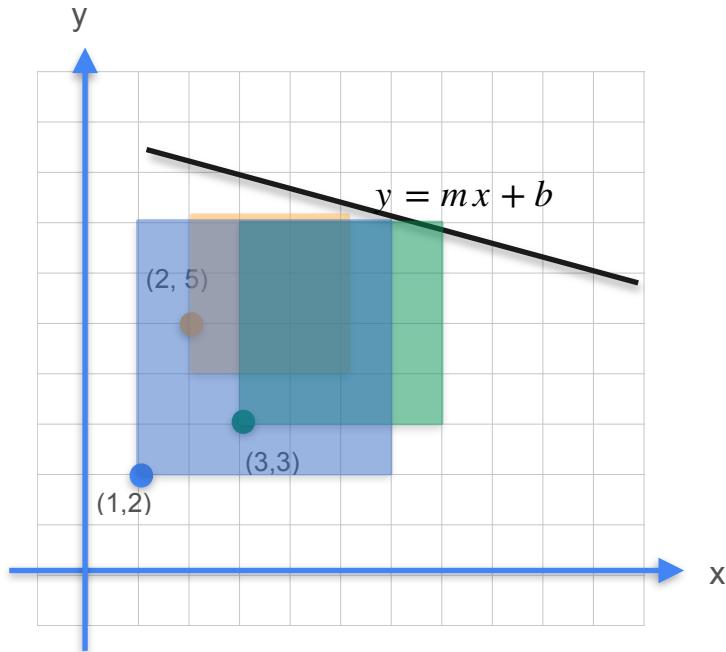
# Gradient Descent



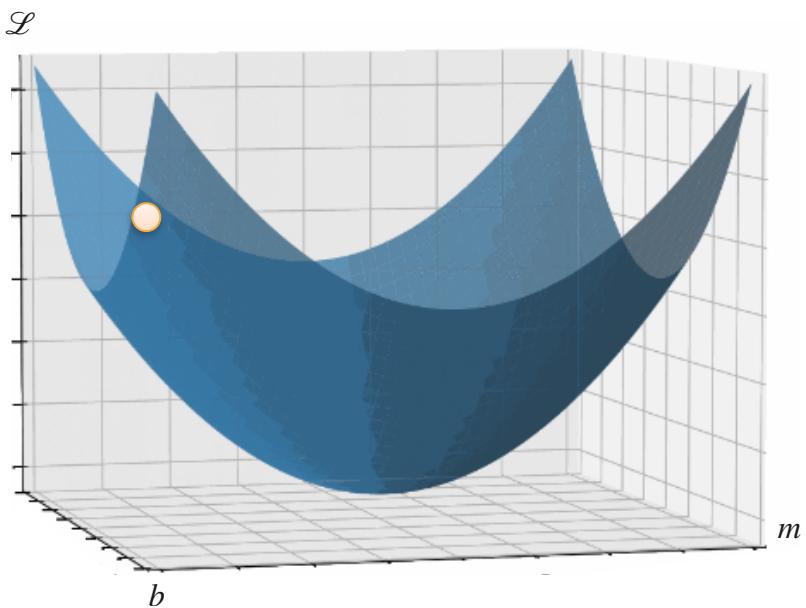
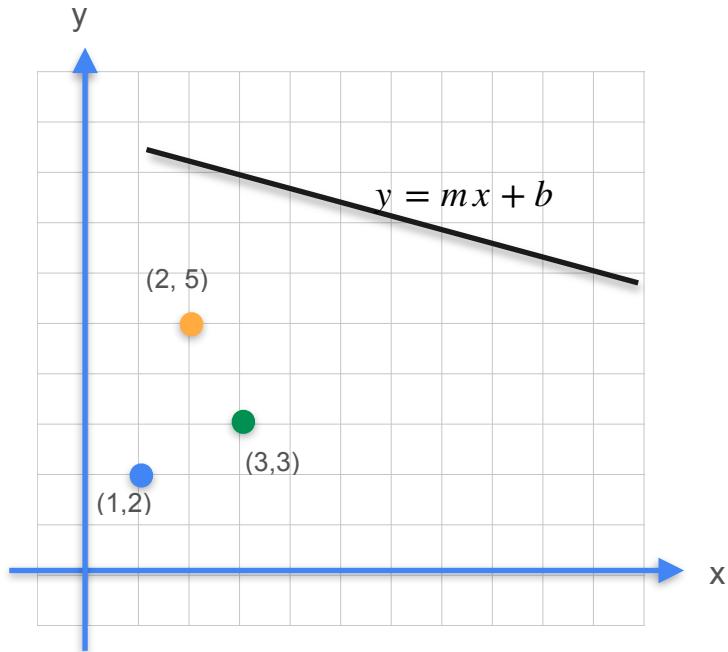
# Gradient Descent



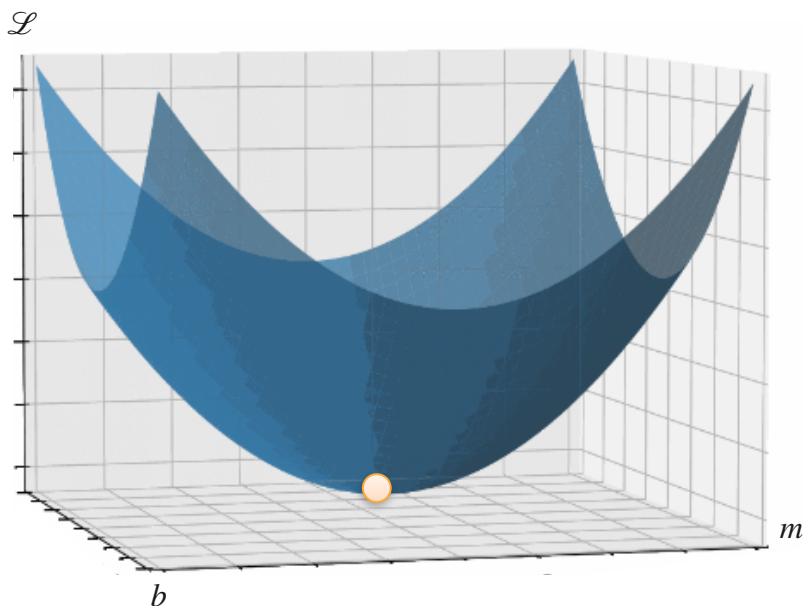
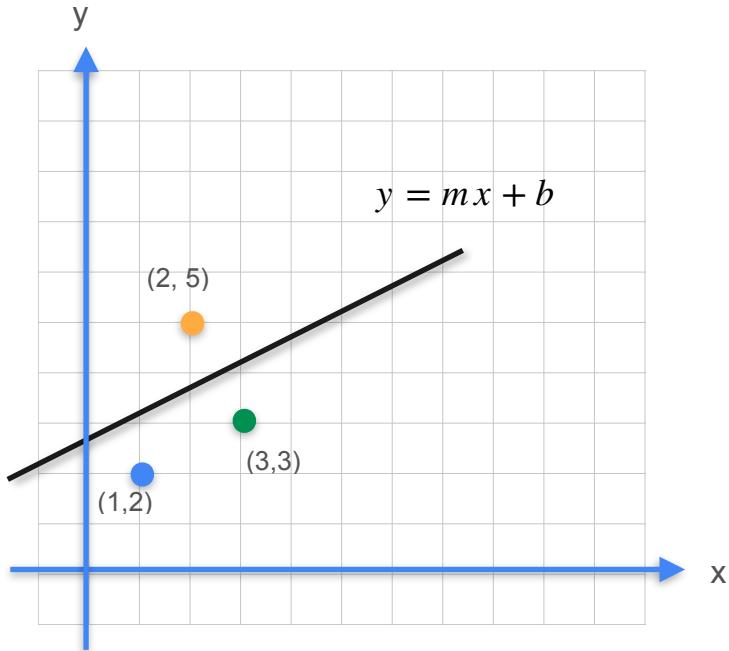
# Gradient Descent



# Gradient Descent



# Gradient Descent



# Another Example

# Another Example



# Another Example



TV advertisement  
budget

# Another Example



TV advertisement  
budget



# Another Example



TV advertisement  
budget



Number of sales

# Another Example

# Another Example

TV budget

Sales

# Another Example

TV budget	Sales
230.1	22.1

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

**Goal:** Predict sales in terms of TV budget

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

**Goal:** Predict sales in terms of TV budget

**Tool:** Linear regression

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

**Goal:** Predict sales in terms of TV budget

**Tool:** Linear regression

$$y = mx + b$$

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

**Goal:** Predict sales in terms of TV budget

**Tool:** Linear regression

$$y = mx + b$$

# Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3



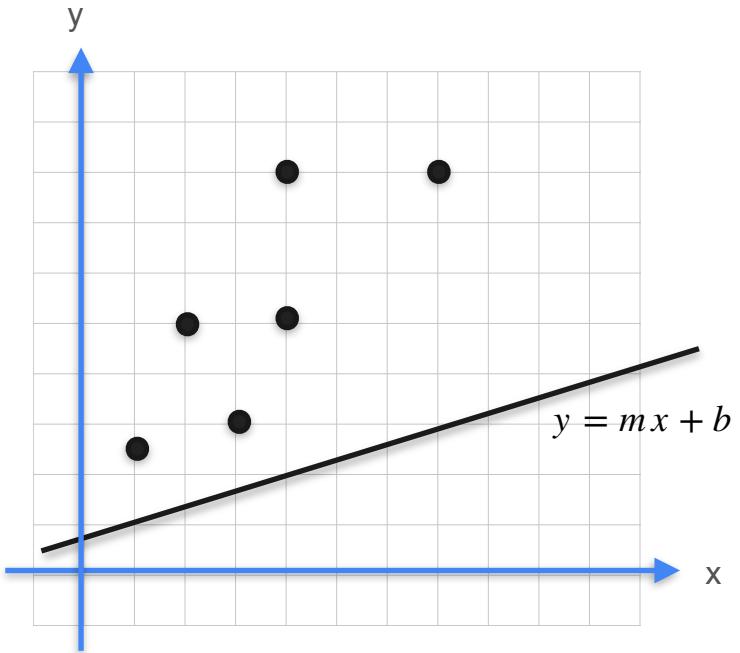
Multiple observations

**Goal:** Predict sales in terms of TV budget

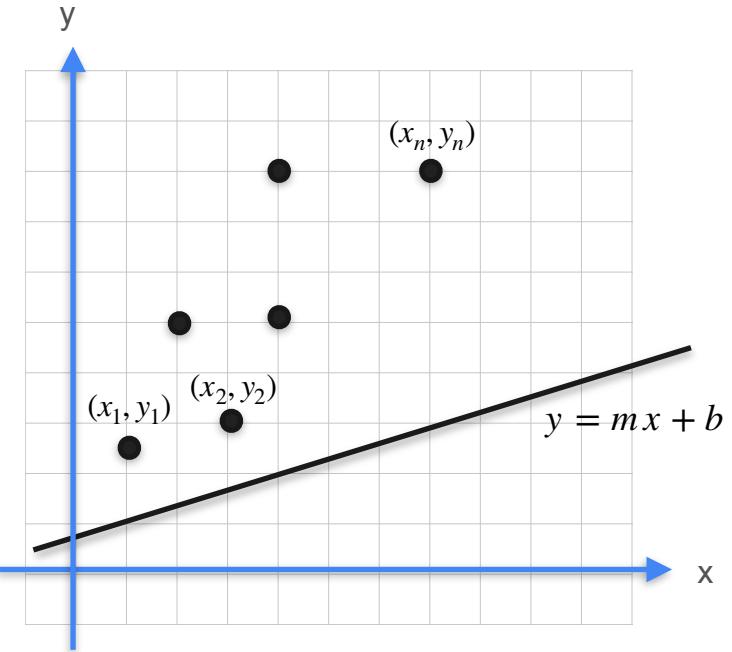
**Tool:** Linear regression

$$y = mx + b$$

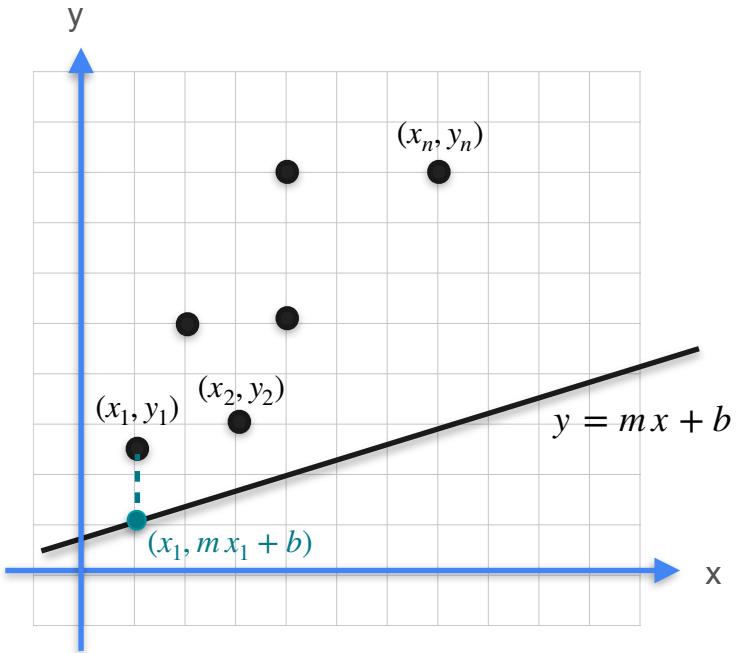
# Gradient Descent



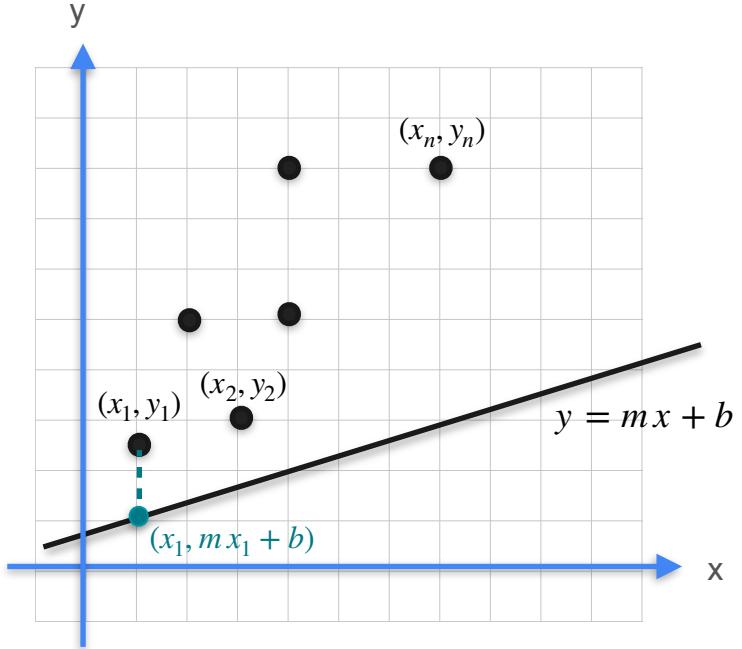
# Gradient Descent



# Gradient Descent

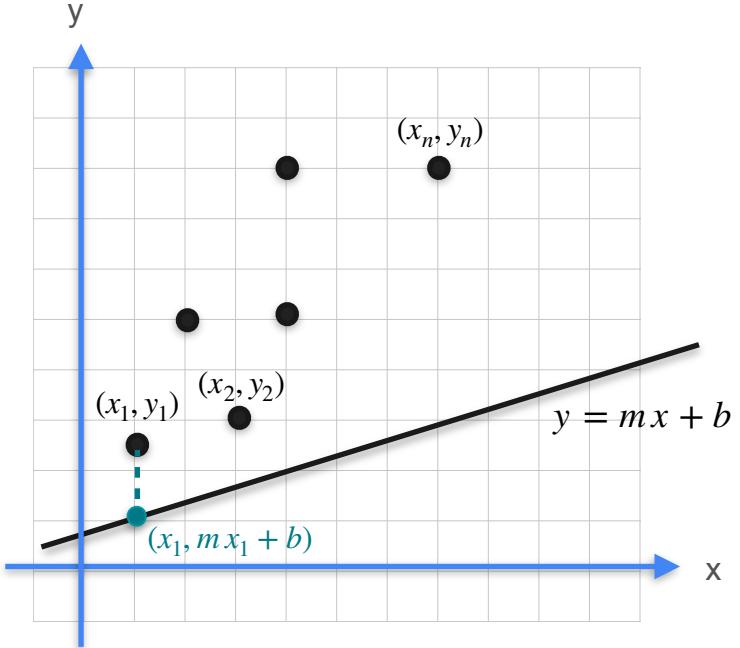


# Gradient Descent



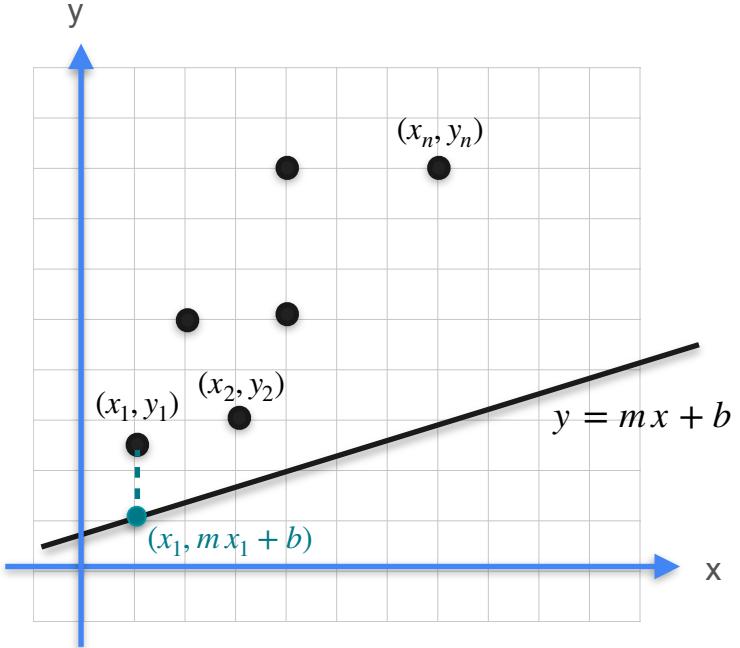
Loss  
↓  
 $mx_1 + b - y_1$

# Gradient Descent



Loss  
↓  
 $(mx_1 + b - y_1)^2$

# Gradient Descent



Cost

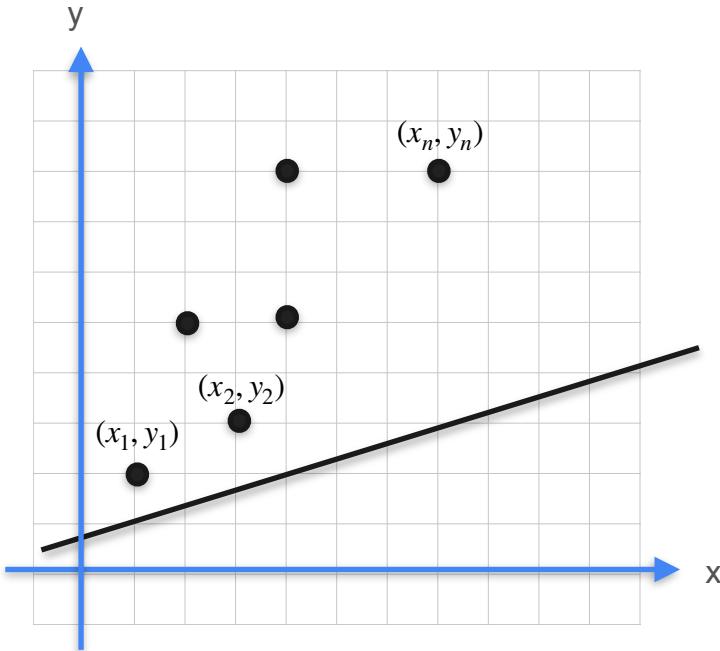


$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

Loss

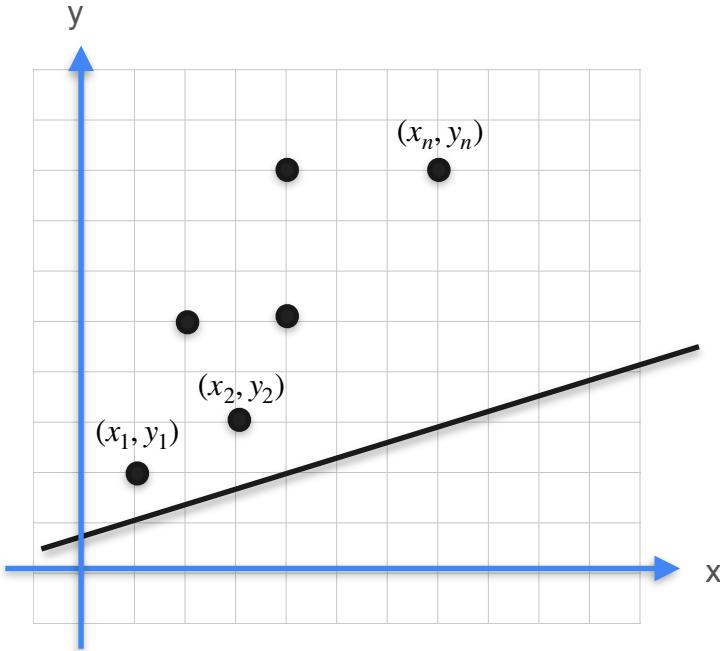


# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} \left[ (mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2 \right]$$

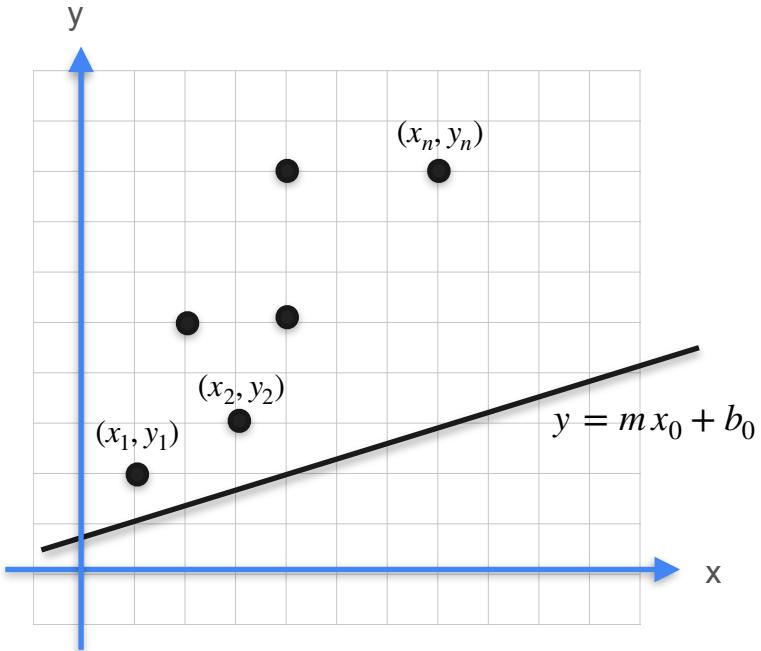
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

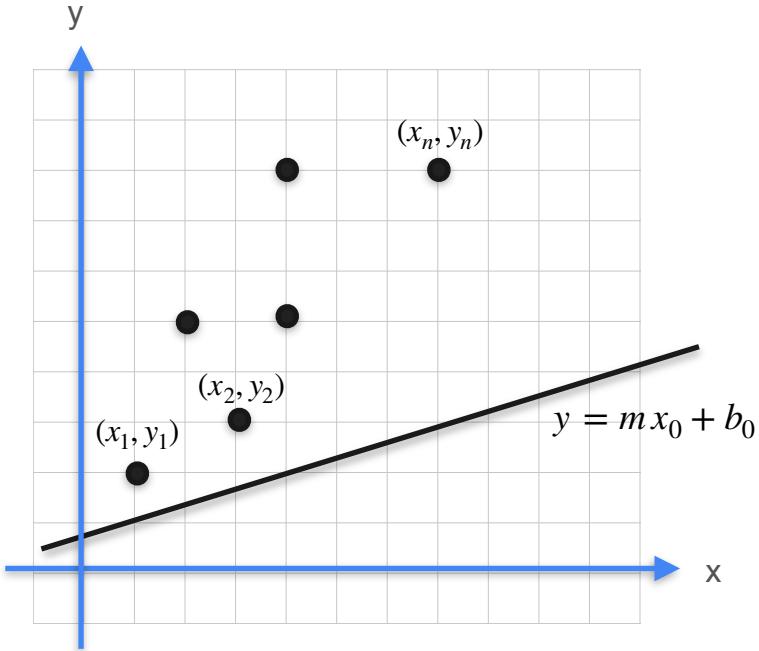
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

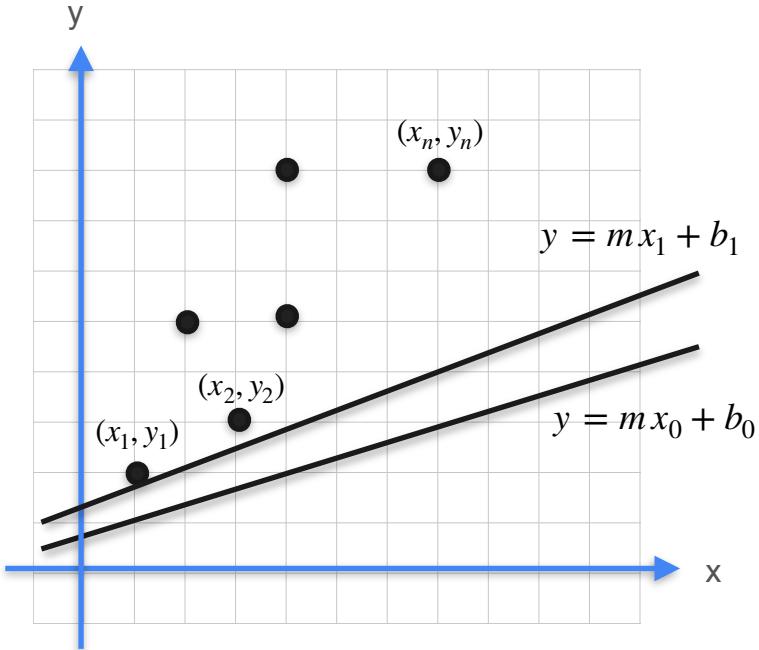
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\hspace{1cm}} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

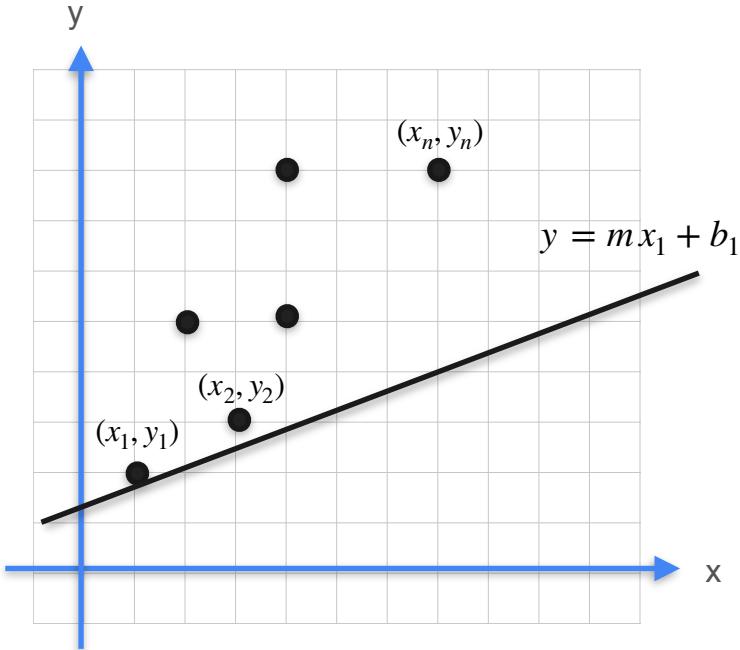
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

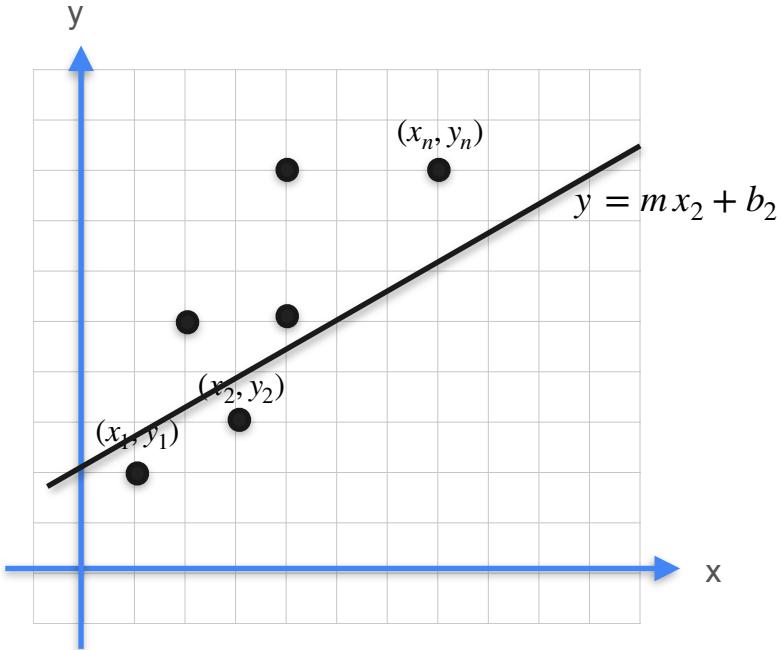
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

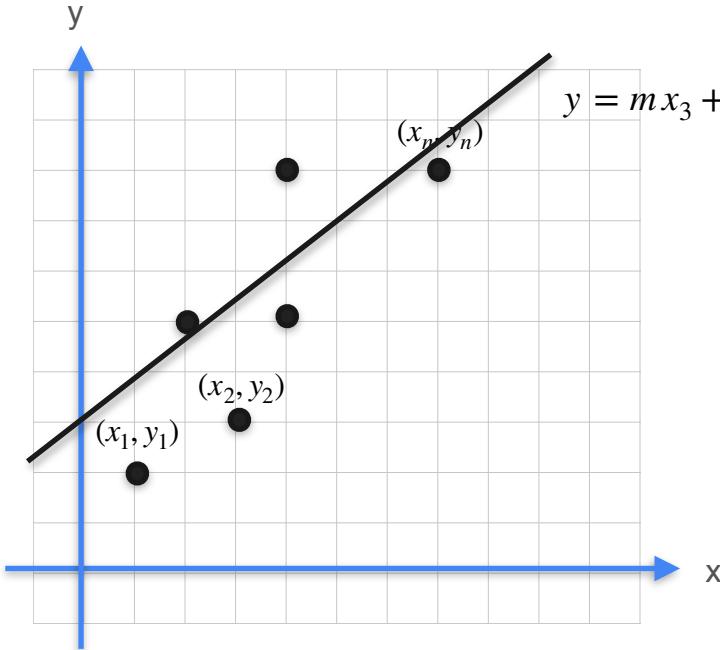
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_1 \\ b_1 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_1, b_1)$$

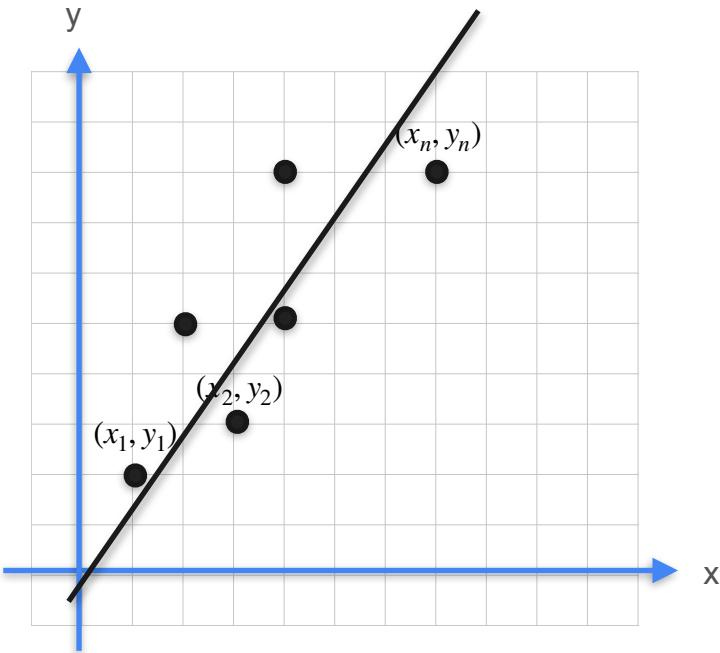
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_2 \\ b_2 \end{bmatrix} \rightarrow \begin{bmatrix} m_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_2, b_2)$$

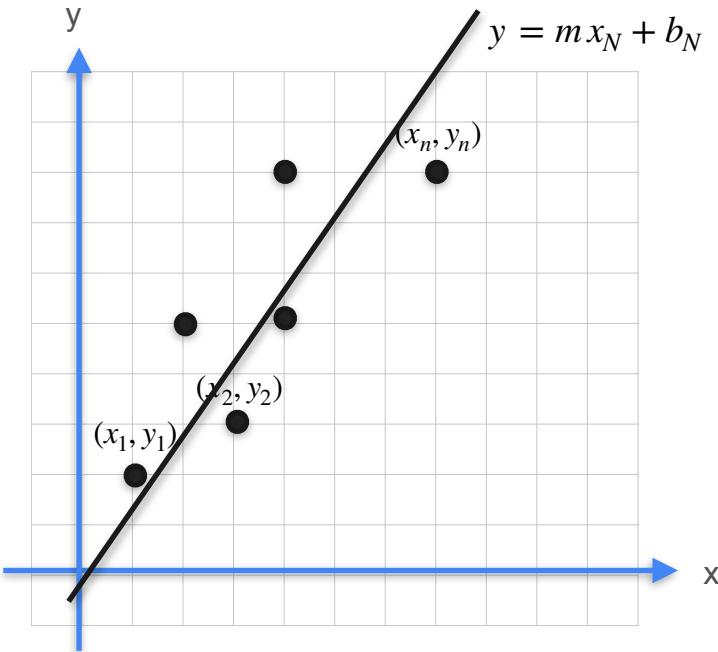
# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_{N-1}, b_{N-1})$$

# Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_{N-1}, b_{N-1})$$



DeepLearning.AI

# Gradients and Gradient Descent

---

## Conclusion