Solution

GIK Institute of Engineering Sciences and Technology

Midterm Examination

ES-341 Numerical Analysis, (Fall 2022)

Maximum marks: 30 **Time allowed:** 60 minutes

Registration no. ____________ **Name** ____________

Section ____________ **Instructor** ____________

Q. 1 (CLO-1):

(a) Use quadratic formula to find the exact root of the following equation: **(2 points)**

$$1.042x^2 - 12.01x + 0.01265 = 0,$$

(b) Use four-digit rounding arithmetic and appropriate formulas to find the most accurate approximation to the root of the above quadratic equation. **(6 points)**

Q. 2 (CLO-1):

From the following data, construct highest degree Newton's divided difference polynomials(forward and backward) to approximate $f(0.501)$. **(8 points)**

x	0.1	0.3	0.4	0.6
$f(x)$	0.904837	0.740818	0.670320	0.548811

Q. 3 (CLO-2):

Use Steffensen's method to find $p_0^{(2)}$ for the following function: **(4 points)**

$$g(x) = 1 + (\sin x)^2, \quad \text{with} \quad p_0^{(0)} = 1$$

Q. 4 (CLO-2):

Find all the intervals containing solution of the following equation. Use Newton's method to compute the smallest negative solution(if any), accurate to within 10^{-5}. **(6 points)**

$$-4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99 = 0$$

Q. 5 (CLO-4): Write Algorithm/MATLAB code for Newton's method in Q4. **(4 points)**

Q1 : $1.042x^2 - 12.01x + 0.01265 = 0$

(a) Using quadratic formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

2

Exact values are

$\boxed{x_1 = 11.52485832}$, $\boxed{x_2 = 0.0010533852}$

(b)

Here
$b = -12.01 < 0$.
So formula for x_1 is

$$x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a} = \frac{12.01 + \sqrt{144.2 - (4.168)(0.01265)}}{2.084}$$

$$= \frac{12.01 + \sqrt{144.2 - 0.05273}}{2.084} = \frac{12.01 + \sqrt{144.1}}{2.084}$$

$$x_1 = \frac{12.01 + 12.00}{2.084} \Rightarrow$$

x1=11.52

3

Since b is negative so we use alternate formula for x_2. i.e

$$x_2 = \frac{-2c}{b - \sqrt{b^2-4ac}} = \frac{-2(0.01265)}{-12.01 - \sqrt{12.01^2 - 4(1.042)(0.01265)}}$$

Solved this part in x_1

$$= \frac{-0.0253}{-12.01 - 12.00} = \frac{-0.0253}{-24.01}$$

$\Rightarrow \boxed{x_2 = 0.001054}$

3

Q2 :

(2)

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$$

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$$

x	$f(x)$	1st DD	2nd DD	3rd DD
0.1	0.904837			
		-0.820095		
0.3	0.740818		0.383717	
		-0.70498		-0.117868
0.4	0.670320		0.324783	
		-0.607545		
0.6	0.548811			

3 2 1

Using the table, Newton's forward divided difference polynomial of degree three is

$$P_3(x) = f(x_0) + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

$$\Rightarrow P_3(x) = 0.904837 - 0.820095(x - 0.1) + 0.383717(x - 0.1)(x - 0.3) - 0.117868(x - 0.1)(x - 0.3)(x - 0.4)$$

Simplification gives

$$\Rightarrow \boxed{P_3(x) = -0.117867x^3 + 0.478011x^2 - 0.995977x + 0.999772}$$

$$\boxed{f(0.501) \approx P_3(0.501) = 0.605947}$$

Newton's Backward DD polynomial is

$$P_3(x) = f(x_n) + b_1(x - x_n) + b_2(x - x_n)(x - x_{n-1}) + b_3(x - x_n)(x - x_{n-1})(x - x_{n-2})$$

$$P_3(x) = 0.548811 - 0.607545(x - 0.6) + 0.324783(x - 0.6)(x - 0.4) - 0.117868(x - 0.6)(x - 0.4)(x - 0.3)$$

Simplifying:

$$\boxed{P_3(x) = -0.117867x^3 + 0.47801x^2 - 0.995976x + 0.999772}$$

$$\boxed{f(0.501) \approx P_3(0.501) = 0.605947}$$

Q3 : $g(x) = 1+(\sin x)^2$, $p_0^{(0)} = 1$

$p_0^{(2)}$ using Steffensen's method.

$$p_n^{(k)} = p_n^{(k-1)} - \frac{\left(p_{n+1}^{(k-1)} - p_n^{(k-1)}\right)^2}{p_{n+2}^{(k-1)} - 2p_{n+1}^{(k-1)} + p_n^{(k-1)}} , \quad k \geq 1$$ .5

k	$p_0^{(k)}$	$p_1^{(k)} = g(p_0^{(k)})$	$p_2^{(k)} = g(p_1^{(k)})$
0	1	1.708073418	1.981273081
1	2.152904629	1.697735097	1.983972911
2	1.873464043		

3

Thus $\boxed{p_0^{(2)} = 1.873464043}$ .5

Q4: $f(x) = -4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99 = 0$

Intervals of solution:

$f(-4) < 0$

$f(-3) < 0$

$f(-2) < 0$] solution exists b/w $[-2,-1]$
$f(-1) > 0$

1

$f(0) > 0$] solution exists b/w $[0,1]$
$f(1) < 0$

1

$f(2) < 0$

$f(3) < 0$

So there are two intervals containing the solution.

Smallest negative interval is $[-2,-1]$.

Newton's Method:

$$p_{n} = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}, \quad f'(x) = -13.86e^{3x} - 6.22e^{2x} + 12.2e^{x}$$

1

n	p_n
0	$p_0 = -2$
1	-1.728959375
2	-1.75610265
3	-1.756408202
4	-1.75640824
5	-1.75640824

3

] $|p_5 - p_4| = 0 < 10^{-5}$

Thus $\boxed{p_5 \approx p = -1.75640824}$

Q5 : Algorithm

To find a Solution to

$$-4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99 = 0.$$

Input : intial approximation p_0, tol (TOL)
max. no. of iterations N

Output : Approximate solution p or message of failure.

Step 1 : set $i = 1$.

Step 2 : while $i \leq N_0$ do steps 3-6.

Step 3 : Set $p = p_0 - \frac{f(p_0)}{f'(p_0)}$ (compute p_i)

Step 4 : If $|p - p_0| < TOL$ then
OUTPUT : p (procedure successful).
STOP

Step 5 : Set $i = i + 1$

Step 6 : Set $p_0 = p$ (update p_0)

Step 7 : OUTPUT (Method failed after N iterations)
Procedure was unsuccessful.
STOP.

4

```
% newton  raphsaon method
clear all
close all
format long
clc
syms x; %to treat x as variable in MATLAB
otherwise values of x will be required
f(x)=-4.62*exp(3*x)-3.11*exp(2*x)+12.2*exp(x)-
1.99;
f1(x)=diff(f(x),x); %derivative of f(x)
p0=input('Enter Initial Guess :');
tol=input('Enter tolerance :');
p=p0-(f(p0)/f1(p0));
i=1;
fprintf('\n n            pn');
fprintf('\n%d          %f',0,p0);
fprintf('\n%d          %f',i,p);
while(abs(p-p0)>tol)
    p0=p;
    p=p0-(f(p0)/f1(p0));
    i=i+1;
    fprintf('\n%d           %f',i,p);
end
fprintf('\nThe required aproximation is %f',p);
```