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Directional relations and frames of reference

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Abstract As an intermediate category between metric and topology, directional relations are as much varied as “right of”, “before”, “between”, “in front of”, “back”, “north of”, “east of”, and so on. Directional relations are ambiguous if taken alone without the contextual information described by frames of reference. In this paper, we identify a unifying framework for directional relations and frames of reference, which shows how a directional relation with its associated frame of reference can be mapped to a projective relation of the 5-intersection model. We discuss how this knowledge can be integrated in spatial query languages.

Keywords Spatial relation · Frame of reference · Directional relation · Projective relation · Spatial taxonomy

1 Introduction

A significant trend of research in Geographical Information Science (GIS) relates to the study of the fundamental concepts governing our perception of objects in space and the increase of computational capabilities, such as assessing spatial relations among geographic objects. Directional relations represent a category of spatial relations that provide information about the spatial arrangement of objects in a scene [1] or locations in an embedding space [2, 3]. Directional relations are also defined as describing positions with respect to a frame of reference [4].

In queries such as “What are those buildings ahead on the right?”, spatial qualifiers like “ahead” and “on the right” correspond to the computation of directional relations. Moreover, to make this computation possible, there are several spatial aspects that are implicit in the query (we can call these aspects the ‘context’ of the query) and that should be made explicit. In previous query example, context information could be the direction of

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movement, the distance from the buildings, and the fact that the person that formulates the query and the listener share the same point of observation. Therefore, directional relations between two objects cannot be expressed independently of a geometric structure called frame of reference.

While topological relations are available in existing spatial query languages (indeed, the computation of topological relations and topological properties of spatial objects is the most prominent feature of spatial query languages), directional relations have not been included so far. This might be ascribed to two difficulties: (1) the definition of models that would be able to describe the directional relation between two extended objects and (2) the definition of models that would be able to describe directional relations under various frames of reference. In fact, most models for directional relations (including both the orientation and distance components) refer to point abstractions (e.g., [5–10]). Some models of directions among extended objects are the direction matrix [11] and the 5-intersection [12]. The direction matrix describes the relation between two objects in an absolute (projection-based) frame of reference. The 5-intersection describes ternary projective relations among objects without being tied to a specific frame of reference. The second difficulty is related to the representation of frames of reference. Direction relations are normally expressed in a variety of frames of reference [13–15]. A spatial query language that includes directions should be able to deal with such a diversity and the ways to switch from one frame of reference to another. The latter issue was defined as 'articulation rules' by Hernández [16]. The lack of formalization of frames of reference has probably been the major barrier for the inclusion of directional relations in spatial query languages. Early studies on the link between projective terms and frames of reference have been conducted by Eschenbach [17, 18] and later by Moratz and Tenbrink [19].

In this paper, we define a model of directional relations in various frames of reference, specifying the main types and subtypes: this outcome can be considered a taxonomy of frames of reference, refining the well-known classification of [20]. The knowledge of directions and frames of reference allows us to define a set of directional operators that can be integrated in a spatial query language. The directional relations are mapped to the 5-intersection model, which precisely gives the geometric definitions and the algorithms to compute relations. The 5-intersection model classifies projective relations between three geometric objects in the plane, distinguishing among 34 ternary projective relations that make up a jointly-exhaustive and pair-wise disjoint (JEPD) set of relations. Among the strengths of the model are the facts that it only uses projective properties and that the acceptance areas of relations depend on the shape and size of the reference objects. The limit of the 5-intersection is that its primitives cannot be directly integrated in a spatial query language without effort. The model is strongly geometric-oriented: with its formal definitions, it can be effectively implemented and optimized at the computational level creating a small set of projective operators that apply to all geometric spatial data types. On the other hand, it does not directly express the typical contextual information that is associated with directional relations. As an attempt to overcome such a limit, in this paper, we define directional relations in various frames of reference with transformation rules to the geometric level and vice versa.

The paper is structured as follows: we start in Section 2 introducing some concepts about frames of reference. In Section 3, we briefly recall the 5-intersection model. In Section 4, we identify the taxonomy for different subtypes of frames of reference and the mapping rules towards the 5-intersection. In Section 5, we describe a prototype architecture and the Java implementation of a computation engine for the defined directional relations. Finally, in Section 6 we draw short conclusions.

2 Frames of reference

Most human reasoning about physical space appears to use a qualitative interpretation of that space. Therefore, purely quantitative methods are less adequate than qualitative methods to effectively represent spatial information [21]. All spatial relations describing the mutual position of objects are usually described as binary relations from human beings. Examples are “the bus stop in front of the university” or “the tree to the left of the building”. Very few exceptions are ternary: notably the relation “between” [22], such as in the expression “the fountain between those two buildings”. The binary nature of directional relations must be combined with the presence of a *frame of reference* to disambiguate the meaning of the relation. In fact, a directional relation has a unique meaning when associated with some contextual information. If the context varies, most often the meaning of the relation becomes ambiguous: the expression “the tree to the left of the university”, which is unambiguous in a given context, becomes ambiguous if extrapolated from its context (see Fig. 1).

In this paper, we refine the classification of frames of reference given in [20]. Our goal is not to give an exhaustive taxonomy, but we aim to identify the most commonly used subcategories and show how the directional relations expressed in them can find a mapping towards the 5-intersection model. Frames of reference are *implicit* or *explicit* structures that are imposed on space [16]. The term implicit refers to a frame of reference which has not been made explicit because all information is expressed in the same frame of reference: the observer performs its reasoning in a given frame of reference without ambiguities. Nonetheless, the frame of reference needs to be made explicit if we need to compare information from different frames of reference, and therefore, the observer is aware of more co-existing frames of reference. The most detailed description of various frames of reference in a cross-linguistic setting is given by Levinson [13] and subsequent refinements by Klatzky [14] and Frank [15]. In [15], frames of reference are described, in analogy to Cartesian coordinate systems, as having three parameters: the origin, the orientation, and the handedness.

Three basic types of frames of reference are distinguished by Retz-Schmidt [20]:

- *intrinsic* frames of reference are established on a reference object that determines the origin of the coordinate system as well as its orientation;
- *extrinsic* frames of reference may also inherit their origin from a reference object; however, their orientation is determined by external factors such as a conventional object used as landmark;
- *deictic* frames of reference involve three objects: a primary object that is in a particular relation with respect to the reference object and the point of view. The orientation is imposed on the reference object as seen by the point of view.

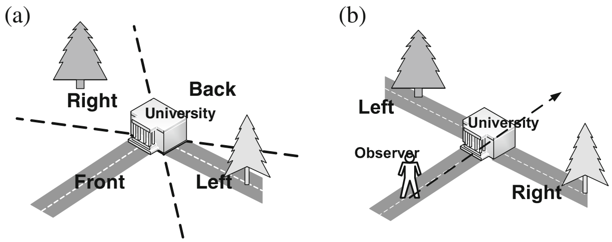


Fig. 1 Different interpretation of the expression “the tree to the left of the university” varying the context: **a** the left side is determined by an intrinsic property of the building; **b** the left side is determined by the point of view of an observer

According to [23], the same types of frames of reference are called intrinsic, absolute, and relative, respectively. Another way to characterize frames of reference is given in [24] and [14] leading to two categories: egocentric (subject-centered) and allocentric (other-centered), depending on whether the origin of the frame of reference is centered inside or outside the observer.

To better illustrate Retz-Schmidt's classification, let us consider the following queries:

1. What is the name of the street in front of the university?
2. What is the building towards north?
3. What is the building to the left of the city hall (from an observer's point of view)?

If we analyze the queries above, we can see that each of them can be associated with a different frame of reference: in the first one, the frame of reference is intrinsic, due to the fact that it is determined by the prominent front of the university building, which is an intrinsic property of the reference object (Fig. 2(a)). In the second one, the underlying frame of reference is extrinsic, since the conventional "north" direction is used as reference for orientation (Fig. 2(b)). The frame of reference used in the third query is deictic: the reference object is the city hall and the spatial relation of the unknown building is expressed with respect to the point of view of the observer (Fig. 2(c)).

Directional relations are used in different contexts: both man-made features, like roads and buildings, or natural features, like mountains and rivers, can have one or more geometric representations. Contextualization of spatial data semantics allows the same underlying data to take multiple forms, and disambiguate spatial concepts based on localized contexts [25]. The processing and representation of spatial information is essential for human cognition, especially for wayfinding and the creation of mental spatial representations. In [26], authors examined how people use verbal route directions given in different frames of reference in real-world navigation, particularly differences or similarities in cognitive load posed by two frames of reference. The influence that language has on the conceptualization of spatial relations, particularly turn directions, has been pointed out in [27]. A comparable approach to ours is proposed in [28], where spatial relations are discussed in the context of moving vehicles on US road networks. Those relations are ternary and are expressed in the form *relation*(X,Y,T), where X and Y are variable terms that refer to either a reference object (e.g., my car) or target object (e.g., the vehicle behind my car) moving on the network at variable time T. One of the scenarios of our paper (see Section 4.1.2), dealing with moving users, is quite similar to the one proposed in [28], though we take into account qualitative time only (past and future) and we define a different set of relations.

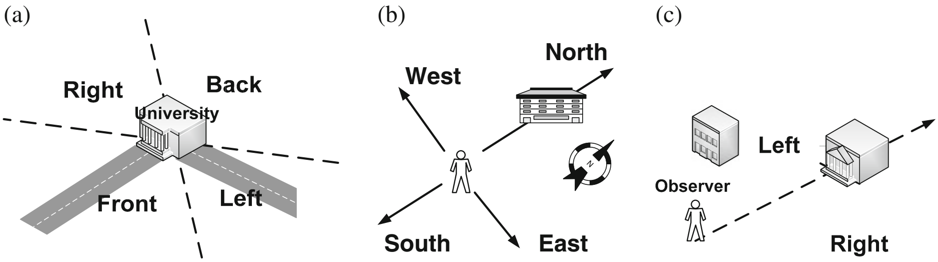


Fig. 2 **a** Intrinsic frame of reference; **b** Extrinsic frame of reference; **c** Deictic frame of reference

We will not consider relations between overlapping objects such as “Milan is in the north of Italy” or “Sweden is in the north of Europe”. These relations can be considered part/whole spatial relations [29], in which the parts are determined by cardinal directions.

3 Projective relations

Projective relations are a category of spatial relations that can be described by projective properties of the space without resorting to metric properties [30, 31]. Projective relations are thus qualitative in nature because they do not need exact measures to be explained. Also, projective relations can add more details to topological relations and, hence, they can serve as a basis for describing relations that are not captured by topology.

A ternary relation among three objects A, B, and C is denoted with $r(A,B,C)$. We assume that the first object A involved in the relation r has the role of *primary object* and the second and third objects have the role of *reference objects*.[1] Therefore the relation $r(A,B,C)$ should be read as “A is in the relation r with B and C”. The order of the reference objects B and C affects the orientation, that is, an orientation from reference object B to reference object C is assumed.

The basic projective invariant is the collinearity of three points: based on this primitive, which has been extended to regions [32], the 5-intersection was coherently built upon ternary relations of the kind $r(A,B,C)$. Three points x,y,z are *collinear* if they lie on the same line. Given an oriented line connecting the two reference points y and z, the plane is subdivided in two half-planes and the line itself is subdivided into three parts. This gives rise to the five projective relations *between* (*bt*), *rightside* (*rs*), *before* (*bf*), *leftside* (*ls*), *after* (*af*). If two of the points are coincident, the three points are trivially collinear, since two points determine a line. If all three points are coincident, they are also trivially collinear because there are infinite lines incident in a point. In the case the two reference points are coincident, we refine the relation *collinear* to the relations *inside* (*in*) and *outside* (*ou*). The set of ternary relations among points *bt*, *rs*, *bf*, *ls*, *af*, *in*, *ou* is a JEPD set of relations, as it was proved in [12].

The definitions for points are a special case of the definitions for regions. The relation *collinear* among regions can be introduced as a generalization of the same relation among points. For a complete discussion about definitions of collinearity for regions, see [32]. If two reference regions B and C have disjoint convex hulls (CHs), then two pairs of common tangents are uniquely defined: the external common tangents and the internal common tangents (Fig. 3(a)). The internal common tangents subdivide the plane into four cones. The external common tangents define the convex hull of the union of the two reference regions: such a convex hull corresponds to the acceptance area of the relation *between*. By intersecting the convex hull with the four cones, we obtain the acceptance areas, which we call *tiles*. The tiles *Between*(B,C), *Rightside*(B,C), *Before*(B,C), *Leftside*(B,C) and *After*(B,C) correspond to the five projective relations *bt*, *rs*, *bf*, *ls*, *af*, respectively, which are called *single-tile* relations. If a primary region A lies in more than one tile, the relation is called *multi-tile*: for example, if A is partly in *Rightside*(B,C) and partly in *Before*(B,C), then we say that *A is partly rightside and partly before of B and C* and we write *rs*:*bf*(A,B,C). As a convention for the notation, the order in which we enumerate the tiles composing a multi-tile relation is always *bt*, *rs*, *bf*, *ls*, *af*. For example, we will always write *rs*:*bf* and not *bf*:*rs*.

[1] We use the terminology of [5, 8, 20]: the primary object is the one to be located and the reference objects are the ones in relation to which the primary object is located.

As a special case, if the two reference regions have not disjoint convex hulls, common tangents cannot be defined and the plane can be partitioned in two areas, corresponding to relations *in* and *ou* (Fig. 3(b)).

The *5-intersection* matrix can be defined by considering the empty/non-empty intersections (indicated with 0 and 1, respectively) of the primary region A with the five acceptance areas:

	$A \cap Leftside(B,C)$	
$A \cap Before(B,C)$	$A \cap Between(B,C)$	$A \cap After(B,C)$
	$A \cap Rightside(B,C)$	

The 5-intersection can have 2^5–1 different configurations. Each configuration corresponds to a projective relation among three regions A, B, and C, where $CH(B) \cap CH(C) = \emptyset$. The matrix with all five values equal to zero does not correspond to a relation. For $CH(B) \cap CH(C) \neq \emptyset$, we consider the following *2-intersection* matrix:

$A \cap Inside(B,C)$	$A \cap Outside(B,C)$

The latter matrix can assume the values (0 1), (1 0), and (1 1). Overall, the model builds a JEPD set of 34 projective relations among three regions of the plane.

4 Identifying directions and frames of reference

We model directional relations as binary relations with an associated frame of reference. The same relations can have a geometric representation in terms of ternary relations of the 5-intersection model. A directional relation r can be defined as a tuple of three elements: two objects, namely, a primary object PO and a reference object RO, and a frame of reference f.

Definition 1. $r = (PO, RO, f)$

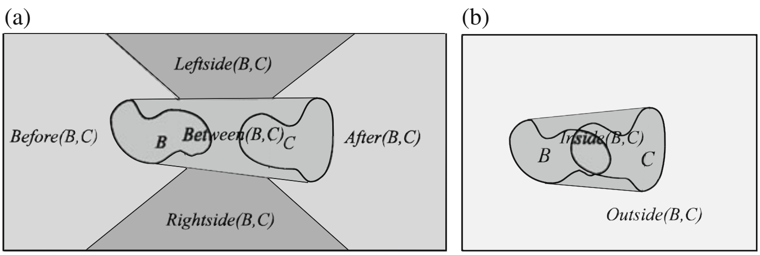


Fig. 3 **a** The partition of the plane into five areas for disjoint convex hulls of reference regions; **b** the partition of the plane into two areas for non disjoint convex hulls of reference regions

Both objects *PO* and *RO* take values from a set of classes *O*, which we assume to have a spatial component. For example, typical objects could belong to the class Building or the class Road. The relation *r* can take values from a subset of possible relations *R*, which depends on the frame of reference *f*. So, for each frame of reference *f*, among other aspects, we define the subset of directional relations that can be used with it. We define a frame of reference *f* made up of four components: *type* (*t*), *subtype* (*st*), *object* (*o*), and a set of *directional relations* (*R**).

Definition 2. $f = (t, st, o, R^*)$

The type can be one in the set (*intrinsic*, *extrinsic*, *deictic*). For each type, we identify several subtypes referring to common application scenarios. The values for the subtype will be specified later on in this section: each subtype corresponds to a different geometric representation in terms of the 5-intersection model. The object *o* is a particular geometric object that, depending on the frame of reference, has a specific role in the scene and is necessary to build the relations. Also the possible values for object *o* will be discussed in detail later on in this section. The set *R** is a subset of the set of directional relations *R* that is associated with the frame of reference. Figure 4 anticipates the results by showing the proposed taxonomy of frames of reference, with the hierarchical structure made up of types and subtypes. The resulting hierarchy is a refinement of the intrinsic-extrinsic-deictic categorization: the distinction in subtypes comes from an analysis of various examples in common scenarios and from those discussed in the literature (e.g., [13, 33]), but it cannot be considered exhaustive nor complete. It would be perfectly possible that other subtypes could be proposed in other peculiar situations.

In order to express a directional relation using the 5-intersection model, we have to identify the formal rules that allow us to perform the mapping from a given frame of reference to its geometric representation. The directional relation assumes the form $r(PO, RO, f)$, with a corresponding projective relation of the 5-intersection model $r'(\mathrm{PO}', \mathrm{RO}'_1, \mathrm{RO}'_2)$. In this correspondence, the object *PO* is always mapped to *PO*′. The reference objects RO'_1 and RO'_2 are determined in various ways, as discussed later on, for each subtype of the frame of reference *f*. In this paper, for each frame of reference, we assume that RO'_1 and RO'_2 have disjoint convex hulls, avoiding thus the degenerate case of the 5-intersection model. The relation *r* is mapped to the corresponding relation *r*′ via a mapping function Φ, which also varies with the frame of reference. We restrict ourselves to *single-tile* relations, that is, relations mapping to a single acceptance region. It is

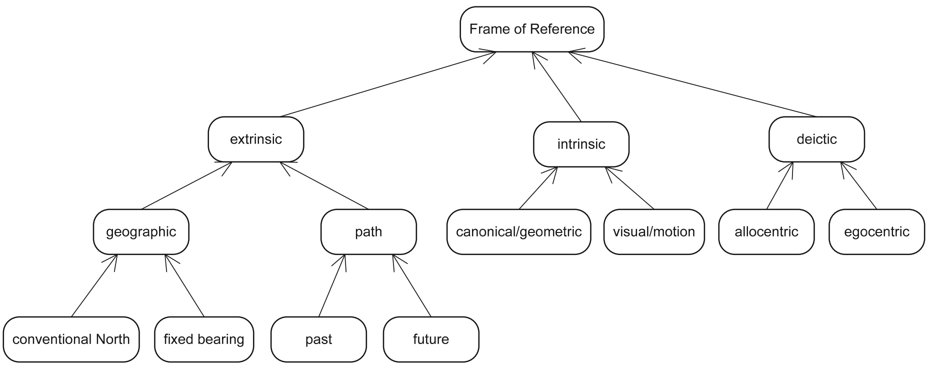


Fig. 4 The taxonomy of frames of reference

straightforward to extend the model to *multi-tile* relations: given two single-tile relations r_1 *and* r_2, the *multi-tile* relation obtained with them can be indicated as r_1: r_2. Multi-tile relations are mapped considering the components individually.

The domain of Φ is the set of relations *R*, while the co-domain of Φ is the set of single-tile relations of the 5-intersection model, denoted by *P* = {*rightside*, *leftside*, *between*, *before*, *after*}.

Definition 3. $\Phi : R \rightarrow P$

The Φ function can be completely defined by enumerating the results in a two columns matrix for each frame of reference. In the following, we discuss the various subtypes of frames of reference.

4.1 Extrinsic frames of reference

For the extrinsic frames of reference, we distinguish two different subtypes, called: *geographic* and *path*.

4.1.1 Geographic subtype

In this subtype, we include directional relations such as "region *A* is north of region *B*". While object *B* acts as reference object for this relation, the direction is determined by external factors (geographic coordinates). The geographic north is a conventional point in the terrestrial surface that determines all other cardinal directions (conventionally, east is defined to be on the right of an observer that looks towards north). Depending on the particular projection in which data are represented, we can have variations of this frame of reference subtype: the north could be represented as a point (Fig. 5(a)) or as a line (e.g., the upper side limit of data bounding box – Fig. 5(b), called north grid line in [34]). Such a conventional geometric object is specified by the object element *o* of the frame of reference.

We label this frame of reference f_{EGN}, where *E* stands for extrinsic, *G* for geographic, and N for North:

Definition 4. $f_{EGN} = (extrinsic, geographic, conventional\_north, R^*_{EGN})$

The set R^*_{EGN} is the following:

Definition 5. $R^*_{EGN} = \{north\_of, east\_of, south\_of, west\_of\}$.

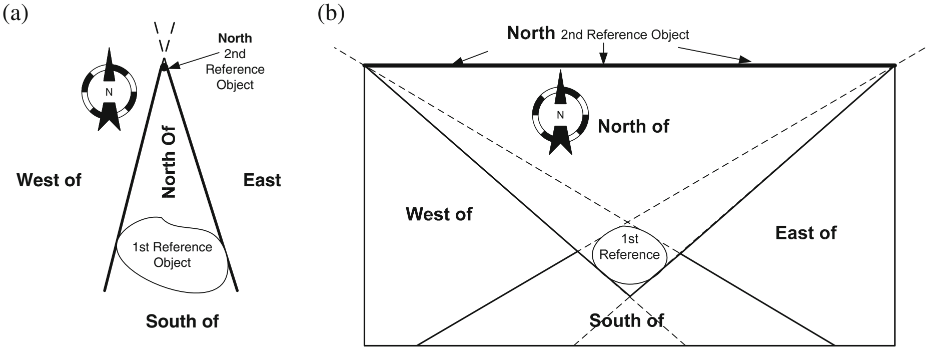


Fig. 5 Extrinsic frame of reference of subtype *geographic*: **a** point as passed object; **b** line as passed object

The mapping at the geometric level takes *RO* as RO'_1 and the *conventional_north* as RO'_2; the Φ_{EGN} function is:

Φ_{EGN}	
r	*r'*
north_of	*between*
east_of	*rightside*
south_of	*before*
west_of	*leftside*
	after

We may assume that in the particular subdivision of the plane that we have in this subtype, the *after* relation of the *P* set does not correspond to any directional relation.

Another situation that we include in the geographic subtype is that of other fixed bearings, other than the conventional north, that are sometimes used to establish an extrinsic frame of reference, such as mountain slopes, prevailing wind directions, river drainages, celestial azimuths [13]. For example, an extrinsic frame of reference can be determined by a slope with directional relations such as 'downhill' and 'uphill', as in the Tzeltal linguistic frame of reference [13]. Also the flow of a river commonly determines the right and left banks.

We label this frame of reference f_{EGB}, where *E* stands for extrinsic, *G* for geographic, and *B* for fixed bearing:

Definition 6. f_{EGB} = (*extrinsic*, *geographic*, *fixed_bearing_object*, R^*_{EGB})

The set R^*_{EGB} is the following:

Definition 7. R^*_{EGB} = {*downhill*, *right_of*, *uphill*, *left_of*}.

The subdivision of the space can be made up in this case by a conventional object that is situated downhill and by a mapping function that can be rewritten as follows:

Φ_{EGB}	
r	*r'*
downhill	*between*
right_of	*rightside*
uphill	*before*
left_of	*leftside*
	after

4.1.2 Path subtype

The *path* subtype describes an extrinsic frame of reference, for which the context is given by successive positions along a linear path. This type of extrinsic frame of reference was described in [16, 20], taking the motion of a reference object to identify contextual factors.

For simplicity, we assume to have a straight path, since a bended path would need a modification of the 5-intersection model.

Examples of relations could be found in expressions such as “the car passed a crossroad” or “you come across a supermarket along the way”. Both primary and reference objects have to be taken along the path. The directional relation is expressed between a primary object and a reference object, but two variations of this subtype can be identified, considering that the mapping at the geometric level can be done by associating the reference object either with the first or the second reference object of the 5-intersection model. The other reference object needs to be found along the path as well.

We can find a temporal interpretation thinking to a moving reference object taken at various time instants. Assuming that one reference object of the projective relation represents the present time position of the moving object, then the other reference object may represent, respectively, a position in the past or in the future. A past or a future position of the reference object is passed to the object element of the tuple (Fig. 6(a-b)). The reference object RO of the directional relation (PO, RO, f) is always the present position. We name these two variations of the path subtype f_{EPP} and f_{EPF}, where E stands for extrinsic, P for path, and P and F for past and future, respectively.

The corresponding tuples are:

Definition 8. $f_{EPP} = (extrinsic, motion, past\_position, R^*_{EPP})$.
Definition 9. $f_{EPF} = (extrinsic, motion, future\_position, R^*_{EPF})$.

Now we define the two sets R^*_{EPP} and R^*_{EPF} of all possible directional relations for the frames above:

Definition 10. $R^*_{EPF} = \{passed, right, left, ahead\}$.
Definition 11. $R^*_{EPF} = \{along, right, behind, left\}$.

The Φ_{EPP} and Φ_{EPF} functions are:

Φ_{EPP}		Φ_{EPF}	
r	r'	r	r'
passed	*between*	*along*	*between*
right	*rightside*	*right*	*rightside*
	before	*behind*	*before*
left	*leftside*	*left*	*leftside*
ahead	*after*		*after*

In both variations, there are two 5-intersection relations that are not used. With respect to the past position, the subtype does not specify what was before the starting point, while,

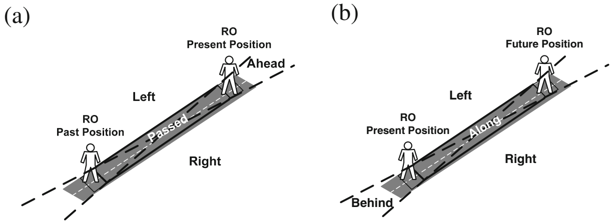


Fig. 6 Extrinsic frame of reference: **a** subtype *path* with past position; **b** subtype *path* with future position

with respect to the future position, the subtype does not specify what will be after the arrival point. Let us better illustrate the above frames of reference with some examples. In the frame of reference f_{EPP}, we can express relations of a moving object with respect to a past position (e.g., the starting point): e.g., "the car (which I am driving) passed a crossroad" or "as we move ahead, we arrive at a gas station". In f_{EPP}, the past position is well fixed, while future positions are underdetermined. In the frame of reference f_{EPF}, we can express relations of a moving object with respect to a future position (well determined). For example, "I'm going towards Rome. Along the way, we meet Florence" and "We left behind Bologna".

The frames of reference f_{EPP} and f_{EPF}, if combined together, would construct a frame of reference very similar to the double-cross calculus [7], obtaining an equivalent for extended regions.

4.2 Intrinsic frames of reference

In this section, we analyze the second type of frames of reference: the intrinsic one. We can identify two different subtypes depending on how we build the reference system: *canonical/ geometric* and *visual/motion.*

4.2.1 Canonical/geometric subtype

The *canonical/geometric* subtype copes with all situations in which there is a reference object with some peculiarities that allow the identification of a prominent front and/or other sides of the object itself. The prominent front may be identified with geometric characteristics of the reference object or by a canonical use of some of its parts. For example, the front side of a building could be identified by an architectural feature, like a façade, or by a functional feature, like its main entrance [35]. Depending on the shape of the reference object, it is possible to identify two geometric parts, back and front, which are assigned to the object o of the frame f. Those two parts correspond to RO'_1 and RO'_2 in the 5-intersection model (Fig. 7(a)). As a special case, the back or front could be a point (Fig. 7(b)).

We name this frame of reference f_{IC}, where I stands for intrinsic and C for canonical/ geometric and the corresponding tuple is:

Definition 12. f_{IC} = (*intrinsic*, *canonical/geometric*, *back-front*, R^*_{IC})

Now we define the set R^*_{IC} of all possible directional relations of f_{IC}

Definition 13. R^*_{IC} = {*inside, rightside, back, leftside, front*}.

The Φ_{IC} function is:

Φ_{IC}	
r	*r'*
inside	*between*
rightside	*rightside*
back	*before*
leftside	*leftside*
front	*after*

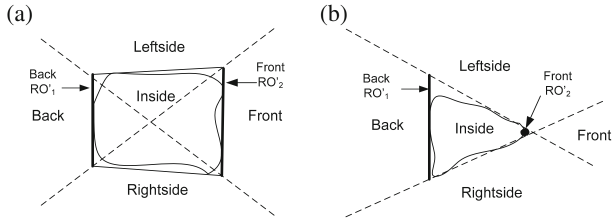


Fig. 7 Intrinsic frame of reference subtype *canonical/geometric*: **a** back-front object; **b** special case

4.2.2 Visual/motion subtype

The *visual/motion* subtype is an intrinsic frame of reference, where the front direction is determined either by the motion of the reference object *RO* [33] or by the direction of vision [35]. It is worthwhile to notice that the direction of motion could be not coincident with the direction of vision. Both cases fall inside egocentric frames of reference. The reference object *RO* corresponds to RO'_1. The acceptance areas can be determined by an angle centered on the front direction (Fig. 8(a)), whose width may be determined by different criteria. In the case of motion, the width of the angle could be the range in which the movement direction can change; in the case of vision, the angle could be the width of view, such as the binocular view angle. The second reference object RO'_2 can correspond to a reference segment, as illustrated in Fig. 8(a). The meaning of such a segment is not uniquely determined: one interpretation in the case of motion could be to delimit a zone where a possible collision can occur during the movement or alert zone (Fig. 8(b)). In the case of vision, we can distinguish a close visual field from a medium/far visual field. The *o* element of the *f* tuple is this reference segment.

We name this frame of reference f_{IV}, where *I* stands for intrinsic and *V* for visual/motion and the corresponding tuple is:

Definition 14. $f_{IV} = (intrinsic, visual/motion, reference\_segment, R^*_{IV})$

Now we define the set R^*_{IV} of all possible projective relations of f_{IV}:

Definition 15. $R^*_{IV} = \{to\_the\_right, back, to\_the\_left, front\}$.

The corresponding Φ_{IV} function is:

Φ_{IV}	
r	*r'*
alert/close	*between*
to_the_right	*rightside*
back	*before*
to_the_left	*leftside*
front	*after*

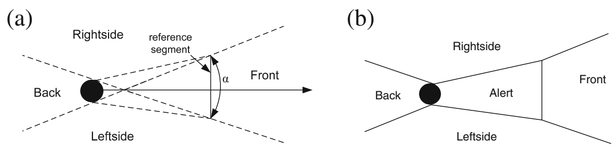


Fig. 8 Extrinsic frame of reference subtype *angle*: **a** identifying Ref.Seg.; **b** acceptance areas

4.3 Deictic frames of reference

In this section, we analyze the last type of frames of reference: the deictic one. We distinguish two different subtypes depending on whether the origin of the frame of reference is centered on the reference object RO or on the point of view. We call the subtypes *allocentric* (Fig. 9(a)) and *egocentric* (Fig. 9(b)). The construction is the same for both frames of reference: we use the RO of the relation tuple as RO'_2 and the point of view, represented by the object o of the f tuple, as RO'_1.

We name these two frames of reference f_{DA} and f_{DE}, where D stands for deictic and A and E mean *allocentric* and *egocentric*, respectively. The corresponding tuples are:

Definition 16. f_{DA} = (*deictic, allocentric, point_of_view,* R^*_{DA}).
Definition 17. f_{DE} = (*deictic, egocentric, point_of_view,* R^*_{DE}).

Now we define the two sets R^*_{DA} and R^*_{DE} of all possible projective relations of f_{DA} and f_{DE}:

Definition 18. R^*_{DA}={*front, right_of, left_of, behind*}.
Definition 19. R^*_{DE}={*between, right_of, back, left_of, after*}.

The Φ_{DA} and Φ_{DE} functions are:

Φ_{DA}		Φ_{DE}	
r	*r'*	*r*	*r'*
front	*between*	*between*	*between*
right_of	*rightside*	*right_of*	*rightside*
		back	*before*
left_of	*leftside*	*left_of*	*leftside*
behind	*after*	*after*	*after*

In the Hausa and other languages [13], for the frame f_{DA}, the relations *front* and *behind* would be exchanged, since in these languages coordinates are translated from the point of view to the reference object, rather than rotated. The Before zone of the 5-intersection model is not mapped by the Φ_{DA} function due to the fact that it is placed on the back of the observer: we assume that, if we focus the attention on the reference object, all objects on the back of the observer lose their meaning. Finally, examples of use of the frame f_{DE} could be "There is a fountain between me and the house" or "The post office is behind you if you look at the church".

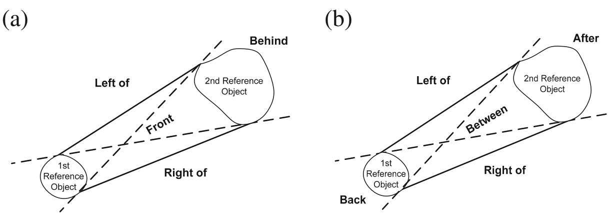


Fig. 9 Deictic frame of reference: **a** subtype *allocentric*; **b** subtype *egocentric*

5 Application

In this section, we outline a possible architecture, our implementation of frames of reference, and query examples.

5.1 Architecture

The proposed architecture adheres to the following guidelines:

- possibility to handle queries on spatial data with spatial operators;
- use of standard data formats to promote syntactic interoperability;
- ability to manage information on the basis of context.

The typical structure of the architecture to promote interoperability is divided into layers: the presentation layer (or user layer) for user interface, the business logic layer (or service layer) to perform operations and the data layer which is used by the previous level to retrieve data to manipulate or to save them in a permanent support. To allow syntactic interoperability, it is necessary to use standards to represent and manage data. In the architecture proposed by the Open Geospatial Consortium (OCG), in the business logic layer there is a WFS (Web Feature Service) to perform data exchange [36]. A WFS contains several interfaces to retrieve and manage vector data. The WFS receives the query and uses the data layer to retrieve the answer. In this kind of 3-tiers architecture, the data storage formats or spatial database software that is used in the data layer is opaque to the client. The client does not have to know the implementation details of the data layer, because it communicates with the WFS service only.

According to the OGC, the query at the geometric level of an interoperable architecture can be expressed by using the Geographic Markup Language (GML) [37], but topological operators only are supported: therefore, an extension of GML is necessary to handle queries with projective operators and the associated frames of reference. When the query reaches the server, the WFS, using the frame of reference expressed within the query, translates application features into geometric types accessing the geometric property of the feature: for example, features like streets, rivers, and railroads will be mapped to their corresponding geometric representations, like polylines and polygons. Directional relations, like “back” or “along”, will be mapped by the WFS according to the corresponding Φ function implicitly contained in the adopted frame of reference and chosen in the Φ repository. Subsequently, the spatial operators will be calculated by a spatial DBMS or by one or more geometric calculation engine. For example, JTS is a Java-implemented calculation engine for topological operators [38]. Regarding projective operators, as shown in the next section, we developed a Java engine capable to compute the 5-intersection relations and all the directional relations with respect to a chosen frame of reference. Then, the data will be sent back to the client for visualization (Fig. 10).

5.2 Implementation

We carried out a Java implementation of the frames of reference and the corresponding operators to compute directional relations. Those operators rely on the geometric operators provided by the 5-intersection model, which have been previously implemented (see [12]). In detail, we extended the JTS “geometry” class that represents geometric objects,

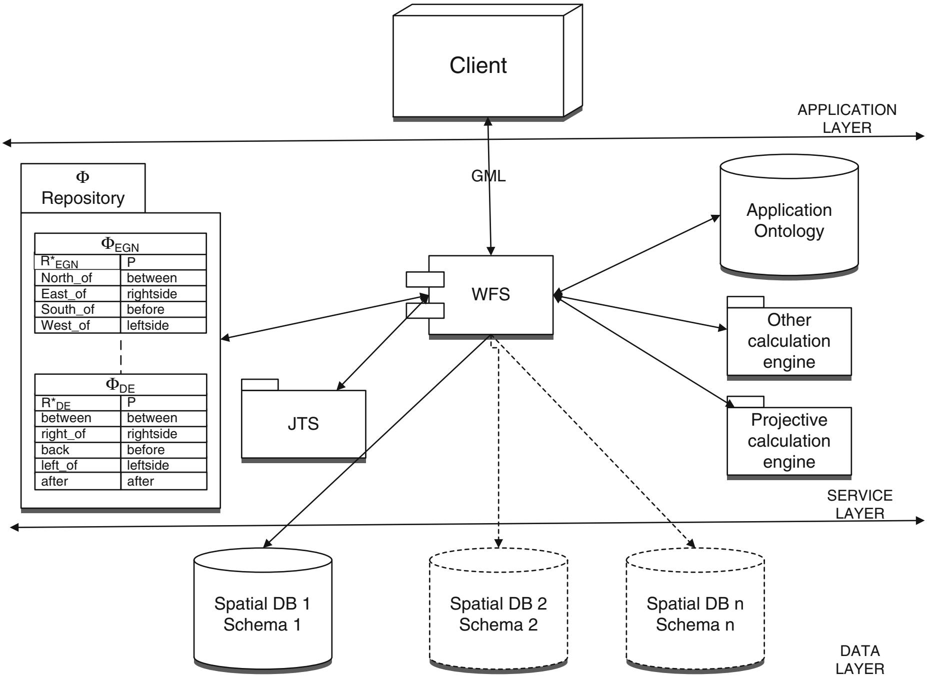


Fig. 10 A prototypical architecture

according to the definitions given in the OGC simple features specifications, adding methods to compute directional relations and, then, we implemented a Java virtual class for the frame of reference called *FoR* and other three virtual classes for the three different subtypes of frames of reference.

The virtual super class *FoR* includes methods as explained below. To illustrate the methods, we will use an instance of *FoR*, called *fr_of_ref*, and two instances, a tree *T* and a university building *U*:

1. The virtual function *Fi* that implements the Φ function and allows us to translate a directional relation to the corresponding 5-intersection relation. This function has to be implemented in the subtype classes because it can be defined only if the type and the subtype are defined.

2. A *relate* function capable of checking if a given directional relation holds between any two given objects. It can also be used to check multi-tile relations such as *after:left_of*. A possible use is: "Is the tree after and left of the university building?"

```
boolean test_rel=fr_of_ref.relate(T,U,"after;left_of");
```

Alternatively, relations can be completely specified in terms of a binary pattern using methods *patt2rel* and *rel2patt* that allow the translation from binary patterns to relations and vice versa.

```
boolean test_rel=fr_of_ref.relate(T,U,"00011");
```

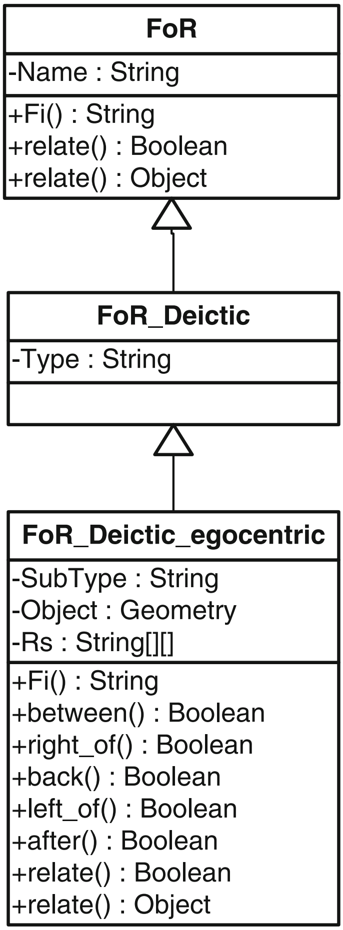
3. A *relate* function that returns all the application objects that are in a given relation with a given object. This function can be used to retrieve objects from a data store filtering those that verify the relation expressed within a query, as we will show in Section 5.3.

The virtual classes representing subtypes have one more attribute containing the corresponding type. For each type of frame of reference, we implemented all the subtypes. As an example, a UML diagram of subtype f_{DE} is given in Fig. 11.

Each subtype has the same structure and includes the following attributes and methods:

1. The subtype of the frame of reference.
2. A geometric object *Object*, as defined in Section 4.
3. The set *Rs* of all possible relations within the frame of reference. Each relation can be completely specified in terms of a pattern: *Rs* is a bi-dimensional array of strings that

Fig. 11 UML diagram of f_{DE}



contains the name of the relations and the relative pattern. For example, for the f_{DE} subtype, *Rs* will look as follows:

```
Rs=new String[5][2];

Rs[0] [0]= "10000";
Rs[0] [1]= "between";
Rs[1] [0]= "01000";
Rs[1] [1]= "right_of";
Rs[2] [0]= "00100";
Rs[2] [1]= "back";
Rs[3] [0]= "00010";
Rs[3] [1]= "left_of";
Rs[4] [0]= "00001";
Rs[4] [1]= "after";
```

4. The mapping function *Fi* that allows to translate a directional relation in the corresponding geometric one calculated by means of the 5-intersection model.
5. A set of Boolean functions that check all the possible *single-tile* relations defined in the frame of reference. Those relations are binary and operate on two application objects by accessing their geometric representation. Referring to the example shown in the introduction, if we want to check if the tree is on the left of the university building from the observer point of view, we have to call the Boolean function *left_of*, which is a method of the *FoR_Deictic_egocentric* class. The function call is the following:

```
boolean test_rel=fr_of_ref.left_of(T,U);
```

5.3 Query example

Let us show how our implementation can be used for answering queries containing spatial operators. We will focus only on the first step of the query processing, which is the retrieving of objects from a data store filtered by a spatial relation. A possible user query could be: “What is the building to the right of the university?”. Thus, we need to retrieve all the objects *Building* from the data store filtered by those that verify the spatial relation *right_of* with the observer location and the university. The relation *right_of* is unambiguous if associated with a frame of reference. This association is carried out automatically by the query interface, based on the context information. Let us suppose that in this case the relation should be interpreted under the f_{DE} frame of reference (instantiated as *fr_of_ref_DE*). The SQL-like statement will be:

*SELECT * FROM Building WHERE fr_of_ref_DE.right_of(Building, id.University)*

This query is expressed at the application level and contains a constant, which is the *id.University*, and a variable *Building*. The *WHERE* clause will invoke the *relate* method of the *FoR_Deictic_egocentric* class with the pattern of the *right_of* relation. It will return all the application objects (buildings) that are in the *right_of* relation with the university in an array called *Collection*. Let us suppose that the *Buildings*[] array contains all the buildings in the data set and *U* is the university. The call to the *relate* function will look as follows:

```
Building Collection[]=fr_of_ref_DE.relate(Buldings[],U,"right_of");
```

As mentioned before, the spatial relation is binary: there is a primary object *Building* that is variable (it will be the result of our query) and a reference object *U*, and it is associated with a particular frame of reference f_{DE}. In this case, the frame of reference contains the observer position in the *Object* attribute. For each *Building*, the *relate* function will invoke the function *relate* of the 5-intersection using the observer position and the university as first and second reference object, respectively. Then, it will evaluate the returned pattern with respect to the one obtained by the Φ function applied to the *right_of* relation: the building with the matching patterns will be added to *Collection*[]. A possible translation of the previous query coded in an extended GML could be the following:

```
<?xml version="1.0" encoding="iso-8859-1"?>
<GetFeature outputFormat="GML2" xmlns:gml="http://www.opengis.net/gml">
<Query typeName="OBJECTS">
 <PropertyName>BUILDING</PropertyName>
 <PropertyName>GEOM</PropertyName>
  <Filter>
    <gml:FoR>
      <TYPE>DEICTIC</TYPE>
       <SUBTYPE>EGOCENTRIC</SUBTYPE>
          <OBJECT>
            <PropertyName>GEOM</PropertyName>
           <gml: Point >
            <coord>
               <X>X coordinate of observer position</X>
               <Y>y coordinate of observer position</Y
            </coord>
            </gml:Point>
          </OBJECT>
      <Right_of>
       <PropertyName>GEOM</PropertyName>
         <gml:Polygon>
          <gml:outerBoundaryIs>
          <gml:LinearRing>
            <gml:coordinates cs="," decimal="." ts=" ">
              …coordinates of the University building..
            </gml:coordinates>
          </gml:LinearRing>
         </gml:outerBoundaryIs>
```

As shown in the example above, the Φ function does not need to be explicitly expressed in the query because once we identify the frame of reference, the associated Φ is known server-side.

6 Conclusions

In this paper, we outlined the development of a framework for expressing directional relations in various well-known frames of reference and their translation to projective relations of the 5-intersection model. In order to reach our aim, we have identified formal rules that allow us to map all frames of reference to the geometric model. Specifically, we have identified a pair of reference objects for each directional relation (and for the associated frame of reference) and a mapping function Φ that provides the binding between directional relations and 5-intersection relations.

The 5-intersection model was not born to satisfy cognitive or linguistic aspects but the geometric need of modeling positions of extended objects in space by using projective invariants. In this sense, it is not essential the name that we assign to the projective relations identified by the model. It is instead the goal of the present article to try to give "meaning" to the 5-intersection relations, by mapping them to typical frames of reference in which the directional relations are able to convey the associated cognitive or linguistics aspects.

The identified frames of reference always map to a valid 5-intersection matrix, avoiding the case of intersecting convex hulls of the reference objects. Therefore, the part of the model for projective relations called 2-intersection was not actually used in the frames of reference discussion. It should be noticed that, in this paper, we do not consider the case of frames of reference made up of overlapping objects, such as a region and one of its parts (the already cited example of "Milan is in the north of Italy"). For these cases, a modified model of directional relations should be developed.

From the application point of view, this paper provides the means of indicating spatial relations on the basis of context: the query interface can build queries that will be processed server-side at a geometric level. The model was implemented as a Java computation engine for projective relations and embedded in a prototypical application for testing.

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