Dissipation in Deforming Chaotic Billiards

A dissertation presented
by

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to
The Department of Physics
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of

Physics

Harvard University
Cambridge, Massachusetts
October 2000

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Abstract

Chaotic billiards (hard-walled cavities) in two or more dimensions are paradigm systems in the fields of classical and quantum chaos. We study the dissipation (irreversible heating) rate in such billiard systems due to general shape deformations which are periodic in time. We are motivated by older studies of one-body nuclear dissipation and by anticipated mesoscopic applications. We review the classical and quantum linear response theories of dissipation rate and demonstrate their correspondence in the semiclassical limit. In both pictures, heating is a result of stochastic energy spreading. The heating rate can be expressed as a frequency-dependent friction coefficient $\mu(\omega)$, which depends on billiard shape and deformation choice. We show that there is a special class of deformations for which μ vanishes as like a power law in the small-ω limit. Namely, for deformations which cause translations and dilations $\mu \sim \omega^4$ whereas for those which cause rotations $\mu \sim \omega^2$. This contrasts the generic case for which $\mu \sim \omega^0$. We show how a systematic treatment of this special class leads to an improved version of the 'wall formula' estimate for $\mu(0)$.

We show that the special nature of dilation (a new result) is semiclassically equivalent to a quasi-orthogonality relation between the (undeformed) billiard quantum eigenstates on the boundary. This quasi-orthogonality forms the heart of a 'scaling

method' for the numerical calculation of quantum eigenstates, invented recently by Vergini and Saraceno. The scaling method is orders of magnitude more efficient than any other known billiard quantization method, however an adequate explanation for its success has been lacking until now. We explain the scaling method, its errors, and applications. We also present improvements to Heller's plane wave method.

Two smaller projects conclude the thesis. Firstly, we give a new formalism for quantum point contact (QPC) conductance in terms of scattering cross-section in the half-plane, of use in open mesoscopic and atomic systems. We derive the maximum conductance through a tunneling QPC coupled to a resonator. Secondly, we numerically model a novel design of coherent atom waveguide which uses the dipole force due to evanescent light fields leaking from an optical waveguide mounted on a substrate.

Contents

Citations to Previously Published Work

Large portions of Chapters **??** and **??**, as well as some of Sections **??** and **??** have appeared in the following two papers:

> "Deformations and dilations of chaotic billiards: dissipation rate, and quasi-orthogonality of the boundary wavefunctions", A. H. Barnett, D. Cohen, and E. J. Heller, Phys. Rev. Lett. **85**, 1412 (2000), `nlin.CD/0003018`;
>
> "Rate of energy absorption for a driven chaotic cavity", A. H. Barnett, D. Cohen, and E. J. Heller, submitted to J. Phys. A, `nlin.CD/0006041`.

The numerical methods of Chapter **??** were used to calculate data appearing in the above papers and in the following:

> "Parametric evolution for a deformed cavity", D. Cohen, A. H. Barnett, W. Bies, and E. J. Heller, submitted to Phys. Rev. E, `nlin.CD/0008040`.

Chapter **??** appears in its entirety as

> "Mesoscopic scattering in the half-plane: how much conductance can you squeeze through a small hole?", A. H. Barnett, M. Blaauboer, A. Mody, and E. J. Heller, submitted to Phys. Rev. B, `cond-mat/0008279`.

Finally, most of Chapter **??** has been published as

> "Substrate-based atom waveguide using guided two-color evanescent light fields", A. H. Barnett, S. P. Smith, M. Olshanii, K. S. Johnson, A. W. Adams, M. Prentiss, Phys. Rev. A **61**, 023608 (2000), `physics/9907014`.

Electronic preprints (shown in `typewriter font`) are available on the Internet at the following URL:

> `http://arXiv.org`

Acknowledgments

Completing this doctoral work has been a wonderful and often overwhelming experience. It is hard to know whether it has been grappling with the physics itself which has been the real learning experience, or grappling with how to write a paper, give a coherent talk, work in a group, teach section, code intelligibly, recover a crashed hard drive, stay up until the birds start singing, and... stay, um... focussed.

I have been very privileged to have undoubtedly the most intuitive, smart and supportive advisor anyone could ask for, namely Rick Heller. Ever since I learned from him what an avoided crossing was (animated in full colour, naturally), I have been stimulated and excited by his constant flow of good ideas. Rick has an ability to cut through reams of algebra with a single visual explanation that I will always admire, and I have learned a great deal of physics from him. He has fostered certainly the most open, friendly, collaborative and least competitive research group in the theory wing of this Department. He has also known when (and how) to give me a little push in the forward direction when I needed it.

Throughout my six years, I was supported for many semesters by the National Science Foundation, and the Institute for Theoretical Atomic and Molecular Physics (ITAMP), through the generosity of my advisor. During my first year I thank the Kennedy Memorial Trust in London for providing my funding.

Rick's other students and post-docs, both past and present, comprise a superb research group. The ability to bounce ideas off so many excellent minds has been priceless. My most intense collaboration has been with Doron Cohen, whose clarity, persistence, ability to create new models, and ability to write a new publication every month, has taught me a lot. (I am still working on the publication per month aspect.)

Michael Haggerty has similiarly influenced me, and I think I can safely say that everyone in the group has benefitted from his generosity and collaboration. In my first couple of years in the Heller group, Adam Lupu-Sax and I swapped many thoughts on numerical methods, while Lev Kaplan prevented me from getting too scarred by the new realm of quantum chaos. Jesse Hersch shared many good times and wacky science experiments with me, and introduced me to a world of electrostatically-levitated chalk particles. Areez Mody has been a companion and goldmine of mathematical knowledge since the year we arrived together. Scot Shaw has been my numerical methods partner-in-crime. Bill Bies was my buddy through thesis formatting hell. It has also been my pleasure to work with (and hang out with) Jiří Vaníček, Axel André, Miriam Blaauboer, Ragnar Fleischmann, Greg Fiete, Allison Kalben, Natasha Lepore, Michael Efroimsky (and his stories), Manny Tannenbaum (and his accents), Sheng Li and Stephan Filipov; also from the earlier days Maurizio Carioli, Allan Tameshtit, Jonathan Edwards, Kazuo Hirai (and his valves), Bill Hoston, Neepa Maitra, Martin Naraschewski and Steve Tomsovic (and his jazz).

Our group's secretary Carol Davis is surely the kindest, coolest and most witty person one could possibly hope to spend a lunch-break gossiping with. She has also helped me out many a time during those pre-presentation panics. I must also thank Mary Lampros for her unique blend of caring and total organisation. She will be sorely missed in this Department.

On the foreign front, I must thank Eduardo Vergini for fascinating email exchanges, and guiding me towards an understanding of the incredible numerical method he invented along with Marcos Saraceno. From the group of Mara Prentiss, Steve

Smith and Maxim Ol'shanii were always there to fill me in on the practice and theory of atom optics.

There are countless others who have been there for me throughout my time as a graduate student. From the world of physics, Meredith Betterton, Joseph Thywissen and Rosalba Perna stand out as both good, caring friends and providers of fascinating physics conversations. From other worlds, Erika Evasdottir and my housemates Vitaly Napadow and John Iversen have been wonderful influences and friends; they have lead me from deconstruction to drumming.

My fascination with the physical world is undoubtedly due to the influence of my father, Ross. He showed me the joys of high-dispersion prisms in sunlight at an early age, and let me spend most of my childhood building Tesla coils and playing with BASIC on our 6502 home computer. Even now, he proofreads my papers. My late mother Pat taught me a love of language, of art, of teaching, and of signs, and it is sad to realise that she cannot be present for this moment in my life. The same realisation holds true for my grandfather Oliver, who is highly responsible for my tinkering nature. My courageous sister Jess has kept me sane during my hardest times via the transatlantic phone system.

Finally, Liz Canner has been my guiding light and love over these last two years. She has seen my best and my worst, and provided support, hugs, and taken me places I never imagined. Even when my emotional and research brains became so hopelessly entwined that I dreamt that the two of us were overlapping eigenfunctions, she still loved me. And she even thought it was cute.

Dedicated to my father Ross,
my late mother Pat,
and my sister Jess.

Chapter 1

Introduction and summary

Structure of this thesis

This thesis falls naturally into four parts, which are relatively independent:

- Study of dissipation rate in deformed chaotic billiards (Chapters 2, 3 and 4),
- Improved numerical methods for quantization of billiards (Chapters **??** and **??**),
- Half-plane scattering approach to mesoscopic conductance (Chapter **??**), and
- Design of an atom waveguide using two-color evanescent light fields (Chapter **??**).

The first two parts form the main body of the thesis, and they are both devoted to the study of billiard systems (hard-walled cavities enclosing a region of free space) in which the classical motion is chaotic. The quantum mechanics of such systems has become known as the field of 'quantum chaos'. The first part probably contains the most significant new physical results; this is reflected in the choice of thesis title. The

second part can be viewed merely as a description of numerical quantum-mechanical calculations that play a supporting role in the first part. However, there will also turn out to be a surprising reciprocal connection, namely that results from the first part will provide a much-needed explanation for the success of a very efficient numerical technique in the second part. The intertwining of these two subject areas had turned out to be one of the most beautiful surprises in this body of research.

The third and fourth parts form essentially separate projects, and can therefore be read independently. However they do share with the rest of the thesis the common theme of wave mechanics: the third presents a new approach to the transport of quasiparticle waves in mesoscopic systems, and the fourth models confined electromagnetic waves to trap and guide atoms (which themselves can be treated as coherent matter waves).

The goals and subject matter of the four parts are sufficiently different to merit individual introductions and summaries, which now follow without further ado.

Chapters 2,3 and 4: Dissipation rate and deformations of chaotic billiards

The dynamics of a particle inside a cavity (billiard) in $d = 2$ or 3 dimensions is a major theme in studies of classical and quantum chaos [37, 23, 4]. Whereas the physics of time-independent chaotic systems is extensively explored, less is known about the physics when such a system is 'driven' (time-dependent chaotic Hamiltonian). The main exceptions are the studies of the kicked rotator and related systems [20]. However, the rotator (with no kicks) is a $d = 1$ integrable system, whereas we are interested in chaotic ($d \geq 2$) cavities.

Driven cavities have been of special interest since the 1970s in studies of the so-called 'one-body' dissipation rate in vibrating nuclei [6, 29, 28, 24, 25]. A renewed interest in this problem is anticipated in the field of mesoscopic physics. Quantum dots [2, 19] can be regarded as small 2D cavities whose shape is controlled by electrical gates. Quasiparticle motion inside the dot can have long coherence (dephasing) times, and enable the semiclassical regime to be approached (many wavelengths across the system).

In Chapter **??** I give tutorial review of the theory of dissipation in general driven ergodic systems, which is quite a young field. The Hamiltonian is controlled by a single parameter x, whose time-dependence will be $x(t) = A\sin(\omega t)$ where A is the amplitude and ω is the driving frequency. In both the classical (Section **??**) and quantum-mechanical (Section **??**) pictures, dissipation is a result of *stochastic energy spreading*. Once this spreading is established, the pictures can be unified [14]. Irreversible growth of energy (heating) is then a result of biased diffusion (a random walk) in energy. I will confine myself to a regime where linear response theory (LRT) is valid. In the quantum case this is known as the Kubo formalism, although the language of the energy spreading picture appears different (I connect the two pictures in Section **??**). The heating rate is given by

$$\frac{d}{dt}\langle \mathcal{H} \rangle \;=\; \mu(\omega) \cdot \tfrac{1}{2}(A\omega)^2, \tag{1.1}$$

where the 'friciton coefficient' $\mu(\omega)$ is related linearly[1] to $\tilde{C}_{\rm E}(\omega)$, the correlation spectrum of the time-dependent fluctuating quantity $-\partial\mathcal{H}/\partial x$. The latter (a generalized 'force' on the parameter x) specifies the random 'kicks' up or down in energy that

[1]The relation depends upon the initial energy distribution.

particles receive. In the quantum case $\tilde{C}_{\rm E}(\omega)$ is the 'band profile' (average off-diagonal shape) of the matrix $\partial\mathcal{H}/\partial x$ in the energy representation. In Section **??** I demonstrate theoretically and numerically the semiclassical equivalence of the classical and quantum versions of $\tilde{C}_{\rm E}(\omega)$, and examine quantum effects beyond the band profile.

In Chapters **??** and **??** we specialize to a system of non-interacting particles inside a billiard whose walls are deformed by the parameter x. One can specify any deformation by a function $D(\mathbf{s})$, where $\mathbf{s}$ specifies location of a wall element on the boundary (surface) of the billiard, and $D(\mathbf{s})x$ is the resulting normal displacement of this wall element. We will be interested in low-frequency driving, meaning $\omega \ll 1/\tau_{\rm bl}$ the mean collision frequency. In the $\omega \to 0$ limit (uniform parameter velocity $\dot{x}$), the heating rate (1.1) is given by the dc friction $\mu(0)$ proportional to $\nu_{\rm E} = \tilde{C}_{\rm E}(\omega \to 0)$. An assumption of uncorrelated collisions (the white noise approximation or WNA) gives an estimate for $\nu_{\rm E}$, which in turn in $d = 3$ leads to the well-known 'wall formula' [6] (from the nuclear application),

$$\mu_{\rm E} \;=\; \frac{N}{\mathsf{V}} m v_{\rm E} \oint D(\mathbf{s})^2 d\mathbf{s}. \tag{1.2}$$

This (ω-independent) estimate of the friction applies to a microcanonical ensemble of N particles with speed $v_{\rm E}$ in a billiard volume V.

We analyze $\mu(\omega)$ numerically in 2D billiard shapes (generalized Sinai, and Bunimovich stadium). We believe this is the first study of frequency-dependent heating rate in billiard systems. The chief discovery (Section **??**) is a class of deformations whose heating rate vanishes in the $\omega \to 0$ limit, like a power-law $\tilde{C}_{\rm E}(\omega) \sim \omega^\gamma$. This holds even for billiards with strong chaos, and goes completely against the WNA prediction. The class of 'special' deformations turns out to be just the class which

preserves the billiard shape. For translations and dilations $\gamma = 4$ and for rotations $\gamma = 2$. This is to be compared to the case of a generic deformation, for which $\gamma = 0$ as the WNA prediction would predict. We give classical explanations for the power-laws (which rely on correlation on short timescales $\sim \tau_{\rm bl}$). Importantly, the special class is manifested in the quantum band profile too. Thus the special nature of dilation, believed to be new in the literature, corresponds to a quasi-orthogonality relation between eigenstates on the boundary, which in turn will be the key to the powerful numerical method of Chapter **??**. We also discuss (Section **??**) non-generic shape-dependent effects (such as marginally-stable orbits) which may alter the power-laws given above.

The goal of Chapter **??** is as follows: given a general deformation $D(\mathbf{s})$, in a given billiard shape, we seek an analytical estimate of $\nu_{\rm E}$ (and hence $\mu(0)$). It is an exact result that $\nu_{\rm E}$ is a quadratic form in the function space of $D(\mathbf{s})$. The WNA fails to take into account that $\nu_{\rm E}$ vanishes for special deformations (which form a linear subspace in $D(\mathbf{s})$). We show that how it is possible to systematically subtract (project out) the 'special' components of a general $D(\mathbf{s})$. Applying the WNA only to the remaining ('normal') component gives an improved estimate of $\nu_{\rm E}$. We analytically and numerically justify this projection procedure, and test the quality of the improved formula. The quality is limited by that of the WNA estimate of the 'normal' component, which relies on the assumption of strongly chaotic motion. However, in the generalized Sinai billiard the improved formula is found to perform much better than the original WNA.

Our work replaces all *ad hoc* corrections which had been introduced [6] to account

for the intuitive result that translations and rotations cause no heating at $\omega = 0$. We thereby clear up some inconsistent habits in the nuclear community (Section **??**). More significantly, the incorporation of the special nature of dilation is entirely new.

Note that the effect of interaction between the particles is not considered. If the mean free path for inter-particle collisions is large compared with the size of the cavity, then we expect that our analysis still applies. (However if the mean free path is much smaller, then we get into the hydrodynamic regime, where viscosity becomes the dominant dissipative effect).

This work was performed in collaboration with Doron Cohen. I also benefitted greatly from use of a classical billiard trajectory code written by Michael Haggerty.

Chapters ?? and ??: Improved billiard quantization methods

The rapid development of electronic computing machines in the last 50 years has had an impact on scientific research whose size is hard [2] to grasp[3]. The interplay between numerical simulations and theoretical models now plays a crucial role in most areas of physics, chemistry, engineering, and other sciences. However, this impact would have been be drastically reduced were it not for the parallel development of efficient numerical algorithms. For instance, the invention of two techniques alone—the diagonalization of dense matrices[21], and the Fast Fourier Transform[39]—has allowed scientists to handle hitherto undreamed-of problems on a daily basis.

[2]Here's another footnote to make sure it's separated from the first

[3]Some idea of the rate of progress of this technology can be gained from such quotes as "Such a machine in the hands of a competent operator can produce 400 full-length products or 1,000 sums during an 8-hour working day"[11], referring to an electromechanical desk calculator typical for scientific use in the 1950s. The diagonalization of an 8-by-8 matrix was a weekend-long task[31].

Quantum chaos [23] is no exception: it has relied heavily on numerical solutions almost since its inception (as did its forebear, classical chaotic dynamics[37]). Billiards in $d = 2$ (or sometimes 3) dimensions have been popular systems for study because use of the free-space Green's function allows formulation as a boundary problem. Thus quantum eigenstates can be calculated at much higher energy than with the traditional (*e.g.* finite element) methods which cover the entire domain. High energies are so sought-after because most of the theoretical predictions involve the semiclassical limit $\hbar \to 0$. In this part of the thesis I will present new and efficient methods for finding these high-energy eigenstates.

The time-independent Schrodinger's equation in such a system is the Helmholtz wave equation,

$$(\nabla^2 + k^2)\psi(\mathbf{r}) \;=\; 0, \tag{1.3}$$

with certain boundary conditions. This problem is common to many other areas of physics and engineering (mesoscopic devices, acoustics, elastodynamics of thin plates, scalar electromagnetics and optics), and there has been some (but not that much) exchange of ideas between those communities and that of quantum chaos. If the boundary conditions are open then we have a scattering problem; if closed, an eigenvalue problem. I will be concerned only with the latter. In Section **??** I present a review, and categorize solution methods dependent on whether the basis does not (Class A) or does (Class B) depend on energy. Only Class B allows formulation as a boundary problem. The pioneering chaotic eigenstate studies of McDonald and Kaufman [34] and Berry and Wilkinson [5] used the Boundary Integral Method [8, 30] (BIM or BEM), while those of Heller [22, 23] used the Plane Wave Decomposition

Method (PWDM, an original technique). Semiclassical quantization methods have also been developed based on boundary matching [3] or the surface of section [7]. All these methods are Class B, and all require an expensive search ('sweep' or 'hunt') in energy-space for zero-determinants of a matrix.

In Chapter **??** I present an original Class B sweep method which is a simplified version of Heller's PWDM. The problems of missing states and sensitivity to basis size choice and matching point density have been solved, and the efficieny increased. The coefficient vector $\mathbf{x}$ of the nearest eigenfunction to a given wavenumber k is given by the largest-eigenvalue (λ_1) solution to

$$[G(k) - \lambda F(k)]\,\mathbf{x} \;=\; \mathbf{0}, \tag{1.4}$$

where the matrix G takes the norm in the domain, and F takes the norm of the boundary condition error (the 'tension'). I show that G can be expressed entirely on the boundary, and discuss improved 'hunt' methods for zeros of λ_1^{-1} (which give the desired eigenwavenumbers k). However, a few diagonalizations of (1.4) are still required per state found.

Of much more significance is Chapter **??**. Here I analyse the 'scaling method' of Vergini and Saraceno [44, 43], which despite being little-understood and little-used, is without doubt the most significant advance in numerical billiard quantization in the last 15 years. Eigenstates are given by the large-λ solutions of

$$\left[\frac{dF}{dk}(k) - \lambda F(k)\right]\mathbf{x} \;=\; \mathbf{0}, \tag{1.5}$$

however, through a certain choice of boundary weighting function an amazing property of F and dF/dk emerges: they are *quasi-diagonal* (have very small off-diagonal

elements) in a basis of the exact eigenfunctions rescaled to all have the same wavenumber k. This allows (1.5) to return up to $N/10$ useful eigenfunctions for a *single* diagonalization, and entirely eliminates the need for 'hunt' procedures. Here N is the matrix size (semiclassical basis size). The relative efficiency over sweep methods is $\sim 10^3$ when there are several hundred wavelengths across the system, and moreover, increases further with increasing k and dimension d! Remarkably, no adequate explanation of the key quasi-diagonality property has been known until now. I give, for the first time, a semiclassical explanation in terms of the 'special' nature of the dilation deformation (from Chapter **??**). I also correct errors in the original authors' derivation [44, 43] of higher-order tension terms.

Both chapters are presented as a practical 'how-to' guide to the diagonalization of d-dimensional billiards, and I hope they may be of use to other communities who solve the Helmholtz eigenproblem. I thoroughly analyse the various types of error in both the sweep and scaling methods, compare results from the two, and discuss the use of real and evanescent plane wave basis sets. For illustration, I use Bunimovich's stadium billiard (a shape known to be classically-chaotic [9]), in which evanescent basis sets have been pioneered by Vergini[43]. Currently the scaling method applies only to Dirichlet boundary conditions. Adequate basis sets for more general shapes is an area in dire need of future research. My work has involved deriving a collection of useful new formulae for boundary evaluation of domain integrals of Helmholtz solutions: these are presented in Appendix **??**.

Two applications of the scaling method are presented in this thesis. The first is the quantum band-profile calculations for Chapters 2–4. The second is an efficient

evaluation of overlaps of eigenstates of a billiard with eigenstates of the same billiard deformed by various finite amounts (Section **??**). The profiles of the resulting matrices can be viewed as local densities of states ('line shapes'), which are analysed in our publication [15]. The diagonalization of the deformed stadium billiard is believed to be new.

During this work, I have benefitted much from fruitful exchanges with Eduardo Vergini. I must thank Doron Cohen for first alerting me to the semiclassical estimation of the band profile of matrix elements on the boundary. Finally, Appendix **??** resulted from collaboration with Michael Haggerty.

Chapter ??: Quantum point contact conductance and scattering in the half-plane

This third part continues the theme of wave mechanics of non-interacting particles. However attention shifts from closed to open systems, namely the transport of quasiparticles (in a 2D electron gas) though a general two-terminal mesoscopic electronic device, or 'quantum point contact' (QPC) [2, 19].

We model the conductance of a QPC, in linear response. If the QPC is highly non-adiabatic or near to scatterers in the open reservoir regions, then the usual distinction between 'leads' and 'reservoirs' breaks down. This situation arises in the recent experimental work of Katine [26] and Topinka [42] in the Westervelt group here at Harvard, where open resonant and scattering geometries were studied. In such systems the Landauer formula [32, 33, 10, 17, 19] for the conductance (including

spin degeneracy),

$$G \;=\; \frac{2e^2}{h}\mathrm{Tr}(t^\dagger t), \tag{1.6}$$

is no longer convenient because no conventional transverse 'lead' states exist between which to define the transmission matrix t. Rather, a technique based on scattering theory in the two-dimensional infinite half-plane is appropriate. We relate conductance to transmission *cross section*, defined as an effective collision size on the reflective boundary of a half-plane (reservoir) region. We also introduce a new half-plane radial basis of 'lead' states in which the usual Landauer formula is recovered.

The relation between the Landauer and the half-plane scattering formalism is expressed by

$$\int_{-\pi/2}^{\pi/2} d\phi\, \sigma_{\mathrm{T}}(k,\phi) \;=\; \lambda \mathrm{Tr}(t^\dagger t), \tag{1.7}$$

where $\sigma_{\mathrm{T}}(k,\phi)$ is the angle-dependent transmission cross section and λ the Fermi wavelength, which I derive for both hard-walled and soft-walled reflective potential barriers.

We analyse an idealized, highly non-adiabatic slit QPC system in the extreme quantum, intermediate, and semiclassical regimes. We derive the counterintuitive result (first due to Heller) that an arbitrarily small (tunneling) QPC can reach a p-wave channel conductance of $2e^2/h$ when coupled to a suitable resonant cavity. We also find that if two or more resonances coincide, the total conductance can in theory reach multiples of this value.

This leads to some thought-experiments on attempting to overcome the maximum conductance $2e^2/h$ per quantum channel. We also discuss reciprocity (left-to-right symmetry) of conductance, and the possibility of its breakdown in a proposed QPC

(which could exhibit 'conductance' quantization) for atom waves[41]. We emphasizes the importance of the *thermal occupation* of states in phase space (as is usual in 2D electron systems), for reciprocity to exist. An analogous atomic QPC in 3D need not have this thermal occupation, thus in this system reciprocity can be broken.

This work has been in collaboration with Areez Mody and Miriam Blaauboer, and at the earlier stages many contributions were made by Adam Lupu-Sax. Joseph Thywissen, and professors Charlie Marcus and Daniel Fisher also contributed via stimulating discussions.

Chapter ??: Waveguides for neutral atoms using evanescent light fields

The fourth and final part is a self-contained proposal for a new design of coherent atom waveguide, using the forces exerted on atoms by near-resonant laser fields. As an independent project, it does not connect directly with other theoretical work in this thesis. However it shares many common themes: the motion of atoms in the trapping potential is a 2D quantum bound mode problem (in a smooth potential), and the optical waveguide bound mode calculation is also similar to this same problem (dielectric constant playing the role of a negative potential). Because the optical 'potential' is not hard-walled, and because the fields are vector rather than scalar, the efficient methods of Chapter **??** or **??** do not apply; rather Finite Elements[18] will be used.

There has been much recent progress in the trapping and cooling of neutral atoms, opening up new areas of ultra-low energy and matter-wave physics [13, 16, 38]. Waveguides for such atoms are of great interest for atom optics, atom interferometery,

and atom lithography. Multimode atom waveguides act as incoherent atom pipes that could trap atoms, transport them along complicated paths or between different environments, or deliver highly localized atom beams to a surface. Single-mode waveguides (or multimode guides populated only by atoms in the transverse ground-state) could be used for coherent atom optics and interferometry [35, 1], as well as a tool for one-dimensional physics such as boson-fermion duality [40, 36, 12] and low-dimensional Bose-Einstein condensation effects [27].

In Chapter **??** we propose a dipole-force linear waveguide which confines neutral atoms up to $\lambda/2$ above a microfabricated single-mode dielectric optical guide. The optical guide carries far blue-detuned light in the horizontally-polarized TE mode and far red-detuned light in the vertically-polarized TM mode, with both modes close to optical cut-off. A trapping minimum in the transverse plane is formed above the optical guide due to the differing evanescent decay lengths of the two modes. This design allows manufacture of mechanically stable atom-optical elements on a substrate.

We find that a rectangular optical guide of $0.8\,\mu$m by $0.2\,\mu$m carrying 6 mW of total laser power (detuning ± 15 nm about the D2 line) gives a trap depth of $200\,\mu$K for cesium atoms ($m_F = 0$), transverse oscillation frequencies of $f_x = 40$ kHz and $f_y = 160$ kHz, collection area $\sim 1\,\mu\text{m}^2$ and coherence time of 9 ms. The laser powers required are orders of magnitude less than those commonly needed for dipole traps. The large tranverse frequencies achieved allow the possibility of atomic single-mode occupation (hence coherent guiding) when fed from a source at cesium MOT temperature ($\approx 3\mu K$). We present design equations allowing optimal parameter choices to

be made. We also discuss the effects of non-zero m_F, the D1 line, surface interactions, heating rate, the substrate refractive index n_s, and the limits on waveguide bending radius. It emerges that lowering n_s is the main goal if large trap depths are desired of order an optical wavelength from the guide surface.

As known in the engineering community, the optical bound mode problem is notoriously hard [18]. We calculate the full vector bound modes for an arbitrary guide shape using two-dimensional non-uniform finite elements in the frequency-domain, allowing us to optimize atom waveguide properties. We chose rectangular guide cross-sections for this optimization, for simplicity. There are many other shapes possible; the fabrication technique will be the determining factor on what is practical.

This work on atom waveguides, an admittedly far-fetched topic for a student of Rick Heller, was in fact a collaboration with the following members of the Prentiss Group: Steve Smith (my principal collaborator), Maxim Olshanii, Kent Johnson, Allan Adams (who introduced me to the problem), and Mara Prentiss. I also benefitted from discussions with Joseph Thywissen and Yilong Lu.

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Appendix A

General transformation of the 1D Fokker-Planck equation

In this Appendix, I show how the drift and diffusion terms $\cdots$

We assume a one-to-one time-dependent mapping $\Omega = \Omega(E, t)$, which can be represented as a (fixed) surface in three-dimensional (E, t, Ω) space (Fig. A.1). $\cdots$

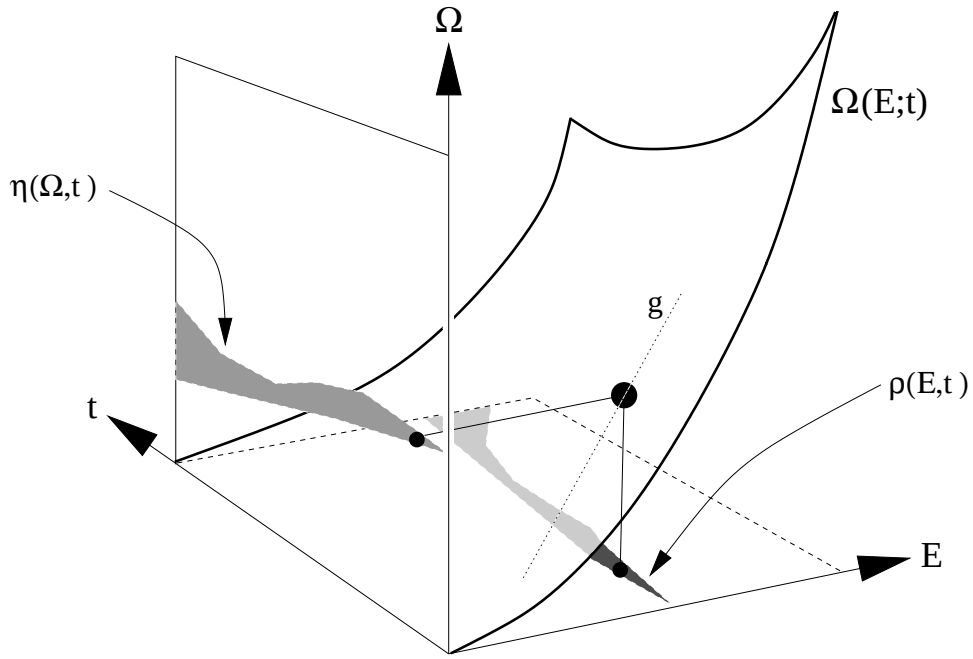

Figure A.1: Projecting a probability density on the surface $\Omega(E,t)$ down to $\rho(E,t)$ in the $E-t$ plane, or across to $\eta(\Omega,t)$ in the $\Omega-t$ plane. The gradient g is also shown.