

Missing Data Imputation

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11.12.2019



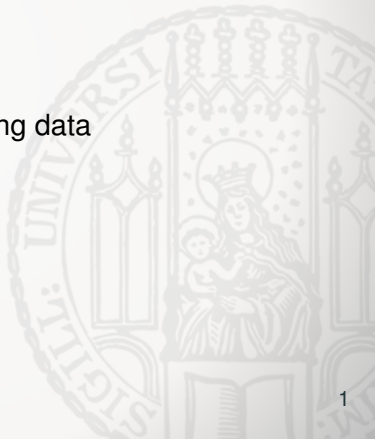
Agenda

Statistical background

Imputation Methods

Tools and libraries for dealing with missing data

Live Demo



Statistical background



Missing data

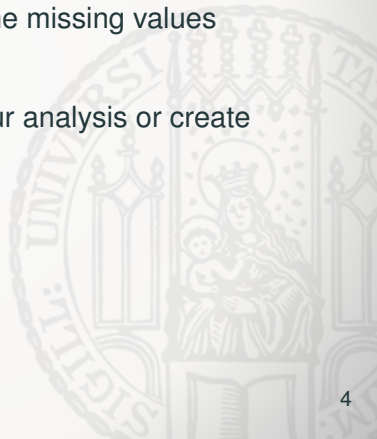
- Very common in practical problems
- Data can be missing for many reasons
 - Survey participant is unreachable or refuses to answer
 - Question does not apply to patient (sex-specific)
 - Subset of sensors not working for a period of time
- Complicates statistical analysis (bias, statistical power, ...)
- Many machine learning models can not handle missing data.

Types of missing data

- Missing completely at random (MCAR): the pattern of missing values is independent of all other covariates (both observed and unobserved)
- Missing at random (MAR): the pattern of missing values depends only on observed covariates
- Missing not at random (MNAR): the pattern of missing values also depends on unobserved covariates

Idea of Imputation

- Use statistical techniques to fill in the missing values
- Make the most of the data we have
- Caution is required! We can bias our analysis or create nonsensical data



Imputation Methods



Complete case analysis

- Row-wise or column-wise deletion
- + Very simple
- + Valid inferences when MCAR
- Biased results on MAR or MNAR
- Too few datapoints (if any) left!
If only 5% missing independently on a dataset with 20 columns, we end up with $0.95^{20} \approx 35\%$ of the original data

Big problem in predictive modelling!

Imputation of constant values

- Substitute mean, median, mode, constant value
- + Simple
- + Retains all observed values
- + Can preserve statistical quantities
- Underestimates standard errors
- May impute unrealistic values



Hot-Deck Imputation

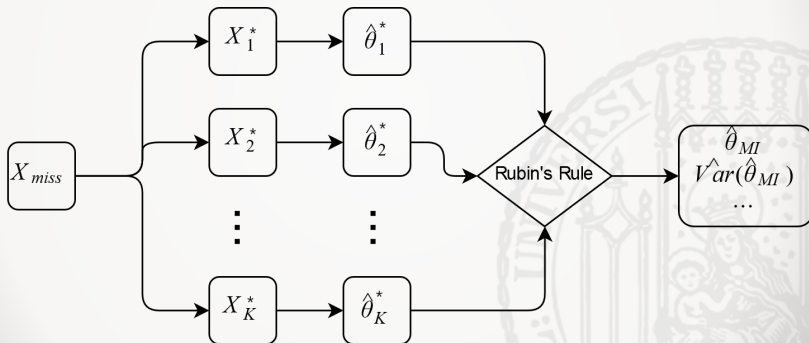
- General idea: reuse values from other observations
- Assumption: identically distributed data points
- "Donor" observations can be chosen in multiple ways
 - Naively: imputing value from randomly chosen observation
 - Distance-based methods
 - Predictive mean matching (Regression & Distance)

$$X = \underbrace{\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ - & x_{32} & x_{33} \end{pmatrix}}_{\text{original design matrix}} \rightarrow \underbrace{\begin{pmatrix} x_{12} & x_{13} \\ x_{22} & x_{23} \end{pmatrix}}_{\text{design matrix}}, \underbrace{\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}}_{\text{response}} \rightarrow \underbrace{\begin{pmatrix} \hat{x}_{11} & x_{12} & x_{13} \\ \hat{x}_{21} & x_{22} & x_{23} \\ \hat{x}_{31} & x_{32} & x_{33} \end{pmatrix}}_{\text{compute distances}}$$

- + Easy to implement and computationally efficient
- Donor values might get reused often
- Underestimates standard errors

Multiple Imputation

- Use given data to quantify uncertainty of imputations



Specification Overhead

- Imputation model: linear regression, random forest, ...
- Predictor vs response variables: leave-one-out, interactions, auxiliary, ...
- Order of variable imputation: random, least/most missing first, ...
- Initial imputations, number of iterations (convergence condition)
- Number of multiply imputed datasets (cycles)

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⇒ Multiple Imputation by Chained Equations (MICE)

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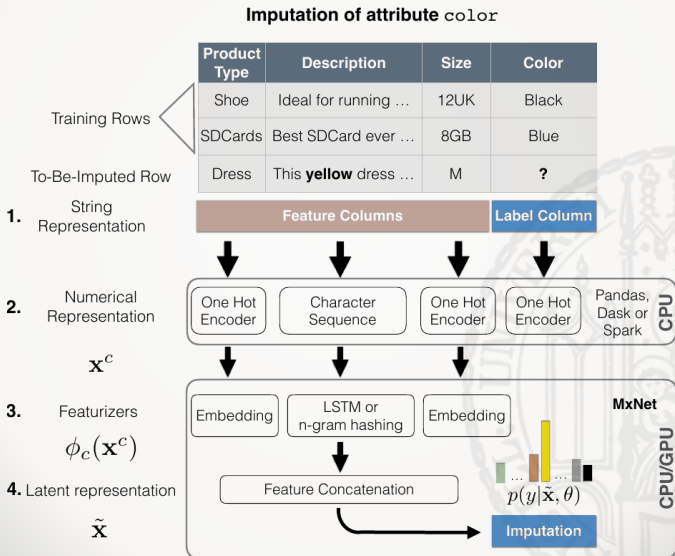
$$\begin{aligned}
 X = \underbrace{\begin{pmatrix} X_{11} & - & X_{13} \\ X_{21} & X_{22} & - \\ - & X_{32} & - \\ X_{41} & X_{42} & X_{43} \end{pmatrix}}_{\text{original design matrix}} &\rightarrow \underbrace{\begin{pmatrix} X_{11} & \bar{X}_{.2} & X_{13} \\ X_{21} & X_{22} & \bar{X}_{.3} \\ \bar{X}_{.1} & X_{32} & \bar{X}_{.3} \\ X_{41} & X_{42} & X_{43} \end{pmatrix}}_{\text{initial imputation}} \rightarrow \underbrace{\begin{pmatrix} \bar{X}_{.2} & X_{13} \\ X_{22} & \bar{X}_{.3} \\ X_{42} & X_{43} \end{pmatrix}}_{\text{design matrix}}, \underbrace{\begin{pmatrix} X_{11} \\ X_{21} \\ X_{41} \end{pmatrix}}_{\text{response}}
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow \begin{pmatrix} X_{11} & \bar{X}_{.2} & X_{13} \\ X_{21} & X_{22} & \bar{X}_{.3} \\ \hat{X}_{31} & X_{32} & \bar{X}_{.3} \\ X_{41} & X_{42} & X_{43} \end{pmatrix} \rightarrow \begin{pmatrix} X_{21} & \bar{X}_{.3} \\ \hat{X}_{31} & \bar{X}_{.3} \\ X_{41} & X_{43} \end{pmatrix}, \begin{pmatrix} X_{22} \\ X_{32} \\ X_{42} \end{pmatrix} \rightarrow \dots \underbrace{\begin{pmatrix} X_{11} & \hat{X}_{12} & X_{13} \\ X_{21} & X_{22} & \hat{X}_{23} \\ \hat{X}_{31} & X_{32} & \hat{X}_{33} \\ X_{41} & X_{42} & X_{43} \end{pmatrix}}_{X_i^*}
 \end{aligned}$$

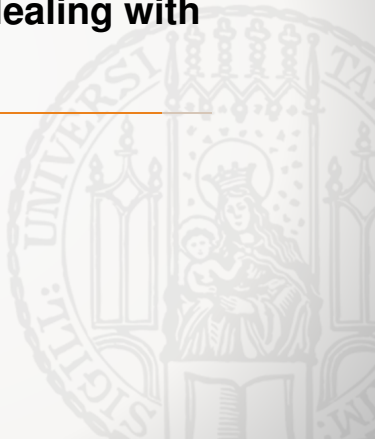
Deep Learning for Imputation

- Recent advances in deep learning improved model performance on a wide variety of NLP tasks
- This can be used to impute textual features or other categorical features
- Example: searching for "yellow dress" on Amazon - textual description might exist, but "colour" is missing in database

scalable, high-precision, language-agnostic, end-to-end pipeline



Tools and libraries for dealing with missing data



Useful libraries

Exploratory data analysis

- General purpose: [pandas-profiling](#)
- Specific to missing data: [missingno](#)

Imputation

- General purpose (\sim [mice](#) in R): [scikit-learn](#)
- Hot-deck with KNNs: [fancyimpute](#)
- Random Forest (\sim [missForest](#) in R): [missingpy](#)
- Imputation of Time Series Data (WIP): [impyute](#)

Live Demo



References

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